

The improvement of crack propagation modelling in triangular 2D structures using the extended finite element method

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Abstract

In this paper, a novel geometric method combined with the piecewise linear function method is introduced into the extended finite element method (XFEM) to determine the crack tip element and crack surface element. Then, by combining with the advanced mesh technique, a novel method is proposed to improve the modelling of crack propagation in triangular 2D structure with the XFEM. The numerical tests show that the accuracy, the convergence, and the stability of the XFEM can be improved using the proposed method. Moreover, the applicability of the conventional multiple enrichment scheme is discussed. Compared with the proposed method, the conventional multiple enrichment scheme has deficiency in mixed mode I and II crack. Finally, a comparative study shows that the performance of the XFEM by using the proposed method to model the crack propagation can be greatly improved.

1 INTRODUCTION

The discontinuity problem is one of the most complex problems in computational mechanics. Therefore, many numerical methods have been developed to solve the discontinuities problem. In the previous studies, the embedded finite element method (E-FEM) is used to describe the discontinuities by embedding a displacement discontinuity model in the element.¹ To model the thermomechanical fracture processes of the concrete, the zero-thickness interfaces, which are obtained by a proper node duplication and update of the finite element connectivity matrix, are employed to capture the cracks.^{2, 3} To model the microstructure, the lattice-particle model is proposed, in which each element of the lattice is considered as a beam element with three degrees of freedom per node, and the discontinuity of the structure is described by the failure of the beam.⁴ In the recent years, the XFEM has become one of the most widely used methods for solving the discontinuity problems.^{5, 6} It employs the partition of unity method and level set method to solve the discontinuous problem. More importantly, it can overcome the distortion of the mesh and achieve the continuous propagation of cracks. Hence, the XFEM had been employed in many fields. Zhou and Yang⁷ employed the XFEM to model the deformation and failure of surrounding rock mass around the underground cavern. Patil et al⁸ used the XFEM to evaluate the elastic properties of heterogeneous materials. Yazdani et al⁹ applied the XFEM to investigate the delamination problem in composite laminates. Based on the XFEM, Shi et al¹⁰ used a kind of reduction technique to model the hydraulic fracture propagation in formations containing water. Kumar et al^{11, 12} introduced the virtual node into the XFEM to simulate kinked cracks in the XFEM and proposed the new enrichments to model dynamic crack response of 2-D elastic solids. Meanwhile, it also comes up many new situations with the perfection of the XFEM. Therefore, many studies are carried out to make an improvement of the XFEM. Laborde et al¹³ proposed a high-order method to obtain the optimal accuracy in the XFEM. Chahine et al¹⁴ introduced a cutoff function into the crack tip enrichment, which can be used to obtain an optimal convergence rate. Xiao and Karihaloo¹⁵ employed a higher order quadrature and statically admissible stress recovery to improve the accuracy of XFEM. From the above studies, it can be recognized that the XFEM still exists some problems in the numerical simulation. Therefore, this paper mainly aims at the improvement of crack propagation modelling. Actually, in the conventional XFEM, it is found that there exist two problems, which can be described as follows:

Problem 1: In the triangle-based XFEM, we discover an interesting phenomenon. The location of the crack tip can affect the propagation of cracks in some cases. It can be illustrated by Figure 1A,B. For the XFEM, the important item is the crack-tip enrichment function $F(r, \theta)$, which is expressed as

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where r, θ are the local polar coordinates defined at the crack tip. It is noticed that the enrichment terms are related with the distance term r . That is to say, the location of the crack tip in the element can affect the numerical results. As shown in Figure 1, the line segment AC in Figure 1A and the line segment BC in Figure 1B have the different length. When we consider the XFEM problems, the two line segments are treated as the different radius r by using the crack-tip enrichment function. Hence, for the same problems, the different solutions can be obtained. Additionally, in general, the propagation of the cracks does not accomplish immediately, and the mesh keeps unchanged during the numerical computation of the XFEM. Hence, it implies that the different crack growth increment leads to the different crack propagation path for the same problem. Therefore, a novel method should be proposed to reduce this effect at the crack tip.

Problem 2: In the numerical computation, the computation of the J-integral always converts the curvilinear integral into the surface integral.⁵ For the numerical computation, the smaller the integral domain, the higher the dispersion of the numerical results. Hence, the integral radius has significant influence on the J-integral. If a satisfied value of the J-integral is obtained, a proper integral radius is necessary. Additionally, the computation should also satisfy the J-integral hypothesis, which means that the integral is closed, and the crack is the straight crack in the integral domain. However, for the crack coalescence problem, when a crack passes through the integral radius of the other crack, the accuracy problem for the computation of the J-integral arises. Moreover, as shown in Figure 1C, if the straight-line portion of the crack at the crack tip domain is too short, the computation of the J-integral cannot satisfy the hypothesis of the J-integral. Therefore, the method should be proposed to decrease the integral radius to calculate the value of J-integral.

Details for the problems in XFEM: A, crack tip at the center of the triangle; B, crack tip near the side of the element; C, the J-integral domain at the crack tip; and D, the multiscale mesh for quadrilateral element

In addition, the triangular mesh is employed in the numerical model when we implement the XFEM. It is known that the triangular element has obvious advantage over other polygons. The triangular element cannot only smoothly deal with the consistency of the displacement between different sizes of meshes but also can avoid the distortion of the mesh. As shown in Figure 1D, if the quadrilateral elements are employed, the boundary problems between coarse meshes and the fine mesh are necessary to conquer. The problem is relatively simple for the virtual element method.^{16, 17} However, it is complex for the XFEM. Zhou and Yang⁷ solved the same problems by combining the displacement projecting method and the displacement loading method in the framework of XFEM, but it leads to the distortion of the mesh when the boundary is not regular. Fortunately, with the development of the computational science, the mesh technology makes considerable advances in recent years, particularly the triangular mesh. It benefits from the optimized Delaunay algorithm and the new mesh method. Actually, the Delaunay algorithm is optimized for proper use in many fields, such as the mapping science, the imaging science, the computational science, etc. Based on the

Matlab, Persson and Strang¹⁸ employed a force-displacement relationship in their algorithm and proposed a new triangular mesh generator for numerical computation. In GIS 3D, the triangular mesh is used to rebuild the texture of the structure.¹⁹ In face recognition, the triangular mesh was applied to do the image processing.²⁰ Therefore, the triangle mesh has a flourishing prospect in the future.

In this paper, based on the triangular mesh, the authors aim to solve the problems mentioned above. First, a novel geometric method is proposed to describe the crack tip elements and the crack surface elements. Then, an advanced mesh technique is developed to improve the modelling of the crack propagation in triangular 2D structures in the framework of the XFEM.

This paper is organized as follows: Section 2 depicts the fundamentals of XFEM. Section 3 illustrates the novel geometric method. Section 4 describes the proposed advanced mesh technique in detail. Section 5 presents the conventional multiple enrichment scheme and its deficiency. Finally, the comparative study is carried out to show the advantage of the proposed method.

2 THE FUNDAMENTALS OF THE XFEM

In this section, a brief introduction to the XFEM and the integral scheme are mentioned. In the XFEM, the Heaviside jump function is used to depict the crack surface faces, and the asymptotic functions are employed to describe the characteristics of stress fields at crack tip. The standard formulation of XFEM takes the following form:

where r , θ are the local polar coordinates defined at the crack tip.

Based on the equilibrium equation of virtual work principle, the discretization of the equilibrium equation using XFEM can be written as

where u is the classical degree of nodal freedom, a_e is the Heaviside function enriched DOF, and b_e is the crack-tip function enriched DOF

and

where $N_{i,x}$ denotes the partial derivative of shape function N_i with respect to x , and $N_{i,y}$ denotes the partial derivative of shape function N_i with respect to y . It is noted that Equations 10 and 11 employ the shifting amendments to take the effects of interpolation into consideration. The details about the shifting amendments can be found in the work conducted by Khoei.²¹

To discuss the accuracy problem, we prescribe the scheme of gauss integral in the XFEM. The circled nodes are ones enriched by the Heaviside function, and the nodes marked with the square are enriched by crack-tip function, as shown in Figure 2A. Then, to calculate the stiffness matrix of the element and the J-integral, the enriched elements are subdivided into subtriangles for numerical integral. Figure 2B is the subdivided scheme for the crack tip elements which is corresponding to the elements labeled by ① in Figure 2A; Figure 2C is the subdivided scheme for the crack surface elements which is corresponding to the elements labeled by ② in Figure 2A; Figure 2D is the subdivided scheme for partly enriched elements by the crack-tip enrichments or crack surface which

is corresponding to the elements labeled by ③ in Figure 2A. For the above three schemes, there are 25 gauss integral points in each subtriangle. Figure 2E is the scheme for the ordinary elements that are corresponding to the elements labeled by ④ in Figure 2A, where only 13 gauss integral points are employed.

A, The enriched nodes in the triangle element; B,C,D,F, the integral scheme for the 3 A NOVEL GEOMETRIC METHOD IN THE FRAMEWORK OF XFEM

In this study, based on the characteristics of the triangle element, a novel geometric method is proposed. In the XFEM, the key step is to describe the crack tips and the crack surface, because they are related to the enrichment functions. Meanwhile, the enrichment functions are the key of the XFEM. In the previous studies, the most common way to describe the cracks was the level set method.²²⁻²⁶ For the description of crack surface, the level set method considers the crack as the level set function; the crack separates the mesh into two parts in a proper range of the mesh in which one part is considered as the upper surface of the crack, and the other part is considered as the lower surface of the crack. For the description of the crack tips, the location of the crack tips must be obtained to determine the nodes that are enriched by the crack tip enrichment. Hence, in the level set method, the additional level set function is necessary, and each crack tip requires an additional level set function, which is complicated in some degree. The more details on the application of the level set method in the XFEM can be found in the work conducted by Stolarska et al.²² In our work, a simple and comprehensible method is proposed to describe the cracks based on the triangular element in the framework of XFEM. It is worthy to note that the proposed method is a geometric method and only needs the geometric information of the mesh and the crack. Hence, it can be easily promoted to all the 2D approaches that need to describe the crack. Meanwhile, the proposed method is not only suitable for the triangular element, but also it can be suitable for other geometric elements, such as the quadrilateral element. For the proposed method, to describe the crack surface, it employs the intersection of line segment to recognize the crack surface element. To describe the crack tip, the area of the element is introduced to judge the location of the crack tip. The details of the proposed method are written in the following subsections.

3.1 The determination of the crack tip elements

As shown in Figure 3, when we know the specific location of crack tip A, by computing the areas, we can judge which type the element belongs to. In Figure 3A, in light of point B,C, D and a crack tip A, the triangle element can be subdivided into three subtriangles $\triangle ABC$, $\triangle ADC$, and $\triangle ABD$. Then, the areas of $\triangle ABC$, $\triangle ADC$, and $\triangle ABD$ can be easily calculated by the coordinates of B,C, D and the crack tip A. When $\triangle ABC$ is the tip element, the following expression can be obtained:

where S denotes the areas of the triangle. However, as shown in Figure 3B,C, when $\triangle ABC$ is not the tip element, Equation 12 cannot be adopted.

image

Figure 3

The sketch of the proposed geometric method: A, the crack tip element; B, crack surface element; C, the standard FEM element; D, a side of an element located at the opposite sides of the crack; E, a side of an element located at the same side of the crack; and F, the intersection of the crack surface and the side of the element

3.2 The determination of the crack surface elements

It is known that the element intersected by the crack can be recognized as the crack surface element. To define the crack surface element, the crack surface and the element sides are considered as a line segment. Then, the following two steps can be used to define the crack surface element.

The definition of the element side intersected by the crack

As shown in Figure 3D, AB is a side of an element, and CD represents the crack. Therefore, the vector \vec{AB} and \vec{CD} are perpendicular. When Equation 13 is satisfied,

The important meaning of the first step is to reduce computation cost at the next step. Therefore, Equation 13 is the necessary condition but not the sufficient condition to find the crack surface elements.

The definition of the intersected element by the crack

In this step, to judge the crack surface element, more vectors relative to the crack surface and the side of element are added into the equation. As shown in Figure 3E, the vectors \vec{AB} and \vec{CD} are perpendicular.

It is noted that the crack surface elements found by Equation 14 contain the tip elements. Therefore, the tip elements should be moved from the crack surface elements in the programming. Furthermore, from the above descriptions, it is easy to see that the novel method rather than the level set method can indeed provide an effective way to describe the crack based on the triangular element.

3.3 Combination of the geometric method with the piecewise linear function in the framework of XFEM

The crack is a straight line or a curve in the real application. Therefore, the original crack can always be approximated by the piecewise linear function.²⁷ Then, it is convenient to acquire the crack-tip elements and crack surface elements by using the novel geometric method. To clearly illustrate the novel method, it is expressed in the following mathematic forms.

For a crack, a specific piecewise linear function is applied to denote it. It can be written as

where x_i , y_i are the x - and y -coordinate of the piecewise point i , respectively. $\phi(x, y, t)$ represents the state of the crack at a certain time t . Therefore, if we want to update the crack after the crack propagation, the original piecewise points can be retained, we only need to update the propagation

segment of the crack. In fact, what we need to do is to make the propagation segment be piecewise and add it into the updated $\phi(x, y, t)$.

For the crack surface elements and the crack-tip elements, we, respectively, use two functions $\psi_1(n_j, t)$ and $\psi_2(n_k, t)$ to denote them. They are written as

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where ψ_1 is the function of crack surface elements, ψ_2 is the function of crack tip elements, ϕ_l denotes the location function of the enriched nodes, n_j and n_k are, respectively, the number of crack surface elements and the crack-tip elements, l denotes the number of enriched nodes, t is corresponding to the state of crack in the $\phi(x, y, t)$, urn:x-wiley:8756758X:media:ffe12918:ffe12918-math-0031 denotes the location parameter of the first node for elements k , x_l and y_l , respectively, denote the x - and y -coordinate of the element node l , and ψ_1 and ψ_2 can be obtained by the proposed method mentioned in Sections 3.1 and 3.2.

Then, to distinguish the specific location of the nodes around the crack and to describe the condition of the crack, a judging criterion is proposed as follows:

It is noteworthy to recognize that $\phi(x, y, t)$ is defined along the positive direction of the x -coordinate to help us strictly define the location of the nodes. Hence, as shown in Figure 4, the crack can be described in the XFEM as in the level set method. In Figure 4, the area containing the crack is divided into four parts that are marked by the sequence of the circled number. The relation describing the four parts can be written as

image

Figure 4

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The description of the crack [Colour figure can be viewed at wileyonlinelibrary.com]

It is noteworthy to note that the assumed propagation direction of the crack is along the direction of the crack tip when we define the crack. The direction is not the final propagation direction, it is only used to help the judgment of Equation 19. Hence, based on the above description, we can clearly define the location of the crack.

Next, the updating of the functions $\phi(x, y, t)$, $\psi_1(n_j, t)$, and $\psi_2(n_k, t)$ is necessary when the crack propagates. The crack function $\phi(x, y, t)$ is a piecewise linear function, when it updates, it means that the crack propagates at one end or both ends of a crack. Because a crack always keeps continuous, we can directly add the propagation segment into the function $\phi(x, y, t)$ based on the characteristics of the piecewise linear function. Hence, it is a relatively simple operation. If the propagation segments of the crack are assumed as $\phi_p(x', y', t)$ at the present crack state, then, the crack at the next time step can be expressed as

where x, x', x'' are the x-coordinate in the corresponding functions, and y, y', y'' are the y-coordinate in the corresponding functions. It is worthy to notice that Equation 21 only expresses the propagation of a crack tip. When the propagation occurs at two tips of the crack, the propagation segments of the crack should be, respectively, added into the crack function. It can be expressed as

where ϕ_{p1} and ϕ_{p2} are the piecewise linear functions at both crack tips of the propagative crack. Therefore, the updating crack can be obtained. Based on the above description, by combining with the proposed method mentioned in Sections 3.1 and 3.2, the updating of the function of crack surface elements ψ_1 and the function of crack tip elements ψ_2 can also be determined. In conclusion, it can be recognized that the novel geometric method is useful to describe the crack.

3.4 The advantage of the proposed geometric method

Compared with the level set method, the advantage of the proposed geometric method is that the proposed method is a local analytical method. For some special cracks, the novel geometric method is very useful. As shown in Figure 5A, to illustrate the deficiency of the level set method, the crack is divided into three parts. For a specific mesh, the crack surface can be described by the level set function. Based on the definition of the crack surface enrichment, as shown in Figure 5B, one side of the crack should be defined as the upper crack surface, and the other side of the crack is the corresponding lower crack surface. However, the level set method is a global analytical method. When the level set method is used to describe the crack surface in Figure 5A, the definition of the crack surface may be misunderstood. Because the lower crack surface in the first part of the crack can be considered as the upper crack surface in the third part of the crack. Hence, some more level set functions should be established to clearly distinguish the crack. If there exist many cracks as in Figure 5A, the level set method may be much troublesome. However, the proposed geometric method is the local analytical method, which only needs the crack surface function and the local location of the line segment. It is easy to describe the crack as in Figure 5A. Therefore, it can be concluded that the proposed geometric method has the advantage over the level set method. In the following sections, by combining with the advanced mesh technique, the feasibility of the proposed geometric method is tested; the result shows that the novel geometric method is applicable.

image

Figure 5

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The advantage of the geometric method: A, the spiral crack in the model and B, the definition of the crack surface enrichment

4 THE ADVANCED MESH TECHNIQUE IN THE FRAMEWORK OF XFEM

4.1 The advanced mesh technique

It is known from Section 1 that the location of the crack tip in the crack tip element can have an effect on the crack tip enrichment of the XFEM. Hence, the authors aim to diminish the r term in the crack tip enrichment to decrease the effect of the location of the crack tip. In addition, the assumption of the straight crack to obtain the J-integral cannot be satisfied if the integral radius is too large. In this paper, the advanced mesh technique is put forward to solve these problems. The

proposed method is valuable for crack propagation problems and large-scale engineering problems, when the mesh size is not small enough to get the satisfactory solution in a proper range. Additionally, it is the common sense that the propagation of the crack is pending when the crack problems are considered. Therefore, if we want to keep the accuracy of the solution all the time, a fine mesh grid, which cause the increase of the computation cost, is necessary in all the cracked parts. When the large-scale problems are encountered, this problem becomes much severe. The fine mesh occupies much computation cost and greatly slows the computational efficiency. However, the proposed mesh technique aims to obtain the accurate results by using a relatively coarse mesh. Therefore, the technique has a great potential in the future.

For the crack propagation problems, the determination of stress intensity factors (SIFs) of crack tips in structures under complicated loading conditions is one of the most important problems.²⁸ Therefore, we seek to improve the computation of the SIF to optimize the propagation of the crack. The approach is inspired by the multiple enrichment scheme method, the automatic remeshing technique, and the hierarchical approach in finite element method.^{15, 28-35} For the multiple enrichment scheme, as shown in Figure 6A, to obtain a higher accuracy in the traditional XFEM, it defines the elements that include the crack tip as the first layer elements. The elements near the first layer elements are called the second layer elements. Additionally, the next layer of elements is defined as the third layer elements in a similar manner. All the three layers of elements are enriched by the crack tip asymptotic field. In this way, the accuracy of the XFEM can get a promotion. The multiple enrichment scheme considers that the promotion is obtained by adding some extra degree of freedoms around the crack tip element. In view of this, in the proposed method, we carefully make good use of these three layers. Instead of the multiple enrichment method, we employ a much fine mesh to substitute the three-layer elements at the crack tip domain as in Figure 6B,C. However, the proposed method is not the remeshing one which has much computation cost. Considering the shape characteristics of the triangle elements, the most used way for the proposed method is based on the mesh substitution. Since the obtained three-layer elements always consists of a hexagon as in Figure 6A. We can employ a prescribed fine mesh as in Figure 6B,C to substitute the original mesh in most cases. However, when the quality of the mesh is not satisfied, the remeshing operation is necessary. In this paper, when the mesh substitution occurs, the original mesh is called “the replaced part,” and the new mesh is “the substitutive part.” As shown in Figure 7, the dash line quadrangle describes the proposed approach, which is illustrated as follows:

Step 1:

The pre-processing. The program needs two input parameters. The input 1 is the initial global mesh for the problem. The input 2 is the standard local mesh for the substitutive part.

Step 2:

The beginning of the program. The crack tips, which need to be replaced, are ascertained. Then, the substitution is carried out.

Step 3:

Check of the mesh quality of the substitutive part. If the mesh quality of the substitutive parts satisfies the demands, the next step is carried out. If the mesh quality of the substitutive parts does not satisfy the demands, the remeshing operation is needed.

Step 4:

The XFEM solver. For the plane stress problem or the plane strain problem, based on the discretization equation of XFEM, which has been described in Section 2, the fracture problem can be solved. Then, the displacement and the stress at the nodes can be obtained. Next, the SIF can be calculated by employing the interaction integral.⁵ Finally, according to the failure criterion of the fracture mechanics, the crack growth condition can be determined.

Step 5:

The crack is updated. If the program reaches the termination conditions, the results are given out. Otherwise, the crack is updated, and Step 2 is carried out.

It is worthy to note that the XFEM is independent of the mesh. The proposed method tracks the crack tip, and substitutes or remeshes the crack-tip domain with a finer mesh. The original mesh remains unchanged.

4.2 The implementation of the program

4.2.1 The Delaunay algorithm coupled with the piecewise-linear force-displacement relations

It is known that the Delaunay algorithm can form non-overlapping triangles to fill the convex hull of the input points.³⁶ Therefore, the Delaunay algorithm is employed. Because of the mesh substitution or the remeshing, the boundary problem of the mesh between the replaced part and substitutive part should be addressed. Hence, the Delaunay algorithm coupled with the piecewise-linear force-displacement relations is employed in this paper. For the algorithm, the connection between two points corresponds to the bars; the points correspond to the joints of the mesh in the algorithm. Hence, each bar has a force-displacement relationship $f(\ell, \ell_0)$. ℓ is the current side length of the mesh, ℓ_0 is the expected side length of the mesh and it will be discussed in Section 4.2.2, and f is the force function. The force function has many choices. One of the types can be expressed as¹⁸

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where k can be treated as the convergent constant. It is used to control the rate of convergence. Actually, the function can always obtain good results when we set $k = 1$. Then, to solve the force equilibrium, it is assumed that the point set p is arranged in the following form of N -by-2 array

where x, y are the x - and y -coordinates of the point set p , respectively. Then, the force vector can be written as

where f_{int} denotes the internal force, and f_{ext} represents the external force (reaction from the boundary). It can be known that $f(p)$ is a piecewise-linear function. Since the Delaunay algorithm changes the topology when the points move. Hence, if the exact mesh is obtained, the global force-displacement can be written as

It is noteworthy that Equation 26 is hard to obtain a satisfied result in the common way because of the discontinuity in the force function. A time-dependent solution scheme is introduced into the equation. It can be expressed as

When the stationary p is found in Equation 27, it is also suitable for Equation 26. At the discretized time $t_n = n\Delta t$, the approximate solution is determined by

If an initial value p_0 is set, Equation 28 can be solved. In addition, to satisfy the boundary condition, all the points going out the boundary are forced to move back to the closest boundary during the update from p_n to p_{n+1} . Hence, if the boundary condition of the nodes obtained from the three-layer elements in the original mesh is known, the algorithm will be feasible.

4.2.2 The principle to generate distance function

In Section 4.2.1, we refer to the desired length of the bars, which reflects the density of the mesh. It is necessary to illustrate the generation of the distance function. Firstly, the boundary of mesh can be denoted by a level set function $g(x, y)$. Moreover, it is easy to set $g(x, y)$ to be a zero level set. For the point $p_0(x_0, y_0)$, the closest point P on the level set is obtained. Hence, $g(P) = 0$, and $P - p_0$ is parallel to the gradient (g_x, g_y) at point P . Therefore, the following expression can be obtained:

Hence, all the points in the range of the boundary have a definite distance. Based on the distance, the mesh density can be defined. Additionally, the sign function can help us to distinguish the direction of the distance. Therefore, through the union, difference, and intersection of the distance set of the points, the demanded mesh density can always be satisfied. If A and B represent the regions which are projected by the different level set function $g(x, y)$, the mathematical operation mentioned above can be expressed as

4.2.3 The vertex-welding algorithm

Through the above description, the substitutive mesh can be obtained. However, when the original mesh is replaced by the substitutive mesh, the node problem arises. Actually, the substitution of the mesh leads to the redundant node problem in the original mesh. Therefore, based on the mesh requirement of the proposed method, we developed a new vertex-welding algorithm in the paper. Firstly, to keep the original grid unchanged, the original grid is duplicated when the vertex-welding algorithm is performed. Secondly, the boundary of the replaced mesh is found using the vertex-welding algorithm in the duplicated original grid. Thirdly, the elements of the replaced mesh are removed in the duplicated original grid. Meanwhile, the nodes of the replaced mesh are also deleted except for the boundary nodes because the replaced mesh shares the same boundary with the duplicated original grid. Fourthly, the substitutive mesh is inserted into the duplicated original grid, and the boundary of the substitutive mesh can be defined. Fifthly, the nodes on the boundary of the duplicated original grid and the nodes on the boundary of substitutive mesh are merged. It is worth noting that the nodes of the substitutive mesh should be shifted and rotated when the substitutive mesh is merged with the duplicated original mesh. The displacement and rotation of the nodes of the substitutive mesh are measured by the centroid of the mesh. Sixthly, the elements of the substitutive mesh are inserted into the duplicated original grid. After that, the redundant nodes,

which are located on the boundaries of the duplicated original grid and the substitutive grid, are erased. Finally, the node information of the element is updated.

4.2.4 The mesh quality

Owing to the substitution operation, the mesh quality may change, and the problem of degenerate triangles may arise, which leads to the mathematical error of the XFEM. Therefore, to assure the accuracy of the computation, after the substitution operation is carried out, the check of the mesh quality is necessary. Researchers commonly assess the shapes of triangles via aspect ratios.^{37, 38} The aspect ratios denote fractions determined by dividing length of edges, altitudes, and angles, etc. One mostly used quality measure is the ratio of the radius of the largest inscribed circle (times two) to the radius of the smallest circumscribed circle of triangles,³⁸ which can be denoted as

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where r_{in} denotes the radius of the largest inscribed circle of the triangles, r_{out} represents the radius of the largest inscribed circle of the triangles, and A , B , and C are the side lengths. An equilateral triangle has $q = 1$, and a degenerate triangle has $q = 0$. As a rule of thumb, if all triangles have $q > 0.5$, the results are good.³⁷ When Equation 36 cannot be satisfied, the remeshing is necessary. A typical operation for the substitution of the mesh is shown in Figure 8A,B. Figure 8C,D shows the values of ratio aspect q . In Figure 8C,D, the horizontal axis is the sequence number of the elements; the vertical axis is the value of q . Figure 8C is the result obtained from the substitution operation. Figure 8D is the result obtained from the remeshing operation. It can be seen from Figure 8C,D that the mesh quality of the substitutive part is very good, and the value of q is larger than 0.6. Therefore, we can say that the proposed method is successful.

image

Figure 8

Open in figure viewerPowerPoint

The mesh quality: A, the original mesh; B, the mesh after the substitution; C, the q value for substitution operation; and D, the q value for remeshing operation [Colour figure can be viewed at wileyonlinelibrary.com]

4.3 The feasibility and convergence of the proposed method

As shown in Figure 9, some numerical examples are illustrated to verify the feasibility and convergence of the proposed method. The classical plane plate problem with a central straight crack is used to verify the accuracy, the efficiency, and the convergence of the method. As shown in Figure 9A-D, a plane plate with a central straight crack is discretized by the adaptive mesh. The side length of the fine mesh part, respectively, takes 0.02, 0.04, 0.06, and 0.08 m, as shown in Figure 9A-D. All the four kinds of mesh are formed in the same way. The dimension of the plane plate is plotted in Figure 9E. A unit uniform force is applied on the plate in the vertical direction when the numerical test is done. Additionally, the novel geometric method mentioned in Section 3 is tested in this subsection.

The different side length of the element in the middle uniform mesh in the plane plate with a central straight crack: A, 0.02 m; B, 0.04 m; C, 0.06 m; D, 0.08 m; and E, the overall dimension of the plane plate [Colour figure can be viewed at wileyonlinelibrary.com]

For the XFEM, the common way to define the accuracy of the method is that the numerical SIF at crack tip is compared with the theoretical solution. The theoretical solution in this plane stress problem can be found in the work conducted by Mohammadi.³⁹ The SIF at the tips of the crack can be expressed as

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where the correction coefficient is urn:x-wiley:8756758X:media:ffe12918:ffe12918-math-0051, a denotes the half length of the crack, and b is the width of the plane plate. The convergent results of SIF obtained by the above meshes are recorded in Table 1. To show the capability of the method to avoid the mesh dependence of the results, two kinds of refined meshes are employed. The substitutive meshes in Figure 6B,C are, respectively, called the “mesh 1” scheme and the “mesh 2” scheme. In Table 1, the values in bracket are obtained by the mesh 2 scheme. It can be seen from Table 1 that the values of SIF obtained by the proposed method are more accurate than those obtained by the traditional method. Moreover, Figure 10 shows the convergence process of the meshes. It is obviously found from Figure 10 that the two mesh schemes have the same convergence. Hence, we can say that the proposed method is of mesh independence. Moreover, compared with the traditional method, the proposed method can obtain the convergent J-integral value in a much smaller integral radius. In other words, it means that the proposed method can be helpful to solve the problems mentioned in Section 1, ie, if the crack is curved at the crack tip, the traditional method cannot satisfy the assumption of the straight crack when the SIF is calculated by the J-integral. While, the proposed method can greatly diminish the integral radius to obtain the convergent SIF and to make the computation of the J-integral satisfy the assumption of the straight crack. Meanwhile, as mentioned in Section 4.1, the fine mesh at the crack tip can diminish the r term in the crack tip enrichment to decrease the effect of the location of the crack tip. Therefore, the proposed method can obtain the more accurate SIF and the smoother crack path.

Table 1. The SIF value for all the meshes

The value of SIF K_I vs the J-integral radius curve for all the four kinds of meshes: A, the result of Figure 9A; B, the result of Figure 9B; C, the result of Figure 9C; and D, the result of Figure 9D [Colour figure can be viewed at wileyonlinelibrary.com]

Figure 11 shows the results of the computation time obtained by the mesh in Figure 9C. The mesh 1 scheme is employed during the computational process. It can be seen from Figure 11A that the proposed method is relatively stable. For the proposed method, as shown in Figure 11B, the increased average time is about 4 seconds, and the maximum time difference is 12 seconds. In some cases, the proposed method is slightly faster than the traditional method. That is to say, the computation time does not obviously increase. Therefore, it can be concluded that the advanced mesh method is feasible.

A, The computation time of the two methods and B, the time difference of the two methods [Colour figure can be viewed at wileyonlinelibrary.com]

5 DISCUSSION ON THE CONVENTIONAL MULTIPLE ENRICHMENT SCHEME

In this section, the familiar multiple enrichment schemes for the XFEM were discussed.^{15, 40-43} In many previous works, it was applied to improve the accuracy of the XFEM algorithm. As shown in Figure 12A, there are three-layer elements enriched by the crack-tip enrichment function. However, because of the unstructured mesh, the works conducted by Bordas et al²⁹ are taken into consideration, as shown in Figure 12B. The radius R is defined as the farthest distance from the three-layer enriched nodes to the crack tip. Within the radius R , the nodes are enriched by the crack-tip enrichment functions. Therefore, in the following section, we consider Bordas' method as the multiple enrichment scheme.

image

Figure 12

Open in figure viewerPowerPoint

The enriched scheme: A, the three-layer enrichment scheme and B, Bordas' scheme for unstructured mesh [Colour figure can be viewed at wileyonlinelibrary.com]

However, we find that the multiple enrichment scheme is not suitable for all the problems. As shown in Figure 13A,B, in the mixed mode I and II crack, it is found that the stress concentration is not located at the crack tip when the multiple enrichment scheme is employed. Obviously, the stress contour at the crack tip is not correct. The correct stress distribution contour for the problem is shown in Figure 13C. Actually, because of the multiple enrichment scheme, the enriched elements cover up the real crack tip. Although the multiple enrichment scheme has the ability to improve the accuracy of the XFEM in some degree.^{15, 40-43} It is considered that the multiple enrichment scheme has a disadvantage in solving the problem of mixed mode I and II crack.

image

Figure 13

Open in figure viewerPowerPoint

The multiple enrichment schemes for XFEM: A, the enriched scheme at the crack tip; B, the horizontal stress distribution at crack tip (unit: Pa); and C, the correct horizontal stress distribution (unit: Pa) [Colour figure can be viewed at wileyonlinelibrary.com]

6 COMPARATIVE STUDY

In this section, a three-point bending (TPB) test is illustrated to prove the feasibility and superiority of the proposed mesh. The dimension of the numerical model is shown in Figure 14A. The location of the crack is defined by γ . When $\gamma = 0$, it corresponds to the pure mode I configuration. Therefore, a value of γ other than zero can result in a mixed mode failure at the crack tip. Actually, we take $\gamma = 0.5$ and $\gamma = 0.75$ in the numerical model. Furthermore, the crack growth region has employed the fine mesh. The material used in the numerical model is concrete, and some parameters were given by John and Shah.³⁴ As shown in Figure 14A, the height of the beam H is 15 m, the length of the crack h is 3.75 m, the span of the beam L is 40 m, and the displacement loading P is applied on the top of the beam. Additionally, the Young's modulus is 32.5 GPa, Poisson's ratio is 0.24, and fracture

toughness is $1 \text{ MPa}\cdot\sqrt{\text{m}}$. The maximum tangential stress criterion is employed as the failure criterion,^{7, 39} which can be expressed as

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where K_I is the SIF for mode I loading, K_{II} is the SIF for mode II loading, and K_{IC} is the fracture toughness for mode I loading. The crack initiation angle with respect to the pre-existing crack (θ_c) is defined as

Figure 14

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A, The sketch of the numerical model; B,C,D,E, the implementation of the proposed method [Colour figure can be viewed at wileyonlinelibrary.com]

When $\gamma = 0.5$, there are 4090 nodes and 7941 elements in the numerical model. When $\gamma = 0.75$, there are 4436 nodes and 8625 elements in the numerical model. To illustrate the superiority of the proposed method, the same crack growth increment is employed in the two models. Meanwhile, when the numerical test is conducted, the displacement loading control method is employed. The loading rate is 0.00005 m per time step.

Figure 14B,C,D,E simply illustrates the implementation of the proposed method in the numerical model with $\gamma = 0.75$. Figure 15A,C shows the crack propagation path for $\gamma = 0.5$ and $\gamma = 0.75$ in the numerical TPB test, respectively. Only half of the beam with the initial crack is depicted in Figure 15A,C. The force vs crack mouth opening displacement (CMOD) curves for $\gamma = 0.5$ and $\gamma = 0.75$ in the numerical TPB test are plotted in Figure 15B,D, respectively. As shown in Figure 15A,C, the thick black line represents the initial crack, the blue line defines the crack propagation path predicted by the linear elastic fracture mechanics,⁴⁴⁻⁴⁶ the green line presents the crack propagation path predicted by multiple enrichment scheme method, and the black line represents the numerical result predicted by the proposed method. It can be observed from Figure 15A,C that the proposed method has the higher accuracy and can obtain the smoother crack path than the multiple enrichment scheme. In addition, it can also be seen from Figure 15B,D that the multiple enrichment scheme has great effect on the crack initiation load; it obtains the lower crack initiation load than the proposed method when γ is 0.5, while it obtains the higher crack initiation load than the proposed method when γ is 0.75. Moreover, the influence is more noticeable for $\gamma = 0.75$ than for $\gamma = 0.5$ in the multiple enrichment scheme. For $\gamma = 0.5$, K_I dominates the propagation of the crack. For $\gamma = 0.75$, K_{II} dominates the propagation of the crack. Therefore, it can be concluded that multiple enrichment scheme can greatly affect the value of K_{II} , and the proposed method has the advantage over the multiple enrichment scheme in the XFEM.

image

Figure 15

Open in figure viewerPowerPoint

The comparison of the two methods: A, the crack propagation path when $\gamma = 0.5$ (unit: m); B, the force vs CMOD when $\gamma = 0.5$; C, the crack propagation path when $\gamma = 0.75$ (unit: m); and D, the force vs CMOD when $\gamma = 0.75$ [Colour figure can be viewed at wileyonlinelibrary.com]

7 CONCLUSIONS

In this paper, a novel geometric method combined with the piecewise linear function method is introduced into the XFEM to determine the crack tip element and the crack surface element. Then, by combining with the advanced mesh technique, a novel method is proposed to improve the modelling of crack propagation in the XFEM. The main conclusions are summarized as follows:

Based on the geometrical feature of the triangle, a novel geometric method is proposed to define the crack tip element and the crack surface element in the XFEM, which is different from the conventional level set method. Then, by combining with the piecewise linear function, the geometric method can be used to describe some complex cracks. It is worthy to note that the geometric method is a local analytical method, and it only needs some geometric information of the mesh and the crack. Hence, the geometric method can also be applied to other numerical methods, which needs to describe the characteristic of the crack.

It is found that the multiple enrichment scheme has a disadvantage in the computation of KII. While, the advanced mesh technique can improve the accuracy of the computation and can obtain the smooth crack path. Additionally, the proposed method can greatly diminish the integral radius to obtain the convergent SIF and to make the computation of the J-integral satisfy the assumption of the straight crack. Meanwhile, the fine mesh at the crack tip can diminish the r term in the crack tip enrichment to decrease the effect of the location of the crack tip. Therefore, the proposed method has an advantage over the multiple enrichment scheme.

The proposed geometric method only needs the geometric information of the mesh and the crack, which can be easily applied to the 3D problems of the XFEM in the future studies. In addition, the advanced mesh technique can also be easily extended to solve the 3D problems of the XFEM. Moreover, for the proposed mesh technique, the substitutive mesh can be in other shapes. Therefore, the proposed mesh technique can still be an effective way to establish the complicated mesh or to create the microstructure for other works in the future studies.