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IFAC PapersOnLine 51-8 (2018) 113–119

Model-Based and Model-Free Optimal Control of a Gas Liquid Cylindrical Cyclone

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Abstract: Optimal control is important in the oil and gas industry, where small changes of process variables may have significant economical, environmental and safety impacts. Finding the optimal setpoint for a controlled variable may not be straight-forward for an operator, motivating the use of automatic optimization methods. A widely used optimal control method is the model predictive control (MPC) method which requires a dynamic model to predict the future behaviour of the system. The MPC satisfies constraints and finds the optimal input to a plant. However, the requirement of a model can make the method difficult to implement and computation time might be too high for real-time implementation. An alternative to MPC is the Extremum Seeking (ES) method. This method aims to find the input that optimizes the output by slowly perturbing the controlled variable. It does not require a model and is less computationally expensive. In this paper we apply an MPC and an ES optimization scheme to a gas liquid cylindrical cyclone in order to optimize the purity of the gas outlet. Simulations show that both methods are able to find a liquid level setpoint that ensures high quality of the gas outlet stream under varying inlet conditions.

Keywords: Optimal control, Adaptive control, Subsea processing

1. INTRODUCTION

Optimal control plays an important role in the offshore production of oil and gas. It can ensure operation within the environmental, product quality and safety constraints of the production facility while economically optimizing production. A small change in process variables, such as pressure and flow rate, may lead to a huge economic payback as reported in Willersrud et al. (2013), where it is shown through simulations that the implementation of a short-term nonlinear model predictive control (NMPC) method increased the yearly revenue from a typical oil production facility with over USD \$16 million (with an oil-price of USD \$100 per barrel).

The offshore oil and gas industry is currently relying on large vessels called gravity separators to separate the produced fluids, i.e., oil, water and gas. The gravity separators are well known and robust, but due to their large size they are not suited for installation in deep waters and in challenging areas such as the arctic regions (Hannisdal et al., 2012). This is why the oil and gas industry is leaning towards more compact separation equipment, like the gas-liquid cylindrical cyclone (GLCC).

The GLCC is a widely used separation device and is currently installed in over 6000 onshore gas production and processing plants around the world (Kristiansen et al., 2016). While popular in onshore production facilities, the GLCC has yet to reach the same popularity in subsea and offshore production and processing facilities.

Subsea separation and gas-liquid separation in particular is described in Hannisdal et al. (2012) as an enabler for (i) more efficient liquid boosting, (ii) longer range gas compression from subsea to onshore, (iii) cost efficient hydrate management, (iv) more efficient riser slug depression and (v) access to challenging field developments. Subsea separation is also considered one of the main enablers for what is referred to as the *Subsea Factory*, an all subsea oil and gas production facility able to produce and deliver oil and gas directly to customers without sending the produced fluids topside for processing (Ramberg et al., 2013).

The quality of the outlet streams of the GLCC are inversely proportional to each other, i.e., we have to reduce the quality of one outlet to ensure improved quality of the other. Research has shown that the separation performance of the GLCC can be improved by control. Extensive research has been performed at the University of Tulsa in the late 90's and early 2000's. In Wang et al. (1998), a dynamic model of the liquid level and gas pressure inside a GLCC is derived and PI and PD controllers are used to stabilize the level and pressure. In Wang et al. (2000) a gain-scheduling like approach is used to stabilize the liquid level when different inflow conditions are present. Another solution, also considering the varying inlet flow, is found

10.1016/j.ifacol.2018.06.364

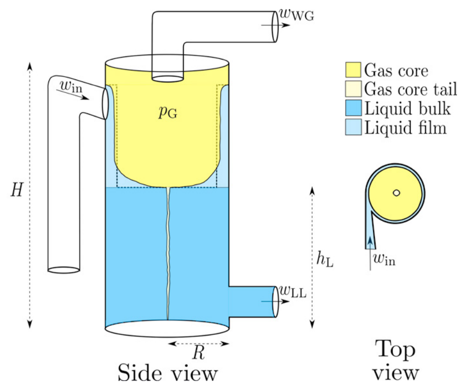

Fig. 1. Schematic of a GLCC.

in Earni et al. (2003) where a feedforward controller is combined with feedback control to counteract the effect of the variations.

These control methods work quite well and are able to stabilize the liquid level and gas pressure. However, the models used in the above papers do not include the separation of gas and liquid. Hence, the control objective is limited to maintaining a constant liquid level and gas pressure. Recently, a more sophisticated model including the separation of gas and liquid was developed (Kristoffersen et al., 2017b). This model includes calculations of the mass flow of gas and liquid between the phases and includes measures of the gas and liquid that is not separated, the so called gas carry under (GCU) and liquid carry over (LCO). The GCU and LCO plays an important role in determining the efficiency of the GLCC.

The model from Kristoffersen et al. (2017b) is sufficiently complex to capture the phenomena of GCU and LCU and sufficiently simple to be used in model-based control solutions. In Kristoffersen et al. (2017a), a feedback linearizing controller that handles transients better than a PI controller is derived. This method requires full state and parameter knowledge, hence an adaptive feedback linearizing controller was derived in Ohrem et al. (2017) to overcome these requirements.

The model has also been incorporated into model predictive control (MPC) schemes. In Kristoffersen and Holden (2017b), a nonlinear MPC scheme using band control was introduced. The band control allows the liquid level to change in order to release a degree of freedom to use for optimization. The results show an increase in GLCC efficiency when the band control algorithm is applied. A linear MPC scheme in combination with an extended Kalman filter has also been developed (Kristoffersen and Holden, 2017a).

MPC is a commonly used and well established optimal control scheme with increasing popularity in the industry (Qin and Bagwell, 2003). The advantages of MPC are many and some are already mentioned, but there are also disadvantages. The first, obvious, disadvantage is that a model of sufficient accuracy is required. If the plant dynamics are complex, such models can be difficult to obtain. Developing models is time-consuming and non-trivial and models will never be exact. Another disadvantage is the computational time required to solve the optimization problem. In some cases this is too long for the scheme to be implemented in an industrial application, especially in the case of nonlinear MPC.

An alternative to MPC are model-free optimization schemes. One such scheme is called Extremum Seeking (ES) (Ariyur and Krstić, 2003). Extremum seeking is an adaptive optimization method that attempts to find the extremum (minimum or maximum) of the reference-to-output map of a plant. Since this map is unknown or uncertain it is necessary to use adaptation to find the setpoint which, in our case, maximizes the output. The method has been around for several decades, but has risen in popularity since the stability proof of the method on general nonlinear systems was given in Krstić and Wang (2000). Applications of ES ranges from finding the optimal gas injection rate in a gas-lifted oil-well (Krishnamoorthy et al., 2016) to optimizing bioreactors (Hsin-Hsiung et al., 1999) and tuning PID controllers (Killingsworth and Krstic, 2006).

In this paper we evaluate the efficiency of the GLCC using both MPC and ES, and investigate whether the more complex model-based optimization scheme performs better than the model-free method. As a lower-level stabilizing controller for the ES case we use the adaptive feedback linearizing controller from Ohrem et al. (2017). For the NMPC we use the scheme presented in (Kristoffersen and Holden, 2017b) with added integral action and a reformulated objective function. PI controllers are used as lower level controllers.

The paper is divided into the following sections: Section II describes the model used in the simulations. Section III presents the MPC and ES strategies as well as the lower level controller. Section IV presents the results and Section V concludes the paper.

2. DYNAMIC MODEL

A GLCC separator is shown in Fig. 1 and is based on the principle of centrifugal separation. The inlet flow of gas and liquid w_{in} is brought into a rotational motion inside the separator due to the tangential inlet and high inlet velocity. The gas and liquid are separated due to their density difference by the centrifugal forces created by the rotational motion. The separation is incomplete, resulting in the accumulation of gas with some liquid droplets, called wet gas (WG), and the accumulation of liquid with some gas bubbles, called light liquid (LL). The accumulated gas creates a gas pressure p_{G}, while the accumulated liquid creates a liquid level h_{L}. The outlet flows of gas and liquid are denoted by w_{WG} and w_{LL}, respectively.

In this paper we consider the dynamic model of a GLCC separator with separation performance presented in Kristoffersen and Holden (2017b), which is an extension of the model presented in Kristoffersen et al. (2017b). The dynamic model consists of four state variables given by:

- $m_{\mathrm{LL,L}}$: accumulated liquid in LL [kg]
- $m_{\mathrm{LL,G}}$: accumulated gas in LL [kg]
- $m_{\mathrm{WG,L}}$: accumulated liquid in WG [kg]
- $m_{\mathrm{WG,G}}$: accumulated gas in WG [kg] .

The dynamics is described by four ordinary differential equations (ODEs) given by

$$\dot{m}_{\mathrm{LL,L}} = \beta_{\mathrm{in}} w_{\mathrm{in}} - \epsilon_{\mathrm{im,L}}(1-\beta_{\mathrm{in}})w_{\mathrm{in}} + \epsilon_{\mathrm{L}}(1-\beta_{\mathrm{WG}})m_{\mathrm{WG,L}} - (1-\beta_{\mathrm{WG,L}})w_{\mathrm{LL}} \quad (1)$$

$$\dot{m}_{\mathrm{LL,G}} = \epsilon_{\mathrm{im,G}}\beta_{\mathrm{in}} w_{\mathrm{in}} - \epsilon_{\mathrm{G}}\beta_{\mathrm{LL}} m_{\mathrm{LL,G}} - \beta_{\mathrm{LL}} w_{\mathrm{LL}} \quad (2)$$

$$\dot{m}_{\mathrm{WG,L}} = \epsilon_{\mathrm{im,L}}(1-\beta_{\mathrm{in}})w_{\mathrm{in}} - \epsilon_{\mathrm{L}}(1-\beta_{\mathrm{WG}})m_{\mathrm{WG,L}} - (1-\beta_{\mathrm{WG}})w_{\mathrm{WG}} \quad (3)$$

$$\dot{m}_{\mathrm{WG,G}} = \beta_{\mathrm{in}} w_{\mathrm{in}} - \epsilon_{\mathrm{in,G}}\beta_{\mathrm{in}} w_{\mathrm{in}} + \epsilon_{\mathrm{G}}\beta_{\mathrm{LL}} m_{\mathrm{LL,G}} - \beta_{\mathrm{WG}} w_{\mathrm{WG}}, \quad (4)$$

where $\dot{m}_{\mathrm{LL,L}}$ and $\dot{m}_{\mathrm{LL,G}}$ are the time derivatives of liquid and gas in the LL, respectively, $\dot{m}_{\mathrm{WG,L}}$ and $\dot{m}_{\mathrm{WG,G}}$ are the time derivatives of liquid and gas in the WG, respectively. The inlet conditions are given by the inlet flow w_{in} and the inlet gas mass fraction β_{in}. The two outlet flows are given by w_{LL} for the liquid outlet and w_{WG} for the gas outlet. The LL and WG gas mass fractions (GMFs) are given by

$$\beta_{\mathrm{LL}} = \frac{m_{\mathrm{LL,G}}}{m_{\mathrm{LL,G}} + m_{\mathrm{LL,L}}}, \quad \beta_{\mathrm{WG}} = \frac{m_{\mathrm{WG,G}}}{m_{\mathrm{WG,G}} + m_{\mathrm{WG,L}}}, \quad (5)$$

where β_{LL} is the gas mass fraction in the LL and β_{WG} is the gas mass fraction in the WG.

The immediate separation factors, $\epsilon_{\mathrm{im,L}}$ and $\epsilon_{\mathrm{im,G}}$, describe the immediate distribution of gas and liquid into the separator volumes as the inlet flow enters the separator. The continuous separation factors, ϵ_{L} and ϵ_{G}, describe the continuous separation of liquid from the WG to the LL and of gas from the LL to the WG, respectively. These separation terms are derived from steady-state separation performance and are highly nonlinear.

In a real process, the liquid level h_{L} and gas pressure p_{G} would be available as measurements and thus, these variables constitutes the controlled variables and are given by

$$h_{\mathrm{L}} = \frac{m_{\mathrm{LL,L}} + m_{\mathrm{LL,G}}}{a} \quad (6)$$

$$p_{\mathrm{G}} = \frac{b m_{\mathrm{WG,G}}}{aH - (m_{\mathrm{LL,L}} + m_{\mathrm{LL,G}})}, \quad (7)$$

where H is the total tank height and a and b are positive constants.

3. CONTROL

Two control structures are considered in this paper. The first utilizes the model-free Extremum Seeking method to find the setpoint for the liquid level in the GLCC that maximises the purity of the gas outlet. The method requires a measurement of the gas mass fraction in the gas outlet and this is assumed available. The liquid level setpoint calculated by the ES method is sent to a lower-level stabilizing controller that calculates the correct valve opening for the liquid outlet valve and hence ensures tracking of the setpoint. The gas pressures is kept at a constant value by another lower-level controller. An adaptive feedback linearizing controller is used for lower-level control of the liquid level and for control of the gas pressure.

In the second control structure a NMPC is used to both optimize and stabilize the liquid level and to stabilize the gas pressure. The NMPC requires measurements of the masses, separation factors, inlet gas mass fraction, inlet flow and the outlet flows. The NMPC, however, calculates the outlet flow rates of liquid and gas that corresponds to an optimal liquid level and a constant gas pressure. These flow rates are sent to lower-level controllers which, in turn, calculates the valve openings. PI controllers are used as lower level controllers in this case.

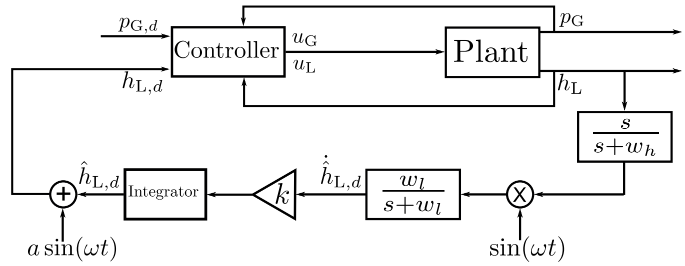

Fig. 2. The non-model based control scheme consisting of the adaptive feedback linearizing controller and the ES scheme.

3.1 Extremum seeking

Extremum seeking is an adaptive method for finding the steady-state input that maximizes (or minimizes) the steady state output from a plant, online and without requiring knowledge of the plant (Ariyur and Krstić, 2003). The output we seek to maximize in this case is the GMF in the gas outlet stream. As input we will use the liquid level. A static mapping between the liquid level and GMF in the gas outlet can be described by

$$f(h_{\mathrm{L},d}) = f^* + \frac{f''}{2}(h_{\mathrm{L},d} - h_{\mathrm{L}}^*)^2 \quad (8)$$

where f^* is the optimal value of $f(h_{\mathrm{L},d})$, the second derivative $|f''| > 0$ is constant and h_{L}^* is the optimal liquid level. All these parameters are unknown, in fact, all we need to know is whether f'' is positive or negative, i.e. we need to know if the map provides us with a minimum or maximum. By perturbing the input to the plant, i.e. the liquid level, with a sinusoid of the form $a\sin(\omega t)$ we get a measure of the unknown gradient $f'(h_{\mathrm{L}}^*)$. The integrator adds to the estimate $\hat{h}_{\mathrm{L}}$. At the maximum, the gradient will be zero and only small variations caused by the perturbations will affect the estimate. A block diagram showing the non-model based control solution is shown in Fig. 2.

When designing the method, key parameters are the amplitude a and frequency ω of the perturbation signal, the cut-off frequencies of the high-pass and low-pass filters, ω_h and ω_l, respectively, and the adaptation gain k. As mentioned in Krstić and Wang (2000), the overall feedback system has three time-scales, from fastest to slowest these are: (i) the plant with the stabilizing controller, (ii) the periodic perturbation and (iii) the high-pass and low-pass filters.

The GMF of the gas outlet and the level of liquid in the GLCC are inversely related, i.e., a low liquid level indicates a high GMF in the gas outlet stream. By slowly changing the liquid level from 1 m to 2.5 m and measuring the GMF with different inlet conditions, we obtain the relation shown in Fig. 3 by curve fitting the measured relationship. It is clear from Fig. 3 that the location of the maximum and the slope of the curve depends greatly on the inlet conditions. In the worst case, when the inlet contains low

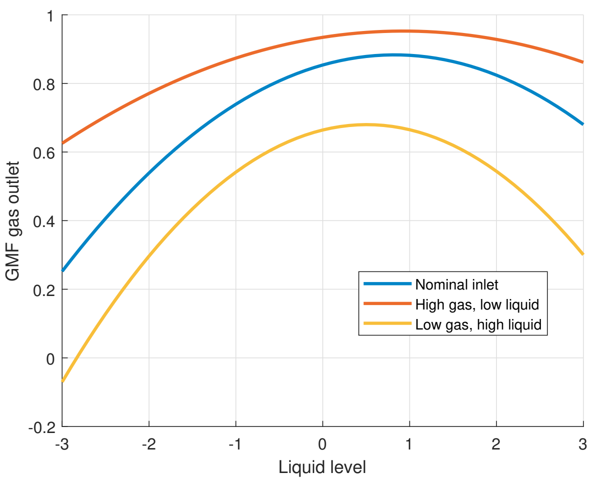

Fig. 3. The efficiency peak and slope of the curve changes with different inlet conditions. The low limit of the liquid level is at 1 m.

amounts of gas and high amounts of liquid, the efficiency peak is below the minimum allowed liquid level of 1 m. The extremum seeking scheme is unaware of this limit and will therefore attempt to decrease the level to an unfeasible operating point.

To address this problem we suggest using a projection based integration scheme instead of a standard integrator when calculating the desired liquid level in the extremum seeking algorithm. The projection based integration scheme utilizes the fact that we know certain limits on the parameter we try to estimate and ensures that the integration stops when the parameter reaches this limit. The update law for the liquid level setpoint then becomes

$$\dot{\hat{h}}_{L,d} = k\gamma \mathrm{Proj}(\hat{h}_{L,d}, \dot{\hat{h}}_{L,d}) \,. \tag{9}$$

As a lower-level stabilizing controller for the ES case we have chosen the adaptive feedback linearizing controller presented in Ohrem et al. (2017). This controller ensures excellent trajectory tracking for both the liquid level and the gas pressure. The controller is briefly presented here and for a full presentation including stability proofs we refer to the original paper (Ohrem et al., 2017).

We first define the error variables for the liquid level and gas pressure

$$\tilde{h}_{\mathrm{L}} = h_{\mathrm{L}} - h_{\mathrm{L},d} \tag{10}$$

$$\tilde{p}_{\mathrm{G}} = p_{\mathrm{G}} - p_{\mathrm{G},d} \tag{11}$$

where $h_{\mathrm{L},d}$ and $p_{\mathrm{G},d}$ are the desired liquid level and gas pressure, respectively. The time derivatives of (10) and (11)

$$\dot{\tilde{h}}_{\mathrm{L}} = \frac{1}{a}\left(f_1 + f_{1,s} - w_{\mathrm{LL}}\right) = \phi_1\left(\theta_1 - \Delta w_{\mathrm{LL}}\right) \tag{12}$$

$$\dot{\tilde{p}}_{\mathrm{G}} = \mathcal{F}\left[f_3 + f_{3,s} - \frac{1}{\eta+1}w_{\mathrm{WG}} + \frac{p_{\mathrm{G}}}{b}\left(f_1 + f_{1,s} - \Delta w_{\mathrm{LL}}\right)\right] \tag{13}$$

where $\phi_1 = 1/a$ is known and $\theta_1 = f_{1,s}$ is an unknown disturbance. $\phi_2 = \left[\mathcal{F}\ \mathcal{F}w_{\mathrm{WG}}\ \frac{\mathcal{F}\xi_2}{b}\right]$ consists of known signals, while $\theta_2 = [f_{3,s}\ \sigma\ \theta_1]$ with $\sigma = 1 - \frac{1}{\eta+1}$ is unknown. The variable $\eta = \frac{m_{\mathrm{WG,L}}}{m_{\mathrm{WG,G}}}$ has its own dynamics, but this is left out in this paper. The variable $\mathcal{F} = \frac{b}{a(H-h_{\mathrm{L}})}$.

The separation flows between the phases are described by

$$\begin{aligned} f_{1,s} &= \epsilon_{\mathrm{L}}(1-\beta_{\mathrm{WG}})m_{\mathrm{WG,L}} - \epsilon_{\mathrm{im,L}}(1-\beta_{\mathrm{in}})w_{\mathrm{in}} \\ &\quad + \epsilon_{\mathrm{im,G}}\beta_{\mathrm{in}}w_{\mathrm{in}} - \epsilon_{\mathrm{G}}\beta_{\mathrm{LL}}m_{\mathrm{LL,G}} \end{aligned} \tag{14}$$

$$f_{3,s} = \epsilon_{\mathrm{im,L}}(1-\beta_{\mathrm{in}})w_{\mathrm{in}} - \epsilon_{\mathrm{L}}(1-\beta_{\mathrm{WG}})m_{\mathrm{WG,L}} \tag{15}$$

and are considered as unknown disturbances to the system.

The inputs to (12) and (13) are the net mass flows of liquid and gas through the GLCC. These are chosen as

$$\Delta w_{\mathrm{LL}} = \hat{\theta}_1 + \frac{1}{\phi_1}k_1\tilde{h}_{\mathrm{L}} \tag{16}$$

$$\Delta w_{\mathrm{WG}} = -\frac{p_{\mathrm{G}}}{b}\hat{\theta}_1 - \frac{p_{\mathrm{G}}}{b\phi_1}k_1\tilde{h}_{\mathrm{L}} + \frac{1}{\mathcal{F}}\left(k_2\tilde{p}_{\mathrm{G}} + \hat{\theta}_2^{\mathrm{T}}\phi_2\right) \,, \tag{17}$$

where $\Delta w_{\mathrm{LL}} = w_{\mathrm{LL}} - f_1$ and is the difference between the outlet liquid flow w_{LL} and the inlet liquid flow f_1. For the gas outlet, $\Delta w_{\mathrm{WG}} = w_{\mathrm{WG}} - f_3$ where w_{WG} is the outlet gas flow and f_3 is the inlet gas flow. The controller gains are $k_1 > 0$ and $k_2 > 0$. Since the actual inputs to the GLCC are the valve openings u_{L} and u_{G} we calculate these as follows:

$$u_{\mathrm{L}} = \frac{w_{\mathrm{LL}}}{A_{\mathrm{L}}\sqrt{\rho_{\mathrm{L}}\max(p_{\mathrm{L}} - p_0, 0)}} \tag{18}$$

$$u_{\mathrm{G}} = \frac{w_{\mathrm{WG}}}{A_{\mathrm{G}}\sqrt{\rho_{\mathrm{G}}\max(p_{\mathrm{G}} - p_0, 0)}} \,, \tag{19}$$

where A_{L} and A_{G} are the cross sectional areas of the liquid and gas valves, respectively, p_0 is the downstream pressure and $p_{\mathrm{L}} = \rho_{\mathrm{L}}gh_{\mathrm{L}} + p_{\mathrm{G}}$ is the pressure at the liquid outlet and p_{G} is the pressure at the gas outlet. The max function ensures a non-negative flow.

The estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ are updated with the following projection based update laws:

$$\dot{\hat{\theta}}_1 = \gamma_1\mathrm{Proj}(\hat{\theta}_1, \phi_1\tilde{\xi}_1) \tag{20}$$

$$\dot{\hat{\theta}}_2 = \Gamma_2\mathrm{Proj}(\hat{\theta}_2, \phi_2\tilde{\xi}_2) \,. \tag{21}$$

The projection operator is defined in (Hovakimyan and Cao, 2010, App. B) as

$$\mathrm{Proj}(x,y) \triangleq \begin{cases} y \ \text{ if } g(x) < 0 \vee g(x) \geq 0 \wedge \nabla g^{\mathrm{T}}y \leq 0 \\ y - \frac{\nabla g \nabla g^{\mathrm{T}} y g(x)}{||\nabla g||^2}, \text{if } g(x) \geq 0 \wedge \nabla g^{\mathrm{T}}y > 0 \end{cases} \tag{22}$$

where the logic symbols $\wedge$ and $\vee$ represents *or* and *and*, respectively, and $g(x)$ is a smooth function given by

$$g(x) = \frac{(\kappa_x + 1)x^{\mathrm{T}}x - x_{\max}^2}{\kappa_x x_{\max}^2} \,, \tag{23}$$

with κ_x as the projection tolerance bound, $||x||_2^2 \leq x_{\max}^2$ and the gradient $\nabla g(x) = 2\frac{\kappa_x+1}{\kappa_x x_{\max}^2}x$. This adaptive controller achieves local asymptotic stability of the error system (12) and (13).

3.2 Nonlinear Model Predictive Control

MPC is an advanced control method using a prediction model of the plant to achieve optimal control. At each execution time, the MPC optimizes the future performance of the prediction model over a finite horizon and based obtains an optimal input sequence based on constraints and a specified optimal control objective. Optimal control is achieved by applying the first element of the optimal input sequence to the plant and repeating this procedure at the next execution time. MPC combines both the stabilizing task of the controller and the optimization task of the optimizer, while naturally including constraints instead of using a projection method like in the ES scheme. Thus, for evaluating the efficiency of the cascade consisting of the adaptive feedback linearizing controller and the extremum seeking method, a NMPC combining the objectives of both the adaptive controller and extremum seeking optimizer is designed. The MPC computes the flow references to two PI controllers controlling the flow through the outlet valves. A block diagram of the closed-loop system using the NMPC scheme is shown in Fig. 4.

The applied NMPC is an extension of the NMPC presented in Kristoffersen and Holden (2017b) using an augmented nonlinear prediction model to incorporate integral action and a reformulated objective function to overcome the prediction error experienced for high inlet gas mass flows in Kristoffersen and Holden (2017b). The following summary is a brief presentation of the applied NMPC and the reader is referred to Kristoffersen and Holden (2017b) for additional details.

The nonlinear dynamic model (1)–(4) is augmented with two additional integral error states of the controlled variables giving the augmented nonlinear prediction model

$$\dot{\tilde{m}} = \begin{bmatrix} (1) \\ (2) \\ (3) \\ (4) \\ \frac{\tilde{m}_1+\tilde{m}_2}{a} - h_{\mathrm{L},d} \\ \frac{b\tilde{m}_2}{aH-(\tilde{m}_1+\tilde{m}_2)} - p_{\mathrm{G},d} \end{bmatrix} = f(\tilde{m}, w_{\mathrm{in}}, \beta_{\mathrm{in}}, \epsilon_{\mathrm{in}}, \epsilon, w), \quad (24)$$

where $\tilde{m} = [\tilde{m}_{\mathrm{LL,L}}, \tilde{m}_{\mathrm{LL,G}}, \tilde{m}_{\mathrm{WG,L}}, \tilde{m}_{\mathrm{WG,G}}, \tilde{m}_5, \tilde{m}_6]^\top$ is the augmented state vector, $\epsilon_{\mathrm{in}} = [\epsilon_{\mathrm{in,L}}, \epsilon_{\mathrm{in,G}}]^\top$, $\epsilon = [\epsilon_{\mathrm{L}}, \epsilon_{\mathrm{G}}]^\top$, $w = [w_{\mathrm{LL}}, w_{\mathrm{WG}}]^\top$.

The objective of the NMPC is to control the liquid level and gas pressure while optimizing the purity of the outlet products. As there are only two inputs and four control variables, the optimization problem solved by the NMPC is augmented with band control of the liquid level, using slack variables, to free up a degree of freedom enabling optimization of the gas product purity. Moreover, the NMPC uses a different sampling time for the states and inputs. The fast dynamics of the system necessitates a fast sampling time, while the sampling time for the inputs are constrained to a slower sampling time to reduce complexity and enable use industrial applications. Additionally, to further reduce complexity, the separation factors are assumed constant over the prediction horizon.

The NMPC scheme is implemented in MATLAB using CasADi version 3.1.0. CasADi is a framework for automatic differentiation and optimization (Andersson et al., 2012). The nonlinear optimization problem created offline is discretized using the method of multiple shooting and solved online by the NMPC using IPOPT which is an interior-point solver interfaced by CasADi (Wächter and Biegler, 2005).

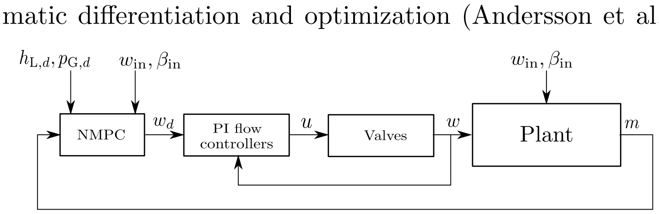

Fig. 4. The NMPC scheme including the PI flow controller.

The part of the cost function of the nonlinear optimization problem optimizing the purity of the outlet products is reformulated from minimizing $(1-\beta_{\mathrm{LL}})$ and $(1-\beta_{\mathrm{WG}})$ to maximizing the purity of the outlet products $m_{\mathrm{LL,L}}$ and $m_{\mathrm{WG,G}}$. This change is motivated by the prediction error experienced for high inlet gas mass flows in Kristoffersen and Holden (2017b) occurring due to model error introduced in $m_{\mathrm{WG,L}}$ by using constant separation factors. The reformulated optimization problem, including band control, is given as

$$\begin{aligned} \min_{\tilde{m},w} \; J = \int_0^T \Big[& \mathrm{q}_1 \Big(\frac{h_{\mathrm{L}} - h_{\mathrm{L},d}}{h_{\mathrm{L},d}}\Big)^2 + \mathrm{q}_2 \Big(\frac{p_{\mathrm{G}} - p_{\mathrm{G},d}}{p_{\mathrm{G},d}}\Big)^2 \\ & + \mathrm{q}_3 \big(\Delta w_{\mathrm{LL},d}\big)^2 + \mathrm{q}_4 \big(\Delta w_{\mathrm{WG,d}}\big)^2 \\ & + \mathrm{q}_5 (\tilde{m}_5)^2 + \mathrm{q}_6 (\tilde{m}_6)^2 \\ & - \mathrm{q}_7 \big(\tilde{m}_{\mathrm{LL,L}}\big)^2 - \mathrm{q}_8 \big(\tilde{m}_{\mathrm{WG,G}}\big)^2 \Big] \mathrm{d}t \end{aligned} \quad (25)$$

$$\begin{aligned} \text{s.t.} \quad & \tilde{m}(0) = \tilde{m}_0 && (26) \\ & \dot{\tilde{m}} = f(\tilde{m}, w_{\mathrm{in}}, \beta_{\mathrm{in}}, \epsilon_{\mathrm{in}}, \epsilon, w) && (27) \\ & s_1 + h_{\mathrm{L}} = h_{\mathrm{L,max}} && (28) \\ & s_2 - h_{\mathrm{L}} = -h_{\mathrm{L,min}} && (29) \\ & h_{\mathrm{L},d} - cH/2 \le h_{\mathrm{L}} \le h_{\mathrm{L},d} + cH/2 && (30) \\ & p_{\mathrm{G,min}} \le p_{\mathrm{G}} \le p_{\mathrm{G,max}} && (31) \\ & w_{d,\mathrm{min}} \le w_d \le w_{d,\mathrm{max}} && (32) \\ & -s \le 0 && (33) \end{aligned}$$

where $\mathrm{Q} = \mathrm{diag}[\mathrm{q}_1, \mathrm{q}_2, \mathrm{q}_3, \mathrm{q}_4, \mathrm{q}_5, \mathrm{q}_6, \mathrm{q}_7, \mathrm{q}_8]$ is the MPC weightss, $h_{\mathrm{L},d}$ is the desired liquid level, $p_{\mathrm{G},d}$ is the desired gas pressure, $w_d = [w_{\mathrm{LL},d}, w_{\mathrm{WG},d}]^\top$ is the desired outlet flows applied as reference values for the PI controllers controlling the outlet valves, $h_{\mathrm{L,max}}$ and $h_{\mathrm{L,min}}$ is the upper and lower limits of the liquid level for the band control scheme, $p_{\mathrm{G,max}}$ and $p_{\mathrm{G,min}}$ is the upper and lower limits for the gas pressure, $w_{d,\mathrm{max}}$ and $w_{d,\mathrm{min}}$ is the upper and lower limits for the outlet flow references, cH is the size of the control band and $s = [s_1, s_2]^\top$ are the slack variables.

The optimal desired outlet flow of gas and liquid computed by the NMPC are applied as references to the respective PI flow controllers controlling the gas and liquid outlet flows, respectively.

4. RESULTS

The ES and NMPC schemes were implemented in a Simulink model of the GLCC and the parameters used in the simulations are shown in Tab. 1. The results of the simulation are shown in Fig. 5, where we see that the level setpoint is reduced. The projection-based integration scheme ensures that the liquid level setpoint is not reduced below the minimum allowed value of 1 m. The scheme is robust towards changes in inlet conditions and the low-level adaptive controller ensures excellent tracking of the liquid level setpoint. The gas pressure tracks its setpoint very well during the whole simulation.

The results from using the NMPC scheme are also shown in Fig. 5. The results show that the NMPC immediately

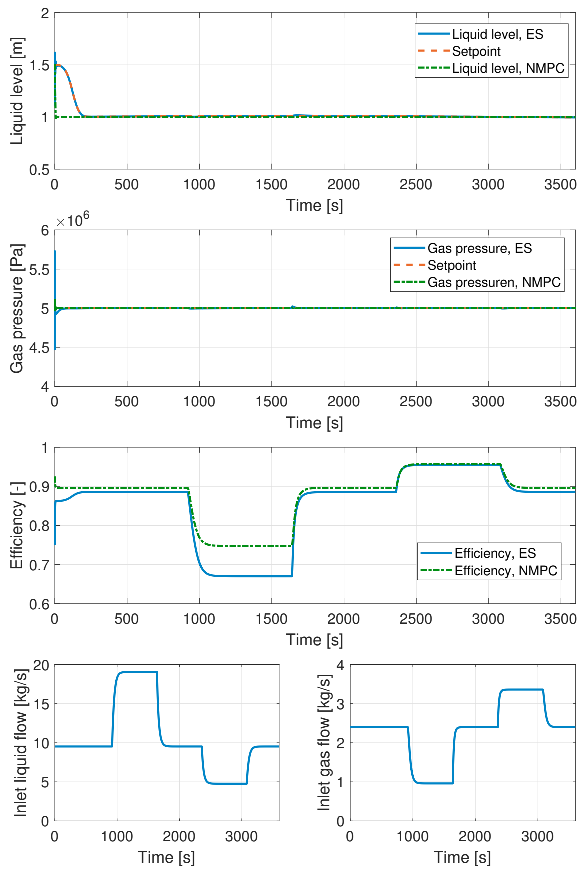

Fig. 5. The results of simulation with both extremum seeking and NMPC.

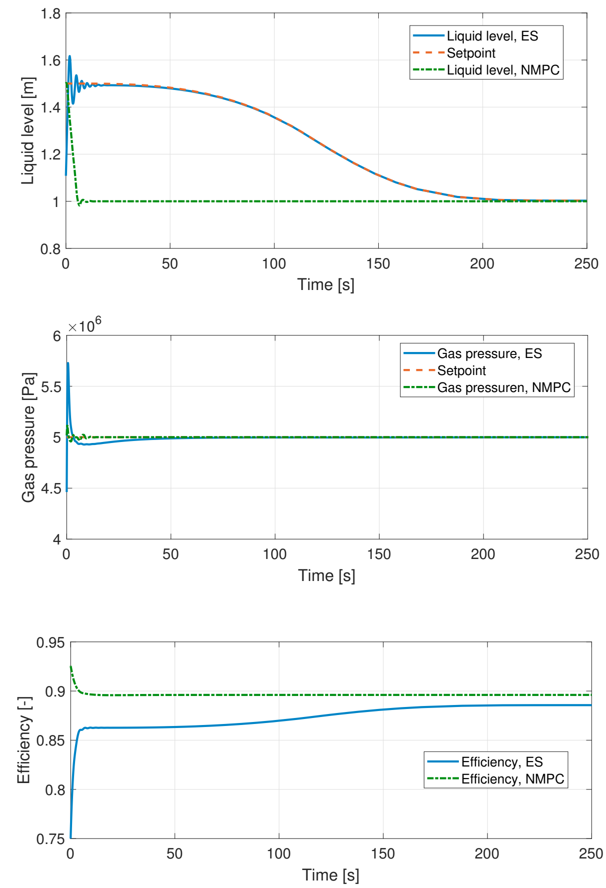

Fig. 6. The first 250 seconds of simulation.

finds the global optimum for the liquid level at the lower constraint of 1 m and quickly moves the liquid level towards this operating point. Simultaneously, as the operating point of the liquid level is optimized, the NMPC achieves offset free control of the gas pressure. The results also show that the NMPC achieves robust control of both controlled variables, with regards to severe changes in inlet conditions. The differences between the methods are more easily observed from Fig. 6 where the first 250 seconds of simulation are emphasized.

To objectively evaluate the performance of the ES and NMPC schemes, the mean and peak efficiency of the GLCC when using the ES and NMPC are shown in Table 2. Perfect efficiency is equivalent to an efficiency of 1. The results from this table show that the NMPC achieves a slightly better mean efficiency than the ES at the expense of increased computational time. The peak efficiencies are approximately equal. The lower mean efficiency of the ES scheme is caused by the slower convergence of the desired setpoint towards the optimum and beacuse the setpoint is slightly perturbed by the sinusouid.

Table 1. Values of parameters used in simulations

Parameter	Value	Description
k_1	$250/\phi_1 \approx 9$	Adaptive controller gain
k_2	10	Adaptive controller gain
γ_1	$5/\phi_1 \approx 139$	Adaptation gain
Γ_2	$20 \times 10^{-5} I_{3\times 3}$	Adaptation gain
ϵ	1	Projection tolerance bound
a	0.01	Amplitude, perturbation signal
ω	0.001	Frequency, perturbation signal
ω_h	0.0001	Frequency, high-pass filter
ω_l	0.0001	Frequency, low-pass filter
k	10	Adaptive integrator gain
Q	$[0, 1\cdot 10^8, 1, 1, 0, 1000, 100, 0]^\top$	NMPC weights
cH	1	Size of the control band
$h_{L,d}$	1.5	Desired/nominal liquid level
$p_{G,d}$	$50 \cdot 10^5$	Desired gas pressure
$h_{L,\min}$	1	Lower limit for the liquid level
$h_{L,\max}$	2	Upper limit for the liquid level
$p_{G,\min}$	45	Lower limit for the gas pressure
$p_{G,\max}$	65	Upper limit for the gas pressure
$w_{d,\min}$	$[0, 0]^\top$	Minimum desired outlet flows
$w_{d,\max}$	$[20, 20]^\top$	Maximum desired outlet flows

5. CONCLUDING REMARKS

The efficiency of a gas liquid cylindrical cyclone has been optimized using two different optimal control methods. The model-free method, utilizing a model-free adaptive controller and a model-free extremum seeking optimization scheme, is able to keep the average efficiency of the GLCC at $\sim 85\%$ with varying inlet conditions and has a peak efficiency of $\sim 95\%$. By using a projection based integrator we are able to ensure that the setpoint does not breach the constraint for minimum liquid level.

Table 2. Mean and peak efficiency

	Mean efficiency	Peak efficiency
Extremum seeking	0.8574	0.9550
NMPC	0.8769	0.9567

The model-based NMPC scheme shows a slightly better mean efficiency with $\sim 87\%$, but the peak efficiency is very similar to that of the ES scheme, namely at $\sim 95\%$. The liquid level constraint is handled at all times and with varying inlet conditions, showing the robustness of the method.

There are, however, pros and cons to both methods. The NMPC requires a model of the plant dynamics and may be sensitive to parameter variations, while the ES scheme is model-free. Obtaining a model might be difficult or time consuming when dealing with complex or large plants. The overall complexity and computational time of the NMPC is much higher than that of the ES scheme. A typical runtime for the NMPC simulation was ∼1.5 hours while the ES simulation took ∼3 min on an Intel Core i7 2.6 gHz computer. The NMPC shows a very fast convergence to the optimum, whereas the ES scheme takes longer. In addition, the NMPC converges to the actual optimal setpoint, but the ES scheme converges to a point very close to the optimum, thus inflicting some loss of efficiency.

As future work we propose doing field or laboratory tests of both methods to properly evaluate whether a model-based approach is better than a model-free approach. If model-free methods proves better, or the potential loss of efficiency can be justified, much time can be saved when developing optimization and control software, as the model-free methods are universal, whereas the model-based methods are designed specifically for each plant they are implemented on.

ACKNOWLEDGEMENTS

This work was carried out as a part of SUBPRO, a Research-based Innovation Centre within Subsea Production and Processing. The authors gratefully acknowledge the financial support from SUBPRO, which is financed by the Research Council of Norway, major industry partners, and NTNU.

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