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Contrasting broadly adopted model- based portfolio risk measures with current market conditions

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Problem Description

The main objective of this thesis is to study the performance of market risk measures for a case portfolio before and during the financial turbulence in 2007-2008. A Monte Carlo simulation model will be implemented to estimate the risk measures. The accuracy of these estimates is to be analyzed and compared with the actual losses of the case portfolio.

Assignment given: 05. January 2009
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Preface

The work on this thesis was carried out at the Department of Mathematical Sciences at the Norwegian University of Science and Technology during spring 2009, leading to the degree Master of Science.

I would like to thank my supervisor Jacob Laading for guidance and constructive feedback, and my student colleagues for interesting discussions.

Trondheim, March 2009
Øystein Sand Koren

Abstract

The last two years have seen the most volatile financial markets for decades with steep losses in asset values and a deteriorating world economy. The insolvency of several banks and their negative impact on the economy has led to criticism of their risk management systems for not being adequate and lacking foresight. This thesis will study the performance of two broadly adopted portfolio risk measures before and during the current financial turbulence to examine their accuracy and reliability.

The study will be carried out on a case portfolio consisting of American and European fixed income and equity. The portfolio uses a dynamic asset allocation scheme to maximize the ratio between expected return and portfolio risk. The market risk of the portfolio will be calculated on a daily basis using both Value-at-Risk (VaR) and expected shortfall (ES) in a Monte Carlo framework. These risk measures are then compared with prior measurements and the actual loss over the period.

The results from the study indicate that the implemented risk model do not give totally reliable estimates, with more frequent and larger real losses than predicted. Nevertheless, the study sees a significant worsening in the performance of the risk measures during the current financial crisis from June 2007 to December 2008 compared with the previous years. This thesis argues that VaR and ES are useful risk measures, but that users should be well aware of the pitfalls in the underlying models and take appropriate precautions.

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Chapter 1

Introduction

1.1 Background

The still ongoing financial crisis, initially referred to as "the credit crunch", began in July 2007 with the loss of confidence in the value of securitized mortgages and a following liquidity crisis. The background for this was the end of a boom in American housing prices and the following collapse of the U.S. subprime mortgage industry. For many years low interest rates and large inflows of foreign funds created easy credit conditions, fueling a housing market boom and encouraging debt-financed consumption. Lending to borrowers who did not meet prime underwriting guidelines (subprime) and the growth of intricate and highly-leveraged financial contracts made a "deadly mix" as the easy initial terms of the debt ended.

Following the collapse of two Bear Stearns hedge funds (USD 1.6 billion) holding subprime mortgage-backed securities in July 2007, liquidity and credit markets dried up. One indicator of the credit crunch was the fast rising spreads between risk-free Treasury bills and the interbank rates that jumped from 0.5% to 2.5% in a few days. The loss of confidence in other banks stopped the interbank market and led to the downfall of several banks depending on short-term funding from the market. Northern Rock saw the first bank-run in Britain in 141 years after it received a liquidity support facility from the Bank of England on September 14, 2007. (The bank was later nationalized in February 2008.) The tightening of credit and volatile financial markets continued through the first half of 2008 with ever higher commodity prices, federal stimulus packages and cuts in interest rates. In March 2008 the first large global investment bank collapsed with the Federal Reserve supported takeover of Bear Stearns by J.P. Morgan.

Until mid-September 2008 the problem was mainly the drying up of credit and liquidity. Stock and commodity markets traded at record levels at the

beginning of the summer on the hope of a decoupling of emerging markets from the American recession. September started with the government takeover of the U.S. home mortgage lenders Fannie Mae and Freddie Mac. A turning point came on September 15; Lehman Brothers, an investment bank, filed for bankruptcy after the Federal Reserve refused to provide any financial backing to a takeover. At the same day the American investment bank Merrill Lynch was acquired by Bank of America due to loss of confidence in the bank's solvency after significant losses on mortgage-backed securities. Together with uncertainty about the liquidity of the large insurer American International Group (AIG) (who was partly nationalized later the same week) the collapse of Lehman Brothers threatened the stability of the whole financial system, leading to a week with dramatic drops in financial markets worldwide. September and October continued with more banks collapsing, large losses in all financial markets, breakdown of the Icelandic economy, temporary bans on the short-selling of financial institutions, government rescue plans and cutting of interest rates. Investors have seen a "flight to quality" with vast amount of cash transferred into "safe" currencies such as USD and JPY, and increased demand for U.S. Treasury bills reducing yields to zero at the beginning of December.

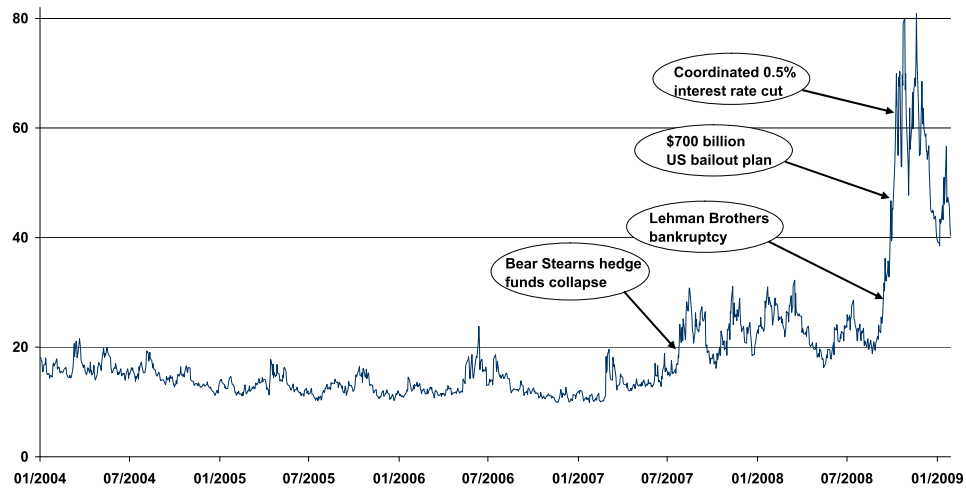


Figure 1.1: 30 days implied volatility of the S&P 500 index (VIX).

The credit crisis then rapidly evolved from America into a global financial crisis that has pulled the world towards a global recession. The crisis is still ongoing and its full implications are yet to be known, but some important questions can be asked: How could all these financial institutions fail? Is something wrong with the regulation of banks and insurance companies? Are the current mathematical models appropriate and do the various risk

measures describe the actual risk accurate enough? This thesis will look at the behavior of risk measures during the two last years, with focus on the periods before and after several of the most important financial events. The risk measures will be compared with earlier measurements and the actual returns over the same periods to check how these events influenced the measures and how useful the measurements were at predicting losses.

1.2 Thesis outline

This thesis consists of eight chapters. First an introduction to and the background of the current financial situation. Secondly, financial risks, the role of risk management and the current bank regulations. Chapter 3 describes the most common measures of market risks such as Value-at-Risk and Expected shortfall and how they can be computed. Chapter 4 takes a closer look at mathematical models for equity and fixed income. Chapter 5 presents the data material used in this thesis, before the implementation of the risk models are described in chapter 6. The results are given in chapter 7 and the thesis is summarized in chapter 8.

Chapter 2

Risk management

"All of life is the management of risk, not its elimination."

Walter Wriston, former chairman of Citicorp

2.1 Financial risks

Financial risk can be defined as the unexpected variability or volatility of returns on assets. The term thus includes both potential gains and losses. Investors and businesses will try to minimize the downside risk while maximizing the upside risk. This thesis will focus mainly on the downside risk. Generally, financial risks are classified into the categories of market risks, credit risks, liquidity risks and operational risks. These categories are not always distinct and the risks may interact with each other.

2.1.1 Market risk

Market risk is the risk of losses owing to movements in the level or volatility of market prices. It can be separated into two classes. Directional risks involve exposures to the direction of movements in financial variables (stocks, interest rates, exchange rates) while non-directional risks consist of exposures to non-linear variables, hedged positions and volatility. Market risk is the main subject of this thesis.

2.1.2 Credit risk

Credit risk is the risk that counterparts may be unwilling or unable to fulfill their promised repayments. The credit losses often occur before the actual default since the mark-to-market value of the debt will change with credit events (debt downgrades) and the credit default swap market. This creates some overlap to market risk. An important form of credit risk is settlement risk. This risk occurs when two payments are exchanged the same day, but

not simultaneous. The counterpart may default after the first institution made its payment. Legal risk is sometimes intertwined with credit risk. A few investors who lose money have the habit of turning to courts to invalidate the transaction. An example is municipalities claiming the investment activity as illegal because it went beyond their power.

2.1.3 Liquidity risk

Liquidity risk takes two forms. Asset-liquidity risk arises when the size of a position results in a transaction being un-executable at prevailing market prices. Funding-liquidity risk refers to the inability to meet payments obligations. This may force bankruptcy or liquidation. Leveraged portfolios which are subject to margin calls are especially exposed to funding-liquidity risk.

2.1.4 Operational risk

Operational risk is the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. This category includes failed settlements and back-office operations, fraud and rogue traders, legal risk and faulty models. Model risk arises from flawed or misspecified valuation models.

2.2 Risk management

Risk management refers to design and implementation of procedures for identifying, measuring and managing financial risks. Industrial corporations usually try to reduce the financial risks so that the firms can concentrate on business. In contrast, financial institutions' main function is to manage financial risks actively.

2.2.1 Derivatives

Derivatives are instruments designed to manage financial risks efficiently. A derivative can be defined as a financial instrument whose value is derived from the values of underlying variables. In contrast to securities, which are issued to raise capital, derivatives are contracts or private agreements between two parties. Derivatives can be categorized into three main groups: Forwards and Futures, Options and Swaps. Each group can again be sub-categorized by the pay-off profile and the type of underlying variable. Most derivatives have low or no upfront cash flow, and the instruments are leveraged. The leverage makes derivatives efficient instruments for hedging and speculations, but the absence of an upfront payment makes it difficult to

assess the downside risk. Used without proper controls derivatives have the potential for creating large losses, hence they should be monitored carefully.

Growth of derivative markets

In 1973 The Chicago Board Option Exchange (CBOE) started trading option contracts on stocks. This marked the beginning of a growth period in which derivatives have become an important part of the financial system. An important alternative to trading derivatives at the exchanges is the over-the-counter market (OTC), which offers tailor made contracts. These contracts can be adjusted to the need of the trader since the contracts are negotiated directly between the buyer and the seller. This opens for a whole new range of products such as credit derivatives, exotic options and interest rate derivatives. A disadvantage with the OTC market is the credit risk that contracts will not be honored [1].

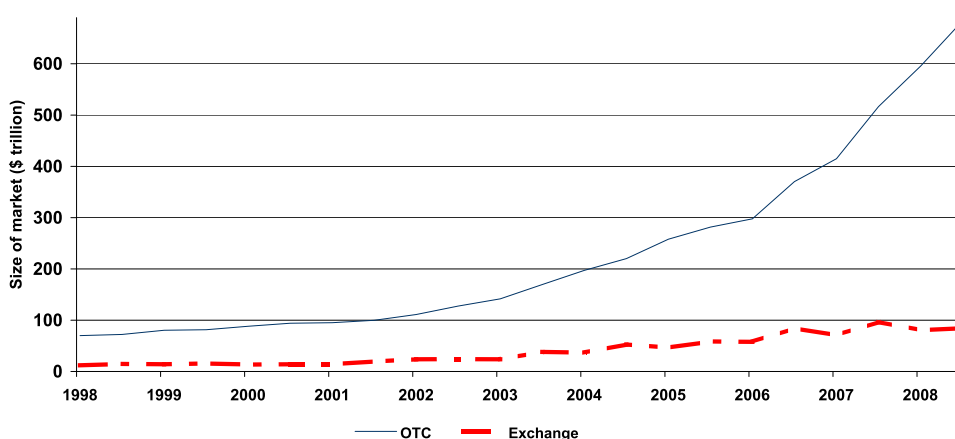


Figure 2.1: Market size of OTC and exchange-traded derivatives.

Measured in the total volume of trading, the OTC market has become much larger than the exchange-traded market. The outstanding volume of derivatives has grown exponentially over the last 15 years, and by June 2008 the notional amounts of outstanding OTC derivatives had reached USD 684 trillion and the exchange traded instruments had reached USD 84 trillion¹. These numbers seem amazingly large compared to the annual gross domestic product of United States at USD 12 trillion in 2005. However the notional amounts do not describe market risks. The gross market value for all OTC contracts was only USD 20 trillion in June 2008, which is 3.0 percent of their

¹Bank for International Settlements collects statistics on derivatives volume, www.bis.org

notional amounts. This number is still misleading because many of these positions were hedging each other. Nevertheless, the size of the derivative market is astonishingly large considering it has existed for only 35 years.

The recent growth of the risk management industry can be traced directly to the increased volatility of financial markets since the early 1970s [2] with events such as: The breakdown of the fixed exchange rate system in 1971, the oil-price shocks in 1973, the U.S. equity collapse in October 1987, equity bubbles such as Japan (1989) and the dot-com boom (2000), the Asian crisis (1997), the Russian default (1998) and today's credit crunch. Each event is unpredictable and surprise market observers with the rapidity of the changes. Financial risk management provides a partial protection against such sources of risk.

2.2.2 Bank regulations

An important part of creating a stable economic environment is to provide a reliable banking system. By nature bank deposits are destabilizing. Depositors are promised to be repaid at any time, however the deposits are often backed by illiquid assets. Many governments guarantee for bank deposit with an insurance fund to eliminate the rationale for bank runs. This guarantee creates the problem of moral hazard. Banks are offered a put option, and get the incentive to take on additional risk. In addition, governments are concerned about systematic risk. Systematic risk is the risk that an institution's failure has a cascading effect on other firms and threatens the stability of the financial system. To control risk-taking activities, regulators force banks to carry minimum levels of capital. This protect the insurance fund, and can serve as a deterrent to unusual risk taking if there is a link between the amount of capital set aside and the level of risk undertaken. At the same time the capital-adequacy levels should not be set too high since banks must earn a competitive rate of return on capital at risk.

A central idea in today's regulations is that the capital a financial institution is required to hold should cover the difference between expected losses over some time horizon and "worst-case losses" over the same time horizon [3]. The worst-case loss is the loss that is not expected to be exceeded with some high degree of confidence (figure 2.2). A measure for worst-case losses is Value-at-Risk (VaR) which will be described in details in the next chapter.

The first international risk-based standard for capital adequacy was the 1988 BIS Accord or Basel I. It introduced a measure for the bank's total credit exposure called Total Risk-Weighted Assets and created a level playing field among international banks. The 1996 Amendment (BIS 98) included the VaR measure and required institutions to hold capital to cover market risks

as well as credit risks. In 2007 the Basel II accord started to be implemented. It includes operational risk and a new calculation of the capital requirement for credit risk which reflects the credit ratings of counterparts.

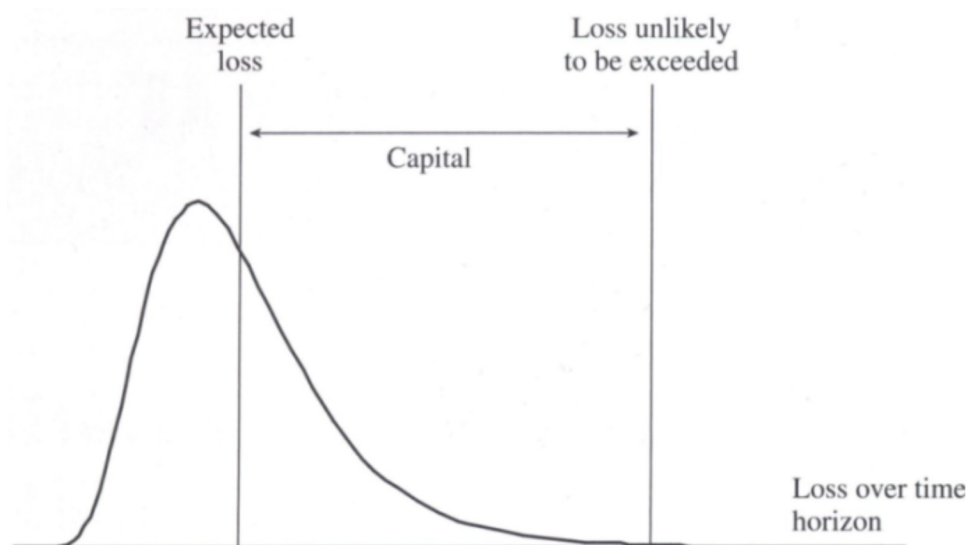


Figure 2.2: The loss probability density function and the capital required by a financial institution.

2.2.3 Portfolio allocation

An important risk management technique to reduce the risk of a portfolio is diversification. By investing in various classes of assets, which do not have perfectly correlated returns, the overall portfolio risk is reduced for a given level of expected return. Modern portfolio theory was pioneered by Harry Markowitz[6] in the 1950s and describes how rational investors will use diversification to optimize their portfolios. The portfolio is seen as a weighted combination of assets with different expected return and volatility. By adjusting the assets' weights an optimal portfolio can be found with the lowest possible level of risk for a given return. All such optimal portfolios make up an efficient frontier in which an investor should choose between according to his risk preference. Several ratios are used to choose among optimal portfolios. The Sharpe ratio measures the risk premium per unit of risk of a portfolio and describes how well the return of an asset compensates for the risk taken. Another popular measure is the Risk Adjusted Return On Capital (RAROC) which look at the expected return over Value-at-Risk.

Chapter 3

Market Risk

The four main types of financial market risks are equity risk, interest rate risk, exchange rate risk and commodity risk. Losses occur from two factors: the volatility of the underlying variable and the exposure to changes in the value of the underlying variable.

3.1 Approaches to risk management of market risk

There are many methods for companies to manage market risks. Some of the most common approaches are described below.

3.1.1 Notional amount

This approach measures risk as the sum of the notional values of the assets in the portfolio. A limit could thus be placed on the total amount allowed. This approach is simple but often not sufficient to control losses. The notional amount does not necessarily describe the risk. One bond can be risk-free while another with the same notional amount can have a high risk. The notional amount and the value of derivatives can differ widely as seen in chapter 2.

3.1.2 Factor-sensitivity

The factor-sensitivity approach combines notional amounts with sensitivity. The method finds parameters that describe the change in portfolio value for a given change in one of the underlying factors. The linear sensitivities are called Beta for exposure to stock-market movements, Duration for exposure to interest rates, and Delta for exposure of derivatives to the underlying asset price. The names of the second-order sensitivities are Convexity for interest rates and Gamma for derivatives. This approach does not consider the volatility of the risk factors nor their correlations. Sensitivity measures are not additive and can not aggregate risks.

3.1.3 Scenario-based

In addition to monitoring the factor-sensitivities it is possible to carry out a scenario analysis. This approach involves calculating the gain or loss on a portfolio under a variety of different scenarios. The scenarios can be generated by a model, historical data or chosen by management. The portfolio can hence be stress-tested with extreme historical market conditions.

3.1.4 Loss distribution

By generating a probability distribution for the portfolio value, both volatility and correlation are incorporated. The loss distribution can be used to measure the risk of losses in a portfolio. An advantage of this method is that it makes sense on all levels of aggregation; it can be compared across portfolios and reflects diversification and netting effects. The problem with this approach is that estimates of the loss distribution are based on historical data and these data might not reflect today's financial environment. Mathematically the loss L on a portfolio value V over a the time period from t to $t + 1$ can be defined as

$$L_{t+1} = -(V_{t+1} - V_t) \quad (3.1)$$

where V_t and V_{t+1} represent the portfolio values on day t and $t + 1$.

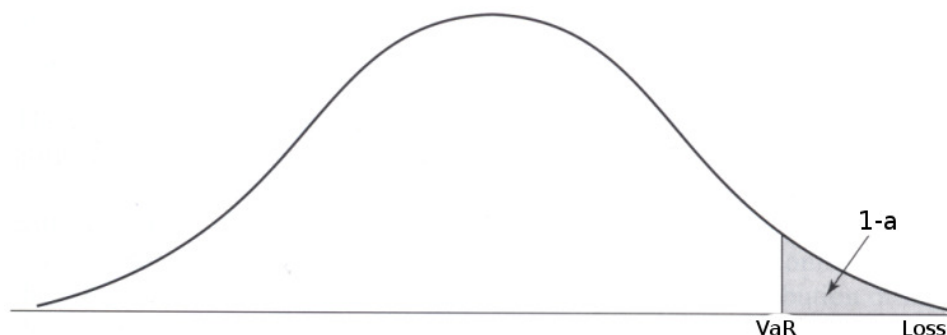


Figure 3.1: Illustration of the loss distribution and Value-at-Risk.

3.2 Risk measures based on the loss distribution

3.2.1 Value-at-Risk (VaR)

Value-at-Risk is an attempt to provide a single number that summarizes the total risk in a portfolio. J.P. Morgan pioneered VaR in the early 1990s and VaR has become a widely used risk measure for financial institutions, fund

managers and corporate treasurers. It is included in the Basel regulatory framework and is hence a standard in setting capital requirements for banks. VaR of a portfolio can be stated as: "We are α percent certain that we will not lose more than L dollars in the next N days."

Definition 3.1 Given the confidence level $\alpha \in (0, 1)$, VaR_α of a portfolio at confidence level α is the smallest amount l such that the probability that the loss L exceeds l is no larger than $(1 - \alpha)$.

$$VaR_\alpha = \inf \{l \in \mathbb{R} : P(L > l) \geq 1 - \alpha\} = \inf \{l \in \mathbb{R} : F_L(l) \geq \alpha\} \quad (3.2)$$

where $F_L(l) = P(L \leq l)$. [4]

This definition tells that Value-at-Risk is a quantile of the loss distribution. The choice of parameters vary with use, but the Basel accord requires banks to use a confidence level $\alpha = 99\%$ and $N = 10$ days when computing capital requirement for market risk. The VaR measure gives an estimate of the probable loss and is thus subject to estimation error. VaR is also subject to model risk due to the modeling of the loss distribution.

3.2.2 Expected shortfall (ES)

A key drawback of VaR is that the magnitude of the potential losses is ignored. Figure 3.1 and 3.2 illustrate two portfolios with the same VaR, but with different risk profiles. A closely related measure is the expected shortfall (ES) or conditional VaR. Expected shortfall answer the question: "If things do get bad, what is the expected loss?"



Figure 3.2: Illustration of an alternative loss distribution and VaR.

Definition 3.2 For a loss L with $E(|L|) < \infty$ and distribution function F_L

the expected shortfall at confidence level $\alpha \in (0, 1)$ is

$$ES_\alpha = \frac{1}{1-\alpha} \int_\alpha^1 q_u(F_L) du \quad (3.3)$$

where $q_u(F_L)$ is the quantile function of the loss distribution F_L . [4]

Expected shortfall is thus related to VaR by

$$ES_\alpha = \frac{1}{1-\alpha} \int_\alpha^1 \text{VaR}_u(L) du \quad (3.4)$$

For continuous loss distributions the expected shortfall can be interpreted as the expected loss given that the loss exceeds VaR

$$ES_\alpha = \frac{1}{1-\alpha} E(L; L \geq q_\alpha(L)) = E(L|L \geq \text{VaR}_\alpha) \quad (3.5)$$

3.2.3 Properties of risk measures

Artzner et al. [5] have proposed four properties that risk measures for capital adequacy purposes should have:

- **Monotonicity:** If a portfolio has systematically lower returns than another portfolio for all states of the world, its risk must be greater.
- **Translate invariance:** Adding an amount of cash K to a portfolio should reduce its risk by K .
- **Homogeneity:** Increasing the size of a portfolio by a factor λ should scale its risk by the same factor.
- **Subadditivity:** The risk of two portfolios should not be greater if they are merged than the sum of their individual risk.

Risk measures satisfying all four conditions are referred to as coherent. It can be showed [5] that VaR fails to satisfy the last property while ES satisfy them all.

3.3 Computing Value-at-Risk

3.3.1 Nonparametric

The nonparametric or historical simulation method constructs historical simulated losses for a portfolio using the empirical distribution of the risk factors. These simulated values indicate the changes in the current portfolio if the historical risk factor changes recur. To estimate VaR and ES the simulated losses are calculated and sorted. With n historical simulations an

estimate of VaR_α is the $n(1 - \alpha)$ largest value, and an estimate for ES_α is the mean of the $n(1 - \alpha)$ largest losses. Figure 3.3 reports the distribution of daily returns for J.P. Morgan in 1994 and illustrate the approach of deriving 1-day VaR with $\alpha = 95\%$ (assuming a constant portfolio over the whole year). The absolute $\text{VaR}_{0.99}$ is the $n(1 - \alpha) = 254 \cdot 0.05 = 12.7\text{th}$ largest loss and is found to be USD 9.6 million by interpolation. The relative (to the mean) $\text{VaR}_{0.99}$ is found by adding the mean return of USD 5 million to the absolute VaR.

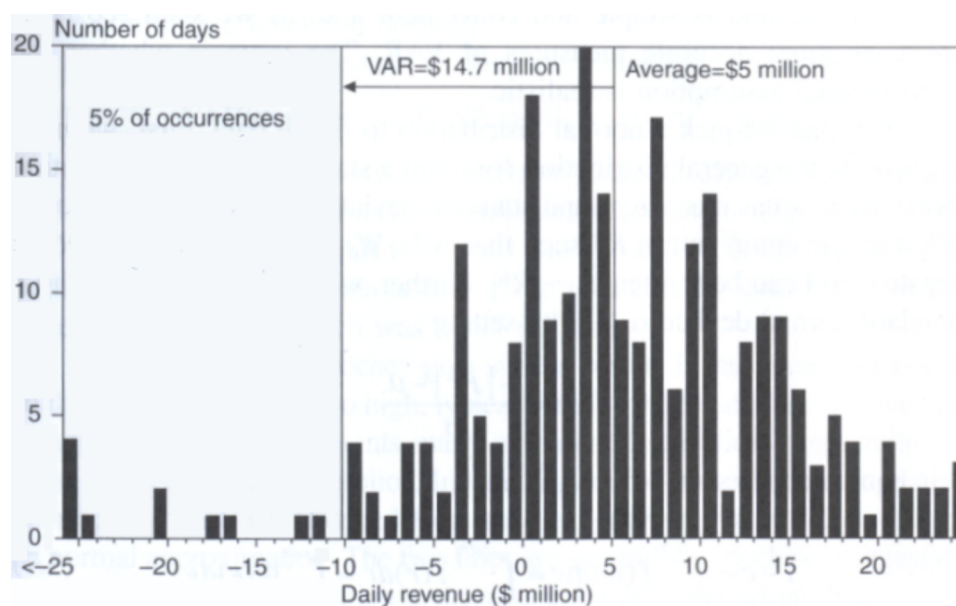


Figure 3.3: Computation of relative nonparametric 95% VaR by J.P. Morgan.

The nonparametric approach is an intuitive and popular method for estimating VaR which makes no assumption about the distribution of the change in risk factors. The estimate is based on a finite number of past observations. As a result the estimates are neither perfectly accurate nor reflecting today's economic environment. Extensions such as adding weights to the observations or using some volatility updating scheme can make the estimations reflect today's market. Applying extreme value theory can smooth the tails and improve the estimations when α is high or if few observations are available.

3.3.2 Parametric

The VaR computation can be simplified if the loss distribution can be assumed to belong to a parametric family. The parametric or model-building approach is based on H. Markowitz portfolio theory [6] and VaR can be derived from the portfolio standard deviation and the confidence factor of the distribution.

The value of a portfolio is depending on d risk factors with value Z_t at time t and can be written as

$$V_t = f(t, Z_t) \quad (3.6)$$

The changes in the risk factors $X_{t+1} = Z_{t+1} - Z_t$ determine the loss of the portfolio over one time period

$$L_{t+1} = -(f(t+1, Z_{t+1}) - f(t, Z_t)) = -(f(t+1, Z_t + X_{t+1}) - f(t, Z_t)) \quad (3.7)$$

If the function f is differentiable, a first-order approximation of the loss becomes

$$L_{t+1} = - \left(\frac{\partial f(t, Z_t)}{\partial t} + \sum_{i=1}^d \frac{\partial f(t, Z_t)}{\partial Z_i} X_{t+1,i} \right) \quad (3.8)$$

A popular choice of distribution for the return on risk factors is the multivariate normal distribution $X_{t+1} \sim N_d(\mu, \Sigma)$, with mean vector μ and covariance matrix Σ . Combined with the linear model it follows that the loss distribution is normal distributed with some mean μ_P and variance σ_P^2 . VaR $_\alpha$ and ES $_\alpha$ can then be calculated as

$$\text{VaR}_\alpha = \mu + \sigma \Phi^{-1}(\alpha) \quad (3.9)$$

$$\text{ES}_\alpha = \mu + \sigma \frac{\phi(\Phi^{-1}(\alpha))}{1 - \alpha} \quad (3.10)$$

where ϕ is the density function of the standard normal distribution. Figure 3.4 illustrates J.P. Morgan's daily returns in 1994. Using $\alpha = 95\%$ and estimating $\hat{\mu} = -5$ and $\hat{\sigma} = 9.2$ this give an absolute VaR of $-5 + 9.2 \cdot 1.645 = 10.2$ million or a relative VaR of 15.2 million.

The parametric approach is simple and convenient, but it has some disadvantages. Parameters are estimated from historical data and may not reflect today's market. Another important issue is whether the distributional assumption is realistic. The linearization of losses is an approximation, particularly when handling derivatives, and can reduce the accuracy of the model.

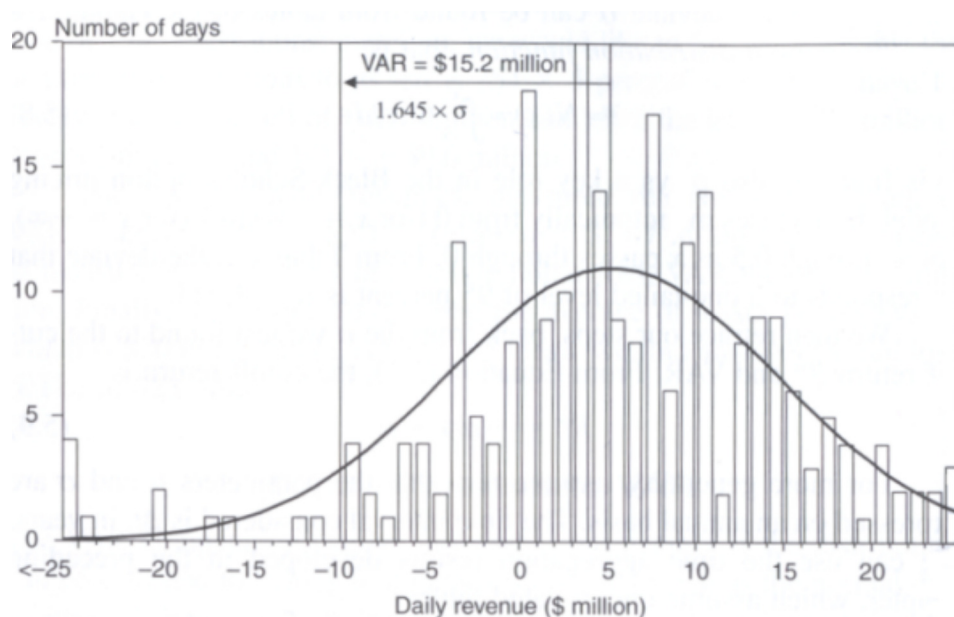


Figure 3.4: Computation of 95% relative VaR assuming normal distributed returns. J.P. Morgan.

3.3.3 Monte Carlo

The parametric approach can be implemented using Monte Carlo simulation to generate the loss distribution of the portfolio. By sampling the return of risk factors X_t from some parametric distribution it is possible to calculate a set of portfolio losses by revaluating the portfolio. The VaR is then calculated as the appropriate percentile of the loss function.

The advantage of Monte Carlo is its flexibility. Each risk factor can be drawn from different distributions and the use of derivatives in the portfolio makes no problems. The Monte Carlo method tends to be computationally slow because the portfolio has to be revaluated many times. Since the method assumes some parametric distribution for the risk factors it might not fit real data perfectly and is thus exposed to model risk.

Chapter 4

Equity and Fixed Income Models

This thesis will use the Monte Carlo approach to compute VaR and ES by simulating price paths for the underlying risk factors of the case portfolio. These simulated returns are sampled from some parametric distribution. This chapter will look into the distributions of equity and fixed income.

4.1 Equity and asset price random walks

4.1.1 Introduction to equity

A stock is a share of ownership in a company. That means that the holder of the stock has a claim on part of the equity and will receive its part of any dividend payments. Usually stocks carry voting rights in corporate decisions. The value of stocks is set by the mechanics of supply and demand, and in theory it is based on the expectation of future generated profits. According to the efficient market hypothesis[7] asset prices must move randomly since the market prices reflect all available information. The price movements are then only reactions to new information which by definition should be random in timing and nature. Given these assumptions the change in asset prices can be described as a Markov process.

4.1.2 A model for stock returns

The most widely used model of stock price behavior is known as the geometric Brownian motion

$$\frac{dS}{S} = \mu dt + \sigma dz \quad (4.1)$$

where $dz = \epsilon\sqrt{dt}$ is a Wiener process and ϵ is a standard normal random variable. The model says that the return on a stock S over a time period dt

can be decomposed into two parts. The expected return μdt is a predictable drift and σdz is a random term. σdz is drawn from a normal distribution with zero mean and standard deviation equal to the volatility times the square root of time. The rationale behind modeling the return on the stock and not the absolute change in the asset price is that the expected percentage return required by investors is independent of the stock prices. The investors will require the same return if the stock price is \$10 or \$50.

4.1.3 Model validation

To test if the geometric Brownian motion is valid for stock prices the time series of the simple daily return $u_t = \frac{S_{t+1} - S_t}{S_t}$ for the General Electric stock from 1991 to 2008 can be used. The expected daily return is estimated to be $\hat{\mu} = 0.000542$ and the daily volatility to

$$\hat{\sigma} = \sqrt{\frac{1}{n-1} \sum_t (u_t - \hat{\mu})^2} = 0.0169 \quad (4.2)$$

By letting dt be one day the normalized return $\frac{u_t - \hat{\mu}}{\hat{\sigma}}$ should have a standard normal distribution.

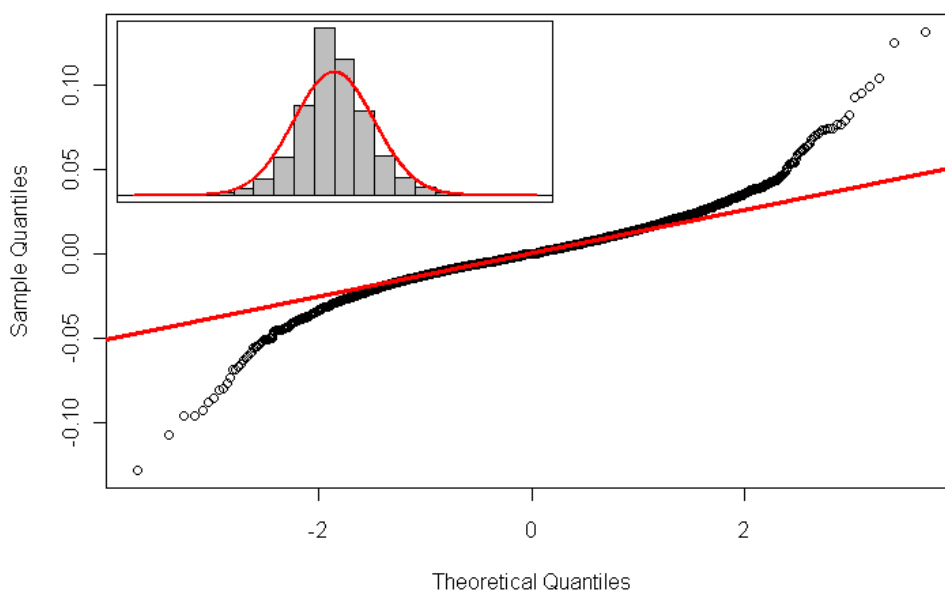


Figure 4.1: QQ-plot and histogram for the stock returns of General Electric.

The QQ-plot in figure 4.1 illustrates that the stock returns are not perfectly normal distributed. The stock returns have heavier tails and for General

Electric the excess kurtosis is 6. By looking at the histogram the choice to use the normal distribution as model for returns might seem clear, but at the same time it is important to keep in mind that market movements have more extreme deviations than assumed in the model.

4.1.4 Ito's lemma

Equation 4.1 describes stock prices as a stochastic variable. An important result regarding the manipulation of random variables was discovered by K. Ito in 1951[8]. It relates the small change in a function of a random variable to the small change in the variable itself and can therefore be seen as the Taylor's theorem for stochastic variables.

Theorem 4.1. (Ito's lemma) *Suppose that the value of a variable S follows an Ito process*

$$dS = a(S, t)dt + b(S, t)dz \quad (4.3)$$

where dz is a Wiener process and a and b are functions of S and t . Ito's lemma shows that any twice continuously differentiable function $f(S, t)$ is itself an Ito process satisfying

$$df = \left(a \frac{\partial f}{\partial S} + \frac{\partial f}{\partial t} + \frac{1}{2} b^2 \frac{\partial^2 f}{\partial S^2} \right) dt + \frac{\partial f}{\partial S} b dz \quad (4.4)$$

where the dz is the same Wiener process as in equation (4.3).

4.1.5 The lognormal property of stocks

Applying Ito's lemma to the earlier model for stock price movements

$$dS = \mu S dt + \sigma S dz \quad (4.5)$$

it follows that a function $f(S, t)$ is described by the process

$$df = \left(\frac{\partial f}{\partial S} \mu S + \frac{\partial f}{\partial t} + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 \right) dt + \frac{\partial f}{\partial S} \sigma S dz \quad (4.6)$$

Use Ito's lemma and equation (4.6) to derive the process followed by the function $f = \ln S$ when S is described by equation (4.5)

$$df = \left(\mu - \frac{\sigma^2}{2} \right) dt + \sigma dz \quad (4.7)$$

This equation indicates that $\ln S$ follows a generalized Wiener process and the change in $\ln S$ is normally distributed with mean $\mu - \frac{\sigma^2}{2}$ and variance $\sigma^2 t$

$$\ln S_t - \ln S_{t-1} \sim \phi \left[\left(\mu - \frac{\sigma^2}{2} \right) dt, \sigma \sqrt{dt} \right] \quad (4.8)$$

This equation shows that the geometric Brownian motion model (4.5) implies that a stock's price at time t given today's price is lognormally distributed. This result will be used later to simulate the stock price over time by rewriting equation (4.8) to

$$S_t = S_{t-1} \exp \left\{ \left(\mu - \frac{\sigma^2}{2} \right) dt + \sigma \sqrt{dt} \epsilon \right\} \quad (4.9)$$

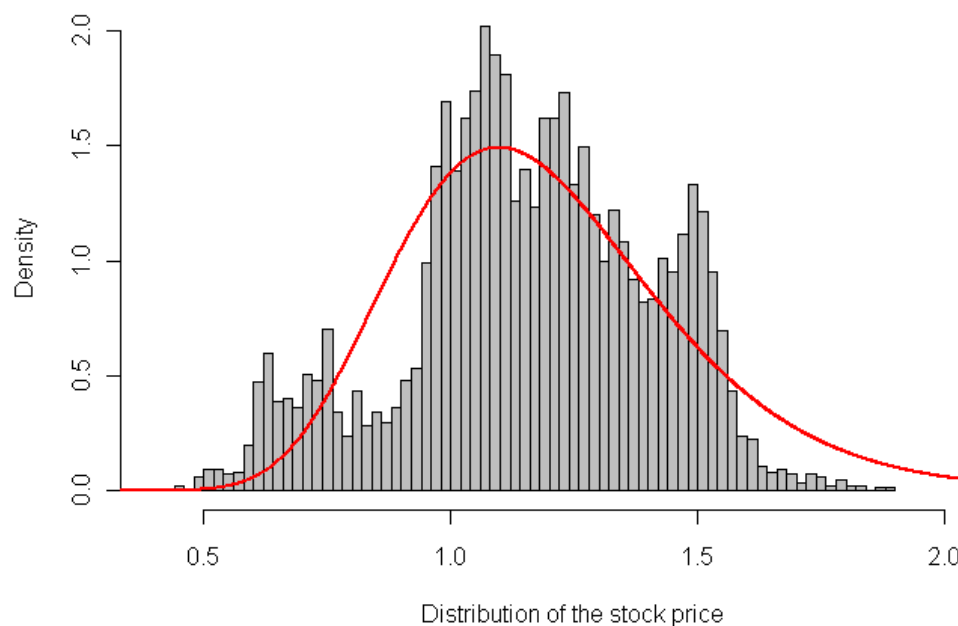


Figure 4.2: Histogram over the 1 year relative stock price of General Electric and the lognormal distribution.

Figure 4.2 plots the stock price of the General Electric relative to the stock a year earlier (250 trading days) and compare this with the assumed lognormal distribution. The figure illustrate that the assumption of lognormal distribution of stock prices is not perfect. There seems to be more frequent large drops in the share price than the model predicts and fewer large increases.

4.1.6 Equity portfolio

Consider a portfolio worth V_t that consist of d stocks with an amount of w_i in stock i ($1 \leq i \leq d$). If X_t is the return vector of the stocks with x_i as the

return on stock i , then the loss in value of the portfolio is

$$L_{t+1} = -(V_{t+1} - V_t) = -\sum_{i=1}^d w_i x_i \quad (4.10)$$

Assuming the returns are multivariate distributed $X_t \sim N_d(\mu, \Sigma)$, then the loss distribution of the portfolio is normally distributed. The expected return and variance of the portfolio loss are given by

$$\mu_P = w^T \mu \quad (4.11)$$

$$\sigma_P^2 = w^T \Sigma w \quad (4.12)$$

4.2 Fixed income

4.2.1 Introduction to fixed income

A fixed income security provides a return in form of fixed periodic payments (coupons) and repayment of the principal at maturity. In contrast to variable income securities such as equity, the payments of a fixed income security are known in advance. Common fixed income securities are corporate and government bonds.

4.2.2 Bond pricing

The value of a bond $B(t, T)$ at time t is depending on several variables[9]: The underlying term structure $r(t)$, the coupon rate and the time of maturity T

$$B(t, T) = e^{-\int_t^T r(\tau) d\tau} \left(Z + \int_t^T K(t') e^{\int_{t'}^T r(\tau) d\tau} dt' \right) \quad (4.13)$$

where Z is the principal and $K(t)$ is the coupon payments. The value of a bond converges to the principal as time to maturity goes to zero $B(T, T) = Z$. The daily return on the bond is neither identically nor independently distributed as the characteristics change over time. To calculate VaR for fixed income it is necessary to simulate the underlying term structure. This complicates the calculations since the changes in bond values are non-linearly related to the interest rates. In addition, the bond value is not depending on one single interest rate (except zero-coupon bonds), but one for each coupon payments.

4.2.3 Stochastic models for interest rates

An often proposed model for the change in interest rates r are the risk-neutral random walk

$$dr = m(r, t)dt + s(r, t)dz \quad (4.14)$$

where $m(r, t)$ and $s(r, t)$ are functions of the interest rate r and time t , and determine the behavior of the interest rate. Interest rates have two properties that need to be described by the models. First the mean reversion property; interest rates appear to be pulled back to a long-run average level over time. Secondly, models for the interest rates should incorporate that interest rates are positive for all practical cases.

Vasicek's model

Several factor models have been suggested for the short-term interest rate. In 1977 Vasicek proposed[10] the following process for r

$$dr = a(b - r)dt + \sigma dz \quad (4.15)$$

where a , b and σ are constants. The model incorporates mean reversion by pulling the short rate to a level b at rate a , but unfortunately the interest rate easily goes negative. The main advantages of Vasicek's model are its simplicity and ease of simulation. The problem with this and other factor models is that they do not describe the dynamic between interest rates with different maturities. For factor models the whole yield curve is simply derived from the short-term interest rate and does not take into account the correlation or volatility of the term-structure. This property is unsatisfying when calculating risk measures for bonds with coupons and a range of maturity dates. To bypass this problem and to simplify the calculation of the risk measures all fixed income included in this thesis will be zero-coupon bonds. Alternatively the LIBOR market model (LMM) could have been used for modeling the whole interest rate curve.

4.2.4 Model validation

Figure 4.3 show the QQ-plot and histogram of the daily yield change for 5-year US Government bonds and illustrate that the assumption of normal distributed changes in the Vasicek model might be appropriate. The interest rate data seems to be closer to normally distributed than the return on stocks from figure 4.1. When simulating market risk over a time horizon of one day the random part will dominate the drift. The large volatility of interest rates seen the last years will increase the domination further. By taking this into account, the close to normal distribution of the change in bond yield in figure 4.3 strengthen the case for a simple model such as Vasicek.

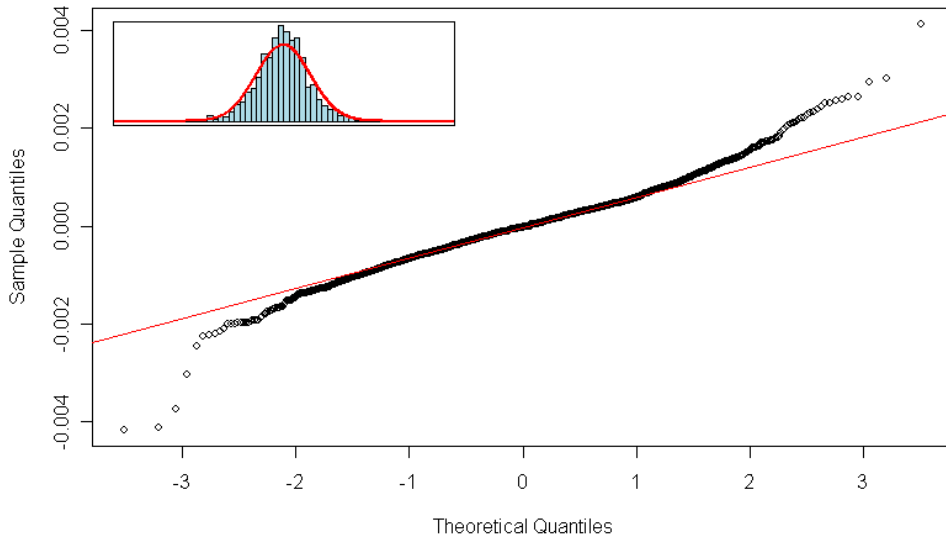


Figure 4.3: QQ-plot and histogram for the daily yield change of 5-year US Government bonds.

4.2.5 Fixed income portfolio

In a large fixed income portfolio where each bond is depending on several interest rates and the bonds have different maturities, some simplifications are necessary when calculating VaR. The value of the different bonds is driven by the same set of risk factors and can be aggregated into a small set of exposures without losing risk information. Cash-flow mapping is a simplification method using interpolation in which the cash-flows from the bonds are mapped into cash-flows occurring on the standard maturity dates. At each of these dates the cash-flow is discounted at the appropriate zero-coupon rate. The Value-at-Risk can then be calculated from the exposure x to and the risk $V = \alpha\sigma^r$ of the selected standard interest rates [2]

$$\text{VaR}_\alpha = \alpha\sqrt{x^T\Sigma x} = \sqrt{(x \times V)^T R (x \times V)} \quad (4.16)$$

where R is the correlation matrix of the selected zero interest rates.

Chapter 5

Data Description

This thesis is based on a case study of risk measures on a single portfolio over a specific time period. The data is thus not necessarily representative for the whole market. This chapter presents the chosen portfolio and describes the different assets.

5.1 Portfolio

The selected portfolio is composed of equity and fixed income assets. For equity, two large indices are included: The S&P 500 index for American stocks and Eurostoxx 50 for European stocks. The fixed income part is represented by American and German 5-year Government bonds. The most important criteria when deciding between the alternative assets were high liquidity, diversification and ability to represent the macro trends of the economies. High liquidity is important as today's financial crisis is in part a liquidity crisis as the name "credit crunch" indicates. When calculating daily risk measures it is essential to have updated prices that reflect the true values. This criterion has eliminated much of the corporate debt and asset-backed security market since both have dried up.

Asset allocation in the portfolio will be set dynamically by choosing the allocation which maximizes the Sharpe ratio $\frac{\mu - r_f}{\sigma}$ according to Markowitz portfolio theory. That is the asset weights that result in the portfolio with largest expected excess return over the risk-free rate divided by the portfolio's volatility. The portfolio will be constrained by the fact that there are no allowance for shorting or leverage. The shorting ban reflects the actual market for financial stocks the second half of 2008 and is in line with rules for mutual funds.

5.1.1 Cash-flow mapping of fixed income

A broad portfolio of fixed income is usually exposed to a high variety of interest rates and maturity dates. For example the US Treasury notes pay semi-annual coupons while the German Government bonds have annual coupons. The 5-year US Treasury note is issued at auctions once a month, and hence all coupon and maturity dates are shifted one month for each auction. After holding the bond for two years it can now be treated as a 3-year bond and so on. In theory the present value of each cash-flow should be calculated by using an interest rate with equal maturity. To simplify, the cash-flows are often interpolated or mapped into a set of maturity dates as described in the previous chapter. To simplify the calculation even further the portfolio in this thesis will be constructed to be mapped into cash-flows that correspond to a 5-year US or German Government bond. Thus the market risk of the bonds' values can be found by simulating the bond yields. Statistics and historical time series for these bonds can be found on the respective central banks' homepages.

5.1.2 Exchange rate risk

The portfolio holds assets denoted in US Dollar and Euro, and is hence exposed to fluctuations in the exchange rate. This exchange risk will be hedged in the portfolio by entering currency swaps. In the thesis this currency hedge will be assumed to be perfect, and the risk measures will only include market risk from changes in equity and fixed income values.

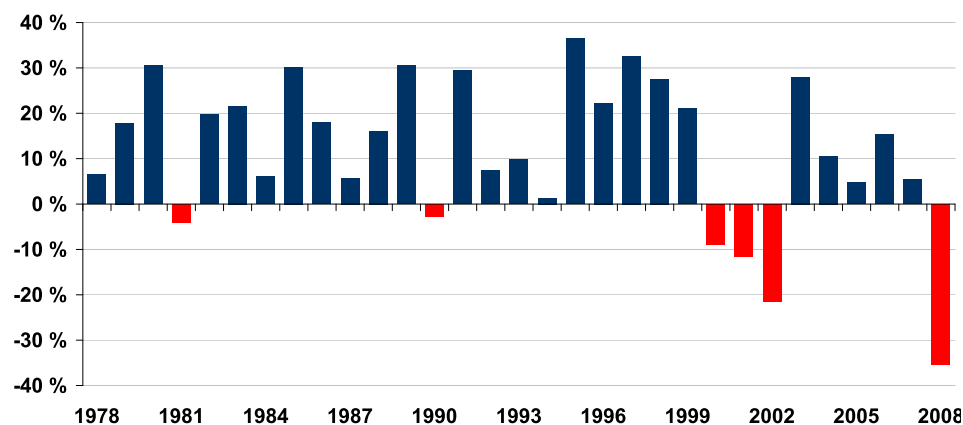


Figure 5.1: Annual return on S&P 500 including dividends.

5.2 S&P 500

The S&P 500 index tracks the prices of 500 large-cap common stocks actively traded on either of the two largest American stock markets, NYSE and NASDAQ. Of the most used American indices, S&P 500 is the broadest and can be considered an indicator for the American economy. The diversification of the index has made it popular among investors. Some Exchange Traded Funds (ETFs), mutual funds and pension funds are designed to mimic the performance of the index, making it one of the most liquid investment assets. In addition a whole range of derivatives are linked to the S&P 500 index, giving more data such as the implied volatility index (VIX) shown earlier in figure 1.1.

Expected return

Figure 5.1 illustrates the annual return on the S&P 500 index the last 31 years. The index has given positive returns in 25 of these years, with large losses only occurring during the dot-com bust/American recession in 2000-2002. Then in the second half of 2008 the losses got substantial larger than anything seen in decades. Before the market fall in 2008, equity seemed to give an almost consistent large return if held over a time horizon of some years. This is reflected in figure 5.2 where the logarithm of the index value is close to linear. From 1977 to 2007 the S&P 500 had a compounded annual growth rate (CAGR) of 12.61%.

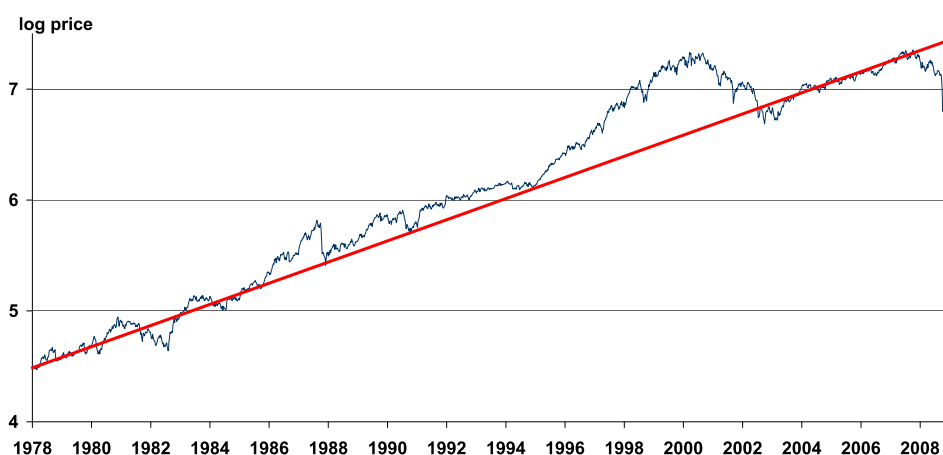


Figure 5.2: Logarithmic price of S&P 500 index adjusted for dividends.

5.3 Eurostoxx 50

Eurostoxx 50 is a stock index of 50 blue-chip companies in the Eurozone and provides a representation of the European stock market. The index covers 12 Eurozone countries with dominance of French and German companies. The Eurostoxx 50 index is not the broadest of the pan-European indices (covers only 57% of the free float market capitalization of the countries represented¹), but it is among the most liquid with a wide range of investment products such as Exchange Traded Funds, derivatives and structured products.

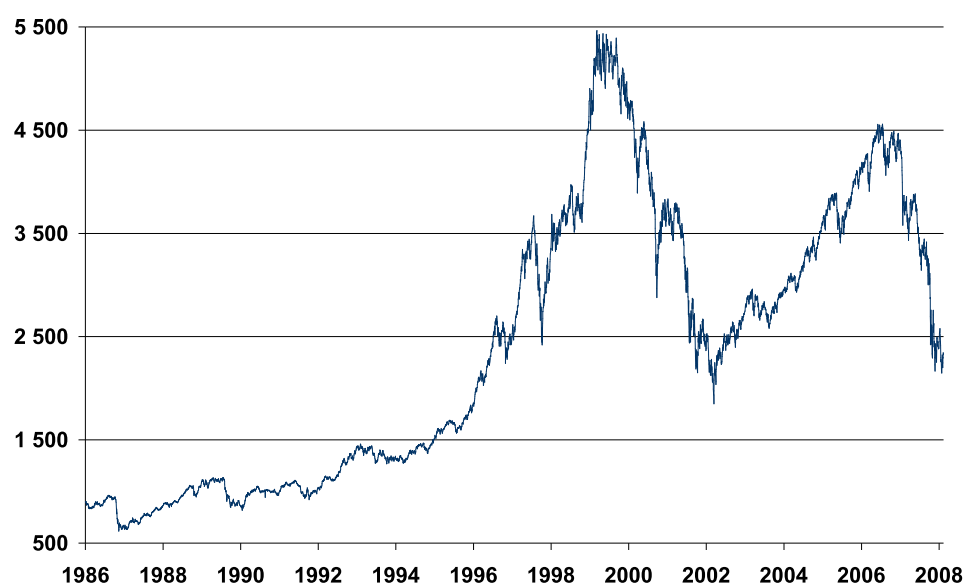


Figure 5.3: Eurostoxx 50 index (1,000 as of 31 December 1991).

Expected return

The time series of Eurostoxx 50 only goes back to 1986 as plotted in figure 5.3. To find the annual growth since 1977, the general performance of European stocks² for the period 1977 to 1986 have been used. Adjusted for dividends this gives a compounded annual growth rate of 11.95% for Eurostoxx 50.

¹According to STOXX Ltd.

²MSCI Europe ex. UK

5.4 US 5-year Treasury note

US Government bonds are the debt financing instruments of the US Federal Government, often referred to as Treasuries. Since the Treasuries are backed by the US Government and the largest economy in the world, they are usually assumed to be risk-free. Treasury securities are very liquid and the financial turmoil the last year has increased the demand for a "safe-haven", pressing down yields on the Treasuries. The yields on the bonds are published every day using an interpolation of bond prices with maturities close to the fixed maturities dates. The 5-year Treasury notes are issued with a par value of \$100. The bond price with T years to maturity and yield γ can be calculated as

$$B(T) = \frac{100}{(1 + \gamma)^T} \quad (5.1)$$

The historical yield γ on 5-year Treasury notes are given in figure 5.4. Since early 80s the yield has trended downwards and is now historical low at two percent. Many critics claim that the Federal Reserve did hold the interest rates too low from 2003 to 2007 and thus inflating the housing prices which is one of the factors leading to today's financial crisis.

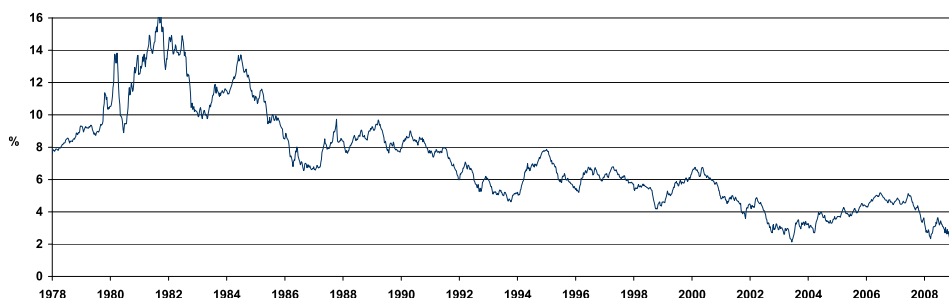


Figure 5.4: Yield on 5-year US Treasury notes.

Expected return

The return on a portfolio of bonds can be splitted between the coupons' payments and the change in the value of the bonds. The upper part of figure 5.6 plots the return a portfolio of 5-year US Treasury notes would have given every year since 1978. During this time period only 1994 gave a negative return. Compared with S&P 500 (figure 5.1) and Eurostoxx 50 the Treasury notes have a lower but more stable return. The compounded annual growth rate was 6.76% from 1978 to 2007.

5.5 German 5-year Federal security

The Bundesbank issues bonds for the German Federal Government which represents the largest economy in the Euro-zone. Until last year, the spread on yields between most Euro countries debt was low, but the recession and the uncertain outlook for some national economies have increased these spreads. Germany with its large economy and prudent fiscal policy has now the lowest interest rate in the Euro-zone. Figure 5.5 plots the historical yield of the 5-year German bond. The trend of the yield looks similar to the American bond, but with lower rates in the 80s. Today's yield is historically low as rates have been cut to stabilize the economy from the shocks of the current financial crisis. The bonds in the portfolio are (similar to the American bonds) assumed to be constructed to in total reflect the structure of a 5-year bond.

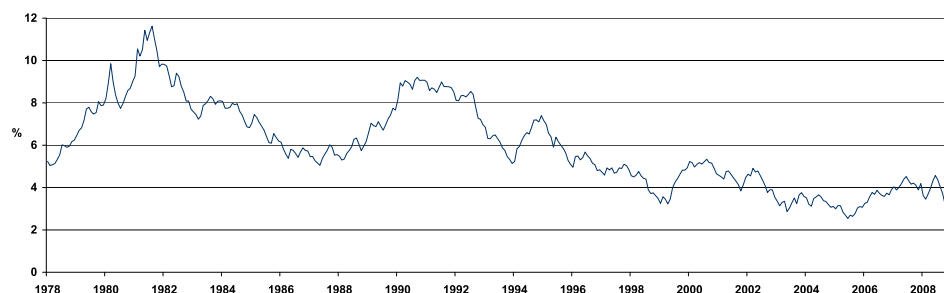


Figure 5.5: Yield on 5-year German Federal securities.

Expected return

The annual return on holding German 5-year bonds the last 30 years is plotted in the lower part of figure 5.6. Four of these 30 years gave negative return. The German bonds have a compounded annual growth rate of 6.40% which is lower than both the equity indices and the American Treasuries. However the German bonds have lower volatility than the equity indices.

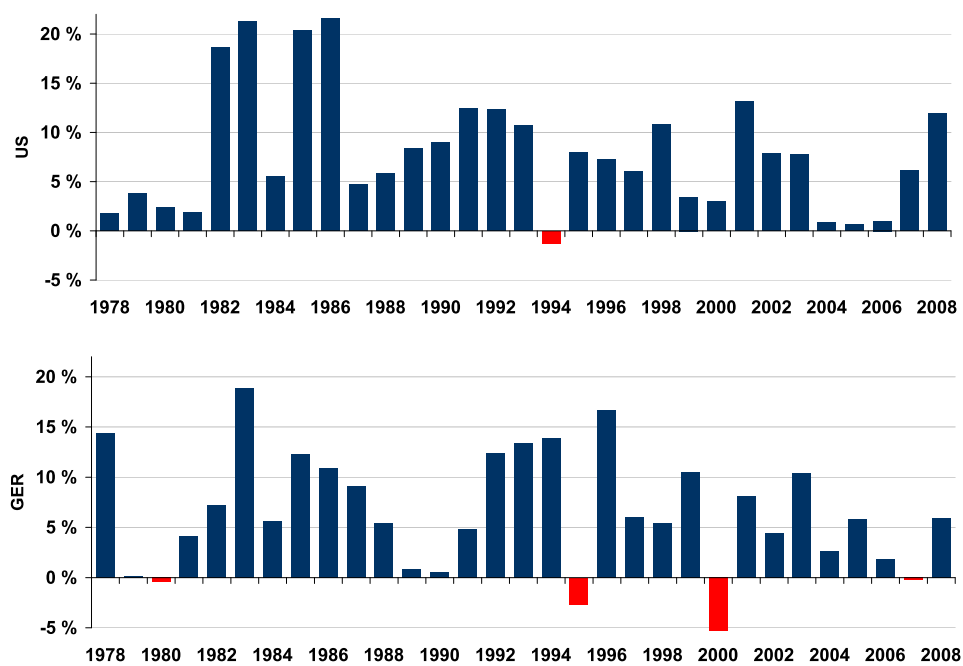


Figure 5.6: Annual return on 5-year American (upper) and German (lower) Government bonds.

Chapter 6

Implementation

This chapter describes how the calculation of Value-at-Risk and expected shortfall will be implemented for the case portfolio. It will take a closer look at the calibration of the underlying models to the observed market and how the dynamic asset allocation is carried out.

6.1 Calibration to market data

When calculating VaR and ES the current levels of volatilities and correlations are important. These parameters describe the size of fluctuations and the linear relationship between the underlying variables. An important feature of volatilities and correlations are their non-constant property. During some periods an asset's volatility may be relatively low, whereas during other periods it may be relatively high. The Exponentially Weighted Moving Average (EWMA) model attempts to keep track of the variations in the volatility and correlation through time. The model is a weighting scheme which gives more weight to recent than to older data. The weights α_i decrease exponentially as data go back through time. Specifically, $\alpha_{i+1} = \lambda\alpha_i$, where λ is a constant between 0 and 1. According to J.P. Morgan [11] the optimal value for λ is 0.94, which resulted in the forecasted variance closest to the realized rate in their study. By using this value for λ the estimates respond moderately fast to new information provided by the daily changes, and data older than a year do not affect the produced estimates. An attractive feature of the EWMA model is its recursive property. To update the estimate only the most recent observation and estimate are needed, thus the estimate is fast to compute and the method needs to store little data.

6.1.1 Volatilities

The formula for updating the volatility estimates is

$$\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1 - \lambda)u_{n-1}^2 \quad (6.1)$$

where u_i is the most recent daily percentage change in the asset S . For the selected portfolio, data from year 2001 are used to initialize the EWMA model giving useable estimates from the start of 2002. Figure 6.1 and 6.2 plot the estimated volatilities for the equity assets and yields on government bonds respectively. The equity indices have steep increases in the volatility from September 2008 with a quadrupling from 20% to almost 80% in just a month. The estimated volatility of S&P 500 follows the implied volatility in figure 1.1 closely, indicating that the EWMA model is reasonable good. The yield volatility of the government bonds starts increasing already in the summer of 2007 when the "credit crunch" began.

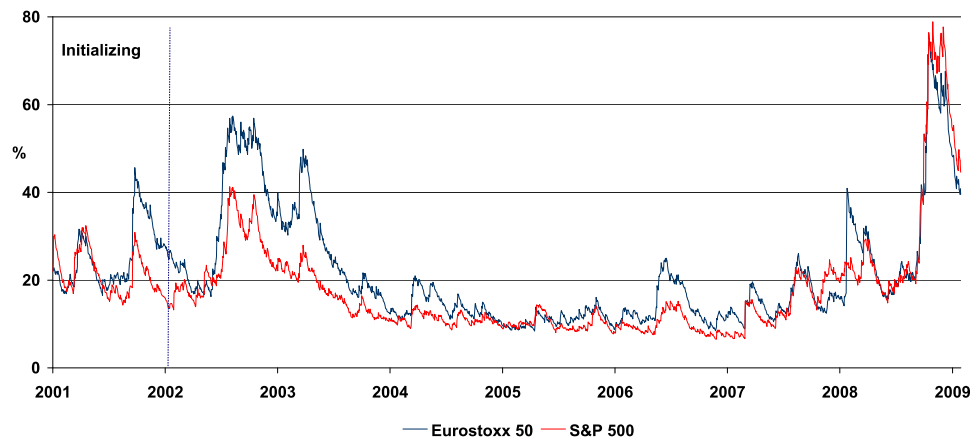


Figure 6.1: Estimated annual volatility of equity assets.

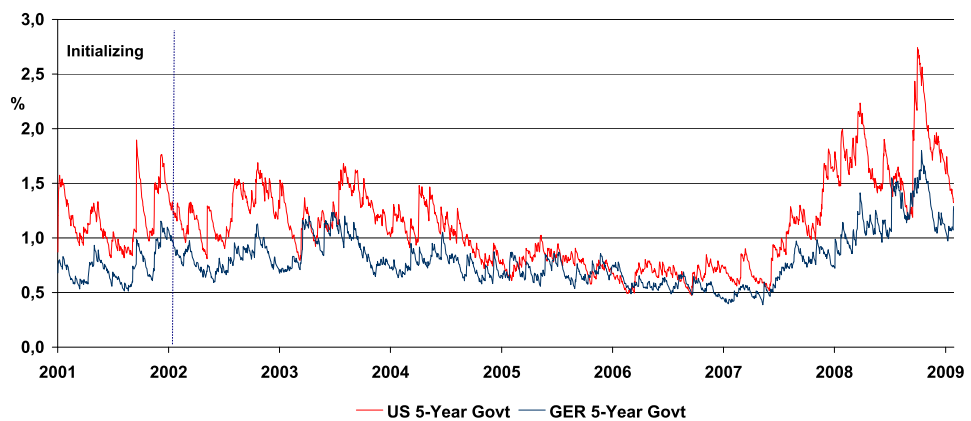


Figure 6.2: Estimated annual volatility of yield on bonds.

6.1.2 Correlation

The estimates for the correlation between the assets are updated in a similar way as with the volatility. First the covariance between the variables X and Y is estimated by

$$\sigma_{XY,n} = \lambda\sigma_{XY,n-1} + (1 - \lambda)x_{n-1}y_{n-1} \quad (6.2)$$

where x_{n-1} and y_{n-1} are the most recent daily percentage change in the variables. The correlation on day n is then found by using the definition

$$\rho_n = \frac{\sigma_{XY,n}}{\sigma_{X,n}\sigma_{Y,n}} \quad (6.3)$$

Figure 6.3 plots the estimated correlation between the S&P 500 and American bond yields. The correlation fluctuates from -0.5 to 0.9 over this period. During both the downturn in 2001-2003 and 2008-2009 the correlation was positive. This can to some degree be explained by central banks lowering interest rates to combat downturns in the economy, and the corresponding fall in equity value. For 2003-2007 interest rates and equity value seems mostly uncorrelated with fluctuations between -0.5 to +0.5.

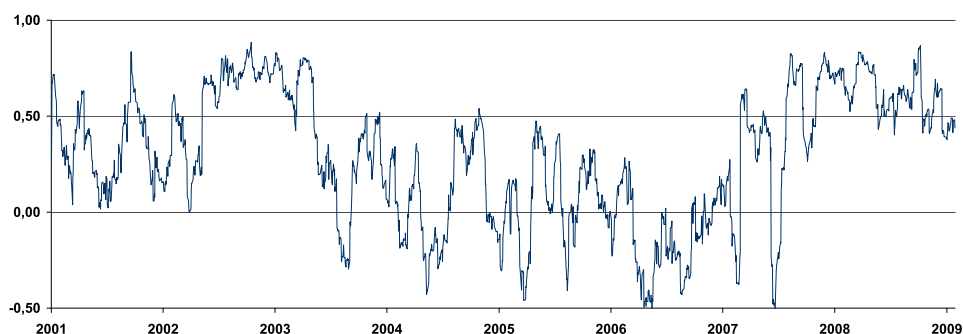


Figure 6.3: Estimated correlation between S&P 500 and yield on 5-year US Government bonds.

6.2 Asset allocation

As described in the previous chapter, the allocation between the assets in the portfolio will be set dynamically according to Markowitz portfolio theory[6]. The risk-return profile of a portfolio can be optimized by adjusting the weights of each asset. An optimal portfolio displays the lowest possible level of risk for a given level of return μ_0 . All optimal portfolios comprise the efficient frontier as illustrated in figure 6.4 for the selected portfolio in

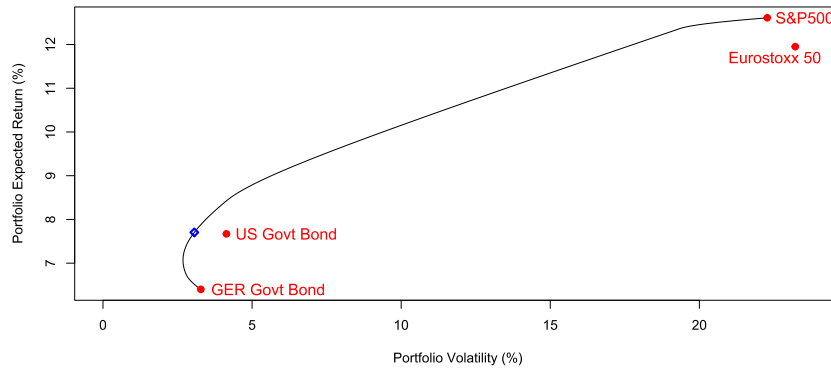


Figure 6.4: The efficient frontier and individual assets January 2nd 2001.

January 2001. All individual assets are to the right of the efficient frontier due to the diversification effect of portfolios. In this thesis the expected return μ on each asset is fixed and set equal to the compounded annual growth rate (CAGR) between 1977 and 2007 found in the previous chapter. The growth in 2008 is not included in the expected return since the main purpose of the thesis is to look at risk measures in 2008 and should not use data that was not available at that time. The standard deviation of the individual assets are estimated with a EWMA model using $\lambda = 0.94$. For the asset allocation the covariance matrix Σ is found for the actual government bonds and not for their yields as done when simulating the risk measures. The weights w for the optimal portfolios in the efficient frontier are calculated using quadratic programming

$$\begin{aligned} \min \quad & w^T \Sigma w \\ \text{s.t.} \quad & w^T \mu = \mu_0 \\ & w \in [0, 1] \end{aligned}$$

where the last constraint prevents shorting and leverage.

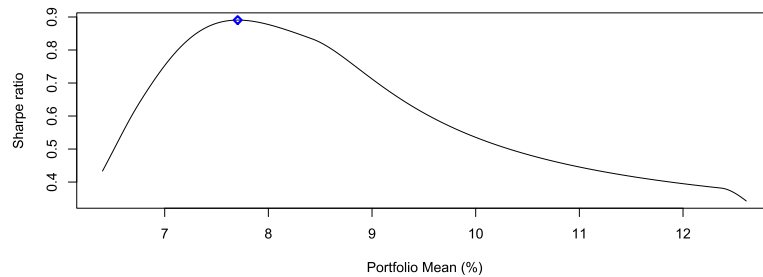


Figure 6.5: Sharpe ratio for the portfolios on the efficient frontier.

In addition to the individual assets, an investor can deposit cash in a bank, earning a risk-free interest rate. For a given risk-free rate, there is only one portfolio on the efficient frontier that combined with cash achieves the lowest risk for any possible return. This portfolio is called the market portfolio and is given as a blue dot in figure 6.4 and 6.5. The market portfolio is found by maximizing the Sharpe ratio $\frac{\mu - r_f}{\sigma}$ over the optimal portfolios comprising the efficient frontier[12]. Figure 6.5 plots the Sharpe ratio for the efficient frontier on January 2nd 2001. The risk-free rate is set equal to the yield on US Treasuries the day of calculation. Figure 6.6 displays the allocation between equity and fixed income for the selected portfolio in 2007 and 2008. Most of the portfolio is allocated to fixed income due to the relatively high return per unit of volatility on government bonds. As equity volatility increased in the end of September 2008, the equity share of the portfolio dropped to around 10%.

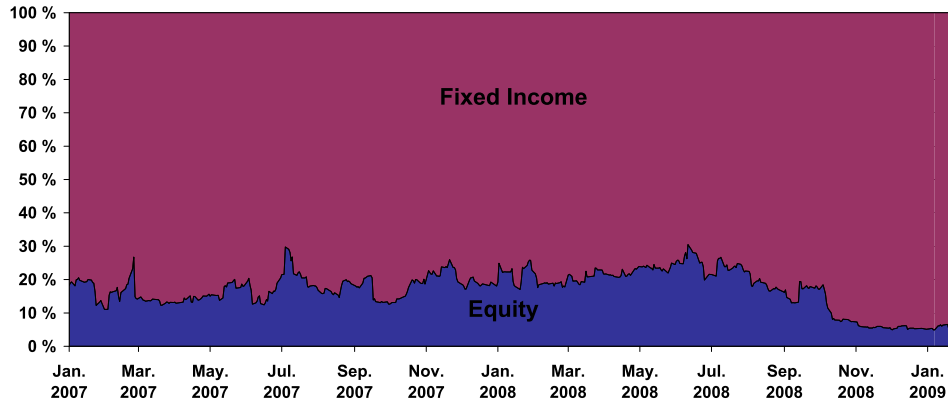


Figure 6.6: Portfolio allocation to equity and fixed income.

6.3 Vasicek parameters

The yields on the two government bonds are simulated using Vasicek interest rate model

$$dr = a(b - r)dt + \sigma dz \quad (6.4)$$

This model has three parameters, σ , a and b , that need to be calibrated. σ represents the volatility of the yields and will be updated with the current estimate from the EWMA model described earlier in this chapter. Parameter a and b represent the speed of the mean reversion and the long-term mean of the yield. These can either be estimated from historical data or from calibrating the model to reflect today's observed yield curve. In order to

use the second approach the parameters would need to be re-estimated for each new day. Since the random part of the model dominates the drift over the short time periods considered in this thesis, the second approach seems unnecessarily computer intensive. Thus b is set equal to 4.5%, the mean US 5-year yield over the last 15 years. The a parameter is set equal to 0.4 by regression of the time series.

6.4 Calculation of risk measures

To generate the loss distribution for the portfolio, both the change in stock values and yields will be simulated repeatedly using the geometric Brownian motion (4.9) and Vasicek's model. As the number of random samples generated of the underlying variables increases, estimates of the risk measures will converge to the "true value" of the given model (Appendix A). Monte Carlo simulation can be computational slow as the convergence rate is of order $O(\sqrt{n})$. The number of simulations is set to 100 000 for each day, as it provides an acceptable trade-off between accuracy and computational time. This number gives a standard deviation of the risk measures of about 0.15% or 5×10^{-6} of the portfolio value.

For each simulation of the underlying variable, the portfolio is revalued and the loss over the time period is calculated. These losses comprise the loss distribution of the portfolio. To estimate VaR and ES the losses are sorted and VaR_α is set equal to the $n(1 - \alpha)$ largest value. For 99% VaR with 100 000 simulations the $\text{VaR}_{.99}$ is the thousandth largest loss. An estimate for $\text{ES}_{.99}$ is the mean of the thousand largest losses.

Algorithm 1 Calculation of VaR and ES

```

 $\alpha \leftarrow$  VaR quantile
 $\Delta t \leftarrow$  time horizon
for  $day = \text{startdate}, \dots, \text{enddate}$  do
  update daily covariance structure  $\Sigma$  using EWMA
  update asset allocation  $w$  based on  $\Sigma$ 
  for  $run = 1, \dots, \text{number of simulations}$  do
     $rand \sim \sqrt{\Delta t}N(\mu, \Sigma)$ 
     $S(t + \Delta t) \leftarrow S(t) \exp\{(\mu_S - \sigma^2/2)\Delta t + rand_S\}$ 
     $\gamma(t + \Delta t) \leftarrow \gamma(t) + a(b - \gamma(t))\Delta t + rand_\gamma$ 
     $Loss[run] \leftarrow w_S(S(t) - S(t + \Delta t)) + w_B \left( \frac{1}{(1 + \gamma(t))^5} - \frac{1}{(1 + \gamma(t + \Delta t))^5} \right)$ 
  end for
   $sort(Loss)$ 
   $VaR[day] \leftarrow \alpha$  quantile of  $Loss$ 
   $ES[day] \leftarrow mean(Loss[0, \dots, \alpha \text{ quantile}])$ 
end for

```

Chapter 7

Results

This thesis focus on the performance of the risk measures during the current financial turbulence. The first part of this chapter examines the risk behavior of the case portfolio during the last two year's financial troubles. In the second part the models to estimate VaR and ES are investigated to check their reliability.

7.1 Risk measures and the current financial crisis

When looking at the performance of VaR and ES there are several factors to examine: How do the risk estimates change over the time period? Does the level of risk increase substantially compared to the levels seen before? Is there a higher frequency of losses exceeding VaR than seen before or compared to the number predicted by the risk measure? Does the average size of the losses exceeding VaR increase and how are they compared to the ES estimate?

7.1.1 Risk measures levels

Figure 7.1 graphs the estimates for $VaR_{0.99}$ and $ES_{0.99}$ in percent of the portfolio value over the last eight years. The first year is used to initialize the parameters and may thus be less accurate. The graph can to some extent be seen as a combination of figure 6.1 and 6.2 with weighting according to the asset allocation in the case portfolio. Since the financial crisis started in the middle of 2007 the risk estimates has increased substantially with up to a fivefold at the end of 2008. In the three years (mid 2004- mid 2007) before the crisis, the level of risk was low and stable between 0.2-0.5% of the portfolio value. Looking at these two periods there is a clear change in the risk levels before and during the financial crisis. By looking at older data one can see that the risk levels were almost as high during peaks in 2001 and 2003 as during the current crisis. Higher allocation to equity can explain

some of the risk, but it also shows that for a diversified portfolio, today's estimated risk is not substantially higher than previous historical heights. In contrast it might seem like it is the low and stable risk during 2005-2007 that is unusual.

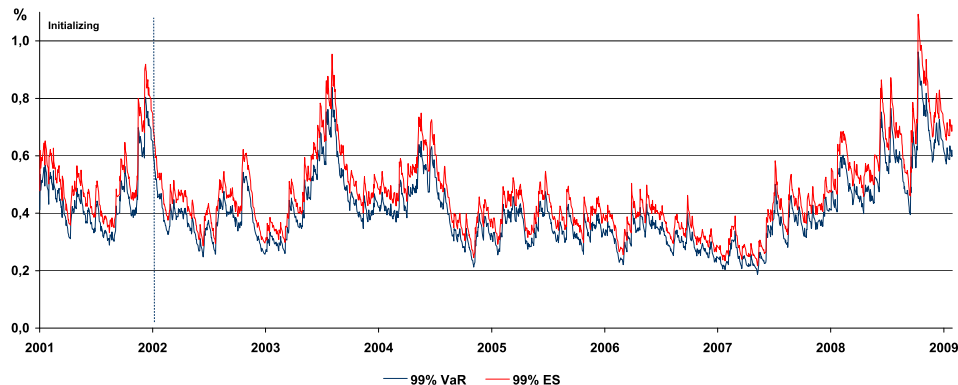


Figure 7.1: Estimated risk measures for the case portfolio the last 8 years.

7.1.2 Frequency and size of large losses

When comparing the performance of risk measures before and during the current financial downturn the frequency and size of losses that exceeds the estimate for Value-at-Risk are two important aspects to examine. January 1st 2005 to May 31th 2007 will be used as the time period before while June 1st 2007 to December 31th 2008 will be used as the time period during the current financial crisis. The choice for these time periods is based on the fact that many market participants in 2007 began to perceive the low volatility of 2005 to 2007 as normal due to globalization and complex financial instruments, which they believed had reduced and spread risk to those who could handle it. Figure 7.2 compares the estimated risk levels with the

<i>Risk measure</i>	<i>Before crisis</i>	<i>During crisis</i>
99% VaR	2.0%	4.3%
95% VaR	8.2%	7.9%
90% VaR	14.3%	12.2%

Table 7.1: Percentage of losses exceeding VaR before and during crisis.

actual losses of the case portfolio over the chosen time periods. For the time period before June 2007 (left of the vertical dotted line) there were 12 losses (in 600 days) that exceeded the estimated VaR which gives a frequency of 2.0% (summarized in table 7.1). For the 393 days during the financial crisis

the number of losses exceeding VaR were as high as 17 or 4.3% of the days, even if the level of VaR estimates was much higher than before. The more than doubling of cases with losses larger than estimated VaR indicates that the VaR measure performs worse during financial turbulence than in stable markets. The fact that losses exceeds $\text{VaR}_{0.99}$ in more than 1% of the cases will be discussed later in this chapter. For $\text{VaR}_{0.95}$ and $\text{VaR}_{0.90}$ there is a small decrease in the frequency of losses exceeding the estimates. It seems like the model get much worse at predicting the extreme losses while a little better at the relative small losses.

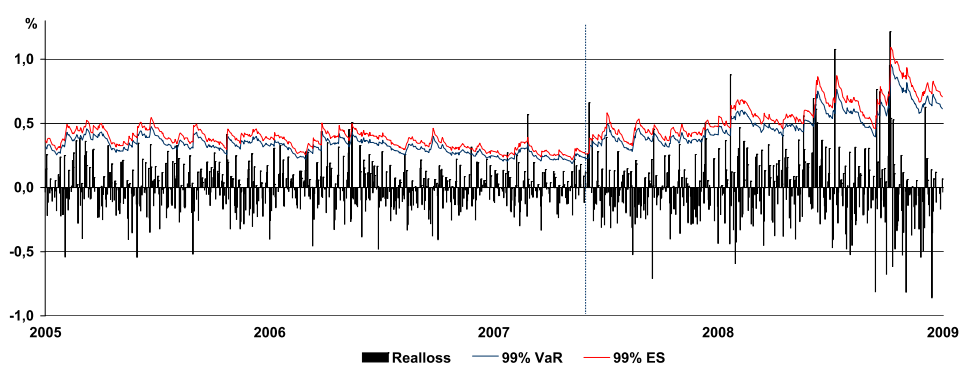


Figure 7.2: Estimated risk measures and actual loss for the case portfolio before and during the current financial downturn.

In figure 7.2 the frequency of really large losses are much higher during the financial crisis than before. In the time period before there is only one loss which is substantially larger than the estimate for expected shortfall (red line) compared to five such losses over the shorter time period during the crisis. Table 7.2 lists the average of losses exceeding VaR relative to the estimate for ES. There is a significant difference between the two time periods with losses 2.5-5 times larger relative to ES during the crisis than before.

<i>Risk measure</i>	<i>Before crisis</i>	<i>During crisis</i>
99% ES	+10.4%	+25.9%
95% ES	+6.4%	+33.5%
90% ES	+7.1%	+31.0%

Table 7.2: Average loss exceeding VaR compared to ES before and during crisis.

7.1.3 Risk during specific events

In addition to the overall risk measure performance before and during the crisis it is interesting to look at the measure's behavior during some specific "game changing" events. This thesis will look at three of the most important events during the current financial crisis. The first event to be examined is the beginning of the "credit crunch" in July 2007 with the collapse of the market for securitized securities. Next event is Lehman Brothers bankruptcy which marked the start of the global financial crisis at September 15th 2008. The last event is the coordinated worldwide interest rate cut in the start of October.

Bear Stearns hedge funds collapse, June-July 2007

The "credit crunch" began in July 2007 with the rapid decline in the market for securitized securities. Most notable for investors were the collapse of two Bear Stearns hedge funds during the week of July 16th, just weeks after they had been bailed out by the bank. Figure 7.3 graphs the development of VaR, ES and the real loss on the case portfolio over June and July 2007. During June and the start of July the risk measures increased after an extreme loss at more than twice the 99% ES estimate at June 7th and several other above or close to 99% VaR. Both the frequency and the size of the losses in the period are unlikely high with three losses exceeding 99% VaR in 42 trading days. The loss at June 7th is so extreme that none of the 100 000 simulated losses are close to the real loss. In July the 99% VaR measure increased to 0.5% for the first time in over three years after doubling in less than a month. At the exact dates of the hedge funds collapse (July 16th) or the bail out (June 22th) there are none extreme movements for the case portfolio. This is not surprising since both events should have come after large losses already incorporated by the equity and fixed income markets.

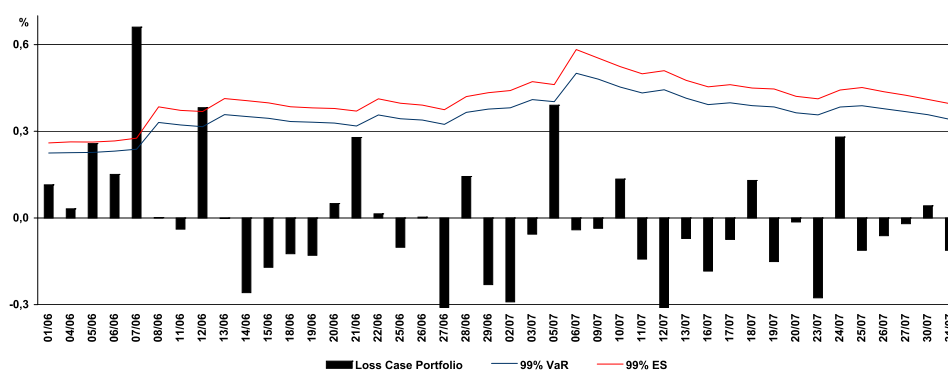


Figure 7.3: Risk measures and real losses during June and July 2007.

Bankruptcy of Lehman Brothers, September 15th 2008

In the week of September 15th 2008 the "credit crunch" deteriorated to become a full blown global financial crisis. The week was dominated by the insolvency of several large American financial institutions, large drop in equity and commodity values and havoc in the corporate bond market. On September 15th Lehman Brothers filed for bankruptcy while Merrill Lynch, another investment bank, was acquired by Bank of America. Then on the next day American International Group, a large insurance corporation, got insolvent after a downgrade of its debt, and needed to be saved by the US Government. The week continued with market money funds falling below \$1 leading to large redemption pressure and a new ban on the short-selling of financial institutions. On September 19th the US Secretary of the Treasury proposed a \$700b US financial bail out plan. The next weekend (September 21th) the two remaining American investment banks, Goldman Sachs and Morgan Stanley, converted to bank holding companies to get easier access to capital.

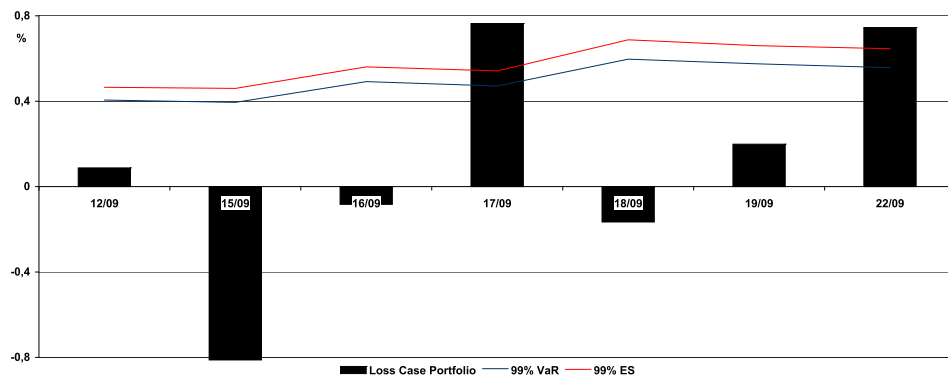


Figure 7.4: Risk measures and real losses during the middle of Sept.08.

Figure 7.4 plots VaR and ES estimates together with the real loss on the portfolio for the time period in September 2008. Both the risk measures increase during the week by about 50% due to the large fluctuations in the underlying variables. On September 15th the equity markets decreased by 4-5%, but the case portfolio nevertheless made a huge gain of 0.81% due to a 42 basis point reduction in the yield on American 5-year Treasuries, which had an asset allocation of 55%. The portfolio made losses exceeding both 99% VaR and ES two times later that week. On September 19th the value of both equity and fixed income changed more than the risk measures, but cancelled due to movements in opposite directions. If including changes in both directions, four out of seven days sees fluctuations larger than 99% expected shortfall which should never happen. Still two out of seven for the

case portfolio is not a good sign for the performance of the risk measures.

Central banks coordinated 0.5% interest rate cut, October 8th 2008

The two first weeks in October 2008 saw the most volatile financial markets in centuries. More and more banks went insolvent, Governments staged rescues and fiscal stimulus, and economies collapsed with large outflows from small currencies. On October 3rd 2008 the \$700b US financial bail out plan was approved. Then on the week of October 6th the Icelandic banking system broke down with a nationalization of the banks and a drop of more than 50% for the currency. The week continued with a coordinated 0.5% interest rate cut by the central banks of the world's largest economies on October 8th. Equity markets fluctuated wildly with large losses until October 13th when they rallied with record gains of more than 11%, the largest American gain in 75 years.

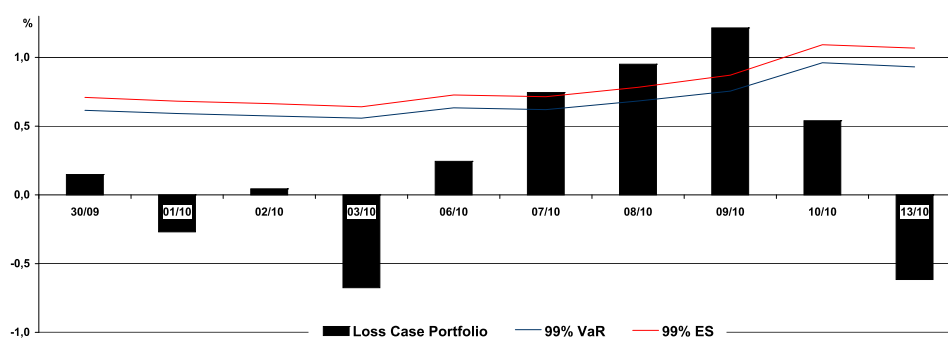


Figure 7.5: Risk measures and real losses during the first part of October 2008.

Figure 7.5 graphs the 99% VaR and ES estimates together with the loss on the case portfolio in the beginning of October 2008. During these weeks both two risk measures increased to their highest level recorded in this study due to the extreme fluctuation in the financial markets. The risk levels were then about five times larger than in the spring of 2007. For the case portfolio the losses exceeded 99% VaR three days in a row, where the loss on October 9th is the largest during the eight years studied in the thesis. If assuming independency the probability of three such losses in a row is one in a million. In addition all these losses also exceed the expected shortfall estimate which is even more unlikely. Even two days before this on October 3rd the portfolio changed value more than the 99% ES estimate, but in the opposite direction. These results more than indicate that the model based risk measures studied fails in such extreme markets as seen during this time period.

7.2 Accuracy of the risk measure models

The first part of this chapter focused on risk measures and the current financial turbulence. In addition to look at the performance during a time period or specific events, it is important to examine the overall risk measure performance. Are the underlying models and assumptions accurate enough to give good estimates for the risk of large losses? To investigate overall performance a look at statistics over a long time period, which at least includes a whole economic cycle, is needed. In this thesis the data from January 1st 2002 to January 28th 2009 (1 745 days) are easily available, includes about one business cycle, and are thus used for the analysis.

7.2.1 Value-at-Risk

The definition of VaR says that the probability that losses are larger than VaR_α is equal to $1 - \alpha$. By assuming independency, the quality of Value-at-Risk estimates could be tested by looking at how often the real losses exceed the estimated VaR. For good estimates the losses exceeding should only be 1% for $\text{VaR}_{0.99}$, 5% for $\text{VaR}_{0.95}$ and so on. Table 7.3 lists the number

<i>Risk measure</i>	<i>Case portfolio</i>	<i>S&P 500</i>
99% VaR	2.8%	2.2%
95% VaR	7.6%	6.5%
90% VaR	12.4%	12.3%

Table 7.3: Percentage of losses exceeding VaR during 2002-2009.

of losses which exceed the estimated Value-at-Risk for three different levels (99%, 95% and 90%) and for both the case portfolio and an equity index (S&P 500). In all cases the frequency of large losses are a lot higher than the model predicts. For 99% VaR the frequency for the case portfolio is 2.8 times higher than expected. The probability of this many independent large losses (49 or more out of 1745 days) given that the model is correct can be calculated using a binomial distribution

$$P(X \geq 49) = 1 - \int_0^{49} \binom{1745}{x} 0.01^x 0.99^{1745-x} dx = 3.51 \cdot 10^{-10}$$

Corresponding for 95% and 90% VaR the probability of registered number (or more) of losses exceeding VaR is $1.67 \cdot 10^{-6}$ and $4.38 \cdot 10^{-4}$. These numbers are very unlikely and indicate that the model to estimate the risk measures is not perfect. There are three main sources of risk in the estimation of the risk measures which will be presented next.

Assumption of normal distribution

As seen in chapter 4 the underlying variables are modeled using an assumption of normality. Figure 4.1 and 4.3 illustrate that this assumption is good when there is low fluctuations, but not for the large deviations that the risk measure tries to estimate. Data for stock returns and interest rate changes have heavier tails than the normal distribution. An example of how bad it can work out is the 3.47% loss on S&P 500 at February 27th 2007 (figure 7.6). It was the largest fall of the index in almost four years with a move of 8.25 times the EWMA estimated standard deviation, a highly unlikely number which according to the normal distribution assumption only should happen once every $5 \cdot 10^{13}$ year (the age of the universe is estimated to $\sim 14 \cdot 10^9$ years). Table 7.3 included both the case portfolio (with a major-

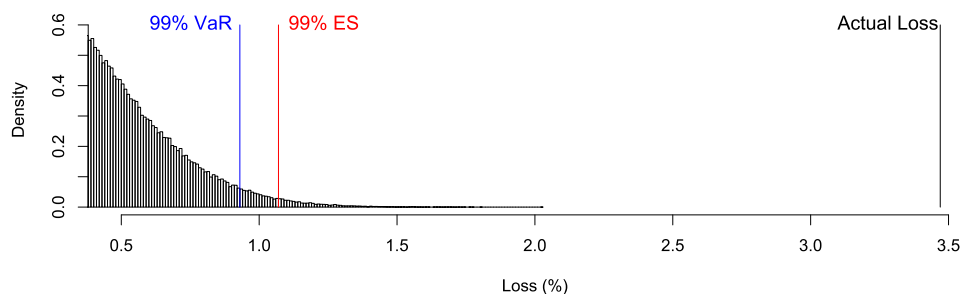


Figure 7.6: Right tail of simulated loss of S&P 500 at Feb. 27th 2007.

ity allocation to bonds) and the equity index S&P 500. In both cases the number of large losses is too high. This indicates that neither asset modeling is perfect.

Volatility estimation

To find the volatility to use for the next time period, historical data on fluctuation were weighted exponential. This method is backward looking by only using observed data for the last year, and can be compared to driving using only the rear window. When volatility increases the estimation will lag behind and underestimate the change. Each new larger loss will "surprise" the model. For most of the time period between 2002 and 2009 the EWMA estimate for S&P 500 has been (marginal) lower than the consensus of option market participants (VIX-index).

Correlation estimation

It is hard to make good estimates for correlation between financial assets. As seen in figure 6.3 the EWMA estimation fluctuates widely and changes

from negative to highly positive in just a couple of days. All the large and fast changes make the estimates less reliable as predictors for the correlation over the next period.

7.2.2 Expected shortfall

Expected shortfall can be interpreted as the expected loss given that the loss exceeds VaR. The quality of the ES estimates can be tested by assuming independency between each day and comparing all losses that exceed VaR with the ES estimate. The average of the losses relative to the ES estimate (in percent) is used since both the VaR and ES estimates change daily.

<i>Risk measure</i>	<i>Case portfolio</i>	<i>S&P 500</i>
99% ES	+12.7%	+10.3%
95% ES	+13.7%	+9.9%
90% ES	+14.4%	+7.1%

Table 7.4: Average loss exceeding VaR compared to ES during 2002-2009.

Table 7.4 lists how large the losses that exceed the VaR estimate are in average compared to the ES estimate. Overall the large losses are about 10% higher in average than the expected shortfall estimates predict, with higher differences for the case portfolio (mostly bonds) than for the equity index. These numbers are not surprisingly high when the discussion on heavy tails of the underlying variables and figure 7.6 are taken into account.

Chapter 8

Conclusion

The thesis has studied the performance of two market risk measures for a case portfolio before and during the financial turbulence in 2007-2008. The aim has been to examine these risk measures to see if the accuracy of the estimates changes during the financial crisis and if the common framework for such models proves reliable. In order to evaluate the risk measures, a Monte Carlo model to generate daily estimates has been built based on the most widely used models for the underlying variables. For stock price behavior, the geometric Brownian motion has been implemented while changes in interest rates has been simulated used Vasicek's model. To calibrate the model an exponential weighted moving average approach has been used in line with the original RiskMetrics technical document for the Value-at-Risk measure.

The results indicate that the implemented risk model do not give totally reliable estimates. The frequency of large losses on the case portfolio is higher than the estimates for Value-at-Risk predict. Losses exceed the 99% VaR more than twice as frequent as predicted even before the financial crisis sets in. The average of these losses exceeding 99% VaR is more than 10% higher than the relative expected shortfall estimate. The main weakness of the model is the assumption of normal distributed changes in the underlying variables. The risk measures examine the extreme movements and are thus affected by the fact that data for both equity and interest rates have heavier tails than the normal distribution. Other factors that influence the reliability of the risk estimates are the calibration of volatility and correlation, both using historical data. Specially the correlation structure is tricky since it fluctuates wildly in normal markets and then often converges in turbulent times.

The risk measures are not perfect in normal markets, nevertheless this study shows a significant worsening in their performance during the current finan-

cial crisis from June 2007 to December 2008 compared to the previous years. The frequency of losses exceeding 99% VaR more than doubled to 4.3% of the trading days. For 95% and 90% VaR the frequency actual fell marginal during the crisis, indicating that there were no more large losses, just that the loss amount of large losses had risen. This is reflected in the quality of the expected shortfall estimates. During the crisis the losses exceeding VaR were much larger relative to the ES estimate than the time period before. For the 99% level the large losses increased from being 10% larger than the ES estimate in average to 26% larger during the crisis. For the 95% and 90% levels this increase was even larger with up to a five fold increase.

This study has taken a closer look at the behavior of the risk measures and financial markets at several short time periods close to specific events that have influenced the progress of the financial crisis. Due to the choice of extraordinary events, there is no surprise that the estimates for the risk measures increase during all periods or that there is several large losses (and gains) exceeding both Value-at-Risk and expected shortfall. More surprisingly is the extent of some losses and their value compared to the risk measures. Examples such as a loss 2.4 times the 99% ES estimate and three losses in a row exceeding the 99% ES level are both extremely unlikely.

During the current financial turmoil the risk measure estimates have increased by up to five times what their value was in the spring of 2007. As Value-at-Risk plays an important part in the regulations of financial institutions, and as the risk measure increases the institutions are then required to hold more capital. With such a rise in the capital adequacy many institutions would need to raise capital or reduce their exposure to risky assets. These moves might again increase volatility and magnify losses, thus increase risks further.

Value-at-Risk and expected shortfall have become an important part of today's risk management industry. The risk measures are used daily in a range of different settings, from setting the capital requirement for banks and limits for traders to asset allocation of portfolios. Both measures are intuitive, can easily be explained to non-experts and the output nicely summarize many different risks into one number. Due to the many uses and simple output, users should be well aware of the pitfalls in the underlying models.

Further Work

As seen earlier, the risk model implemented is not perfectly reliable when compared to the real losses occurring on the case portfolio. To improve the model the underlying variables could be simulated based on distributions with heavier tails than the normal distribution and thus reflect market data better. For volatility calibration the use of implied volatilities might improve the risk estimates if market participants are better at predicting future volatility than historical data.

To examine the performance of the risk measures further during the financial crisis, it would have been interesting to look at risk estimates over different time horizons. For regulatory purposes the market risk horizon is usual ten days while companies might be more interested in a three month horizon since they report quarterly. The horizon of the implemented model could be expanded by a simple scaling, but a volatility model with mean reversion such as a GARCH model might be more appropriate.

In the choice of case portfolio there are unlimited numbers of possibilities. A proposal for extension would be to make the fixed income part more complex by adding bonds with more cash flows and thus depending on the interest rates curve instead of one point. Another extension could be to remove the foreign exchange hedge and thus simulate the exchange rate between Euro and US Dollar, adding one more underlying variable. For the asset allocation of the portfolio it might be more natural to maximize a ratio such as the risk adjusted return on capital (RAROC) than the sharpe ratio, since RAROC is closely related to the Value-at-Risk measure.

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Appendix A

Monte Carlo simulation

Monte Carlo simulation is a commonly used tool in risk management and to price financial derivatives. The method repeatedly generates n random samples of the underlying variable and estimates the value of an asset by averaging over the generated values and payoffs. The convergence of the method can be slow as it is of order $O(n^{1/2})$, but with ever-increasing computer power and more efficient algorithms the method has become essential in today's quantitative finance. An advantage of the methods is that the convergence does not depend on the dimensionality and it is thus popular for high-dimensional problems.

A.1 Monte Carlo integration

Consider a function f and a uniformly distributed random variable $x \in [0, 1]$ then the integral of f can be expressed as an expectation of the function value

$$\theta = E[f(x)] = \int_0^1 f(x)dx$$

where θ is an unbiased estimator. If x_1, \dots, x_n are independent random uniform variables, then $f(x_1), \dots, f(x_n)$ are i.i.d. random variables. An approximation of the expectation is then

$$\hat{\theta} = E[f(x)] \approx \frac{1}{n} \sum_{i=1}^n f(x_i)$$

By the strong law of large numbers, it follows that the approximation is convergent with probability one

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n f(x_i) = \int_0^1 f(x)dx = \theta$$

This approach to approximating integrals is called the Monte Carlo integration. The variance of the Monte Carlo estimate can be calculated using

the result from the central limit theorem. As the number of replications increases, the standardized estimator converges to the standard normal distribution

$$\frac{\hat{\theta}_n - \theta}{\sigma_n/\sqrt{n}} \rightarrow N(0, 1)$$

and that the error in the estimate is

$$\hat{\theta}_n - \theta \sim N\left(0, \frac{\sigma_\theta^2}{n}\right)$$

This result holds if σ_θ is replaced with the sample standard deviation s_θ . An asymptotic $100(1 - \alpha)\%$ confidence interval for θ is given by

$$\hat{\theta}_n \pm z_{\alpha/2} \frac{s_\theta}{\sqrt{n}}$$

where $z_{\alpha/2}$ is the $1 - \alpha/2$ quantile of the standard normal distribution.