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Simulation of counterparty risk in the Norwegian financial market

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Problem Description

The aim of this work is to study different methods for estimating counterparty risk. This work will focus on the exposure estimation and will do a comparison between the BIS methodology and an approach based on Monte Carlo simulations.

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Preface

This Master's Thesis makes the end of a five year programme I have attended since August 2001 at the Norwegian University of Science and Technology culminating in the degree of Master of Science and represents the work of 20 weeks. The work process with this study has been an interesting period of my life and I am glad that I got the opportunity to work with interesting topic in my Master Thesis.

This work is in an extension of my project work from the fall 2005 which was about pricing of interest rate derivatives using the Libor Market Model, implemented as a Monte Carlo simulation in the language of C++. That work gave me valuable knowledge in financial engineering and particularly how the use of Monte Carlo methods can be combined with financial theory in order to solve complex financial problems. In this thesis the work has been taken one step further and uses the theory in estimation of counterparty credit risk for interest rate derivatives, where the Libor Market Model has been used to describe the driving factor in the model.

This paper is written for mathematicians with some knowledge in financial theory. That means the reader should be familiar with basic financial option theory, beside that the financial theory used is explained in the paper. Whereby the mathematics used in this paper is at a level such that it should be accessible for a student with some background in mathematics at the university level, however basic statistical knowledge are assumed.

Acknowledgements

I would like to thank my academic supervisor Associate Professor Jacob Laading for providing academic support and guidance throughout the working process. I would also like to thank my fellow students for constructive discussions through the working process. Beside those persons the paper is inspired by the work of many authors and I have tried to give credit where credit is due. Needless to say, any remaining errors are mine alone.

Hans Michael Øvergaard

Abstract

This work will study different methods to estimate counterparty credit risk, where the methods represent both analytical approximation and simulation based method. The somewhat more analytical approximation that will be used is the current exposure method from the Bank for International Settlements and is based on simple add-on factor to the current market value. In the simulation part, Monte Carlo methods will be used. The paper will show that Monte Carlo methods enable estimation of the full exposure distribution as a function of time. From that distribution two measures of exposure will be used. The first use the peak at the 95% percentile and the second uses the concept of effective expected exposure. Those three alternative measures will be tested on six different portfolios. The portfolios are based on real data and represent both private persons, small companies, life insurance, investment bank and some of more academic interest. The estimate of exposure in those portfolios will be estimated with and without the establishment of netting agreements in order to see how that affects the exposure. The numerical results indicate that netting results in reduced exposure. In the comparisons between the different exposure measures the results show that the simulation based method in general estimates a lower exposure, but it depends intently on the construction of the portfolio. Based on those observations the main conclusion is that a simulation based approach is preferable since it enables better risk control within the firm as a consequence of enabling anatomizes of the evolution of exposure through time.

Keywords: Counterparty Credit Risk, Libor Market Model and Monte Carlo simulation

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Introduction

The purpose of this work is to study different methods to model counterparty credit risk for both portfolios and individual derivatives. The work is concentrated around interest rate derivatives with a substantial focus on interest rate swaps. This is motivated by the large amount of interest rate derivatives daily traded in the market. During the last years there has been a dramatic increase in the outstanding quantity of over the counter (OTC) derivatives. Market surveys conducted by the International Swaps and Derivatives Association (ISDA) show outstanding notional amounts of interest rate and currency swaps reaching US$ 25.4 trillion in 1996, US$ 60.4 trillion in 2000 and US$ 201.4 trillion in 2005. This large amounts give the opportunity to transfer risk resulting on substantially improved risk sharing among market participants. But as the volume grows banks have to be more accurately in quantify the credit risk they are facing due to default on counterparty.

In the latest years there has been an increased focus on how to measure counterparty credit risk, which is the potential loss given default on counterparty. This loss is usually quantified in terms of the replacement cost of the defaulted derivatives. The Bank for International Settlements and its Basel Committee on Banking Supervision has provided guidelines in how to measure counterparty credit risk. In 1988 the Basel Committee decided to introduce a capital measurement system commonly referred as the Basel Capital Accord. This accord represented the beginning of a framework provided for measuring counterparty credit risk. The framework is based on simple add-on factors and a minimum capital standard at 8% of the estimated credit amount. This framework has been the only method that banks where allowed to use in measuring credit exposure. However in 1999 the Basel Committee issued a proposal of a revised capital adequacy framework later known as the Basel II accord, [BIS88] and [BIS05b]. With this revised framework banks are allowed to use internal method which often is based on Monte Carlo simulations in order to estimate credit exposure. As this work will show, the Monte Carlo methods permit modelling whole term structure by using sophisticated models for the underlying rate and manages to take the correlation between different derivatives into account.

The growing activity in the market combined with new regulations have resulted in increased research activity the latest years. And the development of new computer technology has given a larger focus on simulation based methods. The evolution of models to estimate interest rate exposure have in someway gone in two different directions the later years, where this work will try to combine those different directions. By this it is meant that the development of interest rate models and the modelling of credit exposure do not have be fully combined. In the area of interest rate models one have the classic models by Vasicek, Cox & Ingersoll &

Ross, Ho & Lee and Hull & White. Those are all one factor models meaning that one factor describe the evolution of the whole yield curve. Some of those models have later been turned into multi factor models which then permit for several factors to describe the evolution of future rates. However in the latest years the models have been turned into so called market models. The market models are method that take the whole term structure in account and gives a complete set of forward rates for the period one are looking at. They are based on the framework to the HJM model by Heath, Jarrow and Morton [HJM92] which gives forward rates in continuous time. Those models have later been modified into discreet time models and is now known as the Libor Market Model (LMM). The model has no analytical solution and hence some kind of numerical algorithm is necessary. In this work the focus will lie on the LMM through the use of Monte Carlo simulations.

While there have been progress in the development of interest rate models there are not that many published articles about exposure of credit risk. One reason for this can be that the only method banks have been allowed to use is the standard method from the Basel Committee, while in pricing banks may use whatever method they want. In the area of risk management there has been written a lot about credit risk and possible ways to estimate default probabilities but there are fewer articles about credit exposure. However some research has been done in the area of credit exposure too. This involves Jeff Aziz and Narat Charupat from Algo research quarterly [AC98] which have done a case study where a Monte Carlo simulation is compared with the BIS methodology. In their paper they do not describe the underlying method used to model the evolution of the underlying rates in the Monte Carlo simulation, but their conclusion is that a simulation based method is to prefer since it enables to take the evolution of the exposure curve in account when measuring counterparty credit risk. Later on Michael S. Gibson at the Federal Reserve Board [Gib05] has gone in more detail in the estimation methodology, where he describes the use of a simple method for the evolution of the underlying rate. This paper will take the next step and use the LMM, which is an advanced method to estimate the rate, and combine the method with estimation of credit exposure. That exposure measure will be compared with the estimate from the BIS methodology for portfolios with and without netting agreements.

1.1 Organization of Topics

This report is organized as follows. This chapter gives an introduction to the calculation of credit exposure and a review of some previous work in the area. The rest of this chapter will present whats coming later in the paper.

Chapter 2 will give an introduction to the mathematical method used in estimation of credit exposure. This involves theory about Monte Carlo simulation with belonging variance reducing technics used to increase the speed of the simulations. This will be followed by theory related to the Libor Market Model which will be used in forecasting of the rates. The end of the chapter will give an introduction to interest rate derivatives, since these are the derivatives that will be used in estimation of counterpart credit risk.

Chapter 3 will after previous introduction of the background theory give a detailed explanation of how to estimate counterpart credit risk. The chapter will start by introducing the field of risk management in some general terms. This will be followed by the essential theory in the Basel regulations. After an introduction to the aim of risk management Section 3.3 will give some formal definitions used in estimation of credit risk. The definitions are based on the Basel Accord but are treated in a more mathematical way. At the end of the chapter those definitions will be used to estimate credit exposure and the text will show how it can be done with both the BIS methodology and by an internal method based on Monte Carlo simulations.

Chapter 4 will describe the data used in this text. This involves both the historic yield data used in estimating of the volatility structure and the test portfolios used when estimating credit exposure. The example portfolios represent six various portfolios which will be used to compare the BIS exposure with the simulated results. Some of these portfolios represent different financial companies while some of them are of more academic interest.

Chapter 5 will summarise the numerical results obtained from estimating of the exposure based on both the BIS methodology and the Monte Carlo method. The results are described by both tables and figures containing plots showing that a simulation based method gives lower exposure and that the exposure can be reduced by introducing netting agreements.

Chapter 6 is the last chapter in this work and contains the conclusion saying that a simulation based method reduce the estimated exposure and that it can be further reduced by introducing netting agreements.

The rest of this work contains Appendix A with the portfolios and at last a bibliography and an index at the end.

2

Background theory

2.1 Monte Carlo methods

Monte Carlo simulation is a comprehensive method to estimate sophisticated probability distributions. The method is highly used for financial estimation such as risk management which include estimation of Value at Risk (VaR) and credit exposure. One reason for using Monte Carlo methods is that it enables estimation of the full distribution with corresponding expectation and variance of a distribution function containing different correlated stochastic movements. This section will explain the principles of Monte Carlo simulations including variance reducing techniques and how it can be used to determine financial problems.

2.1.1 Principles of Monte Carlo

Monte Carlo methods are based on the analogy between probability and volume, where probability of certain event is its volume or a measure relative to the possible outcomes [Gla04]. In practice this identity is used in reverse, calculate the volume of a set by interpreting the volume as the probability For instance one can sample randomly from some discrete path function $g(x)$ and take the fraction of draws that fall in a given set as an estimate of the set's volume. An illustration of how this may be done i given by algorithm 2.1.

Algorithm 2.1: Monte Carlo simulation

input : A discrete path function $g(x)$
output: An estimate for the set's volume $\hat{x}$

for $i = 1$ **to** n **do**
  $x^j = g(x)$ ▷ Simulate a realization of a variable $g(x)$
end
$\hat{x} = \frac{1}{n}\sum_{j=1}^{n} x^j$ ▷ Which as well represent the expectation to $g(x)$.

Monte Carlo methods is often used to evaluate integrals which may often represents probability density functions. If one for instance want to estimate the integral of a function $f(\cdot)$ over the unit interval the integral may be represented as

$$\mu = \int_0^1 f(x)\,\mathrm{d}x, \tag{2.1}$$

which is an expectation to $\mathsf{E}[f(U)]$, where U are independent and uniformly distributed points on the domain $[0,1]$. Then to evaluate the integral (2.1) one draw points $U_1, U_2, \ldots$ independently and uniformly from the interval $[0,1]$ and evaluate the function $f(\cdot)$ at n of those points. By taking the average of those function numbers the Monte Carlo estimate for μ is given by

$$\hat{\mu}_{\text{MC}} = \frac{1}{n}\sum_{i=1}^{n} f(U_i). \tag{2.2}$$

The existence of the solution to (2.2) is given by the strong law of large numbers, see for instance [CB02], which says

$$\hat{\mu} \to \mu \text{ with probability 1 as } n \to \infty,$$

if $f(\cdot)$ is integrable over $[0,1]$. If $f(\cdot)$ is square integrable and one set

$$\sigma_f^2 = \int_0^1 \big(f(x) - \mu\big)^2 \,\mathrm{d}x,$$

the error $\epsilon = \hat{\mu} - \mu$ in the Mote Carlo estimate is approximately normally distributed with mean 0 and standard deviation $\sigma_f/\sqrt{n}$ for large n i.e.

$$\epsilon \sim N\left(0, \sigma_f/\sqrt{n}\right). \tag{2.3}$$

In a setting where μ is unknown the parameter σ_f would often be unknown to, but can be estimated using the sample standard deviation

$$s_f = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\big(f(U_i - \mu_n)\big)^2}.$$

When n is large and as long there is convergence in the distribution the parameter s_f can be replaced with σ_f [Gla04]. In this way Monte Carlo simulation can be used to estimate both the expectation and variance in an unknown distribution.

In financial estimation and the use of risk management one are often more interested in the percentiles than the expectation or variance itself. This often involve estimating for instance the 95% or 99% percentile of the distribution. In a simulation approach this is usually done by order statistics. Where the order statistics of a random sample $X_1, \ldots, X_n$ is defined to be the sample placed in ascending order denoted by $X_{(1)}, \ldots, X_{(n)}$. They then satisfy $X_{(1)} \leq \cdots \leq X_{(n)}$ and in particular

$$\begin{aligned} X_{(1)} &= \min_{1\leq i\leq n} X_i \\ X_{(2)} &= \text{ second smallest } X_i \\ &\vdots \\ X_{(n)} &= \max_{1\leq i\leq n} X_i. \end{aligned} \tag{2.4}$$

Then in order to find the percentile of interest one order the observation according to (2.4) and pick observation X_{np} as the $(100p)$th sample percentile where n is the number of observations.

As seen by (2.3) the convergence rate of the Monte Carlo integration is of order $O(n^{-1/2})$. For one dimensional integral this is not computer efficient when it is compared to the trapezoidal rule which has a known convergence rate of $O(n^{-2})$ for twice continuously differentiable functions. In order to increase the convergence rate the next section will introduce antithetic variates which is a common variance reduction technique.

2.1.2 Antithetic Variates

The use of antithetic variates is done by introducing pairs of replications with negative dependency which will give a faster and wider exploration of the probability domain. As a result of this one should expect a lower variance in the estimate of the expectation than one get without using antithetic variates.

When using this method one should observe that if U is uniformly distributed over the interval $[0, 1]$ then $1-U$ is that too. As a result if one are sampling a path with the variables $U_1, \ldots, U_n$, a second path with the variable $1 - U_1, \ldots, 1 - U_n$ can be generated to. This variables have clearly negative dependency and antithetic in the sense that if one path get large value it is accompanied by a small value on the other. This causes an unusually large or small value from the first path to be balanced by the value computed from the antithetic path and hence resulting in a reduction in the variance when using the middle value of $f(U)$ and $f(U - 1)$.

This techniques can be used to other distribution as well through the inverse transform method of $F^{-1}(U)$ and $F^{-1}(1 - U)$. Since F^{-1} is monotone they are both antithetic and have the distribution F. If the distribution is symmetric, $F^{-1}(U)$ and $F^{-1}(1-U)$ have the same magnitude but opposite sign. This is often the case in a financial setting where the simulations is based on estimating a Brownian motion, see Section 2.1.3, by using random normal distributed variables. The simulation is then done by simulating a sequence of $Z_1, Z_2, \ldots$ independent identically distributed normal, $N(0, 1)$, variables together with the sequence $-Z_1, -Z_1, \ldots$ of *iid* $N(0, 1)$ variables.

The use of Monte Carlo simulations may have several purposes. It can span from finding the expectation, a percentile or the whole probability distribution, but using antithetic variables reduce the variance in all of those estimates.

When looking at a more precise analysis of the objective of estimating the expectation $\mathsf{E}[Y]$ when using an implementation of antithetic sampling compared with independent sampling one shall note that using antithetic variables result in a sequence of pairs of observations $(Y_1, \tilde{Y}_1), (Y_2, \tilde{Y}_2), \ldots, (Y_n, \tilde{Y}_n)$. Here the pairs are independent but each observation is not independent. Through the analysis which is based on [Gla04] the following features will be used:

- The pairs $(Y_1, \tilde{Y}_1), (Y_2, \tilde{Y}_2), \ldots, (Y_n, \tilde{Y}_n)$ are *iid*.
- The variables Y_i and $\tilde{Y}_i$ have the same distribution, but not independent.
- The computer cost of simulation Y_i and $\tilde{Y}_i$ are the same as to compute a sample of $2n$ independent replications.

From the antithetic variates the estimator is the average of all the $2n$ observations,

$$\hat{Y}_{AV} = \frac{1}{2n}\left(\sum_{i=1}^{n} Y_i + \sum_{i=1}^{n} \tilde{Y}_i\right) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{Y_i + \tilde{Y}_i}{2}\right). \quad (2.5)$$

The right expression in (2.5) make it is evident that $\hat{Y}_{AV}$ is the mean of n independent observations

$$\left(\frac{Y_1 + \tilde{Y}_1}{2}\right), \left(\frac{Y_2 + \tilde{Y}_2}{2}\right), \ldots, \left(\frac{Y_n + \tilde{Y}_n}{2}\right).$$

If one then applie the central limit theorem [CB02] one get

$$\frac{\hat{Y}_{AV} - \mathsf{E}[Y]}{\sigma_{AV}/\sqrt{n}} \Rightarrow N(0,1),$$

with

$$\sigma_{AV}^2 = \mathsf{Var}\left[\frac{Y_i + \tilde{Y}_i}{2}\right],$$

where Y is used to indicate a random variable with the common distribution of the Y_i and $\tilde{Y}_i$.

A question is under what conditions and how well the antithetic variates reduce the variance. In this comparison the third feature, the computer cost of simulation Y_i and $\tilde{Y}_i$ are the same as to compute a sample of $2n$ independent replications, are essential. The use of antithetic variables is then better and reduce the variance compared with independent sampling if

$$\mathsf{Var}\left[\hat{Y}_{\mathrm{AV}}\right] = \mathsf{Var}\left[Y_i + \tilde{Y}_i\right] < \mathsf{Var}\left[\frac{1}{2n}\sum_{i=1}^{2n} Y_i\right] = 2\mathsf{Var}[Y_i]. \quad (2.6)$$

The variance on the left hand side in (2.6) can be written as

$$\begin{aligned}\mathsf{Var}\left[Y_i + \tilde{Y}_i\right] &= \mathsf{Var}[Y_i] + \mathsf{Var}[\tilde{Y}_i] + 2\mathsf{Cov}[Y_i, \tilde{Y}_i] \\ &= 2\mathsf{Var}[Y_i] + 2\mathsf{Cov}[Y_i, \tilde{Y}_i],\end{aligned}$$

since Y_i and $\tilde{Y}_i$ have the same distribution. Thus the variance is reduced by using antithetic variates if

$$\mathsf{Cov}[Y_i, \tilde{Y}_i] < 0.$$

Hence the variance is reduced if the variables has negative dependence. This negative dependence is what one achieve when using Z and $-Z$ from the normal distribution with reduced variance as a result.

From previous section it is known that in financial setting the percentile of the distribution is of interest. When using antithetic variates to find a percentile the order statistics is once more used. All $2n$ numbers are sort according to (2.4) and the observation X_{2np} is used as the $(100p)$th sample percentile. This text will not prove the result but refer to an article by Avramids and Wilson [AW98] which has showed that: correlation induction techniques for estimating percentiles can yield worthwhile improvements in estimator accuracy relative to direct simulation.

2.1.3 Simulation of normal distributed variables

Since financial processes often are driven by Brownian motions one need to be able of modeling that drift. A Brownian motion is in brief a stochastic process $\{W(t), 0 \leq t \leq T\}$ with $W(0) = 0$, independent increments and

$$W(t) - W(s) \sim N(0, t-s) \quad \text{for any} \quad 0 \leq s < t \leq T.$$

This then gives

$$W(t) \sim N(0, t), \tag{2.7}$$

for $0 < t \leq T$ [Gla04]. Since most financial processes are model according to (2.7) it is important to have a framework that can draw random normal distributed numbers. This can be done in several ways, but it is usually done by some inversion of the cumulative distribution function. Marsaglia and Bray [MB64] have developed an algorithm that will be used in this text. It is one of the fastest algorithms to draw random normal distributed numbers [Gla04] which is a nice property when one need to draw a lot of numbers. The algorithm is given Algorithm 2.2.

Algorithm 2.2: Marsaglia Bray algorithm for generating normal random variables.

Output: Two independent normal distributed variables Z_1 and Z_2

while $(X > 1)$ **do**
    generate $U_1, U_2 \sim \text{Unif}[0, 1]$
    $U_1 \leftarrow 2U_1 - 1$
    $U_2 \leftarrow 2U_2 - 1$
    $X \leftarrow U_1^2 + U_2^2$
end
$Y \leftarrow \sqrt{\left(-2 \log X\right)/X}$
$Z_1 \leftarrow U_1 Y$
$Z_2 \leftarrow U_2 Y$
return Z_1, Z_2.

2.2 Libor Market Model

2.2.1 Foundation

Classic models such as CIR and Hull-White only use one driving Brownian motion to model the whole evolution of the yield curve. This will as result imply perfect correlation among all forward rates. By the assumption of perfect correlated forward rates one should only observe parallel shifts in historic yield. Just by looking at historic observation, for instance Figure 4.1, one can observe that this is wrong. For U.S government bonds Litterman and Scheinkman [LS91] showed that at least three factors are necessary to describe mayor movement i historic yield. This fact has motivated the development of market models which takes the whole yield structure in consideration when developing future forward rates. One of the first to solve this problem was Heath, Jarrow and Morton and their HJM model [HJM92].

In the framework of HJM the evolution of future rates are given i continuous time. Hence the model can give a complete set of forward rates without restricting to a finite state of rates or factors. This type of approach is some unreal related to what that can be observe in the market which only contains a finite set of maturity dates. Motivated by this unrealism Brace, Gatarek and Musiela developed the BGM model [BGM97]. The model is close related to the HJM framework, but it is based on discrete time instead of continuous time. In the literature many other people are related to the model and hence it is called the Libor Market Model, which will be used in this text.

To give a more mathematical treatment of the Libor Market Model one shall consider a set of maturity dates

$$0 = T_0 < T_1 \cdots < T_M < T_{M+1}, \tag{2.8}$$

to simulate in, which also represent the maturity of the initial data. The term L is used to denote a forward rate for a given period of length δ where the forward rate $L(t, T)$ is the rate set at time t for a period from T to $T + \delta$ which then represent a simple rate. This means if one at time t agree to borrow 1 at time T and repay with interest at time $t + \delta$ the interest will be $\delta L(t, T)$ and hence the pay back became $1 + \delta L(t, T)$ at time $T + \delta$. This simple rate is in contrast to the usual compound interest where the charge is calculated on the sum lent plus any interest that has accrued in previous periods. Hence the previous multiplier $1 + rt$ now becomes $(1 + r)^t$. In a market model the forward rates are usually not seen, but they can be derived from zero coupon bonds with following relation

$$L(0, T) = \frac{B(0, T) - B(0, T + \delta)}{\delta B(0, T + \delta)}. \tag{2.9}$$

In (2.9) the fact that $B(0, 0) = 1$ is used which will as a result give as many forward rates as there are zero coupon bonds.

To generalise the notation it is usual to write $B_n(t)$ for the bond priced at time t with maturing at time T_n, where $0 \leq t \leq T_n$. In the same way $L_n(t)$ is used to denote the forward rate set at time t for the period $[T_n, T_{n+1}]$. In generality the distance between the maturity times in (2.8) do not have to be equidistant. To compensate for this let

$$\delta_i = T_{i+1} - T_i, \quad i = 0, \ldots, M,$$

be the distance between the maturity. With this notation (2.9) can be written as

$$L_n(t) = \frac{B_n(t) - B_{n+1}(t)}{\delta_n B_{n+1}(t)}, \quad 0 \leq t \leq t_n, \quad n = 0, 1, \ldots, M, \tag{2.10}$$

where the relationship only is valid for $t \leq T_n$. To avoid this problem the definition of $L_n(t)$ is extended by setting $L_n(t) = L_n(T_n)$ for all $t \geq T_n$. Since (2.10) gives forward rates based on initial bond prices the equation can be turned around in order to compute bond prices based on forward rates with the following relationship

$$B_n(T_i) = \prod_{j=1}^{n-1} \frac{1}{1+\delta_j, L_j(T_i)}, \quad n = i+1, \dots, M+1. \tag{2.11}$$

With this notation the estimation is required to be done at a tenor date T_i only and not at an arbitrary time. In many settings, including risk estimation, one are often interested in the forward rate at an arbitrary date t. Hence the need of finding a price at time t where $T_i < t < T_{i+1}$, arise. This have lead to the introduction of a right continuous mapping function η which is defined as

$$\eta \colon [0, T_{M+1}) \to 1, \dots, M+1,$$

which then satisfy

$$T_{\eta(t)-1} \leq t < T_{\eta(t)}. \tag{2.12}$$

With this notation Equation (2.11) can be written as

$$B_n(T_i) = B_{\eta(t)}(t) \prod_{j=\eta(t)}^{n-1} \frac{1}{1+\delta_j L_j(T_i)}, \quad 0 \leq t < T_n, \tag{2.13}$$

where $B_{\eta(t)}(t)$ is the current price of the shortest bond.

2.2.2 Estimation

Last section said nothing about how the forward rate are estimated in the Libor Market Model, only gave some of the formal notation.

To estimate the forward rate one seek a model which describe the arbitrage free dynamic by a stochastic differential equation on the following form,

$$\frac{\mathrm{d}L_n(t)}{L_n(t)} = \mu_n(t)dt + \sigma_n(t)^\top \,\mathrm{d}W(t), \quad 0 \leq t \leq T_n, \quad n = 1, \dots, M, \tag{2.14}$$

where W is a d-dimensional Brownian motion. The parameters μ_n and σ_n represents respectively the expectation and volatility and can in general depend on both current rates $\big(L_1(t), \dots, L_M(t)\big)$ and the current time t. When using the forward measure, which is related to the the usual risk neutral measure Brace, Gatarek and Musiela [BGM97] showed that

$$\mu_n(t) = \sum_{j=\eta(t)}^{n} \frac{\delta_j L_j(t) \sigma_n(t)^\top \sigma_j(t)}{1+\delta_j L_j(t)} \tag{2.15}$$

satisfy necessary no arbitrage conditions. A substitution of (2.15) into (2.14) gives

$$\frac{\mathrm{d}L_n(t)}{L_n(t)} = \sum_{j=\eta(t)}^{n} \frac{\delta_j L_j(t) \sigma_n(t)^\top \sigma_j(t)}{1+\delta_j L_j(t)} \,\mathrm{d}t + \sigma_n(t)^\top \,\mathrm{d}W(t), \quad 0 \leq t \leq T_n, \tag{2.16}$$

A solutions to (2.16) may not be found analytical. This means in order to find a solution it has to be done by some numerical method where Monte Carlo simulation is the most common used.

This text will assume a time grid on the form $0 < t_i < \cdots < t_m < t_{m+1}$, where the simulation will be done. The time grid can be arbitrary compared to the maturity dates when using the η-function (2.12), but it is sensible to include the tenor dates $T_i, \dots, T_{M+1}$ among the simulation dates. In order to simulate from (2.16) it is sensible to use a first order Euler discretization which gives

$$\begin{aligned}\hat{L}_n(t_{i+1}) &= \hat{L}_n(t_i) + \mu_n\big(\hat{L}_n(t_i), t_i\big)\hat{L}_n(t_i)[t_{i+1} - t_i] \\ &\quad + \hat{L}_n(t_i)\sqrt{t_{i+1} - t_i}\,\sigma_n(t_i)^\top Z_{i+1},\end{aligned} \tag{2.17}$$

where

$$\mu_n\big(\hat{L}_n(t_i), t_i\big) = \sum_{j=\eta(t)}^{n} \frac{\delta_j \hat{L}_j(t)\sigma_n(t)^\top \sigma_j(t)}{1 + \delta_j \hat{L}_j(t)},$$

and $Z_1, Z_2, \dots$ are independent $N(0, 1)$ random vectors in $\Re^d$. Using (2.17) may produce negative rates, to avoid that (2.16) can be discretization in the following way

$$\begin{aligned}&\hat{L}_n(t_{i+1}) = \hat{L}_n(t_i)\times \\ &\quad \exp\left(\left[\mu_n\big(\hat{L}_n(t_i), t_i\big) - \frac{1}{2}\|\sigma_n(t_i)\|^2\right][t_{i+1} - t_i] + \sqrt{t_{i+1} - t_i}\,\sigma_n(t)^\top Z_{i+1}\right).\end{aligned} \tag{2.18}$$

Application of (2.18) is equivalent to apply an Euler scheme to $\log L_n$, which may be seen as an approximation of L_n by geometric Brownian motion over $[t_i, t_{i+1}]$. In case of deterministic volatility this method is attractive since L_n is close to lognomal in this case [Gla04].

An implementation of (2.17) or (2.18) are typical given by the pseudo code in Algorithm 2.3 and produce output on the form given by Table 2.1 when compensate for arbitrary time grid.

Algorithm 2.3: Simulation of forward rates in the Libor market model.

input : Initial forward rates L
output: Next forward rates

for $t = 1$ **to** $m + 1$ **do**
  for $n = \eta(t)$ **to** $M + 1$ **do**
    $L_n(t_{t+1}) = L_n(t_t) + \mu_n\big(\hat{L}_n(t_i), t_i\big)\hat{L}_n(t_i)[t_{i+1} - t_i] + \hat{L}_n(t_i)\sqrt{t_{i+1} - t_i}\,\sigma_n(t_i)^\top Z_{i+1}$
  end
end

2.2.3 Adjustments

The LMM is a general model which allows for some choices and modification related to the implementation. This section will provide some modification when the model is used in risk management related to the Norwegian bond market. The dynamics of the LMM is represented

$L(t_0, t_0, t_1)$					
$L(t_0, t_1, t_2)$	$L(t_1, t_1, t_2)$	$L(t_1^*, t_1^*, t_2)$			
$L(t_0, t_2, t_3)$	$L(t_1, t_2, t_3)$	$L(t_1^*, t_2, t_3)$	$L(t_2, t_2, t_3)$		
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	
$L(t_0, t_{n-2}, t_{n-1})$	$L(t_1, t_{n-2}, t_{n-1})$	$L(t_1^*, t_{n-2}, t_{n-1})$	$L(t_2, t_{n-2}, t_{n-1})$		
$L(t_0, t_{n-1}, t_n)$	$L(t_1, t_{n-1}, t_n)$	$L(t_1^*, t_{n-1}, t_n)$	$L(t_2, t_{n-1}, t_n)$	$\dots$	$L(t_{n-1}, t_{n-1}, t_n)$

Table 2.1: Example of output structure from the Libor Market Model when the η function is used to get a start time between the initial maturity times.

by (2.14), where the simulations have been concentrated around the Euler scheme given by (2.17). The covariance matrix, σ, can be based on either implied volatility or historic volatility based on yield which will be used in this text. Traditionally the implicit volatility are higher than historic volatility but in estimating of credit exposure both are allow to use. Either one choose implied or historic volatility the implementation can be done using a decomposition of the covariance matrix into eigenvalues and eigenvectors on the following form

$$\mathbf{\Sigma} = \lambda_i e_i, \quad i = 1, \dots, n,$$

where n is the number of observations in $\mathbf{\Sigma}$. For bonds containing several maturities this will then give a dramatic decrease in the computer time. Litterman and Scheinkman [LS91] showed that tree principal components describes the major movements in US bond. But as shown in [Øve05] tree components is not a enough to describe the major movements in the Norwegian market. In Section 4.1 and particularly Figure 4.3 this is shown by figures containing Norwegian yield data. Hence an implementation using principal component do not necessary reduce the over all computer time. An implementation using eigenvalues and eigenvectors may in this case only introduce more numerical error than using the the full covariance matrix as its given. From that reason the implementation in this text will use full covariance matrix.

2.3 Interest rate derivatives

This work is about counterparty credit risk and how it can be estimated with a strong focus on interest rate derivatives. Before introducing the risk estimation this section will describe the behaviour of a financial derivatives and how the payoff profile look in order to better understand the exposure in the following chapters.

2.3.1 Caps, floors, and swaps

In order to explain caps and floors one will first need to know what caplet and floorlet are. Caplet and floorlet can be compared with call and put options from the asset theory. While a call option gives a payment if the stock price goes above some fixed level a caplet gives a payment if the interest rate is above some level K. This is then written as

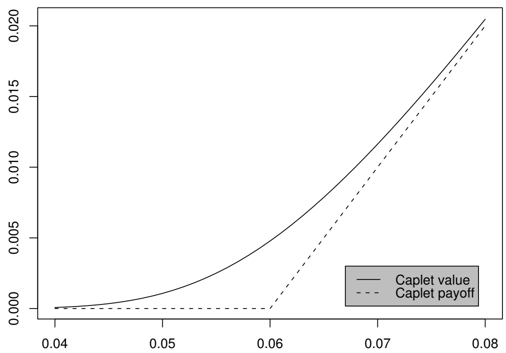
$$\text{Caplet} = \delta \max\big(L(t,T) - K, 0\big),$$

where δ is time between t and T and $L(t,T)$ is the forward rate for accurate period. As the δ parameter indicate the payment is for a period between two tenor dates where the equation reflect this fact since the payment has to be multiplied with the length of the period. The size of δ is usually below one which means a caplet is only valuable in a fraction of a year. From the equation one can see that the payment level is determined at time t, since the forward rate then will be known, but the payment is usually paid at time T.

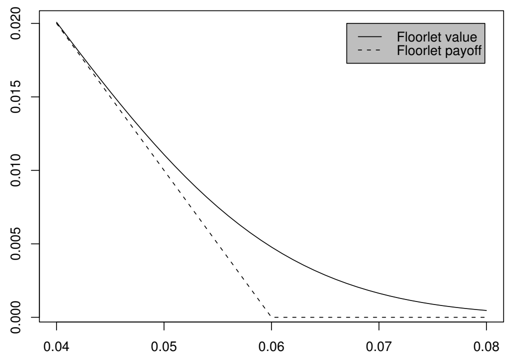
An floorlet has the same properties as a caplet, but gives a payment if the rate goes below some level K. This will then give a payment function in the following form

$$\text{Floorlet} = \delta \max\big(K - L(t,T), 0\big),$$

where δ and $L(t,T)$ has the same properties as for the caplet. The payment form a floorlet has the same properties as the caplet and gives a payment for a time period at time T. This payment is illustrated in Figure 2.1 which also shows the payment structure of caplet combined with their values as a function of the rate.



(a) Caplet.



(b) Floorlet.

Figure 2.1: Illustration of the payoff structure combined with the value for a caplet and floorlet as a function of the rate.

The normal use of caplet and floorlet is to ensure that the rate do not goes above or below some fixed level for a longer time period, which can be several years. In order get this insurance for a upper level in the rate one have to buy a portfolio of single caplets. This collection of single caplets is known as a cap with a payoff function of

$$\sum_{j=1}^{n} = \delta_j \max\big(L(t_j, t_j + \delta_j) - K, 0\big),$$

where n is the number of time periods in the interval. Then in order of evaluate both the value and exposure of this cap one have to sum the value of each single caplet. In the same way a collection of floorlets is known as a floor and is evaluated in the same way as a cap.

As seen a cap gives a payment if the rate goes above some level, while a floor gives payment if the rate goes below some fixed level. Something between is a change of floating against fixed rate formally known as an interest rate swap which will be abbreviated to swap. A swap is an agreement between two counterparty to exchange a series of cash flows on prearranged dates in the future where one part get fixed rate while the other get floating rate. And as the reader may already have understood, a swap is collection of single swaplet. The payoff in each period would then be the difference between the floating and fixed rate which gives the following payoff function

$$\text{Swap} = \delta\big(L(t,T) - K\big).$$

An graphical illustration of this exchange is given in Figure 2.2.

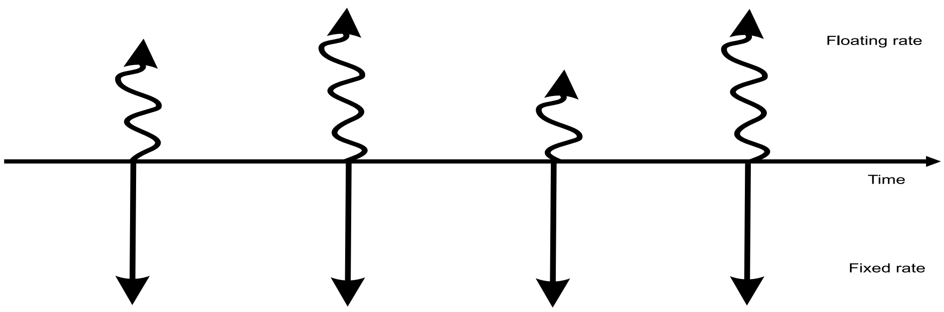


Figure 2.2: Illustration of change with following cash flows in a swap agreement

When one enter into an swap agreement this is usually done with an initial cost of zero for both counterparty. In order to achieve this the fixed rate K has to be at the level where the future forward rates are. A swap with this property is called a par swap, and the value of the fixed rate K for which the swap has zero value is called the par swap rate. When the swap start immediately this often abbreviated to just the swap rate.

2.3.2 Put-call parity

From asset theory there is well known that there exist a parity between the price of a put and call option. In the same way there exist a parity between the price of a swap, cap and floor. This is not an important result for the estimation of counterparty credit risk, but more important in the area of risk management since it can be used to construct a portfolio with the wonted risk profile. In the estimation of derivative priced in Chapter 5.1 one can observe that the parity hold. Here the parity is stated with out proof, but refer to [HK00] for a reader interest in a mathematical treatment of the parity.

A portfolio of long cap and a short floor with $r_c = r_f$ and market rate r has a cash flow of

$$\max(r - r_c, 0) - \max(r_c - r, 0) = r - r_c.$$

This is the same as the put-call parity which says

$$\text{cap} - \text{floor} = \text{swap}.$$

As a remark, without this parity it would have been arbitrage opportunities which in general do not exist.

3

Counterparty Risk

This chapter describes the treatment of counterparty credit risk, which late will be define as the loss caused by default on counterparty, and this Chapter describes have this can be estimated.

3.1 Risk management

Traditionally firms have used options to hedge risk in stock movements and swaps to get rid of changes in interest and foreign exchange rates. But for banks and other financial institutions making money on others risk it is not always possible to hedge all their credit risk. As a result financial institution need to understand the different sources of risk they are facing and know how they can be measured those. This intimate that counterparty credit risk is not the only type of financial risk that financial institutions are exposed to and hence the risk managers have to be capable of measure and controlling many different types of risk. Usually the financial risk is separated into four groups: credit risk, market risk, liquidity risk and operational risk.

The first source of risk to be delineated is market risk which usually is defined as the risk of loss due to movements in market prices and volatility. When banks have open positions one know its market values to day but future values are uncertain which may result in losses. For instance if a bank owns a financial paper worth 100 today its value tomorrow can be 90 and hence cause a loss of 10. Without going into detail it can be mention that market risk usually is estimated using the Value at Risk (VaR) method. The method is based on estimating market scenarios for a period of some few mounts and then estimating the potential loss. From this simulation one estimate the 99% percentile or an other percentile of interest and use it as a measure of the market risk. If the former loss of 10 is given as the value in the 99% percentile it means that in one of 100 days one have the potential loss of 10.

The second risk term, liquidity risk, is used for both market liquidity risk and founding liquidity risk. For the first type an economic loss would arise if a transaction cannot be conducted at market priced due to the size of the required trade relative to normal trading. This type of liquidity risk should be a part of the VaR estimation when estimating the market risk, but is often forgotten [Jor05]. The second type of liquidity risk, founding risk, arise when institution are unable to meet its obligations because of an inability to liquidate assets or obtain adequate funding.

Operation risk is by the Basel Committee [BIS05b] defined as the risk of loss resulting from inadequate or failed internal processes, people and systems, or from external events. This is in effort to measure unexpected events such as the collapse of Barings Bank in 1995 and the attack on the World Trade Center in 2001.

The last and probably larges source of financial risk is credit risk. It is usually defined as the uncertainty in counterparties ability to meet its financial obligations. This uncertainty is often related to companies default probability. CreditMetrics has provide a framework for simulations of changes in credit rating for different firms. The framework are in daily use to estimate the default probability for different firms and business sector, but the framework do not says anything about the size of the potential loss. This potential loss will in the remaining text be denoted as credit exposure. An illustration of the size of this risk category can be found by looking at the 2005 Annual report for Norway's largest financial institution DnBNOR. It shows that the credit risk account for 65% of estimated risk capital before diversification effects, clearly showing the size of this risk category.

The risk categories defined above do not necessarily fit into one separate category. For instance a swap between a speculator and a bank may result in a wrong way trade. If the bank losses money on the swap credit risk is not an issue but if the bank gain on the swap it is at the expense of the speculator. If the loss for the speculator is large this could lead the speculator to default and hence make credit risk for the bank and in this way the swap lead to double default effects. For the bank this this type of transactions are fare more dangerous than when the counterparty is a hedger since it will make the default risk smaller. Lager financial institution all over the world and all banks in the European Economic Area, including Norway, are liable of monitor this risk through the Basel II regulations.

3.2 Basel regulations

In an attempt to avoid bankruptcy, as a worst case, among the financial institution they are by the Basel regulations set out by the Basel Committee on Banking Supervision been given guidelines in how to measure different financial risk and regulations about minimum capital requirement in order to live by a financial loss. The Committee is part of the Bank for International Settlements (BIS) which is an international organization which fosters international monetary and financial cooperation and serves as a bank for central banks. Currently the bank have 55 member central banks including Norway.

The Basel II accord is a revised framework of the former Basel (I) accord, commonly referred as the Basel Capital Accord. The new accord allow for more individual decision among financial institution in the determination of what method to use, but the new accord is more specific about what to estimate. This is done through the three following pillars, which the Basel II accord is based on.

- Pillar 1: Calculations of total minimum capital requirements for credit, market and operational risk.
- Pillar 2: Supervisory oversight of the minimum requirements and other capital issues.

- Pillar 3: Disclosure requirements providing market discipline on bank capital adequacy.

This work will concentrate on the first pillar and the calculation of credit risk since is represent the largest source of financial risk.

3.3 Credit risk

3.3.1 Credit risk definition

In the remaining text the credit risk definitions which follows in this section will be used. They are all based on the Basel Capital Accord [BIS05a] but will here be given in more mathematical terms.

Counterparty Credit Risk (CCR) is define to be the risk that a counterparty to a transaction can default before the final settlement of the transaction's cash flows. If the transaction or portfolio of transactions with a counterparty has a positive economic value at the time of default an economic loss will occur. In generally all positions that give rise to CCR share the characteristics that they generate credit exposure.

To estimate CCR the Basel Accord refer to different probability measures for which the credit risk is calculated. For this work only the distribution of market value and distribution of exposure make sense to use among all proposed distributions. In some sense the names reflect the outcome of the distribution. They are based on the fact that exposure always are larger than zero since an economic loss do not occur if the market value are negative, which means the market value can go below zero. In more mathematical terms $f(m)$ is defined to be the distribution of market values. Since the market value can be both positive and negative the function $f(m)$ is in fact a mapping from $\mathbb{R}$ to $\mathbb{R}$ i.e.

$$f(m) :\to \mathbb{R}. \tag{3.1}$$

This is in contrast to the exposure which only account for positive numbers. From this $f(e)$ is defined to be the distribution of exposure and since it only can be positive it is in fact a mapping into $\mathbb{R}^+$, where $\mathbb{R}^+$ is the non negative real numbers. By follow the notation from (3.1) it can be written as

$$f(e) :\to \mathbb{R}^+.$$

Often the distribution of exposure are defined by setting the non positive values in the market distribution equal zero i.e.

$$f(e) \sim \max\big(f(m), 0\big).$$

hence there exist a function ξ such that

$$\xi : f(e) \to f(m).$$

With these probability distributions we are capable of measuring CCR.

The first measure which is used in estimation of CCR is *Current market value* (CMV). It is used to denote the market value of the transactions within a netting agreement set with a counterparty and is estimated at time zero only. In measuring CMV both positive and negative values are used which means the value is estimated under the distribution of market values. The estimation of CMV are often done by using the method of Black 76 [Bla76]. This method will count for the volatility in future forward rates. But in practice the forward rates are known deterministic at time zero gives a non stochastic market value. In this text the last method is used because of its simplicity. Close related to CMV one have the *Current exposure*, which often is denoted as the replacement cost. It is defined as the cost of replacing the transaction if the counterpart defaults assuming there is no recovery of value. This means it is calculated by taking the larger of CMV and zero.

The next exposure measure which is close related to current exposure is *Expected exposure* (EE) often mention as credit exposure. The EE shall reflect the fact that market value change according to time t and is defined as the mean average of the distribution of exposures at any particular future date before the longest-maturity transaction in the netting set. This gives

$$\mathrm{EE}(t) = \int_0^\infty e_t f(e_t)\,\mathrm{d}e_t, \tag{3.2}$$

where the expected exposure has been give a t parameter to reflect its time dependency.

An other important exposure measure is *Potential future exposure* (PFE). It is defined as the maximum amount of exposure expected to occur on a future date with a high degree of statistical confidence. For instance the 95% PFE is the level of potential exposure that is exceeded with only 5% probability. Implicitly it can be defined as the value that is not exceeded at the given level p [Jor05]. If the position with value $V(t)$ has a distribution $f(e_t)$ at time t, the PFE(t) is given by

$$1 - p = \int_{\mathrm{PFE}}^\infty f(e)\,\mathrm{d}e. \tag{3.3}$$

In terms of simulations PFE is represented by the 95% highest observed value from the order statistics. When estimating (3.2) and (3.3) netting and collateral have to be taken in consideration. That will not change the definition and is treat in detail in Section 3.3.2.

Maximum potential future exposure (MPFE) is define to be the peak of PFE over the life time of the portfolio. In mathematical terms this gives

$$\mathrm{MPFE} = \max\bigl(\mathrm{PFE}(t)\bigr), \quad 0 \le t \le T, \tag{3.4}$$

which means the MPFE is given by the largest value of all PFE.

An other measure of CCR that will be used in this text is Effective EPE. It is established to capture rollover risk and is the amount by which expected positive exposure is understated when future transactions with a counterpart are expected to be conducted on an ongoing basis, but the additional exposure generated by those future transactions is not included in calculation of expected positive exposure [BIS05b]. It is defined as

$$\mathrm{Effective}EE_{tk} = \max(\mathrm{Effective}EE_{tk-1}, EE_{tk}), \tag{3.5}$$

where the exposure is calculated recursively and current date is denoted as t_0.

The two last measure of CCR that will be used in this text is derived from the previous defined measure. The first is *Expected Positive Exposure* (EPE) which is the weighted average over time of the expected exposures.

The last measure of CCR is *Effective Expected Positive Exposure* (Effective EPE). It represents the weighted average over time of effective expected exposure over the first year, or over the time period of the longest-maturity contract in the netting set where the weights are the proportion that an individual expected exposure represents of the entire time interval. Hence it is estimated as

$$\text{Effective EPE} = \frac{1}{n}\sum_{i=1}^{n} \text{EE}_i, \tag{3.6}$$

if all contracts in the netting set mature after one year. If the contracts mature before one year EPE is the average of expected exposure until all contracts in the netting set mature. Hence Effective EPE is computed as a weighted average of Effective EE

$$\text{Effective EPE} = \sum_{k=1}^{\min(1year,maturity)} \text{Effective}EE_{tk} \times \Delta t_k,$$

where the weights $\Delta t_k = t_k - t_{k-1}$ allows for the case when future exposure is calculated at dates that are not equally spaced over time. An illustration of some of the risk measure is given in Figure 3.1.

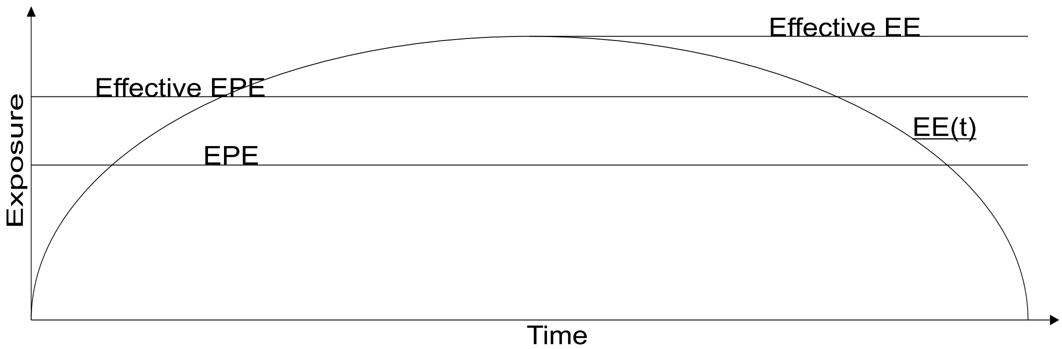


Figure 3.1: Illustration of how EE and effective EE can vary with time and how Effective EE looks when it is estimated recursively starting at t_0.

3.3.2 Treatment of exposure

Before estimating CCR one should know that exposure is highly dependent on contract type and that netting and collateral may reduce the total exposure.

As said an interest rate swap is a agreement between two counterparties to exchange a fixed interest toward a floating interest, where the market value of the swap contract is given as the discounted difference between the fixed and floating rate. The market value of the swap will change according to the state of the market and remaining payments. Since the floating rate can go both up and down the belonging market value can be both positive and negative. If the market value is negative for one part it is positive for the counterparty and the other way around if the market value is positive. Hence if one part default the other part are espoused to an economic loss which make the risk of a swap two sided.

In contrast to swap which is an obligation to exchange payments there exist option type of derivatives. They give the right to change value with a counterparty but is no obligation. In terms of exposure it means that only the option buyer are adversely affected by default on counterparty. For instance a contract of long option is always non negative for the option buyer and non positive for the option writer. Hence only one part are exposed for an economic loss which make the exposure one sided in contrast to swap which gave raise for two sided exposure, related to the put call parity. Exposure may also depend on the features of any embedded option. If the option is of American style the holder of an in the money swap may want to exercise early if the credit rating of its counterparty starts to deteriorate. In this way the exposure decreases relative to an equivalent European option. As the numerical examples shows this can be used to reduce the total exposure, determine that the contract are part of a netting agreement.

In Section 3.3.1 position containing netting and collateral where not take in consideration when defining EE and PFE by (3.2) and (3.3). The introduction of netting and collateral will here demonstrate that the previous define EE and PFE can be reduced and hence a reduction in the over all credit exposure. As a remark and as mention before this can be done with out changing the previous definitions since we only has to change the way we add the different exposure.

This part will focus minor on the distribution but concentrate around the expectation in the distribution. The term $V(t)$ will be used to denote the value at time t which is given by

$$V(t) = \int_{-\infty}^{\infty} m f(m) \, \mathrm{d}m.$$

By using previous notation this gives EE for an uncollateral position equals

$$\mathrm{EE}(t) = \max\big(0, V(t)\big). \tag{3.7}$$

To reduce the potential loss in case of default on counterparty it is by the Basel Accord allow to collateralize the position. The ISDA [ISD05] describe collateral as a contract that is one the side of the main agreement, where the aim of the contract is in the event of default on the primary transaction to reduce the potential loss. In the case of default on counterparty the collateral receiver has recourse to the collateral asset and can thus indirectly make good any loss suffered. By accounting for a collateral worth $C(t) \geq 0$ the net exposure in (3.7) are given by

$$\max\big(0, V(t) - C(t)\big). \tag{3.8}$$

By comparing (3.8) with (3.7) one clearly observe that collateral can reduce the total exposure to a counterparty.

An other method to reduce the exposure is to net contracts with different exposure profiles as shown in Figure 3.2. When talking of netting there are at least two types of netting agreements. The first is netting novation which is an agreement between counterparty to combine different cash flows into one single payment on daily bases. The second type, close out netting, is a bilateral agreement whereby all contracted but not yet due obligations and claims on each other will be accelerated and terminated immediately if default or another termination event occur. In the case of default the gross market value of all contracts are added up and one single net payment is owed by the counterparty that has a negative net portfolio value. This is the type of netting agreement that will be considered in this text, since it is one of the most important credit risk mitigation tools in the market [Fra01].

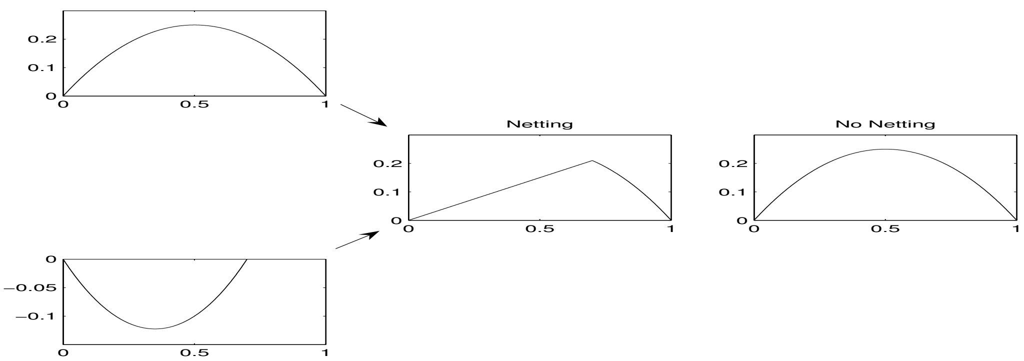


Figure 3.2: Illustration of how netting agreement can reduce the exposure for a portfolio. The left figures shows market value for two different swaps while right figures shows the exposure profile with and without netting agreement.

To describe this in mathematical terms one can consider a position containing plural agreements with market values given by,

$$V_1(t), V_2(t), \dots, V_n(t), \tag{3.9}$$

at time t. If there is no netting agreement the total exposure is given as the sum of the individual exposure in following form

$$\text{Total exposure} = \sum_{i=1}^{n} \max\big(0, V_i(t)\big). \tag{3.10}$$

If one then assume the position in (3.9) to be part of a net portfolio the total exposure is given as the sum of the individual values, which is written as

$$\text{Total exposure} = \max\left(0, \sum_{i=1}^{n} V_i(t)\right). \tag{3.11}$$

From the expression in (3.10) and (3.11) it should be clear that

$$\max\left(0, \sum_{i=1}^{n} V_i(t)\right) \leq \sum_{i=1}^{n} \max\big(0, V_i(t)\big), \tag{3.12}$$

with equality only if all transactions are perfect correlated. As (3.12) then shows, a netting agreement can reduce the overall exposure dramatic.

At last both collateral and netting can be combined to reduce the exposure. Hence the total exposure are expressed as

$$\text{Total exposure} = \max\left(0, -C(t) + \sum_{i=1}^{n} V_i(t)\right),$$

which can be used in estimation of both EE and PFE.

3.4 Estimation of credit exposure

For interest rate derivatives the credit exposure is a function of the future rates, which in general has an stochastic behaviour, of the remaining payments in the contract. Figure 3.3 shows how the exposure profile can be as a function of the time. It shows how the mean value change according to time and how each time has it own probability distribution which depends on future rates and remaining payments. This means in order to estimate the exposure at any given time the forward rates form every time has to be determine, which indicate the computation job can be enormous.

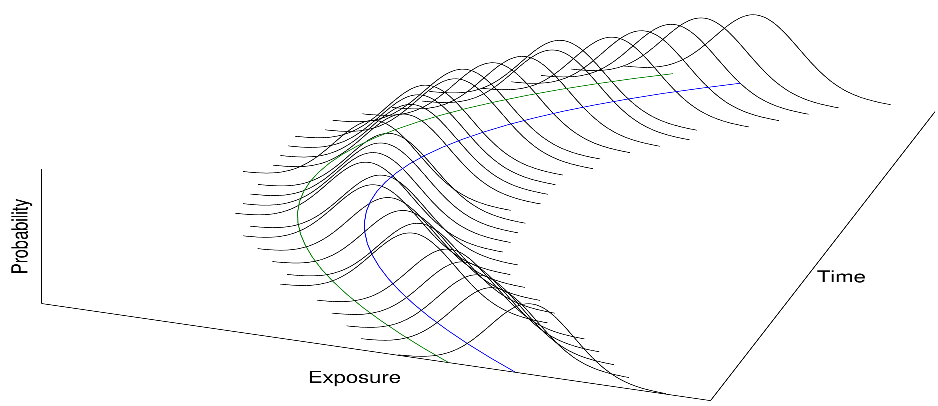


Figure 3.3: Illustration of how the exposure is dependent on time. The Figure shows the mean value change according to time and has an independent probability distribution at each time.

3.4.1 The BIS methodology

The BIS methodology is an analytical approximation, equal for all firms, and do not require any numerical simulation. Banks that decide to use the BIS methodology can choose between the current exposure method and the standard method, where this text only treat the current exposure method [BIS88] and refer to [BIS05a] for readers interested in the standard method.

When estimating credit exposure with the BIS methodology one shall note that actual exposure and thus both the potential and total exposure are defined at current time, $t = 0$, only. To estimate the exposure in the BIS methodology the fact that the total exposure, know as the credit equivalent amount (CEA), for a derivative position consist of two parts actual exposure (AE) and potential exposure (PE) is used. In the current exposure method the AE(0,t) are equal previous defined current market value, which easy can be estimated, while the potential exposure(PE) is calculated using a pre-defined static add-on factor. The potential exposure is then expressed as

$$PE = CF \times N, \tag{3.13}$$

where N is notional and CF is the add on factor which in general depends on security type and time to maturity. Table 3.1 summarize the factor by maturity date and type of underlying. By combining (3.13) with the actual exposure the CEA can be written as

$$CEA = AE(0, t) + PE(t). \tag{3.14}$$

To estimate (3.14) for a portfolio the PE in (3.13) is to be estimated as a sum of the individual add on amounts. This means multiply the notional principal amount by the appropriate add on factor for the transaction and then take the sum all transactions.

Residual Maturity	Interest Rates	FX and Gold	Equities	Precious Metals Except Gold	Other Commodities
< 1 year	0.0%	1.0%	6.0%	7.0%	10.0%
1-5 years	0.5%	5.0%	8.0%	7.0%	12.0%
> 5 years	1.5%	7.5%	10.0%	8.0%	15.0%

Table 3.1: BIS credit conversion factors [BIS05a].

If a netting agreement is established the PE in (3.14) has to be multiply with a netting factor NF. Equation (3.14) is then written as

$$CEA = AE(0, t) + NF \times PE(t), \tag{3.15}$$

where the netting factor, NF, is defined as

$$NF = 0.4 + 0.6 \times NGR, \tag{3.16}$$

where the factors 0.4 and 0.6 are values that have been set by the Basel Committee based on market observations. The last term in (3.16) is the Net to Gross Ration, NGR, which is equal the level of net replacement cost divided by the level of gross replacement cost for transactions subject to legally enforceable netting agreements. Using (3.11) and (3.10) it is expressed as

$$NGR = \frac{\max\left(\sum_{i=1}^{n} V_i(t), 0\right)}{\sum_{i=1}^{n} \max\left(V_i(t), 0\right)}. \tag{3.17}$$

If netting agreement is not permitted the NGR factor is equal one which also is the case when non of the transactions cancel each other as a result of the the numerator being equal the denominator in (3.17). When $NGR = 1$ the NF factor becomes equal one and the CEA is the sum of each transactions CEA, hence (3.14) becomes equal (3.15).

In a wider setting the CEA is used to measure necessary capital reserve. That is done by multiply the CEA by a 8% capital to exposure ration and a risk factor (RF) depending on type of counterparty. Capital reserve is then given as

$$CR = CEA \times 0.08 \times RF,$$

where the RF are given by Table 3.2.

Type of Counterpart	Risk Weight
Organization for Economic Cooperation and Development (OECD) governments	0%
OECD banks and public sector entities	20%
Corporate and other counterparties	50%

Table 3.2: Risk weights factors

The benefit of this model is its simple arithmetic, but it fail to take the different derivatives market correlation in account when measuring credit exposure.

3.4.2 Internal method

A more advanced method for estimating CCR in portfolios of interest rate derivatives is through using Monte Carlo simulations where the Libor Market Model is used to model underlying evolution of the rates. The idea is to estimate the whole exposure distribution at any given future time by estimation the rates and estimate the values of the remaining payments for the derivatives.

This text will use two different method for estimating PFE. The Basel Accord state that PFE is to be estimated as a multiple of the Effective EPE. This method which is denoted as Exposure amount (EAD) is then given as

$$\text{EAD} = \alpha \times \text{Effective EPE} \tag{3.18}$$

where the Effective EPE is given by (3.6) which is a mean value of the Effective exposure measured from (3.5). The alpha multiplier in (3.18) is set equal 1.4 by the Basel Committee based on best market practice. Banks may seek approval form their supervisors to compute internal estimates of alpha subject to a floor of 1.2. But supervisors also have the discretion to require a higher alpha value. Hence the value of 1.4 sounds like a proper value and will be used in this text.

The Monte Carlo simulations enabling estimation of any arbitrary percentile in the probability distribution. This means that in some seance information get lost by using the alpha multiplier is used instead of an direct simulation of the exposure percentile. To avoid this type of information loss this text will also use direct simulation of the 95% percentile. The measure of exposure that will be used is peak in the percentile which is denoted as MPFE and is found by using (3.4). Figure 3.4 shows the relation between this direct simulation of the exposure and the use of the alpha multiplier.

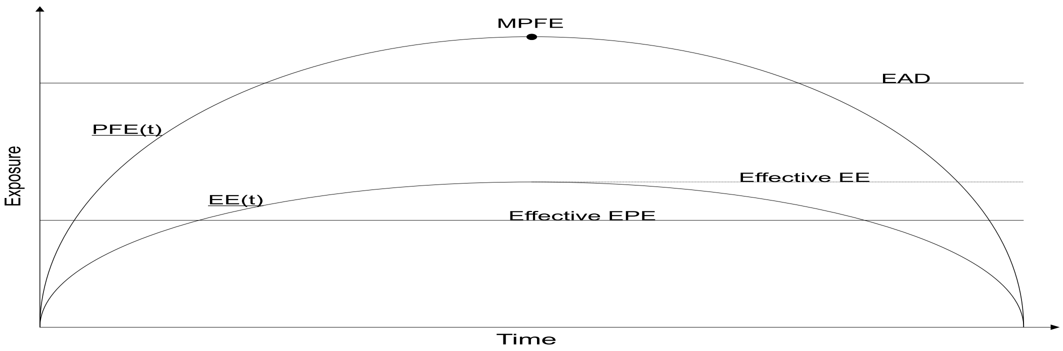


Figure 3.4: Illustration of how EE and effective EE can vary with time and how effective EE looks when it is estimated recursively starting with t_0.

Distribution of exposure

Since the exposure is a function of the remaining stochastic forward rates for the period one have to be able of estimating their distribution. When estimating CCR the interested are the potential loss given default at time t. This imply that the counterparty was alive at $t-1$ and not yet taken by financial distress. Hence the interest is to estimate the loss given alive at $t-1$ and not at $t=0$. An illustration of this is given in Figure 3.5. In mathematical terms this means the forward rates F_t, which is estimated by the Libor Market Model, has to be condition on the previous rates F_{t-1} i.e.,

$$f(F_t|F_{t-1}), \tag{3.19}$$

where F_t is the contracted at time t. This means for every point on the time line the estimation of the exposure it have to be done by using the conditional distribution. In this way exposure have the be estimated using the marginal distribution of F_t which is given by

$$f(F_t) = \int_{-\infty}^{\infty} f(F_t, F_{t-1})\, \mathrm{d}F_{t-1}. \tag{3.20}$$

An illustration of this dependency is given in Figure 3.5. As Equation (3.20) shows each point has it own probability distribution which make the distribution dependent on time and illustrated by previous Figure 3.3

To generalise this for an arbitrary time in order to estimate EE(t) and PFE(t) by using the definition given by (3.2) and (3.3) this gives

$$\mathrm{EE(t)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (F_t - \text{Fixed rate}) f(F_t, F_{t-1})\, \mathrm{d}F_t\, \mathrm{d}F_{t-1} \tag{3.21}$$

and

$$\mathrm{PFE(t)} = \int_{-\infty}^{\infty}\int_{\mathrm{PFE}}^{\infty} (F_t - \text{Fixed rate}) f(F_t, F_{t-1})\, \mathrm{d}F_t\, \mathrm{d}F_{t-1} \tag{3.22}$$

Both (3.21) and (3.22) can be impossible to solve for a general distribution and job has to be done by some simulation method.

A simulation method will typical consist of a program containing two loops. The outer loop will control the different times where the exposure is estimated while the inner loop estimate the outcome based on previous time rates. Algorithm 3.1 shows how this can be done.

Algorithm 3.1: Estimation of exposure

input : Initial forward rates L_0
output: Next forward rate

Effective $EE(0) = 0$

Looping exposure time
for $t = 1$ **to** T **do**
  Looping MC simulations
  for $i = 1$ **to** n **do**
    Draw $L_{(t,i)}$ based on L_{t-1}
    $V = \text{fixed rate} - L_{(t,i)}$
    $EE_{(t,i)} = \max(0, V)$
  end
  $L_{(t)} = \frac{1}{n}\sum_{i=1}^{n} L_{(t,i)}$
  $EE(t) = \frac{1}{n}\sum_{i=1}^{n} EE_{(t,i)}$
  Effective $EE(t) = \max\big(\text{Effective } EE(t-1), EE(t)\big)$
end
$EPE = \frac{1}{T}\sum_{i=1}^{T} EE_i$
Effective $EPE = \frac{1}{n}\sum_{i=1}^{n} \text{Effective } EE(i)$

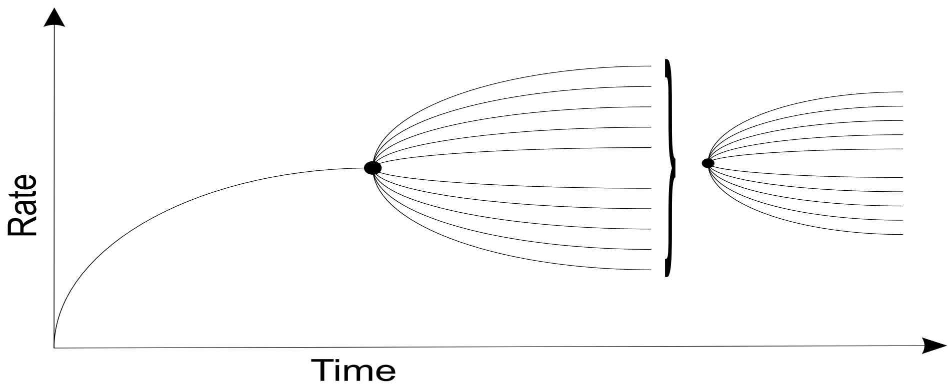


Figure 3.5: Illustration

Calculating exposure in a market model

Previous part treated the exposure in general terms without saying anything about how the term floating minus fixed rate i.e., $F_t - \text{Fixed rate}$, was calculated and F_t was the forward rate contracted at time t. From Section 2.2 it is know that a forward rate is the rate determine at time t for a period $[t^*, t^* + \delta]$. As a result there are in practice not a single forward rate for the whole period but the period consist several rates covering different sub intervals. When calculating exposure this have to be taken in consideration and it is sensible to let $F_t(i)$ denote the forward rate contracted at time t for the time interval i to $i + 1$.

The second thing to take in consideration is that the time interval given by i to $i + 1$ do not necessarily fit by the time period of the derivatives. For instance a swap is typical given as quarter paying swap with a fixed level of r. As an example one can think of a n period quarterly coupon paying swap with a fixed level of r. This swap commits the seller to pay the buyer at each future quarter $i \leq n$ the previous quarters short rate $F_t(i-1)$ in return for a fixed payment of r. As a result the swap owner actually pays the difference $[r - F_t(i-1)]/4$ if positive, and otherwise receives $[F_t(i-1) - r]/4$ at time t [DS03], where the factor 1/4 reflect the fact that the swap pays each quarter and would have to be changed for contracts of other lengths.

Those facts makes the Libor Market Model powerful. It enables estimation of the forward rates for all time periods within the rage of the initial time grid. But for longer time periods the time grid do not contains a enough points related to the pay frequency of the swaps.A native method to compensate for this is by linear interpolation. The motivation for doing this is in order to better reflect the changes in the forward rates from time to time. The interpolation is done with a first order approximation. Form a mathematical point of view a higher method could have been used, but by looking at historical rates in Figure 4.1 that is not rational. The curvature for the short term rates are greater than the long term rates but for the short term rates the points are close and the potential error becomes small. For the long rates the curvature of the rates are nearly flat indicating that a first order approximation is appropriate to approximate that part. This method can not be used for the last time period since there are no points to interpolate between. In practice one then have two choices either one can choose to use the same rate in for the whole period or put the last point forward in time. It will be as much wrong use both method since the curvature is rather flat, but this text will use the last method in order to avoid equal rates for the whole period. An in this way the LMM is powerful since it can be implemented taking this thing in consideration.

As a final remark on calculating credit exposure one should note that the exposure has to be calculated as a sum of the individual payments. To consider a swap the exposure is those a sum of the individual differences between the fixed and floating rate i.e.,

$$\text{Exposure} \propto \sum_{i=1}^{n} (r - F_i(t))dt, \tag{3.23}$$

where dt is the time difference between $i - 1$ and i. The exposure on interest swap can thus change according to the state future market prices. An upstate in one part of the market can be cancelled by a down state in a other part of market. Using equation (3.23) in its simple form do not take in consideration that the payments has to be discounted to the time where the exposure is estimated. By taking that in consideration the exposure can be written as a recursive equation in the following way,

$$\begin{aligned}
E_n =& (r - F_t(n-1)) \cdot dt \\
E_{n-1} =& (r - F_t(n-2)) \cdot dt + E_n/(1 + F_t(n-2) \cdot dt) \\
&\vdots \\
E_1 =& (r - F_t(2)) \cdot dt + E_t(2)/(1 + F_t(2) \cdot dt) \\
E_0 =& E_t(1)/(1 + F_t(n-1) \cdot dt).
\end{aligned} \tag{3.24}$$

Using Equation (3.24) this can be written as an algorithm in the way given by Algorithm 3.2, where interpolation is not taken i consideration.

Algorithm 3.2: Estimation exposure

input : Forward rates
payoff $= 0$
for $t = 1$ **to** T **do**
  $e \leftarrow e + (r - F_t(i-1)) \cdot (t_i(i) - t_t(i-1))$
  $e \leftarrow \frac{e}{1+F_t(i-1)\cdot(t_t(i)-t_t(i-1))}$
end
return *payoff*

After a introduction in the theory of counterpart credit risk estimation the next chapter will describe the data that will be used to test the different methods in Chapter 5.

4

Data description

This chapter describes the data used in both calibration of the volatility structure and an introduction to the test portfolio used when estimating the exposure. The first section describes the structure of the initial data, while the second part gives an overview of the test portfolios.

4.1 Calibration data

The volatility in this report is estimated using historic and not implied volatility. The volatility is estimated form historic Norwegian zero coupon yield provided by DnBNOR, where the data span a time period from 1 October 1998 to 29 August 2005 representing 1758 trading days. Within this time period there are eight different maturity dates cover the range of 3, 6, 9 months and 1, 2, 3 and 10 years. Figure 4.1 shows the evolution of those historic yield. As the Figure shows, some of the well known characteristic of interest rate behaviour are represented. This involve both parallel shift and twisting of the yield curve. As said in Section 2.2 this behaviour is a important motivation for using a market model instead of traditional one factor interest rate models. A result of this is that a few number of factors not are enough to describe the major movements in the historic yield. This is even more clear in Figure 4.3 containing a plot and a table of the principal components of the covariance matrix. The Figure 4.2 shows no sign of a breaking point that can indicate a few number of factors is a enough to describe the correlation structure. This lack of structure in the principal components have motivated the use of full covariance matrix in this paper.

Term structure data have normally the property that short term rates have greater volatility than long term rates. Figure 4.5 contains a plot with the covariance of the historic yield combined with a table of the main diagonal values. The plot shows the expected behaviour with high volatility for short term rates and low volatility for long term rates. On the other hand the plot confirms the results from the principal components analysis in the fashion that rates are correlated.

As a last characteristic of the rate one should look at the level of the rates. The initial rates in this text is shown in Figure 4.6 which also is the last observation covariance matrix. The plot shows that the rates has increasing trend. This tendency is reasonable when interest rates is known to have some kind of positive mean reverting level. One should thus expect long term swap to be on a higher level than current short time rate and on the other hand derivatives that have short time to expire typical are at a higher level resulting a large current exposure.

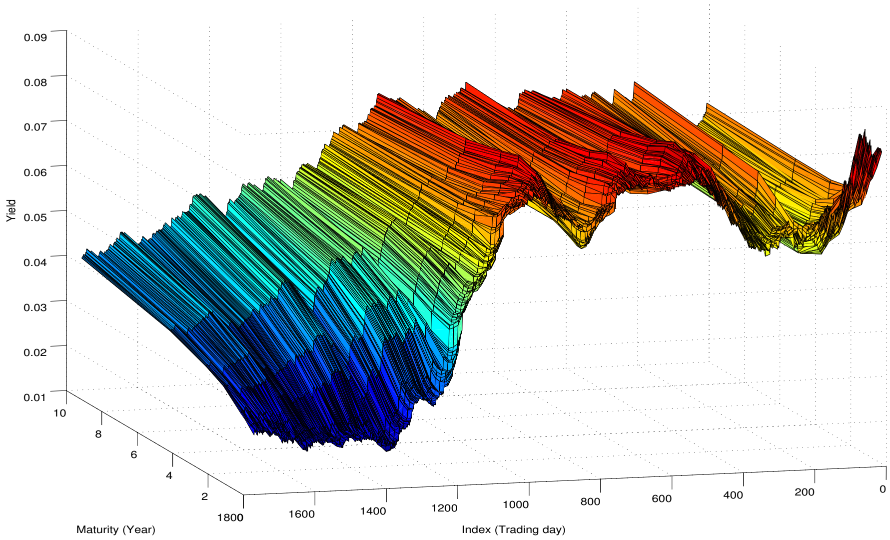


Figure 4.1: Evolution of historical yield as a function of maturity and entering time.

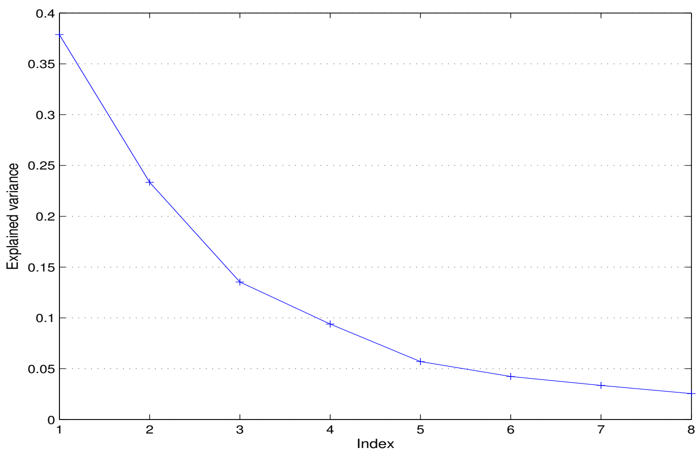
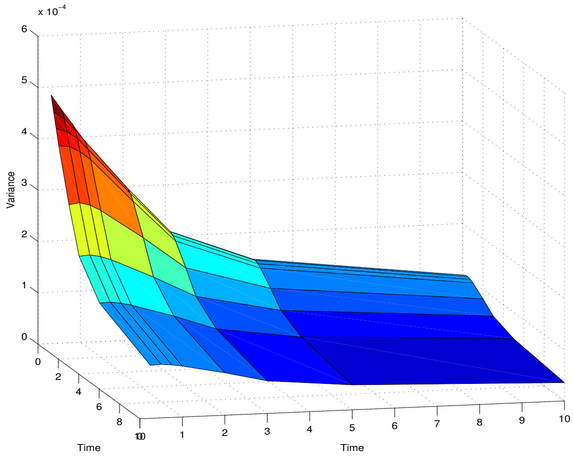


Figure 4.2: Principal components of the covariance matrix as a function of its index in decreasing order.

Factor	Explained variance	
	Notional	Accumulated
1	0.3787	0.3787
2	0.2335	0.6122
3	0.1353	0.7475
4	0.0939	0.8414
5	0.0570	0.8985
6	0.0424	0.9409
7	0.0336	0.9744
8	0.0255	1.0000

Table 4.1: Principal components values together with accumulated sum.

Figure 4.3: Principal components of the covariance matrix represent in both plot and table form.



Diagonal element	Variance ($\times 10^{-4}$)
1	4.880
2	4.445
3	4.093
4	3.706
5	2.293
6	1.262
7	0.693
8	0.356

Figure 4.4: Correlation structure.

Table 4.2: Diagonal values

Figure 4.5: Evolution in historic yield based on daily return for Norwegian NIBOR rates. The values for the main diagonal is represented in the belonging table.

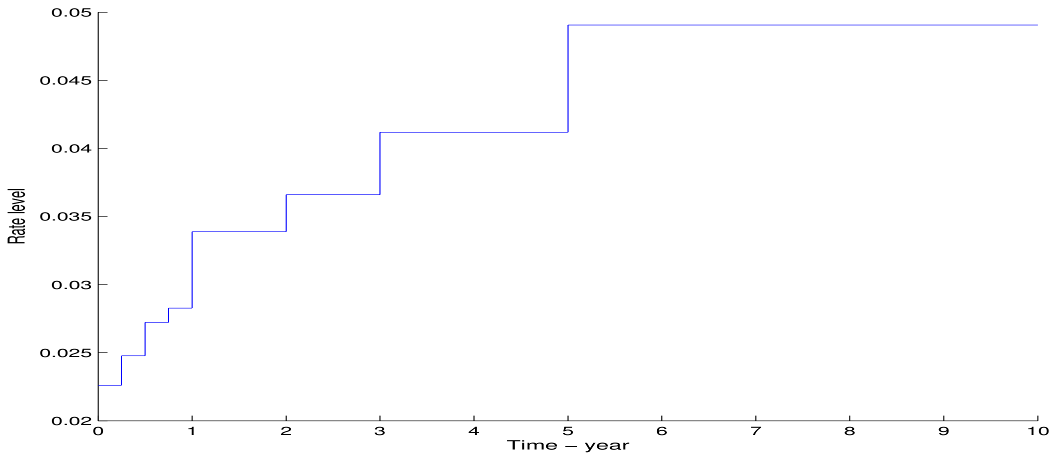


Figure 4.6: Evolution of the initial forward rates in the simulations, which also represents the last observation used in estimation of the covariance matrix.

4.2 Example portfolios

This section will describe all portfolios but only the first portfolio is given in detail. The remaining can be found with complete numbers in Appendix A.

The purpose of this work is to study different methods to estimate credit exposure, where the main focus is to compare the BIS exposure and different estimates from a Monte Carlo simulation. In the comparison portfolios related to various business sector combined with some portfolios that is a bit more of academic interest will be used. All portfolios are based on real data provided by DnBNOR but modified in order to match the stated criterion. In this way all portfolios will reflect sensible market behaviour in terms of the rate and notional level. Before introducing the portfolio there is a remark to the date format used in the notation. From Section 4.1 it is known that the last forward rate is form 29 August 2005. Due to some simplification in the implementation this rate will be stated as the rate of 1 January 2006. This modification will not affect the results since the work is concentrated around relative time to maturity.

The first portfolio that will be considered is a single swap which is given in Tables 4.3 and A.1. This is a 10 years quarter paying swap with a initial price close to zero, which then gives an initial exposure close to zero. This swap represented by Portfolio 1 can be seen as portfolio belonging to small company for instance in the establishment phase which only have one interest rate paying loan that they wish to secure. In some way the swap can also be seen as a belonging to private person securing the rate on the debt related to the apartment.

The aim of using this swap is to see how the BIS exposure behaves compared to simulated exposure profile for a small company with only one swap which gives no possibility of netting effect. As a reminding even though small often has a greater default probability it do not affect the credit exposure and make the use of this swap sensible.

Product	CCY	Start	Maturity	Notional	Rate	Pay freq.
IR-swap	NOK	2006-04-01	2016-04-01	100000000	0.0400	Q

Table 4.3: Portfolio 1

The second portfolio is a more mixed portfolio containing both sold and bought swaps. Most of them are quarter paying swap but some of them are annual and semiannual paying derivatives. The portfolio can bee seen as collection of interest rate derivatives for a smaller firm with some swaps both ways combined with caps and floors in a way that they get the wanted risk profile.

Portfolios 3 and 4, Tables A.3 and A.4, must in some way be seen together. They both consist of long term swap, but in the first of those portfolios all swap are positive correlated while the second one has swaps which is both sold and bought giving rise to negative correlation among the derivatives. The first of those can then typical be against a life insurance firm or a company in real estate. A life insurance company normally has long term obligation to its policy holder. They have an obligation to their customer in order to pay a known amount every year. In this way they hedge some of this risk by entering into long term swap agreement. A company in real estate normally has long term debt on their flats where the interest payments are secured by a swap contract. Hence the portfolio can represent both those types of companies. As said this portfolio has to be seen together with Portfolio 4, which here is based on academic interest. Sine the portfolio contains the same swap and only differ whether they are sold and bought they can be used to illustrate netting effect. The portfolios will not necessarily have the same current market value since of the swap can be of different sign but this negative correlation one should expect the peak exposure to be lower.

Portfolio five is a mixed portfolio containing both swaps, caps and floors. However the special with this portfolio is that none of the derivatives have maturity that is longer than five years and the rates are at a higher level than current state. In this way the portfolio represent for instance a firm with a mixed interest rate portfolio that is terminating the contracts to is counterparty. But an other interesting point with portfolio is to examine how the different method behaves when the contracts are at higher level than current state.

The sixth and last portfolio that will be considered in this text can be seen as interest rate portfolio belonging to an larger investment bank and is constructed by combining Portfolios 1-6. In this way the portfolio will consist of several different swap agreement at different levels and different maturity dates. The swap are both sold and bought which gives rise for netting effect in the sense that they are negative correlated. Beside just swap this portfolio consist of both caps and floors with different strike levels. In this way the portfolio represent different transactions which is typical for an investment bank.

5

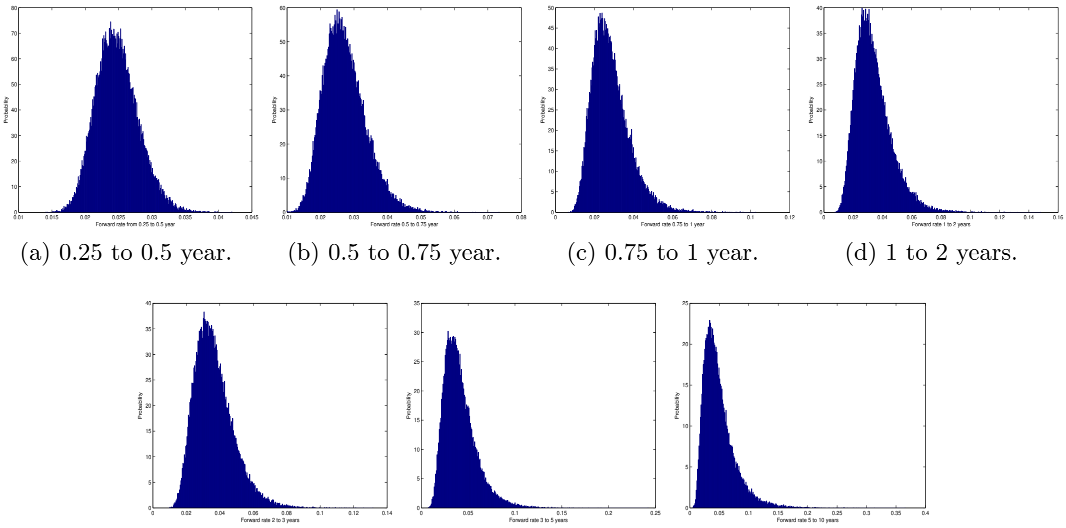
Numerical results

This chapter describes the numerical results from the Monte Carlo simulations. First section shows some result related to simulation of forward rates and derivative pricing using the LMM, while the second section gives the result related to estimation of credit exposure using the exposure measures as defined in Chapter 3.

5.1 Interest rate simulation

From Section 2.2 it is known that the Libor market model makes it possible to simulate the whole forward rate curve with corresponding distribution, expectation and variance for each different rate interval. An illustration of that simulation is given in Figure 5.1, showing the distribution of the forward rate for all different time periods. In the simulation a log-Euler discretization, Equation (2.18), with 50 000 simulations has been used, and the plot shows the LMM gives a complete distribution for all periods with a peak around the initial forward rates. Hence the result shows the model enables simulation of the volatility for the forward rates, which then can be used to pricing interest rate derivatives and estimation of credit exposure.

Results related to pricing of interest rate derivatives with focus on cap, floor and swaps are shown in Table 5.1, where the prices are measured with both the Euler and log-Euler discretization. In Table 5.1 also a column with the difference, in absolute value, between the two discretization are presented. As the table shows there are only minor differences between those two method, making both methods suitable for pricing interest rate derivatives. Hence in this paper both method could have been used, however the log-Euler discretization is used in order to avoid possible negative rates. When the numerical results for these derivative prices are seen together with the strike level and the corresponding volatility for the period, the derivative prices shows reasonable levels. Based on those results the Libor market model is as an appropriate method for estimation of financial derivatives. A last characteristic of the simulation that is worth looking at, is the convergence rate of prices. Plot 5.2 shows trace plots for both the mean value and the variance for one of the swaps. The plot shows that the mean value has converged after about 15 to 20 thousands iterations, which means there are no use to simulate 50 000 iterations which have been used for pricing purpose. Based on this observations only 20 000 simulations will be used in the purpose of estimating counterparty credit risk.



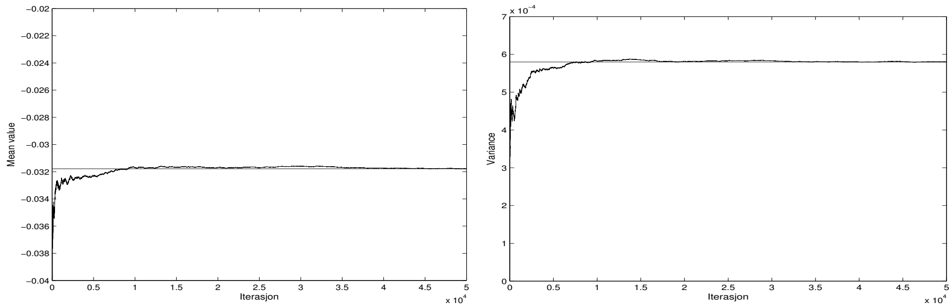
(a) 0.25 to 0.5 year. (b) 0.5 to 0.75 year. (c) 0.75 to 1 year. (d) 1 to 2 years.

(e) 2 to 3 years. (f) 3 to 5 years. (g) 5 to 10 years.

Figure 5.1: Distribution of estimated forward rates for different time intervals where ALU the rates are determine at time zero. The x-axis gives the forward rates while the y-axis represent the probability. All estimations are done by employ log Euler discretization with 50 000 simulations.

Type	Strike Level	Log Euler	Euler	Difference
Cap	0.035	0.03600	0.03372	0.00228
Cap	0.045	0.00582	0.01500	0.00918
Cap	0.055	0.00634	0.00586	0.00048
Floor	0.035	0.00498	0.00493	0.00005
Floor	0.045	0.01909	0.02014	0.00105
Floor	0.055	0.04292	0.04490	0.00198
Swap	0.035	0.03101	0.02878	0.00223
Swap	0.045	-0.00265	-0.00514	0.00249
Swap	0.049	-0.03011	-0.03178	0.00167
Swap	0.055	-0.03657	-0.03903	0.00246

Table 5.1: Simulated price for some common derivatives with start after two years and a final maturity after seven years, where all derivatives have four payments a year.



(a) Trace mean value. (b) Trace variance.

Figure 5.2: Convergence rate of the mean and variance in the LMM illustrated by trace plot of cumulative mean and variance for a quarter paying swap with start at two years and maturity after seven years, with a strike level at 4.9%.

5.2 Credit exposure

5.2.1 Portfolios credit exposure

The analysis will start by looking at a plain vanilla interest rate swap. This swap, which has been denoted as Portfolio 1 and representing a small company or a private person, has a finial maturity of ten years and a current market value just below zero. The BIS methodology is calibrated towards firms having larger portfolios containing plural derivatives with different exposure profiles. Since this portfolio only consist of a single swap one should not be surprised when the results shows that the BIS estimate gives an exposure that is beneath the simulated value. This is most easily seen in Figure 5.8 showing the exposure profile for the different methods. As the plot shows the MPFE estimate is about two times the size of the BIS estimate and the EAD measure is about two and a half time as large as the BIS estimate. Those values shows how the BIS methodology underestimates the somewhat more true exposure from the Monte Carlo simulation, when the portfolio consist of a single swap only. An interesting observation is that the MPFE measure is above the EAD measure. This will be treated in detail at the end, but it is a result coming from the fact that the EAD is estimated based on the concept of effective expected exposure. As a last remark, the consideration of netting agreement for this portfolio has no effect since a single swap do not give raise to any netting effects and hence the numbers are equal for the portfolio with and without netting agreement.

Beside just study the values, the whole exposure distribution is given in Figure 5.7. As the plot indicate the estimated exposure reach its maximum value around three years, where the profile increasing before that time and decrease to zero after the peak is reach. This profile is not unrealistic when the nature of a swap is taken in consideration. At time zero the market value of a swap are usually close to zero making it into a par swap. But as time goes the uncertainty in future rates becomes larger resulting in higher exposure. On the decreasing side of the peak the numbers of remaining payments fell, giving fewer payments that are exposed to changes in the rates. This behaviour can be seen in the distribution plot since the tail of the distribution gets smaller, resulting in more probability mass around the mean value. An remark for this swap is that the exposure has been calculated in a regime where the forward rates are at a low level and with a increasing trend. In this kind of regime the swap rate are higher than the rate at current time, hence this can explain some of the fall in the exposure curve.

For larger firms looking at only on single swap is not that interesting, since their portfolios often consist of several different derivatives. To reflect that fact Portfolio 2, which was constructed to represent a smaller company, have both caps, floor and sold and bought swaps all with different maturity, strike level and notional amount.

For the first portfolio the BIS estimate was below the Monte Carlo estimate when the portfolio only consist of a single swap. For this diversified portfolio the results are the other way around, the BIS estimate is above the simulated value. Here the ration between MPFE and BIS is 0.37 in the net portfolio against 2.3 in the previous. When the simulation of exposure is based on peak in the 95% percentile, the MPFE measure, the ration to BIS is about 0.4. This shows how the current exposure method is to conservative and overestimates the real underlying exposure for a larger portfolio. On the other hand if the portfolio is not assumed to be part of a netting agreement the BIS estimate but above the EPE estimate and below EAD estimate. Those values should be seen together with the distributions given by Figures 5.7 - 5.9. The first represent the full distribution while the two others shows the time profiles of the exposure measures. In opposite to Portfolio 1, Portfolio 2 has a current exposure that is above zero and a flat exposure profile when the portfolio is part of a netting agreement and a decreasing exposure profile when it is not part of a netting agreement. Since the add-on factor in the BIS exposure is meant to compensate for potential future exposure, Portfolio 2 shows it works against it purpose with that kind of exposure profile. This portfolio then clearly shows some of the weakness with BIS methodologies ability to take the exposure curvature in account when the portfolios have a positive current exposure, containing netting effects and a no changing exposure profile.

For the next examples Portfolios 3 and 4 would have to be seen in some connection. Portfolio 4 was presented as belonging to a life insurance firm or a company in real estate while Portfolio 3 is a modification of Portfolio 4 in other contain both sold and bought positions.

The plots in Figures 5.7 and 5.8 containing the exposure profiles for the net setting shows quite different exposure profiles for those two portfolios, as to expect. Portfolio 3 has zero current exposure but after a couple of years the exposure profile is familiar with the one for the single swap presented in Portfolio 1. This in contrast to Portfolio 4 which have an almost decreasing exposure profile the whole time. To explain this behaviour one have to look at the construction of the portfolios. The portfolio is constructed in the way that the life insurance companies continuously updates the swaps agreements. In this way the swaps that mature will be replaced by new one. This will as a result give some expensive swaps, in the portfolio, which came from the time where the rate where contracted at a higher level. This will then increase the exposure at time zero which is seen in Portfolio 4. On the other hand when the sign on some of those swaps are changed, the exposure are zero at current time as is the result for Portfolio 3. An other feature the plot shows is how those two portfolios with the same magnitude of notional exposure can have quite different exposure profiles. Based on the MPFE measure Table 5.2 shows that Portfolio 3 has an exposure equal 0.010 and Portfolio 4 has an exposure 0.015 related to the notional amount. On the other hand since the current exposure for these two method is quite different it affect the BIS measure in the way that

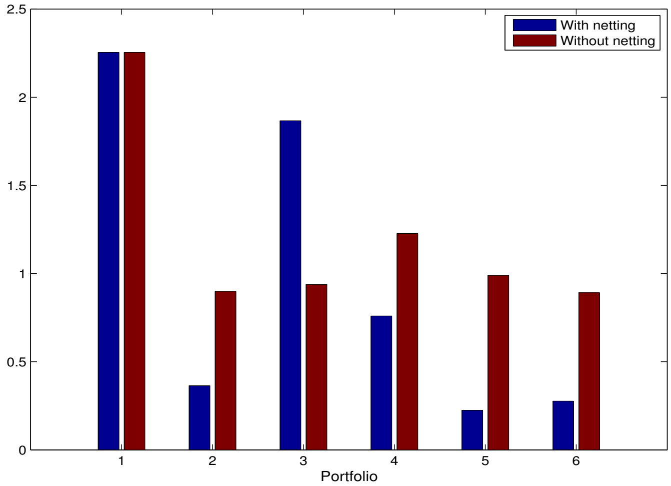
Portfolio 3 has an exposure of 0.008 towards Portfolio 4 with 0.020. To summarise a bit Portfolio 4 shows how the BIS methodology becomes to restrictive by estimating a to high exposure for companies that continuously update their swaps portfolios, which in reality gives a decreasing exposure profile. On the other hand Portfolio 3 which have the same notional amount shows how the BIS methodology do not enable to take the different correlation in consideration.

So far the portfolios have consisted of derivatives carried out under today's low rates and with maturity up to ten years. Portfolio 5 which represent a firm that had withdrawal from its counterparty and had only some few remaining derivatives left, where they consist of both cap, floor and swaps with a maximum of five year to maturity. As the exposure plot show the characteristic with this portfolio is that it has a decreasing exposure profile for both the net and unnet setting. This behaviour is not unrealistic since all the derivatives are deep in the money and hence the derivatives will in practice always give raise for CCR. The only thing that reduce the real exposure in this portfolio is when the number of remaining payments decrease. From the behaviour in Portfolio 4 and its decreasing exposure profile it is known that the current exposure method over estimates the true credit exposure for this kind of exposure profile, which once more is confirmed with this portfolio.

The last example is the investment bank represented by Portfolio 6. The exposure profile for this portfolio has the same characteristics as for portfolio 1. It has a low current exposure with a increasing exposure whereby it after three to four years decrease to zero when all derivatives have matured. However while the BIS estimate was below the simulated estimate in Portfolio 1 the results is the other way a round for this portfolio. Hence due to netting effects the estimates with the BIS methodology is beyond the results from the simulated approach. This means for an investment bank the current exposure method do not enables to take the effect of netting in account when estimating the exposure.

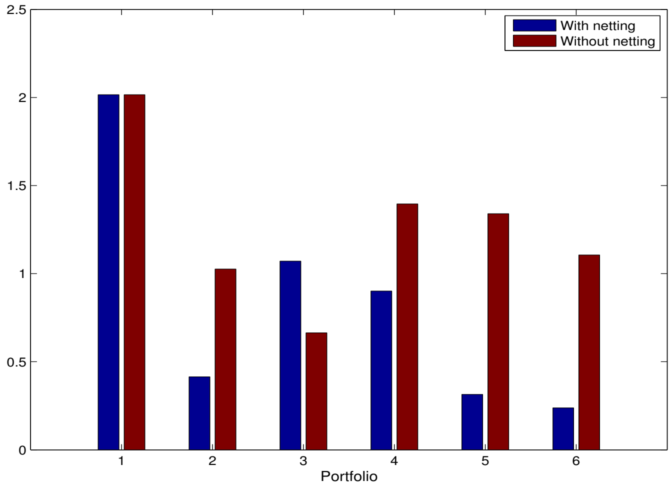
Portfolio	With net agreement			Without a net agreement		
	BIS	MPFE	EAD	BIS	MPFE	EAD
1	0.0150	0.0338	0.0302	0.0150	0.0338	0.0302
2	0.0156	0.0057	0.0065	0.0178	0.0160	0.0183
3	0.0057	0.0106	0.0061	0.0126	0.0118	0.0084
4	0.0195	0.0148	0.0176	0.0197	0.0241	0.0275
5	0.0402	0.0090	0.0127	0.0411	0.0406	0.0550
6	0.0196	0.0054	0.0047	0.0225	0.0200	0.0249

Table 5.2: Exposure relative to its notional value. The estimates are done with current exposure measure denoted as BIS, direct simulation of the maximal value at the 95% percentile in the exposure distribution denoted as MPFE and the concept of effective expected exposure with an α-factor of 1.4 denoted as EAD. All estimates are done with and without netting agreement.



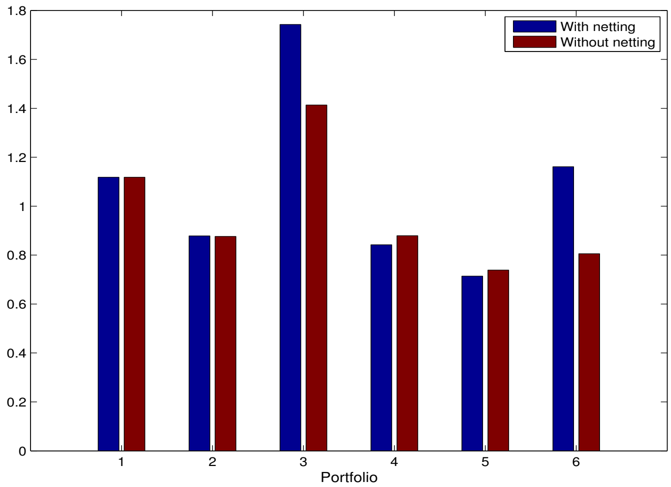
Portfolio	Netting agreement Yes	No
1	2.2541	2.2541
2	0.3639	0.8993
3	1.8670	0.9387
4	0.7591	1.2272
5	0.2247	0.9902
6	0.2766	0.8910
Mean(1-6)	0.9576	1.2001
Mean(2-6)	0.6983	0.9893

Figure 5.3: Comparison of direct simulation of the 95% percentile, denoted as MPFE, and the corresponding BIS exposure. The values are estimated for a setting with and without the establishment of netting agreement.



Portfolio	Netting agreement Yes	No
1	2.0155	2.0155
2	0.4143	1.0260
3	1.0713	0.6641
4	0.9011	1.3959
5	0.3146	1.3403
6	0.2382	1.1057
Mean(1-6)	0.8258	1.2579
Mean(2-6)	0.5879	1.1064

Figure 5.4: Comparison of simulation of exposure based on the method where the mean value in the exposure distribution is multiplied with an alpha factor of 1.4, denoted as EAD, and the corresponding BIS estimate. The values are estimated for a setting with and without the establishment of netting agreement.



Portfolio	Netting agreement Yes	No
1	1.1184	1.1184
2	0.8785	0.8764
3	1.7427	1.4134
4	0.8424	0.8791
5	0.7143	0.7388
6	1.1613	0.8058
Mean(1-6)	1.0763	0.9720
Mean(2-6)	1.0679	0.9427

Figure 5.5: Comparison of exposure based on the simulation of EAD, which is based on the concept of effective expected exposure with an α-factor of 1.4, and the direct simulation of the 95% percentile which is denoted as MPFE. Hence the plot shows the ration of EAD to MPFE. The values are estimated for a setting with and without the establishment of netting agreement.

5.2.2 Comparison of exposure measure

While the previous section described the exposure related to the different portfolios this part gives a more systematic comparison between the different exposure measures.

Figure 5.3 shows a comparison between the peak exposure in the 95% percentile and the current exposure measure. The result shows that the BIS methodology estimates less exposure than the Monte Carlo approach does in four out of six portfolios. The estimation of exposure for Portfolios 1 and 3 gave a BIS estimate that was beneath the estimate from than the Monte Carlo simulation, when the estimation where done with the establishment of netting agreement. Portfolio 1 which represented the small company, with an single swap contract, clearly shows how the current exposure method is to conservative in the way that it estimates a to high exposure compared with the simulated estimate. Portfolio 3 which is the modified life insurance portfolio have been constructed to have a low market value at time zero combined with derivatives which later gives a high potential exposure. As a result of this the BIS methodology estimates a small exposure since the current market value are low while the Monte Carlo method managed to estimate peak which arise after some years. In the setting where netting agreements is not permitted Portfolios 1 and 4 have a BIS estimate that is beneath the Monte Carlo estimate. For Portfolio 1 the explanation is obvious since one swap do not give raise to any netting effects. But for Portfolio 4, the life insurance portfolio, the BIS methodology do not manage to take the continuous new derivatives, making a decreasing exposure profile in, consideration. This then shows how the BIS method do not enables to take the structure of the exposure in account. And the result shows that for larger portfolios the reduction with a netting agreement can be enormous. This results is confirm by estimation done by the Financial Supervisory Authority of Norway[1] indicating that a simulation based method reduce the necessary capital reserve of approximately 35%.

Since the MPFE measure is not permitted by the Basel Committee, which says that the estimation should be based on the concept of effective expected exposure, it is of interest to compare the EAD measure with the BIS measure. Results for this is given in Figure 5.4. For the portfolios with a netting agreement the results are at the same magnitude as for the method based on the MPFE measure. Hence the explanation is the same as for the other method and once more confirm the results that a simulation based method estimates a lower exposure compared with the BIS measure. However for the position without netting agreements the results are a bit different. In that setting only the modified life insurance portfolio, i.e. Portfolio 3, had a BIS measure that was beneath the Monte Carlo based. Hence this shows some of the same problem as observe previous, in the way that BIS methodology estimates to high exposure related to the underlying real exposure when the exposure profile decrease substantially over time.

A more detailed analysis of the two Monte Carlo measure is given i Figure 5.5 which shows a comparison between those two estimates. As the figure indicate the two method are in mean value almost equal, but some differences do exist. Those differences is a direct result of the curvature of the exposure curve as previous explained. Since effective expected exposure is estimated recursively the shape will obvious have great influence on the estimate which the results confirms. When the peak in the exposure are near time zero the MPFE estimate lies below the EAD estimate and vice versa when the peak are far away out in time.

The distinction between the two estimate can also be explained by the alpha multiplier of 1.4 and the percentile used in the direct simulations. The alpha multiplier of 1.4 used in this paper is based on the proposal from the Basel Committee. But the Basel Accord [BIS05a] discuss both higher and lower estimate for alpha value. As it is said “because of high correlation market values and high exposure to general wrong way risk supervisors have the direction to require a higher alpha value.” But in the other end the Basel Accord allow for own estimates for the alpha value subject to a floor of 1.2. Based on that fact an alpha value of 1.4 is not unrealistic. In the estimation based on a direct simulation of the peak exposure the 95% percentile have been used. This quantile has in some way been used based on a best qualified guess. The 90% or 99% percentile could for instance have been used. However using a percentile of 95% would count for 95 out of 100 potential losses. The use of an other quantile should in some sense be determine from the banks wanted risk profile and corresponding credit rating. However in these examples a percentile of 95% is not unreasonable, and turned the other way around one can say that an alpha factor of 1.4 will account for losses up to a confidence level of 95%.

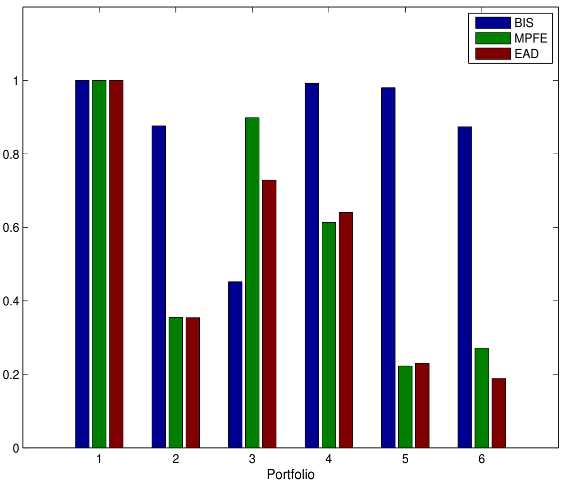
5.2.3 Netting effects

As seen all simulations have been done with and without establishment of netting agreement. To discuss the results in more detail Figure 5.6 shows a comparison of the portfolios with and without netting agreement where the figure is given for all portfolios and for all three

[1] Press release 14/2006

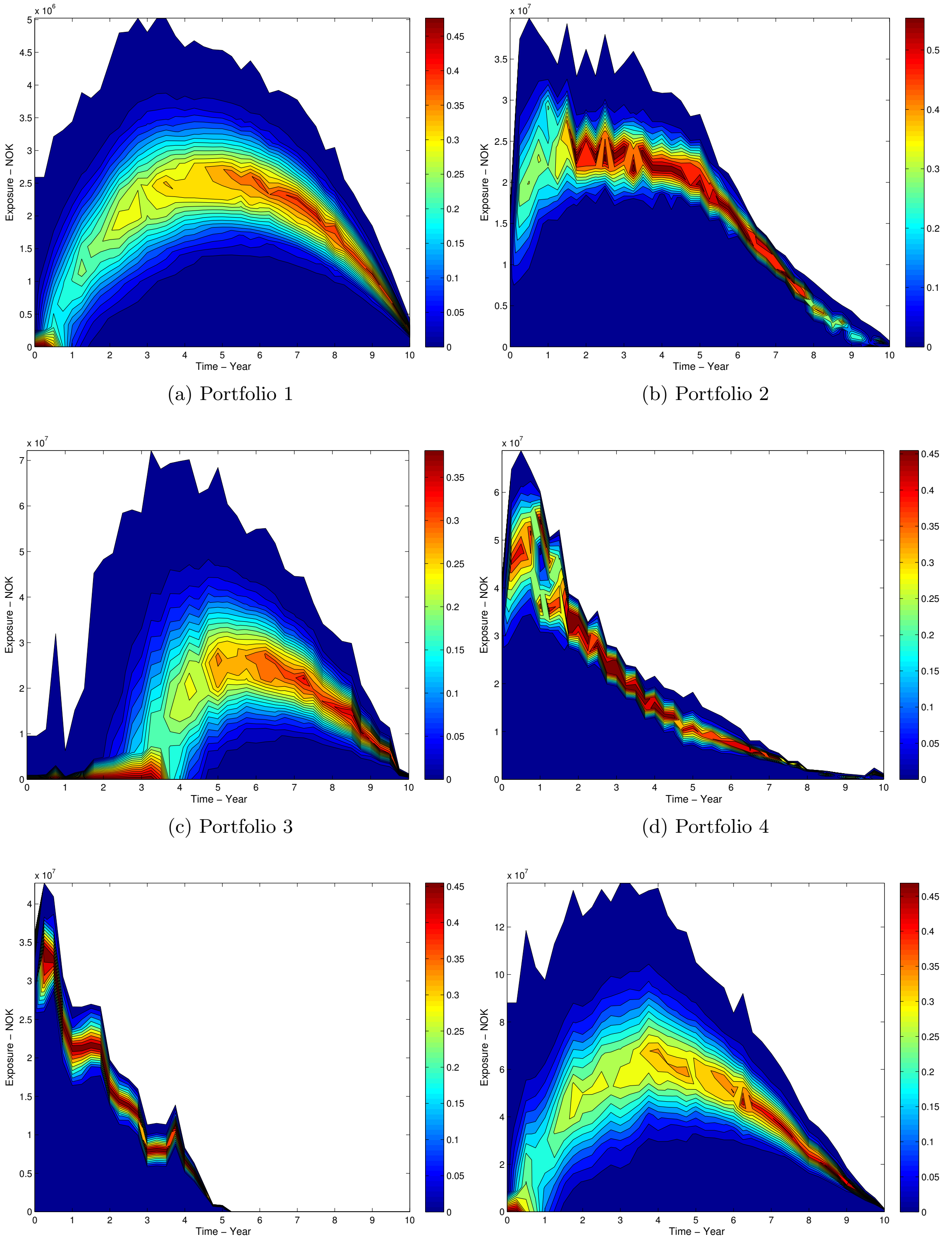
measures used to estimate the exposure. A number below one indicate that netting agreement reduce the total exposure for the portfolio.

Once more there are no use to discuss Portfolio 1 since it only consist of one swap and in that case do not give any raise to netting effect, and gives a ration exactly equal one. But for the other portfolios the results are some different. By looking at the mean values in the Figure one observe that the BIS methodology estimate a less reduction in the exposure caused by netting effects compared with the other measures. One more characteristic with this results is that Portfolio 3 perform different than the other portfolios for all estimation measures. For this portfolio the BIS methodology estimate a reduction in the exposure caused by netting that is greater then all the other methods does. This was know as the modified life insurance portfolio in order to compare netting effect related to the exposure from the life insurance example in Portfolio 4. These two example then shows some of the weakness with the BIS methodology in handling netting effects when the portfolios are constructed in special ways. For the other portfolios the simulation based method gives a quite lower exposure estimate when the portfolio is in a netting agreement. For instance for the investment bank portfolio the ration between the BIS estimate and the EAD estimate is below 0.2. In mathematical terms this means that the simulation based method better manages to take the derivatives different correlation in consideration when measuring counterparty credit risk. But it also confirm the assumption from Chapter 3 saying that netting agreements should result in lower exposure.



Portfolio	BIS	MPFE	EAD
1	1.0000	1.0000	1.0000
2	0.8764	0.3547	0.3538
3	0.4518	0.8986	0.7288
4	0.9923	0.6139	0.6406
5	0.9803	0.2224	0.2301
6	0.8739	0.2713	0.1882
Mean(1-6)	0.8625	0.5601	0.5236
Mean(2-6)	0.8349	0.4722	0.4283

Figure 5.6: The figure shows the effect caused by netting the derivatives in the portfolios. The numbers represent the fraction between the exposure in the setting where netting agreement is established to the setting where all derivatives stands alone. A number below one means there is a reduction in the exposure when netting agreement is established. This is estimated for current exposure measure denoted as BIS, direct simulation of 95% percentile denoted as MPFE and the concept of effective expected exposure with an α-factor of 1.4 denoted as EAD.

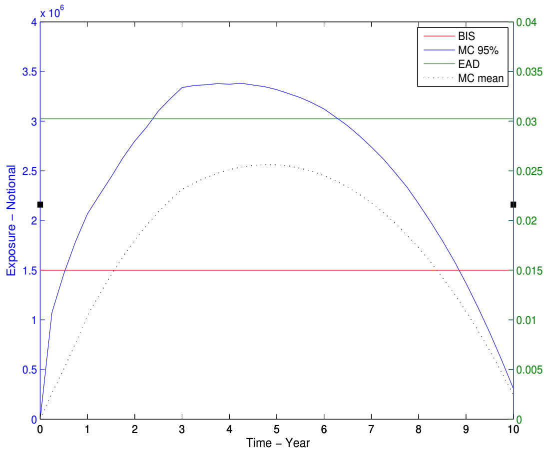


(a) Portfolio 1 (b) Portfolio 2

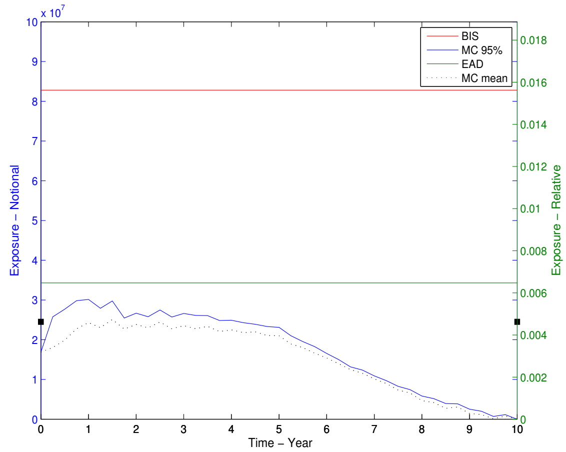
(c) Portfolio 3 (d) Portfolio 4

(e) Portfolio 5 (f) Portfolio 6

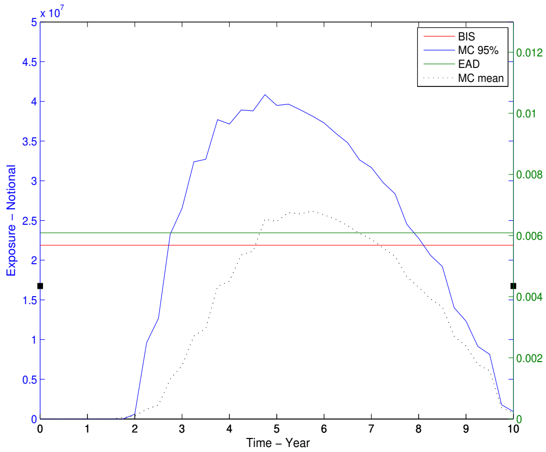
Figure 5.7: The plot shows the distribution of exposure values as a function of time for the different portfolios. The outline in the plot represents the potential max and minimal values and the colour bar represent the belonging probability of the exposure at each time. For all portfolios the estimation has been done under the establishment of netting agreement.



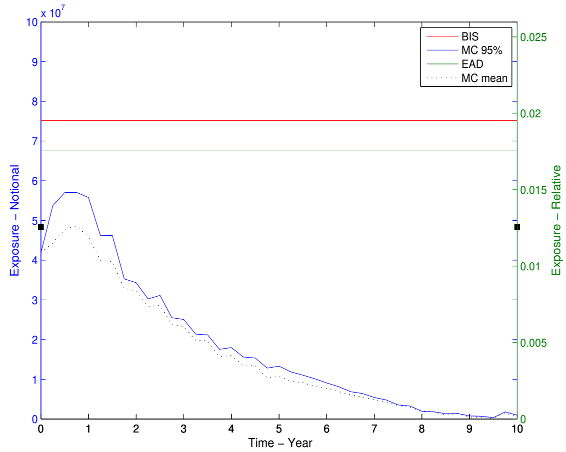
(a) Portfolio 1



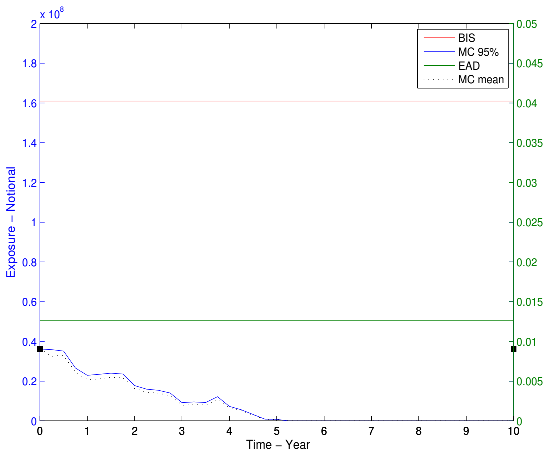
(b) Portfolio 2



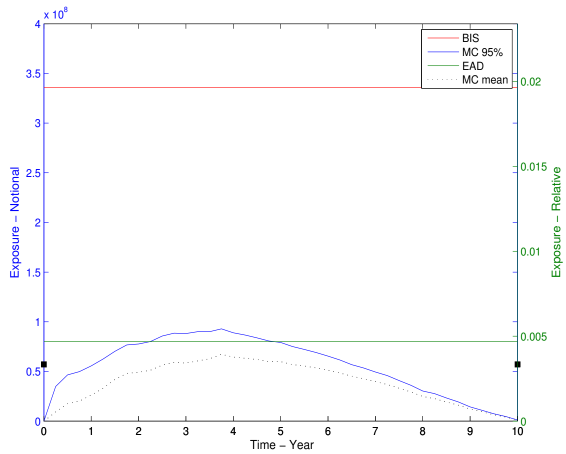
(c) Portfolio 3



(d) Portfolio 4

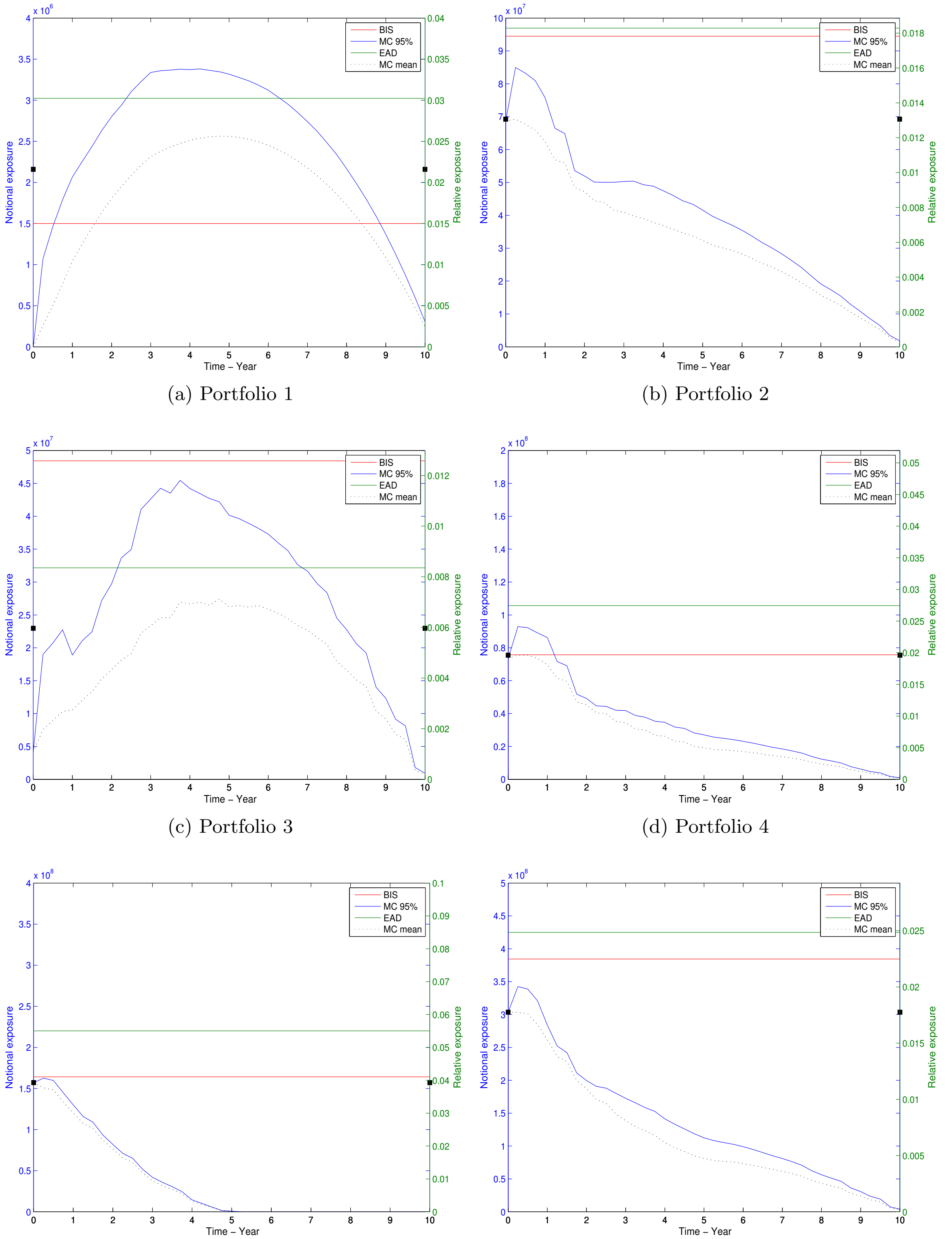


(e) Portfolio 5



(f) Portfolio 6

Figure 5.8: Evolution of the 95% percentile and the mean value in the exposure distribution as a function of time. The red line denoted as BIS shows the BIS estimate for the potential exposure by using the current exposure method. The green line shows the EAD measure which is based on the concept of effective expected exposure with an α-factor of 1.4, where the effective expected exposure is given by the dots on the y-axis. All values are estimated with the establishment of netting agreement.



(a) Portfolio 1 (b) Portfolio 2

(c) Portfolio 3 (d) Portfolio 4

(e) Portfolio 5 (f) Portfolio 6

Figure 5.9: Evolution of the 95% percentile and the mean value in the exposure distribution as a function of time. The red line denoted as BIS shows the BIS estimate for the potential exposure by using the current exposure method. The green line shows the EAD measure which is based on the concept of effective expected exposure with an α-factor of 1.4, where the effective expected exposure is given by the dots on the y-axis. All values are estimated without the establishment of netting agreement.

6

Conclusion

The purpose of this work has been to study different methods for estimating of counterparty credit risk, where the effort has been concentrated around how a Monte Carlo implementation of the Libor market model can be used to estimate credit exposure. The results have shown how this enables estimation of the full exposure distribution for portfolios with and without netting agreements. From those distributions, the report has shown how an estimate for the potential exposure can be estimated in two ways. The first, MPFE, was estimated by taking the peak observation in the 95% percentile, while the second, denoted as EAD, was based on multiplying the effective expected exposure with an α-factor of 1.4. Previously the Bank for International Settlements has proposed the current exposure method, also denoted as the BIS methodology, in order to estimate CCR. That method, which is based on simple arithmetic, gave an estimate of potential future exposure based on the current exposure only.

First it start shown how the Libor market model can be used to price interest rate derivatives. Those results suggested fair prices of the derivatives, showing that the model is suitable for pricing interest rate derivatives. Another quality of the LMM which the simulation showed was the ability to estimate the distribution of all forward rates. Those properties have motivated the use of the LMM to model the evolution of the rates combined with Monte Carlo simulation to estimate counterparty credit risk for portfolios of interest rate derivatives.

Based on the numerical values related to the estimation of CCR they indicate that a method based on Monte Carlo simulation estimate a less exposure related to the the BIS methodology. However on the other hand the portfolio related to the small company gave a BIS estimate that was below the simulated value and the same results were seen for the academic example in Portfolio 3. Those portfolios show how the BIS methodology can underestimate the exposure when the portfolio has non-linear behaviour. In this way the Monte Carlo simulation enables better risk management in the form that one can control the evolution of the whole exposure curve. When the different exposure measures from the Monte Carlo simulation are compared, there are minor differences. The choice of exposure measurement unit in the internal method approach will then not influence the business volume of the estimated exposure, but use of the MPFE measurement would give improved risk management capabilities even though banks not official allowed using the MPFE measure.

The various methods only differ in the way that they manage to estimate the underlying exposure and hence they do not reduce the real underlying exposure. On the topic of real exposure reduction the results show that netting the derivatives in a portfolio can significantly reduce the total exposure. Based on the EAD measure the results indicate that a netting agreement can reduce the exposure with about 50%. But there are differences; while the life insurance portfolio gave just a small reduction because of highly correlated derivatives, the investment bank portfolio gave a large reduction due to a much smaller degree of variation. For these types of portfolios, the simulation-based shows how it explicitly managed to take the correlation to when measuring exposure. This would not be possible under the alternative method.

The results in this report confirms both previous research and theory. Essentially this involves the results in [AC98] from J. Aziz and N. Charupat which found that a simulation based method often gives a lower estimate of exposure relative to the BIS methodology. However one of their main comments is that "the BIS does not take into account the evolution of the exposure through time, the resulting reserve can be either too low or too high, depending on the nature of the transaction," which is what the results in this paper also have shown. In terms of netting effects, the results have confirmed the assumption from Chapter 3 saying that netting agreements should result in lower exposure.

Hence this paper confirms their conclusion to recommend financial institution to use an internal method based on Monte Carlo simulation, since it gives a more accurate estimate of the exposure and enables better risk control with in the firm. Most important, a sophisticated underlying model for this type of exposure measurement has been implemented and shown great accurately.

6.1 Further research

To take this work further there are at least two obvious extension. The first is to improve the estimation techniques used in this work. This should involve a systematic selection of the points where the exposure is estimated, but should be coupled with effort to minimize the unavoidably reduction in accuracy. A reduction in points will reduce the computer time giving faster estimates of the exposure, research that could have been carried out to such an effect, is a detailed exploration of which points on the time axis influence the exposure most. An-other natural candidate for future research that would have been interesting to carry out, is a combination with the default probabilities of counterparties. In that way one can estimate not only the credit exposure, but also the expected loss on each counterparty.

A

Portfolios

The following tables represent the portfolios used when estimating the different exposure profiles. The first portfolio consist of only one swap while the remaining are a collection of many different swaps and other derivatives. Portfolio 6 is a combination of the portfolios 2-5, given in Tables A.2 to A.5, and hence not written twice. A detailed description and a motivation for each portfolio is given i Section 4.2.

The following notation is used

- **Product** represent the type of interest rate derivative.
- **CCY** represent the currency of the underlying interest.
- **Start** gives the start time of the derivative.
- **Maturity** gives the date of the final maturity of the derivative.
- **Notional** represent the derivatives notional amount whereby the sing represent sold or bought position.
- **Rate** is the fixed rate belonging to the derivative.
- **Pay freq.** is the payment frequency of the derivative. Q represent quarter paying derivative, S represent semi annual payments and 1Y represent one payment a year.

Product	CCY	Start	Maturity	Notional	Rate	Pay freq.
IR-swap	NOK	2006-04-01	2016-04-01	100000000	0.0400	Q

Table A.1: Portfolio 1.

Product	CCY	Start	Maturity	Notional	Rate	Pay freq.
IR-cap	NOK	2006-07-02	2016-08-02	100000000	0.0500	Q
IR-floor	NOK	2006-07-02	2015-08-02	150000000	0.0300	Q
IR-swap	NOK	2006-07-12	2015-08-02	-150000000	0.0427	Q
IR-swap	NOK	2006-04-01	2011-04-01	-100000000	0.0710	Q
IR-swap	NOK	2006-04-01	2016-04-01	100000000	0.0400	Q
IR-swap	NOK	2007-01-02	2015-08-02	-300000000	0.0430	S
IR-swap	NOK	2007-01-02	2015-08-02	150000000	0.0450	1Y
IR-swap	NOK	2007-01-02	2015-08-02	-300000000	0.0430	S
IR-swap	NOK	2007-01-02	2016-08-02	-300000000	0.0430	1Y
IR-swap	NOK	2006-01-02	2016-01-02	200000000	0.0250	Q
IR-swap	NOK	2006-01-02	2015-01-02	200000000	0.0427	Q
IR-swap	NOK	2006-01-01	2011-01-01	-290000000	0.0255	Q
IR-cap	NOK	2006-01-01	2011-01-01	250000000	0.0450	Q
IR-floor	NOK	2006-01-01	2011-01-01	250000000	0.0250	Q
IR-swap	NOK	2006-01-01	2011-01-01	-120000000	0.0395	1Y
IR-swap	NOK	2006-01-11	2012-01-11	300000000	0.0390	Q
IR-swap	NOK	2006-07-11	2013-07-11	300000000	0.0390	Q
IR-cap	NOK	2006-01-11	2012-01-11	200000000	0.0490	Q
IR-cap	NOK	2006-07-11	2013-07-11	200000000	0.0430	Q
IR-floor	NOK	2006-07-02	2015-08-02	150000000	0.0300	Q
IR-swap	NOK	2006-07-02	2015-08-02	150000000	0.0427	Q
IR-swap	NOK	2006-07-02	2015-08-02	150000000	0.0429	Q
IR-swap	NOK	2006-07-02	2008-08-02	100000000	0.0409	Q
IR-cap	NOK	2006-01-01	2010-08-02	250000000	0.0450	Q
IR-floor	NOK	2006-01-01	2015-08-02	290000000	0.0380	Q
IR-swap	NOK	2006-01-01	2010-08-02	250000000	0.0450	Q

Table A.2: Portfolio 2.

Product	CCY	Start	Maturity	Notional	Rate	Pay freq.
IR-swap	NOK	2006-07-02	2015-08-02	150000000	0.0427	Q
IR-swap	NOK	2006-04-01	2011-04-01	100000000	0.0710	Q
IR-swap	NOK	2006-04-01	2015-04-01	100000000	0.0400	Q
IR-swap	NOK	2007-01-02	2015-08-02	100000000	0.0430	S
IR-swap	NOK	2007-01-02	2015-08-02	150000000	0.0420	1Y
IR-swap	NOK	2007-01-02	2015-08-02	300000000	0.0430	S
IR-swap	NOK	2007-01-02	2015-08-02	300000000	0.0430	1Y
IR-swap	NOK	2006-01-02	2015-01-02	200000000	0.0450	Q
IR-swap	NOK	2006-01-02	2015-01-02	200000000	0.0427	Q
IR-swap	NOK	2006-01-01	2011-01-01	300000000	0.0315	Q
IR-swap	NOK	2006-01-01	2011-01-01	250000000	0.0395	1Y
IR-swap	NOK	2006-01-11	2012-01-11	300000000	0.0390	Q
IR-swap	NOK	2006-07-11	2013-07-11	300000000	0.0390	Q
IR-swap	NOK	2006-07-11	2009-08-02	300000000	0.0450	Q
IR-swap	NOK	2007-01-02	2010-08-02	300000000	0.0550	S
IR-swap	NOK	2006-04-01	2016-04-01	300000000	0.0400	Q
IR-swap	NOK	2006-07-02	2007-08-02	200000000	0.0527	Q

Table A.3: Portfolio 3.

Product	CCY	Start	Maturity	Notional	Rate	Pay freq.
IR-swap	NOK	2006-07-02	2015-08-02	-150000000	0.0427	Q
IR-swap	NOK	2006-04-01	2011-04-01	-100000000	0.0710	Q
IR-swap	NOK	2006-04-01	2015-04-01	100000000	0.0400	Q
IR-swap	NOK	2007-01-02	2015-08-02	-100000000	0.0430	S
IR-swap	NOK	2007-01-02	2015-08-02	150000000	0.0420	1Y
IR-swap	NOK	2007-01-02	2015-08-02	-300000000	0.0430	S
IR-swap	NOK	2007-01-02	2015-08-02	-300000000	0.0430	1Y
IR-swap	NOK	2006-01-02	2015-01-02	200000000	0.0450	Q
IR-swap	NOK	2006-01-02	2015-01-02	200000000	0.0427	Q
IR-swap	NOK	2006-01-01	2011-01-01	-300000000	0.0315	Q
IR-swap	NOK	2006-01-01	2011-01-01	-250000000	0.0395	1Y
IR-swap	NOK	2006-01-11	2012-01-11	300000000	0.0390	Q
IR-swap	NOK	2006-07-11	2013-07-11	300000000	0.0390	Q
IR-swap	NOK	2006-07-11	2009-08-02	-300000000	0.0450	Q
IR-swap	NOK	2007-01-02	2010-08-02	-300000000	0.0550	S
IR-swap	NOK	2006-04-01	2016-04-01	300000000	0.0400	Q
IR-swap	NOK	2006-07-02	2007-08-02	-200000000	0.0527	Q

Table A.4: Portfolio 4.

Product	CCY	Start	Maturity	Notional	Rate	Pay freq.
IR-swap	NOK	2007-01-02	2009-08-02	100000000	0.0640	S
IR-swap	NOK	2006-04-01	2007-04-01	100000000	0.0810	Q
IR-swap	NOK	2006-01-01	2010-01-01	-150000000	0.0689	1Y
IR-swap	NOK	2005-01-02	2008-08-02	-100000000	0.0620	1Y
IR-swap	NOK	2006-07-11	2009-07-11	300000000	0.0680	Q
IR-swap	NOK	2006-07-02	2010-08-02	-150000000	0.0765	Q
IR-swap	NOK	2006-07-02	2010-08-02	150000000	0.0678	Q
IR-swap	NOK	2007-01-02	2009-08-02	100000000	0.0650	1Y
IR-swap	NOK	2007-01-02	2008-08-02	150000000	0.0650	S
IR-swap	NOK	2006-07-11	2009-07-11	-100000000	0.0680	Q
IR-swap	NOK	2006-07-02	2010-08-02	-150000000	0.0765	Q
IR-swap	NOK	2006-01-02	2008-08-02	-200000000	0.0750	S
IR-swap	NOK	2006-01-01	2011-01-01	100000000	0.0659	Q
IR-swap	NOK	2007-01-02	2009-08-02	-100000000	0.0540	S
IR-swap	NOK	2006-04-01	2010-04-01	100000000	0.0640	Q
IR-swap	NOK	2006-07-02	2007-08-02	150000000	0.0727	Q
IR-swap	NOK	2006-07-02	2010-08-02	150000000	0.0427	Q
IR-swap	NOK	2006-04-01	2011-04-01	-100000000	0.0710	Q
IR-cap	NOK	2006-01-01	2010-08-02	150000000	0.0650	Q
IR-cap	NOK	2006-01-11	2008-01-11	200000000	0.0700	Q
IR-cap	NOK	2006-07-11	209-07-11	100000000	0.0690	Q
IR-floor	NOK	2006-01-01	2010-08-02	150000000	0.0450	Q
IR-floor	NOK	2006-01-11	2008-01-11	200000000	0.0390	Q
IR-floor	NOK	2006-07-11	209-07-11	200000000	0.0400	Q
IR-cap	NOK	2005-01-02	2008-08-02	100000000	0.0820	1Y
IR-cap	NOK	2006-07-11	2009-07-11	150000000	0.0790	Q
IR-floor	NOK	2006-07-02	2010-08-02	150000000	0.0460	Q
IR-floor	NOK	2006-07-02	2010-08-02	150000000	0.0410	Q

Table A.5: Portfolio 5.

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