EMD-Prony for Phasor Estimation in Harmonic and Noisy Condition

Jalal Khodaparast[1], Olav Bjarte Fosso[1] and Marta Molinas[2]

***Abstract*— Wide area monitoring systems (WAMSs) enables the real-time monitoring of power system dynamics by bringing together new developments in the field of measurement, communication and computing. Measurements of voltage and current phasors are recorded by phasor measurement units (PMUs) installed across a wide area power system and time tagged at the point of measurement using a common time reference. Estimates of Phasor, Frequency and rate of change of frequency (ROCOF) are duties of PMU in every installed bus. For this purpose, the Prony algorithm is one of the promising method since its phasor estimates are calculated adaptively based on estimated frequency. For fundamental phasor estimation, the Prony algorithm with the order of one is suitable but its performance in terms of accuracy is diminished when non-fundamental components interfere in the measured signal. These components can be eliminated from fundamental phasor estimates by increasing the Prony's model order. In order to specify adaptively the order of the Prony algorithm, the Empirical Mode Decomposition (EMD) method is proposed in this paper to be combined with the Prony algorithm as EMD-Prony. The EMD decomposes a signal into finite Intrinsic Mode Functions (IMF) based on the number of modes in the measured signal. The number of IMFs is utilized by Prony to extend its model to higher order and so purifies the fundamental phasor estimates. In addition, EMD is also used as a pre-processor to filter the noise from the input signal of Prony. Finally, the proposed method can estimate phasors accurately under noisy and harmonic conditions.**

I. Introduction

Due to the lack of recommended specific algorithms to estimate phasors in IEEE Std. C37.118, phasor estimation has attracted a lot of attention recently. Fast and precise estimation is also necessary for accurate decision in power system control. In the context of phasor estimation, there are several initiatives that investigate algorithms for phasor estimation [1], [2]. An adaptive filter is suggested in [3] to estimate phasors. A new algorithm based on Fast Recursive GaussNewton (FRGN) is introduced in [4] to extract frequency and phasor simultaneously. To remove DC component, an improved Fourier Transform is applied in [5]. Moreover, an energy operator based on a shift angle is proposed in [6] to estimate amplitude during dynamic condition. The Prony algorithm is also a promising method that is proposed in [7], [8] for phasor estimation where estimates are calculated adaptively based on estimated frequency [9].

*This work was supported by ERCIM

[1]Jalal Khodaparast and Olav Bjarte Fosso are with the Department of Electric Power Engineering, Norwegian University of Science and Technology, 7491 Trondheim, Norway jalal.khodaparast@ntnu.no, olav.fosso@ntnu.no

[2]Marta Molinas is with the Department of Engineering Cybernetics, Norwegian University of Science and Technology, 7491 Trondheim, Norway marta.molinas@ntnu.no

The Prony algorithm approximates the main signal by exponentially damped sinusoidal signals. This algorithm is able to determine the values of frequency, damping factor, amplitude and phase of the main signal. By this algorithm, frequency and damping factor parameters are calculated in the first step and consequently amplitude and phase parameters are obtained in the second step. Despite the promising performance of the Prony algorithm in phasor estimation, the accuracy of phasor estimates is challenging under both noisy and harmonic conditions. To address the challenge of noise, multi-channel Prony is proposed in the literature [10]–[12]. Moreover, a robust Prony method to handle noise of measurement is proposed in [13] in which structural weighted least square is replaced with traditional least square. On the other hand, the order of Prony is also a challenging point [14], which is set experimentally lower than $N/2$, where N is sample number per fundamental cycle [15]–[17]. If the order of Prony is set to one ($L = 1$) in harmonic conditions, accuracy of phasor estimates will decrease due to interference of non-fundamental components. If the order of Prony is set more than one ($L > 1$) in non-harmonic conditions, the complexity of the calculations will increase. Therefore, accuracy and speed of the phasor estimation procedure by Prony relies on appropriate determination of order. In order to determine the order of Prony, Empirical Mode Decomposition (EMD) is suggested to be combined with Prony. Meanwhile, EMD can also be used as a denoising filter to increase the accuracy of Prony during noisy conditions.

EMD is a fundamental part of Hilbert Huang Transform (HHT) proposed by Huang [18]. It is designed to provide appropriate performance with nonlinear and non-stationary signals. In the first step of HHT, EMD decomposes data into a finite and limited number of components (Intrinsic Mode Functions (IMF)) that constructs an orthogonal basis for input signal. These IMFs actually represent oscillatory modes, which have equal number of zero crossing and extrema where their envelopes have zero mean value. The core of EMD is a sifting procedure that is an iterative algorithm with a defined stopping criteria [19]. The goal of the sifting procedure is to extract the highest frequency component from other frequencies. Finally, the sifting procedure is stopped when there is no more IMFs in residue. Thus, the decomposition of a nonlinear signal into L empirical modes is obtained by EMD [20].

II. First-Order Prony algorithm

Consider a complex version of the general sinusoidal quantity:

$$s(t) = \frac{1}{2}(p\ e^{j2\pi f_1 t} + p^* e^{-j2\pi f_1 t}) \tag{1}$$

where $p = ae^{j\phi}$ is the phasor of the signal while a and ϕ are the amplitude and the phase angle of $s(t)$. f_1 is the frequency of the signal. Assume that the signal $s(t)$ is sampled at N samples per cycle and can be expressed as:

$$\begin{pmatrix} s[0] \\ \vdots \\ s[n] \\ \vdots \\ s[N-1] \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ \vdots & \vdots \\ Z_1^n & Z_1^{-n} \\ \vdots & \vdots \\ Z_1^{(N-1)} & Z_1^{-(N-1)} \end{pmatrix} \begin{pmatrix} p \\ p^* \end{pmatrix} \tag{2}$$

$$\mathbf{S} = \mathbf{J}\ \mathbf{P}$$

where $Z_1 = e^{j2\pi f_1 T}$ and T is sampling period. The best estimate of the phasor (amplitude and phase) is obtained as:

$$\mathbf{P}_{estimated} = (\mathbf{J}^{tr}\mathbf{J})^{-1} J^{tr}\mathbf{S} \tag{3}$$

where tr is the transpose operator. According to equation (2), the phasor is estimated based on Z_1 that is extracted from the roots of the characteristic equation. This equation is in terms of the new parameters, a_1 and a_2, given by:

$$F(z) = (z - Z_1)(z - Z_1^*) = z^2 + a_1 z + a_2 \tag{4}$$

The characteristic equation coefficients are found from:

$$\begin{pmatrix} s[0] \\ s[1] \\ s[2] \\ \vdots \\ s[N-1] \end{pmatrix} = \begin{pmatrix} s[-1] & s[-2] \\ s[0] & s[-1] \\ s[1] & s[0] \\ \vdots & \vdots \\ s[N-2] & s[N-3] \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \tag{5}$$

$$\mathbf{S} = \mathbf{Q}\ \mathbf{A}$$
$$\mathbf{A} = (\mathbf{Q}^{tr}\mathbf{Q})^{-1}\mathbf{Q}^{tr}\mathbf{S} \tag{6}$$

Therefore, the coefficients of the linear prediction model are calculated based on equation (5). By inserting these coefficients in equation (4), roots of the polynomial can be extracted. The frequency and the rate of change of frequency are obtainable in this step. Finally, the amplitude and the phase are obtained by equation (2).

III. Frequency analysis of Prony

Frequency analysis is an appropriate tool to get insight into the performance of the Prony algorithm. To examine the performance of Prony for higher order (L ¿ 1), consider an input signal as:

$$s(t) = \sum_{k=1}^{10} e^{jk\omega_0} \tag{7}$$

where equation (7) is the sum of different frequency components. The Prony algorithm with $L = 1$ and $L = 5$ are applied to the test signal given in equation (7) and the results are shown in Fig.1. According to this figure, in the First-order Prony ($L = 1$), there is just one output. Therefore, all the frequencies of the input will appear in the output with $Amplitude = 1$. In this case, harmonic components may be critical for the accuracy of the estimated fundamental phasor if a low-order Prony is used. However, this problem can be solved using a higher order of Prony, for example a fifth-order ($L = 5$). The results when $L = 5$ are shown in Fig.1. According to this figure, there are five outputs in this extended Prony and every frequency will appear in a specific output. The interference between harmonics is prevented. To solve the challenge of choosing the order, this paper proposes the Empirical Mode Decomposition (EMD) as a tool for determining the order.

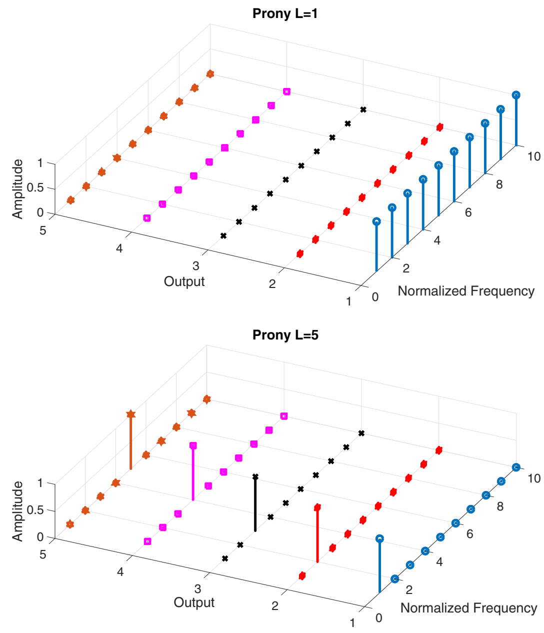

Fig. 1. Amplitude response of Prony with $L = 1$ and $L = 5$

IV. Empirical Mode Decomposition (EMD)

Empirical Mode decomposition (EMD) is a time domain algorithm for separating a non-linear and non-stationary signal into its individual components. The separation procedure is done by a sifting process that extracts the highest frequency component first. The sifting process serves two purposes: to eliminate riding waves, and to make the wave profiles more symmetric. According to the sifting process, extremes are extracted at the first step. Once the extremes are identified, all the local maxima are connected by a cubic spline line as the upper envelope (E_{max}). Repeat the procedure for the local minima to produce the lower envelope (E_{min}). The mean value (m) of upper and lower envelopes is calculated and extracted from the main signal. This process is repeated as many times as required to make the extracted signal to satisfy the criteria of an Intrinsic Mode Function (IMF) [21]. Based on the definition of IMF, it has only one extreme value between two zero crossings and has a zero mean value. Finally, the sifting procedure is stopped when there is no more IMFs in the residue. A successful termination criteria is proposed in [20]. This criteria guaranties globally small fluctuations in the mean and prevents locally large excursions. This stopping index is implemented by definition of two new parameters as:

$$a(t) = \frac{E_{max} - E_{min}}{2} \qquad \sigma(t) = |\frac{m(t)}{a(t)}| \tag{8}$$

Sifting procedure is stopped when:

$$\begin{aligned} \sigma(t) < \theta_1 \quad & during\ (1-\alpha)\ of\ total\ duration \\ \sigma(t) < \theta_2 \quad & during\ (\alpha)\ of\ total\ duration \end{aligned} \tag{9}$$

where typical values are: $\alpha = 0.05$, $\theta_1 = 0.05$ and $\theta_2 = 0.5$. After the EMD procedure, the given signal, $s(t)$, can be written as a summation of the individual IMFs and the residue as:

$$s(t) = R + \sum_{k=1}^{L} IMF_k \tag{10}$$

where R is the residue and L is the number of IMFs. The number of IMFs is used in this paper for extending the Prony algorithm to higher orders. Additionally, the EMD can be used as a denoising block before applying the Prony algorithm. In order to denoise the corrupted signal, EMD is applied and then every sifted IMF is evaluated by Hurst index. Hurst is an statistical measure to distinguish non-random from random signal [22]. It is a measure of tendency of time series to regress to a long term equilibrium. There are different methods to estimate the Hurst index like as Rescaled Range Analysis, Average Wavelet Coefficient, Detrended Fluctuation Analysis and Geweke-Porter-Hudak spectral estimator. In this paper, Hurst index (HI) is estimated by Rescaled Range Analysis. To do this, the given IMF is split into M shorter time spans and a rescaled range is obtained for each of them. Consider the first IMF entitled IMF_1 and different time spans as: IMF_{11},IMF_{12},IMF_{13},...,,IMF_{1m},...,IMF_{1M}. For each span, the cumulative series is calculated as:

$$C_m = \sum_{N_m} (IMF_{1m} - mean(IMF_{1m})) \tag{11}$$

where N_m is the number of samples in every time span. Thereafter, the difference between the maximum and the minimum value of C_m is obtained as the range value (R_m). Rescaled range is calculated by dividing R_m over its standard deviation as:

$$RR_m = R_m / \sqrt{\frac{1}{N_m} \sum_{N_m} (IMF_{1m} - mean(IMF_{1m}))^2} \tag{12}$$

Next, the average of the rescaled range (RR_m) over all time spans is calculated. By plotting the logarithm of the average rescaled range and the logarithm of length of time span, its slope will be Hurst index (HI). It is worthy to note that this procedure is done for each IMF

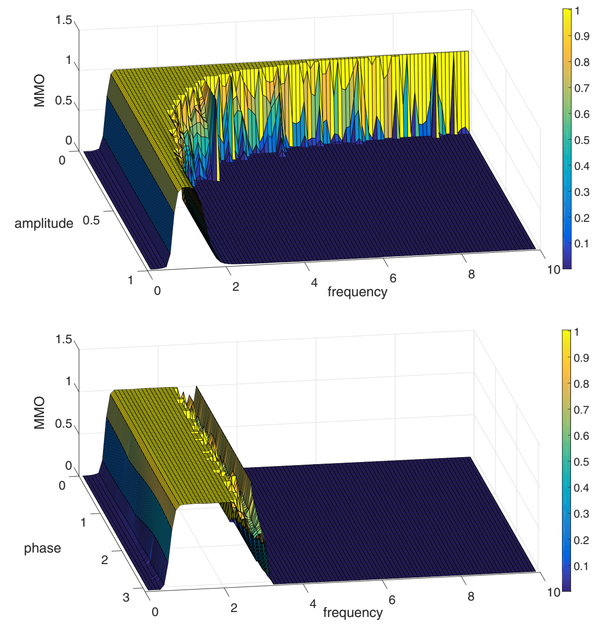

Fig. 2. Mode Mixing Occurrence (MMO)

(IMF_1,IMF_2,IMF_3,...,,IMF_k,...,IMF_L) and the corresponding HI is compared by threshold value (0.5). IMF with $HI > 0.5$ is not considered for reconstruction (because it is uncorrelated) of signal and the denoised signal is obtained by adding the remaining IMFs. Finally, the purified signal is sent to Prony for further analysis. There are still two challenges regarding application of EMD with Prony. These are explained and solutions proposed in next subsections.

A. *Mode mixing in EMD*

Mode mixing may be a challenge in the EMD procedure. When a non-fundamental component has a frequency closely spaced to the fundamental frequency, the EMD may be unable to decompose the components to individual IMFs. In this section, the EMD performance of a signal with two components (fundamental component plus non-fundamental component) is examined when the frequency and the amplitude of the non-fundamental component are varied over a specified range. Consider a general signal as:

$$\begin{gathered} s(t) = s_1(t) + s_2(t) \\ s_1(t) = cos(2\pi t) \qquad s_2(t) = a\ cos(2\pi f t + \phi) \\ a = [0-1]; f = [0-10]; \phi = [0-2\pi] \end{gathered} \tag{13}$$

where the fundamental component has constant amplitude and frequency both equal to one while the frequency and amplitude of non-fundamental component changes in range of [0-10] and [0-1], respectively. In order to quantify the mode mixing phenomenon in harmonic condition, the following index named as Mode Mixing Occurrence (MMO), proposed in [23], is used in this section as:

$$MMO = \frac{|mean(IMF_1 - s_1)|}{|mean(s_2)|} \qquad f \leq 1$$
$$MMO = \frac{|mean(IMF_1 - s_2)|}{|mean(s_1)|} \qquad f > 1 \qquad (14)$$

where IMF_1 is the first extracted IMF from the EMD procedure. The presented signal in equation (13) is applied to the EMD. The results after using the index of equation (14) are shown in Fig.2 (the top sub-figure shows the impact of amplitude (a) and the bottom sub-figure shows the impact of phase (ϕ) variation in the MMO value). According to equation (14) and Fig.3, there are some mode mixing in the vicinity of the fundamental frequency ($f = 1$) (when the frequency of non-fundamental component is near the fundamental component) and when the amplitude of the non-fundamental component is much smaller than the fundamental component. To solve this problem in the proposed method, the EMD is replaced with Ensemble Empirical Mode Decomposition (EEMD) [24]. EEMD is a kind of noise assisted data analysis methods in which white noise is added to the targeted data. The decomposition procedure is done with different noisy data and finally mean (ensemble) of the corresponding IMFs provides the final result. Although EEMD may result in smaller Signal to Noise ratio (SNR), the low SNR does not have impact on decomposition procedure but prevents the mode mixing problem in traditional EMD. However, the computational burden of EEMD is significantly higher than EMD. This is the motivation of proposing a method for detecting mode mixing and then replace the EMD with EEMD in the proposed method.

B. Detection of Mode mixing

To activate the EEMD instead of EMD in the proposed method, it is essential to first detect the mode mixing issue. Since $s_1(t)$ and $s_2(t)$ are unavailable in a real power system (only $s(t)$ is available), the MMO index in equation (14) can not be used. Therefore, a new index based on the accuracy of the phasor estimation is used [25] as:

$$PEE(t) = \sum_{T} |s(t) - \hat{s}(t)| \qquad (15)$$

where T is the fundamental period, $s(t)$ is the main signal and $\hat{s}(t)$ is the recomputed signal from the estimated phasor from the Prony algorithm. To show the performance of this index, a test signal given in equation (16) is considered. Here there are two distant frequencies (50 and 300 Hz) in the main signal before $t = 0.15$ and two closely spaced frequencies (50 and 90 Hz) after $t = 0.15$. This signal and its two frequency components are shown in Fig.3 (top and middle sub-figure).

$$s(t) = 1cos(2\pi 50t) + 0.2cos(2\pi 300t) \qquad f \leq 0.15$$
$$s(t) = 1cos(2\pi 50t) + 0.2cos(2\pi 90t) \qquad f > 0.15 \qquad (16)$$

By applying this signal to the proposed method (EMD-Prony), the index PEE is calculated. The EMD can decompose the main signal into two distinct frequency components

TABLE I

ESTIMATION ERROR BY PRONY AND EMD-PRONY IN NOISY CONDITION

Method Method	Error of estimated amplitude	Error of estimated phase
Prony	0.2851	0.2601
EMD-Prony	0.0171	0.0157

accurately with low phasor estimation error. Therefore, it can be concluded that there is no mode mixing in the signal before $t = 0.15$. However, there is mode mixing in the EMD after $t = 0.15$ detected by high value of the PEE. Therefore,mode mixing phenomenon can be detected by comparing PEE with a threshold value (TR). By detection of the mode mixing problem in EMD, EEMD is replaced in the proposed method.

V. SIMULATION RESULTS

To illustrate the abilities of the proposed method, two subsections are provided here: A) denoising ability of EMD and B) determination of Prony's model order using EMD.

A. Denoising using EMD

In this section, the ability of the EMD to purify the signal from noise is examined. Firstly, the EMD is applied to the signal and then noise is detected by comparing Hurst Index (HI) of the IMFs with threshold (0.5). The IMFs with Hurst Index smaller than 0.5 are removed from signal and finally the purified signal is extracted. Consider the test case as:

$$s(t) = a\ cos(2\pi f_0 t + \phi) + w(t) \qquad (17)$$
$$a = 1, \phi = 0.5, f_0 = 50, N = 150, \sigma^2 = 10^{-3}$$

where $w(t)$ is white noise with the variance σ^2. The EMD is applied to the signal and the simulation result (main signal and its IMFs) are shown in Fig.3. Noisy signal is decomposed into five IMFs with hurst index $HI_1 = 0.3576$, $HI_2 = 0.2743$, $HI_3 = 0.2022$, $HI_4 = 0.3010$ and $HI_5 = 0.7347$, respectively. These values show that the first four IMFs are created as a consequence of noise but the last IMF is a fundamental components. Therefore, the first four IMFs are removed from the signal, and the rest is applied by Prony for phasor estimation. Estimated amplitudes by Prony and EMD-Prony are shown in Fig.3 as well. According to this figure, EMD-Prony shows better performance compared to Prony under noisy conditions. To quantify the performance of these two methods, estimation error of the amplitude and the phase are tabulated in Table I and these results demonstrate that EMD-Prony is more accurate than traditional Prony under noisy conditions.

B. Determination of Prony's model order by EMD under harmonic conditions

In this section, the ability of the EMD to determine the order of Prony, is examined. The proposed method is examined using synthesized signals programmed in MATLAB. Two test cases are considered in this section to examine the performance of both the EMD (test case 1) and the EEMD

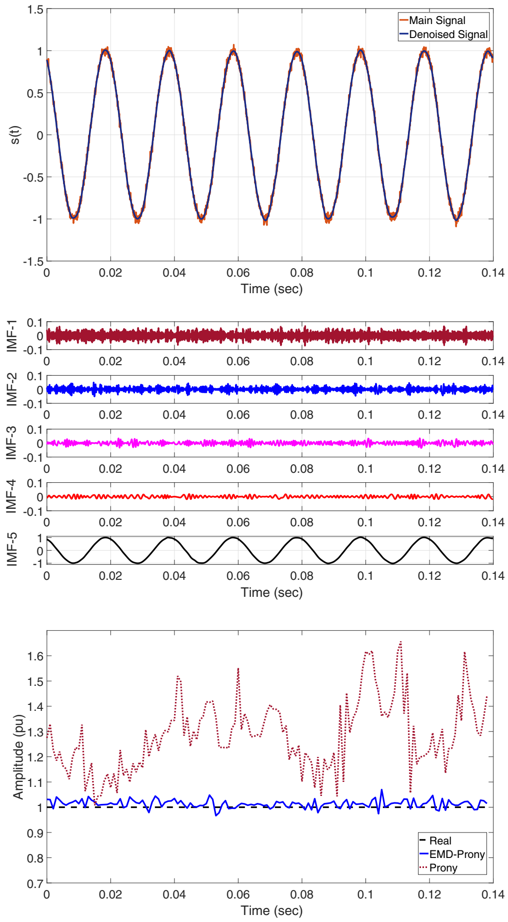

Fig. 3. Main signal, Denoised signal and the different IMFs, Estimated amplitude

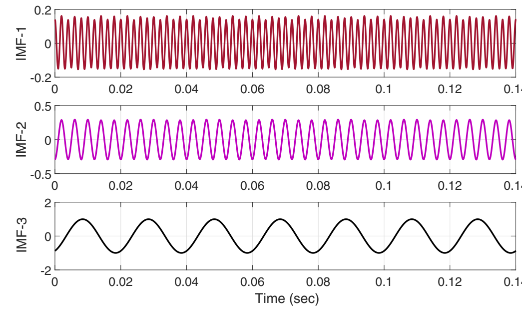

Fig. 4. Extracted IMF by EMD

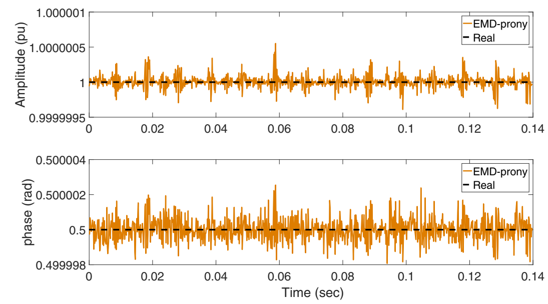

Fig. 5. Amplitude and phase estimates by EMD-Prony (L=3)

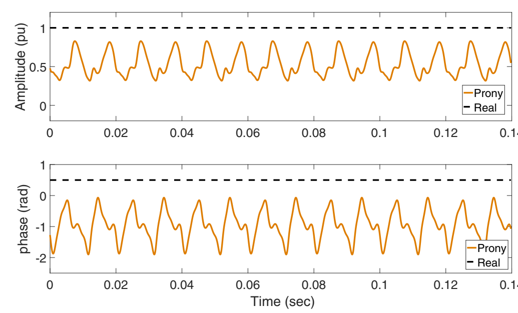

Fig. 6. Amplitude and phase estimates by traditional Prony (L=1)

(test case 2).

$Test\ Case\ 1$: Consider the test case as:

$$s(t) = 1\ cos(2\pi f_0 t + 0.5)$$
$$+0.3\ cos(2\pi(5f_0)t) + 0.15\ cos(2\pi(8f_0)t) \tag{18}$$

where $s(t)$ presents a summation of one fundamental component and two harmonic components, which have frequencies far from each others. EMD is applied to this main signal and the extracted IMFs are shown in Fig.4. According to this figure, the EMD detects that there are three modes in main signal. Therefore, a third-order Prony is formed for phasor estimation. Phasor estimation by EMD-Prony ($L = 3$) and traditional Prony ($L = 1$) are shown in Fig.5 and Fig.6, respectively. According to Fig.6, harmonic components destroyed the accuracy of estimated amplitude and phase because the order of Prony is not set according to the number of present harmonics. However, EMD-Prony overcomes this difficulty by forming a third order model of Prony. With this choice, the amplitude and phase estimations are accurate as shown in Fig.5. In order to compare the performance of traditional Prony and the proposed method, the estimation error of the amplitude and the phase under this condition (test case1) are tabulated in Table II and these results demonstrate that EMD-Prony is more accurate than traditional Prony under harmonic conditions.

$Test\ Case\ 2$: Consider the test case as:

$$s(t) = 1\ cos(2\pi f_0 t + 0.5)$$
$$+0.3\ cos(2\pi(5f_0)t) + 0.15\ cos(2\pi(5.6f_0)t) \tag{19}$$

where $s(t)$ presents a summation of one fundamental component and two harmonic components, which have spectral proximity (250 and 280 Hz). Due to this spectral proximity, EEMD is applied in this case. The EEMD decomposes the main signal into six IMFs, of which the first three IMFs extract the noise, according to the Hurst Index (Table III). The last three IMFs represents the modes since their Hurst Indices are higher than 0.5 (Table III). According to Table III, the order of Prony is fixed to three by the number of IMFs representing the independent modes after the EEMD. The estimation error of the amplitude and the phase under this condition (test case 2) are tabulated in Table IV. The

TABLE II

ESTIMATION ERROR BY PRONY AND EMD-PRONY IN HARMONIC CONDITION (TEST CASE1)

Method	Error of estimated amplitude	Error of estimated phase
Prony	0.4636	1.4553
EMD-Prony	5×10^{-8}	4.8×10^{-7}

TABLE III

EXTRACTED IMF BY EEMD AND THEIR HI IN HARMONIC CONDITION (TEST CASE 2)

IMF	Hurst Index (HI)
IMF_1	0.1739
IMF_2	0.2993
IMF_3	0.3450
IMF_4	**0.5721**
IMF_5	**0.6468**
IMF_6	**0.7927**

results show that the proposed method is more accurate than the traditional Prony.

VI. CONCLUSION

An EMD assisted Prony algorithm is proposed in this paper to estimate phasors (amplitude and phase) under noisy and harmonic conditions. The EMD has two purposes: firstly it denoises the signal and secondly it determines the order of the Prony algorithm. Noise and non-fundamental components decrease the performance and accuracy of Prony. Based on the number of independent IMFs, the traditional Prony is extended to higher order Prony to provide separate output channel for all harmonics. Therefore, it avoids the interference of different harmonics in the output of the fundamental harmonic component. It is demonstrated that EMD is a promising method for denoising the signal and determination of Prony's order to increase the accuracy of Prony algorithm under noisy and harmonic conditions.

REFERENCES

[1] A. G. Phadke and J. S. Thorp, *Synchronized phasor measurements and their applications*. Springer, 2008, vol. 1.

[2] J. Khodaparast and M. Khederzadeh, "Least square and kalman based methods for dynamic phasor estimation: a review," *Protection and Control of Modern Power Systems*, vol. 2, no. 1, p. 1, 2017.

[3] I. Kamwa, A. K. Pradhan, and G. Joos, "Adaptive phasor and frequency-tracking schemes for wide-area protection and control," *IEEE Transactions on Power Delivery*, vol. 26, no. 2, pp. 744–753, 2011.

[4] P. Dash, K. Krishnanand, and R. Patnaik, "Dynamic phasor and frequency estimation of time-varying power system signals," *International Journal of Electrical Power & Energy Systems*, vol. 44, no. 1, pp. 971–980, 2013.

TABLE IV

ESTIMATION ERROR BY PRONY AND EMD-PRONY IN HARMONIC CONDITION (TEST CASE 2)

Method	Error of estimated amplitude	Error of estimated phase
Prony	0.5845	1.2230
EMD-Prony	3.4×10^{-7}	3.02×10^{-6}

[5] S.-H. Kang, D.-G. Lee, S.-R. Nam, P. A. Crossley, and Y.-C. Kang, "Fourier transform-based modified phasor estimation method immune to the effect of the dc offsets," *IEEE Transactions on Power Delivery*, vol. 24, no. 3, pp. 1104–1111, 2009.

[6] X. Jin, F. Wang, and Z. Wang, "A dynamic phasor estimation algorithm based on angle-shifted energy operator," *Science China Technological Sciences*, vol. 56, no. 6, pp. 1322–1329, 2013.

[7] J. A. de la O Serna, "Synchrophasor estimation using prony's method," *IEEE Transactions on Instrumentation and Measurement*, vol. 62, no. 8, pp. 2119–2128, 2013.

[8] J. Khodaparast and M. Khederzadeh, "Dynamic synchrophasor estimation by taylor–prony method in harmonic and non-harmonic conditions," *IET Generation, Transmission & Distribution*, 2017.

[9] J. F. Hauer, C. Demeure, and L. Scharf, "Initial results in prony analysis of power system response signals," *IEEE Transactions on power systems*, vol. 5, no. 1, pp. 80–89, 1990.

[10] L. Fan, "Data fusion-based distributed prony analysis," *Electric Power Systems Research*, vol. 143, pp. 634–642, 2017.

[11] S. Nabavi, J. Zhang, and A. Chakrabortty, "Distributed optimization algorithms for wide-area oscillation monitoring in power systems using interregional pmu-pdc architectures," *IEEE Transactions on Smart Grid*, vol. 6, no. 5, pp. 2529–2538, 2015.

[12] J. Khazaei, L. Fan, W. Jiang, and D. Manjure, "Distributed prony analysis for real-world pmu data," *Electric Power Systems Research*, vol. 133, pp. 113–120, 2016.

[13] J. Zhao and G. Zhang, "A robust prony method against synchrophasor measurement noise and outliers," *IEEE Transactions on Power Systems*, vol. 32, no. 3, pp. 2484–2486, 2017.

[14] X. Xia, C. Li, and W. Ni, "Dominant low-frequency oscillation modes tracking and parameter optimisation of electrical power system using modified prony method," *IET Generation, Transmission & Distribution*, vol. 11, no. 17, pp. 4358–4364, 2017.

[15] N. Zhou, Z. Huang, F. Tuffner, J. Pierre, and S. Jin, "Automatic implementation of prony analysis for electromechanical mode identification from phasor measurements," in *Power and Energy Society General Meeting, 2010 IEEE*. IEEE, 2010, pp. 1–8.

[16] F. Costa, L. de Almeida, F. Wegelin, and E. da Costa, "Recursive prony's method for improving the monitoring of electrical machines," in *Instrumentation and Measurement Technology Conference, 2005. IMTC 2005. Proceedings of the IEEE*, vol. 2. IEEE, 2005, pp. 1498–1502.

[17] D. J. Trudnowski, J. Johnson, and J. F. Hauer, "Making prony analysis more accurate using multiple signals," *IEEE Transactions on power systems*, vol. 14, no. 1, pp. 226–231, 1999.

[18] N. E. Huang, Z. Shen, S. R. Long, M. C. Wu, H. H. Shih, Q. Zheng, N.-C. Yen, C. C. Tung, and H. H. Liu, "The empirical mode decomposition and the hilbert spectrum for nonlinear and non-stationary time series analysis," in *Proceedings of the Royal Society of London A: mathematical, physical and engineering sciences*, vol. 454, no. 1971. The Royal Society, 1998, pp. 903–995.

[19] R. Fontugne, P. Borgnat, and P. Flandrin, "Online empirical mode decomposition," in *Acoustics, Speech and Signal Processing (ICASSP), 2017 IEEE International Conference on*. IEEE, 2017, pp. 4306–4310.

[20] G. Rilling, P. Flandrin, P. Goncalves *et al.*, "On empirical mode decomposition and its algorithms," in *IEEE-EURASIP workshop on nonlinear signal and image processing*, vol. 3. NSIP-03, Grado (I), 2003, pp. 8–11.

[21] N. E. Huang and Z. Wu, "A review on hilbert-huang transform: Method and its applications to geophysical studies," *Reviews of geophysics*, vol. 46, no. 2, 2008.

[22] H. E. Hurst, "Long term storage capacity of reservoirs," *ASCE Transactions*, vol. 116, no. 776, pp. 770–808, 1951.

[23] G. Rilling and P. Flandrin, "One or two frequencies? the empirical mode decomposition answers," *IEEE transactions on signal processing*, vol. 56, no. 1, pp. 85–95, 2008.

[24] Z. Wu and N. E. Huang, "Ensemble empirical mode decomposition: a noise-assisted data analysis method," *Advances in adaptive data analysis*, vol. 1, no. 01, pp. 1–41, 2009.

[25] J. Khodaparast and M. Khederzadeh, "Three-phase fault detection during power swing by transient monitor," *IEEE Transactions on Power Systems*, vol. 30, no. 5, pp. 2558–2565, 2015.