

Time-frequency analysis of acceleration data from ship-ice interaction events

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Abstract

During operations with ships in ice-infested waters, vibrations of the hull occur due to ship-ice interaction. The acting ice load and the induced vibrations depend on the ice regime and ice breaking mechanism. Especially crushing of ice against the hull causes high loads against the vessel. The objective of this study is to analyse the frequency components of ice-induced vibrations, measured with accelerometers placed in the bow section of the ship's hull. The Wigner-Ville distribution, which provides a time-frequency representation, is applied on acceleration data collected on the icebreaker Frej during transit in ice-infested Arctic waters. The resulting time-frequency representations show that the excited frequencies depend on the dominant ice breaking mechanism, on the encounter velocity, and on the position of where the ice interaction against the hull occurs. Furthermore, the natural frequencies of the ship's hull change slightly, depending on the conditions on the sea-ice around the vessel. It is concluded that a system of distributed accelerometers on a ship can provide information about the acting ice breaking mechanism, the ice conditions around the vessel, and about the location along the hull, where the ship-ice interaction occurs. This can be used as an additional tool in monitoring systems during operations in ice-infested waters.

Keywords: Arctic, Ship-ice interaction, Acceleration, Wigner-Ville distribution, Time-frequency analysis, Vibrations, Ice load, Marine technology,

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1 1. Introduction

2 The presence of sea-ice makes operations in the Arctic challenging (Løset
3 et al., 2006). The decay of ice cover in the Arctic region (Lubin & Massom,
4 2006) will lead to new sea-cargo routes between Europe and Asia (Arctic Coun-
5 cil, 2009). Furthermore, hydrocarbon deposits lie possibly under the seabed of
6 the Arctic oceans (Hossain et al., 2014). Technologies have to be developed to
7 support safe and sustainable operations in the fragile environment of the Arctic.
8 Ice management refers to a system that is capable of reducing the ice loads on
9 a protected vessels to endurable levels (Hamilton et al., 2011). An ice manage-
10 ment system can be parted into an ice observer system and a response system
11 (Eik, 2008). The ice observer system requires ice intelligence, which is obtained
12 by collecting and processing information about the ice environment utilising
13 different sensor technologies (Haugen et al., 2011). When a ship operates in ice-
14 infested waters, it is subjected to ice-induced vibrations (Belov & Spiridonov,
15 2012). The properties of the ice-induced vibration change, depending on the ice
16 interaction mechanism and the ice load affecting the vessel (Yue et al., 2009).
17 In laboratory experiments, Sodhi (1991) showed that the ice-induced vibrations
18 clearly depend on the failure mechanism of the ice. Measuring and evaluating
19 the ice-induced vibrations in real-time could therefore be a reliable source of ice
20 intelligence.

21 By measuring vibrations on the Finnish icebreaker Sisu during ice breaking op-
22 erations, Matusiak (1982) showed that the additional ice load excites several
23 natural modes of the ship's hull in the bandwidth 2.75 Hz to 7.5 Hz. For a nar-
24 row, conical structure, equipped with load panels, Yue et al. (2007) showed that
25 the frequency spectrum of the measured ice loads change under different ice con-
26 ditions. They used short-time fourier transformation (STFT) for their analysis.
27 The major drawback of the STFT is that the product of time resolution and
28 frequency resolution is constant. It is desired to have a higher frequency resolu-

29 tion at the lower frequency band, in which the ice-induced vibrations typically
30 occur. Bjerckås (2006) overcame this limitation by using a wavelet transforma-
31 tion. Bjerckås et al. (2007) applied a continuous wavelet transformation on ice
32 load signals recorded during ice-structure interaction of a lighthouse in the Gulf
33 of Bothnia. The authors were able to identify intervals of intermittent crushing
34 of ice against the structure, which was the cause of high ice loads and significant
35 vibrations in the structure.

36 The results from the wavelet transformation can be difficult to interpret, be-
37 cause the resulting spectrum depends on the chosen wavelet function. It is more
38 intuitive to study directly the energy representation of the recorded vibrations
39 and to determine the distribution of the energy content of the vibrations over
40 both frequency and time (Flandrin et al., 2002). The Wigner-Ville distribution
41 is such a joint time-frequency energy density (Boashash, 2003). The vibrations
42 can be recorded with accelerometers placed inside the hull of a ship. Compared
43 to strain gauges or load panels, accelerometers are significantly easier to install
44 and they can be placed anywhere in the ship at little cost and effort.

45 This study shows that data from accelerometers placed in the hull of a ship,
46 in combination with the Wigner-Ville distribution, provides clear time and fre-
47 quency information about ice-induced vibrations. The aim is to provide an
48 additional tool for ice condition assessment. Typical ship-ice interaction events
49 as cause for ice-induced vibrations are introduced. A windowed version of the
50 Wigner-Ville distribution, which allows real-time analysis of ice-induced vibra-
51 tions, is derived. The data from a system of four accelerometers on the ice-
52 breaker Frej is used for frequency analysis by the proposed method. In five
53 scenarios, the frequency patterns during ship-ice interaction are analysed and
54 discussed. Because the icebreaker Frej is a sistership of the Finnish icebreaker
55 Sisu, a comparison with the results of Matusiak (1982) is conducted. It is shown
56 that the variations of the natural frequencies of the ship's hull can be tracked
57 in order to assess the ice conditions.

58 The paper is structured as follows: Section II contains a brief overview of the
59 relevant theory on ship-ice interaction and ice-induced vibrations. Section III

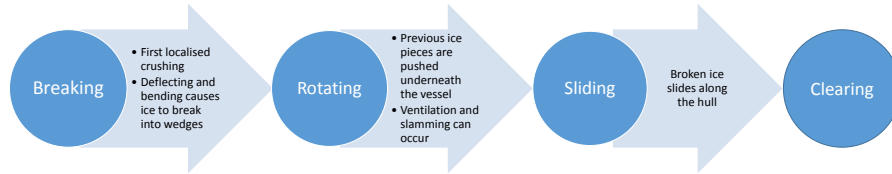


Figure 1: The ice-breaking process under conventional ice conditions.

60 presents the methodology of the time-frequency decomposition based on the
 61 Wigner-Ville distribution. Section IV describes the test setup on the Swedish
 62 Atle-class icebreaker Frej and a method to estimate the impact load during the
 63 encounter of sea-ice. In Section V the recorded acceleration data and the re-
 64 sult from the time-frequency analysis are presented and discussed. Section VI
 65 provides a closer analysis of the excitation of natural modes of the ship’s hull.
 66 Section VII concludes the research results and gives an outlook on future re-
 67 search.

68 2. Ship-ice interaction

69 The effects of ship-ice interaction depend on properties of the ship and prop-
 70 erties of the ice. When a ship interacts with ice, either by ice drifting towards
 71 it or the ship travelling through ice, the ice eventually breaks. The additional
 72 loads a ship experiences during the ice-breaking process depend on the failure
 73 mode of the ice (ISO/FDIS/19906:2010). During ship-ice interaction, three fail-
 74 ure modes play a significant role: Failure due to bending of the ice, failure of
 75 the ice by splitting, and failure of the ice by crushing. The active failure mode
 76 depends on the shape of the structure (e.g., inclination angle) and several state
 77 variables and material properties of the ice (Lu et al., 2015a). State variables
 78 are the encounter velocity, the contact area, and the temperature. Some ice
 79 properties are the density/porosity of the ice, the ice thickness, and the salinity.

80 *2.1. Ice loads due to bending failure*

81 Due to the gravitational force, the structure or vessel pushes the ice down-
82 wards. A vertical force component builds up, and once it exceeds the breaking
83 strength of the ice, the ice fails by radial and circumferential cracking. Su et al.
84 (2010) mentions that, due to the periodic characteristic of the process, domi-
85 nant frequencies in the load by bending failure are identifiable. Yue et al. (2009)
86 identified such regular force patterns for the interaction of ice and a slender ver-
87 tical monopod structures. Bending failure is the dominant failure mechanism
88 for sloping structures (Løset et al., 2006, Ch. 5), such as in the bow of an
89 icebreaker.

90 *2.2. Ice loads due to splitting failure*

91 Besides bending failure, splitting as failure mode can occur especially in
92 level-ice. As in the case of bending failure, a vertical force component builds up
93 and leads to radial and circumferential cracking of the ice. In-plane forces act
94 on the ice floe and on the newly formed crack. Lu et al. (2015b) refers to them
95 as *splitting loads*. They cause an opening of the crack and a propagation of the
96 crack in the ice floe. The propagated crack can extend to a length longer than
97 the structure causing it (Bhat et al., 1991). When the ship propagates into the
98 crack, and thus widens the crack, ice can slide along the hull, which typically
99 cause measurable vibrations.

100 *2.3. Ice loads due to crushing failure*

101 Especially at small contact areas, non-sloped contact areas, and high en-
102 counter speeds, or with increasing ice thickness, crushing failure becomes the
103 dominant failure mode during ship-ice interaction. The contact area of the ice
104 with the ship or structure is changing randomly, which causes a non-uniform
105 pressure distribution (Løset et al., 2006, Ch. 5) and the production of pulver-
106 ized material. Due to the random size of the contact area, the load does not
107 show regular frequency patterns. Instead a rather chaotic pattern is observed
108 (Yue et al., 2009), (Masterson et al., 1999).

109 *2.4. The ice-breaking process*

110 According to Riska (2011), Lubbad & Løset (2011), and Su et al. (2010), the
 111 ice-breaking occurs generally in four steps, as shown in Figure 1: When the ship
 112 encounters an ice floe, the ship’s hull and the ice form a small contact area. Due
 113 to the small contact area ice crushing will be the dominating failure mode. The
 114 ship will proceed further into the ice, which increases the contact area between
 115 the hull and the ice. A vertical force component on the ice increases until the
 116 bending strength of the ice is reached and ice floes are created by the formation
 117 of a bending crack, as described by Riska (2011). The ice pieces start to rotate
 118 and to push against the hull. This can cause an additional slamming load against
 119 the hull of the vessel (Lubbad & Løset, 2011) and an additional hydrostatic load
 120 can occur due to ventilation. This will occur if the gaps between the rotating
 121 ice floe and the water surface do not fill up with sea water fast enough. After
 122 the rotation process, the ice floe slides along the hull of the vessel until it clears
 123 to the sides or gets milled in the propellers of the vessel. If the ice is too strong,
 124 for example in the case of multi-year ice, the necessary vertical force component
 125 will not exceed the strength of the ice. Instead of bending, the ice will then
 126 continue to fail by crushing.

127 *2.5. Ice-induced vibrations*

Matusiak (1982) showed that ship vibrations can be regarded as the motions
 of a forced, damped multi-degree-of-freedom (MDOF) system, given as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{G}\mathbf{x} = \boldsymbol{\tau}(t), \quad (1)$$

with $\mathbf{M} \in \mathbb{R}^{n \times n}$ is the mass matrix, $\mathbf{C} \in \mathbb{R}^{n \times n}$ is the damping matrix, $\mathbf{G} \in \mathbb{R}^{n \times n}$ is the stiffness matrix, and $\mathbf{x} \in \mathbb{R}^n$ is the displacement. Upon an external excitation, such as the impact of sea-ice, the hull shows undamped free-vibrations natural modes, denoted ϕ_i , congregated in the modal matrix $\boldsymbol{\Phi} = [\phi_1, \phi_2, \dots, \phi_n]$. The natural modes of the vessel’s hull can be found by solving (2) under a known load:

$$\ddot{\boldsymbol{\eta}}(t) + \mathbf{C}_g \dot{\boldsymbol{\eta}}(t) + \text{diag}(\omega_i^2) \boldsymbol{\eta}(t) = \text{diag}(M_i^{-1}) \mathbf{f}(t), \quad (2)$$

Table 1: Generalised parameters of the icebreaker Sisu, a sister ship of the icebreaker Frej Matusiak (1982).

Mode	Condition (Ice Thickness)	Natural frequency ω_i	Generalised damping ξ_i	Generalised mass M_i ($kg \cdot m^2$)
1	Open water	2.916 Hz	0.008	0.1180
	30 cm	2.769 Hz	0.053	0.1210
	50 cm	2.703 Hz	0.095	0.1220
2	Open water	5.513 Hz	0.012	0.0201
	30 cm	5.150 Hz	0.054	0.0208
	50 cm	5.078 Hz	0.112	0.0210
3	Open water	7.550 Hz	0.011	0.0047
	30 cm	7.067 Hz	0.046	0.0050
	50 cm	6.594 Hz	0.084	0.0052
4	Open water	9.000 Hz	0.011	0.0036
	30 cm	8.700 Hz	0.044	0.0037
	50 cm	8.312 Hz	0.076	0.0038

where $\mathbf{C}_g = \text{diag}(2\xi_i\omega_i) \in \mathbb{R}^{n \times n}$ is the generalised damping matrix, M_i is the i^{th} generalised modal mass, ξ_i is the damping factor for the i^{th} natural mode, $\mathbf{f}(t) = \Phi^T \boldsymbol{\tau}(t)$ is the generalised force, and $\boldsymbol{\eta} \in \mathbb{R}^n$ is the generalised coordinate vector. Matusiak (1982) showed further that a change in generalised mass can be calculated under the assumption of a velocity and ice thickness independent ship stiffness by

$$M_{i,1} = M_{i,0} \cdot \sqrt{\frac{f_{i,0}}{f_{i,1}}}. \quad (3)$$

128 The generalised parameters have been identified by Matusiak (1982) for the
129 icebreaker Sisu, a sister ship of the icebreaker Frej. They are summarised in
130 Table 1.

131 **3. Time-frequency analysis of acceleration signals**

132 Changes in the ice failure regime cause changes in the frequency characteris-
 133 tic of the induced vibrations, and ice failure against the hull can also excite the
 134 natural modes of the ship’s hull. Following the definition of (Boashash, 2003)
 135 we require the following results from a time-frequency decomposition:

- 136 • Identification of time variations in the signal in order to detect ice inter-
 137 action events.
- 138 • Identification of frequency variations in the signal in order to characterize
 139 the ice failure regime and track the excitation of natural frequencies.

140 *3.1. Definition of a signal and its spectrum*

141 The *spectrum* $S(f)$ of a signal $s(t)$ is given by the *Fourier transformation*

$$\begin{aligned} S(f) &= \mathcal{F}\{s(t)\} \\ &= \int_{-\infty}^{\infty} s(t)e^{-j2\pi ft} dt. \end{aligned} \tag{4}$$

A real signal $s(t)$ exhibits *Hermitian symmetry* between the positive-frequency and negative-frequency components of its spectrum (Ohm & Lüke, 2010, Ch. 3).

$$S(-f) = S^*(f) \tag{5}$$

142 A *stationary signal* $s(t)$ can be written as the sum of cosine-terms and an
 143 offset a_0 (Hoffmann & Wolff, 2014, pp. 102-104), as shown in (6).

$$s(t) = a_0 + \sum_{k \in \mathbb{N}} c_k \cos(n\omega_0 t + \varphi_k) \tag{6}$$

144 Wave elevations of a long-crested irregular sea, for example, can be described
 145 by the sum of harmonic components (Fossen, 2011, Ch. 8.2.3), implying the
 146 assumption of stationarity holds. Ice loads, however, are irregular and highly
 147 fluctuating. They are of *non-stationary* character (Bjerkås et al., 2007).

The idea of Bjerkås (2006) to use a varying time-frequency decomposition in order to identify different ice load situations will be employed further. But instead of looking at the measured signal directly, an energy definition of the signal will be used. An energy distribution approach is directly applicable to non-stationary and random signals (Matz & Hlawatsch, 2003) and intuitive to interpret. The energy of a signal $s(t)$ in the time-domain is

$$E_s = \int_{-\infty}^{\infty} |s(t)|^2 dt, \quad (7)$$

The energy of a signal in the frequency-domain is equivalently

$$E_s = \int_{-\infty}^{\infty} |S(f)|^2 df, \quad (8)$$

where the term $|S(f)|^2$ is the *energy spectral density*. A joint time-frequency energy density $\rho_s(t, f)$ is then implicitly defined by

$$E_s = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho_s(t, f) dt df. \quad (9)$$

Such a joint time-frequency energy density must satisfy the *marginal conditions* (Boashash, 2003), which state that if the time-frequency energy density is integrated along one of the variables, the result will be the energy density of the corresponding other variable, that is,

$$\int_{-\infty}^{\infty} \rho_s(t, f) dt = |S(f)|^2 \quad (10)$$

$$\int_{-\infty}^{\infty} \rho_s(t, f) df = |s(t)|^2. \quad (11)$$

A joint time-frequency energy density, that fulfils the marginal conditions (10) and (11), is the Wigner distribution. The kernel $K_s(t, \tau)$ of the distribution is based on the autocorrelation function of the signal, given by

$$K_s(t, \tau) = s\left(t + \frac{\tau}{2}\right) s^*\left(t - \frac{\tau}{2}\right). \quad (12)$$

The Fourier transformation of the kernel leads further to a time-frequency energy density, denoted $W_s(t, f)$, defined as

$$\begin{aligned} \rho_s(t, f) &= \mathcal{F}\{K_s(t, \tau)\} \\ &= \int_{-\infty}^{\infty} s\left(t + \frac{\tau}{2}\right) s^*\left(t - \frac{\tau}{2}\right) e^{-j2\pi f\tau} d\tau \end{aligned} \quad (13)$$

$$= : W_s(t, f). \quad (14)$$

150 As a quadratic time-frequency distribution, the Wigner distribution leads to
 151 artefacts that compromise the interpretability of the time-frequency distribution
 152 image. An example of this effect is given in the Appendix.

153 3.4. Wigner-Ville distribution

The Wigner-Ville distribution can be defined exactly as the Wigner distribution (13) except for the chosen signal. Instead of the signal $s(t)$, the analytic associate $s_+(t)$ is used, which eliminates the frequency interference terms from the distribution. The analytic associate is defined by

$$s_+(t) = s(t) + j\mathcal{H}\{s(t)\}, \quad (15)$$

where $\mathcal{H}\{s(t)\}$ describes the *Hilbert transformation*, which removes the negative frequency components from the original signal $s(t)$. Due to the Hermitian symmetry (5), no information is lost. The Wigner-Ville distribution is then defined as

$$\begin{aligned} W_{s_+}(t, f) &= \mathcal{F}\{K_{s_+}(t, \tau)\} \\ &= \int_{-\infty}^{\infty} s_+\left(t + \frac{\tau}{2}\right) s_+^*\left(t - \frac{\tau}{2}\right) e^{-j2\pi f\tau} d\tau. \end{aligned} \quad (16)$$

154 The kernel $K_{s_+}(t, \tau)$ is referred to as *instantaneous autocorrelation function*
 155 (Boashash, 2003).

156 3.5. Windowed Wigner-Ville distribution

The signal $s_p(t)$ is time-limited and obtained by multiplying the signal $s(t)$ with a window function $h(t)$. The Wigner-Ville distribution is applied on the

corresponding analytic associates of the signals,

$$\begin{aligned}
 W_{s_{p,+}}(t, f) & \\
 &= \int_{-\infty}^{\infty} h(\tau) s_+ \left(t + \frac{\tau}{2} \right) s_+^* \left(t - \frac{\tau}{2} \right) e^{-j2\pi f\tau} d\tau.
 \end{aligned} \tag{17}$$

157 The window function $h(t)$ causes a frequency smoothing of the time-frequency
 158 distribution (Boashash, 2003, Ch. 2.1.4.3). The use of a window function will
 159 be necessary when the system is used in a real-time system, where a sliding
 160 window function selects the available data for analysis.

161 *3.6. Aspect of the application of the Wigner-Ville distribution on ice-induced*
 162 *acceleration signals*

163 The Wigner-Ville distribution has several positive properties that are bene-
 164 ficial for the analysis of ice induced acceleration signals. Some important prop-
 165 erties are summarised here, as from Flandrin (1999) and Boashash (2003).

- 166 • *Time-shift invariance:* A time shift in the signal s_+ causes the same time
 167 shift in $W_{s_+}(t, f)$. This allows moving time windows for analysing the ice
 168 induced signals in a real-time system.
- 169 • *Frequency-shift invariance:* A frequency shift in the signal s_+ causes the
 170 same shift in frequency in $W_{s_+}(t, f)$. This is needed to identify varying fre-
 171 quency components in the time-frequency distribution of the acceleration
 172 signals.
- 173 • *Instantaneous frequency:* The mean of the Wigner-Ville distribution is the
 174 instantaneous frequency of the signal.
- 175 • *Marginal conditions:* The Wigner-Ville distribution fulfils the marginal
 176 conditions (10) and (11). That allows the calculation of the instantaneous
 177 power (7) and the energy spectrum of the signal (8) from the distribution.
- 178 • *Global energy:* The total energy of the signal can be evaluated with (9).
 179 This ensures that energy-changes caused by the ice interaction can be
 180 identified from the distribution.

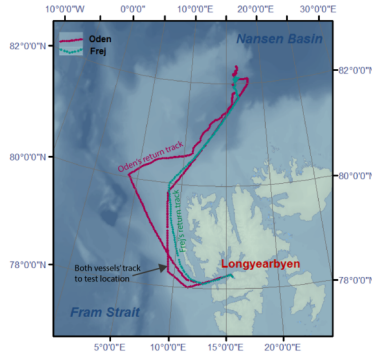


Figure 2: Track of the icebreakers Frej and Oden (Courtesy of Wenjun Lu).

- 181 • *Compatibility with filtering*: A convolution of two signals results in a con-
 182 volution of the Wigner-Ville distributions of these two signals. This prop-
 183 erty ensures that an ice load signal can be filtered, e.g. by a lowpass
 184 filter.
- *Non-stationary signals*: It is shown by Matz & Hlawatsch (2003) that the
 Wigner-Ville distribution can generally also be applied on non-stationary
 random signals $s_r(t)$. Instead of the total energy of the signal, the mean
 energy is obtained by

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_{s_r,+} dt df = E\{|s_r|^2\}. \quad (18)$$

185 4. A measurement system for ice induced accelerations

186 This section describes the measurement setup for collecting ice-induced ac-
 187 celeration data during an Arctic expedition with the Swedish icebreaker Frej in
 188 the Arctic Ocean north of Svalbard in September 2015.

189 4.1. The Oden Arctic Technology Research Cruise 2015

190 The Oden Arctic technology research cruise took place in September 2015
 191 in the Arctic Ocean north of Svalbard with the two icebreakers Oden and Frej.
 192 The track of the expedition is shown in Figure 2, and general information about
 193 the expedition can be found in Lubbad et al. (2016).

Table 2: Technical specifications of icebreaker Frej (Lubbad et al., 2016)

Icebreaker Frej	
Length	107.75 m
Beam	31.20 m
Draft	7.0 m - 8.5 m
Power generation	18.6 MW
Speed in open waters	18 kts
Icebreaking capability	1.2 m at 3 kts
Displacement	7800 t
Hydro. damping d_{11}	$2.14 \cdot 10^4 \text{ kg} \cdot \text{s}^{-1}$

194 Frej is an Atle-class icebreaker, commissioned in 1975. The propulsion system
 195 consists of two fore and two aft propellers and a diesel-electric power-plant with
 196 four diesel generators. Up to three auxiliary generators provide electricity for
 197 the ship. In normal operations the generators for the propulsion system run
 198 at 495 rpm and the auxiliary generators run at 750 rpm. This information is
 199 important since the generators induced a constant vibration into the ship, which
 200 is noticeable in the vibration measurements. Table 2 summarises the technical
 201 specifications of the icebreaker.

202 4.2. Sensor configuration and specification

203 Four inertial measurement units (IMU) from Analog Devices were installed
 204 at various positions of the vessel; see Figure 3. An IMU contains a tri-axis
 205 angular rate sensor and a tri-axis acceleration sensor, as illustrated in Figure 4.
 206 Only the accelerometer data were used in this study.

207 One sensor (*ADIS 16480*) was placed on Deck 9, underneath the bridge
 208 level, on a firm metal beam to serve as a reference sensor. Three sensors (*ADIS*
 209 *16364*) were placed in a storage compartment in the bow, close to the ice in-
 210 teraction zone of the vessel. The sensors in the bow compartment were placed
 211 port, mid-ship, and starboard. The port- and starboard sensors were placed on

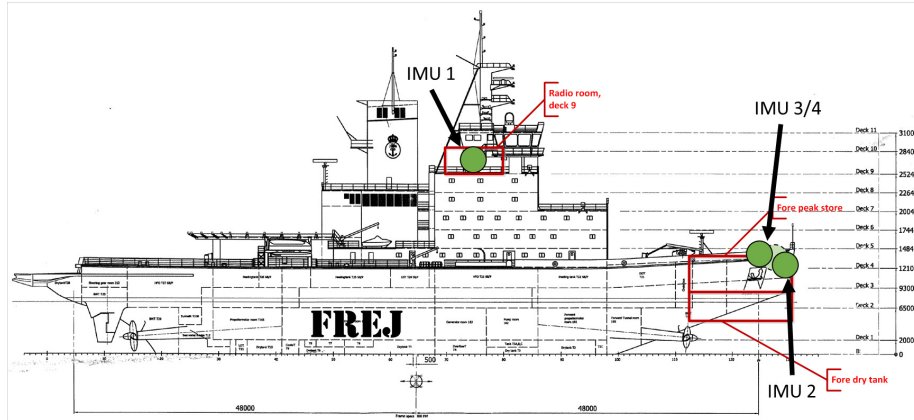


Figure 3: Position of the motion sensors.

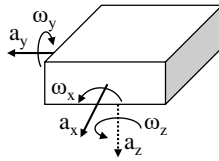


Figure 4: IMU sensor with three accelerometers and three gyros

212 beams directly connected to the hull of the ship. The compartment was heated
 213 to a constant temperature of 18°C, which minimised temperature related biases
 214 on the measurements. The *ADIS 16480* sensor contained additionally a tri-axis
 215 magnetometer, which was not used in this study.

216 A central server collected the data from the individual IMUs and served as
 217 time synchronisation server for the sensor units. To ensure time synchronisa-
 218 tion during the measurements, a real-time clock in each of the sensor units was
 219 continuously synchronised with the GPS-time obtained from the ship systems.
 220 Furthermore, synchronised data from other ship systems such as the GPS sys-
 221 tem, the gyrocompass, and wind sensors, were collected. Images from a 360°
 222 camera system and two 180° cameras were also collected, synchronised and
 223 processed in order to identify the ice conditions around the vessel.

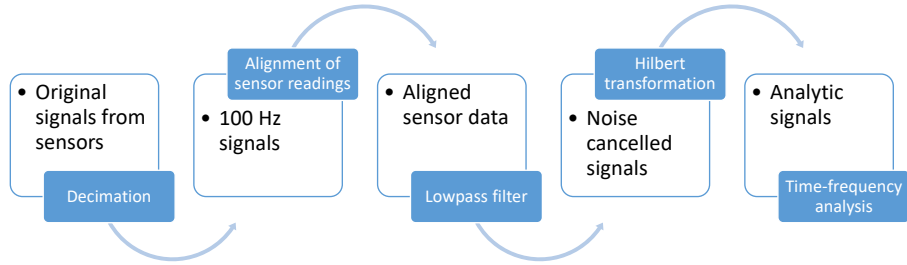


Figure 5: Pre-processing steps for the signals from the accelerometers

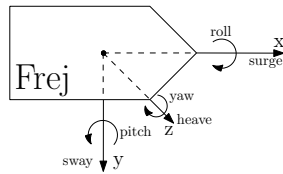


Figure 6: Body-fixed coordinate system definition.

224 4.3. Preprocessing of sensor readings

225 To minimise the influence of sensor noise, to align the sensor readings and to
 226 remove the influence of cross terms in the time-frequency distribution, several
 227 preprocessing steps were executed, as summarised in Figure 5.

228 4.3.1. Decimation of sensor signals

229 The sensors were operated with a sampling rate of 300 Hz. For the ice-
 230 induced vibrations, such a high sampling rate is not required since the ship's
 231 structure damps high frequency vibrations. To achieve a faster processing speed
 232 of the data, the signals were decimated to 100 Hz.

233 4.3.2. Alignment of sensor readings

234 For the alignment of the sensors, a body-fixed ship-wide coordinate system
 235 according to Fossen (2011) is defined, as shown in Figure 6.

At the location of the sensor, the sensor's coordinate system is not aligned with the body-fixed coordinate system. The accelerations measured by any of the sensors in its coordinate system are denoted \mathbf{a}_i^s , and referred to as proper

Table 3: Sensor orientation in relation to the body-fixed coordinate system of the ship

	IMU 1	IMU 2	IMU 3	IMU 4
ϕ	180.18°	-1.06°	2.10°	-0.83°
θ	-0.12°	208.73°	204.86°	203.94°
ψ	0°	0°	-90.00°	90.00°

acceleration. The corresponding acceleration vector in the body-fixed coordinate system is denoted \mathbf{a}_i^b . The orientation-matrix $\Theta_{\mathbf{s},i} = [\phi_i, \theta_i, \psi_i]^T$ states the orientation of each sensor's coordinate system relative to the body-fixed coordinate system. The orientation matrices for the four sensors are given in Table 3. A rotation matrix $\mathbf{R}_{s_i}^b(\Theta_{s,i})$, as the result of a rotation sequence, is applied to transform all sensor readings into the body-fixed coordinate system of the ship (Fossen, 2011), according to

$$\mathbf{a}_i^b = \mathbf{R}_{s_i}^b(\Theta_{s,i})\mathbf{a}_i^s, \quad (19)$$

with

$$\mathbf{R}_{s_i}^b(\Theta_{s,i}) = \mathbf{R}_{z,\psi_i}\mathbf{R}_{y,\theta_i}\mathbf{R}_{x,\phi_i}.$$

236 4.3.3. Lowpass and Hilbert filter

237 The signal has to be converted into an associated analytic signal in order
 238 to minimise the influence of cross terms in the time-frequency distribution. A
 239 lowpass filter removes high frequency noise components above 45 Hz from the
 240 signal. The sensors were calibrated and operated under a constant temperature
 241 in order to avoid a temperature related bias in the measurements.

242 4.4. Impact load estimation from acceleration measurements

Under the assumption that sensor noise, biases, and the earth gravity influence are removed from the measurements, and that the influence of wind and waves can be neglected, the measured acceleration in the body-fixed coordinate system is

$$\mathbf{a}_i^b = \mathbf{a}_{hyd}^b + \mathbf{a}_{prop}^b + \mathbf{a}_{ice}^b + \mathbf{a}_{vib,i}^b, \quad (20)$$

with $\mathbf{a}_{hyd}^b \in \mathbb{R}^3$ describing the global hydrodynamic load, $\mathbf{a}_{prop}^b \in \mathbb{R}^3$ the global propellers' induced load, $\mathbf{a}_{ice}^b \in \mathbb{R}^3$ the global ice induced load, and $\mathbf{a}_{vib,i}^b \in \mathbb{R}^3$ the locally induced vibrations.

With a constant speed and heading just before the impact with an ice feature, it can be assumed that the propeller thrust compensates for the hydrodynamic forces acting against the travel direction of the vessel. If the acceleration is measured at a location with sufficient damping from locally induced vibrations, such that these can be neglected, the measured acceleration in surge direction will consist of the global ice induced impact load:

$$a_{ice,surge}^b = \mathbf{B}_x \mathbf{R}_{s_i}^b (\psi_i) \mathbf{a}_i^s, \quad (21)$$

where $\mathbf{B}_x = (1, 0, 0)$ selects only the surge acceleration. With information about the ship's mass m_{ship} , added mass m_{add} (assumed as 20% of m_{ship} for this study), linear hydrodynamic damping d_{11} for the surge direction, and velocity u_{surge} , the force acting against the vessel in surge direction is

$$F_{surge} = m_{11} (a_{ice,surge}^b + d_{11}^{-1} u_{surge}), \quad (22)$$

with

$$m_{11} = m_{ship} + m_{add}. \quad (23)$$

243 5. Time-frequency distributions of ship-ice interaction events

244 In this section several cases of ship-ice interaction events of the icebreaker
 245 Frej in the Arctic Ocean are analysed in regards to the excited frequencies in the
 246 hull vibration with the help of the Wigner-Ville distribution. The data for the
 247 cases 1 to 4 have been collected on the 26th September 2015 between 11:00 UTC
 248 and 12:00 UTC. The track of the ship is illustrated in Figure 21. The relative
 249 wind speed was measured to be between 1 m/s and 6 m/s in the mentioned
 250 period. The influence of wind loads on the ice are neglected due to the low wind
 251 speeds. An overview of the analysed data is given in Table 4.

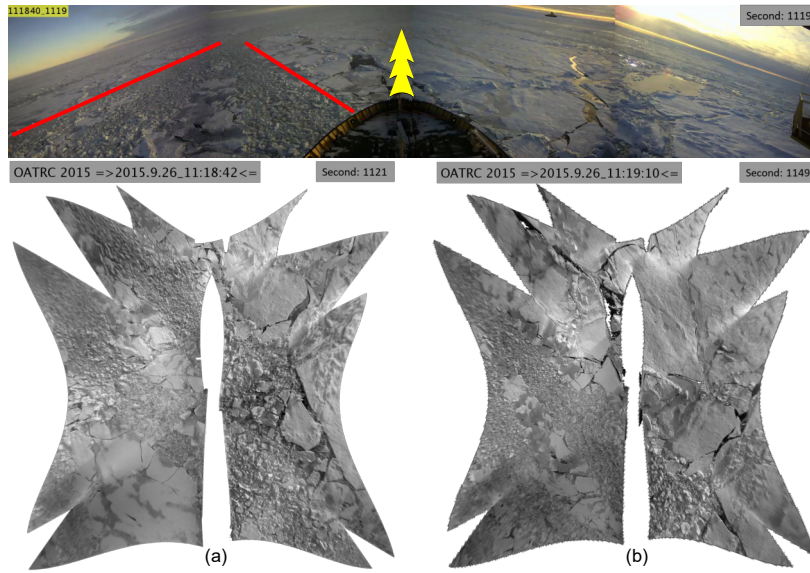


Figure 7: Case 1: Ship leaving a channel of broken ice (a) and transits into unbroken ice (b).

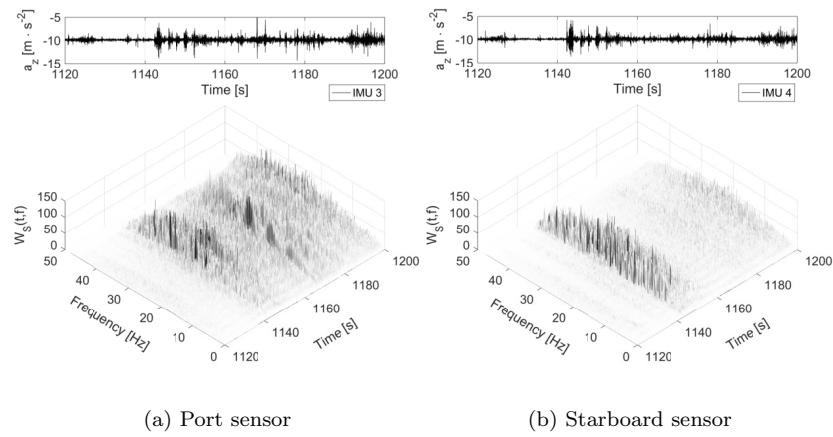


Figure 8: Case 1: Time-frequency distributions of the vertical acceleration.

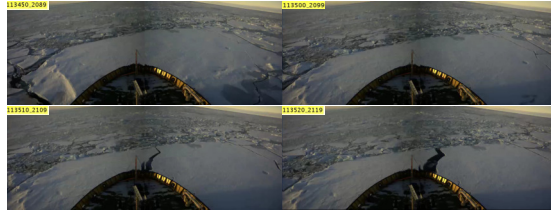


Figure 9: Case 2: Ship encounters an unbroken ice floe which splits upon impact.



Figure 10: Case 2: Crushing ice at starboard bow hull section just before splitting occurs.

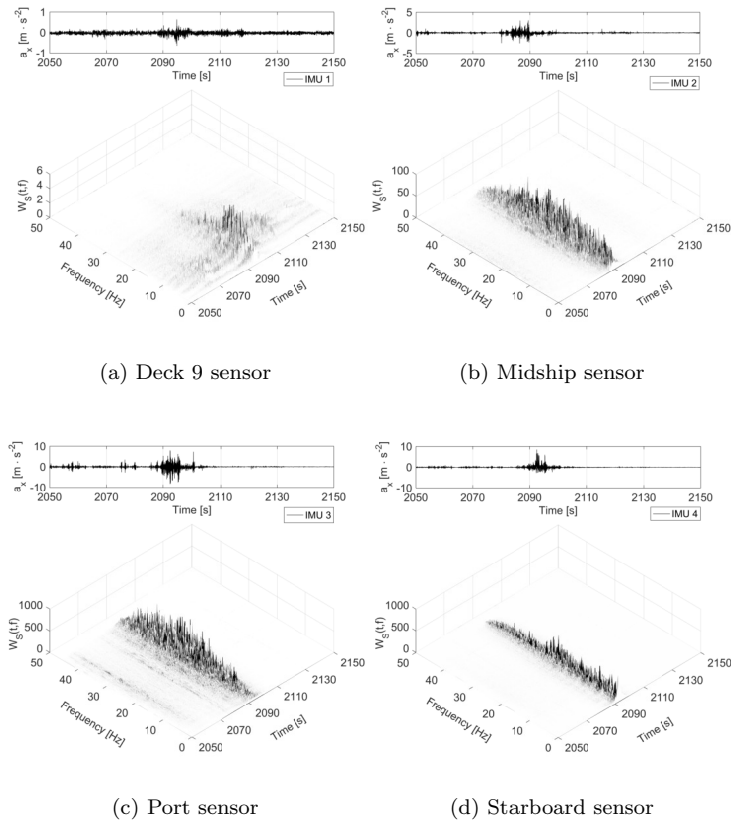


Figure 11: Case 2: Time-frequency distributions of the x-acceleration.

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Second: 910

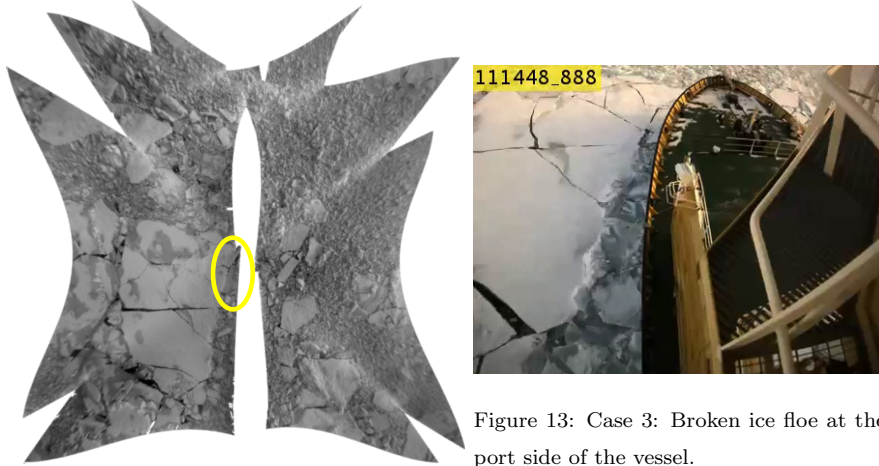


Figure 13: Case 3: Broken ice floe at the port side of the vessel.

Figure 12: Case 3: One piece of ice gets flipped by the ship.

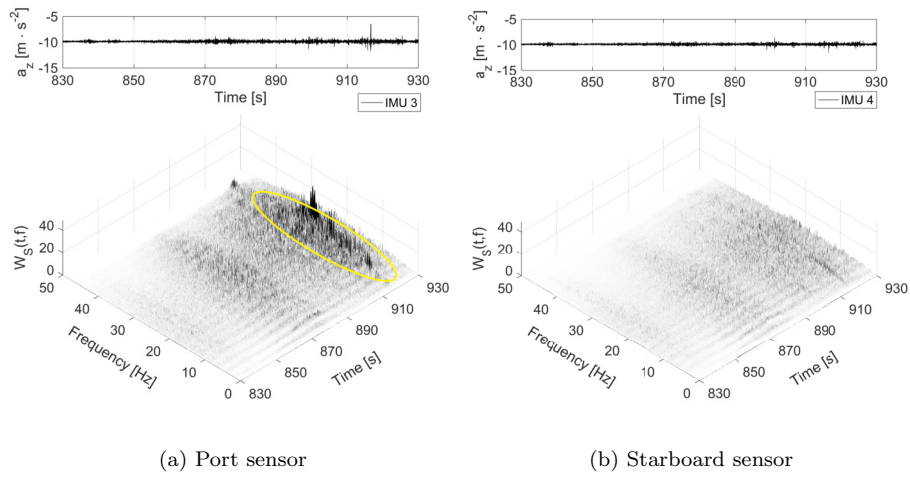


Figure 14: Case 3: Time-frequency distributions of the vertical acceleration.

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Second: 2458

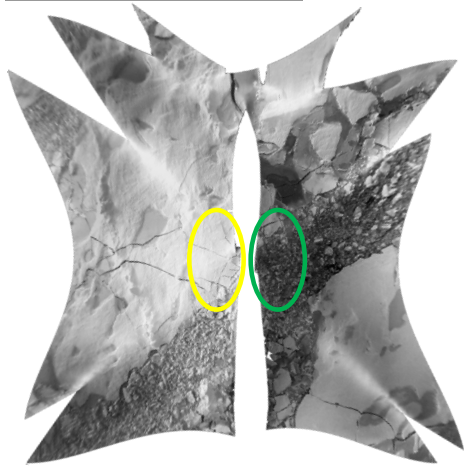


Figure 15: Case 4: Ice fails at port side of the ship (marked yellow) while starboard side encounters broken ice pieces (marked green).

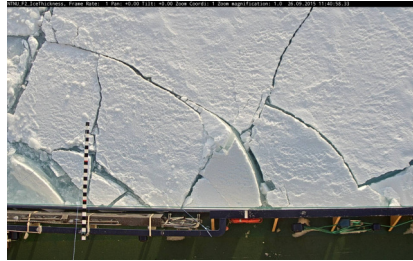


Figure 16: Case 4: Close-up view of yellow marked area in Figure 15. The measuring beam raster is 10 cm per block and has been mounted about 3 m above the ice. The camera is installed 20 m above the ice. Therefore the lengths taken from the measuring beam has to be multiplied by a factor of 1.2.

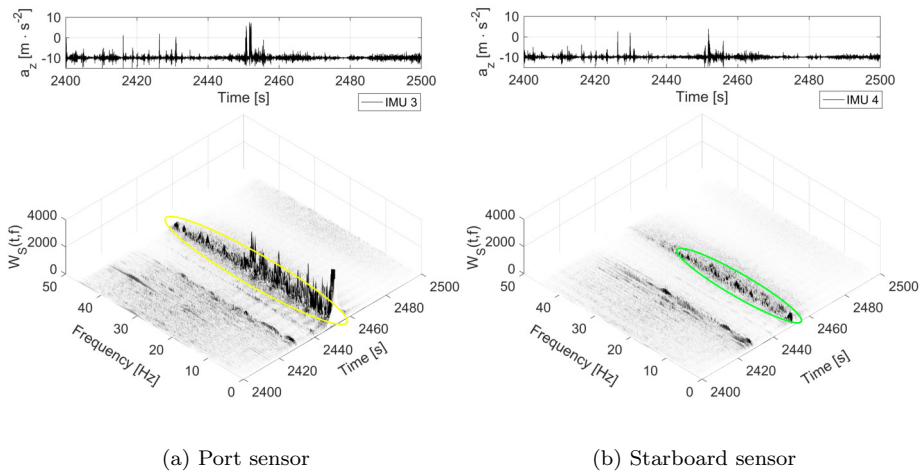


Figure 17: Case 4: Time-frequency distributions of the z-acceleration during one-sided encounter of a large piece of ice in a broken ice field.

Table 4: Overview of analysed measurements

Case	Date	Seconds in measurement	Ice conditions	Ice thickness	Speed over ground
1	26.09.2017 11:19.40	1120 - 1200	Transit from broken ice to unbroken very close pack-ice	0.5 m - 1.0 m	3.58 knots
2	26.09.2017 11:34.10	2050 - 2150	Broken ice, encounter of unbroken ice floe of ~ 100 m diameter	0.7 m	2.97 knots
3	26.09.2017 11:13.50	830 - 930	Broken ice, flipping of ice pieces on port side	0.3 m - 0.5 m	3.74 knots
4	26.09.2017 11:40.00	2400 - 2500	Broken ice on starboard close pack-ice on port	0.5 m - 0.7 m	4.33 knots
5	30.09.2017 10:47.40		Open-water		10.50 knots

252 *5.1. Case 1: Transition from managed ice to unmanaged ice*

253 The icebreaker Frej travelled at a speed of 3-4 knots inside an ice-channel
 254 created by the second icebreaker Oden. At second 1140, Frej moved out of the
 255 ice channel and hit a field of unbroken ice with an angle of about 45° . The ice
 256 was first-year ice with a thickness of 0.5 m to 1 m. The situation is shown in
 257 Figure 7.

258 Figures 8a) and 8b) show the time-frequency distribution of the ship's vertical
 259 acceleration measured by the sensors IMU 3 and IMU 4, which were placed in
 260 the bow section of the vessel. While moving in managed ice (seconds 1100-
 261 1140), significant vibrations in the hull are not noticeable. Upon entering the
 262 unbroken ice field at second 1141, the ice around the vessel failed by crushing
 263 and bending of the ice. The transition into unbroken ice is clearly visible in the
 264 time-frequency distribution due to a significant increase of energy of the signal
 265 over all frequencies. Especially vibrations with frequencies between 15 Hz and
 266 30 Hz have a higher energy. After a few seconds the excitation decreases, but
 267 the energy of the vibrations is still significantly higher than when travelling in

268 the broken ice.

269 The significant excitations upon exiting the ice channel can be connected to a
270 sudden decrease in velocity of the vessel, which was observed. The kinetic energy
271 upon contact with the unbroken ice might have caused a significant amount of
272 crushing prior to bending of the ice. As soon as the ship's speed decreased,
273 bending failure might have become the dominating failure mode again. This
274 created significantly less vibrations than crushing, but still more than in the
275 case of ice-interaction with already broken ice inside the ice channel.

276 *5.2. Case 2: Splitting of a major first-year ice floe*

277 The ship encountered an unbroken ice floe with a characteristic length of
278 about one ship length while travelling at about 3 knots through a field of broken
279 ice. Upon impact the ship got significantly decelerated, nearly to a standstill.
280 After about 15 seconds the ice floe split, and the ship moved ahead into the
281 opening crack. The situation is shown in Figure 9.

282 The resulting time-frequency distributions for the x-acceleration signals of all
283 four sensors are shown in Figure 11. The energy of the vibrations measured
284 by the two sensors placed directly on the hull of the vessel (sensors IMU 3 and
285 IMU 4) is significantly stronger between 2085 and 2010 seconds. The midship
286 sensor IMU 2 captured the vibration over all frequencies but with a tenth of
287 the energy content as the sensors placed directly on hull. The reason is that
288 the sensor IMU 2 was not placed directly on a hull segment but on a bearing
289 beam of the foredeck. The sensor on Deck 9 (IMU 1) could not measure signal
290 components over 20 Hz, which is due to damping of the superstructure.

291 It is notable, that major vibrations occur prior to the actual splitting of the ice
292 floe. The reason might be that, due to the propagating ship, crushing occurred
293 while the overall stress on the ice floe had increased. Upon splitting, the mea-
294 sured vibrations decay as the ship moves forwards inside the crack. Instead of
295 failing, the ice was pushed away from the ship. In Figure 10 the starboard hull
296 section of the bow of the vessel is shown just before the splitting has occurred.
297 Crushing of ice is visible along the hull of the vessel (see yellow marked area

298 in the image) and it might be the cause for the broadband vibrations in the
299 time-frequency distribution.

300 *5.3. Case 3: Port side contact with larger ice piece in broken ice field*

301 While travelling in a field of broken ice, the ship encountered several pieces
302 of a broken ice floe on the port side. The situation is shown in Figure 12. While
303 moving forward, the port section of the hull of the vessel got in contact with
304 one piece of the broken ice floe and flipped it, as shown in the yellow marked
305 area in Figure 13. The first-year ice had a thickness of estimated 30 to 50 cm
306 and the ship travelled at about 3-4 knots.

307 Figure 14 shows the time-frequency distributions for the vertical accelerations
308 measured by the hull mounted sensors. The energy content of the signals is low,
309 because the ship moved in already managed ice. During the time of hull contact
310 with a piece of the broken ice floe, the port mounted sensor (IMU 3) recorded
311 an increase in vibration energy, especially in the frequencies 5 Hz - 15 Hz. The
312 starboard sensor (IMU 4) did not record any significant change in the energy of
313 the vibrations.

314 Since the piece of ice got only in contact with the port section of the hull, it
315 makes sense, that the sensor on this side recorded the vibrations, which could
316 have been caused by sliding of the ice piece along the hull while it flipped. The
317 energy content of the measured vibrations is much lower than in the previous
318 Case 2, where crushing against the hull occurred. The kinetic energy of the
319 impact went into the rotational movement of the ice piece, and no failure of the
320 ice piece occurred.

321 *5.4. Case 4: Bending failure on one shipside*

322 The ship hit unbroken level-ice on the port side while travelling in a channel
323 of broken ice. The first-year level-ice had a thickness of 50 to 70 cm. While
324 propagating forward the unbroken ice at the port side of the ship failed by
325 forming circumferential and radial cracks, as shown in Figure 15 and, in more
326 detail, in Figure 16. The sea-ice consisted of small broken ice pieces at the

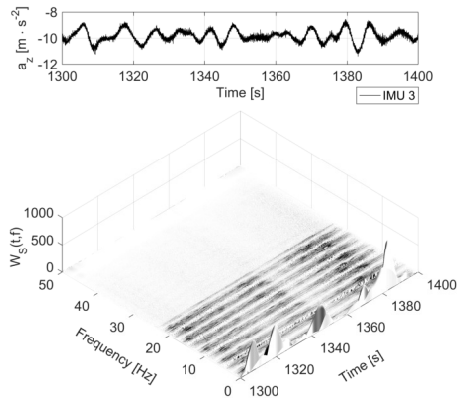


Figure 18: Case 5: Time-frequency distributions of the vertical acceleration (IMU 3)

327 starboard side of the ship.

328 The time-frequency distribution for the vertical accelerations measured by the
 329 hull mounted sensors IMU 3 and IMU 4 are shown in Figure 17. The bending
 330 failure occurred between seconds 2450 and 2460. Both sensor registered the
 331 ice interaction. However, the energy of the measured vibrations on the side of
 332 impact, the port side, is significantly higher. Most energy is contained in the
 333 range of frequencies between 5 Hz and 25 Hz.

334 The high energy content of the measured accelerations must be a result of
 335 the circumferential and radial cracking, as shown in Figure 16. In contrast to
 336 Case 3, where the piece of ice just flipped around without breaking, the ice floe
 337 actually failed against the hull of the vessel, which induced significant broadband
 338 vibrations.

339 5.5. Case 5: Measurements in open water during a severe sea state

340 The ship encountered a storm on the return journey to Svalbard while trav-
 341 elling in open water without any ice interaction. The significant wave height
 342 was estimated to be between 5 and 7 metres. The data was recorded on the
 343 30th September 2015. The resulting time-frequency distribution for the mea-
 344 sured vertical acceleration of IMU 3 is shown in Figure 18. The most obvious

345 property of the time-frequency distribution of the recorded vertical acceleration
346 is the high energy content at the typical wave frequencies below 1 Hz. Further-
347 more, the ship hull's natural frequencies are visible. In contrast to ice-induced
348 vibrations, there are no broadband impact events.

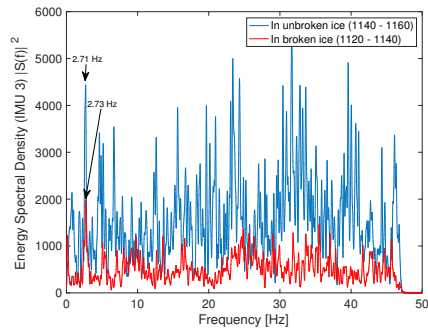
349 **6. Excitation of natural frequencies of the hull**

350 *6.1. Tracking of the first natural frequency during transit in ice*

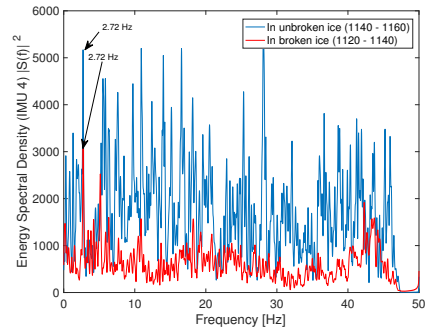
351 In all cases, the natural modes of the hull were excited and the natural
352 frequencies can be seen in the time-frequency distributions. A closer look at the
353 spectral densities for different scenarios is given in Figures 19 and 20.

354 Significant energy is contained in vibrations at around 2.72 Hz-2.75 Hz, 5.02 Hz-
355 5.08 Hz, and 8.05 Hz and 9.85 Hz. The two frequencies around 2.72 Hz and
356 5.02 Hz correspond to the natural frequencies found by Matusiak (1982), see
357 Table 1. The vibrations with 8.05 Hz must be originated in the engines, since
358 the main engines ran normally at 495 rpm, which equals a frequency of about
359 8.25 Hz. The frequency at 9.85 Hz can be another higher natural mode, not
360 present during the measurements on the sistership Sisu in 1982. It has to be
361 noted, that the ship Frej has been significantly modified since it's launch in
362 1975.

363 Matusiak (1982) mentions that the first natural frequency decreases, depend-
364 ing on the ice thickness, when travelling in ice. To investigate this behaviour,
365 the resulting Wigner-Ville distribution was used to automatically determine the
366 first natural frequency in the measured acceleration signal of IMU 1 during tran-
367 sit in an ice field with broken and unbroken ice, as illustrated in Figure 21. The
368 result of the tracking of the first natural frequency is presented in Figure 22,
369 and plotted together with the estimated ice induced impact load. The impact
370 load has been calculated from the measurements of IMU 1 using (22), and the
371 parameters given in Table 2. A lowpass filter, set with a cut-off frequency at
372 7.5 Hz was applied to the acceleration signal. A running average filter has been
373 applied on the resulting load estimate in order to make the load trend better.

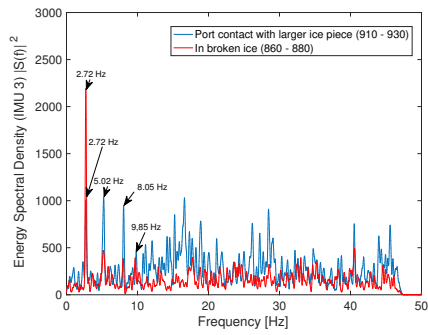


(a) Port sensor

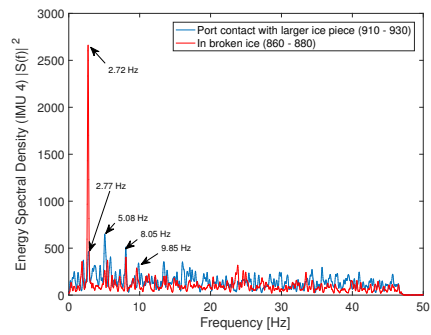


(b) Starboard sensor

Figure 19: Case 1: Energy spectral density of the vertical acceleration during transit from broken to unbroken ice.



(a) Port sensor



(b) Starboard sensor

Figure 20: Case 3: Energy spectral density of the vertical acceleration during port-sided encounter of a larger piece of ice.

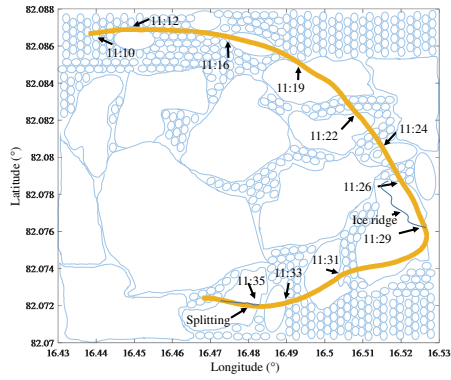


Figure 21: Illustration of encountered icefield

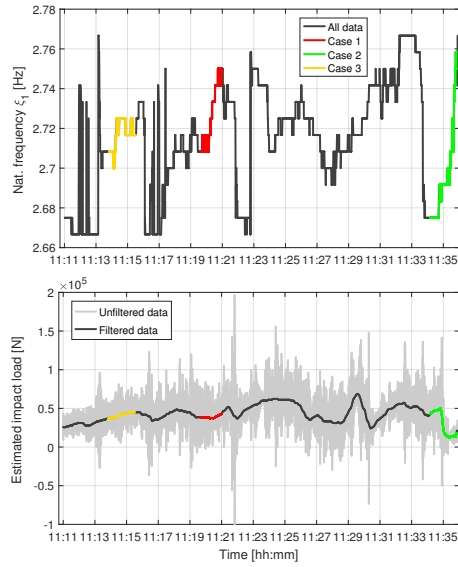


Figure 22: Track of first natural frequency and estimated impact load during transit in icefield

374 The first natural frequency ξ_1 measured by IMU 1 occurs in the frequency band
375 2.67 Hz to 2.77 Hz. It is notable, that a lower first natural frequency (lower
376 than 2.70 Hz) occurs always during an increase in induced ice load and upon
377 impact with an unbroken ice floe. A higher first natural frequency (higher than
378 2.74 Hz) corresponds with transit in broken ice between the unbroken ice floes.

379 7. Conclusion

380 This study proposed the Wigner-Ville distribution for the time-frequency
381 analysis of ice-induced accelerations during transit with an Atle-class icebreaker.
382 The resulting time-frequency spectra motivates the development of accelerom-
383 eter based sensor system for ships operating in ice-infested waters.

384 The interaction with sea-ice induces vibrations into the ship's hull with fre-
385 quency components up to 25 Hz, and especially crushing as failure mode in-
386 duces high-frequency vibration in the hull of the vessel. Natural frequencies of
387 the ship's hull up to 7.5 Hz are globally measurable, and high-frequency vibra-
388 tion are only locally measurable. Measuring ice induced accelerations locally
389 at several points along the ship's hull allows therefore for the localisation of ice
390 action against the ship along the ship's hull. Each failure regimes of the ice
391 induces a unique frequency pattern, which allows for the detection of the acting
392 failure mode during ship-ice interaction.

393 Accelerometers are capable of tracking the excitation of the ship's hull natu-
394 ral frequencies. Depending on the ice-condition around the vessel, the natural
395 frequencies change. Matusiak (1982) claimed that this change in the natural
396 frequencies is due to a change in added mass when an ice floe is attached to
397 the ship's hull. It was shown, that a lower first natural frequency occurs upon
398 impact of the vessel with an unbroken ice floe (Case 3). Just before the split-
399 ting occurred, the ship's bow was well pressing against the unbroken ice floe
400 and the measured first natural frequency decreased to 2.675 Hz. After splitting
401 had occurred, the bow of the vessel moved into the open water inside the split
402 and the first natural frequency increased to 2.760 Hz, which, according to (3)

403 corresponds to a loss of mass of 1.55%. The reason can be, that the additional
404 mass of the ice floe was not in contact with the ship's hull anymore.

405 *7.1. Further work*

406 The shift of natural frequency upon impact with ice floe can be used for an
407 estimation of the ice mass during ship-ice interaction, as already mentioned by
408 Matusiak (1982). With the addition of IMUs on a vessel, a system could be
409 realised, that estimates in real-time the mass of any ice feature the ship get in
410 contact with. By locally tracking high frequent vibrations in the hull, a warning
411 upon occurrence of crushing against the hull can be issued to the crew of an
412 ice-going vessel.

413 Only global and local acceleration measurements during transit were available
414 for this study. Additional work should be conducted in collecting acceleration
415 data during stationkeeping, and analyse if scenarios like accumulation of ice
416 on one side of the stationkeeping vessel can be detected with local vibration
417 measurements in the hull.

418 **Acknowledgement**

419 The authors would like to thank the Research Council of Norway (RCN)
420 for financial support through projects 203471 CRI SAMCoT and 223254 CoE
421 AMOS. Oden Arctic Technology Research Cruise 2015 (OATRC2015) was sup-
422 ported by the ExxonMobil Upstream Research Company and performed by the
423 Norwegian University of Science and Technology (NTNU) through the SAMCoT
424 CRI and in cooperation with the Swedish Polar Research Secretariat (SPRS)
425 and the Swedish Maritime Administration (SMA). Furthermore, the authors are
426 grateful for the extraordinary hospitality and support received from the crews
427 of the icebreakers Frej and Oden and the Swedish Maritime Administration
428 (SMA).

429 The authors would like to thank Professor Sveinung Løset, Professor Raed Lub-
430 bard, Dr. Wenjun Lu, Dr. Øivind Kjerstad, and Thorvald Grimstad for their
431 help, input, and valuable discussions during and after the expedition.

432 **Appendix**

Interference term in the Wigner distribution. An example from (Boashash, 2003) explains the interference effect: A signal with constant frequency f_c is given as

$$s(t) = \cos(2\pi f_c t).$$

The kernel of the corresponding Wigner distribution is given by:

$$\begin{aligned} K_s(t, \tau) &= s\left(t + \frac{\tau}{2}\right) s^*\left(t - \frac{\tau}{2}\right) \\ &= \cos\left(2\pi f_c \left(t + \frac{\tau}{2}\right)\right) \cdot \cos\left(2\pi f_c \left(t - \frac{\tau}{2}\right)\right) \\ &= 0.5 \cdot \cos(2\pi f_c \tau) + 0.5 \cdot \cos(2\pi f_c t), \end{aligned}$$

and the Fourier transformation leads further to the time-frequency energy density

$$\begin{aligned} W_s(t, f) &= \mathcal{F}\{K_s(t, \tau)\} \\ &= 0.25 \cdot \delta(f - f_c) + 0.25 \cdot \delta(f + f_c) \\ &\quad + 0.5 \cdot \cos(4\pi f_c t) \delta(f). \end{aligned} \tag{24}$$

433 The last term in (24) is the interference between the positive- and negative
434 frequency components of the real signal (Boashash, 2003).

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