On modelling of anisotropic undrained strength for non-horizontal terrain

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ABSTRACT: Low plasticity soft clays show pronounced variation in undrained shear strength with the direction of loading. The active undrained shear strength (A) is significantly larger than the direct shear strength (D), which again is significantly larger than the passive shear strength (P). The total stress based NGI-ADP model, available in Plaxis, captures such shear strength anisotropy and works well when applied to embankments on or excavations from a horizontal or almost horizontal terrain. For non-horizontal terrain the direction of the in-situ principal stresses is inclined. This paper presents a simple linear elastic, perfectly plastic ADP model that adds anisotropy induced by initial shear stresses on horizontal and vertical planes to an ADP framework. One model parameter controls the conventional anisotropy related to compression versus extension, while another parameter controls the anisotropy caused by the initial shear stress on horizontal and vertical planes. The model is using total stresses. A plane strain version is presented herein. The formulation is inspired by results from DSS laboratory testing where samples were consolidated under inclined effective stresses before shearing in the same or the opposite direction of the initial shear stress. As expected, the extended model called ADPX shows higher factors of safety when applied to a slope than a conventional ADP model. The paper discusses to what extent this represents a real safety margin that has previously been neglected.

1 ACTIVE, DIRECT AND PASSIVE STRENGTH

1.1 *Background*

Soft clays, and in particular lean soft clays have different undrained shear strengths when sheared on differently oriented planes, Soydemir (1976). Eide and Bjerrum (1973) present early results illustrating the significant difference between undrained strength obtained from triaxial compression and extension tests. Pragmatically the triaxial compression tests provides the plane strain active strength while the extension test provides the plane strain passive strength, Figure 1. The passive strength is for lean Drammen clay about 1/3 of the active strength.

1.2 *Soil models for anisotropic undrained strength*

Several well known effective stress based soil models like the MIT-E3, Whittle (1993), S-CLAY1-S, (Karstunen et al. (2005), and SaniClay, Dafalias (2006) provide an anisotropic undrained shear strength, lower for triaxial extension than for triaxial compression. One challenge with these models is that they require careful calibration of several effective stress based input parameters to provide a specific undrained strength design profile.

The NGI-ADP model in Plaxis, Grimstad et al. (2012), is a conceptually simple, total stress model where the ADP strength is direct input. This is convenient for practical applications since in practice the undrained strength (active or direct) may be the measured and available strength parameter.

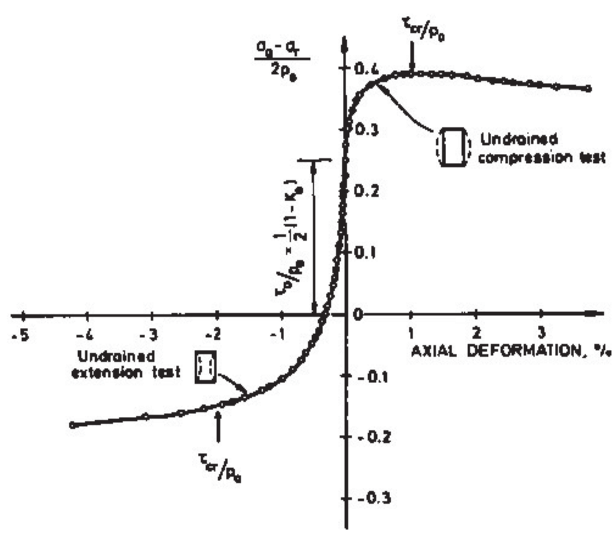

Figure 1. Anisotropic undrained strength Drammen clay (Eide and Bjerrum, 1973).

1.3 *The NGI-ADP model in plain strain*

The model is available in Plaxis for a full 3D stress state, Grimstad et. al (2012). A plane strain version of the NGI-ADP model with nested yield surfaces is illustrated in Figure 2. A vertical y-axis and a horizontal x – axis is used. In the model s_u^A, s_u^P, s_u^{DSS} are the active, passive and direct shear strengths respectively. The initial, vertical and horizontal effective stresses, σ'_{yy0} and σ'_{xx0} , define the starting point for loading by $\tau_0 = (\sigma'_{yy0} - \sigma'_{xx0})/2$. The initial value of $\tau_{xy} = \tau_{xy0}$ is assumed to be zero.

1.4 *Initially inclined principal stresses*

Anisotropy in undrained shear strength may originate from the direction of sedimentation. Anisotropy may also be stress induced. The NGI-ADP has a "K_0 – related" anisotropy, related to τ_0, and implicitly assumes that the initial, major principal stress is vertical. However, in slopes the initial principal stresses will be inclined, with $\tau_{xy0} \neq 0$, Figure 3. The ADPX model opens for "shear induced" anisotropy caused by $\tau_{xy0} \neq 0$.

The "τ_{xy0} - shear induced" anisotropy has been investigated by direct simple shear tests at NGI, where in addition to a vertical effective stress, σ'_{vc}, the sample was left to consolidate (drained) with a horizontal shear stress, τ_c, before sheared undrained to failure in the direction of τ_c , Figure 4. It is demonstrated that the undrained strength is increasing for increasing τ_c, for further loading in the "direction" of τ_c. The direct, undrained, simple shear stress increases by almost 30% for $\tau_c/\sigma'_{vc} = 0.30$ compared to $\tau_c/\sigma'_{vc} = 0$, Andersen (2009).

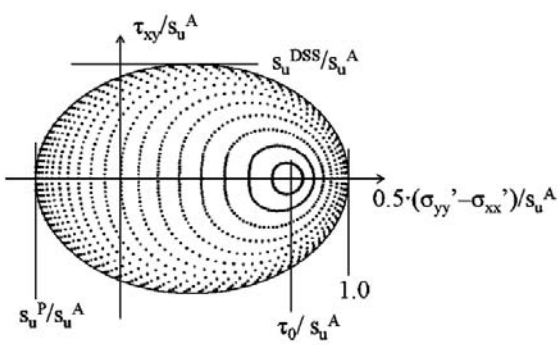

Figure 2. The NGI-ADP yield surfaces, Grimstad et al. (2012).

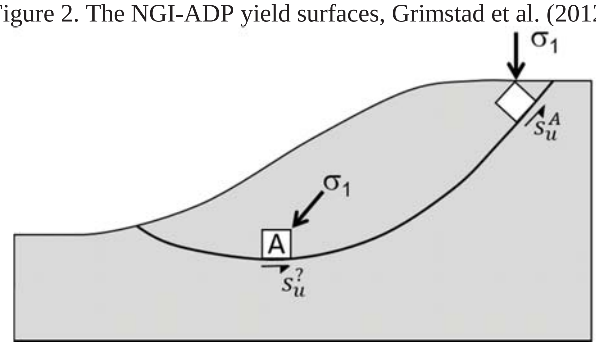

Figure 3. In soli element A the major principal stress is inclined and the shear strength may be closer to s_u^a than to s_u^{DSS}.

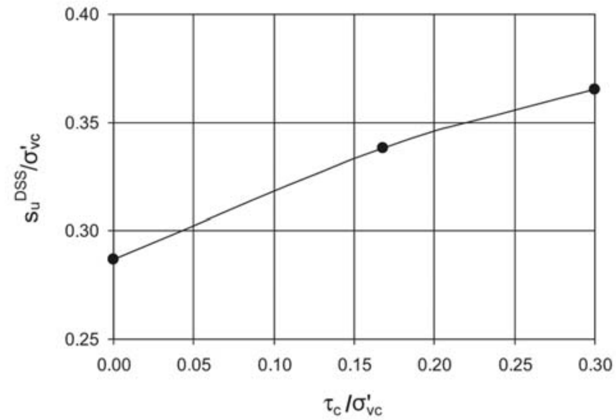

Figure 4. The DSS undrained shear strength increase when consolidated for an inclined, initial effective stress in the direction of undrained loading, Andersen (2009).

2 AN EXTENDED ADP MODEL, ADPX

2.1 *The anisotropic yield surface*

This paper reports on work done by MSc students at NTNU aiming to study the effect of an anisotropy induced by inclined principal, initial effective stresses. A simple linear elastic, perfectly plastic soil model is adopted for this purpose. It is often found that for vertical σ'_1 then s_u^{DSS} is almost $s_u^{DSS} \approx (s_u^A + s_u^P)/2$. Thus a circular yield surface in a $(\sigma_{yy} - \sigma_{xx})/2$ versus τ_{xy} stress space is selected for this study, Equation 1 and Figure 5.

$$F = \sqrt{\left(\frac{\sigma_{yy}-\sigma_{xx}}{2} - \xi \cdot \tau_0\right)^2 + (\tau_{xy} - \eta \cdot \tau_{xy0})^2} - \bar{s_u} \qquad (1)$$

For isotropic conditions, the circular yield surface has its origo in the center. For combined "K_0 – induced" and "τ_{xy0} - induced" anisotropy the center of the circle is suggested to move in the direction of the initial effective stress point, (τ_0, τ_{xy0}). This movement is in the formulation controlled by two dimensionless parameters, ξ and η, both in the interval [0,1]. The circle center is at $(\xi \cdot \tau_0 , \eta \cdot \tau_{xy0})$. Isotropic conditions are given by $\xi = \eta = 0$, while maximum anisotropy is given by $\xi = \eta = 1$. The consequence of the formulation is the anisotropic strengths expressed by Equations 2-5:

$$s_u^A = \bar{s_u} + \xi \cdot \tau_0 \qquad (2)$$

$$s_u^P = \bar{s_u} - \xi \cdot \tau_0 \qquad (3)$$

$$s_u^{DSS1} = \bar{s_u} + \eta \cdot \tau_{xy0} \qquad (4)$$

$$s_u^{DSS2} = \bar{s_u} - \eta \cdot \tau_{xy0} \qquad (5)$$

Where $\bar{s_u} = (s_u^A + s_u^P)/2$. The $\bar{s_u}$ may be given proportional with depth or with $(\sigma'_{yy0} + \sigma'_{xx0})/2$ for plane strain. The model is linear elastic, perfectly plastic with an associated flow rule. The ADPX is implemented as a user defined soil model in Plaxis.

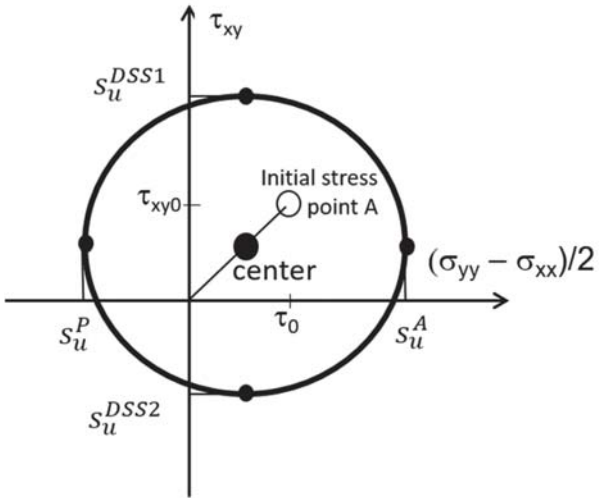

Figure 5. The ADPX yield surface is translated towards point A representing the initial deviatoric stresses. Here: $\xi = \eta = 0.5$.

2.2 *Initial effective stresses and model parameters*

Application of the model requires a known initial effective stress as a starting stress for all integration points. These initial stresses may be computed using an effective stress based model under drained conditions in an initial computational phase. Adding soil weight is one possible procedure. In Plaxis a Hardening Soil (HS) model may be used. A challenge of consistency may occur when the ADPX model is applied in the next computational phase: The ADPX parameters, $\bar{s_u}$, ξ and η must be selected so that the undrained shear strength is consistent with the initial effective stresses. In an adjusted version the ADPX-model it is pragmatically suggested that for normally consolidated clays the resulting shear strength could be limited by a maximum undrained shear strength and a minimum undrained shear strength, Figure 6. For simplicity, the limits may be given by the Mohr Coulomb parameters used during stress initiation, and a parameter *B*, Equations 6 and 7.

$$s_u^{max} = c \cdot cos\varphi + \left(\frac{\sigma'_{yy0}+\sigma'_{xx0}}{2}\right) \cdot sin\varphi \tag{6}$$

$$s_u^{min} = s_u^{max}/B \tag{7}$$

When implemented, these restrictions will to a large degree limit and overrun the parameters $\bar{s_u}$ and ξ. Note that it will indirectly also affect η, but η is given from the input ratio of η/ξ. This restriction (eq. 6) means that effectively the effective friction and effective cohesion become input for undrained strength in the ADPX model. In addition, the ratio η/ξ and the value of *B* position the yield surface within the limiting undrained maximum and minimum strengths, see Figure 6.

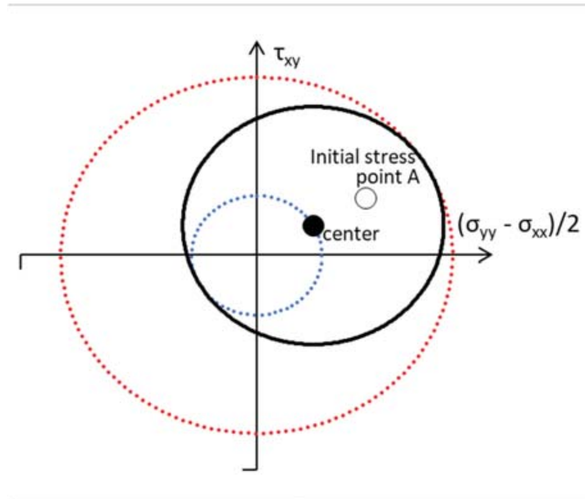

Figure 6. The ADPX yield surface as limited by the maximum and the minimum undrained strength circles around origo. The initial stress is point x. Here: B = 3 and $\xi/\eta = 1$.

In application it is necessary to evaluate the resulting shear strength in key points to ensure that the model provides realistic values for s_u^A, s_u^P, s_u^{DSS1} and s_u^{DSS2}. The friction angle may have to be a bit low to provide measured undrained strengths. Further, current experience indicate that B around 3 and η/ξ between 0.5 and 1 may be realistic.

3 EXAMPLES OF APPLICATION

3.1 *Stress driver testing*

The implemented formulation has been tested by applying strains and studying the resulting stress paths. One such example is given in Figure 7. A plane strain soil element is tested with initial stresses $(\sigma'_{yy0} + \sigma'_{xx0})/2$ = 17,5kPa and $\tau_{xy0} = 19$kPa.

The shear stiffness is 5MPa, $\bar{s_u} = 35kPa$ and $\xi = \eta = 1$.

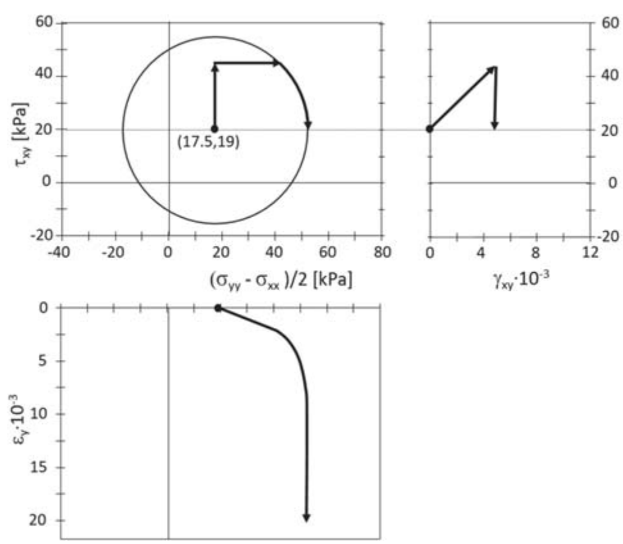

Figure 7. The stress path and stress strain paths obtained from applying shear followed by normal compression.

First, pure shear strain is applied with $\Delta\varepsilon_y = \Delta\varepsilon_x = 0$, $\Delta\gamma_{xy} = 0{,}5\%$.

Next, $\Delta\varepsilon_y = 2\%$, $\Delta\varepsilon_x = -2\%$, $\Delta\gamma_{xy} = 0$ is applied. The resulting stress will slide along the yield surface until a final position is reached where the applied strains are all plastic as given by normality to the yield surface (associated flow). In step 2 the elastic shear strains γ^e_{xy} are unloaded and replaced by plastic shear strains since the total shear strain is kept constant. Step 2 involves a rotation of principal stresses during pure normal compression.

3.2 *Application to slope stability*

A slope at Vestfossen in Norway failed in 1984 during construction of an embankment, a fill, in the slope. The case has been studied for investigating the effect of anisotropy related to inclined, initial effective stresses, i.e. $\tau_{xy0} \neq 0$. Figure 8 shows the slope geometry with sensitive NC clay under a dry crust. The figure also shows the fill that was placed when the slide took place. The failure surface indicated is the one resulting from a simulation. The ADPX model was used for the NC clay. Drained conditions and a Mohr Coulomb model was used for both the fill material and the dry crust. The active undrained shear strength of the NC clay was 15 kPa, increasing in depth by 2.5 kPa pr. meter. This was fitted to Equation 6 using the effective stress parameters $\varphi = 25^0$ and $c' = 2.3$ kPa. For the ADPX model, G = 5 MPa and an undrained Poisson ratio of 0.495 were used. A series of simulations were performed varying B and the ratio η/ξ. Table 1 shows the results in terms of ΣM_{stage} for adding the fill. A value very close to 1.00 should be obtained for a realistic simulation. It is observed that a standard ADP approach with $\eta/\xi = 0$ underestimates the capacity of the slope with respect to carrying the fill. B = 3 and $\eta/\xi = 0{,}75$ is a parameter set that correctly predicts pending failure. The numbers illustrate that taking the shear induced anisotropy due to $\tau_{xy0} \neq 0$ into account as in ADPX, lifts the calculated safety margin by about 10%. This contribution is neglected or ignored in a conventional ADP stability analysis, which corresponds to $\eta/\xi = 0$.

Table 1. Values of ΣM_{stage} obtained when constructing the embankment in the slope at Vestfossen. A value less than 1 shows that too low strength is used in that particular simulation.

B	η/ξ				
	0	.25	.5	.75	1.0
2	.9995	≥ 1.0	≥ 1.0	≥ 1.0	≥ 1.0
3	.9445	.9886	.9994	.9999	≥ 1.0
4	.9140	.9640	.9935	.9992	.9997

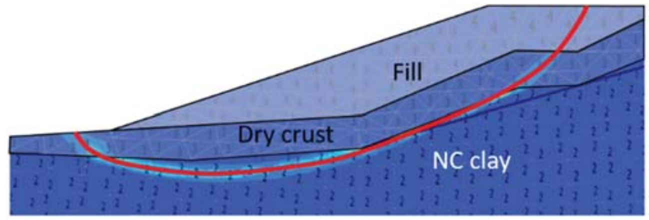

Figure 8. The Vestfossen slope were a fill added in the slope caused failure in the NC clay. Here modelled by using ADPX.

4 DISCUSSION AND CONCLUSION

It is believed that for a slope the initial inclination of the principal stresses increases the undrained shear strength compared to the strength predicted by standard ADP. The ADPX model includes this by a translation of the failure surface in the direction of the initial shear stress on horizontal and vertical planes in stress space. Preliminary results indicate that the calculated safety factor may increase by about 10% using ADPX compared to ADP.

Many aspects affect calculation of slope stability in soft clays using alternative soil models. Anisotropy in strength, sample disturbance, rate dependency, partial drainage and softening are important features that complicates the problem. Still, since the aim is to make realistic soil models and use relevant simulations, it makes sense to include the ADPX type anisotropy in stability evaluations. Whether a 10% strength increase can allow higher slopes, must be discussed in terms of safety levels and required partial material factors calibrated to avoid failure. The factors are given without taking the ADPX contribution to an “upgraded” strength into account.

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