

# Analysis of Epileptic Activity based on Brain Mapping of EEG adaptive time-frequency decomposition

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**Abstract.** The applications of Empirical Mode Decomposition (EMD) in Biomedical Signal analysis have increased and is common now to find publications that use EMD to identify behaviors in the brain or heart. EMD has shown excellent results in the identification of behaviours from the use of electroencephalogram (EEG) signals. In addition, some advances in the computer area have made it possible to improve their performance. In this paper, we presented a method that, using an entropy analysis, can automatically choose the relevant Intrinsic Mode Functions (IMFs) from EEG signals. The idea is to choose the minimum number of IMFs to reconstruct the brain activity. The EEG signals were processed by EMD and the IMFs were ordered according to the entropy cost function. The IMFs with with more relevant information are selected for the brain mapping. To validate the results, a relative error measure was used.

**Keywords:** Brain mapping, Empirical Mode Decomposition, Epilepsy, Signal Analysis.

## 1 Introduction

Richard Caton discovered electrical currents in the brain in 1875 and Hans Berger recorded these currents and published the first human Electroencephalogram (EEG) in 1924 [1]. The analysis of EEG signals has been very useful tool to support the medical diagnosis by extracting those meaningful features that can allow to identify some diseases (for example, Alzheimer or epilepsy) or some disorders (for example, attention-deficit/hyperactivity disorder (ADHD) or autistic spectrum) or some changes in the signals in depth of anesthesia. Nevertheless, the EEG signals are very difficult to analyze in time and frequency due to their non-linear and non-stationary nature [2], [3]. For this reason, EMD

and Hilbert Huang Transform (HHT) have been used to analyze the EEG signals and they have allowed to obtain a better signal representation and to detect instantaneous frequencies (IF) that with other methods are difficult to observe [4]. By this way, the use of linear filters and pre-processing is not necessary. In [5], a method to quantify interaction between nonstationary cerebral blood velocity (BFV) and blood pressure (BP) is proposed for the assessment of dynamic cerebral autoregulation (CA) using HHT. In [6], the authors use Multivariate Empirical Mode Decomposition (MEMD), which allows to analyze multichannel signals directly; in that case, this method was used for a full data-driven analysis to decompose resting-state fMRI (functional Magnetic Resonance Imaging) data into different sub-bands looking for connectivity functions. Our paper has a similar purpose, but instead we use EEG signals and another brain reconstruction algorithm. The use of fMRI implies higher costs due to the equipment required for acquisition and processing of information. Different strategies have been used for the process of reconstruction of Neural Activity from EEG data, but to the best knowledge of the authors, EMD has been used for this purpose only recently. For neural activity reconstruction, an iterative regularized method that explicitly includes space (grounded in a physiological model) and time constraints within the dynamic solution of the EEG inverse problem, is presented in [7].

When medications for focal epilepsy are not effective, it is necessary to use invasive treatments how resective surgery where a part of the brain is removed. First, the sources or brain zones, where the epileptic seizures start, are located and second, the surgery is carefully performed. Sometimes, when the mapping is not the best, it is necessary an additional estimation of the zone that has to be removed. Therefore, intra-cranial electrodes are used and additional surgery is performed [8]. In this work, an improved technique to brain activity reconstruction is presented. This technique is based on data-driven and applies pre-processing stage of the EEG using empirical mode decomposition. The information is classified in frequency bands from IMFs and then a highly accurate brain mapping is performed to locate the active sources. To this end, an entropy cost function is proposed for the optimal selection of IMFs. The entropy is an indicator of the amount of information stored in a more general probability distribution and is a measure of the complexity of the time series [9]. Some previous works have considered the use of entropy to detection of epileptic seizure [10], [11]. This paper is organized as follows: Section 2 gives an introduction to the essential concepts about EMD and EEG signals. The experimental setup is presented in Section 3 and the results obtained with the EEG signals are shown in Section 4. The discussion of the results is presented in Section 5. Finally, some conclusions are given in Section 6.

## 2 Methods

### 2.1 The inverse problem in EEG signals

The following is the forward model of EEG generation:

$$\mathbf{y}(t_k) = \mathbf{M}\mathbf{x}(t_k) + \boldsymbol{\epsilon}(t_k) \quad (1)$$

where  $\mathbf{y}(t_k) \in \mathbb{R}^d$  is the EEG and the neural activity is  $\mathbf{x}(t_k) \in \mathbb{R}^n$ , with  $t_k = kh$  the time at sample  $k$  being  $k = 1, \dots, T$  the number of samples, the sample time is  $h$  and the lead-field matrix  $\mathbf{M} \in \mathbb{R}^{d \times n}$ , which relates the neural activity with the EEG. Different models can be used to simulated the evolution of  $\mathbf{x}(t_k)$  in time. It is possible to formulate an iterative inverse problem [7] based on regularized Tikhonov-Phillips functional, in order to estimate the neural activity  $\hat{\mathbf{x}}(t_k)$  for each measurement  $\mathbf{y}(t_k)$ , as described in:

$$\begin{aligned} \hat{\mathbf{x}}(t_k) = \arg \min_{\mathbf{x}(t_k)} & \|\mathbf{y}(t_k) - \mathbf{M}\mathbf{x}(t_k)\|_2^2 + \lambda_k \|\mathbf{x}(t_k) - \hat{\mathbf{x}}(t_{k-1})\|_2^2 \\ & + \alpha_k \|\mathbf{x}(t_k)\|_1 \end{aligned} \quad (2)$$

where the regularization parameters  $\lambda_k$  and  $\alpha_k$  are computed by generalized cross validation [7].

### 2.2 Empirical Mode Decomposition

The Empirical Mode Decomposition (EMD) is a data-driven time-frequency (T-F) method that allows to analyze multivariate signals in an adaptive way. A nonlinear and non-stationary signal  $\mathbf{y}(t_k)$  can be decompose into a sum of intrinsic mode functions (IMFs) using EMD and these IMFs satisfies two conditions [12]: first, *Zero mean defined by the symmetry between upper/lower envelopes* and second, *The amount of extrema and zero crossings must differ at most by one or be the same*.

$$\mathbf{y}(t_k) = \sum_{i=1}^N \gamma_i(t_k) + \mathbf{r}(t_k) \quad (3)$$

$\gamma_i(t_k)$  is obtained when EMD is applied over  $\mathbf{y}(t_k)$  and where  $i$  is the intrinsic mode function (IMF). The residual is  $\mathbf{r}(t_k)$  and  $N$  is the number of IMFs. The Hilbert transform can be applied to each IMFs and the instantaneous frequency is computed according to equation (4).

$$f_i(t) \triangleq \frac{1}{2\pi} \cdot \frac{d\theta_i(t)}{dt}, \quad (4)$$

being  $\theta_i(t)$  the function phase of each IMF calculated from the analytical signal associated. Finally, the instantaneous frequency can be observed in the Hilbert Spectrum.

### 2.3 Entropy Function for automatic IMF selection

The proposed entropy function is the following:

$$e_i = - \sum_k \|\gamma_i(t_k)\|_2^2 \log(\|\gamma_i(t_k)\|_2^2) \quad (5)$$

It is applied over each IMF  $\gamma_i(t_k)$  where  $e_i$  is the entropy of each IMF, and  $\mathbf{e} = [e_1 \dots e_N]$ . The estimated EEG signal  $\tilde{\mathbf{y}}(t_k)$  from IMFs with highest entropy (chosen automatically) is rebuilt according to the measured entropy  $e_i$ .

$$\tilde{\mathbf{y}}(t_k) = \sum_{i \in O} \gamma_i(t_k) \quad (6)$$

being  $O$  the subset of of IMFs whose entropy  $e_i$  is over a threshold  $\tau_e$  computed as follows

$$\tau_e = \frac{\max e - \min e}{2} + \min e \quad (7)$$

## 3 Experimental setup

The performance of the aforementioned method is evaluated by using simulated and real EEG signals with epileptic activity. The experimental setup is divided in the following tasks:

1. EEG acquisition or simulation ( $\mathbf{y}(t_k)$ ) based on a nonlinear model.
2. Apply EMD on the EEG signal.
3. Optimal selection of IMFs using an entropy based cost function.
4. Reconstruction of a signal  $\tilde{\mathbf{y}}(t_k)$  based on the optimal selected IMFs according to (7).
5. Brain mapping of the neural activity based on the reconstructed signal.
6. Detection of focal origin of Epileptic seizures is performed by locating the source where the seizure is generated.

Four methods are considered for brain mapping comparison to evaluate the performance of the proposed algorithm:

1. Brain mapping ( $\hat{\mathbf{x}}(t_k)$ ) using the EEG database  $\mathbf{y}(t_k)$  without EMD.
2. Brain mapping ( $\hat{\mathbf{x}}_{EMD}(t_k)$ ) using the reconstructed EEG  $\tilde{\mathbf{y}}(t_k)$  obtained from EMD standard decomposition and an entropy based IMF selection.
3. Brain mapping ( $\hat{\mathbf{x}}_{\mathbf{W}}(t_k)$ ) using the reconstructed EEG  $\tilde{\mathbf{y}}_{\mathbf{W}}(t_k)$  obtained from Wavelet Transform using Daubechies wavelet and three decompositions levels, where the level with highest energy is selected for reconstruction of the EEG.
4. Brain mapping ( $\hat{\mathbf{x}}_{\mathbf{WP}}(t_k)$ ) using the reconstructed EEG  $\tilde{\mathbf{y}}_{\mathbf{WP}}(t_k)$  obtained from Wavelet Packets decomposition using Daubechies wavelet and three decompositions levels, where the level with highest entropy is selected for reconstruction of the EEG.

A common procedure to evaluate the performance of brain mapping techniques is by using simulated EEG signals where the underlying brain activity is known. In this case, a measure of the brain mapping quality can be evaluated with the relative error measure [13] as follows:

$$e_s = \sum_k \frac{\|\hat{\mathbf{x}}(t_k) - \mathbf{x}(t_k)\|_2^2}{\|\mathbf{x}(t_k)\|_2^2} \quad (8)$$

$$e_{EMD} = \sum_k \frac{\|\tilde{\mathbf{x}}_{EMD}(t_k) - \mathbf{x}(t_k)\|_2^2}{\|\mathbf{x}(t_k)\|_2^2} \quad (9)$$

$$e_W = \sum_k \frac{\|\tilde{\mathbf{x}}_W(t_k) - \mathbf{x}(t_k)\|_2^2}{\|\mathbf{x}(t_k)\|_2^2} \quad (10)$$

$$e_{WP} = \sum_k \frac{\|\tilde{\mathbf{x}}_{WP}(t_k) - \mathbf{x}(t_k)\|_2^2}{\|\mathbf{x}(t_k)\|_2^2} \quad (11)$$

being  $e_s$  the reconstruction error of the brain mapping estimation  $\hat{\mathbf{x}}(t_k)$  resulting from  $\mathbf{y}(t_k)$ ,  $e_{EMD}$  the reconstruction error of the brain mapping estimation  $\tilde{\mathbf{x}}_{EMD}(t_k)$  resulting from  $\mathbf{y}(t_k)$ ,  $e_W$  the reconstruction error of the brain mapping estimation  $\tilde{\mathbf{x}}_W(t_k)$  resulting from  $\tilde{\mathbf{y}}_W(t_k)$  and  $e_{WP}$  the reconstruction error of the brain mapping estimation  $\tilde{\mathbf{x}}_{WP}(t_k)$  resulting from  $\tilde{\mathbf{y}}_{WP}(t_k)$ .

### 3.1 Simulated EEG signals

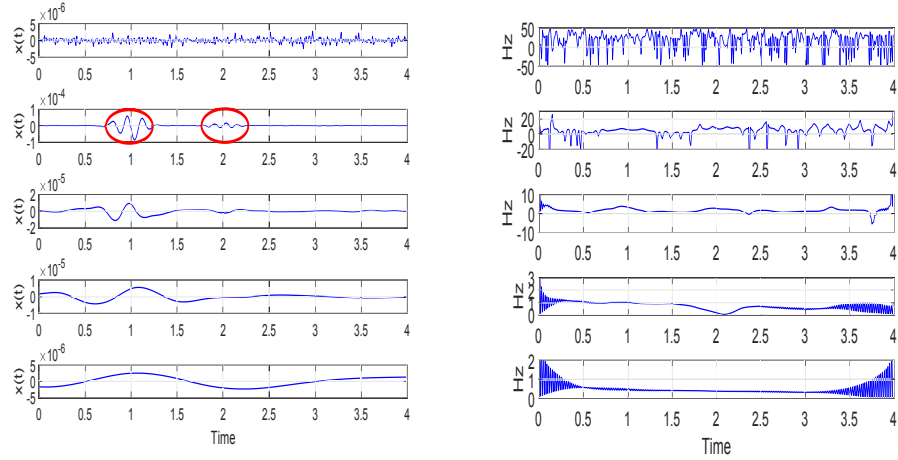
For the simulated database (SD-1) a complex nonlinear model of neural activity is used for EEG generation during an epileptic seizure based on [14] as follows

$$\begin{aligned} \mathbf{x}(t_k) = & \mathbf{A}_1 \mathbf{x}(t_{k-1}) + \mathbf{A}_2 \mathbf{x}(t_{k-2}) \\ & + \mathbf{A}_3 \mathbf{x}(t_{k-\tau}) + \mathbf{A}_4 \mathbf{x}(t_{k-1})^{\circ 2} + \mathbf{A}_5 \mathbf{x}(t_{k-1})^{\circ 3} + \boldsymbol{\eta}(t_k) \end{aligned} \quad (12)$$

being  $\mathbf{A}_1 = a_1 \mathbf{I}_n$ ,  $\mathbf{A}_2 = a_2 \mathbf{I}_n$ ,  $\mathbf{A}_3 = a_3 \mathbf{I}_n$ ,  $\mathbf{A}_4 = a_4 \mathbf{I}_n$  and  $\mathbf{A}_5 = a_5 \mathbf{I}_n$ , where  $\mathbf{I}_n \in \mathbb{R}^{n \times n}$  is an identity matrix and  $a_i \in \mathbb{R}$  are the model parameters which describe the dynamics of the brain activity, where  $\mathbf{c}_{k-1}^{\circ 2}$  denotes the Hadamard Power. The model parameters are set to  $\tau = 20$ ,  $a_1 = 1.0628$ ,  $a_2 = -0.42857$ ,  $a_3 = 0.008$ ,  $a_4 = 0.000143$ ,  $a_5 = -0.000286$ , and  $\|\boldsymbol{\eta}(t_k)\| \leq 0.05$ . The epileptic seizure is simulated at time  $t_k = 0.5$  s by modifying the values of  $a_1$  from 1.0628 to 1.3, while  $a_2$  from  $-0.428$  to  $-1$  over the entire diagonal. The simulated EEG  $\mathbf{y}(t_k)$  is obtained from  $\mathbf{x}(t_k)$  using (1) where  $\boldsymbol{\epsilon}(t_k)$  is set to achieve the Signal-to-Noise Ratios (SNRs) of 0, 5, 10, 15 and 20 dB, the sample rate is 250Hz, and a number of  $d = 128$  electrodes and  $n = 8196$  sources are considered.

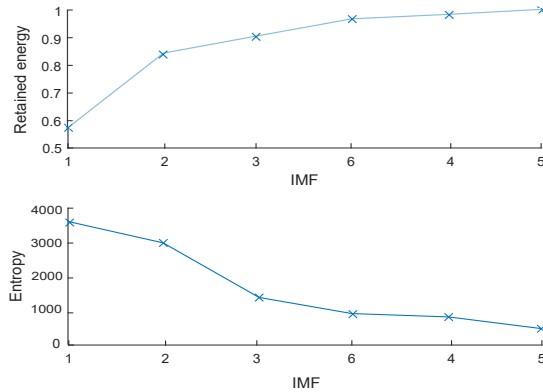
## 4 Results

After analyzing the database with the EMD, we obtained 6 IMFs per channel. In the IMF 2 in Fig. 1, it is possible to observe two areas in red that show how

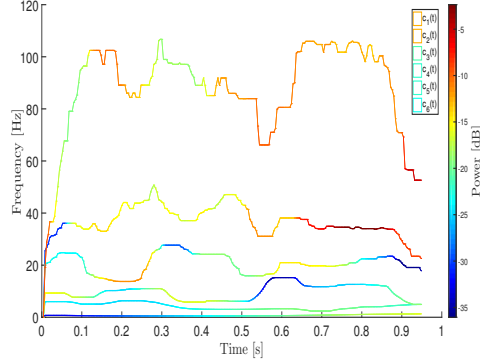


**Fig. 1.** IMF and IF of  $y_s$  for SD-1 using standard EMD

different frequencies (different oscillations) appear in the same IMF. In these IMFs the mode mixing problem is evident. An example of the retained energy and entropy for each IMF is presented in Fig. 2. In this example, the threshold is  $\tau_e = 1930.9$  and then the EEG is reconstructed by using the  $IMF_1$  and  $IMF_2$ . An example of the Hilbert spectrum is presented in Fig. 3, it is possible to see how the instantaneous frequency is changing with time. As expected, it is observed that the highest frequency is in IMF 1.

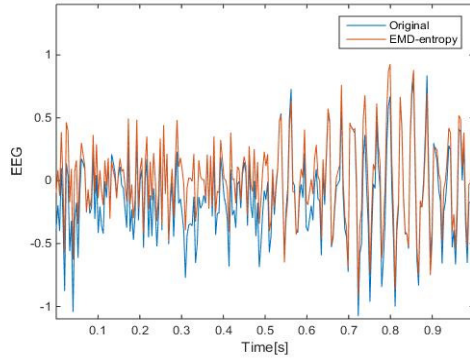


**Fig. 2.** Retained energy and entropy of  $y(t_k)$  for SD-1 using standard EMD



**Fig. 3.** Hilbert spectrum of  $\mathbf{y}(t_k)$  for SD-1 using standard EMD

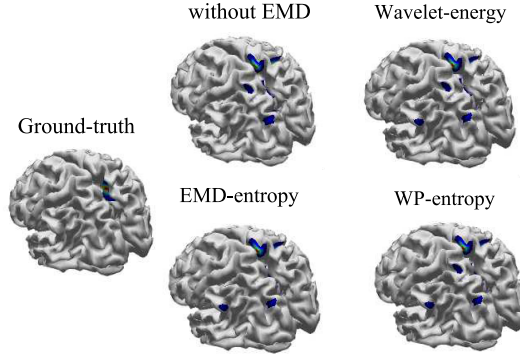
A comparison of the original  $\mathbf{y}(t_k)$  and reconstructed  $\tilde{\mathbf{y}}(t_k)$  signals is presented in Fig. 4. The resulting brain mapping for each method is presented in Fig 5.



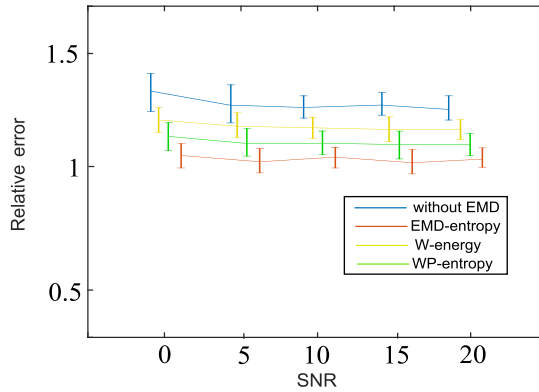
**Fig. 4.** Comparison of simulated  $\mathbf{y}(t_k)$  and optimally reconstructed  $\tilde{\mathbf{y}}(t_k)$  signals for SD-1 by using standard EMD for one channel

Relative error measure is used for evaluation and these results were obtained based on (8)  $e_s = 1.3284$ ,  $e_{EMD} = 1.2942$ ,  $e_W = 1.3106$  and  $e_{WP} = 1.2007$ . Showing that the best result is obtained for the brain mapping computed from the reconstructed neural activity using entropy-based selection of IMFs. An analysis based on 30 trials for each noise condition is shown in Fig. 6.

As shown in Fig. 6, the best results are achieved by the proposed method of EMD decomposition with automatic selection of relevant IMFs based on the entropy measure (EMD-entropy).



**Fig. 5.** Comparison of brain mapping obtained for simulated  $\mathbf{x}(t_k)$ , estimated without EMD  $\hat{\mathbf{x}}(t_k)$  and optimally reconstructed  $\tilde{\mathbf{x}}(t_k)$  neural activity for SD-1



**Fig. 6.** Relative error comparison for SD-1 under several noise conditions

From the above, it can be seen an improvement of the source localization in terms of the relative error. That allows an improvement of epilepsy treatment when a smaller part of the brain needs to be removed.

## 5 Discussion

First, it must be highlighted that this method allowed to reconstruct the brain activity from IMFs with relevant information for thi application. The problem of mode mixing was shown in [4] and therefore, the conclusion was that the EMD does not have a good performance in decomposing and reconstructing the signals with low frequency. In Figures 1 is possible to observe this phenomena. There are methodologies such as the masking signal [15] or Ensamble Empirical Mode Decomposition (EEMD) that can avoid this problem. However, the mode mixing does not disappear completely. When this technique is compared with



strategies very common for this type of application such as Discrete Wavelet Transform (DWT), some factors can affect the performance in epileptic focus localization: the mother wavelet, the level of decomposition, frequency bands, and features. The validation allowed us to calculate the relative error and to affirm that the technique presented provides an accurate detection of sources associated to epileptic seizures.

In Figure 5 is showed the brain activity reconstruction with raw data (without pre-processing) compared with reconstruction using EMD-entropy, wavelet-energy and WP-entropy. The reconstructions are not perfect, but they are very close to the ground-truth. The estimated relative errors allow to conclude that the estimated EEG from EMD-entropy was the lowest and the second lowest error was for WP-entropy (Figure 6). The IMFs and levels for WP are selected automatically, and depending of the EEG the number of IMFs or levels for WP could change, but in either case the sources are located very close to ground-truth. The epileptic seizure was simulated at time  $t_k = 0.5s$  and although time localization was not one of the purposes of this paper, in the Hilbert spectrum is possible to observe that the instantaneous frequencies associated with each IMF have a change in their behavior at exactly this time, therefore in order to automatically detect the beginning of an epileptic activity, an additional analysis of the instantaneous frequency could be performed.

## 6 Conclusions

An automatic detection of actives sources is presented. The method is based on EMD and and entropy function for brain activity reconstruction. This strategy can be used to support medical diagnosis when is necessary the visual observations of EEG signals. The tests carried out with the simulated databases and the calculation of the relative error measure show an excellent performance of the proposed methodology.

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