

APPENDIX A

MOIST AIR THEORY

In developing the mathematical and numerical model we quantified the air stream as being moist air – a mixture of dry air and water vapor - connecting the concentration equations to the temperature problem. As the air stream entering the channel will have some mass fraction or concentration of water vapor, and because the mass flow rate rather than the velocity at the inlet is usually known (which are related through the flow area and mixture density), it becomes important to calculate these values from the model parameters of temperature and relative humidity. The relative humidity is also a very useful quantity to study the condensation as it defines the boundary between the liquid phase and the vapor phase. In the following we will therefore introduce the concept of moist air theory as related to the calculated parameters used in the numerical model.

Assuming that the moist air behaves like an ideal gas we may define a series of humidity quantities that each provides a way to describe the humidity in the moist air. In the present analysis the relative humidity is the most useful quantity, but if we were to include sorption or condensation the moisture content (also called the mixing ratio or humidity ratio) – a ratio of mass – could be used to quantify the actual amount of transfer. Without these effects the moisture content is not affected by temperature and pressure, as is evident by simply looking at a Mollier diagram. For the same moisture content however the relative humidity changes with temperature and pressure. It is defined as follows:

$$\phi = \frac{p_v}{p_{sat}} \quad (\text{A.1})$$

where p_v is the water vapor partial pressure and p_{sat} is the saturation pressure of water vapor.

From this definition it follows that the relative humidity is a quantity defined in the interval $0 \leq \phi \leq 1$ ranging from dry air to saturated air at which point the water vapor partial pressure is equal to the saturation pressure of water vapor. When reaching this state the water vapor in the air is saturated and will begin to condense if the temperature drops further (assuming that the total pressure is constant).

The saturation pressure p_{sat} can be defined using the Clausius-Clapeyron formulation of the vaporization-condensation equilibrium as follows (considering only the gas volume),

$$\frac{dp}{dT} = \frac{h_{fg} M_v p}{RT^2} \quad (\text{A.2})$$

where p is the pressure, T is the temperature, h_{fg} is the latent heat of vaporization, M_v is the molecular weight of water vapor and R is the universal gas constant.

Integrating equation (3.33) and rearranging results in the saturation pressure equation,

$$p_{sat} = p_{ref} \exp \left(\frac{h_{fg} M_v}{R} \left(\frac{1}{T_{ref}} - \frac{1}{T_{sat}} \right) \right) \quad (A.3)$$

where p_{ref} is the atmospheric pressure, T_{ref} is the atmospheric boiling point and h_{fg} is the energy required to transform a given quantity (mol, kg) of a substance from a liquid into a gas at a given pressure.

Using the definition of relative humidity (3.32) and the saturation pressure equation (3.34) we may define the molar fraction of dry air x_a and the molar fraction of water vapor x_v as follows,

$$x_a = \frac{n_a}{n_{tot}} = \frac{p_a}{p} = \frac{p - \phi p_{sat}}{p} \quad (A.4)$$

$$x_v = \frac{n_v}{n_{tot}} = \frac{p_v}{p} = \frac{\phi p_{sat}}{p} \quad (A.5)$$

where n_a and n_v is the amount of dry air and water vapor, respectively, and n_{tot} is the total amount of moist air in mol.

The density of the binary mixture of pure water vapor and dry air at the corresponding partial pressures and molar fractions is calculated through the gas equation of state by the following simple mixture correlation,

$$\rho_m = \frac{p}{RT} (M_a x_a + M_v x_v) \quad (A.6)$$

Based on a kinetic theory approach Tsilingiris (2008) recommends the following expression for the viscosity of moist air,

$$\mu_m = \frac{x_a \cdot \mu_a}{x_a + x_v \cdot \Phi_{av}} + \frac{x_v \cdot \mu_v}{x_v + x_a \cdot \Phi_{av}} \quad (A.7)$$

where μ_a and μ_v are the dynamic viscosities of dry air and water vapor, respectively, and Φ_{av} and Φ_{va} are interaction parameters given as,

$$\Phi_{av} = \frac{\sqrt{2}}{4} \cdot \left(1 + \frac{M_a}{M_v} \right)^{-\frac{1}{2}} \cdot \left[1 + \left(\frac{\mu_a}{\mu_v} \right)^{\frac{1}{2}} \cdot \left(\frac{M_v}{M_a} \right)^{\frac{1}{4}} \right]^2 \quad (A.8)$$

$$\Phi_{va} = \frac{\sqrt{2}}{4} \cdot \left(1 + \frac{M_v}{M_a}\right)^{-\frac{1}{2}} \cdot \left[1 + \left(\frac{\mu_v}{\mu_a}\right)^{\frac{1}{2}} \cdot \left(\frac{M_a}{M_v}\right)^{\frac{1}{4}}\right]^2 \quad (\text{A.9})$$

The thermal conductivity of moist air may be expressed in a similar manner according to an analysis based on the relationship between the thermal conductivity and the viscosity of mixtures as concluded by Tsilingiris (2008)

$$k_m = \frac{x_a \cdot k_a}{x_a + x_v \cdot \Phi_{av}} + \frac{x_v \cdot k_v}{x_v + x_a \cdot \Phi_{av}} \quad (\text{A.10})$$

where k_a and k_v are the thermal conductivity of dry air and water vapor, respectively.

Finally by applying a simple linear mixing equation we can evaluate the specific heat capacity as (Tsilingiris, 2008),

$$c_{pm} = c_{pa} \cdot x_a \cdot \frac{M_a}{M_m} + c_{pv} \cdot x_v \cdot \frac{M_v}{M_m} \quad (\text{A.11})$$

where c_{pa} and c_{pv} are the specific heat capacity of dry air and water vapor, respectively, and M_m is the molecular weight of the mixture.

The moist air theory as defined features as part of the moist air functionality package available with the Heat Transfer module in Comsol Multiphysics and comes with the three functions of which two is used as feature parameters in developing the numerical model:

ht.fluid1.fc(RH, T, pA) returning the corresponding molar water vapor concentration $\left(\frac{\text{mol}}{\text{m}^3}\right)$.

ht.fluid1.fpsat(T) returning the saturation pressure (Pa).

In the above functions RH denotes the relative humidity φ , T the temperature, and pA the absolute pressure p .

REFERENCES

P.T. Tsilingiris (2008). Thermophysical and transport properties of humid air at temperature range between 0 and 100°C. *Energy Conversion and Management*, 49(5), 1098-1110