Energy 159 (2018) 1218–1233

Contents lists available at ScienceDirect

Energy

journal homepage: www.elsevier.com/locate/energy

From linking to integration of energy system models and computational general equilibrium models – Effects on equilibria and convergence

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A R T I C L E I N F O

Article history:
Received 29 October 2017
Received in revised form
15 April 2018
Accepted 21 June 2018
Available online 22 June 2018

Keywords:
Hybrid modeling
Energy system analysis
Economic modeling
Model linking
Integrated models
Stackelberg equilibrium

A B S T R A C T

This paper compares hard-linked and integrated approaches of hybrid top-down and bottom-up models in terms of equilibria and convergence. Four setups where a bottom-up linear programming model is hard-linked with a top-down computable general equilibrium model are implemented. A solution is found by iterating between the two models, until convergence is reached. The same equilibrium solution is found by all hard-linked setups in all problem instances. Next, one integrated model is introduced by extending the computable general equilibrium mixed complementarity model with the Karush-Kuhn-Tucker conditions that represent the bottom-up linear programming model. This integrated model provides the same solutions as the hard-linked models. Also, an alternative integrated model is provided, where the bottom-up model objective is optimized while the top-down model is included as additional constraints. This nonlinear program corresponds to a multi-follower bilevel formulation, with the energy system model as the leader and the general equilibrium players (firms and household) as followers. The Stackelberg equilibrium from this bilevel formulation pareto-dominates the Nash equilibrium from the other model setups in some problem instances, and is identical in the remaining problem instances. Different ways to couple the mathematical models may result in different solutions, because the coupling represents different real-world situations.

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1. Introduction

A challenge in modeling energy policy is to capture energy system effects, impact on the general economy and feedback effects in an adequate way. Different approaches to combine economic modeling with energy system modeling exist in the literature. This paper compares hard-linking approaches with hybrid models implementing full integration of a top-down economy model and a bottom-up energy system model. Top-down and bottom-up models represent two contrasting and wide-spread approaches for quantitative assessment of energy policies [1]. The strengths of one model complement the other model. Grubb et al. described early how economic models assume that no investments are available beyond the production frontier, while engineering models assume widespread potential for investments beyond this frontier [2]. Wene [3] discusses how the two approaches differ in their identification of the relevant system, and thus complement each other, while Böhringer and Rutherford [4] employ the complementarity format to combine the technological explicitness of bottom-up models with the economic comprehensiveness of top-down models.

Our contribution is to compare different ways of combining top-down and bottom-up models using both complementarity formulations and optimization formulations as well as hard linking and full integration. The main contribution is to integrate full-linked hybrid models and compare with hard-linked approaches. The authors are not aware of previous work that investigates this comparison.

Bottom-up engineering models include thorough descriptions of technological aspects of the energy system, including future improvements. They include interactions among the numerous individual energy technologies that make up the energy system of an economy, from primary energy sources, via conversion and distribution processes to final energy use. A solution constitutes a partial equilibrium where energy demand is fulfilled in a cost-

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https://doi.org/10.1016/j.energy.2018.06.146
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optimal fashion. Bottom-up models neglect the macroeconomic impact of energy policies, since they are partial equilibrium models and look only at the energy market. They are also unable to capture the full economy-wide rebound effects. They can easily capture substitution of energy carriers or technologies, but cannot anticipate demand adjustments due to income effects [5].

Top-down computable general equilibrium (CGE) models, on the other hand, describe the whole economy, and emphasize the possibilities to substitute different production factors in order to maximize the profits of firms. The substitution possibilities between energy and other production factors are captured in production functions, which describe changes in fuel mixes as the result of price changes under certain substitution elasticities. Prices are determined by the market clearance conditions that equalize supply and demand for all commodities in the economy, both energy and non-energy alike. The main workhorse in CGE modeling is the constant elasticity of substitution (CES) function. This function generalizes the Leontief function and the Cobb-Douglas function, and is used to model production, consumer utility and trade, usually in nested hierarchies [6]. One challenge is that such production functions can result in violation of basic energy conservation principles. The CES function aggregates economic quantities in a nonlinear fashion, conserving value but not physical energy flows [7]. Top-down representations of technologies can also produce fuel substitution patterns that are inconsistent with bottom-up cost data [8].

While bottom-up models usually emerge from linear programming (LP), CGE models are typically formulated as mixed complementarity problems (MCP), based on the framework of Mathiesen [9]. This modeling exploits the complementarity features of economic equilibrium: 1) Each activity that runs must reach zero profit. If the profit is negative, it will not run. 2) Each good must have a price that clears the market (demand equals supply). The good can be oversupplied only if the price is zero. 3) Consumer utility is assumed to be insatiable, thus every household will spend all its income (the model may include opportunities to save income for future consumption). CGE models are highly nonlinear, and may have more than one solution. Known conditions that are sufficient for uniqueness are highly restrictive. If either the weak axiom of revealed preference (WARP) or gross substitutability (GS) is satisfied by the consumer excess demand function, then a pure exchange economy has a unique equilibrium [10]. For CGE models involving production, Mas-Colell [11] provides sufficient conditions for uniqueness by proving that economies with CES utility and production functions whose elasticities of substitution are greater than or equal to one are guaranteed to have a unique equilibrium in the absence of taxes and other distortions. These conditions are restrictive, and introduction of taxes further complicates formulation of sufficient conditions for uniqueness [12].

There are few examples of models with multiple equilibria. Kehoe provides an overview with numerical examples [12]. According to Dierker [13], the number of equilibria in exchange economies is odd. Whalley and Zhang show tax-induced examples with 3 equilibria in a 2-individual 2-good pure exchange economy [14], and they are able to find 5 equilibria in a 3-individual 2-good pure exchange economy [15]. There are also examples of multiple equilibria in CGE models with production and increasing returns. Mercenier [16] reports two equilibria in a large-scale applied world economy CGE model. Denny et al. [17] find two equilibria while studying tax reforms using a CGE covering the Irish economy. The possibility of multiple equilibria means that convergence of solution algorithms cannot be guaranteed [18]. Mathiesen [19] discusses why theoretical results concerning convergence are few, but for a specific example with linear complementarity problems he is able to proof convergence if one solution exists. The possibility of multiple equilibria prohibits us from studying alternative decomposition methods for the integrated models that relies on convexity, for example Benders decomposition.

Hybrid models aim to combine the technological explicitness of bottom-up models with the economic richness of top-down models [4]. This can be accomplished in different fashions. Wene classifies model linking as (informal) *soft-linking* versus (formal) *hard-linking* [3]. Böhringer and Rutherford [18] do not use the term “hard-linking”, but define three categories: 1) Coupling of existing large-scale models, 2) having one main model complemented with a reduced form representation of the other, and 3) directly combining the models as mixed complementarity problems. This paper adopts the terms *soft-linking* and *hard-linking* as defined by Wene [3], where soft-linking is information transfer controlled by the user and hard-linking is formal links where information is transferred by computer programs without any user judgment. One further step is to *integrate* the models, as in the third category of Böhringer and Rutherford [18]. Integrated models are run as one, instead of exchanging information between separate model runs. Fig. 1 depicts these variants of hybrid modeling.

Fortes et al. [20] use the terms “full-link” and “full-form” to characterize hybrid models. Full-link hybrid models cover all economic sectors, while full-form hybrid models combine detailed and extensive technology data with disaggregated economic structure. Despite the extensive literature on hybrid models, there are few quantitative examples employing full-link and full-form bottom-up and top-down approaches [20].

Soft-linking is the natural way to start, when large-scale standalone models already have been implemented. Early examples are found in Hoffman and Jorgenson [21], who couple an econometric macroeconomic model with a process analysis model of the energy sector, Hogan and Weyant [22], who define a model framework and a solution method which moves through a network of process models, and Messner and Strubegger [23], who combine an energy system model with an economic model consisting of five modules which are solved iteratively. Many contributions focus on specific sectors, for example soft-linking ETEM and GEMINI-E3 focusing on residentials [24] and soft-linking MARKAL and EPPA focusing on transport [25]. Recent examples employ full-link of all economic sectors, for example between TIMES and EMEC [26] and between TIMES and GEM-E3 [20].

Hard-linking has historically been accomplished by narrowing

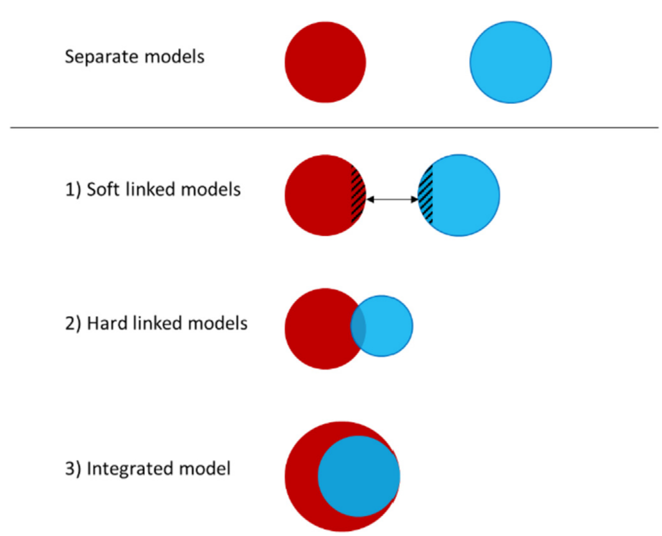

Fig. 1. Hybrid model variants.

the focus in one of the models, usually by aggregating the sectors of the economy. Examples are the ETA-Macro model [27], the MESSAGE-Macro model [28], and MARKAL-Macro (described methodologically by Manne and Wene [29], assessing inter-regional trade of CO_2 emissions [30], and studying long-term carbon reduction scenarios [31]). A recent example is provided by Arndt et al. [32], where the South African TIMES energy system model (SATIM) has been hard-linked to a detailed dynamic CGE model of South Africa (SAGE). However, the information interchange is related to electricity, and does not reflect the full sectoral coverage of the models.

Integration of bottom-up activity analysis into top-down CGE models was demonstrated in a static three-sector two-household sample model by Böhringer [33]. A dynamic extension was given by Frei [34], and a large-scale application was illustrated by Böhringer and Löschel, investigating renewable energy promotion in Europe [35]. The approach was extended by Böhringer and Rutherford [4], and further developed by adding a decomposition approach [18]. The conceptual idea was presented early by Scarf and Hansen [36] in 1973 (page 98), and further demonstrated by Mathiesen [9]. Such integrated hybrid models have focused on one selected sector, to maintain tractability. Most contributions in this category have focused on electricity. Sue Wing describes electric power technology detail in a social accounting framework [37], and studies the cost of limiting CO_2 emissions through carbon taxes [7]. Lanz et al. [8] presents a sensitivity analysis to demonstrate the pitfalls of making simplifying assumptions regarding emission abatement from the electricity sector. Their benchmark model utilizes the decomposition method described by Böhringer and Rutherford [18]. Proença and Aubyn assess feed-in tariffs for the promotion of electricity from renewable sources using a static CGE model of Portugal with integrated representation of the electricity sector [38]. Rausch and Mowers examine energy standards versus carbon pricing in five US policy scenarios toward the electricity sector [39]. Abrell and Rausch extends a multi-country multi-sector general equilibrium model with a bottom-up electricity dispatch model, to include electricity transmission infrastructure expansion [40].

This paper is oriented towards methodology, not policy analysis. The aim is to compare hard-linking and integration. However, in contrast to the integration approaches described previously, full detail is maintained in each model. It is assumed a setting where top-down and bottom-up models already have been implemented separately, and it is desirable to build on existing expertise without developing new models from scratch. This is a realistic starting point in many countries.

The scope has similarities to one previous study by Bauer, Edenhofer and Kypreos [41], which also compares linking of separate models with an integrated model approach.[1] Instead of a top-down CGE model, they consider a Ramsey-type macroeconomic growth model. They conclude that linking the models does not guarantee simultaneous equilibrium at the energy and capital market. A sound coupling requires integrating the models, and solving one very complex non-linear programming problem. Furthermore, integrating the models limits the level of detail and complexity of the energy system model.

Our approach to integration maintains full detail in each model. However, we simplify the representation of the time dimension, and use a static CGE model. We implement various versions of hard-linking and novel approaches to integration, using a stylized bottom-up TIMES model and a static top-down CGE model. One of the described hard-link approaches has been implemented on large-scale stand-alone models, employing a full-link and full-form bottom-up and top-down approach. A policy study based on this implementation is provided in Helgesen et al. [42].

All model reformulations are implemented without any need to change data inputs to the respective models. Demand for energy services are derived from equilibrium solutions of the CGE model, and employed as exogenous input to the bottom-up model. Solutions from the bottom-up TIMES model are then used to adjust the input-output structure describing future energy use in the different economic sectors of the CGE. This is an alternative approach to the CES functions that are routinely used in long term economic models. CES functions are central building blocks of General Equilibrium Integrated Assessment Models, which run future scenarios until year 2100 - for example in the Assessment Reports from the Intergovernmental Panel on Climate Change (IPCC). However, the focus in these reports has shifted from a single-discipline cost-benefit analysis to multi-disciplinary uncertainty analyses [43], as the economic models have important weaknesses. They cannot foresee actions that are profitable but not implemented (for example the energy "efficiency gap" [44]), and technological progress is often modelled as "manna from heaven" in the form of autonomous energy efficient improvement factors [45]. Kaya, Csala and Sgouridis [46] present critical views towards CES functions, claiming that this practice fails to match historically observed patterns in energy transition dynamics and that results are sensitive to parameter choices and the nesting. CES functions tend toward factor share preservation. The authors propose perfect substitution for alternative energy options, physical modeling complementing the economic analysis or applying functions with dynamic elasticity of substitution.

The approach in our paper improves upon the use of CES production functions in the energy sector, by utilizing the physical modeling of the energy system model as suggested [46]. Leontief production technologies with fixed input factors for energy inputs are assumed in the top-down CGE model, and Leontief coefficients are updated based on the bottom-up energy system model.[2]

Research questions for this paper are summarized as follows:

1) How can we integrate stand-alone versions of a top-down economic and a bottom-up energy system model?
2) Will hard-linked and integrated hybrid models produce the same solutions?
3) Will one larger, more complex integrated model be able to run in a similar time scale as two smaller separate hard-linked models?

The results presented are produced from stylized models, but the approach is generic and may be applied to large-scale models, as shown in Helgesen et al. [42]. The authors are not aware of any previous work that compares different implementations of full-linked integrated hybrid models.

The paper proceeds as follows. Our two models are presented in section 2, as well as the two hybrid modeling approaches. Section 3 presents results, demonstrating the interplay between models and comparing results from our hybrid model alternatives. The findings are discussed in section 4, and section 5 concludes the paper.

2. Methods

The purpose of this chapter is to define our mathematical

[1] Bauer et al. [41] define soft-link and hard-link differently from Wene [3], whose definitions we have applied in this paper.

[2] Income elasticities and elasticities of substitution are kept constant, as this is standard practice in CGE modeling, and the current models have no relevant basis for updating the elasticities endogenously.

models and the different hybrid variants we compare. The mathematical programming models of the energy system and of the whole economy are stylized, but general. Firms in the economy optimize their decisions in order to maximize profits, while other actors (for example government or households) similarly maximize their utility. The energy system supplies energy services to fulfil energy demand at the least cost attainable.

A static computable general equilibrium model describes a future economic equilibrium based on expected capital and labor growth. The energy system model calculates the optimal investments to meet the demands for energy services in this future economy. The resulting energy mix from the energy system model is used to update the computable general equilibrium model, resulting in new energy service demands.

This logic is first implemented using hard-linking, automatically iterating between both models until convergence is reached. Next, an integrated model is implemented, where the bottom-up model is represented by its Karush-Kuhn-Tucker conditions. Third, a different variant of the integrated model is implemented, where both models are integrated into one non-linear model.

Integrated models are solved either as a mixed nonlinear complementarity problem (MNCP) or as a nonlinear program (NLP). These variants are justified, since a nonlinear complementarity problem may equivalently be stated as a nonlinear program [47]. We exemplify this here by stating the pure nonlinear complementarity problem in vector form [48]. Given a vector-valued function *F(x)* defined for $x \geq 0$, find a solution that satisfies:

$$F(x) \geq 0, x \geq 0, F(x)^T x = 0 \tag{1}$$

This is often written more compactly as $0 \leq F(x) \perp x \geq 0$ with the perpendicular operator $\perp$ denoting the inner product of two vectors equal to zero. We may now state the nonlinear complementarity problem as a nonlinear program:

$$\min_x F(x)^T x \text{ subject to } F(x) \geq 0, x \geq 0 \tag{2}$$

Any feasible vector x satisfying the two non-negativity conditions must have $F(x)^T x \geq 0$. If there exists a solution satisfying the complementarity condition $F(x)^T x = 0$, it will also be a global minimizer of the nonlinear program. Given the existence of a solution to the complementarity problem, a global minimizer of the nonlinear program will also be a solution to the complementarity problem.

Typical examples of functions *F(x)* are zero profit conditions on production of goods, and market clearing conditions with regards to prices. A firm will not produce a good x if it earns a loss, production must reach zero profit (after paying wages and capital return). Similarly, a supplier will not experience a positive market price on a good in excess supply. A positive price implies market balance between supply and demand.

The models presented are scaled down, and many important real-world aspects or policy issues have been simplified, allowing us to focus on the linking and integration techniques. Nevertheless, the top-down and bottom-up models are general enough to represent large-scale, real world models, and the simplifications do not affect the validity of the analyses that are presented.

2.1. Bottom-up energy system model

Our bottom-up model has been defined and extracted from the TIMES (**T**he **I**ntegrated **M**arkal **E**fom **S**ystem) model generator, which has been developed in the frame of the implementing agreement IEA ETSAP.[3] A TIMES model gives a detailed description of the entire energy system including all resources, energy production technologies, energy carriers, demand devices, and sectorial demand for energy services. The model assumes perfect competition and perfect foresight (can also be used in a myopic mode) and is demand driven. The model finds the cost-minimizing way to fulfil energy service demands over a defined planning period. Yearly demands for heat and electricity are provided exogenously. Our stylized problem structure is depicted in Fig. 2.

Four technologies are available. Electricity can be produced from gaspower or hydropower. Heat can be produced by a gasburner or from electric heating. Only one region, one currency and a yearly timeslice are defined. For simplicity, a discount rate equal to zero is assumed, and discounting is omitted from the formulas.
The mathematical model is defined as follows:

Sets	
T	Time periods in bottom-up model, indexed by t (time) and v (vintage).
P	Processes in bottom-up model, indexed by p. This set includes the subset of production processes P_{prod} (as opposed to supply and demand processes). This set also includes subsets P_c^{in} (processes with commodity c as input) and P_c^{out} (processes with commodity c as output).
C	Commodities in bottom-up model, indexed by c. This set is further divided into natural supplied commodities C_{supply} and produced commodities C_{prod}.
Parameters	
$C_{t,p}^{cap}$	Capacity investment cost in year t and process p.
$C_{t,p}^{fom}$	Fixed operating and maintenance costs in year t for process p.
$C_{t,p}^{act}$	Activity cost in year t for process p.
$C_{t,c}^{prd}$	Production cost in year t for commodity c.
A_p^f	Availability factor[4] for process p.
α_p^{capact}	Capacity factor[4] in process p.
$\phi_{p,c,c'}$	Flow conversion factor in process p from commodity c to c'.
$D_{t,c}$	Demand in year t for commodity c.
$I_{2015,p}^{cap}$	Existing capacity in base year (2015) for process p.
$U_{t,p}^{cap}$	Upper bound on capacity investment in year t for process p.
$salvage_{t,p}$	Salvage value in horizon year (2026) from investment in year t in process p.
L_p	Technical lifetime (number of years) on investment in process p.
$\rho_{t,p}$	Remaining share of capacity from base year ($I_{2015,p}^{cap}$) in year t of process p.
Variables	
$i_{t,p}^{cap}$	Capacity investment in year t in process p.
$x_{t,p}^{act}$	Activity in year t in process p.
$x_{t,c}^{prd}$	Production in year t of commodity c.

Minimize system costs:

[3] The Energy Technology System Analysis Program of the International Energy Agency.

[4] The availability factor and capacity factor could be collapsed into a single parameter in this model, but these parameters are defined individually to maintain the correspondence to: the TIMES formulation.

$$\min_{i^{cap}_{t,p},x^{act}_{t,p},x^{prd}_{t,c}} \left(\begin{array}{l} \sum_{t=2015}^{2026} \sum_{p \in P} (1 - salvage_{t,p}) \left(C^{cap}_{t,p} \bullet i^{cap}_{t,p} \right) \\ + \sum_{v=2015}^{2026} \sum_{p \in P} \sum_{t=v}^{\min(2026, v+L_p-1)} C^{fom}_{t,p} \bullet i^{cap}_{v,p} + \sum_{t=2015}^{2026} \sum_{p \in P} C^{act}_{t,p} \bullet x^{act}_{t,p} + \sum_{t=2015}^{2026} \sum_{c \in C} C^{prd}_{t,c} \bullet x^{prd}_{t,c} \end{array} \right) \tag{3}$$

subject to
CAPACT: Process activity $\leq$ capacity

$$x^{act}_{t,p} \leq \sum_{v=\max(2015,t-L_p+1)}^{t} A^f_p \cdot \alpha^{capact}_p \cdot i^{cap}_{v,p} + A^f_p \cdot \alpha^{capact}_p \cdot \rho_{t,p} \cdot I^{cap}_{2015,p} \quad , \forall t \in [2015, 2026], p \in P_{prod} \tag{4}$$

COMBAL: Use of commodity $\leq$ commodity supply

$$D_{t,c} + \sum_{p \in P^{in}_c, c' \varepsilon C} \frac{x^{act}_{t,p}}{\phi_{p,c,c'}} \leq \sum_{p \in P^{out}_c} x^{act}_{t,p}, \quad \forall t \in [2015, 2026], \quad c \in C \backslash C_{supply} \tag{5}$$

COMPRD: Commodity production must equal corresponding process activity

$$x^{prd}_{t,c} = \sum_{p \in P^{out}_c} x^{act}_{t,p} \quad , \forall t \in [2015, 2026] \quad , c \in C_{prod} \tag{6}$$

CAPUP: Capacity upper bounds

$$i^{cap}_{t,p} \leq U^{cap}_{t,p} \quad \forall t \in [2015, 2026] \quad , p \in P_{prod} \tag{7}$$

The modeling described above makes simplifying assumptions such as: 1) invested capacities are maintained (not depreciated) during their technical life, 2) economical lifetimes different from technical lifetimes are not considered, 3) vintages are not considered, and 4) early retirement is not considered.

2.2. Top-down computable general equilibrium model

A closed economy with production and competitive behavior throughout the economy is considered. A simple nesting structure is employed, where capital and labor are combined using a constant elasticity of substitution (CES) production function. The capital-labor composite is further combined with intermediate goods, using a Leontief production function (see Fig. 3).

In general, the economy is characterized by m firms, producing n goods to h households owning f factors. The stylized economy consists of four firms (or sectors) and one representative household. Each of the firms is producing one good. These goods are gas, electricity (ele), manufacturing (man) and non-manufacturing (non) respectively. The household owns two production factors: labor and capital. The behavior of the agents is modelled based on preferences, technology and budget constraints. The firms are assumed to maximize their profits, due to their production technology and their use of available production factors. The household is assumed to be maximizing its utility by spending its budget earned from its production factors. A Stone-Geary utility function is assumed, which gives rise to a linear expenditure system (a description is provided by Goldberger and Gamaletsos [49] page 364, see Lluch [50] for further references). The economic transactions from the base year are described in a social accounting matrix (SAM), which is shown in Table 1.

To simplify our hybrid implementations and improve readability, we assume positive prices for all goods and factors, and we assume that all four firms are producing in the equilibrium solution (as is the case in the base year). We formulate the CGE as a primal mathematical program, and define our equations with equal signs, instead of oriented inequalities. This allows us, without loss of generality, to simplify the NLP formulation and run the same code in NLP and MCP model setups. The mathematical model is defined as:

Sets

I	Sectors in top-down model, indexed by i and j.

Parameters

KS	Capital endowment (given in the SAM).
LS	Labor endowment (given in the SAM).
$io_{i,j}$	Input-output coefficient, amount input of good i to produce one unit of good j (calculated from the SAM).
σ^F_i	Constant elasticity of substitution (CES) between capital and labor in firm i.
γ^F_i	Distribution factor in CES production function of firm i.
a^F_i	Efficiency parameter in CES production function of firm i.
σ^h_i	Income elasticity of demand for good i.
α^h_i	Household marginal budget share of good i, sum over i equals one.
μ^h_i	Household subsistence level of good i.

Variables

p_l	Price of labor (wage rate) (normalized to one in the base year).
p_k	Price of capital (return to capital) (normalized to one in the base year).
p_i	Price of good i (normalized to one in the base year).
x_i	Production of good i.
h	Household income.
L_i	Use of labor in sector i.
K_i	Use of capital in sector i.
c_i	Consumption of good i.

Zero profit conditions ($\perp x_i$):

$$p_i \cdot x_i = p_l \cdot L_i + p_k \cdot K_i + \sum_{j \in I} io_{j,i} \cdot p_j \cdot x_i \quad , \forall i \in I \tag{8}$$

Market clearing conditions for goods ($\perp p_i$):

$$c_i + \sum_{j \in I} io_{i,j} \cdot x_j = x_i \quad , \forall i \in I \tag{9}$$

Market clearing condition for production factor labor ($\perp p_l$):

$$\sum_{i \in I} L_i = LS \tag{10}$$

Market clearing condition for production factor capital ($\perp p_k$):

$$\sum_{i \in I} K_i = KS \tag{11}$$

Income balance ($\perp h$):

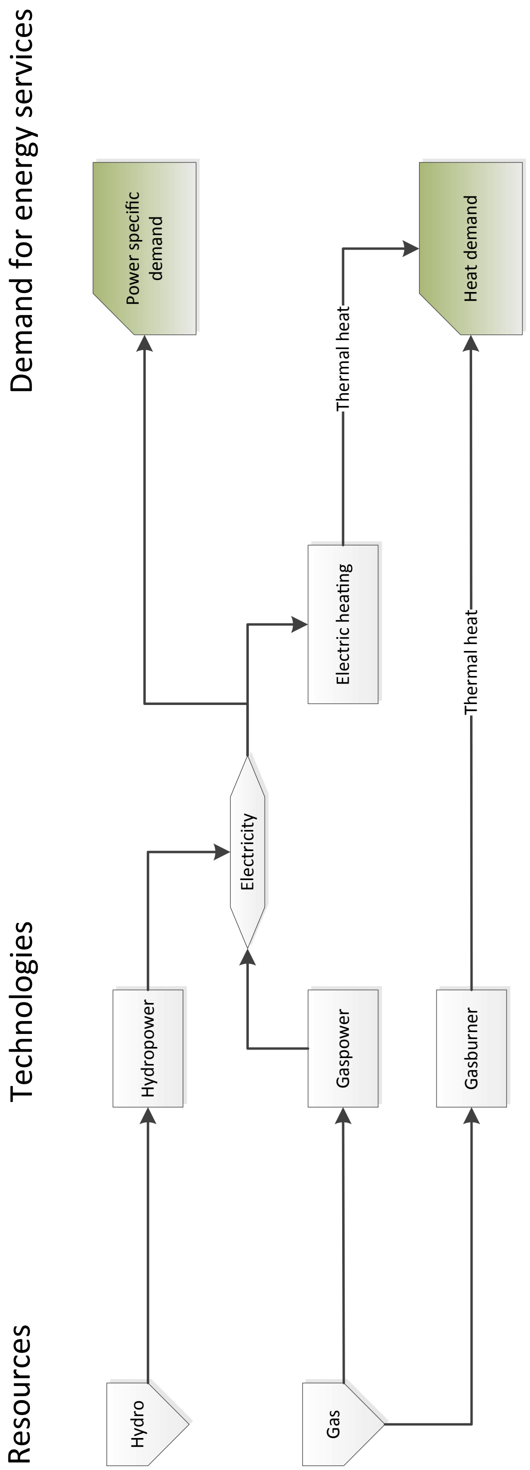

Fig. 2. Structure of the bottom-up model.

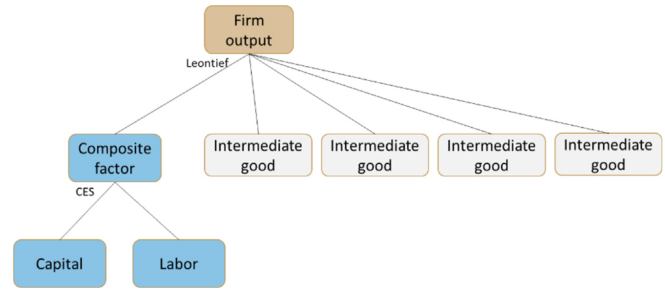

Fig. 3. Nesting structure.

Table 1
Social accounting matrix (SAM).

	gas	ele	man	non	L	K	hou	*Tot*
gas		4	2	3			1	*10*
ele	1	1	7	8			5	*22*
man	1	3	6	26			2	*38*
non	5	10	10	30			92	*147*
L	1	1	5	53				*60*
K	2	3	8	27				*40*
hou					60	40		*100*
Tot	*10*	*22*	*38*	*147*	*60*	*40*	*100*	

$$h = p_k \cdot KS + p_l \cdot LS \tag{12}$$

Household consumption ($\perp c_i$):

$$p_i \cdot c_i = p_i \cdot \mu_i^h + \alpha_i^h \cdot \left(h - \sum_{j \in I} p_j \cdot \mu_j^h \right) \quad , \forall i \in I \tag{13}$$

Firm's use of labor solved explicitly ($\perp L_i$):

$$L_i = \frac{x_i}{a_i^F} \cdot \left(\frac{1-\gamma_i^F}{p_l} \right)^{\sigma_i^F} \left(\gamma_i^{F\sigma_i^F} \cdot p_k^{(1-\sigma_i^F)} + \left(1-\gamma_i^F\right)^{\sigma_i^F} \cdot p_l^{(1-\sigma_i^F)} \right)^{\left(\frac{\sigma_i^F}{(1-\sigma_i^F)} \right)} , \forall i \in I \tag{14}$$

Firm's use of capital solved explicitly ($\perp K_i$):

$$K_i = \frac{x_i}{a_i^F} \cdot \left(\frac{\gamma_i^F}{p_k} \right)^{\sigma_i^F} \left(\gamma_i^{F\sigma_i^F} \cdot p_k^{(1-\sigma_i^F)} + \left(1-\gamma_i^F\right)^{\sigma_i^F} \cdot p_l^{(1-\sigma_i^F)} \right)^{\left(\frac{\sigma_i^F}{(1-\sigma_i^F)} \right)} , \forall i \in I \tag{15}$$

This system is homogenous of degree zero in prices. By Walras's law, one of the equations, against the same number of endogenous variables, is redundant [51]. A consequence is that absolute prices cannot be determined, and all prices are expressed relative to a chosen numeraire. The price of labor p_l is defined as numeraire, and the value is fixed to 1. In the base year, all prices are assumed to be equal to unity.

The modeling makes simplifying assumptions such as: 1) capital and labor are mobile among sectors and exogenously fixed, 2) there are no savings and investments, 3) there is no government, 4) the economy is closed, and 5) the model is static.

2.3. Links between the models

Fig. 4 shows the conceptual coupling between the top-down and bottom-up models. The top-down model calculates a future equilibrium based on exogenous changes (economic shocks), and the

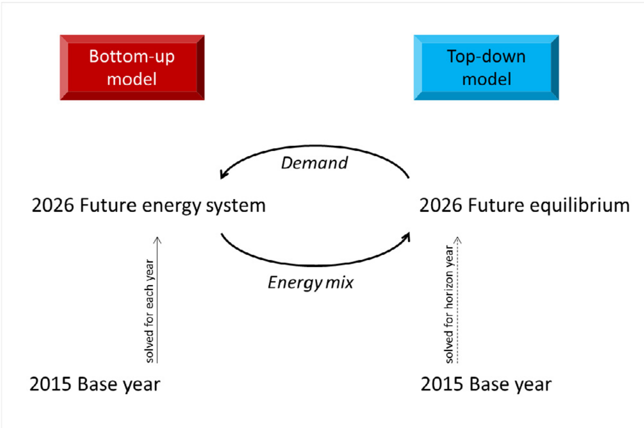

Fig. 4. Model coupling.

future economic equilibrium decides the demand for energy services in the horizon year of the bottom-up model. The static CGE model only calculates the horizon year equilibrium, and we assume for simplicity that demand develops linearly from the base year. A dynamic CGE model would provide demand also in intermediate years. The bottom-up energy system model then calculates the most cost-effective way to supply these energy services. The updated future energy mix is then taken into account by adjusting the input-output structure of the future economic equilibrium.

The bottom-up parameter $D_{t,c}$ for heat and electricity specific demand is calculated from the top-down model:

$$D_{t,c} = D_{2016,c} + D_{2016,c} \cdot \frac{x_{gas} + x_{ele} - x^0_{gas} - x^0_{ele}}{x^0_{gas} + x^0_{ele}} \cdot \frac{(t-2016)}{(2026-2016)},$$
$$\forall t \in [2017, 2026], c \in \{electricitydemand, heatdemand\} \tag{16}$$

There is no direct correspondence (one to one relationship) between demand for energy services in the bottom-up model and the energy commodities in the top-down model. Increased use of gas in the top-down model may correspond to either an increased demand of heat, or an increase of electricity specific demand, in the energy system model. The same logic applies to increased use of electricity in the top-down model. This lack of direct correspondence is a general challenge when we want to link top-down and bottom-up models. For simplicity, we assume that the combined use of gas and electricity in the top-down model gives rise to the same relative increase for heat and electricity specific demand in the bottom-up model.

Furthermore, the top-down parameter $io_{gas,ele}$ (gas input share of the electricity product) is estimated from the bottom-up model:

$$io_{gas,ele} = \frac{x^{act}_{2026,gaspower}}{x^{act}_{2026,electricitydemand}} \tag{17}$$

The gas input share in the top-down model is approximated by the gaspower share of electricity production in the bottom-up model. This relation needs to be calibrated from the problem case that is investigated.

With these equations connecting the models, the parameter input is updated after each model solve, and hard-linked iterations are run until convergence is reached. Convergence is assumed when the relative change from one iteration to the next in 1) total energy system cost, 2) gas input share to electricity sector and 3) projected future demand is below a small tolerance (10^{-6}).

2.4. Reformulation from linear program to mixed complementarity problem

Complementarity problems generalize linear programs (LP), quadratic programs (QP), and convex nonlinear programs (NLPs) [48]. A linear or nonlinear program can be posed as a complementarity problem based on Karush-Kuhn-Tucker (KKT) optimality conditions, by forming the Lagrangian and differentiating. Thus, the bottom-up linear program can be reformulated and expressed as an MCP. The bottom-up linear program expressed as an MCP is presented below.

Dual variables $u^{capact}_{t,p}$, $u^{combal}_{t,c}$, $u^{capup}_{t,p}$ and $v^{comprd}_{t,c}$ are defined for the corresponding bottom-up model constraints. The dual constraints related to variables $i^{cap}_{t,p}$, $x^{act}_{t,p}$ and $x^{prd}_{t,c}$ from the energy system model are provided below. The full bottom-up KKT system is reported in the first seven complementarity conditions of appendix 8.1, listed in equations (A.1) to (A.8).

KKT condition perpendicular to variable $i^{cap}_{t,p}$:

$$(1 - salvage_{t,p})C^{cap}_{t,p} + \sum_{t'=t}^{\min(2026,t+L_p-1)} C^{fom}_{t',p} - \sum_{t'=t}^{\min(2026,t+L_p-1)} A^f_p \cdot \alpha^{capact}_p \cdot u^{capact}_{t',p} + u^{capup}_{t,p} \geq 0 \quad , \forall\ t \in [2015, 2026],\ p \in P_{prod} \tag{18}$$

KKT condition perpendicular to variable $x^{act}_{t,p}$:

$$C^{act}_{t,p} + u^{capact}_{t,p} + \left(\sum_{c \in C_p} -1 + \sum_{c' \in C'_p} \frac{1}{\phi_{p,c,c'}} \right) \cdot u^{combal}_{t,c} + \sum_{c \in C_p} v^{comprd}_{t,c} \geq 0 \quad , \forall t \in [2015, 2026], p \in P_{prod} \tag{19}$$

KKT condition perpendicular to variable $x^{prd}_{t,c}$:

$$C^{prd}_{t,c} - v^{comprd}_{t,c} \geq 0 \quad , \forall t \in [2015, 2026], c \in C_{prod} \tag{20}$$

This MCP reformulation of the bottom-up model may be used for hard-linking the models, in the same way as the LP formulation.

2.5. Integrated mixed complementarity problem formulation

Instead of solving hard-linked models by exchanging model results, all variables and constraints, as well as the linking expressions, may be collected into one integrated model.

Since the CGE model is formulated as an MCP, the bottom-up reformulation gives us the opportunity to collect all variables, equations and complementarity conditions into one integrated MCP formulation. The linking parameters $D_{t,c}$ and $io_{gas,ele}$ are expressed endogenously in this integrated model, instead of being exchanged iteratively between the hard-linked models.

The MCP formulations reflect the reaction curve for each player, and are developed from the KKT conditions. A solution from the integrated MCP model constitutes a Nash equilibrium, where no player may gain from a unilateral change of strategy if the strategies of the others remain unchanged. Each player is assumed to take his decision simultaneously, and each player is assumed to know the equilibrium strategies of the other players.

The integrated MCP-model is provided in appendix 8.1.

2.6. Integrated nonlinear program formulation

The CGE model may also be posed as an NLP problem. By assuming strictly positive prices for all goods and factors, and that all four firms are producing in the equilibrium solution, we can define all equations as equalities and solve the CGE model as an NLP. This assumption is not unreasonable as long as the CGE model is rather aggregated, with few sectors. The NLP formulation of the CGE model may be used for hard-linking the separate models, in the same way as the MCP model.

The CGE model does not have any objective function (the model just solves a system of nonlinear equations in order to find an equilibrium solution.). We may therefore extend the NLP CGE model with the bottom-up variables, equations *and objective function*, and include the affected linking parameters $D_{t,c}$ and $io_{gas,ele}$ using the endogenous mathematical expressions defined in 2.3.

When the NLP CGE model and the bottom-up LP model is fully integrated rather than hard-linked, the resulting model is equivalent to a multi-follower bi-level optimization problem, with the energy system at the upper level and the firms and household at the lower level. The solution from this model will constitute a Stackelberg equilibrium.

The integrated LP-NLP hybrid model is reported in appendix 8.2.

3. Analysis and results

In this section the four hard-linked and the two integrated model setups are introduced, and an instructive test problem is described in detail. The hard-linking convergence is described. All model variants are run over a problem grid defining 2501 problem instances. Equilibrium solutions are compared and convergence results are described.

3.1. Model setups

We implement four variants of hard-linking (alternatives A-D), see Table 2. The bottom-up model is either expressed as a linear programming problem (being solved by the CPLEX solver from IBM), or as a mixed complementarity problem (being solved by the PATH solver from University of Wisconsin - Madison). The top-down model is either expressed as a mixed complementarity problem (being solved by the PATH solver), or as a nonlinear programming problem (being solved by the CONOPT solver from ARKI Consulting and Development).

As explained in the previous section, we have two integrated model setups, see Table 3. The bottom-up and top-down models are run together, by collecting all variables and constraints into an integrated hybrid model. The integrated models are solved by expressing them either as one mixed complementarity problem (being solved by the PATH solver), or as one nonlinear programming problem (being solved by the CONOPT solver).

The six different setups are shown in Fig. 5. All our hybrid models are implemented in GAMS.[5]

In order to demonstrate the dynamic behavior of the models, we run an instructive test problem where we assume that available labor in the CGE model increases by 10% compared with the base year. We also assume that the energy system has unused potential for hydropower electricity production. Thus, the bottom up model invests in hydropower production facilities, and the share of gaspower in the electricity mix decreases (see Fig. 6 in the next section). The bottom-up model is dynamic and solves for each year, while the static CGE model only solves for the future equilibrium in 2026 (see time dimension depicted in Fig. 4). For simplicity we assume that demand for energy services in the bottom-up model grows linearly from the base year to the future demand derived from the CGE model. A dynamic CGE model would provide demand also in intermediate years.

All input parameters are provided in appendix 8.3.

Results from the test problem are shown in the next section,

Table 2
Hard-linked model setups.

Bottom-up\Top-down	MCP	NLP
LP	A	C
MCP	B	D

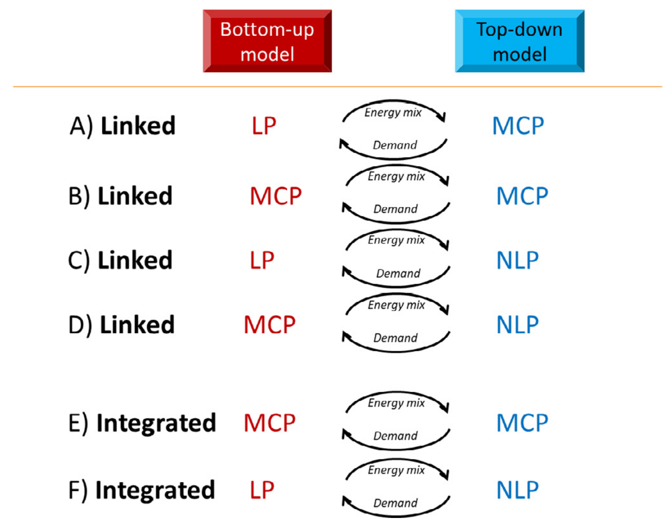

Fig. 5. Hybrid model setups.

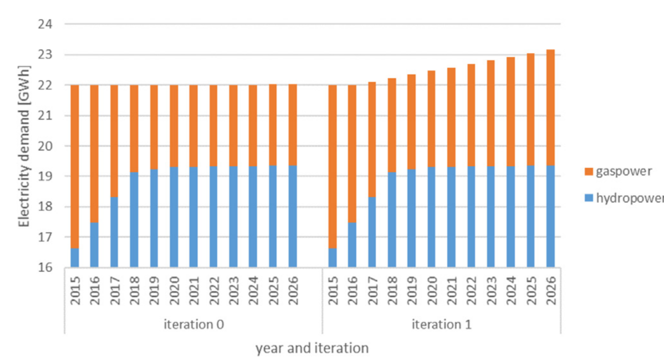

Fig. 6. Bottom-up model response from demand increase in iteration 1. Iteration 0 is initial bottom-up solution.

Table 3
Integrated model setups.

Bottom-up\Top-down	MCP	NLP
LP		F
MCP	E	

[5] General Algebraic Modeling System, see www.gams.com.

Table 4
Relative changes in Social Accounting Matrix from increasing labor supply by 10% [all values in per cent] for iteration 1.

	gas	ele	man	non	L	K	hou	Tot	Price increase	Volume increase
gas		10.1%	10.6%	10.8%			7.7%	10.1%	4.5%	**5.4%**
ele	10.2%	10.1%	10.6%	10.8%			8.2%	10.1%	4.6%	**5.3%**
man	10.3%	10.2%	10.7%	10.9%			8.7%	10.7%	4.7%	5.7%
non	9.1%	9.0%	9.5%	9.7%			9.9%	9.7%	3.6%	6.0%
L	11.2%	11.1%	11.1%	9.9%				*10.0%*	*0%*	*10.0%*
K	12.2%	13.1%	12.2%	7.9%				9.4%	9.4%	*0%*
hou					*10.0%*	9.4%		9.7%		
Tot	10.1%	10.1%	10.7%	9.7%	*10.0%*	9.4%	9.7%		4.7%	5.7%

demonstrating the dynamic interplay between the models. Results from 4 hard-linked hybrid models and 2 integrated hybrid models are compared. Then all 6 hybrid model setups are run over a problem grid where *both* the growth of capital and labor are adjusted in the top-down model. Again, results from our 4 hard-linked and 2 integrated models are compared.

3.2. *Hybrid model interplay*

Let us demonstrate the interplay between the models, by showing in detail what happens in the first iteration of linking the top-down and bottom-up model. The linking dynamics is driven by a labor increase of 10% in the CGE model. The CGE model utilizes the increased labor supply and finds a new equilibrium. Table 4 shows relative changes in iteration 1. Note that the price of labor is defined as numeraire.

The combined volume demand increase for energy (consisting of gas and electricity, shown in bold in Table 4) of 5.3% is transferred to the bottom-up model. The bottom-up response in terms of electricity production is shown in Fig. 6.

The bottom-up model invests in available capacity of hydropower after 2015, but the demand increase from iteration 0 to iteration 1 is supplied from gas power. The 2026 share of gaspower in iteration 1 still decreases compared with the 2015 share in iteration 0. The top-down model needs less gas to produce the same amount of electricity as before. This change triggers a new adjustment of the equilibrium in the top-down model.

When it comes to the final convergence of the linking, Fig. 7 shows the relative increase in household utility by iteration. The initial increase of labor supply results in a relative increase in household utility of 7.1% in 2026 compared with 2015. The subsequent reduction of gas in the future electricity production raises household utility further to an increase of 8.0% compared with 2015.

Since energy production becomes cheaper, the top-down model reallocates resources, and the perhaps surprising effect is that energy demand decreases after the initial increase (see Fig. 8).[6]

Fig. 9 shows the relative prices in 2026 by iteration, having the price of labor as numeraire. All prices are assumed to be equal to unity in the base year. The labor supply increases, so all other prices increase initially. Electricity production becomes cheaper in the bottom-up model, and the gas input in the top-down model decrease during iterations. The relative price of electricity decreases compared to the labor price. Capital becomes the scarce

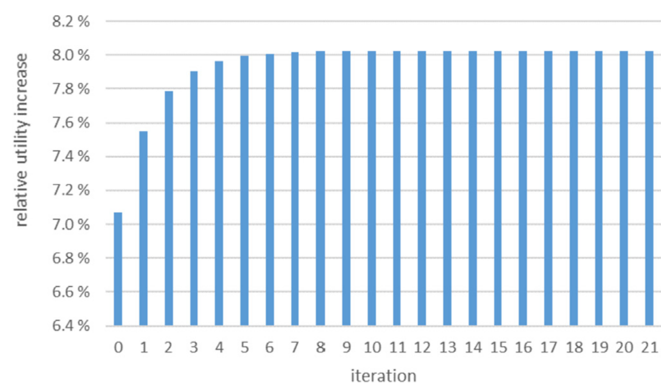

Fig. 7. Relative increase in household utility in 2026 compared with 2015, by iteration.

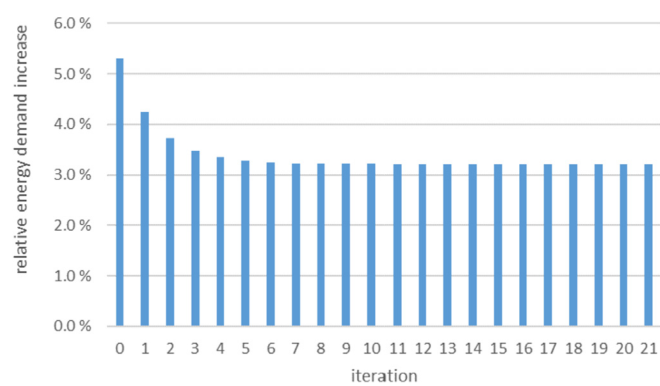

Fig. 8. Relative energy demand increase by iteration.

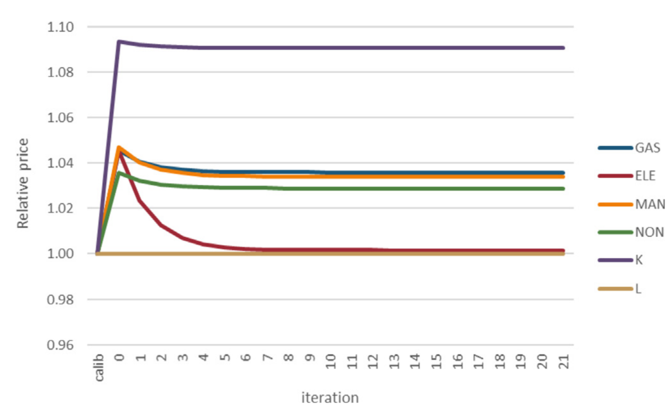

Fig. 9. Relative prices by iteration.

[6] This effect depends on the volume of hydropower potential compared to the growth of the economy. A higher labor (or capital) growth would increase the energy demand further, and exhaust the relative hydropower benefits. Increased use of gas will be required for electricity production, and $io_{gas,ele}$ adjustments will make electricity more expensive (instead of cheaper as seen in Fig. 9). The top-down model will have to reallocate more resources to energy production. The development will be reversed, resulting in decreasing utility and increasing energy demand during iterations following the initial one.

factor with the highest price, while prices of gas, manufacturing and non-manufacturing are grouped in the middle.

An integrated model setup does not produce intermediate solutions from iterations towards a converged solution. Instead the solver knows the whole integrated model, and finds the solution directly. Fig. 10 shows total energy system costs from the linked model setup by iteration, compared with solutions from our two integrated model setups shown as horisontal lines.

The linked energy system costs follow the same pattern as the energy demand shown in Fig. 8. The integrated models directly find solutions with the same level of energy system costs as the linked models. Since the solver can aim for the integrated solution directly instead of solving many intermediate problems, the solution process of the integrated models is much faster than the linked models. (Comparisons of elapsed time for the different models are provided in Table 5.)

Fig. 10 shows that all the models end up with similar energy system costs. A closer inspection of the solutions shows that the integrated LP-NLP model finds a solution with slightly lower costs than the other models (see Fig. 11), but still with increased household utility. This solution pareto-dominates the solution from

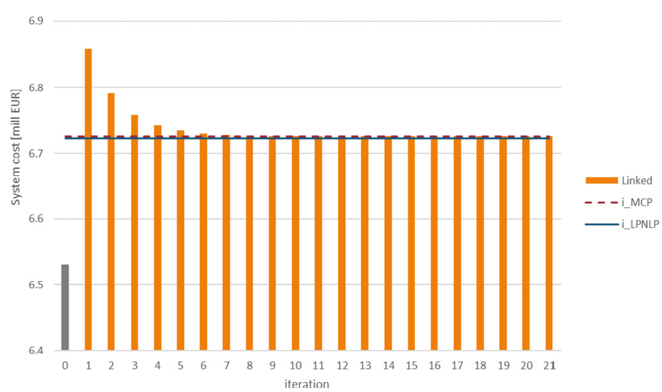

Fig. 10. Energy system costs by iteration, compared with solutions from integrated models.

Table 5
Elapsed time for model variants, solving 2501 problem instances.

Hybrid setup	Model variant	Elapsed (h:m:s)	Solver versions
Hard-linked	LP-NLP	6:41:42	BU: Cplex 12.7.0.0 TD: Conopt 3.17C
Hard-linked	LP-MCP	8:30:23	BU: Cplex 12.7.0.0 TD: Path 4.7.04
Hard-linked	MCP-NLP	8:04:48	BU: Path 4.7.04 TD: Conopt 3.17C
Hard-linked	MCP-MCP	9:46:35	Path 4.7.04
Integrated	NLP	**0:07:15**	Conopt 3.17C
Integrated	MCP	**0:10:31**	Path 4.7.04

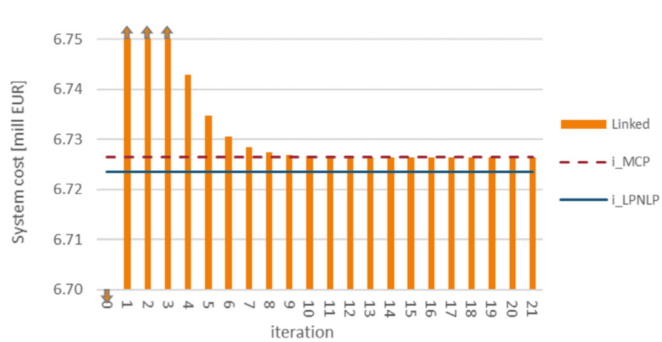

Fig. 11. Energy system costs, showing difference between solutions.

the linked models. The integrated MCP model finds the same solution as the linked models. The differences between the solutions are small in our test problem, energy system cost decreases by 0.04%, while household utility increases by 0.01%. This may seem surprising, but it is important to realize that the integrated MCP and integrated LP-NLP are not identical models. Our integrated MCP model includes the reaction functions of the different players, but the assumption is that their decisions are made simultaneously and there is no first mover advantage. In the integrated LP-NLP model the energy system employs a first mover advantage, and makes its decision before the players in the top-down model, resembling a multi-follower Stackelberg decision process. In our problem, the household follower (in the CGE model) also benefits from lower energy system costs. Thus, the integrated LP-NLP solution pareto-dominates the integrated MCP solution.

The reason for the improvement is increased hydropower investment in the integrated LP-NLP model. Our problem allows considerable investments in new hydropower production from 2016, but available natural resources get exhausted, and after 2020 only small investments are possible. Hydropower investments are decided in the bottom-up model, and have the side effect of affecting the Leontief production function of electricity production in the top-down model. The top-down model observes less use of natural gas in the bottom-up electricity production, which reduces the cost of electricity and consequently demand increases. This demand increase makes the hydropower investment profitable in the bottom-up model.

The linked models and the integrated MCP model do not make the hydropower investment in 2026, because the energy demand in the bottom-up model is too low to make it profitable. In the integrated LP-NLP version, the solver sees the indirect relationships and invests in additional hydropower in 2026. The result is both lower energy system costs and increased household utility.

The hard-linked models are solved separately, and iterates towards an equilibrium. In our test problem, the four hard-linked setups reach the same equilibrium as the integrated MCP.

3.3. Multiple problem instances

A grid of problem instances is defined, where available labor and capital in the CGE model are gradually adjusted. All six model configurations are given the same set of problem instances. Capital is increased by a factor running from 1 to 1.3 (30% increase) in steps of 0.005, while labor is increased by a factor running from 1 to 1.2 (20% increase) in steps of 0.005. This produces 61*41 = 2501 problem instances for our six model setups.

All the hard-linked model configurations find the same solution in every problem instance. They typically also follow the same iteration path – except in 34 out of 2501 problem instances where numerical differences (below the solver tolerance) create an additional iteration.[7] The integrated MCP model finds the same solution as the hard-linked models in every problem instance. As noticed in the previous section, the integrated LP-NLP model finds a different (and improved in terms of lower energy system costs) equilibrium in some problem instances. This is depicted in Fig. 12.

Fig. 13 shows the set of problem instances. Instances where the integrated LP-NLP finds an improved energy system cost solution are colored. This happens in 1067 out of 2501 instances (43%).

The problem instances were solved on a Dell Precision T7600 with two Intel Xeon CPU E5-2650 2 GHz processors using GAMS

[7] Solving the CGE as an MCP problem using the PATH solver compared to solving the CGE as an NLP problem using the CONOPT solver produces one extra iteration in 34 out of 2501 problem instances.

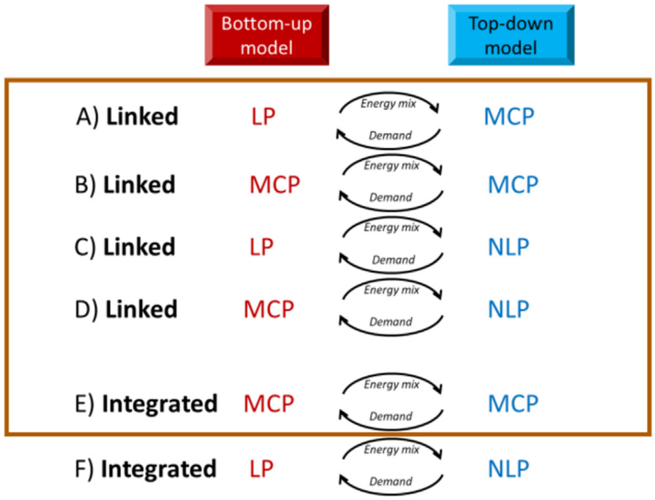

Fig. 12. Model configurations finding the same solutions.

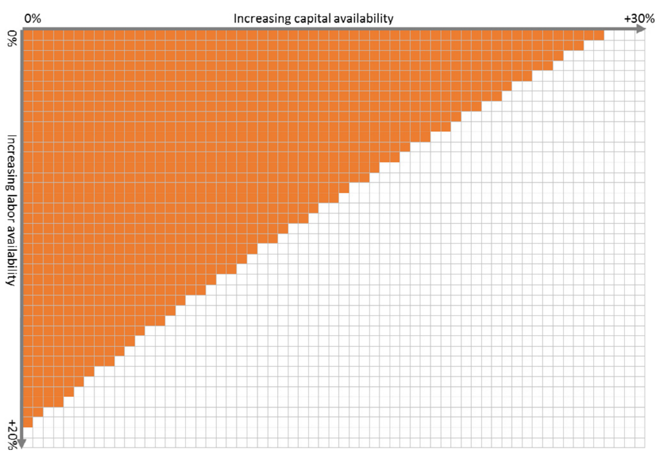

Fig. 13. Problem instances where the integrated LP-NLP model finds improved solution.

version 24.8.3 under Windows 7 SP1 version 6.1.7601 with 32 GB RAM. Computer elapsed time for solving 2501 problem instances are shown in Table 5.

4. Discussion

In this section results are discussed, providing a basis for answering the research questions.

4.1. Equilibria using hard-linking and integration

Four hard-linked and two integrated versions of hybrid modeling have been compared. Our hard-linking method of exchanging primal variable values represents a decomposition of the situation where each player makes his decision simultaneously. The strategies of the other players are signaled through the iterations between the models. All four hard-linked model configurations find the same converged solution in every problem instance. This indicates that the different model formulations are equivalent.

Our integrated MCP version is constructed by extending the CGE model with the KKT conditions from the bottom-up model (represented in the bottom-up MCP reformulation). Thus, an equilibrium problem consisting of each player's KKT conditions together with market clearing conditions is solved, obtaining a generalized Nash equilibrium. In this model each player knows the equilibrium strategies of the other players, and each player makes his decision simultaneously. The integrated MCP finds the same solution as the hard-linked models in all problem instances. This indicates that iterating between linked bottom-up and top-down models will usually produce the same equilibrium solution as the integrated model. This is comforting, since many hybrid approaches consist of soft-linking top-down and bottom-up models. Hard-linking the two models may be seen as a decomposition of the underlying integrated model.

The integrated LP-NLP, on the other hand, finds different solutions. As our results indicate, the integrated LP-NLP and the integrated MCP do not represent the same underlying problem. The integrated LP-NLP formulation corresponds to a multi-follower bilevel problem, with the energy system model as the leader and the CGE players (firms and household) as followers. The leader and the followers play a Stackelberg game, and in some problem instances a Stackelberg equilibrium which differs from the hard-linked and integrated MCP Nash equilibrium is found. Here, the energy system is endowed with a first mover advantage, and the Stackelberg equilibrium represents an improved solution for the energy system (lower system costs). The energy system foresees how the household and firms will react, and is able to decrease the overall energy system cost by making a strategic investment. Interestingly, the CGE household (follower) also profits in the Stackelberg equilibrium, being able to increase its utility. This is due to improved resource utilization enabled by the cost reduction in the energy system. The CGE firms (followers) reach the same zero profit as before, being indifferent between the solutions. Thus, the Stackelberg equilibrium pareto-dominates the generalized Nash equilibrium from the integrated MCP model and the hard-linked models.

The integrated MCP model and the integrated LP-NLP version represent two different situations, the first approach assuming simultaneous decisions and the second a leader-follower formulation. It is interesting that the LP-NLP provides a computationally tractable formulation for a Stackelberg model. In this reformulation, as the energy system can be optimized under the first mover advantage, it manages to reduce the energy system costs by a larger extent than the other setups. In turn, this allows to endow the economy with cheaper energy sources, leading to a general resource efficiency improvement in the whole economic system. The competitive economic setup implies that the benefit of this efficiency improvement is collected by the household. Thus, a lower energy system cost induces a higher household utility level in the integrated LP-NLP model.

Which model that would be preferred, depends on the decision and information structure of the underlying situation. It may be an unrealistic representation to model the energy system as a leader and CGE players as followers. Nevertheless, it is interesting that this produce a pareto-dominant equilibrium with higher value for society. It is an interesting question from a society perspective whether policy measures could be shaped to achieve that equilibrium.

4.2. Hard-linking versus integration: is there a correct choice?

One advantage of linking models, is that the models can be kept separated and intact. The models rely on data collected from different data sources, and often with different product granulation and time resolutions. Bottom-up models focus on quantities and build on national energy balances, while top-down models deal with economic values and build on national accounts. An engineer or an economist starting to work with one of these modeling types has to learn a lot of details in order to run useful analyses. Integrating such models demands combined knowledge and modeling skills from both areas, while linking allows us to retain both models separate and also retain the consistency of each database. This makes linking a natural first step to combine the different areas of expertise.

The integrated approach that has been presented maintains this advantage by merging the formulations of the two problem classes using representations of the linking constraints. The demonstrated approach improves current linking practices, without building new models. The demonstrated integration between the energy system and the whole economy can be implemented across all sectors (full-link). Thus, bottom-up data and expertise could be utilized efficiently. Earlier integrated models, like Böhringer and Rutherford [18] took a different approach, by providing a formulation with a detailed integration of bottom-up technologies in a CGE model, but only for a limited number of sectors and hence not giving a full-link formulation. One of our main contributions is to bring the advantages mentioned above into full-link integrated models.

Hard-linking the models also leads to other challenges. Convergence criteria must be defined and implemented. Programming code enabling linking, control of code execution, logging and error detection needs to be implemented. Cycling may occur during iterations. An integrated hybrid model will allow the solver to handle these kinds of problems, which is a great advantage. A disadvantage is that one integrated model becomes much bigger than the separate models, and thus is harder to solve than solving each model separately.

From the perspective of solution times, integrated models seem at first glimpse better than linked models. This is also confirmed by Böhringer and Rutherford [18] who implemented an efficient decomposition method for their integrated model. Computational time spent by the solver may in theory be either higher or lower with an integrated model compared to a hard-linked model. If both the bottom-up and top-down models are demanding to solve on their own, then linking may be the only feasible way to move forward.

5. Conclusions

We have implemented both hard-linking and integration between a top-down computable general equilibrium model and a bottom-up energy system model. Our main contribution is the development of a full-link integrated model. Our approach is generic, and investigates the possibility to integrate instead of hard-linking hybrid models. Four implementations of hard-linked models and one equivalent integrated full-link MCP hybrid model produced the same solutions in all 2501 problem instances. The integration between the energy system and the whole economy that we demonstrate, can be implemented across all sectors (full-link). Our experiments show that when the solver has knowledge of the full integrated model, time-consuming linking iterations between large-scale models may be avoided as well as avoiding a lot of programming code that otherwise must be customized for model linking. The integrated model maintains the advantages of the linked approach by keeping the CGE and bottom-up formulations and their respective data sets intact, and avoids its computational problems by solving the full model directly.

The work also shows that two closely related implementations of integrated models may find different solutions on the same problem instances. A reformulation into an integrated optimization NLP instead of an MCP, represents a Stackelberg formulation where the energy system has a first mover advantage and the firms and household act as followers. In many cases this integrated LP-NLP model finds a Stackelberg equilibrium that differ from the generalized Nash equilibrium found by the integrated MCP and the hard-linked models. Interestingly the Stackelberg equilibrium pareto-dominates the generalized Nash equilibrium in our test cases.

Further research could provide improved methods to update the production functions based on the bottom-up model. CES functions tend toward factor share preservation, so an alternative might be to update CES factor shares instead of Leontief coefficients. Furthermore, a recursive dynamic CGE model would provide opportunities for information exchange in intermediate years, providing improved coupling along the time dimension.

Acknowledgements

This work was made possible by the financial support of the Norwegian Research Council through the funding of the project Regional effects of energy policy (216513), and by the financial support from Enova SF for the doctoral work of Per Ivar Helgesen.

The authors would like to thank Paolo Pisciella, Ruud Egging and Arne Lind for their valuable comments and suggestions.

8. Appendix

The complete integrated models and input parameters are listed in the appendix.

8.1. Integrated mixed complementarity problem – mathematical formulation

KKT_CAP_INVEST ($\perp i^{cap}_{t,p}$):

$$\begin{aligned}&(1-salvage_{t,p})C^{cap}_{t,p}+\sum_{t'=t}^{\min(2026,t+L_p-1)}C^{fom}_{t',p}\\&-\sum_{t'=t}^{\min(2026,t+L_p-1)}A^f_p\cdot\alpha^{capact}_p\cdot u^{capact}_{t',p}+u^{capup}_{t,p}\ge 0,\\&\quad\forall\ t\in[2015,2026],\ \ p\in P_{prod}\end{aligned}\tag{A.1}$$

KKT_VAR_ACT ($\perp x^{act}_{t,p}$):

$$C^{act}_{t,p}+u^{capact}_{t,p}+\left(\sum_{c\in C_p}-1+\sum_{c'\in C'_p}\frac{1}{\phi_{p,c,c'}}\right)\cdot u^{combal}_{t,c}+\sum_{c\in C_p}v^{comprd}_{t,c}\ge 0,\quad\forall t\in[2015,2026],p\in P_{prod}\tag{A.2}$$

KKT_COM_PRD ($\perp x^{prd}_{t,c}$):

$$C^{prd}_{t,c}-v^{comprd}_{t,c}\ge 0\quad,\forall t\in[2015,2026],\quad c\in C_{prod}\tag{A.3}$$

CAPACT: Process activity $\le$ capacity ($\perp u^{capact}_{t,p}$):

$$x^{act}_{t,p}\le\sum_{v=\max(2015,t-L_p+1)}^{t}A^f_p\cdot\alpha^{capact}_p\cdot i^{cap}_{v,p}+A^f_p\cdot\alpha^{capact}_p\cdot\rho_{t,p}\cdot I^{cap}_{2015,p},\quad\forall t\in[2015,2026],p\in P_{prod}\tag{A.4}$$

COMBAL: Use of commodity $\le$ commodity supply ($\perp u^{combal}_{t,c}$):

$$D_{t,c} + \sum_{p\in P_c^{in}, c' \varepsilon C} \frac{x_{t,p}^{act}}{\phi_{p,c,c'}} \leq \sum_{p\in P_c^{out}} x_{t,p}^{act}, \forall\ t\in[2015, 2016],\ c\in C\backslash C_{supply} \tag{A.5}$$

$$D_{2016,c} + D_{2016,c}\cdot\frac{x_{gas}+x_{ele}-x_{gas}^0-x_{ele}^0}{x_{gas}^0+x_{ele}^0}\cdot\frac{(t-2016)}{(2026-2016)} + \sum_{p\in P_c^{in}, c' \varepsilon C} \frac{x_{t,p}^{act}}{\phi_{p,c,c'}} \leq \sum_{p\in P_c^{out}} x_{t,p}^{act}, \forall t\in[2017, 2026], c\in C\backslash C_{supply} \tag{A.6}$$

COMPRD: Commodity production must equal corresponding process activity ($\perp v_{t,c}^{comprd}$):

$$x_{t,c}^{prd} = \sum_{p\in P_c^{out}} x_{t,p}^{act}, \forall t\in[2015, 2026], c\in C_{prod} \tag{A.7}$$

CAPUP: Capacity upper bounds ($\perp u_{t,p}^{capup}$):

$$i_{t,p}^{cap} \leq U_{t,p}^{cap}\, \forall t\in[2015, 2026], p\in P_{prod} \tag{A.8}$$

Zero profit conditions ($\perp x_i$):

$$p_i\cdot x_i = p_l\cdot L_i + p_k\cdot K_i + \sum_{j\in I} io_{j,i}\cdot p_j\cdot x_i, \forall i\in I\backslash\{ELE\} \tag{A.9}$$

$$p_i\cdot x_i = p_l\cdot L_i + p_k\cdot K_i + \sum_{j\in I\backslash\{gas\}} io_{j,i}\cdot p_j\cdot x_i + \frac{x_{2026,gaspower}^{act}}{x_{2026,electricitydemand}^{act}}\cdot p_{GAS}\cdot x_i, i\in\{ELE\} \tag{A.10}$$

Market clearing conditions for goods ($\perp p_i$):

$$c_i + \sum_{j\in I} io_{i,j}\cdot x_j = x_i, \forall i\in I\backslash\{GAS\} \tag{A.11}$$

$$c_i + \sum_{j\in I\backslash\{ELE\}} io_{i,j}\cdot x_j + \frac{x_{2026,gaspower}^{act}}{x_{2026,electricitydemand}^{act}}\cdot x_{ELE} = x_i, i\in\{GAS\} \tag{A.12}$$

Market clearing conditions for production factor labor ($\perp p_l$):

$$\sum_{i\in I} L_i = LS \tag{A.13}$$

Market clearing conditions for production factor capital ($\perp p_k$):

$$\sum_{i\in I} K_i = KS \tag{A.14}$$

Income balance ($\perp h$):

$$h = p_k\cdot KS + p_l\cdot LS \tag{A.15}$$

Household consumption ($\perp c_i$):

$$p_i\cdot c_i = p_i\cdot\mu_i^h + \alpha_i^h\cdot\left(h - \sum_{j\in I} p_j\cdot\mu_j^h\right), \forall i\in I \tag{A.16}$$

Definition, firm's use of labor ($\perp L_i$):

$$L_i = \frac{x_i}{a_i^F}\cdot\left(\frac{1-\gamma_i^F}{p_l}\right)^{\sigma_i^F}\left(\gamma_i^{F\sigma_i^F}\cdot p_k^{(1-\sigma_i^F)} + \left(1-\gamma_i^F\right)^{\sigma_i^F}\cdot p_l\right)^{\left(\frac{\sigma_i^F}{(1-\sigma_i^F)}\right)}, \forall i\in I \tag{A.17}$$

Definition, firm's use of capital ($\perp K_i$):

$$K_i = \frac{x_i}{a_i^F}\cdot\left(\frac{\gamma_i^F}{p_k}\right)^{\sigma_i^F}\left(\gamma_i^{F\sigma_i^F}\cdot p_k + \left(1-\gamma_i^F\right)^{\sigma_i^F}\cdot p_l\right)^{\left(\frac{\sigma_i^F}{(1-\sigma_i^F)}\right)}, \forall i\in I \tag{A.18}$$

8.2. Integrated nonlinear program — mathematical formulation

Minimize system costs:

$$\min_{i_{t,p}^{cap}, x_{t,p}^{act}, x_{t,c}^{prd}} \left(\begin{array}{l} \sum_{t=2015}^{2026}\sum_{p\in P}(1-salvage_{t,p})\left(C_{t,p}^{cap}\bullet i_{t,p}^{cap}\right) \\ + \sum_{v=2015}^{2026}\sum_{p\in P}\sum_{t=v}^{\min(2026,v+L_p-1)} C_{t,p}^{fom}\bullet i_{v,p}^{cap} + \sum_{t=2015}^{2026}\sum_{p\in P} C_{t,p}^{act}\bullet x_{t,p}^{act} + \\ \sum_{t=2015}^{2026}\sum_{c\in C} C_{t,c}^{prd}\bullet x_{t,c}^{prd} \end{array} \right) \tag{A.19}$$

subject to

CAPACT: Process activity<Roman> = </Roman>capacity

$$x_{t,p}^{act} \leq \sum_{v=\max(2015,t-L_p+1)}^{t} A_p^f\cdot\alpha_p^{capact}\cdot i_{v,p}^{cap} + A_p^f\cdot\alpha_p^{capact}\cdot\rho_{t,p}\cdot I_{2015,p}^{cap}, \forall t\in[2015, 2026], p\in P_{prod} \tag{A.20}$$

COMBAL: Use of commodity<Roman> = </Roman>Commodity supply

$$D_{t,c} + \sum_{p \in P_c^{in}, c' \varepsilon C} \frac{x_{t,p}^{act}}{\phi_{p,c,c'}} \le \sum_{p \in P_c^{out}} x_{t,p}^{act}, \forall t \in [2015, 2016], c \in C \backslash C_{supply} \tag{A.21}$$

$$D_{2016,c} + D_{2016,c} \cdot \frac{x_{gas} + x_{ele} - x_{gas}^0 - x_{ele}^0}{x_{gas}^0 + x_{ele}^0} \cdot \frac{(t - 2016)}{(2026 - 2016)} + \sum_{p \in P_c^{in}, c' \varepsilon C} \frac{x_{t,p}^{act}}{\phi_{p,c,c'}} \le \sum_{p \in P_c^{out}} x_{t,p}^{act}, \forall t \in [2017, 2026], c \in C \backslash C_{supply} \tag{A.22}$$

COMPRD: Commodity production must equal corresponding process activity

$$x_{t,c}^{prd} = \sum_{p \in P_c^{out}} x_{t,p}^{act}, \forall t \in [2015, 2026], c \in C_{prod} \tag{A.23}$$

CAPUP: Capacity upper bounds

$$i_{t,p}^{cap} \le U_{t,p}^{cap} \forall t \in [2015, 2026], p \in P_{prod} \tag{A.24}$$

Zero profit conditions:

$$p_i \cdot x_i = p_l \cdot L_i + p_k \cdot K_i + \sum \sum_{j \in I} io_{j,i} \cdot p_j \cdot x_i, \forall i \in I \backslash \{ELE\} \tag{A.25}$$

$$p_i \cdot x_i = p_l \cdot L_i + p_k \cdot K_i + \sum_{j \in I\{gas\}} io_{j,i} \cdot p_j \cdot x_i + \frac{x_{2026,gaspower}^{act}}{x_{2026,electricitydemand}^{act}} \cdot p_{GAS} \cdot x_i, i \in \{ELE\} \tag{A.26}$$

Market clearing conditions for goods:

$$c_i + \sum_{j \in I} io_{i,j} \cdot x_j = x_i, \forall i \in I \backslash \{GAS\} \tag{A.27}$$

$$c_i + \sum_{j \in I\{ELE\}} io_{i,j} \cdot x_j + \frac{x_{2026,gaspower}^{act}}{x_{2026,electricitydemand}^{act}} \cdot x_{ELE} = x_i, i \in \{GAS\} \tag{A.28}$$

Market clearing condition for production factor labor:

$$\sum_{i \in I} L_i = LS \tag{A.29}$$

Market clearing condition for production factor capital:

$$\sum_{i \in I} K_i = KS \tag{A.30}$$

Income balance:

$$h = p_k \cdot KS + p_l \cdot LS \tag{A.31}$$

Household consumption:

$$p_i \cdot c_i = p_i \cdot \mu_i^h + \alpha_i^h \cdot \left(h - \sum_{j \in I} p_j \cdot \mu_j^h \right), \forall i \in I \tag{A.32}$$

Definition, firm's use of labor:

$$L_i = \frac{x_i}{a_i^F} \cdot \left(\frac{1 - \gamma_i^F}{p_l} \right)^{\sigma_i^F} \left(\gamma_i^{F\sigma_i^F} \cdot p_k^{(1-\sigma_i^F)} + \left(1 - \gamma_i^F\right)^{\sigma_i^F} \cdot p_l^{(1-\sigma_i^F)} \right)^{\left(\frac{\sigma_i^F}{(1-\sigma_i^F)} \right)}, \forall i \in I \tag{A.33}$$

Definition, firm's use of capital:

$$K_i = \frac{x_i}{a_i^F} \cdot \left(\frac{\gamma_i^F}{p_k} \right)^{\sigma_i^F} \left(\gamma_i^{F\sigma_i^F} \cdot p_k^{(1-\sigma_i^F)} + \left(1 - \gamma_i^F\right)^{\sigma_i^F} \cdot p_l^{(1-\sigma_i^F)} \right)^{\left(\frac{\sigma_i^F}{(1-\sigma_i^F)} \right)}, \forall i \in I \tag{A.34}$$

8.3. Input data

Table 6
Energy system parameters with time dimension

Parameter	Unit	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026
Investment cost $C_{t,p}^{cap}$													
Hydropower (50 years lifetime)	kNOK/MW	12200	12200	12200	12200	12200	12200	12200	22200	22200	22200	22200	22200
Gaspower (25 years lifetime)	kNOK/MW	3200	3200	3200	3200	3200	3200	3200	3200	3200	3200	3200	3200
Gasburner (25 years lifetime)	kNOK/GWh/a	46.3	46.3	46.3	46.3	46.3	46.3	46.3	46.3	46.3	46.3	46.3	46.3
Electric heating (25 years lifetime)	kNOK/GWh/a	2100	2100	2100	2100	2100	2100	2100	2100	2100	2100	2100	2100
Fixed operating and maintenance cost $C_{t,p}^{fom}$													
Hydropower	kNOK/MW	205.23	205.23	205.23	205.23	205.23	205.23	205.23	205.23	205.23	205.23	205.23	2300
Gaspower	kNOK/MW	96	96	96	96	96	96	96	96	96	96	96	96
Gasburner	kNOK/GWh/a	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8
Electric heating	kNOK/GWh/a	2	2	2	2	2	2	2	2	2	2	2	2
Variable cost $C_{t,p}^{act}$													
Hydropower	kNOK/GWh	4.37	4.37	4.37	4.37	4.37	4.37	4.37	4.37	4.37	4.37	4.37	4.37
Production cost $C_{t,c}^{prd}$													
Natural gas	kNOK/GWh	130	153	130	130	130	130	130	130	130	130	130	130
Salvage value share in 2026 by investment year													
Hydropower (50 years)	(unitless)	0.76	0.78	0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98
Gaspower (25 years)	(unitless)	0.52	0.56	0.60	0.64	0.68	0.72	0.76	0.80	0.84	0.88	0.92	0.96
Gasburner (25 years)	(unitless)	0.52	0.56	0.60	0.64	0.68	0.72	0.76	0.80	0.84	0.88	0.92	0.96
Electric heating (25years)	(unitless)	0.52	0.56	0.60	0.64	0.68	0.72	0.76	0.80	0.84	0.88	0.92	0.96
Bound on capacity investment $U_{t,p}^{cap}$													
Hydropower	MW	2	0.1	0.1	0.1	0.01	0.01	0.001	0.001	0.001	0.001	0.001	0.001
Demand $D_{t,c}$													
Heat demand	GWh	14.2843	14.2857	(demand in 2026 defined from top-down model, linearly interpolated to 2016)									
Electricity demand	GWh	21.9978	22	(demand in 2026 defined from top-down model, linearly interpolated to 2016)									

Table 7
Energy system parameters by process

Technology	Availability of capacity A_p^f	Capacity to activity conversion factor α_p^{capact}	Technical lifetime L_p (years)
Hydropower	0.95	8.76	50
Gaspower	1	8.76	25
Gasburner	1	1	25
Electric heating	1	1	25

Table 8
Energy system parameters for commodity conversion processes

Technology	Input commodity	Output commodity	Flow conversion factor $\phi_{p,c,c'}$
Gaspower	Natural gas	Electricity	0.4
Gasburner	Natural gas	Heat	0.95
Elheater	Electricity	Heat	1.0
Hydropower	Hydro	Electricity	1.0

Table 9
Input parameters to top-down CGE model.

	gas	ele	man	Non
Capital-Labor substitution elasticity σ_i^F	0.9	0.8	0.9	1.2
Income elasticity of demand σ_i^h	0.7	0.8	0.9	1.2

Remaining parameters for the top-down CGE model

To calculate remaining model parameters, we define all relative prices to be equal to one in the base year, and we define the following intermediate parameters:

K_i^{base} is capital use in sector i in the base year (given in the SAM)
L_i^{base} is labor use in sector i in the base year (given in the SAM)
XD_i^{base} is gross production from sector i in the base year (calculated from the SAM)
C_i^{base} is consumer commodity demand in the base year (given in the SAM)
I^{base} is consumer income in the base year (given in the SAM)

For the Stone-Geary utility function, we define the Frisch parameter (which determines the money flexibility [52]): $\phi = -1.2$

Household marginal budget share of good i (rescaled such that sum over i equals one):

$$\alpha_i^h = \sigma_i^h \frac{C_i^{base}}{I^{base}} \tag{A.35}$$

Household subsistence level of good i:

$$\mu_i^h = C_i^{base} + \frac{\alpha_i^h \cdot I^{base}}{\phi} \tag{A.36}$$

Distribution factor in CES production function of firm *i:*

$$\gamma_i^F = 1 \Bigg/ \left(1 + \left(\frac{K_i}{L_i}\right)^{\frac{-1}{\sigma_i^F}}\right) \tag{A.37}$$

Efficiency parameter in CES production function of firm *i:*

$$a_i^F = XD_i^{base} \Bigg/ \left(\gamma_i^F \cdot K_i^{base\left(\frac{\sigma_i^F-1}{\sigma_i^F}\right)} + \left(1-\gamma_i^F\right) \cdot L_i^{base\left(\frac{\sigma_i^F-1}{\sigma_i^F}\right)}\right)^{\left(\frac{\sigma_i^F}{\left(\sigma_i^F-1\right)}\right)} \tag{A.38}$$

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