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A Multistage Stochastic Optimization Model for Trading on the German Intraday Power Market

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Problem description

As the market share of intermittent renewable power production in Germany grows, the need for short term optimization of power marketing drives trading activity towards the EPEX Intraday power market. However, optimizing trading decisions in the market is hard, for three reasons; firstly, it is assumed that only limited liquidity is available through the Limit Order Book; secondly, unique features of the market complicates the modeling of the stochasticities; and finally, because it is assumed that decisions for consecutive delivery products affect each other.

The motivation for the thesis is to explore the modeling assumptions that are made in the contemporary scientific literature, and evaluate the impact that each assumption has on the profit of the trader. The desired output is a set of recommended modeling assumptions for a hydropower producer with limited energy storage, as well as a rationale for the recommendation.

Based on a thorough review of the contemporary scientific literature, a set of cases is outlined, where each case is related to one specific modeling assumption. For each case, a mathematical model is presented, and implications of the modeling assumption are explained conceptually. Finally, the contribution from each of the recommended modeling assumptions to the trading profit is estimated using scenarios based on an extensive analysis of historical market data.

Preface

This master thesis is written within the field of Applied Economics and Operations Research at the Department of Industrial Economics and Technology Management at the Norwegian University of Science and Technology (NTNU). The thesis is motivated by the increased demand for short term optimization in power markets, and the successful implementation of trading algorithms in other financial markets.

The quality of this thesis is significantly higher thanks to the help of both supervisors. We would like to thank our supervisor Asgeir Tomasgard for making the work with this thesis possible. Despite that this thesis was only tangentially related to his former research, he has asked insightful questions that helped focus our research in productive directions, and his experience with scientific writing has been crucial to the structure of the thesis.

This thesis was not part of an existing research program. Thus, a particularly exciting challenge has been to explore the theoretical background for the thesis and develop the modeling choices during the work with our thesis. The guidance of our co-supervisor Post Doc Gro Klæboe has been vital in this regard. In particular, she has shown genuine interest, invested significant time and given valuable feedback. We are grateful that her employers, Powel AS and Trønderenergi AS, have let her cooperate with us.

Moreover, we would like to thank Marte Fodstad at SINTEF Energi, who took many hours out of her working day to advice us, despite having no formal responsibility to do so. Her insight, enthusiasm and thoughtful feedback has been invaluable to our research.

Abstract

As the German Intraday power market has grown steadily over the last seven years, the academic and commercial interest in mathematical optimization of decision making related to the market has grown. The relevant decisions include both production and trading of 24 hourly delivery products. The trading is organized as a continuous auction implemented as a limit order book for each delivery product. The market opens soon after the clearing of the Day-Ahead market and closes just before delivery. As both the delivery products and the trading decisions for a given delivery product happen sequentially, the decision structure has a *doubly dynamic* trading structure that makes it hard to optimize. Additionally, the liquidity in the market is limited, further complicating the optimization.

Due to the high complexity of the problem, existing papers that attempt to optimize decision making related to the Intraday market make several simplifying assumptions. In order to find the best combination of modeling assumptions to make, an extensive literature review is performed, focusing on the modeling assumptions in each paper. It is found that there is disagreement in the contemporary literature about how to best model the problem. In particular, the relevant papers optimize either order placement in the market, or physical power dispatch, and neglect the other type of decision.

In this thesis, the Intraday Trading Problem (ITP) is defined to include both the order placement and the production optimization. The ITP is further broken down into three subproblems: the price forecasting problem, the cost estimation problem and the strategy formulation problem. Based on the points of disagreement in the related literature, a set of modeling assumptions is defined, mostly relating to the strategy formulation problem. The goal of the thesis is to explore which of the modeling assumptions that are the most profitable to make, and estimate the impact on the objective function of making a given combination of assumptions. Based on the set of modeling assumptions, a proposed benchmark model and six models inspired by the existing literature are defined. In order to test the model, a detailed analysis of historical EPEX Intraday order book data is performed, and a model of the market is developed. The estimation of all the relevant market parameters is a significant expansion compared to the market analyses performed in the existing literature. Small scenario trees are developed based on the market analysis, and the models are tested on a "forest" of semi-optimized, semi-randomized scenario trees. It is found that the proposed benchmark model outperforms the alternative models with a 4-41% premium on the objective value. Finally, the theoretical reasons for the improved performance is discussed, and three avenues for future research are outlined.

Sammendrag

Parallelt med veksten til det tyske Intraday-kraftmarkedet i løpet av de siste syv årene, har interessen for matematisk optimalisering av beslutninger knyttet til markedet vokst innen både akademia og næringslivet. Relevante beslutninger for en trader i markedet inkluderer både fysisk produksjon og budgiving. Handelen er organisert som en kontinuerlig auksjon implementert som 24 ordrebøker; en for hvert timelange leveranseprodukt. Markedet åpner kort etter at Day-Ahead markedet har klarert, og stenger like før leveranse. I og med at både den fysiske produksjonen av leveranseproduktene og budgivingen for hvert enkelt leveranseprodukt skjer sekvensielt, har beslutningene en *dobbelt dynamisk* struktur, som gjør at det er vanskelig å ta optimale beslutninger i markedet. I tillegg er likviditeten i markedet begrenset, noe som ytterligere kompliserer optimeringen.

Grunnet problemets høye kompleksitet gjør den eksisterende optimeringslitteraturen innen emnet flere forenklende antakelser. For å finne det beste settet med modelleringsantakelser, studerer vi den eksisterende litteraturen i detalj, med fokus på modelleringsantakelsene i hver artikkel. Vi finner at de ulike artiklene på emnet har motstridende anbefalinger for hvordan det bør modelleres. Særlig tydelig er skillet mellom artiklene som optimerer allokering av produksjonsressurser, men utelater budgivingsbeslutningene – og de som optimerer budgivingen, gitt at man har en forhåndsbestemt produksjonsplan.

I denne masteravhandlingen defineres Intraday Trading Problem (ITP), som omfatter både budgiving og produksjonsbeslutninger. ITP dekomponeres videre i tre delproblemer: prisprognoseproblemet, kostnadsestimeringsproblemet og strategiformuleringsproblemet. Basert på punktene med uenighet i den eksisterende litteraturen defineres et sett av modellantakelser, med hovedfokus på strategiformuleringsproblemet. Målet med avhandlingen er å utforske hvilke av modelleringsantakelsene som er mest lønnsomme å gjøre, og å anslå virkningen på objektivfunksjonen av gitt kombinasjon av antagelser. Basert på settet med modelleringsantagelser, er det utviklet en anbefalt referansemodell og seks alternative modeller som er inspirert av den eksisterende litteraturen. For å teste modellen er en markedsmodell utviklet basert på en detaljert analyse av historiske ordrebokdata fra EPEX Intraday. Estimeringen av alle de relevante markedsparameterne er en betydelig utvidelse av tidligere forskning på området. Videre utvikler vi begrensede scenariotrær basert på markedsanalysen, og modellene testes på en «skog» av semi-optimaliserte, semi-randomiserte trær. Vi finner at de alternative modellene taper med 4-41% relativt til den foreslåtte modellen. Til slutt diskuteres de teoretiske årsakene til at den anbefalte modellen slår de andre, og tre veier for videre forskning skisseres.

Contents

List of Figures

List of Tables

Nomenclature

Sets

Γ The set of virtual reservoirs.

$\mathcal{D}$ The set of delivery products.

$\mathcal{P}$ The set of order levels.

$\mathcal{S}$ The set of scenarios.

$\mathcal{S}_{tk}$ The set scenarios that are in the same nonanticipativity set k in stage t.

$\mathcal{T}$ The set of trading stages.

K_t The set of nonanticipativity sets for stage t.

Indices

δ A delivery product.

γ A virtual reservoir.

σ A scenario.

d A delivery product.

k A nonanticipativity set.

p A price level.

s A scenario.

t A trading stage.

Decision Variables

$f^{in}_{ds\gamma}$ The inflow allocated to virtual reservoir γ for delivery product d in scenario s.

$f^{out}_{ds\gamma}$ The overflow from virtual reservoir γ for delivery product d in scenario s.

$q_{ds\gamma}$ Production quantity for the generator connected to virtual reservoir γ for delivery product d in scenario s.

x^{A}_{ptds} Ask order volume placed at price level p in trading stage t for delivery product d in scenario s.

x^{B}_{ptds} Bid order volume placed at price level p in trading stage t for delivery product d in scenario s.

Support Variables

q_{ds} The total production quantity for delivery product d in scenario s.

$r_{ds\gamma}$ The reservoir volume in virtual reservoir γ for delivery product d in scenario s.

r_{ds} The total reservoir volume for delivery product d in scenario s.

v^{BM+}_{ds} The balancing up volume for delivery product d in scenario s.

v^{BM-}_{ds} The balancing down volume for delivery product d in scenario s.

v^{A}_{ptds} The cleared ask transaction volume at price level p in trading stage t for delivery product d in scenario s.

v^{B}_{ptds} The cleared bid transaction volume at price level p in trading stage t for delivery product d in scenario s.

Functions

$\Pr(\cdot)$ Probability.

$C^{Q}(\cdot)$ The production cost function.

Operators

$\underline{(\cdot)}$ Lower bound.

$\overline{(\cdot)}$ Upper bound.

$\bigcap$ The intersection of several sets.

$\bigcup$ The union of several sets.

$\hat{(\cdot)}$ Decision or parameter that was saved from a previous iteration of a model.

$|(\cdot)|$ Cardinality of a set.

Deterministic parameters

$\underline{\rho}^{BM}$ A lower bound on the probability that the balancing market constraint is adhered to.

$\underline{Q}$ The minimum production volume.

$\bar{Q}$ The maximum production volume.

$\bar{R}^{BM}$ Maximum total balancing market volume over the day.

ρ_s The probability of scenario s.

A_{ptd} The highest possible ask order volume in the order book at price level p in trading stage t for delivery product d.

B_{ptd} The highest possible bid order volume in the order book at price level p in trading stage t for delivery product d.

C^C The transaction cost.

C^Q_γ The production cost for the virtual reservoir γ.

F^{in}_d The inflow for delivery product d.

$R_{0\gamma}$ The initial reservoir volume in reservoir γ.

V^{DA}_d The market commitment from the Day-Ahead market.

Stochastic parameters

Ξ^A_{ptds} The bid limit order clearing indicator at price level p in trading stage t for delivery product d in scenario s.

Ξ^B_{ptds} The ask limit order clearing indicator at price level p in trading stage t for delivery product d in scenario s.

P_{ds}^{BM+} The balancing up price for delivery product d in scenario s.

P_{ds}^{BM-} The balancing down price for delivery product d in scenario s.

P_{ptds}^{A} The price of ask orders at price level p in trading stage t for delivery product d in scenario s.

P_{ptds}^{B} The price of bid orders at price level p in trading stage t for delivery product d in scenario s.

Time parameters

$\underline{t}_0$ Time of gate opening.

$\bar{t}_d^{GC}$ Gate closure time of delivery product d.

Chapter 1

Introduction

As observed by Akersveen and Graabak (2017), the share of renewable energy production compared to consumption in Germany has grown from 5.8% in 2004 to 14.6% in 2015 (Eurostat, 2017). As renewable intermittent production facilities must deal with stochastic production, the need for more short-time optimization is growing (Hassler, 2017). The German Intraday market has potential to cover this need, and it has grown from 5.6TWh in 2009 (EPEX, 2009) to 41TWh in 2016 (EPEX, 2016), an impressive 33% year over year growth rate. While volumes are still small enough that a single large trader can disrupt the price trajectory for a given day, this may not hold true for long. The market is still small compared to the Day-Ahead market (EPEX, 2016), but falling Day-Ahead margins have power producers on the lookout for new market opportunities (Klaboe and Fosso, 2013). Garnier and Madlener (2015) state that *"The more liquid and competitive the intraday market is, the more efficient it is to balance forecast errors intra-daily"*.

However, optimizing trading in the Intraday market is hard and costly. The trading happens continuously, both during normal working hours and throughout the night. Skilled traders are short in supply, and expensive to keep up at night. Prices are volatile, more so than for the Day-Ahead market (Garnier and Madlener, 2014) and subject to unpredictable Urgent Market Messages and changes in weather forecasts. As the market still is young and changing, little theory exists on how to optimize the trading strategy in it.

In comparison, trading in the Day-Ahead market requires only the preparation of one order curve for each hour for each day. The orders are aggregated to a supply curve, and each producer receives the clearing price for the entire market; this removes the incentive to bid above marginal cost for a price taking producer, in contrast with how the pay-as-bid Intraday market forces suppliers to bid strategically. This makes optimal bidding more complicated in the Intraday market than in the Day-Ahead market. Large volumes are traded on the Day-Ahead market, and a producer with competitive marginal cost can be confident that their order will clear. Significant theory exists on how to optimize orders in the Day-Ahead market.

In this thesis, the *Intraday Trading Problem* is defined. While former papers have focused either on the allocation of production throughout the day or on the trading of a given delivery product, this thesis includes both production and trading optimization in the model. As both the delivery products and the trading decisions for a given delivery product happen sequentially, this leads to a *doubly dynamic* decision structure. Furthermore, the Intraday Trading Problem is decomposed into three parts; the *Price Forecasting Problem*, the *Cost Estimation Problem* and the *Strategy Formulation Problem*.

The goal of this thesis is to explore which assumptions that are profitable to make about the Intraday Trading Problem, and in particular the Strategy Forecasting Problem, and to estimate the impact on the objective function of several common combinations of assumptions.

In order to do so, an extensive literature review is performed. It is found that the contemporary literature has focused more on solution methods than on the modeling, and that there are several traders in the market for which no relevant model has been published. Thus, a proposed benchmark model is developed. Inspired by the models in the related literature, six alternative models with different combinations of assumptions are developed. The assumptions focus in particular on the relation between production and trading, on the coordination of decisions across delivery products, on the value of trading strategies that can alter between sales and purchases, and on the modeling of a market with limited liquidity.

The models are tested for a hydropower producer in small problem instances with a realistic market model. The market model is based on a detailed analysis of commercial order book data. Moreover, the rationale for each of the assumptions are explored theoretically.

It is found that our proposed model beats the ones that are based on assumptions found in the contemporary literature by 4-41% in the given problem instances. Some of the alternative models make very different production decisions from the proposed model, or are unable to obtain good prices for their marketed power. Additionally, the benchmark model successfully exploits the production flexibility of the hydropower producer to offer liquidity to the market at a premium, whereas some of the alternative models don't.

The modeling assumptions in the Intraday market is still an underexplored area, and three avenues for future research are outlined. Firstly, there is need for stronger evidence for our findings; do these profit improvements still apply for different traders - maybe with a larger reservoir or production capacity, or different production assets? How large is the gain when the models are back-tested on historical data, or in simulations with different market parameters? Secondly, it is necessary to be able to scale the problem instances if the model is to be applied in practice. A reformulation to a dynamic model seems necessary in this regard, but would require significant work in order to be tractable, as the state space grows very large. Finally, there is still potential for conceptual modeling improvements that expands the applicability of the model to other market situations or risk preferences.

The rest of the thesis is structured in the following way: in chapter 2 the background for the thesis is covered, including an introduction to the EPEX Intraday market and a thorough literature review. The focus of the chapter is to give a good understanding of the aspects that are relevant for modeling choices in the optimization of trading in the EPEX Intraday market as well as intraday production decisions, and to present an overview of the modeling choices in the contemporary literature. In chapter 3 the questions that this thesis attempts to answer are outlined in more detail, and a set of modeling assumptions is introduced. Chapter 4 covers how to express the assumptions mathematically as a multistage stochastic linear program, and outlines one proposed benchmark model and six alternative models based on combinations of the modeling assumptions. Then, the modeling of the market is explored more in-depth in chapter 5, and the parameters of the market model are estimated based on empirical data. Scenarios are generated based on the estimated parameters, and the models are tested on these scenarios. Results for the models in the different cases are compared. In chapter 7 the results are interpreted, and a theoretical explanation of the implications of all of the modeling assumptions is provided.

Chapter 2

Background

In this chapter, the background for this thesis is presented. Key aspects of trading in power markets in general, and EPEX Intraday in particular, are presented in section 2.1. It should be noted that section 2.1 is strongly based on former work by the authors of this thesis (Akersveen and Graabak, 2017), and partly an excerpt from that report. In section 2.2, the related literature is analyzed in-depth. Modeling assumptions are explored with particular detail, and used as a foundation for the rest of the thesis. While this section also is inspired by (Akersveen and Graabak, 2017), it mainly contains original content.

Henceforth, a set of assumptions is created and used throughout the rest of the thesis. In particular, it is assumed that the optimization of the decisions that are relevant to the EPEX Intraday include both trading and production, and that the relevant constraints include physical, legal and trading-related constraints. This is referred to as the Intraday Trading Problem (ITP) for short, though the problem includes production too. The goal of research on the optimization of the EPEX Intraday decisions is to provide practical advice to the relevant decision makers. Thus, the proposed models in the EPEX Intraday optimization research should make assumptions that contribute to the objective function of the relevant decision makers. Additionally, the models should be tractable to solve with a reasonable amount of computational resources. The decision makers are interchangeably referred to as traders or producers, though obviously they may be both at the same time.

For simplicity, the view of a selling trader is taken throughout the entire thesis when explaining a concept unless something else is specified, although both selling and buying is permitted for the model proposed in chapter 4.

2.1 Introduction to power markets and the EPEX Intraday

In this section, key aspects of trading in power markets in general, and EPEX Intraday in particular, are covered. Initially, a rationale for the current power market structure, as well as a brief introduction to three of the physical power markets, is included in sections 2.1.1-2.1.3. Section 2.1.4 covers the main actors in the EPEX Spot markets. In section 2.1.5, legal requirements to the traders in the EPEX Intraday market are covered, and section 2.1.6 introduces limit order book concepts.

Electrical power is a special commodity for several reasons. Production must at all times equal demand; it isn't possible to store in large scale. Even momentary imbalances will affect the frequency and voltage of the power supply, potentially damaging large amounts of expensive electrical equipment. Larger imbalances may cut the power supply entirely, disconnecting large customer groups through no fault of their own.

Demand is highly seasonal, with both annual, weekly and daily cyclic patterns (see figure 2.1.1). Typically, demand is high in winter, on weekdays and in the morning and afternoon, whereas it is low in summer, in weekends and during the night. With increasing penetration of solar power, supply is also becoming more cyclic.

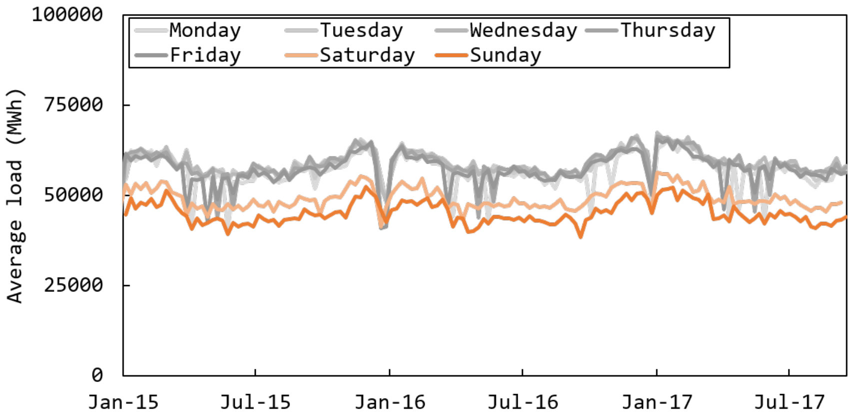

Figure 2.1.1: Average power load per day in the period 2015-01-01 - 2017-09-29 (ENTSO-e, 2017b)

The power grid is the only infrastructure that is well suited to transmit electrical power across large distances. Once a production (or consumption) unit is connected to the grid, the production of that unit becomes physically indistinguishable from the production of other grid-connected units; the power flow is subject to the laws of physics governing the power grid. Thus, consumers will experience the exact same power quality regardless of which vendor they purchase their power from. This makes it harder for consumers to discriminate between producers and vice versa, making electrical power a near-perfect commodity.

Consumption is nearly completely inelastic for low volumes (Malic, 2017), creating a strong incentive for price making producers in an oligopoly to artificially withhold production. In current power markets, the short-term elasticity is low for all volumes as customers usually are shielded from short term fluctuations in the power price. This may change as the adoption of smart meters increases so that consumption can be measured over arbitrarily short time intervals, potentially creating an incentive for demand response that is absent today.

Due to the above-mentioned complications, a complex market structure has been developed to provide power to the customers with high security of supply, high power quality and relatively low prices. In the following paragraphs, we describe the three power markets that are the most relevant for this thesis. While there also exist other, longer-term financial markets, they aren't physical markets - that is, markets where the underlying traded asset is a certificate to produce one unit of energy. All of the following markets are physical energy-only markets, in the sense that only the sale of energy is rewarded. Other desiderata like available production capacity in the case of demand spikes are not rewarded in such markets. An overview of the markets can be seen in figure 2.1.2.

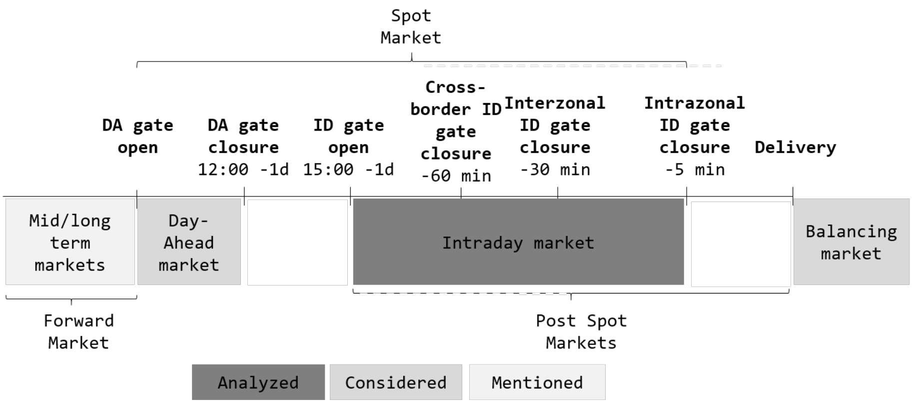

Figure 2.1.2: Power markets timeline

2.1.1 The Day-Ahead Market

The purpose of the Day-Ahead Market is to assign sufficient production capacity to cover demand for each hour of the following day. In this market, producers are asked to place their orders for power certificates for the next day. Each power certificate gives the right and obligation to produce one unit of energy (MWh), constantly distributed over one specific hour on the next day. The period of validity for the power certificate is referred to as the *delivery product*, and there is a separate auction for each delivery product. The prices of the orders for each delivery product are generally expressed as a function of the cleared volume, $p = p(v)$. After gate closure, the orders are sorted in a nondecreasing aggregated supply curve, and the cheapest orders with a total volume equaling the predicted consumption volume for the given hour are cleared. The producers with the cleared orders will receive a compensation equal to the asking price of the ”marginal producer” - that is, the most expensive order that cleared. This is denominated a *pay as cleared-market*, and removes the incentive to order strategically for a producer without market power; the optimal order for the individual producer will equal their marginal cost. The clearing price is also called the *spot price*, as the Day-Ahead market is also referred to as the spot market. Consecutive markets are collectively referred to as *post-spot* markets, although the Intraday market is sometimes included in the *spot markets*. In 2016, 235 MWh were sold in the Day-Ahead market EPEX (2016).

2.1.2 The Intraday Market

In contrast to the Day-Ahead market, the EPEX Intraday market is a *pay-as-bid market* - that is, traders receive their asking price, plus an eventual spread. The Intraday market is a continuous auction, similarly to conventional stock markets, starting shortly after the production plan based on the Day-Ahead orders has been published, and closing 5 minutes before the time of delivery for each delivery product; the latter is referred to as ”gate closure”. The continuous auction is organized through a *limit order book* where traders can place orders digitally. In the last 25 minutes of the Intraday trading window, referred to as the intrazonal Intraday market, only orders placed in the same control area can be matched. The four German control areas can be seen in figure 2.1.3. Note that the gate closure times have been changed recently. The pay-as-bid feature allows each trade to clear independently of other trades, speeding up clearing in the Intraday market. This is suitable for a market that is designed to allow producers to adjust and reoptimize their production plans over shorter time horizons than the Day-Ahead market, but comes at a price; it incentivizes placing orders above marginal cost, and opens for inefficient allocation of production. In the Intraday market, delivery products with both hourly and quarterly duration are traded. 41 MWh were traded in the Intraday market in 2016 (EPEX, 2016). It is possible to place orders that apply to multiple delivery products (blocks) that an eventual buyer will have to buy as a bundle.

2.1.3 The Balancing Market

On a systems level, the rationale for the Balancing Market is to close any real-time discrepancies between load and production. In practice, this implemented by charging market participants in the Day-Ahead and Intraday markets for

Figure 2.1.3: The control areas of Germany (Robinius et al., 2017)

deviations between the individual net production and the commitment made in the former markets (see section 2.1.5 for further explanation). Such deviations may arise if the traders were unable to close their open position in the EPEX Intraday, if forecasts change near gate closure, et cetera. Production capacity is allocated for the Balancing market one week in advance. The price of power in the Balancing market is denominated *imbalance price* (alternatively "reBAP") is volatile compared to the prices of the Day-Ahead and Intraday markets, as figure 2.1.4 suggest. It is calculated as the total net cost of balancing energy divided by the net balancing energy provided (Regelleistung, 2017). There are two imbalance regulations. The Up-regulation arises if the power consumption is greater than the power generation, and the Down-regulation operates if the power consumption is less than the power generation. The imbalance price is very volatile and high on expectation, incentivizing producers to close their position before the Balancing market opens. This cost is usually symmetrical for the Up and Down regulating balance markets (ENTSO-e, 2017a). The imbalance prices are published with a 20 day lag the 20th of every month (ENTSO-e, 2017a). In addition to the economic incentive to avoid the Balancing market, "*The German regulator legally requires Balancing Responsible Parties* [see section 2.1.5] *to minimize their use of the imbalance market to the largest possible degree*" (Malic, 2017).

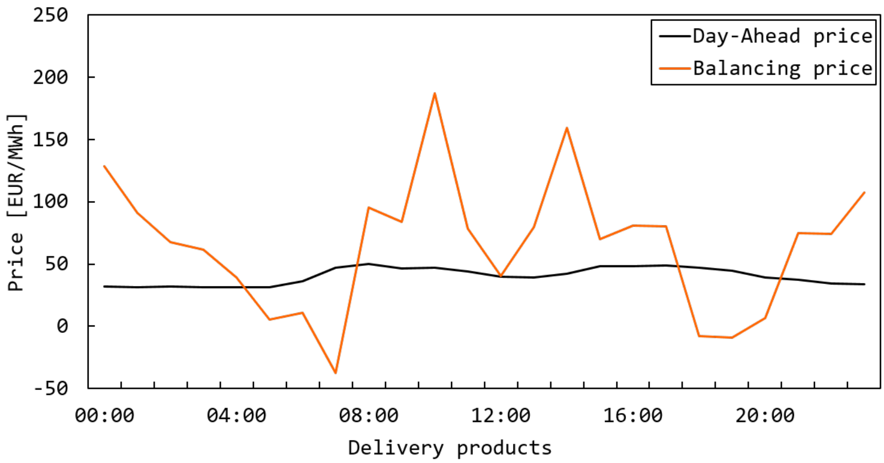

Figure 2.1.4: Comparison of Day-Ahead and balancing prices 2016-12-21. Data from (EPEX, 2017b) and (ENTSO-e, 2017a).

2.1.4 Intraday market participants

Utilities, aggregators, power retailers and purely financial traders are referred to collectively as *traders* in the Intraday market. The traders can be categorized by their production or consumption technology. There are several features that determine the traders' behavior; the storage capacity, production capacity, flexibility of production, marginal production cost and their intertemporal constraints. Producers with small storage capacity relative to production may have one discharge cycle per day, whereas producers with very large storage capacities may have only one discharge cycle annually. The difference between the maximum and minimum production level determines the flexibility in total market commitment. Production inflexibility is likely to yield significant imbalance costs for the trader. For producers, these bounds typically are non-negative whereas for consumers (that is, retailers and demand aggregators) they are negative, but storage units may make it possible to reverse production. Hydropower producers typically model their resource inflow as deterministic over 24-hour decision horizons, whereas other intermittents have stochastic inflow even Intraday. Thermal producers typically have *intertemporal* costs and constraints; that is, costs and constraints related to the ramping (and starting or stopping) of their production. While both start-up costs and ramping times are heterogeneous for different thermal producers (i.e. generally larger for coal producers than gas producers) this is not detailed further here, as thermal producers aren't the main focus of this thesis. The relevant types of traders are summarised in table 2.1.1. In the table, *High/low* means that a parameter may take a high value for some and a low value for others. *(No)* means that something usually isn't true.

Table 2.1.1: Trader types and their trading-relevant features

Trader type	**Storage**	**Production flexibility**	**Energy inflow**	**Production reversibility**	**MPC**
Hydro (Flow-of-the-river)	No	No	Stochastic	No	0
Intermittent w/o storage	No	No	Stochastic	No	0
Power retailer	No	No	Stochastic	No	0
Financial trader	No	No	Deterministic	No	0
Thermal power	No	Yes	Controllable	No	Fuel
Hydropower dams	High/low	Yes	Stochastic	No	VoS
Demand aggregators	Low	Yes	Stochastic	(No)	VoS
Intermittent w/ storage	Low	Yes	Stochastic	Yes	VoS
Pumped hydro	High/low	Yes	Stochastic	Yes	VoS

Here, *MPC* refers to the marginal production cost. *VoS* refers to the *Value of Storage*, which is the alternative cost of storage that is spent during the day. If the long-term production schedule is optimized - typically with a stochastic dynamic program - the Value of Storage is "*the shadow price (...) i.e. the Lagrange multipliers corresponding to the reservoir constraints*" (Javanainen et al., 2005). In addition to the Value of Storage, demand aggregators may face a *comfort cost*, which corresponds to the tradeoff between load flexibility and consumer preferences; if the flexible demand is used to heat office spaces, temperatures near the upper or lower bounds of the agreement between the consumers and the demand aggregator will presumably be less comfortable for the users of the building. Depending on the agreement between the consumers and the demand aggregator, this cost may be implemented as constraints. This cost is not discussed further in this thesis.

Note that the medium- to long term Value of Storage contains assumptions about the short-term production. As illustrated by Javanainen et al. (2005), the expected returns to extra storage are decreasing on the margin - or conversely, if more energy is sold in the short term than originally planned, the value per unit of the end-of-day remaining storage increases. For producers with small enough reservoirs that the Intraday production decisions alter the remaining storage level significantly, the Value of Storage unit cost may therefore be heterogeneous as a function of storage level. This heterogeneity works as a link between the short- and long term optimization of power trading.

Henceforth, *marginal production cost* refers to costs driven by the Value of Storage, fuel or intertemporals. *Alternative costs of future trades* (ACs) refer to the lost option of trading later in the ID trading window if trades are performed now, and may or may not include imbalance costs depending on if the balancing market is included in the model or not. Note that this alternative cost is thus much more short-term than the Value of Storage, as the first considers only the current day, and the other typically has a decision horizon of a year. *Marginal costs* refer to the combination of the ACs and the MPC; for a seller, this is the expectation of the maximum of future prices and marginal production costs.

2.1.5 Post-spot power market regulations

In this section, some of the more relevant regulations that traders in the EPEX Intraday Market need to comply with are discussed. Specifically, market manipulation, insider trading in the light of *Urgent Market Messages* (UMMs), *Balance Responsible Parties* (BRPs) and the *Order-to-Trade Ratio* (OTR) fine are discussed. These topics have been chosen for their relevance to the later modeling choices. According to the EPEX Code of Conduct (EPEX, 2017a), "*Any engagement in, or attempt to engage in, market manipulation on a Physical Power Contract is prohibited.*". Market manipulation includes, but is not limited to, both cooperative collusion and price fixing behavior. The second part is the most relevant for this thesis. In the same document, inside information is defined as non-public information that "*would be likely to significantly affect prices*". One example of this is if the availability status of a generator unexpectedly changes - for instance, if the generator breaks down. In this case, an Urgent Market Message is created, and no trading can be performed until the UMM has been published.

In the EPEX Operational Rules (EPEX, 2017c), Balance Responsible Parties are responsible for the delivery of physical power. As stated in Doorman et al. (2011), "*All market participants interact with the market through a (...) BRP, or are a BRP themselves (...) The BRP is generally obliged to try to (...) comply with this balance in real time*". "This balance" refers to that the *Balancing*

Group (BG) that the BRP manages should have a total production equal to its total commitment from the Day-Ahead and Intraday markets. Furthermore, the paper explains that it is BRPs, and not market participants directly, that pay the imbalance price if the BG as a whole is imbalanced. The BRP will usually organize in such a way that these costs are transferred to the individual market participants in it, however.

The Operational Rules also define the Order-to-Trade Ratio, which is a fine for placing an illegally high number of unmatched orders, to prevent traders from spamming the market with orders. If the number of orders per transaction for a given delivery product over the entire trading window is larger than 50, the trader is alerted. If no orders clear, the OTR is defined as the number of orders. The first four alerts every month are free of charge, after which each OTR costs 100€. While not saying so explicitly, the Operational Rules give the impression that persistently extreme OTRs may cause loss of market access.

2.1.6 Trading in limit order book markets

In limit order books, all transactions consist of a *limit order*, and a matching *market order*. The limit order is placed in the limit order book with the standard order parameters, whereas the market order does not need a price specification as it automatically matches the limit orders with the best available prices in the limit order book. A trader placing a limit order is referred to as a *market maker*, as they provide liquidity to the market (Guo et al. (2017), Cont et al. (2014), Cont and Kukanov (2017)). The term that is used for a trader that places a market order, is *liquidity taker*.

The limit order book is *two-sided*, so both sellers and buyers may place either limit orders, market orders or both in the market. Sell orders and buy orders may also be referred to as ask order and bid orders, and these terms are used interchangeably throughout the thesis.

The price of a market order is set to the volumetric weighted average of the cleared volumes of the matched limit order prices. For limit orders, the clearing process is more complex. Limit orders may clear fully, partly or not at all, and the clearing may happen initially or later. This is determined by the following factors:

1. In order to clear initially, several previously placed bid (ask) limit orders

with higher (lower) or equal price to that of the ask (bid) order must be available in the market. This means that the orders must be in the same control area, or cross-zonal trading must still be allowed. The clearing volume is equal to the smaller of the volume of the ask (bid) order and the volumes available at good enough prices in the market. The transaction price is set to the volumetric weighted average of the cleared volumes of the previously placed orders.

2. If the order didn't clear initially, or cleared only partly, it may clear (fully or partially) in a later stage if it is the ask (bid) order with the lowest (highest) price available in the market, and either a bid (ask) market order is placed or a bid (ask) limit order with a higher (lower) price is placed. If the ask (bid) order is not the one with the lowest (highest) price available in the market, but the new order is large enough to clear all the better orders in the market, the ask (bid) order may clear (fully or partially) anyways. The transaction price will be equal to that of the ask (bid) order itself. The clearing in a later stage requires that it still is open.

3. The probability that a limit order clears is referred to as the *clearance probability*, and the risk that a limit order does not clear is referred to as *execution risk* (Cont and Kukanov (2017), Tsoukalas et al. (2017)).

When placing an order in a limit order book, it is possible to predefine when the order should expire, referred to as *canceling* an order. Order cancellation is also possible to determine dynamically. So-called *Iceberg orders* are large orders that are partitioned into small pieces and each piece is activated only after the former clears, to avoid excessive price impact, but again, this is possible to determine dynamically by placing several consecutive orders. Finally, it is possible to make the activation of an order conditional on the future states of the order book (e.g. "activate the order if transactions occur at prices above X"). For the rest of the thesis, it is assumed that the only such specification that is used is the predetermined order duration.

2.2 Related literature

As mentioned in chapter 1, there are two separate axes of time related to the ITP: the *trading time* and the *production time*. Figure 2.2.1 shows the trading time on the x-axis and production time on the y-axis. In this section, papers covering one or the other of these axes are covered. Papers that apply relevant methods in other contexts (i.e. more conventional markets) are also mentioned. In figure 2.2.1, the double dynamics are illustrated, with the orange dots illustrating a trading stage for a delivery product in the Intraday market.

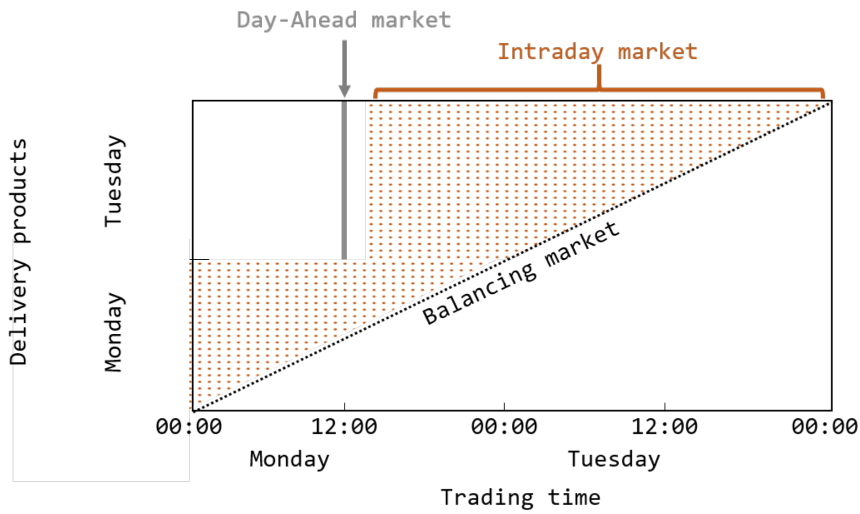

Figure 2.2.1: The double dynamics of 48 delivery products inspired by Belsnes et al. (2016)

The dynamics in trading time resemble those in conventional limit order books; that is, markets where limit orders are placed continuously, and later removed by market orders or limit order cancellations. There is a rich literature on how to model the dynamics in conventional LOB markets and how to trade well in them (see e.g. Alfonsi and Schied (2010) for a review). In the conventional LOB literature, it is typically assumed that the *target inventory* is known in advance, and it is explored how this inventory level can be reached through the placement of sell and buy orders. The optimization problem for the trader is referred to either as an Order Execution Problem, Order Placement Problem or Market Making Problem depending on the specifics of how the decisions are modeled (Guo et al., 2017). Order Placement Problem is the most general of these terms, so it is used throughout this thesis. LOB theory has also been applied to Intraday power markets, including contributions from Garnier and Madlener (2015), Aïd et al. (2015), Tan and Tankov (2016) and Edoli et al. (2016). Skajaa et al. (2015) also share some similarities with these papers, though it is less based on the general limit order book research.

For traders with intertemporals or storage, the dynamics in production time are relevant too; decisions for one delivery product affects the utility of decisions related to other delivery products. The problem of determining how much of a stored resource to spend per stage over a period consisting of several stages is referred to as a Dynamic Resource Allocation Problem. Dynamics in production time is considered by Gönsch and Hassler (2016), Hassler (2017), Löhndorf et al. (2013), Farinelli and Tibiletti (2017) and Engmark and Sandven (2017), among others. For all of these papers, storage allocation is the reason for the dynamics in production time.

For both of these two branches of the literature, we have selected papers that explicitly state that they focus on optimization of decision making, and other forms of decision support, for producers that are marketing their power through the Intraday continuous auction market. Other branches of the literature focus on estimating the value of Intraday trading in addition to the Day-Ahead trading, or multimarket optimization, or market design, but those have not been considered here. The two branches of papers are collectively referred to as ITP papers, though each paper covers a different subset of our definition of the ITP.

To the best knowledge of the authors, no paper as of yet considers both of the *double dynamics*; in fact, the concept itself is a novel contribution in this thesis. For this reason, it is hypothesized that the recommended framework of all of the papers either is in contradiction to, or neglects at least one consideration that has non-trivial impact on the objective function of a large subset of the traders in the market. Put differently, the hypothesis is that there exist several traders in the EPEX Intraday that would be unwise to apply either of the proposed frameworks because each of them systematically overlook at least one relevant concept; in particular, the traders that are the most affected by these double dynamics are unable to optimize Intraday decisions well with any one of the proposed frameworks. It should be noted that each of the papers (except Edoli et al. (2016)) explicitly focus on a subset of the traders in the market, so it should be no surprise that their proposed frameworks are less applicable for other types of traders.

The rest of this section is structured as follows; in section 2.2.1, the modeling in the literature of the relevant decisions for the ITP is outlined. Then, in section 2.2.2, assumptions about the price dynamics and attempts to forecast prices are covered. Section 2.2.3 considers the implications of limited liquidity on the price dynamics more closely. Section 2.2.4 looks into several features that distinguishes the EPEX Intraday from conventional limit order books. In section 2.2.5, the other sections are summarized, and hypotheses for potential improvements to the recommendations of the contemporary literature are proposed.

2.2.1 Decision making and solution methods

The papers focusing on a single delivery product consider the optimal *trading rate* - the change in the production commitment per unit of time. In continuous time, this means that order volumes are infinitesimally small, and thus can be modeled as a rate rather than a set of individual orders. Garnier and Madlener (2015) models time as discrete, so the order volume in each stage (which is the discrete-time equivalent of the trading rate) is considered instead.

It is thus assumed that the desired final commitment is already known; that is, the production quantity (which is interchangeably referred to as a *dispatch* or *dispatch schedule* in the literature) is not optimized in these papers. As Edoli et al. (2016), Tan and Tankov (2016) and Garnier and Madlener (2015) focus on an intermittent producer without storage, this makes sense, as the production isn't flexible and therefore is treated as a constraint. Aïd et al. (2015) is the exception, which awards the trader some thermal power in addition to the wind power. They assume that the trader is a retailer (in addition to being a power producer) which has a target production equal to the consumption of their customers. The thermal production kicks in if the intermittent production falls short of the market commitment. Thus, Aïd et al. (2015) is the only trading focused paper which also includes a (partial) optimization of the dispatch. Furthermore, Aïd et al. (2015) and Garnier and Madlener (2015) allow the trader to deviate from the predetermined dispatch schedule for an imbalance cost (but not to redefine the dispatch schedule).

All of the aforementioned ITP papers assume that the market price is well defined at any given time, and that orders can only be placed at this price. This is similar to the literature on the Order Execution Problem in conventional markets (e.g. Gatheral and Schied (2013), Alfonsi and Acevedo (2014)), and implies that the trader may only use market orders. In the Order Placement Problem and the Market Making Problem (Guo et al. (2017)), the trader is also allowed to place limit orders, which necessarily makes the price a relevant decision variable, but no paper was found investigating the implications for a trader that may post limit orders at prices of their own choice in the Intraday market. It should be mentioned that Garnier and Madlener (2015) accounts for the uncertain clearing of the limit order, without considering the price as a decision variable.

In contrast, the research on the Dynamic Resource Allocation focuses on the dispatch schedule. Thus, the production volume per delivery product is the only decision being made, and no attention is paid to how to acquire the commitment in the limit order book market. Neither order volume, -timing or -price nor the number of orders are modeled as decisions. By allowing only one decision per delivery product, the strategy proposed in the dispatch-focused papers does not adjust to new price information after a production decision has been made - it isn't adaptive in trading time. For instance, it is not specified if the trader should increase production if prices unexpectedly rise, or what to do if an order does not clear. A partial exception in this regard is Engmark and Sandven (2017), who explicitly state that the model should be run several times throughout the day to update the recommended decisions based on the new information, and allow the trader to place orders in three different price segments.

Figure 2.2.2 shows the assumptions that each group of papers make about the axes of decision dynamics. As shown, the trading-focused papers (left chart) consider each delivery product separately, assuming that decisions (illustrated by a dot) are independent across delivery products. The dispatch-focused papers (center chart) on the other hand make only one decision per market per delivery product, and thus ignore the dynamic properties of the continuous auction market structure. To our knowledge, this thesis is the first to consider the full double dynamics of the Intraday Trading Problem (right chart). Note that not all papers include all three markets, and all of the covered papers assume that the Day-Ahead decision has already been made.

The dispatch-focused papers only allow either selling or buying, depending on the direction of the initial *open position* - that is, the difference between the initial commitment from the markets and the production schedule. In contrast, the trading-focused papers allow for the placement of both sell and buy orders, regardless of the current open position.

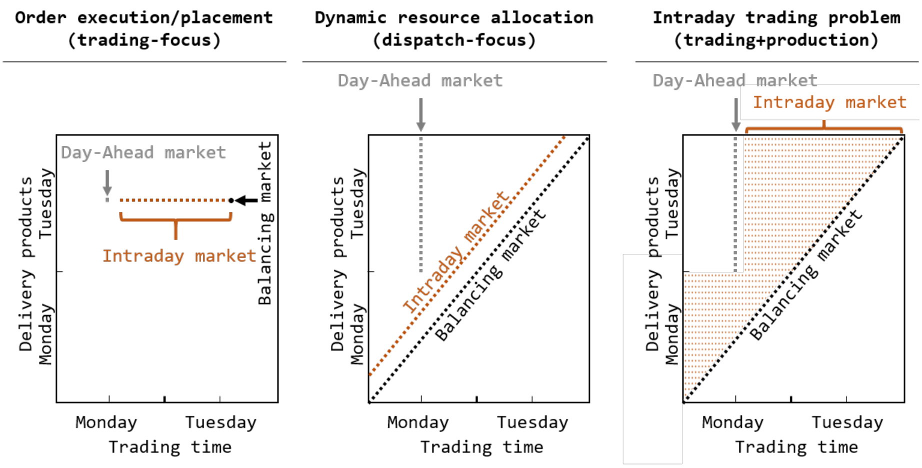

Figure 2.2.2: Modeling of dynamics by different groups of the contemporary literature.

To solve the Order Placement Problem, Garnier and Madlener (2015) use an options valuation approach to find an analytic solution in each node, and then apply backward induction to solve it for all scenarios and stages. Aïd et al. (2015), Tan and Tankov (2016) and Edoli et al. (2016) model cannot apply the same approach as both time and the probability distributions for the stochastic parameters are modeled as continuous. However, all of them are able to find (approximate) closed-form solutions to the *Hamilton-Jacobi-Bellman Partial Differential Equation* (Evans and James, 1989), which is similar to the infamous Bellman equation, but applies in continuous time. Skajaa et al. (2015) take an algorithmic approach where each trader may define acceptance criteria on their own, and then place a market order if the criteria are fulfilled.

Gönsch and Hassler (2016) and Hassler (2017) build on the work of Powell (2011) and solve the dynamic resource allocation problem using approximate dynamic programming (ADP). Furthermore, Hassler (2017) compares the ADP approach with several heuristics, and finds that some of the heuristics show promising results for the heuristics that focus on the available storage level. Löhndorf et al. (2013) combines ADP with stochastic dual dynamic programming, and calls the resulting solution method approximate dual dynamic programming. Farinelli and Tibiletti (2017) uses dynamic programming and solves the convex optimization problem in each node with the Interior Points Method (Boyd and Vandenberghe, 2004). Finally, Engmark and Sandven (2017) uses a stochastic mixed integer program to optimize

2.2.2 Price dynamics and -forecasts

This section covers the assumptions that the ID-related papers make about price dynamics, and how they attempt to forecast prices. First, the relevant concepts are introduced.
Stochastic processes are *"ways of quantifying the dynamic relationships of sequences of random events"*, according to Pinsky and Karlin (2010). Examples of stochastic processes are the *standard Brownian motion*, the *geometric Brownian motion* and the *Ornstein-Uhlenbeck dynamic.* The transition function for the standard Brownian motion has a normally distributed probability density function, whereas the geometric Brownian motion is the equivalent with a lognormal distribution. While the Brownian motions don't necessarily have a non-zero drift coefficient, Ornstein-Uhlenbeck dynamic does. For this process, the drift is assumed to be in the direction of the empirical mean of the process, making it a *mean-reverting process* (Pinsky and Karlin, 2010). All of the aforementioned processes have the *Markov property*, which means that the current state is sufficient to determine the probability distribution for the next stochastic draw (Alexander, 2008b). In contrast, consecutive draws in an *autoregressive process* (AR-N process) are correlated, so that the last N elements of the history of the process are relevant to its future values. A *martingale process* has the martingale property, which means that the expected value of the process at any given time in the future is equal to its current value (Alexander, 2008c).

The *assumption of no arbitrage* is common in financial modeling. It states that *"we cannot make an instantaneous profit from an investment that has no uncertainty"* (Alexander, 2008a). As predictable price drifts could be a source of arbitrage, a common way to prevent arbitrage is to assume that prices are martingales.

There is a fundamental difference between how the two groups of former papers on optimization of trading in the Intraday market considers prices. The papers that focus on the dynamics in trading time consider the price process of only one delivery product.

Garnier and Madlener (2015) and Tan and Tankov (2016) assume that this price follows a Geometric Brownian Motion. Furthermore, the price is assumed to be a martingale.

Edoli et al. (2016) instead considers an Ornstein-Uhlenbeck dynamic, to capture the speed of the mean reversion. The rationale for the mean reversion is that the value of the underlying security for a given trader after gate closure is equal to her marginal cost. Thus, all traders in the market have a privately known benchmark for what price they should be willing to accept. In contrast to conventional financial assets, where the future divided from a security may be highly uncertain, each trader is certain about their marginal production cost, so this benchmark should serve as an anchor point for the price, causing significant mean reversion. In addition to the theoretical argument, it is commonly observed that commodity prices show strong mean reversion.

Aïd et al. (2015) make less restricting assumptions, and consider the three cases where the price is a martingale, submartingale and supermartingale, without constraining the shape of the probability distribution. In this case, the lack of a martingale property is caused by price jumps. Price jumps may for instance be caused by urgent market messages about generator outages, or publications of surprising weather forecasts.

Overall, there are several reasons why it may sometimes be rational to assume non-zero price drift. In addition to the mean reversion and jumps proposed by Edoli et al. (2016) and Aïd et al. (2015), Tan and Tankov (2016) points out that energy futures in general (including ID-prices) often have a negative trend. This is referred to as *contango* and may occur if risk averse buyers pay a premium to secure their supply of energy early (Alexander, 2008c). If markets aggregate information slowly due to noisy data, prices may show autoregressive properties (Dontoh et al., 2003). Moreover, if several market parameters are stochastic, but only there only is one type of financial asset, the outcome space is larger than the number of available bets, which means that the market is incomplete. In such markets, there may not exist a fundamental price at all, let alone one with a martingale property (Staum, 2007). Examples of market parameters that may be modeled as stochastic include the spread, the price volatility and the order depth.

The dispatch-focused on the other hand focus simplify the price process and assumes that there exists only one *representative price* for each delivery product; this may be the market price at the time of a scheduling decision (e.g. Farinelli and Tibiletti (2017)), or the (volume-weighted) average prices over the trading window (e.g. Hassler (2017)) for each delivery product. The prices of the delivery products with early gate closure is known before a decision is made for later delivery products. Thus, the price of past delivery products is used to predict the prices of delivery products with trading windows that are still open.

A variety of forecasting techniques are used; notably, both "technical" autoregressive forecasts (Hassler (2017), Engmark and Sandven (2017)) and "fundamental" techniques based on supply- and demand-forecasts (Löhndorf et al., 2013) appear in the literature. Farinelli and Tibiletti (2017) and Gönsch and Hassler (2016) assume that the price follows a discrete time Gaussian process.

There are several implicit assumptions behind these models; for instance, it is assumed that the representative price is a good approximation of the price that a trader can achieve on average for her trades throughout the trading window. Moreover, it is assumed that the optimal order placement strategy either does not matter much or can be determined independently of the optimization of production schedules, without significant profit loss. This breaks with Rudlang et al. (2014), which states that production volumes should be price dependent. Also, the information about the future price of one delivery product revealed by the current price of the same delivery product is neglected, as the prediction is exclusively based on the prices of other delivery products. This would not make sense if prices were martingales; if the expected future price for one delivery product is equal to the current price for the same delivery product, there is no reason to base predictions on the prices of other delivery products.

Since the "representative price" isn't necessarily close to the current price at the timing of an order placement, there is a considerable risk that an order placed at the representative price won't clear, or that it clears at an unfavorable price. No documentation of the relevance of the representative price was found in the dispatch-focused ITP literature; and to the best knowledge of the authors of this thesis, no strategies were proposed for how to mitigate losses when the representative price is a poor approximation.

2.2.3 Limited liquidity and order book dynamics

Aside from stating the assumption about the stochastic process that the price follows (and its corresponding price drift), the trading-focused papers don't attempt to justify their price assumptions (about volatility or other market parameters) with extensive empirical research. Such an analysis would quickly have lost its value anyways; the growth in liquidity in the market has been so strong historically, that it is not unreasonable to assume that some market parameters have changed since the papers where published. The dispatch schedule-focused papers consider only the single point forecast for each price process, so other market parameters are largely irrelevant. In the following section, other parameters that are usually considered in the conventional LOB research are elaborated on.

In a limit order book, there is significantly more information than only the market price; the volume of orders at a given price and time is referred to as the order depth. The ask (resp. bid) order with the lowest (highest) price is referred to as the best quoted order, and the corresponding price is the best quoted price. The difference between the best quoted prices is called the spread. The spread is seen as a measure of the liquidity of the market, and it is smaller the more liquid the market is - typically only one or two tick marks in liquid markets (Cont et al. (2010)). Typically, the base price that is used as a proxy for some fundamental market price is the mid-price, which is in the middle of the best quotes, one half-spread away from each of them (see e.g. Cont et al. (2010)). If the trading time is discretized into stages, other approximations of the fundamental market price include the volumetric weighted average transaction price, or the high, low, first and last transaction prices for each stage.

2.2.3.1 Limit order premium

Recall that limit orders are placed in the limit order book, whereas market orders remove limit orders from the limit order book when they are placed, as they immediately clear. For a trader, the market order thus presents a certain transaction opportunity; it will clear at whatever price is available. The limit order on the other hand clears if a counterpart finds the price acceptable; otherwise it is left in the limit order book until the end of the trading window without any transaction occurring. The volumes of all the transactions related to a limit order may sum to anywhere between 0 and the stated order volume.

Any clearing between 0 and 100% is referred to as partial order clearing. The advantage of the limit order relative to the market order is that it offers the opportunity to earn better unit profits than what is available in the market at the moment. This can be seen as a premium rewarded for providing liquidity to the market (so-called *market making*), to compensate for the added risk.

In the conventional LOB literature, it is common to assume that the limit order premium is equal to the spread (Guo et al. (2017), Horst and Naujokat (2014), Cont et al. (2010)), and that the spread is narrow. This is a sensible approximation if response times of traders are short relative to the time it takes the mid-price to move; in that case, traders can refresh the details of their limit order so that the price stays close to the best quoted price in the market, and the difference between the clearing prices of a limit and a market order will be equal to the spread (given that the limit order clears). It is also necessary to assume that clearing times are short relative to the time it takes for the mid-price to move, so that limit orders and market orders posted at the same time are comparable; otherwise, liquidity takers will have access to price information that market makers do not, which constitutes a risk for the market makers (Ruchti, Goettler et al. (2009)). Even if the two assumptions don't hold, it is still reasonable to assume that the limit order premium is strongly correlated with the size of the spread. In particular, when liquidity approaches infinity and the tick size goes to zero, the spread vanishes and the limit order premium converges to zero. Thus, the distinction between a market order and a limit order is pointless in a perfectly liquid market.

Note that the spread is modeled as a cost in the Order Execution Problem (similar to the costs in section 2.1.4). Spreads, transaction costs and price impacts are collectively referred to as *trading costs*. In the Market Making Problem, where the trader may place limit orders, the spread may have a positive contribution to the objective function and it is therefore not a cost.

Of the ITP papers, only Garnier and Madlener (2015) and Engmark and Sandven (2017) include an execution risk that implies they allow the trader to place limit orders, and only Engmark and Sandven (2017) allow the trader to determine the prices of the placed orders. This may seem strange, considering that the conventional LOB research find limit orders highly profitable (see e.g. Horst and Naujokat (2014), Guo et al. (2017), Cont et al. (2010), Gonzalez and Schervish (2017)). However, Horst and Naujokat (2014) state that *"To the best of our knowledge, the problem of when to cross the bid ask spread has not been* *addressed in the mathematical finance literature on limit order markets"*. The conventional LOB research has only recently started to tackle the problem, so it is no surprise that it didn't instantly propagate to research on more complex limit order books like the Intraday market.

2.2.3.2 Modeling price impacts from trading

When liquidity is limited enough, orders have a price impact in several ways, and different papers include different subsets of these forms of price impact. The terminology differs somewhat between the papers, but the following list attempts to use the most common terms only.

- *The instantaneous price impact* reflects that the order depth at a given time is less than infinite, so that the average clearing price of a large order is less than or equal to that of a small order. The instantaneous price impact from a given reduces the profitability of the same order. This type of order is modeled both in conventional limit order markets (e.g. by Alfonsi et al. (2012)) and versions of it has been modeled in the Intraday market by Aïd et al. (2015), Tan and Tankov (2016), Garnier and Madlener (2015), Engmark and Sandven (2017) and Skajaa et al. (2015).

- *The transient price impact* reflects that a recent market order has exhausted the liquidity in a market. Given sufficient time, the transient price impact decays and becomes negligible. The transient price impact is discussed by Gatheral and Schied (2013), Cont et al. (2014) and Bouchaud et al. (2004), and references therein.

- *The permanent price impact* arises because traders adapt strategically to the behavior of other traders. Large market orders, limit orders or limit order cancellations shifts the perception the traders have of the demand in the market, altering their trading strategy for the rest of the trading window. If some of the strategic adaptation is temporary, it may add to the transient price impact rather than the permanent price impact. Permanent price impact is modeled by Cont et al. (2014), Gatheral and Schied (2013), Bertimas et al. (1999) and Huberman and Stanzl (2004), among others.

It is commonly assumed that all forms of price impact are in an adverse direction, when adverse is defined as unprofitable for orders in the same direction as the original order. For instance, sell orders lower prices, which makes selling less profitable (both instantaneously and in the future). It is common to assume that each price impact is an increasing function of volume, but the exact shape of the price impact is an ongoing field of research (Huberman and Stanzl (2004), Cont et al. (2014)). However, an assumption of no dynamic arbitrage gives some indication of theoretically sensible shapes; in particular, transient and permanent price impact should be linear in order volume and symmetric for sales and purchases (Huberman and Stanzl (2004), Weber and Rosenow (2005)). Moreover, the decay of the transient price impact should have a convex shape (for instance exponential), else it will give rise to another form of price impact called *transaction triggered price impact* (Alfonsi et al. (2012)). The reason for these constraints is that other shapes of the long-term price impact may create *profitable round trips*; a net zero change in commitment, where the price impact from the early orders makes it possible to get great prices for the later orders in the opposite direction, producing a net profit in expectation. As the instantaneous price impact only affects the price of the same order that causes it, it can never produce arbitrage opportunities on its own. Whether the empirical shapes of the price impacts comply with these theoretical constraints or not in the Intraday market has not been investigated further, but it would be an interesting area of research.

As all price impacts are increasing in order volume, there is a tradeoff between the different forms of price impact; a "one-shot" strategy may eliminate the transient and permanent price impact (since they only affect later orders) but cause great instantaneous price impact; whereas the instantaneous price impact is reduced by distributing the trading activity throughout the trading window.

Both the transient and permanent price impact affect the profitability of future orders; thus, they create path dependencies in the trading optimization problem. In addition to this, the permanent price impact is a simplification that reduces a game theoretical problem to an optimization problem that utilizes that only the system-level response of the traders matter, and not their individual strategic adaptations. In the papers about limit order books, this is handled by assuming that the price impact has a nice shape that allows for a closed-form expression of the optimal control derived from the Hamilton-Jacobi-Bellman partial differential equation.

Of the formerly mentioned ITP papers, Skajaa et al. (2015), Engmark and Sandven (2017) and Garnier and Madlener (2015) include simplified versions of the instantaneous price impact, while Aïd et al. (2015) and Tan and Tankov (2016) include both the instantaneous and permanent price impacts. However, Aïd et al. (2015) uses the term transient price impact to refer to the instantaneous price impact, while Tan and Tankov (2016) refers to it as a volume penalty. The other papers don't include any price impact.

2.2.3.3 Implications of multiple correlated price processes

When the price processes of the consecutive delivery products are positively correlated, movements in one of the prices convey information about the development of the other prices. If trading of one delivery product has a price impact on the given delivery product, one should therefore expect to have some price impact on the other delivery products too. The rationale is that traders who observe increased supply (demand) of one delivery product should expect similar changes the supply (demand) of other delivery products in the same direction, and adjust their trading strategies accordingly. Since the *cross-impact* is caused only by the changed expectations of other traders, it is exclusively permanent and possibly transient (Mastromatteo et al. (2017), Bertimas et al. (1999)).

Recall that non-standard shapes of the transient and permanent price impacts may cause dynamic arbitrage opportunities. In a similar way, an assumption of no dynamic arbitrage constrains the shape of the cross-impact. In particular, it should be linear as a function of volume and symmetrical for ask and bid orders, just like the conventional transient and permanent price impacts (Schneider and Lillo, 2016). Additionally, the *cross-impact matrix* should be a symmetrical matrix, so that a round trip involving several delivery products will have a net zero price impact on all of the delivery products (Mastromatteo et al. (2017), Bertimas et al. (1999)). Note that the cross-impact matrix does not necessarily have to equal the price correlation matrix, though it may be a reasonable assumption; for instance Mastromatteo et al. (2017) assumes that *"have assumed that the impact matrix has the same eigenvectors as the correlation matrix itself (...) prevents arbitrage opportunities and price manipulation strategies"*.

None of the ITP papers model any form of cross-impact, as none of them simultaneously include several delivery products and a model of the limit order book simultaneously.

2.2.4 Unique features of the EPEX Intraday market

There are several aspects of the Intraday market that distinguishes it from a conventional LOB. In this section, three of these are covered. Section 2.2.4.1 outlines how former papers have modeled the regulations covered in section 2.1.5. Section 2.2.4.2 focuses on the modeling of the imbalance price, while 2.2.4.3 concerns the production costs.

2.2.4.1 Regulations

In this section, the modeling of the relevant regulations mentioned in section 2.1.5 of the related research is discussed. In particular, the modeling choices related to insider trading, market manipulation, the balancing market and the Order-to-Trade ratio fine are considered.

Neither of the covered ID papers mentions insider trading; however, neither of the models in the papers use information that would count as insider information, so this is justifiable.

Profitable market manipulation opportunities could arise if the transient-, permanent- or cross-price impact is modeled with non-standard shapes that allow for dynamic arbitrage through profitable round trips. However, none of the covered ID papers discuss market manipulation; for Edoli et al. (2016), Gönsch and Hassler (2016), Hassler (2017), Löhndorf et al. (2013) and Farinelli and Tibiletti (2017) this follows naturally from the assumption of no market power; similarly, Garnier and Madlener (2015), Skajaa et al. (2015) and Engmark and Sandven (2017) only include the instantaneous price impact, which never creates profitable round trips on its own; finally, Aïd et al. (2015) and Tan and Tankov (2016) make the standard set of assumptions about the shape of the permanent price impact that avoids arbitrage - thus, market manipulation will always be assumed to be unprofitable. Thus, none of the models permit market manipulation.

Both Hassler (2017) and Garnier and Madlener (2015) attempt to avoid the added complexity of allowing for active use of balancing markets. The former paper claims that use of the balancing markets does not scale to a Nash Equilibrium. However, an incremental increase in the demand for balancing services would increase the price of using them, making it less profitable to rely on the balancing market. It is therefore not clear to the authors of this thesis why such a strategy not would converge to an equilibrium.

Garnier and Madlener (2015) on the other hand apply an asymmetric definition of illegal speculation in the balancing market; they claim that an over-adjustment to an open position of the market amounts to balancing market speculation, whereas failing to close the open position is perfectly OK. Therefore, the residual for the balancing market should always be in the same direction as the original forecast error. It is not explained why the use of the Balancing market in one direction apparently is acceptable, whereas use in the other direction amounts to illegal speculation, ceteris paribus. Without such an explanation, it seems more natural to interpret any large adjustment as active use of the balancing market, regardless of the direction of said adjustment.

To summarize, both papers fail to convincingly argue why use of the Balancing market cannot simply be limited by raising the spread between the imbalance prices in the objective function drastically, and capping the maximum imbalance volume. Alternatively, Aïd et al. (2015) assumes a quadratic imbalance cost, which will make it progressively more expensive to utilize the balancing market and thus avoid too large deviations from schedule.

Finally, none of the papers mention the Order-to-Trade ratio fine. As mentioned, the papers that focus on the dispatch schedule largely don't mention trading altogether, whereas the trading-focused papers place only market orders, which always clear. The omission of the Order-to-Trade ratio fine is thus justifiable. The exception is Garnier and Madlener (2015), which includes the execution risk, and thus opens for the possibility that an order won't clear. However, the paper was written before the current Code of Conduct was written, so the Order-to-Trade ratio was likely introduced at a later point in time (EPEX, 2017a).

To summarize, the balancing market is the only regulatory issue that is covered, and only in some of the relevant papers. However, none of the other regulatory issues seem to be a problem, given the proposed models, so they may be disregarded safely.

2.2.4.2 Imbalance price forecasting

Of the former ITP papers, few attempt to forecast the imbalance price. In particular, Garnier and Madlener (2015), Edoli et al. (2016), Skajaa et al. (2015) and Löhndorf et al. (2013) don't mention it at all. Aïd et al. (2015), Tan and Tankov (2016) and Farinelli and Tibiletti (2017) briefly analyze the sensitivity of the solution to the imbalance price without performing a corresponding empirical analysis to determine realistic values of the parameter. Tan and Tankov (2016) underscores the unprofitability of adjusting imbalances through the balance market: *"In the adjustment market, the bid-ask spread is very wide, which may be interpreted as a penalty imposed on the agents for using this market"*. Gönsch and Hassler (2016) specifically assume that the imbalance price is 2.0 times the representative ID price and Hassler (2017) assume a multiplier of 1.25.

2.2.4.3 Modeling of production costs

As most of the papers about optimization of trading in the Intraday market focus on renewable production, fuel costs are largely irrelevant. Garnier and Madlener (2015), Tan and Tankov (2016) and Edoli et al. (2016) therefore omit production costs altogether, as they focus on intermittent production. The dispatch-focused papers discuss the alternative cost of the Value of Storage. While Farinelli and Tibiletti (2017) assume that the Value of Storage is homogeneous for a given production facility (and Gönsch and Hassler (2016) and Hassler (2017) don't model the value of storage directly), Hassler (2017) finds that heuristics based on the storage level may increase profits. These heuristics are simplified ways to handle that the Value of Storage is a heterogeneous (monotonously non-increasing) function of the storage level. Löhndorf et al. (2013) and Engmark and Sandven (2017) assume that the total Value of Storage in the reservoir is convex and can be approximated by a piecewise linear function. Additionally, Engmark and Sandven (2017) include the start-up cost of the generators, creating path dependencies between consecutive delivery products. Aïd et al. (2015) is the only paper with thermal production, and quadratic production cost is assumed. Skajaa et al. (2015) perform no optimization, and therefore no cost function is included.

2.2.5 Concluding remarks

The existing literature on trading in the Intraday market largely fall in two categories; one is focused on the placement of orders in a continuous auction market with a limit order book, but disregards the impact of optimization of the dispatch schedule and the interdependencies between delivery products; the other is focused on the optimization of the dispatch schedules across all delivery products, but neglects the dynamics of the continuous auction market, and disregards the details of trading in such a market. Both categories of papers have implemented fast solution algorithms that solve the respective problems faster than the chosen benchmarks in each paper.

Of the covered papers in this thesis, no paper models the placement of limit and market orders, and no paper optimizes the placement of multiple orders for multiple delivery products. No paper optimizes both the order placement and the dispatch schedule simultaneously (though Aïd et al. (2015) and Engmark and Sandven (2017) model approximations of this). Only a few papers consider the link between the short- and long term optimization of power trading (i.e. through heterogeneous marginal Value of Storage). Some papers make strict assumptions about the shape of the production costs, which limits their relevance for traders with other cost functions. Moreover, the empirical justification for the assumed market parameters is often lacking. In short, highly efficient solution algorithms have been implemented to solve models that (at least for a significant subset of the traders in the market) are answering the wrong question.

For this reason, it is hypothesized that it is possible to improve the modeling of the trading in the Intraday market by optimizing production and trading simultaneously, by allowing for both sell- and buy orders, and both limit- and market orders. The first improvement implies that the double dynamics are modeled, which is particularly relevant for producers with intertemporal costs or storage reservoirs; the two categories may respectively reduce costs or increase revenues through shifting production to other delivery products, so the production decisions of consecutive delivery products are interconnected. It is believed that this improvement is possible to achieve with only weak assumptions about production costs, and in realistic market assumptions. Moreover, such improvements may be tested and quantified by simulating the market based on empirical data.

Table 2.2.1a: Key features of the most relevant strategy formulation papers

Paper	**Trader type**	**Solution method**	**Decision variable**
Garnier & Madlener (2014)	Intermittent	Options-based	Order placement
Aïd et al. (2015)	Thermal+Wind	Analytic	Trading rate (+dispatch)
Tan & Tankov (2016)	Wind	Analytic	Trading rate
Edoli et al. (2016)	PV	Analytic	Trading rate
Skajaa et al. (2017)	Wind	Logic-based	Order placement
Gönsch & Hassler (2016)	Intermittent+	Numeric: ADP	Dispatch plan
Hassler (2017)	Intermittent+	Numeric: ADP	Dispatch plan
Löhndorf et al. (2013)	Hydro	Numeric: ADDP	Dispatch plan
Farinelli & Tibiletti (2017)	Hydro	Numeric: IPM	Dispatch plan
Engmark & Sandven (2017)	Hydro	Numeric: SMIP	Dispatch (+orders)
This thesis	**Hydro**	**Numeric: MSLP**	**Dispatch+orders**

Table 2.2.1b: Key features of the most relevant strategy formulation papers

Paper	$\lvert\mathcal{T}\rvert > 1$?	$\lvert\mathcal{D}\rvert > 1$?	**Price Impact**	**Buy+ Sell?**	**Limit+ market?**	**Production Costs**
Garnier & Madlener (2014)	Yes	No	IPI	Yes	Limit only	N/A
Aïd et al. (2015)	Yes	No	I+P PI	Yes	Market only	Quadratic
Tan & Tankov (2016)	Yes	No	I+P PI	Yes	Market only	N/A
Edoli et al. (2016)	Yes	No	No	Yes	Market only	N/A
Skajaa et al. (2017)	Yes	No	IPI	Yes	Market only	N/A
Gönsch & Hassler (2016)	No	Yes	No	Sell only	Market only	N/A
Hassler (2017)	No	Yes	No	Either/or	N/A	N/A
Löhndorf et al. (2013)	No	Yes	No	Either/or	N/A	Convex
Farinelli & Tibiletti (2017)	No	Yes	No	Either/or	N/A	Linear
Engmark & Sandven (2017)	(No)	Yes	IPI	Either/or	Limit only	Convex
This thesis	**Yes**	**Yes**	**IPI**	**Yes**	**Yes**	**Convex**

In tables 2.2.1a and 2.2.1b, the key features of the former ITP optimization papers are summarized and compared to this thesis. The "Trader type" column describes which kinds of traders the research is relevant for. Intermittent+ means that the solar and/or wind power plant has co-located storage. The "Solution method" column describes the solution algorithm that is recommended in the paper. *Analytic* means that a closed-form solution is derived for the Hamilton-Jacobi-Bellman PDE. *Logic-based* means that no optimization problem is solved at all. *ADP* is short for Approximate Dynamic Programming (see Powell (2011)), *ADDP* is short for Approximate Dual Dynamic Programming (see Löhndorf et al. (2013)), *IPM* is short for Interior Points Method (see Boyd and Vandenberghe (2004)), *SMIP* is short for Stochastic Mixed Integer Program and MSLP is short for Multistage Stochastic Linear Program. The "Decision variable"-column summarizes the decisions that are made by the models in the paper. Approximately half the papers focus on order placement or trading rates, whereas the other half focuses on the dispatch decision.

The "$|\mathcal{T}| > 1$?" takes the value "Yes" if the paper considers several trading stages per delivery product and "No" otherwise. For Engmark and Sandven (2017) it takes the value "(No)", as several trading stages aren't included in their model, but they recommend that the model is rerun throughout the day. Similarly, the "$|\mathcal{D}| > 1$?"-column asks whether the paper considers multiple delivery products. The "Price Impact" lists the forms of price impact that the paper includes, where "I", "T" and "P" correspond to instantaneous, transient and permanent price impacts respectively. The "Buy+Sell?"-column considers if the paper allows for the placement of both bid and ask orders. If only one type of order is allowed per DP, it is referred to as "Either/Or". The "Limit+Market?"-column similarly considers if both limit and market orders may be placed by the model in the paper. This is often not stated explicitly, but it is implicit in the modeling of e.g. clearance probabilities (which only are relevant for limit orders). The "Production cost" column describes the assumptions that are necessary to make about production costs for the model to work. If costs are approximated as piecewise linear, it is assumed that the underlying cost function is convex.

Chapter 3

Formulating and decomposing the Intraday Trading Problem

Recall from section 2.2 that former papers typically have solved different parts of the *Intraday Trading Problem* (ITP): either the Order Placement Problem, or the Dynamic Resource Allocation Problem. In this chapter, the problem of how to optimize the Intraday production and trading decisions together is outlined. The overall objective is to maximize profit generated from trading- and production decisions for all delivery products with an hourly duration in the EPEX Intraday market on a specific day, for a hydropower producer with a small reservoir. For the rest of the thesis, a novel decomposition of the ITP into three parts is used:

1. *The Price Forecasting Problem* (PFP) is about predicting the future aspects of the market, particularly the prices that the trader may hope to be able to obtain.

2. *The Cost Estimation Problem* (CEP) considers the marginal cost of production for a given power producer.

3. *The Strategy Formulation Problem* (SFP) takes the marginal cost curve and the price forecast as input parameters, and formulates an optimal order placement strategy and a dispatch schedule dynamically.

The rest of this chapter is structured in the following way: in section 3.1, the components of the ITP are outlined further. In particular, the decisions and objectives of each optimization problem is outlined. Section 3.2 outlines the general features of the SFP in more detail. In section 3.3, different versions of the SFP is described for a set of different assumptions.

3.1 The three components of the Intraday Trading Problem

Due to the instantaneous price impact, the price one may hope to obtain in the market is a function of the desired trading volume, and the Price Forecasting Problem could naturally be thought of as a residual demand forecasting problem. The alternative formulation makes it obvious how the decomposition follows the logic of standard microeconomics, where the Price Forecasting Problem is analogous to predicting a demand curve (and possibly other relevant features of the market), the Cost Estimation Problem attempts to establish a supply curve, and the Strategy Formulation Problem is concerned with finding a policy for converging to the equilibrium where the two curves intersect. The main focus of this thesis is on the modeling of the Strategy Formulation Problem, as it is considered the core of the Intraday Trading Problem. The PFP and CEP are playing important supporting roles, providing necessary input parameters to the SFP. Thus, the modeling of the PFP and CEP must be sufficient to evaluate the proposed model for the SFP in realistic conditions.

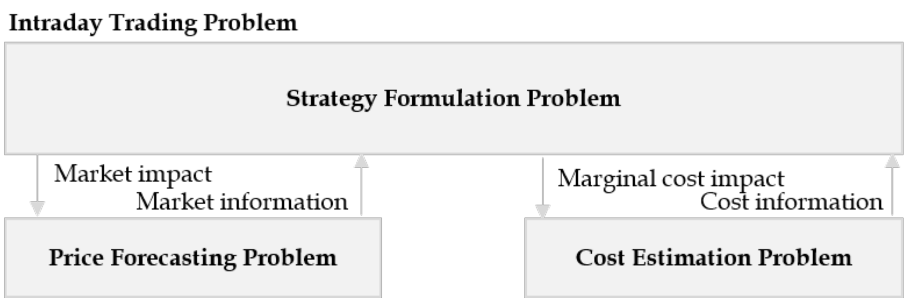

Figure 3.1.1: Intraday Trading Problem breakdown

3.1.1 Requirements to a model of the Price Forecasting Problem

The Price Forecasting Problem consists of determining the market parameters. This may include both deterministic and stochastic parameters, and indeed the act of determining which parameters to model as deterministic, and which to model as stochastic. Examples of such parameters may be the price drift, volatility, and distribution, as well as order depths, magnitudes of the different price impacts, premiums for limit orders and the bid/ask spread, as well as relations between these parameters and correlations between parameters for different delivery products or trading stages. If the outcome space for the stochastic parameters is too large to consider, it may be necessary to create a reduced outcome space that preserves the key properties as well as possible. This is often referred to as scenario reduction in the stochastic programming literature.

The key requirement of a market model is that it preserves the true incentive structure of the trader. In the trading literature, this primarily means to avoid false arbitrage opportunities. Traditionally, this is done by assuming a driftless price process and imposing trading costs - spread, transaction costs, et cetera. Dynamic arbitrage in a market with limited liquidity is avoided by making the standard assumptions about the shape of the price impacts. For instance, Alfonsi and Schied (2010) state that *"any reasonable market impact model should not admit price manipulation strategies in the sense that there are no "round trips" (i.e., trading strategies with zero balance in shares) whose expected trading costs are negative"*. There are several methods that aim at preserving the incentives of the trader in the scenario reduction phase; one example is moment matching, which ensures that the *statistical moments* of the original probability distribution - such as expected value and variance - are preserved (Kaut and Wallace, 2003); another is the minimization of the filtration distance, which is a measure of how different the available information is in each node (de Oliveira et al., 2010). One example of why the statistical moments may matter is that a high price drift may create arbitrage opportunities, and Dupačová et al. (2000)) state that *"Selection of scenarios should respect the no-arbitrage properties"*. For the same reason, it is necessary that some information is revealed in each trading stage; with perfect information, a model may easily find false arbitrage opportunities.

3.1.2 Requirements to a model of the Cost Estimation Problem

The Cost Estimation Problem consists of determining which of the costs described in section 2.1.4 that are relevant for the given producer over short-term decision horizons, and to estimate their magnitudes. This may depend on former trading decisions made by the producer. For instance, if a thermal producer already has committed a significant volume in the Day-Ahead market, start-up costs may be irrelevant because the generators will be running anyways - even if the start-up costs are very large.

Just like the PFP, the key requirement of a production cost model is that it preserves the true incentive structure of the trader. For the hydropower producer in question, it is assumed that the relevant cost driver is the Value of Storage. Moreover, for reasons relating to the coordination between long- and short-term optimization explained in section 2.1.4, the marginal Value of Storage is assumed to be heterogeneous and monotonous non-increasing as a function of storage volume. The reason for the choice of hydropower as the production type is that it makes the double dynamics highly relevant.

3.1.3 Requirements to a model of the Strategy Formulation Problem

The key requirements of the model of the SFP are that it produces high profits in the actual market environment, requiring manageable computational resources. As the liquidity in the market has been increasing fast, the market has changed fast in the past. If the growth continues, it is important that the model not only performs well in the current market environment, but also in similar realistic market environments that may soon be reality. If the decision maker is risk averse, the model of the SFP should take this into account.

3.2 Formulating the Strategy Formulation Problem

As formulated by Akersveen and Graabak (2017), the Strategy Formulation Problem takes the variable costs of production and the price forecast as input parameters, and formulates an optimal order placement strategy for a trader. The goal is to maximize the expected profit from placing orders in the German short-term power markets. The revenue source is cleared sell orders in the Intraday market, less the cost of cleared buy orders - or vice versa, when prices are negative. The relevant costs are transaction costs, production costs, and imbalance costs. Large orders or many consecutive orders may have short- or long-term price impact, reducing the revenue source from cleared orders; thus the market is assumed to be weakly inefficient.

The trader can place market orders and limit orders for a set of delivery products. The order prices, volumes and time of placement and cancellation must be determined, in addition to an optimal dispatch plan. Only cleared orders generate revenues. Order placement and clearing is done continuously within the frame of the Intraday trading windows for each delivery product. The decision horizon is the time from Intraday gate opening until the latest time of delivery of any considered delivery product.

The optimal Intraday order placement strategy depends on the commitment from the Day-Ahead market (and other physical markets such as the reserve markets, though for simplicity they are left out here). If parts of the commitment is not produced, the imbalance is traded in the balancing markets. The production decision is constrained by the upper and lower bounds on the production capacity for a given trader. The produced energy either stems from an inflow of energy, or stored energy. Storage of energy is limited to the storage capacity, and changes in storage levels are subject to conservation of energy. In order to retain the license to trade in the Intraday market, traders must comply with the relevant regulations.

3.3 Developing a set of assumptions for the Strategy Formulation Problem

In this thesis, the set of relevant assumptions to consider is selected based on the assumptions that are made in the contemporary literature. In particular, disagreements in the related literature about which assumptions to make about a concept is used as a heuristic that the topic is worthy of further exploration. The assumptions are described conceptually in the following sections.

3.3.1 Assumption 1: Predetermined or informed production decisions?

The purpose of considering this assumption is to demonstrate that when the target inventory - which in the Intraday power market is referred to as the optimal production quantity - is a function of a stochastic parameter throughout the trading window, there is value in delaying the production decision until more information is available. The producer is assumed to be risk neutral and the market is perfectly liquid. This is illustrated using a trader with some spare production capacity (henceforth referred to as an open position), who is allowed to determine both the order volumes in each trading stage, and the final production. She operates a hydropower plant connected to one reservoir, with a homogeneous marginal production cost. The extension to several reservoirs is fairly straightforward. Any commitment from the Day-Ahead or Intraday market incurs a commitment to produce the equivalent amount of energy, and the Day-Ahead commitment is known in advance. As the balancing market is assumed to always be less profitable than the Intraday market, and both production capacity and order clearing is deterministic, the Balancing market is safely omitted from the model.

3.3.2 Assumption 2: One or several delivery products

The purpose of the case is to show that the decisions made for one delivery product affects decisions related to other delivery products in non-trivial ways, so that simultaneous optimization for several delivery products may increase profits compared to separated optimization of each delivery product. To illustrate this, the initial model is extended to three delivery products. Marginal production costs are assumed to be similar for all DPs, but may be heterogeneous as a function of storage level, because the Value of Storage is a function of the amount of available storage. The cost heterogeneity shows that short-term optimization and long-term planning can be coordinated efficiently.

3.3.3 Assumption 3: One- or two-sided trading?

The purpose of the case is to show that when the target inventory is stochastic, there is value in allowing the model two-sided trading. Even though the initial open position is in a given direction - say, prices are higher than marginal production costs, so the trader wishes to sell - the direction of the open position may flip when prices are stochastic. Thus, the second extension of the original model is to allow the trader to make purchases in addition to sales. The trader is also allowed to place multiple orders throughout the trading window, and the set of orders may be a combination of both ask and bid orders. Transaction costs are introduced, and they are assumed to be symmetric for ask and bid orders, as well as proportional to the clearing volume of each order.

3.3.4 Assumption 4: Limited order depth

The purpose of the case is to show that limited order depth forces the trader to distribute the trading throughout the trading window. With this assumption, some market imperfections are introduced; in particular, the order depth is assumed to be limited. The changes from the perfect market include the introduction of a bid-ask spread, as well as a cap on the maximum clearing volume for a given price in each stage. When order book volumes at the best quoted prices in a stage are exhausted, it is assumed that additional volumes may be available at worse prices. It is assumed that the order depths in a given stage are independent of the decisions of the trader in the earlier stages; that is, the order book is instantly replenished after an order clears, and the transient and long-term price impacts are both zero. While it is still possible to include transaction costs, the presence of the bid-ask spread serves the same purpose and may make it less relevant.

3.3.5 Assumption 5: Limit order premium

The purpose of the case is to show that limit orders broadens the scope of available strategies to the trader; in particular, a trader that is not too risk averse may offer liquidity to the market in exchange for higher expected profits. For this reason, another consequence of the limited order depth is considered; the market making premium. Recall that while the limit order has a potential for higher unit profits than the market order, the clearing of it is uncertain. Only cleared orders have an impact on the objective function. It is assumed that the transaction premium in one stage may be smaller than the change of the best quoted price in the next stage - that is, if prices are 55 €/MWh in stage t, but were only 50 in stage $t-1$, it does not imply that the limit order premium was $\geqslant 5$ in stage $t-1$. However, changes in the base price and limit order premiums are likely correlated.

3.3.6 Assumption 6: Transient and permanent price impact

The purpose of the case is to show that the modeling of the transient price impact in the proposed modeling framework grows overly complex, so that new modeling frameworks are needed to handle this concept. Another consequence of the limited liquidity is the transient price impact following a limit order that depletes the order volumes at the best quotes, and the permanent price impact from strategic competitors in the market. With this assumption, the available order volumes are no longer independent of the decisions of the trader in earlier stages. However, the influx of new orders is still assumed to be independent of the earlier trading decisions, meaning that the permanent price impact still is neglected with the given assumptions.

3.3.7 Assumption 7: Asynchronous gate closure

The purpose of the case is to be able to model the EPEX Intraday market even more realistically. The final assumption recognizes that the actual gate closures of the delivery products happen asynchronously. Thus, trading- and production decisions for the early delivery products must be made without knowledge of the last hours of price information for the later delivery products. The difference from before is therefore that after a few trading stages, the trading windows of the delivery products start closing with a steady frequency of a few stages per gate closure. If there are N stages between two consecutive gate closures, this is equivalent to an assumption of a stage duration of 60/N.

Chapter 4

Mathematical Model

In the following chapter, a model is modularly extended to permit the features that are relevant for each of the cases described in chapter 3. A Multistage Stochastic Linear Program (MSLP) is chosen as the modeling framework. The stochastic processes are modeled in discrete trading time. This framework is assumed more flexible than the ones that some of the former papers (e.g. Aïd et al. (2015), Tan and Tankov (2016)), have proposed, as it can model more constraints and make fewer assumptions about the shape of the cost function. The dispatch-focused papers consider delivery products as the axis of stochasticity rather than trading time, so their modeling choices are less directly comparable.

Such a formulation implies that the time during the trading window is discretized into several stages $t \in \mathcal{T}$, from the initial stage $\underline{t}_0$ to the Gate Closure $\bar{t}^{GC}$. Moreover, the probability distributions of the stochastic variables are discretized, so that the uncertainty can be handled through a set of scenarios $s \in \mathcal{S}$. In the first stage, all scenarios are similar, so that the trader does not know which scenario she is in. Information is revealed gradually in each trading stage, until no uncertainty remains at the end of the trading window. In each stage, the trader may place orders in the market. In the final stage (at Gate Closure), production is determined.

It should be noted that a dynamic framework was also considered, as it is a computationally tractable alternative when there are many stages and the scenario space grows quickly. However, a recourse-based MSLP framework was considered more suitable for the initial exploration and illustrative cases in this

thesis. In particular, the state space of a suitable dynamic program grows so large that significant effort must be put into both the modeling and the solution algorithm of such a framework, making it better suited at a later point when the effects of each of the modeling assumptions are better known.
The units of all parameters or variables are in €, MWh, or €/MWh. For instance, reservoir levels and -inflow are measured in MWh, not in terms of water volumes.

The rest of this chapter is structured in the following way; in sections 4.1.1-4.1.7, a model is formulated and gradually expanded as new assumptions are included. It is recommended to be familiar with the assumptions and problem instances of section 3.3 before reading these chapters. In section 4.2, the gradual changes are summarized as an MSLP, later referred to as the `Benchmark model`. Finally, some sets of model modifications are presented in section 4.3, with the purpose of approximating the models of the related literature.

4.1 Building an Intraday Trading Problem model

Throughout this section, the seven assumptions are presented in the same order that they were introduced in chapter 3.

4.1.1 Assumption 1: Predetermined or informed production decisions?

The objective is to maximize the expectation of the sum of trading profits from all trading stages and minimize production costs, over all scenarios. The expectation is calculated as the weighted sum over all scenarios, where each scenario has the probability ρ_s. Trading profits are the product of prices P_{ts} and placed order volumes x_{ts} (which equal trading volumes). Production costs are the product of unit variable costs C^Q and production volume q_s.

$$\max z = \sum_{s \in \mathcal{S}} \rho_s \Big(\sum_{t \in \mathcal{T}} P_{ts} x_{ts} - C^Q q_s \Big) \tag{4.1.1}$$

Trading incurs a commitment to produce the equivalent volume. The commitment is equal to the sum of the traded volumes in the Intraday market and the

Day-Ahead commitment V_d^{DA}.

$$\text{s.t.} \sum_{t \in \mathcal{T}} x_{ts} - q_s \quad = \quad -V_d^{DA}, \quad s \in \mathcal{S} \tag{4.1.2}$$

Production is limited by the maximum production capacity $\bar{Q}$.

$$q_s \quad \leqslant \quad \bar{Q}, \quad s \in \mathcal{S} \tag{4.1.3}$$

Production is also limited by the sum of the initially available storage R_0 and the inflow F^{in} during the production interval.

$$q_s \quad \leqslant \quad R_0 + F^{in}, \quad s \in \mathcal{S} \tag{4.1.4}$$

Order placement is nonanticipative. The production decision is made with full information, so no nonanticipativity constraints are needed for the production variables. In the nonanticipativity constraints, the set of scenarios $\mathcal{S}$ is partitioned into subsets $\mathcal{S}_{tk}$ for each $t \in \mathcal{T}$. The subsets are mutually inclusive and collectively exhaustive, $\bigcup_{k \in \mathcal{K}_t} \mathcal{S}_{tk} = \mathcal{S}$ and $\bigcap_{k \in \mathcal{K}_t} \mathcal{S}_{tk} = \emptyset$. The scenarios that belong to the same $\mathcal{S}_{tk}$ have identical parameter realization of the stochastic parameters until stage t, thus the decisions should also be identical.

$$x_{ts} \quad = \quad x_{t\sigma}, \quad t \in \mathcal{T}, k \in \mathcal{K}_t, s \in \mathcal{S}_{tk}, \sigma \in \mathcal{S}_{tk} \tag{4.1.5}$$

Sales are nonnegative. From equation (4.1.2), production is also nonnegative without enforcing it as a variable constraint.

$$x_{ts} \quad \geqslant \quad 0, \quad t \in \mathcal{T}, s \in \mathcal{S} \tag{4.1.6}$$

$$q_s \quad free, \quad s \in \mathcal{S} \tag{4.1.7}$$

It is never explicitly enforced that the trader is only allowed to place one order, but the assumptions ensure that the optimal strategy only has exactly one anyways. The reason is that a strategy that is optimal for an infinitesimal, atomic open position scales perfectly when marginal production costs are homogeneous and the market is liquid; neither marginal production costs nor prices are a function of the order volume.

4.1.2 Assumption 2: One or several delivery products?

When several delivery products are introduced, the relevant parameters and variables are given an index $d \in \mathcal{D}$, where d marks one delivery product and $\mathcal{D}$ is the set of all delivery products. Thus, $\{P_{ts}, x_{ts}, q_s, F^{in}\}$ becomes $\{P_{tds}, x_{tds}, q_{ds}, F_d^{in}\}$, and constraints (4.1.2) to (4.1.7) apply for each $d \in \mathcal{D}$.

Moreover, the reservoir constraint (4.1.4) must be adjusted. When there are several delivery products, the initial storage level for a given delivery product is set by a flow constraint. The storage level R_s at the beginning of a delivery hour then becomes a variable r_{ds} for all delivery products except the first, where R_0 is the storage level.

From before, production q_{ds} and inflow F_d^{in} are the factors that may change the storage level r_{ds} (defined as the storage at the beginning of production of delivery product d). In addition to this, it is assumed that water is allowed to spill freely (for instance if the reservoir is full). This is added as an overflow variable f_{ds}^{out}.

$$r_{(d+1)s} - r_{ds} + q_{ds} + f_{ds}^{out} = F_d^{in}, \quad d \in \mathcal{D}, s \in \mathcal{S} \tag{4.1.8}$$

$$f_{ds}^{out} \geqslant 0, \quad d \in \mathcal{D}, s \in \mathcal{S} \tag{4.1.9}$$

The initial storage is equal for all scenarios.

$$r_{0s} = R_0, \quad s \in \mathcal{S} \tag{4.1.10}$$

The outgoing storage level is nonnegative and upper bounded by the reservoir capacity, $\bar{R}^{storage}$.

$$r_{(d+1)s} \leqslant \bar{R}^{storage}, \quad d \in \mathcal{D}, s \in \mathcal{S} \tag{4.1.11}$$

$$r_{(d+1)s} \geqslant 0, \quad d \in \mathcal{D}, s \in \mathcal{S} \tag{4.1.12}$$

Until the final delivery product, the model has an incentive to not spill water, as it can be used in later production. However, spillage during the last delivery product is not penalized as the remaining storage at the end of the day is not included in the objective function. A reward equal to the Value of Storage for remaining water at the end of the production horizon is therefore introduced.

With this modification, the production is still penalized as it reduces the available storage.

$$\max z = \sum_{s \in \mathcal{S}} \rho_s \Big(\sum_{t \in \mathcal{T}} \sum_{d \in \mathcal{D}} P_{tds} x_{tds} + C^Q r_{(\bar{d}+1)s} - C^Q R_0 \Big) \tag{4.1.13}$$

The last term subtracts the value of the initial storage. It consists only of parameters, so it will affect none of the actions. However, the objective value will better reflect the profit throughout the day when the original value of the storage is accounted for.

4.1.2.1 Implications of heterogeneous marginal production costs

In section 2.1.4 it was stated that the Value of Storage is a function of the remaining storage, which is affected by the production decision in the Strategy Formulation Problem. Ideally, the Value of Storage should be recomputed to reflect the new storage level if the assumed dispatch plan is violated significantly, but this is intractable for high-frequency decision making due to the complexity of the problem. In line with the literature on hydropower dispatch optimization, the heterogeneous Value of Storage is piecewise linearly approximated as a function of storage level (see e.g. Farinelli and Tibiletti (2017), Engmark and Sandven (2017)).

For the rest of this thesis, it is assumed that suitable heuristics have been applied and that the marginal Value of Storage accurately reflects the actual alternative cost of production. When MPCs are heterogeneous, the objective function takes the following form.

$$\max z = \sum_{s \in \mathcal{S}} \rho_s \Big(\sum_{t \in \mathcal{T}} \sum_{d \in \mathcal{D}} P_{tds} x_{tds} + C^Q (r_{(\bar{d}+1)s}) r_{(\bar{d}+1)s} - C^Q (r_{0s}) r_{0s} \tag{4.1.14}$$

Here, the function $C^Q(r_{(ds})$ is the average Value of Storage in the reservoir at the beginning of the production interval d. As it is multiplied with the storage level, the term is non-linear and potentially non-convex. In order to handle this, it is linearly approximated. The linearization of the marginal Value of Storage-function is outlined in the following paragraphs.

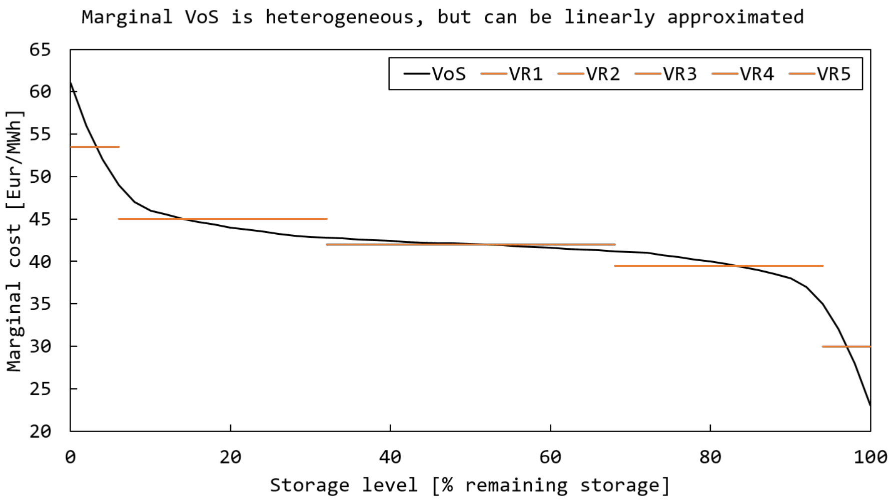

Figure 4.1.1: Hypothetical linear approximation of marginal VoS with virtual reservoirs.

The Value of Storage is linearized by discretizing the reservoir volume into virtual reservoirs with homogeneous marginal Value of Storage per reservoir. Each virtual reservoir is connected to a generator, and is indexed by $\gamma \in \Gamma$. Together, the virtual reservoirs with homogeneous marginal Values of Storage approximate the heterogeneous marginal Value of Storage of the actual reservoir.

$$q_{ds} - \sum_{\gamma \in \Gamma} q_{ds\gamma} = 0, \quad d \in \mathcal{D}, s \in \mathcal{S} \tag{4.1.15}$$

$$r_{ds} - \sum_{\gamma \in \Gamma} r_{ds\gamma} = 0, \quad d \in \mathcal{D}, s \in \mathcal{S} \tag{4.1.16}$$

The virtual production variable is free like the initial production, but the total production is still capped by constraint (4.1.3). All production- and storage-related constraints apply (4.1.8 - 4.1.12) for each virtual reservoir, with one important change:

$$r_{(d+1)s\gamma} - r_{ds\gamma} + q_{ds\gamma} + f^{out}_{ds\gamma} - f^{in}_{ds\gamma} = 0, \quad d \in \mathcal{D}, s \in \mathcal{S}, \gamma \in \Gamma \tag{4.1.17}$$

Note that the inflow parameter F^{in}_d has become a variable $f^{in}_{ds\gamma}$. The reason is that the inflow parameter flows into the original, physical reservoir. When the physical reservoir is modeled as several virtual reservoirs, it is not predefined which virtual reservoir that receive the inflow volume. When the inflow parameter is transformed to a vector of variables - one per virtual reservoir - the model may choose where the inflow should go, and it will choose the virtual reservoir with the highest water value. The total inflow to the virtual reservoirs should equal the inflow to the physical reservoir, and virtual storage should be nonnegative.

$$\sum_{\gamma \in \Gamma} f^{in}_{ds\gamma} = F^{in}_{d}, \quad d \in \mathcal{D}, s \in \mathcal{S} \tag{4.1.18}$$

$$f^{in}_{ds\gamma} \geqslant 0, \quad d \in \mathcal{D}, s \in \mathcal{S}, \gamma \in \Gamma \tag{4.1.19}$$

The non-linear VoS function is approximated by a set of lines.

$$C^{Q}(r_{ds}) \cdot r_{ds} - \sum_{\gamma \in \Gamma} C^{Q}_{\gamma} \cdot r_{ds\gamma} = 0, \quad d \in \mathcal{D}, s \in \mathcal{S} \tag{4.1.20}$$

All of the virtual generators have the same production capacity as the original generator, but equation (4.1.3) still applies, so this is already enforced.

As long as marginal production cost are monotonously non-decreasing as a function of production volume, the model will choose the virtual reservoir with the correct Value of Storage - the cheapest one. Otherwise, integer variables with corresponding constraints can be added to select the correct virtual reservoir. Non-monotonous MPCs may however create multiple equilibria in the intersections between supply and demand, which complicates some of the later cases. Moreover, marginal Value of Storage is monotonously non-increasing as a function of storage level and storage level is monotonously decreasing as a function of production volume, so if Value of Storage is the only cause of the heterogeneous MPC, they should be monotonously non-decreasing. Thus, this is not investigated further.

If the gate closures of the delivery products are synchronous (that is, $\bar{t}^{GC}_{d} = \bar{t}^{GC}_{\partial} = \bar{t}^{GC}$, if $\quad d \in \mathcal{D}, \partial \in \mathcal{D}, d \neq \partial$), as they are with the current formulation, then all storage related decisions (inflow, overflow and production) are made with perfect information. Thus, no nonanticipative constraints are needed.

4.1.3 Assumption 3: One- or two-sided trading?

If it is allowed to both buy and sell, the direction of an order must be specified. Thus, x_{tds} is renamed x^A_{tds}, where the A symbolizes that the order is an ask order. Similarly, x^B_{tds} represents a bid order. Ask orders have a positive profit contribution, while bid orders have a negative profit contribution. A transaction cost C^C is applied to all clearing orders, and it contributes negatively to the objective function for both sales and purchases. The resulting objective function is thus:

$$\begin{aligned} \max z = \sum_{s \in \mathcal{S}} \rho_s \big(\sum_{t \in \mathcal{T}} \sum_{d \in \mathcal{D}} (P_{tds}(x^A_{tds} - x^B_{tds}) - C^C(x^A_{tds} + x^B_{tds})) \\ + C^Q(r_{(\bar{d}+1)s}) r_{(\bar{d}+1)s} - C^Q(r_{0s}) r_{0s} \end{aligned} \tag{4.1.21}$$

The rationale for the transaction cost is to prevent arbitrage. Transaction costs may prevent arbitrage opportunities when there is a weak expected price drift if the double transaction cost from a round trip exceed the expected price difference (Fruth et al. (2017), Alfonsi and Schied (2010)).

Equation (4.1.22) then must be adjusted so that a production commitment can be exited with a bid order x^B_{tds}.

$$\sum_{t \in \mathcal{T}} (x^A_{tds} - x^B_{tds}) - q_{ds} = -V^{DA}_d, \quad d \in \mathcal{D}, s \in \mathcal{S} \tag{4.1.22}$$

By equation (4.1.22), production is no longer positive by definition, so a lower bound is introduced. The lower bound on production may be different from zero, if pumps are installed in connection to the dam. The parameter $\underline{Q}_d$ is therefore introduced for the lower bound.

$$q_{ds} \geqslant \underline{Q}_d, \quad d \in \mathcal{D}, s \in \mathcal{S} \tag{4.1.23}$$

The new order placement variables are non-negative and nonanticipative like the former ones, while the production variable still is free and determined with perfect information.

4.1.4 Assumption 4: Limited order depth

When the available order depth is limited, the modeling of the price changes significantly. In particular, there is no longer one well defined price P_{tds} at which ask and bid orders will clear. Instead, there is a range of prices, where different volumes are available depending on the price. The price range is approximated by a set of order levels $p \in \mathcal{P}$. For each value that the index p may take, there is a corresponding order level P_p, sorted so that prices with lower indices are lower: $P_p \leqslant P_\rho$ if $p \in \mathcal{P}, \rho \in \mathcal{P}, p \leqslant \rho$. The set of buy order and sell order prices may be different, so that the price P_p^A may be different from P_p^B, even for the same value of the p-index.

The difference between P_p and P_{p+1} is necessarily larger than the minimum price tick, and may vary with p, t, d, and s. This is referred to as a *block-shape approximation* of the order book by Tsoukalas et al. (2017) and Wu and Gao (2018), and is used by e.g. Fruth et al. (2017), Alfonsi and Schied (2010) and Horst and Naujokat (2014). To reduce the number of parameters, only the relevant order levels are included. In stage t in scenario s for delivery product d, the relevant prices are P_{ptds}^B and P_{ptds}^A for $p \in \mathcal{P}$. Note that these prices may be different even if they have the same index.

At each order level, a given order volume is available. The volume may differ between trading stages and delivery products, and be different for ask and bid orders (in fact, the available order depth at a given order level is necessarily zero either on the ask or bid side for each order level, since otherwise a transaction would occur and the orders would be removed. The ask order depth at order level p in stage t for delivery product d in scenario s is referred to as A_{ptds}. The corresponding bid order volume is referred to as B_{ptds}. Order placement variables are modified to specify the order price, so that $\{x_{tds}^A, x_{tds}^B\}$ becomes $\{x_{ptds}^A, x_{ptds}^B\}$. The trading profit term in the objective function must therefore include a sum over p too:

$$\begin{aligned}\max z = \sum_{s \in \mathcal{S}} \rho_s \Big(\sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \sum_{d \in \mathcal{D}} (P_{ptds}^B x_{ptds}^A - P_{ptds}^A x_{ptds}^B - C^C (x_{ptds}^A + x_{ptds}^B)) \\ + C^Q (r_{(\bar{d}+1)s}) r_{(\bar{d}+1)s} - C^Q (r_{0s}) r_{0s}\end{aligned} \tag{4.1.24}$$

Observe that the ask order(s) x_{ptds}^A clears with the price P_{ptds}^B, *not* P_{ptds}^A. The reason is that it clears by matching the bid order(s) with volume B_{ptds}. For the same reason, x_{ptds}^B clears with the price P_{ptds}^A. The difference in price between two order levels is referred to as the order level penalty, as it is the penalty that a trader faces for wanting to trade larger volumes than the order depth at the given order level. The instantaneous price impact is a function of both the order depth and the order level penalty. In liquid LOBs, it is assumed that there are orders on every price tick, but this isn't necessarily the case in less liquid markets. For this reason, the term "order level penalty" is not common in the literature. Note that in this thesis, "penalty" refers to a less desirable price - for either a seller or a buyer - and "premium" refers to a more desirable price in either direction, though obviously a buyer desires a discount rather an actual premium.

Note that with no loss of generality, the model in this thesis always specifies the order level that an order is placed at (all orders have a p-index), although this is superfluous when orders clear completely and instantly at the specified price: they could have been placed as market orders - who have no defined price - and the outcome would have been the same. This modeling choice ads no computational complexity, but reduces the decision space and simplifies the syntax.

Order volumes are restricted to the available order depth at a given order level. The constraints for the ask and bid side are perfectly symmetrical.

$$x^A_{ptds} \leqslant B_{ptd}, \quad p \in \mathcal{P}, t \in \mathcal{T}, d \in \mathcal{D}, s \in \mathcal{S} \tag{4.1.25}$$

$$x^B_{ptds} \leqslant A_{ptd}, \quad p \in \mathcal{P}, t \in \mathcal{T}, d \in \mathcal{D}, s \in \mathcal{S} \tag{4.1.26}$$

Note that in reality, ask (bid) orders may have lower (higher) prices than the orders that are available in the market, and a transaction will still occur. Without loss of generality, this inequality is transformed to an equality in the model: only orders with equal prices will match. This significantly simplifies transaction price calculations, and breaks symmetry.

4.1.5 Assumption 5: Limit order premium

When limit orders (above the already available prices in the market) are allowed, the placed and cleared order volume are no longer necessarily equal. The placement of an ask order is still referred to as x^A_{ptds}, whereas the order clearing is referred to v^A_{ptds}. A similar terminology is adopted for bid orders.

In the objective function, order volumes are replaced with transaction volumes.

$$\max z = \sum_{s \in \mathcal{S}} \rho_s \Big(\sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \sum_{d \in \mathcal{D}} \big(P^B_{ptds} v^A_{ptds} - P^A_{ptds} v^B_{ptds} - C^C (v^A_{ptds} + v^B_{ptds}) \big) + C^Q (r_{(\bar{d}+1)s}) r_{(\bar{d}+1)s} - C^Q (r_{0s}) r_{0s} \tag{4.1.27}$$

Constraints (4.1.25) and (4.1.26) remain the same, but the interpretation of A_{ptds} and B_{ptds} is now the *highest possible available volume*. For order levels with worse than or equal prices than the best quote available, the volume is certainly available as it already is in the order book. For better prices, the volume may be made available during the stage, or not. The probability that the volumes are made available during the stage, is falling as a function of the premium (discount) received compared to the best bid (ask) quote.

The parameter $\Xi^B_{ptds} = \{0, 1\}$ is chosen to model the execution risk related to limit orders. If it takes the value 1, it is possible to clear ask orders at the order level p during stage t for delivery product d in scenario s. Otherwise, it is not possible to clear such an order. For order levels with prices below the best quoted price at the beginning of the stage, it is known that Ξ^B_{ptds} is 1 in advance. For better prices, this is not known in advance, so there is an execution risk related to the placement of an order at that price.

$$v^A_{ptds} - \Xi^B_{ptds} x^A_{ptds} = 0, \quad p \in \mathcal{P}, t \in \mathcal{T}, d \in \mathcal{D}, s \in \mathcal{S} \tag{4.1.28}$$

Symmetry still applies.

$$v^B_{ptds} - \Xi^A_{ptds} x^B_{ptds} = 0, \quad p \in \mathcal{P}, t \in \mathcal{T}, d \in \mathcal{D}, s \in \mathcal{S} \tag{4.1.29}$$

The order clearing volume variables inherit the nonanticipative properties of the order placement volume variables from constraints (4.1.28) and (4.1.29). Like order volumes, transaction volumes are non-negative. It is assumed that limit orders never clear in the last trading stage, as it is too late to find a counterparty for the transaction. An illustration of a snapshot of the limit order book is shown in figure 4.1.2. Note how the bid limit orders are placed with the hope that ask orders may be placed at better prices than the current best quoted ask order, which is why the bid limit order levels are shown on the same side as the accessible ask orders, and vice versa.

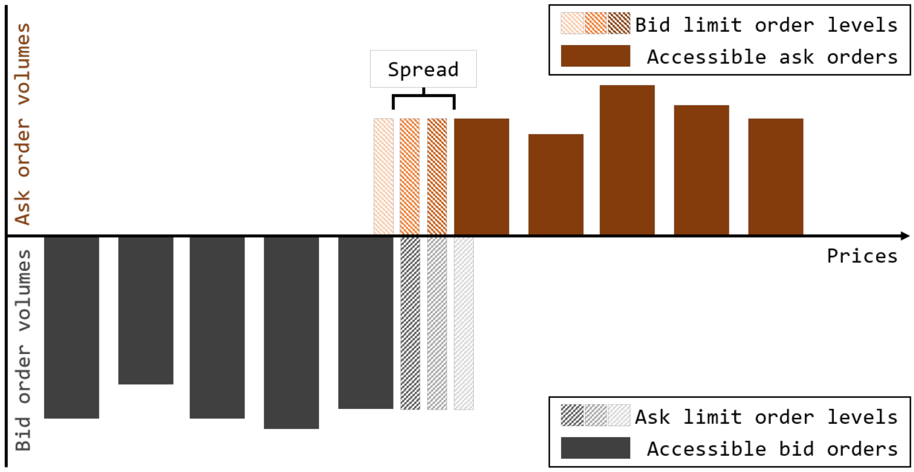

Figure 4.1.2: A block approximation of a hypothetical snapshot of the limit order book

4.1.5.1 Implications of the execution risk

Constraints (4.1.28) and (4.1.29) state that the cleared volume must be lower than the order volume that is placed in the same stage. It is thus implicitly assumed that there are no residual volumes from former stages; orders are canceled by default at the end of each trading stage. This simplifies the modeling somewhat as it reduces the decision space, and it becomes superfluous to model the residual open order volume from former stages. With a dynamic formulation, such an assumption would be a necessity, as the state space would grow with a factor of $\bar{V}_p^{|\mathcal{P}|}$ (where $\bar{V}_p$ is the maximum order volume per order level and $|\mathcal{P}|$ is the number of order levels) if residual open order volumes must be included.

It may seem like the modeling of the Order to Trade Ratio fine becomes relevant at this point, since it is now possible to place orders that have a low probability of clearing. However, this can be solved by preprocessing; if the empirical probability that a given order clears is too low, the probability that is provided to the model may be set to 0. Thus, the model will believe that there is no reason to place such an order (see figure 4.1.3 for a visualization with a hypothetical distribution, and the modified distribution that is provided to the model), and the probability that the Order to Trade ratio is violated approaches zero.

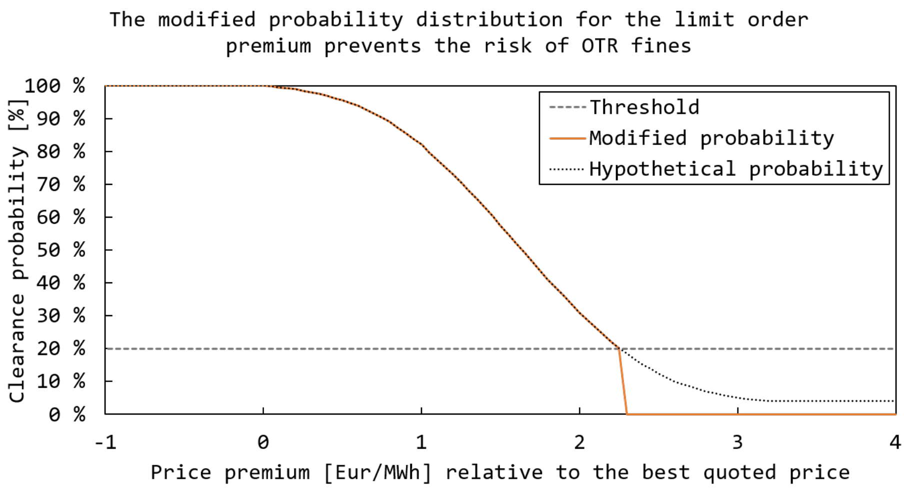

Figure 4.1.3: Hypothetical modified clearance probability for limit orders.

A quick calculation shows that if the lower clearance probability threshold is 20%, even conservative assumptions makes it very hard to fall below the 2% clearing rate where an Order to Trade ratio alert is issued. Specifically, three assumptions are made; 51 orders are placed (which is the lowest number that can trigger and Order to Trade ratio alert, and therefore the number where an alert is the most likely) in all trading windows; all orders are placed at the threshold clearance probability; and the clearing of the orders are statistically independent. In this case, the probability per trading window that less than 2 orders clear and an Order to Trade ratio alert is issued is $(1-20\%)^{51}+(1-20\%)^{50}\cdot 20\% \quad = \quad 1.4\cdot 10^{-5}$. The risk that this happens more than four times, even in months with 31 days (where the number of delivery products is the largest), is clearly negligible - in fact, the probability that the number of Order to Trade ratio alerts is zero is $(1-1.4\cdot 10^{-5})^{31\cdot 24} \quad = \quad 99\%$. Thus, the handling of the Order to Trade ratio constraint is performed in the preprocessing, and no explicit modeling is needed.

If orders don't clear as planned, it may happen that the market commitment after gate closure is infeasible or very expensive to produce. Thus, the balancing market becomes relevant. Down- and up-adjustments in the balancing market are added to the objective function as sales and purchases, respectively.

$$\begin{aligned}\max z = \sum_{s\in\mathcal{S}} \rho_s \Big(\sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} \sum_{d\in\mathcal{D}} (P^B_{ptds} v^A_{ptds} - P^A_{ptds} v^B_{ptds} - C^C(v^A_{ptds} + v^B_{ptds})) \\ + C^Q(r_{(\bar{d}+1)s}) r_{(\bar{d}+1)s} - C^Q(r_{0s}) r_{0s} + \sum_{d\in\mathcal{D}} (P^{BM-}_{ds} v^{BM-}_{ds} - P^{BM+}_{ds} v^{BM+}_{ds}) \Big)\end{aligned} \tag{4.1.30}$$

The adjustment in the balancing market equals the difference between the commitment from the former markets and the production. Equation (4.1.31) replaces equation (4.1.22).

$$\sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} (v^A_{ptds} - v^B_{ptds}) + v^{BM-}_{ds} - v^{BM+}_{ds} - q_{ds} = -V^{DA}_d, \quad d\in\mathcal{D}, s\in\mathcal{S} \tag{4.1.31}$$

The use of the balancing market is restricted with a chance constraint, ensuring that it is at most $\bar{R}^{BM}$ MWh per day with at least $\underline{\rho}^{BM}$ probability.

$$\Pr\{\sum_{d\in\mathcal{D}} (v^{BM+}_{ds} + v^{BM-}_{ds}) \leqslant \bar{R}^{BM}\} \geqslant \underline{\rho}^{BM} \tag{4.1.32}$$

Assuming that there is sufficient liquidity in the market (and flexibility in the production technology) to avoid the balancing market through the use of market orders (even if the prices of the given market orders are poor), the chance constraint can be simplified to a deterministic constraint.

$$\sum_{d\in\mathcal{D}} (v^{BM+}_{ds} + v^{BM-}_{ds}) \leqslant \bar{R}^{BM}, \quad s\in\mathcal{S} \tag{4.1.33}$$

Balancing market adjustment is nonnegative. The adjustment is made with perfect information, so no nonanticipativity constraints are needed.

$$v^{BM-}_{ds}, v^{BM+}_{ds} \geqslant 0, \quad d\in\mathcal{D}, s\in\mathcal{S} \tag{4.1.34}$$

With the current formulation, the structure of each trading stage is illustrated in figure 4.1.4, where LOP is short for limit order premium.

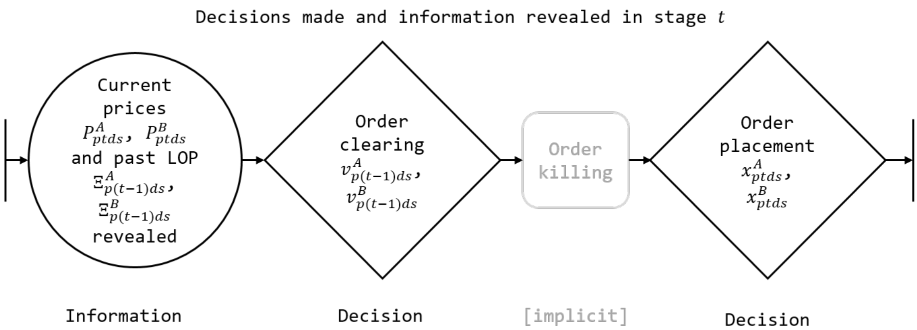

Figure 4.1.4: The structure of a trading stage

4.1.6 Assumption 6: Transient and permanent price impact

It would also be desirable to capture transient and permanent price impact. Former papers in conventional LOB markets do this by generating scenarios with the "unperturbed price", formulating the problem dynamically, assuming a nice perturbation, and then solving the Hamilton-Jacobi-Bellman partial differential equation analytically. As the stages in the MSLP are solved in the same program rather than separately as in a DP, this is not a feasible approach; it would create at non-linear (non-convex) product between the price and the volume in the objective function.

An alternative approach to modeling the transient price impact was therefore attempted; the full order book could be modeled, so that orders that have cleared are permanently removed from the order book. As new orders are added independently, the impact on the price gradually disappears. However, such a model grows vastly complex, with thousands of binary variables; the reader is referred to Appendix A. This is therefore clearly not a tractable approach. Moreover, it would have been unable to capture the permanent price impact, since the placement and canceling of limit orders by other actors in the market was assumed to be independent from the actions of the trader.

It is hypothesized that a dynamic linear model formulation could be able to model the transient and permanent price impact, using the same modeling trick as the former papers (Aïd et al. (2015), Tan and Tankov (2016)), but with a numerical solution method. Such an approach would not only require a reformulation of the entire model; the amplitudes of the two price impacts as well as the decay rate of the transient price impact must also be estimated. Such an estimation is non-trivial, as it is hard to untangle the causality; even if prices fall after a given order is placed, the price fall isn't necessarily caused by the order placement. For these reasons, the approach is considered out of scope for this thesis, though it is recommended as a future area of research.

4.1.7 Assumption 7: Asynchronous gate closure

When gate closures happen asynchronously for different delivery products - for instance, DP11 closes at 10:55, DP12 closes at 11:55 et cetera - the set of trading stages must be defined separately for each delivery product: $t \in \mathcal{T}$ becomes $t \in \mathcal{T}_d$, and $\bar{t}^{GC}$ becomes $\bar{t}_d^{GC}$. Moreover, all production decisions except the one for the last delivery product are no longer made with full information. Thus, nonanticipativity constraints are needed.

$$q_{ds\gamma} = q_{d\sigma\gamma}, \quad d \in \mathcal{D}, k \in \mathcal{K}_t, s \in \mathcal{S}_{tk}, \sigma \in \mathcal{S}_{tk}, t = \bar{t}_d^{GC}, \gamma \in \Gamma \tag{4.1.35}$$

The inflow and overflow variables also need nonanticipativity constraints.

$$f_{ds\gamma}^{in} = f_{d\sigma\gamma}^{in}, \quad d \in \mathcal{D}, k \in \mathcal{K}_t, s \in \mathcal{S}_{tk}, \sigma \in \mathcal{S}_{tk}, t = \bar{t}_d^{GC}, \gamma \in \Gamma \tag{4.1.36}$$

$$f_{ds\gamma}^{out} = f_{d\sigma\gamma}^{out}, \quad d \in \mathcal{D}, k \in \mathcal{K}_t, s \in \mathcal{S}_{tk}, \sigma \in \mathcal{S}_{tk}, t = \bar{t}_d^{GC}, \gamma \in \Gamma \tag{4.1.37}$$

As production q_{ds} is linearly dependent on virtual production $q_{ds\gamma}$, it inherits the nonanticipative property. Likewise, the storage variable $r_{ds\gamma}$ is linearly dependent on the production, inflow and overflow, and inherits the nonanticipative property from them. Balancing market adjustments are nonanticipative.

$$v_{ds}^{BM+} = v_{d\sigma}^{BM+}, \quad d \in \mathcal{D}, k \in \mathcal{K}_t, s \in \mathcal{S}_{tk}, \sigma \in \mathcal{S}_{tk}, t = \bar{t}_d^{GC} \tag{4.1.38}$$

$$v_{ds}^{BM-} = v_{d\sigma}^{BM-}, \quad d \in \mathcal{D}, k \in \mathcal{K}_t, s \in \mathcal{S}_{tk}, \sigma \in \mathcal{S}_{tk}, t = \bar{t}_d^{GC} \tag{4.1.39}$$

4.2 Modeling the Strategy Formulation Problem

As the initial model has undergone substantial changes throughout the last few sections, this section summarizes the changes by presenting the proposed final version of the model.

The final version of the model assumes that one already has a piecewise linearization of the Value of Storage as a function of storage level, and a description of the relevant market parameter. In particular, it is necessary to have a description of the uncertainty in the stochastic parameters. The parameters in the model with an s-index are the scenario probabilities ρ_s, the prices P^A_{ptds} and P^B_{ptds}, and the limit order clearance parameters Ξ^A_{ptds} and Ξ^B_{ptds}. Thus, those are the only parameters that vary between the scenarios, and all other parameters are deterministic. The parameters with a scenario index have known values for each scenario, but the nonanticipativity constraints ensure that the decisions that are made are implementable based only on the information that should be available in the given node.

All former trading-focused ID papers have modeled price as stochastic. This is a natural choice, as deterministic prices create a false arbitrage opportunity for the model, which will dominate its decision making. Limit order clearing has not been modeled as a stochastic process in the ID literature before, but it is common in the conventional LOB literature. It is a necessary feature if one wishes to study the tradeoff between limit and market orders; if the clearing of a limit order is known in advance, there is no such tradeoff as limit orders with certain clearing dominate the market orders. It could also have been interesting to include stochasticity in other parameters; for instance, a wind power producer could have been modeled with the production inflows F^{in}_d as stochastic parameters F^{in}_{ds} and creating a plant with zero marginal production cost. The choice of stochastic parameters is a tradeoff; while there may be some uncertainty around many parameters, it is necessary to limit the number of stochastic processes for computational reasons.

In the final version of the model, it is assumed that there is limited liquidity in the market - limit orders trade at a premium relative to market orders, and large trading volumes are instantaneously penalized. Transient and permanent price impact is negligible, however. Both buy and sell market orders can be placed in all stages of the trading window. The same applies to limit orders, except for the very last trading stage for each delivery product, when it is too late to find a counterparty - the limit order cannot clear at gate closure. Both trading and production is optimized, and production is optimized at gate closure, just before delivery. However, gate closures for the different delivery products are asynchronous, so only the very last production decision is made with perfect information.

The final objective function includes trading profits, transaction costs, changes in the Value of Storage and imbalance costs.

$$\begin{aligned} \max z = \sum_{s \in \mathcal{S}} \rho_s (\sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \sum_{d \in \mathcal{D}} (P^B_{ptds} v^A_{ptds} - P^A_{ptds} v^B_{ptds} - C^C (v^A_{ptds} + v^B_{ptds})) \\ + C^Q (r_{(\bar{d}+1)s}) r_{(\bar{d}+1)s} - C^Q (r_{0s}) r_{0s} + \sum_{d \in \mathcal{D}} (P^{BM-}_{ds} v^{BM-}_{ds} - P^{BM+}_{ds} v^{BM+}_{ds})) \end{aligned} \quad (4.2.1)$$

4.2.1 Physical constraints

Production is bounded from above by the upper production constraint.

$$q_{ds} \leqslant \bar{Q}, \quad d \in \mathcal{D}, s \in \mathcal{S} \quad (4.2.2)$$

Production is bounded from below by the lower production constraint.

$$q_{ds} \geqslant \underline{Q}, \quad d \in \mathcal{D}, s \in \mathcal{S} \quad (4.2.3)$$

Production, storage and inflow are decomposed into virtual reservoirs with homogeneous Value of Storage.

$$q_{ds} - \sum_{\gamma \in \Gamma} q_{ds\gamma} = 0, \quad d \in \mathcal{D}, s \in \mathcal{S} \quad (4.2.4)$$

$$r_{ds} - \sum_{\gamma \in \Gamma} r_{ds\gamma} = 0, \quad d \in \mathcal{D}, s \in \mathcal{S} \quad (4.2.5)$$

$$\sum_{\gamma \in \Gamma} f^{in}_{ds\gamma} = F^{in}_d, \quad d \in \mathcal{D}, s \in \mathcal{S} \quad (4.2.6)$$

Virtual storage may be changed by inflow, outflow or production decisions.

$$r_{(d+1)s\gamma} - r_{ds\gamma} + q_{ds\gamma} + f^{out}_{ds\gamma} - f^{in}_{ds\gamma} = 0, \quad d \in \mathcal{D}, s \in \mathcal{S}, \gamma \in \Gamma \quad (4.2.7)$$

The initial virtual storage is equal for all scenarios.

$$r_{0s\gamma} = R_{0\gamma}, \quad s \in \mathcal{S}, \gamma \in \Gamma \tag{4.2.8}$$

The outgoing storage level is nonnegative and capped by the reservoir capacity.

$$r_{(d+1)s\gamma} \leqslant \bar{R}_{\gamma}^{storage}, \quad d \in \mathcal{D}, s \in \mathcal{S}, \gamma \in \Gamma \tag{4.2.9}$$

The total Value of Storage equals the volume-weighted sum of the aggregated Value of Storages for the virtual reservoirs.

$$C^Q(r_{ds}) \cdot r_{ds} - \sum_{\gamma \in \Gamma} C_{\gamma}^Q \cdot r_{ds\gamma} = 0, \quad d \in \mathcal{D}, s \in \mathcal{S} \tag{4.2.10}$$

4.2.2 Trading constraints

Order depths may not be larger than the maximum available volumes in the market.

$$x_{ptds}^A \leqslant B_{ptd}, \quad p \in \mathcal{P}, t \in \mathcal{T}, d \in \mathcal{D}, s \in \mathcal{S} \tag{4.2.11}$$

$$x_{ptds}^B \leqslant A_{ptd}, \quad p \in \mathcal{P}, t \in \mathcal{T}, d \in \mathcal{D}, s \in \mathcal{S} \tag{4.2.12}$$

Only placed orders may clear, and only if a counterpart matches the order.

$$v_{ptds}^A - \Xi_{ptds}^B x_{ptds}^A = 0, \quad p \in \mathcal{P}, t \in \mathcal{T}, d \in \mathcal{D}, s \in \mathcal{S} \tag{4.2.13}$$

$$v_{ptds}^B - \Xi_{ptds}^A x_{ptds}^B = 0, \quad p \in \mathcal{P}, t \in \mathcal{T}, d \in \mathcal{D}, s \in \mathcal{S} \tag{4.2.14}$$

4.2.3 Connecting trading and production

The adjustment in the balancing market equals the difference between the commitment from the former markets and the production.

$$\sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}_d} (v_{ptds}^A - v_{ptds}^B) + v_{ds}^{BM-} - v_{ds}^{BM+} - q_{ds} = -V_d^{DA}, \quad d \in \mathcal{D}, s \in \mathcal{S} \tag{4.2.15}$$

The use of the balancing market is restricted.

$$\sum_{d \in \mathcal{D}} (v_{ds}^{BM+} + v_{ds}^{BM-}) \leqslant \bar{R}^{BM}, \quad s \in \mathcal{S} \tag{4.2.16}$$

4.2.4 Nonanticipativity constraints

Virtual production, inflow, and overflow adjustments are nonanticipative.

$$q_{ds\gamma} = q_{d\sigma\gamma}, \quad d \in \mathcal{D}, k \in \mathcal{K}_t, s \in \mathcal{S}_{tk}, \sigma \in \mathcal{S}_{tk}, t = \bar{t}_d^{GC}, \gamma \in \Gamma \tag{4.2.17}$$

$$f^{in}_{ds\gamma} = f^{in}_{d\sigma\gamma}, \quad d \in \mathcal{D}, k \in \mathcal{K}_t, s \in \mathcal{S}_{tk}, \sigma \in \mathcal{S}_{tk}, t = \bar{t}_d^{GC}, \gamma \in \Gamma \tag{4.2.18}$$

$$f^{out}_{ds\gamma} \quad f^{out}_{d\sigma\gamma}, \quad d \in \mathcal{D}, k \in \mathcal{K}_t, s \in \mathcal{S}_{tk}, \sigma \in \mathcal{S}_{tk}, t = \bar{t}_d^{GC}, \gamma \in \Gamma \tag{4.2.19}$$

Order placement (and thus -clearing) is nonanticipative.

$$x^A_{ptds} = x^A_{ptd\sigma}, \quad p \in \mathcal{P}, t \in \mathcal{T}, d \in \mathcal{D}, k \in \mathcal{K}_t, s \in \mathcal{S}_{tk}, \sigma \in \mathcal{S}_{tk} \tag{4.2.20}$$

$$x^B_{ptds} = x^B_{ptd\sigma}, \quad p \in \mathcal{P}, t \in \mathcal{T}, d \in \mathcal{D}, k \in \mathcal{K}_t, s \in \mathcal{S}_{tk}, \sigma \in \mathcal{S}_{tk} \tag{4.2.21}$$

The balancing market adjustment is nonanticipative.

$$v^{BM+}_{ds} = v^{BM+}_{d\sigma}, \quad d \in \mathcal{D}, t = \bar{t}_d^{GC}, k \in \mathcal{K}_t, s \in \mathcal{S}_{tk}, \sigma \in \mathcal{S}_{tk} \tag{4.2.22}$$

$$v^{BM-}_{ds} = v^{BM-}_{d\sigma}, \quad d \in \mathcal{D}, t = \bar{t}_d^{GC}, k \in \mathcal{K}_t, s \in \mathcal{S}_{tk}, \sigma \in \mathcal{S}_{tk} \tag{4.2.23}$$

4.2.5 Variable domains

Storage, inflow and overflow are nonnegative.

$$r_{ds\gamma}, f^{in}_{ds\gamma}, f^{out}_{ds\gamma} \geqslant 0, \quad d \in \mathcal{D}, s \in \mathcal{S}, \gamma \in \Gamma \tag{4.2.24}$$

$$r_{ds} \geqslant 0, \quad d \in \mathcal{D}, s \in \mathcal{S} \tag{4.2.25}$$

The production variables are free.

$$q_{ds\gamma} \quad free, \quad d \in \mathcal{D}, s \in \mathcal{S}, \gamma \in \Gamma \tag{4.2.26}$$

$$q_{ds} \quad free, \quad d \in \mathcal{D}, s \in \mathcal{S} \tag{4.2.27}$$

Order placement and -clearing is nonnegative.

$$x^A_{ptds}, x^B_{ptds} \geqslant 0, \quad p \in \mathcal{P}, t \in \mathcal{T}, d \in \mathcal{D}, s \in \mathcal{S} \tag{4.2.28}$$

$$v^A_{ptds}, v^B_{ptds} \geqslant 0, \quad p \in \mathcal{P}, t \in \mathcal{T}, d \in \mathcal{D}, s \in \mathcal{S} \tag{4.2.29}$$

Balancing market adjustment is nonnegative.

$$v^{BM-}_{ds}, v^{BM+}_{ds} \geqslant 0, \quad d \in \mathcal{D}, s \in \mathcal{S} \tag{4.2.30}$$

4.3 Alternative models from the contemporary literature

In this section, the model described in section 4.2 (henceforth referred to as the `Benchmark Model`) is modified so that the decisions approximates the output from models that make different assumptions. These alternative models all lack one or several of the features of the `Benchmark Model`. Note that neither of the alternatives are exact replicas of any given model in the reviewed ITP papers. Together, the models span the range of assumptions that there is disagreement about in the literature, and each of the alternative models explores the contribution to the objective value of one modeling assumption. The alternative models make the following assumptions that `Benchmark model` does not:

1. That the production quantity is predetermined, and a single order is placed. It has order volume equal to the production quantity, and order price equal to the representative price that the production quantity was based on (*Alternative 1A*).
2. That the production quantity is predetermined, before the trading is optimized (*Alternative 1B*).
3. That each delivery product is optimized separately, without coordinating between them (*Alternative 2*).
4. That trading is one-sided; only selling (or buying) is allowed (*Alternative 3*).
5. That only market orders can be placed - no limit orders (*Alternative 4A*).

6. That the trader falsely assumes the market to be liquid; only market orders can be placed, and the instantaneous price impact is neglected (*Alternative 4B*).

This is done to try to quantify the cost of limiting Intraday trading models in different manners. In order to do so, it is necessary to make realistic assumptions about the parameters of the production system and the market, including the production and storage capacities, the inflow of power, the marginal production costs, the Day-Ahead commitment, the imbalance prices and bounds on the use of balancing markets, the bid-ask spread, the limit order premium, the order depths, and the properties of the prices of orders in the market, such as drift, mean reversion, autocorrelations and volatility.

4.3.1 Alternative 1A: Predetermined production, one single order

The abstract of Hassler (2017) states that *"we present a model (...) for the short-term trading of intermittent energy production"*. Similar formulations can be found in the rest of the dispatch-focused ITP literature. However, recall from section 2.2 that these papers optimize only the production decision, and leave out the order placement decisions. There are several assumptions that may make this a reasonable choice, and two of them are explored in this thesis. The first possible interpretation of the dispatched-focused ITP papers is that they assume that the details of the order placement decisions don't matter much for the final profit, as long as the resource allocation of the stored energy is optimal. In this case, a very simple trading strategy is sufficient - for instance, one single order may be placed after the production decision has been made. There are thus two algorithmic steps that must be approximated; the optimization of the dispatch schedule, and then the clearing of the single order.

In order to optimize production, a representative price is chosen. If the price drift is weak, the current price is a good approximation of the representative price. The first stage best ask- and bid prices for each delivery product are therefore assumed to apply for the entire trading window, and the model is run as normally. The value of the objective function will be wrong since the actual price scenarios throughout the trading window aren't used, but the dispatch decisions $\hat{Q}_d, \quad d \in \mathcal{D}$ are saved.

$$q_{ds} = \hat{Q}_d, \quad d \in \mathcal{D}, s \in \mathcal{S} \tag{4.3.1}$$

$$\sum_{p \in \mathcal{P}_d^{bad,B}} x_{ptds}^A = 0, \quad t \in \mathcal{T}, d \in \mathcal{D}, s \in \mathcal{S} \tag{4.3.2}$$

$$\sum_{p \in \mathcal{P}_d^{bad,A}} x_{ptds}^B = 0, \quad t \in \mathcal{T}, d \in \mathcal{D}, s \in \mathcal{S} \tag{4.3.3}$$

In constraints 4.3.2 and 4.3.3 $\mathcal{P}_d^{bad}$ is the set of prices that is worse than the representative price for the given delivery product, where "worse" is defined in different directions for ask and bid orders; P_{pd}^B is in $\mathcal{P}_d^{bad,B}$ if and only if it is lower than the representative price $\hat{P}_d^B$, while $P_{pd}^A \in \mathcal{P}_d^{bad,A}$ iff $P_{pd}^A > \hat{P}_d^A$. Note that $\hat{P}_d^B = \hat{P}_d^A$ if the spread is zero. In the objective function, the price received for all cleared orders are equal to the representative price.

$$\begin{aligned} \max z = \sum_{s \in \mathcal{S}} \rho_s \Big(\sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \sum_{d \in \mathcal{D}} (\hat{P}_d^B x_{ptds}^A - \hat{P}_d^A x_{ptds}^B - C^C (v_{ptds}^A + v_{ptds}^B)) \\ + C^Q (r_{(\bar{d}+1)s}) r_{(\bar{d}+1)s} - C^Q (r_{0s}) r_{0s} + \sum_{d \in \mathcal{D}} (P_{ds}^{BM-} v_{ds}^{BM-} - P_{ds}^{BM+} v_{ds}^{BM+}) \Big) \end{aligned} \tag{4.3.4}$$

One may ask why a single order is approximated with several added constraints and a change to the objective function - why not just place one order? The answer is that the current model assumes an order is withdrawn if it does not clear in the stage when it is placed. Moreover, orders may not clear partially in our formulation, though the single large order would likely clear partially several times. The necessary modifications to allow orders to clear partially and last for several stages are thus significant.

4.3.2 Alternative 1B: Predetermined production, adaptive trading

An alternative way to interpret the papers mentioned in the former section is that they assume that trading can be optimized independently, taking in the optimized production decisions as constraints to the order placement problem. To approximate this process, it is once again necessary to separate between the dispatch optimization and the order placement optimization. The dispatch optimization is performed in the exact same way as described in section 4.3.1.

Once again, constraint 4.3.1 is added to perform the order placement optimization with a production constraint.

With the additional constraint, the base model is rerun on the original price scenarios. The value of the objective function is a realistic approximation of the best feasible profit when dispatch and order placement is optimized separately. Actually, the fact that the representative prices of all delivery products is known before the gate closure of the first delivery product may be an unrealistic assumption, providing information that the dispatch optimization algorithm isn't supposed to have yet. In that case, the objective value would be an optimistic bound on the value of a nonanticipative policy for the given approach.

4.3.3 Alternative 2: Uncoordinated delivery products

Several of the contemporary ID trading papers (notably Garnier and Madlener (2015), Aïd et al. (2015), Tan and Tankov (2016), Edoli et al. (2016) and Skajaa et al. (2015)) optimize trading separately for each delivery product. To approximate this, the base model is run several times consecutively, with only one delivery product per run. After each run, the initial storage parameters $R_0\gamma$ for the next run are set equal to the final storage in the last run.

4.3.4 Alternative 3: Sell-only

Most of the contemporary papers allow for both sales and purchases of power (though not necessarily for the same delivery product). In this section, the benefits from allowing two-sided trading is explored. This is solved by constraining the buy volume to zero.

$$x^B_{ptds} = 0, \quad p \in \mathcal{P}, t \in \mathcal{T}, d \in \mathcal{D}, s \in \mathcal{S} \tag{4.3.5}$$

4.3.5 Alternative 4A: No limit orders

Only Garnier and Madlener (2015) and Engmark and Sandven (2017) allow for the placement of orders with less-than-unity clearance probability. Implicitly, this means that the models in all other papers place exclusively market orders, never limit orders. The assumption that trading is performed through market orders only is approximated by setting the placed order volumes at the limit order levels to zero. The set of limit orders are denoted $\mathcal{P}_{tds}^{LO,A}$ and $\mathcal{P}_{tds}^{LO,B}$ for ask and bid orders, respectively.

$$x_{ptds}^{A} = 0, \quad p \in \mathcal{P}_{tds}^{LO,A}, t \in \mathcal{T}, d \in \mathcal{D}, s \in \mathcal{S} \tag{4.3.6}$$

$$x_{ptds}^{B} = 0, \quad p \in \mathcal{P}_{tds}^{LO,B}, t \in \mathcal{T}, d \in \mathcal{D}, s \in \mathcal{S} \tag{4.3.7}$$

4.3.6 Alternative 4B: False liquidity

Edoli et al. (2016), Gönsch and Hassler (2016), Hassler (2017), Löhndorf et al. (2013) and Farinelli and Tibiletti (2017) include no penalty for large trading volumes. To model the impact of this assumption, the base model is run twice, with slightly different modifications. In the first run, the order depths are modified so that the entire order volume in the market is available at the best prices. As prices are assumed to be better than they actually are, the objective value from this run will be too high. However, the order placement decisions $\hat{x}_{tds}^{A}$ and $\hat{x}_{tds}^{B}$ are saved, and the model is rerun with the original order depths. A constraint is added on the order placement variables, so that they sum to the order placement in the original run. Thus, the impact on the objective value of neglecting the instantaneous price impact can be estimated.

$$\sum_{p \in \mathcal{P}} x_{ptds}^{A} = \hat{x}_{tds}^{A}, \quad t \in \mathcal{T}, d \in \mathcal{D}, s \in \mathcal{S} \tag{4.3.8}$$

$$\sum_{p \in \mathcal{P}} x_{ptds}^{B} = \hat{x}_{tds}^{B}, \quad t \in \mathcal{T}, d \in \mathcal{D}, s \in \mathcal{S} \tag{4.3.9}$$

Note that order volumes are moved to the best price that is already quoted in the market. This is a worse price than the best limit order price. For this reason, it would be desirable to exclude limit orders from this analysis.

4.3.7 Summarizing the alternative models

The relation between the alternative models and the assumptions that each of them make can be observed in table 4.3.1. If a model has a given feature, the corresponding cell in the table says "Yes", otherwise it says "No". Note that while alternative model 1A allows either sales or purchases, both are not allowed for the same delivery product. "AM" is short for alternative model and "BM" is short for the `Benchmark model`. "IMI" is still short for the immediate price impact.

Table 4.3.1: Relation between alternative models and modeling assumptions

~	**AM1A**	**AM1B**	**AM2**	**AM3**	**AM4A**	**AM4B**	**BM**
Informed production	No	No	Yes	Yes	Yes	Yes	Yes
Adaptive trading	No	Yes	Yes	Yes	Yes	Yes	Yes
Coordinated DPs	Yes	Yes	No	Yes	Yes	Yes	Yes
Two-sided trading	(No)	Yes	Yes	No	Yes	Yes	Yes
Accounts for IMI	Yes	Yes	Yes	Yes	Yes	No	Yes
Limit orders	Yes	Yes	Yes	Yes	No	No	Yes

Chapter 5

Numerical simulations

This chapter contains a thorough description of how the parameters of the mathematical programs of chapter 4 are determined. Section 5.1 describes how the limit order book is represented using scenario parameters. Here, a mapping from parameter values to the state of a limit order book is described. Going forward, the set of parameters are split in two; the stochastic parameters and the deterministic parameters, covered in section 5.2 and 5.3 respectively. Note that parameters are given the label deterministic also if they can be determined deterministically conditional on some realization of stochastic parameters, because such parameters don't increase the number of scenarios.

In total, the Intraday market consists of 24 separate markets, one for each delivery product. The markets are open from the day ahead until right before delivery, but only a subset of the total market is considered here. In figure 5.0.1 a categorization of the market can be seen.

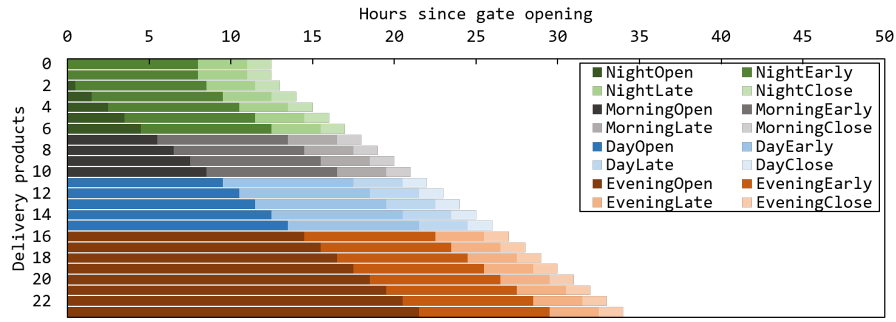

Figure 5.0.1: Categorization of delivery products and trading stages

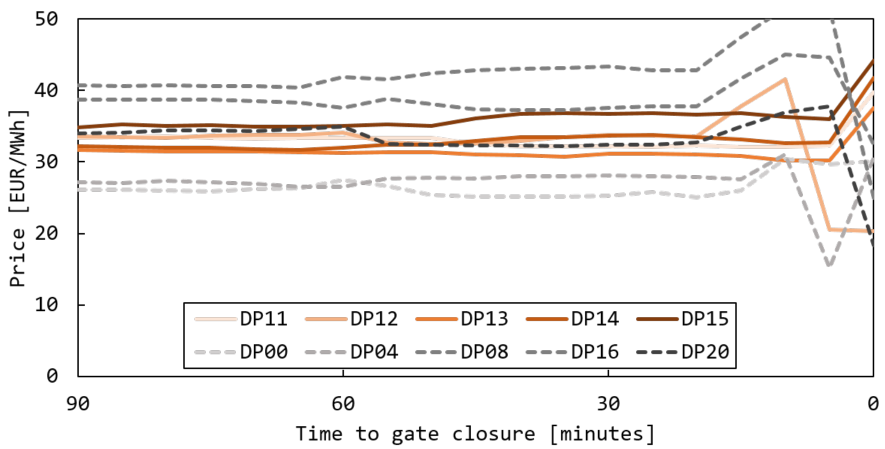

Figure 5.0.2: Average prices for the DayClose category compared to other delivery products in the Close phase.

Figure 5.0.2 illustrates the great diversity in delivery product average prices. The differences seen in this figure advocate using parameters that are specific for the delivery product and the trading stage. However, having a restricted dataset, this would induce a non-negligible probability of overfitting the data. To reduce this risk, delivery products in the same category are treated as similar. Readers should notice that the categorization in figure 5.0.1 is based mostly upon common sense. For instance, it seems natural that the trading patterns of the night delivery products behave similarly as little new information arrives from the gate closure of one delivery product to another and both the demand and supply sides of the market follow similar patterns for the consecutive delivery products. In the future, more effort could be placed on investigating the optimal number of categories to be used and whether there exist more appropriate thresholds than those proposed in figure 5.0.1.

The categorization proposed above yields 16 categories with different sets of parameters. Going forward, we describe how the parameter values have been determined for the `DayClose` category. That is, the parameters of delivery product 11-16 and trading stages later than 90 minutes prior to gate closure are described here. The same logic and analyses can indeed be applied to the other categories as well, but such analyses have not been performed in the work related to this thesis. It cannot be concluded from the analyses presented in this chapter to what extent the same assumptions would fit the other categories well or not.

5.1 A parametric representation of a limit order book

The cornerstone of our limit order book representation, is the stochastic `BasePrice`, corresponding to the market mid-price for a delivery product in a given timeslot. A timeslot is defined as a 5-minute interval of the limit order book, and market parameters can be defined at the beginning, end or over the duration of the timeslot. For instance, the mid-price is defined at the beginning of the timeslot, while the highest transaction price is defined over the duration of a timeslot.

The stochastic price processes take different values for each delivery product. The `BestQuotedPrice`s are defined using the `BasePrice`. The `BestQuotedPrice`s are assumed to be symmetric around the `BasePrice` and have a delivery product-specific price premium relative to the `BasePrice` equal to half the spread of the given delivery product. The orders at at each order level also have a given order volume. The prices of other order levels are determined based on the `BestQuotedPrice`s. While each order level may in reality consist of multiple orders with heterogeneous volumes, this is irrelevant for the given trader; only the total volume available at an order level is decision relevant.

As it should also be possible to place limit orders, some parameters need to be determined for the limit orders to be well defined. In addition to an order volume and a price premium relative to the `BestQuotedPrices`, the limit orders have a less-than-certain clearance probability. These relationships can be visualized as in figure 5.1.1. In this figure, the order book has a base price of 50 €/ MWh and a spread of 1.4 €/ MWh. In the figure, three levels of buy market order levels and two sell market order levels can be seen. Also, each side has three potential limit order levels. In fact, the set of parameters of the figure corresponds to a realization of the intrazonal parameters for DP11 .

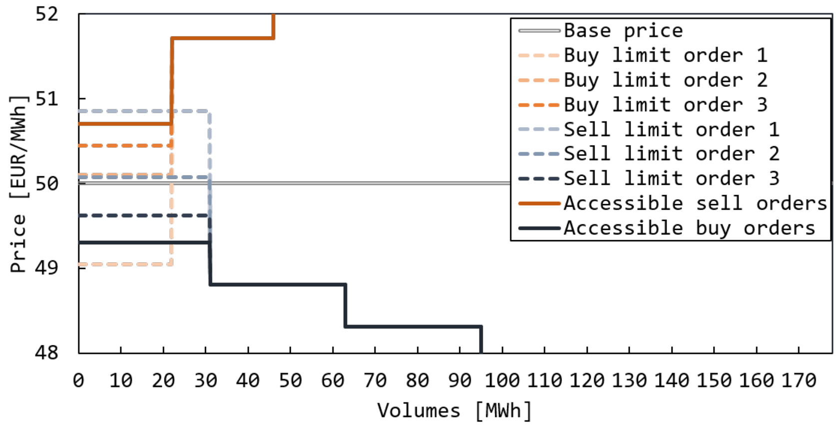

Figure 5.1.1: A hypothetical snapshot of the market situation

5.2 Stochastic processes

Recall from chapter 4 that there are two types of stochasticity in the model; the base price, and the limit order clearing. In this section, the process for determining the probability distributions for both of these types of stochastic processes is described in detail in sections 5.2.1 and 5.2.2. As the scenario space becomes vast, several techniques are used to reduce it in section 5.2.3. In section

5.2.4, a rationale is presented for why the models should be tested on a "forest" of semi-optimized, semi-randomized scenario trees, rather than one optimized scenario tree in line with the norm in stochastic programming. Then, a method for creating such a "forest" of scenario trees is presented.

Table 5.2.1: The market parameters and the modeling assumptions

Symbol	**Parameter**	**Modeling assumptions**
P_{ptds}	Base price	Conditional on the former price, DP, TTGC
$\Xi^A_{ptds}.\Xi^B_{ptds}$	Clearing indicator	Conditional on the spread

In table 5.2.1, the assumptions about the two stochastic parameters are summarized. Moreover, it is observed that the statistical properties of the relevant delivery products are very similar in the relevant phase of the trading window, so the processes are assumed to be identically distributed for all delivery products (see for instance figure 5.0.2. Other figures throughout the chapter also document that the limit order books for the given category behave similarly).

5.2.1 Modeling the base price process

Three aspects of the mid-price process were explored in order to model it; the expected drift in a given state, the volatility, and the distribution of the prediction error. Each of them are explored in turn in this section.

The analyses are based on the historical order book data acquired from Agder Energi Gmbh. For each delivery product of the date range 2016-03-01 - 2017-02-28, orders were processed sequentially to simulate the Intraday market. The applied clearing algorithm can be found in appendix C. The market simulations were then aggregated into five minute blocks containing information about transactions in the five minute block as well as instantaneous order book features such as the prices and volumes of the best available buy and sell orders at the time of block change. Only the data for the relevant delivery products and trading timeslots were considered. From this, the occurrences of price transitions were counted to form a Markov Transition Matrix with the mid-price in one timeslot along each row, and the mid-price in the next timeslot along each column. In this matrix, the prices were discretized into 0.5€/ MWh price bins. Underflow and overflow bins were set to −10€/ MWh and 120€/ MWh respectively.

Table 5.2.2: Section-specific symbols (Modeling the base price premium)

Symbol	Interpretation
p_{ts}	The price in stage t and scenario s
r	The rate of mean reversion
μ	The empirical mean price
ϵ_{ts}	The prediction error in stage t and scenario s
σ	The standard deviation of the prediction error

5.2.1.1 Estimating the price drift

Three potential sources of drift were hypothesized initially; first, if prices were in backwardation or contango (see section 2.2.2) there would be an average trend throughout the relevant part of the trading window; second, autocorrelation could cause the price drift to depend on prices in former stages; finally, mean reversion could cause the expected drift to be negatively proportional to the current deviation from the mean. If none of these sources of drift were confirmed. it would be assumed that the price was a martingale.

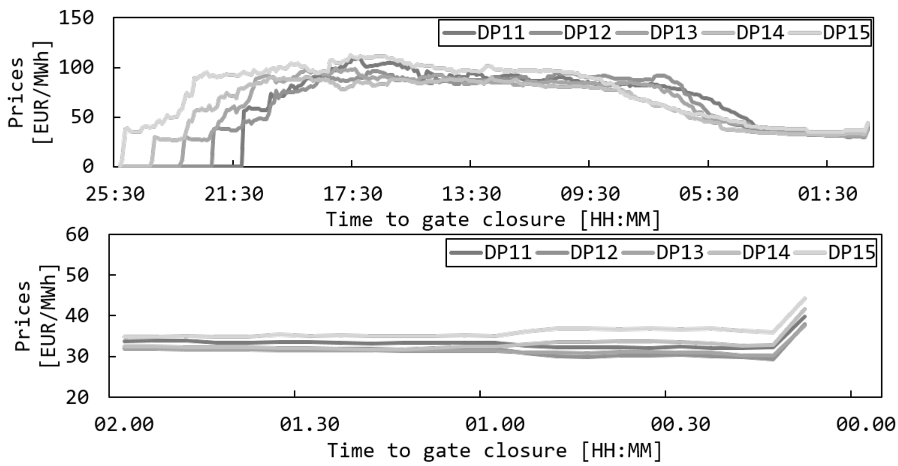

Figure 5.2.1: Average price trajectories of DP11-DP15. No trend is observed. The slight increase in the last timeslot is omitted from the market model, as the case study is limited to a smaller number of stages than the full `DayClose` category.

The average trend throughout the `DayClose` part of the trading window was near-zero (see the lower half of figure 5.0.2), so for this part of the trading window prices don't seem to be in backwardation nor contango on average. The *partial autocorrelation* "*measures the correlation between an observation k periods ago and the current observation, after controlling for observations at intermediate lags*" (Brooks, 2014). The plot of the partial autocorrelation in figure 5.2.2 suggests that only the marginal benefit of including more than one lag when predicting base price is rather low for all the `Day` delivery products, thus the Markov property seems to hold for the base price.

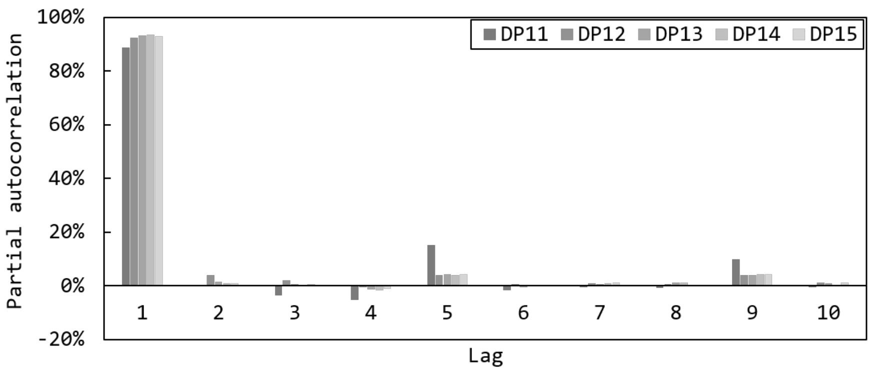

Figure 5.2.2: Partial autocorrelation plot

Figure 5.2.3 shows expected drift in the next stage relative to the current price in the `DayClose` category. As the trend of the regressed line is lower than that of the identity line, one can conclude the price processes do have the mean reversion property. In fact, increasing the current price with 1 EUR/MWh decreases the expected price drift by approximately 27 cents. It should be noted that prices rarely take values above 70 €/MWh, so the available data in that price range is sparse and has received little weight in the regression.

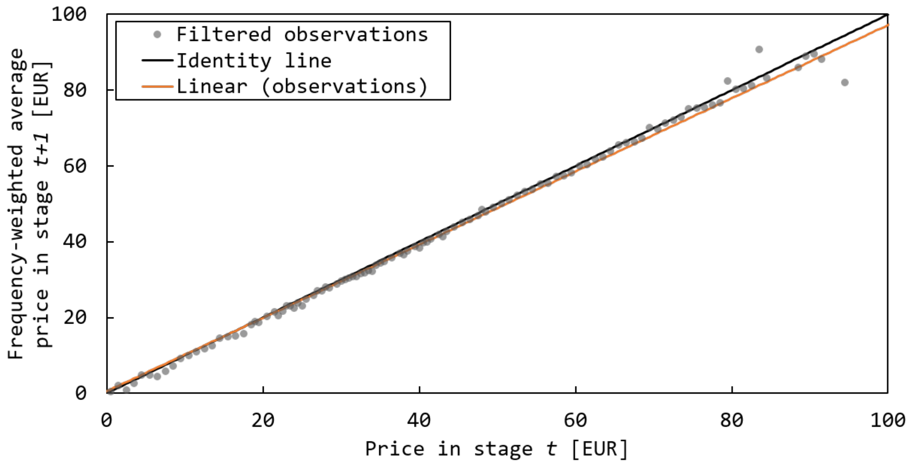

Figure 5.2.3: Base prices are mean reverting.
Observations with an occurrence frequency of less than five are filtered out.

It is therefore assumed that the price is mean reverting and therefore not a martingale. In particular, the price is modeled with equation (5.2.1). In the equation, r is the rate of mean reversion for the price, μ is the empirical mean and ϵ_{ts} is the prediction error.

$$p_{(t+1)s} = p_{ts} + r(\mu - p_t) + \epsilon_{ts} \tag{5.2.1}$$

5.2.1.2 Estimating the price volatility

The goal is then to estimate the variance of the ϵ in equation (5.2.1). Three hypotheses are tested; first, the variance is estimated as a function of time to gate closure. As shown in figure 5.2.4, for the `DayClose` category, the average 45-minute rolling standard deviation of figure 5.2.4 appears constant. The high and fluctuating variance in the rest of the trading window strengthens the case for focusing only on the `DayClose` category. Thus, the first hypothesis is discarded and replaced by a hypothesis stating that the variance can be estimated as a constant function.

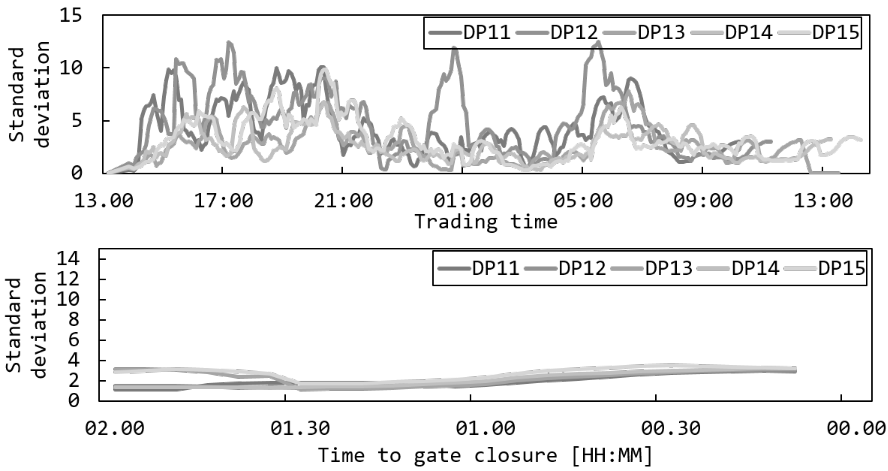

Figure 5.2.4: Average 45 minute rolling standard deviation plots

Moreover, the variance as a function of the price was calculated, to see if the variance as a constant percentage of the price is a better approximation of the historical variance than a constant nominal value. Using the Markov Transition Matrix for the mid-price, the variance in the *frequency-weighted average price in stage t+1* was calculated for all *prices in stage t*, using the predicted price in stage *t+1* as the mean. As one can observe in figure 5.2.5, the variance is not a function of the price.

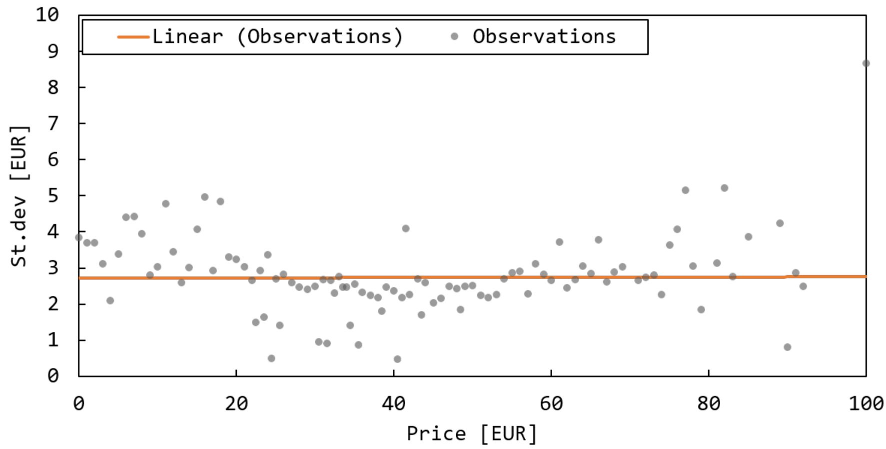

Figure 5.2.5: Volatility is constant as a function of price

For this reason, a constant nominal variance is chosen. The error term in equation (5.2.1) is thus identically distributed and independently drawn for each stage for each scenario, with a mean of zero and a known standard deviation of approximately 2.71 €.

5.2.1.3 Estimating the shape of the error probability distribution

The question then becomes what shape the probability distribution for the error term follows. A new matrix was created, where the initial distributions in the Markov Transition Matrix were shifted to a mean of zero by subtracting the average value along each row. Based on this matrix and the linear regression of the standard deviation, yet another matrix was created by counting the number of transitions that were within 0.2σ intervals. Having standardized the data to a mean of 0 and a standard deviation of 1, the data was smoothed by taking the weighted average over a range of 4€ to reduce noise. Thus, the distribution for 50€ in figure 5.2.6 is the average from 48€ to 52€, weighted for the number of times that the input price has taken the given value. Finally, the data was adjusted to form a probability distribution along each row, by dividing by the sum of the row. If the error term was normally distributed, each row should resemble a standard normal distribution after these adjustments.

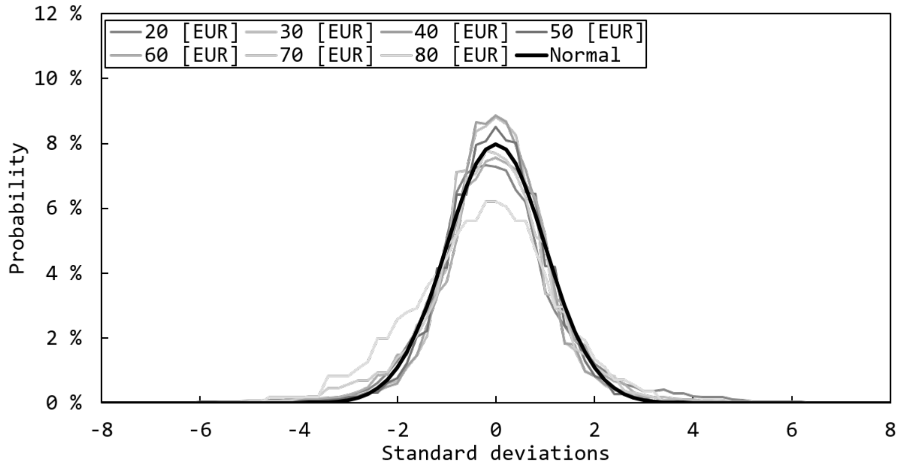

Figure 5.2.6: The shape of the distribution for the prediction error.

As figure 5.2.6 shows, a normal distribution is a good approximation of the price forecast error at a wide range of prices. As the shape, volatility and expectation of the price are known for all initial price levels, a full model for the price dynamics is developed. The initial price in all scenarios for all delivery products is set to 50.

5.2.2 Modeling the limit order parameters

The second type of stochastic processes is the clearance of limit orders. To the authors' best knowledge, no previous research has attempted to model the limit order parameters as accurately as what is done in this thesis - recall from section 2.2.3 that it is common to assume a narrow spread and near-zero clearance probability for premiums that are larger than the spread. Thus, the highest possible premium that a limit order can get is more or less known. As the liquidity is lower in the EPEX Intraday market than most conventional LOBs, this is assumed to be an inaccurate description of the limit order clearance probability. However, the clearance probability for a limit order is assumed to be a univariate probability distribution, with the premium measured as a percentage of the spread as the only input variable. This is in line with the historical literature on the topic (e.g. Guo et al. (2017), Horst and Naujokat (2014), Cont et al. (2010)).

The order clearing mechanism of the Intraday market was simulated using the purchased order book data, in order to derive transaction information. The transactions were aggregated into five minute-blocks. A transition probability matrix was then derived for each side (bid and ask), containing the empirical probability of a transaction occurring within the next five minute block at a given price premium relative to current spread equal to `BestTransactionPricePremium` .

Table 5.2.3: Section-specific symbols (Determining the limit order parameters)

Symbol	**Interpretation**
N	Number of limit order levels
A, B, C, D	Parameters of the exponential regression function.
$\underline{W}$	Rectangle width lower bound
$\underline{\rho}^{Clear}$	Clearance probability lower bound
$\bar{\Delta}P_t$	Highest limit order premium during stage t
P_t^{Spread}	The spread in stage t

For ask limit orders, `BestTransactionPricePremium` corresponds to the highest transaction price of the upcoming five-minute trading stage less the `BestQuotedPrice`, where the `BestQuotedPrice` corresponds to the highest order price of any currently open bid order. Similarly, for bid orders, `BestTransactionPricePremium` corresponds to `BestQuotedPrice` less the lowest transaction price of the upcoming five-minute trading stage. Here, the `BestQuotedPrice` corresponds to the lowest order price of an open ask order.

A reverse cumulative probability matrix for each side (bid and ask) was computed based on the transition matrices. The entries of these matrices can be understood as the probabilities that the `BestTransactionPricePremium` is at least ΔP_t conditional on the input P_t^{Spread}. $\Pr(\texttt{BestTransactionPricePremium}_t \geqslant \Delta P \mid \texttt{DayClose}, P_t^{Spread})$. Then, for some set of uniformly distributed percentile levels $\{0.95, 0.85, 0.75, 0.65, 0.55, 0.45, 0.35, 0.25, 0.15, 0.05\}$ the relative limit order premium $\Delta P / P_t^{Spread}$, corresponding to each percentile level is computed for all values of P_t^{Spread}. The relative limit order premiums were then determined using the occurrence frequency-weighted average over all P_t^{Spread}-values. The output of these analyses can be seen in figure 5.2.7. The figure shows the relation between the relative limit order premium and the probability of clearing. For instance, one can conclude from the figure that a sell order having a price equal to the best quoted buy price plus $0.14 \cdot P_t^{Spread}$ has 80% clearance probability during the next five minutes.

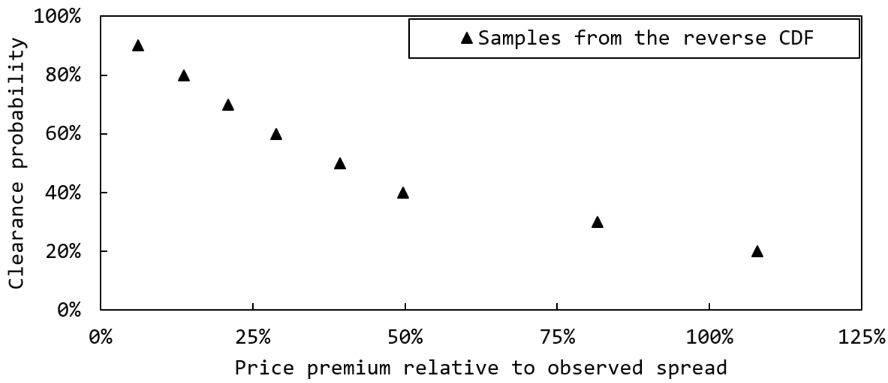

Figure 5.2.7: Reverse cumulative distribution

Next, this discretized function was approximated by a continuous reverse cumulative distribution. The sigmoid function $f(x) = D + \frac{A-D}{1+(\frac{x}{C})^B}$ proved to fit the distribution function for the buy orders well for $A = 0.97, B = 1.38, C = 0.41$ and $D = 0.01$. This distribution was then approximated by a new, discrete set of price premiums where orders could be placed. The set of discrete points should approximate the reverse cumulative distribution as well as possible, while limiting the variable space and the scenario space. $N = 3$ limit order levels was assumed to be a sufficient number.

The problem of determining which subset of limit order price level premiums to include in the model is equivalent to finding the N non-overlapping rectangles bounded by the reverse cumulative distribution and the lines $x = 0$ and $y = 0$ that maximizes the total area covered by the rectangles. This minimizes the area between the rectangles and the reverse cumulative distribution is minimized, thus approximating it well. This can be solved using the mathematical program of equation (5.2.2)-(5.2.5). In this program, the decision variables, x_i, can be interpreted as the x-value of the right-hand side of rectangle $i = 1, ..., N$. Equation 5.2.3 forces the rectangles to be non-overlapping, and ensures that all $x_i > 0$ when combined with equation 5.2.4. Equation 5.2.5 prevents the model from choosing x-values with probability less than some parameter $\underline{\rho}^{Clear}$. The function $f(x)$ must approximate the reverse cumulative distribution.

In order to avoid highly unlikely limit order price levels, the limit order clearance probability is bounded from below at $\underline{\rho}^{Clear} = 20\%$. Lower clearance probabilities would allow higher premiums, but the data foundation for such events is assumed too sparse to take advantage of, and one would risk violating the Order to Trade ratio constraints (see section 4.1.5.1). Also, the minimum rectangle width $\underline{W}$ is set equal to 0.01.

$$\max_{x_1.....x_N} \sum_{i=1}^{N} f(x_i)(x_i - x_{i-1}) \tag{5.2.2}$$

$$x_i - x_{i-1} \geqslant \underline{W}. \quad i = 1.....N \tag{5.2.3}$$

$$x_0 = 0 \tag{5.2.4}$$

$$f(x_N) \geqslant \underline{\rho}^{Clear} \tag{5.2.5}$$

The problem of finding the optimal regression parameters as well as the problem of placing the rectangles were both solved using the Nonlinear Generalized Reduced Gradient method (GRG Nonlinear) of the Microsoft Excel 2016's Solver tool. The solution of program (5.2.2)-(5.2.5) can be seen in figure 5.2.8. The program suggests to include sell limit order price levels at 23%, 55% and 111% spreads over `BestQuotedPrice` for buy orders, having clearance probabilities of 67%, 55% and 20% respectively. Thus, these limit order price levels are applied in the models of this thesis. A similar model for buy orders (see appendix D) suggests to include limit order price premium levels of 18%, 43% and 118% spreads under `BestQuotedPrice` for sell orders, having clearance probabilities 64%, 38% and 20% respectively. The limit order volumes are set equal to the volume of the best quoted buy and sell orders in all stages.

Table 5.2.4: Optimal limit order quantities for the DayClose category

	Price premiums relative to spread	**Clearance probabilities**
Sell orders	23% - 55% - 111%	67% - 55% - 20%
Buy orders	18 % - 43 % - 118 %	64 % - 38 % - 20 %

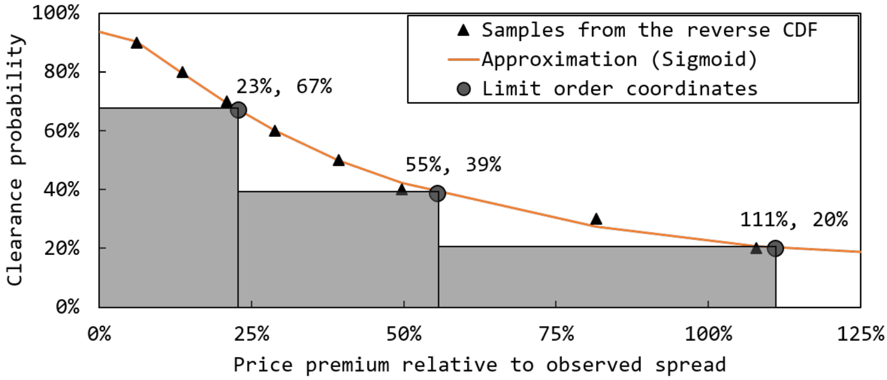

Figure 5.2.8: The buy order limit order quantities.

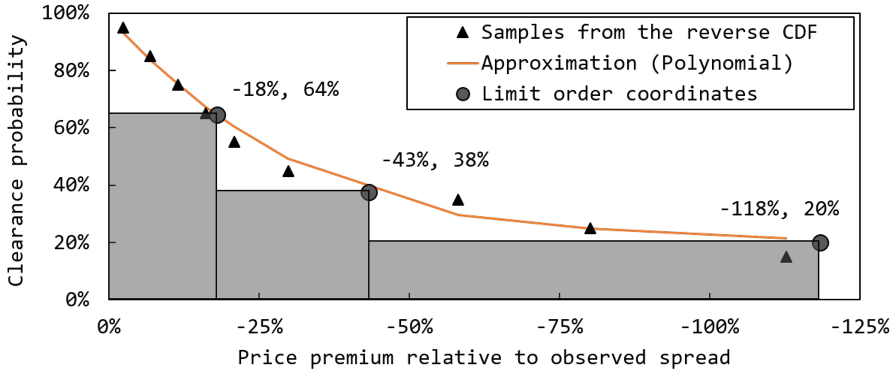

Figure 5.2.9: The sell order limit order quantities.

The resulting prices at the three given premiums then become:

$$P^B_{ptds} = -\texttt{Spread} \cdot \texttt{BuyVolumePremium}_p + P^B_{(p=4)tds}, \quad p \in \{1, 2, 3\} \tag{5.2.6}$$

$$P^A_{ptds} = \texttt{Spread} \cdot \texttt{SellVolumePremium}_p + P^A_{(p=4)tds}, \quad p \in \{1, 2, 3\} \tag{5.2.7}$$

Here, $P^A_{(p=4)tds}$ and $P^B_{(p=4)tds}$ are the best quoted prices on the ask and bid size respectively. Though these parameters add no stochasticity, they are included here as they relate to the limit order processes.

5.2.3 Reducing the number of stochastic processes

So far, the stochastic processes have been described as if they are independent of each other. With five delivery products with one price process each and two limit order clearing processes (one for buy and one for sell), there are 15 stochastic processes in total. If each price may take 130 values and the limit order premium may take 4 values on each side, this means that the number of branches per stage is $130^5 \cdot 4^{10} = 4 \cdot 10^{16}$. While the number of branches per stage is lower later in the trading window after the gate closures of the first delivery products, the scenario space still becomes intractably large. Moreover, the correlations between the delivery products - and between the price process and the limit order clearings on each side - have not been accounted for.

Table 5.2.5: Section-specific symbols (Reducing the number of stochastic processes)

Symbol	Interpretation
η_{tsj}	A draw in a stochastic process.
ϵ_{tds}	The price prediction error of DP d.
σ_ϵ	The volatility of the prediction error.
w_{dj}	The weight of a stochastic draw η_{tsj} for the error term of the price of DP d.
λ_j	The eigenvalue corresponding to eigenvector j.
U_{dj}	The element of eigenvector j corresponding to DP d.
η^A_{ts}, η^B_{ts}	The limit order premiums on the ask and bid side.

In table 5.2.6, the empirical correlations between the price trajectories of the `Day` delivery products are shown. Here, one should notice that the price processes of the different delivery products are strongly correlated (during the relevant part of the trading window) and that correlations of consecutive delivery products are higher than the correlations between non-consecutive delivery products. In order to be able to compute the correlation between price processes of different lengths, a zero-padding transformation has been done to the shorter processes.

The correlation matrix suggests that the price processes of the different delivery products should indeed be modeled as correlated processes.

Table 5.2.6: Delivery product correlations

	DP11	**DP12**	**DP13**	**DP14**	**DP15**
DP11	100%	69%	62%	57%	61%
DP12		100%	72%	63%	66%
DP13			100%	73%	58%
DP14				100%	69%
DP15					100%

Principal Component Analysis (PCA) is a technique for reducing the number of stochastic parameters in a multivariate stochastic process, while keeping as much as possible of the original variation (Alexander (2008a), Alexander (2008b)). A PCA was performed on the correlation matrix, and the results can be found in table 5.2.7.

Table 5.2.7: PCA analysis of the correlation matrix

Principal component	**1**	**2**	**3**	**4**	**5**
Eigenvalues	3.01	0.87	0.48	0.34	0.29
DP11	0.29	0.90	-0.16	0.27	-0.02
DP12	0.46	0.08	0.84	-0.25	-0.14
DP13	0.50	-0.06	-0.30	-0.49	0.65
DP14	0.49	-0.19	-0.42	-0.17	-0.72
DP15	0.47	-0.37	0.04	0.77	0.21

Each principal component is orthogonal to the other principal components by definition. The principal component matrix can thus serve as a map from a set of uncorrelated processes to a set of processes with correlation equal to the ones described in table 5.2.6. If a subset of the principal components are chosen, the proportion of variance explained is equal to the sum of the eigenvalues corresponding to the subset, divided by the sum of all the eigenvalues (Kreinin et al., 1998). A choice of two principal components leads to a proportion of explained variance of $(3.01 + 0.87)/(3.01 + 0.87 + 0.48 + 0.34 + 0.29) = 84\%$, which is considered acceptable. As two principal components are chosen, two independent standard normal draws are made, and the percentile of the error term in the price transition function is a weighted sum of the values of the two draws, and then adjusted to the correct variance.

$$\epsilon_{tds} = \frac{\sigma_\epsilon}{w_{d,1} + w_{d,2}} (w_{d,1}\eta_{ts,1} + w_{d,2}\eta_{ts,2}), \quad p \in \mathcal{P}, t \in \mathcal{T}, d \in \mathcal{D}, s \in \mathcal{S} \quad (5.2.8)$$

The motivation for the use of the cumulative probability functions, is that ϵ_{ptds} and η_{tsj} follow different probability distributions. As the distributions have similar shape, the correlations are conserved through the transformations (Høyland et al., 2003). The first two columns $\mathbf{U}_1$ and $\mathbf{U}_2$ of the principal component matrix and their corresponding eigenvalues λ_1 and λ_2 are used to calculate the weights $w_{d,j}$. The resulting weights are found in table 5.2.8.

$$w_{dj} = \sqrt{\lambda_j} U_{dj}, \quad d \in \mathcal{D}, s \in \mathcal{S}, j \in \{1, 2\} \quad (5.2.9)$$

Table 5.2.8: Weights for the price parameters

Principal component	$w_{d,1}$	$w_{d,2}$
DP11	0.50	0.84
DP12	0.80	0.07
DP13	0.86	-0.05
DP14	0.85	-0.18
DP15	0.81	-0.35

While these weights have no intrinsic interpretation, the pattern of the weights is interesting. In particular, the weights of the first column of the weight-matrix are all in the same direction, and have very similar values (especially for DP12-DP15). This process will therefore contribute the most to the overall drift in the market (across delivery products). On the other hand, the second column of the weight-matrix has a wide range of values in either direction, so it will contribute the most to the difference between the price changes of the delivery products.

For simplicity, it is assumed that the limit order premiums on the same side (bid or ask) are perfectly correlated across delivery products. Moreover, it is assumed that the clearing of both sell- and buy limit orders are explained by the same stochastic processes that drive the price. In particular, the weights for the limit order premiums are found in table 5.2.9. The weights are not based on an extensive empirical analysis, but they have some desirable properties; the bid limit order clearing is negatively correlated with the drift in the market, whereas the ask limit order clearing is positively correlated with the drift. The clearing of both sides is positively correlated with differences in price movements.

Table 5.2.9: Weights for the limit order clearing parameters

Principal component	**1**	**2**
Bid limit order premium	-0.50	0.50
Ask limit order premium	0.50	0.50

Using these weights, the two random standard normal variables are mapped to a random uniform variable. In equations 5.2.10 and 5.2.11, $\mathrm{F}_N(\cdot)$ takes the cumulative distribution of a normally distributed input variable. As the unweighted variance of the η_{tsj}-variables not is equal to the weighted variance, this is adjusted for to get the correct uniform distribution.

$$\eta_{ts}^{LOP,B} = \mathrm{F}_N(2 \cdot (-0.50\eta_{ts,1} + 0.50\eta_{ts,2})), \quad t \in \mathcal{T}, s \in \mathcal{S} \tag{5.2.10}$$

$$\eta_{ts}^{LOP,A} = \mathrm{F}_N(2 \cdot (0.50\eta_{ts,1} + 0.50\eta_{ts,2})), \quad t \in \mathcal{T}, s \in \mathcal{S} \tag{5.2.11}$$

An order at a given limit order premium clears if the clearance probability of the given limit order is higher than the value of the random uniform variable. Thus, the 15 stochastic processes have been reduced to two stochastic processes.

5.2.4 Creating a "forest" of scenario trees

When the scenario space is too large, it is common in the field of stochastic programming to approximate it by a reduced scenario tree. The scenario tree is typically developed using one or several of the techniques described in e.g. Høyland et al. (2003), Dupačová et al. (2003) and Heitsch and Römisch (2009), and the goal is to create the "best" approximation of the initial scenario space that can be solved tractably. "Best" is usually defined as a tree that gives an accurate representation of the incentive structure of the trader; for instance, (Kaut and Wallace, 2003) state that *"We are not concerned about how well the distribution is approximated, as long as the scenario tree leads to a "good" decision. In other words, we are not necessarily searching for a discretization of a distribution that is optimal (or even good) in the statistical sense"*. Common measures of the quality of the scenario reduction includes in-sample and out-of-sample stability (Kaut and Wallace, 2003), filtration distance (de Oliveira et al., 2010), moments of the probability distribution of the stochastic parameters (Høyland et al., 2003), or statistical measures such as the Wasserstein distance or Fortet-Mourier distance (Hochreiter and Pflug, 2007).

Many papers are constrained to selecting only one scenario tree - if the goal is to output a good decision, several scenario trees that result in conflicting recommended decisions are mainly useful to test the out-of-sample stability, and not to produce the recommendation in itself. In this thesis, the recommendation is one level of abstraction higher - it is about how to model the problem of optimal trading, not about how to trade. We are therefore not restricted to only one scenario tree. Actually, performing well in many semi-realistic environments would be an argument in favor of a given model, since the market has been changing fast historically and may continue to develop over the next few years. For this reason, it is preferable to compare the models in many semi-realistic settings, rather than only one stylized scenario tree. In order to ensure that the proposed model is robust to changes in the environment, the models are evaluated in 500 semi-randomized scenario trees that all have some of the desirable properties described in the scenario reduction literature. This also reduces the risk that one model will outperform another simply because of the assumptions in the scenario generation process, and not because it actually is better.

Table 5.2.10: Section-specific symbols (Creating a "forest" of scenario trees)

Symbol	**Interpretation**
η_{tsji}	A realization i of a draw in a stochastic process j
ρ_{tsji}	The probability of the realization η_{tsji}
$\mathrm{E}(\eta_j)$	The expectation of a draw for stochastic process η
$\mathcal{I}_s$	The set of draws i in a scenario s

The scenarios in a scenario tree are described by the values of the underlying random variables η_{tsj}. For each scenario tree, the random variables are restricted to a binomial distribution, similarly to e.g. Garnier and Madlener (2015) and in line with traditional option pricing research (Cox et al., 1979). This gives a branching factor of 4 per stage. In order to create a full forest of scenario trees, some randomization is needed. Therefore, the value in the first branch for each stochastic variable is drawn randomly from a normal distribution. The value of the second branch and the probabilities of the two branches are optimized to approximate the original probability distribution.

Together, the four branches in each stage needs to replicate the as much as possible of the relevant properties of the original probability distribution for all the original stochastic parameters. For a trading model, the first moment of the price (the drift) is obviously important to conserve, to prevent arbitrage. It is assumed that the second moment also is important to conserve. As the sum of probabilities must equal one, there are only two degrees of freedom, so no more than two moments can be exactly matched. The equations for the moment matching process are:

$$\sum_{i\in\{1,2\}} \rho_{tsji}\eta_{tsji} = \mathrm{E}(\eta_j) = 0, \quad t \in \mathcal{T}, s \in \mathcal{S}, j \in \{1,2\} \tag{5.2.12}$$

$$\sum_{i\in\{1,2\}} \rho_{tsji}\eta^2_{tsji} - \mathrm{E}(\eta_j)^2 = \sum_{i\in\{1,2\}} \rho_{tsji}\eta^2_{tsji} = \mathrm{Var}(\eta_j) = 1, \quad t \in \mathcal{T}, s \in \mathcal{S}, j \in \{1,2\} \tag{5.2.13}$$

The index i here represents a draw. As $\eta_{tsj,1}$ is already known at this point and $\rho_{tsj,1} + \rho_{tsj,2} = 1$, the equations are solved with respect to $\eta_{tsj,2}$ and $\rho_{tsj,1}$.

$$\eta_{tsj,2} = (-\eta_{tsj,1})^{-1} \quad t \in \mathcal{T}, s \in \mathcal{S}, j \in \{1,2\} \tag{5.2.14}$$

$$\rho_{tsj,1} = (\eta^2_{tsj,1} + 1)^{-1} \quad t \in \mathcal{T}, s \in \mathcal{S}, j \in \{1,2\} \tag{5.2.15}$$

The probability of a scenario then becomes $\prod_{t\in\mathcal{T}}\prod_{j\in\{1,2\}}\rho_{tsji}, \quad i \in \mathcal{I}_s, s \in \mathcal{S}$, (where $\mathcal{I}_s$ is the set of draws in the scenario), as the underlying random variables are statistically independent. Note that although the moment matching is performed on the underlying random variables, it is the moments of the price processes and limit order premiums that are relevant. However, the multiplication with the weights from the PCA is a linear transformation, and the matching of the first two moments is conserved through linear transformations when the underlying random variables are independent (Høyland et al., 2003).

The output prices are rounded to the closest 0.50 €. Repeating this process for each leaf node in the scenario tree until the tree spans all stages for all scenarios, a scenario tree is constructed by forward iteration. The process is then repeated 500 times, and each of the resulting scenario trees is a semi-realistic, semi-randomized approximation of the original scenario space.

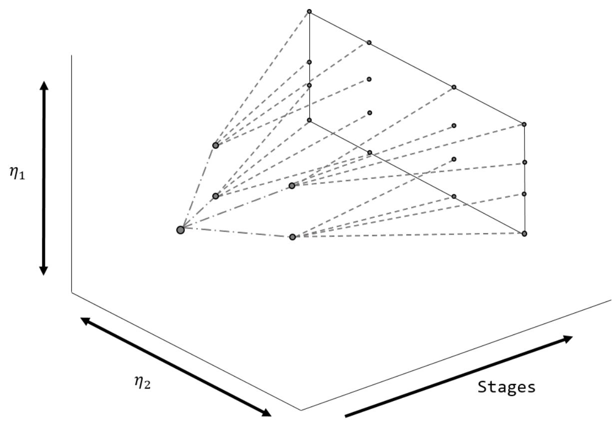

Figure 5.2.10: The scenario structure is non-recombining.
With two binomial stochastic processes, there is a branching factor of 4 per node.

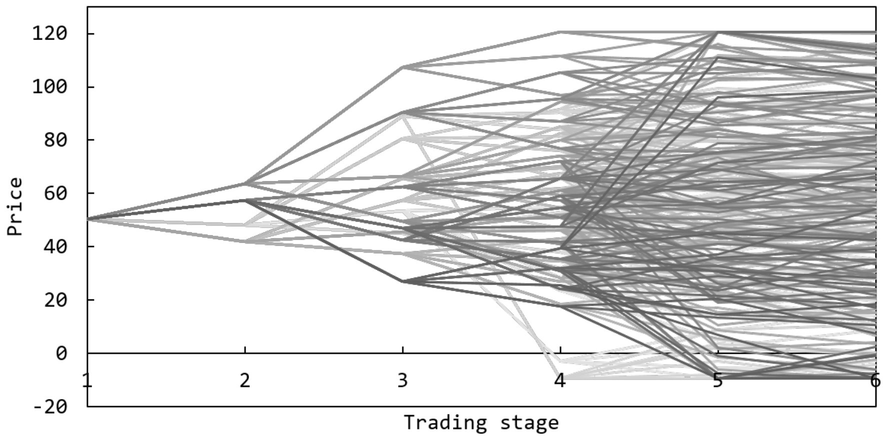

Figure 5.2.11: 1/4th of the price trajectories for DP15 for the first scenario tree. Only one branch from the last stage is included, due to constraints in Excel. Note that scenarios with very high or low prices have lower probability, so the figure exacerbates the volatility of the price.

5.3 Setting the deterministic model parameters

When testing the model, there are two types of deterministic parameters that must be determined; trader specific parameters (section 5.3.1) and market parameters (5.3.2 and 5.3.3). The main difference between these two classes is that in a real-world case, market parameters are similar for all traders, whereas the trader specific parameters differ between traders. Based on empirical data, it is argued that the market parameters should be partitioned into a set of *interzonal market parameters* and a set of *intrazonal market parameters*. The choice of parameter values are meant to be realistic, while at the same time generating easily understandable results.

5.3.1 Trader specific parameters

The trader specific parameters of table 5.3.1 are derived in cooperation with TrønderEnergi AS. TrønderEnergi is a Norwegian renewable energy producer located in the Trøndelag region. The parameters and their interpretations are introduced in chapter 4 and will not be explained in depth here. Together, the parameters of this section are sufficient to model a hydropower producer with pumping capacity and a small reservoir.

Table 5.3.1: Trader specific parameters

Symbol	**Parameter**	**Value**
F_d^{in}	Asset inflow	100 MWh
V_d^{DA}	Day-Ahead Commitment	50 MWh
$\bar{R}^{storage}$	Reservoir bound	1000 MWh
$\bar{R}_0$	Initial storage level	700 MWh
$\underline{Q}, \bar{Q}$	Production bounds	-50, 200 MWh

To account for the nonlinearity created by heterogeneous marginal Value of Storage, five virtual reservoirs with homogeneous marginal production costs are created. Each reservoir has a production cost, storage capacity and initial storage. This is summarized in table 5.3.2.

Table 5.3.2: Virtual reservoirs

Parameter	**VR1**	**VR2**	**VR3**	**VR4**	**VR5**
Production cost	63.5	55	52	49.5	40
Storage capacity	60	260	360	260	60
Initial storage	60	260	360	20	0

5.3.2 Interzonal market parameters

The historical EPEX Intraday order book data of the period 2016-03-01 - 2017-02-28 was used to determine the deterministic market parameters too. An overview of the deterministic market parameters and their modeling assumptions can be seen in table 5.3.3. The following sections attempt to outline in detail how each market parameter can be modeled and why this is a reasonable modeling choice.

Table 5.3.3: The market parameters and the modeling assumptions

Symbol	Parameter	Modeling assumptions
A_{ptd}, B_{ptd}	Order level volumes	Function of TTGC
P_{ptds}	Order level prices	Function of price, TTGC
$P_{ds}^{BM+}, P_{ds}^{BM-}$	Imbalance price	Function of price, DP

In table 5.3.3, "TTGC" is short for time to gate closure.

5.3.2.1 Determining the order level parameters

For each order level p, the MSLP takes in two parameters; the order price P_{ptds} and the accessible order volume A_{ptd} (alternatively B_{ptd}). We have decided to include five order levels for market order placement in our model. With three limit order levels, the total number of order levels is eight. This is assumed to be enough for the model to act like in a real world setting as additional order levels would have less attractive prices. As indices $p \in \{1, 2, 3\}$ are reserved for limit orders, $p \in \{4, 5, 6, 7, 8\}$ are reserved for market orders, where order level 4 represents the best quote in either direction.

The order level prices are determined as in equation (5.3.1 - 5.3.4). From these equations, it is possible to conclude that three sets of quantities must be determined for the order level prices to be well defined; the `Spread`, the `BuyVolumePenalty` and the (`SellVolumePenalty`) .The `Spread` is the difference between the `BestQuotedPrice` on the ask and bid side, whereas the `BuyVolumePenalty` (`SellVolumePenalty`) is the relative price change between the `BestQuotedPrice` on the buy (sell) side and each of the other buy (sell) market order levels. Analyses on the historical data are done in order to determine these quantities.

$$P^B_{(p=4)tds} = P_{tds} - 0.5 \cdot \texttt{Spread} \tag{5.3.1}$$

$$P^A_{(p=4)tds} = P_{tds} + 0.5 \cdot \texttt{Spread} \tag{5.3.2}$$

$$P^B_{ptds} = \texttt{BuyVolumePenalty}_p \cdot P^B_{(p=4)tds}, \quad p \in \{5, 6, 7, 8\} \tag{5.3.3}$$

$$P^A_{ptds} = \texttt{SellVolumePenalty}_p \cdot P^B_{(p=4)tds}, \quad p \in \{5, 6, 7, 8\} \tag{5.3.4}$$

Figures 5.3.1 demonstrate the development of the average spread relative to the transaction price for each of the `Day` delivery products. Here, one can see that there are significant fluctuations throughout the trading window of each delivery product. When considering the `DayClose` category only, the spread looks much more predictable; the mean spread can be accurately predicted by a piecewise stationary function. It is suggested to set the spread parameters equal to the constant levels of such an approximation, one level for the interzonal market and one for the intrazonal market. The average values of the interzonal market are presented in table 5.3.4. f

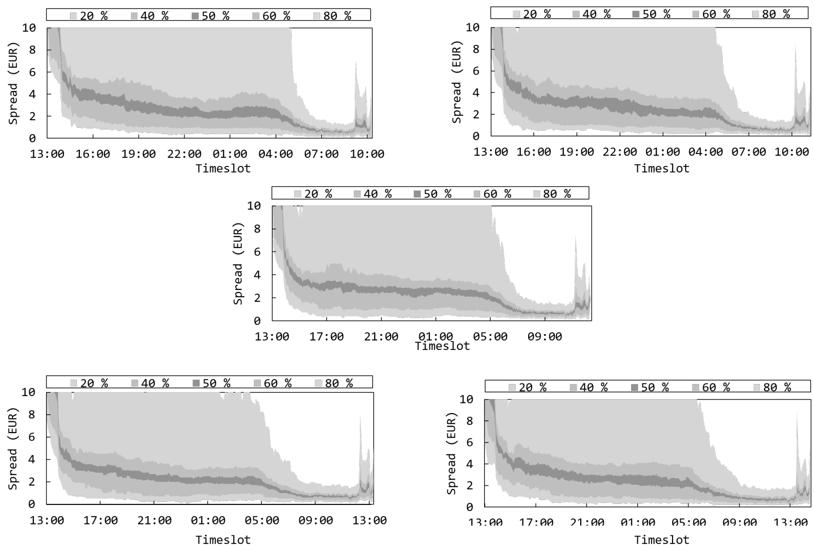

Figure 5.3.1: Spread trajectory percentile plot

Table 5.3.4: Mean spread levels

DP	Interzonal DayClose spread
DP11	0.7 €/MWh
DP12	0.8 €/MWh
DP13	0.8 €/MWh
DP14	0.9 €/MWh
DP15	1.0 €/MWh

Figure 5.3.3 and 5.3.2 shows the trajectories of the average order level prices of the second best, the third best, the fourth best and the fifth best quoted buy and sell order price levels relative to the price of the best order level for each delivery product and each trading stage. These average values constitute the accessible order price premiums relative to the best price of the 2nd to the 5th price level. The premiums can be seen in table 5.3.5 and 5.3.6, whereas the corresponding order volumes can be seen in appendix B. At first sight, the plots look very similar across delivery products. At closer inspection, one can see that there are some differences. The x-axes are slightly changed from delivery product to delivery product. Also, there are some differences in `DayClose` values. These values are summed up in table 5.3.5 - 5.3.8. To cope with a wide range of prices, including negative ones, the relative penalties are multiplied by the initial price of 50€$/MWh$ to form absolute penalties in the model preprocessing. These absolute penalties are applied throughout the trading window and are added to (subtracted from) the best quoted ask (bid) price levels instead of being multiplied with them (as originally shown in equations 5.3.3 and 5.3.4).

Table 5.3.5: Interzonal DayClose average BuyOrderPenalty of the five best accessible buy orders

DP	**1st**	**2nd**	**3rd**	**4th**	**5th**
DP11	100 %	99 %	98 %	97 %	95 %
DP12	100 %	99 %	97 %	96 %	95 %
DP13	100 %	98 %	97 %	95 %	93 %
DP14	100 %	97 %	95 %	93 %	91 %
DP15	100 %	97 %	95 %	93 %	91 %

Table 5.3.6: Interzonal DayClose average SellOrderPenalty of the five best accessible sell orders

DP	**1st**	**2nd**	**3rd**	**4th**	**5th**
DP11	100 %	102 %	106 %	110 %	113 %
DP12	100 %	103 %	110 %	116 %	120 %
DP13	100 %	104 %	109 %	112 %	115 %
DP14	100 %	112 %	119 %	123 %	127 %
DP15	100 %	112 %	124 %	129 %	135 %

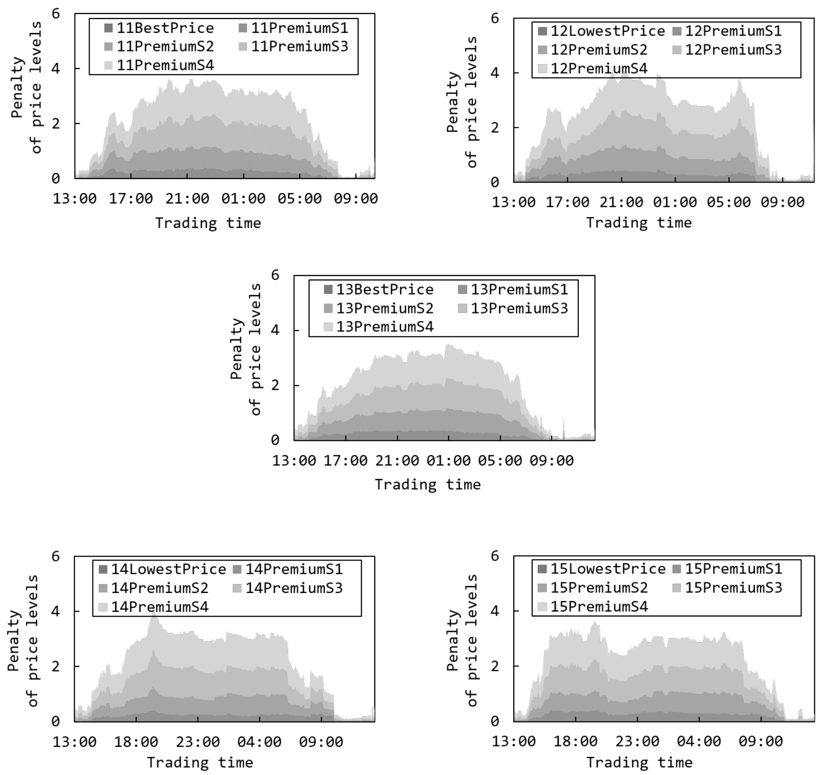

Figure 5.3.2: Sell order level penalties relative to best sell prices

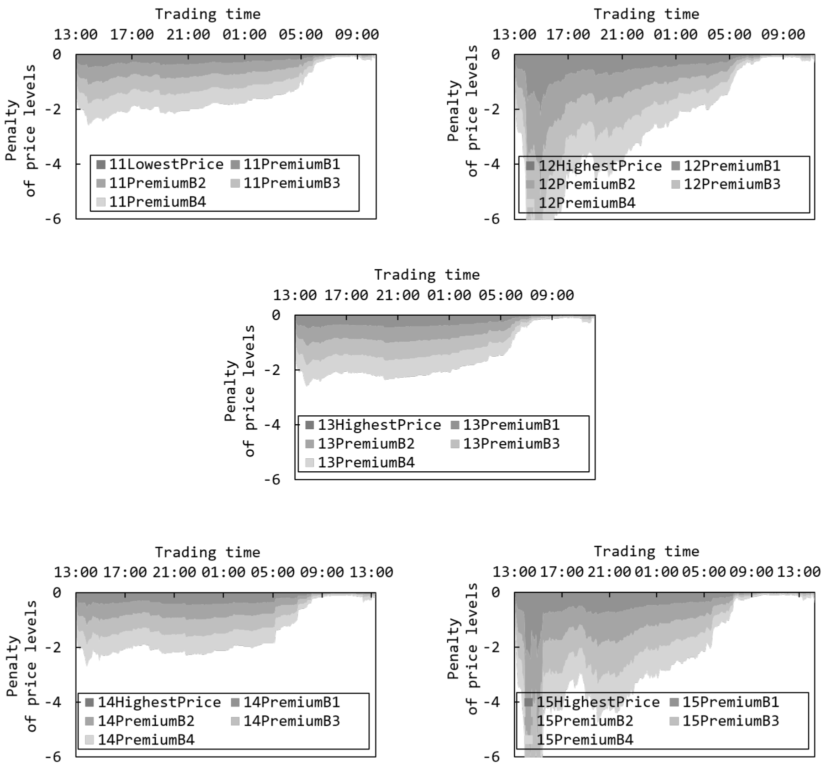

Figure 5.3.3: Buy order level penalties relative to best sell prices

Table 5.3.7: Interzonal DayClose average volumes of the five best accessible buy orders [MWh]

DP	**1st**	**2nd**	**3rd**	**4th**	**5th**
DP11	31	32	32	30	31
DP12	30	31	32	30	31
DP13	30	31	33	32	31
DP14	29	31	33	32	30
DP15	26	30	32	33	29

Table 5.3.8: Interzonal DayClose average volumes of the five best accessible sell orders [MWh]

DP	**1st**	**2nd**	**3rd**	**4th**	**5th**
DP11	22	24	21	23	22
DP12	22	21	20	20	21
DP13	21	23	19	21	22
DP14	22	23	22	22	23
DP15	20	23	22	21	24

5.3.2.2 Determining the imbalance prices

As stated by Wärtsilä (2014), the imbalance prices are very hard to predict; "*it it is near impossible for market participants to manage their imbalance exposure in the spot markets on an informed basis*". Figure 5.3.4 illustrates how a simple, linear regression model with the price of a delivery product at gate closure as input variable would look like. The figure is based on historical EPEX data fetched from ENTSO-e (2017a) and EPEX (2018) in the time period 2014-02-01 - 2017-10-31.

The EPEX Operational Rules prevent traders from trading actively in the imbalance markets (EPEX, 2017c). Thus, it is an important feature of the imbalance price parameters that they do not incentivize imbalance market trading. Recall that two imbalance price parameters are needed per delivery product; one for the + imbalance price and one for the - imbalance price. It is therefore assumed that the imbalance price equals the price in the last trading stage, but with a very large spread, referred to as the imbalance price penalties. These penalties are found by partitioning the dataset of imbalance prices in two - one for when

Table 5.3.9: Proposed imbalance price penalties relative to the close price

Balancing market	Imbalance penalty
+ Imbalance	142.05 €/MWh
- Imbalance	-127.42 €/MWh

the price is higher than the Intraday close price, and one for when it is lower - and then taking the average difference between the close price and the imbalance prices for each set. The resulting imbalance price penalty parameters can be seen in table 5.3.9. Note that this is not the most empirically accurate model, but it incentivizes the model to comply with the present regulations.

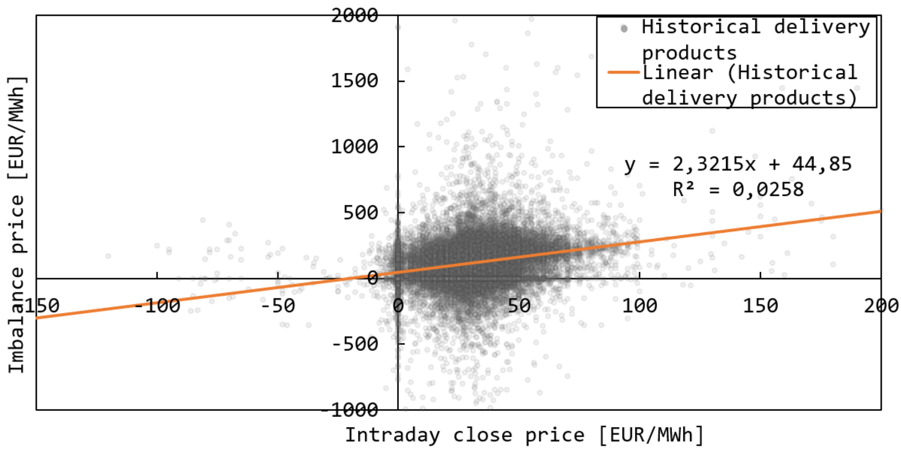

Figure 5.3.4: Imbalance prices vs Intraday close prices

5.3.3 Intrazonal market parameters

Trading across trading zones can only be done until 30 minutes before delivery. In the trading window of the *intrazonal* Intraday market, the liquidity decreases significantly. Thus, the market parameters must be modified correspondingly. One rather trivial way of handling this is by scaling the parameters. For the `DayClose` category, a set of proposed multipliers are presented in table 5.3.10.

These multipliers are based on empirical studies. The price does not seem to change predictably as the time passes interzonal gate closure. However, the average spread doubles (see table 5.3.11) and the order depths are halved (see table 5.3.12-5.3.13). Recall from section 4.1.5 that no limit orders can be placed in the last trading stage of a delivery product. Thus, in models with one stage per hour, it is not necessary to define intrazonal limit order premiums.

Table 5.3.10: Scaling factors used to approximate the intrazonal Intraday market

Parameter	**Scaling factor**
Price	1
Spread	2
Volume penalty	1
Order depths	0.5

Table 5.3.11: Average interzonal and intrazonal spreads for DP11-DP15

DP	**Interzonal `DayClose` spread**	**Last 30 minutes `DayClose` spread**
DP11	0.7 €/MWh	1.4 €/MWh
DP12	0.8 €/MWh	1.5 €/MWh
DP13	0.8 €/MWh	1.7 €/MWh
DP14	0.9 €/MWh	1.6 €/MWh
DP15	1.0 €/MWh	1.7 €/MWh

Table 5.3.12: Average interzonal and intrazonal sell order price levels for DP11-DP15 [MWh]

DP	**Interzonal order volumes**	**Intrazonal order volumes**
DP11	22-24-21-23-22	10-10-11-12-11
DP12	22-21-20-20-21	9-11-11-12-11
DP13	21-23-19-21-22	9-10-11-11-11
DP14	22-23-22-22-23	9-10-10-10-11
DP15	20-23-22-21-24	8-11-10-11-11

The order volumes are presented on the form (1st-2nd-3rd-4th-5th)

Table 5.3.13: Average interzonal and intrazonal buy order price levels for DP11-DP15 [MWh]

DP	**Interzonal order volumes**	**Intrazonal order volumes**
DP11	31-32-32-30-31	15.5-16-16-15-15.5
DP12	30-31-32-30-31	15-15.5-16-15-15.5
DP13	30-31-33-32-31	15-15.5-16.5-16-15.5
DP14	29-31-33-32-30	14.5-15.5-16.5-16-15
DP15	26-30-32-33-29	13-15-16-16.5-14.5

The order volumes are presented on the form (1st-2nd-3rd-4th-5th)

Chapter 6

Results

The `Benchmark model` of section 4.2, as well as the alternatives from section 4.3, have been implemented and evaluated using the Gurobi Optimizer. The models have been run on identical scenario trees, and the parameters are set based on the findings of chapter 5. This chapter is centered around the side-by-side comparison of the outcome of the model runs. These analyses are found in section 6.1. As the main purpose of this chapter is to explore the effects of different sets of modeling assumptions, most focus is placed on interpreting the model output. However, computational and technical studies, as well as an evaluation of the sample space, can be found in section 6.2.

A 64-bit Windows 10 PC with 3.40 GHz Intel® CoreTM i7-6700 CPUs and 32 GB RAM was used to run the models. The models are implemented using the Gurobi Optimizer (Version 7.5.2) with a *Named-user academic license.* In order to generate comparable results, the models are run on scenarios with a similar structure and scope. This is specified in table 6.0.1.

Table 6.0.1: Model parameters

Symbol	Parameters	
$\|\mathcal{T}\|$	Number of trading stages	6
$\|\mathcal{D}\|$	Number of delivery products	5
$\|\mathcal{S}\|$	Number of scenarios per scenario tree	1024
$\|\Gamma\|$	Number of virtual reservoirs	5
	Number of stages per gate closure	1

Each alternative model is solved for 500 scenario trees. Each scenario tree takes between 20 and 100 seconds to solve for each of the alternative models as well as the `Benchmark model`. However, solution times are not in focus here, for two reasons; firstly, because the alternative models are implemented by adjusting the `Benchmark model`, so they aren't necessarily solved as efficiently as a simpler model based on the same set of assumptions could be; and secondly, because the problem instances here are smaller than practical applications would require, and the solution times may not scale proportionally for each model as the problem instances are scaled. Note that "problem instance" here refers to a full scenario tree.

6.1 Decisions and objective values of each model

In section 6.1.1 to 6.1.6, three aspects are shown for each model; the objective value (referred to as the *model performance*) - both in absolute terms and in comparison with the `Benchmark model`; the performance for a wide range of scenario tree percentiles; and the average order volumes placed at each order level. A summary of the performances and decisions of each model is included in section 6.1.7.

The distribution of the expected profit for the `Benchmark model` can be observed in figure 6.1.1. Each scenario tree is counted once. In figure 6.1.2, the profit in scenario percentile 10, 25, 50, 75 and 90 are visualized for all scenarios. As one can see, the model makes a stable profit of 20-40 000 €in most scenarios in most scenario trees, but achieve very high profits in a few scenarios in a few scenario trees.

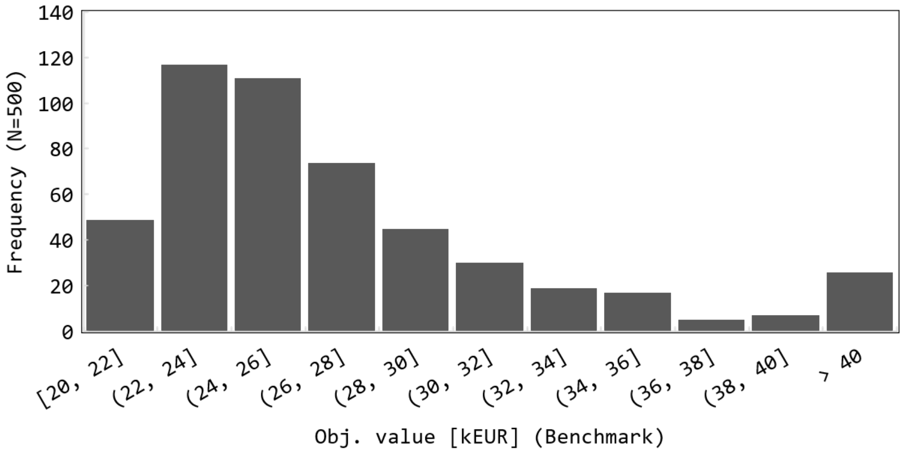

Figure 6.1.1: Distribution for the performance of the Benchmark model (N=500).

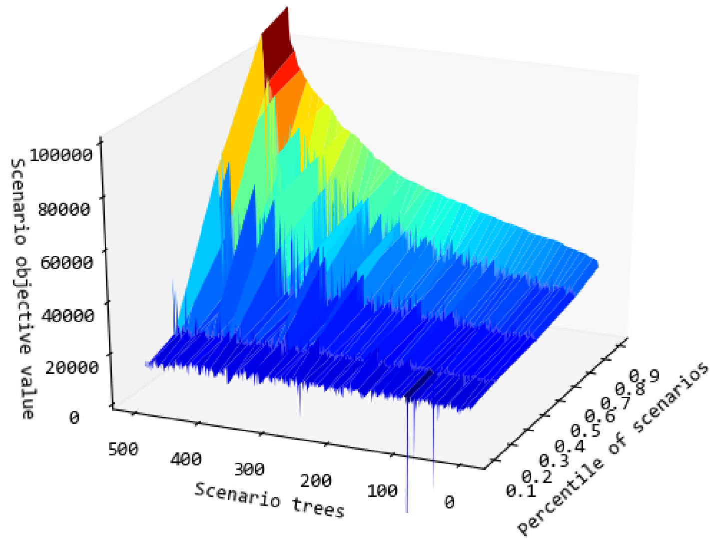

Figure 6.1.2: Profit in 5 selected percentiles of scenarios for all scenario trees.

In figure 6.1.3, the average order volume placed at each order level for DP15 in one of the scenario trees is shown. For order level 1-3 one may see that limit orders have lower clearance probability. A higher order level corresponds to a worse price;. Order level 1 has the best price due to a high limit order premium; while the large order level penalty makes 8 the least attractive order level in either direction.

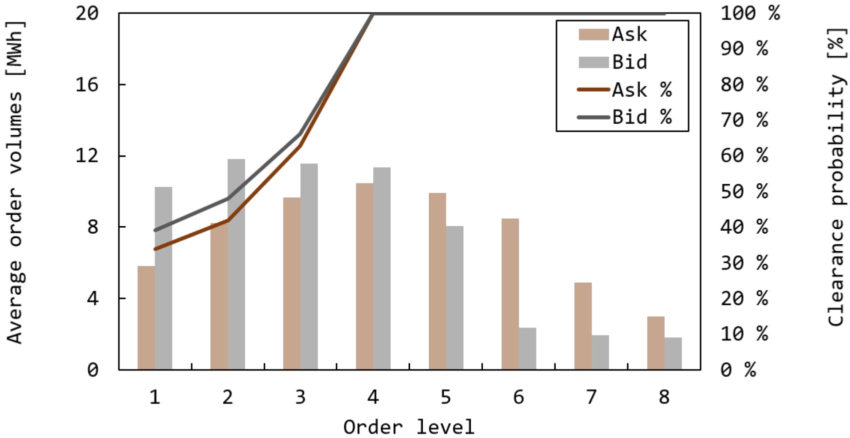

Figure 6.1.3: Average order volumes placed per order level by the Benchmark model for DP15 (N=1).

6.1.1 Alternative 1A: Predetermined production, single-order

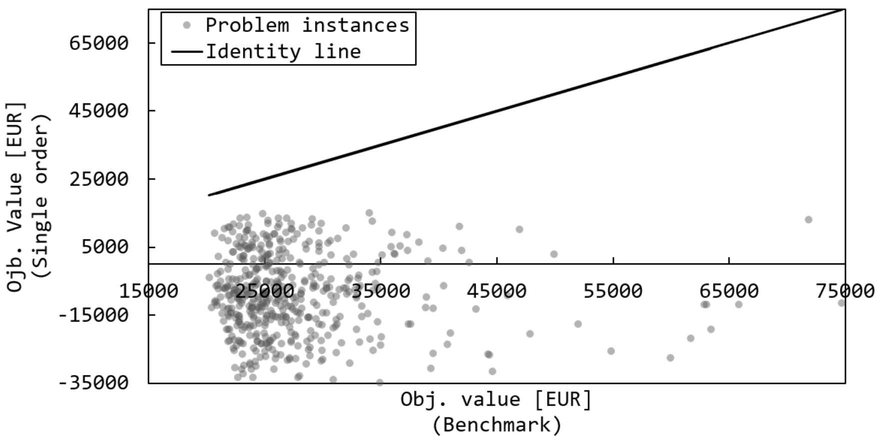

Figure 6.1.4: Predetermined production with a single order placement, vs. Benchmark model (N=500)

In figure 6.1.4 one can observe the performance of alternative model 1A on the vertical axis and the `Benchmark model` on the horizontal axis. The identity line represents the line where the models perform equally well. Dots below the identity line represent scenario trees where the `Benchmark model` outperforms the alternative model. As the alternative model performs poorly in all scenario trees, it seems unlikely that this interpretation of the dispatch-focused ITP papers is correct. Further analysis of this alternative model is therefore superfluous.

6.1.2 Alternative 1B: Predetermined production, adaptive trading

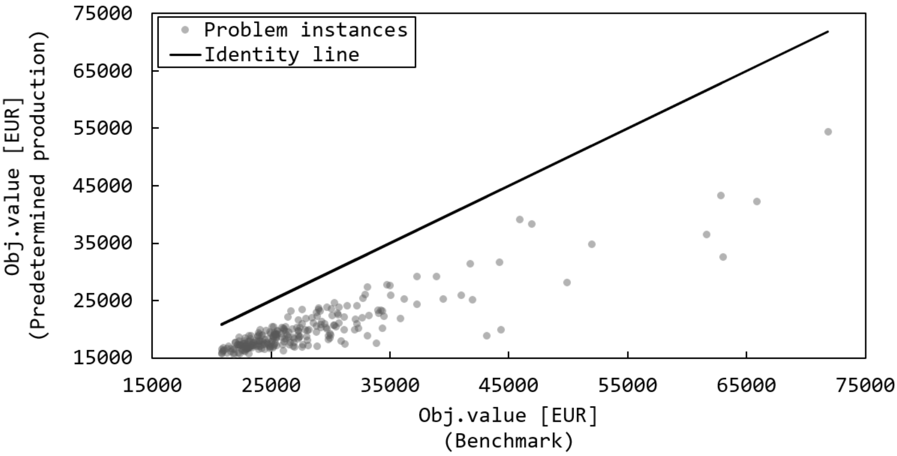

Figure 6.1.5: Predetermined production with adaptive trading strategy vs. Benchmark model (N=500)

As one may see in figure 6.1.5, this interpretation of the dispatch-focused ITP papers seems more reasonable.

Table 6.1.1: The performance of the predetermined production-model at different percentiles

Percentile	**2.50 %**	**10 %**	**50 %**	**90 %**	**97.50 %**	**Mean**
Absolute performance	15807	16549	18579	24214	30378	19739
Relative performance	57 %	63 %	73 %	80 %	83 %	72 %

In table 6.1.1, the performance of the alternative model at different percentiles of the set of scenario trees is shown. It is consistently outperformed by the `Benchmark model`.

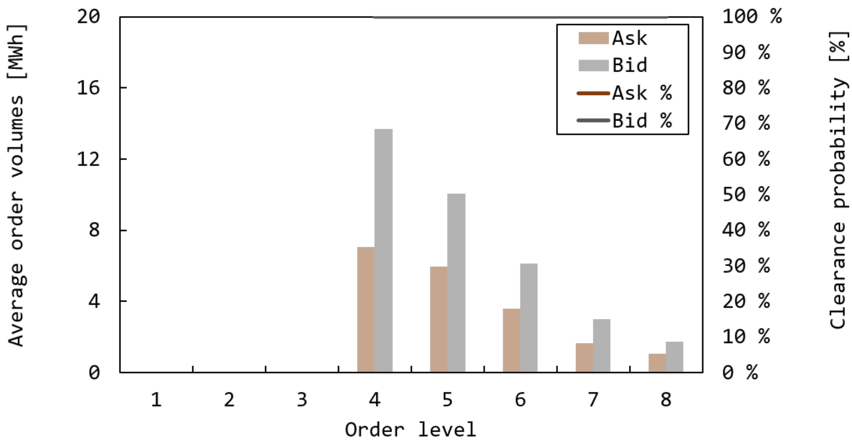

Figure 6.1.6: Average order volumes placed per order level by the Predetermined production model for DP15 (N=1).

In figure 6.1.6 the order placement of the alternative model is shown. Observe how the model avoids limit orders completely, as the uncertain clearing of limit orders is an unacceptable risk for a model with inflexible production.

6.1.3 Alternative 2: Uncoordinated delivery products

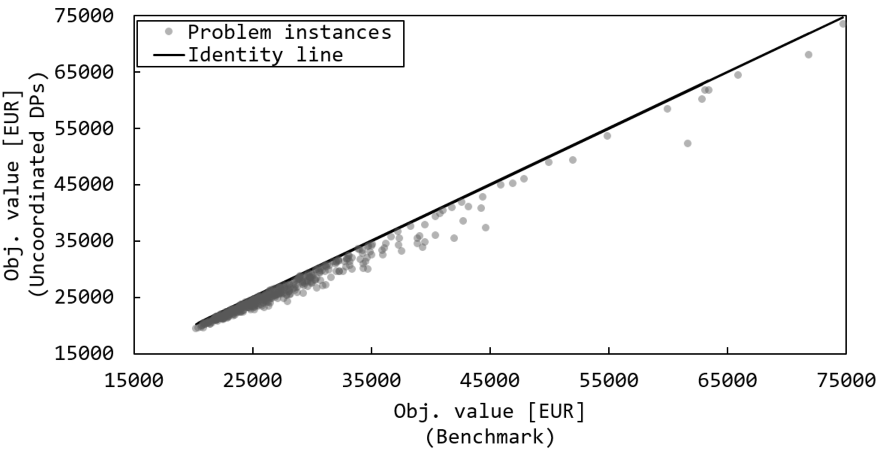

Figure 6.1.7: Uncoordinated delivery products vs. Benchmark model (N=500)

Table 6.1.2: The performance of the uncoordinated DP-model at different percentiles

Percentile	**2.50 %**	**10 %**	**50 %**	**90 %**	**97.50 %**	**Mean**
Absolute performance	20382	21322	24499	32548	45007	26417
Relative performance	89 %	92 %	96 %	98 %	99 %	96 %

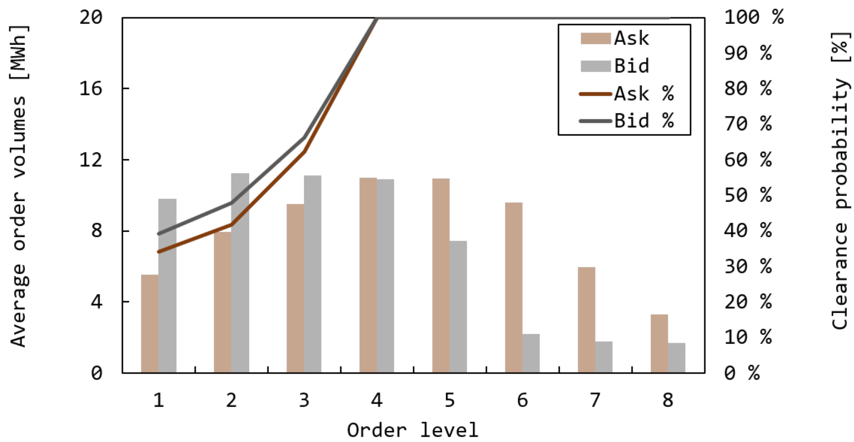

Figure 6.1.8: Average order volumes placed per order level by the Uncoordinated DPs model for DP15 (N=1).

6.1.4 Alternative 3: Sell-only

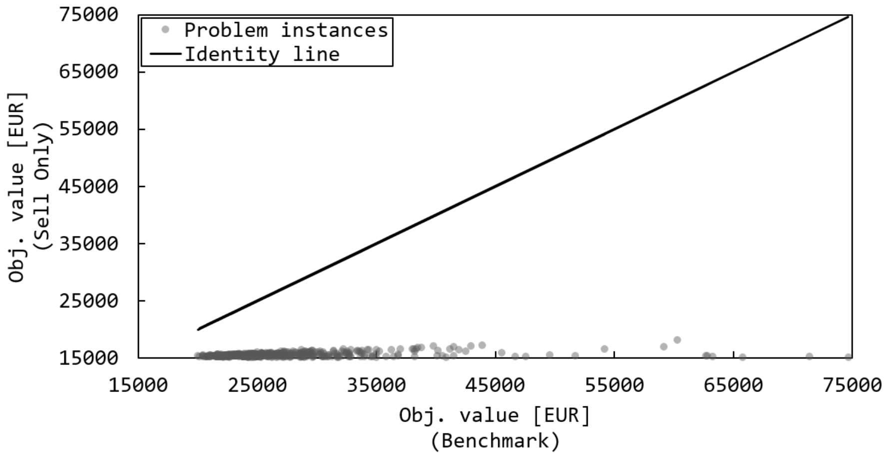

Figure 6.1.9: Sell-only vs. Benchmark model (N=500)

Table 6.1.3: The performance of the sell-only model at different percentiles

Percentile	**2.50 %**	**10 %**	**50 %**	**90 %**	**97.50 %**	**Mean**
Absolute performance	15178	15265	15499	16169	16855	15642
Relative performance	32 %	46 %	61 %	69 %	73 %	59 %

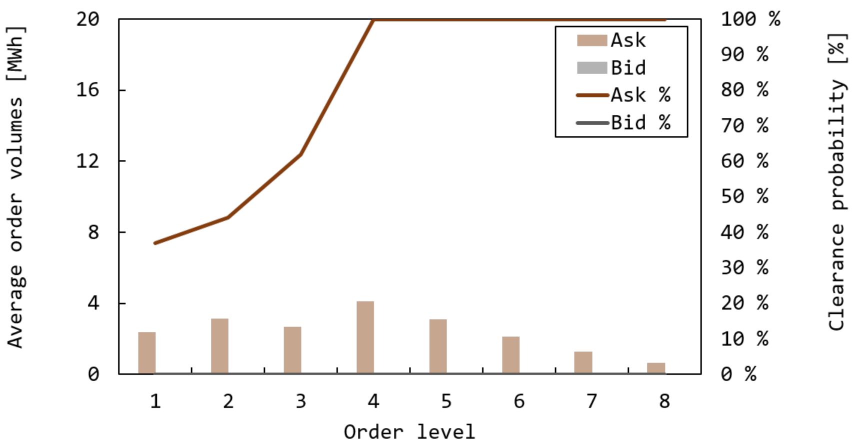

Figure 6.1.10: Average order volumes placed per order level by the Sell only model for DP15 (N=1).

6.1.5 Alternative 4A: No limit orders

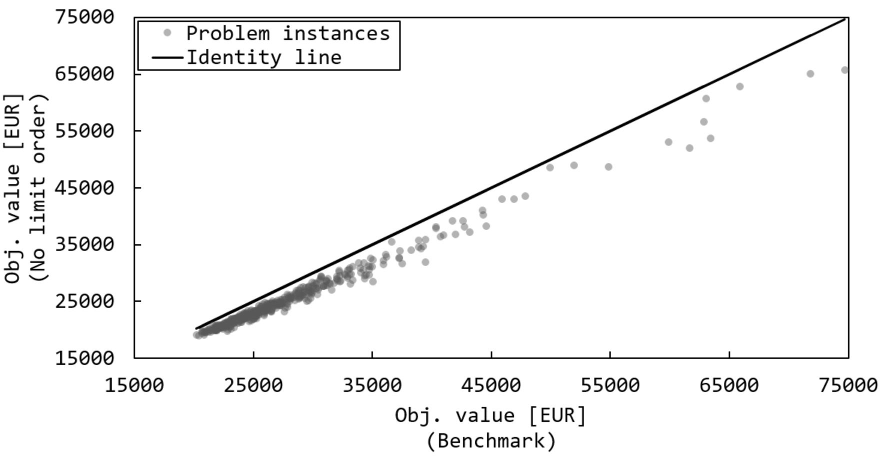

Figure 6.1.11: No limit orders vs. Benchmark model (N=500)

Table 6.1.4: The performance of the no limit orders-model at different percentiles

Percentile	**2.50 %**	**10 %**	**50 %**	**90 %**	**97.50 %**	**Mean**
Absolute performance	19711	20464	23344	30860	42995	25220
Relative performance	86 %	88 %	92 %	94 %	95 %	91 %

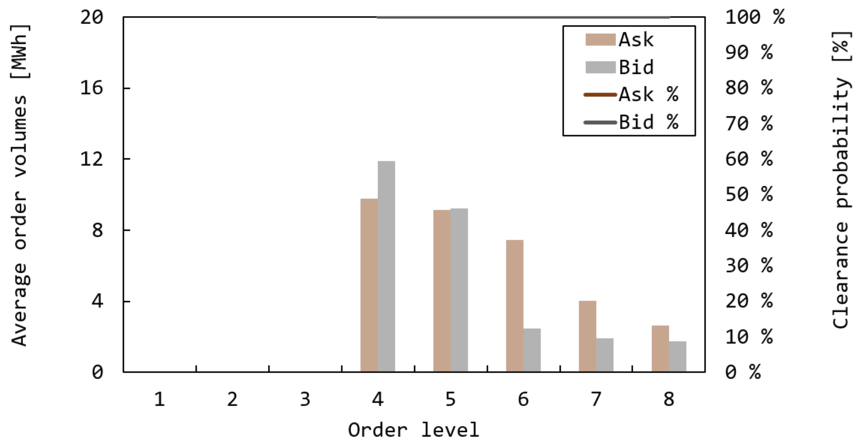

Figure 6.1.12: Average order volumes placed per order level by the No limit orders model for DP15 (N=1).

6.1.6 Alternative 4B: False liquidity

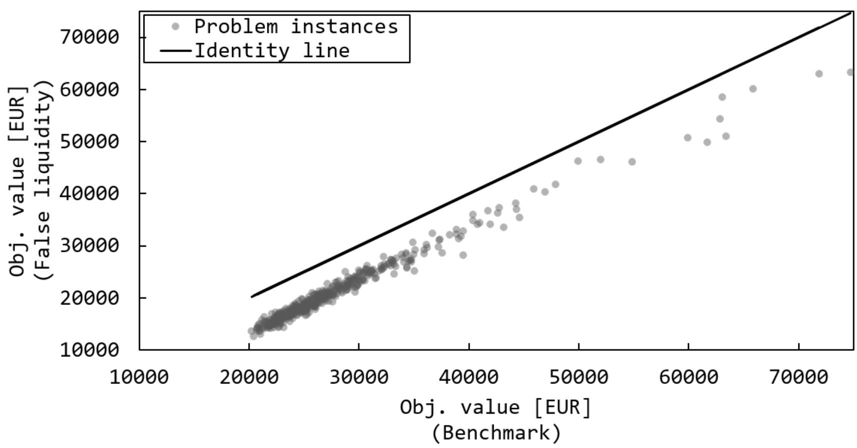

Figure 6.1.13: False liquidity vs. Benchmark model (N=500)

Table 6.1.5: The performance of the false-liquidity at different percentiles

Percentile	**2.50 %**	**10 %**	**50 %**	**90 %**	**97.50 %**	**Mean**
Absolute performance	14298	15293	18988	27597	40283	21082
Relative performance	66 %	69 %	75 %	83 %	87 %	75 %

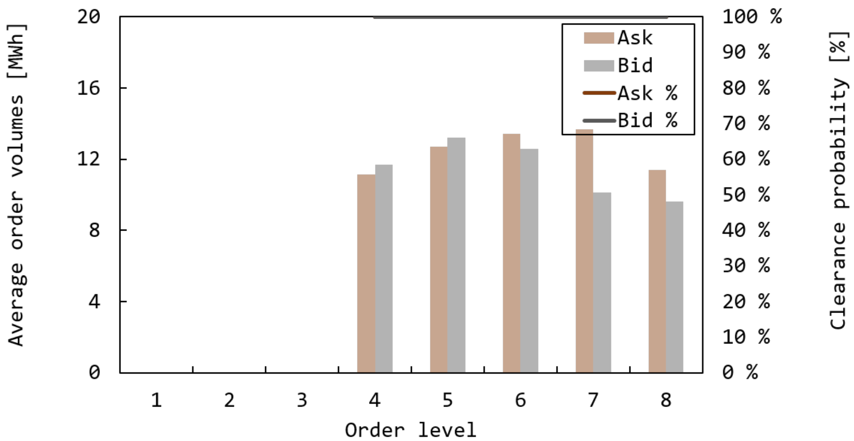

Figure 6.1.14: Average order volumes placed per order level by the False liquidity model for DP15 (N=1).

Note that a slack of 1 MWh was added to the constraint on the order placement in the False liquidity model, as numerical instability sometimes made the problem infeasible. The actual order volumes at the worst order levels should therefore probably be marginally higher.

6.1.7 Summary of the model performances

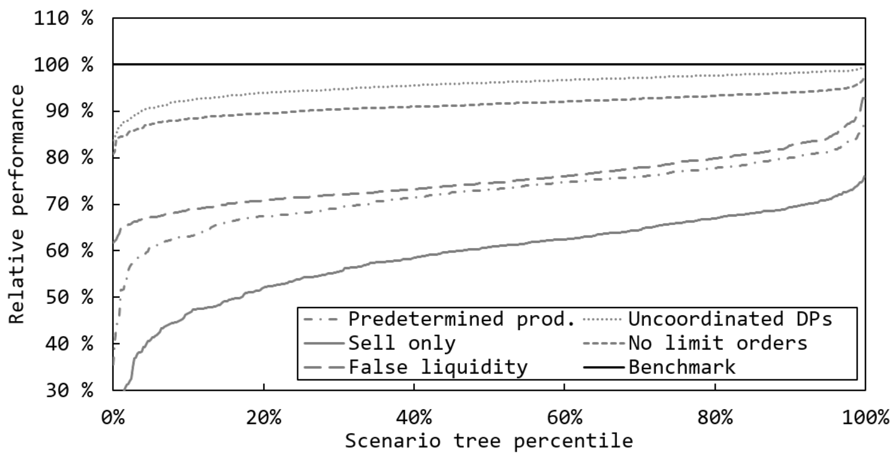

Figure 6.1.15: Comparison of the performance of all the models (N=500)

Figure 6.1.15 shows the performance of all alternative models relative to the `Benchmark model` for all scenario trees, sorted from worst to best. The `Benchmark model` dominates all of the other models in all scenario trees.

Table 6.1.6: Performance and standard deviation of 5 alternative models and the Benchmark model (N=500).

~	**AM1B**	**AM2**	**AM3**	**AM4A**	**AM4B**	**Benchmark**
Avg. obj.value	19 739	26 417	15 642	25 220	21 082	27 673
St.dev	4 353	6 753	461	6 337	7 022	7 137

In table 6.1.6, the average objective value and the standard deviation in objective value is shown for 5 alternative models and the `Benchmark model`. "AMB1" is short for alternative model 1B, predetermined production - and similarly for AM2-AM4B. In table 6.1.7 the same is shown for the relative performance and the standard deviation of the relative performance. Note that this standard deviation is very low for several of the alternative models.

Table 6.1.7: Relative performance and standard deviation of 5 alternative models and the Benchmark model (N=500).

~	**AM1B**	**AM2**	**AM3**	**AM4A**	**AM4B**	**Benchmark**
Average obj.value	72 %	96 %	59 %	91 %	75 %	100 %
Relative st.dev	7 %	3 %	10 %	2 %	5 %	0 %

Table 6.1.8: Roundtrip Ratio and production quantities of 5 alternative models and the Benchmark model for DP15 (N=1).

~	**AM1B**	**AM2**	**AM3**	**AM4A**	**AM4B**	**Benchmark**
RTR [%]	0,3 %	8,3 %	0,0 %	0,5 %	1,0 %	8,8 %
Production [MWh]	-42	111	140	78	80	75

In table 6.1.8, RTR is short for *roundtrip ratio*. It is defined as $|v^A_{ptds} - v^B_{ptds}|/(v^A_{ptds} + v^B_{ptds})$ for each combination of trading stage and scenario, and then averaged over trading stages and scenarios. A market maker that offers limit orders to the market may have a non-zero RTR, but traders that are limited to market orders should not buy and sell in the same stage, as it is equivalent to paying the spread and transaction cost twice, with no benefit to the trader. As all former analyses of the decisions of the models, this table is based on average numbers over all scenarios in the first scenario tree, considering only the last delivery product.

6.2 Computational study

As the small problem instances in this thesis are of an exploratory nature, and therefore significantly smaller than practical applications would require, the computational study is not the main value of the thesis. It is however interesting to attempt to identify the main obstacles to scaling the instance size, given the current formulation.

6.2.1 Calculating the scenario space

If i is the number of branches per stage, n the number of stochastic processes and $|\mathcal{T}|$ the number of stages, the total number of scenarios is $i^{n \cdot |T|}$, assuming synchronous gate closures. If it is desirable to have many stages and a fine-grained probability distribution for each stochastic process, n must be small. Examples of scenario spaces for different assumptions about $|\mathcal{T}|$, n and i can be found in table 6.2.1. The numbers account for the effects of asynchronous gate closures, by reducing the number of stochastic processes in the stages where the trading windows of some of the delivery products are closed. For the PCA-based scenarios, the number of stochastic processes is constant until the gate closure of the last delivery product.

Table 6.2.1: Scenario spaces for different problem instances.

Case	**DPs**	**Stages**	**Branch/DP**	**LOs?**	**Scenarios**
Full day, high frequency	24	360	2080	Yes	$1 \cdot 10^{17599}$
Full day, low frequency	24	30	2080	Yes	$4 \cdot 10^{1393}$
6 stages, binomial price	5	6	2	No	32768
6 stages w/LOs	5	6	32	Yes	$4 \cdot 10^{22}$
6 stages no LOs, w/ PCA	(2)	6	2	No	1024

In table 6.2.1, LO is short for limit order, DP is short for delivery product and PCA is short for principal component analysis. The price is represented by a discretized normal distribution with 130 branches, before it is simplified to a binomial distribution for the last 3 cases. In the last case, the number of DPs is actually 5, but the PCA is used to reduce dimensionality from 5 to 2 stochastic processes. As one can easily observe from the table, the scenario space quickly grows intractably large when the number of DPs, stages and branches per DP grows. This is the rationale for the number of approximations and simplifications that have been performed.

6.2.2 Technical study

Changing the set size parameters may affect the scenario space. The number of variables and constraints grow exponentially as a size of added stages, since more scenarios are required in order to keep some uncertainty throughout the entire trading window.

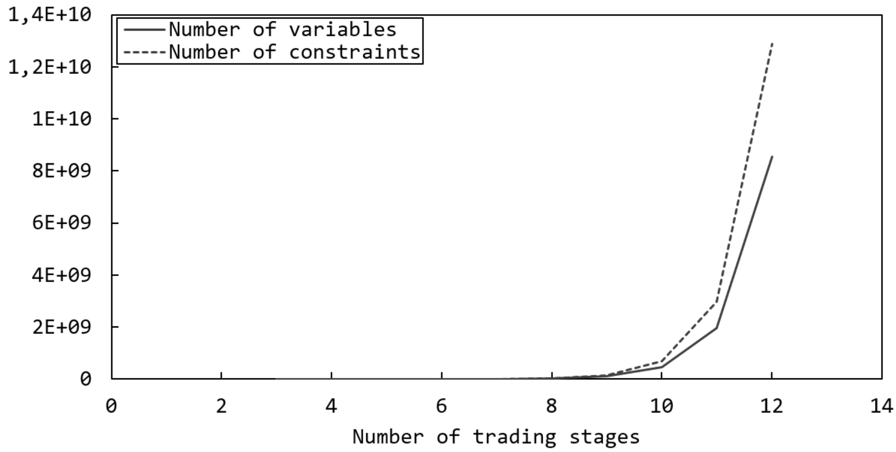

Figure 6.2.1: Problem size as a function of number of trading stages

If it is assumed that two stochastic processes are sufficient to model even more delivery products, the scenario space does not grow as a function of the number of delivery products, so the increase in the number of variables and constraints is only linear.

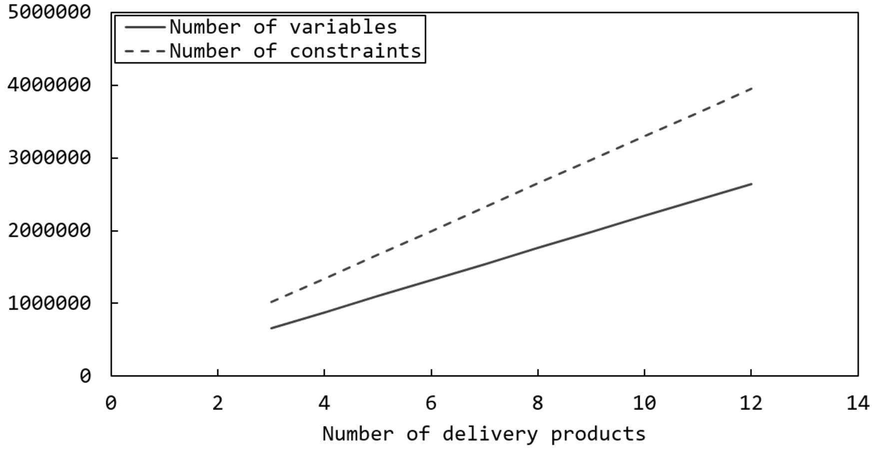

Figure 6.2.2: Problem size as a function of number of deliver products

Similarly, increases in the number of order levels or virtual reservoirs do not increase the scenario space, and the growth in the number of variables and constraints is therefore linear.

Chapter 7

Discussion

In this chapter, the results in chapter 7 are discussed. In section 7.1, the decisions and objective values of each of the alternative models are analyzed, and the reliability of the results is examined. In section 7.2 the applicability of the findings is discussed, and avenues of future research are outlined.

7.1 Interpretation of results

What can be concluded from the results presented in section 6? In this section, this question is decomposed in two parts; first, the link between the decisions and the objective values of each model is discussed, and then the reliability of the results is examined. The interested reader is referred to appendix E for a more in-depth, case-based discussion of the contribution of each of the modeling assumptions that differ between the models in this thesis.

Based on the dispatch-focused branch of the literature, a natural hypothesis could be that the production decision is particularly important to the objective value. An indeed; in tables 6.1.8 and 6.1.6 one may observe that the two models that deviate the most from the recommendation of the `Benchmark model` - namely the Predetermined production and Sell only models - are the two that perform the worst. However, the Uncoordinated DPs model also deviates from the recommendation, while performing almost as well as the `Benchmark model`. The No limit orders and False liquidity models on average produce approximately optimally, but perform worse.

A hypothesis of the reason for the poor performance of the False liquidity model could be that the high trading cost from transactions at poor order levels (see figure 6.1.14) is pulling down the performance of an otherwise good model. This is in line with the conventional LOB literature (see e.g. Mastromatteo et al. (2017)). However, this hypothesis still does not explain why the Uncoordinated DPs model beats the No limit orders model despite the drawback of being unable to coordinate the production decisions for consecutive delivery products.

The final component of the decision making that is underscored in this thesis is the ability to *make the spread* (Cont et al., 2010). By offering liquidity to the market in the form of risky bid and ask limit orders, traders may achieve profitable round trips. In conventional LOBs, banks often take a market making role. However, taking such a role requires that you are somewhat agnostic to your final inventory at the end of the trading window, since the clearing of the limit orders is uncertain. In the Intraday market, banks are not agnostic to the final inventory, since any non-zero final inventory will incur a large imbalance cost for traders without production assets. *Making the spread* is thus hard for traders that don't have flexible production. The low liquidity in the EPEX Intraday makes it a particularly interesting strategy for such producers. In particular, it is a good strategic fit for producers with *gently sloping marginal production costs* - such as the hydropower producer in our case - since the *marginal costs of a deviation from the target inventory* will be lower. Using the analogy to traditional microeconomics (in line with e.g. Aïd et al. (2015), who state that *"the optimal strategy consists in making (...) the forecast marginal cost equal to the forecast Intraday price."*), figure 7.1.1 illustrates this cost. Observe that the colored area for the producer with gently sloping marginal production costs is far smaller than the area for a deviation of similar size for the other producer.

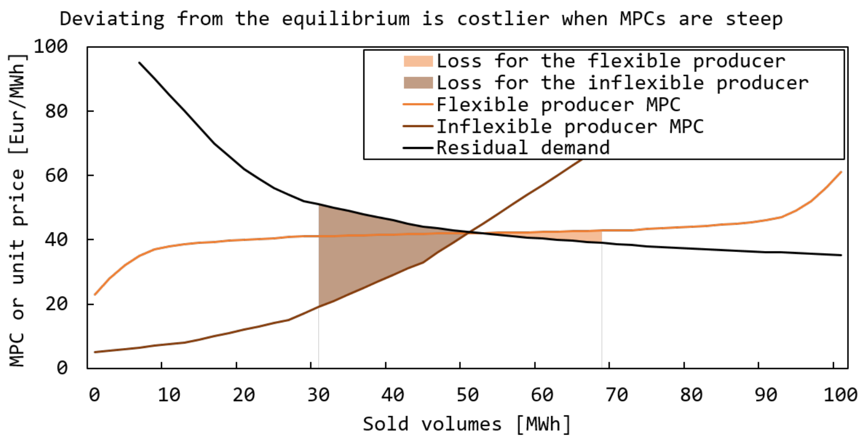

Figure 7.1.1: Marginal costs of deviation from the target inventory for producers with steep and gently sloping marginal production costs.

Making the spread thus requires that the trader is able to place limit orders on both sides of the limit order book, and has gently sloping marginal costs. This is not the case for the False liquidity, Sell only and No limit order models, and the Predetermined production model sacrifices the production flexibility before the trading decisions are made. Thus, only the Uncoordinated DPs model and the `Benchmark model` are able to do this, as observed from the Roundtrip ratio in table 6.1.8.

Thus, three hypotheses for the differences in the objective values of the models have been identified. Together, they explain much of the observed results in this thesis. Note however that there is a risk of inferring too much from the results in this thesis. In particular, the exact objective values of the models and their performances relative to the `Benchmark model` are sensitive to the assumptions in the cases. For instance, the net inflow after subtracting the Day-Ahead commitment has a value of just above 11 k€even if no action is taken by the model, which is "free profit" for all of the models. For this reason, the numbers may not reflect actual Intraday profits for traders in the market.

A common issue in the conventional LOB literature is that it is hard to avoid false arbitrage, and it cannot be categorically excluded in this thesis either.

However, the Sell only model cannot take arbitrage opportunities even if they exist, since it can only trade in one direction. Thus, the difference in the objective value between the Sell only model and the other models is an upper bound on the impact of arbitrage opportunities. However, as further explained in appendix E, the value of two-sided trading has extra value when the target end-of-day inventory is stochastic, so the value of false arbitrage opportunities should be significantly lower than this difference. As one may observe from figure 6.1.9, the nominal difference is lowest in the worst scenarios for the `Benchmark model`. For this reason, it is explored if the main conclusions are very different if only the worst 20% of scenarios are included for each model. As seen in table

Table 7.1.1: The performance of the alternative models in the 20% worst scenarios

~	**AM1B**	**AM2**	**AM3**	**AM4A**	**AM4B**	**Benchmark**
Average obj.value	16 451	21 278	15 250	20 424	15 309	22 114
Relative obj.value	74 %	96 %	69 %	92 %	69 %	100 %

Finally, it should be underscored that even if a strongly superior expected profit for the `Benchmark model` in verifiably realistic market conditions had been observed, it would not necessarily imply that it should be applied by the traders in the market. All former ITP papers have documented that their proposed model scales to practical problem instances, whereas the scalability of the proposed model in this thesis remains to be seen. Moreover, traders with different risk preferences may prefer models with lower variance in the performance, such as the No limit orders model (see table 6.1.6).

7.2 Applicability of the findings

Under which assumptions do the findings apply? In this section, the assumptions behind the results are highlighted, and the potential to remove assumptions is discussed. At first, the assumptions relating to the market are discussed in 7.2.1, and then the assumptions about the trader are discussed in 7.2.2.

7.2.1 Market assumptions

Only the `DayClose` category of the market was explored in the numerical simulations, due to the high similarity between the behavior of the delivery products and the high liquidity in this phase. It may seem trivial to extend the analysis to other categories of the market, as the market parameters like the price, limit order clearance probability and instantaneous price impact may simply be adjusted to values that are representative to other segments. However, some categories may have so quantitatively different market parameters that the market dynamics are qualitatively different, and other concepts are needed to model them well. For instance, the liquidity at night seems to be so low that the market is incomplete, because several market parameters become stochastic (see figure 5.2.1).

The problem instances in the numerical simulations are quite limited, and it would be desirable to test the proposed models on a larger scale. However, as documented in section 6.2, the proposed MSLP framework does not scale because the scenario trees grow too large for the available memory. A reformulation of the `Benchmark model` to a dynamic program may be able to scale to larger problem instances as the memory requirements are lower. However, the state space of such a dynamic program would become very large. With 5 delivery products with 1000 available storage volumes and 250 possible market commitments per delivery product, the *endogenous state space* is already $1000^5 \cdot 250^5 = 10^{27}$. When exogenous state variables are added - such as 5 prices, each of which may take a large number of values - it is obvious that a lot of effort is needed in order to find even approximate solutions to the dynamic program. Considering the strong performance of the Uncoordinated DP model, it may be possible to model each delivery product separately, in which case the dynamic program looks more tractable.

An additional benefit of being able to solve a dynamic formulation of the `Benchmark model` would be the ability to model the transient and permanent price impacts. Similarly to Aïd et al. (2015) and Tan and Tankov (2016), one would be able to differentiate between the unaffected (potentially martingale) price process, and the actual prices that have been affected by the trading behavior of the trader herself. If such price impacts are modeled, additional effort is needed to ensure that the model makes realistic assumptions about the shapes of the price impacts, so that it does not find false arbitrage opportunities.

An implicit assumption in several of the papers in the contemporary ITP literature is that the model not is adapted at scale. As only market orders are used, the market does not converge to an equilibrium as adoption grows, because the liquidity in the market disappears. As our model permits both limit and market orders, large-scale adaption would converge to an equilibrium where the limit order premium reflects the risk preferences of the traders in the market, so this is not an issue with the `Benchmark model`.

7.2.2 Trader assumptions

The `Benchmark model` assumes that the trader is risk neutral, but it is possible to extend the model to a formulation that allows for risk aversion. In order to do so, linear measures of risk, such as the Conditional Value-at-Risk (see e.g. Alexander (2008d)) must be added to the objective function. This may reduce the strength of the preference for the `Benchmark model` over models that take lower risk, such as the No limit orders model.

The choice of parameters for the production system in this thesis may have affected the results. In particular, the number of stages and delivery products, the storage capacity, initial storage, production capacities, Day-Ahead commitment, marginal production cost curve and power inflow were identical for all scenarios. Unlike the other parameters in the model, these parameters are not based on a rigorous empirical analysis. Changing these parameters may change the optimal trading behavior in many ways. For instance, the preference for limit orders falls if marginal production costs are far lower than market prices. In order to test the model for realistic production parameters, it is proposed that the model is applied to the production systems of actual traders in the market.

Chapter 8

Conclusion

In this thesis, the Intraday Trading Problem has been defined. The problem includes both production- and order placement- decisions related to the continuous auction Intraday power market. The goal of the ITP is to maximize the profits of the trader over a decision horizon that spans the part of the trading windows for the delivery products when the market is liquid. The goal of the thesis was one level of abstraction higher; it was to discover which modeling assumptions about the ITP that are the most profitable to make, and to estimate the impact on the objective function of making alternative assumptions that are proposed in the existing literature.

There are four main contributions in this thesis. First, an extensive literature review was performed, producing a set of modeling assumptions that there are disagreements about in the contemporary literature, and a novel definition and decomposition of the ITP was proposed. In particular, this thesis was the first to recognize the double dynamics of the decision structure in the ITP, resulting from the coordinated optimization of the trading and production of multiple delivery products.

Second, a detailed market analysis has been performed. An extensive list of relevant market parameters was included, all of which were estimated empirically. In particular, the modeling of the clearing of limit orders in a market with limited liquidity went beyond the existing literature.

Third, seven models were developed using an MSLP framework, and the models were compared in limited scenario trees based on the market analysis. The proposed `Benchmark model` outperformed all of the alternatives inspired by the existing literature in all scenario trees. The alternative models fell short of the proposed `Benchmark model` by 4-41%. The Sell only-, Predetermined production- and False liquidity models achieve 69-75% of the performance of the `Benchmark model`, whereas the No limit orders- and Uncoordinated DPs models achieve 91-96%.

Fourth, the profit gains of each of the relevant assumptions were discussed through a theoretical lens, and probable explanations of the profit gains were found. The differences in performance can be partly explained by suboptimal production decisions, higher trading costs and inability to "make the spread". One alternative model was discarded due to unrealistically poor performance.

It was noted that the specific percentages of profit gain are sensitive to the parameters of the case. However, the finding that the proposed `Benchmark model` significantly outperforms most of the alternatives inspired by the related literature in the small scenario trees described in section 5 was considered robust, after tests on 500 scenario trees.

Three avenues of future research are proposed. First, the empirical evidence of the findings should be strengthened through backtesting on empirical market data and simulations based on different assumptions. Second, it is necessary to scale the problem instances in order to apply the model in practice. It is hypothesized that a reformulation to a dynamic program could handle larger problem instances, though the resulting state space is vast and would require sophisticated solution methods. If such a reformulation does not work, further tests of the performance of the Uncoordinated DPs model can reveal if it is a viable alternative model. Finally, improved modeling of the transient and permanent price impacts, as well as the inclusion of risk measures in the objective function, would expand the applicability of the model.

Chapter 9

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Appendix

A A model with transient and permanent price impact

In order to capture the transient price impact, a further reformulation of the model proposed in section 4.2 (the `Benchmark model`) is considered. In this appendix, such a reformulation is developed. The reformulation uses integer variables, and is therefore a Multistage Stochastic Integer Linear Program (MSILP). A nonlinear formulation is provided first, and an outline for how to linearize the nonlinear constraints are included at the end.

The assumption behind the model that includes the transient price impact is that the inflow of orders in each stage are parameters A^{new}_{ptds} and B^{new}_{ptds}. The order depth in the stage is a variable, a_{ptds} and b_{ptds}. As the trader places an order in the market, she thus alters the future order depths for the rest of the trading window. The impact on the order book is immediately visible as a given order volume is added to or removed from the limit order book, but as many stages pass, the impact of the one order drowns in the noise of the many the new orders that are added in each stage. In order to model these assumptions, several reformulations to the `Benchmark model` are needed.

In particular, the price indeces p may no longer be defined based on the mid-price in a given stage, since said mid-price is unknown. P^A_{ptds} therefore refers to the same price for all t and s, and one may therefore remove the t, d and s indeces. Moreover, one may define that $P^A_p = P^B_p = P_p$, for simplicity. As $P_p, \quad p \in \mathcal{P}$ refers to a constant set of prices, this set must span the entire range of possible prices. In order to keep the desired granularity of order prices to determine order prices accurately, the set of order levels is therefore much larger than before.

The scenarios also need to be redefined. Based on the inflow of orders A^{new}_{ptds} and B^{new}_{ptds} and the rules of the clearing mechanism in the market, the limit order premium and the mid-price may be uniquely determined. Thus, the stochasticity is in the inflow itself. As there are very many parameters A^{new}_{ptds} and B^{new}_{ptds} in total, the number of stochastic processes should be far smaller than the number of stochastic parameters; it is necessary to define relations between the order inflows in ways that reduces the scenario space while maintaining the decision relevant aspects of the model. This is not considered further here.

In the Benchmark model, it is assumed that transactions only occur when two opposite orders with the same price clear - that is, if the spread between the two orders was zero. In reality, orders may clear if the spread between itself and an opposite order is negative. This is not a concern when one of the parties in every transaction is the trader that the model represents, since it will adjust its decisions accordingly without loss of generality. When order depths in the market are modeled as variables and not parameters, transactions between two external traders also have to be modeled. Thus, this is no longer a valid assumption. It is therefore assumed that orders clear if the spread between the orders is negative, and there exists no pair of opposite orders with a larger negative spread.

In the Benchmark model, the purpose of constraints (4.2.13) and (4.2.14) is to force the cleared volume to equal $\min(x^A_{ptds}, B_{ptd})$ (or $\min(x^B_{ptds}, A_{ptd})$, by symmetry). This is with a binary parameter, since the minimum of the two were either equal to x^A_{ptds} or zero. That is no longer possible, and new variables become necessary to keep account of the order clearing. For simplicity, assume that a limit order has a duration equal to one stage, so that it is canceled immediately after the orders in the next stage arrive. In this case, the clearing of an order is expressed in in equation (A.1).

$$v^A_{ptds} - \min\{x^A_{ptds}, \sum_{\widetilde{p}=p}^{\bar{p}} b_{\widetilde{p}tds}\} - \max\{0, \min\{\sum_{\widetilde{p}=p}^{\bar{p}} B^{new}_{\widetilde{p}(t+1)ds} - \sum_{\widetilde{p}=0}^{p-1} (a_{p(t+1)ds} + A^{new}_{p(t+1)ds}), x^A_{ptds} - \sum_{\widetilde{p}=p}^{\bar{p}} b_{\widetilde{p}tds}\}\} = 0, \quad p \in \mathcal{P}, t \in \mathcal{T}_d, d \in \mathcal{D}, s \in \mathcal{S} \tag{A.1}$$

Here it is assumed that $\mathcal{P} : p \in \mathcal{P} \iff p \in \{0, 1...\bar{p}\}$. Equation A.1 decomposes the cleared volume of the order into two parts, where one part is the immediate clearing in stage t and the other part is the clearing in stage $t+1$. $\min\{x^A_{ptds}, \sum_{\widetilde{p}=p}^{\bar{p}} b_{\widetilde{p}tds}\}$ represents the immediate (partial) clearing of the order x^A_{ptds}, if there is a volume $\sum_{\widetilde{p}=p}^{\bar{p}} b_{\widetilde{p}tds}$ available at the prices that are better than p in stage t. If the available volume at better prices is smaller than the order volume x^A_{ptds}, some of the residual order volume $x^A_{ptds} - \sum_{\widetilde{p}=p}^{\bar{p}} b_{\widetilde{p}tds}$ may clear as new order volumes $\sum_{\widetilde{p}=p}^{\bar{p}} B^{new}_{\widetilde{p}(t+1)ds}$ are placed in the next stage. However, if there exist competing orders at better prices, $\sum_{\widetilde{p}=0}^{p-1}(a_{p(t+1)ds} + A^{new}_{p(t+1)ds})$, then those will have priority. Thus, $\sum_{\widetilde{p}=p}^{\bar{p}} B^{new}_{\widetilde{p}(t+1)ds} - \sum_{\widetilde{p}=0}^{p-1}(a_{p(t+1)ds} + A^{new}_{p(t+1)ds})$ is the new order volume that is available for the given order in the next stage, just before the order x^A_{ptds} is canceled. If either $\sum_{\widetilde{p}=p}^{\bar{p}} B^{new}_{\widetilde{p}(t+1)ds} - \sum_{\widetilde{p}=0}^{p-1}(a_{p(t+1)ds} + A^{new}_{p(t+1)ds})$ or $x^A_{ptds} - \sum_{\widetilde{p}=p}^{\bar{p}} b_{\widetilde{p}tds}$ is negative, the cleared volume in stage $t+1$ is zero. By symmetry, a similar equation can be formulated for a bid order x^B_{ptds}.

The variables for the available order volumes, a_{ptds} and b_{ptds} must be updated for every stage. Inflow of orders A^{new}_{ptds} and B^{new}_{ptds} increase the available order volumes, unless they match with opposite orders and clear. Orders placed by the model, x^A_{ptds} and x^B_{ptds} remove orders from the limit order book. If other traders are allowed to cancel orders, this may be approximated by a decay rate Λ^A_{ptds} and Λ^B_{ptds}. Assuming that the external order canceling takes place at the end of a stage, the outbound order volume from a stage is $\Lambda^B_{ptds} \max\{0, b_{ptds} - \max\{0, \sum_{\widetilde{p}=0}^{p} x^A_{\widetilde{p}tds} - \sum_{\widetilde{p}=p}^{\bar{p}} b_{\widetilde{p}tds}\}\}$. In stage $t+1$, $B^{new}_{p(t+1)ds}$ is added to the outbound balance in t, and potential matching orders $\sum_{\widetilde{p}=0}^{p} A^{new}_{\widetilde{p}tds}$ are subtracted, after accounting for competing orders with priority, $\sum_{\widetilde{p}=p}^{\bar{p}} b_{\widetilde{p}tds}$. Denoting the outbound order volume $\widetilde{b}_{ptds}$, the resulting order depth flow constraint thus becomes:

$$
\begin{aligned}
b_{p(t+1)ds} - \max\{0, \widetilde{b}_{ptds} + B^{new}_{p(t+1)ds} - \max\{0, \sum_{\widetilde{p}=0}^{p} A^{new}_{\widetilde{p}tds} - \sum_{\widetilde{p}=p}^{\bar{p}} \widetilde{b}_{\widetilde{p}tds}\}\} \\
= \quad 0, \quad p \in \mathcal{P}, t \in \mathcal{T}_d, d \in \mathcal{D}, s \in \mathcal{S}
\end{aligned}
\tag{A.2}
$$

$$
\begin{aligned}
\widetilde{b}_{ptds} - \Lambda^B_{ptds} \max\{0, b_{ptds} - \max\{0, \sum_{\widetilde{p}=0}^{p} x^A_{\widetilde{p}tds} - \sum_{\widetilde{p}=p}^{\bar{p}} b_{\widetilde{p}tds}\}\} \\
= \quad 0, \quad p \in \mathcal{P}, t \in \mathcal{T}_d, d \in \mathcal{D}, s \in \mathcal{S}
\end{aligned}
\tag{A.3}
$$

By symmetry, similar equations apply to the ask order depth.

i) Linearizing the min- and max-statements

If the objective function of a problem is strictly increasing in x and A and B are parameters of the problem, a constraint on the form $x = \min\{A, B\}$ can be linearized by replacing it with $x \leqslant A$, $x \leqslant B$ (and by symmetry, $x = \max\{A, B\}$ can be linearized in a resembling way if the objective function is strictly decreasing in x). In this case however, we have nested min- and max-statements, the objective function doesn't pull in the right direction for all of them, and inside some of the min- and max-expressions there are decision variables. Such a linearization would fail in several ways; for instance, the placed order volumes x^A_{ptds} would be very large, since the model would not know that the market rules consider an order placement as a commitment to trade if a counterparty is found, so it would consider the order volume merely as an option to trade; moreover, it may pretend that two external opposite orders with very good prices did not match each other, in order to reserve both the orders for itself. After all, the model has no intrinsic incentive to encertain that the external orders clear in the correct fashion.

For this reason $x = \min\{A, B\}$ must be expressed with two binary variables. If $\mathcal{M}$ is defined so that $\mathcal{M} \geqslant x$ is true for all feasible x and $\delta^A \in \{0, 1\}$, $\mathcal{M}(1 - \delta^A) - a + x \geqslant 0)$ allows $\delta^A = 1$ only when $x \geqslant A$. Using a similar constraint for δ^B, $\delta^A + \delta^B = 1$ states that either $x \geqslant A$ or $x \geqslant B$. Combined with $x \leqslant A$ and $x \leqslant B$, it follows that x must equal $\min\{A, B\}$. As the constraints above apply for $p \in \mathcal{P}, t \in \mathcal{T}_d, d \in \mathcal{D}, s \in \mathcal{S}$, the number of binary variables grows very large if such a formulation is attempted. Moreover, for each binary variable there is one $\mathcal{M}$, so the formulation is very weak. Thus, this approach is deemed intractable. No further work has been done with this model.

B Historical volumes of five best open orders

In the figure grids of figure B.1 and B.2, the development of the volumes of the five best buy and sell orders are shown. For the `DayClose` category, the volumes seem somehow constant.

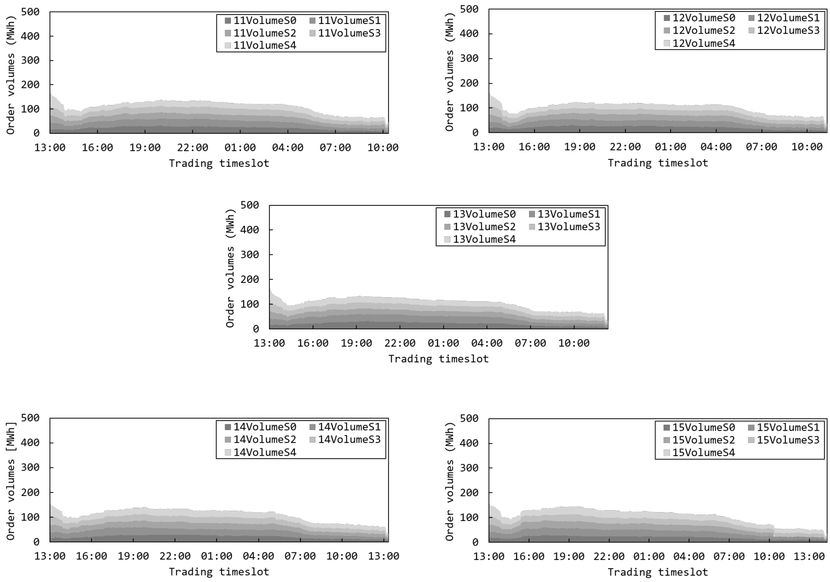

Figure B.1: Sell premium order level volumes plot

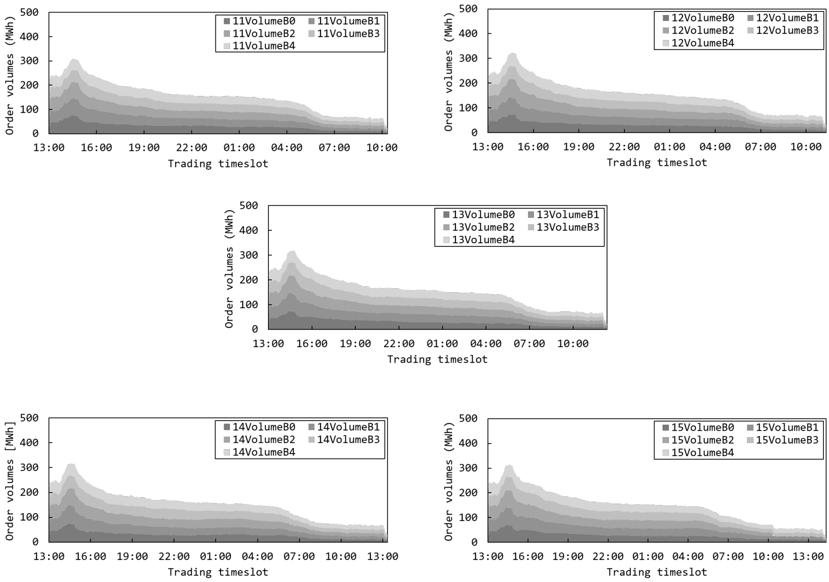

Figure B.2: Sell premium order level volumes plot

C Order clearing algorithm

```
Data: Buy orders, sell orders, sorted chronologically
Result: Set of transactions, DP order book feature list
initialization;
while unopened orders exist do
    CurrentOrder := order Sort opposite side clearable orders by price
    forall Opposite side clearable orders do
        if price of OppositeSideClearableOrder better than price of
          CurrentOrder then
            create transaction;
            reduce volumes of involved orders by transaction volume;
        else
            continue;
        end
    end
    if timeslot of CurrentOrder is later than TimeOfNextSnapshot then
        append market data to RecordedMarketData;
        TimeOfNextSnapshot += minutes(5);
    else
        continue;
    end
    Sort opposite side clearable orders chronologically;
    Append order to opened buy / sell orders;
end
```

Algorithm 1: Order clearing algorithm

An implementation of the pseudocode written in Python 3.6 can be found in the attached file *market.py*.

D A model for determining buy limit order quantities

The program of equation (D.1)-(D.4) maximizes the total area of N non-overlapping rectangles bounded by the x-axis, the y-axis and a cumulative distribution of clearing probabilities as a function of price premiums. The program is very similar to that of (5.2.2)-(5.2.5), but obviously, the cumulative function f(x) is different and some minor changes must be done to the constraints. These model changes are results of the rectangle x-coordinates being negative in the program of (D.1)-(D.4). Specifically, all variables x_i must be non-positive, the minimum rectangle width $\underline{W}$ must be a negative number and the signs in the objective function are changed. The program was executed using Microsoft Excel 2016's Solver tool with the parameters $\underline{W} = -0.01$, $\rho^{CPR} = 0.2$ and $f(x) = Ax^3 + Bx^2 + Cx + D$ with $A = 0.87, B = 2.4, C = 2.28$ and $D = 0.98$.

$$\max_{x_1,\ldots,x_N} \sum_{i=1}^{N} f(x_i)(x_{i-1} - x_i) \tag{D.1}$$

$$x_i - x_{i-1} \leqslant \underline{W}, \quad i = 1, 2, \ldots, N \tag{D.2}$$

$$x_0 = 0 \tag{D.3}$$

$$f(x_N) \geqslant \rho^{CPR} \tag{D.4}$$

E Conceptual explanation of the impact of each assumption

What is the mechanism by which each of the assumptions that have been discussed so far contribute to the objective function? Are the given mechanisms simple enough that most of the profit increase can be captured by simpler models with clever heuristics, or are the decisions altered in ways that are hard to approximate in the pre- or postprocessing of the decisions? In this section, the conceptual explanation of the recommended choices for modeling assumptions provided, and it is argued that the mechanisms are hard to approximate through pre- or postprocessing.

i) Assumption 1: Predetermined or informed production decisions?

In this section, the optimal trading strategies for sell-only traders with homogeneous marginal production costs in a liquid market is explored for different price assumptions and cost levels. A problem instance with only one delivery product and a two-stage trading window is used. The optimal 1st stage decision

is explored for different assumptions about price drift and variance. As both marginal revenue and cost are constant as a function of trading volume until the production capacity is reached, the optimal strategy reduces to either HOLD or SELL (implicitly: the entire open position) in the first stage, depending on whether marginal costs are above or below prices.
Recall that the goal of this section is to show the value of delaying the production decision. If production has already been determined, production costs are unaffected by the trading decisions. In this case, the only concern for the trader is to obtain the best price for the sold volume. The timing of the placed order(s) therefore should be no different for traders with marginal production costs. Thus, if traders with different production costs decide to time their orders differently when they are allowed to determine the production quantity ex post, the incentive to do so must be caused by the value of postponing the trading decision. In this section, the trader is allowed to determine the production quantity ex post.

In table E.1, "1" indicates that an order is placed and "0" indicates that an order is not placed. "[0, 1]" indicates that the trader is agnostic to whether a trade is placed or not, given that she is profit maximizing. Trades in the first stage happen when profits from trading are higher than future expected profits from the same trade volume. Trades happen in the second stage if prices are higher than marginal production costs and the volume hasn't already been sold in the first stage. Since the second stage decision is fairly trivial, the following discussion is exclusively focused on the first stage decision.

Some background information about figure E.1 and table E.1:

- First stage prices are always 50 €/MWh.
- `S-Sub` means that prices are strongly submartingales; they are expected to rise by a wide margin (specifically, 2nd stage prices are 66, 56 or 46 €/MWh with equal probability).
- `W-Sub` means that prices are weakly submartingales; they are expected to rise somewhat (61, 51 or 41).
- `Mar` means that prices have the martingale property; the expectation in the second stage equals the price in the first stage (60, 50 or 40)
- `W-Sup` means that prices are weakly supermartingales; they are expected to fall somewhat (59, 49 or 39).

- `S-Sup` means that prices are strongly supermartingales; they are expected to fall by a wide margin (54, 44 or 34).
- In all scenarios, it is expected that `S-Sub` > `W-Sub` > `Mar` > `W-sup` > `S-Sup` in the 2nd stage.
- Recall from section 2.2 that the assumption of non-martingale prices doesn't necessarily break with the Efficient Market Hypothesis; thus, the market may still be assumed to be liquid and efficient.
- `HV` is a case of weakly supermartingale prices with a very high variance (99, 49 or -1).
- `MC` is short for (expected) marginal cost, and is the probability weighted maximum per scenario of the marginal production cost and the market price in the next stage. For the last stage, as there does not exist any next state, it is equal to the marginal production cost.
- `Low` and `High` denotes the two producers with low (25 €/MWh) and high (45 €/MWh) marginal production cost, respectively.
- `H`, `M` and `L` denotes high, medium and low realizations of the second stage price, respectively, and all scenarios are assumed to be equally likely. Scenarios `H` and `L` are symmetrical around scenario `M` for all price assumptions.

Table E.1: Trades placed by one low cost and one high cost producer in a two-stage problem with three scenarios, for 6 different price expectations.

Price-assumption	**MC - Low**	**MC-High**	**Low= SELL?**	**High= SELL?**
S-Sub	56,0	56,0	0	0
W-Sub	51,0	52,3	0	0
Mar	50,0	51,7	[0, 1]	0
W-Sup	49,0	51,0	1	0
S-Sup	44,0	48,0	1	1
HV	57,7	64,3	0	0

The key takeaways from the table are the following:

- Marginal costs in the first stage equal the expectation of the maximum of the price in the second stage, and the marginal production costs. They

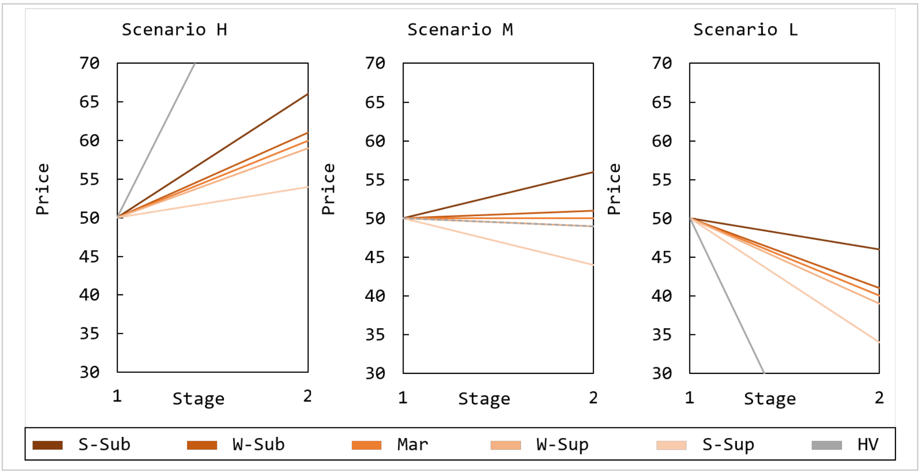

Figure E.1: Prices in stage 1 and 2 for the different scenarios under different price assumptions.

thus incorporate the alternative cost of later trades, as pointed out in section 2.1.4.

- In the first stage, an order is placed if marginal costs are lower than the price. If marginal costs and prices are equal (as it is for the low cost trader when prices are martingales), the trader is agnostic to whether a trade is placed or not.

- The second strategy can be summarized as "sell if you haven't sold already and prices are above marginal production costs". As it is so trivial, it was omitted from the table.

- The only price assumption where the low and high cost producers have strictly different preferences (that is; there is no overlap between the sets of optimal strategies for the two producers) is the weakly supermartingale case.

- The low cost producer is agnostic to the first stage strategy with the martingale price assumption when she is strictly profit maximizing, but it is reasonable to assume that all else equal, the trader will prefer to close an open position early rather than later.

- Even when prices are martingales, the high cost producer has a preference for trading in the second stage over the first stage. Moreover, marginal costs are much higher in the `HV` scenario than in the `W-sup` scenario, even though expected prices are the same. These counterintuitive situations are caused by the same phenomenon, which is explored more in depth in the following paragraphs.

The differing strategies can be explained in the following way: If the marginal production cost is between the extrema of the probability distribution for the price (higher than the lowest possible price and lower than the highest possible price) in the second stage, the trader doesn't capture (all of the) the downside of the variance in the second stage, while capturing some of the upside. If the trader is risk neutral as a function of profits, she will display risk seeking behavior as a function of price (see figure E.2), since the mapping from price to profit is convex. In the weakly supermartingale case, this effect outweighs the expected price drop for the high cost producer, but not for the low cost producer which is outside of the price range. However, in the strongly supermartingale case, prices are expected to fall too much, so both producers opt for the first stage.

In order to explain this, two concepts are introduced. The *option value* is the value of delaying the production decision, absent price drift. The textithold value is the expected profit increase from selling in stage 2 rather than 1. It is thus the sum of the option value and the expected gain from the price drift. In this case, the option of waiting for the high variance stage is outweighed by the expected price drop, so the hold value is negative. Note that by this definition of the hold value, it doesn't by itself determine if trading in the first stage is profitable, only if it is more profitable than trading in the last stage.

Note that the traders in table E.1 sell in the first stage whenever the marginal costs - which include the alternative cost of later trades - are lower than the current prices. When prices and marginal costs are equal, the risk neutral trader is agnostic to the timing of the trade.

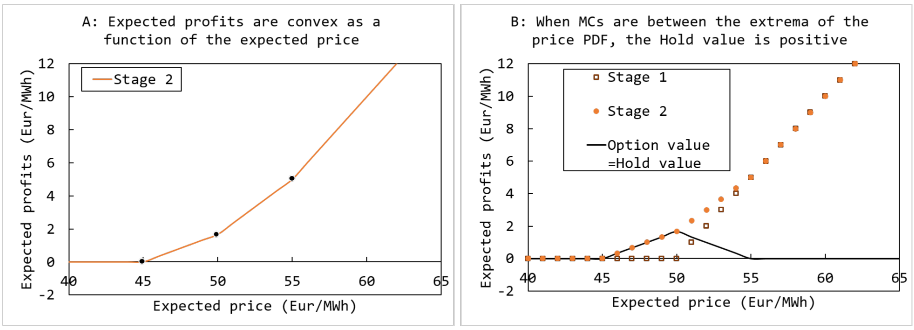

Figure E.2: Profits are a convex function of price.

In figure E.2 part A, marginal production costs are 50 €/MWh, and the price difference between the scenarios is 5 €/MWh. The black dots mark the price levels where one scenario has a price equal to the marginal cost. The reason for the convexity can be explained in the following way: assume that the trader changes her price assumptions so that second stage prices in all scenarios increase with 1€/MWh compared to the initial assumption. For the scenarios where the prices are below marginal production costs, this doesn't affect the profits, as the trader won't sell anyways. For such a change in price assumptions, the expected increased profits is thus equal to the probability that prices are higher than marginal costs in the second stage, multiplied with the change in expected prices. Expected profits thus increase more if it already was likely that prices were higher than marginal costs; if the worst price outcome is higher than marginal costs, the slope of the mapping from expected price to expected profits is 1; if the best price outcome is lower than marginal costs, there is no slope; and finally, if the marginal costs are between the extrema of the probability distribution for the second stage prices, the slope is between 0 and 1. Note that this analysis only applies to a uniform price update across all scenarios; if the shape of the probability distribution for the second stage changes in more complex ways than a shift of the expected value, the analysis also becomes more complex. Also note that if the price probability distribution was continuous, the mapping from price to profit would be continuously differentiable over the domain of the price, rather than piecewise linear.

In figure E.2 part B, the orange dots represent the same function as the orange line in part A. The option value is positive when the marginal production costs are between the extrema of the price probability distribution. Absent price drift, it is largest when marginal production costs equal the expected value of the price. A formal proof is not supplied here, but intuitively, the reason for this is that the risk neutral trader has no relevant information a priori about whether to "SELL" or "HOLD" in the second stage when the expected price is equal to marginal production costs; for all other price expectations, one of the strategies will be superior-in-expectation to the other before the second stage price is revealed. Thus, the value of the information gain is largest when expected prices are equal to marginal costs.

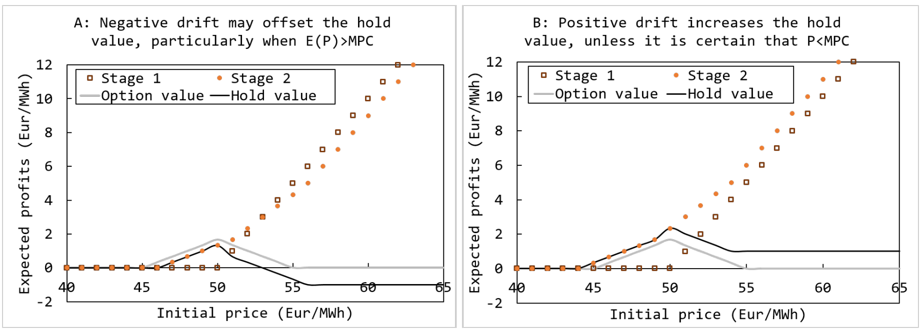

Figure E.3: Price drift may offset or increase the hold value.

Introducing price drift, the benefit of the option value may be offset or amplified by the difference in expected prices. The hold value thus may be negative even if the option value is positive, as illustrated in the left part of figure E.3. In the right part of the figure, the positive price drift creates a positive hold value, particularly when prices are certain to be higher than the MPCs. The resulting HOLD- and SELL-regions in the first stage are illustrated in figure E.4.

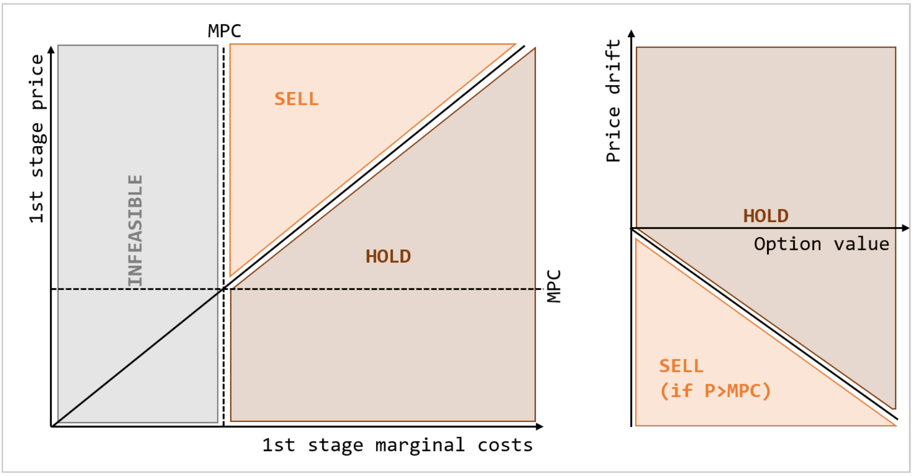

Figure E.4: HOLD- and SELL-regions in two different coordinate systems.

The key takeaway is that optimal order strategies are different for producers with different marginal costs. This shouldn't be surprising, as different objective functions may cause different behavior in general. However, it implies that one producer with heterogeneous marginal costs will have different optimal strategies for the different cost segments of the production portfolio; orders corresponding to the cheap production of one generator may be placed at a different time from orders corresponding to the expensive production of another generator. This is sufficient to demonstrate the necessity of allowing multiple orders to be placed in the market - at least for some price trajectories - when costs are heterogeneous. Note that heterogeneous MPC is not a necessary criterium for distributed order placement; for instance, short-term price impacts may have the same effects.

The reason that order placement is distributed throughout the trading window is that the option to determine production with more information holds positive value, as long as the last-stage prices may fall on either side of the marginal production cost. Thus, the finding that order placement should be distributed throughout the trading window implies that production should be determined as late as possible without violating production ramping constraints.

ii) Assumption 2: One or several delivery products?

For this case, the assumption of a weakly negative price drift (-1 €/MWh) from section i) is kept. The trading windows of the DPs are still two-stage, and they are synchronous in time. The price is known in the first stage and may take one out of three values for each DP in the second stage, and the price processes are uncorrelated. It is assumed that the trader has sufficient storage capacity, so that the open position may be closed through either one of the limit order books associated to the delivery products. The trader still focuses only on selling, as this assumption simplifies the explanations. As observed in section i), the expected price fall creates a weak incentive to sell in the first stage, but other considerations may switch that decision.

The initial prices of the DPs are sorted in chronological order in the column titled "DP1-DP2-DP3". The price difference for a given delivery product between each scenario is denoted "2nd stage price diff" in table E.2. The MPC is set so that it is always lower than the 2nd stage best price, to remove the risk-seeking effect from the convex price-profit mapping in section i): in particular, it is 0 for the `V-Low` and 25 for the `Low` cost producer. Thus, the traders are incentivized to sell in the first stage when there is only one delivery product with the given assumptions.

In table E.2, the effects of several assumptions about initial prices and price volatility is explored for a fixed number of delivery products. However, the number of delivery products, capacity constraints on production or storage, intertemporals and marginal production cost heterogeneity may also alter the resource allocation across delivery products and thus the trading activity across stages. The impact of these considerations is explained qualitatively below the table.

Table E.2: 1st stage decisions for two producers with different homogeneous marginal costs in a two-stage trading window with three delivery products.

DP1-DP2 -DP3	**Max price**	**2nd stage price diff**	**Corre-lation**	**MC-V-low**	**MC-Low**	**V-Low= SELL?**	**Low= SELL?**
50-NA-NA	50	10	0	49.0	49.0	1	1
50-50-50	-:-	10	0	55.7	55.7	0	0
50-30-10	-:-	10	0	49.0	49.0	1	1
50-30-10	-:-	20	0	51.2	51.2	0	0
50-30-10	-:-	20	1	49.0	49.0	1	1

The key takeaways from the table are the following:

- NA is short for not applicable, and refers to DP2 and DP3 in row 1 when only one DP exists. In this case, the trader sells in the first stage due to the negative price drift.
- In row 2, when prices for several DPs are equal initially, several prices may be the highest price in the second stage. Thus, the probability that at least one of the 2nd stage prices beats the 1st stage price is higher, and the marginal cost is higher.
- In row 3, when initial prices are sufficiently different, this effect disappears and only the highest priced DP matters (note that row 2 is identical to the `W-sup` scenario in table E.1.
- In row 4, when the price volatility ("2nd stage price diff") is higher, this may compensate for initial price differences. This effect is separate from the one in section i); as marginal costs for the `V-Low` and `Low` producers are identical, the 2-nd stage price must always be better than MPCs, so the domain of the probability distribution is in the affine part of the price-profit mapping.
- In row 5, when price processes are correlated, the benefit from several delivery products falls. All else equal, a higher correlation means lower potential for cross-DP arbitrage, until only one price process matters when they are perfectly correlated.
- To summarize, the trading decisions are different for different assumptions about the number of DPs, the initial price difference, the volatility of each process and the correlation between the price processes.

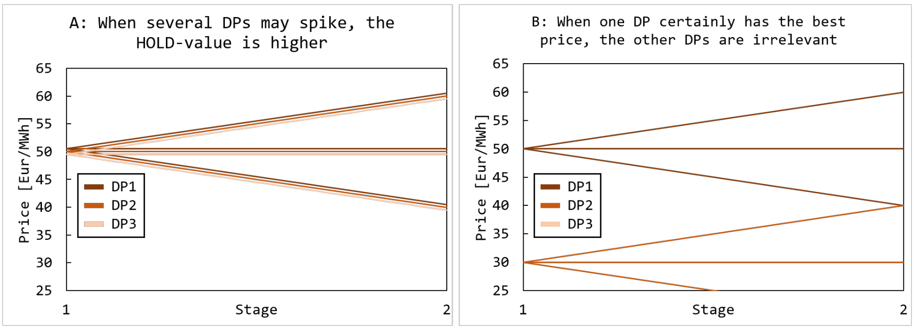

Figure E.5: Price scenarios with low/high initial price differences and high/low 2nd stage variance.

Figure E.5 shows the price trajectories from row 2 and 3 in table E.2) respectively. In the left half, prices start out at 50, and may take the values 59, 49 or 39 in the 2nd stage. In the right half, prices start out at 50, 30 and 10 (below the chart) respectively, and may change with +9, -1 og -11 to the second stage. As one can observe, low initial price difference creates many combinations of scenarios for the three prices where the best 2nd stage price is higher than the best 1st stage price, increasing the HOLD-value in the first stage. The maximum of several martingale processes is not in general a martingale - even if the individual price processes have negative drift, the "Best price"-process may have positive drift.

If the number of delivery products (with approximately similar first stage prices) is increased, the incentive to wait is increased, as it is more likely that at least one price will spike. If storage capacity- or production capacity constraints are tight, the ability to reallocate resources across delivery products is reduced, reducing the ability to sell at will and therefore the incentive to wait for price spikes. If there are non-negligible ramping constraints or ramping costs, changes in production between consecutive delivery products become a relevant consideration, and the trader will be incentivized to smooth production. Similarly, if marginal production costs are heterogeneous and monotonously non-decreasing as a function of production volume, stable production is incentivized because production spikes become more costly.

To conclude, the introduction of multiple DPs increases alternative costs of future trades and thus the HOLD value, until physical constraints become binding or the trading prices of the delivery products or the marginal production costs diverge sufficiently. Thus, not only the allocation of resources between delivery products, but also the optimal trading strategy for each delivery product, is changed when multiple delivery products are taken into account. The number of DPs, initial price differences, volatility per price process and correlations between processes determine how much the HOLD value increases, and the ability to arbitrage between DPs. This has different implications for different types of traders:

- The introduction of multiple delivery products therefore changes the strategy of traders with non-zero but finite storage capacity, heterogeneous marginal production costs or intertemporals (including hydropower producers, intermittent producers with co-located storage and most thermal producers as well as responsive demand aggregators) in non-trivial ways; both towards DPs with higher prices, and towards other trading stages than before. Thus, these types of traders are likely better off with a model that optimizes for several delivery products than a model that focuses on one delivery product only.
- For traders with zero storage capacity, homogeneous marginal production costs and zero intertemporals (including flow-of-the-river hydropower, intermittent producers without co-located storage, purely financial traders, power retailers and other non-responsive demand aggregators) there is no reason to consider several delivery products at once (as long as the market is perfectly liquid), as the optimal strategy can be found by optimizing for each delivery product separately.
- Finally, a theoretical trader with infinite storage capacity, homogeneous production cost and no intertemporals need only consider the delivery product with the best price at any time, and can therefore behave as if there is only one delivery product with prices equal to that of the "Max price" column. How to set a proper Value of Storage for such a producer is a separate question however, as the relevant decision horizon would be very large for an unconstrained reservoir.

iii) Assumption 3: One- or two-sided trading?

Gatheral et al. (2012) find that *"optimal strategies always exist and are non-alternating between buy and sell trades when price impact decays as a convex function of time"* - so one may question why Garnier and Madlener (2015), Aïd et al. (2015), Tan and Tankov (2016) and Edoli et al. (2016) propose models that allow both selling and buying. Would it not be better to have a buy-only and a sell-only model, and then add their decisions together? As Gatheral et al. (2012) state, *"A general trading strategy in which buy and sell trades can alternate should thus be described by adding a nonincreasing strategy and a nondecreasing [one]"* (though both models are only needed if the initial open positions for different delivery products are in different directions, otherwise a sell-only *or* a buy-only model is sufficient for one-sided trading). After all, there is a risk that two-sided trading models discover arbitrage opportunities that arise from the modeling choices, and that don't exist in the real world. The trading behavior of such models will be dominated by the apparent arbitrage opportunities, and hardly provide useful information about optimal trading strategies in reality.

However, Gatheral et al. (2012) assumes that the target inventory is known, and that the trader places only market orders. The first assumption is not true for a flexible producer in the Intraday market - if prices fall, one may wish to reduce production, and if prices spike, one may wish to increase it, so if prices are stochastic, the target inventory is stochastic too (the observant reader will recall that unlike the hydropower producer in this thesis, Garnier and Madlener (2015) and Tan and Tankov (2016) model inflexible intermittent producers. However, the production capacity of the intermittent producer is stochastic, so the argument still applies). The option to adjust production - and by extension, the market commitment - in either direction, thus holds value even if the model finds no arbitrage opportunities. For this reason, the optimal strategy of a two-sided trading model will not necessarily equal the superposition of the optimal strategies of a buy-only and a sell-only model, even absent round-trips that are profitable-on-expectation ex-ante. This effect is illustrated with an example in this section.

For simplicity, only one delivery product is explored, though the arguments can be extended to several delivery products. Recall the `W-sup` case with the weak negative drift from section i). Despite the expected loss from the negative price drift, the high-cost producer chose the "HOLD"-strategy, because the option value was larger than the loss from the drift. For a two-sided trading model, the option value would be of no concern, since it would be able to exit the production commitment in the second stage if prices fall below marginal production costs - as long as the market is liquid, the set of feasible final market commitments cannot be restricted by any first-stage decision, as the exact reverse trade in the second stage will cancel any commitment from the first stage. The option value of buying and selling cancel each other out perfectly, and the two-sided algorithm will attempt to exploit the price drift and arbitrage the market.

If transaction costs are imposed, and they are large enough to make round-trips unprofitable (even with some price drift), the two-sided trading model will no longer arbitrage the market. Moreover, the transaction cost will create an aversion against early trading - similarly to how the option value delays trading for a one-sided trader - to avoid the risk of double transaction costs from potential round trips. In particular, a first stage sell order x will only be placed if:

$$2 \cdot C^c \cdot p^{RT}(x) \quad \leqslant \quad \mathrm{E}(\mathrm{Drift}) \tag{E.1}$$

, where C^C is the transaction cost and $p^{RT}(x)$ is the probability that the optimal production level in stage two is lower than the market commitment x from stage 1 (because stage 2 prices are below marginal production costs), so that a buy order is placed in the 2nd stage an a round trip has occurred. Thus, the left hand side of equation E.1 is the expected unit cost of a (possible) round trip, given a sell order volume x in the first stage. On the right hand side the expected price drift $E(\mathrm{Drift})$ is equal to the expected unit profit from the early trade relative to a later trade. The equation therefore states that during a round trip, you pay the transaction cost twice, but earn the drift once - and when there is no round trip, you only earn the drift by timing the market correctly. To prevent arbitrage, the inequality should never be true for $p^{RT}(x) = 1$, a certain round-trip.

Now, assume a modification of the `W-sup` case where all prices are 1 €/MWh higher in all stages and scenarios, but the trader had to pay a transaction cost of 1 €/MWh (which is far higher than the actual transaction cost, but it is set for illustrative purposes only). The first stage price is thus 51 €/MWh, and the 2nd stage prices are 60, 50 and 40 €/MWh. For the sell-only model, the payoffs are identical to before, so it still chooses a "HOLD" strategy in the first stage. Assume that the initial market commitment already is at the minimum production level, so no purchases are feasible. The buy-only model will thus propose to "HOLD", and the superposition of the sell-only and buy-only algorithm is to "HOLD".

For the two-sided trading model, the unit transaction costs of a round trip are 2 €/MWh and the price drift is 1 €/MWh. According to equation E.1, the model will sell in the first stage until the probability that the 2nd stage price is lower than the MPCs is 1/2. As MPCs are homogeneous until the production constraint and the 2nd stage price is lower than the MPCs in only 1/3 scenarios, the two-sided trading model will sell the entire production capacity in the first stage.

The second assumption in Gatheral et al. (2012), that only market orders are placed, don't have to be true either. If a model allows for limit orders, the trader may choose to provide liquidity to the market using strategies such as *"making the spread"* (Cont et al., 2010), which are profitable on expectation but involve risk.
The optimal strategy for the two-sided trading model is thus not in general equal to the superposition of a sell-only and a buy-only model, even absent arbitrage opportunities. One reason for the difference is that the two different factors that create wait-and-see incentives for the two- and one-sided trading models (transaction costs and option value, respectively) aren't necessarily equally strong. Because the final target inventory is unknown, it may be necessary to model the trading in the Intraday market with a model that is able to adjust the market commitment in either direction. Another reason for the difference is that two sided trading allows for new types of strategies such as "making the spread". The rationale for including both buy and sell orders in the same model is thus that it allows the model to better assess the true option value of delaying a trade, and that it expands the space of possible trading strategies.

iv) Assumption 4: Limited order depth

The `W-sup` price assumption from section i) is once again chosen because the traders have a preference for trading in one stage over the other, but this preference is weak enough to be counteracted by other effects. Recall that the low cost trader had a preference for a "SELL" strategy in the first case, due to the expected negative price drift. Assume that the initial open position was 30 MWh, and that the order depths available at the best quotes in each stage is limited to 20 MWh. More volumes are available, but the volume penalty is at 2 €/MWh. Thus, the first stage price is 50 €/MWh for the first 20 MWh, and then 48 €/MWh after that. The marginal cost in the first stage is equal to the expected price in the second stage, which is 49, so the optimal strategy is to sell 20 MWh in the first stage and 10 MWh in the 2nd stage, with a 10 € higher objective value.

In conclusion, the optimal order placement depends on both the expected drift, order depth, volume penalty and size of the open position. The rationale for modeling the instantaneous price impact is that the trader is better able to assess the true impact of an order clearing on the objective function. While there may exist heuristics that approximate the instantaneous price impact - for instance, one may artificially cap the placed order volume in a given stage - this is one of the less complicating factors among the proposed assumptions, so the gain in terms of model tractability from implementing such a heuristic would be small.

v) Assumption 5: Limit order premium

In this section, two different cases are presented to demonstrate two different aspects related to limit order premiums. At first, assume that the high-cost sell-only producer from section i) is in the final stage of the trading window, and that limit orders now can clear in the last stage. Recall that the marginal production costs are equal to 45 €/MWh, and assume that the best quoted price is 47, and that the volume of the best quoted order is larger than the open position. Also recall that with the assumptions in section i), the trader would have placed a market order to match the best quote.

However, there is now a non-zero probability that limit orders with prices above the best quote will clear, if they are matched during this stage. In particular, there is a 50% probability that a limit order with a low premium relative to the best quote of 2.5 €/MWh will clear, and a 20% probability that a limit order with a high premium of 6.0 €/MWh will clear. The limit orders either clear completely, or not at all.

The unit profit from the market order is $47 - 45 = 2$ €/MWh for the high-cost producer. The expected unit profit from the low limit order is $(2.5 + 2) \cdot 0.50 = 2.25$ €/MWh, and the expected unit profit from the high limit order is $(6 + 2) \cdot 0.20 = 1.6$ €/MWh. The high-cost producer will therefore opt for the low limit order in this case. Alternatively, if the price had been 40 €/MWh, it is trivial to observe that the high limit order is the only one with the possibility of contributing positively to the profits of the high-cost producer.

The low-cost producer with MPCs of 25 €/MWh would however opt for the market order in both of these cases, as the unit profits of the market order are 22 and 15 €/MWh when the price is 47 and 40, respectively. The potential for additional 2.5-6 €/MWh unit profits therefore can't compensate for the risk of no profit at all. Once again, traders with marginal costs closer to the market price are more risk seeking than traders with MPCs far away from the market price (see figure E.6). Correspondingly, for a given producer with homogenous MPCs, the incentive to place limit orders is higher when prices are lower.

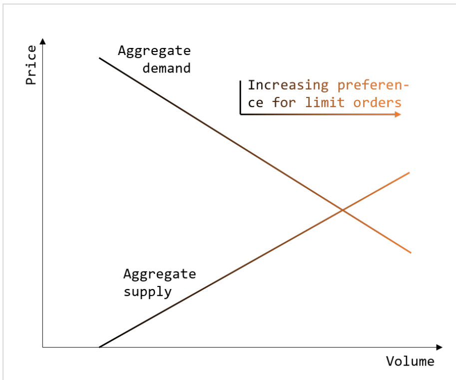

Figure E.6: Costly supply and cost-sensitive demand is the most prone to use limit orders.

So far, the focus has been on one- or two-stage trading windows. Does the trading behavior change notably during the trading window if it is extended to more stages? A three-stage trading window with a weak negative price drift is assumed (see figure E.7), in order to explore this question. All price scenarios are still equally likely. Order depths are still assumed to be larger than the open position, and the trader may place either a market order, low limit order or high limit order (with the same premiums and clearance probabilities as before). As the trader is focused on selling and there may be a spread between the best quoted ask and bid orders, the "price" should now be interpreted as the best quoted bid price rather than the mid-price.

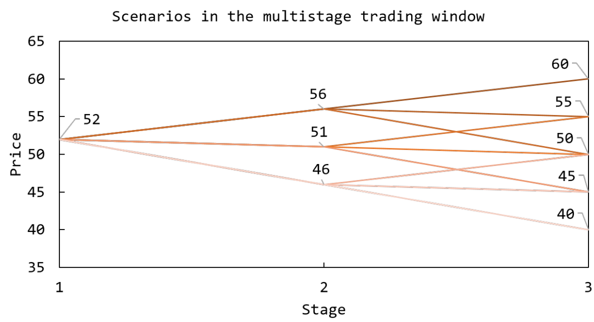

Figure E.7: Price scenarios in a three-stage trading window.

As the incentive to place limit orders is higher when prices are lower, one should expect that the preference for limit orders is stronger in the middle node in the third stage, than in the first stage. However, this turns out to not be the case, as shown in figure E.8.

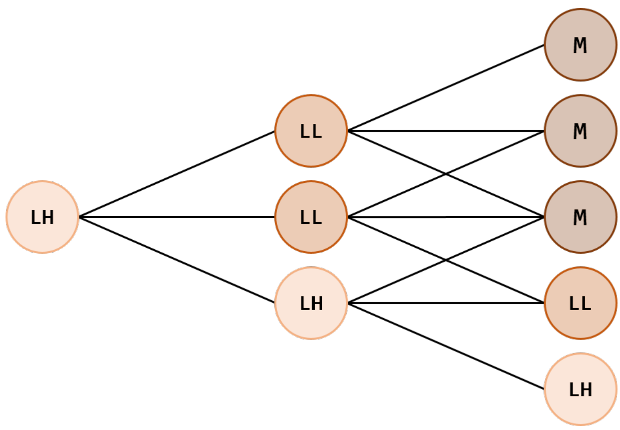

Figure E.8: Optimal order placement in a three-stage trading window.

In figure E.8, `M` symbolizes a market order, `LL` symbolizes a limit order with a low premium, and `LH` symbolizes a limit order with a high premium. From the figure, one can clearly see that within the same stage the incentive to demand a high premium relative to the best quoted price increases as the best quoted price decreases - particularly in stage 3, where there are a range of possible prices. It is also clear that the preference for high premiums is higher earlier in the trading window. The reason is that the alternative cost of future trades drives up the marginal costs. By computing the expected value of the best option in the last stage, one may find the marginal cost in each node by backwards induction (see the attached file LimitVSMarketOrders.xlsx for detailed calculations). The marginal costs can be seen on the left side of figure E.9, while the difference between prices and marginal costs can be seen on the right side of the figure.

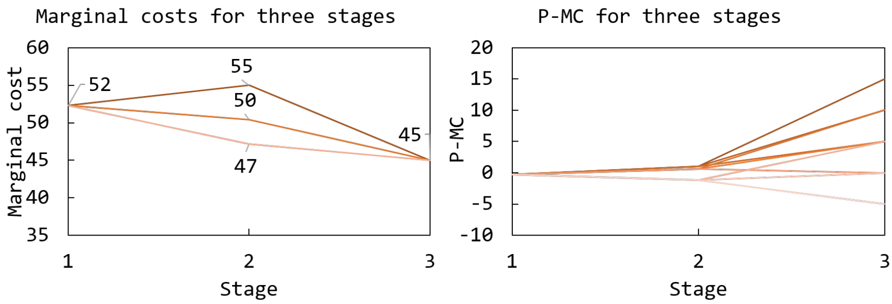

Figure E.9: Marginal costs, and the difference between prices and marginal costs, in a three-stage trading window.

As one can observe from figure E.9, the marginal costs and prices are very similar in all but the last stage. Unless one expects a steep negative price drift, prices can only be slightly above marginal costs that include the alternative cost of future trades. As limit orders are preferred to market orders when prices are close to marginal costs, the trader will prefer limit orders early in the trading window. This is in line with the findings of Guo et al. (2017) in conventional LOBs, who state that *"as time goes by, the optimal strategy shifts from the more aggressive types to the more conservative ones"*.

Note that the analysis above assumes that the open position is small relative to the liquidity in the market. If this is not the case, and the instantaneous price impact is significant, it may be necessary to close more of the open position early in the trading window, making the trader more conservative in the first stages too. For instance, Kumaresan and Krejić (2015) state that *"Non-trivial* *order sizes cannot be executed as a single market order"*. Also, the findings only consider a fixed clearance probability for two possible limit order premiums. In reality, the premium may take any value between the best quote and the maximum price in the market, and the clearance probability for a given premium may fluctuate throughout the trading window. Horst and Naujokat (2014) find that the trader should only place market orders when the spread is low enough (assuming that the spread is a good measure of the highest limit order premium with a high clearance probability).

The rationale for the assumption is that it expands the decision space to include orders with better unit profits when they clear, which in some situations have a higher expected value than the market orders. The optimal premium in a given stage depends on the spread, the marginal production costs of the individual producer, the size of the open position, the number of stages left, the assumptions made about the price dynamics for the rest of the trading window and the assumptions about the liquidity in the market, and it is therefore far from trivial to approximate the preferences for order premiums through the use of heuristics.