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Risk Management through Optimal Procurement Contracting in the Aluminium Remelting Industry

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Thesis Description

For manufacturing industries, activities such as procurement and replenishment are susceptible to uncertain purchase price volatility. Approaches such as long-term contracting and spot procurement can mitigate some risks due to price uncertainty. In this thesis, we develop an optimisation approach to manage risk in a contract procurement portfolio within the aluminium remelting industry. The presented model is a multistage stochastic program in which replenishment decisions are made at various stages along a time horizon. Replenishment quantities are jointly determined by the deterministic material demand and the stochastic price dynamics of the spot market. We minimise an objective function existing of two components: expected costs and a control measure to reflect the risk level, thus leading to multi-objective optimisation. The specific model presented considers both procurement planning and risk hedging. Numerical experiments are conducted to test the effectiveness of the proposed model.

Preface

This master thesis is the result of the final work for my M.Sc. degree in Industrial Economics and Technology Management at the Norwegian University of Science and Technology (NTNU), Department of Industrial Economics and Technology Management. With a specialisation in Managerial Economics and Operations Research, the thesis is a continuation of my Specialization Project prepared during the autumn of 2017.

I have received much appreciated help and guidance in completing this thesis, both from co-students and teachers at NTNU, as well as several industry contacts. I would like to thank my academic supervisor Ruud Egging for providing helpful guidance and valuable points of view. Your feedback has been invaluable during the work on this thesis. In addition, I would like to thank Hans Bjerkaas and Ken Henry Torvanger at Norsk Hydro for valuable discussion and data. Due to limited literature on the aluminium remelting business, their cooperation has been a key part of gaining an in-depth insight into the industry and obtaining realistic input data for the model. My work has greatly benefited from the help and feedback received from all mentioned above.

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Abstract

In today's increasingly turbulent and volatile commodity exchange markets, it is evident that controlling the risks in procurement strategies is an important issue. In this thesis, we develop a portfolio procurement optimisation model handling a Procurement Hedging Strategy Problem (PHSP). The portfolio optimisation problem has been one of the important research fields in modern risk management and there has been a growing interest in Conditional Value at Risk (CVaR) as a financial risk measurement tool in portfolio optimisation. This interest is based on many key advantages of CVaR compared with the more used measures of risk: standard deviation (SD) and Value-at-Risk (VaR).

Aluminium remelters participating in the European scrap market face significant uncertainty in future spot prices. While facing this risk, a remelter selects a procurement portfolio of scrap contracts with different maturities and prices to satisfy the expected material usage. The procurement strategy is also created in accordance with the decision maker's level of risk aversion. The methods described in this thesis will be useful as a decision support tool for determining risk management and hedging strategies in the aluminium remelting business.

Taking the view of a single risk-averse producer, namely Holmestrand Rolling Mill (i.e. Norsk Hydro ASA), we propose a multistage stochastic model for the coordination of inventory and procurement, while considering uncertainty in the spot prices of aluminium. The presented model focuses on minimising a combination of expected cost and CVaR. Although the basis of the model is a standard CVaR approach, the model is further developed in a nested fashion, using recursive constraints, to handle time-inconsistency issues. Numerical results are presented for a six-stage, 256 scenario data instance with a one year horizon, and are based on data from Norsk Hydro and listed indexes on London Metal Exchange.

The model is tested to demonstrate its application as a decision-making tool in practice. An assessment of the applicability and validity of the model is made based on several test instances. Results show that instances of considerable size are challenging to solve due to the model's complexity. Nevertheless, optimal solutions can be found within a reasonable time frame for Holmestrand Rolling Mill. Moreover, findings prove the model's ability to suggest hedging strategies according to the decision maker's level of risk aversion. That is, the results show that hedging with the use of forward contracts reduces the risk in terms of CVaR. The percentage procurement through forward contracts vary between 46.21 % to 60.53 % depending on risk preferences and test cases, demonstrating diversification and hedging. The degree of risk-aversion also influences the maturity of the chosen long-term contracts: the higher the degree of risk aversion, the longer the maturity of the forward contracts. The model also demonstrates the ability to adjust the procurement strategy for different price data by shifting towards long-term contracts for increased price variation and scheduling more material purchase when the average prices are lower.

Sammendrag

I dagens stadig turbulente råvaremarkeder er det åpenbart at risikostyring er et viktig tema i forbindelse med innkjøp og kontraktinngåelse. I denne masteroppgaven utvikler vi en optimeringsmodell som håndterer et Procurement Hedging Strategy Problem (PHSP). Porteføljeoptimering har vært et av de viktige forskningsområdene innen moderne risikostyring, og det har vært en økende interesse for Conditional Value at Risk (CVaR) som et finansielt risikomål. Denne interessen er basert på flere fordeler ved CVaR sammenlignet med de mer brukte risikomålene: standardavvik og Value-at-Risk (VaR).

Aluminiumsmelteverk som deltar i det europeiske skrapmarkedet står overfor betydelig usikkerhet med fremtidige spotpriser. I møte med denne usikkerheten, velger en aluminium omsmelter en innkjøpsportefølje av skrapkontrakter med ulike løpetider og priser for å tilfredsstille forventet materialbruk. Anskaffelsesstrategien er også opprettet i samsvar med beslutningstakerens risikoaversjonsnivå. Metodene og modellen beskrevet i denne oppgaven vil være nyttige som et beslutningsstøtteverktøy for risikostyring og bestemmelse av hedging-strategier (i.e. sikringsstrategier) for bruk i aluminiumsindustrien.

I denne oppgaven observerer vi en enkelt produsent, nemlig Holmestrand Rolling Mill (i.e. Norsk Hydro ASA), og foreslår en fler-stegs stokastisk optimeringsmodell for koordinering av inventar og innkjøp, mens vi vurderer usikkerhet i aluminium spotpriser. Den presenterte modellen fokuserer på å minimere en kombinasjon av forventet kostnad og CVaR. Selv om modellens grunnlag er en standard CVaR-tilnærming, er modellen videreutviklet rekursivt for å håndtere problemer med tidsinkonsekvente løsninger. Numeriske resultater presenteres for et 6-stegs, 256 scenario case med en tidshorisont på ett år, og benytter data fra Norsk Hydro og noterte indekser på London Metal Exchange.

Det utviklede beslutningsverktøyet er testet for å demonstrere dets anvendelse i praksis. En vurdering av funksjonaliteten til det stokastiske programmet er gjort basert på flere testinstanser. Resultatene viser at testinstanser av betydelig størrelse er utfordrende å løse på grunn av modellens kompleksitet. Likevel finner vi optimale løsninger innen rimelig tid for Holmestrand Rolling Mill. Resultatene viser også modellens evne til å foreslå hedging strategier (i.e. sikringsstrategier) i henhold til beslutningstakernes grad av risikoaversjon. Det vil si at resultatene viser at hedging ved bruk av langsiktige terminkontrakter reduserer risikoen for CVaR. Resultatene bekrefter at økt grad av risikoaversjon fører til mindre anskaffelse gjennom spot innkjøp og mer gjennom langsiktige kontrakter, samt tilsvarende lavere CVaR-verdier. Andelen anskaffelse av langsiktige kontrakter med fast pris varierer mellom 46,21% til 60,53%, avhengig av risikograd og tetsinstanser, og viser modellens evne til å sikre og diversifisere. Graden av risikoaversjon påvirker også løpetidene til de valgte langsiktige kontraktene: jo høyere grad av risikoaversjon, desto lengre løpetid får kontraktene. Modellen demonstrerer også variasjon i anskaffelsesstrategi for ulik prisdata ved å skifte til langsiktige kontrakter for økt prisvariasjon og planlegger større materielle innkjøp når gjennomsnittsprisene er lavere.

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Abbreviations

CVaR	Conditional Value-at-Risk
ES	Expected Shortfall
HARP	Hydro Aluminium Rolled Products
HRM	Holmestrand Rolling Mill
LME	London Metal Exchange
MIP	Mixed Integer Program
PHSP	Procurement Hedging Strategy Problem
SI	Sheet Ingots
VaR	Value at Risk

Definitions

Alloy	-	A metal made by combining two or more metallic elements, especially to give greater strength or resistance to corrosion.
Alloying Elements	-	Chemical elements such as aluminium, copper, magnesium, silicon, copper and zinc added in specified or standard amounts to a base-metal to make an alloy.
Casthouse	-	The space in a foundry where a mixture of raw materials is melted down to make an alloy and transformed into ingots
Primary Aluminium	-	Unalloyed aluminium produced from alumina usually by electrolysis, typically with an aluminium content of 99.7% (OEA, 2006).
Recycled aluminium	-	Aluminium obtained through recycling of scrap. Recycled aluminium is also referred to secondary aluminium (OEA, 2006).
Remelter	-	Producer of aluminium alloys, from mainly clean and sorted scrap (OEA, 2006).
Rolled Products	-	The products of rolling, a metal forming process in which metal stock is passed through one or more pairs of rolls to reduce the thickness.

1 Introduction

Sustainable and efficient handling of aluminium has become essential due to the exponential growth in global demand. In the last decades, application areas for aluminium have steadily increased. Aluminium is a flexible material that is commonly used in automotive, packaging and construction industries with substantial recent growth (Norsk Hydro ASA, 2012). This increasing demand for aluminium has outpaced the growth of primary aluminium production and the use of secondary, recycled aluminium materials has therefore become necessary to meet this shortfall.

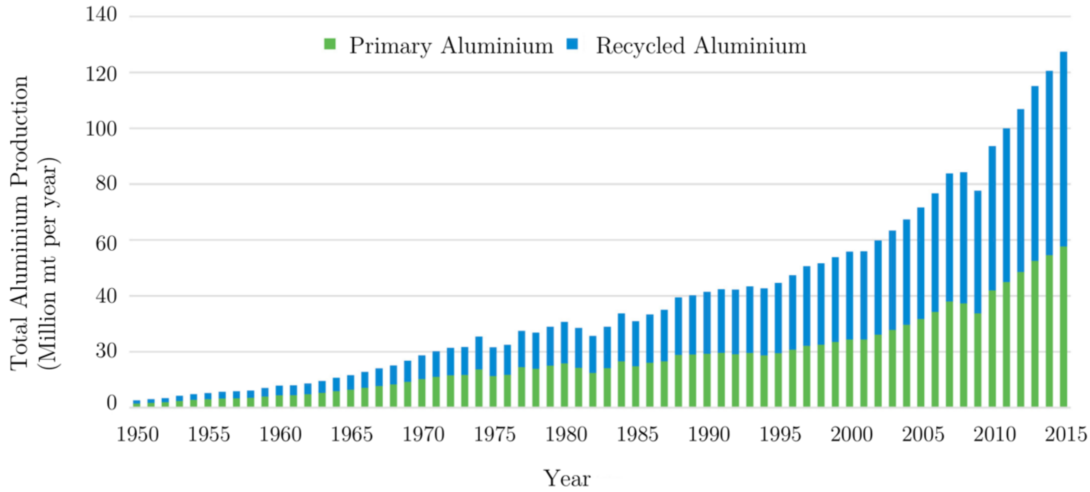

Figure 1.1 Amount of primary and recycled aluminium used globally (IAI, 2017)

The amount of aluminium used globally has been increasing since 1950, illustrated in Figure 1.1, and this trend is projected to continue (Cullen & Allwood, 2013). Aluminium has a number of good qualities that have led to it now being the metal with the fastest growing demand in the world. Perhaps the most important characteristic of the metal is that it can be recycled repeatedly without quality loss using only five percent of the energy required to initially produce it. Aluminium has historically been recycled at a higher rate than most other raw materials. Other advantages include high strength-to-weight ratio, good formability and high corrosion resistance (Soo, et al., 2018).

Figure 1.1 also illustrates the gradual growing share of recycled compared to primary aluminium. Recycling is critical for sustainable development and will provide both environmental and economic benefits. In addition to the energy savings, greenhouse gas and other harmful emissions can be reduced in the future by limiting primary aluminium production while increasing aluminium recycling. In 2012, recycling of post-consumer aluminium saved over 90 million metric tonnes (mt) of emissions and over 100,000 GWh of electrical energy globally compared to primary production (Norsk Hydro ASA, 2012). This amount is equivalent to the annual power consumption of the Netherlands.

The context of this thesis is the Procurement Hedging Strategy Problem (PHSP) faced by Holmestrand Rolling Mill (HRM), owned by Norsk Hydro ASA. Based on their procurement of aluminium scrap, they produce aluminium products in a remelting process. Their procurement activities depend strongly on uncertain and highly fluctuating factors, and decision support is essential in maintaining profitable production. Today, HRM recycles approximately 20,000 metric tonnes (mt) of used scrap aluminium every year, purchased in the European market for scrap, and has a yearly production capacity of 86,000 mt. With a listed aluminium price (LME) of $1770 per metric tonne, the annual revenue value generated by HRM is 152 million USD. These figures illustrate how even small efforts towards the application of Operations Research can lead to significant improvement in the profits realised by HRM.

The foremost target of Hydro in Holmestrand today is to secure HRMs position as the leading recycling rolling mill in Europe. Since the early 1990s, recycled aluminium has been central for Hydro's rolling mill production in Holmestrand. With considerable experience in remelting technology, HRM is positioned as a front-runner for aluminium processing in Norway. However, considerable cost is associated with aluminium scrap purchasing, and the core of HRM's procurement problem is to determine how costs can be reduced through more efficient contracting of recycled aluminium. As aluminium is a commodity traded on the London Metal Exchange (LME), the value over time is easily measured. Pricing of scrap in contracting is often directly dependent on LME listings of aluminium. As a consequence, the price volatility of listed LME Aluminium prices is a source of risk for an aluminium remelter contracting scrap. As scrap materials must be contracted in advance to cover future production needs, the need for decision support rises for the task of creating a scrap procurement portfolio.

Against a backdrop of uncertainty, the scope of this thesis is to present a decision support tool for aluminium scrap contracting while incorporating stochastic aspects. The purpose of considering elements of randomness is to enable viable decisions when information is limited, in order to reduce the risk of generating unfavourable strategies. A multi-stage stochastic model is proposed in order to capture uncertainty in the spot price of aluminium. Minimising the overall cost while managing risk constitutes the objective of the model. Owing to the generality of this model, it is readily applicable to a wider range of procurement contracting problems. However, in order to conduct a concrete computational study, this thesis will primarily deal with aluminium scrap contracting.

To handle risk, we present the Conditional Value-at-Risk concept and combine an analysis that covers its application as a risk measure with a portfolio optimisation problem. More precisely, CVaR optimisation is analysed in the context of hedging a portfolio consisting of scrap procurement contracts. The risk of incurring extreme costs due to price uncertainty is limited through the use of CVaR in a multi-stage stochastic optimisation programme. A nested CVaR implementation is used to handle the problems of time-inconsistency in the model.

The main focus of the thesis is how the producer's risk preferences influence the decision on whether to purchase the scrap using forward contracts in order to secure the price or to purchase the scrap based on variable price contracts depending on the spot price. A multistage stochastic programming problem is formulated to model the producer's contracting decisions with time consistent risk constraints (e.g. Shapiro, 2009). The objective is to minimise the weighted sum of expected cost and the conditional value-at-risk (CVaR) of the future cost. The weights associated with expected cost and the CVaR are used as a measure of the producer's risk preference and by varying the weight one can examine how the producer's decisions change as the degree of risk aversion increases. Degree of risk aversion is further adjusted through the specified confidence level of CVaR.

In brief, the research objective is to gain insight and develop an optimisation model and solution method for the purpose of reducing the cost, as well as managing risk, of scrap procurement contracting at HRM. The type of analysis will provide valuable decision support to aluminium remelters regarding the choice of an optimal hedging strategy according to the company's risk preferences. To the knowledge of the author, a multi-stage stochastic optimisation problem for procurement contracting, utilising CVaR as the risk management tool, has not yet been researched. Furthermore, CVaR has never been analysed in the context of scrap procurement for aluminium remelters. Though this thesis is written for HRM, the application area for the implemented model is broad and the model can easily be modified for other procurement players contracting in the commodity marketplace.

The thesis is organised as follows. Chapter 2 gives a short introduction of Norsk Hydro and HRM in context of the aluminium recycling and remelting industry. This entails an introduction of the price volatility of aluminium listings and the unique features of scrap contracting. Also, the uncertainty factors involved in the contracting process and the initial assessments necessary to map out the scope of the problem are described. Chapter 3 states the portfolio problem at hand in terms of relevant attributes. Aspects of uncertainty, the importance of restricting problem features and the objective of the problem are discussed in greater detail. In Chapter 4, a review of literature relevant to the portfolio problem is given in order to illustrate the differentiating value of the model proposed. This also entails motivation for utilising multi-stage stochastic programming and the implications of its use. Chapter 5 presents the nested stochastic optimisation model. Underlying assumptions of the model are stated along with the complete mathematical formulation and a discussion of essential features. Chapter 6 covers a computational study, including an evaluation and validation of the model in terms of practical value and computational efficiency. Implementation of the mathematical model in commercial software and a presentation of a numerical example is also presented in order to illustrate output, characteristics and application of the model. Finally, the conclusions of the findings followed by directions for future work are presented in Chapter 7.

2 Background

The prominence of international aluminium scrap sourcing is exposing procurement activities to risk, which directly impacts the financial results of a remelter. This chapter provides a description of the aluminium remelting industry in general and the field of risk management and portfolio hedging in relation to HRM. A review of the main phases and associated activities invoked in scrap procurement and aluminium remelting will also be given to clarify central terminology.

2.1 The Aluminium Remelting Industry

Aluminium has a broad spectrum of applications and numerous benefits. Many of these benefits are related to recyclability and remelting. Primary aluminium production is an energy-intensive and time-consuming process. In fact, energy costs constitute a substantial part (i.e. 20-40 percent) of primary production (Norsk Hydro ASA, 2016). Nevertheless, once the aluminium is produced, it can be recycled repeatedly without losing its advantageous properties (Norsk Hydro ASA, 2012). The remelting process requires up to 95 percent less energy than primary production, offering considerable energy savings. In a resource constrained reality, recycling is also essential to sustainable development. It allows resources to be spared and existing scrap to be reduced (Li & Kirchain, 2005).

The term “aluminium scrap” can be defined as recyclable aluminium materials left over from product manufacturing and consumption, such as parts of vehicles and building. Unlike waste, i.e., unusable material, scrap has monetary value and is therefore recovered for recycling. The quantity of recycled aluminium has increased steadily in recent years, and remelters have implemented technologies to reduce harmful emissions from the remelting process (Cullen & Allwood, 2013). Presently, the process of recycling and remelting is increasing faster than primary metal production, as more scrap is available on the market. Currently, around 75 percent of all aluminium ever produced is still in use and creates a resource reservoir for future use (Norsk Hydro ASA, 2012). Also, the infrastructure required for the collection of scrap metals is already well-established and is likely to continue to improve on its own economic merit to provide an increasingly efficient recycling system. Given that Europe depends more on imported primary aluminium than any other continent, the increasing levels of scrap recycling will also reduce Europe's dependency on imports (Djukanovic, 2016). Today, the use of recycled metal is also a strong marketing advantage due to its environmental benefits. In other words, recycling and scrap procurement will continue to be highly relevant in the future.

Even though the quantity of recycled aluminium has steadily increased, the access to aluminium scrap is still restricted. Most of the aluminium produced in recent years has ended up in products with long lifetimes such as vehicles and constructions (Norsk Hydro ASA, 2016), see Figure 2.1. Consequently, it will take a long time for a large amount of the aluminium to be available for recycling. In fact, recycled aluminium scrap can only supply 20-25 percent of the currently increasing global demand for aluminium. The rest must be produced from primary production. In short, aluminium recycling is a significant source of economic, energy and aluminium resource savings, as well as in environmental protection. Consequently, the recycling industry plays a valuable part in the aluminium life cycle and will continue to do so.

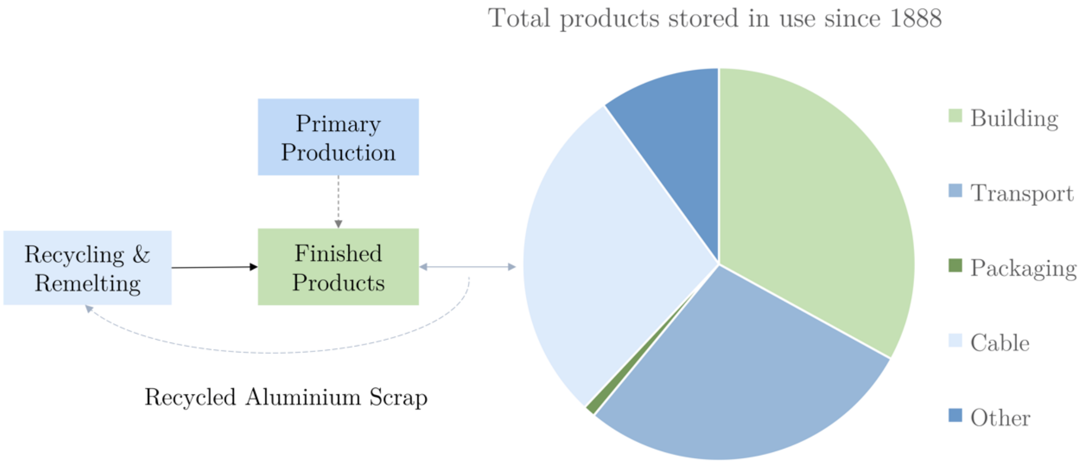

Figure 2.1 The material flow balance of aluminium (Norsk Hydro ASA, 2016)

2.2 Norsk Hydro and HRM

Norsk Hydro ASA, henceforth referred to as Hydro, is a major global supplier of high-quality aluminium for applications in various market segments, such as building, transport, packaging, renewable energy and engineering. Based in Norway, Hydro has a strong presence in Europe and worldwide operations covering every step of the production chain, from the extraction of raw materials to semi-finished products, as well as recycling. Hydro's ambition is to grow faster than the market in recycling and to take a strong position in this part of the value chain. By 2020, Hydro aims to recover 1 million metric tons (mt) of scrap annually. Hydro also has set fixed strategic targets to increase its production of recycled metal, such as reorganising to recycling plants, optimising procurement and processing of scrap, increasing sales of recyclable friendly alloys, and developing closed circuits in cooperation with customers (Norsk Hydro ASA, 2003). Regarding procurement (i.e the process of contracting and sourcing), Hydro has a high potential for cost reduction.

Hydro Aluminium Rolled Products (HARP) is Norsk Hydro's business unit for rolled aluminium products. Rolled products are the results of a metal forming process in which metal stock is passed through one or more pairs of rolls to reduce the thickness and to make the thickness uniform. HARP's products are used in a number of applications in industries such as construction, automotive, printing and packaging. The plant located in Holmestrand, Holmestrand Rolling Mill (HRM), manufactures rolled products almost exclusively based on the remelting of recycled and primary aluminium at its own facilities (i.e. foundry and casthouse). More than 95 percent of their production is exported, primarily to customers in Europe and the factory employs around 400 people with round-the-clock operations (Norsk Hydro ASA, 2017).

Though aluminium has a high scrap value, the benefits of recycling are influenced by the purity level of the scrap (Soo, et al., 2018). The casthouse at HRM has a remelting capacity of 120 million metric tonnes per year and utilises modern remelting furnaces to handle scrap with higher levels of impurities. One of the main difficulties of casting aluminium based on recycled scrap is achieving the correct chemical specifications of the melt with minimal additions of primary aluminium and alloying elements. Primary aluminium is produced from alumina usually by electrolysis, typically with an aluminium content of 99.7%. Alloying elements, on the other hand, are chemical elements such as copper, magnesium, carbon etc., added in specified or standard amounts to a base-metal to make an alloy (OEA, 2006). The addition of primary aluminium is necessary to dilute impurities to an acceptable level, while alloying elements are added for correction if necessary. Since primary aluminium and alloying elements are generally more expensive than scrap materials, the producer will incur additional cost if utilising more primary material and alloying elements than necessary. Consequently, effective management of material inventory and scrap procurement is essential in keeping costs down. Consequently, the application of mathematical modelling and optimisation for better resources utilisation is of great potential. For more detailed description of HRM's production process, the reader can refer to Hovland (2017).

2.3 The Aluminium Scrap Market

This section gives an overview of the aluminium scrap market, where HRM conducts procurement activities, and a discussion on concepts such as scrap pricing, aluminium spot listings and transportation costs.

There are many aspects influencing the value of metals. Though purity and quality are main factors, demand and exchange rates can also have a strong influence on prices (Dabbas, 2007). In general, demand for aluminium is evolving in line with consumption patterns and industrial development (Norsk Gjenvinning, 2016). Scrap is normally priced at a discount to the primary aluminium price to reflect impurities. Along with this discount, the reduced energy costs of remelting and the added values of expensive alloying elements included in the scrap, there is a significant value increase in the remelting process. However, some additional costs are also related to scrap collection and processing.

As aluminium is a commodity traded on the London Metal Exchange (LME), value development can be followed. The aluminium prices for scrap in the European market are driven by the major players, and remain in alignment with the LME indexes. Concerning the aluminium market, aluminium has been traded on the London Metal Exchange (LME) since 1978 (Figuerola-Ferretti & Gilbert, 2010). Today, there exist two LME listings of aluminium; Aluminium High Grade and Secondary Aluminium. Aluminium High Grade is primary aluminium with strict impurity requirements, while Secondary Aluminium include aluminium alloys with a higher impurities acceptance level. Aluminium High Grade, also called LME Aluminium, is unalloyed primary ingots with minimum 99.7% purity and Secondary Aluminium, also called LME Aluminium Alloy, contains valuable alloying elements primary aluminium does not comprise (The London Metal Exchange, 2017). Price volatility of the two listings for the last 2 years can be viewed in Figure 2.2.

Figure 2.2 Volatility of LME listings: High Grade (green) and Secondary (blue)

Aluminium scrap is traded in the international market-place. Price, availability and shipping costs are usually the determining factors in choosing whether to sell scrap in the domestic market or the international market. HRM contracts scrap from suppliers such as Stena Metal, Metallco, Norsk Gjenvinning and S. Norton which are considered some of the major players in the market. The scrap material is transported to HRM mainly from various locations in Norway and England, but also from Sweden, Scotland, Germany and the Netherlands.

A downside of international scrap sourcing is the large costs of transporting scrap from the market in Europe. Table 2.1 displays a selection of shipping costs for the scrap material referred to as OLD from available suppliers around Europe. The different shipping costs data is given by HRM and gives information about weight per load and shipping location. The data can be used to calculate the shipping cost of a contract. All costs are based on a Free Carrier Agreement (FCA) where the seller is responsible for the delivery of goods to a specific destination. In all cases, the buyer assumes all risks and costs after the goods have been delivered at the determined delivery location.

Table 2.1 Exemplification of transportation costs (FCA) to HRM for the material OLD

Location	Price per load [NOK]	Weight per load [mt]	Price [NOK/mt]
Barnsley	7,000	14	500
Brighton	18,000	13	1385
Darlington	9,500	10	950
Eberswalde	19,500	23.5	830
Leeds	10,200	14	729
London	12,700	14	907
Malmø	10,600	28	379
Manchester	9,900	14	707
Moss	3,300	14	236
Oldbury	12,300	12	1025
Portsmouth	14,100	14	1007
Poulton	10,100	14	721
Rotherham	11,400	14	814
Skien	4,200	28	150
Trier	25,000	23.5	1064

2.4 Procurement Contracting and Risk Management

This section gives an overview of HRM's scrap procurement activities by discussing concepts such as price volatility, material usage and procurement hedging. Also included is a detailed description of contracting and pricing activities, as well as an analysis of the main sources of risk related to these topics. Furthermore, elements of uncertainty relevant to the procurement portfolio problem will be discussed. Essentially, we present the LME spot price risk, which is the most important risk that an aluminium remelter face. Other sources of risk relevant to the problem are discussed along with a reasoning of why they fall outside the scope of this thesis. These include uncertain material demand for production and uncertainty in material composition.

Scrap procurement and portfolio hedging

For an aluminium remelter, price risk stems from variations in the value of LME spot positions. The aluminium spot prices have high volatility and will therefore have significant impact on the cost of the production. The core of portfolio hedging is to protect against adverse price movements. Hedging in procurement contracting contributes locking in an agreed profit margin (by fixing procurement costs) and protects inventory value. As scrap must be contracted today to cover later production needs and multiple pricing alternatives exist, strategic planning is necessary in procurement contracting.

Today, long-term contracts constitute about 20 percent of the procured scrap at HRM, while the rest is procured through spot purchases. Historically, HRM only entered spot contracts, with a few exceptions. A contract entered through spot purchasing includes one delivery of scrap at an agreed upon time and is priced based on the spot price (i.e. LME Aluminium Alloy) when entering the contract. Delivery for spot priced contracts are usually in the current month with a maximum delivery time of 2 weeks. However, HRM has recently adopted a new strategy and now wishes to include a bigger proportion of long-term contracts in their scrap procurement portfolio. The motivation for entering more long-term contracts is to be less exposed to changes in the scrap market. By basing the portfolio on spot-contracts only, HRM's stock of scrap has previously been emptied when prices in the scrap market become too high. In such situations production has been solely based on primary material and alloying elements. The long-term contracts aim to prevent this to some extent, as they secure deliveries in periods when the scrap market collapses.

Long-term contracts are often entered as a fixed-price contract where the price is determined at the time of contract signing. This price can then be calculated based on LME Aluminium or LME Aluminium Alloy. Long-term contracts can also adopt a so-called "formula" contract. While the price of spot contracts and fixed-price long-term contracts are fixed once for the entire delivery, formula contracts can be fixed several times. A typical formula contract in 2018 could be 1200 mt divided as 100 mt/month, priced at 75 % of LME Aluminium or priced based on LME Aluminium Alloy. Price fixing times are specified in the contract. An annual contract, as the one exemplified above, will typically be fixed 6-15 times. For practical purposes, HRM does not fix less than 50-100 mt at a time. When referring to deliveries in this thesis, we refer to deliveries in terms of price fixing. The number of physical deliveries in a contract is simply the size of the contract divided by the average weight of a truckload (15-25 mt), so this varies greatly. Furthermore, it should be noted that all scrap materials purchased by HRM are linked to the value of Secondary LME, though the price formula can depend upon either Secondary LME or LME High Grade, depending on what is settled through negotiation.

Given the spot price uncertainties, the fundamental challenge for HRM in terms of scrap procurement is to determine when to enter contracts, as well as contract specifications such as volume, pricing and delivery, given storage limitations and quantity requirement for production. Decisions regarding which long-term contracts to enter are performed on a monthly basis with a planning horizon of 6-12 months with a monthly resolution. Long-term contract decisions are therefore referred to as tactical decisions (Schmidt & Wilhelm, 2000). However, spot contracts are entered on a weekly basis to meet production needs closer to actual production. Note that a forecast of LME listings can be generated based on publically available LME listings on LME's official website (HKEX Company, 2018).

Uncertainty in material composition and usage

Furthermore, unpredictable material usage patterns increase the complexity of the portfolio problem. Material usage can occasionally fluctuate unexpectedly due to a number of reasons. These include, the available levels of internal scrap from own production and varying scrap usage depending on the shift leader in charge of production. Furthermore, risk stems from the fact that sales orders for final products can vary slightly from year to year. A fairly reliable forecast of the scrap requirement for the planning period can however be generated based on material usage statistics from HRM.

HRM has a business plan for the coming year forecasting the monthly material requirement for production. Significant changes in production plans due to variation in sales contracts are handled through spot purchases. The casthouse at HRM needs information about material requirements at least 2 weeks before actual production in order to obtain the relevant scrap material. If this information is not available on time, the casthouse often becomes short of the required scrap. In the latter situation, expensive primary material and alloying elements will replace the deficient scrap, incurring additional costs. However, due to good communication, the casthouse seldom experiences such shortage. Material usage has therefore been neglected as a source of uncertainty to the problem.

Furthermore, for a remelter, uncertainty regarding material composition and impurities of purchased scrap is often a contributing factor to additional costs. However, HRM is advantageously positioned regarding detailed data on material composition due to good routines and usage of recent technology. Material composition is therefore not a significant factor of uncertainty for HRM. Hence, this aspect goes beyond the scope of this thesis and will not be treated further. For a more thorough explanation of why these risks are not significant for HRM, see (Hovland, 2017). In conclusion, to the extent that this study concerns uncertainty, the focus lays on the aspects of price uncertainty only and the consequences it entails. For an overview of the three discussed risk areas, see Table 2.2.

Table 2.2 Relevant risks for this thesis

Type of Risk	Part of the thesis scope
Price Risk	Yes
Material Usage	No
Material Composition	No

2.5 Myopic Optimisation

It should be noted that the basis for this thesis is the local optimisation of material procurement at the HRM casthouse (see Figure 2.3). The initial manner of producing a fixed production plan based on optimal sales orders is not ideal. More optimally, the sales department at HRM should coordinate with the production planning unit and the procurement material unit at the casthouse. What sales contracts to enter should therefore depend on overall profitability for HRM. However, based on HRM present day routines and wishes, this thesis handles the myopic optimisation of procurement optimisation at the casthouse given a fixed production plan from the sales department.

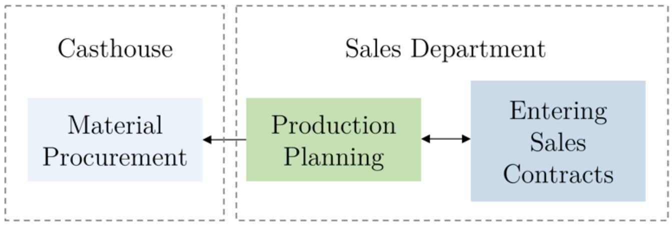

Figure 2.3 Optimisation of material procurement vs. sales contracts

3 Problem Description

This chapter defines the scope of the portfolio optimisation problem as described in the preceding sections and presents a general problem description. The problem will be stated in a general manner, and the timing and extent of decisions relevant to the decision maker will be described. Aspects concerning available information, assumptions and relevant elements of uncertainty will also be accounted for.

This thesis considers the risk management and portfolio planning problem (i.e. the PHSP problem) for an aluminium remelter. More specifically, we take the perspective of the casthouse at HRM, operating in the scrap purchasing market. Overall, it is assumed that HRM's objective is to minimise the expected cost of the contract portfolio while managing risk efficiently throughout the planning horizon. The main task is to establish what contracts to enter during the planning horizon depending on material type, quantity, maturity and pricing, to satisfy the expected material usage. As discussed in Section 2.4, the industry operates with several types of contracts, differing in how price and volume are established, the frequency of the deliveries and the time horizon of the agreement (i.e. maturity of the contract). The contracting decisions are made while seeking adherence with storage capabilities. The problem is further complicated by limited and uncertain information, resulting in planning activities of high complexity. More explicitly, we aim to determine an optimal mix of scrap contracts for HRM, given uncertainty in the future aluminium LME price listings. Figure 3.1 illustrates the portfolio decision problem as presented in this thesis, both in terms of the decision making and the information received throughout the planning horizon. Particularly, the timeline shows the here-and-now decisions, revealed information and recourse decisions. This structure repeats itself a number of times dependent on the number of stages considered.

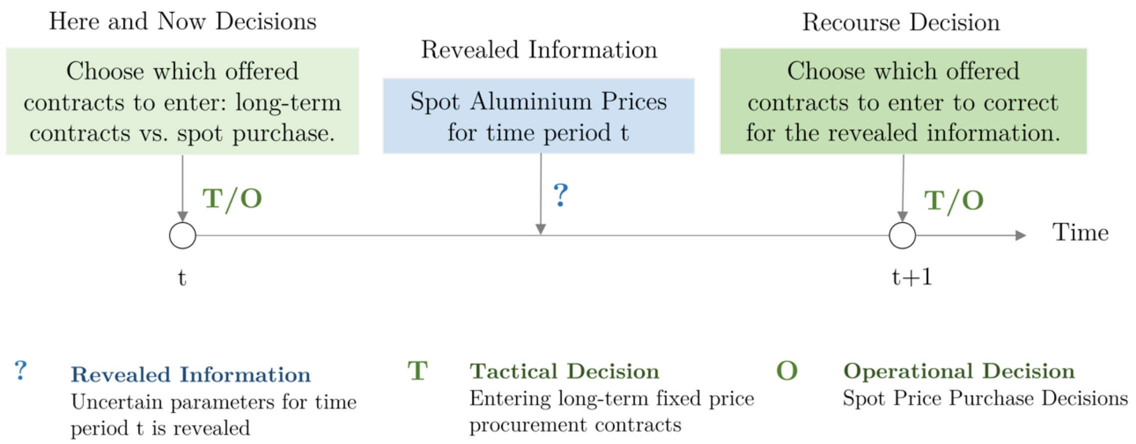

Figure 3.1 Information structure of the portfolio decision problem

Relevant costs to minimise include the purchasing cost of contracts, transportation cost, inventory holding cost as well as the cost of being short of scrap material. In the latter situation, expensive primary material and alloying elements will replace the deficient scrap, incurring additional costs. The value of the scrap inventory at the end of the planning horizon is also considered. In regard to boundary conditions, the problem is restricted by inventory management and a budget on the procurement expenditures. Since HRM has a limited storage capacity, the scrap inventory is restricted and must be regulated. The inventory balance incorporates the purchased scrap in addition to the present inventory, as well as deduction of material for production. The total inventory level must be equal or lower than the storage capacity at all times. In order to compare the amount of storage space required for different scrap types, we consider density specifications. Moreover, the problem is bounded by a procurement budget, where the sum of contract expenditures over the planning horizon must be equal to or lower than the given scrap procurement budget for the considered period. It should also be noted that with a planning horizon of 6-12 months, the time value of money is not negligible. All expenditures during the time horizon must be discounted. Consequently, the problem is sensitive to when costs are incurred. In short, minimising the net present value will favour costs at the end of the horizon over costs today.

All contracting and production decisions are subject to uncertainty about the future and this makes risk important to evaluate and respond to. Uncertain information and subsequent risk are therefore central parts of this problem. In order to incorporate risk management, a basic feature of the problem becomes risk aversion. HRM perceive themselves to be moderately risk-averse and consider price risk to be the most important source of risk. The price risk can be mitigated by purchasing scrap through fixed price contracts, guaranteeing a deterministic expenditure at the cost of foregoing the chance of purchasing the scrap later at a lower price or incurring a lower cost through a variable price contract. By entering a combination of fixed long-term contracts and spot purchases, HRM control their desired level of risk.

Depending on HRM's level of risk preference, we seek to optimise a combination of two objectives; expected cost and price risk. As an optimal procurement portfolio is dependent on the preferences of the decision maker, risk aversion is incorporated in a way that allows the producers to specify desired level of risk aversion. For a risk neutral decision maker, the objective will simply be to minimise the expected future cost of the procurement portfolio. Different types of risk metrics will be discussed in Chapter 4.

Uncertainty may have great impact on the expenditures related to procurement contracting. Therefore, when deciding on what contracts to enter, it is important to incorporate the critical uncertain aspects of the problem. An appropriate representation of the uncertain elements is also of great importance. This thesis incorporates the uncertainty explicitly by representing future uncertain parameters through scenarios. With a random parameter fixed to one of its possible outcomes, a scenario is created representing one possible realisation of the future. For a more detailed explanation of how uncertain information is represented through scenarios, see Section 4.3. The main uncertain elements that are incorporated in this thesis are the spot prices of aluminium listed at LME. See Section 2.4 for a detailed analysis of other risks that are relevant to the problem.

In short, the thesis is intended to provide valuable decision support to aluminium remelters regarding cost reduction and the choice of an optimal hedging strategy according to the company's risk preferences. That is, how much to hedge and when, while reducing overall costs of procurement. Depending on hedging strategy, HRM will opt for a procurement portfolio combining long-term contracts and spot purchases. It is the explicit incorporation of uncertainty that facilitates the advancement of flexible and robust solutions and consequently enables risk management. When new information is revealed concerning uncertain information, the decision maker will adjust his decisions to adapt to the shifting circumstances.

The following chapter introduces theory and literature concerning different aspects of the problem. It covers different risk metrics, aspects related to stochastic programming and other existing literature relevant to this thesis. This is meant to give insights into the theoretic aspects of the problem and highlight the contributions of this thesis.

4 Literature Review

This chapter introduces theory and literature concerning different aspects of the problem presented in Chapter 3. Furthermore, it highlights how this thesis utilises and contributes to existing literature. Section 4.1 presents operations research frameworks developed for the aluminium remelting industry, followed by an introduction to literature on hedging and portfolio management in Section 4.2. Section 4.3 and 4.4 contains a brief overview of relevant theory on the subject of stochastic programming. The sections serve as a framework for later discussion and introduce important terms and concepts used in the model developed in this thesis.

Different financial assessment procedures of risk and examples of financial assessments of portfolios similar to the one we are assessing, are presented in section 4.5. This assessment is set into a context of stochastic programming. Section 4.6 further provides an insight into the subject of multi-objective optimisation. To round up this chapter, Section 4.7 places our work into the context of the existing literature presented, and highlights our contribution to the field of study. It should be noted that some sections are based on the work conducted in Hovland (2017).

4.1 Operations Research in the Aluminium Industry

Though the demand for aluminium is increasing, the competition from alternative materials to substitute aluminium in consumer goods and industrial components has also increased. This has forced the aluminium industry to intensify its efforts to reduce the cost of production either through alternative technologies or through improvements in existing processes by implementing operation research models. This thesis is motivated by the need for innovation in the emerging modern aluminium industry. The implementation of operations research in the aluminium industry can reduce production costs, enhance manufacturing functions and provide higher quality products, which can make a significant economic and sustainable impact (Dutta, Apujani, & Gupta, 2016).

The number of operations research studies conducted in the area of aluminium remelting has increased steadily in the last decades (Dutta, Apujani, & Gupta, 2016). However, application areas for operations research are primarily restricted to optimisation of production planning and scheduling, material blending and some research on supply chain optimisation. As far as the author knows, the use of operations research in procurement contracting is non-existing. For a short literature summary of operation research literature in the remelting industry, see Hovland (2017) and Dutta, Apujani, & Gupta (2016).

4.2 Portfolio Management & Contracting

The topic of this thesis is centralised around contracting and portfolio management, a thoroughly covered research area. As the main focus areas for this thesis is related to price risk, this section is limited to present published research focusing on the same aspect. Though the problem faced by HRM is a procurement problem, it has numerous similarities with sales contracting problems. By consequence, such problems will also be evaluated and considered in this section.

Price risk can be mitigated to a certain extent through forward and futures contracts, as well as other instruments such as swaps and options. Forward contracts have a long history of use by producers of a commodity (e.g. aluminium) as a means to insure, or hedge, against unfavourable variations in the prices of that commodity (for an overview, see Bernstein (1996). The benefit of forward buying and selling of a commodity producing company is to secure a predetermined price and avoid the risk of making a loss should the commodity price change unfavourably. With this approach, the company avoids the expected costs linked with financial distress (Tufano, 1996); (Stulz, 1984). The downside of securing against risk is the lost opportunity of selling or purchasing at potentially more favourable prices in the future. The trade-off between reward and risk is a central element in all decisions under uncertainty.

To exemplify, Schütz & Westgaard (2018) study the optimal hedging decisions for a risk-averse producer within the seafood industry. Using a multistage stochastic programming model, the methods described in this paper are useful in determining hedging strategies in terms of selling salmon in the spot market or through forward contracts. Numerous other applications of contracts with the purpose of hedging are documented for a wide range of commodity producing companies, such as agricultural companies (Tomek and Peterson, 2001), seafood companies (Martínez-Garmendia & Anderson, 1999), oil and energy producers (Wallace and Fleten, 2003), in particular hydro power producers (Fleten et al., 2002; Fleten et al., 2011; Kettunen et al., 2010; Dupuis et al., 2016).

4.3 Representing Uncertainty

Uncertainty is a central part of this thesis and is taken into account through direct use in the problem formulation. This approach is referred to as stochastic programming, which in some cases can be superior in dealing with uncertainty compared to deterministic methods (Wallace S. W., 2003). The aim of stochastic programming is to find a policy or strategy that is feasible for all or almost all of the potential realisations of uncertain data, while minimising or maximising the expectation of a function of random variables. Stochastic programming can also be considered as a tool for discovering all the options in a decision problem. A stochastic recourse model is therefore incorporating flexibility at a price. This important characteristic of flexibility is not present in a deterministic model. It should also be noted that the probability distribution of the random variables, representing the potential outcomes and their corresponding probabilities, is either known or can be estimated in order to model a problem stochastically.

Conversely, a deterministic approach does not incorporate the uncertainty directly in the model, but instead relies on either thorough selection of the input parameters or detailed analysis of the solution using methods such as what-if analysis, sensitivity analysis or scenario analysis (Wallace S. W., 2017). Commonly, the deterministic approach is applied due to its simplicity, as stochastic modelling problems often struggle with an acceptable solution time and tractability when including all relevant aspects of the problem. However, when uncertainty is an important part of a problem, deterministic models can fail to represent reality adequately, and the introduction of stochasticity may be necessary.

Other prominent methods to model uncertainty are chance constrained optimisation, robust optimisation, simulation and real option theory. A short overview of these methods is presented in this section. The chance constrained method, first introduced by Charnes et al. (1958) and Charnes & Cooper (1959), is one of the foremost approaches to solving optimisation problems under uncertainty. The optimisation problem is formulated in such a manner ensuring that the probability of satisfying a particular constraint is above a certain level. However, the chance-constrained method is often difficult to solve. While the chance-constrained method ensures a high confidence level of the solution, robust optimisation pursues a solution that will have an acceptable performance under most realisations of the uncertain parameters. It is a conservative, worst-case oriented methodology. Commonly, no distribution is used for uncertain parameters. Main contributors to the field of robust optimisation include Soyster (1973) and Bertsimas & Sim (2004).

Simulation is often used in combination with optimisation to handle uncertainty. In short, simulation imitates a real-world process or system over time (Banks et al., 2001). The key aim is to evaluate how different input variables affect the system. Main issues in simulation include the attainment of the relevant selection of key characteristics and behaviours of the problem, and the use of simplifying approximations and assumptions within the simulation. Simulation is frequently used when the complexity and tractability of a stochastic model becomes too high.

Moreover, the field of real options theory is also often encountered when modelling uncertainty. In short, an option is an opportunity to make a decision after having observed the outcome of random variables (Wallace S. W., 2010). An option is typically offered at a cost, i.e. the option cost, but will in return provide valuable flexibility. An option should only be purchased if the cost is less than the expected benefit. One important advantage of real options is its usage in very complex options which are hard or impossible to handle in stochastic programming. However, it should be noted that real option theory can only value options (i.e. the option of doing it differently), not find them. Contrary, a major strength of stochastic programming is the ability to create operational flexibility. This is one of the main reasons stochastic programming has been chosen as the best framework for our problem.

4.4 Multistage Stochastic Programming

This section is devoted to the field of multistage stochastic programming which is the foundation for the model developed in this thesis. Multistage stochastic programming is a natural framework for financial long-term planning problems, procurement management and contracting in particular. In short, the major reason for developing multistage stochastic models is to incorporate the flexibility of dynamic decisions to improve the objective. The flexibility is based on the option to change a strategy after the realisation of random variables (Rudloff, Street, & Valladão, 2014).

Multistage recourse problems represent a planning situation where new information is revealed at specific time points during the planning horizon and decisions have to be made repeatedly based on the available information (Higle, 2005). When modelling a dynamic setting, a timeline perspective of decisions is incorporated and a decision at a given stage is taken facing an unknown future. In stochastic programming the term stage is defined as a point in time where new and useful information is revealed (Kall & Wallace, 1994). After decisions are implemented, the following period information is revealed and the procedure is repeated for the following stage (Valladão, Veiga, & Veiga, 2014). The possibility to adapt a solution to updated information when it becomes available is referred to as recourse (Higle, 2005). It should be noted that the recourse in a problem can be classified as either simple, fixed, general or complete, depending on problem characteristics (Birge & Louveaux, 2011). We will only handle what is referred to as general recourse in this thesis. Further information on the other classifications is covered by Birge and Louveaux (1997).

There are numerous options when formulating a multistage stochastic linear problem. It should be noted that the structure of a recourse problem will have implications for potential solution methods, computational demand and feasibility. The information structure can be implicitly represented, as modelled by the scenario tree, see Figure 4.1. Alternatively, but equally valid, the problem can be formulated explicitly for each possible scenario. Constraints are then added to ensure that the information structure associated with the decision process is honoured (Higle, 2005). These constraints are referred to as non-anticipativity constraints, which forces decision variables at a given stage to be equal if their scenarios share the same history (Valladão, Veiga, & Veiga, 2014) see Figure 4.2. This thesis uses a combination of the two approaches. A special focus of this chapter is therefore put on explaining the difference between these two formulations.

A scenario tree is an organised distributional representation of the stochastic variables and the way in which they may progress over the time periods included in the problem (Higle, 2005). A path in the tree is referred to as a scenario and a policy or strategy is defined as the set of decisions for all scenarios and stages. In other words, the tree is structured based on predicted sequences of events, that is, the way in which a state may develop depending on outcomes for uncertain parameters. It is the enumeration of all possible combinations of outcomes that allows us to represent the scenarios in a tree.

Figure 4.1 displays a scenario tree with 4 scenarios, 3 stages and 6 time periods. The nodes indicate the state of the process at decision points and arcs indicate the realisations of uncertainty before the subsequent stage, see Figure 4.1. A path from the initial root node to any leaf node (i.e nodes in the last time period) represents a scenario, a complete course of events that is considered during the planning. A model formulation based on the exemplified scenario tree in Figure 4.1 is commonly referred to as an implicit formulation of the stochastic problem and is recognised as a node formulation or compact form representation.

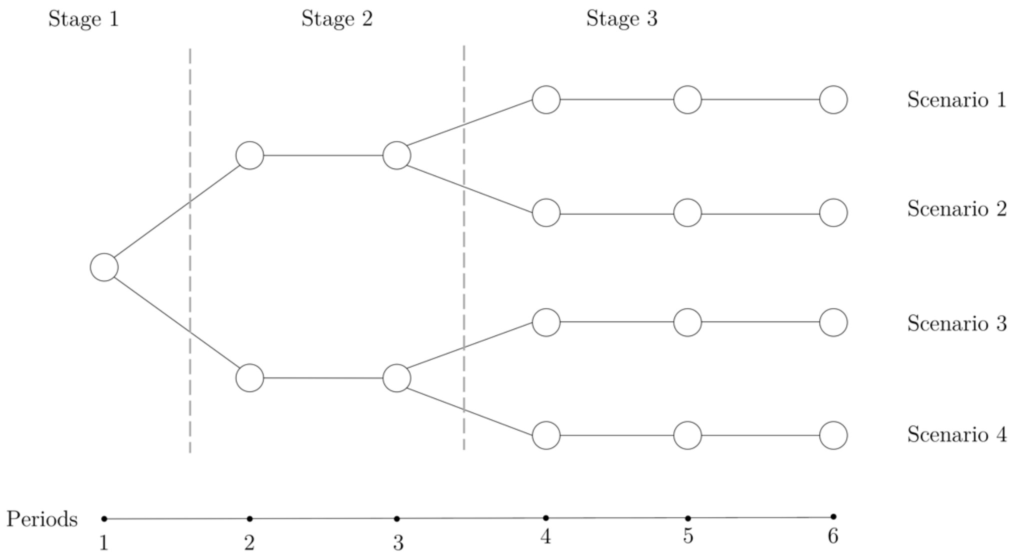

Figure 4.1 Implicit scenario tree exemplification with three stages

As the paths divide, unique realisations develop and consequently the state in each node is directly reliant on all previous and potential following nodes. Furthermore, each possible outcome is associated with an occurrence probability. All possible realisations of the succeeding stage and their corresponding probabilities are known, though, it is not known what realisation will occur. Finding a present decision that will ensure the best expected results given all forecasted outlooks and adapting to each of the potential scenarios is an essential part of solving recourse problems.

As represented in Figure 4.1, multiple scenarios go through each node in all but the nodes in the last stage, meaning that they share common stochastic parameters and have equal choices made for all decisions in these stages. This is known as the non-anticipativity requirement. An alternative explicit formulation, also called an extensive formulation or scenario formulation (Higle, 2005) is based on Figure 4.2. The explicit formulation of a multistage model is generalised as follows.

$$\min \ z = \sum_{\omega \in \Omega} p_\omega \sum_{t \in T} c x_\omega^t \tag{1}$$

$$\sum_{j=1}^{t} A_\omega^{tj} x_\omega^j \leq b_\omega^t, \qquad t \in T, \omega \in \Omega \tag{2}$$

$$x_\omega^t - \ x_n^t = 0, \qquad t \in T(n), \omega \in \Omega(n), n \in \mathcal{N} \tag{3}$$

$$x_\omega^t \geq 0, \qquad t \in T, \omega \in \Omega \tag{4}$$

In this formulation, x_ω^t represent the decisions made in time period t and scenario ω. All potential scenarios ω are specified by the set Ω, containing every scenario in the tree. Each scenario is weighted with their respective probability of occurrence p_ω in the objective. Further, by introducing a set of envelopments, $\mathcal{N}$, indexed by n, the constraints in Equation (3) ensure that each decision x_ω^t in time period $t \in T(n)$ is equal in all scenarios given by $\omega \in \Omega(n)$. Consequently, the non-anticipativity constraints enforce the right relationship between decisions and information structure. This is illustrated in Figure 4.2, where the non-anticipativity constraints are represented by the same coloured envelopment and time period. For more information on stochastic models and modelling of uncertainty see Hovland (2017), Higle (2005), Birge & Louveaux (2011) and Kall & Wallace (1994).

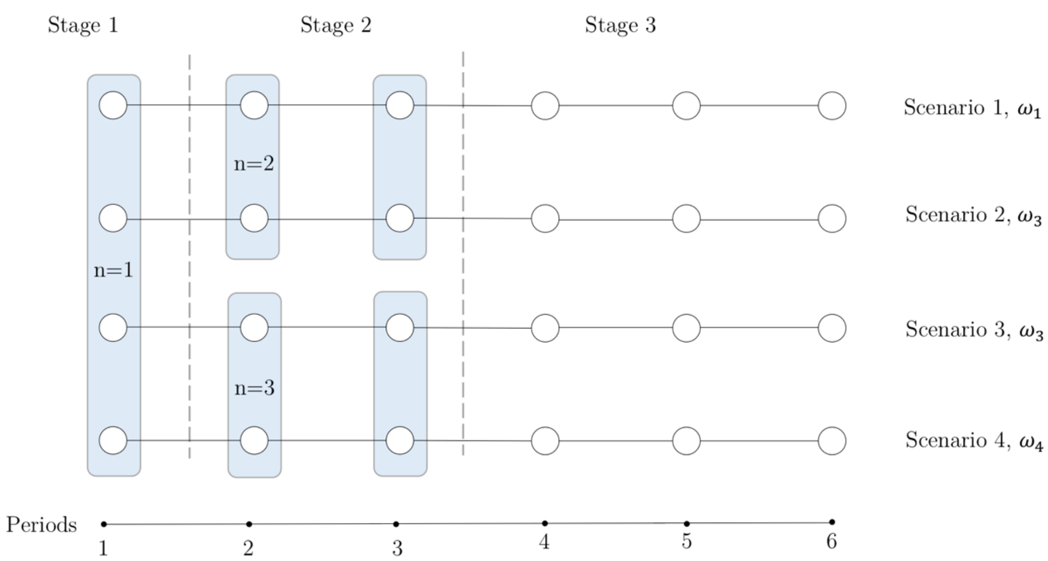

Figure 4.2 Explicit scenario representation with non-anticipativity constraints

4.5 Risk Management and Stochastic Modelling

The concept of risk aversion is often encountered in problems modelled through stochastic programming. This section will give a brief introduction to relevant research related to risk management and stochastic optimisation.

Risk management

In this section, we discuss different considerations related to risk management from the point of view of an aluminium remelter. Firstly, we define risk management as the limitation and control of the risks confronted by an organisation due to market volatility exposure (Krapels, 2000). In order to employ risk management correctly, an organisation must identify its risk factors and further evaluate the exposure to these risk factors. Subsequently, there is a need to prioritise and to resolve how the risks should be controlled. Depending on organisational goals and attitude towards risk, some risks should be reduced, some should be eliminated and some should be disregarded. It is important to note that the return will be limited if the goal is to eliminate all risk exposure (Fleten, Bråthen, & Nissen-Meyer, 2011). However, the fundamental aspect of proper risk management is to be conscious of all the risks that the organisation faces and to constantly measure and control them in a way that is coherent with the goals and risk attitude of the organisation. For a risk-averse producer seeking predictability of future costs, a risk management program with considerable hedging is suitable. For a risk-neutral producer that can handle a greater standard deviation in costs, a risk management program with less hedging is desired.

The two main goals of a hedging strategy are to reduce the standard deviation of future cost for better decision and budgeting support, and to insure against major shortfalls. A natural hedging strategy can be seen as the maximum degree of risk that the producer is able to undertake, under the assumption that it is not speculating (Fleten, Bråthen, & Nissen-Meyer, 2011). A natural hedge gives a strategy with highest uncertainty in future cost and the highest possible target shortfalls, but also the highest upside potential. Consequently, a natural hedge is the same as not hedging at all. A natural hedge strategy will therefore be appropriate for producers with a low degree of risk aversion. Alternatively, a static strategy regulates a standard deviation reduction and the protection against target shortfalls. This is determined based on the amount of shorting on forward contracts and by the time horizon (i.e. contract maturity) of these contracts. One main issue when planning a static hedging strategy is therefore to determine the proportions and time horizon in order to meet the producer's risk preferences.

Risk aversion and expected utility theory

In economics and finance, risk aversion is a description of an investor's attitude when exposed to uncertainty. When faced with two investments with a similar expected return but with different risk levels, a risk averse investor will always prefer the low-risk investment. It is generally believed that people are risk-averse, and that they need a premium to take part in a risky decision (Kall & Wallace, Stochastic Programming, 1994).

Expected utility is the standard framework for modelling investor choices with risk aversion and was first considered by Von Neumann and Morgenstern in 1944. Von Neumann and Morgenstern (1944) derived conditions on an individual's preferences that are consistent with an expected utility function. Using expected-utility theory, risk aversion is modelled based on a concave utility function over wealth, depicted in Figure 4.3. The diminishing-marginal-utility-of-wealth theory of risk aversion is intuitive, and aids in explaining risk aversion to large-scale risk and how people are approximately risk neutral when stakes are small. However, accordion to Rabin (2000), the utility-of-wealth function is an unlikely explanation for risk aversion, except when the stakes are very large. In short, expected-utility theory is a useful and adequate model of risk aversion for many purposes, though it is not applicable to risk attitudes over modest stakes (Rabin, 2000).

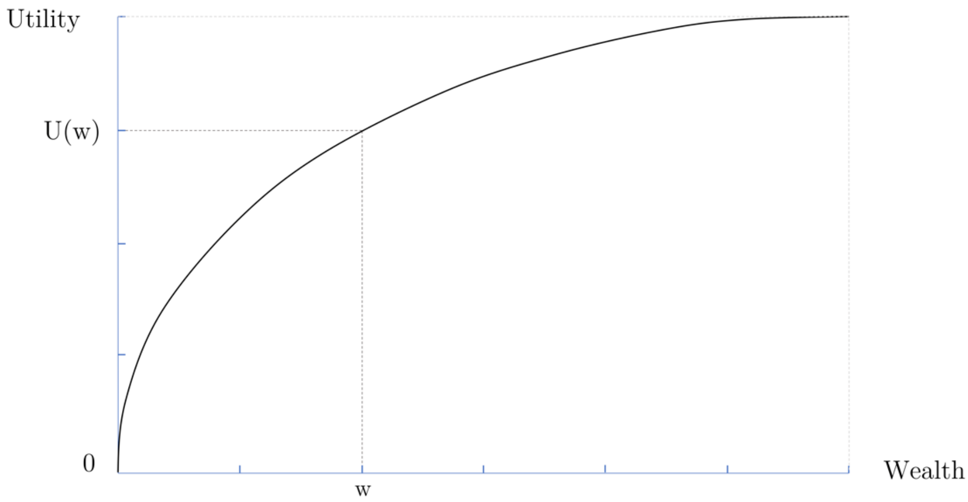

Figure 4.3 Example of a typical concave utility function representing risk aversion

Risk measurement and modelling

Minimisation of expected losses or maximisation of expected gains leads to decisions that are optimal on average while possible risks are neglected. In some circumstances, this is not an acceptable goal. The recent tendency is to explicitly incorporate risk monitoring and control. There are various types of risk and the choice of a suitable risk definition depends on the context, on the decision maker's preferences and the company goals (Kozmík, 2014). To reflect risks in a stochastic model formulation, it is necessary to quantify them through risk measures by assigning real values to the random outcomes. Furthermore, as for the risk-neutral expected value criterion, risk measures should not depend on individual realisations of stochastic data, but on decisions and the probability distribution. Popular examples of risk measures include Standard Deviation, Value at Risk and the Conditional Value at Risk. Hedging with use of forward contracts will reduce the risk in terms of such risk measures (Fleten, Bråthen, & Nissen-Meyer, 2011).

Risk aversion: Expected Shortfall

Modelling of risk depends on the problem and available data. An elementary approach to incorporate risk, resulting in a piecewise linear model, is to minimise expected shortfall (Kusy & Ziemba, 1986). As the decision maker perceives risk as the potential for downside losses, shortfall is defined as profit underperformance relative to pre-set financial performance target at various periods. A way of incorporating this in a model is to progressively penalise shortfall in the objective. This way of modelling operational risk has been very successful in asset and liability models (Ziemba & Mulvey, 1998).

To exemplify this approach, Fleten et. al (2002) adoption of shortfall is presented. They discuss a risk management model for a hydropower producer operating in a competitive electricity market. Taking the view of a single risk-averse producer, Fleten et. al (2002) propose a stochastic programming model for the coordination of production with hedging through the forward and option market. They define shortfall as profit underperformance relative to pre-set profit targets at various periods. Risk is then progressively penalised in the objective function through shortfall costs in the form of a piecewise linear cost function as shown in Figure 4.4. The objective function is thus understood as a utility function that reflects the level of risk aversion. The approach of Fleten et. al (2002), yields a piecewise linear concave objective function in profit, interpreted as a utility function that reflects risk aversion (see Figure 4.4).

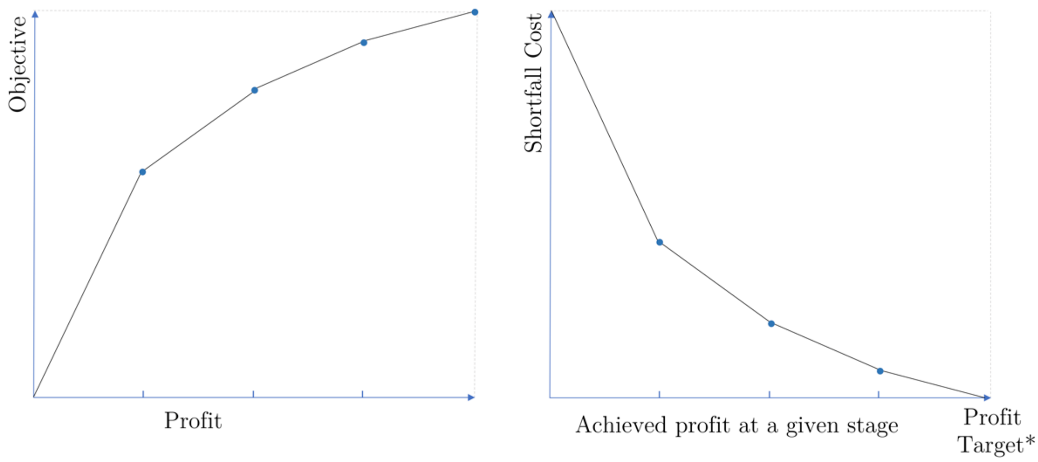

Figure 4.4 Piecewise linear concave utility function and shortfall cost function

Similarly, Valladão, Veiga, & Veiga (2014) guides optimal policies by including in the objective a penalty for highly leveraged debt positions in multistage linear stochastic programming model for optimal corporate debt management. As Fleten et. al. (2002), they propose a piecewise linear function that increasingly penalises the excess leverage based on a sequence of threshold targets. These target values correspond to critical leverage levels established by debt managers. In the objective function, they impose a cumulative penalty for violating each one of the leverage targets in each scenario, at each time period. However, it should be noted that a model formulation using convex piecewise linear penalty in the objective requires particular problem insight for the specified cost targets for each stage. Also, linear cost functions must be specified, which requires knowledge about the relative cost of shortage.

Value at Risk and chance constraints

Value at Risk (VaR) is widely used in financial mathematics and financial risk management and measures the risk of loss. It estimates how much a set of investments could drop in value with a given probability, assumed normal market conditions, within a fixed time frame. In a financial context, VaR for a given portfolio and time period can be defined as the loss that will not be exceeded with a given probability α. In other words, it is a threshold loss value. Typical values for α are 0.9, 0.95 and 0.99. However, it should be noted that the measure does not contain information about the expected loss. The curve in Figure 4.5 represents a hypothetical profit and loss probability density function with a mean of zero. The figure illustrates an asset with a one-month VaR of t %, representing a 5% chance of the asset declining in value by t % during the one-month time frame.

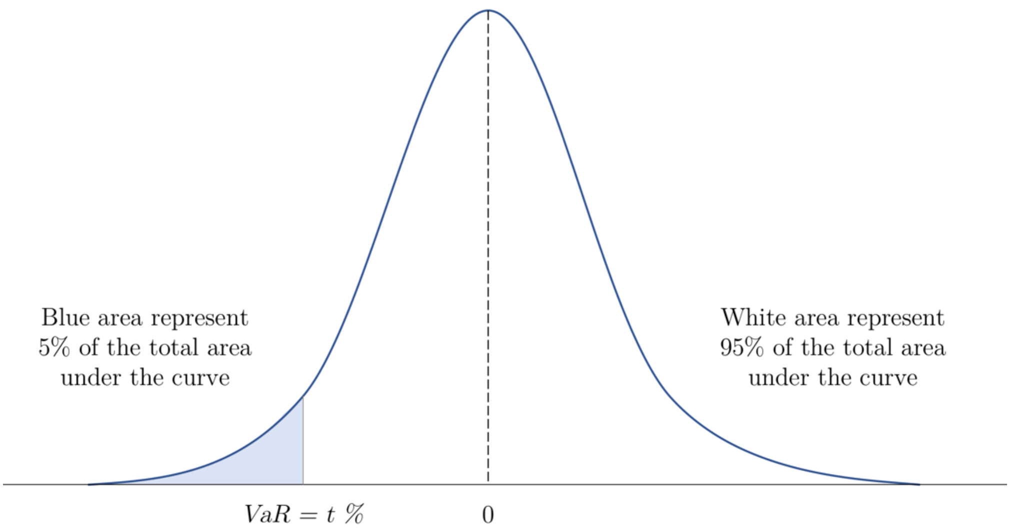

Figure 4.5 Profit-and-loss probability density function

VaR can be formulated as a chance constraint, also referred to as a probabilistic constraint. The chance constrained method is one of the major approaches to solving optimisation problems under various uncertainties. It is a formulation of an optimisation problem that ensures that the probability of meeting a certain constraint is above a certain level. In other words, it restricts the feasible region so that the confidence level of the solution is high. As described in Section 4.3, the chance-constrained method is, however, often difficult to solve. The mathematical formulation of VaR as a chance constraint is given in Equation (5) and illustrated in Figure 4.6, where $F_\xi(t) = Pr\{\xi \leq t\}$ is the cumulative distribution function. This is the probability that the random variable ξ will take a value less than or equal to t.

$$VaR_{1-\alpha}(\xi) = inf\{t\colon\ F_\xi(t) \geq\ 1-\alpha\} \tag{5}$$

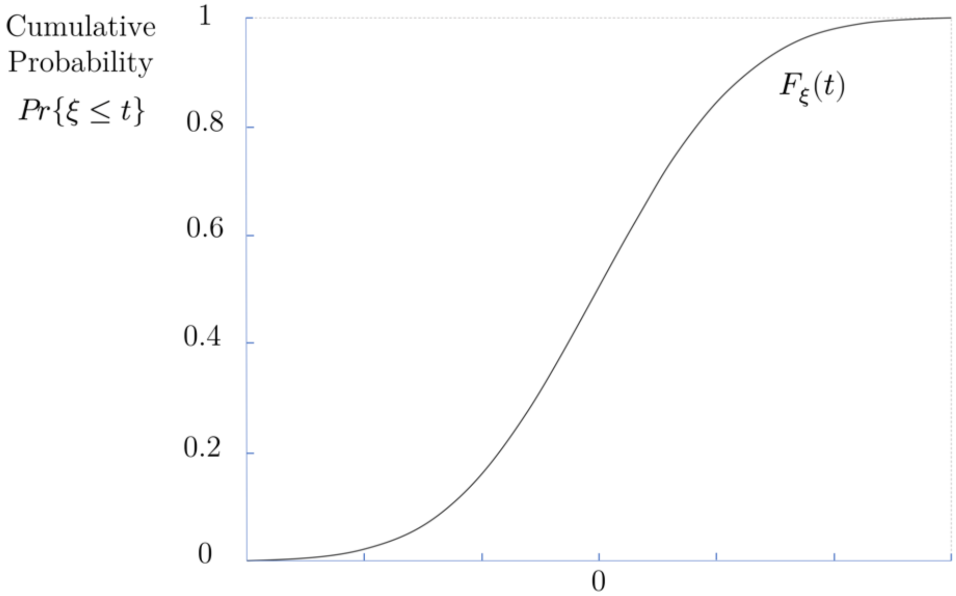

Figure 4.6 Cumulative probability distribution

Conditional Value at Risk in Stochastic Programming

The concept of conditional value at risk (CVaR) serves as an extension of VaR and calculates the average of the losses that occurs beyond the VaR cut-off point in the distribution (see Figure 4.7). CVaR was first introduced by Rockafellar and Uryasev (2000) and is a risk assessment method often used to reduce the probability that a portfolio will incur sizeable losses. This is performed by assessing the probability that a specific loss will exceed the value at risk. For a given portfolio, time period and probability α, CVaR can be defined as the expectation of the losses under the condition that they will exceed VaR (see e. g. Zenios, 2008, for more detailed information). Mathematically, CVaR is derived by calculating a weighted average between the value at risk and losses exceeding the value at risk. More specifically, the CVaR risk measure computes the expected shortfall below the specified quantile level. CVaR is also known as mean shortfall, expected shortfall, mean excess loss or average value at risk. When measuring loss, the smaller the value of the CVaR, the better.

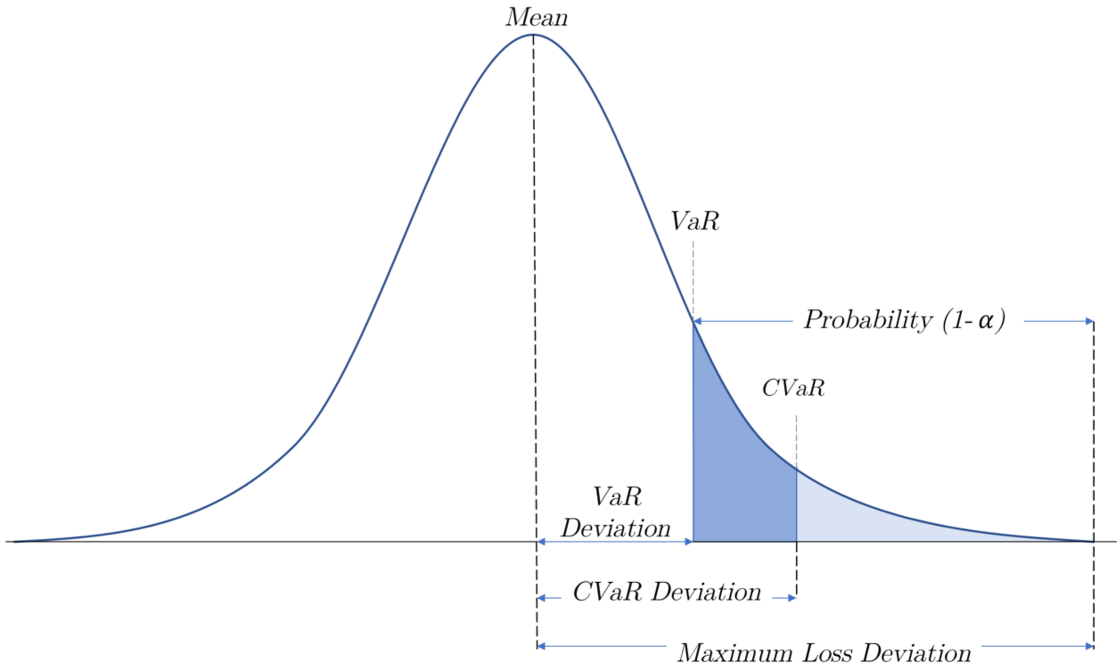

Figure 4.7 VaR and CVaR representation for a frequency-of-loss distribution

CVaR is a popular risk measure in stochastic programming and has a linear implementation. As CVaR provides information about the expected loss of a portfolio in the worst α % of the cases, it is a value that can be utilised in optimisation. Pflug (2000) defines CVaR via an optimisation problem, based on Rockafellar and Uryasev (2000) in Optimization of Conditional Value-at-Risk. Several case studies showe that risk optimisation with the CVaR performance function and constraints can be done for large portfolios and a large number of scenarios with relatively small computational resources (Uryasev, 2000). The mathematical formulation can be viewed in Equation (6), where $[f(x,\xi) - a]^{+} = max(f(x,\xi) - a,\ 0)$ and a is $VaR_{\alpha}(x)$.

$$CVaR_{\alpha}(x) = \ \min_{a} \left\{ a + \frac{1}{1-\alpha}\ E[f(x,\xi) - a]^{+} \right\} \tag{6}$$

The following formulation is a portfolio optimisation problem represented as a minimisation problem for negative returns based on the definition of Pflug (2000). The formulation is a direct reproduction of the formulation in Chapter 1, page 8, of the publication *Probabilistic Constrained Optimization: Methodology and Applications* (Pflug, 2000).

$$
\begin{aligned}
\min \quad & a + \left(\frac{1}{1-\alpha} \sum_{s \epsilon S} p^s z_s \right) & & \\
\text{s.t} \quad & z_s \geq -\, x^T \xi_s - a & s \epsilon S & \\
& x^T \xi_s \geq \ \mu & & \qquad (7) \\
& x^T 1 \ \geq \ 1 & & \\
& z_s \ \geq \ 0 & s \epsilon S & \\
& x \ \geq \ 0 & &
\end{aligned}
$$

We let $\xi_s = (\xi_1, \ldots \xi_k)$ be the random return of asset categories $1, \ldots k$ and let $x = (x_1, \ldots x_k)$ be the investments in these categories. We also assume that the total budget is 1. The portfolio return is represented by $x^T \xi_s$ and the aim is to minimise CVaR of the asset returns. The expected return must also exceed a pre-specified expected return μ. It should be noted that the optimal a is $VaR_\alpha(-\, x^T \xi_s) = \ - VaR_{1-\alpha}(x^T \xi_s)$ and we can reformulate it as a maximisation problem.

In risk management, research has mostly focused on extreme risks. For example, by focusing on VaR measures to constrain expected losses at a given level of confidence. But, although VaR is a standard for risk monitoring in the financial sector, it may not capture correctly the portfolio diversification benefits. Consequently, CVaR, which measures the weighted average of the tail events for a given fractile, is theoretically preferable (Uryasev, 2000). Furthermore, since it can be formulated using linear programming (Rockafellar & Uryasev, 2000), CVaR constraint portfolio optimisations have gained popularity (Kettunen, Salo, & Bunn, 2010).

Conditional value at risk in multistage stochastic problems

Time consistency is a requirement for ensuring optimal decisions in risk averse multistage stochastic programming problems (Rudloff, Street, & Valladão, 2014). However, using CVaR in stochastic programming for multistage problems is often faced by time consistency issues. Time consistency can occur if optimal decisions depend upon scenarios that with certainty cannot happen in the future (Shapiro, 2009). Similar to non-anticipativity, which forces identical decisions for scenarios sharing the same past, time consistency requires that optimality and feasibility should not depend on unrealizable scenarios. In other words, if we let the optimal solution of the multistage stochastic problem be computed, we let the optimal decision be assigned to the corresponding variables on the path from the root to node n in an intermediate stage and let the problem be solved for the subtree proceeding from node n. If the optimal values of the subtree problem conform with those computed on the whole problem, the solution is time consistent, otherwise it is time inconsistent.

Al-Baali, Gradinetti and Purnama (2014) explain time inconsistency differently. If the implementation is based on a future model run with updated information of uncertain parameters, an optimal policy is time consistent only if the future planned decisions are truly going to be implemented. A time inconsistent model does not incorporate the value of the correct recourse variables in the different stages, and this produce distortions when defining optimal decisions in earlier stages. The property of time consistency does normally not hold for risk-averse optimality problems. See Shapiro (2009) and Rudloff et al. (2014) for further discussions of why time-inconsistent stochastic programs can produce unsatisfactory policies.

However, a number of proposals have been put forward to extend the concept of risk measures to handle multistage stochastic optimisation while confronting the issue of time consistency. Pisciella et al., (2016) and Schütz & Westgaard (2018) use a nested CVaR-implementation to ensure time consistent optimal decisions. The nested multistage CVaR definition iteratively solves a convex combination of performance and risk in the last stage, using it as the performance measure for the previous stage. The approach is based on the research by Philpott and de Matos (2012) and Rudloff et al. (2014). The concept has been adopted from dynamic programming and has been used by several academics in a stochastic programming framework, see Philpott and de Matos (2012), Ruczcynski (2010) and Rudloff et. al (2011). Schütz & Westgaard (2018) model is the first to use multistage stochastic programming with a time-consistent risk measure in the objective function to study how the hedging decisions of a commodity producer depend on the producer's risk preferences. Other approaches include multi-period CVaR risk measure and a formulation with a sum of CVaR risk measure, both following the notion of Pflug & Römisch (2007). The models allow specification of different risk aversion coefficients and confidence levels at each stage. For more theory on coherent risk measurement to multiple time periods, the reader is also referred to Densing (2007). In a multi-period setting, Densing also builds upon a recursive definition over time to ensure time consistency.

4.6 Multi-Objective Optimisation

As this thesis handles a problem centred upon the recognised cost-risk trade-off, multi-objective optimisation becomes a relevant topic. The main aim of this subsection is to provide a brief and general overview of the multi-objective optimisation field.

Incorporating risk aversion in a model often involves the simultaneous optimisation of several objectives. These problems are called Multi-Objective Optimisation Problems (MOPs) (Jaimes, Zapotecas-Martínez, & Coello Coello, 2009). In single-objective optimisation, it is possible to determine a superior solution among any given pair of solutions and we usually obtain a single optimal solution. Conversely, in multi-objective optimisation this is not straightforward. The method most commonly adopted in multi-objective optimisation to compare solutions is the one called Pareto dominance relation. Instead of attaining a single optimal solution, this approach leads to a set of alternatives with different trade-offs among the objectives. These solutions are referred to as non-dominated or Pareto optimal solutions.

In the multi-objective optimisation procedure, two tasks can be distinguished, namely: i) finding a set of Pareto optimal solutions, and ii) select the most preferred solution of the set (Jaimes, Zapotecas-Martínez, & Coello Coello, 2009). Since Pareto optimal solutions are mathematically equivalent, the latter task requires a decision maker who can provide subjective preference information to select the best solution in a particular instance of the multi-objective optimisation problem.

4.7 Relevance and Contribution of this Thesis

This chapter has studied contributions of operations research within the aluminium remelting industry, and further investigated literature on risk management and incorporation of uncertainty in optimisation. To the best of our knowledge, there exists no published literature on stochastic modelling incorporating risk aversion from the perspective of an aluminium remelter. It also becomes clear from the reviewed literature that most operational research utilised by remelters today is limited to topics excluding procurement and inventory management. Hence, it is noted that the development of a model handling procurement portfolio optimisation while incorporating risk management contributes to existing literature.

In short, this thesis is the first to use multistage stochastic programming with a time-consistent risk measure to study hedging strategies based on the decision maker's risk preferences. The basis of the developed model is the flexibility created when incorporating uncertainty through the multistage stochastic structure. That is, the potential impact of possible outcomes is balanced in the valuation. Additionally, this thesis contributes to theory on coherent risk measures for multistage stochastic problems by utilising existing theory on a new problem, within a new industry. Lastly, it should be noted that even though the problem formulation is intended for HRM, the model's area of application can easily be extended for other procurement problems and utilised by other players operating in commodity markets.

5 Mathematical Model

This chapter presents the mathematical model for the Procurement Hedging Strategy Problem (PHSP) handled in this thesis, namely a nested multistage stochastic model. Assumptions and limitations used in the construction of the model are discussed, as well as a comprehensive representation of necessary sets, definitions and constraints. As the model builds upon the procurement portfolio problem developed in Hovland (2017), the improvements and modifications from this model are also emphasised.

In order to explicitly capture uncertainty in the model and to facilitate the development of flexible solutions, a multistage stochastic framework is used in this thesis. By utilising a multistage information structure, the decision maker can adjust his contracting decisions to adapt to the new information that is revealed during the planning horizon. Furthermore, several ways to handle risk in multistage optimisation are presented in this thesis, see Chapter 4. Based on an evaluation of the presented options with the industry partner and supervisor, the representation of uncertainty using CVaR has been decided as the preferred approach. CVaR does not require detailed cost information and further gives flexibility in managing cost. Also, CVaR is a financial performance measure widely used by practitioners and has experienced an increased relevance in the operations research field in recent years (Hafsa, 2015). Hence, it is considered the most appropriate risk measure for this problem.

CVaR is incorporated in the objective function through the linear programming formulation proposed by Rockafellar and Uryasev (2002). The cost-risk trade-off is implemented through two contrasting components: the first measuring the expected cost and the second measuring risk through CVaR. The objective is to minimise the weighted sum of expected costs and CVaR with respect to costs over the planning horizon. Central to the approach is a technique for portfolio optimisation which calculates VaR and optimises CVaR simultaneously. However, as the nature of this problem is multistage, the implementation of CVaR will result in issues with time inconsistency. This is discussed in Section 4.5. To prevent time inconsistency issues, a nested CVaR formulation based on Schütz & Westgaard (2018) is used in this thesis to handle risk-aversion. The nested approach is utilised due to the successful application in Schütz & Westgaard (2018), a problem with numerous similarities to ours.

The set of decision variables in the model represents implementable policies and further includes auxiliary variables defining the inventory, costs and risk modelling. Solving the model should return the binary decision variables, which completely specify the hedging strategy by denoting what contracts to be entered and thereby the amount that should be traded at any point in time within the planning horizon. The selection of contracts includes the two most common contract types utilised by HRM. Both are based on fixed volume agreements, but differ in how the price is established, and thereby in how they affect the risk profile and predictability. The contracts can either be priced as fixed price long-term contracts dependent on the value of LME High Grade or as variable price contracts dependent on the spot value of Secondary LME and priced for each delivery. Spot purchases, i.e. contracts with one immediate delivery, are modelled through variable price contracts. Fixed price long-term contracts give full predictability in terms of costs, avoiding the risk of incurring additional costs if the spot price rises during the contract period. Spot purchasing and adjustable price contracts expose HRM to the spot market volatility, thus providing less predictability. For both contract types, the price formula included is a fixed percent of the LME spot price as discussed in Section 2.4. This percentage remains constant throughout the planning horizon. An overview of the data input and output for the model is illustrated in Figure 5.1.

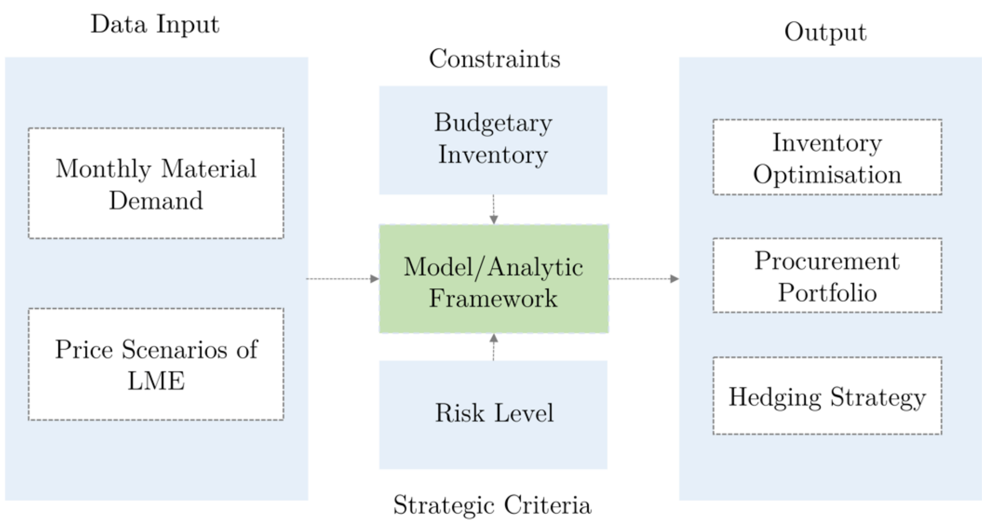

Figure 5.1 Overview of model data input and outputs

The producer's optimal procurement portfolio is computed based on input of future scenarios. Based on estimated parameters, the scenario tree accounts for the unique characteristics of the two stochastic spot prices: LME High Grade and Secondary LME. The model uses the scenario tree to optimise the procurement portfolio while accounting for the risk constraints. This provides optimal purchasing decisions at discrete time steps of the planning horizon, as well as a contingency plan. Figure 5.2 illustrates the scenario tree structure. The total number of scenarios in the tree will depend on the number of scenarios that are generated in each stage (indicated by the dotted lines) and the number of stages.

The information structure and time horizon of the implementation is motivated by the standard length of long-term contracts. As long-term contracts normally are entered with a maturity of 6 to 12 months, a time horizon of one year (with a monthly resolution) will constitute the basis for this thesis. As detailed decisions regarding spot purchase through short-term contracts are entered on a weekly basis, short term before production, spot purchase will have contracting and delivery within a month, with maximum delivery time of two weeks. Consequently, with a monthly resolution, the spot purchases are modelled with immediate delivery. It should also be noted that the mathematical model provides the opportunity to incorporate more detailed contracting options in the early stages if desired.

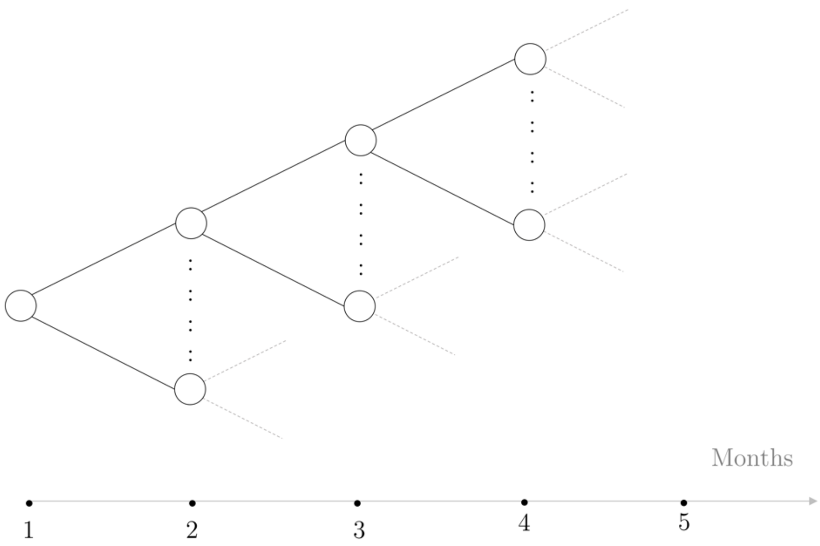

Figure 5.2 Structure of the multistage scenario tree used in the optimisation model

The main differences and extensions between Hovland (2017) and the model presented in this thesis are linked to the addition of risk aversion. This addition has led to a necessary re-structuring of the problem adapting a recursive approach with inspiration from dynamic programming. The most complex part of the model is the recursive constraints implementing CVaR in the model. In addition, necessary adjustments have been made to the inventory balance constraints. The changes have been implemented to better capture HRM's fundamental challenges in procurement contracting. Budget constraints, contract pricing and contract fulfilment constraints remain unchanged, as developed in Hovland (2017).

5.1 Definitions

This section gives a formal definition of the sets, indices, constants and variables used in the mathematical formulation of the stochastic programming model proposed in this thesis. Sets, deterministic and stochastic data are denoted by upper-case letters, whereas indices and variables are denoted by standard lower-case letters. Unless otherwise indicated, all quantities are referred to in weight given in metric tonnes (mt), while prices and costs are given in USD/mt. A complete overview of the model without explanations can be found in Appendix A.

5.1.1 Sets

$C(n)$	Set of children nodes (successors) of node n, $n \in N$.
K	Set of representative contracts k considered during the planning horizon.
K^F	Set of fixed price contracts k based on Secondary LME, $K^F \subset K$.
K^I	Set of initial contracts k entered before the planning horizon.
K^V	Set of variable price contracts k based on LME High Grade, $K^V \subset K$.
M	Set of materials m.
N	Set of event nodes n of the scenario tree.
N_i	Set of event nodes at stage i in the scenario tree.
N^K	Set of event nodes n of the scenario tree where contracts are offered.
$N(s)$	Set of nodes belonging to the path forming scenario s, $N(s) \subseteq N$.
S	Set of scenarios s representing the stochastic outcomes.
$S(n)$	Set of scenarios passing through event node n of the scenario tree, $S(n) \subseteq S$.
T	Set of time periods t of the planning horizon.
T_k^D	Set of delivery time periods associated with initial contract k.
T_{kn}^D	Set of delivery time periods associated with contract k entered at node n.

5.1.2 Indices

i	Stage of the scenario tree, $i = 1 \ldots I$.
k	Contract index, $k \in K$.
m	Material index, $m \in M$.
n	Event node index for the scenario tree, $n \in N$.
s	Scenario index, $s \in S$.
t	Time period index, $t \in T$.

5.1.3 Parameters, constants and coefficients

A_m^F	Fixed percent of LME High Grade for material m for fixed price contracts.

A_m^V	Fixed percent of Secondary LME for material m for variable price contracts.
B	Total budget for scrap purchase for the entire planning horizon.
C_{mt}^S	Cost per mt of being short of material m in time period t.
C_k^T	Total transportation cost in contract k.
D_{mt}	Required scrap quantity of material m in time period t.
H_m	Inventory holding cost per mt of material m.
I_m^0	Initial inventory level of material m.
L_k	Number of deliveries in contract k.
M_k	Material included in contract k.
P^n	Conditional probability of reaching node n from its predecessor.
Q_k	Tonnage per delivery in contract k.
Q^S	Total available storage space for the scrap material.
R_t	Discount rate in time period t.
$\bar{T}$	The last time period of the planning horizon.
$T(n)$	The time period of node n.
V_m	Density parameter [m^3/mt] for scrap material m.
W	Weight factor for the residual scrap value at the end of the time horizon.
α	Confidence level (percentile) for VaR and CVaR.
λ	Weight for HRM's risk preference, $\lambda \in [0,1]$.

5.1.4 Stochastic data

C_{ts}^F	Spot value of LME High Grade in time period t and scenario s.
C_{ts}^V	Spot value of Secondary LME in time period t and scenario s.

5.1.5 Decision variables

c_{kns}	Cost generated from entering contract k at node n in scenario s.
i_{mts}	Inventory level of material m at the beginning of period t in scenario s.
o_{in}	Objective function value at stage i and node n of the scenario tree.
q_{mts}	Quantity of material m used for production in time period t and scenario s.
x_{knmt}	Quantity purchased of material m through contract k at node n for delivery in time period t.
x_{kmt}^I	Quantity purchased of material m through initial contract k for delivery in time period t.
y_{in}	Cost exceedance (shortfall) with respect to CVaR at stage i and node n of the scenario tree.
z_{in}	Auxiliary variable for modelling CVaR, also representing VaR.
δ_{kn}	Binary variable, 1 if a contract k is entered at node n, 0 otherwise.

5.2 Mathematical Formulation

In this section, we provide the mathematical formulation for the Procurement Hedging Strategy Problem. A multistage stochastic programming formulation for the hedging problem is formulated with a weighted minimisation objective reflecting a cost-risk trade-off. The main decisions focus on how much of the required scrap should be purchased in the spot market and how much through long-term contracts, and at what point in time. The model output will be the optimal hedging strategy for HRM given the generated price scenarios and HRM's risk preference, i.e. degree of risk aversion. The rest of this section is divided into subsections covering the objective function and constraints respectively.

5.2.1 Objective function

The objective is to minimise the weighted sum of expected costs and CVaR with respect to procurement costs over the planning horizon. The first part of the objective function (i.e. inside the square bracket) is the total expected cost of contract purchases, inventory and scrap shortfall for the entire planning horizon. The second part of the objective (i.e. the last parenthesis) is part of the nested CVaR implementation. The reason for only including the costs from stage one and weighted average of stage two in the objective, is the recursive nature of the model. As the objective value, o_{in}, at a particular node n and stage i is calculated while incorporating the objective values at all succeeding nodes, the objective at stage one will account for costs of the whole scenario tree. The same applies to the shortfall variables y_{in}. Consequently, it is sufficient to only include the weighted average of shortfall at the nodes of stage two.

$$\begin{aligned} \min \quad (1-\lambda) \Bigg[\frac{1}{|S|} \sum_{s \in S} \Bigg(& \sum_{k \in K} \sum_{n \in N_1} c_{kns} \\ & + \sum_{m \in M} H_m i_{m,1,s} + \sum_{m \in M} C^S_{m,1} (D_{m,1} - q_{m,1,s}) \Bigg) \\ & + \Bigg(\sum_{n \in N_2} P^n o_{2,n} \Bigg) \Bigg] + \lambda \Bigg(z_{2,1} + \frac{1}{1-\alpha} \sum_{n \in N_2} P^n y_{2,n} \Bigg) \end{aligned} \tag{8}$$

The weighting of risk is modelled based on the decisions-maker's degree of risk aversion determined through the weight factor $\lambda \in [0,1]$, where $\lambda = 1$ signifies a risk-averse producer and $\lambda = 0$ signifies risk neutrality. Because this thesis comprises a cost minimisation problem, CVaR at the α confidence level is defined as the expected cost of the $(1-\alpha)100\%$ scenarios that provide the highest costs. With a $\alpha\%$ probability, the costs will not exceed VaR, where VaR is an endogenous variable. The risk managing part of the objective aims to minimise the cumulative cost of a given future time period in the $(1-\alpha)100\%$ worst cost scenarios.

The expected cost of contract purchases accounts for the average cost over all scenario outcomes. Furthermore, the inventory term is calculated by multiplying the periodic inventory level with the inventory cost (i.e. cost of storing one mt for one time period). The primary function of including inventory costs is to emphasise that storing ties up working capital and also incurs costs (Michalski, 2008). The shortfall term represents the cost of being short of scrap materials. The shortfall cost is based on the expensive alternative of using primary aluminium and alloying elements instead of scrap. It should be noted that since HRM has this alternative, the material requirement will always be satisfied and the use of backlogging is unnecessary.

It is also important to note that the cost expression is not an accurate expression of expected future costs as the objective function only includes cost elements relevant to the decision-making process. Consequently, no fixed costs are included in the model objective. Furthermore, the time value of money is considered. All cost expenditures during the time horizon are discounted in the model. Subsequently, the problem is dependent on when costs are incurred. In short, minimising the net present value will favour costs at the end of the horizon over costs today. As no discounting is necessary in stage one, the discount factor is only included in the recursive cost formulations in (10) and (11).

5.2.2 Constraints

In this subsection follows a detailed description of the model in terms of its constituting constraints. Firstly, the necessary recursive CVaR constraints are presented. Secondly, the expected cost of contracting decisions accounted for in the objective function is further regulated through contract fulfilment and expenditure constraints. Lastly, the contracting of scrap material is limited by inventory restrictions, material demand and a purchasing budget. Below, a mathematical representation of these relations are presented.

CVaR constraints

Constraints (9) are the necessary nested CVaR constraints. As discussed in section 4.5, the nested multistage CVaR takes into account the potential cost exceedance of each decision. The two auxiliary variables, y_{in} and z_{in}, are defined at each stage, where z_{in} plays the same role as VaR at the optimal solution of the problem. Figure 5.3 illustrates how the VaR, z_{in}, is associated with the objective variables. To exemplify, the shortfall on node 2 and 3 becomes; $y_{2,2} = o_{2,2} - z_{2,1}$ and $y_{2,2} = o_{2,3} - z_{2,1}$, respectively. That is, the shortfall on all children nodes (successors) of node 1 is calculated with the same local VaR, $z_{2,1}$. This is true for all stage nodes in the scenario tree.

$$y_{i+1,\acute{n}} \;\geq\; o_{i+1,\acute{n}} - \; z_{i+1,n} \qquad i = 1 \ldots I-1, n \in N_i, \acute{n} \in C(n) \tag{9}$$

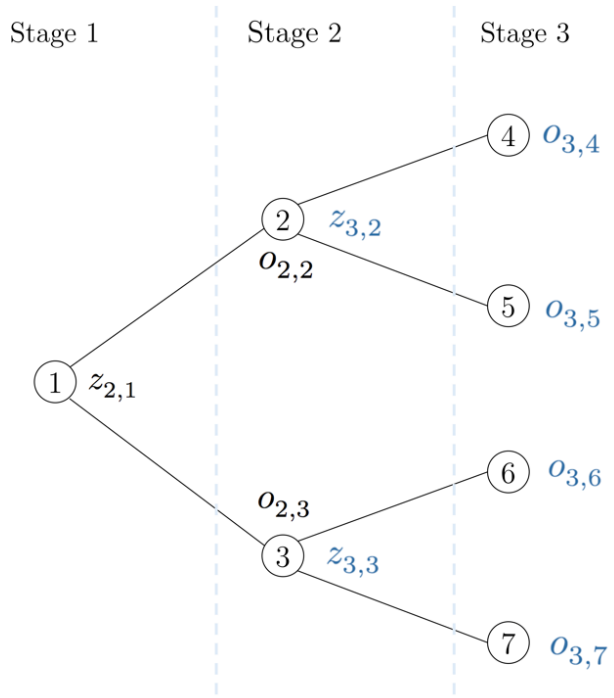

Figure 5.3 Relationship between nested CVaR variables

The recursive formulation for calculating the objective function at a given node in the scenario tree is given by Constraints (10). The objective value at a node is calculated by incorporating the objective values of all succeeding nodes. In a nested fashion, we iteratively solve a convex combination of cost performance and risk of the next stage and use it as the performance measure for the previous stage. The approach is based on the research by Philpott and de Matos (2012), Rudloff et al. (2014) and Schütz & Westgaard (2018), and is borrowed from dynamic programming.

$$\begin{aligned} o_{in} = \; & (1-\lambda)\Bigg[\frac{1}{|S(n)|}\sum_{s\in S(n)} R_{T(n)}\Bigg(\sum_{k\in K} c_{kns} + \sum_{m\in M} H_m i_{mT(n)s} \\ & + \sum_{m\in M} C^S_{m,T(n)}(D_{mT(n)} - q_{mT(n)s})\Bigg) + \sum_{\acute{n}\in C(n)} P^{\acute{n}} o_{i+1,\acute{n}}\Bigg] \\ & + \lambda\Bigg(z_{i+1,n} + \frac{1}{1-\alpha}\sum_{\acute{n}\in C(n)} P^{\acute{n}} y_{i+1,\acute{n}}\Bigg) \quad i = 2 \dots I-1, n \in N_i \end{aligned} \tag{10}$$

Constraints (11) calculate the cost in the last stage for each scenario. The last term also incorporates the residual value of the material at the end of the time horizon, where $\bar{T}$ represents the last time period of the planning horizon. A term that controls the end of horizon effects is included to avoid strange model behaviour. In order to value the material inventory at the end of the time horizon, the spot price of Secondary LME is used (i.e. C^V_{tn} in the last time period and for the relevant scenario).

$$
\begin{aligned}
o_{In} = R_{\bar{T}}\Big(\sum_{k\in K} c_{knS(n)} + \sum_{m\in M} H_m i_{m\bar{T}S(n)} + \sum_{m\in M} C^S_{m\bar{T}}\big(D_{m\bar{T}} - q_{m\bar{T}S(n)}\big) \\
- \; W \sum_{m\in M} C^V_{\bar{T}S(n)}(i_{m\bar{T}S(n)} - \; q_{m\bar{T}S(n)})\Big) \qquad n \in N_I
\end{aligned}
\tag{11}
$$

Contract fulfilment

All signed contracts must be fulfilled, which is ensured by (12) and (13). (12) handles the fulfilment of contract decisions made during the planning horizon, while (13) quantifies the set of existing contracts K^I.

$$x_{knmt} = Q_k \delta_{kn} \qquad k \in K, n \in N^K, m \in M_k, t \in T^D_{kn} \tag{12}$$

$$x^I_{kmt} = \; Q_k \qquad k \in K^I, m \in M_k, t \in T^D_k \tag{13}$$

The binary variable δ_{nk} is equal to 1 if contract k is entered in time period t and scenario s, and equal to 0 if not. A signed contract k comprises a specified material M_k and a specified tonnage to be delivered Q_k. Note that the total tonnage is uniformly distributed for multiple delivery contracts and Q_k represents the distributed delivery value. Contracts can include either one or several deliveries, where T^D_{kn} defines the set of delivery time periods comprised in contract k entered at node n. It should also be noted that all offered contracts are assumed to be fulfilled within the planning horizon.

Contract expenditures

As described in Section 2.4, HRM operates with several different types of contracts, each of which must be modelled differently. The set of contracts, K, is therefore divided into subsets, where K^F represents all fixed price contracts and the set K^V includes all contracts where the price is variable. Common for both contract types is that the contract price is based on the stochastic outcomes of LME, while other contract characteristics are deterministic. In a fixed price contract $k\epsilon K^F$, HRM and the supplier agree upon a fixed price per metric tonne, C^F_{ts} based on the spot price of LME High Grade (i.e. LME Aluminium) at the time of entering the contract. The cost of entering a fixed price contract is modelled in (14). Note that the price determined when entering the contract holds the entire contract period when the contract contains multiple deliveries. The cost of entering contract k at node n with the realisation of scenario price s is represented by c_{kns}. The cost is calculated by multiplying the binary variable δ_{kn} by the spot price of Secondary LME $C^F_{T(n)s}$ and further multiply by the contracted tonnage Q_k and number of deliveries L_k for contract k. A fixed percentage A^F_m, agreed upon before the planning period, is also multiplied with the price. C^T_k represents the fixed transportation cost for the tonnage included in contract k.

$$c_{kns} = \delta_{kn}(L_k Q_k A^F_{M_k} C^F_{T(n)s} + \ C^T_k) \qquad n \in N^K, k \in K^F, s \in S(n) \tag{14}$$

The second type of contract depends upon the spot price of Secondary LME (i.e LME Aluminium Alloy). In variable price contracts involving multiple deliveries, the price will be adjusted for each delivery. The cost generated from variable price contracts is calculated in (15). The classical spot price contracts with one delivery are modelled through (15) as well. These contracts will have material delivery in the same month as entering the contract. It should also be noted that each constraint in (15) represents the cost of entering a specific contract for a specific price scenario outcome.

$$c_{kns} = \ \delta_{kn}(Q_k A^V_{M_k} \sum_{t\epsilon T^T_{kn}} C^V_{ts} + C^T_k) \qquad n \in N^K, k \in K^V, s \in S(n) \tag{15}$$

In (15), the pricing is very similar to fixed price contracts, however, a sum of the LME Secondary spot price in each delivery period is included. The spot price for one constraint remains constant for scenario $s\epsilon S(n)$ and varies depending on the time period $t \in T^D_{kn}$. Figure 5.4 depicts an example contract k entered at node n in time period t with delivery in time periods $T^D_{kn} = \{t+1, t+2\}$. The relevant spot prices for the contract cost c_{kns} in the resulting price scenario s is then represented by $C^V_{t+1,s}$ and $C^V_{t+2,s}$.

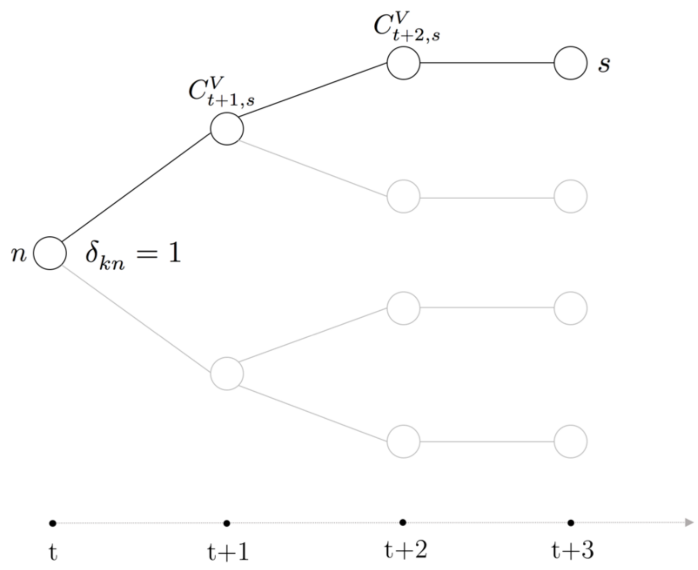

Figure 5.4 Illustration of parameter relationships for variable price contracts

Inventory balance

Constraints in (16) and (17) are the inventory balancing constraints representing the material inventory at the beginning of period t in scenario s. The inventory level for the next time period $i_{m,t+1,s}$ depends upon the inventory level from the previous time period i_{mts}, the quantity usage q_{mts} of material m in time period t and scenario s, and the total quantity delivered $\sum_{k \epsilon K} \sum_{n \epsilon N^K} x_{knm,t+1} + \sum_{k \epsilon K^I} x^I_{km,t+1}$ of material m in time period $t+1$. All deliveries are made at the beginning of specified time period t. Constraints (16) represent the initial inventory constraints.

$$i_{mts} = I^0_m \ + \sum_{k \epsilon K} \sum_{n=1} x_{knmt} + \sum_{k \epsilon K^I} x^I_{kmt} \qquad m \in M, t = 1, s \in S \tag{16}$$

$$i_{m,t+1,s} = i_{mts} - q_{mts} + \sum_{k \epsilon K} \sum_{n \epsilon N(s)} x_{knm,t+1} + \sum_{k \epsilon K^I} x^I_{km,t+1} \quad m \epsilon M, t \in T \backslash \{\bar{T}\}, s \in S \tag{17}$$

Constraints in (18) ensure that the quantity q_{mts} of material m used for production in time period t and scenario s is less than or equal to the minimum of the material demand D_{mt} and the inventory level i_{mts} for this time period.

$$q_{mts} \leq \ min(i_{mts}, D_{mt}) \qquad m \in M, t \in T, s \in S \tag{18}$$

Figure 5.5 illustrates how the inventory variables are associated with quantity used for production and quantity purchased through contracts for a specific scenario. When a contract is entered at a particular node, the variable δ_{kn} becomes non-zero, and deliveries contained in the contract are quantified through the quantity purchase variable x_{knmt} which is further included in the inventory balance.

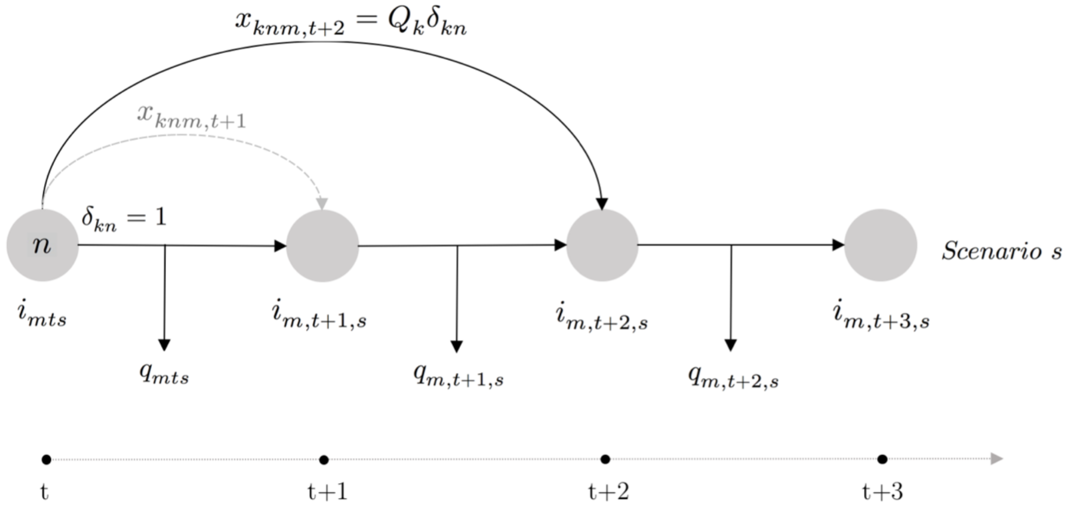

Figure 5.5 Relationship between inventory and quantity variables

In (19), the level of inventory i_{mts} is restricted by the storage capacity, where Q^S is the total available storage. The density parameters V_m ensures that the volume relationship between materials is correct.

$$\sum_{m \epsilon M} V_m i_{mts} \leq \quad Q^S \qquad t \in T, s \in S \tag{19}$$

Constraints in (20) represent the total budget for material purchase for the entire planning horizon. The total contract expenditure $\sum_{n \epsilon N(s)} \sum_{k \epsilon K} c_{kns}$ must be within the total budget for contract purchase for all scenario outcomes.

$$\sum_{n \epsilon N(s)} \sum_{k \epsilon K} c_{kns} \leq \; B \qquad s \in S \tag{20}$$

Non-anticipativity

Non-anticipativity constraints preserve the dynamic structure of the model by stating the equality of variables across different scenarios when they share the same history, or, equivalently, are associated with the same node in the event tree. This guarantees implementable optimal policies. In short, they force decisions that are based on the same information to be equal across the scenario tree and enforce the relationship between stages, periods and scenarios. The non-anticipativity constraints are represented in a concise manner in Equation (21).

$$\frac{1}{|S(n)|} \sum_{s' \epsilon S(n)} (i_{mts'}, q_{mts'}) = (i_{mts}, q_{mts}) \qquad m \in M, t \in T, n \in N, s \in S(n) \tag{21}$$

Variable constraints

$$\begin{aligned}
&c_{kns} \geq 0 \quad k \in K, n \in N^K, s \in S(n)\\
&i_{mts} \geq 0 \quad m \in M, t \in T, s \in S\\
&q_{mts} \geq 0 \quad m \in M, t \in T, s \in S\\
&x_{knmt} \geq 0 \quad k \in K, n \in N^K, m \in M, t \in T^D_{kn}\\
&x^I_{kmt} \geq 0 \quad k \in K^I, m \in M, t \in T^D_k\\
&o_{i+1,n} \geq 0 \quad i = 1 \dots I-1, n \in N_{i+1}\\
&y_{i+1,n} \geq 0 \quad i = 1 \dots I-1, n \in N_{i+1}\\
&z_{i+1,n} \geq 0 \quad i = 1 \dots I-1, n \in N_i\\
&\delta_{kn} \in \{0,1\} \quad k \in K, n \in N^K
\end{aligned} \tag{22}$$

6 Computational Study

In this chapter, the applicability and value of the proposed model (presented in Chapter 5) will be evaluated through a computational study. The implementation is conducted using commercial software and tested on a realistic case. After a brief introduction to hardware and software, the input data of the test case is presented. The analysis that follows will in turn move from technical evaluations of the implementation performance, to a more practical perspective. The practical analysis is mainly focused on how different boundary conditions affect inventory management and hedging strategy.

6.1 Model Implementation

This section briefly describes how the model presented in Chapter 5 is implemented using available commercial software. The optimisation model is implemented in Mosel and solved using FICO Xpress. All calculations and numerical experiments are carried out on a computer powered by an Intel® Core™ i7-7700 CPU clocked at 3.60 GHz with 32.0 GB of RAM. The software used is FICO® Xpress Optimization Suite, with Xpress-IVE version 1.24.18, Xpress Mosel version 4.6.0 and Xpress Optimizer version 31.01.09.

The module mmsheet has been used to acquire input data from Microsoft Excel. The module allows the accessing and modifying of spreadsheet files in different formats from initialisations blocks. An effort has been made to structure the input spreadsheets in a clear and understandable manner to ensure that the model can be used without detailed knowledge of modelling or optimisation. All calculations are done using functions in Xpress, thus eliminating the need for manual pre-solved calculations of the input. In other words, after entering the input data specified into the input spreadsheet and input text file, the Xpress models can be run and relevant output is written to the "Output/Input" tab of the "Run Bar" in Xpress.

6.2 Problem Instances

In this section, an overview of input parameters and construction of the test cases are presented. Spreadsheets in MS Excel are used to generate the test instances. The spreadsheets are written in a manner that is suited for Xpress.

Scenario generation
In terms of scenario generation, the unknown parameters are the LME spot prices. These spot prices are directly observed from the market. Historical Prices (Secondary LME and LME High Grade) are displayed in Figure 6.1.

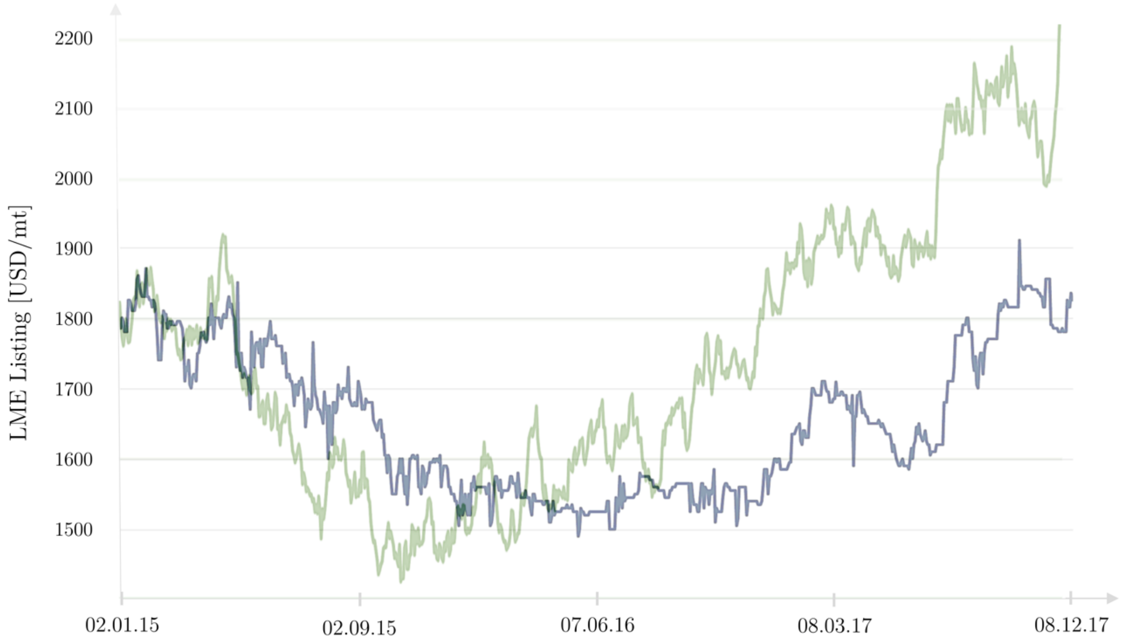

Figure 6.1 LME listings: High Grade (green) and Secondary (blue)

Examining the historical prices from London Metal Exchange during the period January 2015 to December 2017, two characteristic periods stand out: Between January 2015 and June 2016, the two listings overlap and cross each other several times over a steady drop period. Since then, the prices have exhibited an increasing trend, starting early 2017 and lasting all through 2017. We therefore choose three different points in time to study the remelters optimal hedging strategy: the first time period studied, P1, is towards the end of the drop in aluminium prices, starting with the first stage in January 2016. The second problem instance, P2, starts before experiencing a long period of increasing prices, with July 2017 as the start of the planning horizon. The last price development, P3, starts in January 2015, and displays considerable overlap in the two listings.

The expected price development for the test cases is given in Table 6.1 and illustrated in Figure 6.2, 6.3 and 6.4. Testing the model using three problem instances with different price data is completed to illustrate portfolio strategy variation. It should also be noted that a factor of 0.85 is multiplied with the prices of LME High Grade for P2, as this historically was the practice for HRM for this time period. For test case P1, the standard deviation is calculated as the lowest of the three instances at 15.0 USD for Secondary LME and 38.2 USD for LME High Grade. For test case P2 the standard deviation is considerably higher: 78.4 and 81.9 USD for Secondary LME and LME High Grade respectively. For test case P3 the standard deviation is calculated at: 25.7 and 52.5 USD for Secondary LME and LME High Grade respectively. The difference in standard deviation in the three price samples should be considered when comparing procurement portfolio compositions developed by the model. The scenario tree used in the computational study has 6 stages and covers a planning horizon of 12 months. As we have a monthly resolution of the planning horizon, we use average monthly LME listings as basis when creating the scenario tree. More information on the scenario tree is presented later in Section 6.2.

Table 6.1 Expected monthly prices in USD/mt for P1, P2 and P3

	P1		P2		P3	
Stage	Secondary	High Grade	Secondary	High Grade	Secondary	High Grade
1	1560	1479	1623	1618	1819	1808
2	1553	1536	1727	1726	1797	1821
3	1555	1531	1755	1785	1753	1773
4	1545	1564	1815	1811	1790	1817
5	1529	1556	1840	1786	1771	1805
6	1523	1592	1798	1760	1756	1683

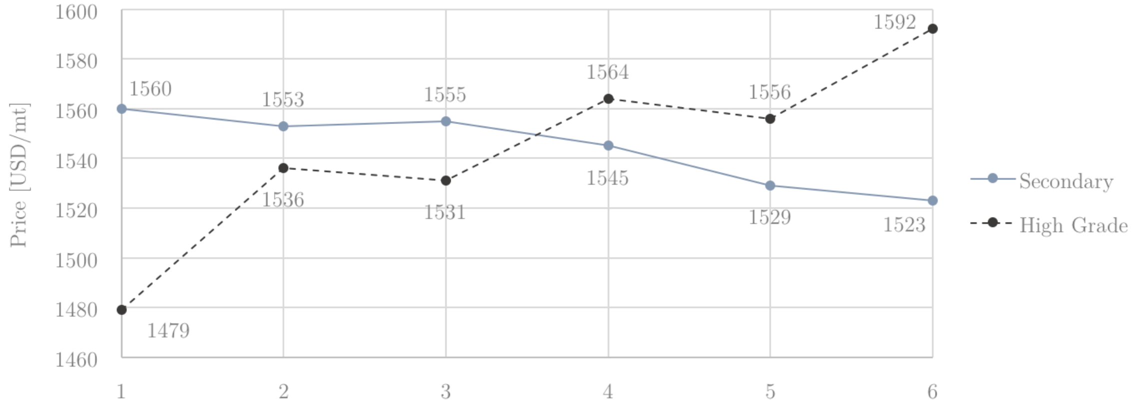

Figure 6.2 Expected price development for P1

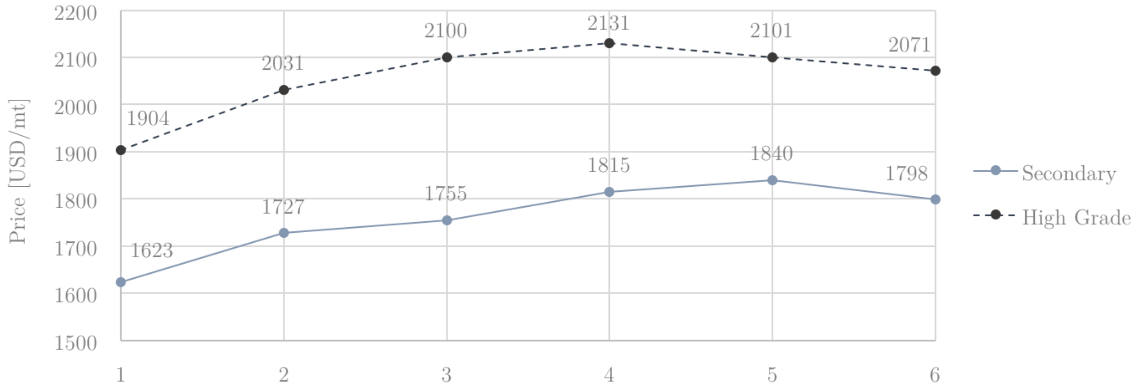

Figure 6.3 Expected price development for P2

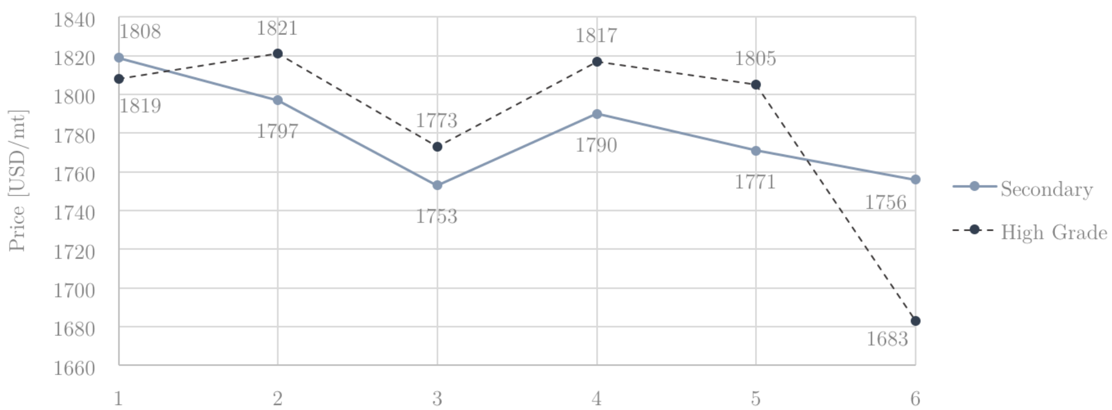

Figure 6.4 Expected price development for P3

Material Demand and Initial Inventory

Material demand for the planning horizon used for all test cases is presented in Figure 6.5 and Figure 6.6. We use the monthly registered scrap consumption data from HRM realised in 2017. For this thesis, all test instances use the three most common scrap materials contracted at HRM; namely Old, Mix1 and Mix2. The period for the consumption volumes matches the planning horizon as given by the scenario tree. Note that both prices and deliveries are only specified by month, not exact dates, as prices and deliveries are aggregated on a monthly basis. Inventory values for December 2016 from HRM are used as basis for initial inventory values in the model, see Table 6.2. As the model is run on historical data, it can also be used as a benchmark against HRM's current hedging strategy.

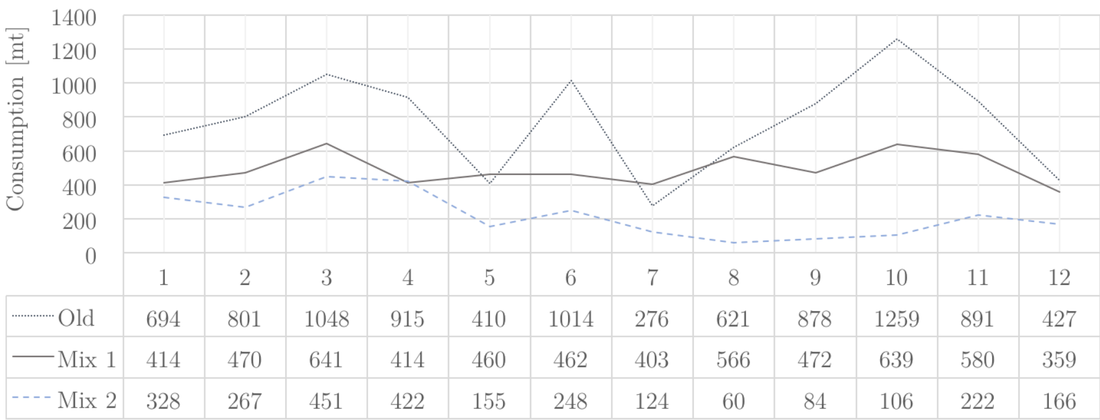

Figure 6.5 Stacked line chart of the material demand for the planning horizon

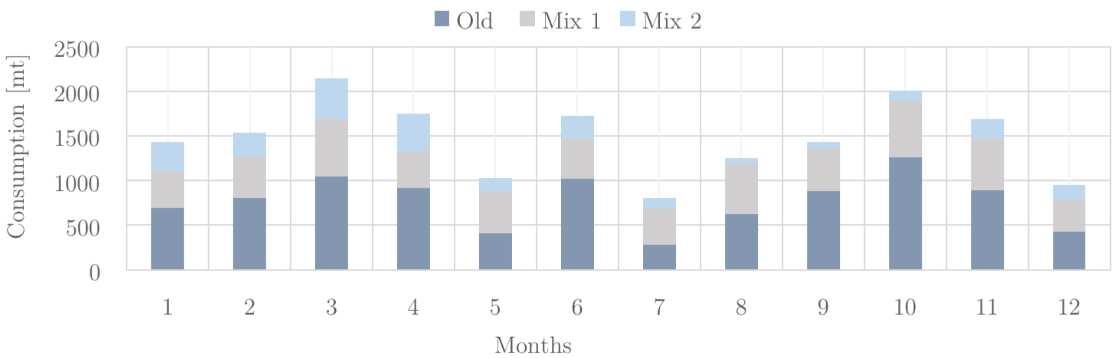

Figure 6.6 Stacked bar chart of the material demand for the planning horizon

Table 6.2 Initial Inventory

Material	Old	Mix1	Mix2
Inventory [mt]	307	127	116

Contract Data

A selection of available contracts is offered during the time horizon. A total number of 81 contracts can be selected at all decision nodes, varying in pricing method, quantity and number of deliveries. For a complete overview of the contract selection, see Appendix B. In the selection, all spot contracts are linked to Secondary LME (Aluminium Alloy) and all long-term contracts to LME High Grade (Aluminium). The selection does not include variable price contracts with more than one delivery (i.e. spot purchases), as investigated in Hovland (2017), since the purpose of this computational study is to illustrate how the model can be used as a hedging tool for HRM. However, it should be noted that the model is implemented in a manner that allows long-term variable price contracts as well. Via SOS1, we ensure that at most one contract is non-zero (entered) at each node for each material. An assortment of initial contracts signed before the planning horizon is also included. The assortment comprises one initial contract for each material. The initial contracts for Mix1, Mix2 and OLD contain 12 deliveries respectively and each delivery comprises 100 mt, 200 mt and 300 mt respectively, see Table 6.3.

Table 6.3 Initial Contracts

Material	Number of Deliveries	Delivery Periods	Delivery Volume [mt]
Mix1	12	1-12	100
Mix2	12	1-12	200
OLD	12	1-12	300

Table 6.4 contains information about the fixed input parameters. It should be noted that all costs are calculated in USD per month. The weight λ specifying the producer's risk preference is increased in steps of 0.1 in the interval [0,1]. The problem is solved for two different CVaR percentiles, α: 0.9 and 0.95. For all test instances, we use a discount rate of 0.5 % per month. Due to similar density of the materials used, we run the model with storage capacity given in weight [mt] instead of volume [m^3]. However, if needed, the model is implemented in a manner that allows density specifications. Inventory cost is set to 3,000 USD/mt. This is a relatively high cost considering purchasing prices, however, this cost is set due to HRM's limited storage ability and need for available storage at all times. Shortage cost is set to 20,000 USD/mt for the first 6 time periods (i.e. the stages of the planning horizon) and reduced to 5,000 USD/mt for the rest of the planning horizon. This reduction is included to allow additional contracting options in the future. Furthermore, all transportation costs are set to zero because the relevant quantities for the materials can be ordered from a single supplier and the freight cost increases linearly with contracted weight. Consequently, the inclusion of transportation cost will not affect the solution for the included test cases. However, the implementation allows the specification of transportation cost for each contract if necessary.

Table 6.4 Parameter Input

Parameter	Value
Storage Capacity [mt]	4,000
Budget [USD]	25,000,000
Shortage Cost in period 1-6 [USD/mt]	20,000
Shortage Cost in period 7-12 [USD/mt]	5,000
Inventory cost, IC [USD/mt]	3000
LME factor 1 for P1, P2 and P3	1.00/0.85/1.00
LME factor 2 for P1, P2 and P3	1.00
Density Parameter [m^3/mt]	1.00
Inventory Weight Factor, W	0.90
Risk Weight Factor, λ	[0,1]
Confidence Level, α	[0,1]

The computational study includes the extension of the planning horizon from 6 to 12 months compared with the test cases in (Hovland, 2017). Furthermore, a greater selection of contracts is available at each node to better reflect the contracting options available for HRM. For all test instances, the 6 first months include the option of entering contracts and the following 6 months is an evaluation of the inventory level and production quantity. Consequently, this study uses 6 stages. That is, the decision maker will have full information about the future prices as of month 6, see Figure 6.7. We use a stylised price scenario tree for the analysis. The scenario tree comprises 3 successors for each node developing as 95%, 100% and 105% of the average price value for the given stage. All scenario outcomes have the same probability. The resulting optimisation problem of 243 scenarios has 260,965 variables and 504,492 constraints before pre-processing versus 41,260 variables and 13,963 constraints after pre-processing.

Stages	1	2	3	4	5	6	
						122	
					41	:	
				14	:	:	
			5	:	:	:	
		2	:	:	:	:	
	1	:	:	:	:	:	Scenarios: 243
		4	:	:	:	:	
			13	:	:	:	
				40	:	:	
					121	:	
						364	
# Nodes	1	3	9	27	81	243	

Figure 6.7 Nodal overview of the scenario tree

6.3 Computational Results

For this computational study, a brief computational efficiency study is conducted. In an economic study, the model is analysed and examined for changes in risk preferences and risk percentiles. Furthermore, a detailed analysis of hedging strategies and procurement portfolio composition is completed for the different price scenarios, followed by a sensitivity analysis performed with respect to the budget and storage restrictions.

6.3.1 Computational Efficiency

In this subsection, model size and time considerations for different test instances are considered. The test instances are extended stepwise and model sizes are recorded. Table 6.5 gives a detailed overview of the number of continuous variables, binary variables and number of equations before pre-processing for the different test instances. The solution time and gap of the test instances are also presented in Table 6.5.

With a test case of 243 scenarios, the gap is reduced to 0.19% after 36.6 hours. The best solution is found already after 1.22 hours. The rest of the time, the solver is working on reducing the gap by improving the lower bound. Figure 6.8 displays the upper and lower bound output for the first 30 minutes of the model run on the test case with 243 scenarios. It can be observed that a lot of solutions are found the first 2 minutes, reducing the gap considerably. Henceforth, solutions are found in a less frequent manner, reducing the gap gradually. For the 81-scenario test case, the best solution is found by the solver after 2.58 hours. An overview of the gap development for the 81-scenario case can be viewed in Figure 6.9 illustrating when the last solution is found and the solvers effort to reduce the gap (i.e. by improving the lower bound) for the rest of the run. After 24 hours, the gap is at 0.02%. For the 27-scenario test instance, the model is solved to optimality after 19.4 hours. The best solution is found after 1.50 hours.

Table 6.5 Computational Efficiency

Scenarios	Stages	Continuous variables	Binary variables	Constraints	Running time	Gap [%]
243	6	231,211	29,484	504,492	36.6 hours	0.19
81	5	69,784	9,801	101,658	24.0 hours	0.02
27	4	20,929	3,240	24,966	19.4 hours	0.00

It should also be noted that all runs are done with scaling. All prices and costs are entered as USD/1000 mt, while volumes are entered in mt, to more quickly reduce the gaps in Xpress. The cost results are therefore multiplied with a factor of 1000 before they are presented in this thesis. The model is also sensitive to input data. Depending on the fixed input, the solution time and gap can increase. However, all the presented results in this thesis have a gap below 1 % unless otherwise stated.

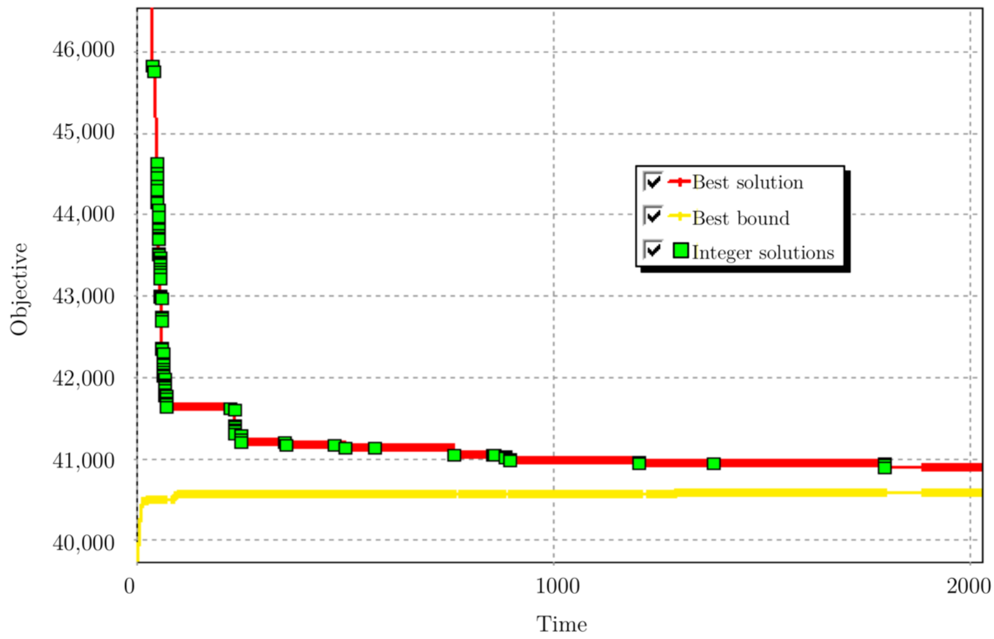

Figure 6.8 Upper and lower bound output for 243 scenarios

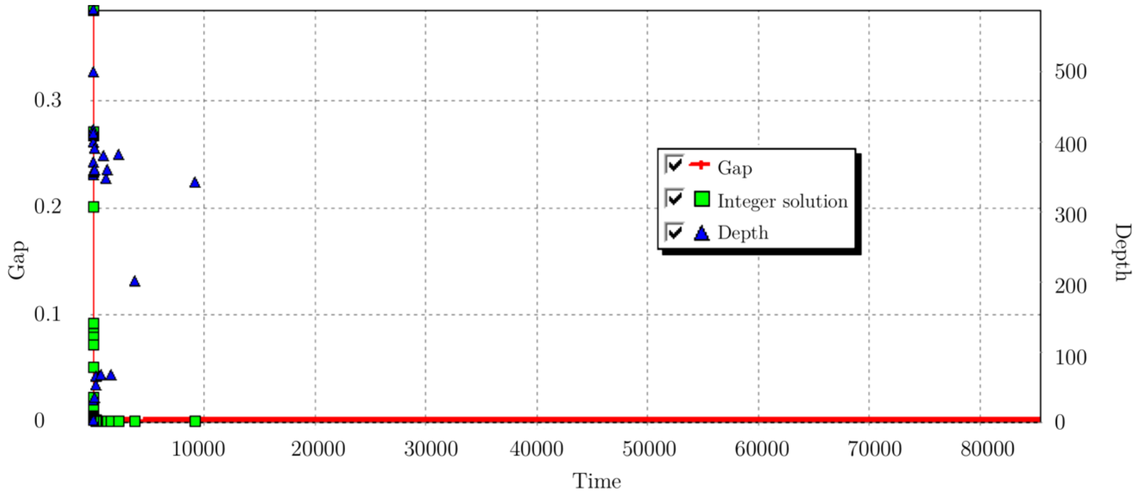

Figure 6.9 Gap advancement for 81 scenarios

6.3.2 Economic Study

For the economic study, all test instances constitute 243 scenarios and 81 contracting options on each decision node. The study includes an analysis of the model's hedging ability by testing for different risk weight factors λ and confidence levels $\alpha = \{0.90, 0.95\}$, also referred to as CVaR percentiles. The analysis also considers model behaviour for the three presented price developments referred to as P1, P2 and P3 (see Section 6.2). It should be noted that, unless otherwise stated, all results given in this economic study are the average solution over all scenarios.

Reducing spot purchase: higher level of risk aversion

The first part of this analysis examines the model solution for different levels of risk aversion. Primarily, we look at how much of the total contracted tonnage is purchased through long-term contracts over the planning horizon. The rest of the contracted material is thus contracted through spot purchases. We see from the results (i.e. Figure 6.10) that the amount of material purchased through long-term contracts increase as the degree of risk-aversion increase. Contrary, the amount purchased in the spot market decrease. While there might be scenarios outcomes in which the solution recommends to not decrease the amount of spot trades, the average spot volumes are clearly decreasing. The results in Figure 6.10 are run on a test case with P1 at a confidence level of 95 %. It should also be noted that for $\lambda = 1$, the percentage amount purchased through long-term contracts increase to 89.5 %. This large growth is not visible in Figure 6.10, as the y-axis is limited to 50 % in order to better illustrate the increase between $\lambda = 0$ and $\lambda = 0.99$.

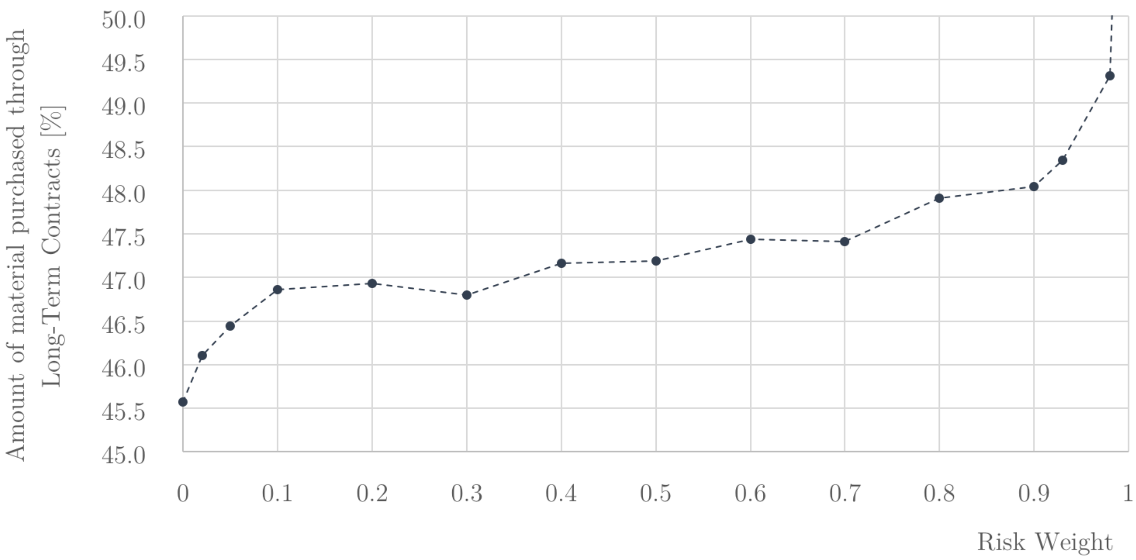

Figure 6.10 Percent of the purchase contracted through long-term contracts

A risk-neutral producer is defined by $\lambda = 0$. The results above indicate that HRM should consider a fairly diversified procurement portfolio from a low degree of risk aversion. That is, the portfolio consists of 45.6 % long-term contracts for a risk neutral producer. The rest of the portfolio consists of spot contracts (i.e. spot purchases). While increasing the degree of risk aversion, a gradual increase in long term contract purchase is observed with an inversely decreasing purchase in spot contracts. The share of long-term purchases increases with 6.8 % between risk neutrality and a risk weight of 99 %. For the extreme case of a 100 % risk averse producer, 89.5 % is invested in long-term contracts. Observe that for this last setting, the expected cost part of the objective is not counted whatsoever. All the entered long-term contracts for $\lambda = 1$ consist of contracts with a maturity of 6 months. This is also the maximum available maturity in the selection of offered contracts.

The results presented here are in line with the presented theory in Chapter 4. A risk neutral model, $\lambda = 0$, determines the values of the decision variables that minimise, over all scenarios, the expected cost along the planning horizon. It does not take into account the variability of the objective function value over the scenarios and does not highlight the possibility of realising some scenarios with very high costs. For our results, the expected cost is lowest with a portfolio consisting of a very similar percentage of spot purchases and long-term contracts. Contrary, a risk averse model, $\lambda > 0$, will also minimise the risk of realising very high costs. That is, the model will minimise the expected value of costs in the worst α-percentile of cases, where α is the confidence level assigned by HRM. When risk is more heavily weighted, long-term contracts will benefit in avoiding the extreme price scenarios. Consequently, the moderate increase of long-term contracts in the portfolio is anticipated. Figure 6.11 displays the percentage share purchased through spot and long-term contracts for the different risk weights through a clustered bar chart and clearly illustrates the discussed trends. The results show a gradual shift towards long-term contracts with longer maturities. This gradual shift is observed for prices in P2 and P3 as well. The same behaviour is also observed for other CVaR criteria. A more detailed analysis of the effect of CVaR is conducted later in this economic analysis.

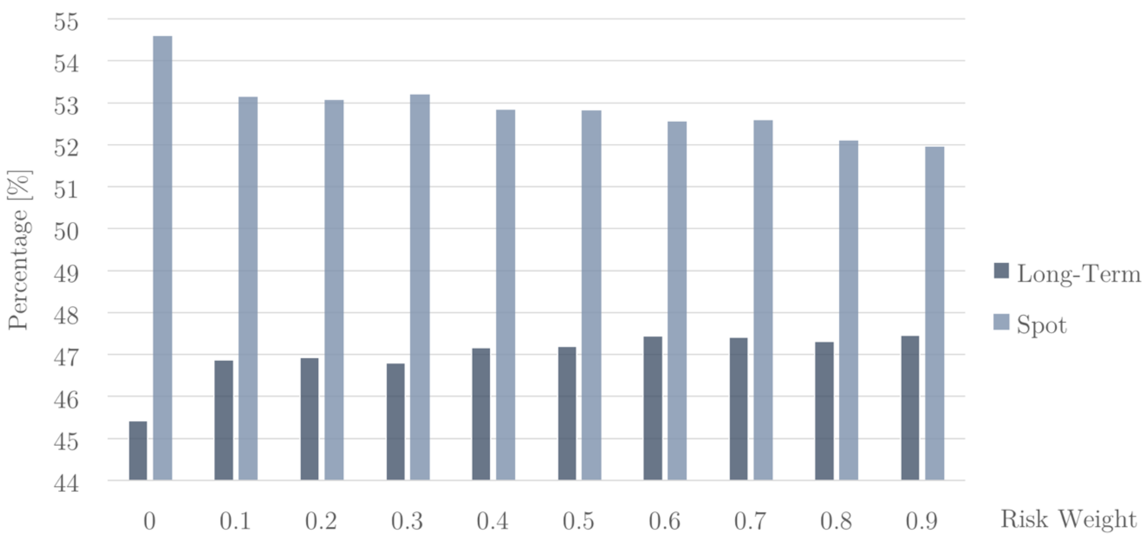

Figure 6.11 Clustered bar chart representation of the portfolio composition

Different CVaR percentiles

The following analysis investigates the effect of different CVaR percentiles (i.e. confidence levels, α) on the solution. When comparing the solution for two CVaR percentiles (see Table 6.6), namely 90 % and 95 %, it is clear that the chosen CVaR percentile also affects the hedging decisions. In general, the higher the CVaR percentile, the less risk-averse HRM needs to be before moving purchase volume from the spot market to long-term contracting. The percentage purchased in long-term contracts increases by 0.4 - 1.7 % depending on the level of risk aversion. Figure 6.12 illustrates the difference between 90% and 95% CVaR for the P1 test case.

Table 6.6 Percentage of long-term contracts for different CVaR percentiles

Test Case		Risk Weight λ					
Price	α	0	0.2	0.4	0.6	0.8	1
P1	0.90	45.57	46.31	46.75	46.64	46.51	98.64
P1	0.95	45.57	46.93	47.16	47.44	47.90	89.46

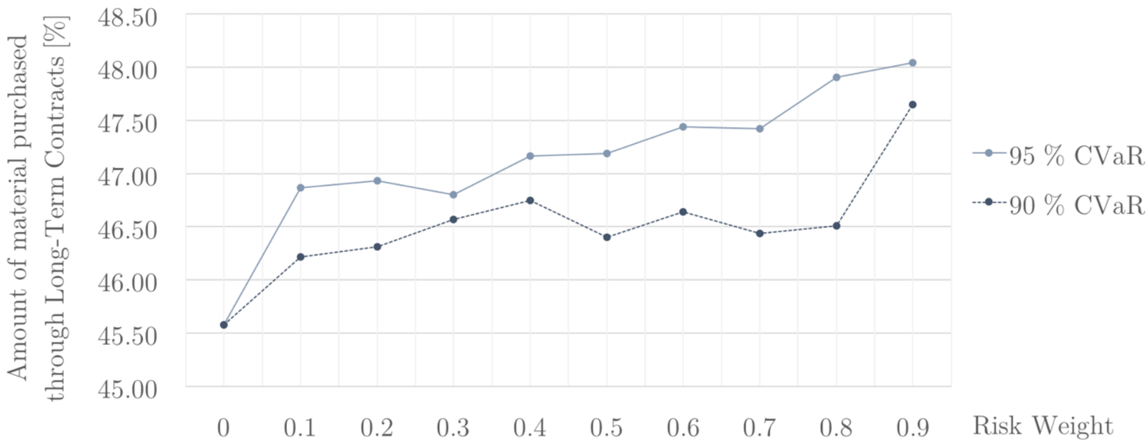

Figure 6.12 Percentage of long-term contracts entered versus risk weight

The increase in long-term contract purchases when increasing the confidence level is expected. With a higher confidence level, the decision maker becomes more risk averse. This is because the CVaR value of the objective will increase and the solver will to a greater extent try to reduce this value through hedging. To better explain, CVaR incorporates the (1 - α) % worst case scenarios. That is, for a 95 % CVaR percentile, 5 % of the worst-case scenarios are considered in the objective. In a practical context, CVaR measures the probability that the procurement portfolio will incur very large costs. If we compare the 5 and 10 % largest losses of a loss distribution, the value of CVaR will be higher for the 5 % case. The solver will then to a greater extent attempt to reduce CVaR (i.e. through long-term contracting).

The only clear exception from the notable tendency in Table 6.6 and Figure 6.12 is for a risk weight of 0 and 1 respectively. That is, for the extreme case of risk neutrality and when the decision maker is 100 % risk averse. For risk neutrality, it is clear that the CVaR value has no effect, as CVaR is no longer valued in the objective. For $\lambda = 1$, the level of long-term contracts entered are in general high at 98.6 % and 89.5 % for 90% and 95 % CVaR respectively. A more detailed analysis is presented in Table 6.7 to illustrate the maturities of contracts entered for this extremity. At 90 % CVaR, 10.53 % of the procurement consists of contracts with a maturity of 3 months, and the rest of the long-term contracts have a maturity of 6 months. Only 1.36 % of the procurement portfolio is spot purchases. At 95 % CVaR, all long-term contracts have a maturity of 6 months, and 10.54 % of the total procurement consists of spot purchases. The total percentage of long-term contracts is therefore bigger for the 90 % percentile than for the 95 % percentile. However, the percentage purchase in long-term contracts with a maturity of 6 months is 89.46 %, which is 1.13 % more than for the 90 % CVaR test case. It can therefore be argued that the degree of hedging is higher for the 95 % CVaR test case.

Table 6.7 Procurement portfolio composition for 90 % and 95 % CVaR with $\lambda = 1$

	Maturity of contract					
α	M2	M3	M4	M5	M6	Spot
0.90	-	10.53	-	-	88.11	1.36
0.95	-	-	-	-	89.46	10.54

However, it is evident that the increase in Figure 6.12 is not even, though there is a clear increasing trend for higher values of λ. The explanation for the irregularities can primarily be explained due to the large category defined as long-term contracts. This category includes contracts with maturity of 2-6 month. Contracts with maturity of 2 months could largely be considered closer to a spot contract as the hedging effect from its usage is much less than for contracts with longer maturities. So even though the total amount purchased through long term contracts does not increase, there can be a shift in the contract assortment towards contracts with longer maturities (see Table 6.7). An analysis of the procurement portfolio in terms of maturities can be viewed later in this economic study. The irregular increase can further be explained by the acceptance of solutions with gaps up to 1 %. If we compare the solution for 90 % CVaR at a risk weight of 0.4 and 0.5 (i.e. the largest decline), the drop is at 0.074 %. This is a small number that could easily be effected by the acceptance of non-optimal solutions. It should also be noted that for some irregular results, a re-run over one day has been made, producing noticeable changes in the results. However, due to the limited time available for this analysis, a one day run for each point in the graph was not possible.

Objective cost versus risk weight

In risk management, mathematically equivalent solutions are differentiated based on the decision makers degree of risk aversion. By varying a risk weight factor λ, the decision makers can construct an objective plot and thus choose the optimal objective function value corresponding to their subjective risk preferences. The plot for test case P1 is displayed in Figure 6.13, clearly showing the reduction in cost while the degree of risk aversion increases. The graph is showing total expected costs on the y-axis (i.e. objective value), versus the risk weight on the x-axis. This relationship is clearly visible for all test instances and is a fundamental feature of the model. The discounted cost reduces as the relative importance of risk becomes high. Thus, there is a clear relationship between the risk measure (CVaR) and the expected cost. The plot also represents a set of optimal procurement portfolios that offers the lowest expected cost for a defined level of risk or the lowest risk for a given expected cost.

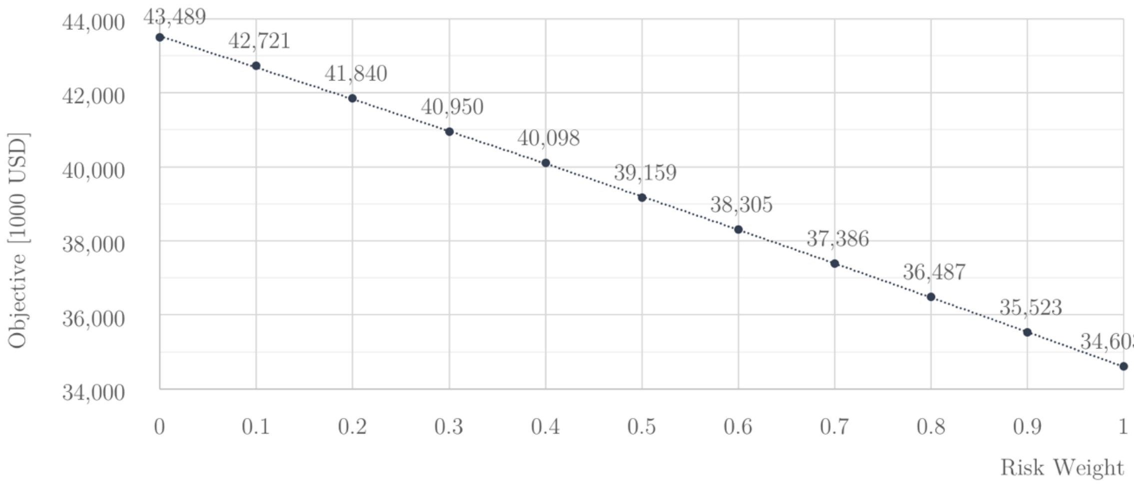

Figure 6.13 Objective plot of the Expected Cost/Risk spectrum

When increasing the weight on CVaR, by increasing λ, the model gives more conservative solutions, reducing the expected costs of the problem. When risk aversion is high, the model recommends what is referred to as a static hedging strategy (see Section 4.5) and the expected cost of the problem becomes lower. This is because the risk part of the objective focuses on the worst-case scenarios of the possible outcomes. By avoiding the worst-case outcomes, the expected cost will drop. Contrary, with a lower level of risk aversion, the model will not put as much weight on unfavourable scenarios, and favour the extra gain from the more favourable scenarios. In other words, the natural hedge gives a strategy with highest uncertainty in future cost and the highest possible target shortfalls, but also the highest upside potential. With a natural hedge (i.e. risk neutrality), one desires to handle a greater standard deviation in costs, in the hope of utilising advantageous scenario outcomes. However, this comes at a greater expected cost.

Moreover, Figure 6.14 illustrates the reduction of CVaR in the objective as the level of risk aversion increases. The decrease is gradual, in line with the reduction of the objective value. The complete reduction between a risk weight of 0.1 and 1 is 9.46 %. For the extreme state of risk neutrality, the value of CVaR becomes distinctly high, as the value is no longer weighted in the objective. The CVaR cost for $\lambda = 1$ is 769 million USD, a value 19 times higher than for $\lambda = 0.05$.

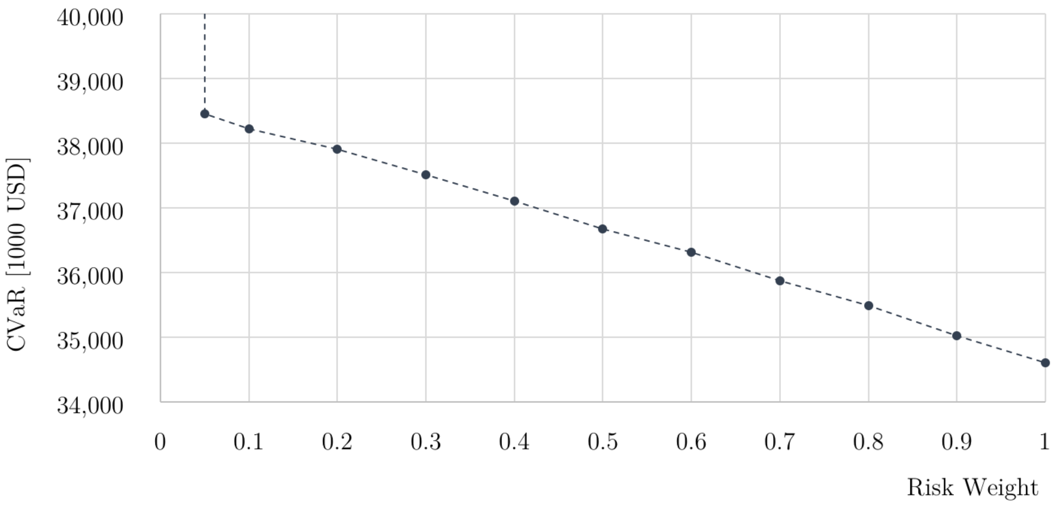

Figure 6.14 CVaR versus risk weight for P1 with a 95 % CVaR percentile

It should also be remarked that the model benefits from the concept of diversification. Diversification is a risk management technique that mixes a wide variety of contracts within a contract collection. A composition constructed of different contracts (i.e. different maturity, quantity, pricing) will, on average, produce lower costs and pose a lower risk than any individual contract found within the procurement portfolio. Optimal portfolios that comprise the objective plot tend to have a higher degree of diversification than the sub-optimal ones, which are typically less diversified. Based on the results for the test cases, the degree of diversification through spot and long-term contracts is clearly present.

Reducing the inventory cost

The effect of reducing the inventory cost is also investigated. While varying the inventory cost from 3000 to 800 and 200 USD/mt respectively, the change in procurement portfolio composition is analysed. The evaluation can be conducted by comparing Table 6.8, Table 6.9 and Table 6.11. The tables display the average procurement portfolio composition for all scenarios for values of 90% and 95% and a selection of weight values (i.e. 0.1, 0.4 and 0.9). The weights show the percentage amount of material that should be purchased through the selection of long term contracts for the planning horizon, differentiated by maturity and spot contracts. For instance, for the test case with risk weight $\lambda = 0.9$ and confidence level $\alpha = 0.95$, in column M4, the data given is the percentage (i.e. average over all scenarios) of the material purchased through contracts with a maturity of 4 months for the whole planning horizon.

Table 6.8 Procurement portfolio composition: IC = 200 USD/mt

Test Case		Maturity of contract					
λ	α	M2	M3	M4	M5	M6	Spot
0.1	0.90	-	0.18	0.48	2.91	55.00	41.43
0.4	0.90	-	-	-	1.29	56.73	41.97
0.9	0.90	-	-	-	0.85	58.32	40.83
0.1	0.95	-	-	-	0.94	58.78	40.27
0.4	0.95	-	-	-	1.02	58.82	40.16
0.9	0.95	-	-	0.37	3.25	56.91	39.47

Table 6.9 Procurement portfolio composition: IC = 800 USD/mt

Test Case		Maturity of contract					
λ	α	M2	M3	M4	M5	M6	Spot
0.1	0.90	0.11	-	-	2.91	51.57	45.41
0.4	0.90	0.28	-	-	0.28	54.61	44.48
0.9	0.90	0.06	0.09	-	0.85	56.22	42.79
0.1	0.95	-	-	-	0.62	54.33	45.05
0.4	0.95	0.63	-	-	0.29	55.26	43.81
0.9	0.95	0.34	-	-	0.29	56.08	43.29

Table 6.10 Procurement portfolio composition: IC = 3000 USD/mt

Test Case		Maturity of contract					
λ	α	M2	M3	M4	M5	M6	Spot
0.1	0.90	-	-	-	-	46.21	53.79
0.4	0.90	-	-	-	-	46.75	53.25
0.9	0.90	-	-	-	-	47.65	52.35
0.1	0.95	-	-	-	-	46.86	53.14
0.4	0.95	0.20	-	-	0.16	46.90	52.73
0.9	0.95	0.59	-	-	-	47.45	51.96

It can be observed that when the inventory cost is reduced, a greater amount is invested in long-term contracts. In general, values increase from about 46-48 % to 53-57 % and 58-61 % for 800 and 200 USD/mt respectively. To exemplify, for a risk weight of 90 % and a confidence level of 95 %, the procurement through long term contracts increases from 48.04% to 56.71 % and 60.53 % when the inventory is reduced from 3000 to 800 and 200 USD/mt respectively. This result is intuitive, as long-term contracts to a larger degree will utilise storage (i.e. all contracts include a fixed amount per delivery). By reducing the storage cost, more long-term contracts will be entered. For all degrees of risk the model results also indicate the longest maturity of contracts (i.e. 6 months), with few exceptions. However, it can be observed that more contracts with a maturity of 5 months are entered (i.e. additional purchase through M5 contracts) when reducing the inventory cost. This transition to enter shorter long-term contracts for certain scenario outcomes can also be explained by the reduced storage cost. Essentially, when storage is cheaper, the M6 contracts are in some scenarios combined with contracts of lower maturities. With that, the model is trying to exploit price developments to a greater extent by entering a more varied contract assortment. However, the increase of long-term contracts with lower maturity is marginal and the percentage is very low compared to the percentage share of M6 contracts. Upon that, it should also be observed that only one contract per material can be entered at each decision node in the scenario tree due to the SOS1 implementation. With this restriction in mind, it is clear that in most scenarios the M6 contract is necessary to cover the material demand, leaving the solver with no option to enter additional long-term contracts with lower maturities. Consequently, M6 contract are largely selected.

It should be noted that when the inventory cost is reduced closer to 0, the solver struggles to reduce the gap between the upper and lower bound. Solutions in Table 6.8 are presented with gaps considerably higher than 1 %, some up to 35 % (i.e. after running the model for 3 days, the best and last solution is found after 2.5 hours. For the rest of the time, the solver attempts to reduce the gap). This increase in gap can be explained due to the increased complexity of the problem. When storing material becomes a cheaper option, it is clear that is it optimal to use more combinations of contract maturities. Smaller percentages of long-term contracts with lower maturities can clearly be observed for lower inventory values (i.e percentages up to 3.25 %). That is, contracts with maturities 2 to 5 months are utilised for specific scenario outcomes. The following analysis incorporates a scenario analysis where particular outcomes are investigated with a focus on material delivery.

Material Delivery - Scenario specific analysis

A detailed investigation on the material delivery plan and shortage is conducted in this analysis. The test case comprises a risk weight of 90 % and a confidence level of 95 %. We present the planned material delivery for three specific scenario outcomes with different characteristics. The scenario outcomes are named S1, S2 and S3 respectively and are presented in Figure 6.15. While prices in S1 demonstrate a clear peak in time period 4, the prices in S2 increase during the course of the planning horizon. Prices in P3 also display an increasing tendency and is the scenario with the highest average prices. An overview of the delivery plan is shown for the three scenarios in Figure 6.16, Figure *6.17* and Figure *6.18* respectively and is represented for each month of the planning horizon. It should be noted that the schedule does not illustrate inventory values, only the material delivery plan based on entered contracts. Initial contracts entered before the planning horizon, are also included. We should also point out that since this is a stochastic model, the solution for one scenario outcome partly has the same solution as other outcomes. That is, due to the structure of the model, there exist equality of solution variables across different scenarios when they share the same history, or equivalently, are associated with the same nodes in the event tree. All solutions but the ones in stage 6 (i.e. decision variables in stages 1-5) are therefore determined while considering multiple scenarios. This should be kept in mind when analysing the outcome of specific scenario outcomes.

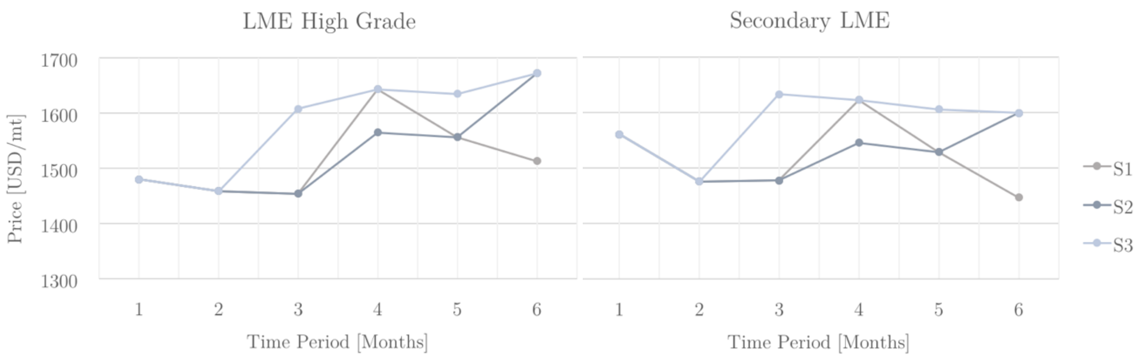

Figure 6.15 Prices for the scenario specific analysis

On average 14.34 contracts are entered for the planning period, where, on average, 60,5 % of the contracted material is through long-term contracts and 39.5 % is spot purchases. It is clear that the model favours spot purchases for the first two time periods and purchase through a mix of long-term and spot contracts in time period 3 to 5. With that, a contract entered through long-term contract has the first delivery the nest time period. For instance, the material with scheduled delivered in time period 4 is contracted in time period 3. Contrary, the spot purchases are contracted in the same time period. For the rest of the planning horizon (i.e. time period 6 to 12), only long-term contracts can be used to cover the material demand.

When comparing the delivery plan for the three scenario outcomes, it is apparent that more spot purchases are made for Mix1 in scenario S3. Based on the shortage graph in Figure 6.20, this scenario is the only one not short of Mix1 in time period 3. Following this purchase is a drop in the securing of long-term Mix1 contracts for periods 5-12, justified by the considerable increase in LME High Grade for periods 3-6. Contrary, most long-term purchases are made for S1, entered in time period 3, where the prices are the lowest and will increase considerably the following time period. An additional spot purchase is made of Mix1 in time period 5, S2. Following, is a lack of long-term Mix1 purchases detected in time period 11. This can be linked to the rising price development after time period 3 for LME High Grade. It should be noted that making an analysis based on price development alone is challenging as inventory and varying material used for production also affects the model solution, making this a complex study. To enrich this analysis, the next part concentrates on an inventory and shortage study for the same three scenario outcomes.

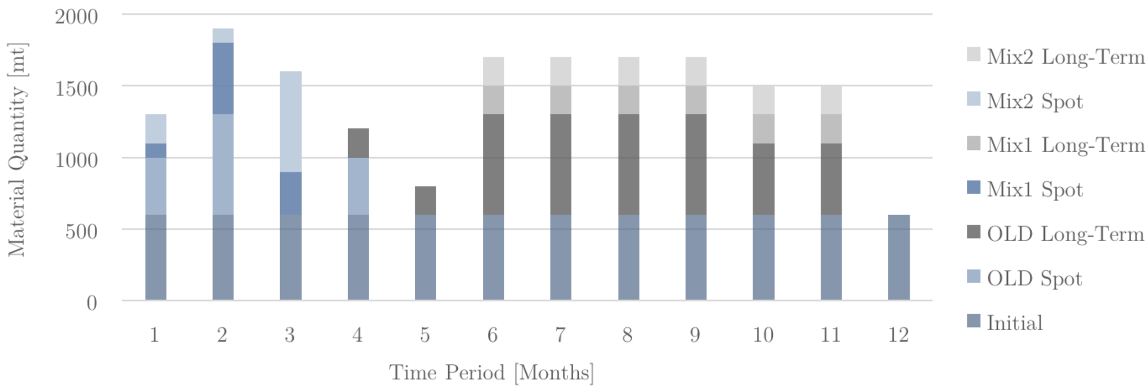

Figure 6.16 Material delivery plan for scenario outcome S1

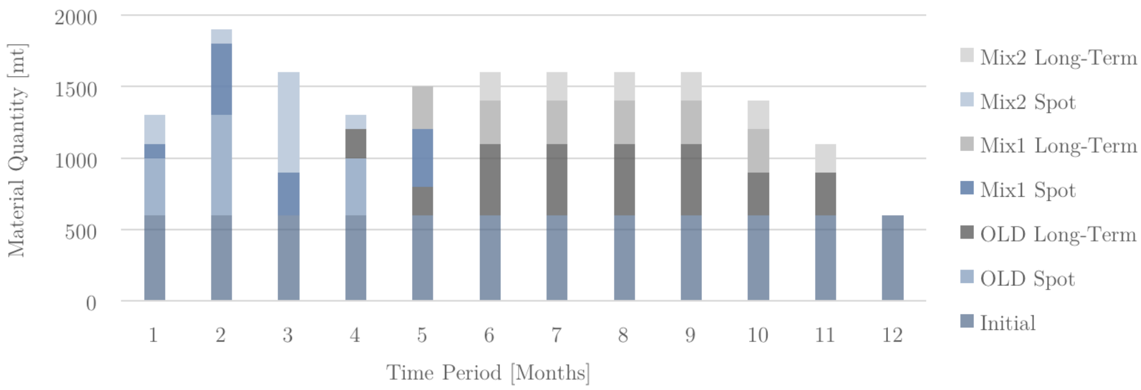

Figure 6.17 Material delivery plan for scenario outcome S2

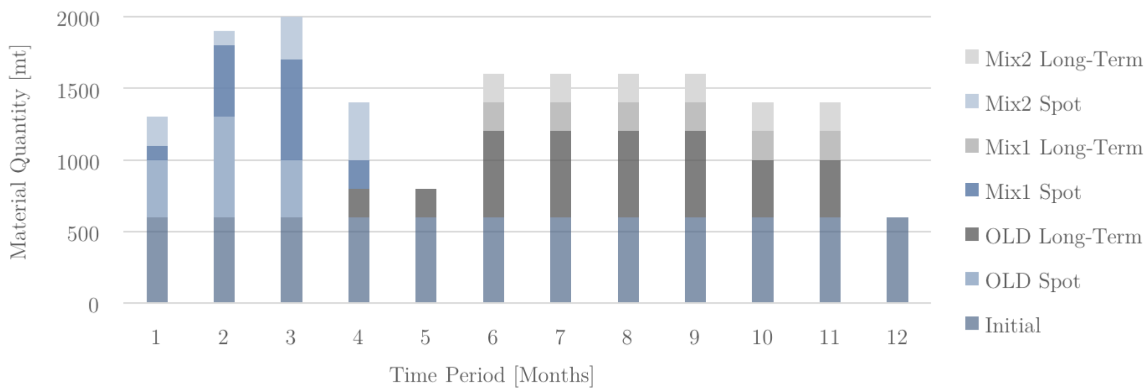

Figure 6.18 Material delivery plan for scenario outcome S3

Figure 6.19 displays the inventory level for the scenario outcomes (i.e. clustered as S1, S2 and S3 respectively) and each material (i.e. stacked). It can be observed that the inventory level for the three selected scenario outcomes are, overall, very similar. For time periods 1 and 2, the inventory level and deliveries are in fact identical for the three scenario outcomes. This is expected, as the presented scenario outcomes share the same stochastic data for these time periods (see Figure 6.15). Due to the information structure of the problem, the complete solution will be identical for all scenario outcomes in time period 1, while the solution in time period 2 can vary slightly (see Figure 6.20). For time period 3, the total inventory level is also similar for the scenario outcomes. However, the delivery and quantity for production vary (i.e see the martial delivery plan and shortage), while the total inventory is kept stable. Looking onward, some differences can be noted. For instance, S1 has the highest inventory for the five last time periods. It is clear from the delivery schedule that S1 also has largest deliveries for time period 8 to 12, so this is expected. Another example is the difference in inventory value in time period 5. In time period 5, the delivery scheduled for S2 is highest, and consequently S2 also has the highest inventory. In general, it is easy to see a relationship between the material delivery plan and the inventory levels.

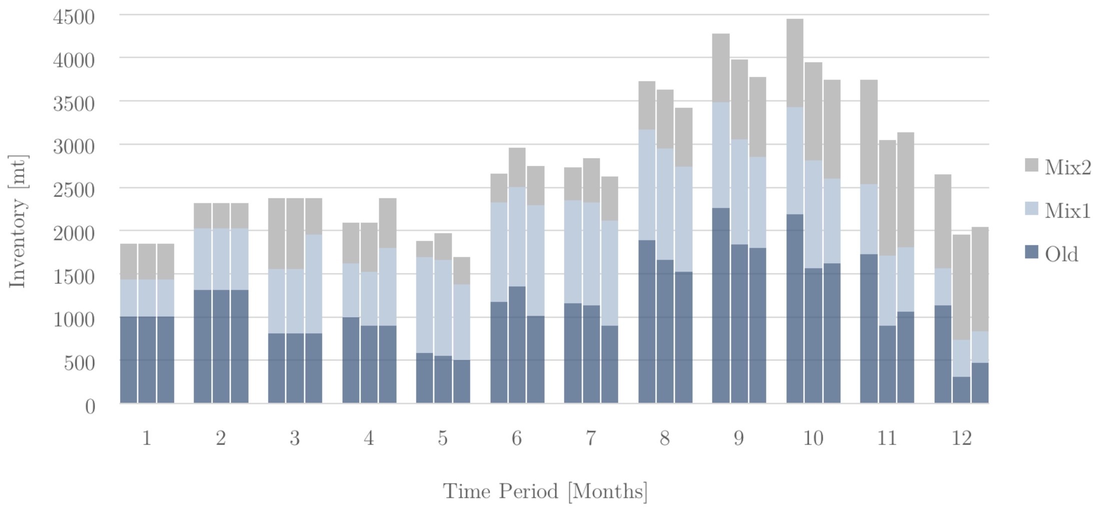

Figure 6.19 Inventory for the scenario outcomes S1, S2 and S3 (clustered respectively)

Figure 6.20 displays the shortage for the specific scenario outcomes. It is clear that the optimal solution includes shortage in all of the scenario outcomes. Numerous explanations can support this result. Firstly, the observed shortage in time periods 10, 11 and 12 for scenarios S1, S1 and S2 respectively, can be explained due to the limited flexibility given for later time periods. As a long-term contract must be entered in time period 6 or earlier to cover the second half of the horizon. With a fixed-delivery for all time periods after this point in time, an expensive oversupply of material would be necessary to cover all demand. It can therefore be more cost effective to accept some shortage. It is therefore clear that the model relies on the initial contracted volume and inventory to a large degree, and undertakes some shortages. The shortage cost for time period 7-12 is set lower than the shortage cost for time period 1-6 precisely for this reason.

Furthermore, due to the large variety in quantities included in the contract selection (i.e. multiples of hundreds), it can be cheaper to purchase less than the required material demand rather than purchasing too much. Accordingly, the use of alternative primary material and alloying elements are used. In our results, this is a likely explanation for all shortages below 100 mt (e.g. all shortage for Mix2). A model with more contracting options (i.e. higher distribution of quantities and more contract nodes) could possibly produce solutions with less cost and less shortage, however, this would also affect the complexity of the problem and the solution time would likely increase considerably. Heuristics might then be required to solve the problem within reasonable time. Nevertheless, all material at HRM today is contracted in larger quantities (i.e. 50-100 mt) and smaller contract purchases are not common practice in the industry due to transaction and transport costs. These types of shortages are therefore expected.

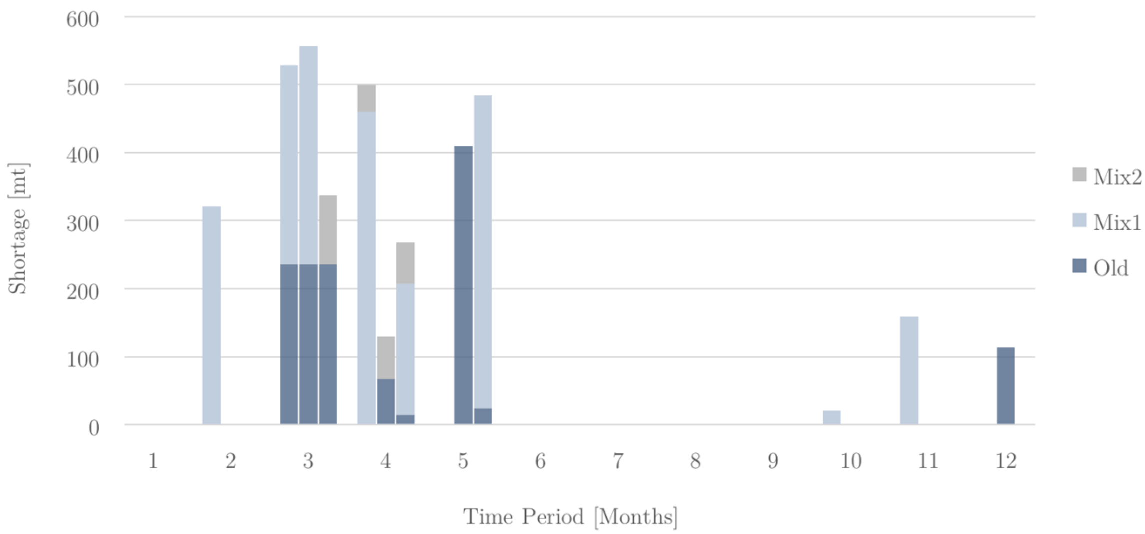

Figure 6.20 Shortage for the scenario outcomes S1, S2 and S3 (clustered respectively)

Some major shortages are also present. Most of these shortages are observed in time period 3, 4 and 5 for Old and Mix1. If we look at the material demand for the planning horizon, presented in Figure 6.6, it is clear that the highest demand occurs in time period 3, with a continuing high demand for time period 4. The solver therefore decides to utilise alternative material for Old in time period 3 for all the presented scenario outcomes and combines different shortages for Mix1 in the scenario outcomes in time period 2, 3, 4 and 5. Upon that, time period 3 has an unusual high material demand of both Old and Mix1 and the available material contracts do not support such large quantities (see Appendix B). A larger selection of contracts with higher contract volumes could reduce these peak shortages but would again increase the complexity of the problem. However, the contract assortment has been created in agreement with historical purchasing quantities made by HRM and it could be argued that the shortage is in line with current practice.

Today, HRM relies on primary material and alloying elements to a large degree even though scrap is a cheaper alternative in terms of purchasing cost. This is because the usage of primary material and alloying elements gives more security in terms of product quality (see Section 2.2). Currently HRM's has a practice of using more than 50 % of primary aluminium and alloying elements in the remelting process depending on final product. The rest constitutes scrap. Further, when there is not enough scrap material in stock, up to 100

% of their production can be based on alternative material. The results of this analysis show a significant improvement to such outcomes as none of the scenario outcomes comprise a shortage higher than 49 %. This shortage occurs in a time period with unusually high demand (i.e. Old in time period 3). In order to remove shortage completely, a larger selection of contracts must be constructed with a higher distribution of quantities available. As discussed above, this will result in a more complex problem that is likely to be difficult to solve to optimality within reasonable time. If larger scrap contract agreements were utilised, the consideration of scrap availability would also have to be included.

Comparing the Price Instances

The following analysis focuses on comparing model results for the different price instances: P1, P2 and P3. Through an analysis testing for different confidence levels and risk weights, it is clear that the solution does not change considerably for the three price instances. For the presented analysis, a confidence level of 95 % is used with a risk weight factor of 0.9, as a higher level of risk aversion evidently gives more variation in the solution between the three price instances. The expected price development for the cases is given in Table 6.1, and is illustrated in Figure 6.2, Figure 6.3 and Figure 6.4. Testing the model using three problem instances with different price data is completed to illustrate procurement strategy variation. Results are presented in Table 6.11, showing the average number of signed contracts for the same risk weight and percentile. Table 6.12 shows how the entered contracts differ in terms of contracted tonnage in long-term and spot contracts given in percent.

Table 6.11 Average number of signed contracts for P1, P2 and P3 ($\lambda = 0.9$, $\alpha = 0.95$)

		Time Period						
Test Case		1	2	3	4	5	6	Total
P1	Long-term	-	-	-	0.15	2.01	-	14.84
	Spot	3.00	3.00	3.00	2.82	0.85	0.01	
P2	Long-term	-	-	0.11	0.63	1.96	-	13.94
	Spot	3.00	3.00	2.78	2.00	0.46	-	
P3	Long-term	-	-	-	0.11	2.03	-	14.43
	Spot	3.00	3.00	3.00	2.52	0.77	0.02	

Table 6.12 Percentage of the material contracted in spot and long-term contracts

Test Case	P1	P2	P3
Long-term	56.9 %	61.3 %	56.2 %
Spot	43.1 %	38.7 %	43.8 %

From the results in Table 6.11, a shift can be observed in time period 3 where we sign 2.78 spot contracts against 0.11 long term contracts for test case P2. This differs from the other price instances, where 3 spot contracts are entered. This modification can be explained by the observation that both spot values clearly increase during the planning horizon in P2. When Secondary LME is increasing, it cheaper to cover material demand through fixed price long-term contracts. Contrary, P1 and P3 have decreasing spot prices. Another explanation supporting the extra purchase at stage 3 is the registered standard deviation for the sample prices. For test case P2 the standard deviation is considerably higher for both spot prices: 78.4 and 81.9 USD (see Section 6.2). With a higher standard deviation, it is expected that we see a shift towards long-term contracts. This is also confirmed in Table 6.12, where it is clear that the results of test case P2 have the highest procurement through long-term contracts of 61.3 %, which is 4.4 % and 5.1 % higher than for P1 and P3 respectively. Further, for P1, a total amount of 8803 mt is contracted, for P2, 8592 mt is contracted and for P3, 8795 mt is contracted. Following, 14.8, 13.9 and 14.4 contracts are entered for the three test cases. That is, most material is purchased for P1. This is expected, as P1 has the lowest average price of the three price scenarios with a 33.3 % and 14.0 % lower average price of long-term and spot respectively compared to P2, and a 15.7 % and 15.3 % lower average price of long-term and spot respectively compared to P3.

Budget and storage sensitivity analysis

The model is also tested for budget and storage sensitivity respectively, by conducting several runs on the P1 instance with a confidence level 95 % and a risk weight of 80 %. Figure 6.21 displays a comparison of objective function values after tightening the budget capacity constraints stepwise.

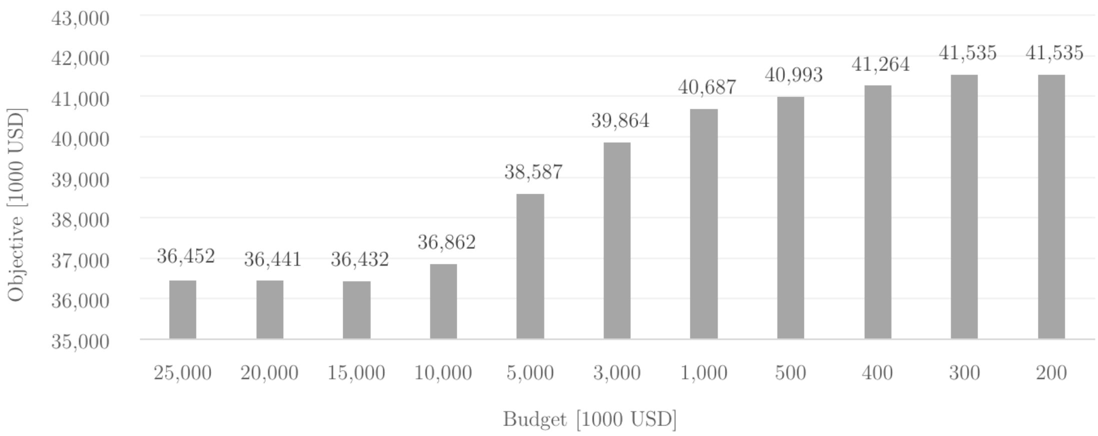

Figure 6.21 Variation in objective value when tightening the budget constraints

For all of the test instances, the budget is set to a value of 25 million USD. However, the constraint is not binding and the budget must be reduced below 10 million USD for the objective value to increase significantly (see Figure 6.21). Figure 6.22 presents a comparison of objective function values when tightening the storage capacity constraints stepwise. It is clear that significant reduction in the objective value can be achieved while increasing the storage up to 1,800 tonnes. With a storage of less than 1000 mt, the problem becomes infeasible due to the fixed initial inventory and contracts entered before the planning horizon. It should be noted that the storage is measured in weight and not volume for this analysis (see Table 6.4). In a further investigation, a 13 % increase in the objective value is observed when reducing the budget by a factor of 50 (i.e. from 10,000 to 200 USD). Likewise, we observe a 33 % increase in the objective when the storage capacity is reduced with a factor of 1.64 (i.e. from 1800 to 1100 mt). This indicates that the storage capacity is a bottleneck in the value chain to a greater degree than the budget capacity.

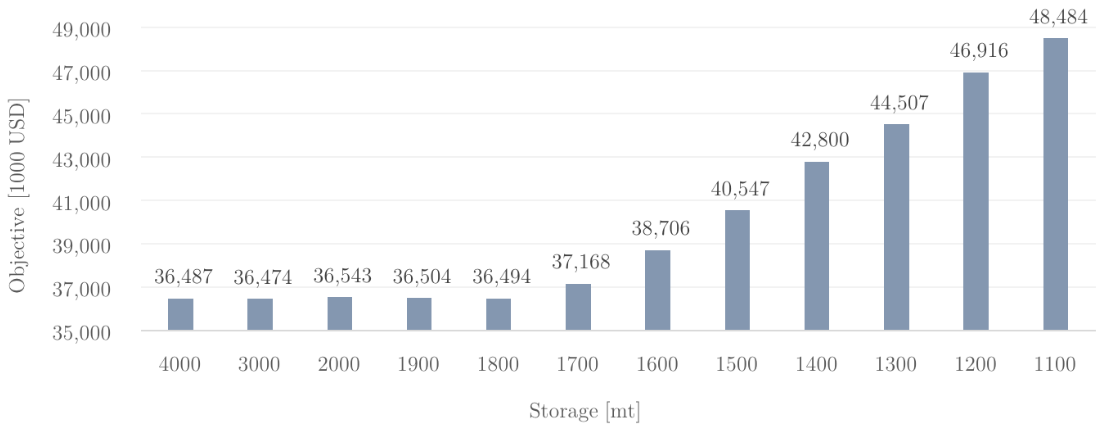

Figure 6.22 Variation in objective value when tightening the storage constraints

Comparing the results with current practice

Today, long-term contracts constitute about 20 percent of the procured scrap at HRM, while the rest is purchased on the spot market through fixed price contracts. Historically, HRM only entered spot contracts, with a few exceptions. If we compare the purchase through long-term contracts for the different inventory cost values, it is clear that this value influences the solution greatly. If we further compare the solution with historical practice at HRM, it is clear that the choice of a higher inventory cost reflects their wishes to limit inventory. The restricted purchase through long-term contracts can be explained through the relatively high inventory cost of 3000 USD/mt per month. With high storage costs, it is expensive to store the surplus material resulting from long-term contracting. It can also be observed that when inventory cost is reduced, the model recommends a much higher procurement through long-term contracts. Especially, if HRM can increase their storage and allow for higher inventories, costs can be reduced. It is also clear from the storage sensitivity analysis that the storage capacity is central in optimal scrap procurement and inventory management.

7 Concluding Remarks

We present in this thesis a multistage stochastic programming model for determining the optimal hedging decisions of an aluminium remelter. We have used the model to propose contracting strategies with different risk characteristics. In general, the results in this thesis confirm what intuition suggests; hedging with the use of forward contracts significantly reduces the risk in terms of CVaR. That is, when increasing the producer's level of risk aversion, the amount procured though spot purchases decrease and a larger share of the procurement goes towards the lower risk alternative of fixed price long-term contracts.

Hedging decisions are complex, requiring a decision on both when to enter into a contract, for how long and the volume traded. The model presented in this thesis can provide valuable decision support to the choice of an optimal hedging strategy according to the company's risk preferences. That is, deriving static hedging positions. The model can also be used for scenario and sensitivity analysis where risk preferences are assessed. This can also help the decision maker determine a suitable risk level.

Results in this thesis demonstrate that HRM should consider a fairly diversified procurement portfolio from a low degree of risk aversion. As the degree of risk aversion increases, HRM should reduce its exposure in the spot market and enter more forward contracts. Overall, the percentage procurement through forward contracts increases by 43.89 % when the risk level goes from risk neutrality to 100 % risk averse (at 95 % CVaR). The degree of risk aversion also influences the maturity of the chosen long-term contracts: the higher the degree of risk aversion, the longer the maturity of the forward contracts.

The percentage procurement through spot purchases vary between 39.47 % and 53.79 % depending on the test case, showing a clear benefit from diversification and hedging. The model also demonstrates procurement strategy for different price data by shifting towards long-term contracts for increased variation in the prices and schedule more material purchase when the average prices are lower. Furthermore, storage capacity has been detected as a bottleneck in the problem and the model solution is significantly affected by inventory cost. Essentially, when storage is cheaper, the model is trying to exploit price developments to a greater extent by entering a more varied contract assortment and increasing the total amount invested in long-term contracts.

The presented evaluation is static, whereas HRM in reality would make hedging decisions continuously. A natural extension to this work is therefore to utilise the model in a more dynamic setting. This could be done by evaluating the model in a rolling horizon environment, and study how such a dynamic setting might affect the hedging strategies. Another path for future research is to investigate price forecasting and scenario generation, as this is an important prerequisite for this model to be purposefully utilised. Conducting an analysis with longer runs, and having a stricter acceptance level for the gap, would also be useful in analysing the efficiency and ability of the model.

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Appendix A: Mathematical Model

Sets

$C(n)$	Set of children nodes (successors) of node n, $n \in N$.
K	Set of representative contracts k considered during the planning horizon.
K^F	Set of fixed price contracts k based on Secondary LME, $K^F \subset K$.
K^I	Set of initial contracts k entered before the planning horizon.
K^V	Set of variable price contracts k based on LME High Grade, $K^V \subset K$.
M	Set of materials m.
N	Set of event nodes n of the scenario tree.
N_i	Set of event nodes at stage i in the scenario tree.
N^K	Set of event nodes n of the scenario tree where contracts are offered.
$N(s)$	Set of nodes belonging to the path forming scenario s, $N(s) \subseteq N$.
S	Set of scenarios s representing the stochastic outcomes.
$S(n)$	Set of scenarios passing through event node n of the scenario tree, $S(n) \subseteq S$.
T	Set of time periods t of the planning horizon.
T_k^D	Set of delivery time periods associated with initial contract k.
T_{kn}^D	Set of delivery time periods associated with contract k entered at node n.

Indices

i	Stage of the scenario tree, $i = 1$ ... I.
k	Contract index, $k \in K$.
m	Material index, $m \in M$.
n	Event node index for the scenario tree, $n \in N$.
s	Scenario index, $s \in S$.
t	Time period index, $t \in T$.

Parameters, constants and coefficients

A_m^F	Fixed percent of LME High Grade for material m for fixed price contracts.
A_m^V	Fixed percent of Secondary LME for material m for variable price contracts.
B	Total budget for material purchase for the entire planning horizon.
C_{mt}^S	Cost per mt of being short of material m in time period t.
C_k^T	Total transportation cost in contract k.
D_{mt}	Required scrap quantity of material m in time period t.
H_m	Inventory holding cost per mt of material m.

I^0_m	Initial inventory level of material m.
L_k	Number of deliveries in contract k.
M_k	Material included in contract k.
P^n	Conditional probability of reaching node n from its predecessor.
Q_k	Tonnage per delivery in contract k.
Q^S	Total available storage space for the scrap material.
R_t	Discount rate in time period t.
$\bar{T}$	The last time period of the planning horizon.
$T(n)$	The time period of node n.
V_m	Density parameter [m^3/mt] for scrap material m.
W	Weight factor for the residual scrap value at the end of the time horizon.
α	Confidence level (percentile) for VaR and CVaR.
λ	Weight for HRM's risk preference, $\lambda \in [0,1]$.

Stochastic Data

C^F_{ts}	Spot value of in LME High Grade time period t and scenario s.
C^V_{ts}	Spot value of Secondary LME in time period t and scenario s.

Decision variables

c_{kns}	Cost generated from entering contract k at node n in scenario s.
i_{mts}	Inventory level of material m at the beginning of period t in scenario s.
o_{in}	Objective function value at stage i and node n of the scenario tree.
q_{mts}	Quantity of material m used for production in time period t and scenario s.
x_{mknt}	Quantity purchased of material m through contract k at node n for delivery in time period t.
x^I_{kmt}	Quantity purchased of material m through initial contract k for delivery in time period t.
y_{in}	Cost exceedance with respect to CVaR at stage i and node n of the scenario tree.
z_{in}	Auxiliary variable for modelling CVaR, also representing VaR.
δ_{kn}	Binary variable, 1 if a contract k is entered at node n, 0 otherwise.

Objective Function

$$\min \quad (1-\lambda)\left[\frac{1}{|S|}\sum_{s\in S}\left(\sum_{k\in K}\sum_{n\in N_1} c_{kns} + \sum_{m\in M} H_m i_{m,1,s} + \sum_{m\in M} C^S_{m,1}(D_{m,1} - q_{m,1,s})\right) + \left(\sum_{n\in N_2} P^n o_{2,n}\right)\right] + \lambda\left(z_{2,1} + \frac{1}{1-\alpha}\sum_{n\in N_2} P^n y_{2,n}\right)$$

Nested CVaR Constraints

$$y_{i+1,\acute{n}} \;\geq\; o_{i+1,\acute{n}} - \; z_{i+1,n} \qquad i = 1 \ldots I-1, n \in N_i, \acute{n} \in C(n)$$

$$o_{in} = \; (1-\lambda)\left[\frac{1}{|S(n)|}\sum_{s\in S(n)} R_{T(n)}\left(\sum_{k\in K} c_{kns} + \sum_{m\in M} H_m i_{mT(n)s} + \sum_{m\in M} C^S_{mT(n)}(D_{mT(n)} - q_{mT(n)s})\right) + \sum_{\acute{n}\in C(n)} P^{\acute{n}} o_{i+1,\acute{n}}\right] + \lambda\left(z_{i+1,n} + \frac{1}{1-\alpha}\sum_{\acute{n}\in C(n)} P^{\acute{n}} y_{i+1,\acute{n}}\right) \qquad i = 2 \ldots I-1, n \in N_i$$

$$o_{In} = \left(R_{\bar{T}}\sum_{k\in K} c_{knS(n)} + \sum_{m\in M} H_m i_{m\bar{T}S(n)} + \sum_{m\in M} C^S_{m\bar{T}}(D_{m\bar{T}} - q_{m\bar{T}S(n)}) - \; W\sum_{m\in M} C^V_{\bar{T}S(n)}(i_{m\bar{T}S(n)} - \; q_{m\bar{T}S(n)})\right) \qquad n \in N_I$$

Contract Fulfilment

$$x_{knmt} = Q_k \delta_{kn} \qquad k \in K, n \in N^K, m \in M_k, t \in T^D_{kn}$$

$$x^I_{kmt} = \; Q_k \qquad k \in K^I, m \in M_k, t \in T^D_k$$

Contract Expenditures

$$c_{kns} = \delta_{kn}(L_k Q_k A^F_{M_k} C^F_{T(n)s} + \; C^T_k) \qquad n \in N^K, k \in K^F, s \in S(n)$$

$$c_{kns} = \; \delta_{kn}(Q_k A^V_{M_k}\sum_{t\epsilon T^T_{kn}} C^V_{ts} + C^T_k) \qquad n \in N^K, k \in K^V, s \in S(n)$$

Inventory Balance

$$i_{mts} = I_m^0 \ + \sum_{k\epsilon K}\sum_{n=1} x_{knmt} + \sum_{k\epsilon K^I} x_{kmt}^I \qquad m \in M, t = 1, s \in S$$

$$i_{m,t+1,s} = i_{mts} - \ q_{mts} + \sum_{k\epsilon K}\sum_{n\epsilon N(s)} x_{knm,t+1} + \sum_{k\epsilon K^I} x_{km,t+1}^I \qquad m\epsilon M, t \in T\backslash\{\bar{T}\}, s \in S$$

$$q_{mts} \leq \ min(i_{mts}, D_{mt}) \qquad m \in M, t \in T, s \in S$$

$$\sum_{m\epsilon M} V_m i_{mts} \ \leq \ \ Q^S \qquad t \in T, s \in S$$

$$\sum_{n\epsilon N(s)}\sum_{k\epsilon K} c_{kns} \leq \ B \qquad s \in S$$

Non-anticipativity

$$\frac{1}{|S(n)|} \sum_{s'\epsilon S(n)} (i_{mts'},, q_{mts'} \ ) = \ (i_{mts,}, q_{mts}) \qquad m \in M, t \in T, n \in N, s \in S(n),$$

Variable Constraints

$$i_{mts} \ \geq 0 \quad m \in M, t \in T, s \in S$$

$$q_{mts} \geq 0 \quad m \in M, t \in T, s \in S$$

$$x_{kmnt} \geq 0 \quad k \in K, m \in M, n \in N^K, t \in T_{kn}^D$$

$$x_{kmt}^I \geq 0 \quad k \in K^I, m \in M, t \in T_k^D$$

$$c_{kns} \geq 0 \quad k \in K, n \in N^K, s \in S(n)$$

$$o_{i+1,n} \geq 0 \quad i = 1 \ldots I-1, n \in N_{i+1}$$

$$y_{i+1,n} \geq 0 \quad i = 1 \ldots I-1, n \in N_{i+1}$$

$$z_{i+1,n} \geq 0 \quad i = 1 \ldots I-1, n \in N_i$$

$$\delta_{kn} \in \{0,1\} \quad k \in K, n \in N^K$$

Appendix B: List of Offered Contracts

Table B.1 List of Offered Contracts

Contract	Material	Type	Deliveries	Volume
1	Old	1	2	200
2	Old	1	2	300
3	Old	1	2	400
4	Old	1	2	500
5	Old	1	3	200
6	Old	1	3	300
7	Old	1	3	400
8	Old	1	3	500
9	Old	1	4	200
10	Old	1	4	300
11	Old	1	4	400
12	Old	1	4	500
13	Old	1	5	200
14	Old	1	5	300
15	Old	1	5	400
16	Old	1	5	500
17	Old	1	6	200
18	Old	1	6	300
19	Old	1	6	400
20	Old	1	6	500
21	Old	2	1	100
22	Old	2	1	200
23	Old	2	1	300
24	Old	2	1	400
25	Old	2	1	500
26	Old	2	1	600
27	Old	2	1	700
28	Mix1	1	2	200
29	Mix1	1	2	300
30	Mix1	1	2	400
31	Mix1	1	2	500
32	Mix1	1	3	200
33	Mix1	1	3	300
34	Mix1	1	3	400
35	Mix1	1	3	500
36	Mix1	1	4	200
37	Mix1	1	4	300

38	Mix1	1	4	400
39	Mix1	1	4	500
40	Mix1	1	5	200
41	Mix1	1	5	300
42	Mix1	1	5	400
43	Mix1	1	5	500
44	Mix1	1	6	200
45	Mix1	1	6	300
46	Mix1	1	6	400
47	Mix1	1	6	500
48	Mix1	2	1	100
49	Mix1	2	1	200
50	Mix1	2	1	300
51	Mix1	2	1	400
52	Mix1	2	1	500
53	Mix1	2	1	600
54	Mix1	2	1	700
55	Mix2	1	2	200
56	Mix2	1	2	300
57	Mix2	1	2	400
58	Mix2	1	2	500
59	Mix2	1	3	200
60	Mix2	1	3	300
61	Mix2	1	3	400
62	Mix2	1	3	500
63	Mix2	1	4	200
64	Mix2	1	4	300
65	Mix2	1	4	400
66	Mix2	1	4	500
67	Mix2	1	5	200
68	Mix2	1	5	300
69	Mix2	1	5	400
70	Mix2	1	5	500
71	Mix2	1	6	200
72	Mix2	1	6	300
73	Mix2	1	6	400
74	Mix2	1	6	500
75	Mix2	2	1	100
76	Mix2	2	1	200
77	Mix2	2	1	300
78	Mix2	2	1	400
79	Mix2	2	1	500
80	Mix2	2	1	600
81	Mix2	2	1	700