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A Stochastic Dynamic Programming Approach to the Bidding Problem in the Intraday Electricity Market

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Preface

This thesis concludes our Master of Science at the Norwegian University of Science and Technology (NTNU) with specialization in Applied Economics and Optimization under the department of Industrial Economics and Technology Management. It is a continuation of the preliminary project *Bidding in Elbas*, by Bovim and Næss (2017).

We would like to thank our supervisor Professor and Director of CenSES Asgeir Tomasgard as well as co-supervisors Professor Stein-Erik Fleten and PhD Candidate Ellen Krohn Aasgård at NTNU for their valuable guidance. The privilege of discussing the topic with three experienced academics within the field have created interesting discussions of great complexity. We appreciate the helpful feedback we have received.

We would also like to thank TrønderEnergi for providing data to perform a realistic market analysis. A special thanks is given to Gunnar Aronsen from TrønderEnergi for his guidance and way of providing insights into industrial challenges.

Trondheim, 11^{th} of June, 2018

Abstract

The purpose of this thesis is to provide flexible market participants decision support in Elbas, in such a manner that they can take advantage of energy surpluses or deficits in the market after the spot clearing. A mathematical optimization model is constructed, integrating market dynamics and optimal scheduling into a multisequential resource allocation problem. Variable production costs and unit commitment costs are included, while market prices are modelled as stochastic processes with the Markov property. The objective is to *maximize expected profits* in Elbas, determining the optimal timing and volumes of bids.

Stochastic Dynamic Programming (SDP) is utilized due to its strength in solving sequential decision problems. The continuous double auction in Elbas is modelled as discretized time steps, where new bids can be made at the beginning of each period. The real world problem consists of 24 hours of production, each regarded as individual *products* subject to trade in each time step of the auction. *The horizon of the model corresponds to the time period where at least one product is available for trade in Elbas, namely 34 hours*. The products have individual production commitments from the spot clearing, resulting in different remaining production capacities available for trade. Hence, the state space is given by 24 variable production capacities and 24 variable market prices. The state space is discretized, and the resulting state space increases exponentially with the number of products.

To handle the dimensional challenge of the state space, an *approximated* version of the problem is implemented applying aggregation of 6 products into a product block. The size of the state space is consequently reduced from 17×10^{31} to 16×10^{4}. The discretized time steps in the auction are chosen to correspond to the length of a product block of 6 hours. The time period for *trade* extends over a longer time horizon than the time period of *production*, so the problem is implemented with *4 products* subject to trade in *6 time steps*. Optimal sequential decisions are found for the *approximated* problem by a backward recursion algorithm, evaluating the expected value (EV) of each decision. A time dependent policy for decision making is constructed, mapping all possible states of the *approximated* problem to a unique bidding decision in each time step. The policy is applied as a *heuristic* on the real world problem.

All parameters necessary to initialize the model are considered constant throughout the auction period, while market prices and remaining production capacities are variables described by

the model state space. The model is run only once a day, even though stochastic, exogenous market prices arrive sequentially. During the day, the model collects the true market prices, and translates them into the corresponding state representation. Prices taking on any continuous value can can be understood and interpreted by the model - even spikes. At the arrival of new information the policy serves as a *contingency plan* over the modelling horizon without further computations needed.

In order to evaluate the constructed model, both the value function and policy are investigated. To evaluate how well the model solves the real world problem, a procedure to find statistical upper bound (UB) and lower bound (LB) of the optimal solution is presented. An adjustment heuristic to apply on the policy is found. This is to ensure a *feasible* lower bound to the maximization problem. To observe how policies respond to fluctuating market prices, policies for a number of days within selected test instances are constructed utilizing SDP. The main finding from the policy analysis is that the SDP model finds incentives for *arbitrage trading*, before it stabilizes production volumes according to production costs and market prices before the close.

The opportunities for a market participant to increase profits by strategic bidding in Elbas, are increasing as the volumes and frequencies of trades increase. This development is expected to continue by the introduction of an integrated market with most of Europe (XBID). As the frequency of trades increases, quick reactions to the market is essential. Utilizing software as decision support for trading, serves as a first step to automatic trading.

Sammendrag

Hensikten med denne masteroppgaven er å tilby fleksible markedsdeltakere beslutningsstøtte i Elbas, på en slik måte at de kan dra nytte av energioverskudd og -underskudd i markedet etter spotklarering. En matematisk optimeringsmodell er utviklet, og integrerer markedsdynamikk og optimal produksjonsplanlegging til et multisekvensielt ressursallokeringsproblem. Variable produksjonskostnader og oppstartskostnader er inkluderet, mens markedspriser er modellert som stokastiske prosesser med Markovegenskapen. Formålet er å *maksimere forventet profitt* i Elbas ved å bestemme optimale budtidspunkt og budvolum.

Stokastisk dynamisk programmering (SDP) er undersøkt ettersom metoden ofte fungerer godt på sekvensielle problemer. Den kontinuerlige doble auksjonen i Elbas er modellert med diskretiserte tidssteg, der nye bud gjøres i begynnelsen av hver tidsperiode. Det realistiske problemet består av 24 produksjonstimer, der alle er ansett som individuelle *produkter* tilgjengelige for handel i alle tidssteg i auksjonen. *Horisonten til modellen korresponderer til tidsperioden der minst ett produkt er tilgjengelig for handel i Elbas, det vil si 34 timer.* Produktene har individuelle produksjonsforpliktelser fra spotklareringen, som resulterer i ulike gjenværende produksjonskapasiteter tilgjengelig for handel. Dermed er tilstandsrommet gitt av 24 variable produksjonskapasiteter og 24 variable markedspriser. Diskretisering av tilstandsrommet resulterer i et tilstandsrom som vokser eksponentielt med antallet produkter tilgjengelig for handel.

For å håndtere dimensjonsutfordringer ved tilstandsrommet er en approksimert versjon av problemet implementert ved å aggregere 6 produkter i produktblokker. Størrelsen på tilstandsrommet er redusert fra $17\times^{31}$ til 16×10^4. De diskretiserte stegene i auksjonen er valgt slik at de korresponderer til lengden på en produktblokk på 6 timer. Tidsperioden for *handel* går over en lengre tidshorisont enn tidsperioden for *produksjon*, og problemet er dermed implementert med 4 *produkter* tilgjengelige for handel i 6 *tidssteg*. Optimale sekvensielle beslutninger er funnet for det *approksimerte* problemet ved hjelp av en algoritme som benytter bakoverrekursjon, som evaluerer forventningsverdien knyttet til hver beslutning. En tidsavhengig beslutningsregel er laget. Den knytter alle mulige tilstander i det *approksimerte* problemet til en unik beslutning i hvert tidssteg. Beslutningsregelen er benyttet som en *heuristikk* på det realistiske problemet.

Alle parametere nødvendig for å initialisere modellen, er antatt konstante gjennom auksjonspe-

rioden, mens markedspriser og gjenværende produksjonskapasiteter er variable beskrevet av tilstandsrommet til modellen. Modellen er kjørt en gang om dagen, selvom stokastiske, eksogene markedspriser ankommer sekvensielt. Utover dagen samler modellen inn sanne markedspriser og oversetter disse til korresponderende tilstandsvariable. Alle mulige kontinuerlige priser kan bli forstått og håndtert av modellen - til og med ekstremverdier. Når ny informasjon ankommer fungerer beslutningsregelen som et *oppslagsverk* uten at flere beregninger er nødvendig.

For å kunne evaluere modellen er både verdifunksjonen og beslutningsregelen undersøkt nærmere. For å si noe om hvor godt modellen løser det realisetiske problemet er en fremgangsmåte for å bestemme øvre og nedre grense til den optimale løsningen presentert. En justeringsheuristikk for å bruke sammen med beslutningsregelen er funnet. Dette sikrer en lovlig nedre grense på maksimeringsproblemet. For å observere hvordan beslutningsreglene responderer til svingninger i markedspriser er beslutningsregler for dager innen hver testinstanse funnet ved bruk av SDP. Hovedfunnet fra analysen av beslutningsregelen er at SDP-modellen finner insentiver til å *arbitrasjehandle*, før den stabiliserer produksjonsvolumet med tanke på produksjonskostnader og markedspriser innen stengingen av Elbas.

Mulighetene for en markedsdeltaker til å øke profitt gjennom strategisk budgivning i Elbas øker når volumet og hyppigheten av handler øker. Denne utviklingen er forventet å fortsette når det integrerte markedet med mesteparten av Europa (XBID) introduseres. Ettersom at hyppigheten av handler øker, er det behov for raske reaksjoner i markedet. Bruken av software som beslutningsstøtte for budgivning inngår som et første steg mot automatisert handel.

Contents

Glossary ix

Abbreviations x

1. Introduction 1

2. The Power Market and Power Production 4

2.1. Deregulated Markets and Market Coupling 5

2.1.1. Market Areas and Power Prices 6

2.1.2. Roles and Responsibilities 7

2.2. The Spot Market 8

2.3. Elbas - The Intraday Market 9

2.3.1. The Market Structure 9

2.3.2. Bidding Strategies 11

2.4. The Regulating Power Market 12

2.5. Power Production and Resource Management 12

2.5.1. Hydro Power Production 13

2.5.2. TrønderEnergi and the Industrial Case 14

3. Literature Review 16

3.1. Power Production and Trading 16

3.2. Resource Allocation 19

3.3. Uncertainties 19

3.4. Price Modelling 20

3.5. Optimization Models 22

3.5.1. Dynamic Programming 23

3.5.2. The Curses of Dimensionality 24

3.5.3. Dynamic Programming Approaches 24

3.6. Value Function and Bounds 26

4. Problem formulation 29

4.1. Notation 29

4.2. Problem Description 32

4.3. Assumptions . . . 33
4.4. Modelling . . . 34
4.4.1. The Double Time Dimension . . . 34
4.4.2. The Resource Allocation Problem . . . 36
4.4.3. Market Prices . . . 39
4.5. Mathematical Model . . . 44
4.5.1. State Space . . . 45
4.5.2. Decision Variables . . . 47
4.5.3. Exogenous Information Process . . . 47
4.5.4. Transition Function . . . 48
4.5.5. The Contribution and Objective Function . . . 48
4.5.6. The Three Curses of Dimensionality . . . 49
4.5.7. Aggregation . . . 53
4.6. Solution Method . . . 57

5. Computational Study **63**
5.1. Software Description . . . 63
5.2. Test Instances . . . 64
5.2.1. The Effect of Spot Prices and Water Values . . . 65
5.2.2. The Effect of Initial Spot commitment . . . 66
5.2.3. The Resulting Test Instances . . . 68
5.3. The Contingency Plan . . . 70
5.4. Bounds . . . 73
5.4.1. Lower Bound . . . 73
5.4.2. Upper Bound . . . 74
5.4.3. Adjustment Heuristic . . . 75
5.4.4. Analyzing the Properties of the Bounds . . . 77
5.5. Policy Trends . . . 81

6. Concluding Remarks **88**
6.1. Discussion . . . 88
6.2. Conclusion . . . 89
6.3. Future Research . . . 92

6.3.1. Stochasticity 93

Appendices **96**

A. Compact Mathematical Model **96**

B. From the Industrial Partner, TrønderEnergi **97**

C. Space Size of Original Problem **98**

References **100**

Glossary

area price The spot price in a market area.

available area Market area connected with sufficient available transmission capacity for the trade in question.

balance agreement Formal agreement with Statnett to gain trading access in the Norwegian wholesale electricity markets.

bid matrix Matrix containing each participant's required price for buying or selling different amounts of energy.

bid period The point in time when a commitment in the power market is made.

close Bid deadline prior to a production period.

continuous double auction Auction with immediate execution where both buyers and sellers submit offers.

Elbas Intraday market facilitated by Nord Pool.

market area Area with a common spot price set by the area's demand and supply.

natural filtration Time resolution of information arrival under realistic circumstances.

Nord Pool Market operator.

periodic double auction Auction with institutional trade determination and both buyers and sellers submit offers over some finite time horizon.

policy Maps state S_t to a unique decision $\mathbb{X}_t$.

production period Period of physical delivery of power.

system price Power price after spot clearing if unlimited transfer capacities and no congestion.

the regulating power market Final balance settlement.

the spot market A day-ahead market facilitated by Nord Pool.

Abbreviations

AC average cost

ADDP Approximate Dual Dynamic Programming

ADP Approximate Dynamic Programming

CI confidence interval

DP Dynamic Programming

EV expected value

HPP hydro power plant

LB lower bound

MC marginal cost

MDP Markov Decision Process

NAC non-anticipativity constraint

SDDP Stochastic Dual Dynamic Programming

SDP Stochastic Dynamic Programming

SO system operator

SRMC short-run marginal cost

TE TrønderEnergi

TSO transmission system operator

UB upper bound

1 Introduction

Over the last few decades, the Nordic and Baltic countries have coupled their energy markets into a common, deregulated market operated by Nord Pool. Their objective is to obtain a free market between market areas to increase efficiency and liquidity, as well as to create a more secure power supply. Energy production and consumption have resulted in increased concentration of greenhouse gases in the atmosphere, while energy demand is still increasing. Hence, renewable energy sources are introduced, continuously reducing the carbon footprint from the energy mix.

Belsnes et al. (2016) states that: "The behaviour of the market is expected to become much more volatile due to the transition toward more renewable power production in the energy systems." An increasing amount of renewable energy sources, specifically the increasing share of wind and solar power, makes actual production hard to predict, while the power system requires predictability and stability to ensure power balance and frequency control at all times. The result is an increasing amount of power trades in the *balancing markets* closer to the time of production, as the actual production capacities become more certain. While the largest volumes are still traded the day ahead of production, the frequency of trades and the volumes traded in the intraday market, Elbas, are steadily increasing. The opportunity to profit from Elbas trades are therefore more present now than before, and is assumed to be even more essential in the future as the share of renewables continues to increase.

Due to the lack of interest in the intraday market in the industry, existing research within mathematical optimization have so far focused more on technical aspects related to production and optimal scheduling rather than accurate modelling of the intraday market dynamics (Selasinsky, 2014). Löhndorf et al. (2013) consider the opportunity to profit in the intraday market as small, and that the market is only a way for a power producer to improve the production schedule from the initial spot commitment. Jiang and Powell (2015) do emphasize the opportunity to profit in the intraday market only, by buying energy at a low cost time, storing it in a battery and selling it at a higher price. However, Jiang and Powell (2015) only consider the opportunity to profit related to the times of charging and discharging, and do not cover the continuous double auction mechanism that is present in the intraday market. This introduces an additional timing dimension to the bidding problem in Elbas, of which this thesis contributes to cover.

Power producers with predictable production do normally not have a need to balance their commitments from the spot clearing, as all information relevant for optimal scheduling is available at the spot deadline. However, as long as there exists market participants with uncertain production, e.g. wind or solar energy producers, forecast errors will create energy deficits or surpluses in the market after the spot clearing. This creates opportunities for power producers with flexibility in production, as they can earn profits by offering their flexibility in the market. The industry already has well established mathematical optimization models and strategies for longer term planning and spot market bidding, but is starting to realize that increased volumes traded intraday introduce a potential to increase profits if modelled correctly. Today, bidding in the intraday market is motivated by a cost-minimization perspective and mainly based on individual operators' experiences (appendix B). Having access to optimization tools that ensures an expected profit will make it more attractive to participate in Elbas. The industry requests research on the topic of accurate intraday trading, as a measure of being proactive to changes and better positioned to maximize profits in the future (appendix B).

The purpose of this thesis is to provide flexible market participants decision support in Elbas, in such a manner that they can take advantage of energy surpluses or deficits in the market. The objective is to construct a stochastic optimization model for a sequential bidding problem, according to the market structure of continuous double auctions in Elbas. The study also strive to achieve an integrated model for accurate market dynamics and technical aspects related to production costs. Important aspects of the study include to gain in-depth insight of the problem structure and further to evaluate and handle the problem size.

This study contributes to the field of power production and trading in general, and Elbas participation specifically. The thesis provides increased insight to the opportunities to profit in the intraday market, when an increasing proportion of the energy mix originates from unpredictable energy sources. It is essential for power producers to adhere to these changes to remain competitive. Comparing to trivial bidding strategies applied today, the mathematical optimization model developed in this thesis provides decision support to gain an extra profit above marginal cost whenever the market allows to. Consequently, the model in this thesis contributes to change the cost-minimizing perspective of today, to a profit-maximization in the future.

The main contribution to literature from this study, is how the mathematical optimization model developed in this thesis emphasizes the stochastic price process of the continuous auctions.

This introduces a second dimension of time in the Elbas bidding problem, as an addition to the traditional scheduling problem. This study makes out a base for future research, shedding light on the potential to profit in Elbas, in addition to present modelling challenges that must be handled to accurately model the problem.

Energy production and trading is a complex industry, so the basic information necessary to read this thesis is included in chapter 2. Both market structure and operational aspects are covered. In chapter 3, related literature is presented, emphasizing the gap that this study contributes to fill. Some optimization theory is also presented, as to better understand the methods utilized in this thesis. Chapter 4 presents the modelling choices and the resulting mathematical optimization model constructed in this study, while chapter 5 present how the optimization model is utilized in practice and what value the model can provide a power producer. Finally, chapter 6 concludes the study, followed by interesting aspects subject for future research.

2 The Power Market and Power Production

As any other commodity, electricity may be sold and bought, but in distinction to other goods, electricity cannot easily be stored. The power market is complex in its structure to ensure exact balance between supply and demand, and power to be delivered at a certain point in time may be traded several times in several markets prior to its production. Hence, the power that is transmitted and utilized on different levels in the electricity grid - including the central, regional and distributional levels - has often gone through a process of being traded on the wholesale market, i.e. between producers and suppliers, brokers, large industrial companies and other large market agents. There are multiple power markets a power producer can choose to participate in. The scope of this thesis concerns the intraday market Elbas, making a few aspects from the day-ahead market and balancing market relevant to discuss.

The spot market is a market for selling and buying power the day before the actual production hour, while the *intraday market* Elbas (Electricity Balance Adjustment System) provides an opportunity to regulate ones commitments up until one hour prior to production (close). A market participant can choose to *up-regulate* the generator by producing more power, and *selling* it. Oppositely, one can *down-regulate* the generator and produce less in cases where it is more beneficial to *buy* power from other participants. All imbalance remaining between demand and supply in the grid are then settled at the actual time of production in the regulating power market. An overview of this is shown in fig. 1, as a time line up until the hour of production.

For the scope of this thesis, production is considered constant within an hour, and is referred to as a production hour. A more general term is production period, not restricting the length of production. All hours are given by *standard time*, which is the Central European Time (CET), utilizing a 24 hour clock. The production hour h in fig. 1 represents all production hours during day 1 from hour 1 to 24.

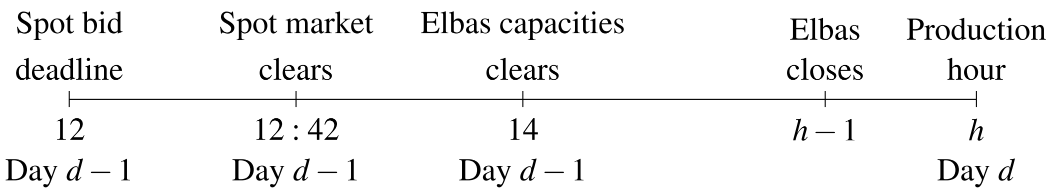


Figure 1: Time line overview of day-ahead and intraday power markets

Section 2.1 introduces the overall regulations affecting participants in the power markets, and their resulting opportunities and responsibilities. A brief introduction to how the spot clearing and the final balance settlement is done is described in section 2.2 and section 2.4 respectively. A more comprehensive review of Elbas is provided in section 2.3. Finally, section 2.5 addresses the physical aspects concerning power production, and introduces the main factors relevant for optimal scheduling and power trade.

2.1 Deregulated Markets and Market Coupling

In *deregulated* electricity markets, the power price is determined by demand and supply. The objective is to obtain something close to a *perfect market*, which will maximize social surplus. With free flow of power between countries, the dispatch of power production at facilities with different associated costs will assure that the best price for society is obtained, across country borders. Areas with energy surplus will be able to sell power to areas with deficits. If there are no transmission and distribution constraints, the power price will be equal for all participants according to basic economic theory, which is the objective of coupled markets (Wangensteen, 2012).

Integrating markets across country borders, assures a diversity in the power sources supplying the grid, which assures a better *security of supply*. Relying too much on a single or too few energy sources, the supply will be highly sensitive to changes in weather conditions, fuel prices or other factors essential for that specific energy source. With a combination of energy sources and geographical placements, the total supply is less affected by single variations. Power producers with flexible production can offer their flexibility in the market to power producers with unpredictable production. An increased number of market participants makes the market closer to a perfect one as the market clearing gets more efficient.

The Nordic and Baltic countries have deregulated power markets, and *coupled* them into a common market facilitated by Nord Pool (Wangensteen, 2012). 380 companies from 20 different countries trade under Nord Pool's operation (NordPool, 2016b), where market clearing, settlement and services in day-ahead and intraday markets are some of their responsibilities. The liquidity is still considered low in the intraday market, due to few participants and low frequencies of trades. However, the development over the last years has shown increased production volumes and transmission capacities, and even more diverse energy sources supplying the grid (NordPool, 2017a). Continuously introducing more renewable energy sources to the energy mix indicates that this development will continue the years to come.

2.1.1 Market Areas and Power Prices

Even though the market is coupled between the Nordic and Baltic countries, available transmission capacities (ATC) restricts the volumes traded and results in *congestion*. In practice, it means that bottlenecks in the transmission system creates market imperfections. When free flow between producers and consumers is not possible, the result is loss of social surplus. Hence, market areas are constructed such that each area functions as a market place with a common power price, whereas power trades between the market areas may be affected by congestion. The market areas in the Nordic and Baltic power market are illustrated in fig. 2.

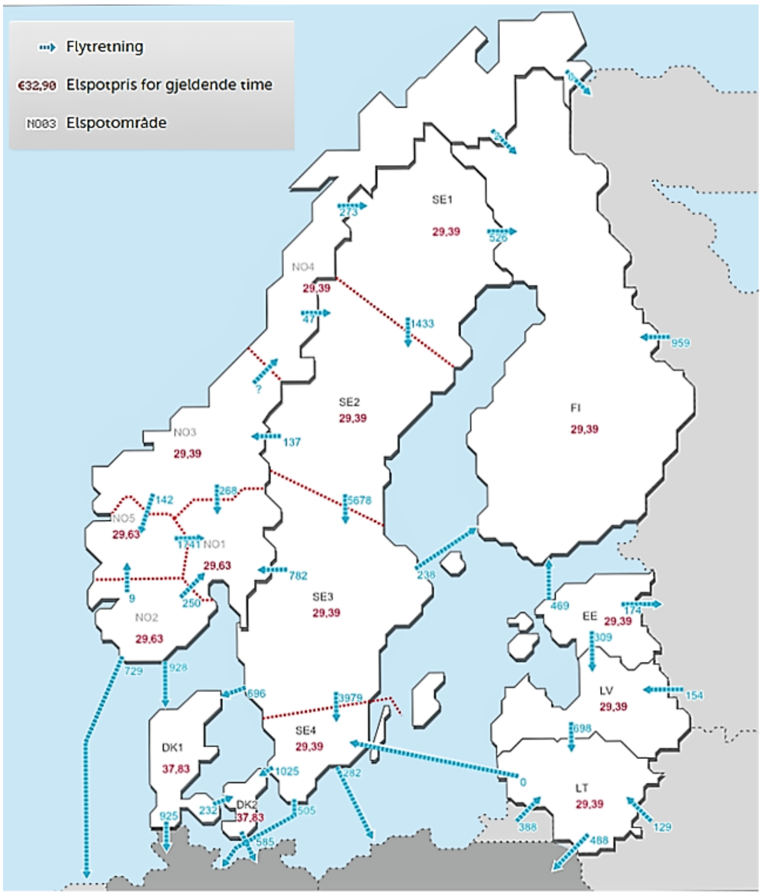


Figure 2: Market areas and related spot prices and power flows. Snapshot from Statnett (2017a), illustrating the 15^{th} of November 2017, at 23:59.

The system price refers to the power price that would have been the result after a spot clearing if there were unlimited transfer capacities and no congestion. The area price is lower than the system price if there is a power surplus in the area, while it is higher than the system price in market areas with deficits. As long as there is available transmission capacity, power will flow from a low price area to a high price area.

In theory, all market areas in Nord Pool are connected. The Nord Pool power markets are constructed such that the net amount of energy exported/imported is correct according to the commitment of each market area. In order to trade with market participants in other areas, there must be available transmission capacity between the areas. This is referred to as available areas in this thesis.

2.1.2 Roles and Responsibilities

To better understand how the different types of power markets works, it is useful to know what roles and responsibilities the participants in the power market have.

Production companies are responsible for selling active power to the market, while the grid companies operates, maintains and invest in the electricity network. Both of these may act as a supplier to the end user. While these market participants have certain responsibilities, the end users are independent in the sense that they control their own consumption pattern. While they may be affected through pricing of power, there is no direct control of their consumption behavior.

This leads to an important responsibility of balancing production and demand at any point in time. The security of supply is maintained by a system operator (SO). All participants in the wholesale market of electrical power in Norway are *obliged* to sign a balance agreement with Statnett, Norway's transmission system operator (TSO). The agreement's objective is to ensure that any unbalance subject to regulation in the regulating market is a result of forecast errors only. The power exchange is facilitated by a market operator. In the Nordic power market, Nord Pool is responsible for both financial and physical trade in the day-ahead and intraday markets.

2.2 The Spot Market

Most of the power traded in Nord Pool is traded the day ahead of production, in the spot marked. In 2016, the total volume traded in the spot market was 391 TWh, which amounts to more than 77% of the total volume traded by Nord Pool and exceeds 98% of the total volume if excluding the UK day-ahead market (NordPool, 2016b).

The day ahead of production, the deadline for submitting bids in the spot market is at CET 12. The market is organized as a sealed bid periodic double auction (Selasinsky, 2014) where each participant in the spot market delivers a bid matrix to the market operator Nord Pool, containing what amount of energy they are willing to sell *and* buy given specific prices. Nord Pool matches all bids concerning the same production hour, evaluating the intersection between willingness to buy and sell in the market throughout the following day (NordPool, 2017c). This is a complex, non-trivial problem, solved by a mixed integer optimization algorithm aiming to maximize social welfare. They create 24 market crosses, simplistically illustrated in fig. 3, one for each production hour. Short-term contracts of purchases and sales for each production hour the following day is committed.

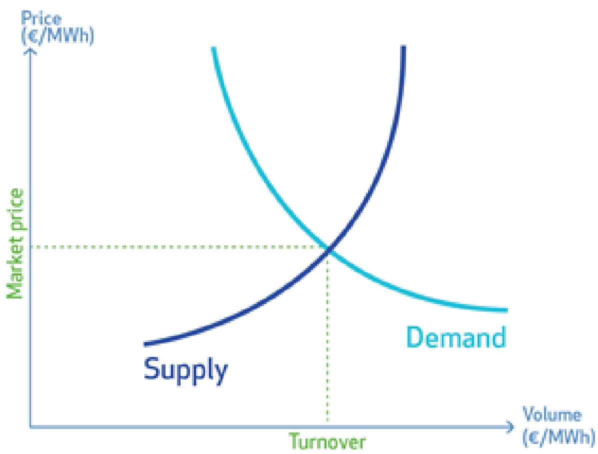


Figure 3: An illustration of how the spot hourly prices are set by where the accumulated bid curves for buy price and sell price meet. Snapshot from NordPool (2017c)

Revenues are calculated as volume dispatched to a market participant times the market price in the given hour (Boomsma et al., 2014). When the spot market clears at some point after 12:42, the hourly spot prices and trades for the following day are announced simultaneously to the market. Due to congestion in the grid, these prices will differ between market areas (Wangensteen, 2012).

2.3 Elbas - The Intraday Market

In general, both volumes and frequency of trades in Elbas are characterized by the tradition of Elbas being a platform to create balance in the grid. Three market makers have a joint responsibility of providing fundamental liquidity in Elbas (NordPool, 2018), required to quote a maximum *bid-ask spread*, but liquidity is still considered low. However, over the period from 2013 to 2015, it is observed a power increment of almost 60% and almost 50% more trades in Elbas (Scharff and Amelin, 2015). This is related to the increasing share of renewable energy sources in the energy mix. The Cross-Border Intraday Initiative (XBID project) creates a integrated intraday market across Europe (XBID, 2018), to increase market liquidity and efficiency as a measure to improve balancing opportunities for producers with intermittent energy sources. Hence, the volumes traded in Elbas are expected to increase over the years as further renewable development continues (Deloitte, 2015), and moreover, a larger market will be available for trade.

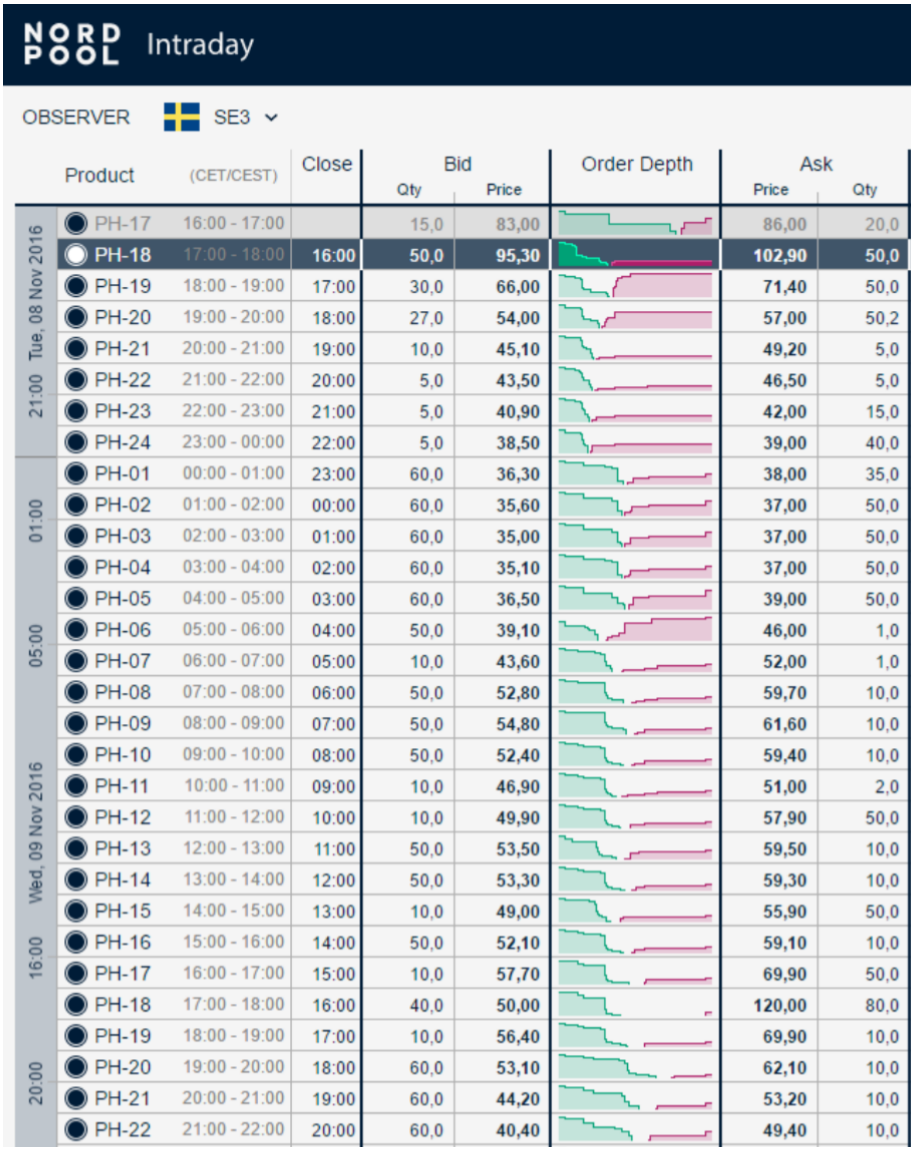
To ease the reading of this report, a specification of terms should be done concerning *time* in the intraday market. As already described, a production period refers to the point in time where the physical delivery of power is to happen. However, in Elbas, this commitment may be performed at any point in time prior to delivery. For the scope of this thesis, the term bid period refers to the time when the *commitment* is made, hence when the financial trade takes place.

2.3.1 The Market Structure

Unlike the spot market, Elbas does not have a market clearing. New trades happen continuously up until the close of the production hour. It is a first-come, first-served market, organized as a continuous double auction (Selasinsky, 2014). Market participants submit their bids to an *open order book*, where offers to sell and buy energy are separated into two columns. One can only see offers from participants located in an available area.

A segment of the market information screen in Nord Pool's trading client is shown in figure 4. Each production hour is listed in different rows, with the corresponding close and present best bid/ask offers in the market. Notice that the observer is set to be located in market area SE3, hence only offers from available areas for SE3 will show on the screen.

The highest offer to buy energy is referred to as the best *bid*, while the lowest offer to sell is



Product	(CET/CEST)	Close	Bid Qty	Bid Price	Order Depth	Ask Price	Ask Qty
PH-17	16:00 - 17:00		15,0	83,00		86,00	20,0
PH-18	17:00 - 18:00	16:00	50,0	95,30		102,90	50,0
PH-19	18:00 - 19:00	17:00	30,0	66,00		71,40	50,0
PH-20	19:00 - 20:00	18:00	27,0	54,00		57,00	50,2
PH-21	20:00 - 21:00	19:00	10,0	45,10		49,20	5,0
PH-22	21:00 - 22:00	20:00	5,0	43,50		46,50	5,0
PH-23	22:00 - 23:00	21:00	5,0	40,90		42,00	15,0
PH-24	23:00 - 00:00	22:00	5,0	38,50		39,00	40,0
PH-01	00:00 - 01:00	23:00	60,0	36,30		38,00	35,0
PH-02	01:00 - 02:00	00:00	60,0	35,60		37,00	50,0
PH-03	02:00 - 03:00	01:00	60,0	35,00		37,00	50,0
PH-04	03:00 - 04:00	02:00	60,0	35,10		37,00	50,0
PH-05	04:00 - 05:00	03:00	60,0	36,50		39,00	50,0
PH-06	05:00 - 06:00	04:00	50,0	39,10		46,00	1,0
PH-07	06:00 - 07:00	05:00	10,0	43,60		52,00	1,0
PH-08	07:00 - 08:00	06:00	50,0	52,80		59,70	10,0
PH-09	08:00 - 09:00	07:00	50,0	54,80		61,60	10,0
PH-10	09:00 - 10:00	08:00	50,0	52,40		59,40	10,0
PH-11	10:00 - 11:00	09:00	10,0	46,90		51,00	2,0
PH-12	11:00 - 12:00	10:00	10,0	49,90		57,90	50,0
PH-13	12:00 - 13:00	11:00	50,0	53,50		59,50	10,0
PH-14	13:00 - 14:00	12:00	50,0	53,30		59,30	10,0
PH-15	14:00 - 15:00	13:00	10,0	49,00		55,90	50,0
PH-16	15:00 - 16:00	14:00	50,0	52,10		59,10	10,0
PH-17	16:00 - 17:00	15:00	10,0	57,70		69,90	50,0
PH-18	17:00 - 18:00	16:00	40,0	50,00		120,00	80,0
PH-19	18:00 - 19:00	17:00	10,0	56,40		69,90	10,0
PH-20	19:00 - 20:00	18:00	60,0	53,10		62,10	10,0
PH-21	20:00 - 21:00	19:00	60,0	44,20		53,20	10,0
PH-22	21:00 - 22:00	20:00	60,0	40,40		49,40	10,0

Figure 4: Example of the market information screen from Nord Pool's web based trading client. Snapshot from NordPool (2016a).

referred to as the best *ask*. These will be located at the top in the order book, as the offers are sorted in descending and ascending order, respectively. As this section (2.3.1) covers double auction theory, it is natural to use the correct terminology concerning *bid* and *ask* offers. Note, however, for the remainder of this thesis, a *bid* refers to any offer in the power market, both for buying and selling energy.

As long as there exists a *bid-ask spread*, there will be no trade. Once there exists a bid price higher than - or equal to - the ask price, a trade is carried out between the two, at the price that occurred first in the order book. Hence, the price in the offers are *limit* prices, and only the first occurring participant pay as bid (Selasinsky, 2014). The counterpart gets either the limit price or a *better* price for the trade.

In addition to a limit price, the offers in Elbas contain information about what quantity the participant is willing to trade, and which production hour the trade is subject to. The offer may concern one single production hour, or several consecutive production hours (block offer). A *fill* order will accept trades not fulfilling the whole commitment, while the left out quantity often remains in the order book. An *all or nothing* order requires the counterpart to accept the full quantity.

2.3.2 Bidding Strategies

Market participants in Elbas have different objectives for their participation. Intermittent energy producers often seek to obtain balance, as a measure to avoid a more expensive balance settlement in the regulating power market (section 2.4). Others may seek to improve their production schedule from the spot clearing, to reduce production costs. Traditionally, these have been the main objectives of participants in Elbas.

However, flexible power producers may also participate regardless of their initial spot clearing, offering to be the counterpart of any trade covering their own marginal cost for up- or down regulation. Indications from the industry (TrønderEnergi, section 2.5.2) is that this strategy is regarded as non-trivial to implement, and that the expected profits have been considered too small to cover the initial investments required to construct optimal bidding strategies. A rule of thumb could be to offer flexibility at a limit price corresponding to a fixed premium above marginal cost. The main draw back of such a strategy is argued that market dynamics are not considered, and potential profits not maximized.

As liquidity in Elbas increases, the potential profits from offering flexibility in the market become more attractive, introducing the need for stochastic models accurately describing market prices throughout the continuous auction in Elbas. Mathematical optimization software will provide decision support to implement non-trivial bidding strategies, and introduce a potential to fully automate trades in the future.

2.4 The Regulating Power Market

All market participants are obliged to plan and provide hourly balance, so one cannot *plan* to be unbalanced as a mean of financial gain (Statnett, 2017b). However, participating in *Elbas* is not required, so participants may choose to settle any imbalance due to forecast errors in the regulating market. In that case, they choose *passive participation* in the regulating market, rather than participating in the intraday market.

Tariffs for up-regulation and down-regulation are set by the TSO, where a two-price mechanism works as an incentive to ensure balance up front. The objective is that participants should always be worse off settling their balance in the regulating power market rather than plan their production and consumption sufficiently ahead. The two-price mechanism is presented in table 1.

Table 1: The two-price mechanism (Engmark and Sandven, 2017)

	Upward regulation	Downward regulation
Production deficit	Pay BM price	Pay spot price
Production surplus	Receive spot price	Receive BM price

2.5 Power Production and Resource Management

Electricity cannot be stored in large quantities after it is produced, and in fact not even all energy sources can be stored. Wind, for instance, must be utilized at the same time as it appears. Water in reservoirs, coal or gas may be stored, but have costs related to start-ups. There are different factors that affect the production costs, uncertainties and alternative gain when evaluating power production from different technologies and energy sources.

In general however, power producers face some sort of costs related to production and facilities, which may include:

- *Fixed costs* concerning e.g basic storage and maintenance, minimum staff etc.
- *Semi fixed costs* e.g extra units must start or stop, extra staff requirements etc.
- *Marginal costs* appearing as a direct effect of each unit of increased production, e.g resource consumption, efficiency of power production plant etc.

In a power trade situation, variable costs are evaluated to what volumes are profitable to trade. Short term production costs reflects the area under the marginal cost (MC) curve in fig. 5, or by multiplying average cost (AC) with the production level. In addition, semi-fixed costs apply if the volume produced exceeds some specific level.

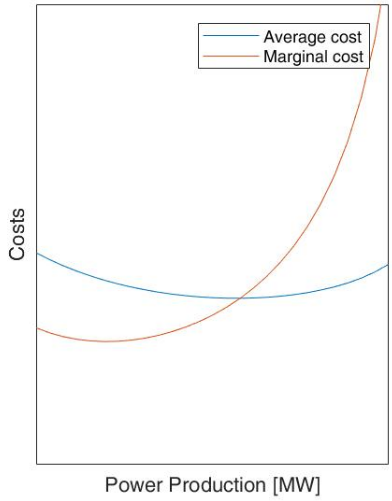


Figure 5: Marginal and average costs as a function of production

For the scope of this thesis, a hydro power producer is basis of analysis, hence evaluating costs in this study will be according to hydro power production. A brief introduction to hydro power production is therefore presented to ease the reading of the following analysis. However, any power producer with costs associated to the production as described above, should find an equivalent analysis applicable.

2.5.1 Hydro Power Production

Hydro power is a well established and renewable energy source. According to Statkraft (2017), hydro power amounts to 99% of the total power production in Norway, while the same number in the world is approximately 17%. An important advantage of hydro power is the flexibility of time of production (Statkraft, 2009). A reservoir acts as a natural storage of energy, where water can be used for production now or stored to satisfy demand later on.

There are no direct costs associated with hydro power production. However, one normally refer to the *opportunity cost* of water, since it is a scarce resource. Utilizing it today, may be at the expense of future opportunities. Moreover, the opportunity cost is referred to as the

MC of hydro power production, affected by *the water value* and production efficiency. As a hydro power producer, knowing the value of the water in a reservoir is important in order to perform optimal scheduling. One can obtain the water value from long-term optimization models, aiming to allocate the water in an optimal manner. If a reservoir is likely to overflow, the water value will be close to zero, while if the reservoir is about to run empty, the water value will increase rapidly. The water value is two-faceted, as it depends on both the operation of a hydro power plant, and market prices (Faanes et al., 2016).

2.5.2 TrønderEnergi and the Industrial Case

The industrial partner of this study is TrønderEnergi (TE), who is a power producer located in the market area NO3. They produce about 2.1 TWh per year, most of it from hydro power production and about 200 GWh from wind power (TrønderEnergi, 2017). This is only a fraction of the power traded in Nord Pool in 2016: 505 TWh (NordPool, 2017b). Today, TE have models that consider their whole portfolio of power production plants, evaluating production plans according to uncertain factors such as water inflow and power prices. However, these models are concerned with long-term scheduling, and do not model the intraday market.

The industrial case as base for the analysis in this thesis is TEs hydro power plant *Søa*. The power plant consists of one single turbine and generator. The reservoir receives water from two natural lakes: Vasslivatnet and Søvatnet, but for all modelling purposes, it is treated by TrønderEnergi as a *single* water source. Technical specifications of the power plant is listed in table 2.

Table 2: Technical specifications of the hydro power plant Søa (TrønderEnergi, 2018).

Yearly Production	195 GWh
Installed Capacity	37 MW
Turbine	Francis
Head	260,00-279,83 m.o.h

Maximum production does not equal the installed capacity of the generator. The capacity limit of the machinery at Søa is a variable given by *hydraulic head*, L^{head}, in [m.o.h.].

$$I^{max} = f(L^{head}). \tag{1}$$

The hydraulic head refers to the height of the water, i.e. the water level in the reservoir relative to the generator. The lower the head, the lower is the maximum obtainable power, due to less pressure forces. Figure 6 shows how the maximum power increases linearly with changes in head from lower to higher values.

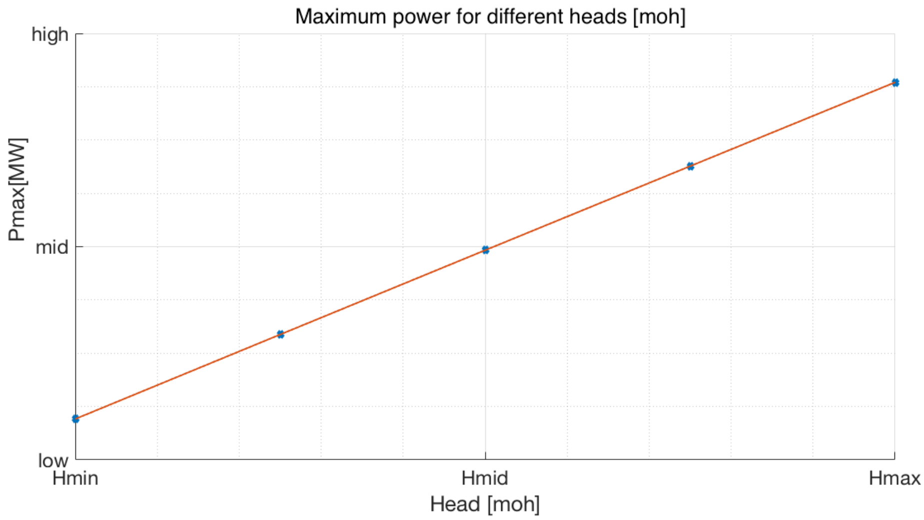


Figure 6: Maximum power for different heads

This is a result of the fact that production efficiencies decrease with lower head. In addition, a lower head also reflects less water in the reservoir. This increases the water value. Both increased inefficiencies and higher water values cause an upward shift in the MC of production.

3 Literature Review

The purpose of this chapter is to introduce literature relevant for the context of this study, to shed light on how this thesis differentiates from existing literature. In section 3.1, an overview of established literature within the field of energy production and trading is presented, to establish the gap in existing literature. In addition, the bidding problem in Elbas has relations to inventory theory, where demand or prices may be uncertain at the time a decision affecting the inventory level is to be made. Inventory problems with similar structure as the bidding problem at hand are presented in section 3.2, while section 3.3 and section 3.4 covers literature that emphasizes exogenous information and uncertainties. Benefits and challenges of mathematical optimization methods are discussed in section 3.5, while section 3.6 present how bounds are utilized to evaluate how accurate a model solves a problem.

3.1 Power Production and Trading

A variety of studies have been conducted within the field of energy production and energy trading. There often exists a way to store energy, either as electricity in a battery (Salas and Powell, 2018; Zhou et al., 2017; Jiang and Powell, 2015) or to hold back energy resources such as fuel or water (Mo et al., 2001; Séguin et al., 2017; Tandberg and Vefring, 2012; Quan et al., 2014; Brelin and Lien, 2017; Bertsimas et al., 2017). Storability introduce flexibility concerning *timing* and *volume* of *production*. This thesis is concerned with flexibility related to timing and volumes of *trades*, as the intraday market in reality is 24 continuous auctions rather than a simultaneous market clearing as in the spot market.

While Boomsma et al. (2014) and Barth et al. (2006) discuss sequential bidding strategies across multiple power markets, most literature like Aasgård et al. (2018) and Anderson and Philpott (2002) focus on participation in the spot market only. Löhndorf et al. (2013) do consider *intraday* decisions for hydro storage systems, integrating bidding and storage decisions in the formulation. Also Jiang and Powell (2015) present a model for intraday bidding, though this model does not include production costs. What neither of the aforementioned models include is how the intraday market have continuously changing prices within a day. Löhndorf et al. (2013) assume there are little incetives for the power producer to trade in the intraday market, hence all bids in the intraday market are determined simultaneously once the spot clearing is announced. Additionally, Jiang and Powell (2015) focus on optimal bidding rather than production, as there are no production opportunities in the storage system they consider. The model in this thesis aims at integrating operational aspects with detailed bidding strategies in the intraday market.

Multiple studies discuss decisions that are made sequentially in time, where each decision affects both present and future opportunities to profit. However, there is a significant difference in the time horizon considered in the power planning literature, and consequently a difference in the level of detail included. A longer time horizon typically requires a coarser time resolution to ensure a manageable complexity of the problem in question. Tandberg and Vefring (2012) present a generation planning problem with a 60 week planning period and a *weekly* time resolution, while Séguin et al. (2017) manages *daily* time resolution as their planning horizon extends to 31 days only. Zhou et al. (2017), whom solve their problem over a weekly time horizon, are able to introduce a *five minute* time resolution where new considerations are made. In general, only a few models aim at optimizing profits with only one day as the modelling horizon, evaluating continuously changing prices with the opportunity to time bids to the optimal execution time.

The time horizon and time resolution are modelling choices that present a trade-off between tractability and accuracy. A way to handle this, which is well known within hydro power scheduling (Gjelsvik et al., 2010), is to have supplementary models with different horizons. In that way, long-term models include long term parameters and the resulting actions affects the medium-term models, which output again serves as input to a short term model. The long term model concerns aspects relevant for multiple years at a time, but does not aim at solving the day-to-day, or hour-to-hour, production scheduling problem. In addition, there are multiple uncertain factors that depend on future, exogenous information. Short term models can include detailed and (now) deterministic information that was not available or known when long or medium term models were run. Water inflow is typically an important factor, but difficult to predict exactly with small time resolutions far ahead in the future. Parameters from the medium term model ensures that decisions today adhere to the optimal long term scheduling.

Short-term models, both hydro power scheduling and other scheduling problems, usually concerns a horizon of up to two weeks. However, these models mostly cover trading opportunities in the spot market (Salas and Powell, 2018; Nadarajah and Secomandi, 2017), and a possible balance settlement in the regulating market. In addition, several studies focus on optimal production rather than optimal bidding, so power prices are estimated on a daily or longer basis (Tandberg and Vefring, 2012; Séguin et al., 2017). Few existing studies concerns the possibility to gain profits by accurately modelling of the continuous double auction in the intraday market (Selasinsky, 2014), as most studies emphasize technical aspects of power systems or power plants. The main assumption seems to be what Löhndorf et al. (2013) state, that it exists little incentives for a power producer to participate in the intraday market. However, an increasing interest in the market is developing as the need of flexible energy reserves arises. As the volumes become larger, potential profits also increase.

Jiang and Powell (2015) present a problem of grid-level storage with the opportunity to buy, store and sell energy, where bids are submitted until an hour prior to production. Their strategy is referred to as *energy arbitrage*, exploiting variations in energy prices. The main distinction from the model in this thesis, is that the arbitrage opportunity exists due to the possibility to buy and store energy at one point in time, while selling it at a better price later. This is due to the lack of production in the problem. However, this thesis aims at modelling the intraday market with continuously changing prices, which both increases the dimensions and complexity of the bidding problem, but also introduces an additional opportunity for arbitrage trading. Short term contracts may be bought and sold multiple times prior to the time of production or consumption.

Jiang and Powell (2015) do not have to include start-up and shut-down effects, which is an important aspect conserning power production. Typically, there is a certain cost related to these activities (Nilsson and Sjelvgren, 1997), which affects the optimal schedule. Hence, obtaining an optimal generation dispatch and production schedule requires to solve both a loading problem and a unit commitment problem. Nilsson and Sjelvgren (1997) states the importance of including startup costs of hydro power units, but also sheds light on the increased complexity it causes. Séguin et al. (2017) and Hjelmeland et al. (2018) suggest different methods to model the unit commitment problem for hydro power scheduling, both utilizing heuristics. Barth et al. (2006) present a model with sequential clearing of both the day-ahead and the intraday market, where start-up and shut-down effects are included in the spot clearing. A redispath is then performed in the intraday market. Due to how the value function computation is implemented for the problem in this thesis, startup and shutdown effects are modelled precisely.

3.2 Resource Allocation

Resource allocation concerns the problem of distributing scarce resources on competitive alternatives, often to maximize profit or production, or to minimize costs. Bertsimas et al. (2017) states that: "Dynamic resource allocation (DRA) problems are problems where one must assign resources to tasks over some finite time horizon." While it may not be intuitive that the bidding problem is related to resource allocation problems, it has similarities. As long as there is either a restricted resource, or restricted production or storage capacity, *the trades committed in Elbas must be allocated to competitive hours of production, and be submitted at competitive times*. This is referred to as the double time dimension of the Elbas bidding problem in this thesis.

Salas and Powell (2018) present an algorithm based on resource allocation, for stochastic control of complex energy storage networks co-located with a wind farm. A similar problem is studied by Zhou et al. (2017) and Jiang and Powell (2015). As storages have limited capacity, the energy available for trade is scarce and must be traded at the hours most profitable. In addition, since transmission lines have a restricted capacity, the accumulated commitment for a specific hour of delivery must be within specified limits. Hence, bids should only be done at the times prior to production where it is most profitable.

The main objective of Jiang and Powell (2015)'s study is to obtain an optimal bidding strategy in the hour-ahead electricity market, where a trade decision at one point in time affects both current profits and opportunities to profit from trades in the future. The energy in the battery is a scarce resource that is allocated to different hours of delivery. Water reservoirs of HPPs are considered natural storages of energy, also subject to optimal allocation. However, this thesis assumes constant reservoir levels throughout the modelling horizon, due to relatively small production volumes comparing to the reservoir volume. This is due to machinery capacities, which are considered the scarce resource in this thesis, also introducing a dynamic resource allocation problem.

3.3 Uncertainties

The bidding problem in Elbas is affected by several uncertain factors. Depending on the time horizon and main objective of the model, different factors are considered uncertain.

Zhou et al. (2017) develop two heuristics for electricity generation and storage management, one deterministic model and one with stochastic wind and electricity prices. Though the stochastic model contains a more accurate description of the problem and solves to optimality on all considered instances, the deterministic heuristic solves to a near optimal solution and is significantly faster. Another way of solving a similar problem is presented by Salas and Powell (2018). Demand is included as a stochastic variable, and the outcome space is decreased using simulation. The problem is solved deterministically at first, as a way to benchmark the approximate algorithm. This thesis constructs a model that can be solved to optimallity. However, the modelled problem is a heuristic version of the original problem, partly due to aggregation of the outcome space of the stochastic prices.

Multiple studies include simulation of water inflow to hydro power production systems (Pereira and Pinto, 1985; Quan et al., 2014; Séguin et al., 2017; Hjelmeland et al., 2018). A distinction between the problem presented in this thesis and the studies treating water inflow as a stochastic variable, is the time horizon of the models. Evaluating short term horizons, such as a few hours or days, the water inflow is more predictable and considered deterministic.

An influential uncertainty in the intraday bidding problem, and the primary focus in this thesis, is the intraday price fluctuations. This is the scope of the next section.

3.4 Price Modelling

This thesis is concerned with optimization in an electricity market. As mentioned in the Preface, this thesis is a continuation of a project (Bovim and Næss, 2017), where the scope is to understand and model the market.

This section revisits the main findings on the topic of price modelling, because when evaluating the value of the program it is important to be aware of all assumptions and simplifications made and evaluate the effect of them. Therefore some of the most important findings of Bovim and Næss (2017) are repeated here for convenience.

Jiang and Powell (2015) concider intraday trading as an opportunity to profit from price changes in the hour-ahead market. They argue that the price process is better described by historical data, where one should not assume any specific distribution of the price process. This is in contrast to Bertsimas et al. (2017) and Nadarajah and Secomandi (2017) whom assume a normal distribution and a Markov Decision Process (MDP), respectively. The exogenous price process in this thesis is assumed to have the Markov property, with a transition matrix constructed from historical data.

Jiang and Powell (2015) include in their model that not all bids are accepted in the intraday market, which deviates from the model in this thesis. This thesis assumes a representation of the market willingness to trade from the stochastic price process developed by Bovim and Næss (2017), hence any bid placed in the market in accordance with the price model is assumed accepted by a counterpart.

Correlation with Spot Prices

The price model in Bovim and Næss (2017) models the stochastic process of *Elbas* prices by investigating *spot* prices. This is based on an assumption of correlation between spot prices and Elbas trade levels. Faria and Fleten (2011) found a high correlation between spot prices and the average Elbas prices from historical data. The relationship between the spot and Elbas prices have been investigated in multiple studies (Olsson and Söder, 2008; Skytte, 1999), and an interesting question is to what degree Elbas prices the following day may be extrapolated by the spot market clearing. Jaehnert et al. (2009) indicate no correlation between the spot and the balancing prices.

To build the price model (Bovim and Næss, 2017), an empirical approach that differs from the standard parametric approach (Pflug and Pichler, 2016), is used. This is done due to relatively good availability of historical data, which supports the decision of using empirical data as a robust basis. Pflug and Pichler (2016) describe this as the non-parametric approach to scenario generation.

One of the analytic results found from time series analysis (Bovim and Næss, 2017), was that autocorrealtion with time lag 1 is observed to be significant, in contrast to points in time of higher lags. Moreover, under the assumption that spot prices in one hour are strongly correlated with that of the previous hour, the Markov property becomes suitable for price modelling. Literature regarding the Markov Decision Process (MDP) is described below.

The Markov Property

A common factor for many of the methods utilized for spot price modelling is that they utilize a crucial property, namely the Markov property. If a process is Markovian, with a known process distribution and known current state, one is able to predict the probability of the system ending up in all any state in the next step.

Investigating the opportunity to model energy prices under the assumption of the Markov property is for instance performed by Olsson and Söder (2008) to describe discontinuous behaviour of the balancing market prices. They propose a method based on seasonal autoregressive integrating moving average (SARIMA). Kongelf and Overrein (2017) model the regulating markets using quantile autoregression to provide probabilistic forecasts for the market prices. Moreover, Barlow (2007) and Bakke et al. (2016) make use of multi-state Markov chains in a regime switching model. In the first, a continuous-time Markov chain entails an arrival density based on a Poisson process. In the latter, each different state is associated with a price process utilized within the regime.

3.5 Optimization Models

What distinguishes this thesis from most studies on the field of power production and trading, is that the optimization model in this thesis evaluates intraday bidding as a sequential decision problem in itself, investigating potential economic incentives to participate in Elbas for a power producer with flexible power production. Both prices and operational costs are included, where the prices are considered stochastic and continuously changing throughout the modelling horizon.

Traditional scenario tree generation, such as the method presented by Séguin et al. (2017), is not as suitable for a continuous auction market. The reason is illustrated in fig. 7, where one can observe how the number of scenarios, i.e. the size of the scenario tree, increases exponentially with the number of sequential decisions where stochastic parameters affect the outcome. Barth et al. (2006) manages to model three sequential decisions only, which are related to wind uncertainties rather than price fluctuations. The bidding problem in Elbas potentially has more than 30 sequential stochastic events. The number of scenarios becomes potentially too large to handle in a single optimization. The number of sequential decisions, each affecting the current and

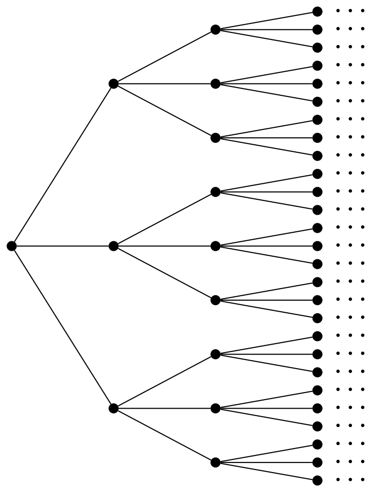


Figure 7: Scenario Tree

future possibility to profit, makes Dynamic Programming (DP) attractive for the bidding problem in Elbas. In addition, assuming a Markov Decision Process (MDP) where the probability of a *future event* is solely described from the *present state attained from the previous event*, indicates that the problem structure fits into the dynamic programming framework. Section 3.5.1 present the basic elements of DP, where the main challenges on the approach are elaborated on in section 3.5.2. Then, section 3.5.3 addresses how DP is utilized in literature.

3.5.1 Dynamic Programming

According to Dixit and Pindyck (1994), DP is a powerful tool when treating multistage problems with uncertainty, and decision variables at each stage. It breaks the problem down so that each decision x_t involves only two components, with the objective to obtain the optimal *policy* for decision making. This is stated by *Bellman's principle of optimality* (Bellman, 1957): "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."

In this case, the direct value obtained from a decision at a time t represents the first component, and the expected value of all future states and corresponding decisions, represents the second component of consideration. The time steps t corresponds to the stages in a dynamic program, the state S_t of the system in each stage is given by price developments, and the probability, $\mathfrak{P}$, that $S_{t+1} = s'$ given the current state is given by $\mathfrak{P}(S_{t+1} = s'|S_t, x_t)$.

The value recursion of a state is then given by Equation 2, which is also referred to as the *Bellman equation*.

$$V_t(S_t) = \min\big(C_t(S_t, x_t) + \sum_{s' \in \mathcal{S}} \mathfrak{P}(S_{t+1} = s' | S_t, x_t) V_{t+1}(s')\big) \tag{2}$$

3.5.2 The Curses of Dimensionality

Though dynamic programming is suitable when there is a multistage sequential decision problem, Powell (2011a) explains three common reasons why dynamic programming cannot be used. The sizes of the state space, outcome space and the action space all contribute to increased complexity of DP, and are subject to the curse of dimensionality. In this thesis, the state, action and outcome spaces are handled as discrete levels, divided into a finite number of that is sufficient to describe the problem in question.

Notice how the number of stages is *not* included in the curses of dimensionality in dynamic programming. This is a great advantage compared to other stochastic optimization methods, at least as long as it is possible to contain the other dimensions within reasonable limits (Dixit and Pindyck, 1994).

Another advantage is that the output is in the form of a policy, suited to construct a *contingency plan*. It is easier to simulate when a policy is obtained, thus evaluation of the output from a dynamic program is easier to evaluate than output from scenario tree formulations. Even if the structure of the problem has limited flexibility, and an optimal policy is hard to compute, there exist heuristic approaches to obtain a policy.

3.5.3 Dynamic Programming Approaches

Most real world problems have uncertainties, introducing stochastic processes to the modelling framework. Stochastic Dynamic Programming (SDP) utilizes transition probabilities and expected future values to solve the dynamic program and find optimal decisions. Zhou et al. (2017) develops a stage and state-dependent policy, utilizing a heuristic SDP approach on an electricity generation and storage problem. What makes the Elbas bidding problem in this thesis more complex is that a continuous auction is evaluated, i.e. continuously changing prices, introducing a question of *timing* of bids in addition to optimal production scheduling.

Séguin et al. (2017) present a short-term hydro power planning model, solving the loading problem and the unit commitment problem in two separate phases. The multistage first phase problem is solved utilizing SDP to estimate future value of the water in the end of the horizon, where the stochastic variable is water inflow. This thesis aims at modelling a problem horizon so short that the reservoir level is assumed constant, while the prices are continuously changing as a stochastic process. As there is a price for each product, the dimensions of the exogenous process in this thesis become extremely large. However, by aggregating dimensions, this thesis is able to solve an approximated version of the bidding problem in Elbas utilizing SDP.

Other studies handle the curses of dimensionalities by simulating the stochastic processes rather than spanning the whole outcome space in the state space. Salas and Powell (2018), Jiang and Powell (2015) and Nadarajah and Secomandi (2017) solves problems related to energy trading and storage utilizing methods referred to as Approximate Dynamic Programming (ADP) algorithms. While each of the studies develops distinct heuristics, they are all based on the same iterative simulation algorithm known as ADP, where only one state in each time step is visited for a specific sample realization.

Both SDP and ADP algorithms require discretization of the state space, while a third method avoids this. Pereira and Pinto (1985), and Zou et al. (2018) utilizes a Stochastic Dual Dynamic Programming (SDDP) algorithm, also based on simulation of stochastic processes. It is a common method to handle stochastic water inflow in long term hydro power scheduling models (Mo et al., 2001), but is established as ineffective when solving multistage stochastic integer problems which is essential to include effects of starts and stops of a HPP. Zou et al. (2018) present an extension of the SDDP algorithm, which is able to solve this problem utilizing Lagrangian cuts, but the algorithm is considered too time consuming to solve to optimality (Hjelmeland et al., 2018). In addition, Pereira and Pinto (1991) states that the SDDP approach require stagewise independence of the stochastic process. Löhndorf et al. (2013) solve a multistage problem for hydro storage systems with a model they refer to as Approximate Dual Dynamic Programming (ADDP). The method utilizes an approach similar to SDDP, but with some ideas from ADP so that the model does not require stagewise independence of the stochastic process to obtain convergence of the algorithm.

Application of DP requires discretization of the state space. If possible, in the sence that the curses of dimensionality can be overcome, it is desirable to solve problems using exact methods, such as SDP. If it is necessary to compromise on discretization accuracy in order to apply the exact method, one would typically proceed with heuristic methods, such as ADP or SDDP. An SDP model is developed to solve the Elbas bidding problem in this thesis, as a first step towards a fully representative model of the bidding problem in Elbas.

3.6 Value Function and Bounds

Developing a mathematical formulation of a real world problem and solving it heuristically, does not automatically result in an optimal solution for the original problem. There is a trade-off between tractability and accuracy when problems are computationally difficult to solve exactly. A sub-optimal solution might be considered *good enough* if it is of importance to obtain solutions quickly, rather than precisely for instance. The challenge is to determine how far from optimality a sub-optimal solution is, i.e. how big is the sub-optimality gap (fig. 8), of a heuristic model.

Bounds hold information about what size the optimal expected value can take. Figure 8 shows how the optimal expected value lies between the upper and lower bound. One typically do not know exactly where. If the difference between the lower bound (LB) and upper bound (UB) - i.e. the *optimality gap* - is sufficiently small, Brown et al. (2010) state that the candidate policy used to determine the LB is ”good enough” - and one can stop searching for better policies.

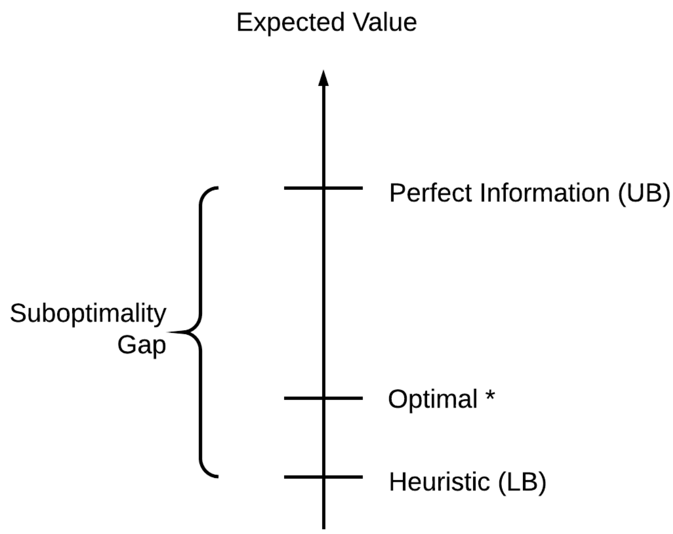


Figure 8: Upper and lower bound relative to optimal value

The lower bound (LB) represents a feasible and potentially sub-optimal solution to the original problem. The feasible policies in the DP cannot depend on information that is unavailable under the natural filtration at the time of decision making. Balseiro and Brown (2016) states that the feasible policies must be non-anticipative, which for the problem in this thesis means that all *future stochastic parameters* must be considered unknown at the time of decision making. The upper bound (UB) is an optimal or infeasible value. It is either equal to, or greater than (maximum problem) the optimal value (see fig. 8).

Brown et al. (2010) describe a general technique to determine UB on optimal expected value values in SDPs. They utilize the *dual approach* where the first step is to relax some or all the NACs. One type of relaxation discussed by Brown et al. (2010),Brown and Smith (2011) and Balseiro and Brown (2016) is information relaxation. The upper bound obtained from full information relaxation is often weak, and hence Brown et al. (2010) impose a *penalty*, as the second step of the dual approach. A *penalty* can be introduced with the purpose of lowering the UB, by compensating for the benefit of the extra information (Balseiro and Brown, 2016). The penalty is *dual feasible* if "... it does not penalize any policy that is non-anticipative; the penalties may however, punish policies that do not satisfy the NAC." (Brown et al., 2010).

The dual approach is applied by Nadarajah and Secomandi (2017), with a dual penalty corresponding to the extra gain of the information relaxation. An ideal penalty would penalize all gain and the optimal solution is found, but this is computationally difficult to obtain.

Brown et al. (2010) discusses three different types of information relaxation, where the difference is to what degree information is known before originally revealed by . In the most extreme case of *perfect information relaxation* it is assumed that the market participant knows all states, and that there is no stochastics. All NACs are removed under full information relaxation. This relaxation simplifies the problem greatly, as it becomes deterministic. The problem is solved in hindsight, optimizing over a case where the price path scenario is known from the beginning. Balseiro and Brown (2016) work exclusively with this form of relaxation. In accordance with Balseiro and Brown (2016), perfect information bounds are often called "hindsight bounds" and have been successfully used to analyze heuristic policies in several applications in operations research.

The two latter relaxation procedures investigated by Brown et al. (2010), are moderations of the first, where only *some* NACs are relaxed rather than all. The perfect information case provides a weaker bound than the two latter. Moreover, the smallest relaxation provides the tightest bound. In conclusion, the UB is often weak and it can be tightened both by moderating the relaxation and by penalizing the benefit of additional information.

Brown et al. (2010) state that Monte Carlo simulation is a method commonly used to evaluate dynamic programming models, since it is easy to simulate realizations of complex dynamic systems with one ore more stochastic state spaces. Monte Carlo simulation can be used to simulate feasible solutions, which will provide a LB on the expected optimal value. Later in the same study, Brown et al. (2010) also point out that the upper bound is of a format that is convenient for Monte Carlo simulation. In this thesis Monte Carlo simulation of stochastic information is applied both when determining UB and LB (see section 5.4). The UB is solved deterministically for each sample path.

When using simulation to investigate bounds, numerous simulations are performed. Since the absolute optimal value in each scenario is found when investigating UBs, it means that taking the expected value over the results, with equal weighting, represents an upper bound estimate (Brown and Smith, 2011).

Similarly, one can determine the *statistical* LB by weighing the objective values associated with feasible solutions over numerous scenarios.

4 Problem formulation

This chapter introduces the mathematical optimization model developed in this study to solve the bidding problem in Elbas. Initially, notation is presented in section 4.1. It is based on that of Powell (2011a), with some exceptions. In section 4.2 the problem is described. Following, a set of modelling assumptions are presented in section 4.3. Moreover, the light is shed on the physical aspects of the problem in section 4.4, where complexity is evaluated and modelling assumptions discussed. The mathematical model is formulated in section 4.5. Finally, section 4.6 focuses on how the problem is *solved* under the modeled framework and algorithms emphasize how the optimization program is implemented.

4.1 Notation

Indices

t	Time step
h	Production period
s	State
k	Discretized price level
l	Price vector
m	Discretized capacity level
n	Capacity vector
o	Discretized decision level
g	Decision vector

Sets

$\mathcal{T}$	Set of time steps t in the planning horizon.
$\mathcal{H}_t$	Set of production periods h available for trade in time step $t \in \mathcal{T}$.
$\mathcal{S}_t$	Set of states S_t
$\mathcal{S}_t^{\mathbb{I}_t}$	Set of states S_t containing capacity vector $\mathbb{I}_t \in I_t$. $\quad \mathcal{S}_t^{\mathbb{I}_t} \subset \mathcal{S}_t$

$\mathcal{S}_t^{\mathbb{P}_t}$	Set of states s containing price vector $\mathbb{P}_t \in P_t$. $\mathcal{S}_t^{\mathbb{P}_t} \subset \mathcal{S}_t$
$\mathcal{S}_t^{\mathbb{X}}$	Set of post-decision states $S_t^{\mathbb{X}}$.
$\mathcal{P}'$	Set of discretized price levels P_k, $k \in \{1, \cdots, \overline{P'}\}$.
$\mathcal{P}_t$	Set of price vector states $\mathbb{P}_l$, $l \in \{1, \cdots, \overline{P}_t\}$, in time step $t \in \mathcal{T}$
I'	Set of discretized capacity levels I_m, $m \in \{1, \cdots, \overline{I'}\}$
I_t	Set of capacity vector states $\mathbb{I}_n$, $n \in \{1, \cdots, \bar{I}_t\}$ in time step $t \in \mathcal{T}$
$\mathcal{C}_n^{MC}$	Set of marginal costs of production for capacities $\mathbb{I}_n \in I_t$
$\mathcal{C}_n^{AC}$	Set of average costs of production for capacities $\mathbb{I}_n \in I_t$
$\mathcal{X}'$	Set of discretized decision levels X_o, $o \in \{1, \cdots, \overline{X'}\}$.
$\mathcal{X}_t$	Set of decision vectors $\mathbb{X}_g$, $g \in \{1, \cdots, \overline{X}_t\}$ in time step $t \in \mathcal{T}$.
Ω_t	Set of possible exogenous information realizations W_t.
Π	Set of policies π.

State Variables

S_t	State in time step $t \in \mathcal{T}$.
$S_t^{\mathbb{X}}$	Post-decision state, transitioning from state S_t by decision $\mathbb{X}_t$.
$\mathbb{P}_t$	Vector of prices in time step $t \in \mathcal{T}$
$\mathbb{P}_t^S$	Vector of *sell* prices in time step $t \in \mathcal{T}$. Derived from $\mathbb{P}_t$.
$\mathbb{P}_t^B$	Vector of *buy* prices in time step $t \in \mathcal{T}$. Derived from $\mathbb{P}_t$.
$p_{t,h}$	Price level for production period $h \in \mathcal{H}_t$.
$\mathbb{I}_t$	Vector of capacities in time step $t \in \mathcal{T}$.
$i_{t,h}$	Capacity level for production period $h \in \mathcal{H}_t$.

Decision Variables

π	Policy for decision making, mapping state S_t to a decision $\mathbb{X}_t^{\pi}$
$\mathbb{X}_t^{\pi}(S_t)$	Decision given by policy $\pi \in \Pi$, given state $S_t \in \mathcal{S}_t$ in time step $t \in \mathcal{T}$.

$\mathbb{X}_t^S$	Vector of volumes *sold* in time step t. Derived from $X_t^{\pi}(S_t)$.
$\mathbb{X}_t^B$	Vector of volumes *bought* in time step t. Derived from $X_t^{\pi}(S_t)$.
$x_{t,h}^S$	Volume *sold* in time step $t \in \mathcal{T}$, for product period $h \in \mathcal{H}_t$.
$x_{t,h}^B$	Volume *bought* in time step $t \in \mathcal{T}$, for product period $h \in \mathcal{H}_t$.
$\mathbb{C}_n^{MC}$	Marginal cost of production for production capacity $\mathbb{I}_n \in \mathcal{I}_t$
$\mathbb{C}_n^{AC}$	Average cost of production for production capacity $\mathbb{I}_n \in \mathcal{I}_t$
β_t	Number of start-ups and shut-downs of generator in production plan, in time step $t \in \mathcal{T}$.
$\alpha_{t,t+1}$	Difference in number of starts and stops from step t to step $t+1$

Stochastic Parameters and Variables

$\mathbb{W}_t$	Exogenous information vector arriving between time steps $t-1$ and $t \in \mathcal{T}$
$\mathbb{M}$	Transition matrix for stochastic price process

Parameters

$\overline{T}$	Number of time steps $t \in \mathcal{T}$.
$\overline{H}_t$	Number of production periods $h \in \mathcal{H}_t$.
$\overline{S}_t$	Number of states $S_t \in \mathcal{S}_t$.
$\overline{P}_t$	Number of price vectors $\mathbb{P}_l \in \mathcal{P}_t$
$\overline{I}_t$	Number of capacity vectors $\mathbb{I}_n \in \mathcal{I}_t$
$\overline{X}_t$	Number of decision vectors $\mathbb{X}_g \in \mathcal{X}_t$
$\overline{P'}$	Number of price levels $P_k \in \mathcal{P}'$
$\overline{I'}$	Number of capacity levels $I_m \in \mathcal{I}'$
$\overline{X'}$	Number of decision levels $X_o \in \mathcal{X}'$
L^{WV}	Water value.
L^{head}	Head in reservoir.
$L^{EURtoNOK}$	Currency exchange rate from EUR to NOK.

D	Date of production
C^{SRMC}	Short run marginal cost of coal.
$\mathbb{L}_{\mathbb{I}}^{spot}$	Initial spot commitment
$\mathbb{L}_{\mathbb{P}}^{spot}$	Initial spot prices
C^{ss}	Semi fixed start-up and shut-down cost.
τ	Time elapsed from the first date within the data set investigated to *determine the price model*, until date of production, D.
p_k^{LM}	Mean price $[EUR/MWh]$, of price level $k, k \in \mathcal{P}'$

4.2 Problem Description

The purpose of this thesis is to develop a stochastic optimization model as a tool for decision support when bidding in the intraday market, Elbas. To maximize profit, a market participant in the energy markets must bid strategically. Strategic bidding entail allocating resources in a cost-effective manner, whilst considering the potential to take advantage of good market opportunities. Situations where there is a deficit or surplus of production occurring after the spot deadline, have an impact on the market prices and create opportunities to profit from premiums, or excess returns, beyond covering the marginal cost of production.

The profit from trading in Elbas is determined by the market prices, the volumes traded and the resulting change in production costs. The market price is regarded as a stochastic process from the spot market clears and up until the time of production, with individual processes for each production hour. Production costs are related to total power production and unit commitment, and will differ between the production hours according to the initial production allocation from the spot clearing. Hence, optimal bidding must consider the production hours as individual products.

This means that each day, there are 24 different production hours, which can be regarded as 24 products subject to trade in Elbas. Any participant can bid on these products in a continuous double auction. The time horizon of the auctions differs between the products. The market opens at the spot clearing the day ahead of production, but has a individual bid deadline at the close for each production hour. The market prices are considered stochastic for this time horizon. Participants have the option to place bids at any time they want, and as many times they want. Each bid contains information about what volume $[MWh]$ of the individual product the bidder is willing to buy or sell for a specified price $\left[\frac{EUR}{MWh}\right]$. If no counterpart accepts the bid, there will be no trade.

Another factor affecting the optimal bidding strategy, is related to resource allocation. Each time a product is traded, the traded volume affects the available production capacity level on the machinery for the corresponding production hour. One cannot commit to produce more power than the specified maximum output of the generator, or commit to reduce production more than turning off the generator. Consequently, all trades affect future opportunities to trade in the continuous market, and expected future prices must be considered as alternatives to the market prices available at the time.

Considering the resource allocation problem, optimal bidding must regard trade volumes and resulting production costs of 24 different products, all of them with an individual stochastic price process affecting the optimal timing to place bids for each product. This is referred to as the double time dimension of the bidding problem in Elbas.

4.3 Assumptions

1. The stochastic price model built on results from Bovim and Næss (2017) is a representative model of the willingness to trade in Elbas.
2. Spot prices and Elbas prices are correlated.
3. Spot prices have the Markov property.
4. Bids are placed once in the beginning of a bidding period.
5. Constant water value, reservoir head and currency exchange rate over the modelling horizon.
6. Bids placed in accordance with the SDP model presented in this thesis will be accepted in the market.
7. One can only sell *or* buy power - not both - associated with a specific product in a specific bid period.

8. Due to the two-price mechanism (section 2.4), market participants will always be worse off settling their balance in the regulating market

4.4 Modelling

This section focuses on presenting the physical aspects of the problem. The problem is concerned with how a market participant in Elbas can allocate resources optimally when considering price uncertainties. The problem differs from comparable ones in the literature, since it deals with a double time dimension. This aspect increases complexity and is presented initially (section 4.4.1) to shed light on the problem structure. Moreover, a discussion involves how to model the scarce resource (section 4.4.2), in such a manner that redundant constraints are left out. Finally, the stochastic model, applied to describe market prices, is included in section 4.4.3.

4.4.1 The Double Time Dimension

A difference between periodic auctions, such as the Nordic spot market, and continuous auctions such as Elbas, is the flexibility in timing of bid placing and acceptance. In this study, a continuous clearing is modelled with a double time dimension, where timing of bids are discretized into bid periods t while timing of production are related to production period h. As introduced in section 4.2, each production period can be regarded as products subject to trade in the *beginning* of each bid period.

Recall the time line presented in fig. 1, section 2. One cannot trade a product after the associated production period's close. Hence, the set of *available products* $\mathcal{H}_t$ decreases when moving forward through the bid periods $t \in \mathcal{T}$. This is illustrated i fig. 9, where available products are white. The total, initial *number of products, when all products are available*, is $\overline{H}$. The last production period is denoted H.

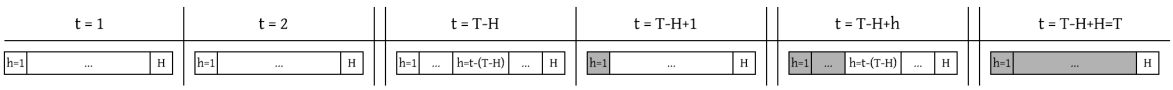


Figure 9: Generalized overview of valid production periods, $h \in \mathcal{H}_t$, to optimize over bid period $t \in \mathcal{T}$. Note that T correspond to the notation $\overline{T}$.

By discretizing time into bid periods of lengths equal to production periods, all products are available for trade in the beginning. Hence, $\overline{H}_t = \overline{H}$ until the bid period passes the first production period's close. $\overline{H}_t$ keeps decreaseing, until only the last product H is available for trade in the last bid period $\overline{T}$. Note that the effect of having a continuous auction modelled by multiple discretized bid periods, is that the bids cannot be placed at *any* moment in time, but the model still allows multiple sequential bids.

Mathematically, the number of available products $\overline{H}_t$ is given by:

$$\overline{H}_t = \begin{cases} (\overline{T} - t) + 1 & \text{if } t \geq (\overline{T} - \overline{H}) + 1 \\ \overline{H} & \text{otherwise} \end{cases} \tag{3}$$

Figure 10 illustrates how the two time dimensions are linked, illustrated with a case of 6 bid periods and 4 production periods. It emphasizes that as time t proceeds, the number of available products decreases. Note that the indices h are linked to specific products throughout this problem. Hence, low numbers are phased out as time proceeds. This is done to ease the discussion about production periods.

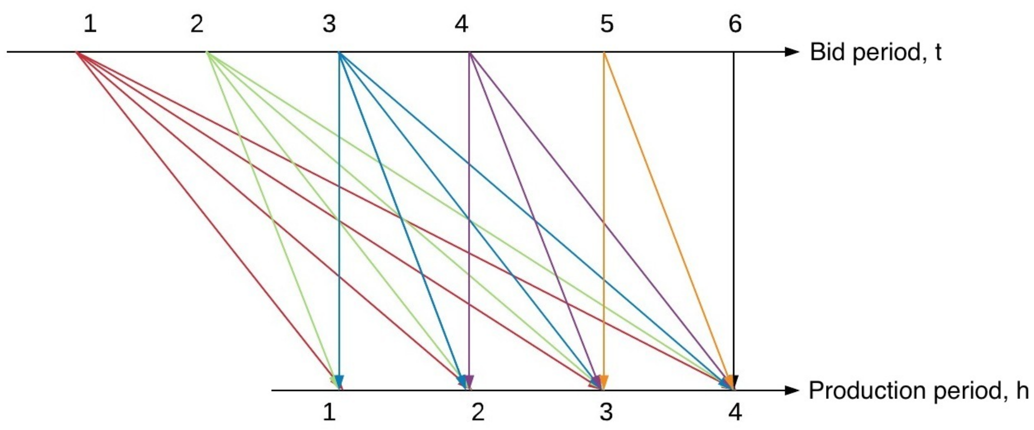


Figure 10: The double time dimension

Figure 10 illustrates the increased complexity the introduction of the continuous double auction introduces, as decisions regarding all products are done multiple times in sequence. In the Elbas bidding problem approached in this thesis, all 24 products are subject to trade for a duration of between 10 to 34 hours, which implies that the overall dimensions are large.

4.4.2 The Resource Allocation Problem

Two important aspects restricting production opportunities in a power plant, hence also power producers' trade opportunities in Elbas, are evaluated in this thesis. The first is capacity of machinery, related to the maximum *power* output the power plant is able to produce. The second is the availability of an energy source, related to the maximum *energy* the power plant can produce within a given time horizon. Both of these are related to resource allocation, making the bidding problem in Elbas a multi-stage resource allocation problem, where decisions made in one stage possibly have an impact on future opportunities to trade. This section aims at discussing the effect from machinery capacities and scarce energy sources on the bidding problem in Elbas.

Machinery Capacities

The generator capacity has a finite maximum and minimum limit given by eq. (4)

$$I^{min} \leq i_{t,h} \leq I^{max} \tag{4}$$

The remaining capacity $\mathbb{I}_t$ on the generator is a scarce resource, with elements $i_{t,h}$ associated with each production period h. The remaining capacity $i_{t,h}$ of the generator in each production period h, restricts trade volumes of that product, as final production Q_h cannot exceed capacity limits on the generator. This is shown in eq. (5) and eq. (6), where Q_h is given by the sum of trades and initial spot commitment for production period h, directly affecting the remaining capacity on the generator.

$$Q_h = \sum_{t \in \mathcal{T}} \left(x^S_{t,h} - x^B_{t,h} \right) + L^{spot}_h \tag{5}$$

$$i_{t,h} = I^{max} - Q_h \tag{6}$$

The remaining capacities for all products in bid period $t+1$, are affected by the remaining capacities and volumes traded in bid period t. Remaining capacities can both decrease and increase, according to whether the trades in bid period t are up- or down-regulations of production. Equation (7) summarizes the power balance, while fig. 11 illustrates it as an inventory balance.

$$\mathbb{I}_{t+1} = \mathbb{I}_t - \mathbb{X}^S_t + \mathbb{X}^B_t \tag{7}$$

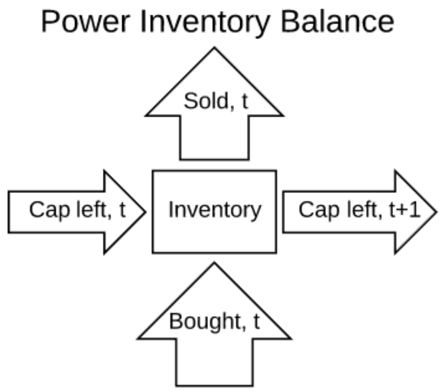


Figure 11: Power balance for trades, with remaining machinery capacity $\mathbb{I}_t$ illustrated as inventory

The bidding problem in Elbas is based on a initial production plan from the spot commitment, namely $\mathbb{I}_1 = \mathbb{L}_{\mathbb{I}}^{spot}$. Changes in power production, i.e. rescheduling, are determined for all production periods $h \in \mathcal{H}_t$ in each bid period $t \in \mathcal{T}$. The market participant decide what time the machinery is run and to what degree. For illustration purposes, fig. 12 shows the problem for a given bid period t' where 4 products are available for trade. The y-axis describes the

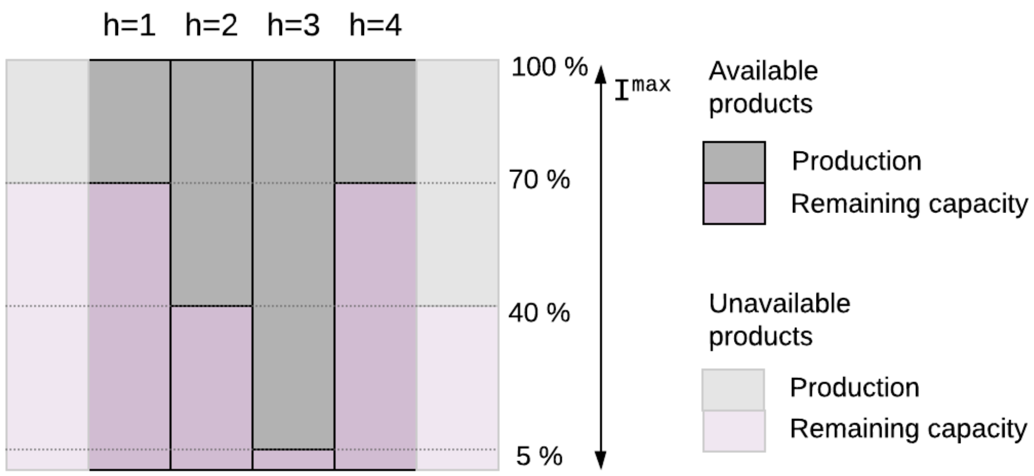


Figure 12: Production plan in bid period t'

remaining capacities of the generator as percentage of I^{max}. The columns marked *h=1, ..., h=4* are associated to products $h \in \mathcal{H}_t$. Product $h = 1$, has a remaining capacity of 70% of maximum capacity. The remaining capacity is $40\%, 5\%$ and 70%, respectively for the products *h=2, h=3* and *h=4*.

Two *unavailable* products are also included in fig. 12, illustrated by more transparent colors. Though the market participant cannot bid on unavailable products, they affect optimal bids for *available* products due to semi-fixed costs related to start-up and shut-down effects. As can be seen of the example in fig. 12, the generator is running in all consecutive production periods. This implies that there are no costs related to start-up or shut-down of the machinery in the current production plan in bid period t'.

Note that the unavailable product to the *right* in fig. 12 represents the first production period outside of the modelling horizon, namely in day $D+1$. There is no way of knowing the actual production plan for this product at the beginning of the modelling horizon for day D, as the spot clearing for day $D+1$ happens one day later on. Hence, the unavailable product is assumed to get a spot commitment equal to the corresponding period in day D, i.e. production period 1 in day D. This is due to seasonality effects of demand within a day, assuming similar effects the day to come.

This thesis consider *discretized levels of production*, hence also discretized levels $i_{t,h} \in I'$ of remaining capacity. The levels are generalized to consider 4 percentage values of the maximum capacity, I^{max}.

$$I' = \{0.05, 0.40, 0.70, 1.00\} \cdot I^{max}. \tag{8}$$

Where maximum capacity is initialized at the beginning of the planning horizon, by *hydraulic head* in the reservoir, denoted L^{head}.

$$I^{max} = f(L^{head}). \tag{9}$$

Reservoir Level

Most literature within the field of hydro power production, consider the amount of water in the reservoir as a variable affected by production and stochastic water inflow. However, when the planning horizon concerns only one day of production, only a negligible amount of water is utilized even at maximum production, due to strict capacity constraints on the machinery. Inflow is also limited over such a short time period.

Consider fig. 13, illustrating three different reservoir levels. The red lines show maximum ($\overline{L}^{head}$) and minimum ($\underline{L}^{head}$) regulated limits of the reservoir. Unless the reservoir is already almost full or empty at the beginning of the planning horizon, the reservoir cannot overflow

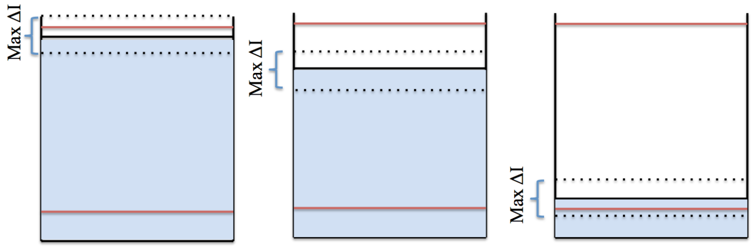


Figure 13: Illustrates the relationship between water level in reservoir and maximum changes due to production.

or run empty. If the reservoir is almost full or empty at the *beginning* of the horizon, the corresponding water value will be extremely low or high, respectively. This ensures the correct incentives for trading the following day. Hence, it is considered redundant to model the reservoir level as an inventory constraint in addition to machinery capacities. By keeping the tightest constraint and leaving the redundant one out, a smaller dimensioned problem with the same properties is obtained.

Referring to the change in reservoir level within the modelling horizon as $\Delta L^{head}(q)$, eq. (10) shows the mathematical relation between reservoir level, water consumption q, inflow U and the ground surface G of the reservoir.

$$\frac{U-q}{G} = |\Delta L^{head}(q)| \ll \left(\overline{L}^{head} - \underline{L}^{head}\right) \tag{10}$$

If the net water consumption is sufficiently small and the ground surface is large, production has little impact on the reservoir level. As long as the reservoir level and water volumes are included in the *initialization* of the mathematical optimization model, any *changes* happening during the planning horizon are considered negligible (see item 5, section 4.3).

4.4.3 Market Prices

The decisions made in Elbas introduce a *change* in revenues and costs from the initial spot commitment. Calculating profits in Elbas must therefore be with a perspective of alternative costs. The intuitive way to gain a profit, is to sell power at a price higher than the marginal cost of production, including any semi-fixed start/stop cost that may occur. However, if it is cheaper to *buy* power than to *produce* the committed volumes from the spot clearing, an additional way to profit is to buy power in Elbas and decrease production costs accordingly.

Production costs related to power plants in general, and Søa specifically, is described in section 2.5. This section aims at introducing market aspects. It makes also the basis for some of the assumptions of this study, including items 1 to 3 and items 6 to 7 in section 4.3.

This thesis is a continuation of a preliminary project performed by Bovim and Næss (2017), where the project scope is to analyze the intraday market and model a stochastic price process describing the *willingness to trade*. First, a summary of how the price process is found in the preliminary project is presented, followed by adjustments made in this thesis to obtain a more compact price process formulation.

Price Process from Preliminary Study

Spot price data and data regarding capacity of transmission lines for different products make up the basis for the price model. The purpose of the latter is to account for existing bottlenecks on transmission lines. Spot prices, rather than Elbas prices, are modelled due to availability of data and the assumed correlation to Elbas prices (section 3.4).

Based on data from numerous days, two time series are obtained. One for *sell* prices and one for *buy* prices. The time series are built by mapping hourly historical spot data together with corresponding historical capacities to determine a pattern, or a process, of prices in available areas over time. The price associated with the maximum available price area is referred to as a sell price, whilst the lowest is referred to as a buy price. Time series analysis shows that there is correlation between data points. For instance, there is a significant auto correlation with a time lag of one hour. The correlation with the previous data point supports what literature suggests: namely that the Markov property holds for energy prices. Hence, a Markov process is chosen to represent the stochastic prices.

Adjustment of the time series is performed in three steps. The purpose is to isolate the process and remove externalities, in order to formulate a general process. The first step is to normalize the data set with respect to short-run marginal cost (SRMC) of coal, because coal prices have a great impact on spot prices. Secondly, a linear trend is observed in the data set, and de-trending is performed in order to extract it. Finally, a probability integral transformation is performed. The latter transformation requires finding a distribution that fits the data set well, and hence the price data is transformed into numbers between zero and one. A Kernel distribution is found to be a good fit to the data set. The good availability of data motivates building the Markov transition matrix utilizing empirical methods based on the data. In order to count transitions, the continuous data set is discretized into levels. Hence, the matrix represents the probability of moving from a price level in time step t to any other price level in time step $t+1$.

Numerous process representations are found and evaluated. The differences between the models concern three areas: Firstly, what in sample data they are based on. Secondly, what step-dependency to model. Finally, it is a matter of how many levels to discretize prices into.

The evaluation and comparison of the models is performed using Monte Carlo simulation and analysis of long-term behaviour. The chosen model is found by weighing desirable properties (Bovim and Næss, 2017). The chosen in sample data is data from Jan 2013-Dec 2016. The step-dependency is one, due to analysis of auto-correlation. Hence, it is a typical Markov processes. The chosen number of discretization is 5 levels, where each level contains equally many data points; i.e. there is an equal probability of being in each level. The five price levels form a price *grid*. Note that the entire process is performed for both sell and buy prices separately, resulting in two different transition matrices with different discretization. *The 5 price levels make up the set of discretized prices*. One set for sell and one for buy prices.

To reduce the number of variables, it is desired to investigate the possibility to model these prices as one variable only. This is what is done in the *compact price process formulation*.

Compact Price Process Formulation

Figure 14 shows the two time series associated with the historical maximum and minimum prices representing the willingness to sell (yellow line) and buy (blue line) as observed from TE's perspective in an arbitrarily chosen period in 2014.

It can easily be seen that the two time series are correlated. They are functions of common external factors, but differ due to bottlenecks on transmission lines. One common price, P^{avg}(red line in fig. 14), is defined in order to represent the market price level, and hence it can be used as a benchmark to determine both sell and buy prices. The dashed lines in fig. 14 describe the mean values of the time series. The notation P^S, P^B and P^{avg} is used in this section to represent the mean value of sell, buy and average prices, respectively.

In order to utilize P^{avg} to map prices back to sell and buy price, one must know *how* they are

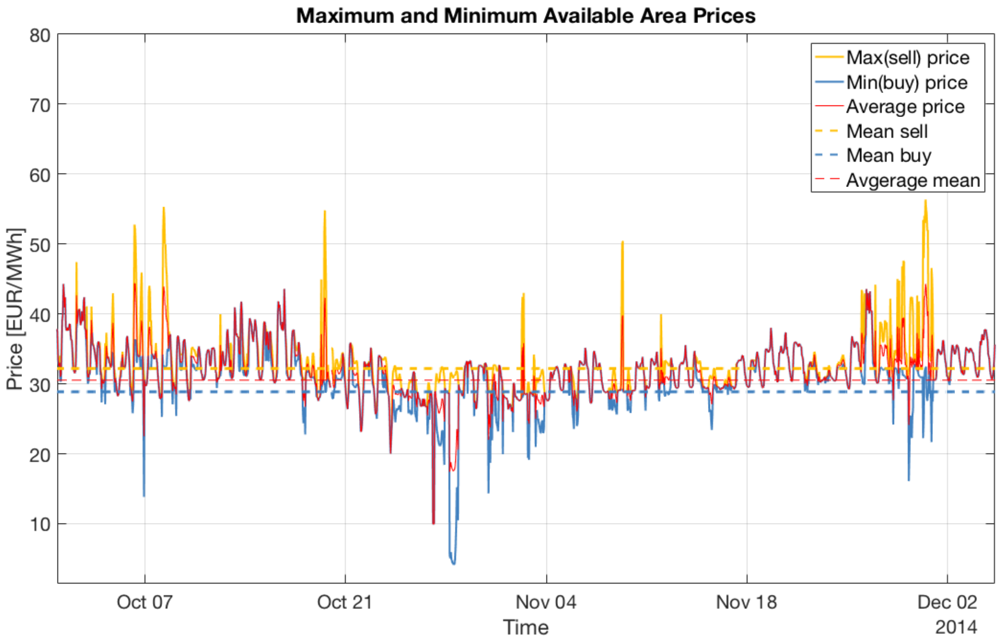


Figure 14: Time series of sell, buy and middle price over time as seen from NO3. Average values are found to determine the average deviations from the middle price level.

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related. An analysis is performed to determine the percentage wise deviation from average price, P^{avg} relative the two remaining *mean* prices, P^S and P^B. The percentage wise deviation is symmetric, and referred to as x. The x is found as a parameter describing the whole period 2013-2016. To calculate an appropriate percentage, x, that describes the deviation from the average price, P_t^{avg}, eqs. (11) to (15) describe the procedure. If

$$P^{avg} = \frac{P^S + P^B}{2} \tag{11}$$

and

$$P^S = P^{avg} \cdot (1 + x) \tag{12}$$

$$P^B = P^{avg} \cdot (1 - x) \tag{13}$$

then,

$$x = \frac{P^S}{P^{avg}} - 1 \tag{14}$$

$$x = 1 - \frac{P^B}{P^{avg}} \tag{15}$$

Under the modelling assumption that the average price describes the general price level, a new transition matrix is developed. Only this time, the average time series make up the basis. *The* *number of discretizations is held equal to five*, but now there is one common set rather than two.

The associated transition matrix describing the stochastic, exogenous price process, P^{avg}, is given by

$$\mathbb{M}^{small} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0.6638 & 0.1831 & 0.0753 & 0.0419 & 0.0359 \\ 0.2217 & 0.4135 & 0.1858 & 0.0985 & 0.0805 \\ 0.0701 & 0.2643 & 0.3276 & 0.1796 & 0.1583 \\ 0.0359 & 0.0094 & 0.3216 & 0.3653 & 0.1625 \\ 0.0077 & 0.1146 & 0.0899 & 0.3151 & 0.5625 \end{pmatrix} \end{array} \tag{16}$$

, with the equilibrium (Kirkwood, 2015):

$$\hat{\Pi} = [\hat{\Pi}_1 \quad \hat{\Pi}_2 \quad \hat{\Pi}_3 \quad \hat{\Pi}_4 \quad \hat{\Pi}_5] = [0.2 \quad 0.2 \quad 0.2 \quad 0.2 \quad 0.2] \tag{17}$$

This equation is the only place where the Π is related to long-term behaviour, and is utilized mostly because it is a commonly used symbol on this measure. Note that the long-term behaviour imply equal probability of the system being in all states after a long time, which is in accordance with the modelling objective of discretization. Note that the probabilities making up the transition matrix, $\mathbb{M}^{small}$, are independent of time and date. The exact same matrix can be utilized at different points in time. On the contrary, the discretized price grid of 5 levels may be shifted, even though the *transition probabilities* remain the same. What values the grid consists of depend on time and date, and have to be calibrated according to the SRMC of coal, C^{SRMC}, and time elapsed, τ, since since the date of the first point in the data set. For this thesis that date is the 1^{st} of January 2013 00:00:00. τ describes the time elapsed between that date and the investigated day of production, D.

Note that even though the transition matrix $\mathbb{M}^{small}$ remains unchanged within a period of time, it may become outdated eventually. One would typically *not* update the transition matrix depending on the day, but as time evolves, the model might not be as representative, and it is desirable to recalculate the matrix.

After introducing the average price representation, it is sufficient to model price as *one* variable, $\mathbb{P}_t$, in the model. The sell and buy prices can be derived from $\mathbb{P}_t$ in the following manner:

$$\mathbb{P}_t^S = \mathbb{P}_t \cdot (1 + x) \tag{18}$$

and

$$\mathbb{P}_t^B = \mathbb{P}_t \cdot (1 - x). \tag{19}$$

Where x is the deviation (eq. (15)) from the average price level P^{avg} and now from the variable $\mathbb{P}_t$, that represents the general price level in the market. Equation (20) shows that the 5 levels in the grid associated with the average price, P^{avg}, are functions of τ and C^{SRMC}. Moreover, $p_k^{LM}[\frac{EUR}{MWh}]$ represents the mean of price level k in the grid.

$$\mathcal{P}' = \{p_1^{LM}(\tau, C^{SRMC}), p_2^{LM}(\tau, C^{SRMC}), p_3^{LM}(\tau, C^{SRMC}), p_4^{LM}(\tau, C^{SRMC}), p_5^{LM}(\tau, C^{SRMC})\} \tag{20}$$

Market Stochasticity

Another perspective to consider is how the model accounts for power deficit and surplus, i.e. the power status in the market. If there is power *surplus*, demand is low and participants see a market that is willing to sell power at relatively low prices. Oppositely, if there is a power deficit, participants are likely to be willing to pay relatively high prices to obtain power. One say that the market will be either *short* or *long*, meaning that the market will have a power deficit or surplus, respectively. If the market is in balance and there is no excess or deficit of power the market is neutral. The model assumes that the user is able to *pick* side to a trade, in order to optimize production plan and maximize profits. It is assumed that the price model is a correct representation of market price levels, and that if placing bids at the given level, one will have the bid accepted. It is further modeled that one cannot alternate between selling and buying *within* a bid period, whilst expecting to get a premium for each trade. It would allow arbitrage opportunities creating incentives for infinite amounts of trades within each bid period t. It is modelled that one can only sell *or* buy at a certain point in time. However, the model opens for fluctuations in trade from one bid period to another, which can be motivated by arbitrage opportunities.

4.5 Mathematical Model

In this section, a dynamic program is built from scratch, with a main objective to handle dynamics of a continuous auction. The problem structure is such that the number of variables may introduce a challenge, but DP makes out an attractive approach due to its strength in sequential decision making. Hence, the mathematical model implemented in this thesis utilizes Stochastic

Dynamic Programming (SDP). The model consists of a state space with states and post-decision states, decision variables, an exogenous information process, a transition function, and a contribution and objective function. These are described in a general manner at first (sections 4.5.1 to 4.5.5). Afterwards, the light is shed on a well-known challenge of dynamic programming: the curses of dimensionality (section 4.5.6), followed by measures to handle an exponentially increasing number of variables (section 4.5.7).

Following this section, the algorithm applied to solve the mathematical formulation is described in section 4.6. A more compact formulation of the mathematical model is presented in Appendix A

4.5.1 State Space

To describe the mathematical *state* of a dynamic system, a set of state variables are utilized. As stated by Powell (2011b), "A state variable is the minimally dimensioned function of history that is necessary and sufficient to compute the decision function, the transition function, and the contribution function."

Different representations of the state spaces have been investigated, with an overlying objective of keeping the number of states low in order to avoid the curses of dimensionality.

The state space is defined by eq. (21):

$$\mathcal{S}_t : \mathcal{P}_t \times \mathcal{I}_t \tag{21}$$

, where $\mathcal{P}_t$ represents the prices in the market and $\mathcal{I}_t$ represents the remaining capacity of the generator, for all available production periods $h \in \mathcal{H}_t$. The state variables are therefore described as *vectors* containing information about price and remaining capacities in all available production periods, as shown in eq. (22) and eq. (23).

$$\mathbb{P}_t = [p_{t,1}, \dots, p_{t,H_t}] \tag{22}$$

$$\mathbb{I}_t = [i_{t,1}, \dots, i_{t,H}] \tag{23}$$

Making up the state

$$S_t = (\mathbb{P}_t, \mathbb{I}_t). \tag{24}$$

The discretized levels to describe price and capacity state variables, are part of sets as described in eqs. (25) to (26). The specific levels were presented in eq. (8) in section 4.4.2 and eq. (20) in section 4.4.3.

$$p_{t,h} \in \mathcal{P}' \tag{25}$$

$$i_{t,h} \in \mathcal{I}' \tag{26}$$

Where, $\mathbb{P}_t$ and $\mathbb{I}_t$ belong to the sets as described in eq. (27) and eq. (28).

$$\mathbb{P}_t \in \mathcal{P}_t \tag{27}$$

$$\mathbb{I}_t \in \mathcal{I}_t \tag{28}$$

Note that the set, $\mathcal{P}_t$ and $\mathcal{I}_t$ depend on t, whilst the sets $\mathcal{P}'$ and $\mathcal{I}'$ do not. The sets $\mathcal{P}_t$ and $\mathcal{I}_t$ decreases with t, because the number of production periods available for trade decreases. The set of discretized capacity levels on the machinery $\mathcal{I}'$, and discretized price levels for the stochastic price model $\mathcal{P}'$, are the same through the entire time horizon.

Post-Decision State Space

Due to the structure of this problem, it is convenient to introduce a *post-decision state*. It is the state that the system exist in *immediately* after a decision is made, but *before* exogenous information has arrived, and the system has evolved to the next state. The post-decision state contain information about the level of remaining capacity, see eq. (29). Figure 15 illustrates the timing of decisions relative to the arrival time of exogenous information (see section 4.5.3).

$$\mathcal{S}_t^{\mathbb{X}} : \mathcal{I}_{t+1} \tag{29}$$

Note that the symbols X_t, I_t, P_t and W_t in fig. 15 correspond to the vectors $\mathbb{X}_t, \mathbb{I}_t, \mathbb{P}_t$ and $\mathbb{W}_t$,

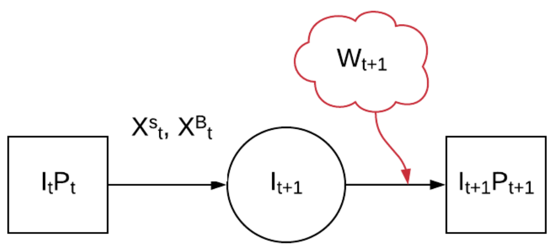


Figure 15: Timing of events

respectively. The circle containing I_{t+1} corresponds to the post-decision state $S_t^{\mathbb{X}}$, while the square boxes correspond to states S_t and S_{t+1}.

4.5.2 Decision Variables

The problem specific decisions are part of a set:

$$\mathbb{X}_t \in \mathcal{X}_t. \tag{30}$$

The decision variables are the part of the problem that are *possible to control*, in contrast to the market price level. The set of actions, $\mathcal{X}_t$, contains all possible inventory changes that correspond to the changes in the state variables, I_t to I_{t+1}. The decision variables are represented by the bid volumes $\mathbb{X}_t = [\mathbb{X}_t^S, \mathbb{X}_t^B]$ for selling and buying power respectively. The decisions concern *when* to trade, *what type of trade* (sell or buy) and *what volumes* to trade for a set of different products.

The decision $\mathbb{X}_t$ is determined by a policy $\pi \in \Pi$. The optimal policy maps all states $S_t \in \mathcal{S}_t$ to a specific decision $\mathbb{X}_t^\pi(S_t) \in \mathcal{X}_t$. The policy is time dependent, which is illustrated by the time step index t at the decision $\mathbb{X}_t^\pi$.

Note that Π is *not* related to the equilibrium $\hat{\Pi}$ presented in section 4.4.3. Only the policy notation Π is utilized for the rest of this thesis.

4.5.3 Exogenous Information Process

The exogenous information, $\mathbb{W}_t$, contain a description of applied processes that are out of the market participants control.

$$\mathbb{W}_t \in \Omega_t \tag{31}$$

, where Ω_t is the most general description of the set of exogenous information.

The notation convention utilized is similar to that of Powell (2011b) in the way that any variable indexed by t is known at time t. Immediately after a decision is made, but before new exogenous information becomes available, the system is in a *post-decision state* which is written as $S_t^{\mathbb{X}}$. The process sequence is described by that of Powell (2011b) as follows:

$$(S_0, \mathbb{X}_0, S_0^{\mathbb{X}}, \mathbb{W}_1, S_1, \mathbb{X}_1, S_1^{\mathbb{X}}, \mathbb{W}_2, ..., S_t, \mathbb{X}_t, S_t^{\mathbb{X}}, \mathbb{W}_{t+1}, ...S_{\overline{T}}).$$

In the modelled case, eq. (32) holds.

$$\mathbb{W}_t = \mathbb{P}_t \tag{32}$$

Where $\mathbb{P}_t$ represents state variable of the exogenous stochastic price process. In step t, the price $\mathbb{P}_t$ is known, and all $\mathbb{P}_{t+1}, \mathbb{P}_{t+2}, \mathbb{P}_{t+3}, \ldots$ are unknown.

4.5.4 Transition Function

The transition function, S^M, describes how the system evolves through time from one state S_t to state S_{t+1}.

$$S_{t+1} = S^M(S_t, \mathbb{X}_t, \mathbb{W}_{t+1}) \tag{33}$$

The system state will evolve from time step t to the next time step $t+1$ as seen from eqs. (34) to (36):

$$S_t = [\mathbb{P}_t, \mathbb{I}_t] \tag{34}$$

$$S_t^{\mathbb{X}} = [\mathbb{I}_{t+1}] \tag{35}$$

$$S_{t+1} = [\mathbb{P}_{t+1}, \mathbb{I}_{t+1}] \tag{36}$$

$\mathbb{I}_{t+1}$ is a controllable state variable uniquely given by the decision $\mathbb{X}_t$ made in step t. However, the price $\mathbb{P}_{t+1}$ is given by the exogenous information $\mathbb{W}_{t+1}$, and is not known at time t when the decision $\mathbb{X}_t$ is made. Hence, the decision determines the post-decision state $S_t^{\mathbb{X}} \in \mathcal{S}_t^{\mathbb{X}}$, while the exogenous information determines the state $S_{t+1} \in \mathcal{S}^{\mathbb{I}_{t+1}}$.

4.5.5 The Contribution and Objective Function

The *contribution function* is the direct contribution to the objective value by making a decision $\mathbb{X}_t^{\pi}$ when in state S_t. Recall that volumes sold $\mathbb{X}_t^S$ and volumes bought $\mathbb{X}_t^B$ are derived from $\mathbb{X}_t^{\pi}$.

$$C_t(S_t, \mathbb{X}_t) = \mathbb{P}_t^S \bullet \mathbb{X}_t^S - \mathbb{P}_t^B \bullet \mathbb{X}_t^B - \left(\mathbb{C}^{AC}(\mathbb{Q}_{t+1}(\mathbb{X}_t)) \bullet \mathbb{Q}_{t+1}(\mathbb{X}_t) - \mathbb{C}^{AC}(\mathbb{Q}_t) \bullet \mathbb{Q}_t \right) - C^{ss} \cdot \alpha_{t,t+1} \tag{37}$$

The contribution in time step t is given by the *changes* in revenues and costs provided within the step. The constituents in eq. (37) are the revenues or cost of selling or buying power, subtracted the *change* in average cost (AC) of production as a function of - and multiplied by - production, and in addition *changes* in semi fixed start and stop costs, C^{ss}, from one step to the next. Note that all constituents in eq. (37), except the last term, are vectors containing elements for each available production period.

Since the contribution for any time step t is dependent on decisions made at every time step $t \in \mathcal{T}$, a backward recourse function is calculated at each time step, estimating the *value* V_t of being in state S_t by eq. (38). The last term is associated with future expected value (EV) of making decision $\mathbb{X}_t$.

$$V_t(S_t) = \max_{\mathbb{X}_t \in \mathcal{X}_t} \left(C_t(S_t, \mathbb{X}_t) + \mathbf{E}^{\mathbb{W}_{t+1}}[V_{t+1}(S_{t+1}|S_t, \mathbb{X}_t, \mathbb{W}_{t+1})] \right) \tag{38}$$

A challenge is that the AC introduces non-linearities in the contribution function (eq. (37)), making eq. (38) non-convex and hard to solve. Hence, it is solved by a *inner problem*, which is a simple look-up algorithm breaking the problem up into smaller problems with fixed variables. The decision $\mathbb{X}_t$ is fixated for each $\mathbb{X}_t \in \mathcal{X}_t$, which correspondingly fixates $\mathbb{Q}_{t+1}$, solving all possible values of the objective function in eq. (38). The optimization is concerned with picking the solution that results in the greatest value. It is of great importance that the inner problem solves quickly, as it is computed once for each possible state $S_t \in \mathcal{S}_t$ in all time steps $t \in \mathcal{T}$.

The associated *objective function* should find the optimal policy π so that the EV of all time steps $t \in \mathcal{T}$ is maximized.

$$V_0^* = \max_{\pi \in \Pi} \mathbf{E}^{\pi} \{ \sum_{t \in \mathcal{T}} C_t^{\pi}(S_t, \mathbb{X}_t^{\pi}(S_t)) | S_0 \} \tag{39}$$

4.5.6 The Three Curses of Dimensionality

1. State Space Size

In dynamic programming one loop over equation eq. (38) $\overline{S}_t$ times to compute the $V_t(S_t)$ for all possible values of S_t. This is referred to as the dimensional curse. This section will shed light on the dimensional challenge and the size of the problem. The set of discretized price levels, $\mathcal{P}'$, is modelled to have the size $\overline{P'} = 5$ (section 4.4.3). There are 4 discretization levels of capacities on the generator, hence $\overline{I'} = 4$ (section 4.4.2). The number of levels is a trade-off between being large enough to represent the state space precisely and low enough to avoid the curses of dimensionality.

The size of the state space depends on the dimensions of its constituents. The dimensions of the price and capacity state spaces are given by eq. (40) and (41). When creating the stochastic dynamic program, the minimally dimensioned state variables are sought.

$$\overline{P}_t = \overline{P'}^{\overline{H}_t} \tag{40}$$

$$\overline{I}_t = \begin{cases} \overline{I'}^{\overline{H}_t} & \text{if } t \leq (\overline{T} - \overline{H}) + 1 \\ \overline{I'}^{\overline{H}_t} \cdot \overline{r}_{h=t-(\overline{T}-\overline{H})-1} \quad = \quad \overline{I'}^{\overline{H}_t} \cdot 2 & otherwise. \end{cases} \tag{41}$$

The size of the price state, $\overline{P}_t$, decreases when t grows, in the same manner as the production periods, $h \in \mathcal{H}_t$. For instance, in the last bidding period, only prices associated with the last product make up the price state. This is because it makes little sense to assign prices regarding products in the past. In the price vector, there is one element assigned to all available production periods, and as time passes the number of elements in the vector decreases.

The size of the capacity state, $\overline{I}_t$, decreases in a similar manner for increasing t. However, note that when making decisions concerning unit commitment, one must consider the *running status* of *neighbouring* product periods. It is sufficient to know whether or not the machinery is running, and not necessary to know the exact capacity level associated with it. The factor, $r_h(t)$, and the corresponding size $\overline{r}_h$ is introduced to account for the binary running property and can be derived from the capacity state, $\mathbb{I}_{t-1}$. When r_h is 1, it indicates that the machinery is committed to run for product h. The motivation to introduce $\overline{r}_h$ is that *it holds sufficient amount of information and has smaller dimensions than the capacity state*. Hence, calculation of state space $\overline{I}_t$ has a term that includes effects of the previous product period. In the steps t where all products h are available, one utilize information that is certain regarding the spot commitment. The running status of the spot commitment is an input to the model. The state space for capacity decreases slower than the price state space due to the binary dependency of the neighbouring product period.

The size of the entire state space depends on the dimensions of its constituents. Moreover, the dimension of $\overline{S}_t$, is given by

$$\overline{S}_t = \overline{P}_t \cdot \overline{I}_t. \tag{42}$$

When considering the problem described, there are $\overline{H} = 24$ products and the number of time periods is $\overline{T} = 34$. The states of the problem are illustrated in fig. 16.

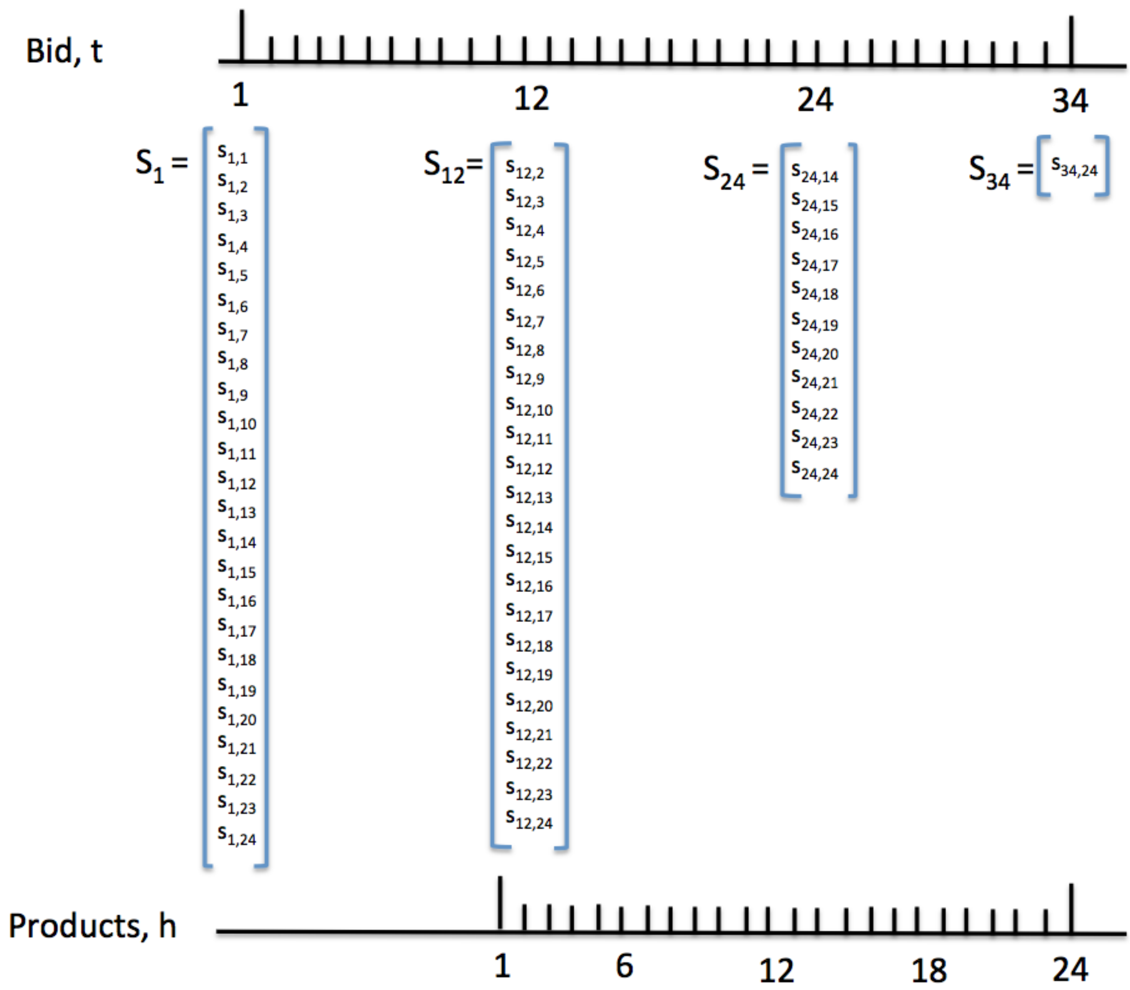


Figure 16: State vector S_t

Each element $s_{t,h} = p_{t,h} \cdot i_{t,h}$ in the vector S_t can take combinations of values from the sets $\mathcal{P}'$ and I'. The size of the state space in the first steps where all product periods are available, is as given by

$$\overline{S}_1 = \overline{P'}_1^{\overline{H}_1} \cdot \overline{I'}_1^{\overline{H}_1} = 5^{24} \cdot 4^{24} = 1.678 \cdot 10^{31}. \tag{43}$$

The decreasing size of $\mathcal{H}_t$ has an impact on the size of the state space both with regards to the price state, $\mathbb{P}_t$, and the capacity state, $\mathbb{I}_t$.

Table 12 in appendix C summarizes the state space size, $\overline{S}_t$, for all steps t from 1 to 34.

Note that the size of one step is extremely large, but also note that the increment from one to multiple steps does not does not grow as quickly.

2. Action Space Size

In addition to the state space, the *action space* is a dimension to consider. (Powell, 2011a) refer to the action space as the feasible region. The action space takes the size of the decision space in this model, which coincide with the size of the capacity state variable. It builds on the assumption (item 7 in section 4.3) is that one *either* sell or buy a product at a given bidding step in time (though one may fluctuate between selling and buying from one bidding step to the next). This results in a one to one relationship between the capacity state and the decision or action space.

The size of the action space in the first step is therefore

$$\overline{X}_1 = \overline{I}_1 = \overline{I'}^{\overline{H}_1} = 4^{24} = 2.815 \cdot 10^{14}. \tag{44}$$

Table 12 in appendix C also summarizes the action space size, $\overline{X}_t = \overline{I}_t$, for all steps t from 1 to 34.

3. Outcome Space Size

The outcome space has a direct link to the state variable $\mathbb{P}_t$. The possible price states origin from the outcome space and hence the outcome space in the first step $t = 1$, has the size

$$\overline{W}_1 = \overline{P}_1 = \overline{P'}^{\overline{H}_1} = 5^{24} = 5.960 \cdot 10^{16} \tag{45}$$

Table 12 in appendix C also summarizes the outcome space size, $\overline{W}_t = \overline{P}_t$, for all steps t from 1 to 34.

4.5.7 Aggregation

Note that the state space of the problem is large, and hence computationally difficult to solve. It is also worth noticing that *the dimension of the problem increases* exponentially *with* $\overline{H}_t$ *and* proportional *with* $\overline{T}$. This illustrates the well known *curses of dimensionality* in DP models, but also the strength in handling sequential decisions well.

The curses of dimensionality have been addressed by researchers by using *aggregation* to reduce the size of the state space (Powell, 2011a). The idea is to take the full problem, then aggregate it and solve it exactly before disaggregating it.

The problem described under section 4.2 is referred to as the *original problem*, and is summarized on the first line in table 3.

Table 3: Description of problem versions

Problem, time resolution	Number of bid periods	Number of product periods
Original	34	24
Aggregated	6	4

A model of aggregated dimensions, hereafter referred to as *the aggregated or the approximated* version of the problem (line 2 in table 3), is defined due to the curses of dimensionality associated with the original problem. The mathematical model is generic and can, in theory, be applied to both problem versions. In practice, one would need very strong software to be able to model an SDP of the original problem.

Hence, modelling the aggregated problem is the scope of this section. A discussion about how the aggregated model contributes in reality and in the original problem framework is found in section 5.

Hours are clustered into blocks, and in stead of having 24 products there are 4, and in stead of having 34 bidding hours, there are 6 of them. Figure 17 illustrates how products become

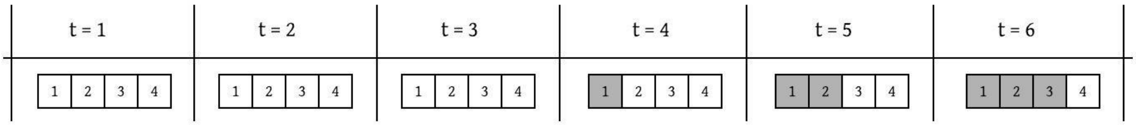


Figure 17: Problem specific overview of valid production periods, $h \in H_t$, to optimize over given bid periods $t \in T$.

unavailable for the chosen approximated modelling parameters. Recall fig. 10, which also describes the double time dimension for an approximated problem. The timing blocks start to run first, leaving the last 4 in parallel to the production blocks. Since neighbouring hours are characterized by the same parameters and see a similar market (due to the Markov property), clustering hours into blocks make sense. Placing block-bids is not unusual behaviour for market participants.

The two first periods are prior to production in the considered horizon, and hence trades can be placed for *all* products. The same goes for bidding hour 3, since the bid is placed in the beginning of the block. For bidding times *t=4* to *t=6* it is only possible to trade in the production hours *h = t-2* up to *h = 6*.

Aggregated Space Size

The aggregated problem is easier to approach using DP, than the full problem. The approximation greatly decreases the computation time.

With the size specified in table 3, the aggregated state vector, S_t with elements $s_{t,h} = p_{t,h} \cdot i_{t,h}$ take the form illustrated in fig. 18. The elements $p_{t,h}$ and $i_{t,h}$ are still elements from the sets $\mathcal{P}'$ and I', respectively. The difference from the *original problem* is the number of bid and production periods. For the aggregated problem, the size of the state space in step $t = 1$ is

$$\overline{S}_1 = \overline{P'}_1^{H} \cdot \overline{I'}_1^{H} = 5^4 \cdot 4^4 = 160000. \tag{46}$$

For the aggregated problem, where both bidding hours and production hours are aggregated, the state space per step decreases as shown in table 4. The column on the far right is found by applying eq. (42). The second column shows the change in number of available products for changes in t. Column three describes the number of discretized capacity levels in step t.

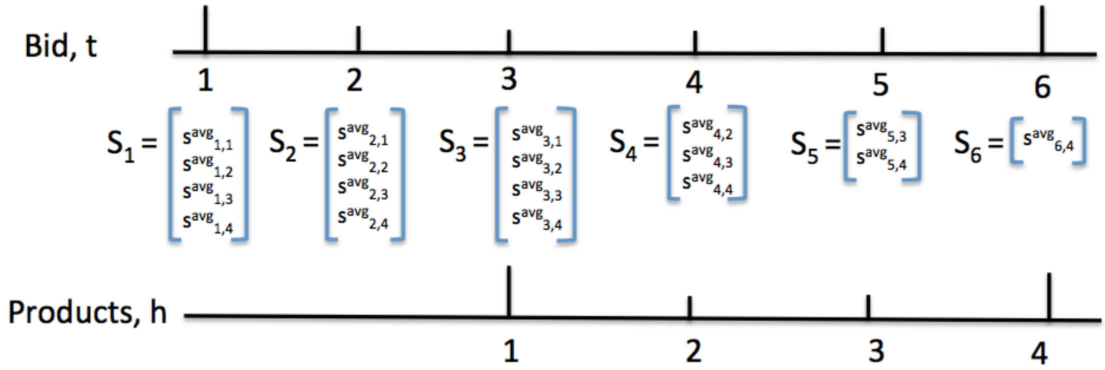


Figure 18: Aggregated state vectors, S_t

Column four describes the values the running status used to calculate the capacity state as given by eq. (41). Column 6 describes the capacity space size, $\overline{I}_t$, which for this thesis corresponds to the action space size $\overline{X}_t$. Column 7 describes the price space size, $\overline{P}_t$, which for this thesis corresponds to the outcome space size $\overline{W}_t$. The last column shows the full state space size $\overline{S}_t$, for all steps t.

Table 4: State space calculation

Bid block $[t]$	$\overline{H}_t$	$\overline{I'}_t$	$\overline{r}_{h=t-(\overline{T}-\overline{H})}$	$\overline{P'}_t$	$\overline{I}_t$	$\overline{P}_t$	$\overline{S}_t$
1	4	4	-	5	256	625	160000
2	4	4	-	5	256	625	160000
3	4	4	-	5	256	625	160000
4	3	4	2	5	128	125	16000
5	2	4	2	5	32	25	800
6	1	4	2	5	8	5	40

For comparison, the state space of the *original problem* in step $t = 1$ is $1.049 \cdot 10^{26}$ times larger than that of the *aggregated problem*.

Aggregated Price

In order to determine the transition probabilities for the *aggregated problem*, some additional steps are performed in the process of constructing the transition matrix (see section 4.4.3). The The raw data in the time series for P^{avg} is originally in an hour-by-hour format. When aggregating, the data points in the time series are grouped into blocks of 6 and 6 hours, where the new price is an average. The price grid is calibrated according to the day of production, D, and the associated time elapsed τ. In addition, it is adjusted for the existing downward sloping trend and the SRMC of coal, C^{SRMC}, corresponding to the input.

Hence, a new transition matrix $\mathbb{M}^{small}$ for the aggregated time series is found. In order to apply the process to *vector* transitions, a larger matrix $\mathbb{M}$ with dimensions $\overline{P}_t \times \overline{P}_t$ is derived from $\mathbb{M}^{small}$.

4.6 Solution Method

This section aims at describing how the mathematical formulation in section 4.5 is implemented. Algorithms used to solve the problem are illustrated. It connects Stochastic Dynamic Programming (SDP) with backward recursion, solving Bellmann's equation in the *inner problem*, and the resulting construction of a contingency plan applicable for the day in question.

When utilizing dynamic programming, the important structure to exploit is that all relevant information to make a decision is present in the state of the system. In a specific time step one does not require information about the past to make an optimal decision. However, Bellman's equation (section 3.5.1) regards both the present and the expected *future* value of any decision. Hence, a backward recursion algorithm is utilized, starting to calculate at the end of horizon at time step $\overline{T}$. It is possible to solve this problem as no future decision exists, and the expected value of any decision is given by that decision alone. Note that all possible states are evaluated to make sure there exists a specific decision $X_t^\pi(S_t)$ for all $S_t \in \mathcal{S}_t$. As the stochastic dynamic program is solved for each time step, the future values V_{t+1} are included in the objective function as an expected value with respect to the transition probabilities in $\mathbb{M}$ from the prices $\mathbb{P}_t$ to the prices $\mathbb{P}_{t+1}$.

The model only needs to run once a day, and is still capable of considering exogenous price information that occurs between time steps. Algorithm 1 shows SDP approach; how the state space and related production costs are initialized by daily input parameters, how all time steps are computed in backward recursion, and that the objective function in each state is solved by *the inner problem*. See the algorithm for the inner problem in algorithm 2.

The output from algorithm 2 is sent back to algorithm 1, and stored as the optimal policy for the state computed.

Algorithm 1: Stochastic Dynamic Programming with Backward Recursion

Input : Date of production, D

Spot commitments, $\mathbb{L}_{\mathbb{I}}^{spot}$

Spot prices, $\mathbb{L}_{\mathbb{P}}^{spot}$

Water value, L^{WV}

Reservoir head, L^{head}

Currency exchange rate, $L^{EURtoNOK}$

SRMC coal, C^{SRMC}

Transition matrix, $\mathbb{M}$

Output : Optimal policy $\pi : \mathbb{X}_t^{\pi} \quad \forall \quad t \in \mathcal{T}$

▷ *Initializations* $t = \overline{T}$

$\mathcal{X}_t \leftarrow$ Calculates set of decision vectors $\mathbb{X}_g \quad \forall \quad t \in \mathcal{T}$

$\mathcal{P}_t \leftarrow$ Calculates set of valid price vectors $\mathbb{P}_l \quad \forall \quad t \in \mathcal{T}$

$I_t \leftarrow$ Calculates set of valid capacity vectors $\mathbb{I}_n \quad \forall \quad t \in \mathcal{T}$

$\mathcal{S}_t : \mathcal{P}_t \times I_t \quad \forall \quad t \in \mathcal{T}$

$\mathcal{C}_n^{AC} \leftarrow$ Calculates set of average costs $\mathbb{C}_n^{AC}$ corresponding to $\quad \forall \quad \mathbb{I}_n \in I_t$

Initialize expected value at end of horizon: $\mathbf{E}(V_{\overline{T}+1}) = 0$

for $t \in \mathcal{T}$ *by* -1 **do** ▷ *Backward recursion*

  for $S_t \in \mathcal{S}_t$ **do** ▷ *The Inner Problem*

$$V_t(S_t) = \max_{\mathbb{X}_t \in \mathcal{X}} \left(C_t(S_t, \mathbb{X}_t) + \mathbf{E}^{W_{t+1}}[V_{t+1}(S_{t+1}|S_t, \mathbb{X}_t, W_{t+1})] \right)$$

$$\mathbb{X}_t^*(S_t) = \arg\max_{\mathbb{X}_t \in \mathcal{X}_t} (V_t(S_t))$$

    $\mathbb{X}_t^{\pi}(S_t) \leftarrow \mathbb{X}_t^*(S_t)$

  end

  if $t > 1$ **then** ▷ *Expected value calculation*

    for $\mathbb{I}_t \in I_t$ **do** ▷ *For each post-decision state in* $t-1$

      for $\mathbb{P}_{t-1} \in \mathcal{P}_{t-1}$ **do** ▷ *For each price vector in* $t-1$

$$\mathbf{E}_{t-1}^{W_t}\left[V_t(S_t \in \mathcal{S}_t^{\mathbb{I}_t} | S_{t-1} \in \mathcal{S}_{t-1}^{\mathbb{P}_{t-1}})\right] = \sum_{S_t \in \mathcal{S}_t^{\mathbb{P}_t}} \left(V_t(S_t) \times \mathbb{M}_{\mathbb{P}_{t-1}, \mathbb{P}_t} \right)$$

      end

    end

  end

end

Some additional notation is introduced to ease the reading of algorithm 2 for the inner problem.

Indices and Sets

i	Current node, state S_t
j	Future node, post-decision state $S_t^{\mathbb{X}}$
$\mathcal{N}_i$	Set of available nodes j transitioning from node i. $\mathcal{N}_i \subseteq \mathcal{S}_t^{\mathbb{X}}$

The time step index t is omitted, and instead of speaking of current and future time steps, we use the notation of current and future nodes. Note that while the current node holds information about both $\mathbb{P}_t$ and $\mathbb{I}_t$, the future node contains information about $\mathbb{I}_{t+1}$ and expected future value only. Hence, it represent the post-decision state $S_t^{\mathbb{X}}$ for each respective decision $\mathbb{X}$. The loop for evaluating all nodes i, is initiated in algorithm 1 and computed in algorithm 2.

Algorithm 2: Inner Problem in DP

Input : Current node i corresponding to state $S_t = (\mathbb{P}_t, \mathbb{I}_t)$ from SDP algorithm

Vector of future expected value elements $\mathbf{E}^{\mathbb{W}}[V_j] \quad \forall \quad j \in \mathcal{N}_i$

Output : Optimal Decision, $\mathbb{X}_i^*$

Value of making optimal decision, V_i

β_i ▷ *Count the number of start and stops in node i*

$\Delta\mathbb{I}_j = \mathbb{I}_i - \mathbb{I}_j$ ▷ *Find the change in remaining capacity from node i to j*

$\mathbb{X}^S_{logical} = logical(\Delta\mathbb{I}_j \geq 0)$ ▷ *Determine trade type associated with transition to node j.*

$\mathbb{X}^B_{logical} = logical(\Delta\mathbb{I}_j \leq 0)$

for $j \in \mathcal{N}_i$ **do** ▷ *Iterative node search through valid future nodes*

  β_j ▷ *Count the number of starts and stops in node j*

  $\alpha_{i,j} = \beta_j - \beta_i$ ▷ *Compare the number of starts and stops in node i and j.*

  for $h \in \mathcal{H}_j$ **do** ▷ *Investigate production blocks*

    if $\mathbb{X}^S_{logical}(h) == 1$ **then** ▷ *Sell product h*

      $\mathbb{X}^S(h) = |(\Delta\mathbb{I}_j(h))|$

    end

    if $\mathbb{X}^B_{logical}(h) == -1$ **then** ▷ *Buy product h*

      $\mathbb{X}^B(h) = |(\Delta\mathbb{I}_j(h))|$

    end

  end

  $V_{i,j} = C_j(j, \mathbb{X}) + \mathbf{E}^{\mathbb{W}}[V_j]$ ▷ *Determine value of transition to node j*

  $\mathbb{V}_i(j) = V_{i,j}$ ▷ *Put objective value into vector*

end

$V_i = \max(\mathbb{V}_i)$ ▷ *Determine the maximum and hence optimal value*

$\mathbb{X}_i^* = \arg\max_{\mathbb{X} \in \mathcal{X}}(\mathbb{V}_i)$ ▷ *Determine associated decision value*

All future nodes $j \in N_i$ are looped through, where the EV of going to each node $j \in \mathcal{N}_i$ is evaluated in relation to the corresponding *contribution function* at node i. This evaluation correspond to the value function in eq. (38). A great advantage of this method is given by the fact that for a given transition from node i to node j, all variables are fixed. In a market where one can only sell or buy at a given time (see section 4.4) the trade must correspond to the change in capacity and hence a decision is uniquely given by a transition between states.

As there are a limited amount of possible transitions in the model, there are a limited amount of possible decisions X_j. This is due to the fact that $\mathcal{N}_i \subseteq \mathcal{S}_t^{\mathbb{X}}$, so that the maximum size of $\mathcal{N}_i$ is given by the number of capacity combinations $\overline{I}_{t+1}$, rather than the full state space.

The expected value increases throughout the backward recursion. It describes the future uncertainties, but since they are included as an expected value, the inner problem becomes a set of deterministic problems for each $j \in \mathcal{N}_i$, where the optimization involves selecting the node j - or equivalently a policy π - providing the greatest value $V(\mathbb{X}_t^{\pi})$.

Figure 19 illustrates what the problem structure looks like. The symbols X, I and P correspond to the vectors $\mathbb{X}_t, \mathbb{I}_t$ and $\mathbb{P}_t$, respectively. The time steps in SDP are related to the square boxes

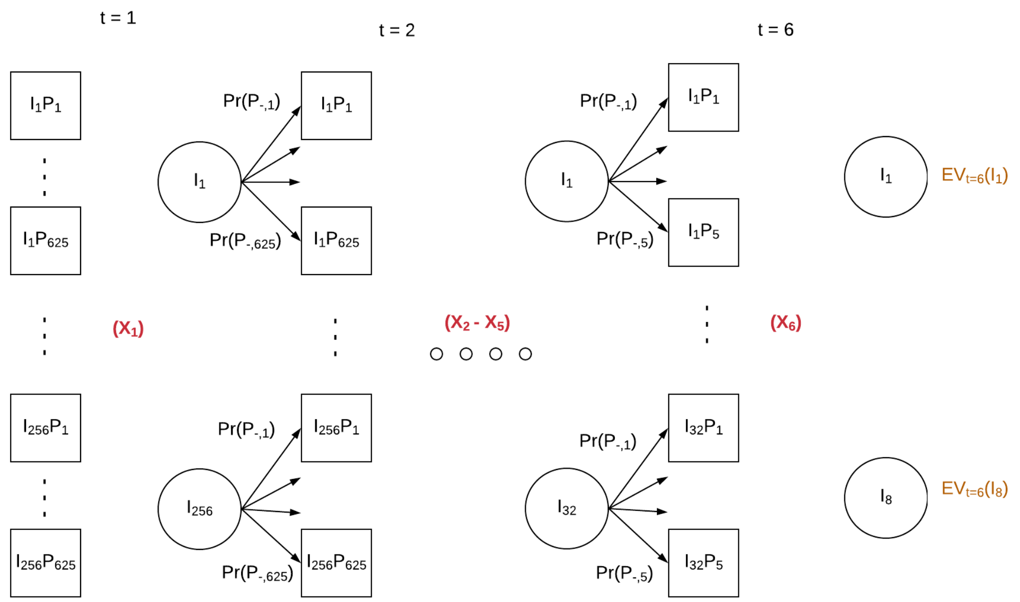


Figure 19: Decision tree

in fig. 19, containing information about what state $S_t = (\mathbb{I}_t, \mathbb{P}_t)$ the system is in, i.e what the machinery capacities and prices associated to each production block are. The backward recursion algorithm (algorithm 1) assigns each box with an optimal decision $\mathbb{X}_t^{\pi}(S_t)$ through solving Bellman's equation for that state in the inner problem (algorithm 2). The decision determines what the future state variable $\mathbb{I}_{t+1}$ will be, but has no effect on the future state variable $\mathbb{P}_{t+1}$. Hence, post-decision states $S_t^{\mathbb{X}}$ are illustrated by circles containing information about capacity $\mathbb{I}_{t+1}$ only. They are each connected to $\overline{P}_{t+1}$ square boxes referred to as the set $\mathcal{S}^{\mathbb{I}_{t+1}} \subset \mathcal{S}_{t+1}$ of all states containing the same capacity vector $\mathbb{I}_{t+1}$. Recall that the number $\overline{P}_t$ refers to the number of stochastic outcomes of the price vector $\mathbb{P}_t$ in time step t. Hence, by backward recursion, value of the post-decision state represents the expected value of the decision $\mathbb{X}_t$, determined by the probabilities and already calculated values of being in states connected to the post-decision state. Each square box represents a node i, while the circles represent the nodes $j \in \mathcal{N}_i$.

When making decisions in a node i, the production plan and associated production costs are affected. Even though AC values are used *several* times, they are only calculated *once* - during the SDP initialization in algorithm 1). Using this precalculation makes it possible to utilize an *exact* function to determine AC, rather than using an approximated linearization to decrease computation time. Therefore, each node j can be assigned an AC in the initialization. The value of going to node j is given by the EV of the post-decision state calculated in algorithm 1.

Determining the amount to trade implicates determining when to run the machinery and not. The semi fixed costs are associated with these changes. It is not necessary to dedicate a new state variable to represent the running state of the system, because it is implicitly given by the level of capacity of the generator, $\mathbb{I}_t$. If there is production in an hour, i.e. the level of capacity of the machinery is different from I^{max}, the machinery runs.

Notice how the inner problem does not include any information about other states in the same time step. A great advantage of this structure is that the inner problems can be computed in parallel instead of traditional sequential looping, decreasing running time significantly (see section 5.1).

5 Computational Study

The SDP model developed in this thesis solves the *aggregated problem* (see table 3) to optimality, under a set of assumptions (section 4.3). The policy to the aggregated problem can be applied to solve the *original problem*, with some adjustments. This policy is not guaranteed being the optimal one for *the original problem*. However, the possibly sub-optimal policy makes up a heuristic candidate SDP policy to the original problem.

The purpose of this chapter is to present an overview of how the stochastic dynamic program is utilized in practice, what characterizes the decisions provided by the policies and what value the model can provide a user. The lack of comparable historical data of TrønerEnergi's Elbas trades, introduces a challenge to retrieve a benchmark for analysis. Hence, bound analysis of the mathematical optimization program is more of interest, with an underlying assumption of few or no trades today. This applies to analysis of policies as well, where the reference point of analysis is how optimal behaviour is in a deterministic case.

Even though the problem is aggregated, the problem is quite complex, and therefore the need to utilize a high performance computing (HPC) cluster emerges. A description of the utilized software is presented in section 5.1, including running times of the model. Following, a set of instances is presented in section 5.2. Section 5.3 explains how the contingency plan is utilized. Afterwards, (section 5.4), a bound analysis is performed by Monte Carlo simulation of exogenous information on the set of instances. Lastly, a comparison of the stochastic and the deterministic model is presented in section 5.5.

5.1 Software Description

All test instances of the mathematical programming models are solved using MATLAB R2017a. MATLAB is run through the high performance computing cluster *Solstorm* to handle the large number of computations required due to the size of the state space of the problem (section 4.5.1). Solstorm is developed for the Department of Industrial Economics and Technology Management (IØT) at NTNU, to handle computing algorithms in large scale optimization models (Ganglia, 2018). The computing cluster is accessed on a remote server through an SSH client.

In theory, all nodes in a time step can be computed in parallel to decrease running time (section 4.6). While most computers support parallel computing by more than one *worker* , the number of parallel processes is restricted by technical specifications of the computers' central processing unit (CPU). Solstorm consists of multiple *computing nodes* with different CPUs (Solstorm, 2018). Note that a computing node must not be confused with a node in the SDP. The SDP model in this thesis is solved through a HP dl160 G5 server, with two Intel Quad Core E5472 (3.0 GHz, 16Gb RAM, 72Gb SAS 15000rpm) processors. The number of workers utilized to compute parallel processes is 30. Some extra overhead computation time is added due to the number of workers, but it is highly rewarded by the decreased time of solving each time step. See table 5 for detailed running time results, for sequential and parallel computing respectively.

Table 5: Model Running Time

Modelling Phase	Running Time	HPC Running Time
Initialization	12.42 sec	16.53 sec
Starting Parallel Pool	-	38.55 sec
Stage $t = 6$	5.02 sec	5.43 sec
Stage $t = 5$	7.97 sec	6.97 sec
Stage $t = 4$	184.38 sec	25.46 sec
Stage $t = 3$	574.23 sec	75.62 sec
Stage $t = 2$	669.99 sec	89.12 sec
Stage $t = 1$	683.24 sec	89.13 sec
Total	2,137.25 sec	346.81 sec

In addition, utilizing a CPU cluster is of great advantage for testing purposes, as multiple test instances can run simultaneously.

5.2 Test Instances

The large amount of data resulting from computing each day, makes general and representative testing of the model less straight forward. Each day has its own input parameters affecting the outputs, and different combinations of input will affect the model outputs in different ways. To present the results of the model and analyze general trends, a number of instances is selected based on a classification of the data set. The assumption is that due to modelling choices (section 4.4), days within the same classification should produce similar, or comparable, outputs. Our set of historical data is from the period January 2013 to August 2017.

Outputs of the stochastic dynamic program refers to both objective values and the policies extracted from computations.

5.2.1 The Effect of Spot Prices and Water Values

As stated in section 2.5.1, the water value of a reservoir is a result of a complex optimization algorithm, regarding the alternative value of saving the water for later production. While the future market price for energy highly affects the water values, Søa is a relatively small reservoir with less flexibility, resulting in volatile water values mostly affected by the reservoir inflow at all times. Hence the marginal cost of production is volatile and not always as correlated with the spot price as a bigger reservoir would be. This is observed in fig. 20. The same figure show plots of the difference, or *delta* Δ, between the average spot price of each day ($L^{spot}_{\mathbb{P},D}$) and the corresponding water value (L^{WV}_{D}), both as a function of time and in a sorted, increasing order.

$$\Delta = L^{spot}_{\mathbb{P},D} - L^{WV}_{D} \tag{47}$$

Figure 21 shows a close up version of fig. 20. Since the profit of the stochastic dynamic program

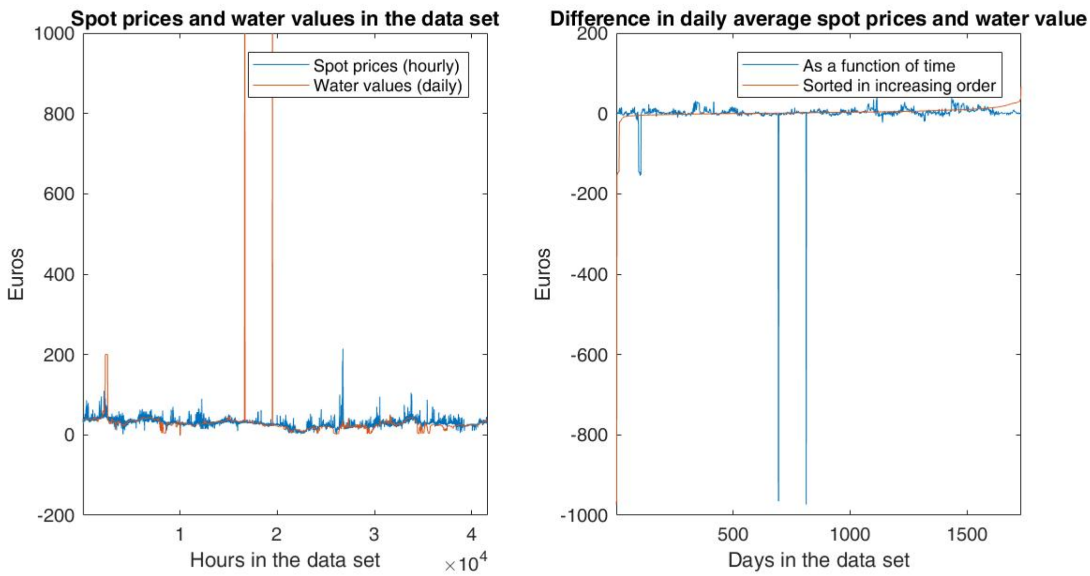


Figure 20: Spot prices and water values, and the corresponding difference

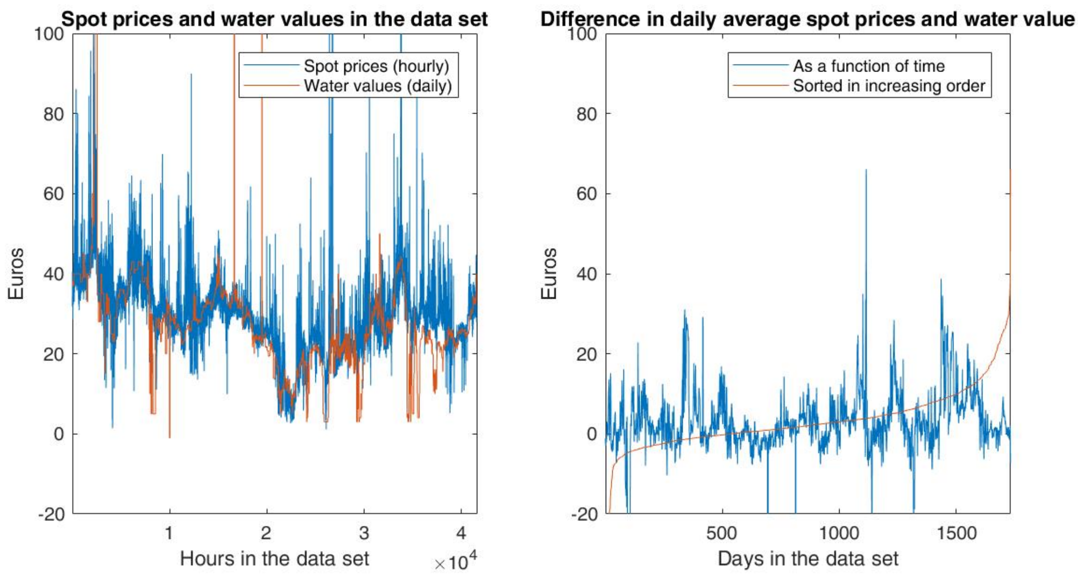


Figure 21: Close-up of spot prices and water values, and the corresponding difference

is strongly related to the difference in Elbas prices and the marginal cost of production, and the Elbas prices are discretized by a grid related to spot prices (Bovim and Næss, 2017), the output of the model will be affected by large differences between spot prices and water values as input parameters.

To represent days with different delta values, the sorted data set is split into three parts as listed in table 6, each containing an equal number of days. The test instances ensure that days within all three delta intervals are included in the analysis.

Table 6: Δ intervals

-	Δ_{min}	Δ_{max}
Δ_1	-973.255	0.323
Δ_2	0.324	4.189
Δ_3	4.200	66.103

5.2.2 The Effect of Initial Spot commitment

Another important aspect of the input parameters is the production volumes initially committed from the spot clearing. As the machinery has a limited production capacity, the initial commitment restricts what volumes are available for trade in Elbas. As stated in section 4.5.6, the number, $\bar{I}_t$, of possible capacity vectors is large. To keep the analysis as general as possible, but still representative for the real problem, the days in the data set is categorized by the initial spot unit commitment R, rather than remaining capacities $\mathbb{L}_{\mathbb{I}}^{spot}$. R is a vector of binary variables, that denotes whether or not machinery is committed in the spot market.

By including the block before and after the modelling horizon, it is possible to categorize the data set into $2^6 = 64$ parts, each representing a combination of 6 production blocks and the associated unit commitments ($[0,1]$). The following list illustrates by a few examples how the unit commitment changes from 1 to 64.

Some Unit Commitment Categories

1	: [0 0 0 0 0 0]
2	: [1 0 0 0 0 0]
30	: [1 0 1 1 1 0]
63	: [0 1 1 1 1 1]
64	: [1 1 1 1 1 1]

The number of days with each initial unit commitment is illustrated in fig. 22. There is also a bar graph of how many days within each Δ interval that have the different unit commitments. Notice how the most common unit commitments are related to production for either all (64) or none (1) of the production blocks. In addition, unit commitment 30 is common, where the production blocks representing night time are the only ones without a spot commitment. Note that the days within the interval with largest Δ are almost all from the category of unit commitment 64, which indicates that the generator is running for all production blocks. This is natural, as a large Δ indicates a higher spot price than the water value, so a power producer is likely to get dispatched in the spot clearing.

To evaluate a representative selection of days, the model is tested on the parameters most often occurring. Hence, a selection of days with unit commitment categories $R_1 = 1$, $R_2 = 30$ and $R_3 = 64$ will be further investigated.

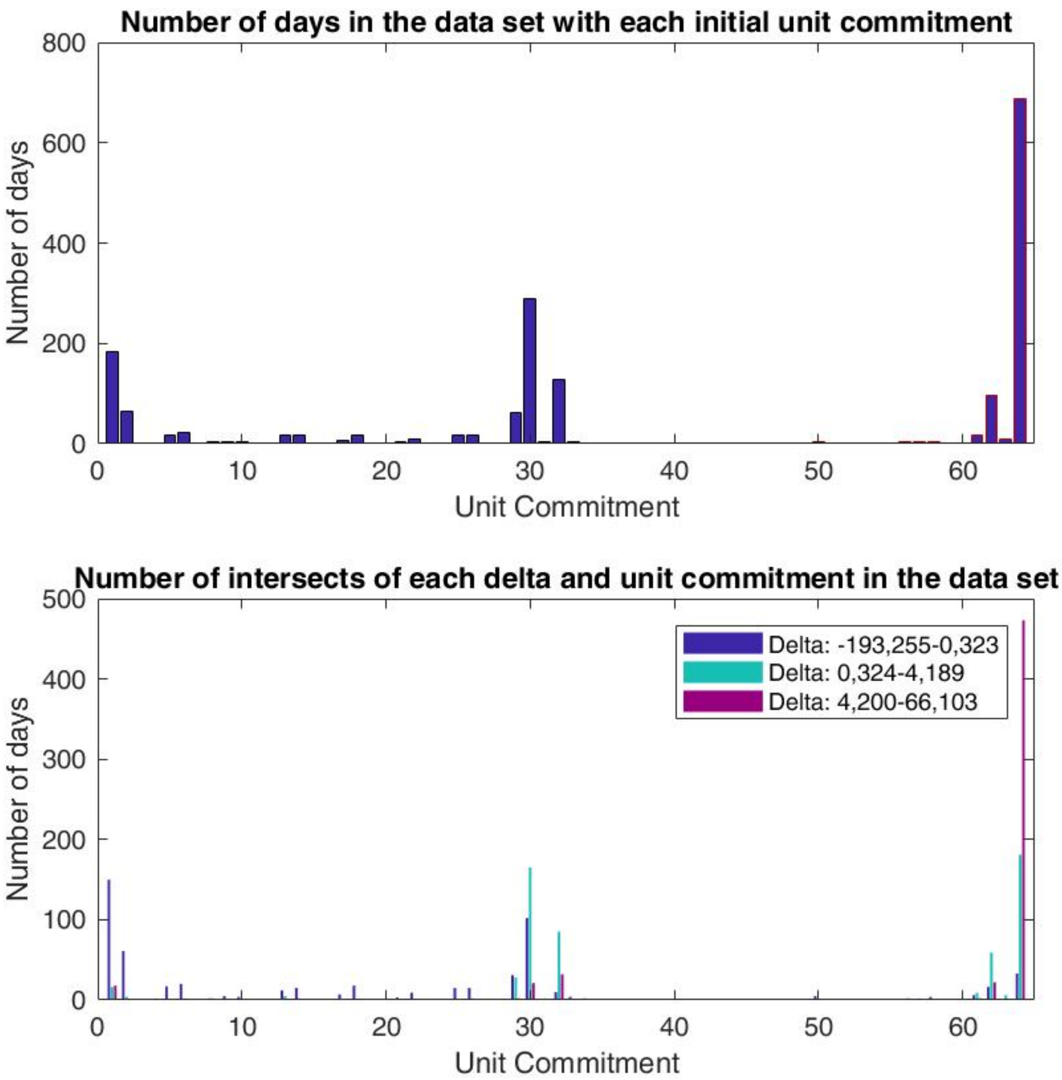


Figure 22: Number of days with each unit commitment category (1-64)

Table 7: Unit Commitments investigated

-	Unit Commitment
R_1	$[0,0,0,0,0,0]$
R_2	$[1,0,1,1,1,0]$
R_3	$[1,1,1,1,1,1]$

5.2.3 The Resulting Test Instances

Combining the two categories of Δ and R, each divided into three instances, the resulting instances for testing are summarized in table 8.

Table 8: Test instances

-	$[-973.25, 0.323)$	$[0.323, 4.189)$	$[4.189, 66.103)$
$[0,0,0,0,0,0]$	$R_1\Delta_1$	$R_1\Delta_2$	$R_1\Delta_3$
$[1,0,1,1,1,1]$	$R_2\Delta_1$	$R_2\Delta_2$	$R_2\Delta_3$
$[1,1,1,1,1,1]$	$R_3\Delta_1$	$R_3\Delta_2$	$R_3\Delta_3$

As the stochastic price model is constructed utilizing data from the year 2013 to 2016, it is preferable to not test the model on these years. Hence, only days for 2017 can be tested. It is beneficial to test on days of the same year, so that the results will be comparable. This section avoids comparing general results related to the policy on days in different seasons, as that may affect the result and make it less observable how Δ and R really affects the policy computed. The objective is to select test instances comparable to each other.

To avoid random results, preferably several days within each test instance should be computed. However, some instances does not have any days with that specific instance, or only a few. As can be seen in table 9, this is the case for intersections between either a large Δ and a unit commitment with few production blocks with initial production commitment, or a low Δ and a unit commitment for most production blocks. As stated in section 5.2.2, the few occurrences of these intersections are natural due to the demand/supply clearing i the spot market.

The dates listed in table 9 makes out the test instances for analysis in this thesis.

In a world of perfect information and no uncertainties, a market participant would choose to place bids similar to a trivial knapsack problem, first choosing the trade with the best price, then the second best, and so on, until production capacities restricts further trade. However, prices are considered stochastic in a realistic scenario, so one cannot know for sure what future opportunities to trade will be. SDP accounts for not knowing the future outcomes, comparing the gain of a certain decision up against the expected value (EV) of future opportunities.

This section focuses on the EV of participating in Elbas utilizing the SDP model constructed in this study on *the original problem*.

Table 9: Dates within each test instance, selected for evaluation

-	Δ_1	Δ_2	Δ_3
R_1	17-Jun-2017	27-Jun-2017	-
	25-Jun-2017	28-Jun-2017	-
	13-Jul-2017	-	-
	16-Jul-2017	-	-
R_2	18-Jul-2017	05-Jun-2017	14-Jun-2017
	-	10-Jun-2017	-
	-	26-Jul-2017	-
	-	31-Jul-2017	-
R_3	20-Jul-2017	17-May-2017	18-May-2017
	-	25-May-2017	26-May-2017
	-	21-Jun-2017	02-Jun-2017
	-	10-Jul-2017	22-Jun-2017

5.3 The Contingency Plan

Figure 19 in section 4.6 describes how the SDP looks and is followed by a description of how it is *constructed*. This section aim at describing how the model output, the policy, is *utilized*. Figure 24 show utilization of a policy and emphasizes how the uncertainty in the prices result in a growing tree of realizations.

As the policy maps an optimal decision to each state of the system in all time steps, the policy works as a look-up table, or a *contingency plan*. When the user inserts necessary information for decision making, the policy is constructed by SDP, and provides decision support as a contingency plan throughout the day. The model must be rerun, only if the input parameters changes. This is the strength of SDP and construction of a policy, the contingency plan has an optimal decision regardless of the exogenous information arriving, not known at the time of policy construction. Figure 23 illustrates how the model is initialized with input parameters and constructs a contingency plan utilizing the SDP framework (algorithm 1), only once at the beginning of the modelling horizon. See section 4.4 for an explanation of the input parameters and their effect on the stochastic dynamic program. After the construction phase, the decisions throughout the day are retrieved from the contingency plan in accordance to the state the system is in at all times, as illustrated in fig. 23. Figure 24 is suitable to describe how the contingency plan is utilized

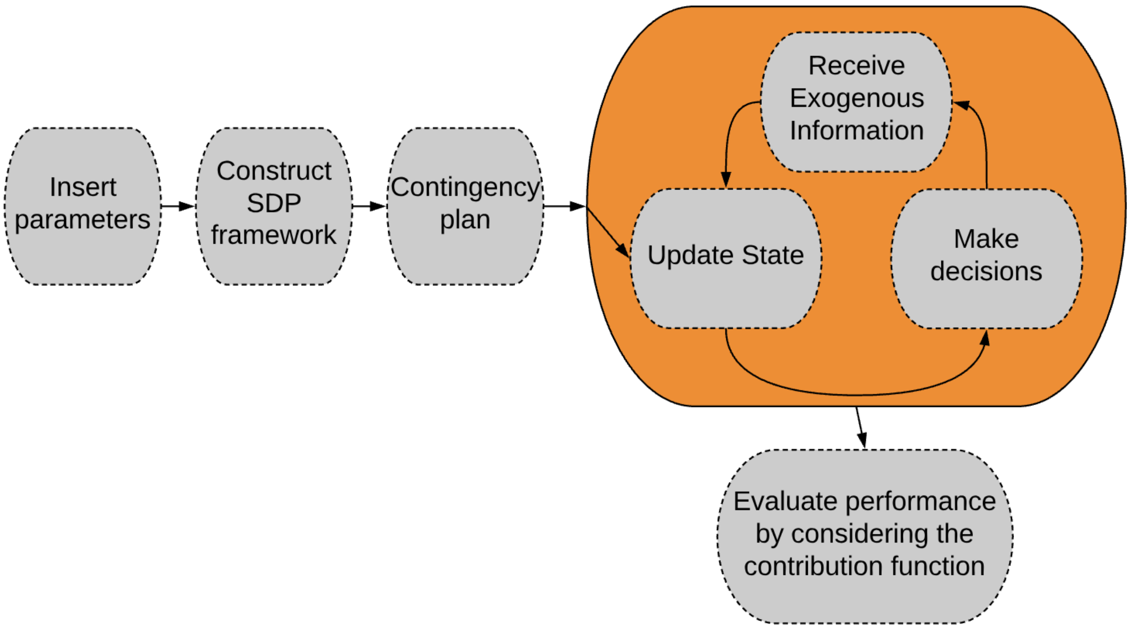


Figure 23: Float chart illustrating the steps in the approach from the construction of an SDP policy to the application including determination of associated optimal decisions.

in practice. The symbols I and P correspond to the vectors $\mathbb{I}$ and $\mathbb{P}$, respectively. A step-wise description is presented below. By the symbols and notation used regarding decision trees by Powell (2011a), a decision node has square shape and circle nodes represent outcome nodes where new information arrives. The circle nodes contain the post-decision state information, but price information is still unknown. Hence, a key take away is that the user can decide how to evolve from a squared node to a circular node, but from there, the outcome is unknown until the time of information revelation. For the aggregated problem, the time resolution is 6 hours per time step. Decisions are made in the beginning of the time block. The utilization description is as follows:

First, the spot price is revealed the user, along with spot commitment in the planning period. In addition, it is of interest to know the production plan to product blocks adjacent to the first and last blocks, but outside of the planning horizon (see: fig. 12). Based on this knowledge, the initial state, S_1, is determined and the corresponding optimal decision can be made by checking the associated node in the look-up table. The square node on the far left in fig. 24 represent the initial state. Based on the decision, one transit to the corresponding post-decision state, the node marked: $I^*(P')$. I^* describes the optimal level of capacity to evolve to from the previous step.

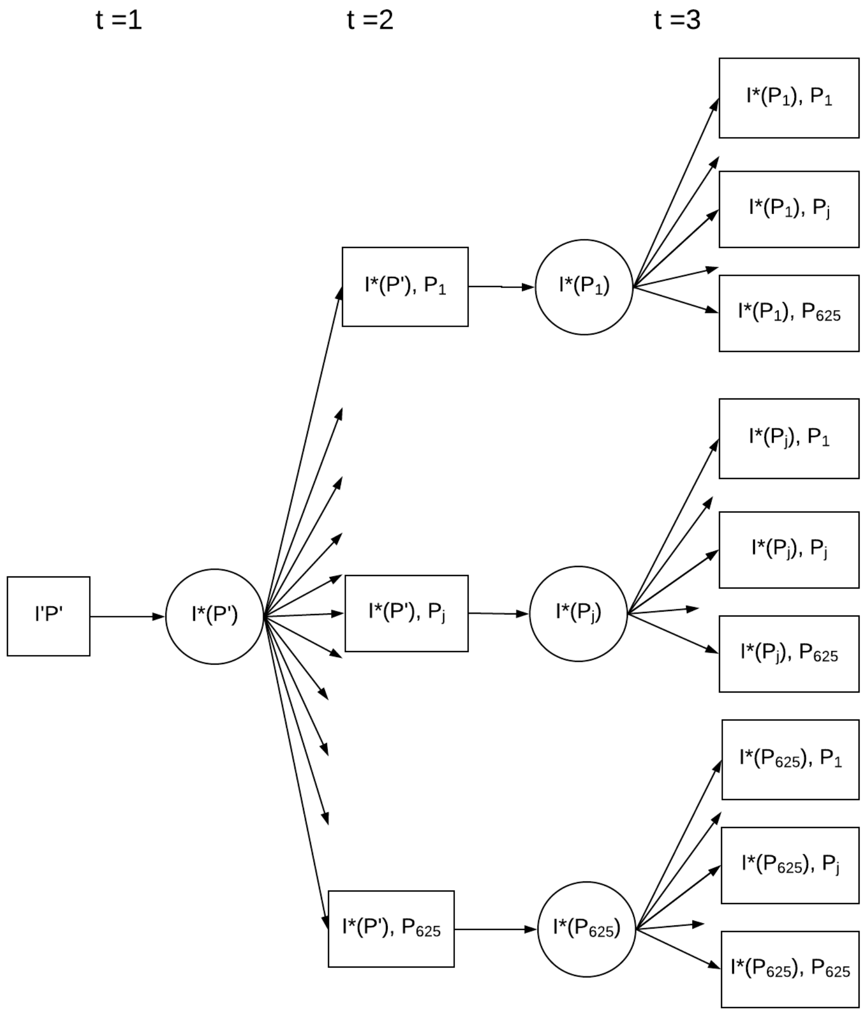


Figure 24: DP set up

During the 6-hour period, market price data becomes knowledge to market participants. Based on prices of realized trades between the points of decisions, i.e. before the beginning of the time block $t+1$, the new price state is found as the average price. The price information is exogenous, and *forces* transition to a new state along with the *chosen* I^* and the price occurrence. After this, the procedure repeats itself until end of horizon is reached. Figure 24 illustrates the three first steps, but in testing 6 steps have been utilized.

As mentioned in the introduction to this section, the aggregated policy does not necessarily provide a valid solution in the original problem. To handle this, an *adjustment heuristic* can be applied when disaggregating state variables for price and capacity.

5.4 Bounds

This section aims at evaluating the *gap* between bounds of the optimal expected value in the *original* problem. The procedures to find both an LB (section 5.4.1) and an UB (section 5.4.2) are presented. In addition, it is elaborated on how solutions in the aggregated problem can be *infeasible* in the original problem. Along comes an explanation of an adjustment heuristic that can be applied to ensure feasibility when determining the LB (section 5.4.3). The optimal expected objective value depends on the set of input parameters, which are used to initialize the model. Hence, it makes little sense to discuss an overall LB and UB. The bounds corresponding to the different test instances are calculated and discussed in section 5.4.4.

It is desirable to look for reasons to why the optimality gap differs for different test instances. However, even with a limited set of input parameters that are more or less correlated, it is non-trivial to extract general conclusions.

5.4.1 Lower Bound

The procedure to determine the LB is presented in this section, and fig. 25 emphasizes the steps. Numerous simulations are run using Monte Carlo simulation, in order to develop the

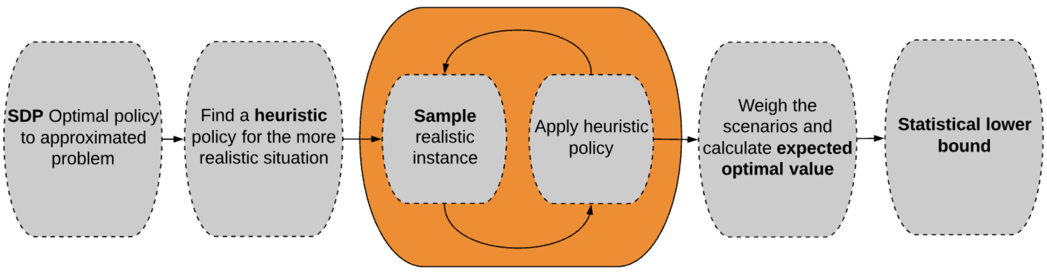


Figure 25: Approach to determine statistical lower bound.

statistical LB. In each simulation the aggregated SDP is applied. The SDP is the same for all simulations, and hence the policy is only developed once. When applying the SDP, information is collected, aggregated and the policy is applied. In order to ensure that the solution is feasible, the solution must be disaggregated and evaluated under the realistic constraints. Moreover, this entails modifying the solution by applying a heuristic, *if* the solution is infeasible.

The model is applied in a manner that it could have been applied in reality to solve the *original problem*. Prices are simulated in accordance with natural filtration, i.e. hourly resolution.

The model collects and interprets information between bid periods, t,in order to determine the average price that makes up the aggregated market price state, $\mathbb{P}_t$. Similarly, capacity information is aggregated. By solving the original problem utilizing the heuristic policy multiple times under stochastic prices, a set of feasible objective values can be found. By weighting all scenarios equally and taking the expectation of the objective values, the statistical LB is found. It is a lower bound because all the solutions are feasible and potentially sub-optimal in their associated scenarios.

The formula is shown in eq. (48), where Ψ here refers to the number of realized scenarios. Each scenario is denoted $\psi \in \Psi$. The objective value V_0 is as defined in eq. (39). Note that since this solution is potentially sub-optimal, the star is removed. The subscript ψ denote what scenario V corresponds to.

$$LB = \frac{1}{\Psi} \sum_{\psi=1}^{\Psi} V_{0,\psi} \tag{48}$$

5.4.2 Upper Bound

When determining the upper bound (see fig. 26), the method of relaxing *all* non-anticipativity constraints (NACs), (see section 3.6) is utilized.

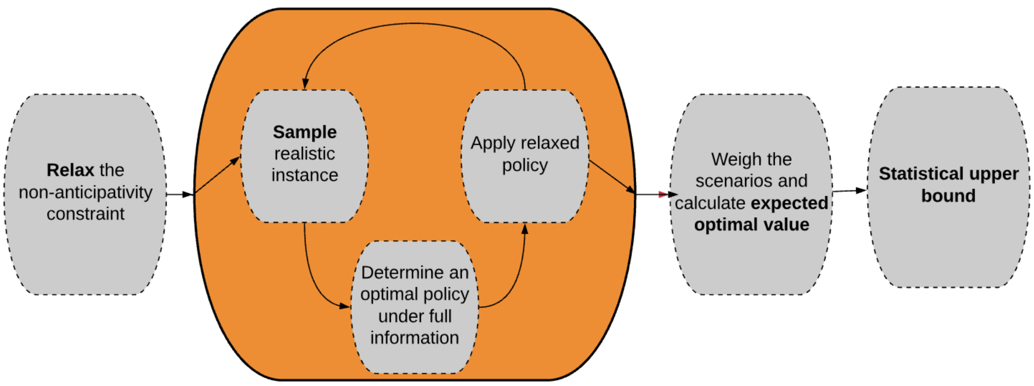


Figure 26: Approach to determine statistical upper bound.

In practice, the effect is that all price information is revealed immediately and simultaneously. When all stochasticity is removed the problem becomes deterministic. This implies that a new policy must be found for *each* simulation. Note the additional step in the orange circle in fig. 26. However, it is easier to solve the deterministic than the stochastic dynamic program. Note that the functions in the deterministic program shown below are similar to those of the stochastic dynamic program, only without stochasticity. In the deterministic program the state space is reduced to

$$\mathcal{S}_t = I_t \tag{49}$$

With the state $S_t = \mathbb{I}_t$. The decision is still $\mathbb{X}_t$, but there is no exogenous information $\mathbb{W}_t$. The transition function is

$$S_{t+1} = S^M(S_t, \mathbb{X}_t). \tag{50}$$

Since prices were certain for the stochastic dynamic program within a step, the contribution function is the same, hence $C_t(S_t, \mathbb{X}_t)$(eq. (37)). However, the Bellman's equation differs, since the stochasticity is removed:

$$V_t(S_t) = \max_{\mathbb{X}_t \in \mathcal{X}} \left(C_t(S_t, \mathbb{X}_t) + V_{t+1}(S_{t+1}|S_t, \mathbb{X}_t) \right) \tag{51}$$

and

$$V_0^* = \max_{\pi \in \Pi} \left\{ \sum_{t \in \mathcal{T}} C_t^\pi(S_t, \mathbb{X}_t^\pi(S_t)) | S_0 \right\} \tag{52}$$

Moreover, the method of finding the UB entails using Monte Carlo simulation of exogenous price information with resolution in accordance with the *original problem*. As for determining the LB, the information regarding state variables is aggregated. Based on the aggregated simulated price path a deterministic program can be solved using backward recursion. For the given capacity level, an optimal solution in found by applying the optimal relaxed policy. This provides an optimistic upper bound, because it optimizes over a deterministic horizon by making use of information that is unavailable under realistic circumstances, without penalizing it. Applying the same argument to each scenario, weighing all scenarios equally and calculating the expected value result in the statistical UB given by

$$UB = \frac{1}{\Psi} \sum_{\psi=1}^{\Psi} V_{0,\psi}^*. \tag{53}$$

Ψ is the total number of scenarios ψ. The objective value V_0 is as defined in eq. (52). The subscript ψ denote what scenario V corresponds to.

5.4.3 Adjustment Heuristic

An important point to make, is that in order to know by certainty that the *lower bound* is valid, the solution must be *feasible*. In this case it would entail that the capacity constraint on the

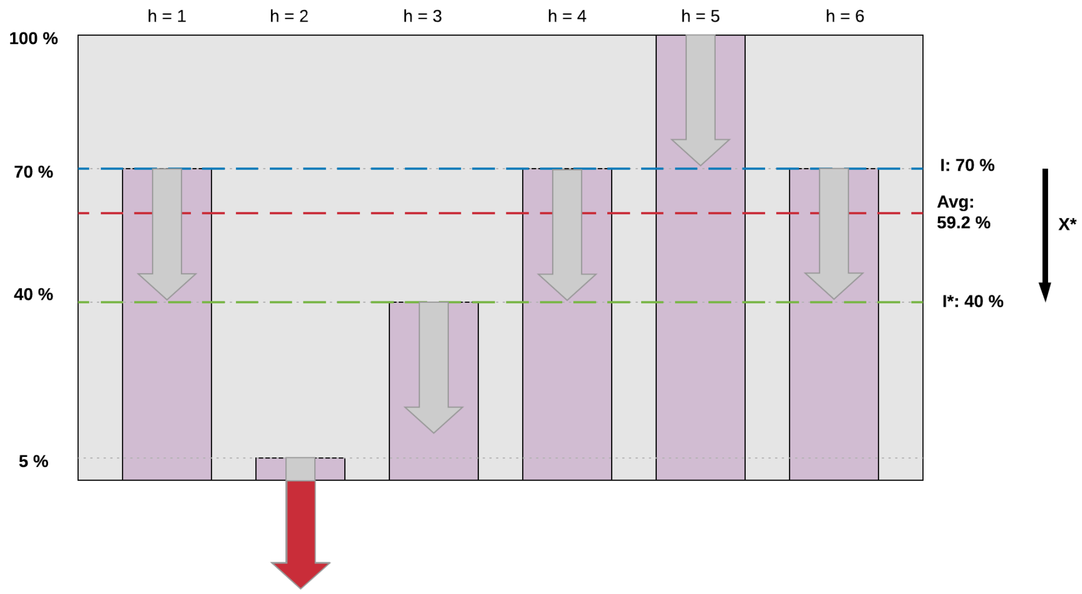


Figure 27: Capacity levels associated with block of 6 products. Illustrates the impact of making decision based on aggregation.

generator is not violated in any production hour and that the price one assume to get acceptance at is not too optimistic.

The aggregated SDP suggests the up- or down-regulation in step t that is evaluated to be the *optimal solution in the aggregated problem*. However, this solution may be sub-optimal and even infeasible in the realistic problem. To obtain a feasible solution, an *adjustment heuristic* that can capture when constraints are violated, is used. Moreover, it is used to modify or penalize the objective function according to the deviation from feasibility. There are different ways to develop such a heuristic. One method is described.

In the aggregated problem, the capacity of the generator is an average of remaining capacities of the generator for 6 products within a product period. Figure 27 illustrates a block of 6 periods.

One product block, h^{avg}, in time step, t, in the aggregated problem takes the value $i_{t,h}^{avg'}$, which correspond to the *closest* capacity discretization level to the actual mean

$$i_{t,h}^{avg} = \frac{1}{6}\sum_{h=1}^{h=6} i_{t,h}. \tag{54}$$

Note that h is the general symbol for a product period in this thesis. h^{avg} *is only introduced here to emphasize that products in the aggregated model are really derived from its constituents in* *the disaggregated problem.*

Figure 27 represents an example situation. Note that for illustration purposes the capacity levels vary a lot between hours. Normally, products within the same block tend to have similar production commitments, due to seasonality effects within a day. All the purple bars correspond to the remaining capacity level for a product h. The aggregated capacity level is the average value: $i_{t,h}^{avg} = 59.2\%$. Hence $i_{t,h}^{avg'} = 70\%$ since it is the closest level of remaining capacity. Moreover, it is found from applying the policy that the next optimal decision is to *buy* $X^* = \mathbb{X}_t$ amount of power. The SDP will interpret that the system moves from a capacity level of $I = \mathbb{I}_t = 70\%$ to $I^* = \mathbb{I}_{t+1}^* = 40\%$. In this disaggregated version, that is obviously not the case, and in fact this trade would lead to a *violation* of the capacity constraint for the generator corresponding to product $h = 2$. Notice the red arrow in fig. 27.

In general, the model suggests an optimal decision of up- or down-regulation of the generator for *all* 6 products. One or more of the products may violate the capacity constraint, and hence the suggested solution is infeasible. A way to handle it is to only reward the feasible actions in the objective function, by performing an *if-then*-check within the optimization algorithm.

When applying the aggregated SDP, *prices* are also aggregated and the average price is found. Based on this aggregation, the model provides a candidate solution, before an adjustment heuristic is applied to find decisions that do not violate the capacity constraint. To determine the contribution function eq. (37), the prices are *disaggregated* and multiplied by the trade, $\mathbb{X}_t$, associated with the corresponding product, h. This way, the contribution function only rewards feasible trades under realistic prices.

5.4.4 Analyzing the Properties of the Bounds

The analysis is performed on the set of instances as described in section 5.2, table 9. Among all the simulated scenarios, there is a spread in the set of optimal values. Figure 28 illustrates how upper and lower bounds differs between days in different test instances, while the numerical results are summarized in table 10.

Note, however, that due to time constraints the adjustment heuristic have not been applied to adjust the disaggregated variables nor the contribution function. However, with the purpose of illustrating and explaining how a further analysis would be carried out, the invalid results from a lower bound that may be too optimistic are shown. In the case where the adjustment heuristic is applied, one would expect the gap to increase. This is due to a the fact that the lower bound is expected to be lower, when not rewarding infeasible trades.

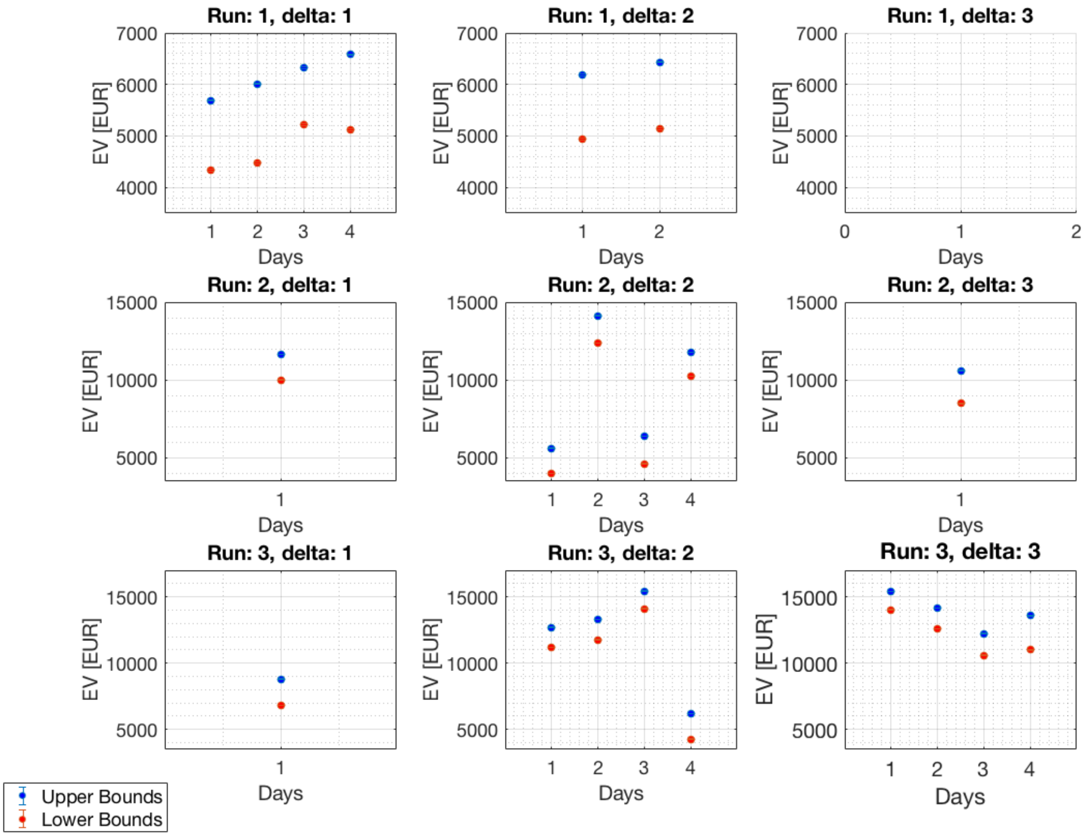


Figure 28: Plot of bounds within different group instances

In fig. 28 the points come in pairs of an UB(blue) and a LB(red) illustrated as points along the same vertical line. The expected optimal value is measured in euros on the y-axis. Note that the scale of the y-axis varies with different spot commitment running plans, denoted R. On days when there is initially no spot commitment, and all machinery is turned off, the expected optimal value is typically lower at the end of the planning horizon. In the opposite case, the expected optimal value is generally higher. Note however that some days' expected objective values stand out from the remaining days within the group. Note for instance test instance number 4 within the $R = 3, \Delta = 2$.

The variations within group instances imply that these factors alone (R and Δ) are *not sufficient* to describe the properties of the objective value.

The output's fit to the normal distribution is investigated by utilizing a Q-Q (quantile-quantile)

Table 10: Bound analysis

Instance	UB +/- CI [EUR]	LB +/- CI [EUR]	Gap absolute[EUR]	Gap [%]
R_1, Δ_1: 17-Jun-2017	5689.017+/-15.068	4328.694+/-19.203	1360.323	23.911
R_1, Δ_1: 13-Jul-2017	6010.473+/-16.028	4474.299+/-20.264	1536.174	25.558
R_1, Δ_1: 25-Jun-2017	6318.896+/-16.028	5220.507+/-21.218	1098.389	17.383
R_1, Δ_1: 16-Jul-2017	6581.512+/-18.478	5112.873+/-20.635	1468.639	22.315
R_1, Δ_2: 27-Jun-2017	6182.804+/-16.395	4935.521+/-19.779	1247.283	20.173
R_1, Δ_2: 28-Jun-2017	6435.204+/-16.143	5145.572+/-19.945	1289.632	20.040
R_2, Δ_1: 18-Jul-2017	11630.794+/-16.927	10012.799+/-19.834	1617.996	13.911
R_2, Δ_2: 05-Jun-2017	5617.297+/-15.907	4005.604+/-21.394	1611.694	28.692
R_2, Δ_2: 26-Jul-2017	14134.334+/-17.711	12365.418+/-21.467	1768.916	12.515
R_2, Δ_2: 10-Jun-2017	6385.155+/-19.253	4618.464+/-23.107	1766.691	27.669
R_2, Δ_2: 31-Jul-2017	11775.906+/-16.3456	10244.461+/-21.706	1531.444	13.004
R_2, Δ_3: 14-Jun-2017	10612.381+/-14.727	8486.146+/-21.359	2126.234	20.035
R_3, Δ_1: 20-Jul-2017	8803.935+/-16.263	6802.303+/-22.235	2001.633	22.736
R_3, Δ_2: 25-May-2017	12699.704+/-14.735	11183.343+/-19.511	1516.362	11.940
R_3, Δ_2: 21-Jun-2017	13314.389+/-16.393	11755.589+/-18.785	1558.800	11.708
R_3, Δ_2: 17-May-2017	15435.951+/-15.121	14103.131+/-21.466	1332.820	8.635
R_3, Δ_2: 10-Jul-2017	6178.693+/-17.366	4265.507+/-19.893	1913.186	30.964
R_3, Δ_3: 26-May-2017	15411.265+/-15.988	14028.760+/-21.575	1382.505	8.971
R_3, Δ_3: 22-Jun-2017	14147.053+/-15.172	12594.976+/-20.271	1552.077	10.971
R_3, Δ_3: 18-May-2017	12231.042+/-14.189	10595.073+/-21.468	1635.970	13.376
R_3, Δ_3: 02-Jun-2017	13578.567+/-15.460	10991.603+/-21.432	2586.964	19.052

plot, in order to describe the properties of the statistical bounds, such as confidence interval (CI). In general, all instances tested by utilizing a Q-Q plot to evaluate fit to the normal distribution. They are found to be sufficient fits to the normal distribution. In all cases, the LB is a better fit than the UB. Figure 29 show an example plot of the Q-Q plots. If the points fall close to the linear line, it indicates that the points are close to normally distributed. Since it is quite representative, the remaining plots are left out of the report.

The bounds are estimated by evaluation of Ψ = 10,000 simulations, by the formulas given in eq. (48) and eq. (53). Since the bounds are statistical it is of interest to know how spread out the different constituents are within the sampled scenarios. Confidence intervals of the bounds are

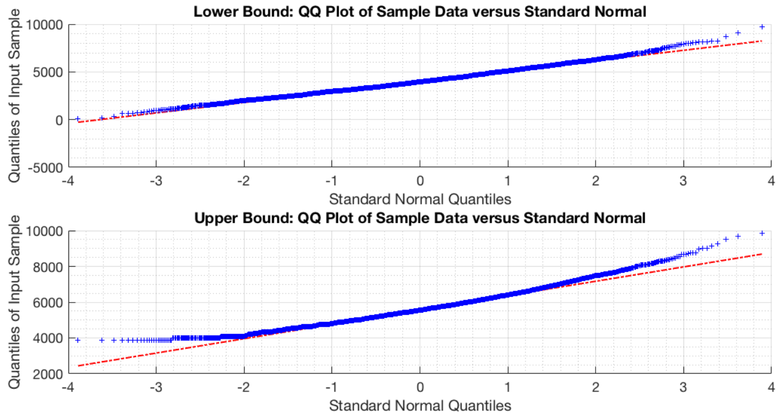


Figure 29: Q-Q plot for the 5^{th} of June 2017.

found under the assumption of a normal distribution. They are used to measure the spread of the expected objective values making up the statistical bound. For a certain test instance, the CIs is quite tight based on the set of Ψ simulations. Hence, input parameters are of great importance for the output.

A 95% CI is calculated on both upper and lower bounds (see table 10). The CI is *small relative to the gap* between upper and lower bound. The CI is plotted in the fig. 28, but due to its relatively small size it cannot be observed. Figure 30 shows the CI for an UB. The test instance is the 18^{th} of July when the statistical UB is $11630.794[EUR] + / -$ the CI of $16.927[EUR]$. Note that since the normal distribution is found to be a good fit, the CI is symmetric around the bound.

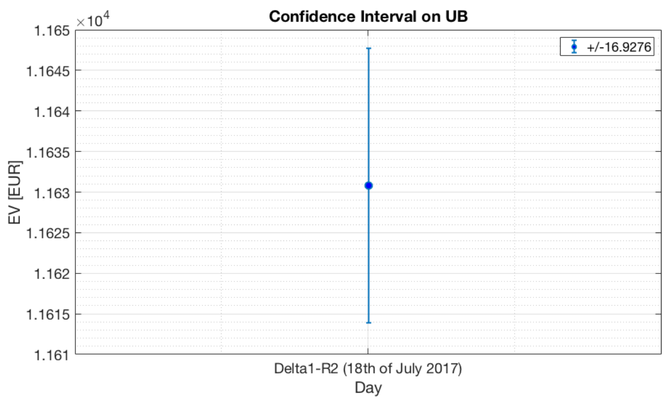


Figure 30: Confidence interval

The percentage-wise gaps are calculated as a percentage of the UB. An interesting observation is that both the maximum and minimum percentage gap lies within the group R_3, Δ_2. Moreover, the gaps are quite large. Different methods can be applied to tighten the upper bound or modify the heuristic in order to find feasible policies that in a manner closer to the optimal. This is future discussed under section 6.3.

5.5 Policy Trends

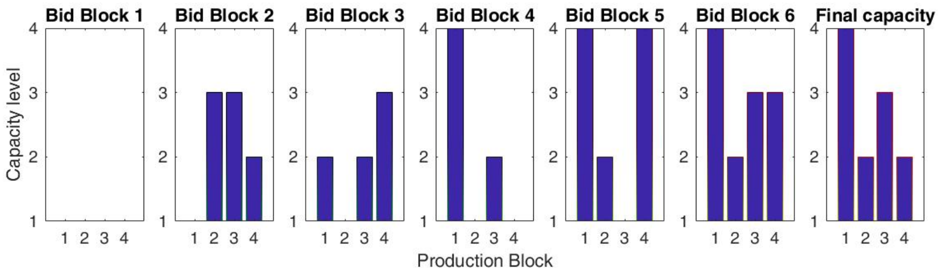
The policy constructed from the SDP algorithm on an instance, consists of large amounts of data difficult to summarize or illustrate precisely. However, trading trends may be observed by evaluating the average behaviour over time. As the objective of this thesis is to develop a model optimizing *expected profits* for a market participant, the scope of interest is to observe how the policy makes a market participant behave over numerous scenarios of exogenous information. In this section, policies constructed with SDP is compared to the case of deterministic prices.

An important aspect to remember is the number of input parameters inserted for each day tested. Though the test instances of this analysis are constructed in such a way that there is some control of the parameters, each parameter affects the policy in different ways. Each of them alone may pull the results in one direction, while combined with another parameter, the opposite effect might occur. A too detailed analysis of the differences in policies are therefor not conducted, but trends for the overall model are emphasized. The main results from this analysis are related to how the model tends to behave in a *stochastic world in comparison with a deterministic world*, and how *average trade volumes evolves* during the modelling horizon.

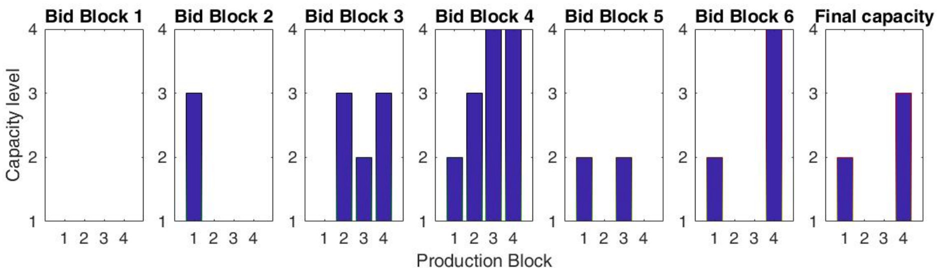
A stochastic solution will be either worse off or equal to a deterministic solution. This applies to the policy by observing how a stochastic policy reacts to fluctuating market prices. While a deterministic model knows for sure what future exogenous information will be, a stochastic model must consider EVs of the future. Hence, stochastic solutions tend to be have lower variance than a deterministic one, depending on how sensitive the stochastic model is to varying stochastic information.

To ease the reading of the following analysis, a simple representation of the results is shown in fig. 31, illustrated by two dummy results of the same day with different simulated price paths. The purpose of showing these figures is to illustrate how the policy may differ in each simulation, according to how sensitive the model is to price fluctuations. Input parameters are held constant, only stochastic exogenous information is simulated.

There are 6 bid blocks, each with a capacity state given by the 4 production blocks' capacity levels at the time. The seventh block indicates the final capacities at the time of production. The capacity levels correspond to the discretization of state space (section 4.5.6), of which the capacity levels are discretized into 4 levels. Remember that capacity level 1 represent full production, while capacity level 4 represents that the machinery is not running. Moreover, the change in capacity from one bid block to the next uniquely corresponds to the volume traded in the previous bid block.



(a) Example simulation 1



(b) Example simulation 2

Figure 31: Illustration result from a specific simulation. Capacities for all production blocks in each bid block.

Simulating 10,000 price paths, the result is 10,000 different trades and capacities, for each product in each bid block. Only the first bid block will always have the same state, as this represents the initial spot commitment before any trade have been carried out. To determine expected results, a counting of occurrences across all simulations is computed. Using the two dummy examples in fig. 31, the result for bid block $t = 3$ is presented in table 11.

All simulations result in a capacity level 2 for production block 3, and capacity level 3 for production block 4. 50% of the simulations results in capacity level 1 for production block 1 and 2, and the other 50% result in capacity level 2 for the same production blocks. Notice how the sum in each column is the same as the number of simulations. Over 10,000 simulations of

Table 11: Bid Block 3 - Counting intersections of production block h and discretized capacity level $I_m \in I'$ over two simulations.

	$h=1$	$h=2$	$h=3$	$h=4$
$m=1$	1	1	0	0
$m=2$	1	0	2	0
$m=3$	0	1	0	2
$m=4$	0	0	0	0

exogenous information, some of these intersections may have 7,000 occurrences, resulting in a *share* of 70% of simulations with the same result for that specific intersection. These *shares* are plotted and analyzed below.

All instances are tested, where the days in table 9 are run with their respective input parameters from historical data. Price paths are simulated and the model is solved as described in sections 5.4.1 and 5.4.2, to obtain stochastic and deterministic solutions, respectively. Overall trends differentiating the stochastic and deterministic solutions are retrieved, similar within all test instances. Hence, the discussion will be based on results from one day only, for illustration purposes.

The stochastic case and the deterministic case for 17-Jun-2017 are presented in figs. 32 and 33 respectively, the difference illustrates how perfect information affects the policy. Similar as fig. 31, there are 6 bid blocks plus one for final production, and a bar for each of the 4 production blocks in each bid block. *The difference is what the bar sizes indicate*. Each color belongs to a specific capacity level, and the y-axis indicates how large *share* of the 10,000 simulations resulted in each respective capacity level - for that specific production block in that specific bid block. Hence, the y-axis in figs. 32 and 33 is a result of counting of intersections in the policies, as illustrated in table 11. The maximum share is 100%.

17-Jun-2017 has an initial *spot commitment* associated to the highest capacity level, namely no production and full capacity for all production blocks $h \in \{1,2,3,4\}$. For the stochastic case (fig. 32), notice how all production blocks changes to a 100% share of the lowest capacity level in bid block $t = 2$, then a 100% share of the highest capacity level in bid block $t = 3$. *This is solely arbitrage trading*, taking advantage of market opportunities. This indicates that all simulations result in selling maximum production the first bid hour, then buying it all back

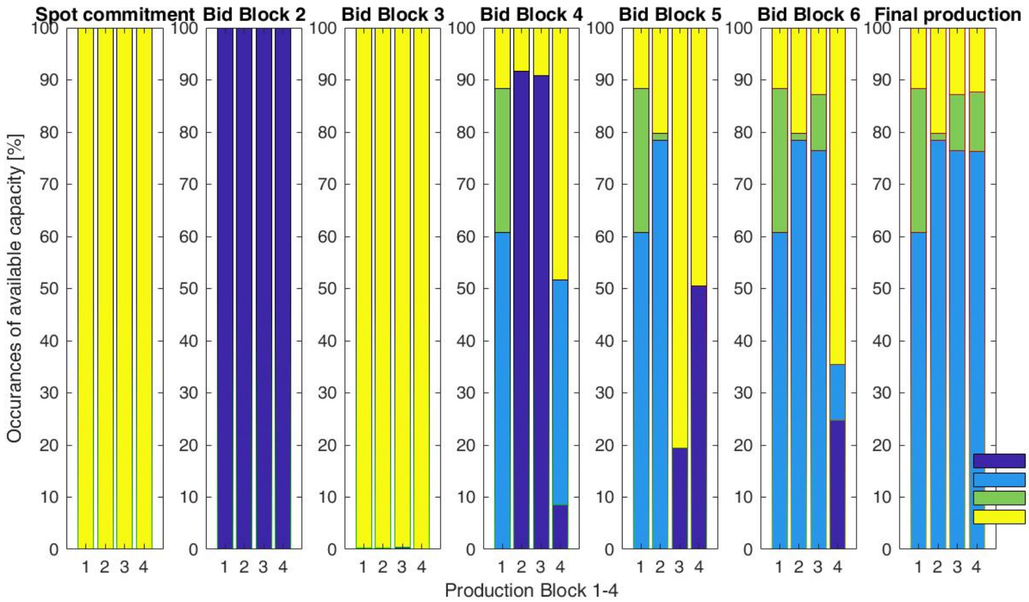


Figure 32: Solving a stochastic dynamic program for the 17^{th} of June 2017. Share of simulations resulting in capacity levels 1-4. Illustration of all production blocks in each bid block.

the next. The policy for these bid blocks is not affected by different price realizations. As all simulations have the same initialization $\mathbb{P}_1$, given by historical spot prices 17-Jun-2017 $L_{\mathbb{P}}^{spot}$, the model will always *expect* the same outcome in bid hour $t = 2$, though the *realized* prices $\mathbb{P}_2$ will differ.

This is in contrast to the deterministic case in fig. 33, where the initialization of the model includes the full price path $\mathbb{W}_t \quad \forall \quad t \in \{1,2,3,4,5,6\}$. Hence, in a significant share of the simulations, the policy suggest to *wait for a better opportunity later*. This is observed in the figure, as the share of capacity levels in bid hour $t = 2$ is not 100% on either of the the levels. Most simulations trigger trades to sell full production at first, and then buying it back, but a significant share of the simulations does not trade anything of products $h = [1,3,4]$ the first bid block. Similar results are seen for bid block $t = 3$, indicating that eventual bids held back the first bid hour, were put in the market in bid hour $t = 2$ instead. This indicates how *the deterministic model is able to catch nuances in the price scenarios that the stochastic model cannot do*.

Later bid blocks have more varying results from the simulations, applicable to the stochastic case as well as the deterministic. The last opportunity to trade product $h = 1$ is in bid block $t = 3$, and the last opportunities to trade products $h = [2,3,4]$ are in bid blocks $t = [4,5,6]$,

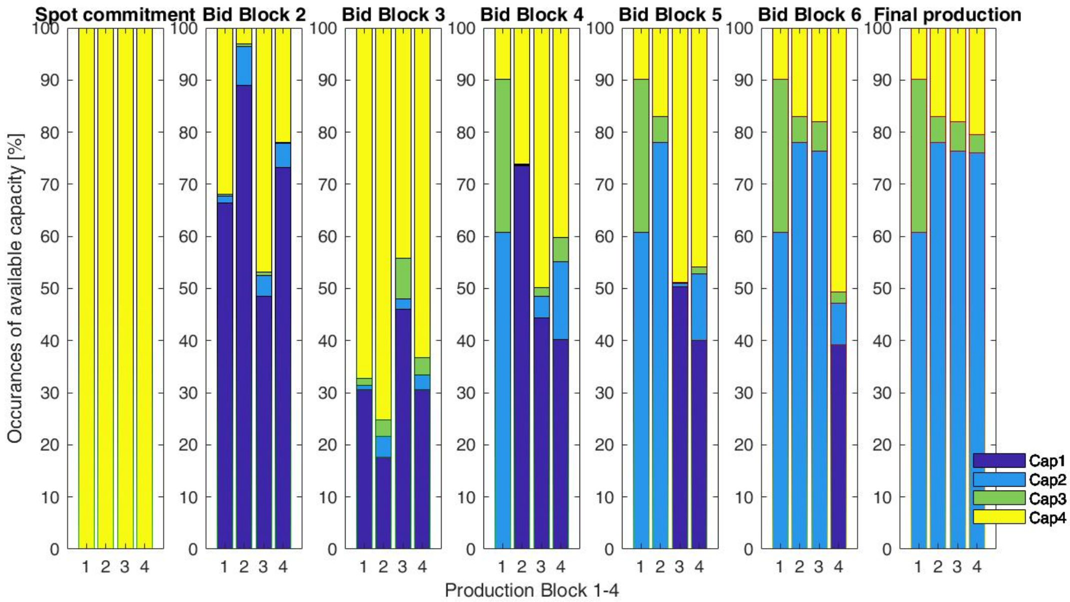
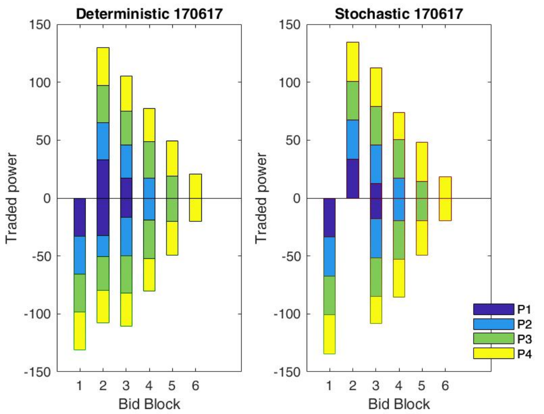


Figure 33: Solving a deterministic dynamic program for the 17^{th} of June 2017. Share of simulations resulting in capacity levels 1-4. Illustration of all production blocks in each bid block.

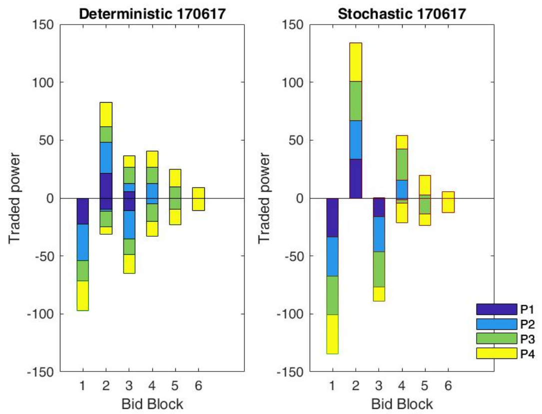
respectively. Hence, *new commitments will be binding*, which seems to have an impact on the resulting capacity levels. While the deterministic model easily can trade at the most profitable times without any risk, this indicates that also the stochastic model expects to be able to up- or down regulate production to optimal levels within the close, in addition to make arbitrage profits up front.

An interesting result is that, in general, it is not necessarily a relation between initial spot commitment and final production. This indicates that the *market prices tend to be higher than marginal cost of production*, until production reaches certain levels. At final production, the remaining capacities are all at the three upper levels, and never at capacity level 1. This indicates that utilizing full capacity of the generator is expensive, and that market prices in general do not cover the production costs associated to maximum production. This corresponds well to the marginal cost curve, rapidly increasing for high production levels. Note that *final production results are similar in the stochastic case and the deterministic case*. Though *future* market prices are stochastic, the market participant is always guaranteed acceptance in the market at the *current* price level. Hence, both the deterministic and the stochastic model have equal information about production block h' the last bid block before the close of h', and equal opportunities of up- or down regulation before the close. The fact that final production varies in different simulations, indicates how market prices determine which production level is optimal.

To illustrate the policy and the resulting *trades*, fig. 34 shows average trade volumes over the simulations. Each bar color represents a production block, and the size of it the volume traded for the respective bid block. A positive volume refers to selling power, while a negative volume refers to buying power. Each of these are calculated separately when computing the average trade volumes. To the left (a), simulations where no trade took place is not included in calculating the average. Hence, it represents the average volume traded, *if there is a trade*. This means that even if there is only 1 simulation resulting in selling power for a specific production block in a specific bid block, the volume illustrated will be equal to that trade. To the right (b) is a total average over all 10,000 simulations, and the same case would be negligible.



(a) Only simulations where a trade occur is weighted.



(b) Each simulation equally weighted.

Figure 34: Average trade volumes, selling and buying considered separately.

Notice that average volumes traded *if there is a trade*, are quite similar in the stochastic and deterministic case. The exception is that there are no selling trades occurring in bid block 2 for the stochastic case, while the volumes are almost symmetrical for selling and buying in the deterministic case. This confirms the results from figs. 32 and 33, that the stochastic case will do the same decision the first two bid blocks, regardless of simulation outcomes. The deterministic model rather customizes its trade pattern for each specific simulated price path. Both cases have the largest volumes traded early, while the volumes decrease as a production block's close is getting nearer. This is related to the effect observed in figs. 32 and 33, where remaining capacity typically alternates between the upper and lower capacity levels, but stabilizes somewhere in between for final production volumes.

The picture changes somewhat when observing the illustration to the right (b), of the total average volumes. Notice how in the stochastic case, the bars are almost of the same size as in (a) the first three bid blocks, though only for either buying or selling power. This distinguishes from the deterministic case, where apparently the trades are more equally likely to happen as a selling trade or a buying trade, and the average volumes decreases. This is what we observed in figs. 32 and 33, that the stochastic case is less sensitive to price fluctuations than the deterministic case, so that all simulations results in the same capacity level, or volume traded, for the first few bid blocks.

Though 17-Jun-2017 is utilized as a day for illustration in this thesis, the days do not all have the same policy. However, the main result from analyzing the policies is how they all have the same dynamic when comparing the deterministic and the stochastic case, of which is a result in itself. The variance of decisions in a deterministic world is more volatile and sensitive to price variations than in a stochastic world, as a stochastic model will remove random sampling effects to a greater extent as it optimizes depending on EVs.

Decisions made by deterministic DP varies more than that of SDP. This is because SDP removes random sampling effects to a greater extent as it optimizes depending on EVs.

6 Concluding Remarks

6.1 Discussion

This section aims to briefly discuss some qualitative aspects of how the model provides decision support to increase expected profits for a market participant, and sheds light on the model result's validity for the real world problem.

Implementation of New Software

A concern from the industry is that potential profits in Elbas are not large enough to cover the resulting costs from implementing new strategies of intraday bidding. In order for an optimization model to add value to a market participant, the cost-benefit relationship for participating in Elbas must be taken into consideration. Initial investments in software and user training increase the threshold to enter, in addition to potential continuous costs such as increased need of human resources. This study contributes to reduce the threshold concerning software *development*, but the software interface must be designed in such a way that the user can utilize it in the intended way. The user must be able to gather and feed the correct input parameters, and interpret the output - hence willingness to learn is an important aspect. The benefit of a output in the format of a contingency plan is that it is easy to interpret. The plan contains information about what bids the user should place at given points in time, determining both price and volume for each trade. The most desirable case would still be to couple the model with the folders containing the correct market information, excluding the risk of human error.

In addition, there are also some long-term opportunities that an optimization model contributes to. As the traded volumes in Elbas increases, with a higher frequency of trades, it becomes more of a challenge to keep trading by experience as operators do today, as the decisions become less trivial due to increased complexity and large amounts of data to interpret. In such a case, having already implemented an optimization model with limited need of human interaction, the next step into developing a trading robot is not far away. As this seems to be necessary in the future, a market participant should rather be proactive and position one self beneficially. In that case, the continuous costs related to Elbas participation will even decrease from today's situation, as less human interaction is needed.

How the Model Adresses the Real World Problem

As today's participation in Elbas is at a minimum, and with a cost minimizing objective, any increased profit above marginal cost contributes to extra gain respective of today. The model developed in this thesis is a heuristic approach to the *original problem*, creating possible sub-optimal policies as both bid blocks and production blocks are aggregated, and all dimensions discretized. Though this restricts the action space, i.e. the theoretically optimal decisions, the model provides decision support improving today's situation significantly. Moreover, trades often concern several consecutive hours of production to cover start and stop costs of the generator, so aggregating production blocks into 6 hour blocks is not far from what is actually done in the *original problem*.

The model in this thesis has an incentive for purely arbitrage trading up until the close of each production hour. This clearly distinguishes from how trading in Elbas is done today. Though this model might over-estimate the arbitrage opportunities with respect to what volumes are possible to trade at the price paths simulated, the fact that the model suggest to trade at all indicates that the prices exceeds marginal cost, i.e. is profitable to carry out. A premium above marginal cost should be possible to obtain, of which increases today's profits.

While this model is developed based on the hydro power plant (HPP) Søa in TEs portfolio, the same modelling procedure can be generalized for similar power plants. This model considers only a part of a portfolio, but with the objective of being a first step towards holistic perspective. In order to account for the entire portfolio, a similar model must be built for *all* its constituents separately. This will provide a heuristic solution to the portfolio.

6.2 Conclusion

Traditionally, Elbas has been a market with low frequencies of trade, and the volumes traded have not been considered attractive for a market participant with low volatility in production capacities. Though there exists an opportunity to profit, even from small volumes, the initial investment in time and human resources to develop tools for optimal bidding strategies, has made the threshold to enter the Elbas market too high for many market participants with flexible energy sources (appendix B).

This thesis is a contribution to the field of power production and trading, investigating the opportunities to model the intraday market as a multisequential decision problem. The market today is quite different from what is expected in the future, but early research makes it possible for participants to be proactive. At the time of completion of this study, a joint initiative to integrate large parts of Europe into a joint intraday market, is about to go live. The objective is to ensure better liquidity, market efficiency and a more secure power supply. By introducing high price areas to the market, flexible power producers with low marginal costs of production face an opportunity to gain, an opportunity this thesis investigates.

This study develops a mathematical optimization model for the bidding problem in Elbas, treating each of the 24 production periods as products subject for trade in continuous double auctions. Aggregation of 6 production hours into a production block is utilized to handle the dimension sizes of state, action and outcome spaces. The model also considers production costs associated with the power plant. The problem is modeled as a MDP, applying SDP to construct an exact solution to the aggregated version of the original problem.

Ideally, the model developed in this study would have been able to solve the original problem to optimality. However, modelling choices are made as a trade-off between tractability and accuracy. The study discovers how the number of production hours makes the dimensions of the bidding problem in Elbas grow exponentially, which confirms DP theory. Aggregation of production hours is done, so that the number of *products* the model evaluates decreases from 24 production hours to 4 production blocks. It is argued that this not necessarily is a too coarse time resolution, due to the common use of block bid. However, evaluating valid upper and lower bounds of the model must be conducted before one can conclude. The main challenge of aggregation is discussed in section 5.4.3, which concludes that aggregated production blocks introduce the possibility of infeasible decisions for certain production hours within a production block. An adjustment heuristic must be implemented to ensure that operators do not conduct infeasable trades.

The stochastic price process utilized in this study is assumed to be correct, hence all bids placed according to the price model will be accepted. The stochastic model does not face any uncertainties related to being *accepted* in the market, and will always have the opportunity of up- or down regulation before the close. The only uncertainty is related to what market prices the model expects for future trades, which overcomes the risk of high production costs. This is evident from policy analysis, where the model tends to carry out arbitrage trading, before it suggest a final production at the close that is in relation to production costs.

This study presents a first step to provide decision support for bidding in Elbas, where more accurate market dynamics are in focus. It is natural to question whether DP is a suitable optimization approach for the bidding problem in Elbas, as the curses of dimensionality is a challenge when developing the model within that framework. However, the main objective of this thesis is to emphasize that Elbas is a continuous auction, not having to make all decisions at only one or two points in time. As traditional scenario trees will explode when increasing the number of time steps, the benefit of DP becomes more evident. Section 6.3 elaborates on extensions to the model presented here, which still are within the DP framework.

6.3 Future Research

The master thesis opens new doors into different directions of research. Some suggestions for continuations of the study are presented below. Some limitations have already been pointed out in the thesis. This section aims at pointing out limitations and challenges and suggests the first steps to consider in a research continuation. It suggests how the optimization model can account for infeasibility when disaggregating, as an alternative to the adjustment heuristic found in this thesis. In the case of bound analysis, methods are suggested to tighten the upper bound. Moreover, modelling different levels of discretization and aggregation, and handling curses of dimensionality, are discussed. At last, the handling of stochasticity is discussed.

Heuristics

The challenge introduced by the utilization of average capacity states, is that the model may suggest *infeasible decisions*. There are several ways one could approach this issue. Firstly, one could develop a robust heuristic that optimizes while accounting for the bottlenecks in the *original problem*. Namely, a model that never exceeds any of the capacity constraints after disaggregating the block. For instance, the two products within the block with the least potential to up- and down-regulate make up the *two* new capacity states, rather than the average capacity level utilized here. This would implicate introduction of an *additional state variable*, and hence increase the state space and conflict with the curses of dimensionality.

Since the number of states is already a challenge in the stochastic dynamic program, introducing more would incentivice the investigation of other optimization techniques. Two heuristic approaches that are less sensitive than the SDP to increasing state space are ADP or SDDP(see section 3.5). These methods are not exact, but utilizes simulation to investigate a desirably representative set of scenarios through multiple simulations. With a robust, but valid, method of this kind, there is a great chance that both the upper and lower bounds are weak.

A difference between this method of an additional state variable and the one where the infeasible solution is modified by an adjustment heuristic, is that this model provides feasible solutions straight away. There is a direct link between suggested actions and the performed actions, and hence future opportunities are weighted more realistically. The latter is a desired property. However, the robust modelling may create weak bounds due to the conservative utilization of resources.

Penalty and Moderate Relaxation

It can be tough to find a tight gap. In general, one strives towards finding a method that is computationally manageable. One can find a tighter UB by moderating the information relaxation (see section 3.6). For instance, a method of revealing information regarding only one or a few steps ahead may provide a smaller violation of constraints. A method utilized by Nadarajah and Secomandi (2017) can be investigated. Namely, a method allowing a one step look-ahead, revealing information one step- before it is known. After relaxing constraints, a method to tighten the bound is to penalize the benefit of additional information.

Impact of Time Resolution

Moreover, an interesting direction for future research can be to analyze how to solve the problem with a *different time resolution*. In order to extract benefits from periods that deviate from the average value, one would need a model with more fine-meshed time resolution. It could entail a higher frequency of decision making or product blocks containing less products. The latter is more difficult, due to the exponential growth in state space associated with it. It could be interesting to analyze if ADP or SDDP are suitable under this moderated aggregation. It may be an important part of the future research to analyze the trade-off between tractability and accuracy when developing models. One of the positive sides that comes from finer resolution is that the infeasibility associated with average state values decreases, and hence smaller adjustments must be made.

6.3.1 Stochasticity

The stochastic price model found by Bovim and Næss (2017) is assumed to be a good representation of the willingness to pay in the market. It is built from a set of assumptions of which some, but far from all, literature agree upon. For instance, the correlation between Spot and Elbas prices. If there are weaknesses in the price model, it will constrain the possible gain from utilizing the optimization tool. In future research, one should strive towards a more comprehensive study of the underlying market price process.

Moreover, an interesting point to investigate is the potential to discretize the state variables of capacity and price differently. Are the four levels of capacity the most suitable ones? What is the gain from having one or several more levels, compared to the downside? How does it apply to prices?

Market Depth

A limitation of the model is that is does not differentiate what *volumes* one can trade at a given price. One can choose what volume to trade, at the given market price level. Under the assumption of high liquidity this holds, but otherwise, unrealistic trades may be initiated by the model. In a continuation of this model one could wish to specify how the price changes with different volumes.

Market Demand and Bid Acceptance

The price model is developed to model the actual *willingness to pay*. Without information about whether the market is short or long, it must, somehow, be accounted for what type of market it is. It is modelled here that the participant can place bids according to the price level and have them accepted. In order to model the market demand in a future research, a binary state variable based on a stochastic process could be utilized.

The price model serves the purpose of modelling the price *level* in the market. As modelled in this thesis, there is a spread between the sell and buy prices. Moreover, the participant could *choose* whether to up- or down-regulate the generator, depending on what is found to be more profitable, in expectation. When uncertainties of bid acceptance are introduced, the main difference from the existing model is that the participant does *not fully control what new capacity level,* $\mathbb{I}_{t+1}$*, to proceed to*. The direction of change in remaining capacity, $\Delta\mathbb{I}$, is then *forced by external factors*, even though the volume is still a matter of choice to be optimized.

This *matter of choice* could be replaced by a state variable containing information about what bids the market participant is likely to get accepted. It could model whether the market is short or long. It would introduce a lower level of controllability from the user side, and hence also be a more realistic model. The state variable could either be a stochastic process, or a process evolving as a function of other states and parameters in the system. Introduction of this additional market stochastic would lead to an increasing state space, and the dimensional curse is again an issue to work around.

The process modelling probability of bid acceptance could be modelled as another stochastic process, independent of the rest of the system. Otherwise, it could be modelled as a function of other factors, if one analyzed tendencies in the market. These factors are likely to be linked to price, water value etc. In a case where additional exogenous information arrive, the eq. (32) is extended to

$$\mathbb{W}_t = (\mathbb{P}_t, \mathbb{W}^M_{t-1}) \tag{55}$$

Where $\mathbb{P}_t$ still represents the exogenously given price process, whilst $\mathbb{W}^M_t$ describes the market status (long, short) associated with the probabilities of bid acceptances.

The market status either indicates that the market is short or long. In practice it can also be neutral, but this is omitted here for simplicity.

$$\mathbb{W}^M_t = [\delta^{M,Sell}_t, \delta^{M,Buy}_t] \tag{56}$$

Where the market is long, short or neutral:

$$\delta^{M,Sell}_t + \delta^{M,Buy}_t \leq 1 \tag{57}$$

given that $\delta^{M,i}_t$ is a vector of $\overline{H}_t$ elements where each element $\delta^{M,i}_{t,h} \in \{0,1\}$. Thus,

$$\delta^{M,Sell}_{t,h} = \begin{cases} 1 & \text{if marked is short (power deficit) in product hour } h \in \mathcal{H}_t \text{ as seen from time } t \in T \\ 0 & \text{otherwise} \end{cases} \tag{58}$$

$$\delta^{M,Buy}_{t,h} = \begin{cases} 1 & \text{if marked is long (power surplus) in product hour } h \in \mathcal{H}_t \text{ as seen from time } t \in T \\ 0 & \text{otherwise} \end{cases} \tag{59}$$

Recall from fig. 15 that the exogenous information arrives *after* making the decision $\mathbb{X}_t$, and before arriving at the new state. The information in this case regards both the new market price levels for step $t+1$ and the market demand status regarding the current step t. A difference is that the exogenous information would also have an impact on what post-decision state one end up in, as it is no longer purely a choice.

The model is based on the assumption that the participant can optimize over its own production plan and disregard uncertainties about having their bids accepted, as long as it corresponds with the price levels given by the price model. This point of view can be argued based on the origin of the price model, namely that it represents the *willingness to trade*. The difference between these two is mainly the ability to profit purely from trades, regardless of production capacity. In the last perspective the possibility of the optimal bids not being accepted is present.

In conclusion, there are numerous interesting directions that a future research can proceed in. A natural first step would be to implement the adjustment heuristic in order to get a feasible LB.

Appendices

A Compact Mathematical Model

State space:

$$\mathbb{P}_t \in \mathcal{P}_t \tag{60}$$

$$\mathbb{I}_t \in I_t \tag{61}$$

$$\mathcal{S}_t : \mathcal{P}_t \times I_t \tag{62}$$

Post-decision state:

$$\mathcal{S}_t^{\mathbb{X}} : I_t \tag{63}$$

Decisions:

$$\mathbb{X}_t \in \mathcal{X}_t \tag{64}$$

Exogenous information:

$$\mathbb{W}_t \in \Omega_t \tag{65}$$

Transition function:

$$S_{t+1} = S^M(S_t, \mathbb{X}_t, \mathbb{W}_{t+1}) \tag{66}$$

Contribution function:

$$C_t(S_t, \mathbb{X}_t) = \mathbb{P}_t^S \bullet \mathbb{X}_t^S - \mathbb{P}_t^B \bullet \mathbb{X}_t^B - \Big(\mathbb{C}^{AC}\big(\mathbb{Q}_{t+1}(\mathbb{X}_t)\big) \bullet \mathbb{Q}_{t+1}(\mathbb{X}_t) - \mathbb{C}^{AC}\big(\mathbb{Q}_t\big) \bullet \mathbb{Q}_t\Big) - C^{ss} \cdot \alpha_{t,t+1} \tag{67}$$

Value function:

$$V_t(S_t) = \max_{\mathbb{X}_t \in \mathcal{X}} \big(C_t(S_t, \mathbb{X}_t) + \mathbf{E}^{\mathbb{W}_{t+1}}[V_{t+1}(S_{t+1}|S_t, \mathbb{X}_t, \mathbb{W}_{t+1})]\big) \tag{68}$$

Bellman's equation:

$$V_0^* = \max_{\pi \in \Pi} \mathbf{E}^{\pi} \big\{ \sum_{t \in \mathcal{T}} C_t^{\pi}(S_t, \mathbb{X}_t^{\pi}(S_t)) | S_0 \big\} \tag{69}$$

B From the Industrial Partner, TrønderEnergi

TrønderEnergi Kraft AS har i lengre tid hatt et tett samarbeid med NTNU og dette er ett samarbeid som er valgt og nedfelt i strategien til morselskapet TrønderEnergi AS.

Nærheten til NTNU og andre undervisningsinstitusjoner forenkler denne type samarbeid mellom vår bedrift og dette tilfellet NTNU, ved institutt for Industriell Økonomi og Teknologi ledelse.

I dette tilfellet har veiledningen vært gjort av undertegnede som jobber innenfor gruppen som har ansvaret for operativ kjøring av langtidsmodeller for produksjonsplanlegging, det vil si prising(fastsettelse av vannverdi) i TrønderEnergi Kraft AS.

Oppgaven som det har blitt jobbet med er ≪Budgivning i Elbas-markedet≫. Til nå har budgivingen i dette markedet vært gjort på en enkel måte internt og formålet med oppgaven var å undersøk om denne oppgaven kan løses ved bruk av stokastiske optimeringsmodeller som bruker markedsdata og interne data fra oss. Hensikten med dette er å øke egen inntjening på egen agering i Elbas-markedet, samt forberede oss for en mer volatil framtid som vi tror kommer med ytterligere økning i kapasiteten på fornybar produksjon.

En av hensiktene bak det strategiske valget som TrønderEnergi AS har gjort er å gi studenter relevant og bransjespesifikke problemstillinger knyttet til kraftbransjen. Oppgaven det har blitt jobbet med representerer en helt annen retning knyttet til budgivning i Elbas enn den som benyttes i dag. Innsatsen som blir lagt ned av studentene, sammen med veiledningen som de får fra sine respektive ressurspersoner innenfor NTNU og SINTEF miljøet gir oss verdifull innsikt i problemet. Problemet er ganske komplekst og den kombinerte prosjektet/master i dette tilfellet kan betraktes som en grundig og vel gjennomført jobb med den hensikt å lage en metodikk på budgivning i Elbas med stokastiske optimeringsmetoder. Oppgaven kan også betraktes som en mulighetsstudie som kvalitet og omfang med god margin overgår det som ville vært muligheten å gjøre innenfor rammen av de ressursene vi har tilgjengelig internt.

Kompleksiteten i både prosjekt- og masteroppgaven har også vært av en slik art, at studentene på ett tidspunkt oppnår ett kunnskapsnivå hvor de gir opplæring av undertegnede. Kort sagt er det å veilede studentoppgaver enn vinn-vinn situasjon for TrønderEnergi AS, men som krever innsats og dedikasjon fra alle parter involvert i prosessen for å kunne høste denne gevinsten.

Med hilsen

Gunnar Aronsen

Senior kraftanalytiker

Trønder Energi Kraft AS

Avd. handel

C Space Size of Original Problem

Table 12 show the size of the original problem. The first row describes bid blocks $t \in \mathcal{T}$. The last row for the three columns on the right is the full size of the respective state spaces, and is a sum of the state spaces in steps t. The third column describes $\bar{r}_{h=t-(\overline{T}-\overline{H})}$, and is written as $\bar{r}'$ to save space in the table. The size of $\bar{r}_{h=t-(\overline{T}-\overline{H})}$ is explained in section 4.5.1.

Table 12: State space calculation

t	$\overline{H}_t$	$\overline{I'}_t$	$\overline{r}'$	$\overline{P'}_t$	$\overline{I}_t$	$\overline{P}_t$	$\overline{S}_t$
1	24	4	-	5	$2.81475 \cdot 10^{14}$	$5.96046 \cdot 10^{16}$	$1.67772 \cdot 10^{31}$
2	24	4	-	5	$2.81475 \cdot 10^{14}$	$5.96046 \cdot 10^{16}$	$1.67772 \cdot 10^{31}$
3	24	4	-	5	$2.81475 \cdot 10^{14}$	$5.96046 \cdot 10^{16}$	$1.67772 \cdot 10^{31}$
4	24	4	-	5	$2.81475 \cdot 10^{14}$	$5.96046 \cdot 10^{16}$	$1.67772 \cdot 10^{31}$
5	24	4	-	5	$2.81475 \cdot 10^{14}$	$5.96046 \cdot 10^{16}$	$1.67772 \cdot 10^{31}$
6	24	4	-	5	$2.81475 \cdot 10^{14}$	$5.96046 \cdot 10^{16}$	$1.67772 \cdot 10^{31}$
7	24	4	-	5	$2.81475 \cdot 10^{14}$	$5.96046 \cdot 10^{16}$	$1.67772 \cdot 10^{31}$
8	24	4	-	5	$2.81475 \cdot 10^{14}$	$5.96046 \cdot 10^{16}$	$1.67772 \cdot 10^{31}$
9	24	4	-	5	$2.81475 \cdot 10^{14}$	$5.96046 \cdot 10^{16}$	$1.67772 \cdot 10^{31}$
10	24	4	-	5	$2.81475 \cdot 10^{14}$	$5.96046 \cdot 10^{16}$	$1.67772 \cdot 10^{31}$
11	24	4	-	5	$2.81475 \cdot 10^{14}$	$5.96046 \cdot 10^{16}$	$1.67772 \cdot 10^{31}$
12	23	4	2	5	$1.40737 \cdot 10^{14}$	$1.19209 \cdot 10^{16}$	$1.67772 \cdot 10^{30}$
13	22	4	2	5	$3.51844 \cdot 10^{13}$	$2.38419 \cdot 10^{15}$	$8.38861 \cdot 10^{28}$
14	21	4	2	5	$8.79609 \cdot 10^{12}$	$4.76837 \cdot 10^{14}$	$4.1943 \cdot 10^{27}$
15	20	4	2	5	$2.19902 \cdot 10^{12}$	$9.53674 \cdot 10^{13}$	$2.09715 \cdot 10^{26}$
16	19	4	2	5	$5.49856 \cdot 10^{11}$	$1.907355 \cdot 10^{13}$	$1.0458 \cdot 10^{25}$
17	18	4	2	5	$1.37439 \cdot 10^{11}$	$3.8147 \cdot 10^{12}$	$5.24288 \cdot 10^{23}$
18	17	4	2	5	34359738368	$7.62939 \cdot 10^{11}$	$2.62144 \cdot 10^{22}$
19	16	4	2	5	8589934592	$1.52588 \cdot 10^{11}$	$1.31072 \cdot 10^{21}$
20	15	4	2	5	2147483648	30517578125	$6.5536 \cdot 10^{19}$
21	14	4	2	5	536870912	6103515625	$3.2768 \cdot 10^{18}$
22	13	4	2	5	134217728	1220703125	$1.6384 \cdot 10^{17}$
23	12	4	2	5	33554432	244140625	$8.192 \cdot 10^{15}$
24	11	4	2	5	8388608	48828125	$4.096 \cdot 10^{14}$
25	10	4	2	5	2097152	9765625	$2.048 \cdot 10^{13}$
26	9	4	2	5	524288	1953125	$1.024 \cdot 10^{12}$
27	8	4	2	5	131072	390625	51200000000
28	7	4	2	5	32768	78125	2560000000
29	6	4	2	5	8192	15625	128000000
30	5	4	2	5	2048	3125	6400000
31	4	4	2	5	512	625	320000
32	3	4	2	5	128	125	16000
33	2	4	2	5	32	25	800
34	1	4	2	5	8	5	40
-	-	-	-	-	$3.28387 \cdot 10^{15}$	$6.70552 \cdot 10^{17}$	$1.86315 \cdot 10^{32}$

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