OPTIMAL ONLINE CONFIGURATION AND LOAD-SHARING IN A REDUNDANT ELECTRIC POWER SYSTEM FOR AN OFFSHORE VESSEL USING MIXED INTEGER LINEAR PROGRAMMING

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ABSTRACT

This paper proposes several schemes for optimal online configuration and load sharing for a shipboard power system of a typical offshore vessel, having a number of varying capacity gensets. Different methods are presented for optimal online scheduling and minimization of specific fuel oil consumption using mixed integer linear programming. The simulations for optimization of specific fuel oil consumption for three different scheduling methods demonstrate their properties.

INTRODUCTION

Electric propulsion of marine vessels has been popular for more than three decades due to high flexibility, availability, and reliability of the systems. Diesel engines have been the main prime movers for propulsion of marine vessels for the last century, where medium-speed and high-speed diesel engines are most common to drive a generator to meet the electrical power demand. Typically, 2 - 10 diesel gensets of equal or varying capacity are installed to ensure the necessary power capacity and sufficient redundancy. Due to strict redundancy regulations for dynamic positioning (DP) vessels, typically DP2 or DP3 vessels [1], the diesel gensets often run at low loading for long periods of time due to a discrepancy between environmental load condition (power demand) and requirements for spinning reserve. The low loading of the gensets lead to low efficiency and increased hazardous emissions from the gensets [2]. Even though the emissions from the marine industry are much smaller compared to the other industries, there are strict requirements set by the International Maritime Organisation to reduce these emissions [3]. With new developments in technology, the complexity of electrical installations on marine vessels is also increasing to meet today's and future requirements in fuel economy and reduced environmental footprint.

For a power system having many power generating units, the optimal generator scheduling becomes very important. The shipboard power system has many similarities and some differences with the land-based islanded smart microgrids. For a land-based smart grid, the load profile is more predictable, and it is easier to do optimal generator scheduling [4]. The load profile of a marine vessel, on the other hand, may change rapidly due to variable load demands from propulsion (and potentially drilling system or other onboard large consumers) due to wave motions, wind gusts, or variable operation of heavy consumers, including low-load operation in calm conditions. This makes it difficult to achieve optimal scheduling for a power system with few gensets. In [5], the authors have formulated a problem of optimal power generator scheduling for a shipboard power system as a discrete time Markov decision process, whereas the authors of [6] have used a mixed integer linear programming (MILP) approach for the specific fuel oil consumption (SFOC) minimization by considering various operating scenarios. The authors of [7] propose a method for real-time optimal scheduling of shipboard power generation with diesel gensets and ESS.

The optimal scheduling of gensets for power generation on marine vessels is a relatively new topic with limited research at-

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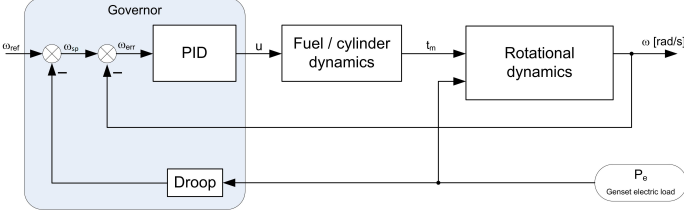*Address all correspondence to this author.

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FIGURE 1. Block diagram of the rotation control system connected in droop control.

tempts [7]. Generator scheduling by using an integer programming approach was proposed in the 1960's, where the author of [8] used this approach for solving a scheduling problem by considering operational characteristics and costs associated with starting and shutting down electric generators. The MILP has been proposed as a solution to perform integer optimization by connecting/disconnecting gensets [9].

The objective of this paper is to propose several methods for optimal genset scheduling and SFOC minimization, and to demonstrate the properties of these methods in simulation studies for an offshore vessel with many gensets of varying capacity. Our toolbox will be optimization by the MILP method due to its availability of efficient and robust linear programming based solvers, easier problem formulations, modelling flexibility, and general acceptance by the industry.

PROBLEM FORMULATION
Speed control

The governor is the diesel engine speed controller. It has the objective to keep engine speed, and thus electric frequency, within an acceptable range. According to the regulations [10], the allowed frequency variation, according to main class, is $\pm5\%$ under steady load conditions, e.g. $57-63$ [Hz], and $\pm10\%$ under transient load conditions. Typically, the governor is a PID controller, and thus the inner control loop as shown in Figure 1 is typically applied to control the engine speed. A governor can typically operate in a so-called *droop mode* or in an *isochronous mode* [2, 11].

Droop speed control

To include droop, the value of active power P of the genset should be normalized such that the droop-percent value can be used directly. Then the corrected setpoint frequency $\omega_{sp,j}$ to the governor of a Genset j is

$$\omega_{sp,j} = \omega_{ref,j} - k_j p_j \qquad (1)$$

where $\omega_{ref,j}$ is the per-unit no-load reference frequency typically set by the Power Management System (PMS), k_j is the droop constant, $p_j := \frac{P_j}{P_{b,j}}$ is the per-unit supplied active power, P_j is the supplied active power, and $P_{b,j}$ is the rated 100% active (base) power value (typically $P_{b,j} = S_{b,j}\cos\varphi_{b,j}$, where $\cos\varphi_{b,j}$ is the rated power factor). Figure 1 shows a typical block diagram.

Consider a plant of M parallel gensets operated in droop mode, let $\mathscr{I} = \{1, 2, \ldots, M\}$ be their index set, and let $u_j = \omega_{ref,j}$ be considered a control input by the PMS. Also, let $c_j \in \mathbb{I} := \{0,1\}$ be a discrete state dictating if the genset is connected ($c_j = 1$) or disconnected ($c_j = 0$) from the bus. Assume that in steady state, the frequency of Genset j equals the setpoint into the speed governor, that is, $\omega_j = \omega_{sp,j}$. In addition, the parallel connected gensets must all have synchronized frequencies equal to the bus frequency ω_{bus}, while those not connected should equal their reference u_j. From the droop curves, we get

$$\omega_1 = u_1 - c_1 k_1 p_1 \qquad (2)$$
$$\vdots$$
$$\omega_M = u_M - c_M k_M p_M$$

where the droop gains k_j are typically set to $3-5\%$.

In addition, the sum of the supplied active powers must balance the demanded power by the bus, i.e., $\Sigma c_j P_j = P_{load}$. By normalizing the bus active load P_{load} by dividing it by the system base $P_{b,bus}$, equal to the total installed (not connected) capacity [kVA] on the bus [12], this gives the equations we will use for calculation of active power sharing between M parallel connected gensets,

$$\sum_{j\in\mathscr{I}} \frac{c_j P_{b,j}}{P_{b,bus}} p_j = \frac{P_{load}}{P_{b,bus}}. \qquad (3)$$

Problem Statement

This paper considers a marine power plant with a large number of small capacity gensets sharing load based on droop control for an offshore vessel. The online optimal power load sharing is investigated by studying three scheduling methods; scheduling by minimizing online capacity, scheduling with redundancy margin for largest connected genset, and scheduling with penalties on running time and connection/disconnection. The mathematical formulations for optimization of SFOC are thereafter developed using the MILP approach. The simulations are carried out in MATLAB, and the results are compared for a specific case study.

SCHEDULING PROBLEM USING MILP

The scheduling problem is an optimal allocation of resources (online gensets in the marine vessel) to activities (load demand) over an interval of time. In this paper, we go stepwise through some of the scheduling methods for electric power generation on marine vessels, gradually increasing the complexity and comparing the results. We assume that we have a plant of M gensets of individual capacity $S_{b,j}$.

For simplicity in simulations, we consider a marine power plant having five gensets of individual capacity varying from 300kW to 700kW. The random load profile similar to a typical load profile of an offshore vessel is used for demonstrating the

algorithms by simulations in MATLAB.

Scheduling minimizing online capacity

The simplest scheduling problem considers only one objective with constraints. As a first approach, our objective is to minimize the number of connected gensets under the constraint that sufficient capacity is maintained on the bus, given the present load demand.

Given the genset connection vector $c \in \mathbb{I}^M$, then the number of gensets is $N = 1_M^\top c$, where $1_M \in \mathbb{R}^M$ is a vector of ones. Typically, the gensets have an optimal loading around 80% of their maximum continuous rating (MCR). Considering the optimal loading factor as γ, we can write $P_{opt,j} = \gamma P_{b,j}$, where e.g. $\gamma = 0.8$. We want to minimize the number of online gensets N. The constraint is to have enough capacity, according to

$$P_b^\top \Gamma c \geq P_{load}, \tag{4}$$

where $P_b = \mathrm{col}(P_{b,1}, ..., P_{b,M})$ and $\Gamma = \mathrm{diag}(\gamma_1, \gamma_2, ..., \gamma_M)$. Each γ_j can here be chosen as the optimal loading value for each individual genset. In addition, we want a minimum of $N_{\min}$ gensets to be connected, e.g., $N_{\min} = 1$ for all time to meet the load demand of the essential loads. This motivates the following simple MILP problem:

$$\min_c \quad 1_M^\top c \tag{5}$$
$$\text{s.t.} \quad -P_b^\top \Gamma c \leq -P_{load}$$
$$-1_M^\top c \leq -N_{\min}$$

It should be noted that minimizing the number of gensets will favorize using the largest gensets. This may not always be beneficial. An alternative is therefore to introduce a weighted cost function that takes value according to the genset capacity. This will ensure to find an optimal configuration of online gensets with the tightest capacity to the prevailing load, thus to some degree favorizing using the smaller gensets within the constraints. We then reformulate the MILP problem to minimize the online capacity according to

$$\min_c \quad \frac{P_b^\top}{|P_b|_\rho} c \tag{6}$$
$$\text{s.t.} \quad -P_b^\top \Gamma c \leq -P_{load}$$
$$-1_M^\top c \leq -N_{\min}$$

where $\rho \geq 1$ is some norm (e.g., $\rho \in \{1, 2, \infty\}$). We want to investigate if different norms give different results and advantages of using it. In this paper, we are only considering $\rho = 1$. The other cases will be investigated in future work.

The simulations are carried out for this scheduling problem of minimizing online capacity using MILP over the interval of 1700s, where we used $\Gamma = 0.8I$. The optimization algorithm runs every 10s, and the results are shown in Figure 2. Due to the choice for Γ, this gives an inherent margin of at least 20%. The running time of each genset, and the number of online gensets at any instant are shown in the figure. The optimization tries to keep the online capacity of the power plant such that the load demand is approximately at 80% of the genset capacities. The more small gensets that are available in the power plant, the tighter this envelope can be maintained. The frequency of connection/disconnection of the gensets is very high, which can easily be seen from the plot of running time for each genset.

Scheduling with redundancy margin

For the simple genset scheduling problem above, if any of the online genset fails then the load is shared among the remaining online gensets until the next genset is connected. During this time, the online gensets might be overloaded. To overcome this issue, we consider the redundancy margin for the largest connected genset and reformulate the problem. Accordingly, we include the constraint that if the largest genset fails, there shall still be enough capacity to supply the demand without overloading the other online gensets. The largest connected genset is given by

$$P_{b,\max} = \max_j (P_b \circ c) = |\mathrm{diag}(P_b)c|_\infty, \tag{7}$$

where '$\circ$' means the Hadamard elementwise vector product. It should be noted that a diesel-genset typically has some additional margin with a maximum overload value of 110% of rated capacity $P_{b,j}$. In our case, we disregard this additional 10% overload margin. Then, introducing this redundancy constraint is the same as adding a margin of $P_{b,\max}$ to the bus load, that is, the updated constraint becomes

$$P_b^\top \Gamma c \geq P_{load} + P_{b,\max}, \tag{8}$$

where we in this case will typically set $\Gamma = I$ since the redundancy margin will ensure to reduce the power on each genset well below their rated values. However, since we do not know beforehand which gensets that will be selected by the optimization, and from that, what is the largest sized connected genset, this constraint is not realizable as a linear inequality constraint.

Let $\varepsilon_j \in \mathbb{R}^M$ be the j'th unit axis vector, so that $\varepsilon_j^\top x$ selects the j'th element of vector $x \in \mathbb{R}^M$. A workaround is then to add several redundant constraints to ensure that the capacity margin is larger than whatever connected genset that fails. This is achieved by increasing the number of constraints from the single constraint above to the M constraints

$$P_b^\top \Gamma c \geq P_{load} + \varepsilon_1^\top \mathrm{diag}(\Gamma P_b)c$$
$$\vdots$$
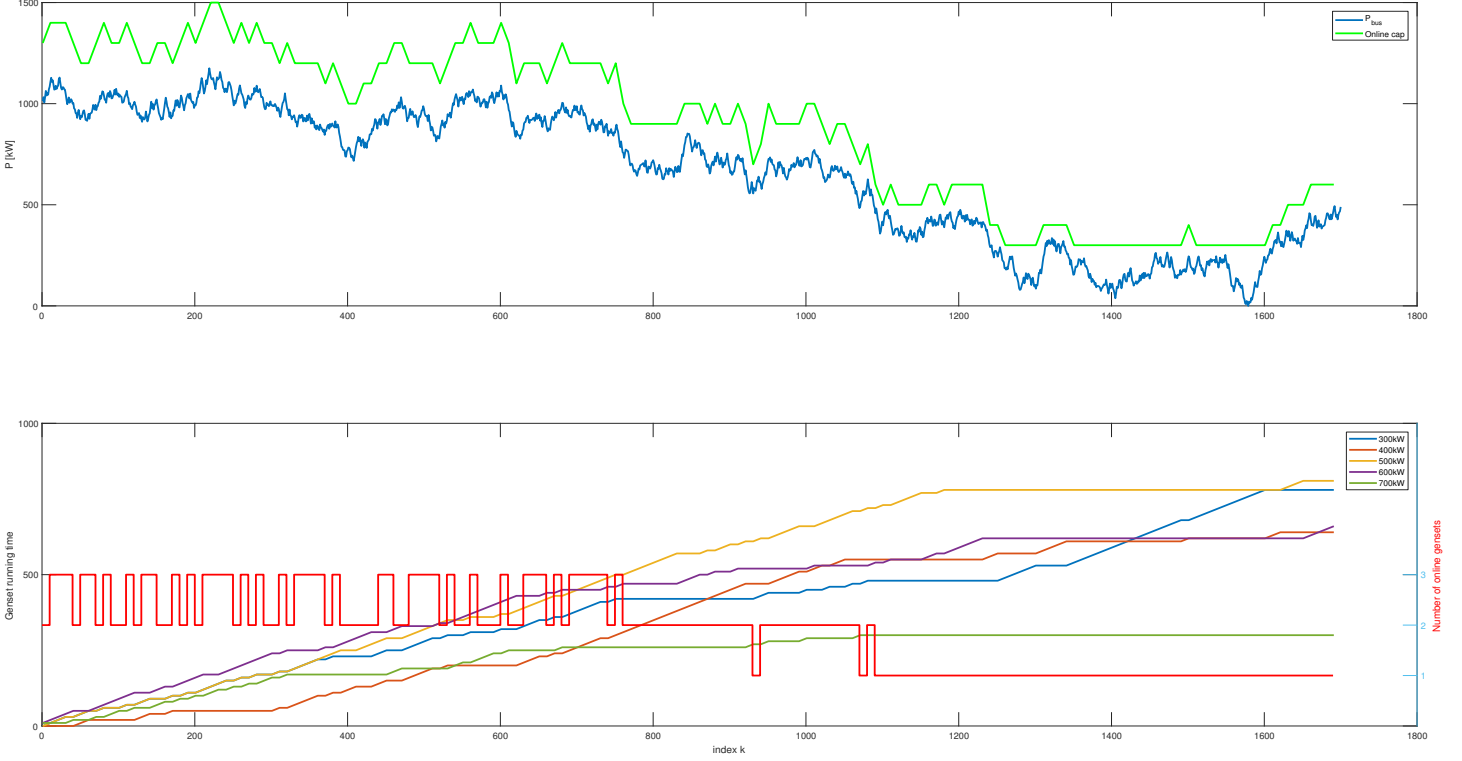$$P_b^\top \Gamma c \geq P_{load} + \varepsilon_M^\top \mathrm{diag}(\Gamma P_b)c$$

FIGURE 2. Scheduling minimizing online capacity, using $\Gamma = 0.8I$.

This can be rewritten according to

$$\left(\varepsilon_1^\top \operatorname{diag}(\Gamma P_b) - P_b^\top \Gamma\right) c \leq -P_{load}$$

$$\vdots$$

$$\left(\varepsilon_M^\top \operatorname{diag}(\Gamma P_b) - P_b^\top \Gamma\right) c \leq -P_{load}$$

Here, the term $\left(\varepsilon_j^\top \operatorname{diag}(\Gamma P_b) - P_b^\top \Gamma\right)$ removes the j'th element of the vector $-P_b^\top \Gamma$. In matrix form, we can write this as

$$A_{ineq} c \leq b_{ineq},$$

where

$$A_{ineq} = \begin{bmatrix} \varepsilon_1^\top \operatorname{diag}(\Gamma P_b) - P_b^\top \Gamma \\ \varepsilon_2^\top \operatorname{diag}(\Gamma P_b) - P_b^\top \Gamma \\ \vdots \\ \varepsilon_M^\top \operatorname{diag}(\Gamma P_b) - P_b^\top \Gamma \end{bmatrix} = \operatorname{diag}(\Gamma P_b) - 1_M \otimes P_b^\top \Gamma \quad (9)$$

$$b_{ineq} = -1_M P_{load}, \tag{10}$$

and $\otimes$ is the Kronecker product. This gives the following MILP

problem:

$$\min_c \frac{P_b^\top}{|P_b|_\rho} c \tag{11}$$

$$\text{s.t.} \quad A_{ineq} c \leq b_{ineq} \tag{12}$$

$$-1_M^\top c \leq -N_{\min} \tag{13}$$

The simulation results for the scheduling with redundancy margin for the largest connected gensets are shown in the Figure 3, where we used $\Gamma = I$. Comparing figures 2 and 3, we can see that by including redundancy margin, the reserve power available in the power plant increases, and the load is shared by more gensets as compared with the simple scheduling case. The frequency of connection/disconnection during the simulation interval is less as compared with the simplest scheduling case, which can easily be seen from the plot of the running time of each genset as well as from the plot of available power.

Scheduling with a penalties on running time and connection/disconnection

The load on an offshore vessel varies continuously, which might cause frequent connect/disconnect of a genset. In addition, by using scheduling for minimizing online capacity or scheduling with redundancy margin for largest connected genset, some gensets might be connected all the time, increasing the wear and tear of these gensets, and some might not be connected at all leav-
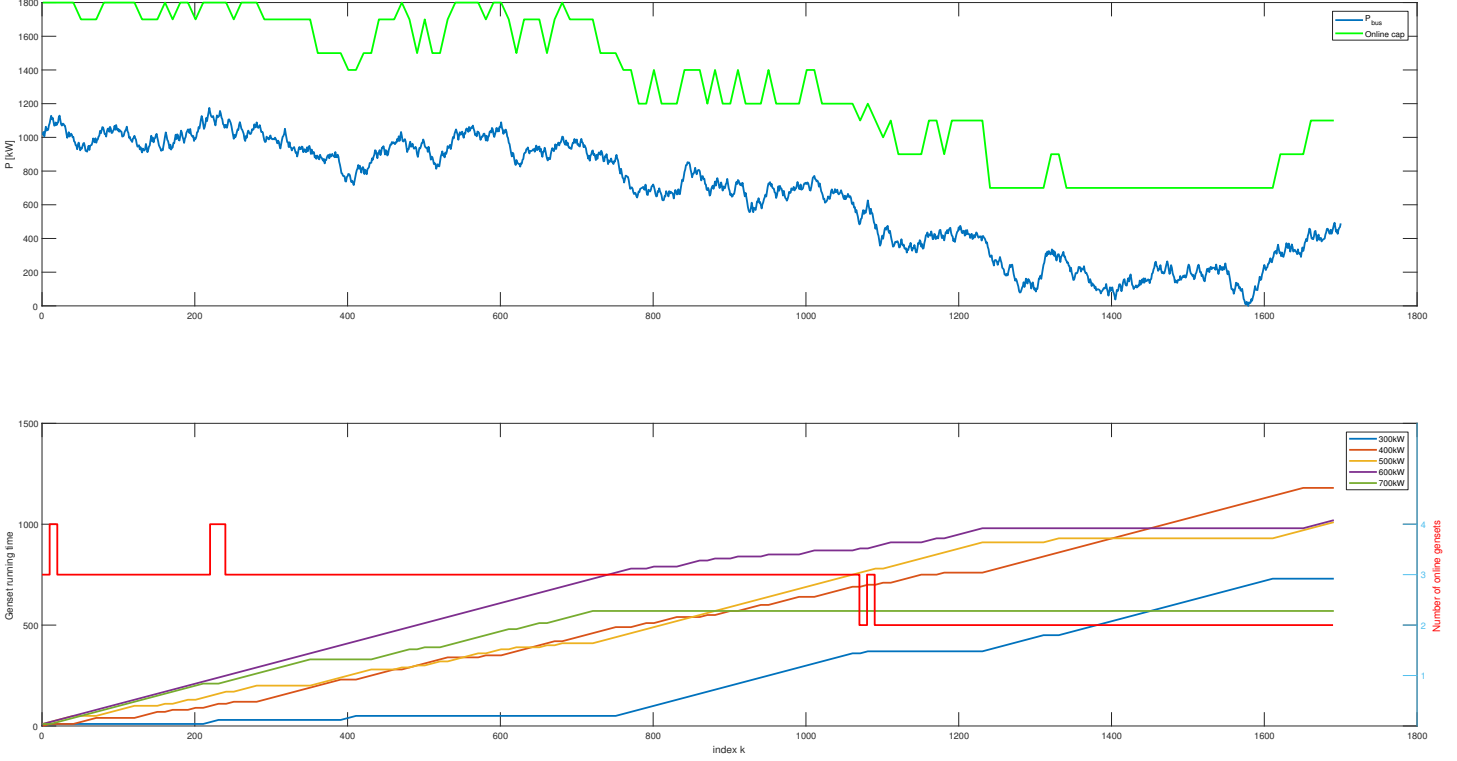
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FIGURE 3. Scheduling with redundancy margin for largest connected genset, using $\Gamma = I$.

ing them unused. To overcome this problem, we introduce additional constraints. To balance the running time for the gensets, we introduce the penalty on running time of a genset. Additionally, to avoid too frequent connection/disconnection of the gensets, a penalty on connection/disconnection is introduced.

Let a signal counting running time for each genset be

$$d_j(k) = d_j(k-1) + c_j(k-1)T_s, \quad d_j(0) = 0,$$

where k is the present time index and T_s is the periodic execution time of the optimization. On vector form, we then get

$$d(k) = d(k-1) + T_s c(k-1), \quad d(0) = 0, \quad (14)$$

for $k = 1, 2, 3, \ldots$. At any instant k of optimization, $d(k)^\top c$ sums up the running time for the connected gensets. Minimizing this term ensures to balance which gensets are being used over time.

Similarly, to minimize the connection/disconnection, let s_j^{on} be a signal adding up the cost of connecting Genset j and s_j^{off} adding up the cost of disconnecting Genset j. We can write it as

$$s^{on} := \text{col}(s_1^{on}, \ldots, s_M^{on}), \quad s^{off} := \text{col}(s_1^{off}, \ldots, s_M^{off}).$$

Let $\Delta c_k = c(k) - c(k-1)$ be the change in connection from one instant to the next, and decompose $\Delta c_k = \Delta c_k^+ - \Delta c_k^-$ where the first term is 1 at the $+1$ elements and the second term is 1 at the -1 terms. Then we get the accumulated cost of connecting and disconnecting gensets by

$$s^{on} = W_{connect} \sum_{i=1}^{k} \Delta c_i^+, \quad s^{off} = W_{disconnect} \sum_{i=1}^{k} \Delta c_i^-.$$

With relative weights between the different terms, we include these signals in the minimization. This motivates the updated MILP optimization problem

$$\min_c \left(w_1 \frac{P_b^\top}{|P_b|_\rho} + w_2 \frac{d}{|d|_\rho + \varepsilon} + w_3 \frac{s^{on}}{|s^{on}|_\rho + \varepsilon} + w_4 \frac{s^{off}}{|s^{off}|_\rho + \varepsilon} \right)^\top c \quad (15)$$

s.t. $A_{ineq}c \leq b_{ineq}$

$\quad -1_M^\top c \leq -N_{\min},$

where $\varepsilon > 0$ is a small constant included to avoid division by zero, and the constraints include the redundancy requirement in equations (9) and (10).

The simulation results for the simple case of scheduling with penalty on running time ($\Gamma = 0.8I, w_1 = 10, w_2 = 100, w_3 = w_4 = 1$) are shown in Figure 4 and with penalty on connection/disconnection ($\Gamma = 0.8I, w_1 = 10, w_2 = 1, w_3 = w_4 = 100$) are shown in Figure 5. If the penalty on running time is more than the penalty for connection/disconnection then the frequency
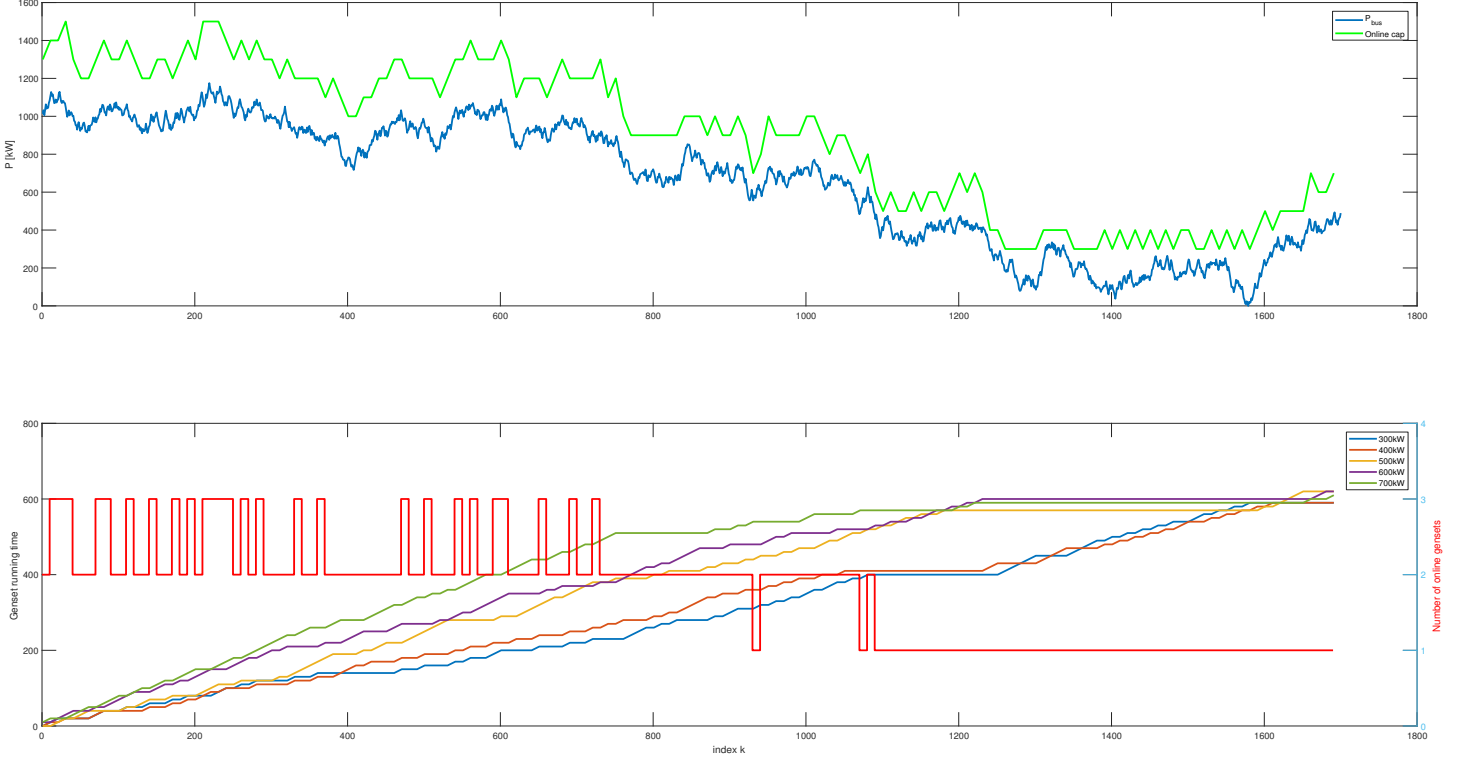
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FIGURE 4.   Scheduling with penalties on running time than connection/disconnection, using $\Gamma = 0.8I, w_1 = 10, w_2 = 100, w_3 = w_4 = 1$.

of connection/disconnection increases (can be seen from subplot for available power) and vice-a-versa. Comparing the simulation results for the three scheduling methods, we can say that the running time and/or the number of connection/disconnections in the system can easily be tuned according to requirements by introducing suitable penalties and selecting appropriate weights.

LOAD-SHARING WITH MINIMIZATION OF SFOC

Since optimizing both the scheduling (c vector) and the load-sharing (p vector) generally leads to a nonlinear program, we aim to look for some simpler formulations where we still can use the MILP method. One such simplification is to perform the optimization as a two-stage optimization. In Stage 1 we perform the scheduling above to assure enough online capacity, given load and redundancy constraints. Then, given the determined connect/disconnect policy c resulting from Stage 1, we perform a Stage 2 optimal load sharing to determine p.

SFOC curves and FOC

Suppose that the SFOC curves for each genset is given by

$$f_{SFOC,j} = h_j(p_j) \qquad (16)$$

where $h_j(\cdot)$ is a convex curve, typically with a minimum of approximately $190\,\mathrm{g/kWh}$ around $75-80\%$ genset load. This curve is given by distinct $(p_j, f_{SFOC,j})$ pairs measured as steady-state values in laboratory testing of the engine. This results in a piecewise linear (PWL) curve. Accordingly, assume the curve $f_{SFOC,j}(p_j)$ for Genset j is populated by the $m_j + 1$ points,

$$p_j = \{p_{j,0}, p_{j,1}, \ldots, p_{j,m_j}\}, \quad p_{j,k-1} < p_{j,k}$$
$$f_{SFOC,j} = \{z_{j,0}, z_{j,1}, \ldots, z_{j,m_j}\}$$

where we assume that the interval $[p_{j,0}, p_{j,m_j}]$ is the allowable power region for Genset j for which the SFOC curve is defined. We also assume that $z_{j,0} = \max_k \left(z_{j,k}\right)$.

Correspondingly, we define the linear curve coefficients by

$$\left.\begin{array}{l} a_{j,k} = \frac{z_{j,k}-z_{j,k-1}}{p_{j,k}-p_{j,k-1}} \\[2mm] b_{j,k} = z_{j,k-1} - a_{j,k}p_{j,k-1} \end{array}\right\}, \qquad k = 1,2,\ldots,m_j \qquad (17)$$

Then we get the m_j linear curves for the Genset j

$$h_{j,k}(p_j) = a_{j,k}p_j + b_{j,k}, \qquad (18)$$

and the combined convex PWL curve is expressed by

$$f_{SFOC,j} = h_j(p_j) = \max_{k=1,\ldots,m_j} h_{j,k}(p_j) = \max_{k=1,\ldots,m_j} \left(a_{j,k}p_j + b_{j,k}\right).$$
$$(19)$$

This curve can further be fitted to a polynomial curve

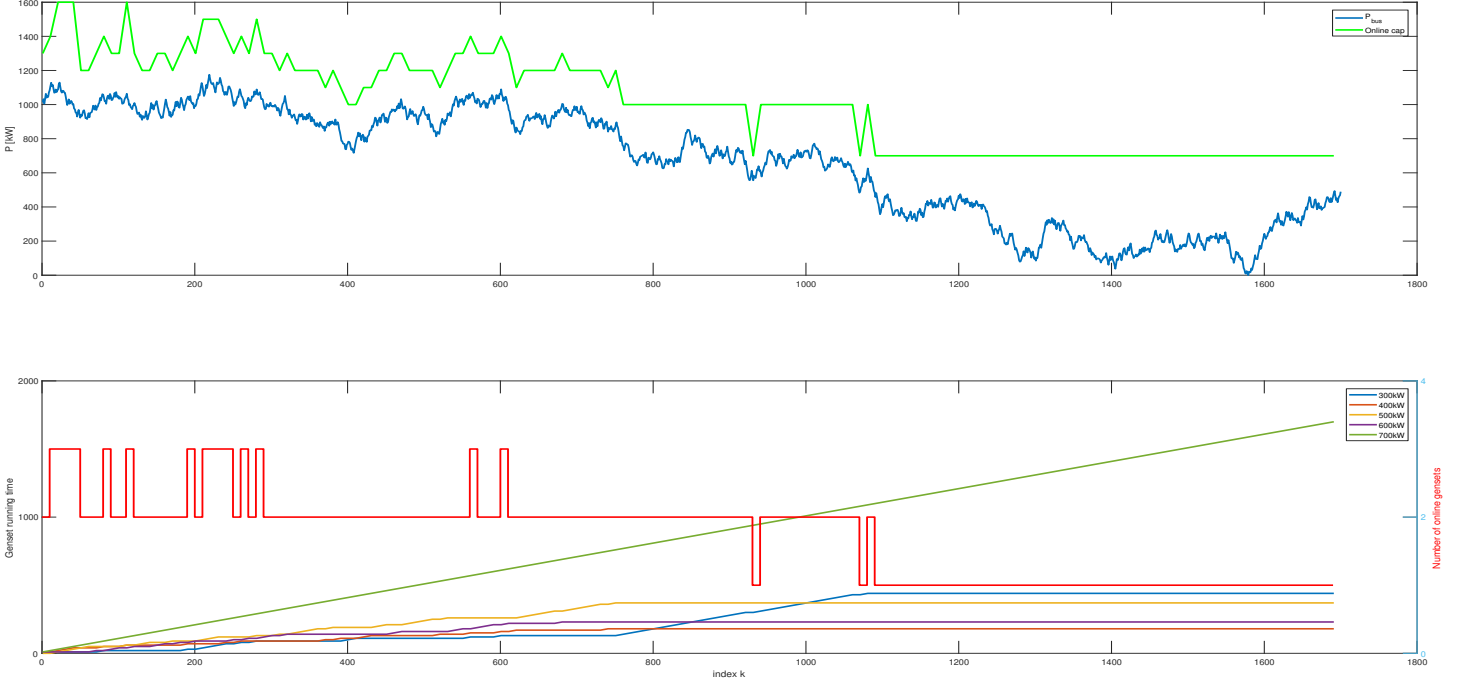$$h_j(p_j) = \sum_{k=0}^{m} a_{j,k}p_j^k, \qquad (20)$$

FIGURE 5. Scheduling with penalties on connection/disconnection than running time, using $\Gamma = 0.8I, w_1 = 10, w_2 = 1, w_3 = w_4 = 100$.

but in our case it will be easier to run the optimization directly on the PWL curve. Collecting the M fuel curves in one vector gives

$$h(p) := \text{col}\,(h_1(p_1), h_2(p_2), \ldots, h_M(p_M))\,. \qquad (21)$$

The instantaneous (steady state) fuel oil consumption (FOC) is now approximated by

$$f_{FOC,j} = f_{SFOC,j}P_j = h_j(p_j)P_j$$
$$f_{FOC} := \text{col}\,(f_{FOC,1}, f_{FOC,2}, \ldots, f_{FOC,M}) = h(p)^\top P$$

For a disconnected but running genset, assume it consumes a constant idle fuel $f_{0,j}$, and collect these into

$$f_0 := \text{col}\,(f_{0,1}, f_{0,2}, \ldots, f_{0,M})\,.$$

Then, for a given connection status at given time instance k, and assuming no gensets are physically stopped, the approximate instantaneous fuel consumption becomes

$$FOC = |c \circ f_{FOC}|_1 + |(1_M - c) \circ f_0|_1 = c^\top [f_{FOC} - f_0] + 1_M^\top f_0. \qquad (22)$$

Online optimization of specific fuel consumption

In the optimization, we aim to minimize the fuel consumption per power unit produced (i.e., the SFOC), given enough capacity online. For a maximum of M gensets being connected or disconnected according to the vector c, this implies minimizing

the instantaneous cost

$$\min_{c,p} \quad J_{SFOC}(c,p) = \sum_{j=1}^{M} c_j h_j(p_j) = c^\top h(p) \qquad (23)$$

where the SFOC curves $f_{SFOC,j} = h_j(p_j)$ are assumed to be convex continuous curves. Given the knowledge of c, then (23) is an optimization problem of $N = 1_M^\top c$ separable convex functions, where we assume each SFOC function is a PWL function according to (19). This can be solved as a new LP problem. Minimization of a single PWL SFOC curve for an online genset is, according to [13], done by

$$\min_{p_j, \mu_j} \quad \mu_j \qquad (24)$$
$$\text{s.t.} \quad a_{j,k}p_j + b_{j,k} \le \mu_j, \qquad k = 1, 2, \ldots, m_j$$
$$P_{b,j}p_j = P_{load}$$
$$p_{j,0} \le p_j \le p_{j,m_j}$$

where μ_j is a scalar auxiliary variable. Including the connection status $c_j \in \{0, 1\}$ allows us an optimal solution where $p_j = 0$ for disconnected gensets. This is achieved by

$$\min_{p_j, \mu_j} \quad c_j \mu_j \qquad (25)$$
$$\text{s.t.} \quad a_{j,k}p_j + b_{j,k} \le \mu_j, \qquad k = 1, 2, \ldots, m_j$$
$$c_j p_{j,0} \le p_j \le c_j p_{j,m_j},$$

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where we see that for a disconnected genset j, $c_j = 0$, the feasible solution is $p_j = 0$ and $\mu_j \geq b_{j,k}$ free. Let

$$a_j := \begin{bmatrix} a_{j,1} \\ \vdots \\ a_{j,m_j} \end{bmatrix} \in \mathbb{R}^{m_j}, \qquad b_j = \begin{bmatrix} b_{j,1} \\ \vdots \\ b_{j,m_j} \end{bmatrix} \in \mathbb{R}^{m_j} \qquad (26)$$

be the coefficient vectors of the PWL curves corresponding to each genset, and let

$$A := \begin{bmatrix} a_1 & 0_{m_1 \times 1} & \cdots & 0_{m_1 \times 1} \\ 0_{m_2 \times 1} & a_2 & & 0_{m_2 \times 1} \\ \vdots & & \ddots & \vdots \\ 0_{m_j \times 1} & 0_{m_j \times 1} & \cdots & a_M \end{bmatrix}, \quad E := \begin{bmatrix} -1_{m_1} & 0_{m_1 \times 1} & \cdots & 0_{m_1 \times 1} \\ 0_{m_2 \times 1} & -1_{m_2} & & 0_{m_2 \times 1} \\ \vdots & & \ddots & \vdots \\ 0_{m_j \times 1} & 0_{m_j \times 1} & \cdots & -1_{m_M} \end{bmatrix}$$

$$b := \begin{bmatrix} b_1 \\ \vdots \\ b_M \end{bmatrix}, \quad p_{\min} = \begin{bmatrix} p_{1,0} \\ \vdots \\ p_{M,0} \end{bmatrix}, \quad p_{\max} = \begin{bmatrix} p_{1,m_1} \\ \vdots \\ p_{M,m_M} \end{bmatrix}$$

Then the overall Stage 2 LP optimization problem becomes

$$\min_{p,\mu} \quad c^\top \mu \qquad (27)$$

$$\text{s.t.} \quad Ap + E\mu \leq -b \qquad (28)$$

$$(c \circ P_b)^\top p = P_{load} \qquad (29)$$

$$c \circ \left(p_j^- - \Delta p \right) \leq p \leq c \circ \left(p_j^- + \Delta p \right) \qquad (30)$$

$$c \circ p_{\min} \leq p \leq c \circ p_{\max} \qquad (31)$$

where:

(28) together with the minimization of the μ vector in (27) ensures the minimization of the PWL SFOC curves.

(29) ensures that the sum of supplied power balances the load on the bus.

(30) ensures a maximum rate of change, where p_j^- is the power value when the optimization is initiated and Δp is set according to a maximum rate of change over the optimization period. This constraint can be combined with (31) by setting $p_{\min,j} = p_j^- - \Delta p$ and $p_{\max,j} = p_j^- + \Delta p$.

(31) ensures that a disconnected genset is set to zero power, and a connected genset is constrained within its minimum and maximum levels.

Simulations

The algorithms are illustrated by simulations in MATLAB. For simulation purpose, it is assumed that the specific fuel consumption of gensets varies from a minimum of around $205 g/kWh$ to a maximum of around $245 g/kWh$ [14]. Three simulations are carried out for the optimization of the SFOC with the simple scheduling with online capacity minimization, scheduling with redundancy on largest connected genset, and simple scheduling with penalties on connection/disconnection and running time of the genset, respectively. The configuration is again the power plant with five gensets of unequal capacities.

The simulation results for the optimal SFOC for the scheduling minimizing online capacity case are shown in Figure 6. The first subplot shows the load profile and online capacity of the power plant. The second subplot shows the genset running time for the given load profile and variation of the SFOC cost during the simulation. In the third subplot, we are plotting the capacity utilization of the online gensets at a given time instant and also the total number of online gensets.

The simulation results for the optimal SFOC for the scheduling with redundancy margin are shown in Figure 7, and for the scheduling with penalties on running time and connection/disconnection of the genset are shown in Figure 8, where we have chosen large value for $w_2 = 100$ to illustrate the effect on balancing the running time between the gensets.

Comparing the simulation results for these three scheduling cases, we see that the optimal SFOC cost increases with the increase in the number of constraints for the system.

CONCLUSION

The optimal scheduling of the power generation and load sharing for a marine vessel, using MILP formulations, were studied in this paper. The scheduling algorithms for three online scheduling methods, scheduling with online capacity minimization, scheduling with redundancy margin for the largest connected genset, and simple scheduling with penalties on running time and connection/disconnection of gensets, as well as online optimization of SFOC, were developed. The simulation cases demonstrated the properties of each proposed method. For the scheduling with minimization of online capacity, the tightest envelope of available power based on prevailing load condition was achieved. This was extended to also account for redundancy to loss of largest genset. Then we showed how one can include penalties for running hours and connection/disconnection cost. The minimization of running hours ensured that the utilization of the different gensets was more balanced. Finally, we showed how to ensure optimal load sharing in a second stage optimization after the scheduling had been determined in the first stage. The load sharing was based on minimizing the instantaneous SFOC among all connected gensets based on PWL parameterizations of the SFOC datasets.

In future work, we will consider nonlinear formulations of these problems, as well as how this can be done in an event-based approach. We also aim to verify the methods on more realistic case studies.
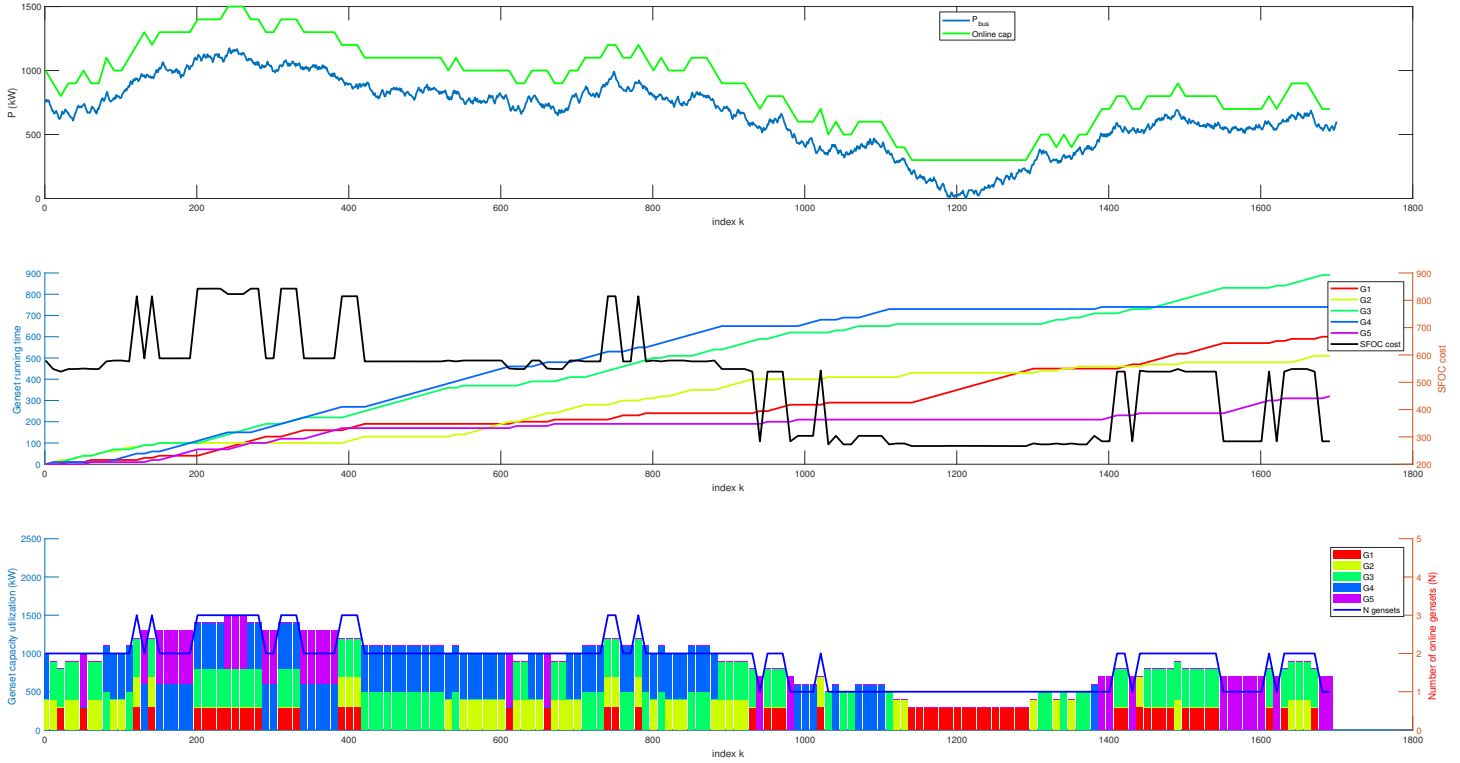
FIGURE 6. The optimization of SFOC for scheduling minimizing online capacity, using $\Gamma = 0.8I$.

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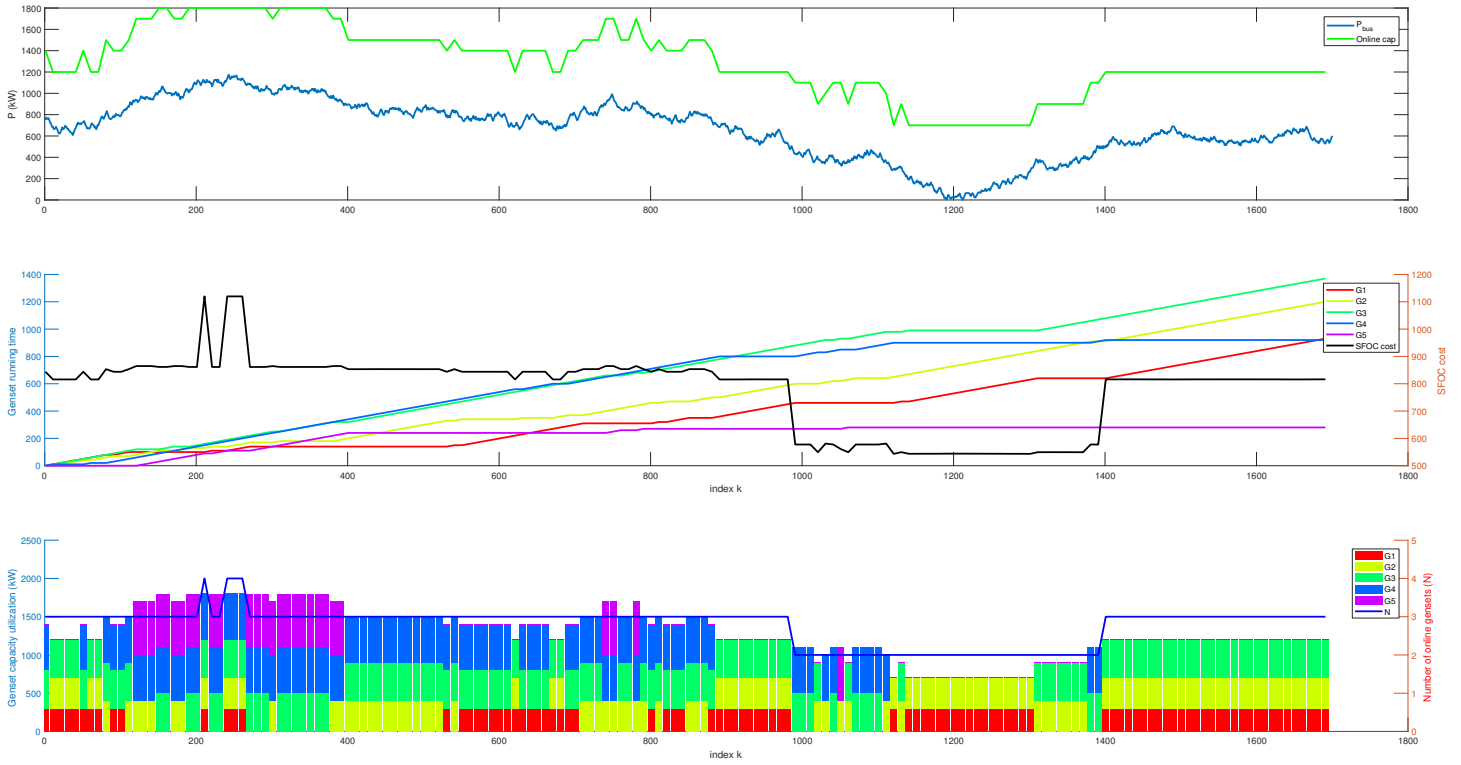
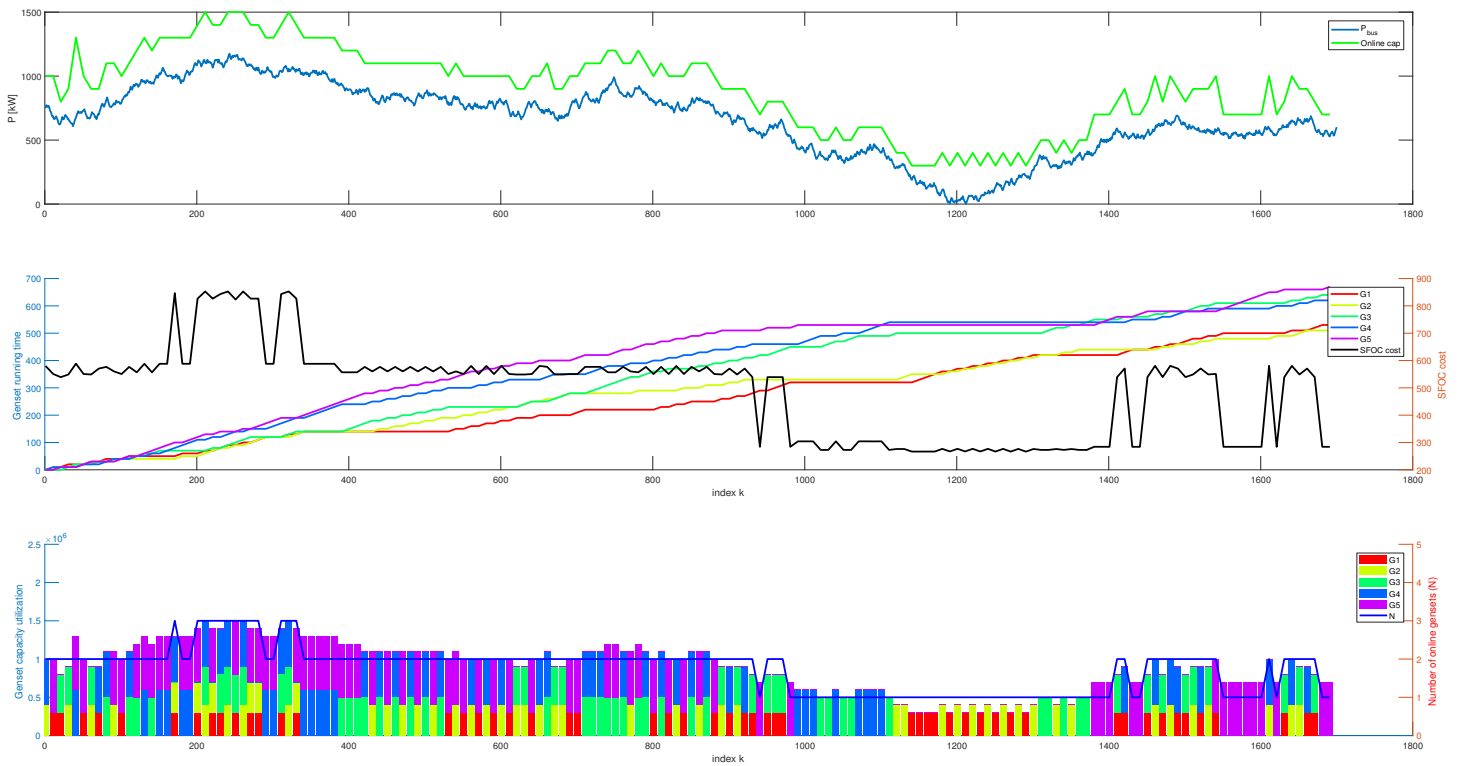**FIGURE 7.** The optimization of SFOC for scheduling with redundancy margin, using $\Gamma = I$.

FIGURE 8. The optimization of SFOC for scheduling with penalties on running time and connection/disconnection of the genset, using $\Gamma = 0.8I, w_1 = 10, w_2 = 100, w_3 = w_4 = 1$.