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Prospective and Efficient Techniques for Model Reduction in Reliability Calculations

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PROBLEM DESCRIPTION

Prospective and Efficient Techniques for Model Reduction in Reliability Calculations

In the project done in the autumn 2012, methods for formal network reduction for a purpose of reliability calculations were investigated. Standard techniques as DC load flow were used to identify affected area after outages. Reliability calculations require a large number of cases investigated and it is urgent to effectively identify the cases that have to be investigated more in-depth as well as to keep the model at a minimum size for each case.

In the MSc-project the focus will be to go deeper into a number of methods, to generalize the descriptions to also cover reactive power/voltage and to test different criteria for choosing affected components. From former work it is experienced that several indices have to be combined to robustly identify the affected area.

The work shall therefore at least cover but not necessarily be limited to:

- A description defining the concepts and the needs
- Description and discussion of techniques and methods used for such studies
- Develop a prototype tool to investigate the methods
- Demonstrate the techniques and discuss strengths and weaknesses of the chosen methods
- Conclude and recommend further steps and need for research

The tool to be used in the studies will be Matlab.

Olav Bjarte Fosso

Supervisor / Subject teacher

ABSTRACT

A reliable electric power supply is essential for modern society. Recently, severe blackouts worldwide have attracted attention to reliability studies in power system planning and operation. The relevance of the traditional N-1 criterion has been discussed, and much focus has been directed towards developing satisfactory probability based reliability tools.

Goodtech Project & Services has developed a methodology for calculation of online power delivery reliability for use in power system operation and planning. The method, based on Markov models, analyzes the entire network for a large number of fault combinations, a useful approach for relatively small networks. However, the computation time increases polynomial with increasing system size. Because the impact of an outage has a limited geographical extent, it is desirable to reduce the system to be simulated, so that it only includes the affected area.

The objective of this project is to develop methods for identifying the components that can be considered influenced by a fault. The focus of the pre-study was to evaluate which post-processing method best suited for developing a reduced network system. In the pre-study, power flow results from the standard DC load flow were used. The main focus of this thesis has been on developing and implementing fast methods for obtaining the necessary power flow data needed in order to use the post-processing methods.

Three approaches have been investigated and tested, namely AC load flow based on the fast decoupled load flow with compensation techniques for obtaining the post-contingency power flows, DC load flow with compensation techniques and the efficient bounding method.

The key principle of the compensation methods is that the effect of outages can be calculated by introducing simple compensation terms, thus avoiding the need to rebuild and factorize the system matrices for every contingency case. The bounding method is based on the principle of sensitivity factors and the fact that given knowledge about changes inside a boundary certain conclusions can be made regarding the changes outside it, thus eliminating the need for studying the entire system.

The method based on the fast decoupled load flow is the only method that gives the possibility to provide an exact solution, and is also the only method that includes reactive power and voltage magnitudes. It is therefore recommended used in cases where a high degree of accuracy is important, or if reactive power and voltages are of interest.

Tests show that the DC load flow is fastest. The accuracy is assumed sufficient for most intended purposes, and should therefore the preferred choice in most cases.

Bounding methods are especially useful in highly meshed grids, and if only the largest changes are of interest.

SAMMENDRAG

En pålitelig strømforsyning er viktig i et moderne samfunn. Mange land har blitt rammet av svært omfattende strømbrudd de siste år, noe som har satt pålitelighetsanalyser for bruk i drift og planlegging av kraftnettet i søkelyset. Nyttigheten av det tradisjonelle N-1 kriteriet har blitt diskutert, og mye fokus har blitt rettet mot utvikling av verktøy for beregning av leveringssikkerhet basert på sannsynlighetsberegninger.

Goodtech Project & Services har utviklet en metode for beregning av leveringssikkerhet under drift og planlegging. Metoden baserer seg på Markov-modeller og simulerer hele kraftnettet for en rekke feilkombinasjoner. Dette er en nyttig tilnærming for relativt små kraftnett der samtidige feil kan påvirke hverandre, men en utfordring med metodikken er at beregningstiden øker polynomisk med størrelsen på kraftnettet. Et utfall har imidlertid normalt sett en begrenset geografisk påvirkning, noe som gjør det mulig å begrense beregningene til et mindre område rundt feilen.

Målet i dette prosjektet er å utvikle metoder for identifisering av de komponenter som kan regnes som påvirket av en gitt feil i kraftnettet. I forstudiet til denne hovedoppgaven var fokuset rettet mot hvilke etterbehandlingsmetoder som var mest hensiktsmessig når en skal finne ut hvor stor utbredelse en feil har i et vilkårlig nett. Arbeidet baserte seg på standard DC lastflyt. Hovedfokuset i denne oppgaven har vært rettet mot å implementere effektive metoder for å oppnå de nødvendige lastflytdataene som er nødvendig for å bruke de nevnte etterbehandlingsmetodene.

Tre metoder har blitt implementert og testet, en AC versjon basert på dekoblet lastflyt og en basert på DC lastflyt, hvor begge nyttiggjør seg av effektive kompensasjonsteknikker for å oppnå lastflyten i feilsituasjon, samt en metode basert på en lineær «bounding» metode.

Hovedprinsippet bak kompensasjonsteknikkene er at effekten av utfall kan bli beregnet svært kjapt ved å introdusere enkle kompensasjonsledd, noe som fjerner behovet for å bygge og faktorisere systemmatrisene for hvert utfallstilfelle.

«Bounding» metoden baserer seg på sensitivitetsfaktorer og det faktum at gitt kunnskap om endringer innenfor en grense så kan en gjøre visse slutninger om endringene utenfor denne grensen, og dermed fjerne behovet for å gjøre beregninger på hele systemet.

Metoden som baserer seg på dekoblet lastflyt, er den eneste som muliggjør nøyaktige resultat, samt den eneste metoden som inkluderer både reaktiv effekt og spenningsnivå. Den er derfor anbefalt brukt i tilfeller hvor detaljnivået er høyt eller reaktiv effekt og spenningsnivå er av interesse.

Metoden basert på DC lastflyt er den mest effektive algoritmen. Detaljnivået er antatt tilstrekkelig i de fleste tilfeller, og er derfor anbefalt brukt i normaltilfeller.

Bounding metoden er spesielt nyttig i sterkt maskede nett, og hvis bare de største endringene er av interesse.

PREFACE

This thesis is the final work in my Master of Science in Electrical Power Engineering at Norwegian University of Science and Technology (NTNU) the spring of 2013.

I wish to thank my supervisor Professor Olav Bjarte Fosso at the Department of Electrical Power Engineering, for his contribution with constructive comments and valuable literature. His guidance has undoubtedly saved me hours of headache and frustration.

I would also like to thank Goodtech Project & Services and my co-supervisors Trond Tollefsen and Arne Brufladt Svendsen, for introducing me to the field of power system reliability, and giving me this opportunity. The fact that I find the topic very interesting has raised my motivation and eased the work

Writing this thesis has been interesting, but wouldn't have been the same without my fellow students. Thank you all, especially Kristin, Ingri, Raghav, Henrik, Runa, Gjert and Sigurd for all the laughs we've shared; you have made the office a great place to be.

Robert Pedersen

Robert Fossmark Pedersen
Trondheim, Norway
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NOMENCLATURE

P_i = Active power delivered to bus i
Q_i = Reactive power delivered to bus i
S_i = Apparent power delivered to bus i
V_i = Voltage at bus i
θ_i, θ_j = Voltage angles at bus i and j
δ_i, δ_j = Sensitivity indices
$G_{ij} + jB_{ij}$ = (i, j)th element of bus admittance matrix $[G] + j[B]$
I_i = Current entering bus i
Y_{ij} = The admittance matrix
f_l = Power flow in line l
PI = Performance index
FDLF = Fast decoupled load flow
$\mathcal{N}$ = The set of all branches within a power system
$\mathcal{V}$ = The set of all branches that are considered “influenced” by a fault
α = Tolerance limit in terms of absolute value of MW change
β = Tolerance limit in terms of percentage change relative to initial power flow
$\mathcal{A}_\alpha$ = A subset of $\mathcal{N}$, where all branches with a change larger than α MW are included (different from base case)
$\mathcal{B}_\beta$ = A subset of $\mathcal{N}$, where all branches with a change larger than β % are included (different from base case)
$\mathcal{X}_{\alpha,\beta}$ = A subset of $\mathcal{N}$, defined as the intersection between $\mathcal{A}_\alpha$ and $\mathcal{B}_\beta$

$$\mathcal{X}_{\alpha,\beta} = \mathcal{A}_\alpha \cap \mathcal{B}_\beta$$

1. INTRODUCTION

The power system is a critical infrastructure to modern society. Its security and reliability have enormous and far-reaching effects on national economy. Due to several recent blackouts worldwide, reliability of supply has gained focus and it is considered increasingly important for electric power system planning and operation [1-4].

The traditional deterministic N-1 security criterion has been put under the spotlight, and alternatives have been investigated [5-7]. The most important tools for transmission system operators (TSOs) are traditionally power flow calculations and dynamic modeling of the power system. Much research has been focused on developing suitable probabilistic reliability tools.

Goodtech Project & Services has developed such a tool for calculation of delivery reliability. The program, PROMAPS[1], can be used both in real time, connected to the SCADA[2] system, or offline, as a planning tool [8]. The program can run simulations for large systems; however, the calculation time will increase along with the size of the system.

Though relatively rare, multiple-event-contingencies are in most cases responsible for the largest disturbances in the power system [9]. Thus, contingency analysis tools must be able to study the effect of multiple contingencies happening in an overlapping manner. However, the number of possible multiple contingencies in a large power system is exceedingly high. The number of potential double contingencies is proportional to the number of branches squared, adding a new dimension for every fault level.

It is generally assumed that effects of an outage in the power system has a limited geographical extent [10, 11]. Simultaneous independent faults in northern and southern Norway will very unlikely have any influence on each other. Thus, it is not necessary to study the effect of such a fault combination, as it will not differ from the effect of each single contingency.

The scope of this project is to develop methods for defining the range for which a fault influences on the system. Special attention is directed towards implementing efficient methods.

Three different approaches has been implemented and tested, and promising results have been achieved for all.

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[1] PROMAPS – Probability and reliability methods applied to power systems

[2] SCADA - *Supervisory control and data acquisition*

The first approach is based on the fast decoupled load flow [12], where the outages are accounted for using fast compensation techniques, thus avoiding the need to rebuild and factorize the B matrices. This approach is the only approach that involves both active and reactive power, along with voltage magnitudes.

The second approach is based on the DC power flow, where outages are accounted for using similar compensation techniques as for the AC version. The DC version does not include reactive power flow or voltage magnitudes, but is significantly faster than the AC version.

The last approach is the efficient bounding method, where normally only a small part of the network needs to be solved to establish the set of influenced branches [13].

In the pre-studies for the thesis, criteria for determining when components should be considered influenced by a fault was discussed and tested. The results are reviewed in this report and are used when conducting tests of the flow algorithms.

Chapter 2 revolves around the importance of reliability analyses, along with power system theory in which the work in this project is based upon.

In chapter 3, the methodology is presented. This includes a description of how the compensation methods and the efficient bounding method are used in this project. In addition, a discussion regarding the definition of an affected component based on the work conducted in the pre-studies is included.

The main findings are presented in chapter 4, followed by a discussion in chapter 5.

The conclusion and suggestions for further work are outlined in chapter 6 and chapter 7 respectively.

2. THEORY

In section 2.1, an overview of power system reliability and security analysis is given. This is not explicitly used in this thesis, but is included with the intention of putting this thesis in perspective.

The subsequent sections of this chapter are used to provide an insight into the theory the work in this thesis is based on.

2.1.Reliability

2.1.1. Definition

A power systems' sole purpose is to supply customers with electrical energy as economically as possible, and with an acceptable degree of security and quality [9, 14].

Interruptions in the supply of electricity can occur at any time, and may last from fractions of a second, to many hours, or even days. Reliability of a component or a system is defined as the probability of adequate operation, for the time intended and under the operating condition intended [15]. The loss of supply can either be caused by disturbances to the system, or the unavailability of adequate resources [16].

In power system operation, reliability is often divided into these two subclasses, adequacy and security.

Adequacy is associated with static conditions, such as the existence of sufficient facilities, and is normally analyzed using power flow analysis. It includes both the ability to generate enough power, and to transmit and distribute the power to satisfy both the consumer's demand along with operational constraints.

Security is associated with the system's response to disturbances, such as loss of generation or transmission units. It includes both transient and steady state response, and can be analyzed through dynamic or static analyses. Security may be defined as the probability or the system's operating point remaining in an acceptable state, given the probabilities of changes in the systems and its environment [14, 17].

2.1.2. Reliability cost and reliability worth

No matter the effort, power systems can never be secure in the absolute sense. Even though it is an unavoidable truth that disturbances will happen [17], efforts can be made to minimize the frequency and duration of such events. By investing in redundant generating and transmission capacity, the consequence of outages may be significantly reduced.

The socio-economic cost of power outages can become very high. It is however a difficult task to determine how much should be invested in extra reliability, and how much that extra reliability is worth.

The relation between cost and reliability can be seen in Figure 1.

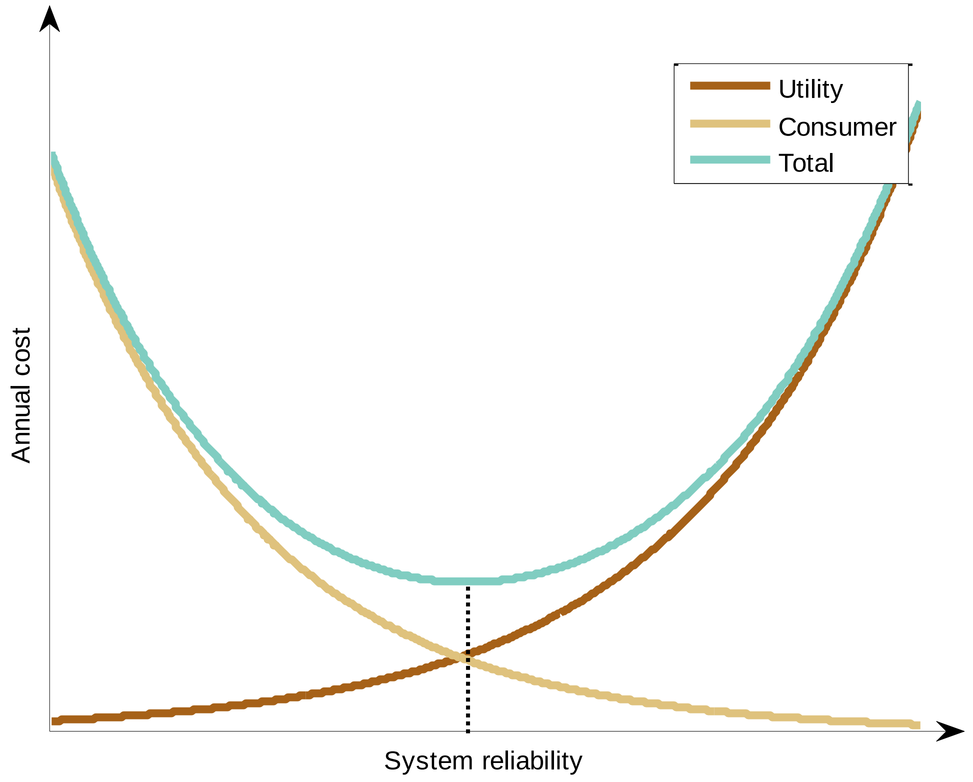

Figure 1: Total reliability cost

The objective when planning and operating the system is to minimize the total cost. Perhaps the greatest difficulty for utilities is to provide an objective function that reflects the true costs and benefits of each alternative.

2.1.3. Reliability criteria

A reliability criterion is an indication of how much a system can be stressed without an unacceptable risk of entering an unacceptable state. A result of the deregulation of the power market is increased utilization of the main transmission grid. Since increased utilization of the system entails increased system stress levels, reliability criteria determine the balance between reliability and allowable system utilization [5]. Some of the most used criteria for reliability classification are described below.

2.1.3.1. Deterministic

Deterministic reliability assessment has the great advantage of not requiring complex probabilistic calculations. The deterministic reliability criteria require that certain classes of failures shall not result in unacceptable operating conditions. Acceptable operating conditions are specified in terms of thermal equipment ratings, voltage limits and avoidance of uncontrolled cascading and instability.

A typical approach is to operate the system in accordance with the N-1 criterion, which implies that the loss of any single primary component must not cause any loss of load, instabilities or cascading phenomena. The N-1 criterion is based on the assumption that the probability of a single contingency is one order of magnitude higher than the joint probability of all other two or more simultaneous (independent) contingencies [8, 18].

Though relatively rare, multiple-event-contingencies are in most cases responsible for the largest disturbances in the power system. Thus, contingency analysis tools must be able to study the effect of multiple contingencies happening in an overlapping manner. However, the number of possible multiple contingencies in a large power system is exceedingly high.

Utilities may add selected multiple element outages of particular concern and exclude some single element outages that can be handled by routine operator action such as switching or generation redispatch, should the outage occur [5]. This criterion ensures that the system should successfully withstand any preselected contingency. In this case, the probability of the contingency is only into account when selecting contingencies, i.e. which ones are likely to occur.

The relevance of the N-1 criterion has been discussed recently, for different reasons. On the one hand, several blackouts worldwide the last decade has led to a discussion about whether the criterion ensures necessary reliability level or not [5, 6]. On the other hand, the increasing focus on efficient utilization of the transmission grid has led to a questions about whether or not the N-1 criterion is too conservative, seen from an economical perspective [7].

2.1.3.2. Probabilistic

System behavior is stochastic in nature, and it is therefore reasonable to base reliability assessment on probabilistic techniques. Measures of system reliability can be derived from the frequency, duration and severity of unacceptable operating conditions. This is the basis for statistics of actual operating reliability produced by many utilities. While deterministic assessments and criteria deal with individual events and the severity of these events, probabilistic reliability assessment procedures are required to incorporate the frequency and duration aspects of the reliability problem in a quantitative manner [5].

The main benefit of the probabilistic operational security criterion is that economic aspects are included in a way that is not possible in the deterministic approach. Corrective and preventive actions may be performed with the objective of minimizing the sum of the congestion costs and the expected interruption costs. A good discussion on the economic aspects are presented in reference [7].

Goodtech Project & Services has developed a computer program for on-line calculation of delivery reliability in the Norwegian main electrical power grid. The method is based on unit Markov component models build together to complete system models by Kroenecker products and reduced by aggregation of similar states. Details on the mathematical method are presented in reference [19].

Current version of the computer program is able to calculate reliability indices for the Norwegian power grid within minutes. However, the computation time increases polynomial with increasing system size, making it hard to implement for large systems, for instance all of Europe.

2.1.4. Security concepts and terminology

Security is the freedom from risk or danger. Power systems, however, can never be secure in this absolute sense. From a control perspective, the objective of power system operation is to keep the electrical flows and bus voltage magnitudes and angles within acceptable limits, despite changes in load or available resources [9, 17].

The aim to operate the system at lowest cost, with the guaranteed avoidance or survival of emergency conditions, means operating the system as close as possible to its security limits. A power system is in an emergency condition of varying severity when operating limits are violated. Reference [20] classifies power system security levels as in Figure 2. The arrowed lines represent involuntary transitions between levels 1 to 5 due to contingencies.

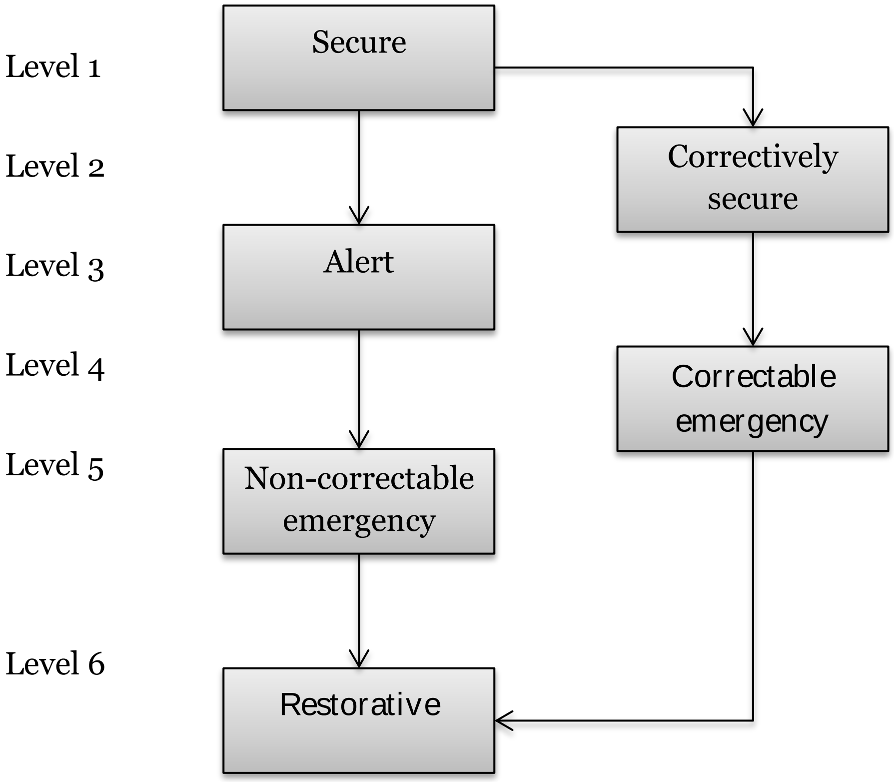

Figure 2: Power system static security levels [20]

Level 1:	All load supplied. No operating limits violated. In the event of a contingency, there will be no violations.
Level 2:	All load supplied. No operating limits violated. Any violations caused by a contingency *can* be corrected by appropriate control action without loss of load
Level 3:	All load supplied. No operating limits violated. Some violations caused by a contingency *cannot* be corrected without loss of load.
Level 4:	All load supplied, but operating limits are violated. These *can* be corrected by appropriate control action without loss of load.
Level 5:	All load supplied, but operating limits are violated. These *cannot* be corrected without loss of load.
Level 6:	No operating limits violated, but loss of load has been suffered.

Level 1 or level 2 are the systems normal operating states. Level 3 is acceptable if the likelihood of the contingencies is small. The selection of operating state depends on the utility's operation policy, and which of the reliability criteria they use. Using a deterministic criterion, only state 1 and state 2 are acceptable.

The upward transitions in the model are more complex. Departure from level 4 requires corrective rescheduling. Normally, the rescheduling is not an optimization with respect to economy, but more directed on the efficiency for removing the problem. Therefore, the security level after corrective rescheduling is not known and can be in any of the levels above in the model. After the rescheduling a fine tuning of the system ought to take place. This is an optimization procedure, which includes contingency evaluation. The correct security level is then found, while minimizing the cost and maintaining the operating criterion [9].

2.1.5. Security assessment

Power system security assessment involves practices designed to keep the system operating when components fail. An overall objective is to make the power system able to remain in an operating state after any credible contingency. There must be enough rotating reserves and reserve generation to make up for loss of generation units, and enough transmission capacity to make up for the power flow displacement resulting from loss of transmission units.

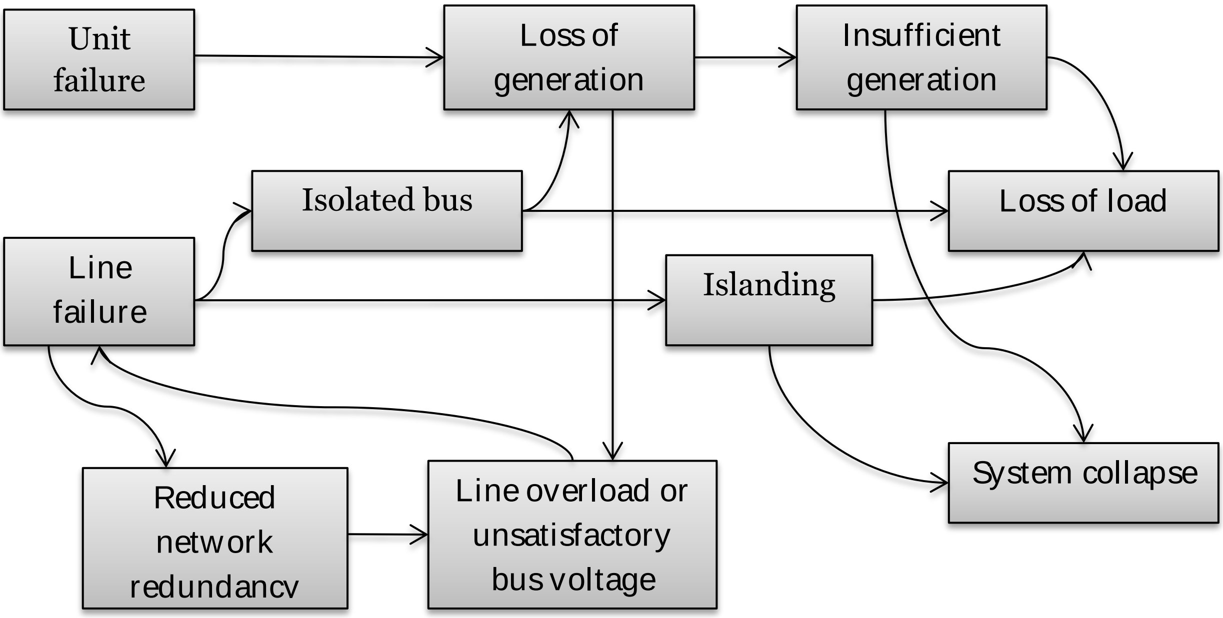

Figure 3: Possible consequences of component breakdowns

Security assessment has two functions. The first is violation detection in the actual system operating state. In its simplest form, this just entails monitoring actual flows, voltages, etc., and comparing them against prespecified limits. The second, much more demanding, function of security assessment is contingency analysis [20].

2.2. Contingency analysis

A contingency analysis is carried out with the purpose of identifying all contingencies causing violation in steady state. It gives the operators an indication of what might happen to the power system in the event of an unplanned or unscheduled equipment outage.

The idea is that if forewarned, the operator can take some action, before or after the event, to avoid system problems such as cascading effects and power losses. Those contingencies that, if they occurred, would create steady-state emergencies must be identified and ranked in order of severity. The power system operator and/or an automated security-constrained scheduling function can then respond to each insecure contingency case, usually in decreasing order of severity, by [20]:

a) Altering the pre-contingency system operating state to mitigate or eliminate the emergency resulting from the contingency, or
b) developing a control strategy that will alleviate the emergency, should it occur, or
c) deciding to do nothing, on the basis that the post-contingency emergency is small and/or very unlikely.

2.2.1. Definition of contingencies

Contingency analysis is performed on a list of credible contingencies. Each contingency to be modeled must be specified separately.

Typical contingencies in the power system consist of outages of transmission components, such as transmission lines, transformers, substation buses or generation units.

Contingencies can be classified as internal of external. Internal causes may be insulation breakdown or relay failures, whereas external contingencies are caused by environmental effects, such as lightning, weather conditions or objects coming in contact with the equipment. Common for all is that they are considered to be unscheduled, random events that the operators do not expect, but have to be prepared for.

Even though most power systems are designed in accordance with the N-1 criterion, operators must still be alert and play an active role if need be. There is a great difference between the ideal, planned system and the system in operation. For instance, load patterns can shift in unforeseen ways, generator outages can necessitate transmitting power over long distances, or construction can be delayed [17].

2.2.1.1. Multiple contingencies

Multiple contingencies are defined as the overlapping occurrence of several independent contingent events. Though relatively rare, multiple-event-contingencies are in most cases responsible for the largest disturbances in the power system. Thus, contingency analysis tools must be able to study the effect of multiple contingencies happening in an overlapping manner.

However, the number of possible multiple contingencies in a large power system is exceedingly high. The number of potential double contingencies is proportional to the number of branches squared, adding a new dimension for every fault level.

2.2.2. Contingency selection

It is important that the computation time is short, so that the conditions have not changed too much when the analysis is finished. Load flows are the most time consuming process in a contingency analysis. Therefore, contingency selection procedures offer the greatest computational savings [21]. The goal of the contingency selection is to identify the contingencies that can potentially cause system violations. First, it must be determined what voltage levels should be included. Second, the geographic extent of the model must be determined, normally a difficult task. A common practice has been to model the system to the extent real-time measurements data is available to support [17]. However, this area can be large, and the simulations can be time consuming, at least if a full AC-load flow is desired.

The majority of outages does not cause overloads or voltage violations and may therefore be omitted from the studies. It is however not an easy task to determine which outages that are not necessary to include. Many operators choose a list of contingencies that they want to study, based on intuition and experience [17]. A possible pitfall is that a contingency that they consider safe may in fact present a problem for the system.

2.2.3. Contingency ranking

Contingency selection techniques are useful in calculations of system reliability. A common approach is to divide the selection problem into two parts. First, a performance index (PI) that measures the system stress is defined. Second, a method for predicting the change in PI when an outage occurs is developed [22].

$$PI = \sum_{l=1}^{L} W_l \left(\frac{P_l}{P_l^{Lim}} \right)^{2n} \tag{2.1}$$

P_l = The megawatt flow of line l
P_l^{Lim} = The megawatt capacity of line l
n = An integer
L = Number of lines in the system
W_l = A real, constant weighting coefficient

The performance index has a small value when all line flows are within their limits and a high value when there are line overloads. The objective of the contingency screening method is to identify the critical outages, thus the PI itself is not significant. The effect of an outage can be found by evaluating the change in PI, i.e. the change in system stress for the particular outage. In most cases, PI provides a good measure for ranking contingencies in terms of severity. However, in some cases where one branch is overloaded while several other branch flows decrease, the PI may decrease in value and fail to recognize the overload [11, 13, 15, 23].

2.3.Power flow analysis

This section deals with the steady-state analysis of interconnected power systems during normal operation. Power flow studies, commonly referred to as load flow studies, are the basis of power system analysis and design [2]. They are necessary for planning, operation and economic scheduling and control of existing systems, as well as planning for the future. The objective is to determine the magnitude and angle of voltages at each bus and active and reactive power flow in each line.

The solution to the load flow begins with identifying the known and unknown variables in the system. The quantities that must be determined are voltage magnitude $|V|$, phase angle θ, real power P and reactive power Q.

The system buses are generally classified into three types. At a load bus, the active and reactive powers are specified. At a generator bus, also called voltage-regulated bus, the active power and voltage magnitude are specified. The buses are also referred to as P-Q and P-V buses respectively, from the known variables. The voltage and phase angle are specified for one arbitrarily generator bus, referred to as the slack bus.

The current entering bus i in a general system can be written as

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j \tag{2.2}$$

The complex power at bus i is

$$P_i + jQ_i = V_i I_i^* \tag{2.3}$$

Separating the real and imaginary parts, and using rectangular form,

$$P_i = |V_i| \sum_{j=1}^{n} |V_j| \left(G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)\right) \tag{2.4}$$

$$Q_i = |V_i| \sum_{j=1}^{n} |V_j| \left(G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)\right) \tag{2.5}$$

These equations must be solved using special techniques. The Newton-Rhapson method is not used in this project, but provides the basis for the other techniques, and is therefore included in the following section.

2.3.1. Newton-Rhapson

The N-R method is an algorithm for solving simultaneous nonlinear equations with equal number of equations and unknowns. Expanding equations (2.4) and (2.5) into a Taylor's series, and neglecting higher order terms, a linear set of equations is achieved.

Let $\Delta P_i + j\Delta Q_i$ be the mismatches between the scheduled power delivery $P_i^{sp} + jQ_i^{sp}$ and the calculated values, then

$$\Delta P_i = P_i^{sp} - |V_i| \sum_{j=1}^{n} |V_j| \left(G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j) \right) \quad \textbf{(2.6)}$$

$$\Delta Q_K = Q_i^{sp} - |V_i| \sum_{j=1}^{n} |V_j| \left(G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j) \right) \quad \textbf{(2.7)}$$

Expressed in terms of a Taylor's series, this becomes

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = J \cdot \begin{bmatrix} \Delta\theta \\ \Delta|V| \end{bmatrix} \quad \textbf{(2.8)}$$

where J is the Jacobian matrix.

$$J = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} = \begin{bmatrix} \dfrac{\partial \Delta P}{\partial \theta} & \dfrac{\partial \Delta P}{\partial |V|} \\ \dfrac{\partial \Delta Q}{\partial \theta} & \dfrac{\partial \Delta Q}{\partial |V|} \end{bmatrix} \quad \textbf{(2.9)}$$

Solving for the voltage angle and magnitude,

$$\begin{bmatrix} \Delta\theta \\ \Delta|V| \end{bmatrix} = -J^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}, \quad \textbf{(2.10)}$$

The convergence rate is typically fast, but the Jacobian matrix must be recalculated for each iteration, a time consuming operation for large systems.

2.3.2. Fast decoupled power flow solution

Numerical methods are generally most efficient when they take advantage of the physical properties of the system being solved [12]. FDLF is a fast solution method that exploits the loose physical connection between MW and MVAR flows in the transmission system. Due to the high X/R ratio, the active power transfer P, is mainly dependent on the phase angles θ, whereas the reactive power transfer mainly depends on the voltage magnitude $|V|$. Therefore, a reasonable simplification is to neglect the $\frac{\partial \Delta P}{\partial |V|}$ and $\frac{\partial \Delta Q}{\partial \theta}$ terms from the Jacobian matrix, giving two separate equations.

$$[\Delta P] = \left[\frac{\partial \Delta P}{\partial \theta} \right] [\Delta\theta] \quad \textbf{(2.11)}$$

$$[\Delta Q] = \left[\frac{\partial \Delta Q}{\partial |V|} \right] [\Delta|V|] \quad \textbf{(2.12)}$$

Further simplification can be justified for practical power systems [12].

$$\cos(\theta_K - \theta_n) \approx 1; \;\; G_{Kn}\sin(\theta_K - \theta_n) \ll B_{Km}; \;\; Q_K \ll B_{KK}V_K^2 \tag{2.13}$$

Thus, good approximations to (2.11) and (2.12) are:

$$[\Delta P] = [V \cdot B' \cdot V][\Delta\theta] \tag{2.14}$$

$$[\Delta Q] = [V \cdot B''][\Delta|V|] \tag{2.15}$$

The matrices B' and B'' are constant matrices that consist solely of network admittances, thus they only need to be calculated and inverted once. Taking the left-hand V terms on to the left-hand side of the equation, and setting right-hand $V = 1\,pu,$ the final decoupled load-flow equations become

$$[\Delta P/V] = [B'][\Delta\theta] \tag{2.16}$$

$$[\Delta Q/V] = [B''][\Delta|V|] \tag{2.17}$$

The method converges slower than Newton's method, but requires considerably less time per iteration, and a solution is obtained very rapidly. This technique is very useful in contingency analysis where numerous outages are simulated [24].

2.3.3. DC load flow

The DC load flow is a simplification of the AC power flow, and looks only at active power flows, neglecting voltage support, reactive power and transmission losses. Its solutions are non-iterative, reliable and unique [2], which gives it considerable analytical and computational appeal. The AC load flow is simplified to a linear circuit problem, making steady state analysis of the power system very efficient.

The DC load flow is based on the same assumptions as FDLF, further simplified by the assumptions that all voltages are equal to 1 pu.

$$[P] = [B'][\Delta\theta] \tag{2.18}$$

This gives

$$P_{km} = \frac{\theta_{\mathrm{k}} - \theta_m}{x_{km}} \tag{2.19}$$

The DC load flow has been widely used in power system planning and operating problems, including contingency analyses.

2.3.4. Optimal power flow

Optimal power flows are variations of other power flows, in which certain controllable variables are adjusted in order to minimize an objective function, typically the cost of production and transmission. Instead of having a fixed level of power injections, a set of constraints can be given and an objective function to compare and select the optimal solution.

Optimal power flow solutions have not been used in this report.

2.4. Compensation techniques

2.4.1. Purpose

In contingency analyses, a large number of contingency cases are studied. Compensation techniques can be used in order to avoid full load flow calculations for all contingency cases.

Compensation permits a network solution to be updated to reflect the effect of network branch and/or bus changes, using the triangular factors of the original network matrix and thus avoiding time-consuming re-factorization.

The compensation techniques are very effective if the modifications of the system are not permanent, and if only few components are affected. The method utilizes already available factors of the base case network matrix. The new situation is reflected by using the “Inverse Matrix Modification Lemma”, IMML.

2.4.2. Principle

The techniques can be used for both active and reactive power flow calculations. In this thesis, both active and reactive power are considered, thus the problem description is as follows.

$$(B' + \Delta B') \cdot \theta = P \tag{2.20}$$

$$(B'' + \Delta B'') \cdot |V| = Q \tag{2.21}$$

For all incidence-symmetric modifications of the base case matrix, the equations can be written as:

$$(B' + M \cdot \delta y \cdot M^t) \cdot \theta = P \tag{2.22}$$

$$(B'' + M \cdot \delta y \cdot M^t) \cdot |V| = Q \tag{2.23}$$

Only the active power formulas are described below. The reactive compensation follows the same pattern. In this project, post-compensation has been utilized, thus this approach is described in detail below. Pre-compensation and mid-compensation are described in reference [15].

$$\theta = \left(I - B'^{-1} \cdot M \cdot c \cdot M^t\right) \cdot B'^{-1} \cdot P \tag{2.24}$$

M is a $n \ x \ m$ matrix containing +1 and -1 in the places of the outaged line.

n is the number of buses in the system

m is the number of lines that are modified for each case.

c is defined as follows:

$$c = (\delta h^{-1} + M^t \cdot H^{-1} \cdot M)^{-1} \tag{2.25}$$

For a single branch outage, c can be written as follows

$$c = \left(\delta h^{-1} + Z_{ii} + Z_{jj} - Z_{ij} - Z_{ji}\right)^{-1} \tag{2.26}$$

Where $Z = H^{-1}$

The calculation can conveniently be divided into the following steps:

1. Perform the network solution

$$\theta^0 = B'^{-1} \cdot P \tag{2.27}$$

2. Calculate the compensation vector

$$\Delta\theta = -B'^{-1} \cdot M \cdot c \cdot M^t \cdot \theta^0 \tag{2.28}$$

3. Find the new angles

$$\theta = \theta^0 + \Delta\theta \tag{2.29}$$

Step 1 includes a forward substitution on the base case matrix. If the vector P is constant, this step can be omitted since θ^0 is known in advance.

Step 2 is performed from the right hand side to the left.

For a simple DC load flow, the description above is sufficient. For an AC load flow however, some additional comments must be made.

As the AC load flow is a non-linear iterative scheme, the compensation must follow iterative steps as shown in Figure 5. The mismatch is calculated according to equation 2.30.

$$\Delta S = \frac{\vec{V}^0 \cdot \left(Y \cdot \vec{V}^0\right)^* - S_{inj}}{|V^0|} \tag{2.30}$$

The figure shows the procedure for running a contingency analysis using AC compensation techniques.

2.5.Distribution factors

Distribution factors, or linear sensitivity factors, show the approximate change in line flow for changes in generation or loss of lines. There are mainly two types of distribution factors, generation shift factors and line outage distribution factors.

The line outage distribution factor is defined as [25]:

$$d_{l,k} = \frac{\Delta f_l}{f_k^0} \tag{2.31}$$

$d_{l,k}$ = Distribution factor when monitoring line l after an outage of line k
Δf_l = Change in MW flow on line l
f_k^0 = Original flow on line k before it was opened

If the original power on line l and k is known, the post-fault flow on line l with line k out can be approximated as

$$\hat{f}_l = f_l^0 + d_{l,k} \cdot f_k^0 \tag{2.32}$$

$\hat{f}_l$ = Flow on line l with line k out
f_l^0, f_k^0 = Pre-outage flows on lines l and k respectively

The derivation of the distribution factor $d_{l,k}$ will be provided in appendix A.1. By pre-calculating the distribution factors, calculating the new line flows in the network after a fault is fast and simple, and the procedure can be repeated for all lines. The sensitivity factors can be considered correct as long as the network topology is not altered due to e.g. switching.

2.6.Bounding methods

The bounding methods proposed by Brandwajn [13] uses an adjustable region around the outage to solve for the outage case overloads. The method was originally applied to the linear power flow, but has subsequently been extended for AC network analysis.

By dividing the network into subsystems, the computation of all variables van be avoided. The three subsystems, shown in Figure 4, are defined as follows:

N1 = the subsystem immediately surrounding the outaged line

N2 = the external subsystem that shall not be solved in detail

N3 = the set of boundary buses that separate N1 and N2

2.6.1. Efficient bounding method

An outage can be simulated by a pair of appropriately scaled injections ΔP_k and ΔP_m. It can be shown that the change in angular spread in the system N2 cannot exceed the maximum change in the angular spread between the boundary nodes in N3 [13]. The proof of the angular spread criterion and derivation of ΔP_k and ΔP_m is shown in appendix B.

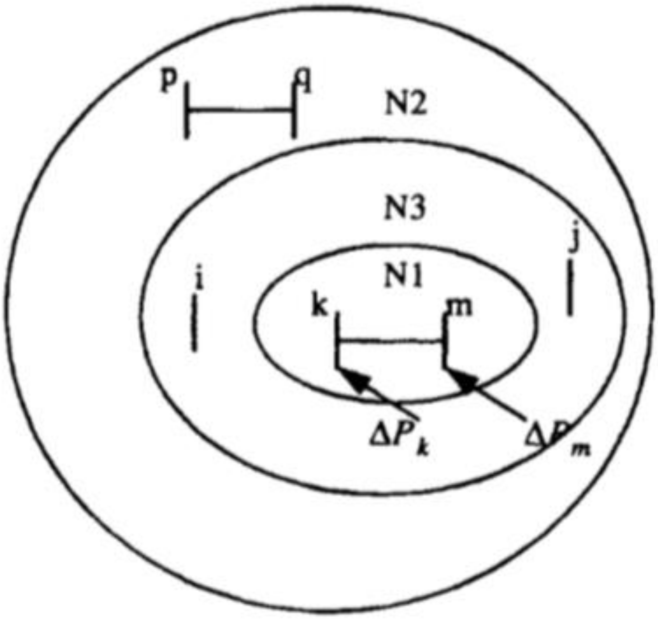

Figure 4: Layers used in bounding analysis (Wood, 1996, p.433) [11]

For a line p-q with initial flow f_{pq}^0, there is a maximum amount that the flow on p-q is allowed to change. This can be determined by thermal limitations or by some other criteria, discussed in section 3.2. Suppose that, for line p-q, the flow is limited by the upper and lower limits f_{pq}^+ and f_{pq}^-. Then, the maximum allowable change can be given as

$$\Delta f_{pq}^{max} = \min\{(f_{pq}^+ - f_{pq}^0), (f_{pq}^0 - f_{pq}^-)\} \tag{2.33}$$

From equation 2.16, equation 2.34 can be achieved

$$\Delta f_{pq} = \frac{1}{x_{pq}}(\Delta\theta_p - \Delta\theta_q) \tag{2.34}$$

Thus, the maximum change in phase angle can be expressed as

$$(\Delta\theta_p - \Delta\theta_q)^{max} = \Delta f_{pq}^{max} \cdot x_{pq} \tag{2.35}$$

Reference [13] develops the theorem that

$$\left|\Delta\theta_p - \Delta\theta_q\right| < \left|\Delta\theta_i - \Delta\theta_j\right| \quad \textbf{(2.36)}$$

Where i and j are any pair of nodes in N3, $\Delta\theta_i$ is the largest $\Delta\theta$in N3, and $\Delta\theta_j$ is the smallest $\Delta\theta$ in N3.

The right hand side, $\left|\Delta\theta_i - \Delta\theta_j\right|$ provides an upper limit to the maximum change in angular spread across any circuit in N2. By combining equations 2.35 and 2.36, we obtain

$$\Delta f_{pq}^{max} \cdot x_{pq} < \left|\Delta\theta_i - \Delta\theta_j\right| \quad \textbf{(2.37)}$$

A binary search can determine the set of endangered branches in N2. All circuits in N2 are safe from overload if the value of $\left|\Delta\theta_i - \Delta\theta_j\right|$ is less than the smallest value of $\Delta f_{pq}^{max} \cdot x_{pq}$, over all pairs p-q, where p-q corresponds to the buses at the ends of circuits in N2. If this condition fails, N1 must be expanded and new values can be calculated. If only a few branches are violates the criteria, the load flow can be explicitly calculated for these branches. Note that the only information achieved when the criterion is reached, is that there are no violations of flow limits in N2. It may, or may not be violations within N1.

Using the sparse adjacency matrix, A, and adding 1's on the diagonal, the subset $nodes^{n+1}$ can be extended one tier from $nodes^{n}$ using:

$$nodes^{n+1} = (A + I) \cdot nodes^{n} \quad \textbf{(2.38)}$$

or

$$nodes^{n+1} = (A + I)^{n+1} \cdot nodes^{0} \quad \textbf{(2.39)}$$

$nodes^{n+1}$ will be a sparse matrix where all non-zero elements represent nodes that are connected to $nodes^{n}$. The first one has proven to be most computational efficient when expanding the boundary several times. When extending the boundary this way, the nodes that have already been included in a previous boundary, must be explicitly removed from the new boundary, in order to avoid calculating the sensitivity factors of these nodes as well.

$$boundary^{n+1} = (nodes^{n+1} - nodes^{n}) \quad \textbf{(2.40)}$$

The bounding methods have traditionally been used in order to determine all branches that are potentially endangered following an outage. Thus, the maximum change is given as

$$\Delta f_{pq}^{max} = f_{pq,limit} - f_{pq} \quad \textbf{(2.41)}$$

Where $f_{pq,limit}$ is the maximum thermal capacity of line pq.

In this project, the purpose of the analyses is not to identify endangered branches, but rather identify those that are affected by a fault.

As discussed in section 3.2, different criteria can be chosen for Δf_{pq}^{max}.

The change in boundary phase angles $\Delta\theta_i$ and $\Delta\theta_j$ can be found using equation A.10.

$$\delta_{i,nm} = \frac{\Delta\theta_i}{P_{nm}}$$

Where $\delta_{i,nm}$ is a sensitivity factor, defined and derived in appendix A.

The complete non-sparse column k of the inverse, can be found by performing a forward and backward substitution to solve:

$$B' \cdot C_n = N_n \tag{2.42}$$

Where N_n is a null vector except for unity in position n and C_n is the desired inverse matrix column. By exploiting the sparsity of N_k the normal work can be halved [26].

As can be seen from the above equations only two columns, C_n and C_m, of the inverse matrix must be calculated in order to obtain the sensitivity factors, for a single branch outage.

2.6.2. Complete bounding method

The complete bounding method has not been implemented in this project, but a brief description is provided for the sake of completeness. The complete bounding method can detect both active power flow violations and bus voltage limit violations. The method is based upon the efficient bounding method described above, and the fast decoupled load flow. Consequently, the method includes reactive power considerations.

Having obtained the first active power solution using the efficient bounding method, the Q mismatches results only from the change in angular spread across the lines, keeping the voltage yet unchanged. The branch contribution to the bus Q-mismatch can be approximated with the first derivative of the reactive flow through the branch.

$$\frac{dQ_i}{d\theta_{ij}} = G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij} \tag{2.43}$$

A measure K of the bus Q-mismatch sensitivity to the incremental angular spread is determined by summing the absolute values of the sensitivities of branches connected to:

$$K_i = \sum_{\epsilon} \left| \frac{dQ_i}{d\theta_{ij}} \right| \tag{2.44}$$

Where ϵ is the set of branches connected to bus i.

By selecting buses with a sensitivity K larger than L, a subset of network buses that can possibly have a Q-mismatch larger than the tolerance T, is established.

$$L = \frac{T}{\Delta\theta^{max}} \tag{2.45}$$

Where T is the pre-defined bus Q-mismatch tolerance.

Using the sensitivity K, a very conservative estimate of the Q-mismatch is established, and a second independent criterion is needed.

One such criterion is using the branch incremental reactive losses

$$\Delta Q_{ij}^{loss} = \Delta I_{ij}^2 X_{ij} \approx 2P_{ij}^o \Delta P_{ij} X_{ij} \approx 2P_{ij}^o \Delta\theta_{ij} \tag{2.46}$$

For buses that violate both criteria, the voltage is computed explicitly.

3. METHOD

Contingency analyses give the possibility to study changes in power flow, voltages and angles for different outage scenarios. The contingency analyses return invaluable information, but common practices return a description of the system as a whole and are not intended used to set boundary conditions for other, more time consuming operations, with the 0exception of bounding methods.

The purpose of this study is not to identify endangered components, but rather identify the branches that are influenced by an outage. Special attention has been devoted to implementing fast and efficient algorithms.

Necessary power flow results have been achieved using fast compensation techniques for both AC and DC load flows. Only the DC version of the bounding method has been implemented in this project.

3.1.Simulation tool

The power flow simulations conducted in this project are based on Matpower, a free MATLAB® based tool for simulating power flow and optimal power flow problems, created by Ray D. Zimmerman, Carlos E. Murillo-Sànches and Deqiang Gan of PSERC at Cornell University. It is possible to run several types of load flow simulations, for instance Newton-Rhapson, fast decoupled load flow, DC power flow, DC optimal power flow and AC optimal power flow.

The calculations in this project are based on modified version of the BX-version of the fast decoupled load flow, and the DC load flow.

In Matpower, factorization of the B' and B'' matrices are carried out using LU factorization with partial pivoting, satisfying

$$L \cdot U = P \cdot B \tag{3.1}$$

Where P is a row permutation used to achieve numerical stability.

LU factorization with full pivoting can be used to reduce the number of fill-ins. By including an additional column permutation matrix, Q, significant speed-up is achieved.

$$L \cdot U = P \cdot B \cdot Q \tag{3.2}$$

For 1000 outages on the 2736 bus Polish system, the performance of the fast decoupled load flow is improved by 92% percent, nothing else changed.

In the DC version, the B matrix is only used once per power flow calculation. Therefore, pre-factorization of the B matrix was not explicitly carried out in Matpower[3]. By introducing an explicit LU factorization with complete pivoting, computational savings of 6% was achieved.

Different approaches for obtaining the permutation matrices are discussed in reference [27]. The MATLAB documentation provides no information regarding the algorithm used to achieve these.

3.2.Definition of an effected component

In order to determine if a component is influenced by a fault, appropriate criteria must be established. This topic was thoroughly discussed in a previous report [28], while the main reflections are presented in this section.

An intuitive approach is that any line or bus that has changes in either flow or voltage more than a given tolerance limit should be considered affected by the outage. A change can be measured in physical quantities like MW or Volts, or as a relative change compared to some reference value. Tolerance limits can be defined in a similar manner, as physical quantities or as a relative change. Some of the most intuitive approaches is discussed first, followed by suggestions for more accurate selection methods.

3.2.1. Absolute change in active power

Change in power flow is given in terms of the absolute value of the difference between the initial and the new power flow. The set of branches that satisfy the condition that $|\Delta P_{i,MW}| \geq \alpha$, is considered affected by an outage.

That is, all lines l, in the network $\mathcal{N}$, that due to a fault have a change in flow larger than α, will be included in the subset $\mathcal{A}_\alpha$.

$$\mathcal{A}_\alpha = \{l \in \mathcal{N} | \Delta P_{i,MW} \geq \alpha\} \tag{3.3}$$

$\Delta P_{i,MW}$ = Change in power flow in line i, given in MW.
α = Tolerance limit, given in MW

Depending on the objective of the study, this method can be a sufficient criterion for defining which branches are affected. A drawback is that the tolerance for flow change will be independent of initial loading and branch ratings, thus setting a tolerance limit that is neither too high for the low capacity network, nor too low for the high capacity network can be difficult.

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[3] MATLAB performs a general triangular factorization using LU factorization with partial pivoting by default.

3.2.2. Percentage change in active power

Change in power flow is given in terms of a percentage of the original flow in the same line. The tolerance limit, β, will be set to some percentage of the initial power flows, and all lines with a larger change will be included in the subset $\mathcal{B}$.

$$\mathcal{B}_\beta = \left\{ l \in \mathcal{N} \,\middle|\, \left(\left| \frac{\Delta P_i}{P_{i,0}} \right| \cdot 100\% \right) \geq \beta \right\} \tag{3.4}$$

$P_{i,0}$ = Initial power flow in line i, given in MW
β = Tolerance limit, given as a percentage

This approach includes the lines that are most influenced, relative to their own initial state. A disadvantage is that a branch with very low initial power flow may be included even though the change in MWs is small, and a highly loaded line may not be included even though the change in MW is quite high.

3.2.3. Change relative to the faulted line flow

Change in power flow is given as a percentage, relative to the initial flow in the faulted line. The tolerance limit will be a dynamic number, dependent on the loading of the faulted line.

$$\left| \frac{\Delta P_i}{P_{k,0}} \right| \cdot 100\% \geq \varepsilon_{P,k} \tag{3.5}$$

$P_{k,0}$ = Initial power flow in faulted line, k
$\varepsilon_{P,k}$ = Tolerance limit, given as a percentage of initial flow in the faulted line

As this approach returns the flow change as a percentage of the original, the result will illustrate how the original flow in the branch is redistributed after the fault. This approach has not been found applicable if a number of contingencies are considered simultaneously.

3.2.4. Change relative to branch limits

Change in power flow is given as a percentage, relative to the branch limits of each individual line.

$$\left| \frac{\Delta P_i}{P_i^{Lim}} \right| \cdot 100\% \geq \varepsilon_{P,lim} \tag{3.6}$$

P_i^{Lim} = The MVA capacity of line i
$\varepsilon_{P,lim}$ = Tolerance limit, given as a percentage of the branch capacity

The selection criterion is based on the change in stress for every line, thus taking into account the fact that different branches have different limits.

3.2.5. Reactive power flow

The reactive power flow is included in the AC power flow, and all the above mentioned criteria for active power flow can be used in a similar manner for the reactive power flow. The change in reactive power flow can give indications regarding voltage conditions in the system.

3.2.6. Voltage drops

It is important that the voltage in the power system is kept within its nominal range. Large voltage drops are often caused by high reactive power transfer. It is, however, not sufficient to monitor the reactive power flow to determine which buses that will have low voltage situations. A bus may be supplied with the same active and reactive power flow, but still have a much lower voltage after a fault, than before. The reason is that the voltage on a bus closer to the fault may decrease, but still be within acceptable limits, while a few lines away from the fault, where the initial voltage was lower, the same voltage drop can cause the voltage limits to be violated. Possible ways to determine which buses are affected by an outage are to include all buses where the voltage is outside the acceptable range, all buses where the voltage change is larger than some tolerance limit, or a combination of the two.

3.2.7. Combinations

Combining the benefits of the different criteria above can help in creating a robust and accurate selection method.

One approach is to include all lines where the change in power flow is more than both a given percentage and an absolute value, thus eliminating some of the drawbacks from the stand-alone methods. For a line with low initial loading, the absolute value will define the lower limit, whereas the percentage value will be limiting for lines with high loading. Mathematically, this condition can be expressed as

$$\mathcal{X}_{\alpha,\beta} = \mathcal{A}_{\alpha} \cap \mathcal{B}_{\beta} \tag{3.7}$$

$\mathcal{X}_{\alpha/\beta}$ is the set of branches in the system that has a load change larger than both α MW and β %. For a heavy loaded line, it will normally only be included if the change is larger than β %, even though the change is larger than α, in absolute value.

The set of affected branches is denoted $\mathcal{V}$, and can be given in terms of any of the above criteria, or by some other factors. $\mathcal{V} = \mathcal{X}_{5MW/10\%}$, defines the set of affected branches as all branches with a flow change larger than 5 MW and larger more than 10% relative to the initial flow. This notation is used later in this report.

3.3. Load flow results

The methods for defining the area of interest are based on load flow studies of the power system. The underlying principle is to first run a power flow calculation for the base case, with no faulted lines or buses. Thereafter, new load flows are run for a number of contingency cases, and the pre- and post-contingency flows and voltages are compared. Three methods have been implemented, one based on the FDLF, one based on the DC load flow, and the last based on the efficient bounding method.

3.4.AC load flow with compensation

This is the most thorough and detailed method, and the only method where an exact solution can be obtained. Figure 5 shows the procedure of running a contingency analysis using the method based on the fast decoupled load flow. The method provides the possibility of obtaining a fully converged power flow solution, and also the ability of studying voltages and reactive power.

Initially, the B' and B'' matrices are built and factorized for the base case, and unless multiple simultaneous outages are studied, these will not be altered throughout the study. The iteration process shown in Figure 5 will continue until the maximum power mismatch is less than a chosen limit, after which the next contingency in the list is chosen.

When all power flow results are achieved for all contingency cases, the results are analyzed. For each contingency case, the new power flows are compared towards the selected criterion, and the influenced branches are identified and stored.

3.4.1. Multiple simultaneous contingencies

Suppose the operator wants to study a set of double contingencies where 100 contingencies involve one particular branch in combination with others. In that case it will be faster to rebuild and factorize the B matrices with that single branch outaged, and consider the other contingencies as single branch faults. This procedure is used in all cases where one single branch is included in a large set of double contingencies. If the operator wants to study only a few double contingencies including each branch, it might be faster to use the compensation terms for both contingencies. This has not been implemented in this project, but follows the same pattern as described in section 2.4.2, for single branch outages.

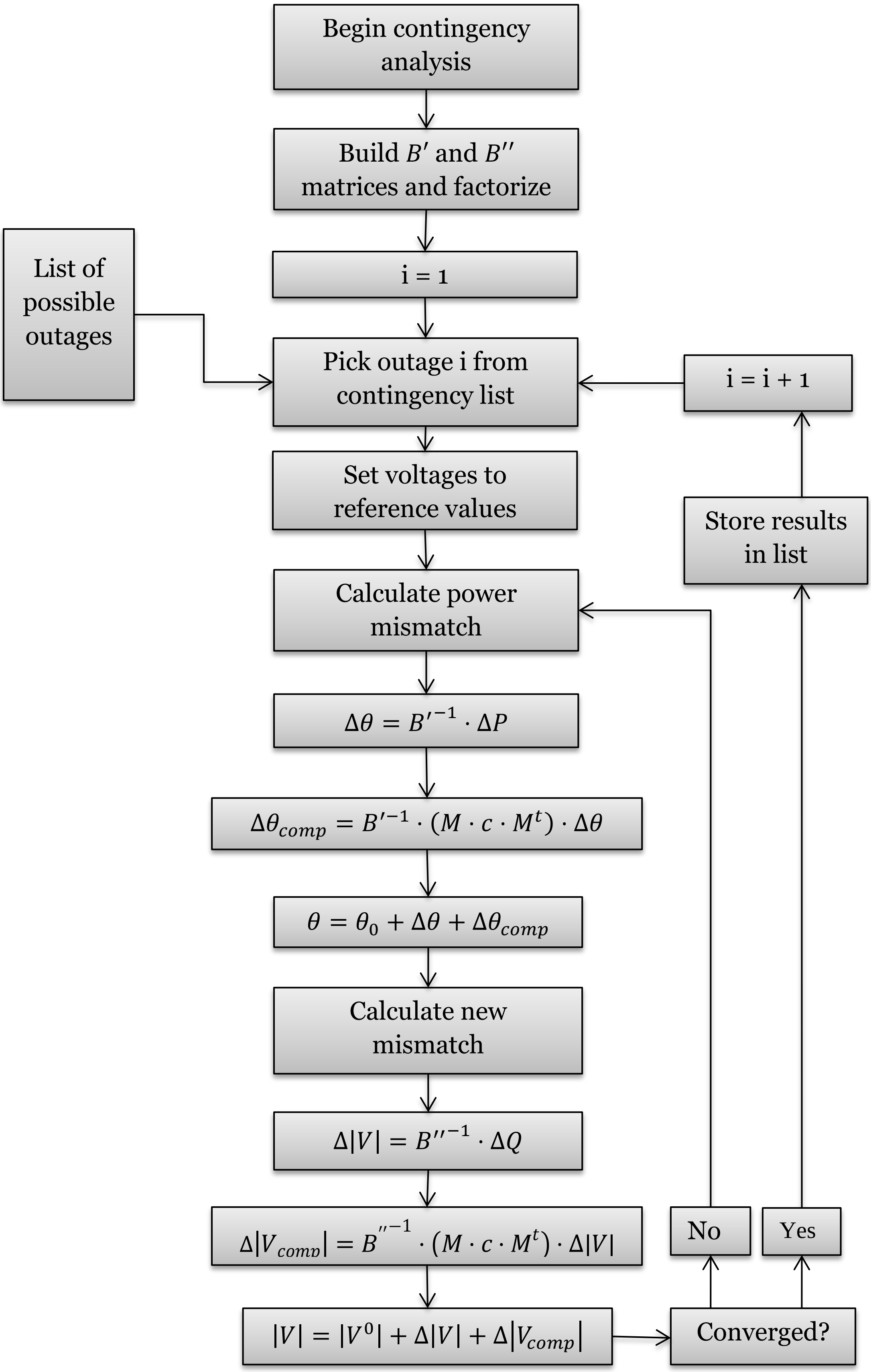

Figure 5: Flow chart for contingency analysis using AC power flow with compensation terms

3.1.DC load flow with compensation

The process of running a contingency analysis based on the DC load flow is similar to that of the AC version, but does not involve iterations.

The principle of operation is the same as above, but there is only one B matrix that needs to be operated on. These features make the DC algorithm significantly faster than the AC version. The method does not have the ability to study voltages or reactive power.

Bounding method

The efficient bounding method described in section 2.6 has been implemented and tested in this project.

The procedure for running contingency analyses using bounding methods is shown in Figure 6.

In regular contingency analyses, the maximum allowable change Δf_{pq}^{max} for each branch is calculated only once, after the base case power flow is solved. This is possible because the post-contingency flows are only compared against static flow limits, such as the thermal capacity. In this project, tests have been conducted where the change is compared to pre-contingency flows, either in the faulted branch, or the pq-branch itself, thus it is necessary to know which branch is faulted before Δf_{pq}^{max} is calculated.

The initial boundary can be set to include only the nodes connected to the faulted branch, or use a set of buses a given number of tiers from the fault. After this, the voltage angles are calculated and possible limit violations are searched for. If any are found, the boundary is expanded and new angles are calculated, if not, the results are stored and the next contingency is chosen.

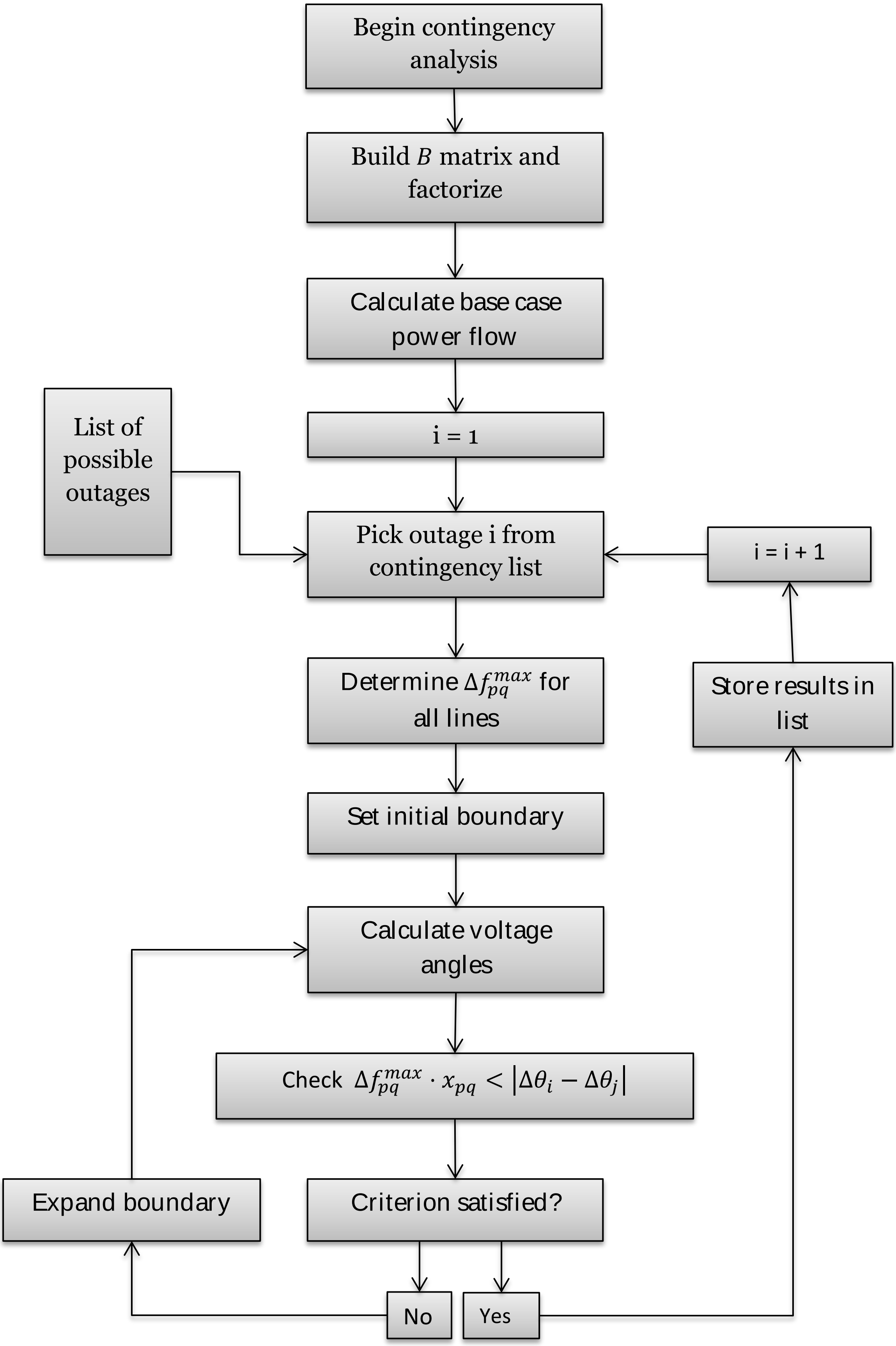

Figure 6: Flow chart for contingency analysis using the efficient bounding method

3.2. How to determine which cases should be included in level 2 studies?

The selection of which cases that should be studied further is important, and difficult. It is hard to predict which cases will have largest influence on the system, however, since faults generally are assumed to have a limited geographical extent, potential faults on lines far away from the original fault can be omitted from further studies. This approach is simple, but may result in critical double contingencies being ignored.

In this project, the branches that are selected for future studies are those with the highest initial power flow. For the selected branches, all possible double contingencies involving these branches are studied.

In reality, only a selection of double contingencies will be studied, and it’s highly unlikely that an operator would want to study all double contingencies involving any specific branch. However, the purpose of doing it this way in the project has been to investigate the efficiency of the computation method.

3.3. Simplifications and assumptions

Certain simplifications and assumptions have been made to ease the implementation of the different methods. The intention of this project has not been to create a production grade program, but create and test various methods for defining the area influenced by fault.

All contingencies that would result in isolation of a component or parts of the system have been left out of the study. There are many considerations that must be taken if isolation occurs, for instance the creation of a new slack bus. This work has been left out of this project.

The reactive power limits of generators have not been enforced in the AC studies in this project. This could be done by inserting large elements on the diagonal of B'', for the generators that needs be changed from PV to PQ nodes.

Shunt capacitors on buses, and line charging susceptances are neglected, and all tap ratios are assumed to be unity.

3.4. Simulations

The simulations in this project have been conducted with the intent of gathering and comparing data regarding computational efficiency and accuracy of the various methods.

Testing of efficiency has, unless otherwise noted, been conducted on a model of the Polish power system during summer peak. The model has 2736 buses, 3504 branches and 420 generators. The list of contingencies that are studied contains 3000 branches.

Testing of accuracy has, unless otherwise noted, been conducted on the standard IEEE 30-bus test system. When testing the accuracy, none of the above mentioned simplifications are made, with the exception of not including isolation cases.

The testing of computational efficiency when studying double contingencies has been conducted on the standard IEEE 300-bus system.

4. RESULTS

This chapter presents the findings of this project. The results of the efficiency testing are presented for all methods, followed by the accuracy results.

4.1. Compensation techniques compared to Fast Decoupled Load Flow

The benefit of the compensation techniques described in section 2.4 is that time-consuming re-factorization is avoided for every contingency case. The method is especially good when running analysis of very many contingency cases on the same base system.

The iteration process of the compensation techniques involves calculation of the compensation terms and is therefore a bit more time consuming than the iteration process of the FDLF. However, relative to the total computation time this difference is small and can be considered negligible.

Because the iteration process of the compensation is similar to that of the FDLF, the only computational savings are in the first part of the process. Since more iterations take more time, illustrated in Figure 7, the relative benefit of the compensation techniques is largest when running few iterations. On the other hand, the absolute benefit in terms of seconds saved is unaffected by the number of iterations.

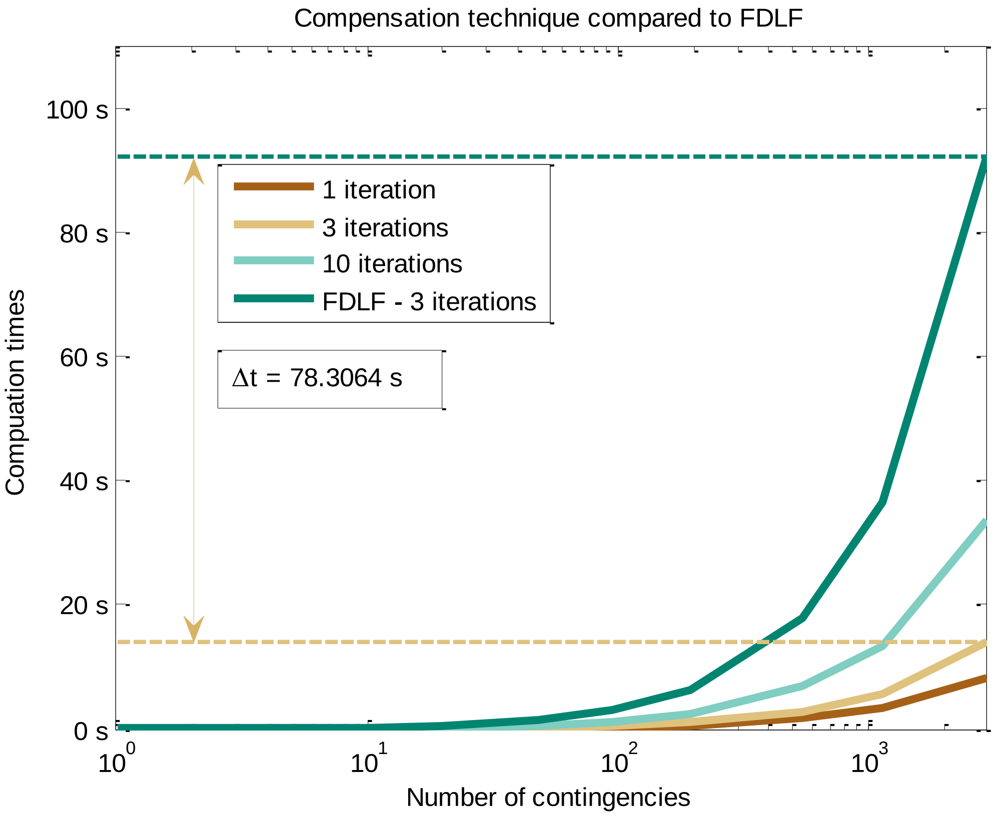

Figure 7: Comparison of computation times, Polish power system - Logarithmic X-axis

For the Polish power system, the factorization of B' and B'' takes approximately 14 milliseconds[4]. This means that if all 3000 possible single contingencies are to be studied using FDLF, the total time spent on LU-factorization will approximately 40.83 seconds. For 3000 contingency cases, the creation of the B' and B'' matrices and factorization takes 24.29 seconds, whereas the time spent on 3 iterations of the decoupled load flow on all cases is only 9.23 seconds. Figure 7 and Figure 8 illustrates the relationship between the number of contingencies and computation time for the FDLF and compensation technique with different number of iterations. The purely linear relationship between number of contingencies and computation time for the fast decoupled load flow, illustrated in Figure 8, is due to the fact that the FDLF conducts the exact same calculations for all cases, except for loading and structuring system data which is only done once.

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[4] Intel(R) Core(TM) i5-3317U CPU @1.70GHz. Memory 4.0 GB. Win-8 64bit, Matlab R2013A

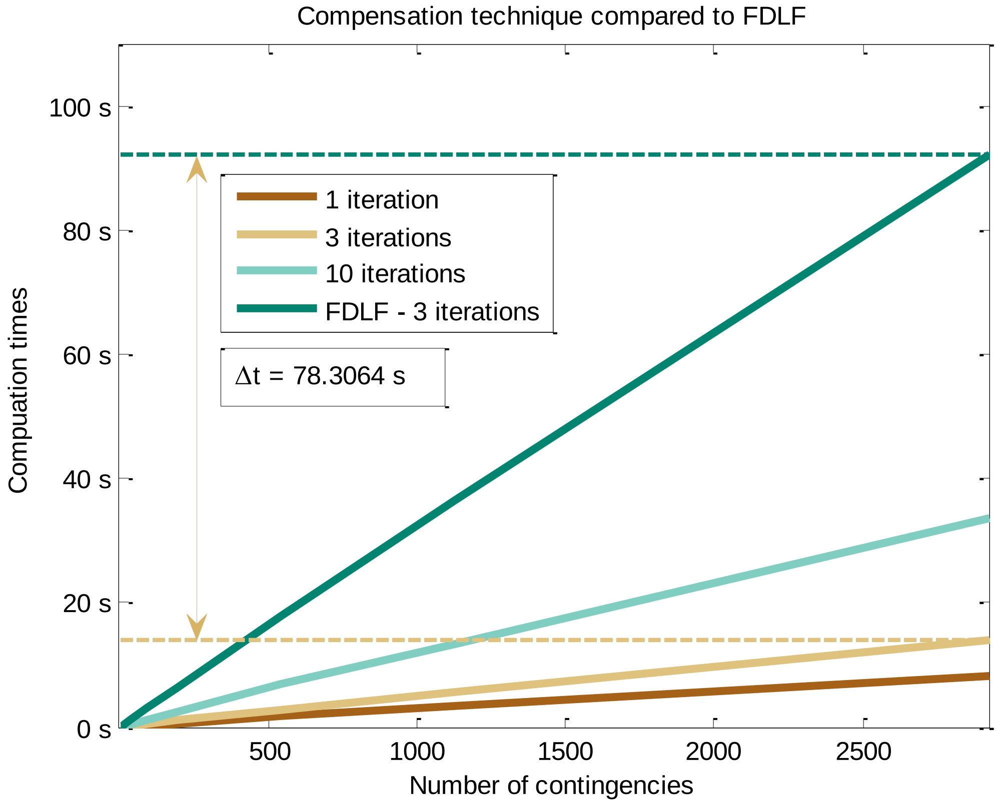

Figure 8: Comparison of computation times, Polish power system – Linear axes

The savings achieved by utilizing compensation techniques, illustrated in Figure 7, increase rapidly as the number of contingency cases increase. For 3000 contingencies, the computation time will be approximately 15% of the fast decoupled load flow with 3 iterations.

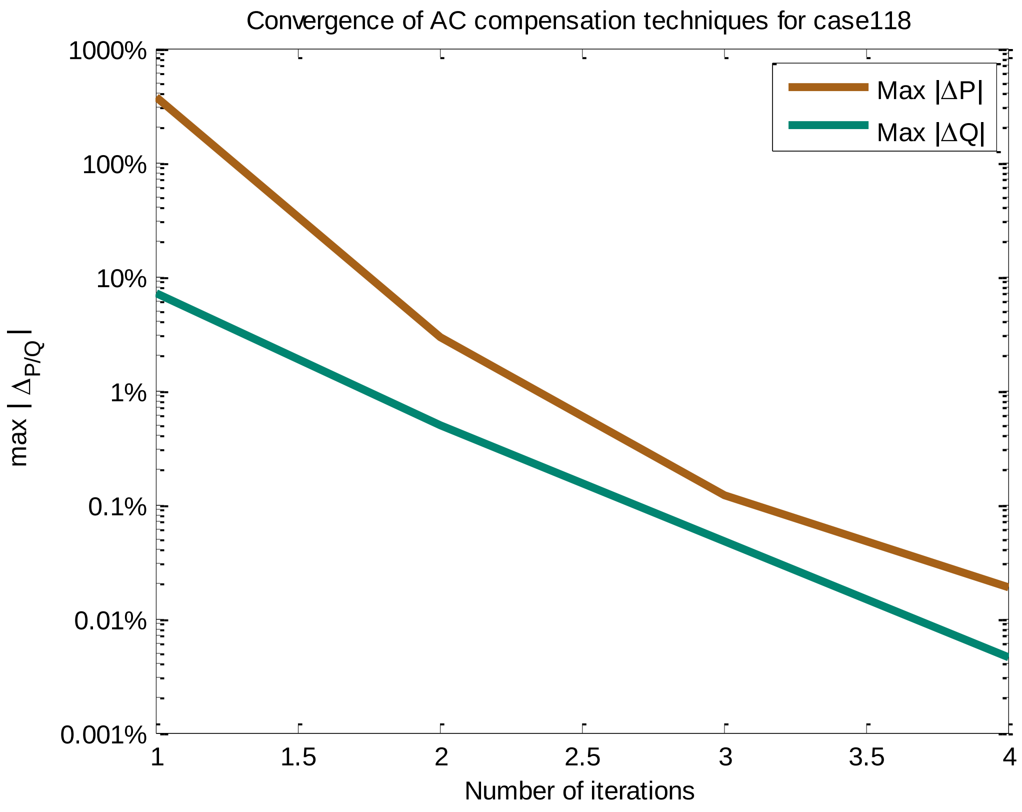

Figure 9: Convergence characteristic for FDLF for IEEE 118-bus system

More iterations yield more accurate calculations, but also, as can be seen from Figure 7, a more time consuming process. Thus a balancing is necessary to determine how many iterations will be best fit for this purpose. Figure 9 illustrates the convergence characteristics of the FDLF for the IEEE 118 bus test system with a flat start, summarized in Table 1. The convergence rate of the FDLF is fast, and after 3 iterations, the mismatch is much less than 1% for both active and reactive power flows.

Table 1: Convergence characteristics of FDLF for IEEE 118 bus test system

Iterations	max$\|\Delta P\|$	max $\|\Delta Q\|$
1	366.9 %	7.188 %
2	2.957 %	0.502 %
3	0.121 %	0.048 %
4	0.018 %	0.005 %

By recognizing that the purpose of the study is not to calculate the exact power flow solution, but rather identify change, it can be assumed that a mismatch of 1% may be sufficiently accurate.

4.2.Compensation techniques compared to DC load flow

In cases where voltage issues are not of interest, approximate DC solutions can be used. As with AC power flow, compensation techniques offer significant computational savings in the DC case.

Table 2 summarizes the computation times for the AC and DC analyses. The results show that the compensation techniques offer a 90% time reduction when using a single iteration on all 3000 contingency cases on the Polish system. The reduction is relatively smaller for the DC case, about 80%, because only the B' matrix need be operated on, while both B' and B'' are included in the AC versions.

Table 2: Calculation time, Polish 2736 bus system, 3000 contingencies

	AC		**DC**	
	FDLF	Compensation	Regular	Compensation
1 iteration	85.42 s	8.33 s	16.83 s	3.32 s
3 iterations	92.47 s	14.16 s		
10 iterations	110.29 s	33.57 s		

Table 2 illustrates why it is desirable to use a DC model when studying a large number of contingency cases. A single branch fault on average takes only 1.1 milliseconds for the DC version, and almost 5 milliseconds for the AC version with 3 iterations. Thus, almost 5 times more cases can be studied using the DC model, compared to the AC version, using the same amount of time. This is especially suited if multiple contingencies are to be studied, as the number of cases to study can be vast.

4.3.Multiple contingencies

The number of contingency cases increases very rapidly when multiple outages are included in the study. If all possible double contingencies are to be studied, the number of cases will be

$$C = \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \textbf{(4.1)}$$

C is the number of contingency cases
n is the number of lines in the system

As can be seen, the number of contingency cases increases quadratic. For the Polish power system, consisting of 3500 lines, the number of cases will be more than 6 million. The memory required to store only the active power flow results will be almost 160 GB in this case, and the computation time will be, with the DC load flow algorithm used in this project, more than 1.5 hours.

Methods for selecting multiple contingencies have been discussed 3.2 and 2.2.3. In this project, "level 2" analyses have been conducted on the lines where the pre-fault active power flow was largest. A study of all single contingency cases is carried out, and by using the pre-fault power flow, a list of branches that are to be studied further is selected. A suitable length of this list must be determined based on the time available and on how thorough the study needs to be.

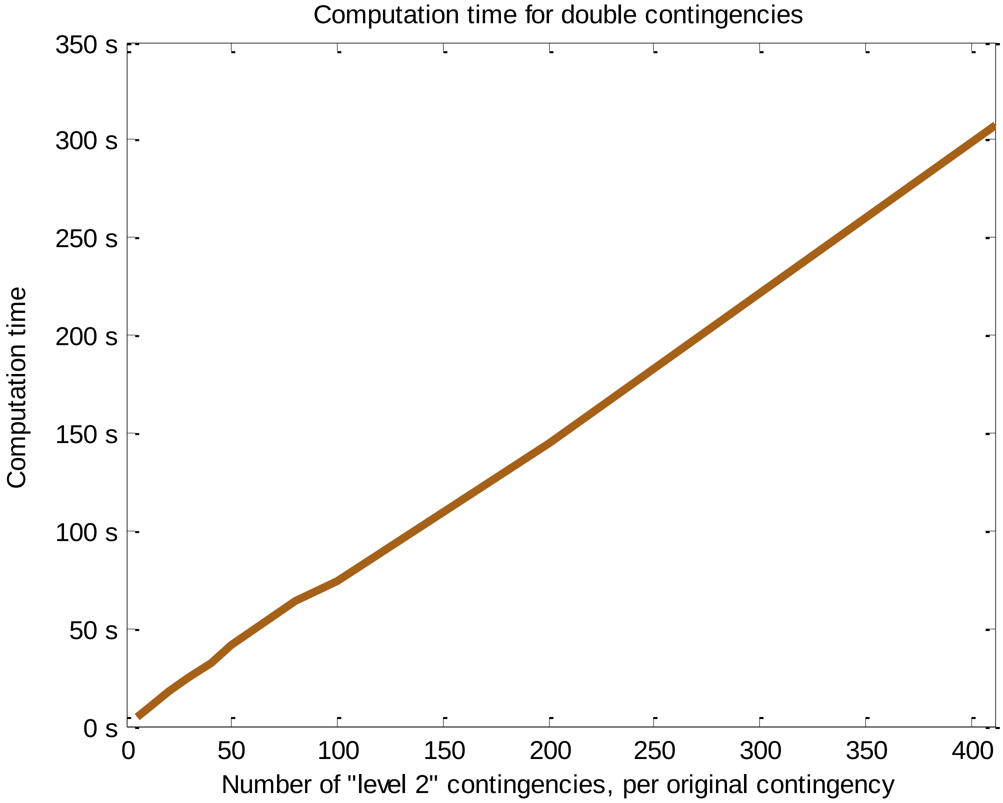

Figure 10: Computation time for double contingencies in IEEE 300-bus system, with AC power flow, 5 iterations

For a selection of branches, new matrices are created and factorized and subsequently used as basis in the compensation methods. Since the process of conducting a single "level 2" study is similar to a full "level 1" study, the computation time is directly proportional with the number of "level 2" studies. That is, if 100 lines are to be studied further, the computation time will be 100 times higher than if only single branch faults were studied.

Figure 10 illustrates the linear relationship between the number of "level 2" branches and computation time for the IEEE 300 bus system when 5 iterations are used. In this system, there are 411 possible single branch faults. The x-axis of Figure 10 represents the number of lines that are selected for further studies. For each of these lines, new base case matrices are created and factorized, and then the procedure for single line outages are repeated.

If all possible double contingencies are studied in the 300 bus system, a total of 85 000 load flows must be calculated. Only one power flow calculation should be conducted per double branch fault, by skipping all previously calculated cases when going through the list of selected “level 2” branches. I.e. when all double contingencies including the first branch has been calculated, that branch will be skipped when calculating double contingencies for the second branch. This has not been implemented in the algorithm used to obtain the results in this report, thus some double contingencies are calculated twice. In the extreme case when all double contingencies are being studied, *all* cases are actually calculated twice. In the 300 bus system, 170 000 load flows are calculated, instead of 85 000. The simulation times, illustrated in Figure 10 are therefore not representative for an optimally coded algorithm, but rather the algorithm used in this project. If an optimal algorithm is used, the slope of the curve in Figure 10 will decrease, as the overlap will increase when increasing the number of lines to include.

4.4.Accuracy of the DC load flow

It is important that lines that are considered affected by an outage when using the AC representation of the system is also considered affected when using the DC representation. Figure 11 illustrates the deviation between the active power flow achieved when running AC and DC studies of all possible contingencies on the IEEE 30-bus test system. The dotted line shows the average difference, whereas the bars illustrate the maximum and minimum deviation relative to the individual branch limits.

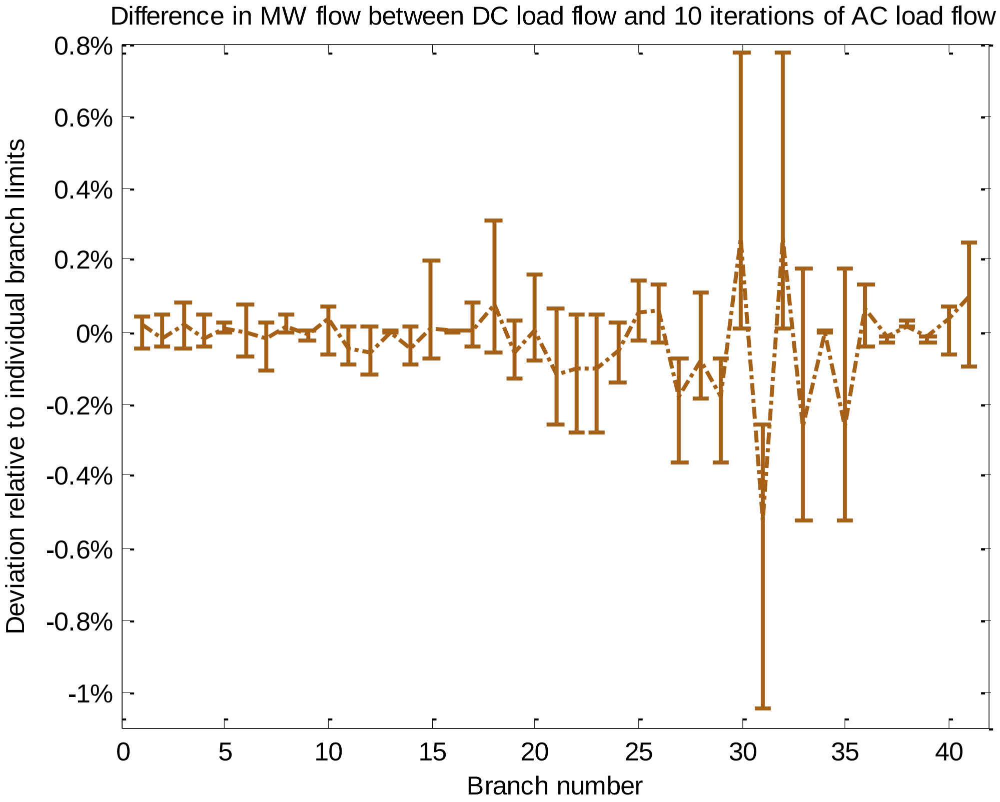

Figure 11: Deviation between active power flow results from AC and DC calculations, relative to branch limits

As can be seen from the figure, the maximum deviation between the AC and DC load flow results is approximately 1% relative to branch limits, corresponding to 0.15 MW. Similar results are achieved for other sample systems.

Even though the DC load flow tends to give satisfactory results in this project, this might not be the case for all systems and in all situations. In systems with a low X/R ratio, the accuracy of the DC load flow decreases, and in such systems it might be necessary to use a regular AC load flow. It can also be noted that in systems with low X/R ratio, the convergence of the FDLF is worse, thus more iterations may be needed in order to achieve the desired accuracy. As the purpose of this project is not to calculate accurate load flows for all contingency cases, but rather identify the size of the system influenced by a fault, a simplified DC load flow might be satisfactory, nonetheless.

4.5.Criteria for identification of influenced components

The applicability of the different criteria discussed in section 3.2 was investigated in the pre-study for this thesis [28]. In summary, the results show that no single criterion can be considered sufficient in all cases.

The results of the pre-study showed that an adequate selection criterion can be obtained by setting limits for the absolute change in MW, and the relative change in percentage. Further enhancement is achieved by including a criterion based on the individual branch limits.

4.5.1. Identification of influenced components

Due to the approximate nature of the DC load flow, it will always be possible to select filtering criteria that will result in different lines being included in the subset, compared to the AC solution. An extreme case is illustrated in Figure 12. If, after a fault on the line 18-19, a subset where all lines with a power flow change of more than 2MW is to be selected, the blue lines will be included in both the AC and DC analysis. The brown lines will only be included in the DC version, whereas the purple line is only included in the AC version.

Table 3: Active power flow before and after a fault on line 18-19. Lines with a change more than 1 MW are included.

		P_0 [MW]		P_1 [MW]		ΔP [MW]	
From	To	AC	DC	AC	DC	AC	DC
10	20	5.9154	5.5635	11.8837	11.7000	5.9683	6.1365
15	18	9.1648	9.3365	3.2127	3.2000	-5.9521	-6.1365
18	19	5.8680	6.1365	0.0000	0.0000	-5.8680	-6.1365
19	20	-3.6541	-3.3635	-9.5000	-9.5000	-5.8459	-6.1365
12	15	9.4768	9.7557	6.1991	6.5484	-3.2776	-3.2073
4	12	-1.6717	-1.2698	-3.7486	-3.2569	-2.0768	-1.9871
12	16	9.2639	9.3218	11.2960	11.4485	2.0321	2.1266
16	17	5.6843	5.8218	7.6774	7.9485	1.9931	2.1266
10	17	3.3698	3.1782	1.4024	1.0515	-1.9674	-2.1266
23	24	7.0847	7.4719	9.0039	9.4946	1.9192	2.0228
15	23	-8.8059	-8.5281	-6.9199	-6.5054	1.8860	2.0228
4	6	22.5031	21.2582	24.3337	22.9524	1.8306	1.6942
22	24	-2.0968	-2.9098	-3.8754	-4.8813	-1.7786	-1.9715
9	10	5.7893	4.7994	7.2136	6.0965	1.4243	1.2972
6	9	5.7893	4.7994	7.2136	6.0965	1.4243	1.2972
21	22	-19.7769	-20.4165	-20.8844	-21.6487	-1.1075	-1.2322
10	21	-2.2332	-2.9165	-3.3366	-4.1487	-1.1034	-1.2322

Even though the resulting subsystem differs a lot from the AC to the DC version, it can be argued that the DC version is sufficiently accurate in this situation by keeping in mind that the purpose of this study is to identify change, not the exact power flow. There is no single correct criterion to determine if a component in the system is influenced by a fault, and the tolerance limits are based on a professional discretion rather than exact physical limits or accurate calculations. By recognizing that there will always be some lines that are just above, or just below a given limit, it must be assumed that the limits are decided in such a way that it is not critically important that lines that are very close to the limit are included in the subset of lines that are considered affected.

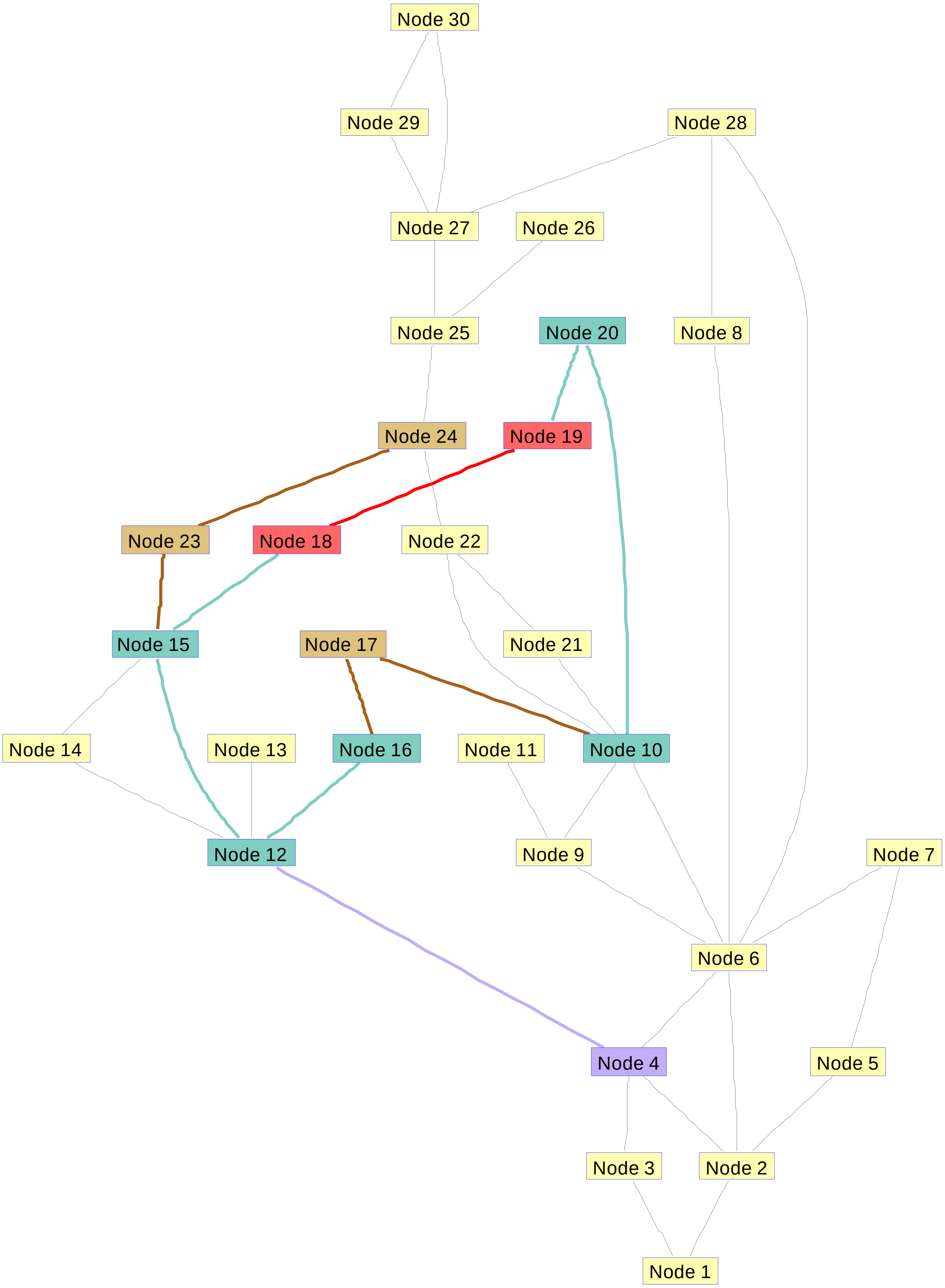

Figure 12: Difference between AC and DC load flow. $\mathcal{V} = \mathcal{A}_{2MW}$

4.6.Voltage issues

Voltages that are outside the system's operating limits can damage equipment or potentially cause blackouts. Since such conditions are not detected by the DC load flow, an AC load flow is necessary in order to study voltage profiles and changes.

The result of a fault on line 21-22 is shown in Figure 14. As can be seen, using the same criterion as in the above section, the set of branches is identical for the AC and DC power flow. However, a DC power flow would not detect the voltage violations on nodes 19 and 20, marked with orange color.

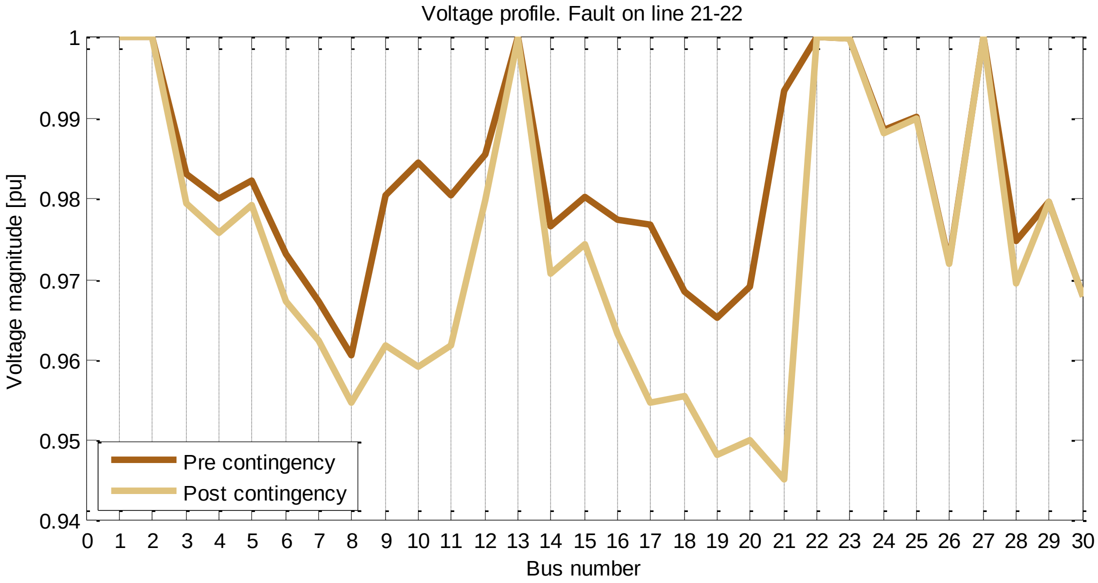

Figure 13: Voltage profile - Fault on line 21-22

Figure 13 shows a large change in voltage on buses 9-11 and 16-21. This is due to a large generator at bus 22 that gets disconnected from the large load at bus 21. The result is higher currents on many of the surrounding lines, increasing voltage drops. The line with the highest change in reactive power losses is between nodes 10-22, shown in Figure 15.

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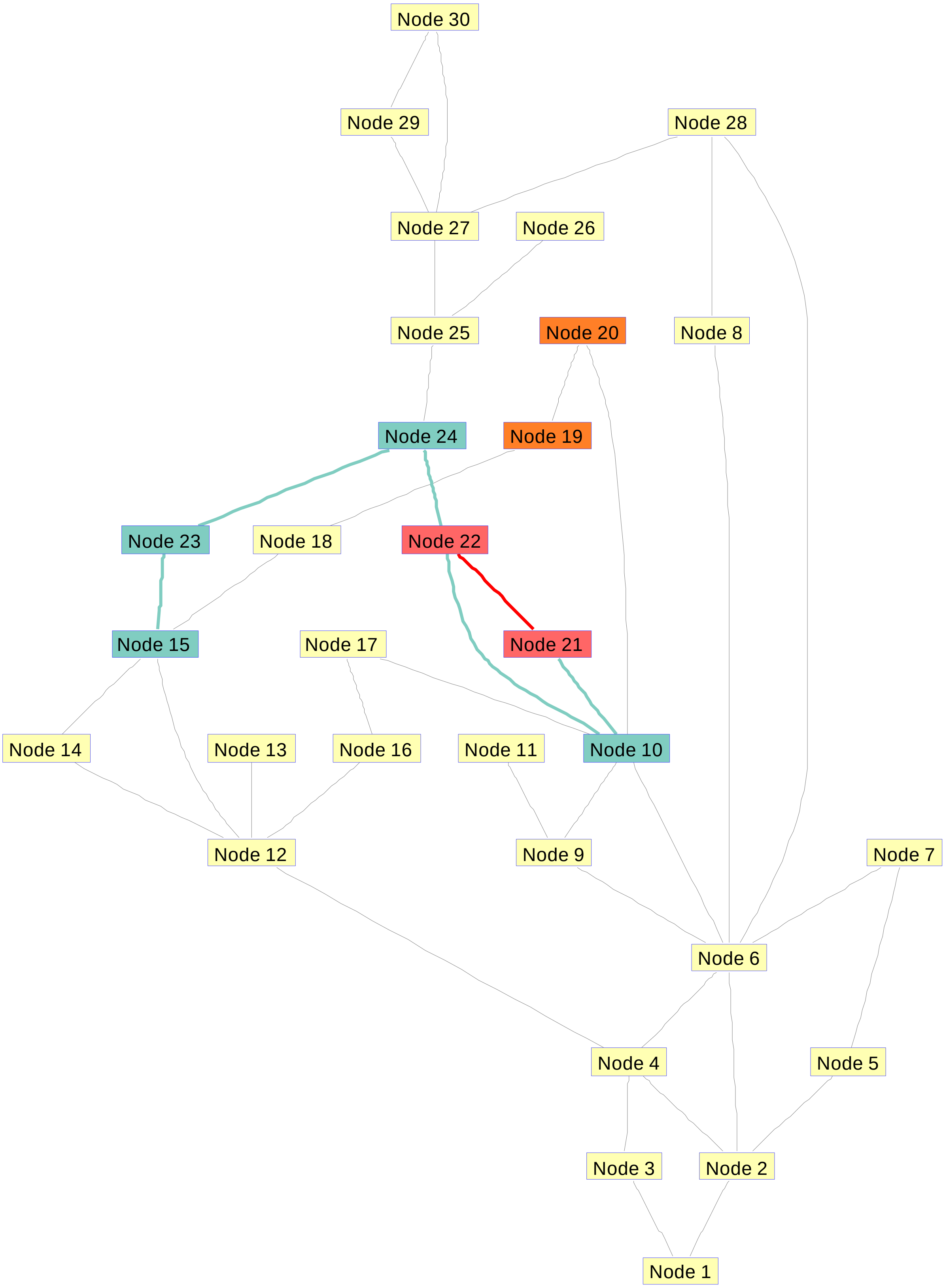

Figure 14: Fault on line 21-22. $\mathcal{V} = \mathcal{A}_{2MW}$. $V_{19\,\&\,20} < 0.95$ **pu**

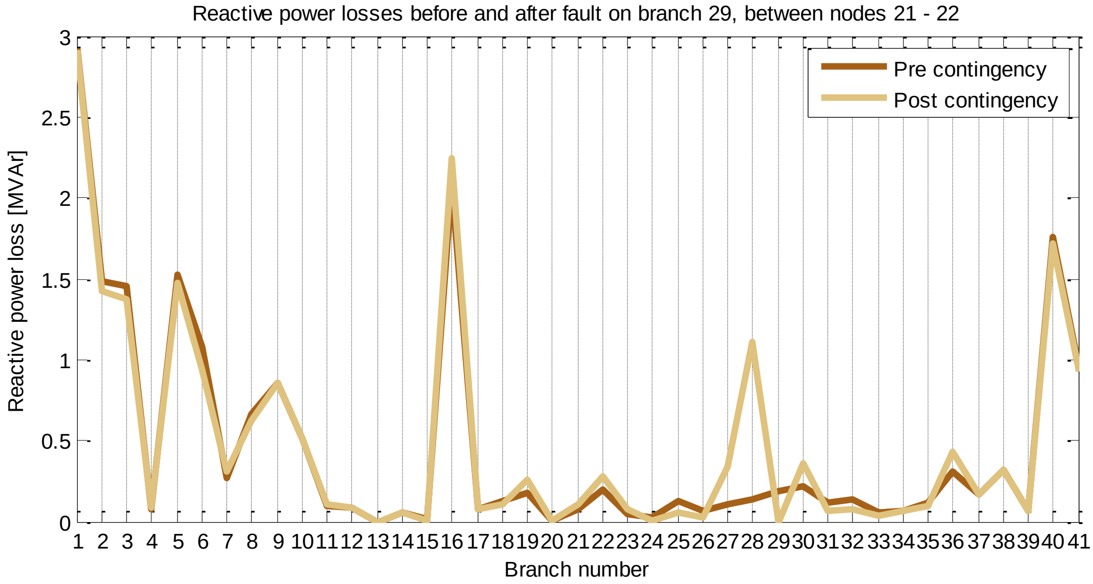

Figure 15: Reactive power losses - Fault on line 21-22

4.7.Bounding methods

The main principle of the bounding methods is to calculate the change in angle difference at the boundary nodes, and check if that angle difference might lead to violations of some limits outside the boundary system.

$$\Delta f_{pq} = \frac{1}{x_{pq}}(\Delta\theta_p - \Delta\theta_q) \qquad \textbf{(4.2)}$$

4.7.1. Low reactance issues

A possible weakness of the bounding methods occurs when there are large differences in per-unit reactances. As can be seen from the above equation, the potential flow change is inversely proportional to the per unit line reactance. This can provide problems in systems where certain lines have either a very low reactance, or a very high voltage.

The per-unit base impedance, Z_{base}, is given as

$$Z_{base} = \frac{V_{base}^2}{S_{base}} \qquad \textbf{(4.3)}$$

For a system with $S_{base} = 100MVA$, and $V_{base} = 400kV$, the base impedance will be

$$Z_{base,400kV} = \frac{(400kV)^2}{100MVA} = 1600\Omega \qquad \textbf{(4.4)}$$

For a short line on a voltage level of $400kV$, the per-unit reactance will be very low, whereas a long line on a lower voltage level will have a much higher per-unit reactance. For the Polish system used in this report, the highest reactances are more than 7000 times the value of the smallest ones. The corresponding difference in Ohms is only a factor of 320, because of different voltage levels.

$$6.3 \cdot 10^{-5} \leq X_{pu} \leq 0.453$$

This means that for a given angle difference, the apparent possible change in power flow is 7000 times higher on some lines, than others.

When checking the angle difference against the acceptable limits outside the boundary, a line with reactance less than 10^{-3} pu acts almost like a short-circuit. The result is that even though the angle difference at the boundary is very low, it can't be said with certainty that there is no violation at the low-reactance line. If no special attention is given to these low reactance lines, the results of the bounding methods are often that the boundary must be expanded to include the entire network.

Occurrences of low reactance problems have been handled by setting a minimum value of $x_{pq} = 10^{-3}$ pu when checking for potential violations.

It must be noted that these low reactance values most likely do not represent the actual reactance values of physical lines in the power system, but rather lack of accurate data. The validity of this assumption is supported by noting that 17% of all the lines in the Polish system are listed with the exact same reactance of $8.3 \cdot 10^{-4}$ pu, all at a base voltage of 110kV.

4.7.2. Angular spread characteristics

The bounding methods are most efficient when a small part of a large system is influenced by a fault. A high degree of connectivity results in only a small part of the system being influenced by a fault, while the opposite is normally the case for a weakly connected grid. Since the impact of a fault decreases with the distance from the fault, so does the change in angular spread between the boundary nodes (or the other way around). Figure 16 shows the maximum angular spread for a fault in two different systems. The average connectivities of the highly and weakly meshed systems are 4 and 2.73 respectively.

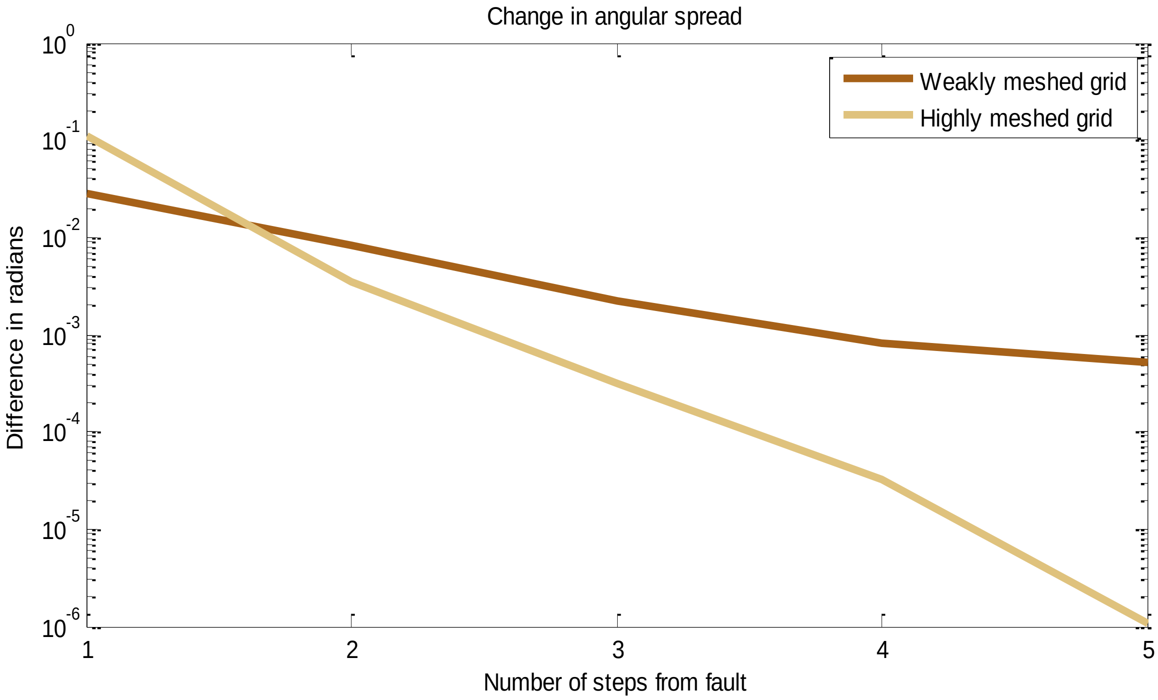

Figure 16: Characteristics of the change in angular spread after a fault

If the boundary needs to be expanded many times, the efficiency of the bounding methods is drastically reduced. This is because the number of $\delta_{i,km}$'s (see appendix A) that must be calculated increases rapidly, in addition to the computational effort related to finding the new boundary nodes.

4.7.3. Initial boundary

Although many different criteria can be used to determine branches that are influenced by a fault, in most cases studied, the boundary must be expanded to include at least 3 tiers from the fault in order to meet the desired criteria. By using an initial boundary that includes all nodes k tiers from the fault, instead of starting with the closest nodes, several steps of the calculations can be skipped. The initial boundary nodes are found using the procedures described in section 2.6.1.

After the initial boundary is set and angle changes are calculated, it might be necessary to expand the boundary. The algorithm for this has been designed in a way that makes it easy to obtain a list of nodes and branches that are enclosed by this boundary, and is not optimal with regards to computational efficiency. It is likely that the largest potential for increasing computation speed lies in optimization of this algorithm. The code is included in appendix C.

By selecting an initial boundary 3 tiers away from the fault, instead of using the closest nodes, and assuming that the boundary must, on average, be expanded until it is 5 tiers from the fault, the computation time is nearly halved, when studying the Polish system.

Calculation of the sensitivity factors $\delta_{i,km}$ is the second most time demanding operation when using the bounding methods. Calculation of a single sensitivity factor is fast, but when the boundary becomes large, a large set of factors must be calculated, thus the time demanded increases rapidly.

$$\delta_{i,km} = \frac{(X_{ik} - X_{im})x_l}{x_l - (X_{kk} + X_{mm} - 2X_{km})} \tag{4.5}$$

4.7.4. Contingency list

When only a few contingency cases are of interest, only the necessary columns of $X = B^{-1}$ are calculated in order to achieve $\delta_{i,km}$, as described in section 2.6. However, when a large set of contingency cases are to be studied, it is generally faster to calculate all columns of the inverse and use the appropriate ones when needed. For 3000 contingency cases in the Polish system, this approach leads to an 18% reduction.

Figure 17 shows the computation time for simulating 3000 contingencies in the Polish power system, as a function of number of steps from the fault. If a chosen criterion is reached after an average of 5 steps from the faulted line, it takes 7.3 seconds to run all contingency cases.

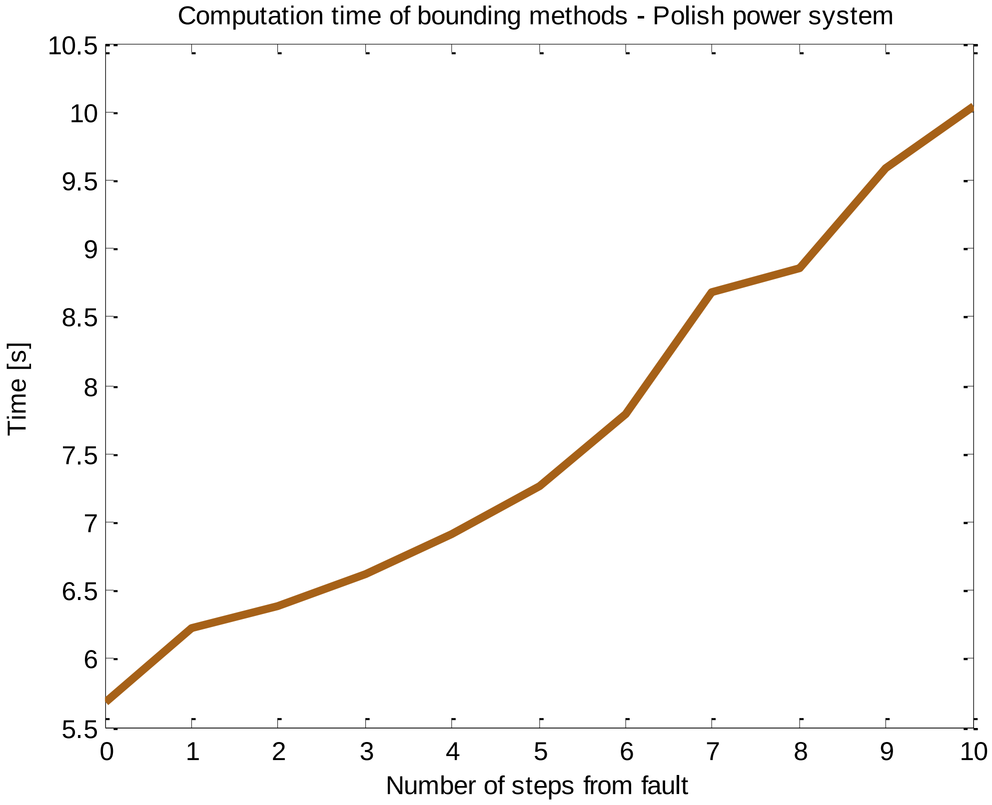

Figure 17: Computation time when running 3000 contingency cases, with different number of max steps from the fault location.

4.7.1. Bounding criteria

As opposed to other techniques, the computation time of the bounding method is dependent on the criteria for when a component is considered influenced by a fault. If the tolerance limits of the criteria are strict, a large number of branches will be considered affected by a fault, and thus the subset within the boundary will become large and the computation time will increase.

From Figure 17, it is evident that choosing a very strict tolerance limit leads to high computation times. The bounding methods are therefore most efficient when only the largest changes are of interest, and only a few lines are considered affected by a fault.

4.7.2. Bounding method compared to regular load flows

The bounding method has shown a tendency of being more conservative than the other methods. In many cases, if the exact results from the AC power flow show that the influenced lines, given some criterion, are all within a certain number of steps from the faulted line, the bounding method includes one or two more steps.

Another characteristic of the bounding method is that the expansion of the boundary goes in all directions, thus sometimes including a large number of lines that are not necessarily affected by the fault. The affected line furthest from the fault location will determine the number of expansions.

This is illustrated in figure 18, where the blue parts of the network is considered affected an outage of line 2-6, while the subsystem obtained when using the bounding method is shown in brown. Note that the subsystem also includes the blue lines. As can be seen from the figure, the affected node furthest from the fault location is node 3. In order to reach this node, two lines must be traversed from either node 2 or 6. This means that all nodes that can be reached by expanding the boundary 2 steps will be included in the subsystem.

In cases where parts of the system are highly meshed, and other parts have a radial structure, this might give unsatisfactory results. Suppose a system is highly meshed close to metropolitan areas, whereas it has a radial structure going out from this area, for instance along the coast. If a fault occurs in between two such network topologies, the fault will have a large influence on the radial side, while only a small part of the highly meshed grid will be influenced. Then, if the bounding method is used, many steps is needed to include all affected parts of the radial system, thus including a large set of branches in the meshed grid, that would otherwise be characterized as unaffected.

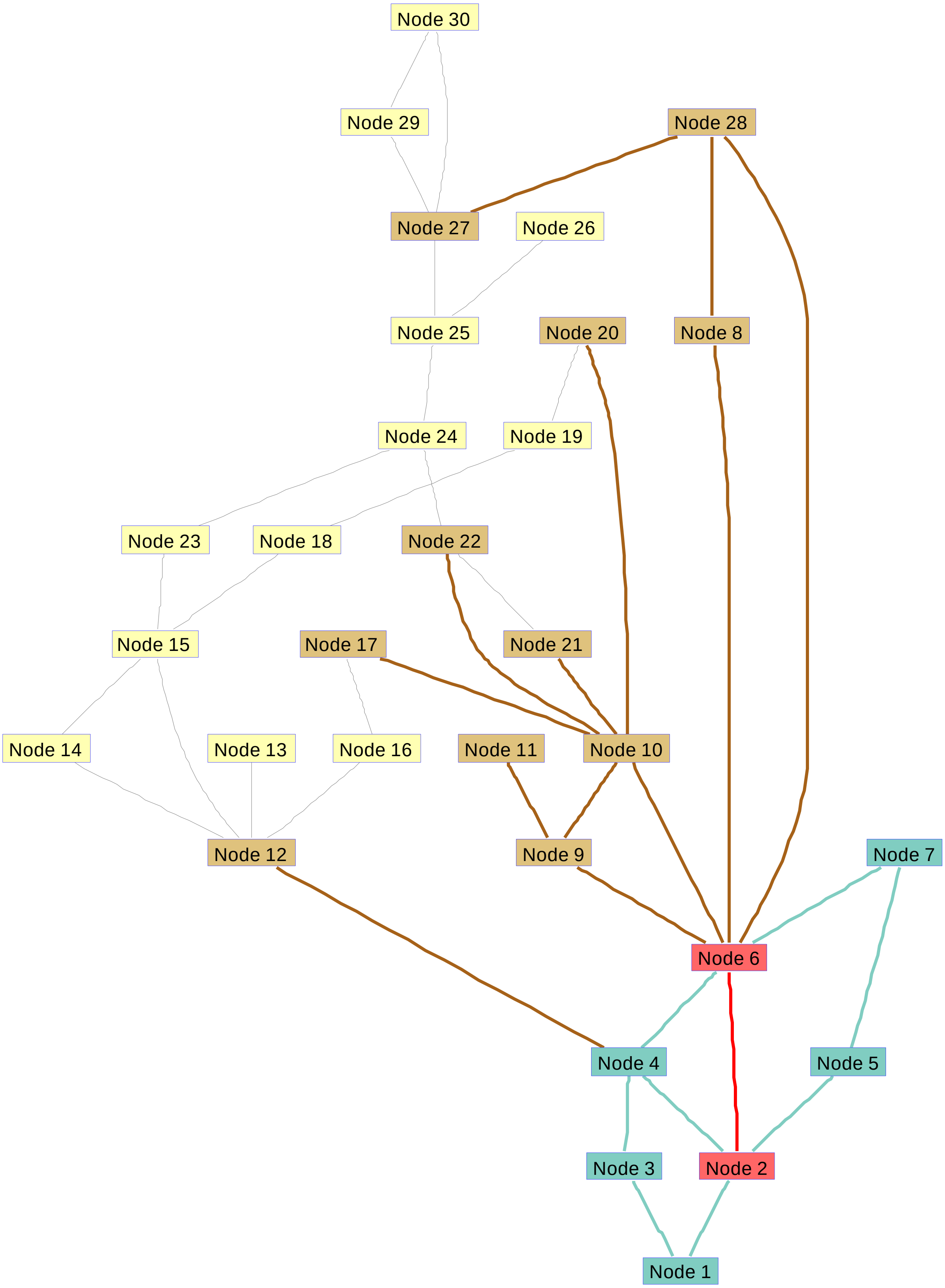

Figure 18: Blue lines are considered affected when using AC the power flow. Brown lines are included in the subsystem when using the bounding method. $v = x_{2MW,5\%}$

5. DISCUSSION

5.1. Computation time

There is a significant reduction in computation time when using compensation techniques instead of building and factorizing new matrices for each case in a contingency study. The methods are especially good when running analysis of many single contingency cases on the same base case system.

An important question when analyzing the effect of outages is to decide the level of accuracy needed in the calculations. If a very detailed and accurate solution is needed, the computation time will be much larger than if a rough estimate is sufficient.

Table 2 shows that the total computation time for analyzing 3000 different single contingencies in the Polish power system is approximately 14 seconds, using 3 iterations. The total time increases by 2.8 seconds per extra iteration needed. Thus, if one additional iteration is needed in order to achieve the desired accuracy, the computation time will increase by 20%. Assuming that there is a fixed amount of time available, the number of contingencies that can be studied is reduced by almost 20%.

There should be a relation between the desired level of accuracy, and the criterion for which a component is to be considered influenced by a fault. If only large changes are of interest, it is not necessary to have very accurate power flow calculations, as the large changes will be detected regardless. On the other hand, if even the smallest of changes are of interest, the power flow calculations must be accurate.

If an approximate DC solution is sufficient, the computation time is reduced to 3.32 seconds, for the same number of contingencies. That means that almost 5 times as many contingencies can be studied using the DC version, compared to the AC version with 3 iterations. This is especially useful if multiple simultaneous contingencies are to be studied.

The bounding method is most efficient when only a small part of the system needs to be included in the subsystem. If the boundary must be expanded many times, the efficiency is significantly reduced, as shown in Figure 17. The computation time depends on the selected criterion for when a component is considered influenced by a fault. If only the most influenced branches are of interest, the subsystem will be small, and the computations efficient.

Comparisons show that the bounding method is slower than the DC compensation method, regardless of the size of the subsystem. It is assumed that this will not be the case when the bounding methods are implemented in a more efficient way. It is believed that the bounding method will prove more efficient when only large changes are of interested, whereas the compensation techniques will prove most efficient when larger parts of the system will be considered influenced.

5.1.1. Multiple contingencies

In this project, studies of multiple contingencies have been conducted using AC power flow. A selection of branches has been selected for further studies. For each of these lines, new B matrices have been built and factorized, and the procedure for single branch contingencies has been used on each of these cases. This gives the linear relationship, shown in Figure 10, between the number of selected branches and computation time, since the same calculations are done for each case.

Note that this will not be the case if the number of selected branches is high, relative to the number of branches in the system, due to the overlap, discussed in section 4.3. When an outage of the last branch on the list is the base case, all the lines previously used as base cases, can be skipped. If all double contingencies are to be studied, for the last branches on the list, there will be few remaining possibilities. In such cases, it might be faster to expand the compensation terms to account for double contingencies instead of rebuilding and factorizing the B matrices for these last branches. This has not been tested, as it is unlikely to occur in practice.

The number of double contingencies that are to be studied must be decided based on the time available and the security requirements.

5.2.Accuracy

As the purpose of the work is not to determine the exact post-contingency power flow, but rather identify changes, it can be assumed that some accuracy can be sacrificed for the sake of computational efficiency.

All the proposed methods have proven sufficiently accurate for the intended purposes. In systems with a low X/R ratio, it might be desirable to run an AC load flow instead of relying on the approximate DC load flow.

It will always be possible to select criteria that will result in different lines being included in the subset of influenced branches for AC and DC load flow. Even though the subsystems differ, both may be considered accurate enough, using the same argument as above.

In systems with large deviations in the per-unit reactance values, the bounding method does not perform well. If any x_{pq}, is very low, the criterion will not be satisfied, unless line p-q lies within the boundary.

It has been noted that the reactance values in the test systems are not representing the true reactance values, but rather lack of data.

6. CONCLUSION

The scope of this project has been to develop and test methods for defining subsystem of the power system that is considered influenced by one or several faults. Post-processing techniques were investigated in the pre-study. In this thesis special attention has been directed at the performance of different methods for achieving the necessary power flow results.

Three approaches, based on the fast decoupled load flow, the DC load flow and the efficient bounding method have been implemented and tested,

The AC version based on the FDLF can be used to analyze the effect of all single branch outages in a large power system in less than 15 seconds with the modest computing power available. The AC approach is the only method that provides information regarding reactive power flow and voltage magnitudes, and should thus be chosen if these need to be analyzed. It is also the only approach where an exact solution is achievable, and should therefore be chosen in cases where a detailed analysis is necessary. This level of accuracy is, in the general case, considered unnecessary for the intended purposes of this study.

The method based on the DC load flow is almost 5 times faster than the AC version, and should thus be the preferred over the AC version in cases where voltages and reactive power flows are not of interest. It is also more suited for calculating multiple contingencies due to its superior efficiency. The DC power flow is assumed sufficiently accurate for the intended purposes.

The efficiency of the bounding method decreases fast if the boundary must be expanded many times, and is thus most effective when only the largest changes are of interest. It has certain deficiencies, for instance cases where there are large variations in reactance, or when fault occur in between meshed and radial parts of the system. It is assumed that, if implemented more efficiently, the bounding method will be faster than the DC based method. It is recommended that the bounding method is used in cases where only the largest changes are of interest, and preferably in a highly meshed grid.

There is no correct measure for determining if a component is influenced by a fault. It is recommended that a combination of the absolute change given in pu and the percentage change is used. The percentage change can be given relative to the pre-contingency power flow in either the faulted line or the line itself, and/or relative to thermal limits.

It is the operator that must determine what an influenced component is. If too many components are included, no network reduction will be achieved, however, if too few are included, the inaccuracy of the study will increase.

One of the most important parts of the study is to determine which contingencies should be included, and which ones shall be disregarded. A suitable list of contingencies to study must be created, based on the time available and on how thorough the study needs to be.

7. FUTURE WORK

In this project, studies of double contingencies have been carried out on the lines where the pre-fault active power flow was largest. A study of all single contingency cases is carried out, and by using the pre-fault power flow, a list of branches that are to be studied further is selected. Techniques, such as the implementation of performance indices, should be incorporated, in order to make a more suitable selection.

It is believed that if the areas influenced by two independent single faults do not overlap, the effect of the two faults occurring simultaneously will not need to be studied. This is however dependent on the criterion for when a component is considered affected. It is recommended that the level of dependency is investigated in future studies.

To keep focus on the techniques and algorithms, certain simplifications have been made to reduce the complexity of the coding, as discussed in section 3.3. These simplifications do not impact the results in this report much, but need to be removed in order to make the methods more robust for real case studies. It must be possible to study the effects in the event of isolation, or when e.g. reactive power limits are reached. The latter one can easily be achieved by minor modifications of the B'' matrix, whereas handling of isolation is more complicated.

Double contingencies have only been studied using the AC power flow in this project. It is of interest to implement techniques for studying double contingencies using both the DC load flow, and the bounding method. Both are assumed more efficient than the AC version.

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A. LINEAR SENSITIVITY FACTORS

The derivation of the linear sensitivity factors in this appendix are based on Appendix 11A in reference [11].

Suppose line k connecting bus n and m is disconnected. Using equation 2.19 for the DC power flow, the following relationships between angles, reactances and power injections can be achieved

$$\begin{aligned} \Delta\theta_n &= X_{nn}\Delta P_n + X_{nm}\Delta P_m \\ \Delta\theta_m &= X_{mn}\Delta P_n + X_{mm}\Delta P_m \end{aligned} \quad \textbf{(A.1)}$$

In the following derivation, the following definitions are made

$\theta_n, \theta_m, P_{nm}$	Exist before the outage, where P_{nm} is the flow on line k from bus n to bus m.
$\Delta\theta_n, \Delta\theta_m, \Delta P_{nm}$	The incremental changes resulting from the outage
$\tilde{\theta}_n, \tilde{\theta}_m, \tilde{P}_{nm}$	Exist after the outage.

The outage modeling criteria requires that the incremental injections ΔP_n and ΔP_m equal the power flowing over the outaged line after the injections are imposed. Then, if the reactance is x_k

$$\tilde{P}_{nm} = \Delta P_n = -\Delta P_m \quad \textbf{(A.2)}$$

Where

$$\tilde{P}_{nm} = \frac{1}{x_k}\left(\tilde{\theta}_n - \tilde{\theta}_m\right) \quad \textbf{(A.3)}$$

Then

$$\begin{aligned} \Delta\theta_n &= (X_{nn} - X_{nm})\Delta P_n \\ \Delta\theta_m &= (X_{mm} - X_{mn})\Delta P_n \end{aligned} \quad \textbf{(A.4)}$$

And

$$\begin{aligned} \tilde{\theta}_n &= \theta_n + \Delta\theta_n \\ \tilde{\theta}_m &= \theta_m + \Delta\theta_m \end{aligned} \quad \textbf{(A.5)}$$

Giving

$$\tilde{P}_{nm} = \frac{1}{x_k}\left(\tilde{\theta}_n - \tilde{\theta}_m\right) = \frac{1}{x_k}(\theta_n - \theta_m) + \frac{1}{x_k}(\Delta\theta_n - \Delta\theta_m) \quad \textbf{(A.6)}$$

Or

$$\tilde{P}_{nm} = P_{nm} + \frac{1}{x_k}(X_{nn} + X_{mm} - 2X_{nm})\Delta P_n \tag{A.7}$$

Then, using the fact that $\tilde{P}_{nm}$is set equal to ΔP_n

$$\Delta P_n = \left[\frac{1}{1 - \frac{1}{x_k}(X_{nn} + X_{mm} - 2X_{nm})}\right] P_{nm} \tag{A.8}$$

It is shown in appendix X that the compensating power injections after an outage of line l, between node k and m can be expressed as

$$\Delta P_n = \left[\frac{1}{1 - \frac{1}{x_l}(X_{nn} + X_{mm} - 2X_{nm})}\right] P_{nm} \tag{A.9}$$

A sensitivity factor δ can be defined as the ratio of the change in phase angle θ, anywhere in the system, to the original power P_{km} flowing over a line km before it was dropped. That is,

$$\delta_{i,nm} = \frac{\Delta\theta_i}{P_{nm}} \tag{A.10}$$

If neither n nor m is the system reference bus, two injections, ΔP_k and ΔP_m, are imposed at buses k and m respectively. This gives a change in phase angle at bus i equal to

$$\Delta\theta_i = X_{in}\Delta P_{kn} + X_{im}\Delta P_m \tag{A.11}$$

Using the relationship between ΔP_k and ΔP_m, the resulting δ factor is

$$\delta_{i,nm} = \frac{(X_{in} - X_{im})x_l}{x_l - (X_{nn} + X_{mm} - 2X_{nm})} \tag{A.12}$$

If either k or m is the reference bus, only one injection is made. The resulting δ factors are

$$\delta_{i,nm} = \frac{X_{in}x_k}{(x_k - X_{nn})} \text{ for } m \ = \ \text{ref} \tag{A.13}$$

$$\delta_{i,km} = \frac{-X_{im}x_k}{(x_k - X_{mm})} \text{ for } n \ = \ \text{ref} \tag{A.14}$$

If bus i itself is the reference bus, then $\delta_{i,nm} = 0$, since the reference bus angle is constant.

A.1. Distribution factors

The expression for the distribution factor, $d_{l,k}$, discussed in section 2.5 is

$$\begin{aligned} d_{l,k} &= \frac{\Delta f_l}{f_k^0} = \frac{\frac{1}{x_l}\left(\Delta\theta_i - \Delta\theta_j\right)}{f_k^0} \\ &= \frac{1}{x_l}\left(\frac{\Delta\theta_i}{P_{nm}} - \frac{\Delta\theta_j}{P_{nm}}\right) \\ &= \frac{1}{x_l}\left(\delta_{i,nm} - \delta_{j,nm}\right) \end{aligned} \qquad \textbf{(A.15)}$$

Where the sensitivity factors are defined in equations A.4 – A.6.

B. INCREMENTAL ANGLE CRITERION

The derivation incremental angle criterion is based on the appendix in the article "Efficient bounding method for linear contingency analysis" by Brandwajn [13].

Suppose the voltage angles of boundary nodes i and j, $\Delta\theta_i$ and $\Delta\theta_j$ are such that

$$\begin{aligned}\Delta\theta_i &> \Delta\theta_f \\ \Delta\theta_j &< \Delta\theta_f\end{aligned} \tag{B.1}$$

For all nodes $f \epsilon N3$. In other words, boundary nodes i and j have the highest and lower voltage angles within N3.

Theorem: The spread in voltage angle across any external branch pq, where p and $q \epsilon N2$, is smaller than $|\Delta\theta_i - \Delta\theta_j|$, i.e.

$$|\Delta\theta_p - \Delta\theta_q| < |\Delta\theta_i - \Delta\theta_j| \tag{B.2}$$

For all p and $q \epsilon N2$

Lemma: For any external node e, the following inequalities are always satisfied:

$$\Delta\theta_e < \Delta\theta_i \tag{B.3}$$

$$\Delta\theta_e > \Delta\theta_j \tag{B.4}$$

Proof: Suppose that inequality B.3. is not satisfied and there exists a node e' such that

$$\Delta\theta_{e'} > \Delta\theta_i \tag{B.5}$$

And, with no loss of generality, suppose that

$$\Delta\theta_{e'} > \Delta\theta_e \tag{B.6}$$

For all $e \epsilon N2$.

This implies

$$\Delta\theta_{e'} > \Delta\theta_e \tag{B.7}$$

For all e in the union of $N2$ and $N3$.

Inequality B.7. implies that all flows leaving node e' must be negative. Because of strict passivity of the external subnetwork, and Kirchoff's current law, the sum of all flow in branches incident to node e' must be equal to zero. This implies that the spreads in voltage angles across all incident branches are equal to zero, i.e. all neighboring nodes of e' have voltage angles equal to $\Delta\theta_{e'}$.

By repeating this reasoning to the neighboring nodes of e', and then to their neighbors and so on, one must conclude that the coltage angle at node i is equal to $\Delta\theta_{e'}$, which contradicts inequality B.5. Thus, inequality B.3 is proven.

The proof of inequality B.4. is very similar to that of inequality B.3. Thus, the lemma is proved.

The theorem follows directly from the above lemma because all voltage angles in the external subnetwork are within the maximum and minimum voltage angles in the boundary.

C. MATLAB IMPLEMENTATION

In section 2.3, different approaches for obtaining necessary data regarding power flows and voltages were described.

Three different approaches have been used in this project, DC load flow, AC load flow and linear bounding methods.

The DC load flow with compensation is implemented in *contAnalDC.*

The AC load flow with compensation is implemented in *contAnalLevel2*. This function includes the possibility of running level 2 analyses, as explained in section 3.4.1.

The efficient bounding method is implemented in *boundingList*.

The main functionalities of the techniques are illustrated in the scripts below, whereas operations that are already included in function in Matpower are excluded.

Functions for comparing results, creating plots and visualizing faults, storing results etc. have not been included.

A list of variable names used in the MATLAB scripts:

casedata	Name of power system.
cList	List of all contingencies to study
cont	The current contingency, selected from cList. I.e. cont = cList(i)
mpc	A struct containing all information about the power system, such as power generation and load, impedances and limits
max_it	The maximum number of iterations to run in the AC studies
level_2_length	The number of initial faults to include in level 2 studies

contAnalDC( )

```
function [resVa, success, t1] = ...
    contAnalDC(casedata, cList,varargin)


%% Load and initiate file:

mpc = loadcase(casedata);

namesAndSimplifications;

t0 = tic;

%% initial voltage angle
Va0 = bus(:, 9) * (pi/180);  % bus(:,9) is the voltage angle in degrees

%% build B matrices and phase shift injections

[B, Bf, Pbusinj, Pfinj] = makeBdc(baseMVA, bus, branch);

%% compute complex bus power injections (generation - load)
%% adjusted for phase shifters and real shunts
Pbus = real(makeSbus(baseMVA, bus, gen)) - Pbusinj - bus(:, 5) / baseMVA;

%% "run" the power flow
Va = Va0;

%% Factorization:

Bt = B([pv; pq], [pv; pq]);
[L, U, P, Q] = lu(Bt);

%%
Va([pv; pq]) = Q * (U \ (L \ (P * ((Pbus([pv; pq]) - B([pv; pq], ref) *
Va0(ref))))));

nb = size(bus,1);
nc = length(cList);

resVa = zeros(nb,nc + 1);  % (Number of buses) x (number of outages + 1)
resVa(:,1) = Va;
k = 0;
success = zeros(nc,1);

while k < nc
    k = k + 1;

    cont = cList(k);          % Contingency

    lf = branch(cont,1);      % Line from
    lt = branch(cont,2);      % Line to

    M = sparse([lf lt],1,[1 -1],nb,1);

    dy = -B(lf,lt);
```

```
    M = M([pv;pq]);

    invB_M = Q * (U \(L \(P * M )));

    z = -M.' * invB_M;          %% M^t*H^-1*M

    c = 1/(1/dy + z);

    dVa = c * invB_M * M.' * Va([pv;pq]);

       if (max(Va([pv; pq]) + dVa) < 360 && min(Va([pv; pq]) + dVa) > -360)

            success(k) = 1;
            resVa([pv;pq],k+1) = Va([pv; pq]) + dVa;

       end

end

resVa([ref0;pv0;pq0],:) = resVa([ref;pv;pq],:) * 180 / pi;
t1 = toc(t0);

end
```

doc_ contAnalLevel2.m

contAnalLevel2.m is a script designed to run “level 2” analyses. It is based on the function contAnal.m, where only single contingencies are considered. Only the “level 2” version is included in the appendix.

This script contains the necessary information needed to use the contAnalLevel2()-function.

```
%% Documentation - contAnalLevel2
%
% This script will first run a decoupled load flow with the maximum number
% of iterations, given in the input, as max_it. Default is 10 iterations.
%
% A list of the branches with the highest initial power flow will be
% created. The length of this list is by default 10, but can be altered by
% setting the input parameter level2_size.
%
% After this, a list of contingencies will be studied. By default, this
% will be all possible single line contingencies. This can also be altered
% by changing the input parameter cList.
%
% Contingencies that cause isolation will not be included in this study.
%
% When the single contingency cases are studied, level 2 cases must be
% studied.
% Now, the matrices must be updated explicitly, one by one, and factorized.
% This is done, and the second level contingencies are studied.
%
% The result matrices are stored in a Map, using the following structure:
%
% Each result matrix (for each level2 - case) are saved as cells in the
```

```
% variables ResVm_cell, ResVa_cell. Like this:
% level_2_idx = iterates through the level2_list
% ResVm_cell{level_2_idx} = resVm;
% ResVa_cell{level_2_idx} = resVa;
%
% Res_Vm_Map = containers.Map({level2_list}, ResVm_cell);
% Res_Va_Map = containers.Map({level2_list}, ResVa_cell);
%
% The identifiers can be obtained using the command "keys".
% identifiers = keys(Res_Vm_Map);
%
```

contAnalLevel2( )

```
function      [res_Vm_Map,      res_Va_Map,      success,      t1]      =
contAnalLevel2(casedata,cList,max_it,level_2_length)

%% Load and initiate file:
alg = 2;
mpc = loadcase(casedata);

namesAndSimplifications;

%% Make copies:
bus0 = bus;
branch0 = branch;
gen0 = gen;

%% Check if level_2_length is to big
if level_2_length > nl
    level_2_length = nl;
end

t0 = tic;
%% Initial voltages
% In namesAndSimplifications, the voltages are set to V = 1pu, angle = 0.
% This may be changed later.

Va0 = Va;
Vm0 = Vm;

V0 = Vm(:) .* exp(1i* Va(:) / 180 * pi);
V = V0;

%% Series admittance, Ys

Ys = STAT./(BR_R + 1i * BR_X);      % y, series admittance (p.u.)

%% Ybus - makeYbus()

[Ybus, Yf, Yt] = makeYbus(baseMVA, bus, branch);

%% Sbus

Sbus = makeSbus(baseMVA, bus, gen);

%% B-matrices
```

```
Bmatrices;

%% Run first Fast Decoupled Load Flow.
% This is done to create a reference, and for selection of level 2
% contingencies.

tol = 1e-8;

converged = 0;
ii = 0;

mis = (V0 .*conj((Ybus)*V0) - Sbus) ./ Vm;

P = real(mis([pv;pq]));
Q = imag(mis(pq));


while (~converged && ii < max_it)
    ii = ii + 1;

    %% -----  do P iteration, update Va  -----

    dVa = -Qp * ( Up \  (Lp \ (Pp * P)));

    %% update voltage
    Va([pv; pq]) = Va([pv; pq]) + dVa;
    V = Vm .* exp(1j * Va);

    %%-----  do Q iteration, update Vm  -----
    dVm = - Qpp * ( Upp \ (Lpp \ (Ppp * Q)) );

    %% update voltage
    Vm(pq) = Vm(pq) + dVm;
    V = Vm .* exp(1j * Va);

    %% evalute mismatch
    mis = (V .* conj(Ybus * V) - Sbus) ./ Vm;
    P = real(mis([pv; pq]));
    Q = imag(mis(pq));

    %% check tolerance
    normP = norm(P, inf);
    normQ = norm(Q, inf);

    if normP < tol && normQ < tol
        converged = 1;
        break;
    end
end


    resVm_cell = cell(level_2_length+1,1);
    resVa_cell = cell(level_2_length+1,1);

%% Achieve the actual power flows:
% Use the Matpower function pfsoln.
```

```
%
[busBase, genBase, branchBase] = pfsoln(baseMVA, bus, gen, branch, Ybus,
Yf, Yt, V, ref, pv, pq);

[~, max_idx] = sortrows(abs(branchBase),-14);    % Sort in ascending order.
14 = PF
level_2_list = max_idx(1:level_2_length);
% level_2_idx is the index matrix of all branches that are to be studied
% further.
% It is the set of branches with the highest initial branch flow, before
% fault.

%% Run contingency analysis
% First, run for the base case:

level_2_count = 0;
while level_2_count <= level_2_length
    level_2_count = level_2_count + 1;
    branch(:,11) = branch0(:,11);

    if level_2_count > 1    %% If level 2 analysis is started
        %% Series admittance, Ys
        branch(level_2_list(level_2_count-1),11) = 0;
        STAT = branch(:,11);
        Ys = STAT./(BR_R + 1i * BR_X);     % y, series admittance (p.u.)

        %% Ybus - makeYbus()

        [Ybus, Yf, Yt] = makeYbus(baseMVA, bus, branch);

        %% Sbus

        Sbus = makeSbus(baseMVA, bus, gen);

        %% B-matrices
        Bmatrices;
    end


    nc = length(cList);     %% Number of contingencies
    success = zeros(1,nc);  %% A vector with info regarding convergence
    resVm = ones(nb,nc);
    resVa = zeros(nb,nc);
    k = 0;                  %% Iterator
    m = 0;
% k is an iterator that runs through the contingency list. The output from
% this function is a list of voltage magnitudes and angles for all
% contingency cases. However, if some of the cases does not converge, these
% cases must be omitted from the output. Thus, a variable to keep track of
% column is needed. This is the m variable.

    %% For a list of contingencies...

    while k < nc
    k = k + 1;
    cont = cList(k);

    Va = Va0;
```

```
Vm = Vm0;
V = V0;

%% Calculate compensation terms

if cont ~= 0
    bf = branch(cont,1);
    bt = branch(cont,2);

    %%--- M-vector ------------------

    M = sparse([bf bt],1,[1 -1],nb,1);
    Mp = M([pv;pq]);
    Mq = M(pq);

    %%--- cp / cq -------------------

    dyp = -Bp0(bf,bt);
    dyq = -Bpp0(bf,bt);

    % Pre-calculate Mp*H^-1.
    invBp_Mp = Qp * (Up \(Lp \(Pp * Mp )));
    invBpp_Mq = Qpp * (Upp \(Lpp \(Ppp *Mq )));

    zp = -Mp.' * invBp_Mp;          %% M^t*H^-1*M
    zq = -Mq.' * invBpp_Mq;         %% M^t*H^-1*M

    cp = 1/(1/dyp + zp);
    cq = 1/(1/dyq + zq);

    Ybus2 = Ybus - M*Ys(cont)*M';


else
    M = sparse(nb,1);
    Mp = M([pv;pq]);
    Mq = M(pq);
    cp = 0;
    cq = 0;
    Ybus2 = Ybus;
end

%% Initial mismatch
mis = (V0 .*conj((Ybus2)*V0) - Sbus) ./ Vm;

P = real(mis([pv;pq]));
Q = imag(mis(pq));


%% Solver

tol = 1e-8;

converged = 0;
ii = 0;

while (~converged && ii < max_it)
```

```
        ii = ii + 1;

        %% -----  do P iteration, update Va  -----

        dVa = - Qp * (Up \ (Lp \ (Pp * P)));
        dVaComp = cp * invBp_Mp * Mp.' * dVa;
        Va([pv;pq]) = Va([pv;pq]) + dVa + dVaComp;

        %% -----  do Q iteration, update Vm  -----

        dVm = - Qpp * (Upp \ (Lpp \ (Ppp * Q)));
        dVmComp = cq * invBpp_Mq * Mq.' * dVm;
        Vm(pq) = Vm(pq) + dVm + dVmComp;

        V = Vm .* exp(1j * Va);

        %% -evalute mismatch
        mis = (V .*conj((Ybus2)*V) - Sbus) ./ Vm;

        P = real(mis([pv;pq]));
        Q = imag(mis(pq));

            %% check tolerance
        normP = norm(P, inf);
        normQ = norm(Q, inf);

        if normP < tol && normQ < tol
            converged = 1;
            break;
        end
    end

        if (max(Vm) < 2 && min(Vm) > 0.5) && ...
                (max(Va) < 360 && min(Va) > -360) && ...
                (sum(isnan(Vm))+sum(isnan(Va))) == 0

            success(k) = 1;
            m = m + 1;
            resVm([ref0;pv0;pq0],m) = Vm([ref;pv;pq]);
            resVa([ref0;pv0;pq0],m) = Va([ref;pv;pq]) * 180 / pi;
        end
    t1 = toc(t0);

    end

    resVm_cell{level_2_count} = resVm;
    resVa_cell{level_2_count} = resVa;

end

%% Store output as map. Easy to identify which nodes are which.

res_Vm_Map = containers.Map([0; level_2_list], resVm_cell);
res_Va_Map = containers.Map([0; level_2_list], resVa_cell);

end
```

boundingList( )

```
function  [boundaryNodes,  branchListOut,  t_all]  =  boundingList(casedata,
cList, start_steps, criterion, cri_tol, cri_tol2, cri_tol3, varargin)

%% Define names and set default arguments:
% Removed from appendix, for readability

%% read data
mpc = loadcase(casedata);

% For use in NewIntToExt()...
branch_numbers = mpc.branch(:,1:2);


%% add zero columns to branch for flows if needed
if size(mpc.branch,2) < QT
  mpc.branch    =    [    mpc.branch    zeros(size(mpc.branch,    1),    QT-
size(mpc.branch,2)) ];
end


[bus, branch, gen, busNum0, type, type0] = ...
    NewExtToInt(mpc.bus, mpc.branch, mpc.gen);

%% get bus index lists of each type of bus

[ref, pv, pq] = deal(type.ref, type.pv, type.pq);
baseMVA = mpc.baseMVA;
%% generator info
on = find(gen(:, GEN_STATUS) > 0);      %% which generators are on?
gbus = gen(on, GEN_BUS);                %% what buses are they at?


Va0 = bus(:, VA) * (pi/180);

%% build B matrices and phase shift injections
[B, Bf, Pbusinj, Pfinj] = makeBdc(baseMVA, bus, branch);

%% compute complex bus power injections (generation - load)
%% adjusted for phase shifters and real shunts
Pbus = real(makeSbus(baseMVA, bus, gen)) - Pbusinj - bus(:, GS) / baseMVA;

%% "run" the power flow

Va = Va0;
%% update angles for non-reference buses
Va([pv; pq]) = B([pv; pq], [pv; pq]) \ (Pbus([pv; pq]) - B([pv; pq], ref) *
Va0(ref));

%% update data matrices with solution
branch(:, [QF, QT]) = zeros(size(branch, 1), 2);
branch(:, PF) = (Bf * Va + Pfinj) * baseMVA;
branch(:, PT) = -branch(:, PF);
bus(:, VM) = ones(size(bus, 1), 1);
bus(:, VA) = Va * (180/pi);

%% Set x-values:
```

```
nl = size(branch,1);
nb = size(bus,1);
bf = branch(:,F_BUS);
bt = branch(:,T_BUS);

stat = branch(:, BR_STATUS);          %% ones at in-service branches
b = stat ./ branch(:, BR_X);          %% series susceptance
tap = ones(nl, 1);                    %% default tap ratio = 1
idx = find(branch(:, TAP));           %% indices of non-zero tap ratios
tap(idx) = branch(idx, TAP);          %% assign non-zero tap ratios
b = b ./ tap;
x = 1./b;
%

%% Find boundary and limits

Adj = sparse(bf, bt, 1, nb, nb);
Adj_mat = Adj + Adj' + speye(nb);

nc = nnz(cList);

nodeListOut = zeros(nb,nc);
branchListOut = zeros(nl,nc);

%% LUPQ factorization of B

[L, U, P, Q] = lu(B([pv; pq], [pv; pq]));


t1 = tic;
count = 0;
while count < nc
    count = count + 1;
    cont = cList(count);
% bn = boundary Nodes
    bn = zeros(2,1);
    bn(1) = bf(cont);
    bn(2) = bt(cont);
    node_vec = sparse(bn,1,1,nb,1);


    limit_x = x;
    limit_x(limit_x < 1e-2) = 1e-2;       %% To avoid horrible convergence
in the below calculations:


    f0 = branch(:,PF) ./ baseMVA;                            %% f_pq^0
    if criterion == 1            %% Thermal limits:
        dfmax = (branch(:,RATE_A) - abs(branch(:,PF))) / baseMVA;
    elseif criterion == 2        %% Change relative to initial fault flow:
        dfmax = cri_tol * abs(branch(cont,PF)) / baseMVA;
    elseif criterion == 3          %% Absolute value of active power flow
change:
        dfmax = cri_tol / baseMVA;
    elseif criterion == 4        %% Active change relative to line limit:
        dfmax = cri_tol * branch(:,RATE_A) / baseMVA;
    elseif criterion == 5
```

```
        dfmax1 = cri_tol * ones(nl,1) * abs(branch(cont,PF)) / baseMVA;
        dfmax2 = cri_tol2 * ones(nl,1) / baseMVA;
        dfmax3 = cri_tol3 * abs(branch(:,PF)) / baseMVA;
        dfmax = max([dfmax1, dfmax2, dfmax3],[],2);
    end

    dfxmax = dfmax .* limit_x;                %% delta f_pq^max*x_pq
    limit = min(abs(dfxmax(dfxmax ~= 0))) ;


    t0 = tic;

    k = start_steps;    %% Indicator for number of steps from fault nodes
    while k > 1
        node_vec = Adj_mat * node_vec;
        k = k - 1;
    end


    bn = bus(node_vec > 0);

%% Find boundary and limits

    success = 0;    %% Stop when success = 1;

    visitedNodes = bn;      %% Don't want to check several times...
    visitedBranches = cont;

    t0 = tic;


    if bf(cont) ~= ref
        N_bf = sparse(bf(cont),1,1,nb,1);
        C_bf = Q * (U \(L \(P * N_bf([pv;pq]))));
    end
    if bt(cont) ~= ref
        N_bt = sparse(bt(cont),1,1,nb,1);
        C_bt = Q * (U \(L \(P * N_bt([pv;pq]))));
    end

    k = start_steps;
    col_num = 0;            %% Column in output matrices
    theta_log = zeros(30,1);

    while success == 0 && length(visitedNodes) < nb && k < max_it
        k = k + 1;  % Number of steps from fault line

        % For every round, a new set of boundary nodes are found
        nbN = nnz(bn);          %% Number of boundary nodes
        delta = zeros(nbN,1);

        for ii = 1:nbN
            if bn(ii) == ref
                delta(ii) = 0;

            elseif bf(cont) == ref
                delta(ii)         =         (C_bt(bn(ii))*x(cont))/(x(cont)-
C_bt(bt(cont)));
```

```
            elseif bt(cont) == ref
                delta(ii)        =        -(C_bf(bn(ii))*x(cont))/(x(cont)-
C_bf(bf(cont)));

            else
                delta(ii) = (C_bf(bn(ii))-C_bt(bn(ii)))*x(cont)/ ...
                    (x(cont)-(C_bt(bt(cont))+C_bf(bf(cont))-
2*C_bf(bt(cont))));
            end
        end

        delta_max = max(delta);
        delta_min = min(delta);

        theta = abs((delta_max-delta_min)*f0(cont));
        theta_log(k) = theta;
        if theta <= limit
            success = 1;
            col_num = col_num + 1;
        else

            [nodeList, branchList] = branchesFromNodes(bus,branch,bn);


            nvn = nnz(visitedNodes);        %% Number of visited nodes
            nvb = nnz(visitedBranches);

            nodeList = nodeList(~ismember(nodeList,visitedNodes));

            branchList = branchList(~ismember(branchList,visitedBranches));

            bn = nodeList;


            visitedNodes(nvn+1:nvn+length(bn)) = bn;
            visitedBranches(nvb+1:nvb+length(branchList)) = branchList;
            visitedBranches = unique(visitedBranches);
            if visitedBranches(1) == 0
                visitedBranches(1) = [];
            end
        end

    end
            nodeListOut(1:length(visitedNodes),count) = visitedNodes;
            branchListOut(1:length(visitedBranches),count)             =
visitedBranches';
tt = toc(t0);
end
t_all = toc(t1);

[bus0,  gen0,  branch0]  =  NewIntToExt(bus,  branch,  gen,  busNum0,  type,
type0,branch_numbers);
ref0 = type0.ref;
pv0 = type0.pv;
pq0 = type0.pq;

bus_temp([ref0; pv0; pq0],1:2) = bus([ref; pv; pq],1:2);
```

```
boundaryNodes = zeros(size(nodeListOut));


for jj = 1:nc
    for ii = 1:nnz(nodeListOut(:,jj))
        boundaryNodes(ii,jj) = bus0(bus_temp(:,1) == nodeListOut(ii,jj));
    end
end

boundaryNodes(boundaryNodes ~= 0) = nonzeros(sort(boundaryNodes));
branchListOut(branchListOut ~= 0) = nonzeros(sort(branchListOut));

end
```