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An Analytical Framework for Valuation and Risk Management in Nordic Hydropower Production

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Problem Description

In this thesis, we modify a valuation model for reservoir hydropower plants. Further, by deriving Greeks from the valuation result, we demonstrate how the model can be applied to hedge the hydropower production against price risk. We apply the model in a case study using data from a Norwegian reservoir hydropower plant.

Preface

This master's thesis has been written at The Norwegian University of Science and Technology (NTNU) during the spring of 2018. The thesis is the concluding part of our five year master's degree program within Industrial Economics and Technology Management. We have both specialized in Investment, Finance and Economic Management.

This study is a part of the project *Investment under uncertainty in the future energy system: The role of expectations and learning [InvestExL]*, which is a four year cooperation project between NTNU, Hydro Energi AS and the Norwegian Research Council, in addition to other European universities. Associate Professor Verena Hagspiel is head of InvestExL.

We will like to thank our supervisor, Associate Professor Maria Lavrutich, for great support every week during the process of writing this thesis. She has been motivating and passionate about our thesis. We will also like to thank Professor Stein-Erik Fleten for providing us with valuable insight to the complex field of hydropower operations and risk management.

At last, we will thank Line Hagman and Knut-Harald Bakke at Hydro Energi for providing us with useful material in addition to helpful comments.

Executive Summary

With an increasing share of intermittent renewable energy production in the Nordic power market, it is a common belief that the volatility of the electricity price will increase. Therefore, there is a demand for energy resources that can balance the supply. Reservoir hydropower serves as the most appropriate renewable energy resource for balancing purposes. The hydropower companies must be able to manage price risk if their aim is to avoid profit losses in the volatile market. In order to increase the competence in the industry in relation to these challenges, we propose a model for valuation and risk management of reservoir hydropower production.

Existing valuation and risk management models often require significant computational time. With severe and unexpected changes in the market, companies may take on big losses while waiting for output from these models. In contrast, our model involves a tractable analytical framework, which leads to low computational time and still good results.

The thesis contributes to the literature by providing theoretical insight to risk management from an analytical standpoint. Further, the tractable and presentable model contributes to the industry by increasing competence of risk managers. To the best of our knowledge, we are the first to provide an analytically tractable framework for risk management of hydropower production.

We solve the valuation problem as a continuous time stochastic control problem. The operational boundaries are handled by introducing penalty functions in the objective function, and a linear discharge strategy is introduced to retain the analytical framework. In order to provide insights for risk management, we relate price parameter sensitivities of the valuation result to hedging using forward contracts.

We demonstrate the model performance using a case study of a Norwegian hydropower plant. We find that the valuation model yields a reasonable result for valuation of a reservoir hydropower plant. Further, our results show that the hydropower producer may avoid severe profit losses if applying the hedging model before a downward shock to the electricity price.

Sammendrag

Med en økende andel av ukontrollerbar, fornybar kraft på det nordiske kraftmarkedet, forventer forskere og kraftbransjen at volatiliteten i elektrisitetsprisen vil øke. Derfor er det etterspørsel etter energikilder som kan balansere tilbudet. Vannkraft er den beste fornybare, regulerbare energikilden. Likevel må vannkraftselskap håndtere prisrisikoen for å unngå inntektstap i det volatile markedet. For å øke kompetansen i industrien knyttet til disse utfordringene, presenterer vi en modell for verdsettelse og risikostyring av magasinkraftverk.

Eksisterende verdsettelses- og risikostyringsmodeller krever ofte lang kjøretid. Ved store og uventede endringer i markedet, kan selskapene påta seg store tap mens de venter på at modellene skal kjøre. Modellen vår innebærer derimot et analytisk håndterbart rammeverk som fører til at modellen har kort kjøretid.

Oppgaven bidrar til litteraturen ved å gi teoretisk innsikt i risikostyring fra et analytisk ståsted. Videre bidrar modellen til industrien ved å øke kompetansen til risikostyrere. Så vidt vi vet, er vi de første som gir et analytisk rammeverk for risikostyring av vannkraftproduksjon.

Vi løser verdsettelsesproblemet som et kontinuerlig stokastisk kontrollproblem. De operasjonelle grensene håndteres ved å innføre straffefunksjoner i objektivfunksjonen, og en lineær utslippsstrategi fra magasinet blir introdusert for å beholde det analytiske rammeverket. For å minimere risiko, finner vi deriverte av verdifunksjonen med hensyn på prisparametere og relaterer disse til forwardkontrakter.

Videre utfører vi en casestudie for å vise hvordan modellen fungerer for det norske markedet. Vi finner at modellen gir et realistisk resultat for verdsettelse av et magasinkraftkraftverk. Til slutt finner vi at vannkraftprodusenten kan unngå alvorlig tap av fortjeneste ved å hedge med vår modell i forkant av et negativt sjokk for elektrisitetsprisen.

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1 Introduction

The European power market is under transition. Ambitious climate goals have put green electricity generation on the agenda for several of the European countries, and we can already observe an increasing share of renewables in the energy mix. For instance, 71 percent of all new power capacity under construction in Norway comes from wind power[1]. Wind power and other renewable resources often have an intermittent nature, i.e. they come with limited storage possibilities and the production is difficult to predict. It is therefore a common belief that an increasing share of renewable and intermittent electricity generation may cause a more volatile electricity price (Möbius and Müsgens, 2015; Wozabal et al., 2016; Masoumzadeh and Alpcan, 2018). Also, the increasing share of intermittent electricity generation leads to a higher demand for flexible renewable resources. One of such resources is reservoir hydropower, that offers the possibility to store water. The hydropower plants have low start-up and shut-down costs, hence they are well suited for balancing purposes (Hirth, 2016). The hydropower producers can plan production with respect to expectations of future electricity price and inflow, which are their two most important risk factors (Fleten et al., 2010). However, a more volatile electricity price may increase the cash flow risk for the hydropower producer. This implies that risk management will play a crucial role for the hydropower plants. In order to meet the challenges related to price uncertainty, we propose a model for valuation and risk management of reservoir hydropower plants.

Nowadays, hydropower companies use models based on stochastic or linear programming in order to optimize production and planning decisions, as well as risk management strategies. Such models provide detailed results, but do often

[1] These are the projects registered and approved by The Norwegian Water Resources and Energy Directorate (NVE) by the end of 2017 (NVE, 2018).

require significant computational time. Companies report running times of approximately two hours per system[2]. If there are severe and, most importantly, unexpected changes in the market, the companies may take on substantial losses while waiting for the output from such models. To accommodate rapid market changes, we therefore see a need for faster models, especially when it comes to hedging. The model we provide is computationally efficient due to an analytical framework and thus provides a contribution in this manner.

The contribution of this thesis is threefold. First we present an analytically tractable approach for valuation of a reservoir hydropower plant. Second, we demonstrate how the model can be applied to risk management in hydropower production. Lastly, we apply the model to a case study. The valuation part of the model is a continuous-time stochastic control model, based on Ernstsen and Boomsma (2018), where we assume that the logarithm of the electricity price follows a one-factor mean-reverting Ornstein-Uhlenbeck (OU) process. We provide detailed steps of the derivations which are not included in the previous contribution, we also correct typos and pitfalls done by the authors. Further, we extend the model by computing derivatives, referred to as Greeks, from the valuation result, and demonstrate how the valuation model is applicable for hedging the hydropower production against price risk. The case study is performed using data from a Norwegian hydropower plant.

Hydropower plants operating in Norway participate in the Nordic electricity market. Nord Pool serves as the leading market for physical delivery in northern Europe. Here, power producers participating in the spot market bid in their supply curve for each hour the following day. Electricity suppliers, on the other hand, bid in their expected demand for the same hours. The hourly day-ahead spot price is then set at the price that balances the supply and demand. The electricity supply is uncertain because it depends on unpredictable weather factors such as very cold weather, fluctuations in precipitation and melt water. The demand is also uncertain since it is largely based on consumer behavior. Thus, modeling the evolution of the electricity spot price is not a straight-forward task as it contains seasonality over the year, as well as within a week and within days. Furthermore, differences between production and consumption might cause price shocks since electricity cannot be easily stored.

It has been shown that simple stochastic processes as for example the Geomet-

[2]Running time of long-term models. Based on interviews with Norsk Hydro.

ric Brownian Motion (GBM), will perform poorly for valuation in commodity markets (Dixit and Pindyck, 1994). A common approach is therefore to model commodity prices as mean-reverting processes, with the argumentation that in the long run, the law of supply and demand will push prices back to the long-term mean (Lucia and Schwartz, 2002; Benth et al., 2010). The literature provides many different approaches to simulate electricity prices. Deng (1999) suggests different specifications of mean-reverting processes, which include jump-diffusion, stochastic volatility and regime switching. Thompson et al. (2004) present a model with periodic variation of the electricity price over the day, including shocks, while Davison et al. (2002) use a model with periodic variation over the year when considering only peak days. It is important to note that these contributions have different goals and tailor their price processes accordingly. Ernstsen and Boomsma (2018), who use the model primarily for valuation, finds that the difference in the valuation result between a one factor OU process and an OU process with jumps is one percent. Therefore, we find a one factor mean-reverting process sufficient for our valuation model.

When it comes to valuation methods, a well-known approach is to compute discounted cash flows (DCF), where the expected cash flow is discounted to todays value by an appropriate discount rate. On the simplest form, the expected cash flow is only an estimate based on planned operations and expected prices. An improvement of the simple DCF-approach is to explicitly account for uncertainty by including one or more stochastic processes, and solve an associated optimization problem subject to operational constraints. Such an approach is applied by Ernstsen and Boomsma (2018) and adopted in this thesis. Our model optimizes the discharge in every time step based on the expectation of future electricity prices and a seasonal inflow function. In this way, the valuation model is also an optimal-operation model (Tseng and Barz, 2002). This can be seen as a contribution to the real option literature in the broad sense, since the model considers optimal operation decisions taking into account expectations for the future. Real options have been widely studied over the recent years, and some of the contributions to hydropower operations and valuation are Tseng and Barz (2002) and Thompson et al. (2004). These valuation models involve analytically non-tractable algorithms, and some of them computationally heavy methods. In contrast, since we apply a linear discharge strategy, we are able to keep analytically tractable results and low computational time. Kjærland (2007) also uses a real option approach to evaluate investment opportunities in Norwegian hydropower. However, in contrast to our model, this model does not include production decisions.

For reservoir hydropower producers, production planning is a part of the risk management since they are able to adjust production according to expectations of price and inflow (Hongling et al., 2008). The field of production planning has received a lot of attention in the literature over the recent decades. The majority of the contributions use solution approaches based on linear programming, stochastic dynamic programming or stochastic dynamic dual programming. For example, the Norwegian research organization SINTEF, has developed the power market software EFI's Multi-area Power-market Simulator (EMPS)[3], which is a tool for long-term forecasting and planning in electricity markets (see Wolfgang et al. (2009)). Other such models are One Area Power Market Simulator (EOPS) (SINTEF, 2018a), ProdRisk (SINTEF, 2018b) and European Model for Power system Investment with Renewable Energy (EMPIRE) (Brovold et al., 2014), which all are long-term models for analyzing the power sector, mainly based on linear and stochastic dynamic optimization.

The models above heavily rely on sophisticated numerical algorithms and are therefore inferior to our model when it comes to computational time. This implies that the power producers relying on these models may lack flexibility in quickly reacting to market changes. In volatile markets, the value of the flexibility to adapt to changes may be large (Thompson et al., 2004). During the time it takes to run a conventional risk management model, the hydropower company may already incur substantial losses due to the unexpected market movements. The model we present has a running time of a few seconds, which is considerably shorter than the stochastic dynamic programming models, and may therefore reduce losses. Also, fast models are especially suitable for stress-tests, back-testing of trading strategies and scenario analysis (Berger et al., 2016).

To keep the model analytically tractable, we introduce a number of assumptions. The operational boundaries for the reservoir hydropower plant in our model are handled by introducing penalty functions in the objective function. Accordingly, we use a linear discharge strategy from the reservoir. We assume that the inflow is determined by a function with seasonal behavior. The inflow function we propose is a further extension from the case of Ernstsen and Boomsma (2018), who assume constant inflow.

[3] EFI is the abbreviation for Elektrisitetsforsyningens Forskningsinstitutt, which is the previous name of SINTEF.

We provide detailed derivations of the model to make it transparent and easy for risk managers to replicate. The model has a solid theoretical foundation, and may therefore be used to get general insights on valuation and risk management in electricity markets. The analytical framework implies that mathematical operations can be performed easily, e.g. the derivation of sensitivities with respect to various factors. To the best of our knowledge, we are the first to provide an analytically tractable model for risk management of hydropower portfolios.

The rest of this thesis proceeds along the following lines. The remainder of this section gives an overview of the related literature concerning risk management of hydroelectric power plants. The full model and solution method is presented in Section 2. In Section 3 we perform a case study with data from a Norwegian reservoir hydropower plant. Section 4 contains discussion, Section 5 concludes and provides suggestions for further research.

1.1 Risk management in hydropower production

This section is dedicated to relevant aspects of risk management for hydropower producers and related literature. Many of the hydropower and hedging related contributions after the 1990's have been motivated by deregulation of the electricity market. After the deregulation, power producers got a reason to hedge since the electricity price was determined by supply and demand in a liberal market, rather than being decided by the government.

Hydropower companies are able to manage their exposure to price risk by trading financial instruments available on Nasdaq OMX Commodities, such as forwards, futures and options, or by entering over-the-counter (OTC) contracts for physical delivery. The instruments are designed to reduce price risk over time horizons ranging from days up to three years. The forward contracts are sold to a standardized size of 1 Megawatt hour (MWh) and guarantee the delivery of power for a specified future time period[4]. Forward contracts prescribe physical delivery, while futures are financially settled (Benth et al., 2008). The contracts are settled with the system price as a reference. We will not distinguish any further between the contract types, and refer to both types as forwards.

The contributions on hedging of hydropower operations agree that correct trad-

[4]This is equivalent to financial swaps.

ing of forward contracts reduces price risk for the company. For example, Fleten et al. (2010) show that hedging with forward contracts reduce risk in terms of risk measures such as value-at-risk (VaR), conditional value-at-risk (CVaR) and standard deviation of revenue. Byström (2003) finds that short-term hedging of electricity spot prices with electricity futures reduces variability of the portfolio returns. Giacometti et al. (2011) propose a stochastic multi-stage portfolio model for a hydropower producer, where the goal is to maximize profit for the producer, and reduce the economic risks connected to the fact that electricity spot and forward prices are highly volatile. The authors show that forward contracts can be used for hedging purposes when considering only one source of uncertainty, i.e price risk.

According to Fleten et al. (2010), a hydropower producer is naturally hedged if there exists a negative correlation between inflow and electricity price. That is, when there is low inflow to the reservoirs, the prices will be higher, and vice versa. However, other factors than inflow also affect the electricity price and it is not possible to be completely naturally hedged. Hence, many of the Norwegian hydropower companies have departments for risk management. Sanda et al. (2013) have analyzed hedging trends in electricity commodity markets based on data and hedging policies from twelve Norwegian hydropower companies. They found evidence of widespread risk management practice in Norwegian electricity companies, where the most common hedging products were electricity contracts such as forwards or futures. For most of the companies studied, hedging constituted substantial profits, which makes hedging an interesting topic not only for risk management purposes, but also for speculation.

The hedging approach where the risk manager chooses the portfolio that minimizes risk measures such as VaR and CVaR, is discussed in many hedging related contributions. See Boroumand et al. (2015) for an introduction of these risk measures. For example, Bjerksund et al. (2000) discuss risk management in the electricity market within the VaR-concept. Arguing that the constant volatility assumption often is violated for commodities, they adopt a three-factor model for market risk. They show that a model taking more factors than just electricity price into account performs better as a tool for risk management. Following this argumentation, one-factor price processes have not traditionally been used in risk management. Nevertheless, by differentiating the hydropower value with respect to the price process parameters, we can relate the risk in the price process to hedging.

The approach of making a portfolio close to neutral to market changes by computing derivatives from the valuation and hedge based on these, is well known from financial theory. The derivatives are often referred to as Greeks. Fleten and Wallace (2009) demonstrate how stochastic programming can be used to solve a hydro-scheduling case, and how such a model can be employed in delta-hedging[5] of the electricity portfolio. They define delta as the change in the value of a hydropower plant when there is a one unit shift in the forward curve. The delta is approximated from the average unit shift in the value of the power plant resulting from a unit shift in the forward curve.

A prerequisite for applying our model for delta-hedging is that the forward curve is incorporated in the price process such that differentiating with respect to the forward price is possible. For storable commodities, it is common to model the forward price as a function of the spot price, as in McDonald (2002). This relationship is often referred to as a *cost-of-carry* relationship since it accounts for the benefit of physically holding the commodity (convenience yield) and storage costs. Contributors have argued that the cost-of-carry relationship is weaker for electricity than for other commodities since it is not possible to buy electricity for storage (Koekebakker and Ollmar, 2005; Benth et al., 2008). Nevertheless, Nord Pool is a market characterized by a high share of hydropower with large storage reservoirs. Botterud et al. (2010) therefore argue that the theory of storage costs and convenience yield is still relevant in the Nordic electricity market. The approach is criticized by Weron and Zator (2014), who find that behavior of convenience yield only has limited support in data. It would have been possible to substitute the spot price in our valuation model with the cost-of-carry relationship. However, we find this step too controversial. Another, more novel approach to incorporate the forward curve, and thereby obtain a delta-hedging strategy, is to introduce a time dependent mean in the price process as in Clewlow and Strickland (2000). This extension, however, may come at the expense of the analytical tractability of this model. Therefore, we have left incorporation of the forward curve and subsequent calculation of delta for further research. We choose to focus on hedging the sensitivities to the price process parameters. In this way, we stay consistent with the one-factor price process in the valuation model of Ernstsen and Boomsma (2018).

Several contributors have performed case studies on risk management based on

[5]Delta is a common Greek. In the stock market, delta is defined as the change in the option price resulting from a change in the price of the underlying stock.

data from Norwegian hydropower plants. Mo et al. (2001) present a tool where risk management and production scheduling is integrated into one model. We have adopted the idea of a combined approach in this thesis. In the model of Mo et al. (2001), the risk level is controlled by setting revenue targets, where the objective function is penalized if the target is not reached. The producer faces price and inflow uncertainty, which result in a large stochastic optimization problem. The authors test the model for a Norwegian hydropower company. Based on the same optimization model, but with improved models of spot price extremes and uncertainty of future prices, Kristiansen (2006) solves the same risk management problem on a more realistic case for the same company. He finds that the expected income decrease with increasing penalty of the objective function, but the optimum is relatively flat. In both cases, the model is solved by a combination of stochastic dynamic programming and stochastic dual dynamic programming. According to Mo et al. (2001), the running time is 3 to 4 hours for the test case, while Kristiansen (2006) reports a running time of 15 to 20 hours for the updated case[6]. In contrast, the model we present outputs a hedging strategy within a few seconds, and may contribute to reduce substantial losses during the running time of stochastic optimization models.

[6]The running time of the stochastic optimization models can be improved by access to better computers over time.

2 Model

We start by introducing the optimization problem faced by a hydropower producer. The stochastic control problem is solved using the approach of Ernstsen and Boomsma (2018) that involves penalty functions and a linear control strategy. We let the logarithm of the electricity price follow an OU-process, which captures seasonal behavior of the electricity price. Further, we extend the work of Ernstsen and Boomsma (2018) by allowing the inflow to follow a function accounting for the seasonal behavior related to rainfall and snowmelt. We provide detailed steps of the derivations making the model transparent and easy to replicate. We also state mistakes found in the previous contribution. At last, we show how the analytical tractability of the model can be exploited to build a risk management framework for a hydroelectric power plant based on Greeks.

2.1 The optimization problem

In this section, we present the optimization problem faced by a profit maximizing hydropower producer operating a one-reservoir hydropower plant. Electricity is produced by exploiting potential energy in water led through pipes from the reservoir to the turbine. We assume that the hydropower producer is a price taker in a complete market, with no possibilities to influence the electricity price. The value of the hydroelectric plant, V, is equal its future expected profit discounted by the rate r. The producer faces the following stochastic control maximization problem where $H(L_t, v_t)$ is the instantaneous production, P_t is the current electricity price and L_t is the current water level in the reservoir. Subscript t indicates the time from the starting point. The overall optimization problem is given as

$$V(P,I,L)=\max_{v}\left(\mathbb{E}\left[\int_0^{\infty}e^{-rt}P_tH(L_t,v_t)\mathrm{d}t|P_0=P,I_0=I,L_0=L\right]\right) \tag{2.1}$$

$$\mathrm{d}P_t=\mu_P(P_t)\mathrm{d}t+\sigma_P(P_t)\mathrm{d}Z_t^P \tag{2.2}$$

$$\mathrm{d}I_t=\mu_I(I_t)\mathrm{d}t+\sigma_I(I_t)\mathrm{d}Z_t^U \tag{2.3}$$

$$\mathrm{d}L_t=(I_t-v_t)\mathrm{d}t \tag{2.4}$$

$$L_{min}\leq L_t\leq L_{max} \tag{2.5}$$

$$v_{min}\leq v_t\leq v_{max}. \tag{2.6}$$

Equation (2.2) is the price process, where μ_P is the drift term and σ_P is the volatility term. The price process will be further discussed in the next section. Equation (2.3) is the process for the stochastic inflow, I_t, with drift and volatility terms, μ_I and σ_I. $\mathrm{d}Z_t^P$ and $\mathrm{d}Z_t^U$ are increments of Wiener processes. Constraint (2.5) provides limits for minimum and maximum allowed water levels, L_{min} and L_{max}. Constraint (2.6) gives limits for minimum and maximum allowed discharge rates, v_{min} and v_{max}, i.e. how much water that can be led through the pipes to the turbines[1]. Equation (2.4) is the storage constraint, implying that the change in water level equals the difference of inflow and discharge rate.

We assume that the production from the hydropower plant is given by the following linear relationship

$$H(L,v)=\eta_1 v_t+\eta_0 \tag{2.7}$$

$$\eta_1=\eta\rho g h. \tag{2.8}$$

Where η_1 is the energy equivalent and η_0 is a constant to account for water led outside of the turbine. We assume the overall efficiency of the system, η, is constant, ρ is the density of water, g is the acceleration due to gravity and h is the net drop in meters from L_{max} to the turbine.

[1] We refer to constraints (2.5) and (2.6) as the operational constraints.

2.2 The price process

In this section, we present the stochastic electricity price process. A common way of modeling commodity prices is to define the logarithm of the price as $X_t = log(P_t)$, such that the price is given as $P_t = e^{X_t}$ (Schwartz and Smith, 2000; Lucia and Schwartz, 2002). Similar to Ernstsen and Boomsma (2018), we define the logarithm of the electricity price as $X_t = log(P_t + M)$, such that

$$P_t = e^{X_t} - M, \tag{2.9}$$

where M is the lower bound for the electricity price. The lower bound was introduced to account for periods with possible negative electricity prices. Negative prices may occur in periods with great amounts of unpredictable electricity generation from renewable resources such as wind, especially combined with not having the flexibility to quickly reduce conventional production. For some hours, negative prices have been the case in Denmark (Ernstsen and Boomsma, 2018). The probability of having negative electricity prices increases with increased share of intermittent renewable production (Götz et al., 2014). Negative prices are not observed in the price areas of Norway. Therefore, we will assume $M = 0$ in the case study in this thesis. However, we keep M in the derivations such that the model is able to handle any future occurrences of negative prices.

Ernstsen and Boomsma (2018) solve the valuation problem for three spot price processes; a GBM, a mean-reverting OU-process, and an OU-process with jumps. As expected, they found that the GBM process was inappropriate to evaluate the flexibility within the hydropower plant (value of storage possibility), while the mean-reverting OU and OU with jumps processes were better suited for capturing this flexibility. For the valuation purpose, the OU-process with jumps only showed a marginally different result from the regular OU-process compared to the GBM. Therefore, to simplify computations, we have assumed a one-factor mean-reverting OU-process[2].

[2] We are aware that this process is not able to represent discrete changes due to new information that has more than marginal effect on the electricity price (price spikes). However, the use of such model enables us to capture some of the important properties of the electricity price, in particular the tendency to revert to a long-run level (Fleten et al., 2010). The inclusion of jump process is left for further research.

The stochastic differential equation (SDE) of the mean-reverting process is given as

$$\mathrm{d}X_t = \kappa(\theta - X_t)\mathrm{d}t + \sigma \mathrm{d}Z_t^P. \tag{2.10}$$

The first term of the SDE is the drift term, which is dependent on the current electricity price, where $X_t = log(P_t + M)$. $\kappa > 0$ is a constant determining the speed of mean reversion of the process. The mean reversion level of the process, θ, is defined as

$$\theta = \alpha - \frac{\sigma^2}{4\kappa}, \tag{2.11}$$

where $\alpha > 0$ is a constant and κ is the speed of mean reversion. The second term of (2.10) is the volatility term, where $\sigma \geq 0$ is a constant, representing continuous changes in the electricity price caused by development in the market, hereby changes in supply and demand, changes in economic environment and other new information causing marginal changes in the price.

The SDE in (2.10) has the solution

$$X_t = e^{-\kappa(t-s)}X_s + \theta(1 - e^{-\kappa(t-s)}) + \sigma \int_s^t e^{-\kappa(t-v)}\mathrm{d}Z_v^P, \tag{2.12}$$

for $t > s$, and from (2.12) it follows that the electricity price at time t is given as

$$P_t = (P_s + M)^{e^{-\kappa(t-s)}} e^{\theta(1-e^{-\kappa(t-s)})} e^{\sigma \int_s^t e^{-\kappa(t-v)}\mathrm{d}Z_v^P} - M. \tag{2.13}$$

As can be seen, the mean reversion level for the price process is given by

$$e^{\theta} - M. \tag{2.14}$$

The stochastic price process presented will be used when solving the valuation and risk management problem for the hydroelectric plant.

2.3 Penalty functions

In this section, we present a solution approach to the optimization problem which enables us to solve the problem analytically. As Ernstsen and Boomsma (2018), we relax the upper and lower constraints in (2.5) and (2.6) by introducing penalty functions for the discharge rate and water level. The penalty functions, N_1 and N_2, are given as the following quadratic functions

$$N_1(L) = \Theta_1 L + \Theta_2 L^2 \tag{2.15}$$

$$N_2(v) = \theta_1 v + \theta_2 v^2. \tag{2.16}$$

The coefficients, Θ_1, Θ_2, θ_1 and θ_2, in the penalty functions are chosen based on the parameters $\tilde{P}_v$ and $\tilde{P}_L$, such that the marginal profit required to exceed the limits for the water level is $\frac{\partial}{\partial v}\tilde{P}_L H(L,v) = \eta_1 \tilde{P}_L$, and the marginal profit required to exceed the limits for the discharge rate is $\frac{\partial}{\partial v}\tilde{P}_v H(L,v) = \eta_1 \tilde{P}_v$. I.e., we solve the following set of equations

$$\begin{aligned} \tilde{P}_L \eta_1 &= \frac{\partial N_1(L_{min})}{\partial L_{min}}, \quad -\tilde{P}_L \eta_1 = \frac{\partial N_1(L_{max})}{\partial L_{max}} \\ \tilde{P}_v \eta_1 &= \frac{\partial N_2(v_{min})}{\partial v_{min}}, \quad -\tilde{P}_v \eta_1 = \frac{\partial N_2(v_{max})}{\partial v_{max}}, \end{aligned} \tag{2.17}$$

and obtain the coefficients

$$\Theta_2 = -\frac{\tilde{P}_L \eta_1}{L_{max} - L_{min}}, \tag{2.18}$$

$$\Theta_1 = \frac{\tilde{P}_L \eta_1 (L_{max} + L_{min})}{L_{max} - L_{min}}, \tag{2.19}$$

$$\theta_2 = -\frac{\tilde{P}_v \eta_1}{v_{max} - v_{min}}, \tag{2.20}$$

$$\theta_1 = \frac{\tilde{P}_v \eta_1 (v_{max} + v_{min})}{v_{max} - v_{min}}. \tag{2.21}$$

This means that the penalty functions reach maximum for $L = (L_{min} + L_{max})/2$ and $v = (v_{min} + v_{max})/2$. We choose $\tilde{P}_L = 4S$ and $\tilde{P}_v = P_0 + 4S$, where S is an estimate of the standard deviation of the electricity spot price. See Appendix E.2 for calibration of S.

The penalty functions are added to the objective function and the discounted expected profit of the relaxed problem is then expressed as

$$\mathbb{E}\bigg[\int_0^\infty \bigg(e^{-rt} P_t H(L_t, v_t) + N_1(L_t) + N_2(v_t)\bigg) \mathrm{d}t | P_0 = P, I_0 = I, L_0 = L\bigg]. \tag{2.22}$$

By replacing the original boundaries with penalty functions, we theoretically al-low the water level and the discharge rate to exceed the boundaries, and the value of the hydropower plant may therefore be higher than in the original problem. Ernstsen and Boomsma (2018) emphasize that this issue can be managed by tuning the parameters $\theta_1, \theta_2, \Theta_1$ and Θ_2. Also, they argue that the overestimation is counterbalanced by restriction to the linear control strategy, which reduces the value of the hydroelectric plant.

2.4 A linear discharge strategy

In this section, we derive the optimal discharge rate from the hydropower model and present the linear discharge strategy applied to solve the valuation problem.

The problem we have presented involves uncertainty and a dynamic decision structure. We consider continuous time and infinite time horizon, i.e. a continuous time stochastic control problem. Øksendal (2000) gives an extensive explanation of the solution strategy to stochastic control problems. Following the procedures, the associated partial differential equation, which is often referred to as the Hamilton Jacobi Bellman (HJB)-equation, of the original stochastic problem is

$$\begin{aligned}
&\mu_P(P)\frac{\partial}{\partial P}\tilde{V}(P,I,L) + \frac{1}{2}\sigma_P(P)^2\frac{\partial^2}{\partial P^2}\tilde{V}(P,I,L) \\
&+ \mu_I(I)\frac{\partial}{\partial I}\tilde{V}(P,I,L) + \frac{1}{2}\sigma_I(I)^2\frac{\partial^2}{\partial I^2}\tilde{V}(P,I,L) \\
&+ \rho\sigma_I(I)\sigma_P\frac{\partial^2}{\partial I \partial P}\tilde{V}(P,I,L) + \Theta_1 L + \Theta_2 L^2 \\
&+ \max_v \Big((I-v)\frac{\partial}{\partial L}\tilde{V}(P,I,L) + \theta_1 v + \theta_2 v^2 + PH(L,v)\Big) - r\tilde{V}(P,I,L) = 0,
\end{aligned} \tag{2.23}$$

where $\tilde{V}(P,I,L)$ is the value function for the relaxed problem and ρ is the correlation between the price and inflow processes.

To simplify the computations, Ernstsen and Boomsma (2018) use a constant inflow rate when solving the optimization problem. In reality, the future inflow to a reservoir is stochastic, and hydropower planners take this stochasticity into account for both short-term and long-term production planning. For the valuation purpose, the use of an average inflow rate does not necessarily have considerable effects on the results, as the valuation of future operation is done over an infinite horizon. Nevertheless, the seasonal behavior of inflow is of great importance in Norway, where we have considerable amounts of snow melting every spring. Since we want the model to be relevant for Norwegian hydropower producers, we introduce a time dependent inflow function, f_t. The storage constraint from (2.4) is now given by

$$\mathrm{d}L_t = (f_t - v_t)\mathrm{d}t. \tag{2.24}$$

With the arbitrary inflow function, the HJB-equation reduces to

$$\begin{aligned}&\mu_P(P)\frac{\partial}{\partial P}\tilde{V}(P,L) + \frac{1}{2}\sigma_P(P)^2\frac{\partial^2}{\partial P^2}\tilde{V}(P,L) + \Theta_1 L + \Theta_2 L^2\\&+\max_v\Big((f-v)\frac{\partial}{\partial L}\tilde{V}(P,L) + \theta_1 v + \theta_2 v^2 + PH(L,v)\Big) - r\tilde{V}(P,L) = 0.\end{aligned} \tag{2.25}$$

Note that the value function is now only dependent on the electricity price and the water level.

Further, we assume that the discharge rate is linear with respect to the electricity price and water level, which is similar to Ernstsen and Boomsma (2018)

$$v_t = d_1 + d_2 P_t + d_3 L_t. \tag{2.26}$$

In contrast to other approaches relying on numerical algorithms and Monte-Carlo approaches, e.g. Carmona and Ludkovski (2010), the explicit linear discharge strategy enables us to keep analytical tractability and short computational time.

Applying the first order condition of the maximization problem stated in (2.25), and using the production function given, the optimal discharge rate is

$$v^* = \frac{-\theta_1 - \eta_1 P + \frac{\partial}{\partial L}V(P,L)}{2\theta_2}. \tag{2.27}$$

In Equation (2.27), there is a typo in the paper of Ernstsen and Boomsma (2018), as they do not include multiplication by P in the η_1-term. The error only seems to appear in this equation, but is important to acknowledge, in order to not confuse the reader.

The linearized value of water is inserted into the optimal discharge of (2.27), which is set equal to (2.26). By collecting the terms and rearranging, we get

$$d_1 = \frac{-\theta_1 + (\frac{\partial}{\partial L} - \overline{P}\frac{\partial^2}{\partial L \partial P} - \overline{L}\frac{\partial^2}{\partial L^2})V(P,L)|_{P=\overline{P},L=\overline{L}}}{2\theta_2}, \tag{2.28}$$

$$d_2 = \frac{-\eta_1 + \frac{\partial^2}{\partial L \partial P} V(P,L)|_{P=\overline{P}, L=\overline{L}}}{2\theta_2} \tag{2.29}$$

and

$$d_3 = \frac{\frac{\partial^2}{\partial L^2} V(P,L)|_{P=\overline{P}, L=\overline{L}}}{2\theta_2}. \tag{2.30}$$

Using the linearized discharge rate, it follows that the value of the hydropower plant can be written as

$$\begin{aligned} V(P,L) = \mathbb{E}\Big[\int_0^\infty e^{-rt}\Big(& [d_3^2\theta_2 + \Theta_2]L_t^2 + [d_2^2\theta_2 + d_2\eta_1]P_t^2 \\ & + [2d_2\theta_2 + \eta_1]d_3 P_t L_t + [2d_1 d_2 \theta_2 + d_1\eta_1 + d_2\theta_1 + \eta_0]P_t \\ & + [2d_1 d_3 \theta_2 + d_3\theta_1 + \Theta_1]L_t + d_1^2\theta_2 + d_1\theta_1\Big)\mathrm{d}t\Big]. \end{aligned} \tag{2.31}$$

We now substitute the discharge strategy from (2.26) into the storage constraint in (2.24), and get

$$dL_t = d_3\Big(\frac{f_t - d_1 - d_2 P_t}{d_3} - L_t\Big)\mathrm{dt}. \tag{2.32}$$

Ernstsen and Boomsma (2018) demonstrate that the water level can be written as

$$L_t = L_0 e^{-d_3 t} + \int_0^t e^{-d_3(t-s)}(f_s - d_1 - d_2 P_s)\mathrm{d}s. \tag{2.33}$$

A lemma proving this is replicated in Appendix B.2 for the sake of convenience.

At last, the new expression for the water level is substituted into the expression for discharge rate in (2.26) to obtain

$$v_t = d_1 + d_2 P_t + d_3\big(L_0 e^{-d_3 t} + \int_0^t e^{-d_3(t-s)}(f_s - d_1 - d_2 P_s)\mathrm{d}s\big). \tag{2.34}$$

Now, the only unknowns are the constants d_1, d_2 and d_3 in the linear discharge function. These are derived by inserting the first, second and mixed order partial derivatives of V into Equations (2.28)-(2.30), and thereby solve the equations with respect to the constants. The partial derivatives are summarized in Corollary 1, and the constants are summarized in Corollary 2. Detailed derivations are found in Appendix C.1.

Corollary 1

The first partial derivative of the value of the hydropower plant with respect to water level is given as

$$
\begin{aligned}
\frac{\partial}{\partial L}V(P,L)|_{P=\overline{P},L=\overline{L}} &= (2d_1d_3\theta_2 + d_3\theta_1 + \Theta_1)\int_0^\infty e^{-(r+d_3)t}\mathrm{d}t \\
&+ (2d_2\theta_2 + \eta_1)d_3\int_0^\infty e^{-(r+d_3)t}\mathbb{E}(P_t|P)\mathrm{d}t \\
&+ 2(d_3^2\theta_2 + \Theta_2)\int_0^\infty \overline{L}e^{-(r+2d_3)t}\mathrm{d}t \\
&+ 2(d_3^2\theta_2 + \Theta_2)\int_0^\infty e^{-(r+d_3)t}\int_0^t e^{-d_3(t-s)}(f_s - d_1)\mathrm{d}s\mathrm{d}t \\
&- 2d_2(d_3^2\theta_2 + \Theta_2)\int_0^\infty e^{-(r+d_3)t}\int_0^t e^{-d_3(t-s)}\mathbb{E}(P_s|P)\mathrm{d}s\mathrm{d}t.
\end{aligned}
\tag{2.35}
$$

The second order partial derivative is given as

$$
\frac{\partial^2}{\partial L^2}V(P,L)|_{P=\overline{P},L=\overline{L}} = 2(d_3^2\theta_2 + \Theta_2)\int_0^\infty e^{-(2d_3+r)t}\mathrm{d}t = \frac{2(d_3^2\theta_2 + \Theta_2)}{r + 2d_3} \tag{2.36}
$$

and the mixed partial derivative with respect to water level and electricity price is given as

$$
\begin{aligned}
\frac{\partial^2 V(P,L)}{\partial L \partial P}|_{P=\overline{P},L=\overline{L}} &= d_3(2d_2\theta_2 + \eta_1)\int_0^\infty e^{-(d_3+r)t}\frac{\partial}{\partial P}\mathbb{E}(P_t|P)\mathrm{d}t \\
&- 2d_2(d_3^2\theta_2 + \Theta_2)\int_0^\infty e^{-(r+d_3)t}\int_0^t e^{-d_3(t-s)}\frac{\partial}{\partial P}\mathbb{E}(P_s|P)\mathrm{d}s\mathrm{d}t.
\end{aligned}
\tag{2.37}
$$

Corollary 2

The constant d_3 is found from

$$
d_3 = \frac{1}{2}(-r + \sqrt{r^2 + 4\frac{\Theta_2}{\theta_2}}), \tag{2.38}
$$

where $r, \theta_2, \Theta_2 \in \mathbb{R}$.

Further, d_2 is found from

$$
d_2 = \frac{-\eta_1 + k_{d_2}}{2\theta_2 - a_{d_2}}, \tag{2.39}
$$

where $\eta_1, k_{d_2}, \theta_2, a_{d_2} \in \mathbb{R}$.

Finally, d_1 is found from

$$d_1 = \frac{-\theta_1 + k_{d_1} - \overline{P}\frac{\partial^2 V}{\partial L \partial P} - \overline{L}\frac{\partial^2 V}{\partial^2 L}}{2\theta_2 - a_{d_1}}, \tag{2.40}$$

where $\theta_1, k_{d_1}, \theta_2, a_{d_1} \in \mathbb{R}$. $\overline{P}$ is the average electricity price over the calibration period, and $\overline{L}$ is the average water level.

See Appendix C.1 for computation of the constants k_{d_2}, a_{d_2}, k_{d_1} and a_{d_1}.

We reveal another typo of Ernstsen and Boomsma (2018). In the first term of (2.37) there is missing a constant d_3, which is corrected in Corollary 1 above. Since (2.37) is a part of the expression for d_1, which again is used in Equation (2.31), we clearly see that inclusion of d_3 would change the valuation result. However, we do not know if the authors have corrected the typo when computing the final value of their hydropower plant.

2.5 Solving the linearized control problem

In this section, we solve the final valuation problem. We have shown how to obtain the constants in the discharge function. With the discharge rate from Equation (2.34), we can now write the value of the hydroelectric plant as

$$\begin{aligned}
&V(P,L) = \mathbb{E}\Big[\int_0^\infty e^{-rt} P_t(\eta_1 v_t + \eta_0)\mathrm{d}t | P_0 = P, L_0 = L\Big] \\
&= \eta_1 d_2 \int_0^\infty e^{-rt}\mathbb{E}[P_t^2 | P_0 = P]\mathrm{d}t \\
&+ (\eta_1 d_1 + \eta_0)\int_0^\infty e^{-rt}\mathbb{E}[P_t | P_0 = P]\mathrm{d}t \\
&+ \eta_1 d_3 L_0 \int_0^\infty e^{-(r+d_3)t}\mathbb{E}[P_t | P_0 = P]\mathrm{d}t \\
&+ \eta_1 d_3 \int_0^\infty e^{-rt}\int_0^t e^{-d_3(t-s)}\mathbb{E}[P_t(f_s - d_1) - d_2 P_t P_s | P_0 = P]\mathrm{d}s\mathrm{d}t.
\end{aligned} \tag{2.41}$$

To compute V, we need to derive the expectation, $\mathbb{E}(P_t|P_s)$, the second moment, $\mathbb{E}(P_t^2|P_s)$, and the autocovariance, $\mathbb{E}(P_t P_s)$, of the electricity spot price process.

The marginal distribution of the OU-process is the Normal distribution,

$$\begin{aligned} X_t &\sim N\big(\mathbb{E}(X_t|X_s), Var(X_t|X_s)\big) \\ &= N\big(X_s e^{-\kappa(t-s)} + \theta(1-e^{-\kappa(t-s)}), \frac{\sigma^2}{2\kappa}(1-e^{-2\kappa t})\big). \end{aligned} \tag{2.42}$$

Since $P_t = e^{X_t} - M$, we apply a lemma presented in Ernstsen (2016) to derive the expectation, the second moment and the autocovariance of the price process in Corollary 3. We replicate the lemma in Appendix B.1 for the sake of convenience. The proof of Corollary 3 is presented in Appendix C.2.

Corollary 3
Let $P_t = log(X_t + M)$, where X_t follows an OU-process. Then the expectation of the price process is given as

$$\mathbb{E}(P_t|P_s) = (P_s+M)^{e^{-\kappa(t-s)}} e^{\theta(1-e^{-\kappa(t-s)})+\frac{\sigma^2}{4\kappa}(1-e^{-2\kappa(t-s)})} - M, \tag{2.43}$$

where the long-term expected spot price is

$$\mathbb{E}(P_t|P_s)_{t\to\infty} = e^{\alpha} - M. \tag{2.44}$$

The second moment of the price process is

$$\begin{aligned} \mathbb{E}(P_t^2|P_s) &= (P_s+M)^{2e^{-\kappa(t-s)}} e^{2\theta_t(1-e^{-\kappa(t-s)})+\frac{\sigma^2}{\kappa}(1-e^{-2\kappa(t-s)})} \\ &- 2M(P_s+M)^{e^{-\kappa(t-s)}} e^{(\alpha-\frac{\sigma^2}{4\kappa})(1-e^{-\kappa(t-s)})+\frac{\sigma^2}{4\kappa}(1-e^{-2\kappa(t-s)})} + M^2, \end{aligned} \tag{2.45}$$

and the autocovariance is

$$\begin{aligned} \mathbb{E}(P_s P_t|P_0) &= \mathbb{E}(e^{X_t+X_s}) - M\mathbb{E}(e^{X_t}) - M\mathbb{E}(e^{X_s}) + M^2. \\ &= P_0^{e^{-(\kappa t+\kappa s)}} e^{(\theta+\frac{\sigma^2}{4\kappa})(2-e^{-\kappa t}-e^{-\kappa s})+\frac{\sigma^2}{4\kappa}\left(2e^{-\kappa(t+s)}(e^{2\kappa s}-1)\right)} \\ &- MP_0 e^{-\kappa t} e^{\theta(1-e^{-\kappa t})+\frac{\sigma^2}{4\kappa}(1-e^{-2\kappa t})} \\ &- MP_0 e^{-\kappa s} e^{-\kappa s} e^{\theta(1-e^{-\kappa s})+\frac{\sigma^2}{4\kappa}(1-e^{-2\kappa s})} + M^2. \end{aligned} \tag{2.46}$$

We assume $M = 0$,[3] and insert the obtained expressions into (2.41) to get

$$
\begin{aligned}
&V(P,L) = \mathbb{E}\Big[\int_0^\infty e^{-rt}P_t(\eta_1 v_t + \eta_0)\mathrm{d}t|P_0 = P, L_0 = L\Big] \\
&= \eta_1 d_2 \int_0^\infty e^{-rt}\Big(P_0^{2e^{-\kappa t}} e^{2\theta(1-e^{-\kappa t})+\frac{\sigma^2}{\kappa}(1-e^{-2\kappa t})}\Big)\mathrm{d}t \\
&+ (\eta_1 d_1 + \eta_0)\int_0^\infty e^{-rt}\Big(P_0^{e^{-\kappa t}} e^{\theta(1-e^{-\kappa t})+\frac{\sigma^2}{4\kappa}(1-e^{-2\kappa t})}\Big)\mathrm{d}t \\
&+ \eta_1 d_3 L_0 \int_0^\infty e^{-(r+d_3)t}\Big(P_0^{e^{-\kappa t}} e^{\theta(1-e^{-\kappa t})+\frac{\sigma^2}{4\kappa}(1-e^{-2\kappa t})}\Big)\mathrm{d}t \\
&+ \eta_1 d_3 \int_0^\infty e^{-rt}\int_0^t e^{-d_3(t-s)}(f_s - d_1)\Big(P_0^{e^{-\kappa t}} e^{\theta(1-e^{-\kappa t})+\frac{\sigma^2}{4\kappa}(1-e^{-2\kappa t})}\Big)\mathrm{d}s\mathrm{d}t \\
&- d_2\eta_1 d_3 \int_0^\infty e^{-rt}\int_0^t e^{-d_3(t-s)}P_0^{e^{-(\kappa t+\kappa s)}} e^{(\theta+\frac{\sigma^2}{4\kappa})(2-e^{-\kappa t}-e^{-\kappa s})}\cdot \\
&e^{\frac{\sigma^2}{4\kappa}\left(2e^{-\kappa(t+s)}(e^{2\kappa s}-1)\right)}\mathrm{d}s\mathrm{d}t.
\end{aligned}
\tag{2.47}
$$

Now, the value of the hydropower plant can be found by applying numerical integration methods to solve the integrals in Equation (2.47).

2.5.1 Interpretation of water values

Scientific contributions often refer to water values when explaining production strategies. The water value is defined as the expected future marginal value of stored water. In other words, the value of getting one extra unit of water in the reservoir (Wangensteen, 2012). In our case, $\frac{\partial V}{\partial L}$ can be referred to as the marginal value of water. The linear production strategy described in Section 2.4 corresponds to assuming that the marginal value of water is linear in price and water level. If we consider the constants, d_1, d_2 and d_3, we see that d_1 enters the valuation result in (2.47) two times. The first corresponds to the constant term of the discharge rate, while the second represents the decrease in value caused by the negative effect of the decrease in water level on the future discharge rate. The same holds for d_2, which also enters two terms. The first represents that the increase in price has a positive effect on the discharge rate, while the second term represents the negative effect on the value as a result of decrease in future discharge rate (Ernstsen and Boomsma, 2018). Summarized, one can say that if

[3] In the case study, we assume no negative prices, hence we set $M = 0$. The complete expression of the valuation result including M is found in Appendix D.1.

the production planner decides to produce, the profit increases at that time point, while the producer looses the opportunity to produce with the same water in a future point in time. According to Ernstsen and Boomsma (2018), d_3 is the speed of mean reversion of the water level and represents how the discharge rate changes when the water level deviates from the mean[4].

2.6 A framework for risk management

One of the goals of this paper is to demonstrate how the analytically tractable model presented in the previous section can be applied in risk management. In this section, we demonstrate how we can compute derivatives from the valuation result given in (2.47) and emphasize how risk managers can apply these to hedge the production portfolio against price risk.

According to Eydland and Wolyniec (2003), hedging achieves two things. First, it removes risk. Second, it limits exposure to modeling assumptions. The value of the hydropower plant is the present value of all future expected cash flows, where the future electricity price is uncertain. Hence, there is need for a risk manager to trade contracts to reduce risk of low operational profit for the company[5]. Furthermore, we have assumed a one-factor mean-reverting price process in our valuation model, where the parameters must be estimated. Therefore, the valuation result also is uncertain because of uncertainty in the parameters. Thus, to provide a hedging strategy, we relate the risk in the parameters of the price process to forward contracts.

Since the model we have presented is analytically tractable, it is possible to directly differentiate the value of the hydropower plant, V, with respect to the parameters of the price process presented in Section 2.2. The hedging strategy we present is a way of mapping risk in the parameters to forward contracts using the derivatives, which we refer to as Greeks. According to Fleten and Wallace (2009), the portfolio value depends in principle on all futures that together constitute the forward curve. That means, if the forward curve goes to zero, then V goes to zero, and the other way around. If V is simply the sum of long positions in all forward contracts, positive derivatives must be offset by selling (short position)

[4]The mean reversion level of the water level is given by the relationship $\frac{f-d_1-d_2P_t}{d_3}$.
[5]We disregard uncertainty in inflow.

contracts, while negative derivatives must be hedged by buying contracts (long position).

2.6.1 Deriving Greeks

In the following, we present the relevant Greeks that can be used for risk management of the hydropower production. We start with the derivative with respect to the starting price of the price process, P_0,

$$\frac{\partial V}{\partial P_0}. \tag{2.48}$$

Since the forward curve starts at the current spot price, a change in P_0 is equivalent to a parallel shift of the short end of the forward curve. An upward shift of the forward curve will increase the value of the hydropower plant, and this risk may be hedged by shorting forward contracts.

The second component we consider is the derivative with respect to the speed of mean reversion, κ,

$$\frac{\partial V}{\partial \kappa}. \tag{2.49}$$

The derivative implies a change in the value of the plant when the speed of mean reversion changes. Considering the extreme scenario, where κ goes to infinity, there will be no need for hedging since deviations from the long-term mean would immediately be counterbalanced. In the opposite case, if κ approaches zero, we would get a pure Wiener process with no mean reverting behavior. It follows that the speed of mean reversion affects the expected deviations from the mean. Hence, for low speed of mean reversion, hedging will be more essential since shocks will have a more long-lasting effect.

The effect of a change in the starting price, P_0, also depends on the speed of mean reversion, κ. Due to the features of the price process, in the long run the price will revert to the long-term mean with the speed of mean reversion. The risk in P_0 will be related to the nearest future if the speed of mean reversion is high. To keep a neutral position with respect to this risk, the risk manager should therefore buy or sell a portfolio of near-term products. That is, put most weight in the contracts with the nearest time to maturity and less in the less near contracts.

It may also be beneficial to hedge the risk in the volatility parameter[6], σ,

$$\frac{\partial V}{\partial \sigma}. \tag{2.50}$$

The derivative shows how the value of the plant changes as a result of a unit change of the volatility parameter. To hedge the risk in the volatility parameter itself, one should invest in contracts corresponding to the derivative with respect to the parameter. Turning to the extreme scenarios, a volatility parameter approaching infinity implies that the price process have infinitely many spikes, and it would be impossible to estimate the price in the next time step. On the other hand, volatility reaching zero implies a deterministic price function, with no need for hedging at all. Hence, the magnitude of the volatility says something about the importance of hedging.

Lastly, we consider the risk in the parameters θ and α. To get an impression of what the parameters represent, we repeat the long-term properties of the price process. The mean reversion level of the process is $e^{\theta} - M$, while the expected future price converges to $\mathbb{E}(P_t|P_s)_{t\to\infty} = e^{\alpha} - M$, due to the skewness of the volatility term (Ernstsen and Boomsma, 2018). That is, θ is the parameter determining the mean reversion level, while α is the parameter which determines the long-term expected mean.

The risk in θ can be related through the derivative

$$\frac{\partial V}{\partial \theta}. \tag{2.51}$$

Since the value of the hydropower plant is the value of all production over an infinite horizon, it will be strongly dependent on the mean reversion parameter. A change in θ will be equivalent to a shift of the whole forward curve. Further, because the value of the hydropower plant depends on all contracts that together constitute the forward curve, hedging the risk in the long-run mean on its own will simply mean buying or selling all non-overlapping contracts, i.e. the whole forward curve. When the risk in the long-term mean is seen in relation to the risk in the other parameters, and the hedge of these, it may be sufficient to trade contracts in the long-end of the forward curve in order to hedge the risk in θ. If, for example, the short-term risk is hedged by neutralizing the sensitivities of P_0

[6]The derivative with respect to the volatility parameter must not be confused with Vega, which is a common Greek corresponding to a change in the implied volatility of the forward curve.

and κ, then the rest of the risk in θ could be managed through buying or selling long-term contracts with maturities of 1-3 years.

Finally, the risk in the long-term expected mean is related to the derivative with respect to α,

$$\frac{\partial V}{\partial \alpha}. \tag{2.52}$$

As for θ, the total risk in α may be hedged by trading non-overlapping contracts over the whole forward curve. We notice that if the risk manager has invested in contracts such that the payoff is neutral to the risk connected to κ and σ, then the risk in the mean reversion level, θ, will only be related to the risk in α . It follows that if we hedge α[7], the risk in the mean reversion level will be totally hedged. Thus, in this case additionally hedging of θ will be redundant[8].

The complete expressions of the Greeks are comprehensive and found using a mathematical software. The final derivatives are found in Appendix D.2.

2.6.2 Payoff from forward contracts

At last, we consider the payoff from trading forward or futures contracts. The payoff is dependent on the traders position and the spot price. If the power company sells a forward contract, it will profit from the hedge if the agreed forward price is higher than the spot price at the time of delivery[9]. The payoff from a short position is given by

$$\text{payoff}_{\text{SP}} = F - P_t. \tag{2.53}$$

The buyer of a forward or futures contract is long in the contract. The profit from a long position increases with the spot price, and the payoff is given by

$$\text{payoff}_{\text{LP}} = P_t - F, \tag{2.54}$$

where P_t is the current spot price, and F is the price of the forward contract.

[7]Recall that θ is defined as $\theta = \alpha - \frac{\sigma^2}{4\kappa}$.

[8]We would like to thank Professor Stein-Erik Fleten at NTNU for valuable comments on the interpretation of the derivatives.

[9]For futures contracts, the difference in the forward price and the spot price is settled daily through a mark-to-market settlement, which covers profit or loss from day-to-day changes in the daily closing price. In addition to a final settlement, which starts on the expiry date. Throughout the final settlement period, the member is credited/debited an amount equal to the difference between the spot market price and the futures contracts final closing price (NASDAQ, 2018).

3 Case Study

In this section, we perform a case study of the model presented in Section 2 considering a Norwegian reservoir hydropower plant. We start by presenting the system studied, and data related to it. We measure the performance of the valuation model by comparing simulations to realized time series for the power plant considered. Further, we compute the value of the hydropower plant and compare it with two benchmarks for the value of the same plant. At last, we demonstrate a numerical example of the hedging model.

3.1 The power system

The power plant studied is located in the western part of Norway. It is one of the largest hydropower plants in Norway, with an installed capacity of approximately 400 MW. The total system consists of several reservoirs and creek intakes. However, to be able to apply the one-reservoir model, we simplify the system by accumulating the main reservoir and the intake reservoir, and treat them as one large reservoir. We assume that the reservoir has the shape of a cylinder with random end surfaces. That is, it may have any random surface, but the surface area a is constant between the regulating limits. The parameters for the power system are given in Table 3.1, while Figure 3.1 presents an illustration of the system.

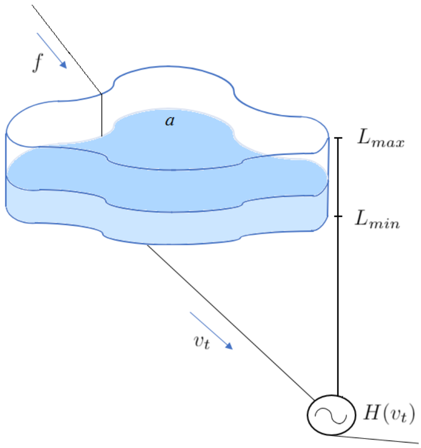

Figure 3.1: Illustration of system

a	$\frac{L_{min}}{a}$	$\frac{L_{max}}{a}$	$\frac{v_{min}}{N_{secs}}$	$\frac{v_{max}}{N_{secs}}$	$\frac{\eta_0}{N_{hours}}$	$\frac{\eta_1 N_{secs}}{N_{hours} \cdot 10^6}$ [1]	η
m^2	m	m	m^3/s	m^3/s	MW	MW/m^3	-
$31 \cdot 10^6$	1030	1040	0	40.605	0	9.2107	0.9028

Table 3.1: System parameters

With the assumption regarding the shape of the reservoir, we calculate the surface area, a, from the known height and volume. The minimum and maximum storage levels allowed in the reservoir, given as L_{min} and L_{max} are defined as the net height from lowest regulation height (LRV), and highest regulation height (HRV), to the turbine. These limits are usually set by the government[2] due to environmental concerns, e.g. in order to avoid flood and to protect fish and other living organisms in the reservoir. Recall that v_{min} and v_{max} represent the minimum and maximum boundaries for discharge rate, respectively. The minimum discharge rate is set by the government due to environmental concerns. The maximum discharge rate stems from the maximum generation capacity, and is derived from (2.7), where $\eta_1 = \eta \rho g L_{max}$, is the energy equivalent from (2.8) and η is the total efficiency of the production facility[3]. In this case study, η is set to 0.9028[4], meaning

[1] $N_{hours} = 8760$ is the number of hours in a year, and $N_{secs} = 3600 \cdot N_{hours}$ is the number of seconds in a year.

[2] In particular, The Norwegian Water Resources and Energy Directorate (NVE).

[3] We define the production facility as the turbine in series with the generator. The total efficiency is the multiplied efficiency of the turbine and the generator. We disregard friction losses in the pipe.

[4] The total efficiency varies from 0.8936 to 0.9124, we assume constant average efficiency.

that we have a total energy loss of almost 10 % through the system. Further, the constant η_0 from the production curve (2.7) is a scale parameter to adjust for water flow, which must be led outside the pipe to ensure minimum flow in the river for environmental reasons. If the reservoir has a high altitude, a significant river flow outside of the pipe is equivalent to a substantial loss of potential energy, hence it will result in high costs for the company. The power plant considered has in fact one of the reservoirs in Norway at the highest altitude. Therefore, they have been granted no restriction on minimum water flow outside of the pipe, thus, η_0 is zero.

3.1.1 Estimating the inflow function

We estimate an inflow function based on hourly inflow to the reservoir over the period from February 20th 2008 to February 20th 2018. It can be seen from the historical data of Figure 3.2 that the inflow vary substantially over the year, but with a clear seasonal behavior. The large fluctuations can be explained by snow melting during the spring together with fluctuating precipitation throughout the year.

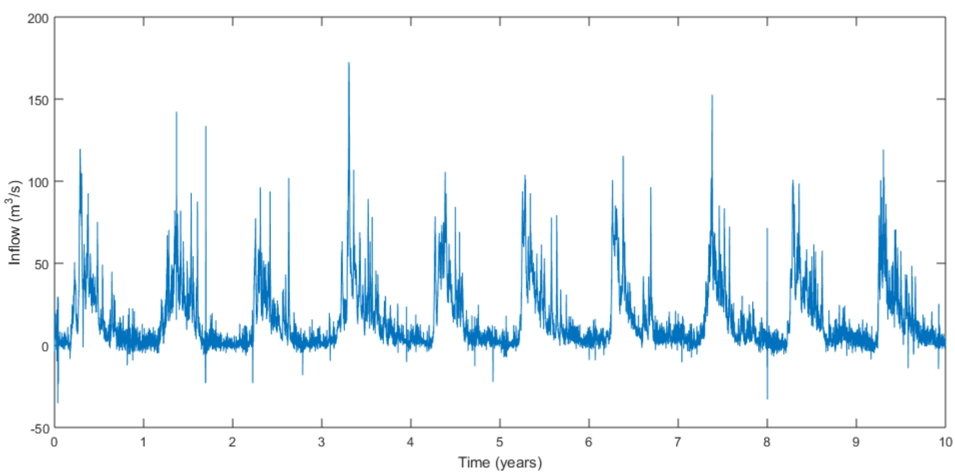

Figure 3.2: Historical inflow to the reservoir from Feb. 2008 to Feb. 2018

From the data given, a periodical function seems to be the best fit. We simulate inflow for 30 years, by repeating the ten year historical data three times. This is done because we assume that the value of the hydropower plant is close to zero after 30 years.

It is worth mentioning that some of the historical measurements are negative. This may be explained by wind that causes swells in the water which disturb the measurements. In reality, negative inflow only occurs through evaporation, which might be significant in some countries, but not in Norway[5]. We set the negative measurements to zero to avoid a biased inflow function. Further, we fit a sinusoidal deterministic function to the data, on the form given by (3.1).

$$f_t = a_1 sin(b_1 t + c_1) + a_2 sin(b_2 t + c_2) + a_3 sin(b_3 t + c_3) + a_4 sin(b_4 t + c_4) + a_5 sin(b_5 t + c_5) \quad (3.1)$$

The choice of having a five-term sinusoidal function is based on a trade-off between goodness of fit and simplicity. Further details are given in Appendix E.3. The resulting fitted curve is shown in Figure 3.3, with the explicit function given as

$$f_t = 131.1 sin(0.0697t + 1.295) + 18.79 sin(6.2744t - 0.9245) + 9.645 sin(12.5488t - 2.903) + 115 sin(0.0750t - 1.867) + 5.919 sin(18.8232t + 1.748). \quad (3.2)$$

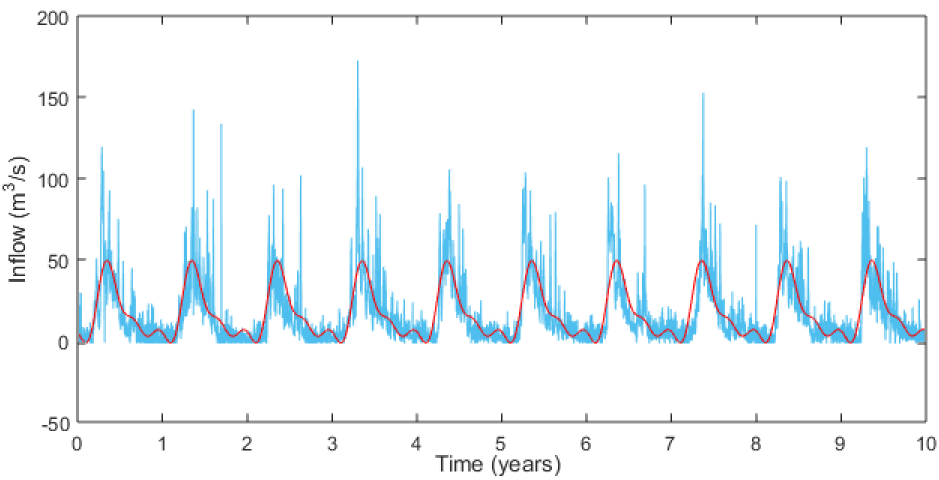

Figure 3.3: Historical inflow data with fitted sinusoidal curve for the period Feb. 2008 to Feb. 2018

The fitted function does capture the seasonal variations, but does not vary significantly from year to year. This follows from the repetitive nature of the historical inflow, as well as the chosen sinusoidal behavior. Note that the spikes in the his-

[5]Evaporation from reservoirs located in Norway is minimal due to the cold climate.

torical inflow are not captured by the deterministic function. This means that we are unable to simulate the highest spikes in the water level and discharge rate. However, it is clear that a seasonal inflow function fits better to the historical data than a constant function, which is used by Ernstsen and Boomsma (2018).

3.2 Valuation

In this section, we focus on valuation of the hydroelectric plant. We start with a simulation of the OU electricity spot price process. Further, we simulate the water level, which is used as input for the final simulation of the discharge rate. We discuss the simulation results and compare them to historical data. Finally, we compute the value of the hydroelectric plant and compare it with benchmarks for the value of the same plant.

3.2.1 Simulation of the electricity price

For simulation of the price process, we collect hourly day-a-head system spot prices from Nord Pool for the period January 1st 2013 to February 15th 2018. The price process considered is a mean reverting OU-process, where the parameters of the process are summarized in Table 3.2. The starting price for the OU-process, P_0, is set to the average electricity price over the calibration period. The asymptotic mean of the price process is denoted by α, κ is the speed of mean reversion, i.e. how strong the system reacts to perturbations from the mean, whereas σ represents the volatility of the process. The parameters are calibrated by linear regression of the log-returns of the electricity price, see Appendix E.1 for further details. The exogenously given discount rate, r, in the valuation problem is defined such that the first 30 years cover 99 % of the discounted value,

$$\int_0^{30} e^{-rt}dt = 0.99 \int_0^{\infty} e^{-rt}dt. \tag{3.3}$$

Solving (3.3) yields a discount rate of $r = 0.1535$. Ernstsen and Boomsma (2018) claim to use the same equation. However, they end up with $r = 0.2062$, which seems like a careless mistake. The valuation result is strongly dependent on the discount rate. Using a too high discount rate, Ernstsen and Boomsma (2018) underestimates the value of their hydropower plant by approximately 15 %[6].

P_0	κ	σ	θ	α	r
NOK	-	-	-	-	-
252.99	174.38	6.1761	5.4863	5.5413	0.1535

Table 3.2: Parameters of price process

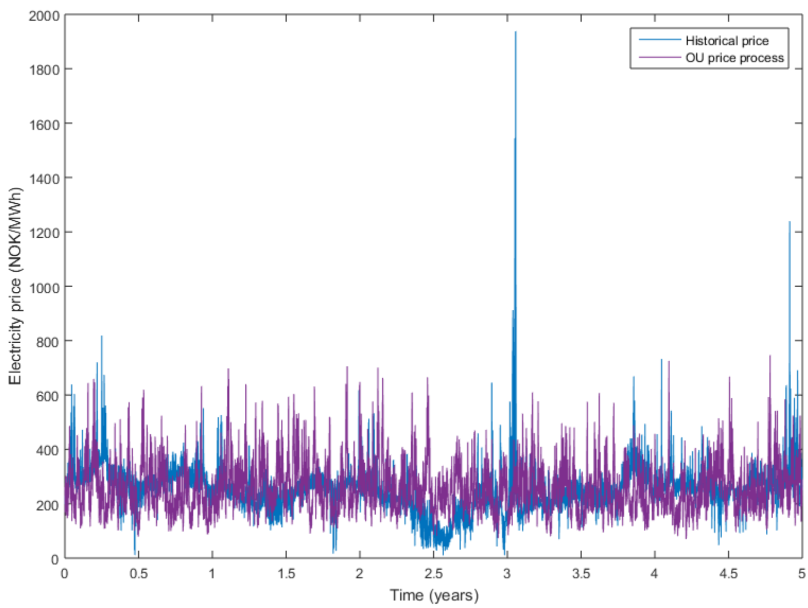

Figure 3.4: Simulated hourly electricity price and historical electricity price for the period Jan. 2013 to Feb. 2018

Figure 3.4 shows the OU-process together with the observed hourly electricity prices for the calibration period. The OU-process captures the long-term fluctuations of the price and is concentrated around the same mean as the historical prices. Hence, we conclude that the price process is suited for valuation of the power plant, which is one of the main goals of this thesis.

[6]The result is found running the model with the parameters from Ernstsen and Boomsma (2018).

3.2.2 Simulation of the water level

With the inflow function and price process defined, we can now use Equation (2.33) to simulate the water level of the reservoir. Figure 3.5 shows simulations of the water level for both variable and constant inflow, together with the historical observed water level. The statistics from the simulations are summarized in Table 3.3. The range is defined as the difference between the highest and the lowest observations. The RMSE is the average root mean squared error, which is a measure for goodness of fit between the simulations and the historical data. From the statistics, we can see that the simulation of water level using variable inflow captures considerably larger range in water level than the one using a constant inflow function. Additionally, calculating the RMSE, we see that the simulation with variable inflow performs substantially better than the simulation with constant inflow. When measuring the error as a percentage of the range of the historical inflow, we find that the simulations have errors of 19.2 % and 31.9 % respectively. It comes clear that the inclusion of the seasonal inflow function is an improvement from the case of Ernstsen and Boomsma (2018), who assumed constant inflow. Thus, we will use the seasonal inflow function in the remainder of this thesis.

	Range (m)	RMSE (m)	RMSE as % of total range
Historical inflow	10.08	-	-
Simulation variable inflow	8.503	1.94	19.2
Simulation constant inflow	1.500	3.22	31.9

Table 3.3: Statistics for water level

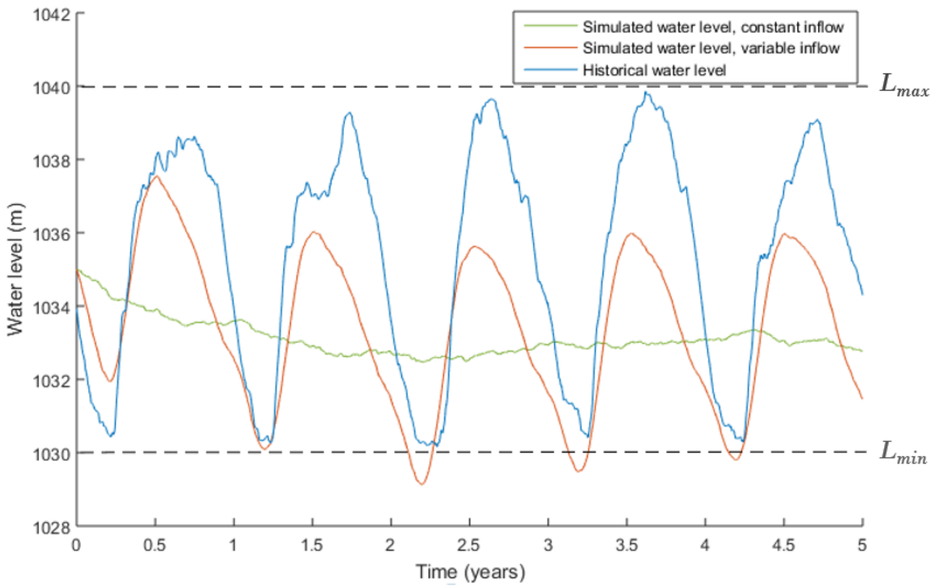

Figure 3.5: Comparison of simulations and historical water levels

We note that in the simulation with seasonal inflow, the boundary L_{min} is violated for some years. This can be explained by the features of the model, where penalty functions were introduced to relax the upper and lower bounds of the operational constraints. Water level below the lower bound indicates that the model overestimates the value of the hydropower plant. A solution to this problem may be to tune the parameters Θ_1 and Θ_2 in the penalty function. Also, recall that in reality, the inflow is stochastic. The fitted inflow function is not able to capture all the spikes of the historical inflow. Including stochastic inflow in the model may yield a larger range of the water level in the simulations. However, adding stochastic inflow would increase the complexity of the overall problem, which implies increased computational time.

3.2.3 Simulation of the discharge rate

The discharge rate is dependent on water level, inflow and electricity price through the linear relationship in (2.34). Figure 3.6 shows the simulated discharge rate for five years.

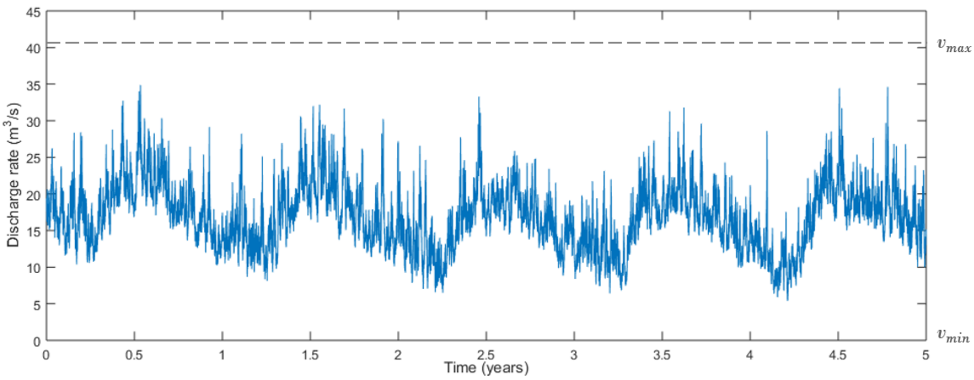

Figure 3.6: Simulated discharge rates for five years

From Figure 3.6 we can clearly see a seasonal behavior of the discharge rate that coincides with the water level. We also notice that the simulated discharge rate always is kept within the boundaries for minimum and maximum discharge. In fact, the discharge rate never reaches the maximum or the minimum production limits. This can, as for the water level, be explained by the choice of the seasonal inflow function, which does not capture all the peaks in the historical inflow. Another reason may be that the parameters θ_1 and θ_2 in the penalty function are chosen such that a discharge strategy close to the boundaries is suboptimal. Decreasing the marginal penalty, $\tilde{P}_v$, for violating v_{min} and v_{max} would yield a discharge strategy closer to the boundaries. However, this will also imply that the probability of violating the operational constraints increases.

We compare the simulation to the historical discharge rate by taking the average discharge rate for every week, in a five year period. Figure 3.7 displays the comparison. The statistics are summarized in Table 3.4.

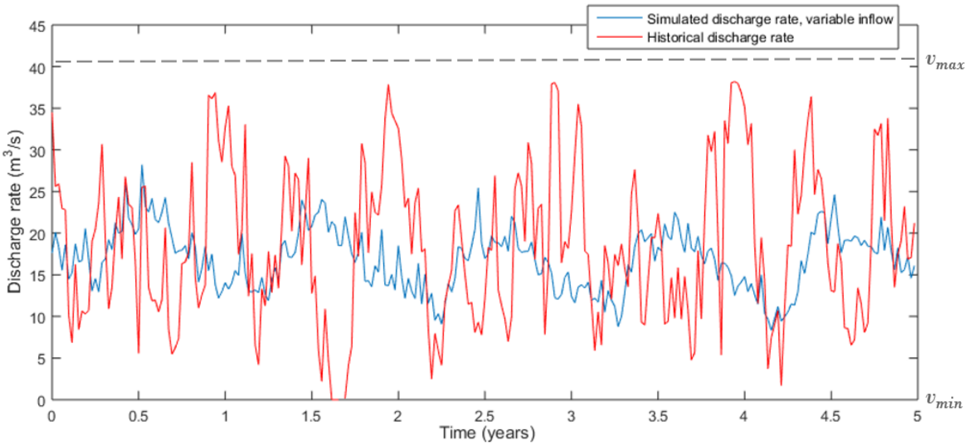

Figure 3.7: Historical weekly average discharge rates from Feb. 2013 to Feb. 2018, compared with simulation of the discharge rate.

	Range (m^3/s)	Mean (m^3/s)	Median (m^3/s)	RMSE (m^3/s)	RMSE as % of total range
Historical discharge rate	38.20	18.64	17.78	-	-
Simulation variable inflow	21.93	16.57	16.53	10.57	27.7

Table 3.4: Statistics for weekly average discharge rates

The historical average discharge rate has significantly larger range than the simulated one. This is not surprising since the inflow in reality is stochastic, and the producer may consider other factors when making its production decisions. We find that the RMSE of the discharge rate simulation is 10.57 m^3/s. As a percentage of the range of the historical inflow, the error is 27.7%.

The intuitive strategy of a power producer will be to optimize its whole portfolio of operations in order to maximize profits. Our model optimizes the value of the power plant alone, not considering that the owners might have other power plants in its portfolio or that the company may operate in other markets than the spot market[7]. The valuation model does not take these factors into account, and it

[7]Other markets could be intraday or balancing markets in addition to long-term contracts traded OTC.

is, therefore, not surprising that the simulated discharge strategy does not match the historical observations. However, the mean and median from the simulated discharge are sufficiently close to the observed ones, as seen in Table 3.4. This indicates that the model may still be suitable for valuation of the particular plant, which is the purpose of this model.

3.2.4 Summary of simulations

The simulations cannot solely be discussed separately. Figure 3.8 shows the electricity price, inflow, water level and discharge rate simulations for the same five year period.

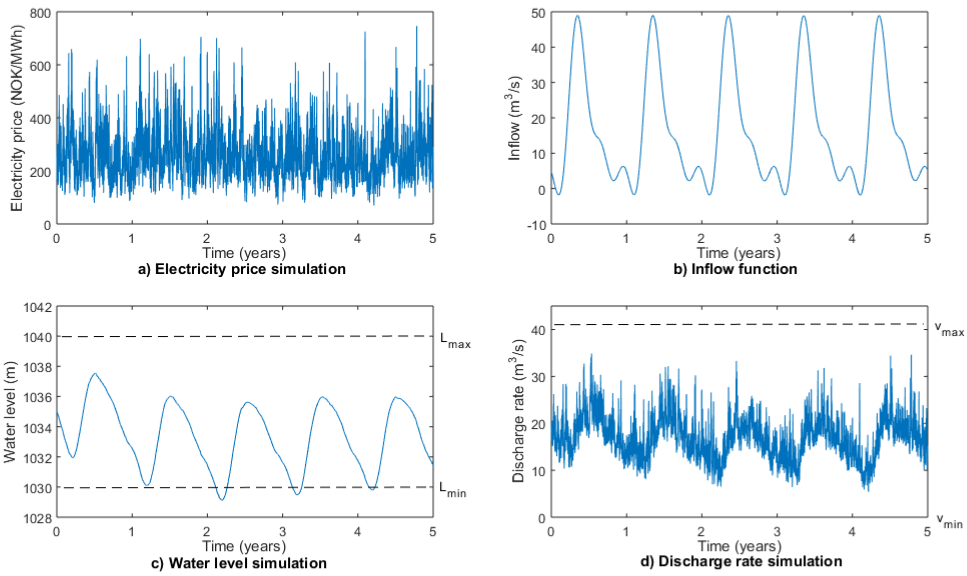

Figure 3.8: All simulations together

First, we see that the seasonality in water level is strongly dependent on the seasonality in inflow. This is obvious as the water level increases with higher inflow. The discharge rate also seems to follow the same seasonality, i.e. the model proposes a strategy where the hydropower plant should discharge when the water level is high. We can also recognize some of the peaks from the electricity price in the discharge rate simulation, though the trend of the discharge rate is determined by the water level.

3.2.5 Numerical result to the valuation problem

In this section, we will discuss the result from the valuation model. Based on the price process and the system parameters, we find that the value of the hydropower plant is 2321 million Norwegian kroner (MNOK)[8]. To get a grasp on the validness of the result, we have compared it with two different benchmarks. The first benchmark, V_{EP}, is based on the company's reported yearly expected production and the second, V_{AR}, is based on the company's annual reports.

V_{model}	V_{EP}	V_{AR}
MNOK	MNOK	MNOK
2321	2562	1551

Table 3.5: Comparison of valuation results

The value of the expected production is an estimate of the discounted cash flow of the plant's operations. The hydropower company reports a yearly production from the plant of 1570 Gigawatt-hours (GWh). If we assume that all production is sold on the spot market to the average electricity spot price from the calibration (P_0 from Table 3.2), we get a yearly revenue, $\overline{PQ}$, of 397 MNOK. In (3.4), we discount the same yearly revenue by the discount rate r from Section 3.2.1 over a period of 30 years, to get V_{EP}.

$$V_{EP} = \int_0^{30} \overline{PQ} e^{-rt} dt \tag{3.4}$$

$$V_{AR} = \int_0^{30} \overline{\pi} e^{-rt} dt \tag{3.5}$$

According to annual reports, the company has a total production of approximately 10 Terawatt-hours (TWh) per year, which of this power plant makes up 15.7%. We assume the plant contributes to the total Earnings Before Interests and Taxes (EBIT) with the same share. Further, we assume the same EBIT, $\overline{\pi}$, every year and then discount it over 30 years to get V_{AR}. This benchmark is naturally lower than the first one as it includes operation costs. Conclusively, we find that the output from the model is placed between the two benchmarks. As we do not account for costs, the most natural is to compare with the first benchmark. The model result is closest to the benchmark V_{EP}, hence it seems to be a realistic result.

[8]Note that we disregard taxes and operational costs in the valuation.

3.3 Hedging

This section is dedicated to a case study of the hedging model. First, we incorporate taxes and present an upper limit for the hedge ratio. Further, we present the case with the related price parameters. Thereafter, we present the resulting Greeks and related hedging approaches. Finally, we present a numerical example to evaluate the performance of hedging the risk in one of the parameters.

3.3.1 Incorporating taxes

In order to perform a case analysis in the Norwegian market, we need some information regarding taxes. The Norwegian hydropower companies must pay taxes to the government from their operating revenue. The revenue from physical production enforces both corporate tax and a resource rent tax. The latter is a tax imposed due to the fact that the resources used for the electricity production, i.e. water resources, is of common value for the country rather than for the industry alone. Per 2018, the corporate tax in Norway is 23% and the tax on resource rent is 35.7% (KPMG, 2018). Revenues from financial contracts are only subject to corporate taxes, and we can therefore derive an upper limit for the hedge ratio, x, from the calculation of the change in after-tax-profit from a unit increase in the spot price (Sanda et al., 2013).

$$\begin{aligned}
&\text{Increase after-tax profit of production} = \text{Decrease after-tax profit financial contracts}\\
&1 - 0.357 - 0.23 = -(x - 0.23x)\\
&\Rightarrow x = -0.536
\end{aligned} \tag{3.6}$$

The derivations above imply an upper hedge ratio (short position) of 53.6%. Ratios above this will be disadvantageous for the hydropower company because an increase in the spot price will lead to a higher decrease in the after-tax profit from the financial contract than the increase in the after-tax profit from the physical production.

3.3.2 Results

To account for new information, the hedging strategy should be dynamically updated to minimize risk. The closest upcoming period of time is most likely to be more dependent on the last days than on the mean over a long period. Thus, the input parameters for the hedging case must be calibrated over a shorter period of time than if the model is used only for valuation purposes.

We calibrate price parameters based on hourly spot prices for twelve weeks ranging from February 19th to May 13th 2018. The parameters and their standard deviations are presented in Table 3.6. Note that the parameters cannot be observed directly, they are results of the chosen model and are estimated based on observed historical prices. In this case, the parameters are updated weekly to see their evolution over time.

	P_0	κ	σ	θ	α
Week	NOK	-	-	-	-
1	351.66	1209.53	8.7288	6.0586	6.0743
2	367.14	1663.46	14.969	6.1126	6.1463
3	341.85	714.862	7.1417	6.0094	6.0272
4	352.72	817.619	5.1859	5.9678	5.9760
5	377.96	1299.98	5.8388	5.9964	6.0030
6	380.59	844.610	4.2372	6.0063	6.0116
7	371.59	1179.79	3.6308	5.9725	5.9753
8	376.07	1293.53	3.2357	5.9557	5.9577
9	327.01	611.110	5.3508	5.9160	5.9277
10	326.57	1166.72	4.2039	5.8596	5.8634
11	262.02	966.983	9.9537	5.7308	5.7564
12	192.17	779.834	27.898	5.3198	5.5693
Standard deviation	55.985	306.72	6.9932	0.20935	0.15294

Table 3.6: Parameters of price process calibrated for twelve weeks

The standard deviation is a measure of parameter volatility. The estimated parameters and their standard deviations confirm the importance of updating the parameters and Greeks to retain hedged positions. In particular, the standard deviation of κ confirms the findings of Barlow et al. (2004). Their results showed that there is a large uncertainty related to the estimation of the speed of mean reversion since it may vary considerably over time.

Figure 3.9 visualizes the evolution of the parameters over the weeks. We observe that there are severe market changes towards the end of the twelve-week period. We find the last two weeks interesting to analyze further, hence we focus on these weeks in a numerical example in Section 3.3.4.

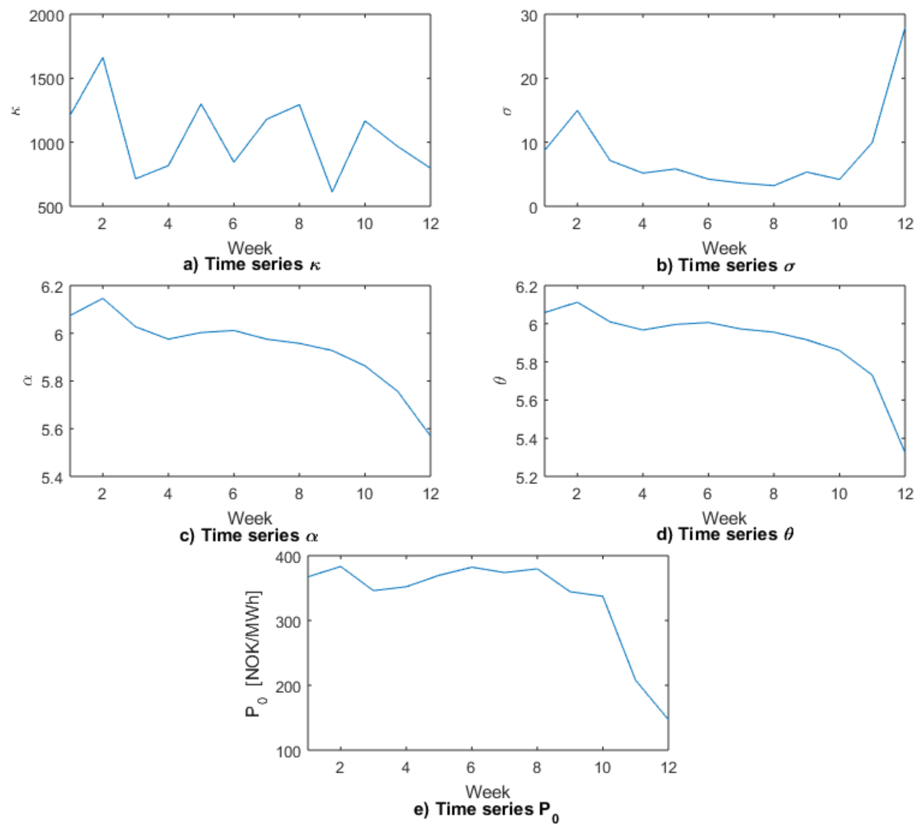

Figure 3.9: Time series for parameters of the price process

We now relate the parameters in the model to hedging instruments through the derivatives. The computed value of the hydropower plant and the associated derivatives for the twelve weeks are given in Table 3.7. Note that the derivatives have different units. The derivatives with respect to κ, σ, θ and α are measured in NOK, whereas the derivative with respect to the starting price, P_0, is measured in MWh.

$Week$	V	$\frac{\partial V}{\partial P_0}$	$\frac{\partial V}{\partial \kappa}$	$\frac{\partial V}{\partial \sigma}$	$\frac{\partial V}{\partial \theta}$	$\frac{\partial V}{\partial \alpha}$
-	NOK	MWh	NOK	NOK	NOK	NOK
1	$3.901 \cdot 10^9$	2293	$-8.213 \cdot 10^4$	$2.260 \cdot 10^7$	$4.279 \cdot 10^9$	$4.279 \cdot 10^9$
2	$4.259 \cdot 10^9$	1603	$-1.172 \cdot 10^5$	$2.600 \cdot 10^7$	$4.673 \cdot 10^9$	$4.672 \cdot 10^9$
3	$3.727 \cdot 10^9$	3816	$-1.451 \cdot 10^5$	$2.876 \cdot 10^7$	$4.082 \cdot 10^9$	$4.082 \cdot 10^9$
4	$3.511 \cdot 10^9$	3416	$-6.208 \cdot 10^4$	$1.916 \cdot 10^7$	$3.875 \cdot 10^9$	$3.875 \cdot 10^9$
5	$3.602 \cdot 10^9$	2146	$-3.217 \cdot 10^4$	$1.448 \cdot 10^7$	$3.988 \cdot 10^9$	$3.988 \cdot 10^9$
6	$3.633 \cdot 10^9$	3353	$-4.243 \cdot 10^4$	$1.637 \cdot 10^7$	$4.042 \cdot 10^9$	$4.042 \cdot 10^9$
7	$3.509 \cdot 10^9$	2491	$-1.676 \cdot 10^4$	$1.044 \cdot 10^7$	$3.969 \cdot 10^9$	$3.969 \cdot 10^9$
8	$3.458 \cdot 10^9$	2287	$-1.116 \cdot 10^4$	$8.436 \cdot 10^6$	$3.945 \cdot 10^9$	$3.945 \cdot 10^9$
9	$3.354 \cdot 10^9$	4502	$-1.073 \cdot 10^5$	$2.678 \cdot 10^7$	$3.684 \cdot 10^9$	$3.684 \cdot 10^9$
10	$3.095 \cdot 10^9$	2489	$-1.922 \cdot 10^4$	$1.066 \cdot 10^7$	$3.519 \cdot 10^9$	$3.519 \cdot 10^9$
11	$2.861 \cdot 10^9$	2958	$-1.328 \cdot 10^5$	$4.158 \cdot 10^7$	$3.298 \cdot 10^9$	$3.257 \cdot 10^9$
12	$2.724 \cdot 10^9$	3467	$-7.430 \cdot 10^5$	$4.158 \cdot 10^7$	$3.228 \cdot 10^9$	$3.228 \cdot 10^9$

Table 3.7: Value of the plant and its derivatives for twelve weeks

We see that the computed value of the hydropower plant is higher than the valuation result in Section 3.2.5 for all weeks. The reason for this is that the parameters presented above are estimated based on weeks when the average electricity price or the volatility is higher in the weeks considered than over a whole year.

The value of the hydropower plant is sensitive to changes in the parameters. For example, a unit shift in P_0 is equivalent to an increased expected value of the hydropower plant corresponding to increased production of 2293 MWh for the first week. Then, hedging the risk related to the starting price corresponds to selling (shorting) 2293 standardized contracts. As emphasized in Section 2.6, the effect of a change in the starting price is closely related to the speed of mean reversion. A change in the starting price only has a short-term effect on the price process. Most risk is, therefore, in the nearest term. To keep a neutral position with respect to this risk, the risk manager should short a portfolio of near-term products.

Turning to the sensitivity in the speed of mean reversion, κ, we see from Table 3.7 that the value of the hydropower plant decreases when κ increases. We recall that a high speed of mean reversion implies that the price process moves faster towards the mean after spikes. Thus, with a lower speed of mean reversion, the valuation model exploits high prices over a longer time. The price process allows for larger deviations from the mean in the upward direction than in the downward direction[9]. Due to this skewness, an increasing κ would be most to the expense

[9]Since we have set $M = 0$, negative prices will not occur.

of the positive shocks. Thus, it is reasonable that the value of the power plant is decreasing with increasing κ.

Further, we see that a unit change in the volatility, σ, will increase the value of the hydroelectric plant substantially. We recall that the valuation model takes into account the flexibility within the reservoir plant. The reservoir hydropower producers can benefit from producing in periods with high prices, and withhold the production when the price is low, hence, they are able to exploit higher volatility. However, higher volatility means higher risk for the producer. In some cases, there is a need for production even when the price is low. One example could be that in a period of considerably high inflow, the alternative to discharging water is a flood over the dam. This implies that even if the expected value of the power plant increases with increasing volatility, a higher volatility also implies higher risk, which matches the traditional view on risk and return in finance.

From Table 3.7, we see that the value of the power plant is highly sensitive to changes in the mean reversion parameter, θ, for all weeks. This is reasonable since the mean reversion level reflects the long-term price. Hedging the total risk related to θ will mean to sell all non-overlapping contracts, i.e. the whole forward curve. We also note that the derivatives with respect to θ and α are equal in some of the weeks. The result confirms the interpretations from Section 2.6.1, where we argued that in most cases it will be sufficient to either hedge the risk in θ or α.

3.3.3 Relating Greeks and standard deviations

We conclude the interpretation of the Greeks by relating them to the standard deviations of the parameters. Table 3.8 is a summary of the Greeks with the standard deviation of the corresponding parameters for the first week in the twelve week period. The hedging amount in the last row is the product of the Greek and the standard deviation of the respective parameter[10].

[10]The product is equivalent to the derivative of V with respect to one standard deviation of the corresponding parameter.

	P_0	κ	σ	θ	α
Derivative of V w.r.t. parameter	2293	$-8.213 \cdot 10^4$	$2.260 \cdot 10^7$	$4.279 \cdot 10^9$	$4.279 \cdot 10^9$
St. dev. of parameter	55.985	306.72	6.9932	0.15294	0.20935
Hedging amount one st. dev.	$1.284 \cdot 10^5$	$-2.519 \cdot 10^7$	$1.581 \cdot 10^8$	$6.544 \cdot 10^8$	$8.958 \cdot 10^8$

Table 3.8: Derivatives for week 1, standard deviation of parameters and hedging amount of one standard deviation change in the parameter[11]

The risk manager should be aware of the differences between the standard deviations when deciding a hedging strategy, in order to hedge the right exposure of the parameter volatility. Because of the property of our valuation model, where the cash flows are discounted to infinity, the Greeks are the required amount of hedging in order to offset the risk from today until infinity, given a *unit change* in the respective parameters. The hedging amount defined above, on the other hand, is the total amount which needs to be hedged from now until infinity to offset the risk of a *standard deviation change*. The parameters vary substantially in absolute values. For example, a unit change in α will have considerably larger effect on the value than a unit change in κ. Therefore, the product could be a better measure of exposure, since it takes the standard deviation into account. For instance, to hedge the risk of one standard deviation change in the mean reversion parameter, θ, is equivalent to hedging 16.8% of the estimated value of the plant in the same week. In comparison, hedging a unit change of the same parameter would mean hedging 109.7% of the value of the plant.

3.3.4 Hedging performance

This section provides a specific example of the hedging performance using the model. From the price parameters in Table 3.6, we see that week 12 has a considerably higher volatility parameter than the other weeks. Southern parts of Norway experienced a flood that week, which might explain the downward shock in the electricity price. We therefore find it interesting to investigate how a hedging strategy following the lines described in Section 3.3.2 would perform compared to a non-hedged cash flow during the mentioned week[12].

To the best of our knowledge, we are the first to approach the risk management problem in hydropower production from the analytical angle. Therefore, we present

[11] All units for the derivatives are in NOK, expect of the derivative of P_0 which is denoted in MWh. The same holds for the hedging amount.

[12] A non-hedged position is defined as selling all the power to spot price.

a numerical example where we evaluate the strategy of hedging the risk in the starting price, P_0. We leave investigation of the other Greeks for further research.

Figure 3.10 illustrates the development in the price of a monthly forward contract with delivery in May[13]. We clearly see a downward movement through the first weeks of May, with a drop at the 11th[14].

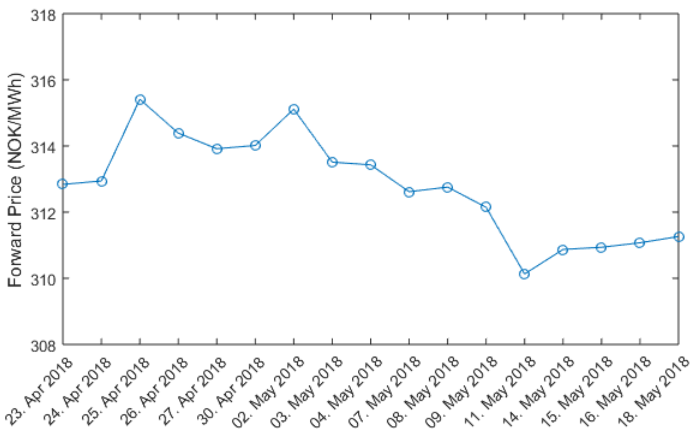

Figure 3.10: Price of a monthly forward contract with delivery in May[15]

Consider the decision of a risk manager on Monday 7th of May. Based on our model, she calibrates parameters of the price process using hourly spot prices from week 11. Further, she runs the valuation and obtain the subsequent derivatives with respect to the parameters. Figure 3.11 shows simulations for the electricity price, water level and discharge rate for the first upcoming days based on the same parameters. In addition, we note that the predetermined inflow function is at its top level in the beginning of the week, which is reasonable when considering the first weeks of May in the Norwegian climate.

[13]Delivery in May means exercise from May 1st to May 31st.

[14]Recall that week 11 and 12 in our case study correspond to the weeks starting at Monday 30th of April and Monday 7th of May respectively.

[15]We note that in addition to Saturdays and Sundays, both 10th and 17th (and 21st) of May are non-trading days in Norway.

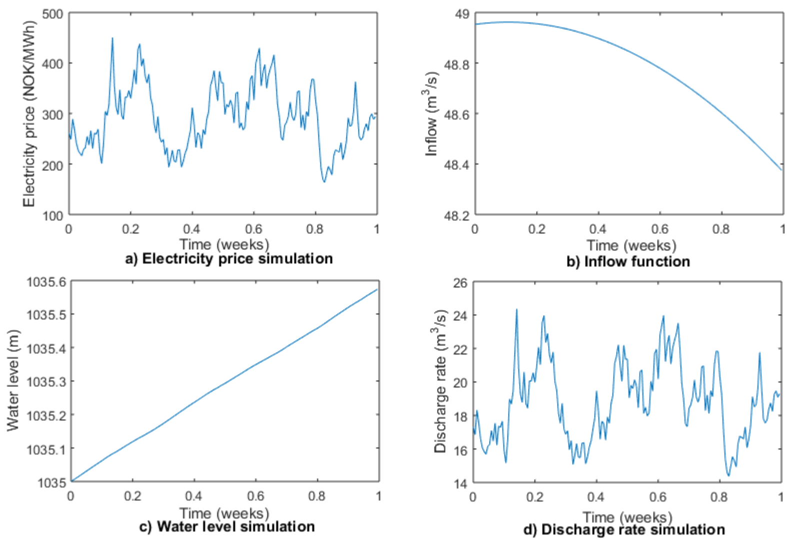

Figure 3.11: Simulations for week 12

The risk manager would like to hedge the risk in P_0, the starting price of the price process. From Table 3.7, we have $\frac{\partial V}{\partial P_0} = 2958$ MWh for week 11. We know that because of the speed of mean reversion, prices will be pushed back to the mean reversion level relatively fast, thus the risk in P_0 should be hedged by selling contracts with short time to maturity. We suggest placing 70% in contracts with maturity in one day, 20% with maturity in two days and 10% with maturity in three days[16].

In Section 3.3.1 we found that the upper hedge ratio should be 53.6% due to taxes. The risk manager must take previous hedged positions into account such that the upper hedge ratio is not violated. In this example, we assume that the hedge ratio is 0% before 7th of May. The model output suggests a total production of 4774.5 MWh the next day, and with an upper hedge ratio of 53.6%, the maximum hedged production the next day is 2559 MWh.

At the 7th of May, the daily fix price of forward contracts with delivery the next day is 282.46 NOK/MWh, the daily fix price of contracts with delivery 9th of May is 247.73 NOK/MWh[17], while contracts with delivery 10th of May trades at 206.85 NOK/MWh. Table 3.9 summarizes the prices and the cash flow results of hedging

[16] This is a suggested strategy, the choice of contracts is up to the risk manager.

[17] On Monday 7th of May there where no traded contracts with delivery the 9th of May. The daily fix price is set as the average of bid and ask prices.

the risk in P_0 compared to the cash flow without hedging.

Day	Forward price	Cash flow without hedging	Cash flow with hedging	Profit from hedging
	NOK	NOK	NOK	NOK
8th of May	282.46	1 236 563	1 224 285	-12 279
9th of May	247.73	728 023	1 392 155	664 132
10th of May	206.85	713 335	1 053 168	339 833
Total	-	2 677 922	3 669 608	991 686

Table 3.9: Comparison of cash flow with and without hedging for the period May 8th to May 10th 2018

The numerical results show that the power producers would be able to earn almost a million NOK more the next three days if hedging the risk in P_0, compared to a non-hedged cash flow.

When the risk manager takes a short position during a period when the spot price drops more than expected, as in this case, it is natural that the company gains profit. Correspondingly, a short position would lead to losses in a period with a positive shock. We note that for the closest day (8th of May), where 70% of the sensitivity in P_0 is off-set, the hedged portfolio performs worse than the non-hedged strategy. This reminds us of the original concept of hedging, as hedging is not meant for gaining profit per se, but to reduce risk. Hence, as the risk manager decides its position before the spot price is observed, there is a possibility of losing potential gains from upward price movements.

4 Discussion

The increasing share of renewable and intermittent capacity in the Nordic electricity market implies that Norwegian reservoir hydropower will be of great importance for balancing purposes the upcoming years. If the prediction about more uncertain electricity prices is correct, risk management will play a crucial role for hydropower companies in order to avoid downside risk.

This thesis focuses on valuation and hedging of price risk for reservoir hydropower plants. The model presented is analytically tractable. The analytical property makes the model suitable for deriving Greeks for risk management within short computational time. Hence, the model is convenient for hedging in volatile markets when companies can benefit from reacting quickly to market changes. Another advantage of the model is the transparency due to the analytical property. In contrast to other complex *black box*-type of models often used by hydropower companies, all steps of the calculations are visible for the risk managers. Also, with fast models, we can easy perform back-testing and scenario analysis.

We are aware that analytical tractability comes with restrictive assumptions. A more advanced optimization model would be able to take e.g. stochastic inflow into account, but not without compromising the running time. In the following, we will present a critical review of the assumptions, focusing on three main areas. First, we discuss relevant aspects regarding the spot price and hedging, before we move on to discussing the modeling assumptions. At last, we comment on expectations for the future power market.

4.1 Hedging uncertainties in the spot price

It is a challenging task to model the electricity price dynamics. The electricity market is incomplete, and illiquid, even though an increased efficiency of the market is observed over the last years (Smith-Meyer and Gjolberg, 2016). The estimates of the price process are uncertain, and the hedge ratios we propose are meant for guidance. The accurate hedging strategy is up to the risk manager to decide.

The parameters of the price process are calibrated for a specific period, thus, the hedging results are parameter dependent and the parameters must be updated to hedge dynamically. To obtain the best results, we urge frequent hedging based on the model with updated parameters. In order to calibrate the most suitable parameters for the hedging period, we recommend using data from a time period as short as possible, while still giving stable parameters. On the other hand, if only the valuation part of the model is of interest, the parameters should be estimated using data from a longer period such that the seasonal properties are captured.

The risk manager must consider transaction costs when trading forward contracts. However, we do not include these in our model. When neglecting transaction costs, the optimal hedging strategy will be to hedge dynamically. The transaction costs will in practice imply that the hedging frequency decreases.

Finally, it is important to note that forward prices are set with the system price as reference, while the power company sells power on the spot market to the area price. Thus, the price risk is not completely hedged by trading forward contracts, as there still is a risk regarding the difference between area price and spot price. This risk is not considered in the model we provide, but it is possible to hedge it by trading *Electricity Price Area Differentials* (EPAD) available at NASDAQ OMX Commodities.

4.2 Modeling assumptions

This thesis focuses on risk management concerning price risk. Risk management is a challenging task because, in reality, there are more risk factors than the electricity price. Inflow and future weather conditions are stochastic properties, and besides price risk, the volume risk is an important risk factor for a hydropower producer. Production planners take stochastic inflow into account, resulting in stochastic optimization problems for power scheduling. The production planning is a way of limiting exposure to inflow risk. However, it is difficult to hedge inflow risk in the same way as price risk, since there are no tradable products for hedging it.

As seen in Section 3.2.2, introducing variable inflow yields more realistic simulations compared to simulations with constant inflow, which was the case in the paper of Ernstsen and Boomsma (2018). We fitted a five-term sinusoidal function to the historical inflow data of the specific reservoir. The choice of number of sine-terms will depend on the model runners desired accuracy. It is also possible to keep the stochastic inflow process from the original valuation problem in Section 2.1, but this will increase the complexity and be at the expense of the tractability of the model. Also, while forecasting inflow is of great importance to scheduling models, a fitted function which captures seasonal effects will be sufficient for the valuation purpose.

Contributors have shown that in markets dominated by hydropower, such as in Norway, there is a negative relationship between electricity price and inflow (Bye and Bruvoll, 2006). We have neglected correlation between price and inflow in our study. This is in accordance to results of Fleten and Wallace (2009), who found that including correlation did not seem to have a significant effect on the valuation results. However, they could not rule out that it may have an effect on hedging. Further research is needed to investigate this relationship.

To solve the optimization problem in Section 2.1 analytically, we relaxed the operational constraints by introducing penalty functions. In theory, this allows the plant to operate outside of the given restrictions, although with a cost. In our case study, the lower boundary for water level is violated for some periods. A reason to this could be low inflow, and that the objective function was not penalized enough. A too low water level must be avoided because of the environmental impacts it might cause. One solution to the problem is to tune the parameters in the penalty function. On the other hand, the discharge rate is within its boundaries the whole simulation period.

We apply a linear production function, and thereby assume constant efficiency. This is a simplification, since in reality the efficiency of the turbine changes with the discharge rate. However, for the case we consider, the range in efficiency only deviates one percent from the average in each direction, and we therefore find the production function accurate enough. Also, Ernstsen and Boomsma (2018) emphasize that an overestimation of the value due to the penalty functions discussed above may be counterbalanced by the restriction to a linear discharge rate.

We assume a cylindrical reservoir with random end surfaces. This is a simplification in comparison to real shapes of reservoirs, but it is necessary in order to keep the model analytically tractable. Further, for systems with more than one reservoir, the reservoirs must be accumulated in order to apply the valuation model. This simplification may lead to a loss in accuracy of results. We, therefore, point out that the model is best suited for valuation of relatively simple systems.

At last, like Thompson et al. (2004) and Ernstsen and Boomsma (2018), we assume no start-up and shut-down costs. In reality, they exist, but are small relative to the other variables affecting the profit. Start-up and shut-down costs for hydropower plants are scarcely studied in literature. Some of the contributors are Bakken and Bjørkvold (2002), who state that the costs increase with older equipment. Tseng and Barz (2002) find that some studies overestimate the value of power plants when neglecting start-up and shut-down costs, but that the estimation error is lowest for hydropower plants.

4.3 The future power market

The future power market is uncertain. We experience development and planning of new power plant facilities based on intermittent resources such as wind power. Also, new interconnecting transmission cables are under construction, which may influence the Nordic market structure.

Förssund and Hjalmarsson (2010) argue that a rapid increase of wind power generation in the market will lead to more volatile prices, but also to reduction in the spot market electricity price. The reason is that the increased wind power capacity may easily lead to more production than matched by the increase in demand over time. Therefore, if the target is to keep the average electricity price at the current level, they claim that export possibilities out of Nord Pool should be expanded. Additionally, new technologies might improve storage possibilities of intermittent power, hence keep prices more stable. However, it is uncertain when these will be available for conventional use.

Intermittent power generation, transmission cables and storage possibilities are some of the factors which will influence the Nordic market structure in the future. Risk management has been a major concern for power producers since the deregulation of the electricity market. When the market is under transition and subject to various future uncertainties, the power producers have to face new challenges. Therefore, risk management will not be less important for the upcoming years.

5 Conclusion

In this thesis we present an analytically tractable model for valuation and risk management in hydropower production. We improve an existing valuation model by correcting pitfalls and by including seasonal inflow. Further, we extend the model by incorporating a framework for risk management. This is done by relating derivatives of the valuation result to hedging with forward contracts.

The model is computationally efficient due to its analytical property, which makes the hydropower producer able to react quickly to market changes in the transient European market. Also, we provide detailed steps of the derivations which makes the model tractable and easy to replicate.

Comparing to benchmarks for the value of the hydropower plant, we find that our model yields a realistic valuation result. Simulations show that the water level and the discharge rate operate within its given boundaries most of the time, where the water level simulation follows the historical data to a great extent. Considering risk management, we find that hedging the risk in the starting price of the price process before a downward shock in the electricity price prevents profit losses for the company.

Our thesis extends the literature by providing theoretical insight to risk management from an analytical standpoint. Further, the tractable and presentable model contributes to the industry by increasing competence for risk managers. To the best of our knowledge, we are the first to provide an analytically tractable framework for risk management of hydropower production.

Since this is a first attempt to apply an analytical valuation model for risk management, much has been left for further research. For further work, we suggest a more comprehensive analysis of the hedging performance of all the derivatives.

The model may be extended in several directions. For instance, one may investigate cross effects of parameters by deriving mixed partial derivatives. Stochastic inflow and correlation between price and inflow may be be included, and finally, one could include a spot-forward relationship to investigate if the model is appropriate for delta-hedging.

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A Nomenclature

r	Exogenous discount rate
η_0, η_1	Coefficients in production function
η	Total efficiency of system
κ	Speed of mean reversion parameter in price process
θ	Mean reversion level parameter of price process
α	Asymptotic mean parameter of price process
σ	Volatility factor in price process
M	Lower boundary for electricity price
S	Standard deviation of price process
X_s	Starting value for logarithm of electricity price
P_s	Starting value for electricity price (NOK/MWh)
v_{min}, v_{max}	Minimum and maximum discharge rate from the reservoir (m^3/s)
L_{min}, L_{max}	Minimum and maximum water levels (m^3)
Θ_1, Θ_2	Coefficients in penalty function for water level
θ_1, θ_2	Coefficients in penalty function for discharge rate
$\tilde{P}_v$	Marginal penalty for exceeding discharge rate limits (NOK/MWh)
$\tilde{P}_L$	Marginal penalty for exceeding water level limits (NOK/MWh)
$\overline{P}$	Average electricity price (NOK/MWh)
$\overline{L}$, L_0	Average water level (m^3)
d_1, d_2, d_3	Constants in discharge function
k_{d_1}, a_{d_1}	Constants in d_1
k_{d_2}, a_{d_2}	Constants in d_2
a	Surface area of reservoir (m^2)
V, V_{model}	Valuation result from model (NOK)
V_{EP}	Valuation benchmark based on expected production (NOK)
V_{AR}	Valuation benchmark based on annual reports (NOK)
F	Forward price (NOK/MWh)

B Lemmas

These lemmas are replications of lemmas presented in Ernstsen (2016).

B.1 Moment generating functions

Let $X \sim N(\mu, \sigma^2)$, then $\mathbb{E}(e^{uX}) = e^{u\mu + \frac{1}{2}u^2\sigma^2}$.

Proof. As we have $\mathbb{E}(e^{uX}) = \mathbb{E}(e^{u(\mu+\sigma Z)})$, it is enough to show that $\mathbb{E}(e^{uZ}) = e^{\frac{u^2}{2}}$ for $Z \sim N(0,1)$. Now as $ux - \frac{x^2}{2} = -\frac{(x-u)^2}{2} + \frac{u^2}{2}$, we have

$$\int_{-\infty}^{\infty} e^{ux} \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \mathrm{d}x = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{ux - \frac{x^2}{2}} \mathrm{d}x = e^{\frac{u^2}{2}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-(x-u)^2}{2}} \mathrm{d}x = e^{\frac{u^2}{2}}. \tag{B.1}$$

as we recognize the density of the normal distribution with mean u and variance 1. □

B.2 Storage level

Let $(Z_t)_{t\geq 0}$ be a Brownian motion, $(Y_n)_{n\geq 0}$ be independent and identically distributed (i.i.d.) random variables and $(N_t)_{t\geq 0}$ a Poisson process with intensity λ and jump times $(T_n)_{n\geq 1}$. Define the compound Poisson process $J_t = \Sigma_{n=1}^{N_t} Y_n$. Assume U_t has the dynamics

$$\mathrm{d}U_t = \kappa(\hat{\alpha}_t - U_t)\mathrm{d}t + \sigma \mathrm{d}Z_t + \mathrm{d}J_t, \tag{B.2}$$

then

$$U_t = U_s e^{-\kappa(t-s)} + \kappa \int_s^t e^{-\kappa(t-v)} \hat{\alpha}_v \mathrm{d}v + \sigma \int_s^t e^{-\kappa(t-v)} \mathrm{d}Z_v^U . + \Sigma_{n=N_s+1}^{N_t} e^{-\kappa(t-T_n)} Y_n. \tag{B.3}$$

Proof. Define $L_t = e^{\kappa t} U_t$, then

$$\begin{aligned} \mathrm{d}L_t &= e^{\kappa t}\mathrm{d}U_t + U_t \mathrm{d}(e^{\kappa t}) \\ &= \kappa(\hat{\alpha}_t - U_t) e^{\kappa t}\mathrm{d}t + \sigma e^{\kappa t}\mathrm{d}Z_t^U + e^{\kappa t}\mathrm{d}J_t. \end{aligned} \tag{B.4}$$

Thus, for $t \geq s$

$$L_t = L_s + \kappa \int_s^t \hat{\alpha}_v e^{\kappa v} \mathrm{d}v + \sigma \int_s^t e^{\kappa v} \mathrm{d}Z_v + \Sigma_{n=N_s+1}^{N_t} e^{\kappa T_n} Y_n. \tag{B.5}$$

Now as $U_t = e^{-\kappa t} L_t$ it follows that

$$\begin{aligned} U_t &= U_s e^{-\kappa(t-s)} + \kappa \int_s^t \hat{\alpha}_v e^{-\kappa(t-v)} \mathrm{d}v \\ &+ \sigma \int_s^t e^{-\kappa(t-v)} \mathrm{d}Z_v + \Sigma_{n=N_s+1}^{N_t} e^{-\kappa(t-T_n)} Y_n. \end{aligned} \tag{B.6}$$

□

C Proofs of corollaries

C.1 Proofs of Corollary 1 and 2: Defining constants in linear discharge function

In this section, we provide detailed steps of deriving the constants d_1, d_2 and d_3 from Section 2.4. This includes finding the partial derivatives of the value function, V, w.r.t. price and water level, which all are based on (2.31). For transparency of the model and to facilitate further replication, we introduce constants to simplify the expressions.

Proof.

Finding d_3 in linearized discharge rate

We start out by evaluating d_3 using equation (2.30). First, (2.33) is substituted into (2.31). Further, we apply the chain rule to find the partial derivative of (2.31) w.r.t. water level

$$\begin{aligned}\frac{\partial}{\partial L}V(P,L)|_{P=\overline{P},L=\overline{L}} &= (2d_1d_3\theta_2 + d_3\theta_1 + \Theta_1)\int_0^\infty e^{-(r+d_3)t}\mathrm{d}t \\ &+ (2d_2\theta_2 + \eta_1)d_3\int_0^\infty e^{-(r+d_3)t}\mathbb{E}(P_t|P)\mathrm{d}t \\ &+ 2(d_3^2\theta_2 + \Theta_2)\int_0^\infty \overline{L}e^{-(r+2d_3)t}\mathrm{d}t \\ &+ 2(d_3^2\theta_2 + \Theta_2)\int_0^\infty e^{-(r+d_3)t}\int_0^t e^{-d_3(t-s)}(f_s - d_1)\mathrm{d}s\mathrm{d}t \\ &- 2d_2(d_3^2\theta_2 + \Theta_2)\int_0^\infty e^{-(r+d_3)t}\int_0^t e^{-d_3(t-s)}\mathbb{E}(P_s|P)\mathrm{d}s\mathrm{d}t.\end{aligned} \tag{C.1}$$

From this, the second order partial derivative w.r.t. water level is

$$\frac{\partial^2}{\partial L^2}V(P,L)|_{P=\overline{P},L=\overline{L}} = 2(d_3^2\theta_2+\Theta_2)\int_0^\infty e^{-(2d_3+r)t}\mathrm{d}t = \frac{2(d_3^2\theta_2+\Theta_2)}{r+2d_3}. \quad \text{(C.2)}$$

(C.2) is inserted into (2.30) to get

$$d_3 = (\frac{\Theta_2}{\theta_2}+d_3^2)\int_0^\infty e^{-(2d_3+r)t}\mathrm{d}t. \quad \text{(C.3)}$$

Integrating (C.3) yields the following quadratic equation

$$d_3^2 + rd_3 - \frac{\Theta_2}{\theta_2} = 0, \quad \text{(C.4)}$$

with solution

$$d_3 = \frac{1}{2}(-r+\sqrt{r^2+4\frac{\Theta_2}{\theta_2}}). \quad \text{(C.5)}$$

Finding d_2 in linearized discharge rate

We move on to computing d_2 using equation (2.29). The mixed partial derivative of (2.31) w.r.t. water level and electricity price is

$$\begin{aligned}\frac{\partial^2 V(P,L)}{\partial L \partial P}|_{P=\overline{P},L=\overline{L}} &= d_3(2d_2\theta_2+\eta_1)\int_0^\infty e^{-(r+d_3)t}\frac{\partial}{\partial P}\mathbb{E}(P_t|P)|_{P=\overline{P}}\mathrm{d}t \\ &- 2d_2(d_3^2\theta_2+\Theta_2)\int_0^\infty e^{-(r+d_3)t}\int_0^t e^{-d_3(t-s)}\frac{\partial}{\partial P}\mathbb{E}(P_s|P)_{P=\overline{P}}\mathrm{d}s\mathrm{d}t,\end{aligned} \quad \text{(C.6)}$$

where $\overline{P} = P_0$. Deriving the mixed parital derivative, we find a mistake in the paper of Ernstsen and Boomsma (2018), where multiplication by d_3 is missing in the first term.

To integrate (C.6), we need the partial derivative of the expectation of the price, which with the OU price process is

$$\frac{\partial}{\partial P}\mathbb{E}(P_t|P_s) = (P_s+M)^{(e^{-\kappa_P(t-s)}-1)}e^{\theta(1-e^{-\kappa_P(t-s)})+\frac{\sigma^2}{4\kappa_P}(1-e^{-2\kappa_P(t-s)})-\kappa_p(t-s)}. \quad \text{(C.7)}$$

To simplify expressions, we define the constants q_1 and q_2 accounting for the first and the second integral terms of (C.6) respectively

$$q_1 = \int_0^\infty e^{-(r+d_3)t}(P_s+M)^{(e^{-\kappa_P t}-1)}e^{\theta(1-e^{-\kappa_P t})+\frac{\sigma^2}{4\kappa_P}(1-e^{-2\kappa_P t})-\kappa_p t}\mathrm{d}t \quad \text{(C.8)}$$

and

$$q_2 = \int_0^\infty e^{-(r+d_3)t} \int_0^t e^{-d_3(t-s)} (P_s + M)^{e^{-\kappa_P s} - 1} e^{\theta(1-e^{-\kappa_P s}) + \frac{\sigma^2}{4\kappa_P}(1-e^{-2\kappa_P s}) - \kappa_P s} \mathrm{d}s\mathrm{d}t. \tag{C.9}$$

Using (C.8) and (C.9) we can write (C.6) as

$$\frac{\partial^2 V(P,L)}{\partial L \partial P}|_{P=\overline{P},L=\overline{L}} = d_3(2d_2\theta_2 + \eta_1)q_1 - 2d_2(d_3^2\theta_2 + \Theta_2)q_2. \tag{C.10}$$

To simplify further, we want to write (C.10) on the form

$$\frac{\partial^2 V(P,L)}{\partial L \partial P}|_{P=\overline{P},L=\overline{L}} = a_{d_2} d_2 + k_{d_2}. \tag{C.11}$$

Therefore, we define the constants

$$a_{d_2} = 2\theta_2 d_3 q_1 - 2(d_3^2\theta_2 + \Theta_2)q_2 \tag{C.12}$$

and

$$k_{d_2} = d_3 \eta_1 q_1. \tag{C.13}$$

Finally, (C.11) is inserted into (2.29) and rearranged to obtain

$$d_2 = \frac{-\eta_1 + k_{d_2}}{2\theta_2 - a_{d_2}}. \tag{C.14}$$

Finding d_1 in linearized discharge rate

At last, we compute d_1. As for d_3, we start out from the partial derivative of V w.r.t. storage level (C.1).

We define the constants q_3 and q_4 as

$$\begin{aligned} q_3 &= \int_0^\infty e^{-(r+d_3)t} \mathbb{E}(P_t|P) dt \\ &= \int_0^\infty e^{-(r+d_3)t} ((P_s + M)^{e^{-\kappa_P t}} e^{\theta(1-e^{-\kappa_P t}) + \frac{\sigma^2}{4\kappa_P}(1-e^{-2\kappa_P t})} - M) \mathrm{d}t \end{aligned} \tag{C.15}$$

and

$$
\begin{aligned}
q_4 &= \int_0^\infty e^{-(r+d_3)t} \int_0^t e^{-d_3(t-s)} \mathbb{E}(P_s|P) ds dt \\
&= \int_0^\infty e^{-(r+2d_3)t} \int_0^t e^{d_3 s} ((P_s + M)^{e^{-\kappa_P s}} e^{\theta(1-e^{-\kappa_P s}) + \frac{\sigma^2}{4\kappa_P}(1-e^{-2\kappa_P s})} - M) \mathrm{d}s \mathrm{d}t,
\end{aligned}
\tag{C.16}
$$

such that (C.1) can be written as

$$
\begin{aligned}
&\frac{\partial}{\partial L} V(P, L)|_{P=\overline{P}, L=\overline{L}} = (2d_1 d_3 \theta_2 + d_3 \theta_1 + \Theta_1) \int_0^\infty e^{-(r+d_3)t} \mathrm{d}t \\
&+ (2d_2\theta_2 + \eta_1) d_3 q_3 + 2(d_3^2 \theta_2 + \Theta_2) \int_0^\infty \overline{L} e^{-(r+2d_3)t} \mathrm{d}t \\
&+ 2(d_3^2\theta_2 + \Theta_2) \int_0^\infty e^{-(r+d_3)t} \int_0^t e^{-d_3(t-s)} (f_s - d_1) \mathrm{d}s \mathrm{d}t \\
&- 2d_2(d_3^2 \theta_2 + \Theta_2) q_4.
\end{aligned}
\tag{C.17}
$$

Similar as for the mixed partial derivative, we want to write equation (C.17) on the form

$$
\frac{\partial}{\partial L} V(P, L)|_{P=\overline{P}, L=\overline{L}} = a_{d_1} d_1 + k_{d_1}. \tag{C.18}
$$

After solving the integrals of (C.17), the constants in (C.18) are defined as

$$
\begin{aligned}
k_{d_1} &= \frac{d_3\theta_1 + \Theta_1}{r + d_3} + (2d_2\theta_2 + \eta_1) d_3 q_3 + \frac{2(d_3^2\theta_2 + \Theta_2)\overline{L}}{r + 2d_3} \\
&+ 2(d_3^2\theta_2 + \Theta_2) \frac{I_{avg}}{d_3} (\frac{1}{r + d_3} - \frac{1}{r + 2d_3}) - d_2 2(d_3^2\theta_2 + \Theta_2) q_4
\end{aligned}
\tag{C.19}
$$

and

$$
a_{d_1} = \frac{2d_3\theta_2}{r + d_3} - \frac{2(d_3^2\theta_2 + \Theta_2)}{d_3} (\frac{1}{r + d_3} - \frac{1}{r + 2d_3}). \tag{C.20}
$$

Finally, (C.18) is inserted into equation (2.28), and d_1 is given as

$$
d_1 = \frac{-\theta_1 + k_{d_1} - \overline{P} \frac{\partial^2 V}{\partial L \partial P} - \overline{L} \frac{\partial^2 V}{\partial^2 L}}{2\theta_2 - a_{d_1}}, \tag{C.21}
$$

where we know $\frac{\partial^2 V}{\partial L \partial P}$ from (C.11) and $\frac{\partial^2 V}{\partial L^2}$ from equation (C.2). □

We compute q_1, q_2, q_3 and q_4 by numerical integration.

C.2 Proof of Corollary 3: Deriving expectation, second moment and auto covariance of price process

C.2.1 Expectation of the price process

Proof. The stochastic diffusion term of the price process is normally distributed

$$\sigma \int_s^t e^{-\kappa(t-v)} \mathrm{d}Z_v^P \sim N(0, \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa(t-s)})). \tag{C.22}$$

By applying Lemma 1 we can write

$$\mathbb{E}(e^{\sigma \int_s^t e^{-\kappa(t-v)} \mathrm{d}Z_v^P}) = e^{\frac{\sigma^2}{4\kappa}(1-e^{-2\kappa(t-s)})}, \tag{C.23}$$

and obtain the following closed form expression for the expectation of the price process

$$\begin{aligned}
\mathbb{E}(P_t|P_s) &= (P_s + M)^{e^{-\kappa(t-s)}} e^{\theta(1-e^{\kappa(t-s)})} \mathbb{E}(e^{\sigma \int_s^t e^{-\kappa(t-v)} \mathrm{d}Z_v^P}) - M \\
&= (P_s + M)^{e^{-\kappa(t-s)}} e^{\theta(1-e^{-\kappa(t-s)})} e^{\frac{\sigma^2}{4\kappa}(1-e^{-2\kappa(t-s)})} - M \\
&= (P_s + M)^{e^{-\kappa(t-s)}} e^{\theta(1-e^{-\kappa(t-s)}) + \frac{\sigma^2}{4\kappa}(1-e^{-2\kappa(t-s)})} - M.
\end{aligned} \tag{C.24}$$

□

C.2.2 Second moment of the price process

Proof. We start out from

$$\begin{aligned}
P_t^2 &= (P_s + M)^{2e^{-\kappa(t-s)}} e^{2\theta(1-e^{-\kappa(t-s)})} e^{2\sigma \int_s^t e^{-\kappa(t-v)} \mathrm{d}Z_v^P} \\
&- 2M(P_s + M)^{e^{-\kappa(t-s)}} e^{\theta(1-e^{-\kappa(t-s)})} e^{\sigma \int_s^t e^{-\kappa(t-v)} \mathrm{d}Z_v^P} + M^2.
\end{aligned} \tag{C.25}$$

Since we have that $2\sigma \int_s^t e^{-\kappa(t-v)} dZ_v^P \sim N(0, \frac{2\sigma^2}{\kappa}(1 - e^{-2\kappa(t-s)}))$, we can apply Lemma 1 and write

$$\mathbb{E}(e^{2\sigma \int_s^t e^{-\kappa(t-v)} dZ_v^P}) = e^{\frac{\sigma^2}{\kappa}(1-e^{-2\kappa(t-s)})},. \tag{C.26}$$

Thus,

$$\begin{aligned}
&\mathbb{E}(P_t^2|P_s) = (P_s+M)^{2e^{-\kappa(t-s)}} e^{2\theta(1-e^{-\kappa(t-s)})}\mathbb{E}(e^{2\sigma\int_s^t e^{-\kappa(t-v)}\mathrm{d}Z_v^P}) \\
&- 2M(P_s+M)^{e^{-\kappa(t-s)}} e^{\theta(1-e^{-\kappa(t-s)})}\mathbb{E}(e^{\sigma\int_s^t e^{-\kappa(t-v)}\mathrm{d}Z_v^P}) + M^2 \\
&= (P_s+M)^{2e^{-\kappa(t-s)}} e^{2\theta(1-e^{-\kappa(t-s)})+\frac{\sigma^2}{\kappa}(1-e^{-2\kappa(t-s)})} \\
&- 2M(P_s+M)^{e^{-\kappa(t-s)}} e^{\theta(1-e^{-\kappa(t-s)})+\frac{\sigma^2}{4\kappa}(1-e^{-2\kappa(t-s)})} + M^2.
\end{aligned} \tag{C.27}$$

□

These steps of the derivations are not included in Ernstsen and Boomsma (2018). Also note that the last two terms disappear when $M = 0$.

C.2.3 Autocovariance of the price process

We proceed to compute the autocovariance of the price process.

Proof. With the definitions of P_t and P_s, we can write

$$\begin{aligned}
&\mathbb{E}(P_sP_t|P_0) = \mathbb{E}\big((e^{X_t}-M)(e^{X_s}-M)\big) = \mathbb{E}\big(e^{X_t+X_s} - Me^{X_t} - Me^{X_s} + M^2\big) \\
&= \mathbb{E}(e^{X_t+X_s}) - M\mathbb{E}(e^{X_t}) - M\mathbb{E}(e^{X_s}) + M^2
\end{aligned} \tag{C.28}$$

Since X_t and X_s are normal variables, the sum of the two must be univariate normal, i.e. $X_t + X_s \sim N(\mu_t + \mu_s, \sigma_t^2 + \sigma_s^2 + 2\rho_{t,s}\sigma_t\sigma_s)$, where $\rho_{s,t}$ is the correlation coefficient of the processes X_t and X_s. From (2.42) and the normal property of X, we already have the mean of the processes

$$\mathbb{E}(X_t|X_0) = \mu_t = X_0e^{-\kappa t} + \theta(1-e^{-\kappa t}) \tag{C.29}$$

and

$$\mathbb{E}(X_s|X_0) = \mu_s = X_0e^{-\kappa s} + \theta(1-e^{-\kappa s}). \tag{C.30}$$

Thus, the sum is

$$\mu_t + \mu_s = X_0(e^{-\kappa t} + e^{-\kappa s}) + \theta(2 - e^{-\kappa t} - e^{-\kappa s})). \tag{C.31}$$

Further, $\sigma_t \sigma_s \rho_{t,s}$ can be computed from the covariance, because

$$2\rho_{s,t}\sigma_t\sigma_s = 2Cov(X_t, X_s). \tag{C.32}$$

Knowing that X_t and X_s are OU-processes, the covariance is given as

$$Cov(X_t, X_s) = \frac{\sigma^2 e^{-\kappa(t+s)}}{2\kappa}(e^{2\kappa min(s,t)} - 1), \tag{C.33}$$

where the proof of the covariance is stated below. In the following, we will assume $s < t$. Now, the variance of the normal $X_t + X_s$ can be computed as

$$\begin{aligned}
\sigma_{bi}^2 &= \sigma_t^2 + \sigma_s^2 + 2\rho_{s,t}\sigma_t\sigma_s \\
&= \sigma_t^2 + \sigma_s^2 + 2Cov(X_t, X_s) \\
&= \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa t}) + \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa s}) + \frac{\sigma^2}{\kappa}e^{-\kappa(t+s)}(e^{2\kappa s} - 1) \\
&= \frac{\sigma^2}{2\kappa}\bigg(2 - e^{-2\kappa t} - e^{-2\kappa s} + 2e^{-\kappa(t+s)}(e^{2\kappa s} - 1)\bigg).
\end{aligned} \tag{C.34}$$

Further, we can compute the expectation of the log normal $e^{X_t + X_s}$ by applying Lemma 1.

$$\begin{aligned}
&\mathbb{E}(P_s P_t | P_0) = \mathbb{E}(e^{X_t + X_s}) - M\mathbb{E}(e^{X_t}) - M\mathbb{E}(e^{X_s}) + M^2 \\
&= e^{\mu_t + \mu_s + \frac{1}{2}\sigma_{bi}^2} - Me^{\mu_t} - Me^{\mu_s} + M^2 \\
&= e^{X_0 e^{-\kappa t} + X_0 e^{-\kappa s} + \theta(1 - e^{-\kappa t}) + \theta(1 - e^{-\kappa s}) + \frac{\sigma^2}{4\kappa}\left(2 - e^{-2\kappa t} - e^{-2\kappa s} + 2e^{-\kappa(t+s)}(e^{2\kappa s} - 1)\right)} \\
&- Me^{X_0 e^{-\kappa t} + \theta(1 - e^{-\kappa t})} - Me^{X_0 e^{-\kappa s} + \theta(1 - e^{-\kappa s})} + M^2 \\
&= P_0^{e^{-\kappa t}} P_0^{e^{-\kappa s}} e^{\theta(2 - e^{-\kappa t} - e^{-\kappa s}) + \frac{\sigma^2}{4\kappa}\left(2 - e^{-2\kappa t} - e^{-2\kappa s} + 2e^{-\kappa(t+s)}(e^{2\kappa s} - 1)\right)} \\
&- MP_0 e^{-\kappa t} e^{\theta(1 - e^{-\kappa t}) + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa t})} \\
&- MP_0 e^{-\kappa s} e^{\theta(1 - e^{-\kappa s}) + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa s})} + M^2. \\
&= P_0^{e^{-(\kappa t + \kappa s)}} e^{(\theta + \frac{\sigma^2}{4\kappa})(2 - e^{-\kappa t} - e^{-\kappa s}) + \frac{\sigma^2}{4\kappa}\left(2e^{-\kappa(t+s)}(e^{2\kappa s} - 1)\right)} \\
&- MP_0 e^{-\kappa t} e^{\theta(1 - e^{-\kappa t}) + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa t})} \\
&- MP_0 e^{-\kappa s} e^{-\kappa s} e^{\theta(1 - e^{-\kappa s}) + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa s})} + M^2.
\end{aligned} \tag{C.35}$$

With M=0, we get

$$\begin{aligned}
&\mathbb{E}(P_sP_t|P_0) = \mathbb{E}(e^{X_t+X_s}) = e^{\mu_t+\mu_s+\frac{1}{2}\sigma_{bi}^2} \\
&= e^{X_0e^{-\kappa t}+X_0e^{-\kappa s}+\theta(1-e^{-\kappa t})+\theta(1-e^{-\kappa s})+\frac{\sigma^2}{4\kappa}\left(2-e^{-2\kappa t}-e^{-2\kappa s}+2e^{-\kappa(t+s)}(e^{2\kappa s}-1)\right)} \\
&= P_0^{e^{-\kappa t}}P_0^{e^{-\kappa s}}e^{\theta(2-e^{-\kappa t}-e^{-\kappa s})+\frac{\sigma^2}{4\kappa}\left(2-e^{-2\kappa t}-e^{-2\kappa s}+2e^{-\kappa(t+s)}(e^{2\kappa s}-1)\right)} \\
&= P_0^{e^{-(\kappa t+\kappa s)}}e^{(\theta+\frac{\sigma^2}{4\kappa})(2-e^{-\kappa t}-e^{-\kappa s})+\frac{\sigma^2}{4\kappa}\left(2e^{-\kappa(t+s)}(e^{2\kappa s}-1)\right)}.
\end{aligned} \tag{C.36}$$

□

C.2.4 Proof of covariance

Proof. For the covariance, we have

$$\begin{aligned}
Cov(X_t, X_s) &= \mathbb{E}(X_t - \mathbb{E}(X_t))\mathbb{E}(X_s - \mathbb{E}(X_s)) \\
&= \mathbb{E}\Big[\int_0^t \sigma e^{\kappa(u-t)}\mathrm{d}W_u \int_0^s \sigma e^{\kappa(u-s)}\mathrm{d}W_u\Big].
\end{aligned} \tag{C.37}$$

If $t > s$

$$\begin{aligned}
&Cov(X_t, X_s) = \sigma^2 e^{-\kappa_p(t+s)}\mathbb{E}\Big[\int_0^s e^{\kappa u}\mathrm{d}W_u(\int_0^s e^{\kappa v}\mathrm{d}W_v + \int_s^t e^{\kappa u}\mathrm{d}W_u)\Big] \\
&= \sigma^2 e^{-\kappa(s+t)}\Big(\mathbb{E}(\int_0^s e^{\kappa u}\mathrm{d}W_u)^2 + \mathbb{E}(\int_0^s e^{\kappa u}\mathrm{d}W_u \int_s^t e^{\kappa u}\mathrm{d}W_u)\Big).
\end{aligned} \tag{C.38}$$

The integrals, from 0 to s and s to t, are independent and so the last term equals zero. Now, using Ito isometry, we are left with

$$Cov(X_t, X_s) = \sigma^2 e^{-\kappa(t+s)}\int_0^s e^{2\kappa u}\mathrm{d}u = \frac{\sigma^2 e^{-\kappa(t+s)}}{2\kappa}(e^{2\kappa s}-1). \tag{C.39}$$

For $t < s$ we use the same solution approach to obtain

$$Cov(X_t, X_s) = \frac{\sigma^2 e^{-\kappa(t+s)}}{2\kappa}(e^{2\kappa s}-1). \tag{C.40}$$

Therefore, in general

$$Cov(X_t, X_s) = \frac{\sigma^2 e^{-\kappa(t+s)}}{2\kappa}(e^{2\kappa min(s,t)} - 1). \tag{C.41}$$

□

D Complete expressions

D.1 Complete expression for the value of the plant

The complete expression for the value of the plant is given as

$$\begin{aligned}
&V(P,L) = \mathbb{E}\Big[\int_0^\infty e^{-rt} P_t(\eta_1 v_t + \eta_0)dt | P_0 = P, L_0 = L\Big] \\
&= \eta_1 d_2 \int_0^\infty e^{-rt}\Bigg(P_0^{2e^{-\kappa t}} e^{2\theta(1-e^{-\kappa t})+\frac{\sigma^2}{\kappa}(1-e^{-2\kappa t})} \\
&- 2MP_0^{e^{-\kappa(t-s)}} e^{\theta(1-e^{-\kappa(t-s)})+\frac{\sigma^2}{4\kappa}(1-e^{-2\kappa(t-s)})} + M^2\Bigg)\mathrm{d}t \\
&+ (\eta_1 d_1 + \eta_0)\int_0^\infty e^{-rt}\Bigg(P_0^{e^{-\kappa t}} e^{\theta(1-e^{-\kappa t})+\frac{\sigma^2}{4\kappa}(1-e^{-2\kappa t})} - M\Bigg)\mathrm{d}t \\
&+ \eta_1 d_3 L_0 \int_0^\infty e^{-(r+d_3)t}\Bigg(P_0^{e^{-\kappa t}} e^{\theta(1-e^{-\kappa t})+\frac{\sigma^2}{4\kappa}(1-e^{-2\kappa t})} - M\Bigg)\mathrm{d}t \\
&+ \eta_1 d_3 \int_0^\infty e^{-rt}\int_0^t e^{-d_3(t-s)}(f_s - d_1)\Bigg(P_0^{e^{-\kappa t}} e^{\theta(1-e^{-\kappa t})+\frac{\sigma^2}{4\kappa}(1-e^{-2\kappa t})} - M\Bigg)\mathrm{d}s\mathrm{d}t \\
&- d_2\eta_1 d_3 \int_0^\infty e^{-rt}\int_0^t e^{-d_3(t-s)}\Bigg(P_0^{e^{-(\kappa t+\kappa s)}} e^{(\theta+\frac{\sigma^2}{4\kappa})(2-e^{-\kappa t}-e^{-\kappa s})} e^{\frac{\sigma^2}{4\kappa}\left(2e^{-\kappa(t+s)}(e^{2\kappa s}-1)\right)} \\
&- MP_0 e^{-\kappa t} e^{\theta(1-e^{-\kappa t})+\frac{\sigma^2}{4\kappa}(1-e^{-2\kappa t})} - MP_0 e^{-\kappa s} e^{-\kappa s} e^{\theta(1-e^{-\kappa s})+\frac{\sigma^2}{4\kappa}(1-e^{-2\kappa s})} + M^2\Bigg)\mathrm{d}s\mathrm{d}t.
\end{aligned} \tag{D.1}$$

D.2 Derivatives of the value of the plant

The complete expression for the derivative of V with respect to the starting price, P_0, is given by

$$\begin{aligned}
\frac{\partial V}{\partial P_0} &= \eta_1\, d_2 \int_0^{\infty} \mathrm{e}^{-rt}\Bigg(2\,\frac{(P_0+M)^{2\,\mathrm{e}^{-\kappa\,t}}\,\mathrm{e}^{-\kappa\,t}}{P_0+M}\mathrm{e}^{2\,\theta\,\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{\kappa}} \\
&- 2\,\frac{(P_0+M)^{\mathrm{e}^{-\kappa\,t}}\,\mathrm{e}^{-\kappa\,t}M}{P_0+M}\mathrm{e}^{\theta\,\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa}}\Bigg)\,\mathrm{d}t \\
&+ \eta_1\,(Ld_3+d_1)\int_0^{\infty}\frac{\mathrm{e}^{-(r+d_3)t}\,(P_0+M)^{\mathrm{e}^{-\kappa\,t}}\,\mathrm{e}^{-\kappa\,t}}{P_0+M}\mathrm{e}^{\theta\,\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa}}\,\mathrm{d}t \\
&+ \eta_0\int_0^{\infty}\frac{\mathrm{e}^{-rt}\,(P_0+M)^{\mathrm{e}^{-\kappa\,t}}\,\mathrm{e}^{-\kappa\,t}}{P_0+M}\mathrm{e}^{\theta\,\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa}}\,\mathrm{d}t \\
&+ \eta_1\,d_3\int_0^{\infty}\int_0^{t}\frac{\mathrm{e}^{-rt}f(s)\,\mathrm{e}^{-d_3\,(t-s)}\,(P_0+M)^{\mathrm{e}^{-\kappa\,t}}\,\mathrm{e}^{-\kappa\,t}}{P_0+M}\mathrm{e}^{\theta\,\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa}}\,\mathrm{d}s\,\mathrm{d}t \\
&- \eta_1\,d_2\,d_3\int_0^{\infty}\int_0^{t}\mathrm{e}^{-rt}\mathrm{e}^{-d_3\,(t-s)}\Bigg(\frac{(P_0-M)^{\mathrm{e}^{-\kappa\,t}+\mathrm{e}^{-\kappa\,s}}\left(\mathrm{e}^{-\kappa\,t}+\mathrm{e}^{-\kappa\,s}\right)}{P_0-M} \\
&\cdot\mathrm{e}^{\theta\,\left(2-\mathrm{e}^{-\kappa\,t}-\mathrm{e}^{-\kappa\,s}\right)+\frac{\sigma^2\left(2-\mathrm{e}^{-2\,\kappa\,s}-\mathrm{e}^{-2\,\kappa\,t}+2\,\mathrm{e}^{\kappa\,(-t+s)}-2\,\mathrm{e}^{-\kappa\,(s+t)}\right)}{4\kappa}} \\
&- M\Bigg(\frac{(P_0-M)^{\mathrm{e}^{-\kappa\,s}}\,\mathrm{e}^{-\kappa\,s}}{P_0-M}\mathrm{e}^{\theta\,\left(1-\mathrm{e}^{-\kappa\,s}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,s}\right)}{4\kappa}} \\
&+ \frac{(P_0-M)^{\mathrm{e}^{-\kappa\,t}}\,\mathrm{e}^{-\kappa\,t}}{P_0-M}\mathrm{e}^{\theta\,\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa}}\Bigg)\Bigg)\,\mathrm{d}s\,\mathrm{d}t.
\end{aligned} \tag{D.2}$$

The complete expression for the derivative of V with respect to the speed of mean reversion, κ, is given by

$$
\begin{aligned}
\frac{\partial V}{\partial \kappa} &= \eta_1\ d_2 \int_0^\infty \mathrm{e}^{-rt}\Bigg(-2\ (P_0+M)^{2\,\mathrm{e}^{-\kappa\,t}}\ t\mathrm{e}^{-\kappa\,t}\ln(P_0+M)\,\mathrm{e}^{2\,\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{\kappa}} \\
&+ (P_0+M)^{2\,\mathrm{e}^{-\kappa\,t}}\Bigg(\frac{\sigma^2\left(1-\mathrm{e}^{-\kappa\,t}\right)}{2\kappa^2} + 2\,\theta t\mathrm{e}^{-\kappa\,t} - \frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{\kappa^2} \\
&+ 2\,\frac{\sigma^2 t\mathrm{e}^{-2\,\kappa\,t}}{\kappa}\Bigg)\mathrm{e}^{2\,\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{\kappa}} \\
&+ 2\ (P_0+M)^{\mathrm{e}^{-\kappa\,t}}\ t\mathrm{e}^{-\kappa\,t}\ln(P_0+M)\,M\mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa}} \\
&- 2\ (P_0+M)^{\mathrm{e}^{-\kappa\,t}}\ M\Bigg(\frac{\sigma^2\left(1-\mathrm{e}^{-\kappa\,t}\right)}{4\kappa^2} + \theta t\mathrm{e}^{-\kappa\,t} - \frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa^2} \\
&+ \frac{\sigma^2 t\mathrm{e}^{-2\,\kappa\,t}}{2\kappa}\Bigg)\mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa}}\Bigg)\,\mathrm{d}t \\
&+ \eta_1\ (Ld_3+d_1)\int_0^\infty \mathrm{e}^{-(r+d_3)t}\Bigg(-(P_0+M)^{\mathrm{e}^{-\kappa\,t}}\ t\mathrm{e}^{-\kappa\,t}\ln(P_0+M) \\
&\cdot\mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa}} + (P_0+M)^{\mathrm{e}^{-\kappa\,t}}\Bigg(\frac{\sigma^2\left(1-\mathrm{e}^{-\kappa\,t}\right)}{4\kappa^2} \\
&+ \theta t\mathrm{e}^{-\kappa\,t} - \frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa^2} + 1/2\,\frac{\sigma^2 t\mathrm{e}^{-2\,\kappa\,t}}{\kappa}\Bigg)\mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+1/4\,\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{\kappa}}\Bigg)\,\mathrm{d}t \\
&+ \eta_0\int_0^\infty \mathrm{e}^{-rt}\Bigg(-(P_0+M)^{\mathrm{e}^{-\kappa\,t}}\ t\mathrm{e}^{-\kappa\,t}\ln(P_0+M)\,\mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa}} \\
&+ (P_0+M)^{\mathrm{e}^{-\kappa\,t}}\Bigg(\frac{\sigma^2\left(1-\mathrm{e}^{-\kappa\,t}\right)}{4\kappa^2} + \theta t\mathrm{e}^{-\kappa\,t} - \frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa^2} \\
&+ \frac{\sigma^2 t\mathrm{e}^{-2\,\kappa\,t}}{2\kappa}\Bigg)\mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa}}\Bigg)\,\mathrm{d}t \\
&+ \eta_1\ d_3\int_0^\infty\int_0^t \mathrm{e}^{-rt}f(s)\,\mathrm{e}^{-d_3\,(t-s)}\Bigg(-(P_0+M)^{\mathrm{e}^{-\kappa\,t}}\ t\mathrm{e}^{-\kappa\,t}\ln(P_0+M) \\
&\cdot\mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+1/4\,\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{\kappa}} + (P_0+M)^{\mathrm{e}^{-\kappa\,t}}\Bigg(\frac{\sigma^2\left(1-\mathrm{e}^{-\kappa\,t}\right)}{4\kappa^2} \\
&+ \theta t\mathrm{e}^{-\kappa\,t} - \frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa^2} + \frac{\sigma^2 t\mathrm{e}^{-2\,\kappa\,t}}{2\kappa}\Bigg)\mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa}}\Bigg)\,\mathrm{d}s\,\mathrm{d}t \\
&- \eta_1\ d_2\ d_3\int_0^\infty\int_0^t \mathrm{e}^{-rt}\mathrm{e}^{-d_3\,(t-s)}\Bigg((P_0-M)^{\mathrm{e}^{-\kappa\,t}+\mathrm{e}^{-\kappa\,s}}\left(-t\mathrm{e}^{-\kappa\,t} - s\mathrm{e}^{-\kappa\,s}\right)
\end{aligned}
$$

$$
\begin{aligned}
&\cdot \ln\left(P_0 - M\right) \mathrm{e}^{\theta\left(2-\mathrm{e}^{-\kappa\,t}-\mathrm{e}^{-\kappa\,s}\right)+\frac{\sigma^2\left(2-\mathrm{e}^{-2\,\kappa\,s}-\mathrm{e}^{-2\,\kappa\,t}+2\,\mathrm{e}^{\kappa\,(-t+s)}-2\,\mathrm{e}^{-\kappa\,(s+t)}\right)}{4\kappa}} \\
&+ \left(P_0 - M\right)^{\mathrm{e}^{-\kappa\,t}+\mathrm{e}^{-\kappa\,s}} \left(\frac{\sigma^2\left(2-\mathrm{e}^{-\kappa\,t}-\mathrm{e}^{-\kappa\,s}\right)}{4\kappa^2} + \theta\left(t\mathrm{e}^{-\kappa\,t}+s\mathrm{e}^{-\kappa\,s}\right) \right. \\
&+ \frac{\sigma^2\left(2\,s\mathrm{e}^{-2\,\kappa\,s}+2\,t\mathrm{e}^{-2\,\kappa\,t}+2\,\left(-t+s\right)\mathrm{e}^{\kappa\,(-t+s)}-2\,\left(-s-t\right)\mathrm{e}^{-\kappa\,(s+t)}\right)}{4\kappa} \\
&\left. - \frac{\sigma^2\left(2-\mathrm{e}^{-2\,\kappa\,s}-\mathrm{e}^{-2\,\kappa\,t}+2\,\mathrm{e}^{\kappa\,(-t+s)}-2\,\mathrm{e}^{-\kappa\,(s+t)}\right)}{4\kappa^2} \right) \\
&\cdot \mathrm{e}^{\theta\left(2-\mathrm{e}^{-\kappa\,t}-\mathrm{e}^{-\kappa\,s}\right)+\frac{\sigma^2\left(2-\mathrm{e}^{-2\,\kappa\,s}-\mathrm{e}^{-2\,\kappa\,t}+2\,\mathrm{e}^{\kappa\,(-t+s)}-2\,\mathrm{e}^{-\kappa\,(s+t)}\right)}{4\kappa}} \\
&- M\left(- \left(P_0 - M\right)^{\mathrm{e}^{-\kappa\,s}} s\mathrm{e}^{-\kappa\,s} \ln\left(P_0 - M\right) \mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,s}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,s}\right)}{4\kappa}} \right. \\
&+ \left(P_0 - M\right)^{\mathrm{e}^{-\kappa\,s}} \left(\frac{\sigma^2\left(1-\mathrm{e}^{-\kappa\,s}\right)}{4\kappa^2} + \theta s\mathrm{e}^{-\kappa\,s} - \frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,s}\right)}{4\kappa^2} \right. \\
&\left. + \frac{\sigma^2 s\mathrm{e}^{-2\,\kappa\,s}}{2\kappa} \right) \mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,s}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,s}\right)}{4\kappa}} \\
&- \left(P_0 - M\right)^{\mathrm{e}^{-\kappa\,t}} t\mathrm{e}^{-\kappa\,t} \ln\left(P_0 - M\right) \mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa}} \\
&+ \left(P_0 - M\right)^{\mathrm{e}^{-\kappa\,t}} \left(\frac{\sigma^2\left(1-\mathrm{e}^{-\kappa\,t}\right)}{4\kappa^2} + \theta t\mathrm{e}^{-\kappa\,t} - \frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa^2} \right. \\
&\left.\left.\left. + \frac{\sigma^2 t\mathrm{e}^{-2\,\kappa\,t}}{2\kappa} \right) \mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa}} \right)\right) \mathrm{d}s\,\mathrm{d}t.
\end{aligned}
\tag{D.3}
$$

The complete expression for the derivative of V with respect to the volatility, σ, is given by

$$
\begin{aligned}
\frac{\partial V}{\partial \sigma} &= \eta_1\, d_2 \int_0^{\infty} \mathrm{e}^{-rt} \Bigg((P_0+M)^{2\,\mathrm{e}^{-\kappa t}} \left(-\frac{\sigma\,\left(1-\mathrm{e}^{-\kappa t}\right)}{\kappa} + 2\,\frac{\sigma\,\left(1-\mathrm{e}^{-2\kappa t}\right)}{\kappa} \right) \\
&\cdot \mathrm{e}^{2\theta\left(1-\mathrm{e}^{-\kappa t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\kappa t}\right)}{\kappa}} \\
&- 2\,(P_0+M)^{\mathrm{e}^{-\kappa t}} M \left(-\frac{\sigma\,\left(1-\mathrm{e}^{-\kappa t}\right)}{2\kappa} + \frac{\sigma\,\left(1-\mathrm{e}^{-2\kappa t}\right)}{2\kappa} \right) \\
&\mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\kappa t}\right)}{4\kappa}} \Bigg)\, \mathrm{d}t \\
&+ \eta_1\,(L d_3 + d_1) \int_0^{\infty} \mathrm{e}^{-(r+d_3)t} (P_0+M)^{\mathrm{e}^{-\kappa t}} \left(-\frac{\sigma\,\left(1-\mathrm{e}^{-\kappa t}\right)}{2\kappa} + \frac{\sigma\,\left(1-\mathrm{e}^{-2\kappa t}\right)}{2\kappa} \right) \\
&\cdot \mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\kappa t}\right)}{4\kappa}}\, \mathrm{d}t \\
&+ \eta_0 \int_0^{\infty} \mathrm{e}^{-rt} (P_0+M)^{\mathrm{e}^{-\kappa t}} \left(-\frac{\sigma\,\left(1-\mathrm{e}^{-\kappa t}\right)}{2\kappa} + \frac{\sigma\,\left(1-\mathrm{e}^{-2\kappa t}\right)}{2\kappa} \right) \\
&\cdot \mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\kappa t}\right)}{4\kappa}}\, \mathrm{d}t + \eta_1\, d_3 \int_0^{\infty}\int_0^{t} \mathrm{e}^{-rt} f(s)\, \mathrm{e}^{-d_3\,(t-s)} (P_0+M)^{\mathrm{e}^{-\kappa t}} \\
&\cdot \left(-\frac{\sigma\,\left(1-\mathrm{e}^{-\kappa t}\right)}{2\kappa} + \frac{\sigma\,\left(1-\mathrm{e}^{-2\kappa t}\right)}{2\kappa} \right) \mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\kappa t}\right)}{4\kappa}}\, \mathrm{d}s\, \mathrm{d}t \\
&- \eta_1\, d_2\, d_3 \int_0^{\infty}\int_0^{t} \mathrm{e}^{-rt} \mathrm{e}^{-d_3\,(t-s)} \Bigg((P_0-M)^{\mathrm{e}^{-\kappa t}+\mathrm{e}^{-\kappa s}} \Bigg(-\frac{\sigma\,\left(2-\mathrm{e}^{-\kappa t}-\mathrm{e}^{-\kappa s}\right)}{2\kappa} \\
&+ \frac{\sigma\left(2-\mathrm{e}^{-2\kappa s}-\mathrm{e}^{-2\kappa t}+2\,\mathrm{e}^{\kappa(-t+s)}-2\,\mathrm{e}^{-\kappa(s+t)}\right)}{2\kappa} \Bigg) \\
&\cdot \mathrm{e}^{\theta\left(2-\mathrm{e}^{-\kappa t}-\mathrm{e}^{-\kappa s}\right)+\frac{\sigma^2\left(2-\mathrm{e}^{-2\kappa s}-\mathrm{e}^{-2\kappa t}+2\,\mathrm{e}^{\kappa(-t+s)}-2\,\mathrm{e}^{-\kappa(s+t)}\right)}{4\kappa}} \\
&- M \Bigg((P_0-M)^{\mathrm{e}^{-\kappa s}} \left(-\frac{\sigma\,\left(1-\mathrm{e}^{-\kappa s}\right)}{2\kappa} + \frac{\sigma\,\left(1-\mathrm{e}^{-2\kappa s}\right)}{2\kappa} \right) \mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa s}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\kappa s}\right)}{4\kappa}} \\
&+ (P_0-M)^{\mathrm{e}^{-\kappa t}} \left(-\frac{\sigma\,\left(1-\mathrm{e}^{-\kappa t}\right)}{2\kappa} + \frac{\sigma\,\left(1-\mathrm{e}^{-2\kappa t}\right)}{2\kappa} \right) \\
&\cdot \mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\kappa t}\right)}{4\kappa}} \Bigg) \Bigg)\, \mathrm{d}s\, \mathrm{d}t.
\end{aligned}
\tag{D.4}
$$

The complete expression for the derivative of V with respect to the mean reversion parameter, θ, is given by

$$
\begin{aligned}
\frac{\partial V}{\partial \theta} &= \eta_1 d_2 \int_0^\infty \mathrm{e}^{-rt}\Bigg((P_0+M)^{2\,\mathrm{e}^{-\kappa\,t}} \left(2-2\,\mathrm{e}^{-\kappa\,t}\right)\mathrm{e}^{2\,\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{\kappa}} \\
&- 2\,(P_0+M)^{\mathrm{e}^{-\kappa\,t}} M\left(1-\mathrm{e}^{-\kappa\,t}\right)\mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa}}\Bigg)\,\mathrm{d}t \\
&+ \eta_1\,(Ld_3+d_1)\int_0^\infty \mathrm{e}^{-(r+d_3)t}\,(P_0+M)^{\mathrm{e}^{-\kappa\,t}}\left(1-\mathrm{e}^{-\kappa\,t}\right)\mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa}}\,\mathrm{d}t \\
&+ \eta_0\int_0^\infty \mathrm{e}^{-rt}\,(P_0+M)^{\mathrm{e}^{-\kappa\,t}}\left(1-\mathrm{e}^{-\kappa\,t}\right)\mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa}}\,\mathrm{d}t \\
&+ \eta_1\,d_3\int_0^\infty\int_0^t \mathrm{e}^{-rt} f(s)\,\mathrm{e}^{-d_3\,(t-s)}\,(P_0+M)^{\mathrm{e}^{-\kappa\,t}}\left(1-\mathrm{e}^{-\kappa\,t}\right)\mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa}}\,\mathrm{d}s\,\mathrm{d}t \\
&- \eta_1\,d_2\,d_3\int_0^\infty\int_0^t \mathrm{e}^{-rt}\mathrm{e}^{-d_3\,(t-s)}\Bigg((P_0-M)^{\mathrm{e}^{-\kappa\,t}+\mathrm{e}^{-\kappa\,s}}\left(2-\mathrm{e}^{-\kappa\,t}-\mathrm{e}^{-\kappa\,s}\right) \\
&\cdot\,\mathrm{e}^{\theta\left(2-\mathrm{e}^{-\kappa\,t}-\mathrm{e}^{-\kappa\,s}\right)+\frac{\sigma^2\left(2-\mathrm{e}^{-2\,\kappa\,s}-\mathrm{e}^{-2\,\kappa\,t}+2\,\mathrm{e}^{\kappa\,(-t+s)}-2\,\mathrm{e}^{-\kappa\,(t+s)}\right)}{4\kappa}} \\
&- M\Bigg((P_0-M)^{\mathrm{e}^{-\kappa\,s}}\left(1-\mathrm{e}^{-\kappa\,s}\right)\mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,s}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,s}\right)}{4\kappa}} \\
&+ (P_0-M)^{\mathrm{e}^{-\kappa\,t}}\left(1-\mathrm{e}^{-\kappa\,t}\right)\mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{4\kappa}}\Bigg)\Bigg)\,\mathrm{d}s\,\mathrm{d}t.
\end{aligned}
\tag{D.5}
$$

The complete expression for the derivative of V with respect to the long term mean parameter, α, is given by

$$\begin{aligned}
\frac{\partial V}{\partial \alpha} &= \eta_1\ d_2\ \int_0^{\infty} \mathrm{e}^{-rt} \Bigg(P^{2\,\mathrm{e}^{-\kappa\,t}}\ \left(2 - 2\,\mathrm{e}^{-\kappa\,t}\right) \mathrm{e}^{2\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{\kappa}} \\
&- 2\,P^{\mathrm{e}^{-\kappa\,t}} M\left(1 - \mathrm{e}^{-\kappa\,t}\right) \mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+1/4\,\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{\kappa}} \Bigg)\,\mathrm{d}t \\
&+ \eta_1\ \left(d_3\,L + d_1\right) \int_0^{\infty} \mathrm{e}^{-(r+d_3)t} P^{\mathrm{e}^{-\kappa\,t}}\ \left(1 - \mathrm{e}^{-\kappa\,t}\right) \mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+1/4\,\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{\kappa}}\,\mathrm{d}t \\
&+ \eta_0 \int_0^{\infty} \mathrm{e}^{-rt} P^{\mathrm{e}^{-\kappa\,t}}\ \left(1 - \mathrm{e}^{-\kappa\,t}\right) \mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+1/4\,\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{\kappa}}\,\mathrm{d}t \\
&+ \eta_1\ d_3\ \int_0^{\infty}\int_0^{t} f\mathrm{e}^{-rt}\mathrm{e}^{-d_3\,(t-s)} P^{\mathrm{e}^{-\kappa\,t}}\ \left(1 - \mathrm{e}^{-\kappa\,t}\right) \mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+1/4\,\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{\kappa}}\,\mathrm{d}s\,\mathrm{d}t \qquad \text{(D.6)} \\
&- \eta_1\ d_2\ d_3\ \int_0^{\infty}\int_0^{t} \mathrm{e}^{-rt}\mathrm{e}^{-d_3\,(t-s)} \Bigg(P^{\mathrm{e}^{-\kappa\,t}+\mathrm{e}^{-\kappa\,s}}\ \left(2 - \mathrm{e}^{-\kappa\,t} - \mathrm{e}^{-\kappa\,s}\right) \\
&\cdot \mathrm{e}^{\theta\left(2-\mathrm{e}^{-\kappa\,t}-\mathrm{e}^{-\kappa\,s}\right)+1/4\,\frac{\sigma^2\left(2-\mathrm{e}^{-2\,\kappa\,s}-\mathrm{e}^{-2\,\kappa\,t}+2\,\mathrm{e}^{\kappa\,(-t+s)}-2\,\mathrm{e}^{-\kappa\,(s+t)}\right)}{\kappa}} \\
&- M\Bigg(P^{\mathrm{e}^{-\kappa\,s}}\ \left(1 - \mathrm{e}^{-\kappa\,s}\right) \mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,s}\right)+1/4\,\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,s}\right)}{\kappa}} \\
&+ P^{\mathrm{e}^{-\kappa\,t}}\ \left(1 - \mathrm{e}^{-\kappa\,t}\right) \mathrm{e}^{\theta\left(1-\mathrm{e}^{-\kappa\,t}\right)+1/4\,\frac{\sigma^2\left(1-\mathrm{e}^{-2\,\kappa\,t}\right)}{\kappa}} \Bigg)\Bigg)\,\mathrm{d}s\,\mathrm{d}t.
\end{aligned}$$

E Calibration

E.1 Calibration of the price process

We calibrate the parameters of the price process similar to Ernstsen (2016). We let $(p_i)_{i=1,...,N}$ denote the hourly spot prices for the period and $(x_i)_{i=1,...,N}$ denote the logarithm of the price, such that $x_i = log(P_i + M)$. The corresponding random variables, X_i, satisfy the linear relationship

$$X_{i+1} \stackrel{d}{=} aX_i + m + b\epsilon_i \quad \text{for} \quad i = 1, ..., N-1, \tag{E.1}$$

where $a = e^{-\kappa\Delta}$, $\Delta = 1/N_{hour}$, $N_{hour} = 24 \cdot 365$, $m = (\alpha - \frac{\sigma^2}{4\kappa})(1 - e^{-\kappa\Delta})$, $b^2 = \sigma^2 \frac{1-e^{-2\kappa\Delta}}{2\kappa}$ and ϵ_i is the i.i.d. error term with $\epsilon_1 \sim N(0,1)$. We use ordinary least squares to get the estimators

$$\hat{a} = \frac{\sum_{i=1}^{N-1}(x_{i+1} - \overline{x})(x_i - \tilde{x})}{\sum_{i=1}^{N-1}(x_i - \tilde{x})^2} \tag{E.2}$$

and

$$\hat{m} = \overline{x} - \hat{a}\tilde{x}, \tag{E.3}$$

where

$$\overline{x} = \frac{1}{N-1}\sum_{i=2}^{N} x_i \tag{E.4}$$

and

$$\tilde{x} = \frac{1}{N-1}\sum_{i=1}^{N-1} x_i. \tag{E.5}$$

Now

$$\hat{\kappa} = \frac{-ln(\hat{a})}{\Delta} \tag{E.6}$$

and

$$\hat{b}^2 = \frac{1}{N-1}\sum_{i=1}^{N-1}(x_{i+1} - \hat{m} - \hat{a}x_i)^2. \tag{E.7}$$

Finally, we estimate σ and α as

$$\hat{\sigma} = \sqrt{\frac{2\hat{\kappa}}{1 - e^{-2\hat{\kappa}\Delta}}\hat{b}^2} \tag{E.8}$$

and

$$\hat{\alpha} = \frac{m}{(1 - e^{-\hat{\kappa}\Delta})} + \frac{\hat{\sigma}^2}{4\hat{\kappa}}. \tag{E.9}$$

E.2 Estimation of standard deviation

We estimate the standard deviation for the price process from

$$S = \lim_{t\to\infty}\sqrt{\mathbb{E}(P_t^2) - \mathbb{E}(P_t)^2}, \tag{E.10}$$

with the expectation and the second moment of the price process from (2.43) and (2.45) respectively, we have

$$\mathbb{E}(P_t|P_s) = e^{\theta + \frac{\sigma^2}{4\kappa}} - M = e^{\alpha} - M, \quad t \to \infty, \tag{E.11}$$

and

$$\mathbb{E}(P_t^2|P_s) = e^{(2\theta + \frac{\sigma^2}{\kappa})} - 2Me^{(\theta + \frac{\sigma^2}{4\kappa})} + M^2, \quad t \to \infty. \tag{E.12}$$

E.3 Fitting the inflow curve

The observed data are fitted to a sinusoidal function because of the periodic property of the inflow. An increasing number of sinusoidal terms were tested, where the result of the R^2-test with different number of sinusoidal terms is shown in Figure E.1.

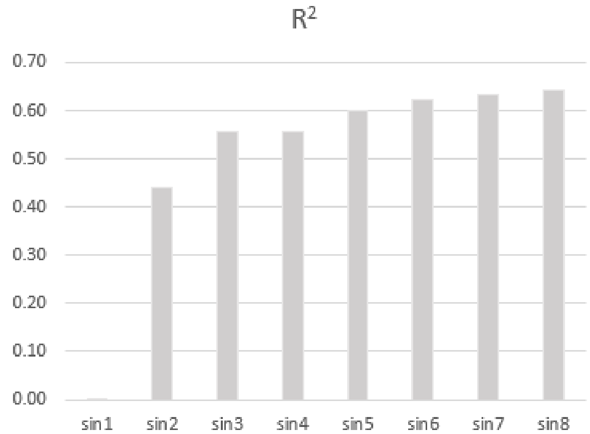

Figure E.1: Result of R^2-test for inflow function

As seen from the figure, all R^2-values are below 0.7 even with eight terms, which means that the fitted function explains less than 70 % of the observed data. Better R^2-values could be obtained using robust fitting, where measurements with extreme deviations from the mean are neglected. However, we are not interested in neglecting the extreme deviations from the inflow, hence normal fitting is a better choice. Because of the complexity of the overall problem, a simple expression is wanted. Here, a five term function is chosen.