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Anomalies and the five-factor model in the Norwegian Stock Market

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Problem description

This thesis will investigate fundamental anomalies in the Norwegian stock market. A part of the study is also to test if Fama & French's five-factor asset pricing model has explanatory power for stocks listed on the Oslo Stock Exchange.

Preface

This master thesis is written for the Department of Industrial Economics and Technology Management at NTNU. A great interest in factor models and stock return regularities arose from writing a project thesis around the same subject. I would especially like to thank my supervisor Einar Belsom for valuable inputs and motivation during this process. Any errors, inaccuracies or miss interpretations are however only my own. A great amount of time is spent mastering Rstudio and ShareLaTeX to perform and present the following thesis.

Abstract

This study investigates stock return regularities related to firm size, value, profitability, investments, momentum, liquidity and estimated β's, in the Norwegian stock market. The research finds significant return patterns related to firm size, liquidity and momentum in the period 1997 to 2017. The presented results further indicate that all investigated anomalies are relevant to some degree in the Norwegian stock market.

The second goal of this thesis is to evaluate how suitable a locally adapted Fama & French five-factor model is for describing Norwegian daily stock returns. The model's performance is scrutinized by comparing test statistics with several other possible risk factor combinations. The risk factors included are related to the momentum, liquidity and β -anomaly, and the statistics for evaluation are retrieved by applying the GRS-test, the Wald-test and risk premium estimation by Generalized Method of Moments. The results indicate that the value and investment -factor are less important for describing variations in Norwegian stock returns and that other risk factor combinations seem more suitable than the proposed five-factor model. Especially a four-factor model including the market, size, liquidity and momentum -factor performs reasonably well in all applied tests, indicating a better fit in a Norwegian setting.

Sammendrag

Denne undersøkelsen gransker avkastninger knyttet til aksjeselskapets størrelse, verdi, lønnsomhet, investeringer, momentum, likviditet og estimerte β'er, i det norske aksjemarkedet. Undersøkelsen finner betydelige avkastningsmønstre relatert til aksjeselskapets størrelse, likviditet og momentum i perioden 1997 til 2017. Resultatene viser videre at alle undersøkte anomalier er til en viss grad relevante i det norske aksjemarkedet.

Et annet mål med denne avhandlingen er å vurdere hvor godt egnet en lokalt tilpasset Fama & French fem-faktormodell er for å beskrive norske aksjeavkastninger. Fem-faktormodellen evalueres ved å sammenligne statistiske resultater mot flere andre potensielle risikofaktorkombinasjoner. De inkluderte risikofaktorene er knyttet til momentum, likviditets og β -anomalien og sammenligningsgrunnlaget hentes ved å anvende en GRS-test, Wald-test og risikopremieestimering ved Generalized Method of Moments. Resultatene indikerer at verdi og investerings -faktoren er mindre viktig for å beskrive variasjoner i norske aksjeavkastninger, og at andre risikofaktorkombinasjoner er muligens bedre tilspasset enn den opprinnelige femfaktormodellen. Spesielt en fire-faktormodell bestående av en markeds, størrelses, likviditets og momentums -faktor presterer akseptabelt i alle anvendte tester, hvilket indikerer at den er egnet i en norsk sammenheng.

Contents

Chapter 1

Introduction

In many international studies, researchers try to explain shortcomings of the Capital Asset Pricing Model (CAPM) introduced by Sharpe (1964) and Lintner (1965). The CAPM is commonly used by investors to understand the relationship between risk and return and assumes that all securities can be described by a relevant market index and its market β . However the model seems to fail when describing several empirically proved regularities in stock returns, which has made a massive incentive to improve and add more factors to the model. While these factors often add explanatory power to the CAPM in empirical analysis, many of them lack a convincing theoretical argument and they are termed "anomalies". The by far most popular model, is the Fama & French (1993) three factor-model (FF3), which adds two factors to describe firm size and value. This model has long served as a benchmark model when investigating returns and multi-factor models across international markets.

Further, researches, such as Titman, Wei & Xie (2004) and Novy-Marx (2013) found evidence that FF3 fails to explain return variations related to company's profitability and investments. Motivated by the new evidence and the dividend discount model, Fama & French (2015b) recently added a profitability and investment -factor yielding the new and improved five-factor model (FF5). Their new model consistently outperformed the old on data from the New York Stock Exchange (NYSE), but the value factor became redundant in presence of the two new factors. To to see how well their new model translates to other markets, Fama & French (2015c) investigated a global model in four international regions, North America, Europe, Asia Pacific and Japan. They conclude that locally adopted models perform significantly better, and that risk patterns in anomalies vary between the international markets. This conclusion has stimulated interest in investigating what risk factors are relevant in a Norwegian setting, as well as to investigate how well these models perform in the Norwegian market.

The most extensive research in the Norwegian market so far is by Næs, Skjeltorp & Ødegaard (2009), where they investigate CAPM and FF3 alongside factors related to liquidity risk, momentum and macroeconomic variables. Their research found that a three-factor model containing a market, size and liquidity -factor was able to describe returns in the Norwegian market. Further they found only weak evidence of macroeconomic variables being priced in the market, and that the most reasonable channel for these effects are through company cash flows. The FF5 has so far only been tested in Hoel & Mix (2016), where they concluded that a two-factor model containing a market and size -factor, give the same amount of information about stock returns as the FF5 in a Norwegian setting. This thesis is very inspired by the work done in these papers, especially the updated work in Ødegaard (2018b) and Ødegaard (2018c) has provided valuable information about the Norwegian stock market and methodology for model testing.

What this research adds to current litterateur, is that it tests the new FF5 alongside factors related to liquidity risk, momentum and the β -anomaly in the Norwegian stock market. This research is also to my knowledge the first to investigate daily return data in a Norwegian factor model setting, which should add some power to the estimations. The research will first investigate anomalies related to the size, value, profitability, investment, liquidity, momentum and β -effect. Then try to find factor combinations which can be added or compete with the FF5 -factors in the Norwegian stock market.

The thesis is structured as follows; the second chapter gives a brief introduction to the most relevant theory and used methodology, while the third chapter presents how the data is collected and sorted. The fourth chapter presents results from investigating anomalies and performance testing of the factor models.

Chapter 2

Theory and methodology

This chapter will introduce the most relevant theory concerning factor models and their performance testing. The first part will introduce the concept of factor models, while later presenting the FF5 and potential additional factors. Further, the last part of this chapter concentrates on methodology for performance testing of the potential factor models.

2.1 Introduction to factor models

For an expected return beta model, one often refers to either a conditional or unconditional model. Cochrane (2009) explains that the difference is simply whether one assumes the coefficients are time varying β_t or constant β. An unconditional model is special case of a conditional model with constant weights. The two different specifications for a single factor model can be illustrated as:

$$E_t[R^i] = R_t^f + \beta_t \lambda_t \tag{2.1}$$

$$E[R^i] = \alpha + \beta\lambda \tag{2.2}$$

The coefficient, β, and the risk premium, λ, are either hold constant or time varying. Equally estimated coefficients β_t for each time period t does not necessary imply an unconditional model. This is only the case if the variance and covariance of the coefficients are constant over time. Researchers such as Gibbons & Ferson (1985), Bollerslev, Engle & Wooldridge (1988) and Jagannathan & Wang (1996) find evidence that risk premiums and implied betas are indeed varying over time. I will primarily investigate the unconditional model specification.

The later presented multi-factor models are arguably variants of the Arbitrage Pricing Theory (APT), developed by Ross (1976) as an alternative to CAPM. The APT assumes that all asset returns can be described by their covariance with several systematic risk factors. These factors can either be derived from macro-variables, valuation theory or statistically proved risk factors in stock returns. It has become common in asset pricing to identify these risk factors from portfolios formed on some state variables which the CAPM fails to explain. The APT factor models can unconditionally be expressed as,

$$E[R^i] = \sum_j \lambda_j \beta_j^i \tag{2.3}$$

The expected return, $E[R^i]$, is the sum of asset i's exposure to several risk premiums λ_j. Further these risk premiums can be expressed through hedged portfolios which represent the extremes of companies sorted on the identified risk state variables. These return time series (risk premiums) are commonly called factor-mimicking portfolios, as they correlate with effects related to the underlying state variable. However, the true state variable may be unknown, as these return patterns or shortcomings of the CAPM are often derived from empirical tests rather than unambiguous theoretical evidence.

2.2 The Fama & French FF5 and factor constructions

The five-factor model directed at capturing the size, value, profitability and investment patterns in average stock return can be seen in equation 2.4,

$$eR_{it} = \alpha_i + \beta_i^b eRm_t + \beta_i^s SMB_t + \beta_i^h HML_t + \beta_i^r RMW_t + \beta_i^c CMA_t + \epsilon_{it} \tag{2.4}$$

eR_{it} is the average excess return for stock or portfolio i in period t, the intercept α_i should be zero for all i, and β_i is each portfolio's exposure to the different factors. eRm_t denotes the excess return of the market index. While "Small-Minus-Big" (SMB), "High-Minus-Low" (HML), "Robust-Minus-Weak" (RMW) and "Conservative-Minus-Aggressive" (CMA) are the risk premiums for simple trading strategies holding long position in a diversified portfolio of respectively small size stocks (S), high value stocks (H), robust profitability stocks (R) or conservative investment stocks (C), while holding short position in a diversified portfolio of big size stocks (B), low value stocks (L), weak profitability stocks (W) or aggressive investment stocks (A).

Constructions of the factor-mimicking portfolios are illustrated in table 2.1, where N denotes the neutral state between the 30th and 70th percentile for value (B/M), profitability

(OP) and investments (INV). All breakpoints are calculated from the sample of selected companies for each year. The factor constructions are possible to perform on finer sorts on the variables (Fama & French, 2015b), but given a small sample, the chosen approach assures that each constructed portfolio is represented by a satisfying amount of stocks.

Table 2.1: Factor Constructions

Sort	Breakpoints	Factor definitions
(2 x 3 sorts on:)	Size: median of selected companies	$SMB_{B/M}$=(SH+SN+SL)/3-(BH+BN+BL)/3
Size and B/M		SMB_{OP}=(SR+SN+SW)/3-(BR+BN+BW)/3
Size and OP		SMB_{Inv}=(SC+SN+SA)/3-(BC+BN+BA)/3
Size and INV		SMB=($SMB_{B/M}$+SMB_{OP}+SMB_{INV})/3
	B/M: 30th and 70th percentiles	HML=(SH+BH)/2-(SL+BL)/2
	OP: 30th and 70th percentiles	RMW=(SR+BR)/2-(SW+BW)/2
	INV: 30th and 70th percentiles	CMA=(SC+BC)/2-(SA+BA)/2

2.2.1 Variables for Fama & French factors

The variables for each characteristic is calculated similar to Fama & French (2015b). The measure for size is the market capitalization (market cap) for stock i by the end calendar year $t-1$. The variable can be calculated by:

$$market\ cap^i_{t-1} = stock\ price^i_{t-1} * outstanding\ shares^i_{t-1} \tag{2.5}$$

This is the standard variable for measuring size effects in stocks, but one could alternatively use the companies book equity or enterprise value. The market caps in this analysis is retrieved from the data source.

The value measure (B/M) is the ratio between book equity and market cap as of end of calendar year $t-1$. The measure is calculated by:

$$B/M^i_{t-1} = \frac{book\ equity^i_{t-1}}{market\ cap^i_{t-1}} \tag{2.6}$$

Intuitively, a low ratio indicate that the market has over-priced the stock, as well as a ratio over 1 indicate that the stock is under-priced.

To evaluate profitability (OP), operating profits[1] is divided by book equity:

$$OP^i_{t-1} = \frac{operating\ profits^i_{t-1}}{book\ equity^i_{t-1}} \quad (2.7)$$

Fama & French (2015b) find the change in total assets as a good proxy for investments (INV), so this effect is calculated as growth of total assets between end of calendar year t-1 and t-2:

$$INV^i_{t-1} = \frac{total\ assets^i_{t-1}}{total\ assets^i_{t-2}} - 1 \quad (2.8)$$

2.2.2 Motivation for HML, RMW and CMA

Fama & French (2015b) motivate the relationship between expected return and B/M-ratio, expected profitability and expected investments through the dividend discount model illustrated in equation 2.9.

$$m_t = \sum_{\tau=1}^{\infty} E(d_{t+\tau})/(1+r)^\tau \quad (2.9)$$

m_t is the share price at time t, while $E(d_{t+\tau})$ is the expected dividends per share in period $t+\tau$ and r is the internal rate of return (approximately the long-term average expected stock return). The total of expected dividends for period $t+\tau$, can be seen as a relationship between expected future earnings $E(Y_{t+\tau})$ and a part of that earnings b being held back by the company. This part b represents a change in the company's book equity, which gives $b = E(dB_{t+\tau}) = E(B_{t+\tau} - B_{t+\tau-1})$ and equation 2.10.

$$M_t = \sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau})/(1+r)^\tau \quad (2.10)$$

In this equation, M_t is the market cap at time t, while $E(Y_{t+\tau} - dB_{t+\tau})$ represents the expected dividends for all stocks in company at time $t+\tau$. If one decides to fix all variables except $E(Y_{t+\tau})$ and the expected return r, higher future earnings implies higher expected returns.

[1] Operating profits: Annual revenues minus costs of goods sold, interest expenses and selling, general and administrative expenses.

Dividing both sides by book equity for time t gives,

$$\frac{M_t}{B_t} = \frac{\sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau}}{B_t} \tag{2.11}$$

If one now fixes every variable except the M_t and expected return, a lower M_t implies a higher expected return. Inversing the left side fraction, this means a higher B/M-ratio implies a higher expected return. At last if one fix every variable except the $(dB_{t+\tau})$ and the expected return, an expected growth in book equity (investments) implies a lower expected return.

Overall equation 2.9 and 2.11 implies there is an interdependence for expected stock returns by the relations between expected future earnings, expected future investments and the B/M ratio. Hou, Xue & Zhang (2014) raise concerns whether this interpretation is correct and on how Fama and French construct their factors. They find through empirical tests that past investments are a poor proxy regarding future investments, as well as that the internal rate of return often correlate negatively with the one-period-ahead expected return. They prove through valuation theory that the value factor should be highly correlated to the investment factor, which substantiates Fama and French's empirical evidence of the factor becoming redundant in the new model.

2.3 Additional factors

The following factors are likely to explain some of the same risk, but potentially compliment the FF5 factors to some degree. There are however several other risk factors which could be interesting to test in this setting, but I have chosen factors which are previously found interesting in the Norwegian stock market.

2.3.1 LIQ

The liquidity anomaly is a stock return regularity which has been investigated in asset pricing theory by researchers such as Acharya & Pedersen (2005) and Liu (2006). Liu (2006) find that a two-factor model containing the market factor and a liquidity factor describes returns related to the size and value -effect quite well. A challenge regarding identifying a stocks liquidity is that its measure can be derived from several dimensions. Næs et al. (2009) used relative spread as a measure, which is under the trading cost dimension. The relative spread is the difference between closing ask and bid price, divided by midpoint price. However, this measure did correlate highly with the size measure (market cap) in their research. This survey will investigate the same measure on daily data. The selected data source did however only contain the daily best bid and daily best ask -price, so the measure is calculated as following each day d,

$$relative\ spread_d^i = \frac{best\ ask_d^i - best\ bid_d^i}{(best\ ask_d^i + best\ bid_d^i)/2} \tag{2.12}$$

The measure is further calculated as the mean of all relative spreads, from start of July year $t-1$ to end of June year t. Zero values during calculations represented missing data points and were excluded from the mean estimate.

An effort to construct this factor corrected for the size effects, such as the Fama & French factors HML, RMW and CMA, substantiates Næs et al. (2009) findings, that the measure is highly correlated with the size of the firm. No companies were found to be small by median and lower than the 30% percentile of relative spread. As a result, the liquidity factor (LIQ) is constructed by forming two portfolios by the 30% and 70% percentiles in relative spread each 1st of July. The zero-investment portfolio LIQ is further formed from having a long position in the illiquid portfolio and short position in the liquid portfolio and sorted annually.

2.3.2 MOM

The momentum factor, is suggested to compliment the FF3 in Carhart (1997) (Carhart model) and Fama & French (2012) and the FF5 in Fama & French (2015a). So, it is originally not a challenging factor, but simply trying to explain returns related momentum not covered by the previous stated factors. The factor is referred to as "Winners-Minus-Loser" (WML), "Up-Minus-Down" (UMD), "Momentum" (MOM) and "Prior 1 Year" (PR1YR) in different studies and its construction is varying. Most notably is the correction for size effects in the studies by Fama & French. The momentum anomaly is credited Jegadeesh & Titman (1993), where they find evidence that buying stocks that have performed well the past 3-12 months and selling stocks that has performed poorly the same period, generate significant positive returns. Næs et al. (2009) found little evidence for the momentum anomaly in the Norwegian stock market in the period 1980 to 2006 on monthly data.

In this research the MOM factor will be constructed similarly to the Fama & French (2015a) and Næs et al. (2009)(called UMD in their research), so corrected for size effects such as the HML, RMW and CMA -factors, but updated monthly rather than annually. The factor's measure is formed at each end of month $m-1$ by the total return calculated between month $m-12$ to $m-2$, referred to as Prior (2-12).

2.3.3 BAB

Frazzini & Pedersen (2014) proposes a Betting Against Beta factor (BAB) which challenge the SMB, HML and momentum -factor in positive average returns and statistical significance. The factor roots from the idea that leverage constrained investors overweight riskier securities in their portfolios, causing them to bid up high beta assets. This implies that the broad use of asset pricing theory to produce high Sharpe ratio[2] portfolios, is contributing to the CAPM's failure. The principle is that low-beta assets are under-priced and high-beta assets are over-priced. The effect is proved to be relevant in the Norwegian market by Korneliussen & Rasmussen (2014). The factor is produced by holding a long position in low beta portfolios and shorting high beta portfolios. Frazzini & Pedersen (2014) estimated betas by the formula,

$$\hat{\beta}_i = \hat{p}\frac{\hat{\sigma}_i}{\hat{\sigma}_m} \tag{2.13}$$

where $\hat{\beta}_i$ is asset i's beta, σ_m the market volatility and $\hat{p}$ their correlation. They used longer time series estimation for the correlation compared to the volatility, as correlations appeared to be more stable over time. This study has not taken this approach due to data constraints, but betas are estimated by rolling regression by the previous 300 trading days and with the market index as explanatory variable. For a beta to be constructed it must have at least 99% valid return data in the period. Frazzini & Pedersen (2014) also corrected for outliers by scaling the estimated beta time series, $\hat{\beta}_t^i$, by the cross-sectional mean β^{XS} by a weight(W) of 0.6:

$$\hat{\beta}_t^i = W * \hat{\beta}_t^i \ + \ (1-W)\hat{\beta}_t^i \tag{2.14}$$

Further the BAB factor is constructed sorting $\hat{\beta}_t^i$ by median each month and forming two portfolios of respectively low $\hat{\beta}_t^i$ and high $\hat{\beta}_t^i$ assets. Asset weights in portfolios are calculated by the ranking of their $\hat{\beta}_t^i$, meaning low betas are ranked from 1 being the highest, and high

[2]Sharpe ratio: the expected excess return per unit of risk.

betas from 1 being the lowest (in their allocated portfolio). This is to amplify those stocks most exposed to the risk factor. The ranking weighted return is the assets ranking divided by the sum of all ranks in that portfolio. Frazzini & Pedersen (2014) describes a slightly different approach, but this produces the same result. The equation for the BAB factor's return is,

$$r_{t+1}^{BAB} = \frac{1}{\hat{\beta}_t^L}(r_{t+1}^L - r^f) - \frac{1}{\hat{\beta}_t^H}(r_{t+1}^H - r^f) \tag{2.15}$$

where r_{t+1}^L is the ranking weighted return in low beta portfolio, r_{t+1}^H is ranking weighted return in high beta portfolio and $\hat{\beta}_t^L$/ $\hat{\beta}_t^H$ is the ranking weighted beta upon construction of each portfolio. The $\hat{\beta}_t^L$/ $\hat{\beta}_t^H$ is used to leverage/de-leverage each portfolio to a beta of 1, which makes the BAB factor a market neutral zero-beta portfolio. Without this correction the long side has very low volatility and the short side has very high volatility. The factor distinguishes from the other factors in this survey which are zero-investment portfolios. The idea is that the BAB factor is self-funding in terms of excess returns. The factor is expected to have positive returns on average, but also to be very sensitive to worsening of funding liquidity such as a credit crisis. The factor in this thesis will be constructed true to the original BAB factor, hence with scaling of betas and with beta-ranking weighted portfolios.

2.4 Evaluating models

This part of the chapter will present methodology for evaluating anomalies and test factor model performance. Some statistics will, however, be further explained when discussing results in chapter 4.

2.4.1 Forming test portfolios

To asses anomalies and factor models one usually creates so-called managed portfolios with scaled returns. Forming portfolios also remove firm specific risk and maintain stationarity in the test assets (Cochrane, 2009; Næs et al., 2009).

Fama & French (2015b) recommends using a 2 x 4 x 4 triple sort to construct portfolios to evaluate their model. This implies forming 32 portfolios sorted on three different characteristics such as 4 different percentiles of value and profitability into 2 size groups. The idea is to disentangle the effects one wish to investigate and test the model on. Preferably for a five-factor model, one would create 3 x 3 x 3 x 3 (81) portfolios to fully isolate each effect added to the market factor. But in most cases these portfolios would be poorly diversified and have low power. An issue is also that some characteristics such as profitability, value and investments are highly correlated, which implies that double sorting would form an uneven allocation of stocks.

With the given number of stocks in this survey's sample, one could at most create 3 x 3 double sorts while retaining power in the portfolios. Ødegaard (2018b) concludes that a well diversified portfolio must contain over 10 stocks. The challenges with a 3 x 3 sort is that these percentiles are very similar to those constructing our factor-mimicking portfolios. This approach could potentially create very high correlation between test assets and explaining factors. In order to maintain power in test assets and investigate anomalies in the Norwegian stock market, I will sort assets into 8 different portfolios based on specific characteristics. Characteristics to be investigated are returns related to the size, value, profitability, liquidity, momentum and β -anomaly. This is very similar to the approach done in Næs et al. (2009).

An important difference in this study compared to Ødegaard (2018b) and Næs et al. (2009), is that all sorting (re-balancing) of portfolios happens at 1st of July. Norwegian companies commonly operate their fiscal year as the calendar year and have a reporting period for their financial statements before 31st of July. It is to the author's understanding that most companies report a lot earlier than July and that the chosen date is reasonable. Anyhow if these models or portfolios are to represent appropriate investment strategies, the investors need to have accessible data.

2.4.2 GRS-test

The Gibbons, Ross & Shanken (1989), GRS , statistic is perhaps the most widely used test to compare asset pricing models. The test is beneficial for retrieving one statistic of a regression's model run on multiple test assets. The idea is that a correctly specified model has a pricing error, the intercept, α_i equal to zero for all test assets i. This is only possible if a combination of the explanatory variables is mean variance efficient. The GRS -test is formulated as:

$$GRS = \left(\frac{T}{N}\right)\left(\frac{T-N-L}{T-L-1}\right)\left[\frac{\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}}{1+\bar{\mu}'\hat{\Omega}^{-1}\bar{\mu}}\right] \sim F(N, T-N-L) \tag{2.16}$$

N is the number of test assets, T the number of observations (days) and L the number of explanatory factors. Further, $\hat{\alpha}$ is an N x 1 vector of estimated intercepts, $\hat{\Sigma}^{-1}$ is the unbiased estimate of the residual covariance matrix, $\bar{\mu}$ is the L x 1 vector of explanatory factor means and $\hat{\Omega}$ is the unbiased estimate of the factor covariance matrix. The test requires that the errors are normally distributed, as well as uncorrelated and homoscedastic. Estimated $\hat{\alpha}$'s close to zero is indicated by a low GRS statistic. Models are not directly comparable in this test, as more factors added to the model is generally expected to decrease the GRS-stat, however the size of the difference give an indication of the added factor's importance. In this survey, the test is performed in Rstudio by implementing the steps described in lectures slides by Diether (2001).

2.4.3 The Wald test for regressors and intercepts

While the GRS-test require that one compare models, a Wald test can be implemented to evaluate the model itself. This by testing if one or a set of coefficients is equal to some value. By testing if a coefficient is equal to zero, one finds if the factor is relevant to explain the test assets return. Cochrane (2009) refers to the test as a "horse race" between explanatory factors. The test is run in two steps. First by estimating the unrestricted regression's model, and then estimating the restricted model where some $\hat{\beta}_j$ or $\hat{\beta}_j{}'s$ are forced equal to zero. The Wald test statistic is formulated for the hypothesis $H_0 : \hat{\beta}_j = 0$ as,

$$W = \hat{\beta}_j{}' var(\hat{\beta}_j)^{-1} \hat{\beta}_j \sim \chi^2(N) \tag{2.17}$$

where $\hat{\beta}_j$ N x 1 vector for a number of N test assets and $var(\hat{\beta}_j)$ is its variance - covariance matrix. Using the same matrix from the unrestricted model for both steps forms the difference in χ^2,

$$W(restricted) - W(unrestricted) \quad \sim \chi^2(\# \; of \; restrictions) \tag{2.18}$$

and this value should not rise much if the tested coefficient is zero. This method can be used both to test estimated coefficients as well as testing the estimated intercepts. In this thesis the test is performed retrieving coefficients/intercepts from the first step of a GMM-estimation and using the linearhypotesis function (car package) in Rstudio.

2.4.4 Two-step regression for evaluating risk premium

To asses which factors exercise a risk premium and if the model has explanatory power, the traditional approach is to perform a two-step regression. The first step is developed by Jensen, Black & Scholes (1972) (BJS- regression), which is an Ordinary Least Squares - regression (OLS) to estimate the coefficients for each factor.

$$er_i^t = \alpha^i + \sum_{j=1}^{j} \beta_j^i f_{jt} + \epsilon_t^i \tag{2.19}$$

er_i^t, the left-hand-side (LHS) of the regression, represent the excess return for selected stock or portfolio *i*. The α^i is the intercept/constant and β_j^i is the exposure to factor f_j of stock or portfolio i. The right-hand-side of the regression is the return time series for each factor f_j in the same time period.

The second step developed by Fama & MacBeth (1973), is the cross-sectional regression where each coefficient β_j^i is used as the explanatory variable. The β_j^i's could either be treated as a constant for each portfolio i during this regression or the estimated coefficient for a moving time period (for example a year).

$$er_i = \lambda_0 + \sum_{j=1}^{j} \lambda_j \beta_j^i + \epsilon^i \tag{2.20}$$

At each t in equation 2.19 the calculated coefficients β_j^i in 2.19 are regressed against the excess return of all portfolios at same t, giving an estimated premium λ_j for the different factors j in that time period. Performing this on all t's, gives estimated premiums, λ_j, time series. The total premium for f_j is simply the mean of each λ_j and each model's error is the mean of the intercept λ_0. Both calculations are illustrated in equation 2.21 and 2.22:

$$E[\lambda_0] = \frac{1}{T} \sum_{t=1}^{T} \lambda_0 \tag{2.21}$$

$$E[\lambda_j] = \frac{1}{T} \sum_{t=1}^{T} \lambda_j \tag{2.22}$$

Futher a t-test, assuming the coefficients are normally distributed, can be run to test if $E[\lambda_0] = 0$ and $E[\lambda_j] > 0$. A model is rejected if the intercept is significantly different from zero and the risk premiums are priced if λ_j is significantly greater than zero. The adjusted R^2 from the second step can also be used to compare the different models.

This approach gives a better estimate than taking the mean of the factor returns, as one includes the covariances between the different assets factor . Cochrane (2009) proposes that Generalized Least Squares (GLS) regression can be used in the second step to deal with residuals in the cross-sectional regression being correlated with each other. However, he suggests that OLS is "pretty darn good" for a robust estimation. The main problem with this two-step estimation is that that the model does not account for the explanatory variables in 2.20 having estimation errors. But these errors are likely to be reduced by using portfolios in the estimation rather than simple stocks (Fama & MacBeth, 1973).

2.4.5 GMM approach

In more recent asset pricing litterateur, the previous explained two-step regression is mapped in a Generalized Method Moments (GMM) -estimation. Most researchers do this in a stochastic discount factor (SDF) framework, but as this research only operate with return time series it allows for a more intuitive beta representation of the models, as proposed by MacKinlay & Richardson (1991). The two different representations should have the same efficiency in GMM -estimation if formulated properly (Jagannathan & Wang, 1996; Jagannathan, Skoulakis & Wang, 2002).

This description of the methodology will cover one asset's excess return, eR_t, at time t and one explanatory variably, f_t, for notational simplicity, but the method easily translates to multi-factor models. To conduct GMM one must first form a set of moment conditions from the model one wants to estimate. The method minimizes these moments by adjusting the parameters in the model. Considering an unconditional linear beta pricing model of the form:

$$eR_t = \alpha + \beta f_t + \epsilon_t \tag{2.23}$$

where α is the intercept, β is the assets exposure to factor f and ϵ_t the error term. It follows from OLS assumptions that the expected value of $E[\epsilon_t] = 0$ and $E[f_t \epsilon_t] = 0$. This allows us to form the following moment conditions:

$$g_T(b) = \begin{bmatrix} E_T(eR_t - \alpha - \beta f_t) \\ E_T[(eR_t - \alpha - \beta f_t) f_t] \end{bmatrix} = E_T \begin{bmatrix} \epsilon_t \\ f_t \epsilon_t] \end{bmatrix} = 0 \tag{2.24}$$

where you have 2N moment conditions for 2N parameters $b' = [\alpha' \ \beta']$ to be estimated and an identified system for N test assets. (It is more precisely a method of moments (MM) estimation when the system is identified.) A mean variance efficient f_t implies that $\alpha = 0$. Exploiting this, and the $\hat{\beta}$ estimate from the the first step allows us to find the risk premium by the moment condition:

$$g_T(b) = \left[E(eR - \hat{\beta}\lambda)\right] = E\left[\alpha\right] = 0 \tag{2.25}$$

This system has N moments for a number of N test assets and K parameters for a number of K factors. $N > K$ leads to over-identifying restrictions and the GMM estimation of λ. A J -test measuring the size of the error (intercept) in the second step GMM can be used to evaluate model fit. The test is formulated as,

$$TJ_T = T\left[g_T(\hat{b})' S^{-1} g_T(\hat{b})\right] \ \chi^2(N-K) \tag{2.26}$$

where a low value of TJ_T and high p-value indicate good model fit.

In this survey, the two different moment functions are minimized using the optimal weighting matrix as proposed by Newey & West (1987) and the two-step GMM procedure proposed by Hansen (1982) (two steps for each function $g_T(b)$). This method is robust in that way that one does not have to assume the errors are independent and identically distributed (iid) and allows for heteroskedasticity and serial correlation (Chaussé & others, 2010). This makes the approach more robust in terms of time variations of variances in the factor time series.

These two steps of GMM -estimations are mapped in the GMM package in Rstudio provided by Chaussé et al. (2010). The codes for moment conditions and implementation are inspired by those provided in Ødegaard (2018b). This two-step method does not, however, account for the error in the estimated $\hat{\beta}$, to do this one has to estimate both λ and β simultaneously. The moment conditions for that estimation is,

$$g_T(b) = \begin{bmatrix} E(eR_t - a - \beta f_t) \\ E_T[(eR_t - a - \beta f_t) f_t] \\ E(eR - \hat{\beta}\lambda) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{2.27}$$

where $b' = [a' \ \beta' \lambda']$ and a is the intercept (not necessary equal to the pricing error in the third equation). This method was less intuitive to map successfully in Rstudio, despite the very simple illustration. For a more extensive introduction to GMM and estimation features I refer to Cochrane (2009) and Jagannathan et al. (2002).

Chapter 3

The data

The dataset used in this thesis is obtained from several sources. This chapter will present choices done when collecting stock and accounting data, while also presenting some statistics regarding this process. Further, the chapter, will elaborate choices regarding portfolio sorting and selection of market factor.

3.1 Stock data

Stock data is collected through TITLON (Financial Data for Norwegian Academic Institutions) between January 1994 to end of May 2017. Metrics collected are adjusted daily prices, market cap, best bid price and best ask price for individual companies listed on the OSE in the period. Companies selected were based on lists provided by the OSE named *Listed companies home-state etc.* (Oslo Børs, 2018b) and *List changes Oslo Børs 1996-2018* (Oslo Børs, 2018a). It was important for the survey to include de-listed companies to avoid survivorship bias[1] in the sample of stocks. Risk free rate time series were collected from Ødegaard (2018a)'s webpage.

Initially it was a challenge to structure the data, as not all companies had identicall names or reported organizational ID in and across the stock and accounting data sources. Name-changes are also common for companies during such a long period. I ended up using companyID as a cross reference to align between the stock data and accounting data. If a company had several companyID's, it will be represented as two different companies in different time periods. Very few stocks were excluded due to poor data. These were only companies that had noted multiple prices for the same day, or price-series under different companyID's for the same day. Some Norwegian companies issue so-called A and B stocks, these stocks had identical companyID, as well as identical accounting data. A and B are usually refer-

[1] Surviorship bias: tendency for failed companies to be excluded from performance studies because they no longer exist.

ring to *common stocks* or *preferred stocks*, and the letter ranking indicate restrictions around subjects as voting rights and dividend payments. Each company's constitution must be investigated to clarify these restrictions, but the easiest choice was to exclude all B stocks in the sample. 472 different companies made it through the initial phase of data collection.

3.1.1 Data dropping

It is usual in asset pricing analyses to exclude penny stocks[2] and stocks with few trading days. This is to avoid exaggerated returns and lack of price data for rarely traded stocks. Ødegaard (2018c) recommends eliminating stocks on three different criteria; a market value under 1 million NOK, a shares price under 10 NOK and stocks with less than 20 trading days before entering the sample. A big challenge investigating Norwegian stock effects, is the modest amount of companies in the market. As I operate with a different data set, I decided to investigate these criteria, with a goal to keep as many stocks as possible in my sample.

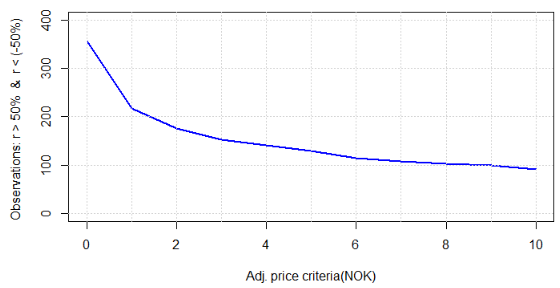

Figure 3.1: Observations exaggerated returns

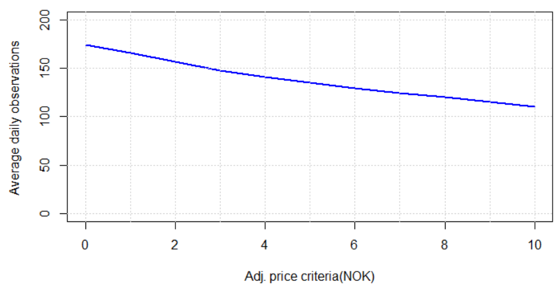

Figure 3.2: Average daily observations

In figure 3.1, I define exaggerated returns as returns larger than $\pm$50%. The graph illustrates that eliminating penny stocks priced under 1 NOK, reduce a significant amount of exaggerated returns in the dataset. From adjusted prices over 2 NOK the curve becomes less steep. Figure 3.2 illustrate how the elimination affect the number of average daily observations. Each elimination step removes almost the same number of observations, and from adjusted prices 2 NOK to 10 NOK, one eliminates an average of 50 observations daily. To keep as many stocks as possible in my sample, I decided to eliminate only stocks with a daily price under 2 NOK. These stocks are eliminated for a year if there is found one observation under the selected criteria, sorting each 1st of July.

[2]Penny stocks: low price stocks

Much of the later analysis involve sorting stocks into portfolios and calculate their average daily returns. To have a valid return, a stock must be priced between two trading days. As not all stocks are priced daily, each portfolio's daily return is a weighted average of those stocks who have valid returns and sorting metrics for that specific day. As a result, each portfolio varies slightly in number of stocks for each time increment (day), but represent the return of those stocks with the correct portfolio characteristics at that specific time. With this strategy one can avoid eliminating stocks based on the trading day criteria. The process is performed by always aligning the return and market cap data with a binary valid return matrix. This is a form of daily sorting which will not be mentioned when further describing sorting annually or monthly by characteristic -variables. Stocks listing/de-listing on the OSE during the period are introduced/removed in the different portfolios continuously.

The last criteria of eliminating stocks with a market cap less than 1000 000 NOK, only removed one stock from the sample for 1 year. All remaining stocks in the sample compared to all listed stocks are illustrated in figure 3.3.

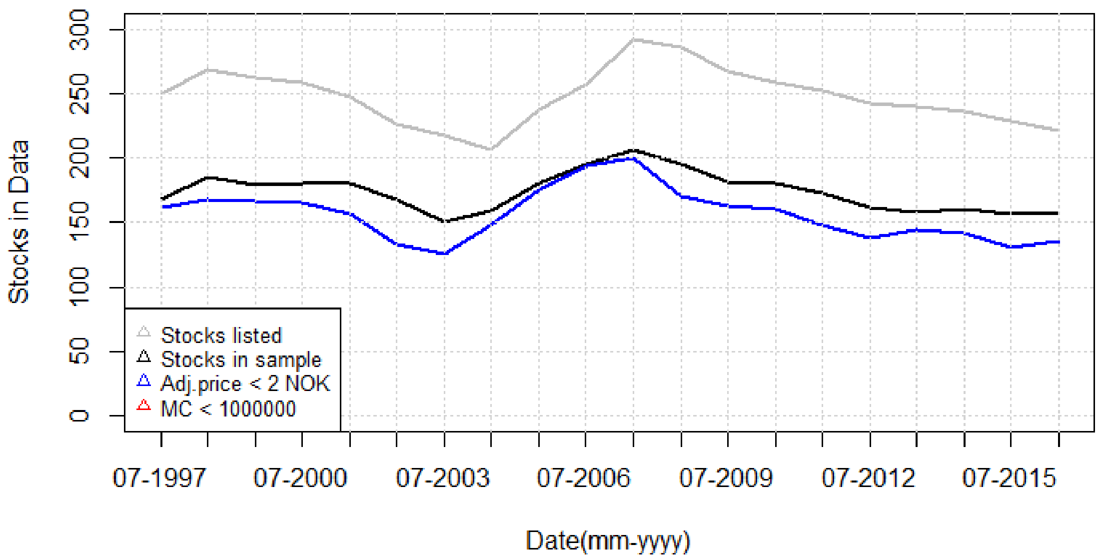

Figure 3.3: Selected stocks vs. total stocks on OSE

3.1.2 Descriptive statistics and further dealing with extreme values

Despite eliminating penny stocks, there was still a significant amount of exaggerated returns in the data which would form misguidance in the later presented analysis. These returns are however interesting for an investor and should be explained by a perfectly suited asset pricing model. They did however complicate this analysis, both when investigating portfolios formed on anomalies and performing the regression analysis. It is common in empirical analysis to perform extreme value treatments such as winsorizing[3], trimming or dropping. Adams, Hayunga, Mansi & Reeb (2017) research found that a majority of papers dealing with OLS- regression and mention outliers used winsorizing. This study has also winsorized the data outside the 0,01% to 99,99% quantile, this to remove the most extreme values in the return data.

Descriptive statistics after the previous steps of excluding whole stocks and winsorizing can be investigated in Table 3.1. The two approaches do increase the normality of the sample's distribution. The original sample was heavy tailed indicated by the high value of the kurtosis.

Table 3.1: Descriptive statistics

	Mean	Std	Min	Max	Skewness	Kurtosis
Original sample	-0.0002	0.026	-2.497	4.363	0.541	891.325
Sample after stock elimination	-0.0001	0.022	-2.319	2.324	-1.524	460.242
Sample after winzoring	-0.0001	0.020	-0.360	0.378	0.316	50.396

3.1.3 Value-Weighted portfolios

Another interesting feature in the Norwegian stock market, is that a very few companies such as Statoil, Norsk Hydro and Telenor has been dominant in terms of size the last decades. These companies return greatly influence the return of value-weighted indexes such as OSEBX (Benchmark Index) and OSEAX (All-share Index). Bhattacharya & Galpin (2011) found that value-weighed portfolios are more common in developed markets compared to emerging markets. A theory is that institutional shareholders and mutual funds may demand value-weighted portfolios because their benchmarks often are value-weighted. But equally-weighted portfolios could potentially outperform value-weighted portfolios, partly because of more exposure to risk factors such as value and size (Plyakha, Uppal & Vilkov, 2012).

Equally-weighted portfolios could potentially significantly change the results of this research. Næs et al. (2009) found that an equally-weighted market index could be used to

[3]Winsorizing is the transformation of statistics by limiting extreme values in the statistical data to reduce the effect of possibly spurious outliers.

explain cross-sectional returns. The value-weighted approach will still be favored in this research since it is more common and that it will reduce each portfolio's exposure of the more extreme risk factor stocks. This will however cause that some big size stocks dominate the return variations of some portfolios, and theoretically give more even returns between portfolios sorted on specific characteristics such as size and value. A portfolio's value-weighted excess return is calculated as,

$$eR_t^{Port} = \sum_{i=1}^{n} \frac{(r_t^i - r_t^{rf}) * market\ cap_t^i}{\sum_{i=1}^{n} market\ cap_t^i} \tag{3.1}$$

where the r_t^i denotes the return for stock i on day t, r_t^f is the risk-free rate and n the number of stocks in the portfolio.

3.1.4 Choice of market factor

The market index for the analysis is the value-weighted return of all stocks in the sample. The main indexes for the OSE are the OSEBX and the OSEAX. OSEBX provides average value-weighted returns for 54 stocks considered to represent the Norwegian stock market. How well the selected stocks equally-weighted (EW) and value-weighted (VW) average returns correlate with the OSEBX, are presented in table 3.2, while excess returns[4] and standard deviation are presented in table 3.3. The value-weighted return for all selected stocks correlate strongly with OSEBX, which is promising for how this sample represent the Norwegian stock market.

Table 3.2: Correlation between selected stocks and OSEBX

	OSEBX	VW-all	EW-all
OSEBX	1	0.976	0.845
VW-all	0.976	1	0.870
EW-all	0.845	0.870	1

Table 3.3: Return for market portfolios

	OSEBX	VW-all	EW-all
Avg excess return	0.0003	0.0004	-0.0003
std	0.0155	0.0140	0.0098

[4]Excess return: return excess of risk free rate.

3.2 Accounting data

Accounting data for year 1994 to 2014 were collected from TITLON, while year 2015-2016 from the private company Proff Forvalt. Metrics retrieved are annual book equity, operating profits and total assets for companies in the sample. It is unfortunate to use two different sources as they may have different reporting conventions and companies in their sample. Proff Forvalt did have a smaller selection compared to TITLON, but the metrics aligned well on visual inspection. It was important for the thesis to present as recent data as possible, but a good sample of accounting data is surly hard to retrieve for Norwegian stocks. There was a great deal of missing reporting, especially at the start of the chosen time period. The accounting data has also been through some elimination steps,

1. Omitted negative book values
2. Omitted INV- variables >10
3. Omitted OP- variables: <(-10) and >10

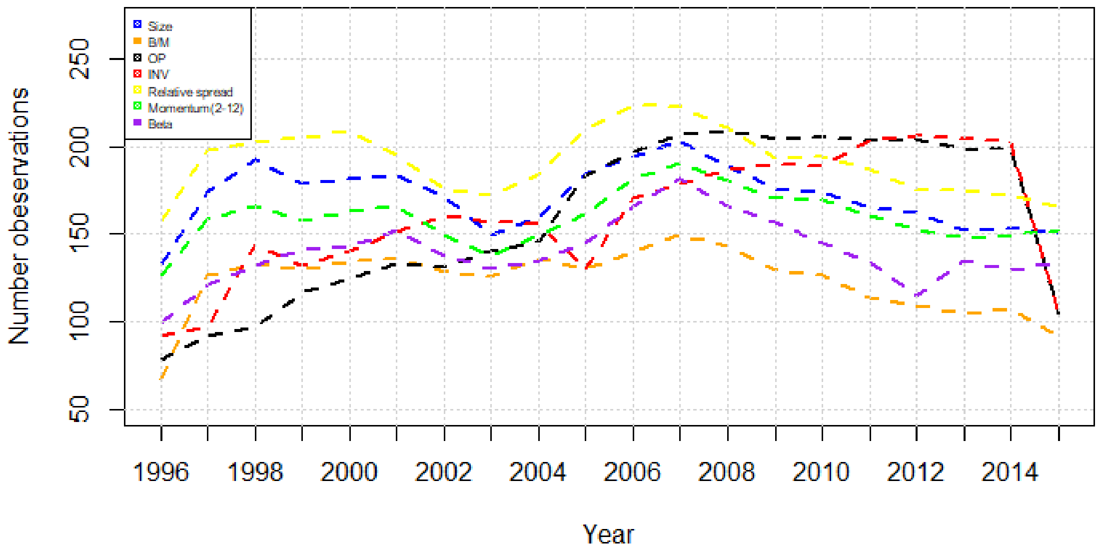

Figure 3.4: Accounting data vs. stock data characteristics

where number 1 is to avoid the OP in equation 2.7 to become very misleading. If the book equity is negative, robust stocks becomes weak stocks . Negative book equity and negative

B/M -ratio is also difficult to interpret. Further number 2 and 3 is to eliminate some extreme outliers in the profitability and investment variables. There were a very few observations of all three cases in the sample.

Figure 3.4 shows how many observations there were in the final accounting data sample compared to characteristics formed by the stock data. The chosen time period for the analysis is from the 1st of July 1997 to the 31st of May 2017, which implies that accounting data is necessary from year 1996 to 2015. There are few observations for the accounting variables in the start of the period (1996-1997), especially the OP -variable has very few observations. All of the accounting measures suffer from changing data source in year 2015.

Chapter 4

Results

This chapter will first investigate portfolios sorted on the selected anomalies related to firm size, value, profitability, investment, liquidity, momentum and estimated β's. Further the results from investigating factor redundancy, and statistics from the GRS and Wald -test are presented. The GMM section tries to identify risk premiums related to the selected risk factors, and an overall impression of all these tests will form an understanding of which risk factors are important in the Norwegian stock market. The last section will point out some interesting findings which demand further explanation.

4.1 Fundamental anomalies in the Norwegian stock market

To evaluate return patterns related to the selected CAPM anomalies, all stocks in the sample are allocated eight portfolios by percentiles of the investigated characteristic. Dependent on which anomaly, portfolios are either sorted annually or monthly and hold constant between each sorting day. These portfolios are also corrected by a binary valid return data matrix as described in section 3.1.1. Excess return or eR indicate return excess of risk free rate in all further results. Additional choices are elaborated in the upcoming subsections.

4.1.1 Size anomaly

Banz (1981) recognized that small stocks tend to yield greater average returns compared to big size stocks and compared to those returns predicted by the CAPM. This size effect is now one of the most empirically tested stock anomalies in the world. Næs et al. (2009) found a significant size effect in the Norwegian stock market in the period 1980-1999, but not significant in the subperiod 2000-2006.

In order to asses this effect for my sample, stocks are allocated to eight different portfolios by their measure of size. The portfolios are sorted each 1st of July between year 1997 to

2017. Table 4.1 show that there has been a significant size effect in the market the last 20 years indicated by the low p-value (1%) of the t-test. The effect is also fairly linear between the eight portfolios as seen in figure 4.1. The volatility of the portfolios is not as linear as expected. This could be a result of the winzoring and elimination process, which also have resulted in a small offset in allocation of stocks, but all portfolios have a mean of over 13 allocated stocks. Inspecting the median of the excess returns, draws a different picture, as small stock have a larger quantity of negative returns compared to the big size portfolios, which indicate that the small portfolios benefit from sudden boosts in daily returns, and investing in them comes at some risk.

Table 4.1: Excess return for portfolios(VW) sorted on size

In the period July 1997- May 2017 stocks are designated to 8 different size portfolios based on market capitalization. The portfolios are re-balanced each 1st of July on market cap as of end of last calendar year. A t.test for the difference between top four and bottom four returns, test whether the return difference is greater than zero.

	Excess return					Number of stocks		
Portf.	Mean	std.dev	min	median	max	min	median	max
1(Small)	0.00105	0.01905	-0.12861	-0.00001	0.16231	5	13	22
2	0.00100	0.01483	-0.09723	0.00027	0.09257	6	16	22
3	0.00063	0.01398	-0.10200	0.00051	0.11769	11	17	24
4	0.00056	0.01374	-0.11974	0.00077	0.19162	8	17	23
5	0.00072	0.01474	-0.10188	0.00073	0.08970	13	18	24
6	0.00050	0.01259	-0.10017	0.00080	0.08918	13	19	24
7	0.00048	0.01343	-0.08830	0.00120	0.07637	16	20	25
8(Big)	0.00034	0.01530	-0.10977	0.00083	0.11266	15	20	24

	Mean port.1-4	Mean port.5-8	Diff	t-test	p-value
Jul1997-May2017	0.00081	0.00051	0.00030	2.35981	0.00916

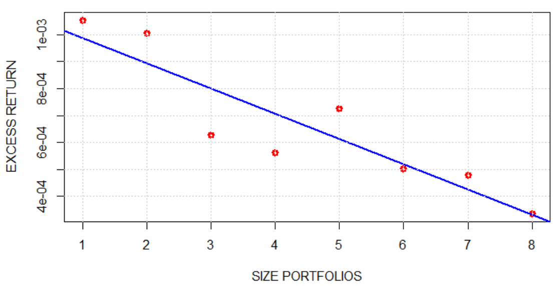

Figure 4.1: Mean eR for size port

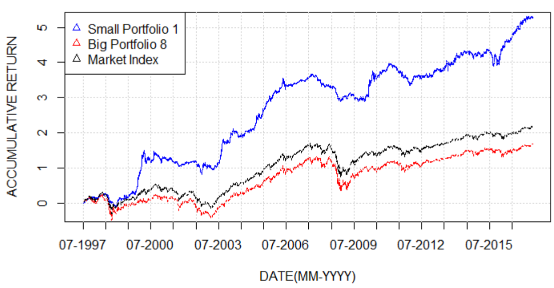

Figure 4.2: Accumulative extreme port

Figure 4.2 shows the accumulative return for the two extreme portfolios in size compared to the market index. The big size portfolio naturally behaves similarly to the value-weighted market index, while the small size portfolio has some extreme increases and generally has a greater gradient. The effect looks especially strong the last two years (2015-2017). (Enlarged accumulative return plots can be inspected in the appendix.)

4.1.2 Value anomaly

The value anomaly, is also empirically proved in several studies as Rosenberg, Reid & Lanstein (1985) and Fama & French (1992). The effect is commonly measured by the ratio between book-price and market cap (B/M). Some studies refer to the effect through the measure of "earnings-to-price" or "cash flow-to-price" (Porta, Lakonishok, Shleifer & Vishny, 1997). Value stocks, which are those stocks with a high B/M-ratio, tend to be under-priced by the market and give superior excess returns. While, glamour stocks, which are overprices by the market (low B/M), tend to yield lower returns. Investors with different value trading strategies have beaten the market's return over the years (Lakonishok, Shleifer & Vishny, 1994).

Næs et al. (2009) did not find evidence of a significant value effect in the Norwegian stock market during the time-period 1980-2006, but the effect was found more significant in the subperiods 1980-1989 and 2000-2006.

Table 4.2: Excess return for portfolios(VW) sorted on value

In the period July 1997- May 2017 stocks are designated to 8 different value portfolios based on B/M -ratios. The portfolios are re-balanced each 1st of July on B/M -ratios as of end of last calendar year. A t.test for the difference between top four and bottom four returns, test whether the return difference is greater than zero.

	Excess return					Number of stocks		
Portf.	Mean	std.dev	min	median	max	min	median	max
1(Low)	0.00018	0.02120	-0.18312	0.00047	0.23287	5	14	18
2	0.00033	0.01877	-0.12658	0.00087	0.12482	7	14	17
3	0.00023	0.01542	-0.09731	0.00055	0.08370	8	14	17
4	0.00048	0.01682	-0.13669	0.00079	0.11138	7	13	17
5	0.00040	0.01622	-0.10438	0.00050	0.11739	6	14	18
6	0.00019	0.01747	-0.12128	0.00015	0.17930	7	14	18
7	0.00039	0.01876	-0.13604	0.00036	0.14284	7	13	19
8(High)	0.00047	0.01589	-0.10727	0.00026	0.12309	5	12	17

	Mean port.1-4	Mean port.5-8	Diff	t-test	p-value
Jul1997-May2017	0.00030	0.00036	0.00006	0.44291	0.32892

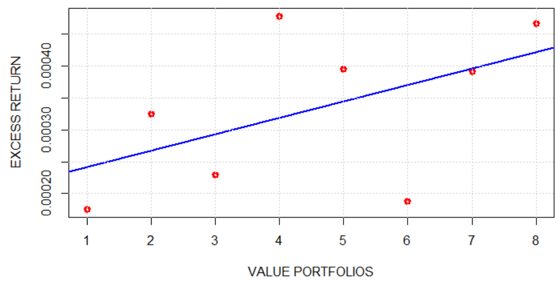

Figure 4.3: Mean eR for B/M port

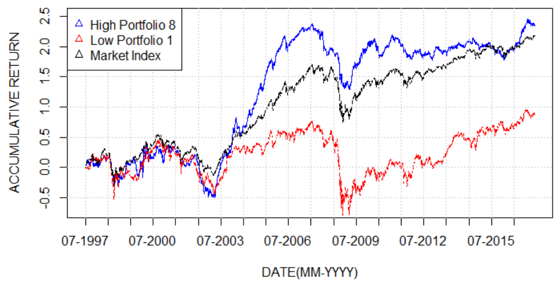

Figure 4.4: Accumulative extreme port.

Table 4.2 shows little evidence of a linear value effect in Norwegian stock market. However there are a tendencies as illustrated in figure 4.3. There is especially a big difference in daily excess returns for the two extreme portfolios (1 and 8). The accumulated return plot in figure 4.4 indicates that the effect is stronger during economical upturns and recessions. Especially low value stocks seem to suffer greatly during recessions. The overall effect seems to have been greater between year 2003 to 2007, which is similar to findings in Næs et al. (2009). The value effect is not visually present since the last recession in 2009.

4.1.3 Profitability anomaly

Novy-Marx (2013) refers to the profitability effect as "the other side of value". As argued by the dividend discount model there is an interdependence between the value, profitability and investments and expected stock returns. Measuring profitability in operating profits, Fama & French (2015b) find that robust profitability stocks outperform weak profitability stocks in average excess returns.

The profitability anomaly proves to be more significant than the value anomaly in the Norwegian stock market, indicated by a low p-value under 6% in table 4.3. There is not a visual linear relationship in figure 4.5, but in general robust companies earn higher average excess returns in the sample. The investigation suffers slightly from lack of accounting data. Portfolio 3 is only described by one stock in a short time period. The plot in figure 4.6 indicate that robust stocks seem to move in line with the market index, while the weak stocks seem to suffer more in average excess returns. The weakest profitability stocks suffer strongly both in the early 2000 recession and between 2007-2009. The effect does not seem to have been present since year 2009.

Table 4.3: Excess return for portfolios(VW) sorted on profitability

In the period July 1997- May 2017 stocks are designated to 8 different profitability portfolios based on profitability. The portfolios are re-balanced each 1st of July on profitability -ratios as of end of last calendar year. A t.test for the difference between top four and bottom four returns, test whether the return difference is greater than zero.

	Excess return					Number of stocks		
Portf.	Mean	std.dev	min	median	max	min	median	max
1(Weak)	0.00014	0.02078	-0.13921	-0.00016	0.15970	4	10	17
2	0.000003	0.01983	-0.10822	-0.000001	0.10452	5	13	19
3	0.00019	0.02054	-0.19634	0.00030	0.17887	1	9	20
4	0.00045	0.01620	-0.08398	0.00060	0.08422	7	12	21
5	0.00050	0.01713	-0.11153	0.00047	0.13754	7	12	19
6	0.00044	0.01731	-0.14435	0.00081	0.13089	7	14	22
7	0.00034	0.01623	-0.10435	0.00048	0.11176	5	14	22
8(Robust)	0.00037	0.01847	-0.13870	0.00015	0.14163	5	11	21

	Mean port.1-4	Mean port.5-8	Diff	t-test	p-value
Jul1997-May2017	0.00020	0.00041	0.00022	1.57441	0.05773

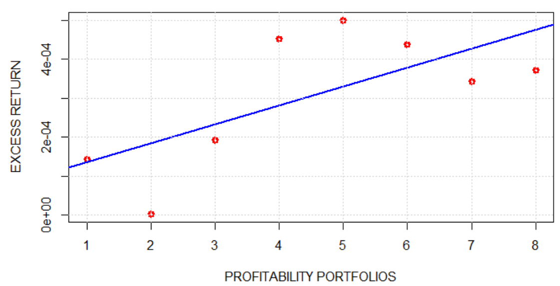

Figure 4.5: Mean eR for OP port

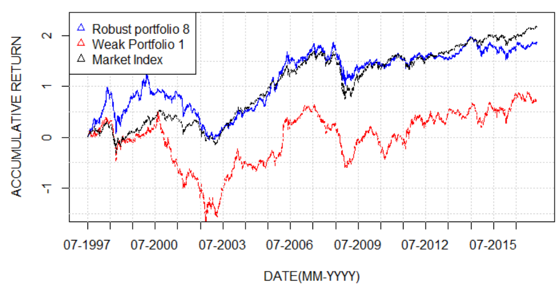

Figure 4.6: Accumulative extreme port.

4.1.4 Investment anomaly

Titman et al. (2004) find that companies with aggressive capital investments, usually generate lower average returns compared to more conservative companies. A company's willingness to invest is usually correlated with previous high returns and robust cash-flow, which makes investments sound intuitively favorable. Increase in capital investments is also often associated with business opportunities and confidence in the company. Nevertheless, a theory is that company managers often fancy the investment opportunity more than their investors. They often publicly announce this opportunity and put their best spin on it to raise capital to cover investment expenditures. However, if the investors under-react, this could potentially generate negative average returns. Fama & French (2015b) find the effect to be strong in stocks on the NYSE, but Hoel & Mix (2016) did not find a significant investment effect in the Norwegian stock market on data between 1991 to 2016.

Table 4.4: Excess return for portfolios(VW) sorted on investments

In the period July 1997- May 2017 stocks are designated to 8 different investment portfolios based on growth in investments. The portfolios are re-balanced each 1st of July on growth in investments as of end of last calendar year. A t.test for the difference between top four and bottom four returns, test whether the return difference is greater than zero.

	Excess return					Number of stocks		
Portf.	Mean	std.dev	min	median	max	min	median	max
1(Cons.)	0.00039	0.02148	-0.15503	0.00021	0.19680	3	10	16
2	0.00049	0.01804	-0.13933	0.00090	0.11942	6	12	17
3	0.00048	0.01596	-0.12782	0.00067	0.08405	5	12	19
4	0.00040	0.01609	-0.12756	0.00074	0.12205	8	13	17
5	0.00032	0.01689	-0.12360	0.00052	0.20982	7	13	17
6	0.00032	0.01793	-0.10739	0.00043	0.10796	8	12	20
7	0.00020	0.01757	-0.11408	0.00048	0.13594	7	15	20
8(Aggr.)	0.00022	0.02062	-0.16642	0.00051	0.12144	7	13	19

	Mean port.1-4	Mean port.5-8	Diff	t-test	p-value
Jul1997-May2017	0.00044	0.00026	0.00018	1.32804	0.09211

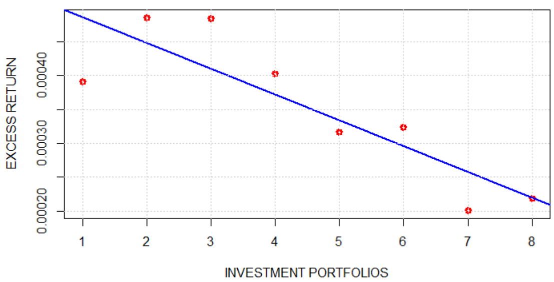

Figure 4.7: Mean eR for INV port

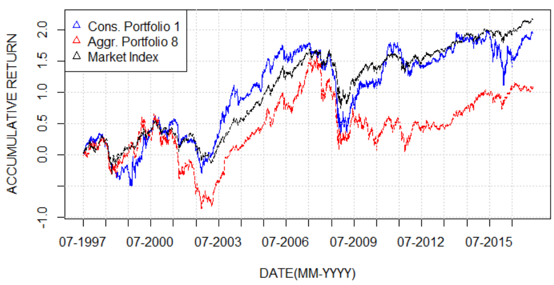

Figure 4.8: Accumulative extreme port.

To investigate the effect further in my sample, stocks are allocated eight different portfolios based on their change in total asset between end of calendar year (t-1) and (t-2), the portfolios are sorted each 1st of July. Table 4.4 illustrates that there are tendencies to an investment effect, with a p-value of the t-test less than 10%. As illustrated in figure 4.7, aggressive investment stocks (port. 5-8) do in general have lower excess returns compared to conservative investment stocks. The effect is fairly linear, but the most conservative portfolio has lower excess return than portfolio 2 to 4. This could be because some outliers in the accounting data have not been eliminated. The accumulative plot in figure 4.8 shows signs of an investment effect in short periods. Conservative investment stocks have especially outperformed aggressive stocks the last 6 years (since 2009). However, during the market up-rise between 2002-2007 aggressive stock did generate strong positive returns, hence the effect is not present.

4.1.5 Liquidity anomaly

This effect was introduced in the theory section. The effect has already been proven significant by Næs et al. (2009) on data between 1980 to 2006. Measured in relative spread, the effect is significant in the Norwegian stock market during year 1997 to 2017 as illustrated in table 4.5. The high liquidity (low relative spread) portfolios 1 to 4, are outperformed by the illiquid stocks in portfolios 5 to 8. The effect is fairly linear between the different sorts as visualized in figure 4.9. The accumulative return plot in figure 4.10, illustrate that the most illiquid stocks (port 8) have stably generated higher excess returns than the most liquid stocks the past 20 years.

Table 4.5: Excess return for portfolios(VW) sorted on liquidity

In the period July 1997- May 2017 stocks are designated to 8 different liquidity portfolios based on prior 1-year average relative spread. The portfolios are re-balanced each 1. of July. A t.test for the difference between top four and bottom four returns, test whether the return difference is greater than zero.

	Excess return					Number of stocks		
Portf.	Mean	std.dev	min	median	max	min	median	max
1(High)	0.00036	0.01536	-0.11048	0.00085	0.11235	17	22	27
2	0.00019	0.01575	-0.13692	0.00118	0.09007	16	21	25
3	0.00060	0.01492	-0.11404	0.00100	0.09420	15	20	25
4	0.00048	0.01251	-0.09991	0.00071	0.08191	13	20	28
5	0.00060	0.01459	-0.12361	0.00054	0.10169	9	17	23
6	0.00066	0.01438	-0.12984	0.00041	0.19667	12	17	29
7	0.00059	0.01218	-0.09127	0.00016	0.22196	9	17	25
8(Low)	0.00084	0.01485	-0.12696	-0.00020	0.17370	4	14	25

	Mean port.1-4	Mean port.5-8	Diff	t-test	p-value
Jul1997-May2017	0.00041	0.00067	0.00026	1.81226	0.03500

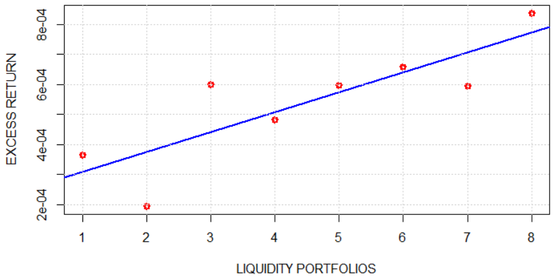

Figure 4.9: Mean eR for liq. port

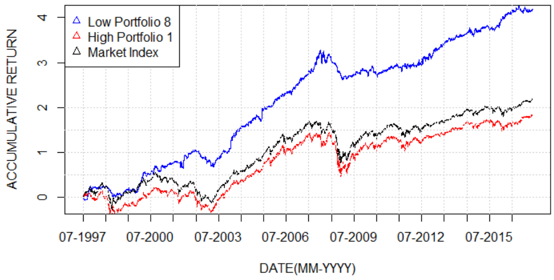

Figure 4.10: Accumulative extreme port.

4.1.6 Momentum anomaly

To assess momentum, stocks are allocated to 8 different portfolios based on their prior (2-12) total return. The portfolios are sorted each month from 1st of July 1997. Results can be investigated in table 4.6, and the effect proves to be the most significant of all investigated anomalies indicated by the low p-value (0,88%). Lowest momentum portfolios show a daily excess return of (-0,034%), while the highest momentum portfolio earn 0,096% daily.

Table 4.6: Excess return for portfolios(VW) sorted on momentum

In the period July 1997- May 2017 stocks are designated to 8 different momentum portfolios based on prior (2-12) months return. The portfolios are re-balanced each month. A t.test for the difference between top four and bottom four returns, test whether the return difference is greater than zero.

	Excess return					Number of stocks		
Portf.	Mean	std.dev	min	median	max	min	median	max
1(Low)	-0.00034	0.02708	-0.18070	-0.00081	0.21579	7	15	21
2	-0.00018	0.02020	-0.13958	-0.00007	0.12433	9	17	24
3	0.00015	0.01843	-0.13720	0.00003	0.20021	10	17	25
4	0.00016	0.01661	-0.10715	0.00037	0.09493	12	18	25
5	0.00039	0.01601	-0.19896	0.00065	0.12085	12	18	24
6	0.00024	0.01559	-0.14238	0.00078	0.12543	12	18	25
7	0.00041	0.01652	-0.10535	0.00110	0.11687	13	18	24
8(High)	0.00096	0.02029	-0.14326	0.00146	0.11730	9	17	23

	Mean port.1-4	Mean port.5-8	Diff	t-test	p-value
Jul1997-May2017	0.00010	0.00054	0.00044	2.37432	0.00881

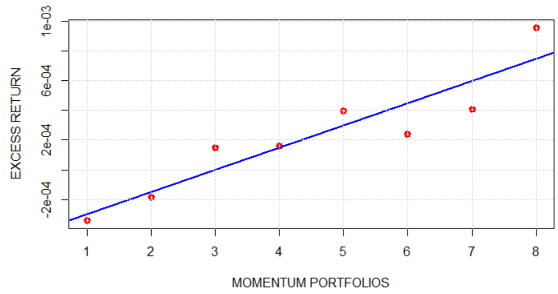

Figure 4.11: Mean eR for mom. port

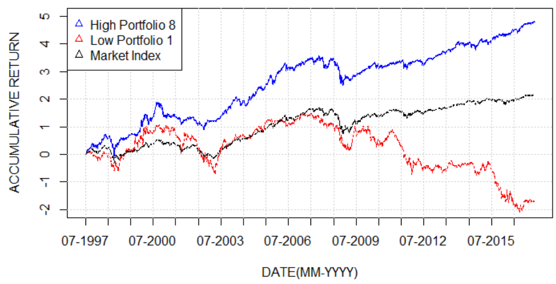

Figure 4.12: Accumulative extreme port.

The effect was not found significant in Næs et al. (2009) between year 1980 and 2006. Illustrated in figure 4.12, the effect in my sample seems to be stronger since year 2009, where the loser and winner portfolio show opposite directions. Figure 4.11 illustrates that the effect is very linear between the sorted portfolios. Also investigating the median of the excess returns show a linear increase in daily excess returns.

4.1.7 β anomaly

The β -effect is evaluated in similar manner. Portfolios are formed each 1st of July, by the measure of stocks estimated β's. β's are estimated by rolling regression of the past 300 days, and a stock is only assigned a β if it has at least 95% valid daily returns in the period. The effect is neither significant nor linear as illustrated in table 4.7 and figure 4.13. There is however a sizable difference in average daily excess return between the low β (0,057%) and high β (0,012%) -portfolio. The volatility is naturally linear from portfolio 1 to 8. Figure 4.14 illustrate that highest β portfolio is outperformed by low β the past 20 years, however high β's performed well in the market up-rise between 2003 to 2007. The accumulative plot also illustrates that the lowest β portfolio has low volatility (flat curve), which gradually exceeds above the market index.

Table 4.7: Excess return for portfolios(VW) sorted on β

In the period July 1997- May 2017 stocks are designated to 8 different β portfolios based on prior 300 days daily return. The portfolios are re-balanced each 1. of July. A t.test for the difference between top four and bottom four returns, test whether the return difference is greater than zero.

	Excess return					Number of stocks		
Portf.	Mean	std.dev	min	median	max	min	median	max
1(Low)	0.00057	0.01089	-0.07593	0.00008	0.10829	6	15	21
2	0.00040	0.01130	-0.06891	0.00036	0.09374	9	16	22
3	0.00045	0.01085	-0.06006	0.00047	0.07745	8	15	21
4	0.00054	0.01326	-0.10044	0.00089	0.14808	10	16	21
5	0.00035	0.01483	-0.12762	0.00078	0.12180	11	15	21
6	0.00034	0.01565	-0.09871	0.00067	0.10415	11	16	20
7	0.00044	0.01894	-0.14041	0.00122	0.11146	11	16	21
8(High)	0.00012	0.02143	-0.11980	0.00050	0.12309	9	15	21

	Mean port.1-4	Mean port.5-8	Diff	t-test	p-value
Jul1997-May2017	0.00049	0.00031	0.00018	1.05046	0.14678

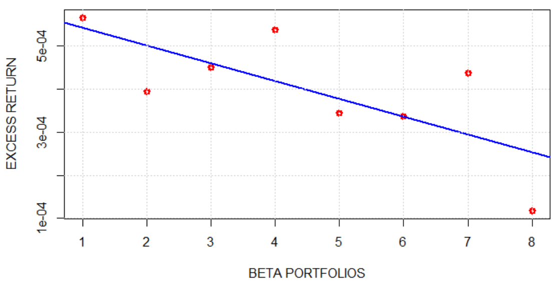

Figure 4.13: Mean eR for β portf

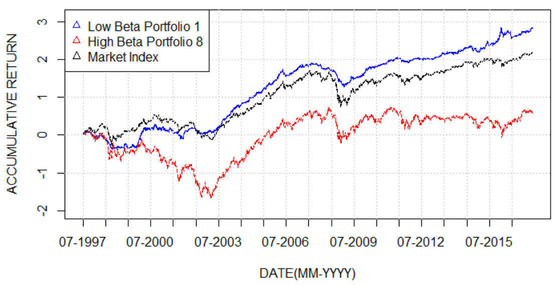

Figure 4.14: Accumulative extreme port.

4.2 Statistics from factor constructions

Investigating the returns for the portfolios used for factor construction in table 4.8, one can identify some of the same results as in the previous section. In panel A, the value effect is not visible for neither small nor big stocks, but there is a notable difference in excess returns between the size groups. In panel B, there is a strong profitability effect for both the small- and big- size groups. It seems that this effect is more linear when corrected for the size of the stocks. Weak big size stocks have an average daily excess return of 0,001%, while robust small stocks have a strong average daily return of 0,09%. The size-INV sort in panel C, show that the investment effect is stronger for big size stocks compared to the small.

The MOM factor's constructing portfolios in panel D, are formed monthly on prior (2-12) total return. As also proved in the previous section, the momentum effect is strong in the Norwegian market. The size-momentum sort shows linear returns for both size groups. Big size stocks with low momentum is the only negative daily excess return of all the value-weighted portfolios, while small size stocks with high momentum outperform all other portfolios with a daily excess return of 0,1%.

The liquidity portfolios in panel E, were not possible to correct for size. It is possible to form liquidity portfolios monthly, as done in Næs et al. (2009), but when constructing the portfolios in the previous section (liquidity anomaly), my research found better spreads in the returns when the eight portfolios were formed annually. As it is to my understanding a goal to find those variations which capture the effect the most, I decided to rather investigate an annually formed factor. I believe it should not make too much of a difference in the further analysis. The liquidity portfolios in panel E, show a linear relationship for all the portfolios constructing the LIQ factor.

The BAB factor constructing portfolios in panel F, is beta ranking weighted and formed monthly by median. This is true to the original factor as proposed by Frazzini & Pedersen (2014). However, it seems that the beta-ranking approach exposes the portfolios for other risk factors and generate an average of negative excess returns. Nevertheless, this investment strategy will benefit greatly from shorting the high β portfolio, which has an average of negative excess returns of (-0,068%). It is important to mention that these are the excess returns before leveraging the low β portfolio and de-leveraging the high β portfolio (see equation 2.15).

Table 4.8: Average returns of portfolios formed for factor constructions

Average excess return for portfolios formed on size and value(A), size and profitability(B), size and investments(C), size and momentum(D), liquidity (E) and β (F), in the period July 1997-May 2017. Portfolios for value, profitability, investment and liquidity are formed yearly from 1st of July. Portfolios for momentum and β are formed monthly. Column breakpoints are the 30th and 70th percentiles of the selected sample, except β's which are sorted by median. Liquidity(E) and β(E) are not sorted into size groups. β's is represented as average excess return before leveraging/de-leveraging of portfolios in panel F.

A: Size -B/M

	LOW	NEUTRAL	HIGH
SMALL	0.00071	0.00079	0.00071
BIG	0.00032	0.00036	0.00034

B: Size-OP

	WEAK	NEUTRAL	ROBUST
SMALL	0.00053	0.00084	0.00092
BIG	0.00001	0.00039	0.00040

C: Size-INV

	CONS.	NEUTRAL	AGGR.
SMALL	0.00072	0.00082	0.00071
BIG	0.00050	0.00040	0.00026

D: Size-Momentum

	LOW	NEUTRAL	HIGH
SMALL	0.00033	0.00064	0.00100
BIG	-0.00016	0.00036	0.00052

E: Liquidity

	HIGH	NEUTRAL	LOW
LIQUIDITY	0.00039	0.00054	0.00080

F: β

	LOW	NEUTRAL	HIGH
BETA	-0.00015	–	-0.00068

Table 4.9: Descriptive statistics factors

	Mean	t-test	p-value	Std	Min	Max	Skewness	Kurtosis
VW_all	0.00043	2.200	0.014	0.014	-0.095	0.096	-0.510	5.607
SMB	0.00042	2.441	0.007	0.012	-0.081	0.069	-0.006	3.293
HML	0.00001	0.061	0.476	0.013	-0.089	0.088	-0.083	2.889
RMW	0.00039	2.260	0.012	0.012	-0.069	0.072	0.003	1.793
CMA	0.00012	0.654	0.257	0.013	-0.072	0.099	0.126	3.158
LIQ	0.00041	1.915	0.028	0.015	-0.114	0.121	0.090	5.645
MOM	0.00067	3.343	0.0004	0.014	-0.109	0.070	-0.286	2.877
BAB	0.00053	2.133	0.016	0.018	-0.108	0.108	-0.209	2.389

Descriptive statistics in table 4.9 show that most factors represent a good investment strategy, especially the BAB and MOM -factor beat the market with high average excess returns. A t-test checking if return time series are greater than zero, reject both the HML and CMA -portfolio as good investment strategies. The market factor, VWall, has the least normal distribution in terms skewness and kurtosis (histograms in appendix for inspection).

Neither of the factors have a perfectly normal distribution, but none of the values for kurtosis or skewness are particularly high. How well the different factors compare to the market can further be investigated in figure 4.16, 4.17, 4.18, 4.19, 4.20, 4.21 and 4.22. It seems that all factors except the BAB, have little response to both market up-turns and recessions and generate stable positive returns. Both the CMA and HML -factor have generated mostly negative returns the last couple of years. The MOM factor benefits greatly from the strong momentum effect towards the end of the investigated period.

Table 4.10 shows the correlation matrix of all factors, including the OSEBX. Most notably is the strong negative correlation between the VWall -index and the SMB (-0,66), LIQ (-0,76) and BAB (-0,45) -factor. The SMB and LIQ -factor correlation is likely because of the value-weighted portfolio approach. Shorting big and liquid -stocks which have strong power in the value-weighted index, will cause negative correlation. However, the BAB-factor which is beta ranking weighted, should theoretically not have this strong correlation. I believe it is a result of the scaling approach in equation 2.14, which influences the leveraging/de-leveraging variable. Another reason could be that the estimated β's are time varying, and since the leveraging/de-leveraging variable is produced by ex-ante data, the BAB factor will not truly be a zero-beta portfolio. The SMB, BAB and LIQ -factor also share a strong positive correlation with each other, and all factors share a negative correlation with the market index.

Table 4.10: Correlation between factors(VW)

	OSEBX	VW_all	SMB	HML	RMW	CMA	LIQ	MOM	BAB
OSEBX	1.00	0.98	-0.68	-0.18	-0.11	-0.06	-0.77	-0.05	-0.48
VW_all	0.98	1.00	-0.66	-0.18	-0.10	-0.05	-0.76	-0.04	-0.45
SMB	-0.68	-0.66	1.00	-0.03	0.09	-0.02	0.67	0.06	0.48
HML	-0.18	-0.18	-0.03	1.00	-0.09	0.07	0.14	-0.11	0.10
RMW	-0.11	-0.10	0.09	-0.09	1.00	-0.06	0.03	0.13	0.09
CMA	-0.06	-0.05	-0.02	0.07	-0.06	1.00	0.03	-0.08	0.01
LIQ	-0.77	-0.76	0.67	0.14	0.03	0.03	1.00	0.03	0.52
MOM	-0.05	-0.04	0.06	-0.11	0.13	-0.08	0.03	1.00	0.12
BAB	-0.48	-0.45	0.48	0.10	0.09	0.01	0.52	0.12	1.00

Figure 4.15: Accumulative returns factors

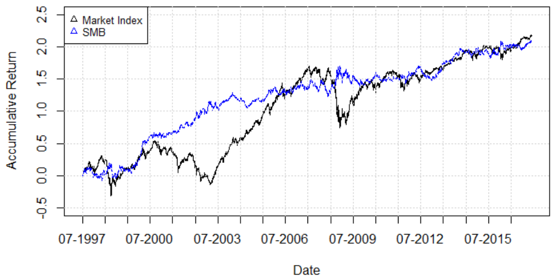

Figure 4.16: SMB vs. market port.

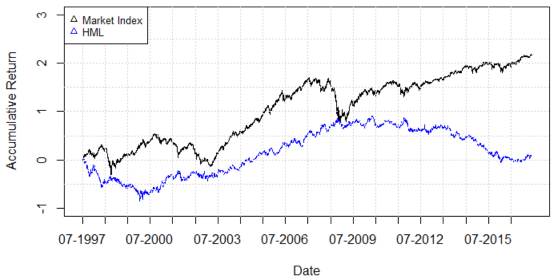

Figure 4.17: HML vs. market port.

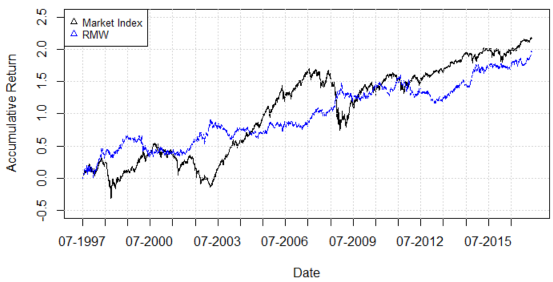

Figure 4.18: RMW vs. market port.

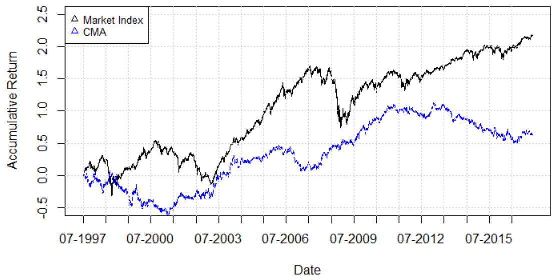

Figure 4.19: CMA vs. market port.

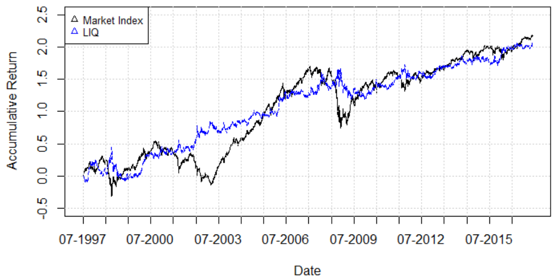

Figure 4.20: LIQ vs. market port.

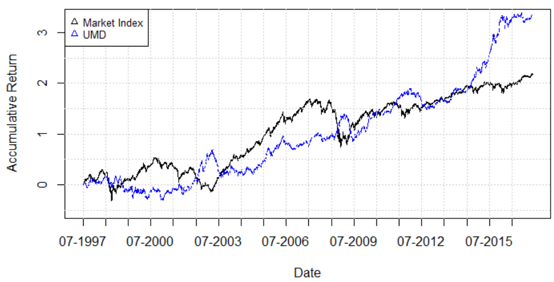

Figure 4.21: MOM vs. market port.

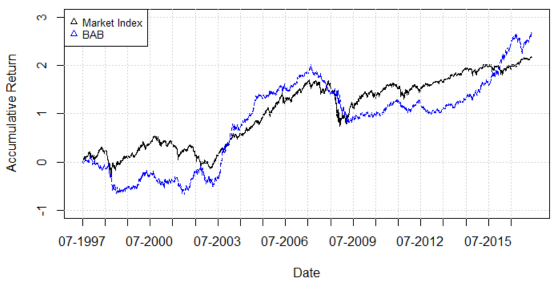

Figure 4.22: BAB vs. market port.

4.3 Investigating factor redundancy

If a factor is redundant, its variations is explained by the other factors in the model. To asses factor redundancy, a method applied in Fama & French (2015b) is to run a regression where each individual factor is used as a regressand being explained by the remaining factors in the model. The results for this approach can be investigated in table 4.11, and as very often in this kind of analysis, the intercept is the crucial statistic for evaluation. A significant intercept greater than zero, indicate that the factor has variations left unexplained by the remaining factors. As an example, the market factor, *eRm*, has 0,08% daily returns left unexplained at a significance level under 1%. The market factor is the foundation of these models, which makes that result somewhat expected. Along the market factor, the SMB, RMW, LIQ and MOM also have a significant intercept. However, the RMW factor seems slightly less important compared to the others with only a significance level under 5% and smaller intercept at 0,04%.

Table 4.11: Investigating factor redundancy

Using seven factors in regressions to explain average daily returns on the eighth. eRm (excess return market) is the value-weighted of all stocks in sample. HML (high minus low) the value factor, SMB (small minus big) the size factor, RMW (robust minus weak) the profitability factor, CMA (conservative minus aggressive), LIQ is the liquidity factor, MOM the momentum factor and BAB (betting against beta) the beta factor.

	eR_m	SMB	HML	RMW	CMA	LIQ	MOM	BAB
Intercept	0.0008***	0.0004***	0.0003	0.0004**	0.0003	0.0005***	0.0006***	0.0001
	(0.0001)	(0.0001)	(0.0002)	(0.0002)	(0.0002)	(0.0001)	(0.0002)	(0.0002)
eR_m		−0.3057***	−0.2710***	−0.1561***	−0.0948***	−0.5599***	−0.0294	−0.0298
		(0.0131)	(0.0210)	(0.0200)	(0.0222)	(0.0126)	(0.0233)	(0.0244)
SMB	−0.3221***		−0.3281***	0.0322	−0.0789***	0.3242***	−0.0215	0.3370***
	(0.0138)		(0.0214)	(0.0207)	(0.0228)	(0.0146)	(0.0239)	(0.0246)
HML	−0.1189***	−0.1366***		−0.0812***	0.0388***	0.0329***	−0.1159***	0.0973***
	(0.0092)	(0.0089)		(0.0133)	(0.0147)	(0.0099)	(0.0154)	(0.0161)
RMW	−0.0771***	0.0151	−0.0914***		−0.0591***	−0.0683***	0.1241***	0.0777***
	(0.0099)	(0.0097)	(0.0150)		(0.0156)	(0.0104)	(0.0163)	(0.0171)
CMA	−0.0384***	−0.0303***	0.0358***	−0.0485***		−0.0031	−0.0767***	0.0119
	(0.0090)	(0.0088)	(0.0136)	(0.0128)		(0.0095)	(0.0148)	(0.0156)
LIQ	−0.5039***	0.2769***	0.0674***	−0.1245***	−0.0069		−0.0279	0.3864***
	(0.0114)	(0.0125)	(0.0203)	(0.0190)	(0.0211)		(0.0221)	(0.0225)
MOM	−0.0108	−0.0075	−0.0973***	0.0926***	−0.0697***	−0.0114		0.1155***
	(0.0086)	(0.0084)	(0.0129)	(0.0122)	(0.0135)	(0.0091)		(0.0147)
BAB	−0.0100	0.1073***	0.0744***	0.0528***	0.0098	0.1440***	0.1052***	
	(0.0082)	(0.0078)	(0.0123)	(0.0116)	(0.0129)	(0.0084)	(0.0134)	
Adj. R^2	0.6353	0.5429	0.0987	0.0470	0.0171	0.6561	0.0454	0.3180

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

The HML factor's returns are redundant, where especially the market factor and size factor have great negative coefficients. All factors seem important to explain the HML in terms of significant coefficients. In contrast, (Fama & French, 2015b) found most of its variations were explained by only the value and investment -factor. Further the CMA factor also has an insignificant intercept, where only the LIQ and BAB - factor as insignificant coefficients. The BAB factor has the lowest insignificant intercept, where both the SMB and LIQ - factor has strong positive coefficients. To asses this perfectly one would have to remove factors with insignificant intercept one by one. The results for the FF5 are presented in the appendix in table A.2. I do not show all results here, but removing the BAB factor made the HML factor's intercept significant at a 10% level with a value of 0,03%, which is not much. Neither of the other possible combinations made the CMA or BAB -factor less redundant. According to this process it seems that a model including the eRm, SMB, RMW, LIQ and MOM -factors covers most variations of stock returns of selected factors in this investigation.

To further investigate factor redundancy, the Wald test is run, to test the hypothesis if estimated coefficients are equal zero. The test is run using an eight-factor model, on all cross-sections of the individual size, value, profitability, investment, liquidity, momentum and β -portfolios, and the methodology is described in section 2.4.3. Results can be inspected in table 4.12, where the size of the chi-squared indicate how important each estimated coefficient is for explaining the individual sorts. A low p-value indicate a rejection of the null hypothesis, hence the factor is important for explaining that specific cross-section.

The methodology is not as successful as one would hope, as only the CMA -coefficient is accepted by the null hypothesis for explaining the momentum portfolios (p-value of 13,7%). However, the size of the chi-squared values in the different testes tells a lot about each factor's importance. For example, in test 1, the market factor proves to be extremely important for explaining all of the cross-sections. The market factor has higher chi-squared values for all sorts compared to the other factors. Further all other factors are seemingly important for explaining portfolios sorted by the same characteristic, for example the size factor is important for explaining portfolios sorted on size (test 2). This could be due to high correlation between test assets and factors sorted on same characteristics, as one would expect when dealing with a small sample of stocks. But hopefully it is because the factor can explain the effects these portfolios represent. Correlation between factors and portfolios of same characteristics are presented in appendix table A.1, which show that the usual suspects, SMB and LIQ, have strong negative correlation with the biggest size and most liquid portfolio. This is likely a consequence of the value-weighted portfolio approach.

Table 4.12: The Wald test for regressors

Model: $eR_{it} = \alpha_i + \beta_i^b eRm_t + \beta_i^s SMB_t + +\beta_i^h HML_t + \beta_i^r RMW_t + \beta_i^c CMA_t + \beta_i^l LIQ_t + \beta_i^m MOM_t + \beta_i^k BAB_t + \epsilon_{it}$

TEST 1: β_{eRm}=0

	Size port.	B/M port.	OP port.	INV port.	Spread port.	Mom. port.	Beta port.
Chisq	42,334.090	14,942.580	9,487.933	9,458.677	61,054.820	12,399.240	12,429.900
p-value	0	0	0	0	0	0	0

TEST 2: β_{SMB}=0

	Size port.	B/M port.	OP port.	INV port.	Spread port.	Mom. port.	Beta port.
Chisq	1,760.017	101.430	85.056	109.928	292.982	103.642	54.063
p-value	0	0	0	0	0	0	0

TEST 3: β_{HML}=0

	Size port.	B/M port.	OP port.	INV port.	Spread port.	Mom. port.	Beta port.
Chisq	66.727	1,033.299	74.212	76.484	31.495	74.630	51.651
p-value	0	0	0	0	0.0001	0	0.00000

TEST 4: β_{RMW}=0

	Size port.	B/M port.	OP port.	INV port.	Spread port.	Mom. port.	Beta port.
Chisq	185.323	95.757	1,428.087	181.343	129.122	146.571	82.963
p-value	0	0	0	0	0	0	0

TEST 5: β_{CMA}=0

	Size port.	B/M port.	OP port.	INV port.	Spread port.	Mom. port.	Beta port.
Chisq	106.420	62.450	45.537	1,577.517	31.423	12.326	19.471
p-value	0	0	0.00000	0	0.0001	0.137	0.013

TEST 6: β_{LIQ}=0

	Size port.	B/M port.	OP port.	INV port.	Spread port.	Mom. port.	Beta port.
Chisq	348.013	68.315	64.241	32.657	3,564.783	36.773	127.123
p-value	0	0	0	0.0001	0	0.00001	0

TEST 7: β_{MOM}=0

	Size port.	B/M port.	OP port.	INV port.	Spread port.	Mom. port.	Beta port.
Chisq	30.868	159.676	43.769	80.056	33.881	3,122.932	75.282
p-value	0.0001	0	0.00000	0	0.00004	0	0

TEST 8: β_{BAB}=0

	Size port.	B/M port.	OP port.	INV port.	Spread port.	Mom. port.	Beta port.
Chisq	60.954	27.903	44.091	19.801	164.863	85.662	916.092
p-value	0	0.0005	0.00000	0.011	0	0	0

The redundant factors (HML,CMA and BAB), do indeed show lower chi-squared values for most sorts in table 4.12. The MOM factor also has some lower chi-squared values, but the factor has a extremely high chi-squared for momentum portfolios and relatively high chi-squared for value portfolios. The MOM factor could potentially prove to be even more important if the redundant HML factor is removed from the model.

4.4 Investigating intercepts for factor combinations

To assess which model combination leaves the minimum of test assets return unexplained, I will further investigate estimated intercepts for factor combinations. The GRS-test, which is described in section 2.4.2, is a test statistic to check if intercepts from multiple regression are jointly zero. I will look at combinations of factors which were found to have most independent variations, as investigated in the previous section. But also some combinations including redundant factors for pure comparison of the test statistics. The results of the GRS-test are present in table 4.13, where model combinations are ranked by the size of their GRS -estimation. The different models are tested on all 56 portfolios formed on the size, value, profitability, investment, liquidity, momentum and β -characteristics. A low GRS-stat and high p-value indicates that one can't reject the null hypothesis of intercepts jointly equal to zero. Intercepts are collected from an one-step OLS regression, but the complete regression statistics will not be presented in this thesis. They would consume to much space and be hard to compare reasonably, however the first step regression of some models will be presented in the appendix.

Some statistics in table 4.13 demand further explanation, the $A|a_i|$ is the absolute value of the estimated intercepts and $A|a_i|/A|\bar{r}_i|$ is the absolute value of the intercept divided by absolute average value of $\bar{r}_i$, which is the average return of test asset i minus the average return of all test assets. The size of the $A|a_i|/A|\bar{r}_i|$ measures how much of the dispersion of average excess returns is left unexplained by the model. It is further important to mention that the GRS-stat is expected to decrease when adding more factors to a model, that leaves the size of the difference more important for comparison. Naturally one can see that a model containing all of the constructed factors has the best GRS-stat in table 4.13 (on top). More importantly one can see several three-factor and four-factor -combinations outperform the traditional FF5.

Table 4.13: GRS-test

GRS-test for factor combinations tested on all 56 portfolios formed on size, value, profitability, investment, liquidity, momentum and β characteristics. The $A|a_i|$ is the absolute value of the estimated intercepts and $A|a_i|/A|\bar{r}_i|$ is the absolute value of the intercept divided by the absolute value of $\bar{r}_i$, which is the average return of test asset i minus the average return of all test assets. The rows are sorted by the rank of the GRS(J) value in first column (ascending order).

	GRS(J)	p_value	$A\|a_i\|$	$A\|a_i\|/A\|\bar{r}_i\|$	$Adj.R^2$
eRm,SMB,HML,RMW,CMA,LIQ,MOM,BAB	0.921181	0.642014	0.000120	0.679684	0.542678
eRm,SMB,HML,RMW,LIQ,MOM	0.934571	0.614021	0.000124	0.706153	0.534396
eRm,SMB,RMW,LIQ,MOM	0.988509	0.499461	0.000134	0.761927	0.529999
eRm,SMB,RMW,LIQ,MOM,BAB	0.992371	0.491311	0.000129	0.734710	0.533615
eRm,SMB,LIQ,MOM,BAB	1.019781	0.434365	0.000133	0.752344	0.527494
eRm,SMB,LIQ,MOM	1.021378	0.431109	0.000138	0.785555	0.523867
eRm,RMW,LIQ,MOM,BAB	1.145805	0.213712	0.000145	0.820730	0.525284
eRm,SMB,HML,RMW,CMA,MOM,BAB	1.161201	0.192933	0.000139	0.788617	0.528321
eRm,RMW,LIQ,MOM	1.163556	0.189883	0.000154	0.874540	0.520941
eRm,LIQ,MOM,BAB	1.181364	0.167909	0.000146	0.829380	0.519426
eRm,SMB,LIQ,BAB	1.181848	0.167339	0.000154	0.876450	0.517722
eRm,SMB,LIQ	1.197579	0.149557	0.000163	0.926654	0.513850
eRm,LIQ,MOM	1.209170	0.137382	0.000157	0.889857	0.515125
eRm,SMB,HML,MOM (**Carhart Model**)	1.230690	0.116792	0.000155	0.880244	0.511403
eRm,SMB,RMW,MOM,BAB	1.244716	0.104722	0.000152	0.864936	0.519174
eRm,SMB,MOM,BAB	1.245891	0.103758	0.000151	0.859131	0.512333
eRm,SMB,MOM	1.292494	0.070875	0.000167	0.947694	0.506526
eRm,SMB,HML,RMW,CMA (**FF 5 Factor Model**)	1.363797	0.037583	0.000175	0.994667	0.513964
eRm,SMB,HML,RMW	1.376222	0.033446	0.000178	1.010262	0.509098
eRm,SMB,BAB	1.394349	0.028127	0.000170	0.964138	0.502413
eRm,LIQ	1.401898	0.026141	0.000184	1.046109	0.505159
eRm,SMB,HML (**FF 3 Factor Model**)	1.425314	0.020748	0.000183	1.039300	0.501921
eRm,SMB	1.464850	0.013861	0.000189	1.075776	0.496487
eRm,BAB	1.730669	0.000627	0.000208	1.183643	0.490120
eRm (**CAPM**)	1.937970	0.000038	0.000250	1.419200	0.480833

Let us compare the two best three-factor combinations, which are models containing either the SMB and LIQ -factor or the LIQ and MOM -factor (added to the market factor). Both combinations significantly outperform the traditional FF5, which is rejected by the low p-value of the GRS-test. The first combination (eRm,SMB,LIQ), has the lowest GRS-stat, which indicate that it can jointly estimate all intercepts closest to zero. However the latter (eRm,LIQ,MOM) has a lower value of $A|a_i|$, $A|a_i|/A|\bar{r}_i|$ and higher $Adj.R^2$ which means it on average has a lower estimated intercept, while also better describing the dispersion of returns between test assets. As seen in previous sections table 4.12, most of all, the MOM factor is important for describing momentum portfolios (high chi-squared), which substantiate the difference in statistics. The LIQ factor is somewhat important for describing similar portfolios as the SMB, which also explains why the second combination (eRm,LIQ,MOM) is not rejected by the null hypothesis. Nevertheless, it seems that both the LIQ and SMB -factor add important variations to the model despite their similarities. A combination of the two models forms the best four factor model (eRm,SMB,LIQ,MOM) in the test.

Comparing the two best factor combinations in the test, which is a model containing all possible factors and a six-factor model without the CMA and BAB factor, indicate that the two excluded factors add very little to the model. Neither of the presented statistics change significantly including the two factors, but only improve by a very small margin. This substantiates results in previous section, that the CMA and BAB -factors are redundant. However, the best four factor model (eRm,SMB,LIQ,MOM), is improved more by adding the RMW and HML -factor, which indicate that these factors are somewhat more important. But a four-factor model containing a market, size, liquidity and momentum -factor seems to perform very well. Other interesting findings in table 4.13 is that neither of the model combinations do improve the $Adj.R^2$ a lot. The CAPM model alone do describe returns relatively good compared to other multi-factor combinations by this measure.

As the last representation did not show how well the models perform on the individual cross-sections, the Wald test is run on the portfolios formed on individual characteristics. Results can be investigated in table 4.14, where the null hypothesis again is whether the intercept is equal zero. The hypothesis is rejected by a high chi-squared value with corresponding low p-value. Models tested are the CAPM, FF3, FF5 and those factor combinations found most interesting by the GRS-test. Test 1 rejects the CAPM model for explaining the size (3,2%), liquidity (0,6%), momentum (2,0%) and β (4,3%) -portfolios. FF3 in test 2, shows that adding the SMB -factor improves chi-squared for the size (28%) portfolios. However, the FF5 model is again rejected for the same sorts as the CAPM in test 3, but has in general lower chi-squared values which is promising. Model 1 containing all possible factors is not rejected by any portfolio sorts, however neither is model 2 nor 3, which has respectively five (eRm, SMB, RMW, LIQ, MOM) and four (eRm, SMB LIQ, MOM) -factors. Leaving out the MOM-factor in model 4, rejects the model for momentum (0,8%) portfolios and increases the chi-squared a lot for portfolios sorted on value, profitability, investments and β -variables. Further removing the SMB factor from model 3 forms model 5, which is rejected at a 10% level for both size (5,2%) and liquidity (9,5%) -portfolios. The results further indicate that a four-factor model containing a market, size, liquidity and momentum factor is adequate for explaining stocks in the Norwegian stock market.

Table 4.14: The Wald test for intercepts

Test 1: CAPM: eRm

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
Chisq	33.632	9.401	6.338	4.854	41.512	19.524	28.785
p-value	0.00005	0.310	0.609	0.773	0.00000	0.012	0.0003

Test 2: FAMA-FRENCH 3: eRm + SMB + HML

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
Chisq	12.198	6.755	8.162	4.583	21.871	21.011	16.627
p-value	0.143	0.563	0.418	0.801	0.005	0.007	0.034

Test 3: **FAMA-FRENCH 5: eRm + SMB + HML + RMW + CMA**

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
Chisq	16.809	5.942	4.574	2.793	21.624	18.178	15.988
p-value	0.032	0.654	0.802	0.947	0.006	0.020	0.043

Model 1: eRm + SMB + HML + RMW + CMA + LIQ + MOM + BAB (All Factors)

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
Chisq	9.785	4.292	5.055	2.001	7.253	10.316	7.509
p-value	0.280	0.830	0.752	0.981	0.510	0.244	0.483

Model 2: eRm + SMB + RMW + LIQ + MOM

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
Chisq	11.012	6.866	5.651	3.552	8.331	10.047	7.948
p-value	0.201	0.551	0.686	0.895	0.402	0.262	0.439

Model 3: eRm + SMB + LIQ + MOM

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
Chisq	8.358	6.980	8.478	4.348	9.191	12.240	8.344
p-value	0.399	0.539	0.388	0.824	0.326	0.141	0.401

Model 4: eRm + SMB + LIQ

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
Chisq	8.700	8.958	10.607	6.014	9.087	20.600	10.355
p-value	0.368	0.346	0.225	0.646	0.335	0.008	0.241

Model 5: eRm + LIQ + MOM

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
Chisq	15.385	7.206	8.231	4.005	13.515	11.537	11.539
p-value	0.052	0.515	0.411	0.857	0.095	0.173	0.173

4.5 Risk premiums by GMM

This section will try to identify risk premiums, λ's, by implementing the two-step (Fama-Macbeth) GMM approach as described in subsection 2.4.4 and 2.4.5. A priced risk premium indicate that the specific risk factor is important for describing the returns related to the effect one investigate (represented by managed portfolios). When dealing with return time series, the estimated λ's can be directly interpreted as the expected return related to the risk of the involved investment strategy (represented by factor-mimicking portfolio). As the J-test is comparable for models with the same number of factors, I will start comparing the FF5 with the best performing five-factor combination (eRm, SMB, RMW, LIQ, MOM) from the previous section. The J-test is measuring the size of the pricing error and a model is rejected by a large chi-squared and corresponding low p-value.

The GMM estimation of the two different five-factor models can be investigated in table 4.15. In this framework neither of the models are rejected by the J-test, which is surprising compared to the previous two tests. The FF5 does produce lower chi-squared values in the J-test for value and investment portfolios, but the alternative five-factor model (in Panel B) has marginal better results for the sorts on liquidity, momentum and β -portfolios. It is however interesting that neither the HML or CMA factor in the FF5 is priced for any of the cross-sections. The market factor is nevertheless priced at 10% significance level for all cross-sections except the investment portfolios. Further the SMB is priced at a significance level of 1% for liquidity and β portfolios, while priced at a 10% level for size and momentum portfolios. And at last the RMW is priced at a 5% level for liquidity and momentum portfolios. Here the alternative model in panel B, performs better as all factors are at least priced for some cross-section. The LIQ factor does however only have a risk premium for liquidity portfolios at a significance level under 10%. Neither of investigated factors, except the market factor (eRm), has a risk premium related to the portfolios formed on value, profitability and investments.

There are some interesting differences between the two estimations in panel A and B. It seems that the eRm, SMB and RMW can explain some risk related to the momentum portfolios in panel A. However, including the MOM factor in panel B, this factor captures most of these variations and neither of the FF5 -factors are priced. The SMB premium for the size portfolios is also smaller and insignificant including the LIQ factor. The premiums apparently become less significant when other factors include some of the same risk effect. I also believe these estimations could lack some power, and that the two-step procedure would benefit from larger cross-sections. Only eight observations for each time increment could be too few for finding significant risk premiums. The J-test is also somewhat biased due to not accounting for the errors in the estimated β's in the second step, and the size of this error seems to be large enough for the results to not be consistent with previous tests.

Table 4.15: Evaluating five factor models by GMM

Two-step GMM -estimation for FF5 (Panel A) and an alternative five factor model containing a market, size, profitability, liquidity and momentum -factor (Panel B). Moment condition for first step is $E[\epsilon_t] = 0$ and $E[f_t\epsilon_t] = 0$, while second step uses estimated factor loadings to solve moment condition $E[\alpha] = 0$. The J-test is measuring the size of the pricing error from the second step.

Panel A: FF5

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
$\lambda(eR_m)$	0.0004**	0.0004*	0.0004*	0.0003	0.0004**	0.0007**	0.0005*
	(0.0002)	(0.0002)	(0.0002)	(0.0003)	(0.0002)	(0.0003)	(0.0002)
$\lambda(SMB)$	0.0005*	0.0002	−0.0002	−0.0017	0.0017***	0.0025*	0.0024***
	(0.0003)	(0.0010)	(0.0013)	(0.0026)	(0.0004)	(0.0015)	(0.0006)
$\lambda(HML)$	0.0029	0.0004	−0.0015	0.0003	0.0015	0.0018	−0.0015
	(0.0018)	(0.0004)	(0.0045)	(0.0017)	(0.0017)	(0.0031)	(0.0016)
$\lambda(RMW)$	−0.0002	0.0008	0.0004	−0.0014	0.0027**	0.0061**	0.0020
	(0.0012)	(0.0013)	(0.0004)	(0.0034)	(0.0013)	(0.0031)	(0.0026)
$\lambda(CMA)$	−0.0001	−0.0012	0.0023	0.0009	−0.0017	−0.0042	−0.0030
	(0.0012)	(0.0012)	(0.0046)	(0.0009)	(0.0014)	(0.0040)	(0.0025)
J(χ^2(3))	1.2141	2.0594	2.8015	0.7817	1.7377	3.7454	0.1712
p_value	0.7496	0.5602	0.4233	0.8538	0.6286	0.2903	0.9821

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Panel B: eRm, SMB, RMW, LIQ, MOM

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
$\lambda(eR_m)$	0.0004*	0.0003	0.0003	0.0004*	0.0004**	0.0002	0.0005*
	(0.0002)	(0.0002)	(0.0003)	(0.0003)	(0.0002)	(0.0003)	(0.0003)
$\lambda(SMB)$	0.0004	−0.0008	−0.0012	0.0008	0.0015***	−0.0011	0.0017**
	(0.0003)	(0.0013)	(0.0020)	(0.0016)	(0.0005)	(0.0026)	(0.0007)
$\lambda(RMW)$	−0.0012	−0.0000	0.0007	0.0022	0.0028**	−0.0010	0.0036
	(0.0014)	(0.0016)	(0.0005)	(0.0036)	(0.0013)	(0.0044)	(0.0036)
$\lambda(LIQ)$	0.0008	0.0010	−0.0002	0.0031	0.0004*	0.0044	0.0008
	(0.0009)	(0.0012)	(0.0026)	(0.0041)	(0.0002)	(0.0050)	(0.0006)
$\lambda(MOM)$	0.0030*	0.0007	−0.0024	0.0016	0.0016	0.0010***	0.0002
	(0.0017)	(0.0010)	(0.0030)	(0.0026)	(0.0015)	(0.0004)	(0.0049)
J(χ^2(3))	1.2466	4.4142	2.2050	2.3579	1.7012	3.4612	0.0444
p_value	0.7418	0.2201	0.5310	0.5015	0.6367	0.3258	0.9975

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

As the estimated risk premiums were found to be very dependent of other factors in the model, I will further investigate the individual factors next to the market index. Table 4.16 show the results for all FF5 risk factors individually tested next to the market factor. Test 1 for the SMB combination shows that the market premium is priced for all sorts at the 10% level (5% for size and liquidity), while the SMB premium is priced at the 1% level for portfolios sorted in size, liquidity and β's. The model is only rejected for momentum portfolios (1,49%) and liquidity (9,58%). This is consistent with previous sections, as there is proof of correlation between variable for size, liquidity and estimated β's. An interesting observation in test 2-4 (table 4.16) is that all of the FF5 factors derived by the dividend discount model (HML, RMW, CMA) are priced for the momentum cross-section, where only RMW is positively related in test 4. There should indeed be some correlation between past operating profits and past momentum resulting in this premium, while the result could also be coincidental. Titman et al. (2004) found that capital investments where related to previous high returns, which could explain the negative relation between momentum and the CMA factor. The RMW is nevertheless the most interesting factor of the three, as it is also priced for size and beta -portfolios.

Further investigating the LIQ, MOM and BAB -factor in table 4.17, the LIQ factor in test 1, produces similar results as the SMB factor in a two-factor setting. The factor's premium is priced for the same portfolios (size, liquidity, beta) as SMB, while the model is also rejected for both the liquidity and momentum cross-sections. The MOM premium (test 2) is again priced for the momentum portfolios, while this factor does not show any signs of pricing cross-sections related to the dividend discount model. Further a very interesting result is that a two-factor model containing the market and BAB -factor is not rejected by the J-test for any of the cross-sections. The factors premium is also priced at a 1% for size, liquidity, momentum and beta -portfolios, while also being the only factor who has premium related to the value cross-section (at 10% level). Furthermore, the BAB factor does seem to fit well with the market factor. Compared to the other two-factor combinations, the market factor has significant risk premiums for all cross-sections when tested next to the BAB factor. This combination did not perform very well in the GRS-test and the BAB factor was also found redundant next to the SMB and LIQ -factor in section 4.3. I believe the J-test shows incorrect chi-squared values as it does not account for errors in the estimated coefficients from the first step, however the BAB factor do prove to be very interesting. The factor is expected to be sensitive to market liquidity as explained in Frazzini & Pedersen (2014), and as seen in figure 4.22 from section 4.2 . It is possible that this factor is more suited in a conditional model specification and fail the classical unconditional tests due to being more varying over time.

Table 4.16: Risk premia for individual Fama & French factors

Two-step GMM -estimation of two-factor models. Moment condition for first step is $E[\epsilon_t] = 0$ and $E[f_t\epsilon_t] = 0$, while second step uses estimated factor loadings to solve moment condition $E[\alpha] = 0$. The J-test is measuring the size of the pricing error from the second step.

TEST 1: $eR_{it} = \alpha_i + \beta_i^b eRm_t + \beta_i^s SMB_t$

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
$\lambda(eR_m)$	0.0004**	0.0003*	0.0004*	0.0004*	0.0004**	0.0004*	0.0004*
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
$\lambda(SMB)$	0.0007***	0.0001	−0.0013	0.0001	0.0012***	0.0004	0.0018***
	(0.0002)	(0.0010)	(0.0010)	(0.0007)	(0.0003)	(0.0006)	(0.0004)
J(χ^2(6))	7.7891	5.9219	4.5247	3.8030	10.7695	15.7890	3.4930
p_value	0.2540	0.4320	0.6060	0.7033	0.0958	0.0149	0.7449

TEST 2: $eR_{it} = \alpha_i + \beta_i^b eRm_t + \beta_i^h HML_t$

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
$\lambda(eR_m)$	0.0003	0.0004*	0.0004*	0.0004*	0.0003	0.0003	0.0002
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
$\lambda(HML)$	0.0000	0.0004	−0.0009	0.0012	−0.0003	−0.0038***	−0.0009
	(0.0011)	(0.0003)	(0.0012)	(0.0012)	(0.0015)	(0.0011)	(0.0013)
J(χ^2(6))	31.9483	4.2465	5.0889	2.9067	38.8616	4.7308	24.8369
p_value	0.00002	0.6434	0.5325	0.8205	0.000001	0.5788	0.0004

TEST 3: $eR_{it} = \alpha_i + \beta_i^b eRm_t + \beta_i^r RMW_t$

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
$\lambda(eR_m)$	0.0004*	0.0003	0.0004*	0.0003	0.0003	0.0005**	0.0001
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
$\lambda(RMW)$	−0.0030***	−0.0001	0.0004	−0.0006	−0.0014	0.0024**	−0.0028*
	(0.0008)	(0.0009)	(0.0003)	(0.0011)	(0.0010)	(0.0010)	(0.0017)
J(χ^2(6))	16.3077	5.9946	3.3983	3.6430	36.7455	10.3835	22.4091
p_value	0.0122	0.4238	0.7574	0.7249	0.000002	0.1094	0.0010

TEST 4: $eR_{it} = \alpha_i + \beta_i^b eRm_t + \beta_i^c CMA_t$

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
$\lambda(eR_m)$	0.0003	0.0003	0.0004*	0.0004*	0.0003	0.0004*	0.0002
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
$\lambda(CMA)$	−0.0008	−0.0006	−0.0004	0.0004	0.0003	−0.0087***	−0.0000
	(0.0010)	(0.0010)	(0.0014)	(0.0003)	(0.0014)	(0.0024)	(0.0018)
J(χ^2(6))	31.2843	5.7284	5.5059	1.6708	38.8258	3.9602	25.2492
p_value	0.00002	0.4543	0.4807	0.9474	0.000001	0.6821	0.0003

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 4.17: Risk premia for additional factors LIQ,MOM,BAB

Two-step GMM -estimation of two-factor models. Moment condition for first step is $E[\epsilon_t] = 0$ and $E[f_t\epsilon_t] = 0$, while second step uses estimated factor loadings to solve moment condition $E[\alpha] = 0$. The J-test is measuring the size of the pricing error from the second step.

TEST 1: $eR_{it} = \alpha_i + \beta_i^b eRm_t + \beta_i^l LIQ_t$

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
$\lambda(eR_m)$	0.0005**	0.0003*	0.0004**	0.0003	0.0004*	0.0004*	0.0003
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
$\lambda(LIQ)$	0.0011***	0.0003	−0.0014	0.0005	0.0005**	0.0006	0.0012***
	(0.0003)	(0.0008)	(0.0008)	(0.0011)	(0.0002)	(0.0012)	(0.0004)
$J(\chi^2(6))$	6.3181	5.5242	4.0604	3.3929	11.2973	16.0231	4.7680
p_value	0.3885	0.4785	0.6685	0.7582	0.0796	0.0136	0.5739

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

TEST 2: $eR_{it} = \alpha_i + \beta_i^b eRm_t + \beta_i^m MOM_t$

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
$\lambda(eR_m)$	0.0003	0.0003*	0.0004*	0.0004*	0.0003	0.0003*	0.0003
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
$\lambda(MOM)$	0.0007	0.0003	0.0011	0.0007	0.0000	0.0009***	0.0033
	(0.0014)	(0.0007)	(0.0011)	(0.0017)	(0.0014)	(0.0003)	(0.0027)
$J(\chi^2(6))$	31.6898	5.7869	4.4563	3.7602	38.8903	5.4755	23.7287
p_value	0.00002	0.4475	0.6152	0.7091	0.000001	0.4844	0.0006

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

TEST 3: $eR_{it} = \alpha_i + \beta_i^b eRm_t + \beta_i^k BAB_t$

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
$\lambda(eR_m)$	0.0004**	0.0004*	0.0004**	0.0004*	0.0004**	0.0004*	0.0004*
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
$\lambda(BAB)$	0.0039***	0.0035*	0.0018	0.0036	0.0032***	0.0046***	0.0017***
	(0.0008)	(0.0019)	(0.0027)	(0.0028)	(0.0006)	(0.0017)	(0.0004)
$J(\chi^2(6))$	6.9415	2.2177	4.9971	2.1873	3.6179	8.2880	3.7400
p_value	0.3263	0.8986	0.5442	0.9017	0.7282	0.2178	0.7118

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

At last testing the proposed four-factor model (eRm,SMB,LIQ,MOM), which did very well in the first-step GRS and the Wald test, the J-test does not reject this model in table 4.18. All factors are at least priced for two cross-sections at a 10% level which is promising, but the model does not have any additional risk premiums related to portfolios sorted on value, profitability and investments. The model should be able to price these effects to be well suited in the Norwegian stock market. It is worth mentioning that these portfolios did not produce significant return patterns for the anomalies, which may reduce the possibility of finding priced factors. Næs et al. (2009) did, for example, not find priced risk factors by testing models on industry portfolios. There is a weak effect related to these portfolios (B/M, OP, INV), but maybe not enough to disentangle the effects one is looking for in these estimations. These portfolios also have the least diversified returns as a result of poor accounting data in the sample. Another reason pointed out, is that the estimations lack power due to small cross-sections only containing eight portfolios. Two-step GMM -estimations of the CAPM, and other three-factor combinations can be investigated in the appendix.

Table 4.18: Factor premiums for model: eRm,SMB,LIQ,MOM

Two-step GMM -estimation of four-factor model containing eRm, SMB, LIQ and MOM. Moment condition for first step is $E[\epsilon_t] = 0$ and $E[f_t\epsilon_t] = 0$, while second step uses estimated factor loadings to solve moment condition $E[\alpha] = 0$. The J-test is measuring the size of the pricing error from the second step.

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
$\lambda(eR_m)$	0.0004**	0.0003*	0.0004*	0.0004	0.0004**	0.0003	0.0005**
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0003)
$\lambda(SMB)$	0.0004	−0.0007	−0.0004	0.0002	0.0009**	−0.0008	0.0015**
	(0.0003)	(0.0013)	(0.0018)	(0.0013)	(0.0004)	(0.0010)	(0.0007)
$\lambda(LIQ)$	0.0015**	0.0009	−0.0014	0.0009	0.0004*	0.0039	0.0006
	(0.0006)	(0.0010)	(0.0023)	(0.0018)	(0.0002)	(0.0033)	(0.0005)
$\lambda(MOM)$	0.0031*	0.0008	0.0001	0.0015	0.0015	0.0010***	0.0038
	(0.0017)	(0.0008)	(0.0021)	(0.0026)	(0.0015)	(0.0003)	(0.0028)
J($\chi^2(4)$)	2.2675	4.3956	4.0321	2.7093	6.0262	3.4760	0.8512
p_value	0.6867	0.3551	0.4017	0.6076	0.1972	0.4815	0.9315

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

4.6 Further discussion of main results

The results from the different regressions analysis are ambiguous to some degree, especially because the GMM approach accept more models compared to the first step Wald and GRS -test. The latter two did however produce more consistent results. What is unambiguous is that the effects found most significant in section 4.1, do fit in a model combination for all tests, and the same effects do have significant risk premiums related to their factor-mimicking portfolios. There are some statistical assumptions for each test already pointed out, which could explain some of the variations in the results. The GRS -test is performed on results from an OLS -regression, while the Wald test benefits from a first step GMM -estimation with more robust standard errors. This should and do not make a great difference in presented results, but the GRS-test could potentially accept more models compared to the Wald test. The two-step GMM does however not account for errors in the estimated β's, which may be a reason for more models being accepted in this part of the analysis.

Further some of the correlations between factors can be suspected to be a result of the data sorting and choice of market factor. There is especially a strong correlation between the eRm, SMB, LIQ and BAB -factor. For example, the BAB-factor is constructed by stocks which have at least 99% valid return data, which leaves out the most illiquid stocks from the constructing portfolios and forms a BAB factor consisting of only stocks which are traded often. This could influence its behavior to some degree. Furthermore, the size and liquidity factors do seem to be very similar in most tests, but also seem to capture some different variations. If this is coincidental or a result of the underlying state variable risk is unknown. However, using a value-weighted market index, which variations are likely to be influenced by a few big size stocks, imply that the estimated β's describe covariance with big size stocks (and potentially liquid stocks) as much as the overall market. This is an issue with investigating the Norwegian stock market, which could be solved by instead using equally-weighted test portfolios and an equally-weighted market index. But this is also not totally favorable as it would increase each portfolio's exposure to other risk factors.

Other interesting findings include the interdependence between the factors motivated by the dividend discount model and the momentum portfolios in the two-step GMM. This is however not two-sided as neither of the factors produce significant risk premiums related to the B/M, INV and OP sorts. The lack of accounting data may leave these portfolios less diversified, while the factor constructing portfolios are better diversified (30% & 70% percentiles). There is reason to believe that the mentioned effects suffer in the analysis due to data constraints. The interdependence between profitability and momentum is proved by an alternative profitability factor (ROE) in Chen, Novy-Marx & Zhang (2011), where the factor is produced by quarterly rather than annual earnings and were found fit to described momentum portfolios. The momentum factor in Fama & French (2015a) (constructed similar to MOM) did however only improve the FF5 for describing momentum portfolios, which is similar to results found in this research.

Chapter 5

Conclusion

This research investigates return patterns related to the size, value, investment, profitability, liquidity, momentum and β -anomaly in the Norwegian stock market. Only stocks sorted on size, liquidity and momentum variables show strong significant return patterns in the investigated period. Of the other investigated CAPM anomalies, return patterns related to the profitability effect seem to be fairly significant. Comparison of extreme portfolio sorts, as well as accumulative return plots indicate that all investigated anomalies have had effects during some subperiod between year 1997 and 2017.

In regards to a locally adapted FF5, the risk factors, HML and CMA, do not improve model performance significantly for any of the investigated cross-sections. Other risk-factor combinations seem to be more fit to describe cross-sectional excess returns in a Norwegian setting. Especially a four-factor model containing the market, size, liquidity and momentum -factor performs reasonably well in all applied tests (GRS, the Wald and two-step GMM). The momentum factor is however most of all important for describing momentum portfolios, and the suggested four-factor model could be slightly improved for profitability and momentum cross-sections by also including the RMW factor. Regarding similar studies, these results substantiate what found in Næs et al. (2009), while the momentum factor becomes more important when investigating more present data. This research also finds the RMW factor more important for describing returns compared to results in Hoel & Mix (2016). This could indeed be a result of the chosen dataset and methodology.

The research further raise concern regarding this sort of analysis in a small market, as some investigated portfolios are for some time periods not properly diversified, which could increase idiosyncratic risk, and may influence the importance of the HML, RMW and CMA -factor. If these results are caused by limited access to accounting data remains unanswered. Another issue not empirically tested, is if the results are sensitive to specific company's weights in portfolio sorts. I do, however, believe the next step in this field is to investigate a conditional model specification, as the investigated effects seem to vary over time. There are surely also several other risk factors, macroeconomic variables and factor construction techniques which could have been tested in this setting. This research could also be improved by better specifications of the GMM -estimation, and forming larger cross-sections in the risk premium analysis.

Bibliography

Acharya, V. V. & Pedersen, L. H. (2005). Asset pricing with liquidity risk. *Journal of financial Economics*, *77*(2), 375–410.

Adams, J. C., Hayunga, D. K., Mansi, S., & Reeb, D. M. (2017). Identifying outliers in finance.

Banz, R. W. (1981). The relationship between return and market value of common stocks. *Journal of financial Economics*, *9*(1), 3–18.

Bhattacharya, U. & Galpin, N. (2011). The global rise of the value-weighted portfolio. *The Journal of financial and Quantitative Analysis*, *46*(3), 737–756.

Bollerslev, T., Engle, R. F., & Wooldridge, J. M. (1988). A capital asset pricing model with time-varying covariances. *Journal of political Economy*, *96*(1), 116–131.

Carhart, M. M. (1997). On persistence in mutual fund performance. *The Journal of finance*, *52*(1), 57–82.

Chaussé , P. et al. (2010). Computing generalized method of moments and generalized empirical likelihood with R. *Journal of Statistical Software*, *34*(11), 1–35.

Chen, L., Novy-Marx, R., & Zhang, L. (2011). An alternative three-factor model. *Available at SSRN 1418117*.

Cochrane, J. H. (2009). *Asset Pricing:(Revised Edition)*. Princeton university press.

Diether, K. (2001). GRS Review, Lecture slides, University of Chicago. `http://faculty.chicagobooth.edu/eugene.fama/teaching/Reading%20List%20and%20Notes/GRS.pdf`. "[Online; accessed 10-February-2018]".

Fama, E. F. & French, K. R. (1992). The cross-section of expected stock returns. *The Journal of finance*, *47*(2), 427–465.

Fama, E. F. & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of financial Economics*, *33*(1), 3–56.

Fama, E. F. & French, K. R. (2012). Size, value, and momentum in international stock returns. *Journal of financial Economics*, *105*(3), 457–472.

Fama, E. F. & French, K. R. (2015a). Dissecting anomalies with a five-factor model. *Review of Financial Studies*.

Fama, E. F. & French, K. R. (2015b). A five-factor asset pricing model. *Journal of financial Economics*, *116*(1), 1 – 22.

Fama, E. F. & French, K. R. (2015c). International tests of a five-factor asset pricing model. *Fama-Miller Working Paper*.

Fama, E. F. & MacBeth, J. D. (1973). Risk, return, and equilibrium: Empirical tests. *The journal of political economy*, 607–636.

Frazzini, A. & Pedersen, L. H. (2014). Betting against beta. *Journal of financial Economics*, *111*(1), 1–25.

Gibbons, M. R. & Ferson, W. (1985). Testing asset pricing models with changing expectations and an unobservable market portfolio. *Journal of Financial Economics*, *14*(2), 217–236.

Gibbons, M. R., Ross, S. A., & Shanken, J. (1989). A test of the efficiency of a given portfolio. *Econometrica: Journal of the Econometric Society*, 1121–1152.

Hansen, L. P. (1982). Large sample properties of generalized method of moments estimators. *Econometrica*, *50*(4), 1029–1054.

Hoel, A. R. & Mix, F. (2016). How suitable is the Fama-French five-factor model for describing German and Norwegian stock returns? Master's thesis, NHH.

Hou, K., Xue, C., & Zhang, L. (2014). A comparison of new factor models. Working Paper 20682, National Bureau of Economic Research.

Jagannathan, R., Skoulakis, G., & Wang, Z. (2002). Generalized methods of moments: Applications in finance. *Journal of Business & Economic Statistics*, *20*(4), 470–481.

Jagannathan, R. & Wang, Z. (1996). The conditional capm and the cross-section of expected returns. *The Journal of finance*, *51*(1), 3–53.

Jegadeesh, N. & Titman, S. (1993). Returns to buying winners and selling losers: Implications for stock market efficiency. *The Journal of finance*, *48*(1), 65–91.

Jensen, M. C., Black, F., & Scholes, M. S. (1972). The capital asset pricing model: Some empirical tests. *Studies in the theory of capital markets*.

Korneliussen, F. G. & Rasmussen, C. (2014). Systematic Risk Factors at Oslo Stock Exchange. Master's thesis, Oslo and Akershus University College.

Lakonishok, J., Shleifer, A., & Vishny, R. W. (1994). Contrarian investment, extrapolation, and risk. *The Journal of finance*, *49*(5), 1541–1578.

Lintner, J. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *The review of economics and statistics*, 13–37.

Liu, W. (2006). A liquidity-augmented capital asset pricing model. *Journal of financial Economics*, *82*(3), 631–671.

MacKinlay, A. C. & Richardson, M. P. (1991). Using generalized method of moments to test mean-variance efficiency. *The Journal of finance*, *46*(2), 511–527.

Næs, R., Skjeltorp, J., & Ødegaard, B. A. (2009). What factors affect the oslo stock exchange. *Norges Bank (Central Bank of Norway), Working Paper*.

Newey, W. K. & West, K. D. (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica*, *55*(3), 703–708.

Novy-Marx, R. (2013). The other side of value: The gross profitability premium. *Journal of financial Economics*, *108*(1), 1–28.

Ødegaard, B. A. (2018a). Asset pricing data at OSE, Webpage. `http://finance.bi.no/~bernt/financial_data/ose_asset_pricing_data/index.html`. [Online; accessed 19-January-2018].

Ødegaard, B. A. (2018b). Empirics of the Oslo Stock Exchange: Asset pricing results. 1980-2017. *University of Stavanger (UiS)*.

Ødegaard, B. A. (2018c). Empirics of the Oslo Stock Exchange: Basic, descriptive, results 1980-2017. *University of Stavanger (UiS)*.

Oslo Børs (2018a). List changes Oslo Børs 1996-2018. `https://www.oslobors.no/ob_eng/Oslo-Boers/Statistic`. "[Online; accessed 09-January-2018]".

Oslo Børs (2018b). Listed companies home-state etc. `https://www.oslobors.no/ob_eng/Oslo-Boers/Listing/Shares-equity-certificates-and-rights-to-shares/Oslo-Boers-and-Oslo-Axess/Listed-companies-home-state`. "[Online; accessed 10-January-2018]".

Plyakha, Y., Uppal, R., & Vilkov, G. (2012). Why does an equal-weighted portfolio outperform value-and price-weighted portfolios?

Porta, R. L., Lakonishok, J., Shleifer, A., & Vishny, R. (1997). Good news for value stocks: Further evidence on market efficiency. *The Journal of finance*, *52*(2), 859–874.

Rosenberg, B., Reid, K., & Lanstein, R. (1985). Persuasive evidence of market inefficiency. *The Journal of Portfolio Management*, *11*(3), 9–16.

Ross, S. A. (1976). The arbitrage theory of capital asset pricing. *Journal of Economic Theory*, *13*(3), 341–360.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of finance*, *19*(3), 425–442.

Titman, S., Wei, K. J., & Xie, F. (2004). Capital investments and stock returns. *Journal of financial and Quantitative Analysis*, *39*(04), 677–700.

Appendix A

Additional Information

A.1 Additional statistics for factors

Table A.1: Correlation between factors and portfolios formed on identical characteristics

Correlation between RHS factors and LHS portfolios constructed by the same characteristics.

Portfolio	cor(sizeport,SMB)	cor(valueport,HML)	cor(profit.port,RMW)	cor(inv.port,CMA)	cor(spread.port,LIQ)	cor(mom.port,MOM)	cor(beta.port,BAB)
1	0.013	-0.295	-0.325	0.263	-0.782	-0.430	0.181
2	0.008	-0.312	-0.366	0.173	-0.595	-0.394	0.081
3	-0.057	-0.212	-0.238	0.065	-0.482	-0.250	-0.024
4	-0.106	-0.147	-0.146	0.002	-0.380	-0.136	-0.156
5	-0.279	-0.129	-0.117	-0.048	-0.298	-0.090	-0.280
6	-0.411	-0.004	-0.047	-0.141	-0.152	0.028	-0.358
7	-0.500	0.149	0.010	-0.202	0.206	0.143	-0.459
8	-0.686	0.130	0.062	-0.206	0.269	0.190	-0.483

Table A.2: Investigating factor redundancy FF5

Using four factors in regressions to explain average daily returns on the fifth. eRm (excess return market) is the value-weighted of all stocks in sample. HML (high minus low) the value factor, SMB (small minus big) the size factor, RMW (robust minus weak) the profitability factor, CMA (conservative minus aggressive).

	eR_m	SMB	HML	RMW	CMA
Intercept	0.0008^{***}	0.0007^{***}	0.0003^{*}	0.0004^{**}	0.0002
	(0.0001)	(0.0001)	(0.0002)	(0.0002)	(0.0002)
eR_m		-0.5958^{***}	-0.3342^{***}	-0.1019^{***}	-0.0910^{***}
		(0.0093)	(0.0174)	(0.0169)	(0.0185)
SMB	-0.7562^{***}		-0.2726^{***}	0.0110	-0.0785^{***}
	(0.0118)		(0.0199)	(0.0191)	(0.0209)
HML	-0.2062^{***}	-0.1325^{***}		-0.0938^{***}	0.0471^{***}
	(0.0107)	(0.0097)		(0.0133)	(0.0146)
RMW	-0.0706^{***}	0.0060	-0.1054^{***}		-0.0677^{***}
	(0.0117)	(0.0105)	(0.0149)		(0.0154)
CMA	-0.0527^{***}	-0.0358^{***}	0.0442^{***}	-0.0566^{***}	
	(0.0107)	(0.0095)	(0.0137)	(0.0129)	
Adj. R^2	0.4762	0.4549	0.0796	0.0247	0.0124

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Figure A.1: Histogram factors: VWall and SMB

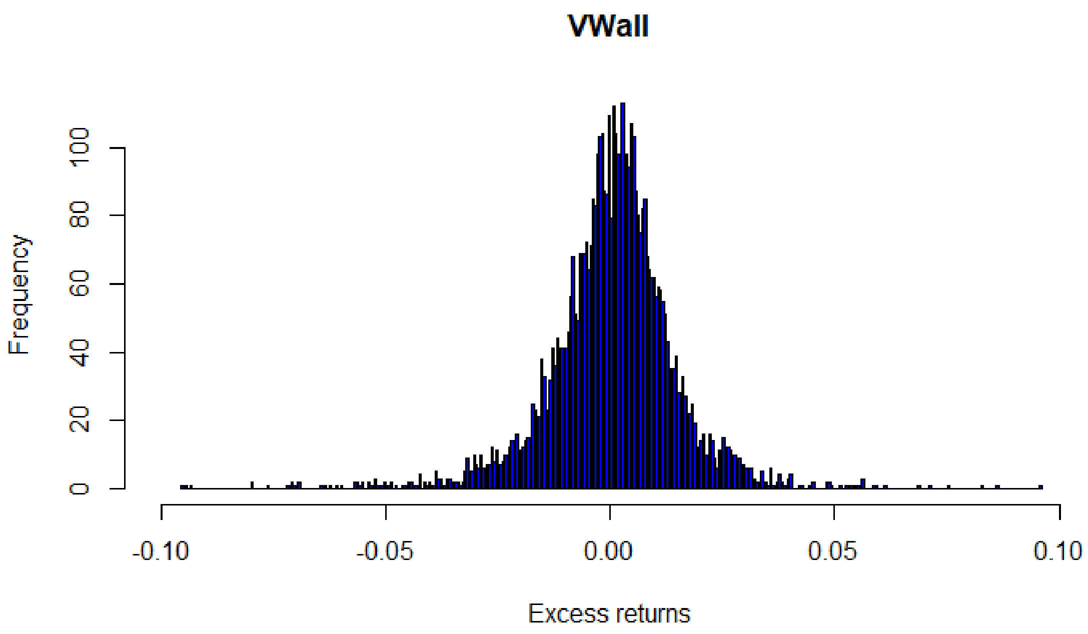

Figure A.2: Histogram VWall

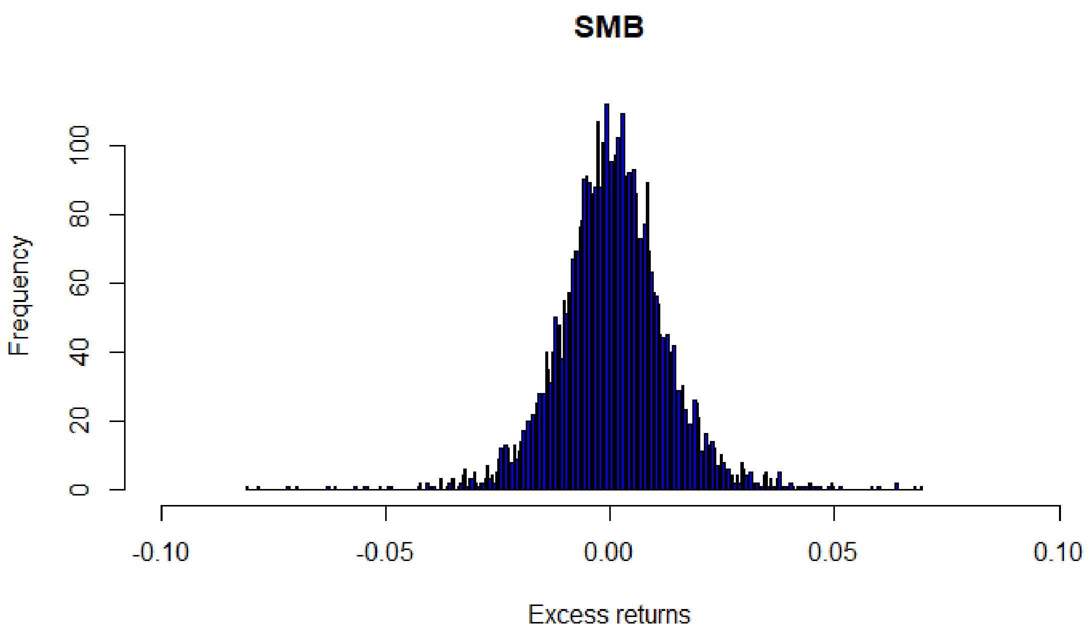

Figure A.3: Histogram SMB

Figure A.4: Histogram factors: HML and RMW

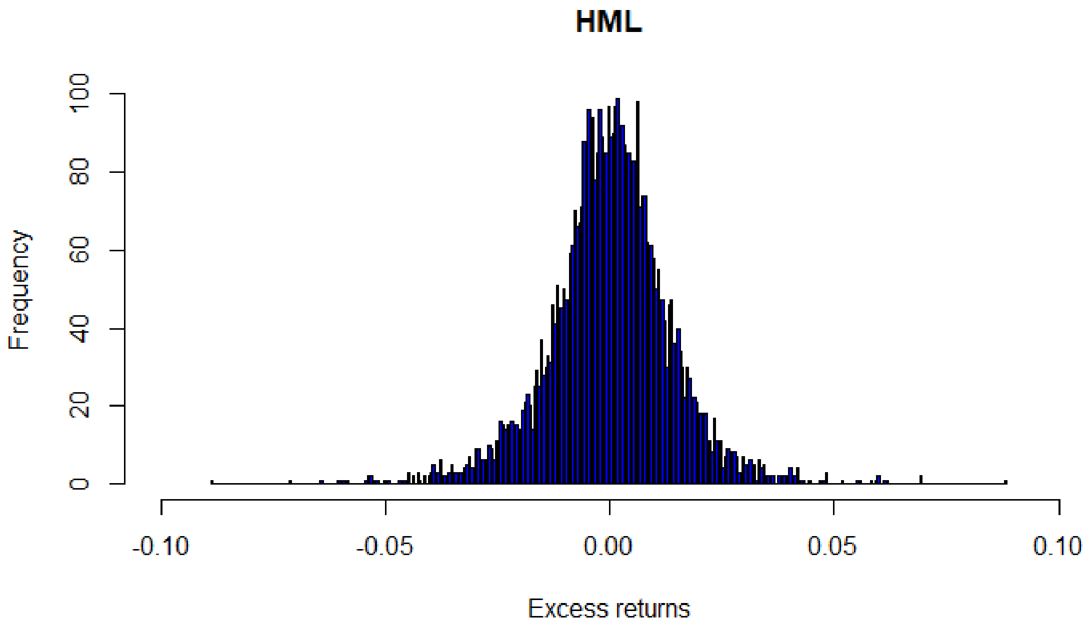

Figure A.5: Histogram HML

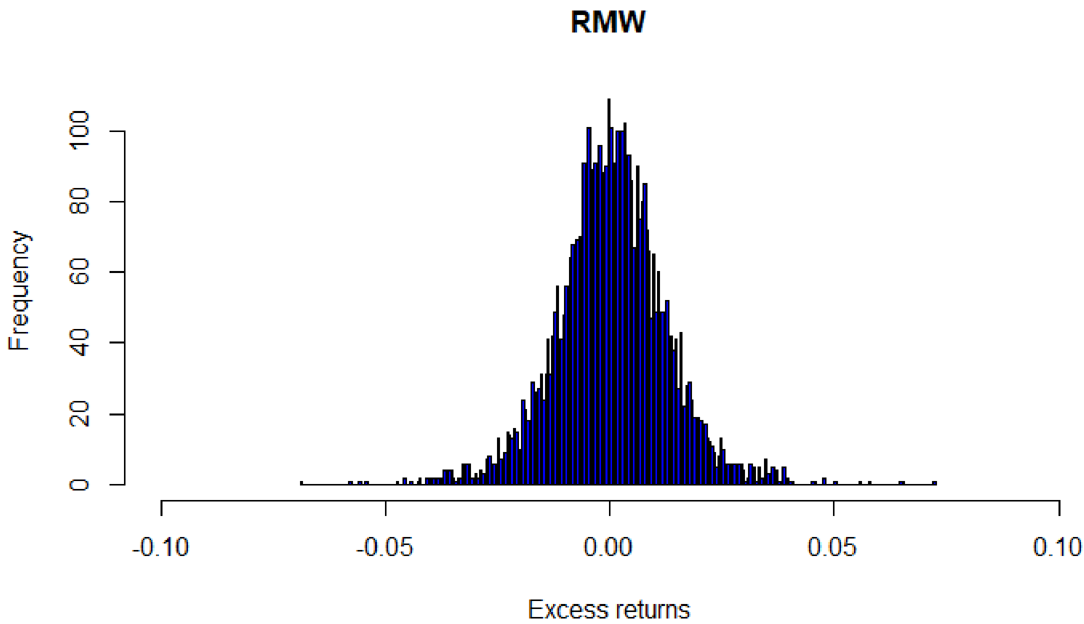

Figure A.6: Histogram RMW

Figure A.7: Histogram factors: CMA and LIQ

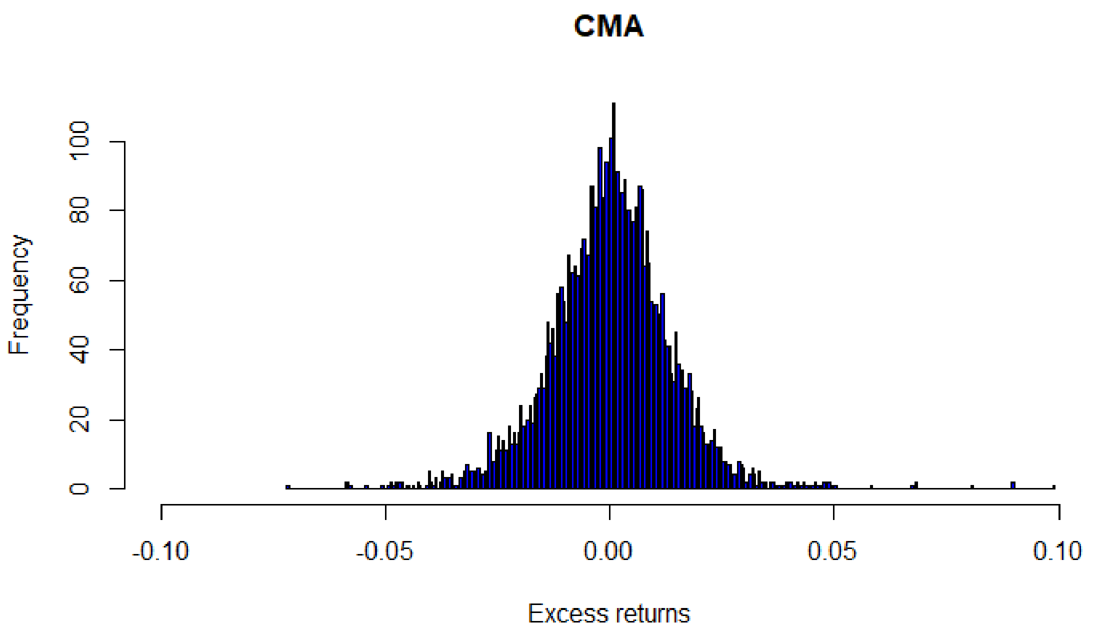

Figure A.8: Histogram CMA

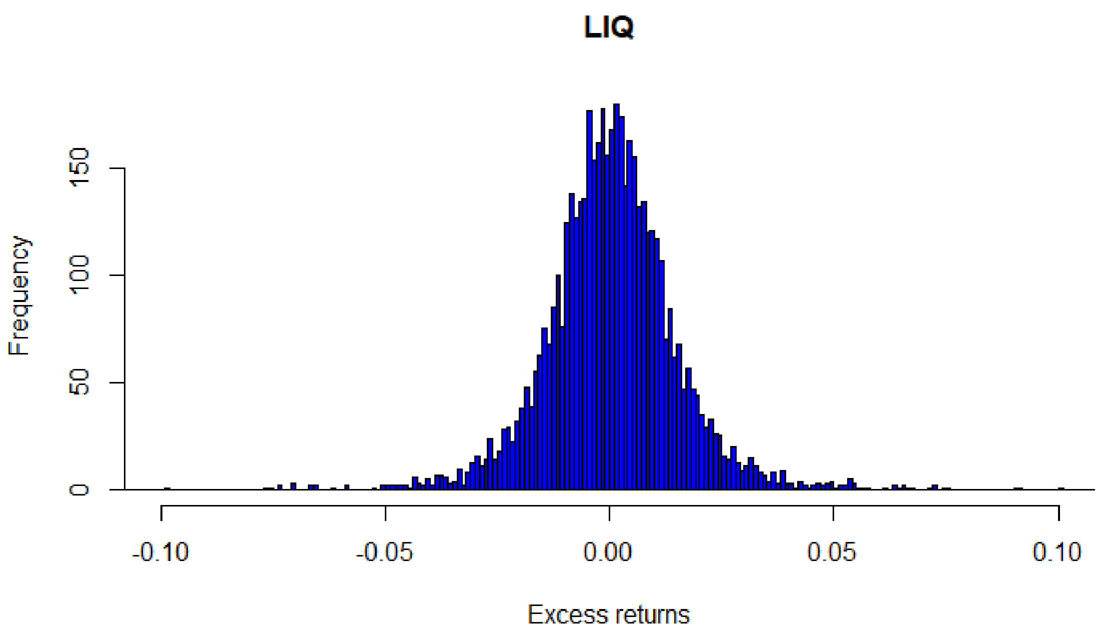

Figure A.9: Histogram LIQ

Figure A.10: Histogram factors: MOM, BAB

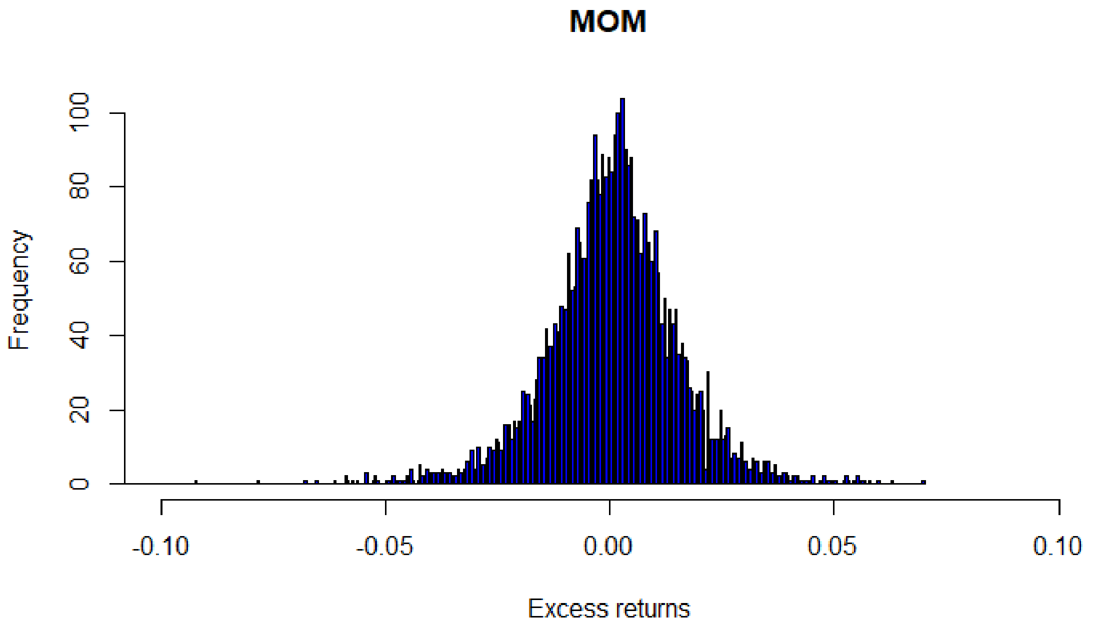

Figure A.11: Histogram MOM

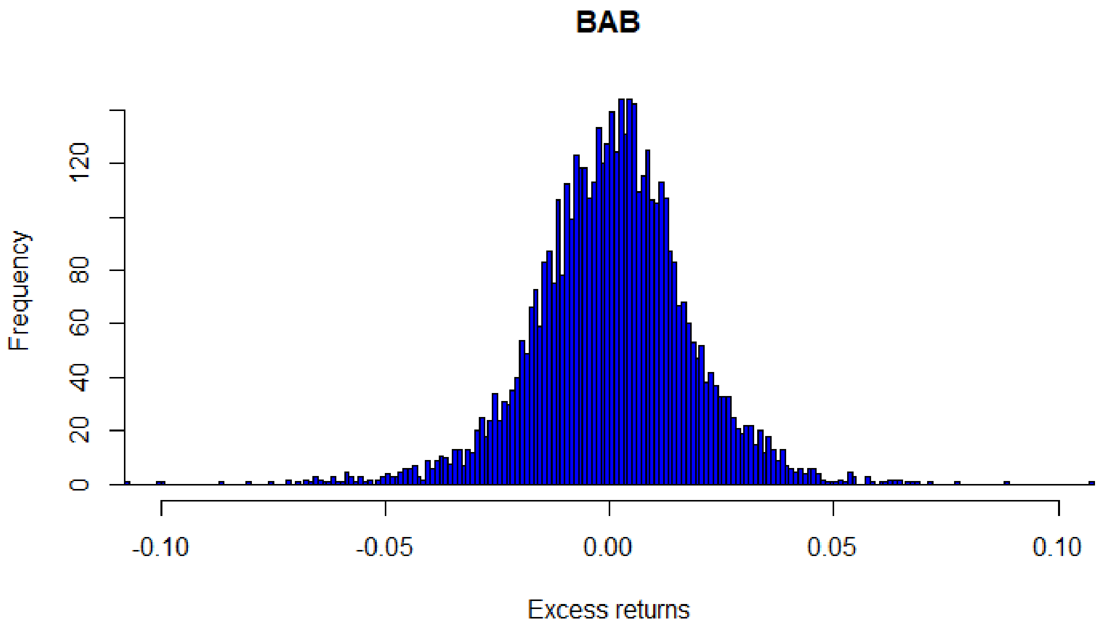

Figure A.12: Histogram BAB

A.2 Enlarged plots from anomaly investigation

Figure A.13: Accumulative return extreme portfolios

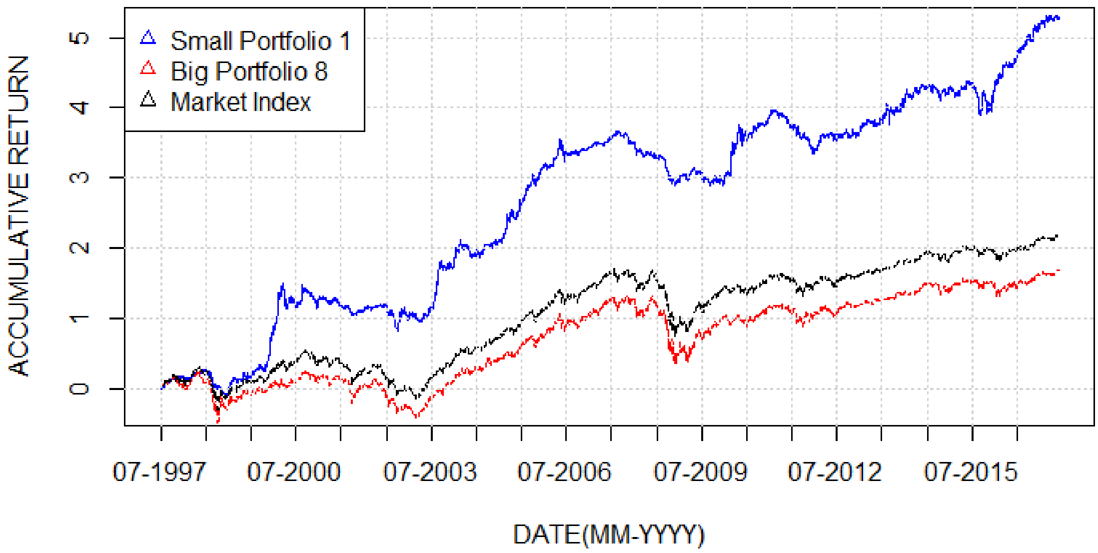

Figure A.14: Size portfolios

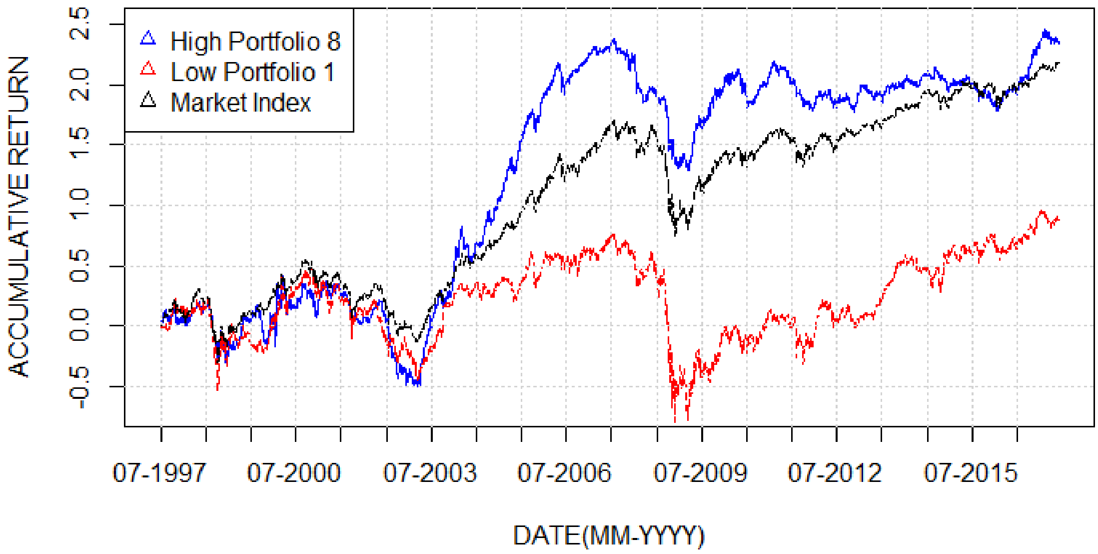

Figure A.15: B/M portfolios

Figure A.16: Accumulative return extreme portfolios

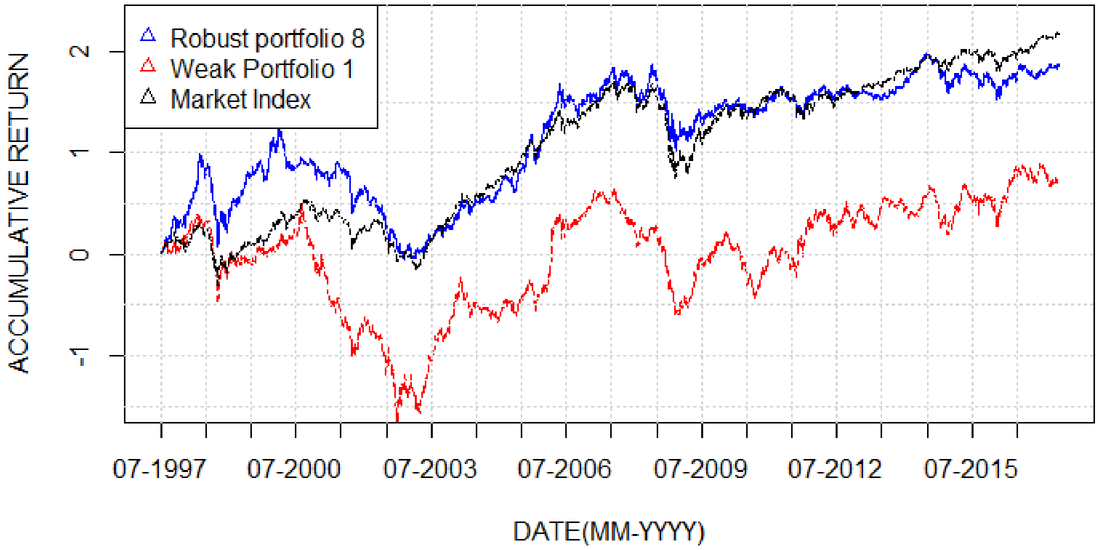

Figure A.17: OP portfolios

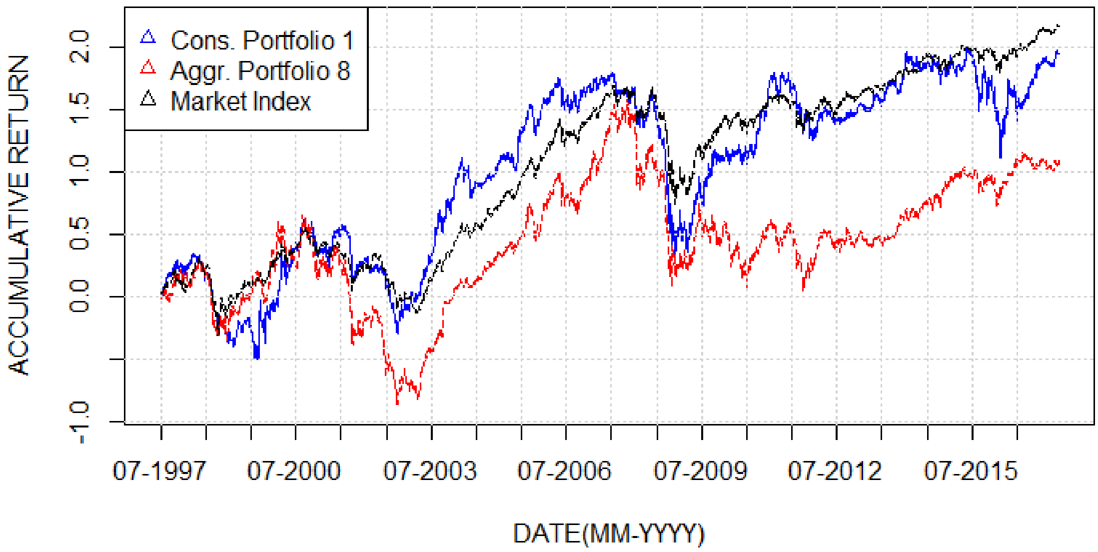

Figure A.18: INV portfolios

Figure A.19: Accumulative return extreme portfolios

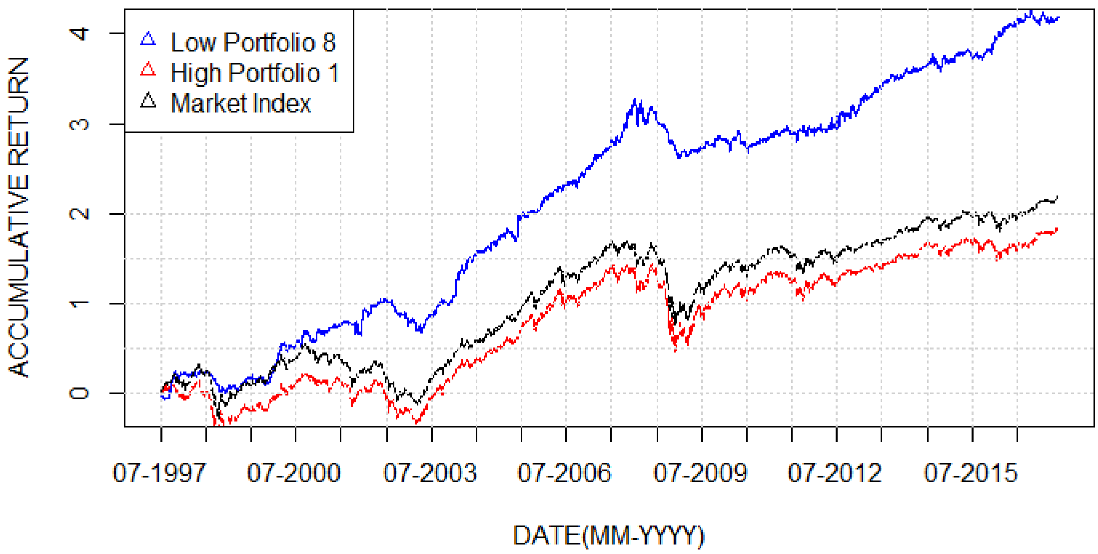

Figure A.20: Liquidity portfolios

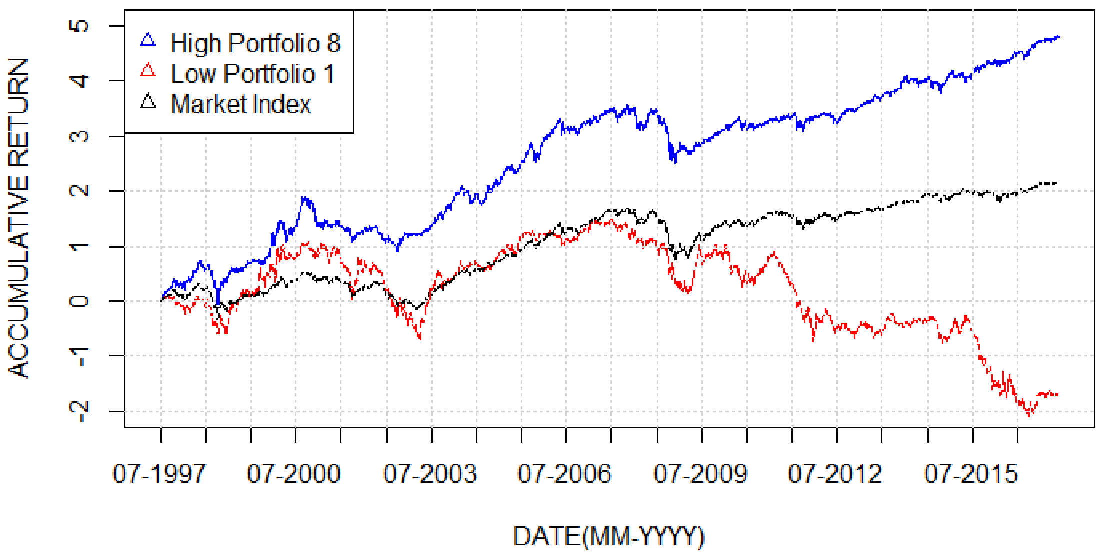

Figure A.21: Momentum portfolios

Figure A.22: Accumulative return extreme portfolios

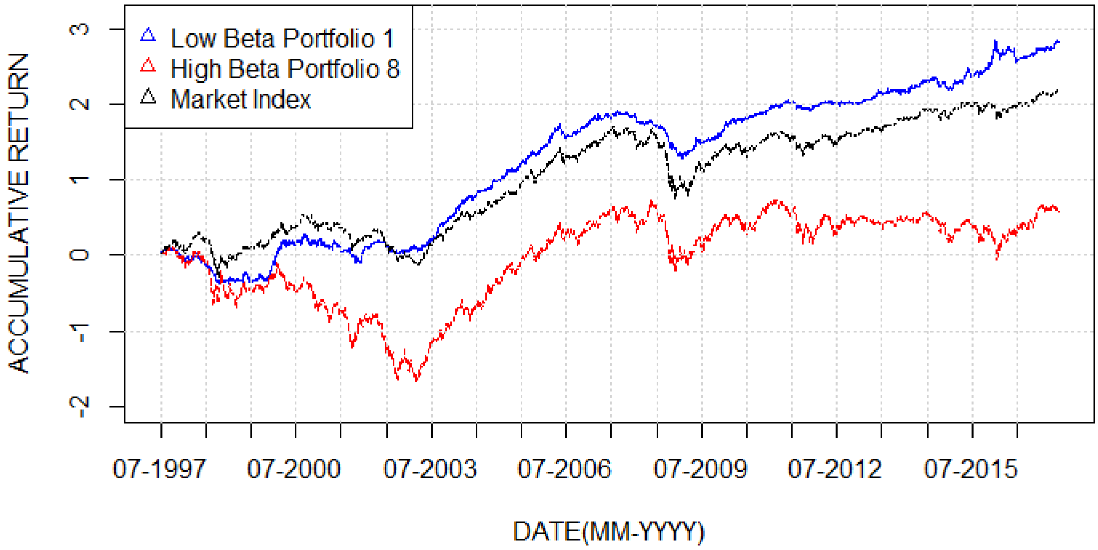

Figure A.23: β portfolios

A.3 First-step OLS regression for important models

Table A.3: First-step regressions of size portfolios

A: CAPM

	1(Small)	2	3	4	5	6	7	8(Big)
Constant	0.0009***	0.0008***	0.0004**	0.0003**	0.0004***	0.0002*	0.0001	−0.0001***
	(0.0003)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0001)	(0.0001)	(0.0000)
eR_m	0.3981***	0.4393***	0.5294***	0.5234***	0.6555***	0.6569***	0.7882***	1.0792***
	(0.0185)	(0.0137)	(0.0120)	(0.0118)	(0.0117)	(0.0087)	(0.0078)	(0.0027)
Adj. R^2	0.0850	0.1710	0.2794	0.2827	0.3855	0.5309	0.6721	0.9705

B: FF3

	1(Small)	2	3	4	5	6	7	8(Big)
Constant	0.0005*	0.0004**	0.0000	0.0000	0.0002	0.0001	0.0001	−0.0001**
	(0.0002)	(0.0002)	(0.0001)	(0.0002)	(0.0002)	(0.0001)	(0.0001)	(0.0000)
eR_m	0.7583***	0.8170***	0.8545***	0.8055***	0.8397***	0.7312***	0.8390***	1.0237***
	(0.0242)	(0.0171)	(0.0147)	(0.0150)	(0.0156)	(0.0119)	(0.0107)	(0.0035)
SMB	0.5960***	0.6305***	0.5787***	0.4893***	0.2979***	0.1271***	0.0821***	−0.0908***
	(0.0274)	(0.0193)	(0.0166)	(0.0169)	(0.0177)	(0.0135)	(0.0121)	(0.0039)
HML	0.1129***	0.0998***	−0.0342***	0.0133	0.0812***	0.0094	0.0228***	−0.0207***
	(0.0190)	(0.0134)	(0.0115)	(0.0118)	(0.0123)	(0.0093)	(0.0084)	(0.0027)
Adj. R^2	0.1641	0.3167	0.4287	0.3874	0.4196	0.5390	0.6751	0.9734

B: FF5

	1(Small)	2	3	4	5	6	7	8(Big)
Constant	0.0005**	0.0004**	0.0000	0.0000	0.0003**	0.0002	0.0001	−0.0001**
	(0.0002)	(0.0002)	(0.0001)	(0.0002)	(0.0002)	(0.0001)	(0.0001)	(0.0000)
eR_m	0.7522***	0.8028***	0.8502***	0.8084***	0.8140***	0.7266***	0.8295***	1.0257***
	(0.0242)	(0.0171)	(0.0147)	(0.0149)	(0.0153)	(0.0119)	(0.0106)	(0.0035)
SMB	0.6061***	0.6275***	0.5823***	0.4992***	0.2939***	0.1286***	0.0829***	−0.0915***
	(0.0272)	(0.0192)	(0.0166)	(0.0167)	(0.0173)	(0.0134)	(0.0119)	(0.0039)
HML	0.0933***	0.0935***	−0.0428***	0.0009	0.0675***	0.0037	0.0143*	−0.0182***
	(0.0190)	(0.0134)	(0.0116)	(0.0117)	(0.0120)	(0.0094)	(0.0083)	(0.0027)
RMW	−0.1476***	−0.0965***	−0.0714***	−0.0664***	−0.1894***	−0.0542***	−0.0920***	0.0242***
	(0.0201)	(0.0142)	(0.0123)	(0.0124)	(0.0128)	(0.0099)	(0.0088)	(0.0029)
CMA	0.0979***	−0.0573***	0.0310***	0.1111***	−0.0867***	0.0079	−0.0074	−0.0040
	(0.0184)	(0.0130)	(0.0112)	(0.0113)	(0.0117)	(0.0091)	(0.0081)	(0.0026)
Adj. R^2	0.1781	0.3249	0.4334	0.4029	0.4484	0.5417	0.6819	0.9738

C: eRm, SMB, LIQ, MOM

	1(Small)	2	3	4	5	6	7	8(Big)
Constant	0.0004	0.0003*	−0.0001	−0.0000	0.0002	0.0000	0.0000	−0.0001*
	(0.0002)	(0.0002)	(0.0001)	(0.0001)	(0.0002)	(0.0001)	(0.0001)	(0.0000)
eR_m	0.8561***	0.9061***	0.9418***	0.9298***	0.9576***	0.8354***	0.9199***	0.9950***
	(0.0284)	(0.0200)	(0.0172)	(0.0172)	(0.0181)	(0.0138)	(0.0123)	(0.0040)
SMB	0.4777***	0.5254***	0.5411***	0.4102***	0.1881***	0.0580***	0.0224*	−0.0639***
	(0.0287)	(0.0201)	(0.0174)	(0.0174)	(0.0182)	(0.0139)	(0.0124)	(0.0040)
LIQ	0.2215***	0.2002***	0.1245***	0.2118***	0.2380***	0.1761***	0.1453***	−0.0586***
	(0.0267)	(0.0187)	(0.0162)	(0.0162)	(0.0170)	(0.0129)	(0.0116)	(0.0037)
MOM	0.0440**	0.0214*	0.0016	−0.0774***	−0.0511***	−0.0024	−0.0278***	0.0181***
	(0.0173)	(0.0121)	(0.0105)	(0.0105)	(0.0110)	(0.0084)	(0.0075)	(0.0024)
Adj. R^2	0.1705	0.3248	0.4343	0.4138	0.4390	0.5554	0.6854	0.9746

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table A.4: First-step regressions of value portfolios

Results from first-step OLS regressions run on eight different value portfolios.

A: CAPM

	1(Low)	2	3	4	5	6	7	8(High)
Constant	−0.0003*	−0.0001	−0.0001	0.0001	−0.0000	−0.0002	−0.0000	0.0002
	(0.0002)	(0.0002)	(0.0001)	(0.0001)	(0.0001)	(0.0002)	(0.0002)	(0.0002)
eR_m	1.1687***	1.0463***	0.8478***	0.9694***	0.9530***	0.9290***	0.9073***	0.6356***
	(0.0137)	(0.0119)	(0.0100)	(0.0101)	(0.0094)	(0.0119)	(0.0140)	(0.0133)
Adj. R^2	0.5928	0.6061	0.5897	0.6480	0.6735	0.5513	0.4563	0.3119

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

B: FF3

	1(Low)	2	3	4	5	6	7	8(High)
Constant	−0.0003	−0.0000	−0.0001	0.0001	0.0000	−0.0003*	−0.0000	0.0000
	(0.0002)	(0.0002)	(0.0001)	(0.0001)	(0.0001)	(0.0002)	(0.0002)	(0.0002)
eR_m	1.0802***	0.9476***	0.8330***	0.9632***	0.9138***	0.9956***	0.9778***	0.8047***
	(0.0182)	(0.0157)	(0.0137)	(0.0139)	(0.0129)	(0.0160)	(0.0179)	(0.0174)
SMB	−0.0747***	−0.0941***	0.0008	−0.0089	−0.0722***	0.0620***	0.0065	0.2036***
	(0.0206)	(0.0178)	(0.0154)	(0.0157)	(0.0146)	(0.0181)	(0.0202)	(0.0197)
HML	−0.2676***	−0.2625***	−0.0890***	−0.0063	0.0121	0.1816***	0.3903***	0.3082***
	(0.0143)	(0.0123)	(0.0107)	(0.0109)	(0.0101)	(0.0125)	(0.0140)	(0.0137)
Adj. R^2	0.6194	0.6389	0.5953	0.6479	0.6753	0.5693	0.5313	0.3792

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

B: FF5

	1(Low)	2	3	4	5	6	7	8(High)
Constant	−0.0002	−0.0000	−0.0001	0.0001	0.0000	−0.0003*	0.0000	0.0001
	(0.0002)	(0.0002)	(0.0001)	(0.0001)	(0.0001)	(0.0002)	(0.0002)	(0.0002)
eR_m	1.0722***	0.9406***	0.8294***	0.9625***	0.9156***	0.9985***	0.9644***	0.7967***
	(0.0182)	(0.0156)	(0.0137)	(0.0140)	(0.0129)	(0.0159)	(0.0178)	(0.0175)
SMB	−0.0687***	−0.1034***	−0.0007	−0.0087	−0.0722***	0.0703***	0.0077	0.2044***
	(0.0205)	(0.0176)	(0.0155)	(0.0157)	(0.0146)	(0.0179)	(0.0201)	(0.0197)
HML	−0.2825***	−0.2541***	−0.0894***	−0.0072	0.0136	0.1716***	0.3784***	0.3009***
	(0.0143)	(0.0123)	(0.0108)	(0.0110)	(0.0102)	(0.0125)	(0.0140)	(0.0137)
RMW	−0.1257***	0.0253*	−0.0172	−0.0080	0.0174	−0.0512***	−0.1291***	−0.0782***
	(0.0151)	(0.0130)	(0.0114)	(0.0116)	(0.0108)	(0.0133)	(0.0149)	(0.0146)
CMA	0.0504***	−0.1115***	−0.0231**	0.0017	0.0023	0.0942***	−0.0104	−0.0051
	(0.0138)	(0.0119)	(0.0104)	(0.0106)	(0.0098)	(0.0121)	(0.0136)	(0.0133)
Adj. R^2	0.6257	0.6454	0.5957	0.6478	0.6754	0.5759	0.5381	0.3825

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

C: eRm, SMB, LIQ, MOM

	1(Low)	2	3	4	5	6	7	8(High)
Constant	−0.0003*	−0.0002	−0.0002	0.0000	0.0001	−0.0001	0.0001	0.0001
	(0.0002)	(0.0002)	(0.0001)	(0.0001)	(0.0001)	(0.0002)	(0.0002)	(0.0002)
eR_m	1.1827***	1.0552***	0.8846***	0.9763***	0.8643***	0.9554***	0.9121***	0.8053***
	(0.0222)	(0.0192)	(0.0162)	(0.0163)	(0.0151)	(0.0186)	(0.0223)	(0.0213)
SMB	−0.0065	−0.0372*	0.0138	−0.0152	−0.0480***	0.0096	−0.1338***	0.0575***
	(0.0223)	(0.0194)	(0.0163)	(0.0165)	(0.0152)	(0.0188)	(0.0224)	(0.0215)
LIQ	0.0227	0.0329*	0.0364**	0.0179	−0.0746***	0.0350**	0.1064***	0.1695***
	(0.0208)	(0.0180)	(0.0152)	(0.0153)	(0.0142)	(0.0175)	(0.0209)	(0.0200)
MOM	−0.0243*	0.0709***	−0.0276***	0.0206**	0.0112	−0.1928***	−0.1573***	−0.0766***
	(0.0135)	(0.0117)	(0.0098)	(0.0099)	(0.0092)	(0.0113)	(0.0135)	(0.0130)
Adj. R^2	0.5929	0.6091	0.5908	0.6482	0.6771	0.5761	0.4756	0.3305

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table A.5: First-step regressions of profitability portfolios

Results from first-step OLS regressions run on eight different profitability portfolios.

A: CAPM

	1(Weak)	2	3	4	5	6	7	8(Robust)
Constant	−0.0002	−0.0004**	−0.0002	0.0001	0.0001	0.0000	−0.0000	−0.0001
	(0.0002)	(0.0002)	(0.0002)	(0.0001)	(0.0002)	(0.0002)	(0.0001)	(0.0002)
eR_m	0.8558***	0.9587***	0.9627***	0.8810***	0.9196***	0.9739***	0.8915***	1.0384***
	(0.0172)	(0.0148)	(0.0157)	(0.0107)	(0.0115)	(0.0108)	(0.0105)	(0.0116)
Adj. R^2	0.3309	0.4557	0.4284	0.5771	0.5623	0.6175	0.5888	0.6164

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

B: FF3

	1(Weak)	2	3	4	5	6	7	8(Robust)
Constant	−0.0004	−0.0005**	−0.0002	0.0001	0.0002	0.0000	−0.0000	−0.0000
	(0.0002)	(0.0002)	(0.0002)	(0.0001)	(0.0002)	(0.0002)	(0.0001)	(0.0002)
eR_m	0.9708***	1.0017***	0.9183***	0.8732***	0.8614***	0.9604***	0.8808***	0.9720***
	(0.0235)	(0.0203)	(0.0216)	(0.0147)	(0.0157)	(0.0149)	(0.0145)	(0.0158)
SMB	0.1786***	0.0683***	−0.0922***	−0.0163	−0.0823***	−0.0310*	−0.0073	−0.0813***
	(0.0266)	(0.0230)	(0.0244)	(0.0166)	(0.0178)	(0.0168)	(0.0164)	(0.0178)
HML	0.0748***	0.0234	0.0485***	0.0090	−0.0647***	0.0247**	−0.0377***	−0.1161***
	(0.0185)	(0.0160)	(0.0169)	(0.0115)	(0.0123)	(0.0117)	(0.0114)	(0.0124)
Adj. R^2	0.3376	0.4565	0.4314	0.5771	0.5657	0.6181	0.5895	0.6235

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

B: FF5

	1(Weak)	2	3	4	5	6	7	8(Robust)
Constant	−0.0002	−0.0002	−0.0001	0.0001	0.0002	0.0000	−0.0001	−0.0001
	(0.0002)	(0.0002)	(0.0002)	(0.0001)	(0.0002)	(0.0002)	(0.0001)	(0.0002)
eR_m	0.9257***	0.9514***	0.8938***	0.8665***	0.8513***	0.9672***	0.8916***	0.9847***
	(0.0224)	(0.0187)	(0.0211)	(0.0147)	(0.0157)	(0.0149)	(0.0144)	(0.0154)
SMB	0.1851***	0.0735***	−0.0850***	−0.0130	−0.0848***	−0.0303*	−0.0093	−0.0908***
	(0.0252)	(0.0211)	(0.0238)	(0.0165)	(0.0177)	(0.0168)	(0.0163)	(0.0174)
HML	0.0309*	−0.0227	0.0192	−0.0009	−0.0687***	0.0288**	−0.0265**	−0.0924***
	(0.0176)	(0.0147)	(0.0166)	(0.0115)	(0.0124)	(0.0117)	(0.0113)	(0.0121)
RMW	−0.4572***	−0.4919***	−0.2831***	−0.0895***	−0.0664***	0.0526***	0.1136***	0.2006***
	(0.0186)	(0.0156)	(0.0176)	(0.0122)	(0.0131)	(0.0124)	(0.0120)	(0.0128)
CMA	−0.0077	−0.0298**	0.0362**	0.0235**	−0.0440***	0.0199*	−0.0035	−0.0807***
	(0.0170)	(0.0142)	(0.0161)	(0.0112)	(0.0120)	(0.0114)	(0.0110)	(0.0117)
Adj. R^2	0.4089	0.5470	0.4603	0.5820	0.5687	0.6195	0.5966	0.6451

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

C: eRm, SMB, LIQ, MOM

	1(Weak)	2	3	4	5	6	7	8(Robust)
Constant	−0.0004	−0.0005**	−0.0001	0.0001	0.0001	0.0001	−0.0001	−0.0001
	(0.0002)	(0.0002)	(0.0002)	(0.0001)	(0.0002)	(0.0002)	(0.0001)	(0.0002)
eR_m	1.0801***	1.0835***	0.9257***	0.9181***	0.8850***	0.9446***	0.9062***	1.0047***
	(0.0274)	(0.0237)	(0.0254)	(0.0172)	(0.0185)	(0.0175)	(0.0170)	(0.0186)
SMB	0.0821***	0.0124	−0.1171***	−0.0456***	−0.0656***	−0.0323*	−0.0057	−0.0497***
	(0.0276)	(0.0239)	(0.0256)	(0.0173)	(0.0187)	(0.0177)	(0.0172)	(0.0188)
LIQ	0.2214***	0.1479***	0.0392*	0.0790***	0.0035	−0.0122	0.0208	−0.0104
	(0.0257)	(0.0223)	(0.0238)	(0.0161)	(0.0174)	(0.0165)	(0.0160)	(0.0175)
MOM	−0.1341***	−0.1084***	−0.0589***	−0.0504***	0.0022	−0.0164	0.0176*	0.0810***
	(0.0166)	(0.0144)	(0.0154)	(0.0104)	(0.0113)	(0.0107)	(0.0104)	(0.0113)
Adj. R^2	0.3535	0.4670	0.4323	0.5810	0.5632	0.6179	0.5889	0.6207

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table A.6: First-step regressions of investment portfolios

Results from first-step OLS regressions run on eight different investment portfolios.

A: CAPM

	1(Cons.)	2	3	4	5	6	7	8(Aggr.)
Constant	−0.0000	0.0001	0.0001	0.0000	−0.0001	−0.0001	−0.0002	−0.0002
	(0.0002)	(0.0002)	(0.0002)	(0.0001)	(0.0001)	(0.0002)	(0.0002)	(0.0002)
eR_m	0.9512***	0.8303***	0.7885***	0.8789***	0.9488***	0.9756***	0.9320***	1.0285***
	(0.0171)	(0.0140)	(0.0117)	(0.0105)	(0.0106)	(0.0118)	(0.0120)	(0.0150)
Adj. R^2	0.3826	0.4130	0.4760	0.5818	0.6157	0.5778	0.5486	0.4851

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

B: FF3

	1(Cons.)	2	3	4	5	6	7	8(Aggr.)
Constant	−0.0002	0.0000	0.0001	0.0000	−0.0001	−0.0001	−0.0002	−0.0003
	(0.0002)	(0.0002)	(0.0002)	(0.0001)	(0.0001)	(0.0002)	(0.0002)	(0.0002)
eR_m	1.0965***	0.9068***	0.7984***	0.8688***	0.9013***	0.9506***	0.9130***	1.0741***
	(0.0233)	(0.0192)	(0.0161)	(0.0144)	(0.0145)	(0.0161)	(0.0164)	(0.0205)
SMB	0.2580***	0.1283***	0.0102	−0.0334**	−0.0573***	−0.0190	−0.0285	0.0905***
	(0.0263)	(0.0217)	(0.0182)	(0.0163)	(0.0164)	(0.0183)	(0.0186)	(0.0232)
HML	−0.0131	0.0182	0.0241*	0.0529***	−0.0863***	−0.0826***	−0.0158	−0.0359**
	(0.0183)	(0.0151)	(0.0126)	(0.0113)	(0.0114)	(0.0127)	(0.0129)	(0.0161)
Adj. R^2	0.3948	0.4169	0.4762	0.5842	0.6203	0.5812	0.5487	0.4874

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

B: FF5

	1(Cons.)	2	3	4	5	6	7	8(Aggr.)
Constant	−0.0002	0.0000	0.0001	0.0000	−0.0001	−0.0000	−0.0001	−0.0002
	(0.0002)	(0.0002)	(0.0002)	(0.0001)	(0.0001)	(0.0002)	(0.0002)	(0.0002)
eR_m	1.1232***	0.9185***	0.8019***	0.8709***	0.9010***	0.9364***	0.8856***	1.0350***
	(0.0214)	(0.0184)	(0.0160)	(0.0145)	(0.0146)	(0.0160)	(0.0159)	(0.0199)
SMB	0.2985***	0.1522***	0.0204	−0.0297*	−0.0580***	−0.0294	−0.0449**	0.0716***
	(0.0241)	(0.0208)	(0.0180)	(0.0163)	(0.0164)	(0.0181)	(0.0180)	(0.0224)
HML	−0.0527***	−0.0084	0.0117	0.0490***	−0.0855***	−0.0780***	−0.0123	−0.0376**
	(0.0168)	(0.0145)	(0.0126)	(0.0114)	(0.0114)	(0.0126)	(0.0125)	(0.0156)
RMW	−0.1427***	−0.1208***	−0.0646***	−0.0161	0.0045	−0.0282**	−0.0881***	−0.1673***
	(0.0178)	(0.0153)	(0.0133)	(0.0121)	(0.0121)	(0.0134)	(0.0133)	(0.0166)
CMA	0.4808***	0.2777***	0.1151***	0.0431***	−0.0080	−0.1359***	−0.2234***	−0.2701***
	(0.0163)	(0.0140)	(0.0122)	(0.0110)	(0.0111)	(0.0122)	(0.0121)	(0.0151)
Adj. R^2	0.4929	0.4673	0.4882	0.5855	0.6202	0.5914	0.5796	0.5252

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

C: eRm, SMB, LIQ, MOM

	1(Cons.)	2	3	4	5	6	7	8(Aggr.)
Constant	−0.0001	−0.0000	0.0001	0.0000	−0.0000	−0.0001	−0.0002	−0.0003
	(0.0002)	(0.0002)	(0.0002)	(0.0001)	(0.0001)	(0.0002)	(0.0002)	(0.0002)
eR_m	1.1430***	0.9638***	0.8318***	0.8691***	0.9137***	0.9855***	0.9345***	1.1496***
	(0.0272)	(0.0225)	(0.0189)	(0.0170)	(0.0171)	(0.0191)	(0.0193)	(0.0241)
SMB	0.2435***	0.0838***	−0.0202	−0.0596***	−0.0211	0.0009	−0.0330*	0.0660***
	(0.0274)	(0.0227)	(0.0190)	(0.0172)	(0.0173)	(0.0192)	(0.0195)	(0.0243)
LIQ	0.0706***	0.1034***	0.0682***	0.0293*	−0.0264	0.0125	0.0267	0.1051***
	(0.0255)	(0.0211)	(0.0177)	(0.0160)	(0.0161)	(0.0179)	(0.0181)	(0.0226)
MOM	−0.1377***	0.0022	−0.0354***	0.0048	−0.0321***	−0.0228**	−0.0189	−0.0840***
	(0.0165)	(0.0137)	(0.0115)	(0.0104)	(0.0104)	(0.0116)	(0.0118)	(0.0146)
Adj. R^2	0.4039	0.4194	0.4783	0.5826	0.6167	0.5779	0.5489	0.4924

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table A.7: First-step regressions of liquidity portfolios

Results from first-step OLS regressions run on eight different liquidity portfolios.

A: CAPM

	1(High)	2	3	4	5	6	7	8(Low)
Constant	-0.0001^{***}	-0.0002	0.0003^{*}	0.0002^{*}	0.0004^{**}	0.0005^{**}	0.0005^{***}	0.0008^{***}
	(0.0000)	(0.0001)	(0.0002)	(0.0001)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
eR_m	1.0805^{***}	0.9190^{***}	0.7514^{***}	0.5463^{***}	0.5334^{***}	0.4057^{***}	0.2507^{***}	0.1342^{***}
	(0.0029)	(0.0092)	(0.0107)	(0.0100)	(0.0127)	(0.0134)	(0.0118)	(0.0149)
Adj. R^2	0.9659	0.6639	0.4946	0.3716	0.2607	0.1551	0.0824	0.0157

$^{***} p < 0.01$, $^{**} p < 0.05$, $^{*} p < 0.1$

B: FF3

	1(High)	2	3	4	5	6	7	8(Low)
Constant	-0.0000	-0.0003^{*}	0.0002	0.0001	0.0001	0.0002	0.0003^{*}	0.0006^{***}
	(0.0000)	(0.0001)	(0.0001)	(0.0001)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
eR_m	1.0288^{***}	0.9580^{***}	0.8504^{***}	0.6819^{***}	0.7340^{***}	0.6459^{***}	0.4406^{***}	0.3054^{***}
	(0.0038)	(0.0127)	(0.0145)	(0.0135)	(0.0168)	(0.0177)	(0.0157)	(0.0202)
SMB	-0.0924^{***}	0.0725^{***}	0.1837^{***}	0.2228^{***}	0.3556^{***}	0.4125^{***}	0.3216^{***}	0.2741^{***}
	(0.0043)	(0.0143)	(0.0165)	(0.0153)	(0.0191)	(0.0200)	(0.0178)	(0.0228)
HML	0.0065^{**}	-0.0144	-0.0353^{***}	0.0479^{***}	-0.0161	0.0252^{*}	0.0352^{***}	0.0843^{***}
	(0.0030)	(0.0099)	(0.0114)	(0.0106)	(0.0132)	(0.0139)	(0.0124)	(0.0159)
Adj. R^2	0.9690	0.6659	0.5094	0.3973	0.3117	0.2220	0.1383	0.0446

$^{***} p < 0.01$, $^{**} p < 0.05$, $^{*} p < 0.1$

B: FF5

	1(High)	2	3	4	5	6	7	8(Low)
Constant	-0.0001	-0.0002	0.0002	0.0001	0.0002	0.0002	0.0003^{*}	0.0006^{***}
	(0.0000)	(0.0001)	(0.0001)	(0.0001)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
eR_m	1.0301^{***}	0.9450^{***}	0.8407^{***}	0.6729^{***}	0.7234^{***}	0.6497^{***}	0.4344^{***}	0.3031^{***}
	(0.0038)	(0.0126)	(0.0146)	(0.0135)	(0.0169)	(0.0176)	(0.0158)	(0.0203)
SMB	-0.0935^{***}	0.0736^{***}	0.1801^{***}	0.2223^{***}	0.3562^{***}	0.4224^{***}	0.3217^{***}	0.2746^{***}
	(0.0042)	(0.0142)	(0.0164)	(0.0153)	(0.0190)	(0.0198)	(0.0178)	(0.0229)
HML	0.0092^{***}	-0.0260^{***}	-0.0374^{***}	0.0418^{***}	-0.0251^{*}	0.0134	0.0303^{**}	0.0818^{***}
	(0.0030)	(0.0099)	(0.0114)	(0.0106)	(0.0132)	(0.0138)	(0.0124)	(0.0160)
RMW	0.0224^{***}	-0.1248^{***}	-0.0521^{***}	-0.0744^{***}	-0.0987^{***}	-0.0593^{***}	-0.0558^{***}	-0.0247
	(0.0031)	(0.0105)	(0.0121)	(0.0113)	(0.0140)	(0.0146)	(0.0132)	(0.0169)
CMA	-0.0100^{***}	-0.0103	-0.0553^{***}	-0.0211^{**}	-0.0115	0.1130^{***}	-0.0088	0.0017
	(0.0029)	(0.0096)	(0.0111)	(0.0103)	(0.0128)	(0.0134)	(0.0120)	(0.0155)
Adj. R^2	0.9694	0.6750	0.5132	0.4026	0.3182	0.2358	0.1411	0.0447

$^{***} p < 0.01$, $^{**} p < 0.05$, $^{*} p < 0.1$

C: eRm, SMB, LIQ, MOM

	1(High)	2	3	4	5	6	7	8(Low)
Constant	-0.0000	-0.0003^{**}	0.0001	0.0000	0.0001	0.0002	-0.0001	0.0001
	(0.0000)	(0.0001)	(0.0001)	(0.0001)	(0.0002)	(0.0002)	(0.0001)	(0.0002)
eR_m	0.9922^{***}	0.9891^{***}	0.9052^{***}	0.7336^{***}	0.8016^{***}	0.7756^{***}	0.9227^{***}	0.8123^{***}
	(0.0043)	(0.0149)	(0.0171)	(0.0159)	(0.0198)	(0.0204)	(0.0140)	(0.0198)
SMB	-0.0730^{***}	0.0612^{***}	0.1640^{***}	0.1669^{***}	0.3225^{***}	0.3259^{***}	0.0049	-0.0828^{***}
	(0.0044)	(0.0150)	(0.0172)	(0.0160)	(0.0199)	(0.0205)	(0.0141)	(0.0200)
LIQ	-0.0565^{***}	0.0434^{***}	0.0701^{***}	0.1107^{***}	0.1023^{***}	0.2275^{***}	0.8097^{***}	0.8770^{***}
	(0.0041)	(0.0140)	(0.0160)	(0.0149)	(0.0186)	(0.0191)	(0.0131)	(0.0186)
MOM	0.0038	-0.0187^{**}	0.0493^{***}	0.0083	-0.0175	-0.1056^{***}	0.0034	0.0186
	(0.0026)	(0.0091)	(0.0104)	(0.0096)	(0.0120)	(0.0124)	(0.0085)	(0.0121)
Adj. R^2	0.9702	0.6666	0.5124	0.4015	0.3159	0.2537	0.5107	0.3346

$^{***} p < 0.01$, $^{**} p < 0.05$, $^{*} p < 0.1$

Table A.8: First-step regressions of momentum portfolios

Results from first-step OLS regressions run on eight different momentum portfolios.

A: CAPM

	1(Low)	2	3	4	5	6	7	8(High)
Constant	−0.0008**	−0.0006***	−0.0003	−0.0002*	0.0000	−0.0002	−0.0000	0.0005**
	(0.0003)	(0.0002)	(0.0002)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0002)
eR_m	1.0636***	0.9821***	0.9782***	0.9243***	0.9029***	0.9033***	0.9561***	1.1072***
	(0.0229)	(0.0150)	(0.0125)	(0.0106)	(0.0100)	(0.0093)	(0.0098)	(0.0133)
Adj. R^2	0.3009	0.4610	0.5494	0.6040	0.6200	0.6549	0.6536	0.5809

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

B: FF3

	1(Low)	2	3	4	5	6	7	8(High)
Constant	−0.0010***	−0.0007***	−0.0002	−0.0002	0.0000	−0.0002	−0.0000	0.0004*
	(0.0003)	(0.0002)	(0.0002)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0002)
eR_m	1.2532***	1.0293***	0.9471***	0.8854***	0.8707***	0.8989***	0.9534***	1.2004***
	(0.0312)	(0.0206)	(0.0172)	(0.0145)	(0.0137)	(0.0127)	(0.0135)	(0.0179)
SMB	0.2692***	0.0574**	−0.0700***	−0.0792***	−0.0569***	0.0015	0.0103	0.1968***
	(0.0353)	(0.0233)	(0.0194)	(0.0164)	(0.0155)	(0.0144)	(0.0153)	(0.0203)
HML	0.2088***	0.0844***	0.0521***	0.0378***	0.0021	−0.0307***	−0.0501***	−0.1131***
	(0.0245)	(0.0162)	(0.0135)	(0.0114)	(0.0108)	(0.0100)	(0.0106)	(0.0141)
Adj. R^2	0.3157	0.4639	0.5523	0.6072	0.6209	0.6555	0.6552	0.5966

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

B: FF5

	1(Low)	2	3	4	5	6	7	8(High)
Constant	−0.0009***	−0.0006***	−0.0002	−0.0002	0.0000	−0.0002	0.0000	0.0004**
	(0.0003)	(0.0002)	(0.0002)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0002)
eR_m	1.2292***	1.0165***	0.9400***	0.8821***	0.8713***	0.9002***	0.9502***	1.1905***
	(0.0309)	(0.0206)	(0.0172)	(0.0145)	(0.0138)	(0.0128)	(0.0136)	(0.0180)
SMB	0.2792***	0.0608***	−0.0681***	−0.0772***	−0.0571***	0.0038	0.0105	0.1946***
	(0.0348)	(0.0232)	(0.0194)	(0.0164)	(0.0156)	(0.0144)	(0.0153)	(0.0203)
HML	0.1757***	0.0695***	0.0439***	0.0323***	0.0029	−0.0332***	−0.0529***	−0.1174***
	(0.0243)	(0.0161)	(0.0135)	(0.0114)	(0.0108)	(0.0101)	(0.0107)	(0.0141)
RMW	−0.3048***	−0.1455***	−0.0795***	−0.0484***	0.0077	−0.0107	−0.0305***	−0.0671***
	(0.0257)	(0.0171)	(0.0143)	(0.0121)	(0.0115)	(0.0107)	(0.0113)	(0.0150)
CMA	0.0661***	0.0152	0.0079	0.0163	−0.0012	0.0271***	−0.0035	−0.0411***
	(0.0235)	(0.0156)	(0.0131)	(0.0111)	(0.0105)	(0.0097)	(0.0103)	(0.0137)
Adj. R^2	0.3358	0.4716	0.5549	0.6085	0.6208	0.6560	0.6556	0.5987

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

C: eRm, SMB, LIQ, MOM

	1(Low)	2	3	4	5	6	7	8(High)
Constant	−0.0005*	−0.0003*	−0.0000	−0.0001	0.0001	−0.0002*	−0.0002	0.0001
	(0.0003)	(0.0002)	(0.0002)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0002)
eR_m	1.2186***	1.0336***	0.9259***	0.8765***	0.8853***	0.9227***	0.9698***	1.2944***
	(0.0321)	(0.0210)	(0.0191)	(0.0168)	(0.0161)	(0.0149)	(0.0152)	(0.0199)
SMB	0.2345***	0.0440**	−0.0657***	−0.0851***	−0.0633***	−0.0022	0.0126	0.1755***
	(0.0324)	(0.0212)	(0.0193)	(0.0170)	(0.0162)	(0.0150)	(0.0153)	(0.0201)
LIQ	0.0649**	0.0582***	−0.0036	0.0072	0.0257*	0.0216	−0.0024	0.0893***
	(0.0302)	(0.0197)	(0.0180)	(0.0158)	(0.0151)	(0.0140)	(0.0143)	(0.0187)
MOM	−0.7842***	−0.5216***	−0.2828***	−0.1203***	−0.0635***	0.0667***	0.2046***	0.3108***
	(0.0195)	(0.0128)	(0.0116)	(0.0102)	(0.0098)	(0.0091)	(0.0092)	(0.0121)
Adj. R^2	0.4755	0.5963	0.5984	0.6168	0.6242	0.6586	0.6846	0.6401

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table A.9: First-step regressions of β portfolios

Results from first-step OLS regressions run on eight different β portfolios.

A: CAPM

	1(Low)	2	3	4	5	6	7	8(High)
Constant	0.0005***	0.0003*	0.0003**	0.0003*	0.0000	−0.0001	−0.0001	−0.0005***
	(0.0002)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0002)
eR_m	0.1578***	0.2829***	0.3927***	0.5764***	0.7457***	0.9169***	1.1636***	1.3201***
	(0.0108)	(0.0107)	(0.0095)	(0.0107)	(0.0107)	(0.0091)	(0.0099)	(0.0111)
Adj. R^2	0.0408	0.1222	0.2555	0.3688	0.4933	0.6700	0.7362	0.7406

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

B: FF3

	1(Low)	2	3	4	5	6	7	8(High)
Constant	0.0004**	0.0002	0.0002	0.0001	−0.0000	−0.0000	−0.0000	−0.0004***
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0002)
eR_m	0.2752***	0.3728***	0.4863***	0.6944***	0.7784***	0.8830***	1.1230***	1.2746***
	(0.0146)	(0.0146)	(0.0129)	(0.0144)	(0.0147)	(0.0125)	(0.0135)	(0.0151)
SMB	0.1932***	0.1473***	0.1513***	0.2074***	0.0403**	−0.0633***	−0.0489***	−0.0933***
	(0.0166)	(0.0165)	(0.0146)	(0.0163)	(0.0166)	(0.0141)	(0.0152)	(0.0171)
HML	0.0404***	0.0332***	0.0413***	−0.0042	0.0562***	0.0134	−0.0741***	0.0460***
	(0.0115)	(0.0115)	(0.0101)	(0.0113)	(0.0115)	(0.0098)	(0.0106)	(0.0119)
Adj. R^2	0.0661	0.1358	0.2716	0.3893	0.4958	0.6715	0.7388	0.7433

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

B: FF5

	1(Low)	2	3	4	5	6	7	8(High)
Constant	0.0004**	0.0002	0.0002	0.0002	0.0000	−0.0000	−0.0000	−0.0004**
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0002)
eR_m	0.2698***	0.3683***	0.4840***	0.6938***	0.7748***	0.8812***	1.1192***	1.2651***
	(0.0147)	(0.0147)	(0.0130)	(0.0144)	(0.0147)	(0.0125)	(0.0135)	(0.0151)
SMB	0.1913***	0.1486***	0.1520***	0.2139***	0.0400**	−0.0656***	−0.0487***	−0.0925***
	(0.0166)	(0.0165)	(0.0146)	(0.0162)	(0.0166)	(0.0141)	(0.0153)	(0.0170)
HML	0.0391***	0.0279**	0.0386***	−0.0141	0.0539***	0.0153	−0.0772***	0.0375***
	(0.0116)	(0.0115)	(0.0102)	(0.0113)	(0.0116)	(0.0099)	(0.0106)	(0.0119)
RMW	−0.0304**	−0.0523***	−0.0263**	−0.0650***	−0.0286**	0.0049	−0.0345***	−0.0917***
	(0.0122)	(0.0122)	(0.0108)	(0.0120)	(0.0123)	(0.0104)	(0.0113)	(0.0126)
CMA	−0.0296***	0.0058	0.0026	0.0681***	−0.0100	−0.0270***	−0.0043	−0.0080
	(0.0112)	(0.0112)	(0.0099)	(0.0110)	(0.0112)	(0.0095)	(0.0103)	(0.0115)
Adj. R^2	0.0681	0.1387	0.2722	0.3978	0.4962	0.6719	0.7392	0.7459

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

C: eRm, SMB, LIQ, MOM

	1(Low)	2	3	4	5	6	7	8(High)
Constant	0.0002	0.0001	0.0001	0.0001	−0.0000	0.0000	−0.0000	−0.0003**
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0002)
eR_m	0.4593***	0.4975***	0.5522***	0.7383***	0.7969***	0.8581***	1.1350***	1.2449***
	(0.0166)	(0.0169)	(0.0151)	(0.0169)	(0.0173)	(0.0147)	(0.0159)	(0.0177)
SMB	0.0595***	0.0542***	0.0907***	0.1855***	0.0030	−0.0531***	−0.0184	−0.0916***
	(0.0167)	(0.0170)	(0.0152)	(0.0170)	(0.0174)	(0.0148)	(0.0160)	(0.0178)
LIQ	0.3240***	0.2225***	0.1306***	0.0704***	0.0612***	−0.0333**	−0.0202	−0.0228
	(0.0155)	(0.0159)	(0.0141)	(0.0159)	(0.0162)	(0.0138)	(0.0149)	(0.0166)
MOM	−0.0064	−0.0084	−0.0066	−0.0570***	−0.0253**	−0.0229**	−0.0324***	−0.0974***
	(0.0101)	(0.0103)	(0.0092)	(0.0103)	(0.0105)	(0.0089)	(0.0097)	(0.0107)
Adj. R^2	0.1387	0.1672	0.2814	0.3954	0.4953	0.6721	0.7369	0.7467

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

A.4 Additional risk premium estimations

Table A.10: Factor premiums for CAPM

Two-step GMM -estimation of the CAPM. Moment condition for first step is $E[\epsilon_t] = 0$ and $E[f_t\epsilon_t] = 0$, while second step uses estimated factor loadings to solve moment condition $E[\alpha] = 0$. The J-test is measuring the size of the pricing error from the second step.

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
$\lambda(eR_m)$	0.0003	0.0003	0.0004**	0.0004*	0.0003	0.0004*	0.0002
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
J(χ^2(7))	31.9533	6.0105	5.5835	3.9412	38.8921	16.6176	25.2492
p_value	0.00004	0.5385	0.5891	0.7865	0.000002	0.0200	0.0007

$^{***} p < 0.01$, $^{**} p < 0.05$, $^{*} p < 0.1$

Table A.11: Factor premiums for FF3

Two-step GMM -estimation of the FF3. Moment condition for first step is $E[\epsilon_t] = 0$ and $E[f_t\epsilon_t] = 0$, while second step uses estimated factor loadings to solve moment condition $E[\alpha] = 0$. The J-test is measuring the size of the pricing error from the second step.

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
$\lambda(eR_m)$	0.0004**	0.0004*	0.0004*	0.0004*	0.0004**	0.0003	0.0004*
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
$\lambda(SMB)$	0.0005**	−0.0000	−0.0010	0.0000	0.0010***	0.0000	0.0018***
	(0.0002)	(0.0010)	(0.0011)	(0.0007)	(0.0003)	(0.0007)	(0.0004)
$\lambda(HML)$	0.0029**	0.0003	−0.0004	0.0012	0.0025	−0.0039***	−0.0008
	(0.0012)	(0.0004)	(0.0014)	(0.0012)	(0.0016)	(0.0011)	(0.0013)
J(χ^2(5))	1.2449	4.1870	4.2818	2.5821	7.2793	4.4941	3.4818
p_value	0.9405	0.5228	0.5096	0.7641	0.2007	0.4807	0.6262

$^{***} p < 0.01$, $^{**} p < 0.05$, $^{*} p < 0.1$

Table A.12: Factor premiums for model: eRm,SMB,LIQ

Two-step GMM -estimation of three-factor model containing eRm, SMB and LIQ. Moment condition for first step is $E[\epsilon_t] = 0$ and $E[f_t\epsilon_t] = 0$, while second step uses estimated factor loadings to solve moment condition $E[\alpha] = 0$. The J-test is measuring the size of the pricing error from the second step.

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
$\lambda(eR_m)$	0.0004**	0.0003	0.0004**	0.0003	0.0004**	0.0004*	0.0003
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
$\lambda(SMB)$	0.0005	−0.0005	−0.0003	−0.0004	0.0007**	0.0006	0.0013*
	(0.0003)	(0.0012)	(0.0016)	(0.0009)	(0.0004)	(0.0010)	(0.0007)
$\lambda(LIQ)$	0.0008	0.0004	−0.0015	0.0012	0.0004*	−0.0010	0.0008*
	(0.0005)	(0.0009)	(0.0012)	(0.0017)	(0.0002)	(0.0029)	(0.0004)
J(χ^2(5))	5.5725	5.3797	4.0374	3.0704	6.9158	15.6978	2.6205
p_value	0.3501	0.3713	0.5440	0.6891	0.2270	0.0078	0.7583

$^{***} p < 0.01$, $^{**} p < 0.05$, $^{*} p < 0.1$

Table A.13: Factor premiums for model: eRm,LIQ,MOM

Two-step GMM -estimation of three-factor model containing eRm, LIQ and MOM. Moment condition for first step is $E[\epsilon_t] = 0$ and $E[f_t \epsilon_t] = 0$, while second step uses estimated factor loadings to solve moment condition $E[\alpha] = 0$. The J-test is measuring the size of the pricing error from the second step.

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
$\lambda(eR_m)$	0.0004^{**}	0.0004^{*}	0.0004^{**}	0.0004^{*}	0.0004^{*}	0.0003	0.0004^{*}
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0003)
$\lambda(LIQ)$	0.0014^{***}	0.0006	−0.0012	0.0009	0.0005^{**}	0.0009	0.0011^{***}
	(0.0003)	(0.0009)	(0.0014)	(0.0012)	(0.0002)	(0.0012)	(0.0004)
$\lambda(MOM)$	0.0030^{**}	0.0007	0.0002	0.0016	0.0002	0.0010^{***}	0.0024
	(0.0015)	(0.0008)	(0.0018)	(0.0019)	(0.0014)	(0.0003)	(0.0027)
$J(\chi^2(5))$	2.2948	4.8150	4.0379	2.7103	11.2782	4.4859	3.9612
p_value	0.8070	0.4389	0.5440	0.7445	0.0461	0.4818	0.5550

$^{***} p < 0.01$, $^{**} p < 0.05$, $^{*} p < 0.1$

Table A.14: Factor premiums for model: eRm,SMB,MOM

Two-step GMM -estimation of three-factor model containing eRm, SMB and MOM. Moment condition for first step is $E[\epsilon_t] = 0$ and $E[f_t \epsilon_t] = 0$, while second step uses estimated factor loadings to solve moment condition $E[\alpha] = 0$. The J-test is measuring the size of the pricing error from the second step.

	Size port	B/M port	OP port	INV port	Spread port	Mom. port	Beta port
$\lambda(eR_m)$	0.0004^{*}	0.0004^{*}	0.0004^{*}	0.0004^{*}	0.0005^{**}	0.0004^{*}	0.0006^{**}
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0003)
$\lambda(SMB)$	0.0008^{***}	0.0002	−0.0009	0.0007	0.0013^{***}	0.0003	0.0019^{***}
	(0.0002)	(0.0010)	(0.0012)	(0.0010)	(0.0003)	(0.0006)	(0.0004)
$\lambda(MOM)$	0.0010	0.0004	0.0007	0.0021	0.0022	0.0009^{***}	0.0042
	(0.0014)	(0.0008)	(0.0013)	(0.0024)	(0.0015)	(0.0003)	(0.0027)
$J(\chi^2(5))$	7.3224	5.6443	4.1512	2.9888	8.7504	5.0238	1.1507
p_value	0.1977	0.3424	0.5279	0.7017	0.1194	0.4130	0.9495

$^{***} p < 0.01$, $^{**} p < 0.05$, $^{*} p < 0.1$