# Modifier adaptation for real-time optimization of a gas lifted well network

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**Abstract:** This work studies the steady-state optimization of a Gas Lift Oil Well Network. The optimization approach used is based on the methodology proposed by (Gao et al., 2016), which is a Modifier Adaptation (MA) with gradient estimation from fitted surfaces. The methodology uses plant data to locally approximate the cost and constraint functions of the plant by quadratic functions and then estimates the plant gradients based on the approximated functions. The proposed scheme is simulated using a phenomenological model of the oil well, which was introduced by (Krishnamoorthy et al., 2016). The optimization results showed that the MA scheme is able to increase production, reaching the plant optimum, despite the presence of plant-model mismatch without any constraint violations. Furthermore, it was able to provide good quadratic approximations of both cost and constraint functions.

## 1. INTRODUCTION

Optimal operation of complex chemical processes is an increasing necessity in face of more competitive global markets. For upstream petroleum industries, model-based optimization, such as Real-time Optimization (RTO), has become a valuable approach to improve daily operations, e.g. maximizing production rates or allocating gas injection to wells. This has been enabled through the increasing number of smart wells being comissioned. In the recent years, these wells have more downhole sensors and show an increasing potential for control applications (Foss, 2012).

Gas lift is an artificial lift method applied when the reservoir pressure is not high enough to lift the fluids from the reservoir to the surface economically. Gas is injected at the well bottom, which reduces fluid mixture density and decreases the pressure at the bottom of the well. As a consequence, the inflow from the reservoir increases. However, there is a trade-off. At first, increasing the gas lift flowrate to its maximum capacity increases well productivity, but larger gas injection flowrates increase frictional pressure drop, decreasing the well production. Therefore, it is necessary to find the optimal injection amount so that the production is maximized overall. Also, process constraints like availability of gas lift or well total gas production capacity need to be taken into account.

Hence, there exists an opportunity to apply RTO strategies in the system to maximize production and avoid constraint violations. Classical RTO approaches use measurements to update the process model (by estimating its parameters) and then optimizes this model in order to drive the plant to an optimal performance, while satisfying the constraints. However, if the model is not an accurate representation of the plant, the optimal operation point calculated by the RTO does not generally coincide with the optimal operation point for the plant (Mendoza et al., 2016). For example, the model can under- or overestimate the value of a particular constraint.

This phenomenon, called plant-model mismatch, is extensively reported in the process engineering literature and several RTO variants have emerged to deal with this problem. Among them, Modifier Adaptation (MA) methods are especially attractive because they guarantee that the model-based optimization problem reaches the plant optimum upon convergence even in the presence of plant-model mismatch (Marchetti et al., 2016).

The basic idea of MA methods is to use correction terms for the cost and constraint functions to update the plant model instead of estimating its parameters. The measurements of the constraints and cost function are used to: (1) calculate bias correction terms to the values of the constraint and cost function predicted by the model; and (2) estimate plant gradients, which are included as a gradient correction term in the model cost and constraint functions.

However, implementing MA methods in real situation can be a challenging problem due to the necessity of estimating the plant gradients (Marchetti et al., 2016). Since the gradient cannot be measured directly, obtaining reliable gradient estimates with noisy measurements can be a challenging task.

In the literature, several alternatives to estimate plant gradients are available (Marchetti et al., 2016). They are

divided in dynamic perturbation methods, which uses transient data, and steady-state (SS) perturbation methods that use only stationary data.

The most simple SS method to estimate steady-state gradients is the Finite-Difference Approximation (FDA) (François and Bonvin, 2013), in which each input is perturbed individually around the current operating point and the corresponding gradient element is measured after the process reaches steady-state. For noise-free process with few inputs, the FDA method provides sufficient accurate plant gradients estimates in acceptable time span. However, the method becomes inefficient for noise-contaminated processes. In addition, this strategy can lead to constraint violations if the operating point is close to a constraint (Mansour and Ellis, 2003).

As an alternative, (Gao et al., 2016) proposed a methodology that uses current and past operating points to obtain a local quadratic approximation of the cost and constraint functions. Afterwards, the quadratic model is used to calculate plant gradients. In comparison to finite-difference calculations, the quadratic approximation captures information from well-distributed points, improving the curvature information and decreasing the influence of noise.

The main contribution of this paper is the application of methodology proposed by (Gao et al., 2016) to a gas lift oil well network. To the authors best knowledge, applying this strategy to handle model uncertainty in oil production optimization has not been reported in the literature before. The results show that despite the presence of model mismatch, the MA scheme is able to drive the model-based optimization to the actual plant optimum in a few iterations.

In the next section, the gas lift oil well network is described and both the model and the optimization problem are discussed. Then the applied modifier adaptation scheme with the gradient estimation algorithm is presented in details. Next, the results of the optimization cycle are shown together with the ability of the proposed strategy to optimize the oil well network in face of plant-model mismatch. Finally, in the last section we present our conclusions.

## 2. GAS LIFT OIL WELL NETWORK MODEL

A simplified flowsheet of the process can be seen in Figure 1. The system can be divided in three sections:

(1) **Wells**: the well network is composed by two wells and each well is composed by two different sections, before and after gas injection;
(2) **Annulus**: a void between the actual product pipeline and the external tubing. Gas is injected in the annulus in order to increase the flow from reservoir;
(3) **Riser**: a pipeline system in which the gas/oil mixture is transported from the wells to the surface. This system includes a manifold that connects the wells to the main pipeline.

The steady-state behavior of the process is modeled as a nonlinear system of equations, which consists in mass balances for oil and gas and relations for calculating

density, flow and pressure. The model considers constant temperatures, frictional pressure drop, ideal gas behavior, and simple linear relations to calculate the reservoir outlet flows. The model results match the results obtained from commercial high fidelity simulators (Codas et al., 2016). For more details to the gas lift well model refer to (Krishnamoorthy et al., 2016).
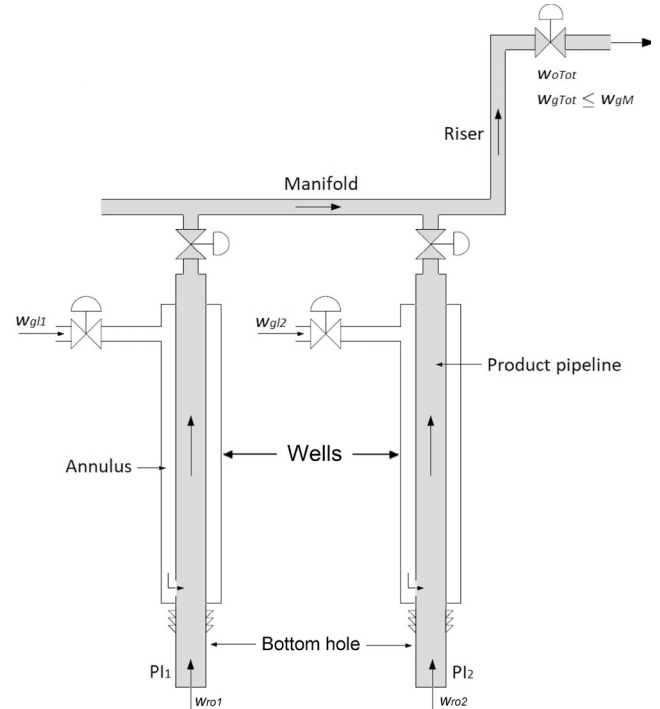


Fig. 1. Network containing two gas lifted wells. Adapted from: (Krishnamoorthy et al., 2016) (adapted).

The optimal operation point of the system is achieved maximizing the profit (i.e. maximizing oil production while minimizing the cost related to compression of the gas for artificial lifting). The optimization problem takes into account processing capacity constraints and constraints on the maximum gas lift flowrates. The model optimization problem for a network with 2 wells can be written as:

$$
\begin{aligned}
\max_{\mathbf{u}=[w_{gl,1},w_{gl,2}]^T} \quad & J := w_{oTot}^2 - 0.5 \sum_{i=1}^{2} w_{gl,i}^2 \\
\text{s. t.} \quad & C := \begin{bmatrix} w_{gTot} \\ w_{gl,1} \\ w_{gl,2} \end{bmatrix} \leq \begin{bmatrix} w_{gM} \\ w_{glM} \\ w_{glM} \end{bmatrix}
\end{aligned}
\tag{1}
$$

Here, $w_{oTot}$ is the total oil production and $w_{gTot}$ the total gas production of the well network; $w_{gl,i}$ are the gas lift flow rate of each well; $w_{gM}$ and $w_{glM}$ are the maximum gas processing capacity of the system and maximum gas lift flowrate for each well, respectively. The total oil and gas production of the well is calculated using the input-output mapping, $[w_{oTot}, w_{gTot}]^T = y(w_{gl,1}, w_{gl,2})$, which represents the steady-state behavior calculated by a model.

The steady-state problem has two degrees of freedom for optimization (i.e. $\mathbf{u} = [w_{gl,1}, w_{gl,2}]^T$): the mass flow rate of gas lift in each well. In order to consider plant-model
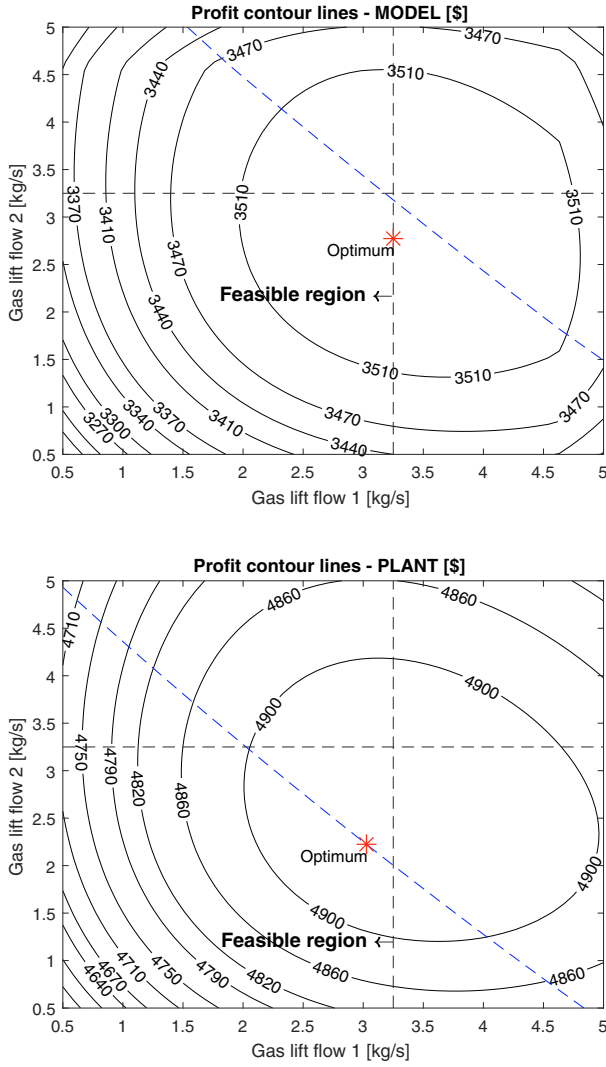
Fig. 2. Profit contour lines for the model and plant, the feasible region is indicated with an arrow. Dashed blue line is the maximum gas production constraint and dashed black lines are the maximum gas lift flow rate for each well

mismatch in the problem two models are developed. One for the plant and one for the optimization layer, which will be referred as "plant" and "model", respectively. The difference between them is the value of the productivity index (PI), which relates the oil flowrate leaving the reservoir, $w_{ro}$, with the difference between well bottom hole, $p_{bh}$, and reservoir, $p_r$ pressures, $w_{ro,i} = PI_i(p_r - p_{bh,i})$, $i = 1, 2$.

The value of $PI$ for the plant is $[7; 7]$ and for the model $[5; 5]$. The profit surface for the model and plant can be seen in Figure 2. The dashed blue line is the constraint of maximum gas production ($w_{gTot} \leq w_{gM}$) and the dashed black lines are the maximum gas lift flow rate for each well. Both constraints delimit the feasible region. The figures show that the model and the plant optimum are different and lie at a different set of constraints.

## 3. MODIFIER ADAPTATION AND GRADIENT ESTIMATION

The methodology applied in this paper was proposed by (Gao et al., 2016). In order to deal with the plant-model mismatch, Modifier Adaptation applies zeroth and first order modifiers (input-affine correction terms) for cost and constraint functions of the optimization problem in a way that, upon convergence, the model-based optimization problem reaches the plant (local) optimum. For more details regarding MA, we refer to (Marchetti et al., 2016).

The zeroth-order modifiers correspond to bias terms representing the differences between plant and model values at the current instant. The first order modifiers represent the difference between the plant gradients and the gradients predicted by the model (also at the current time instant).

Then, after applying the modifiers to the model optimization problem, the modified optimization problem is solved:

$$\begin{aligned}
u_{k+1}^\star = \underset{\mathbf{u}=[w_{gl,1}, w_{gl,2}]^T}{\operatorname{argmax}} \quad & J_{mod} := J(\mathbf{u}) + \lambda_{J,k}^T \mathbf{u} \\
\text{s. t.} \quad & \\
C_{mod} := C(\mathbf{u}) + \epsilon_{C,k} &+ \lambda_{C,k}^T \left( \mathbf{u} - \begin{bmatrix} w_{gl,1,k} \\ w_{gl,2,k} \end{bmatrix} \right)
\end{aligned} \tag{2}$$

where, $J$, $C$ and $w_{gl,i}$ were previously defined in (1). $\lambda_{J,k}$, $\epsilon_{C,k}$ and $\lambda_{C,k}$ are the modifiers that are adjusted to make the optimized model and plant coincide. The subscript $k$ indicates that the values are evaluated at the $k$th iteration. Also, $w_{gl,k}$ are the inputs applied to the system at the $k$th iteration. MA is able to correct the predicted value by the model and also the model surface near the current operation point. It is important to notice that the modifiers are valid only locally, hence the MA schemes needs some amount of filtering to avoid overaggressive corrections that may destabilize the system. The modifiers are defined as:

$$\begin{aligned}
\epsilon_{C,k} &= (I - K_{\epsilon,C})\epsilon_{C,k-1} + K_{\epsilon,C}(C_{p,k} - C_k) \\
\lambda_{C,k} &= (I - K_{\lambda,C})\lambda_{C,k-1} + K_{\lambda,C}(\widehat{\nabla C}_{p,k} - \nabla C_k) \\
\lambda_{J,k} &= (I - K_{\lambda,J})\lambda_{J,k-1} + K_{\lambda,J}(\widehat{\nabla J}_{p,k} - \nabla J_k)
\end{aligned} \tag{3}$$

where, $I$ is the identity matrix of appropriate dimension; $K_{\epsilon,C}$, $K_{\lambda,C}$ and $K_{\lambda,J}$ are are square matrices with values between $[0, 1)$ on the main diagonal and zeros elsewhere; $C_k$, $\nabla C_k$ and $\nabla J_k$ are the model constraints and derivatives of the constraints and cost with respect to $w_{gl,i}$ at the current point; $\widehat{\nabla C}_p$ and $\widehat{\nabla J}_p$ are the estimates of the plant gradients. The plant constraint value, $C_p$, is directly measured.

The modified problem (2) is solved to give the optimal computed input $u_{k+1}^\star$. Then, an input filter, $u_{k+1} = u_k + K_u(u_{k+1}^\star - u_k)$, is used to mitigate the effect of noise and errors in the gradient predictions. Small values of the filter gain represent more conservative update strategy. If the modifiers are unreliable (much noise), it is better to avoid large changes in the setpoints in order to not deteriorate plant performance.

The idea of using a quadratic approximation for gradient estimation comes from interpolation-based trust region methods, which present a relatively fast rate of conver-

gence and also a reliable convergence even in the presence of noisy data (Conn et al., 2009). The main idea of these methods is to construct local quadratic approximations of the cost functions in order to provide a data-based model for the optimization step.

Based on this idea, (Gao et al., 2016) proposed the combination of the quadratic approximation (to estimate plant gradients) with the modifier adaptation method. The step-by-step description of the algorithm can be seen in Figure 3. Note that, before using quadratic approximation to estimate the gradients, the method needs that $n_r = (n_u + 1)(n_u+2)/2-1$ measurements are available (with $n_u$ equals to the number of inputs of the problem). This requirement is imposed due to the number of coefficients that needs to be estimated in the multidimensional quadratic function.

In order to obtain these measurements, finite-difference approximation (FDA) is applied $n_u$ times to obtain the gradient estimates. Afterwards, iterative gradient-modification optimization and gradient estimation (IGMO) is applied until $n_r$ measurements have been generated. IGMO is described in details in (Gao and Engell, 2005). Basically, it optimizes the plant using MA and estimates plant gradients by a different implementation of FDA. This FDA variant uses setpoints of the past iterations to evaluate the gradients as shown below:

$$\widehat{\nabla C}_{p,k} = [S_k]^{-1} \cdot [(C_{p,k} - C_{p,k-1}) \ldots (C_{p,k} - C_{p,k-n_u})]$$
$$\widehat{\nabla J}_{p,k} = [S_k]^{-1} \cdot [(J_{p,k} - J_{p,k-1}) \ldots (J_{p,k} - J_{p,k-n_u})] \tag{4}$$

However, in order to obtain good approximations for the gradients, the inverse of the input matrix $(S_k^{-1} = [(u_k - u_{k-1}), \ldots (u_k - u_{k-1})]^{-1}$ needs to be well-conditioned. If it is not, IGMO has an additional step that perturbs the process in order to improve the input matrix conditioning. Note that in comparison to the FDA, IGMO is not as expensive because it only perturbs the system when necessary.

After obtaining $n_r$ measurements, the gradients are computed by differentiating the quadratic approximations. Clearly, the gradient estimation error depends heavily on the quality of the approximation. Hence, the choice of the points (regression set) used in the estimation of the quadratic approximation is a critical issue of the algorithm. To guarantee that the gradient estimation algorithm uses well-distributed points, the set of all the current and past operation points, which is represented by $U$, is analyzed and divided in three subsets:

(1) $U_{nb}$: Neighborhood of the current point, which is defined by a tuning parameter $\Delta \mathbf{u}$, $U_{nb} := \{\mathbf{u} : \|\mathbf{u} - \mathbf{u}_{current}\| \leq \Delta \mathbf{u}; \mathbf{u} \in U\}$;
(2) $U_{dist}$: Set of points that are sufficiently distant from the current point, but well-distributed and sufficiently scattered to capture the curvature;
(3) $U_{outer}$: which is defined $U_{outer} = U \setminus U_{nb}, U_{dist}$.

Determining $U_{nb}$ is easy and straightforward. In turn, $U_{dist}$ is determined by an optimization problem, which can be seen in details in (Gao et al., 2016). The main idea is to determine at least $n_r$ points by a criterion that minimizes the distance from the current point, while penalizes using

points along the same direction. After obtaining the three subsets, $\mathcal{U} : U_{nb} \cup U_{dist}$ is used to estimate the quadratic approximation. In addition, the methodology proposed by (Gao et al., 2016) has some extra features, which are discussed below.

The methodology imposes a covariance-based constraint to the optimization problem in order to adjust the search space based on previous iterations, for example: if the past setpoints lie on a specific direction, the gradient is more reliably estimated along that direction. The parameter $\gamma$ defines the size of ellipsoid center at the current input, $u(k)$, $B(k) : (u - u(k))^T (cov(\mathcal{U}))^{-1}(u - u(k)) \leq \gamma^2$. Lower values of $\gamma$ avoid setpoints moves in directions in which less data has been collected before, avoiding unnecessary deterioration of the plant performance.

Also, (Gao et al., 2016) propose a switching mechanism between model-based and data-based optimization, which mitigates the effect of inadequate models by using the most accurate model. This enables fast convergence rates based in the model-based optimization, while retaining the accuracy of the data-based quadratic model. The switching is based on a deviation between the plant measurements and model predictions ($\rho_q$ for the quadratic approximation model and $\rho_m$ for the phenomenological model).

An additional step is used to improve the choice of the regression set (the block with colored background in Figure 3). It is related to analyze if the calculated optimum, $u^\star$ lies in the neighborhood of the current input. If so, the algorithm probes the plant in a different point, outside the neighborhood of the current point, in order to improve the quadratic approximation. Then, if the new probing point does not improve the plant cost function, the iteration is considered unsuccessful, the probing point is included in the regression set and the screening algorithm is re-initiated. This step shrinks the regression region and it is similar to the criticality step used in (Conn et al., 2009).

## 4. CASE STUDY SET-UP

The focus of this article is to study plant-model mismatch effects. Therefore, the following assumptions are made: both cost functions and constraints are measured without noise; and only the steady-state behavior of the plant is analyzed. Additionally, the only source of plant-model mismatch is parametric uncertainty due to the lack of knowledge regarding the well productivity index. The values of the model parameter values are not shown here for the sake of brevity. They can be found in (Krishnamoorthy et al., 2016). Table 1 shows the algorithm parameters, described in Section 3 .

The first parameter, perturbation step size, is used in the finite-difference approximation (FDA) to obtain the plant gradients at the initial point. As shown in Figure 3, FDA should be executed only at the first step because in this case study $n_u = 2$. This method requires that a small perturbation ($\Delta h$) is applied in each of the inputs. Then, the measurements of the actual plant are used to estimate the plant gradients. However, there is a trade-off regarding the step size. If it increases, the derivative approximation error increases. On the other hand, if the step size decreases, the errors relative to noise measurements increase
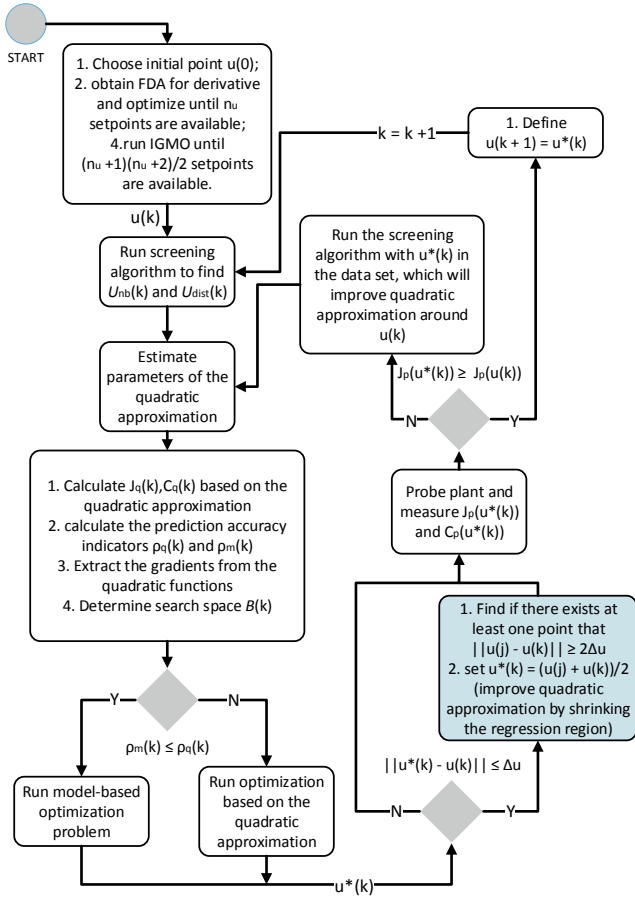
Fig. 3. Flowsheet of MA with gradient estimation via quadratic approximation. Adapted from: (Gao et al., 2016)

(Brekelmans et al., 2005). The value of $\Delta h$ was chosen in a trial-and-error process.

After the initial step, Iterative gradient-modification optimization and gradient estimation (IGMO) is used for calculating plant gradients and optimization. As explained, this approach relies on the conditioning of the inverse of the input matrix to calculate the gradients. If the condition number is lower than $\delta_{IGMO}$, an additional perturbation is introduced around the current setpoint. The third and fourth parameters are used in the screening process explained in Section 3. $\Delta u$ defines the size of the $U_{nb}$ set (past setpoints in the neighborhood of current setpoint) and $\gamma$ is the scaling parameter of the covariance-based constraint. The other parameters are the filters used in the zero- and first-order correction terms that are added to the cost and constraint function of the MA optimization problem. The filter values as well as the screening parameters are also chosen in a trial-and-error process.

## 5. RESULTS AND DISCUSSION

The results of the MA scheme with gradient estimation via quadratic approximation are shown in Figure 4. The figure shows the contour lines of the plant profit surface and the setpoints moves from the initial point, $u(0) = [1, 1]$ [kg/s], to the plant optimum. The setpoints are the solutions

Table 1. Parameters and variables for the scheme

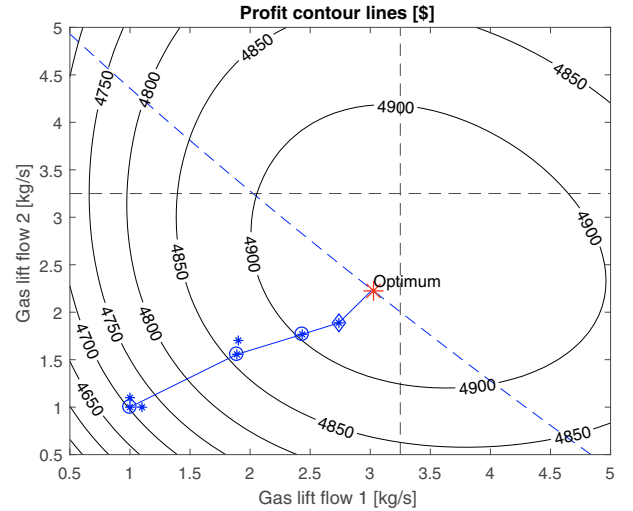| Description | Symbol | Value |
|---|---|---|
| Perturbation step size | $\Delta h$ | 0.1 |
| Conditioning number of input matrix | $\delta_{IGMO}$ | 0.1 |
| Search space parameter | $\gamma$ | 1.9 |
| Screening parameter | $\Delta u$ | 0.3 |
| Constraint bias modifier | $K_{\epsilon,C}$ | 0.7 |
| Constraint gradient modifier | $K_{\lambda,C}$ | 0.5 |
| Cost function gradient modifier | $K_{\lambda,J}$ | 0.5 |
| Input filter | $K_u$ | 0.4 |



Fig. 4. Set-point moves of a noise-free scenario. Circle points indicate setpoints calculated by model-based optimization, and diamond points by data-based optimization. Also, points indicated only with stars represent probing points for estimating plant gradients.

provided by the method described in Figure 3. In addition, around the points of the first and second iteration, the perturbation points to obtain the plant gradients using FDA and IGMO can be seen. The results confirm that the calculated optimum converged to the plant optimum in few iterations.

The next step is the analysis of the quality of the quadratic approximation of both profit and constraints, which can be seen in Figure 5. The figures show the contour lines of the surface generated by the difference between the actual plant constraint/profit surface and the the quadratic approximations at the optimum. The difference is obtained by "subtracting" the plant surface from the quadratic approximation. Note that, near the plant optimum there is no relevant difference between the actual plant surface and the quadratic approximation for both cost function and constraint. As the approximations are only locally accurate, the difference increases in regions that are more distant from the optimum.

In conclusion, despite the fact that the plant and model optima are different (as shown in Figure 2), the MA approach is able to reach the plant optimum, showing robustness to the gradient uncertainty, which comes from the deviation between the quadratic approximation and the plant surface. Furthermore, the optimization method
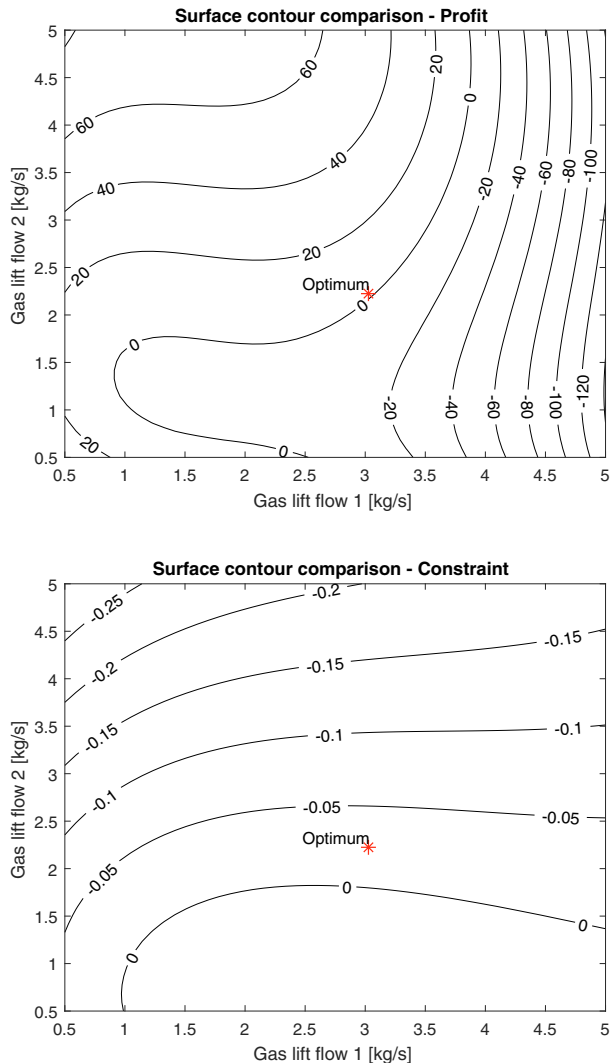
Fig. 5. Difference between the estimated surface (based on quadratic approximation at the optimum) and the actual plant surface. This shows how well the modified and the true profit/constraint match at the optimum.

is able to avoid constraint violations even with the faulty gradient information, which is important for practical applications.

## 6. CONCLUSION

In this work, the application of the methodology proposed by (Gao et al., 2016) in a gas lifted oil well network was studied. Plant-model mismatch was considered as parametric uncertainty regarding the production index of both wells. The simulation results shows the methodology was capable of maximizing the production of the oil well network and guaranteeing that, upon convergence, the model-based optimization problem matches the plant necessary conditions of optimality, reaching the plant optimum. Additionally, the constraints were not violated in any iteration.

The quadratic approximation of the plant cost function and gradient were also obtained successfully. In few iterations, the surfaces approximations were able to provide good estimates of the plant gradients, surpassing the main drawback of the MA-like schemes, which is obtaining plant gradient estimation. The use of quadratic approximation to estimate gradients for MA schemes appears as an interesting alternative to real-time optimization applications in gas lifted oil wells networks.

Further studies, which take noise and structural plant-model mismatch into account, are planned to provide further evidence that the methodology is well suited for handling different sources of plant-model mismatch in oil and gas production systems.

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