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Maximum Covering Location Approach for Solving a Coast Guard Deployment Problem

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Marine Technology

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Maximum Covering location approach for solving a Coast Guard Problem

Background

The coast guard fleet deployment problem is a complex problem. The problem involves determining the location of a coast guard fleet with different vessel capabilities to a specific operation. Measuring effectiveness on a coast guard deployment problem suffers from missions being of non-monetary value, and operation performance is often difficult to estimate due to the lack of operational data and mission requirements.

Facility location problems have been popular within operations research, in particular, the location of fire stations, police stations, and hospitals. One issue with facility locations is that it can be hard to measure the effectiveness of the performance. Although problems like these are often budget restricted which can be a measure of effectiveness, they are still somewhat suffering from being of non-monetary value, because evaluating the safety and welfare of the public can be challenging. A typical class of facility problems that have been used in solving these problems is covering problems. The covering location problem are often categorized as Maximum covering location problems (MCLP) and location set covering problem(LSCP), where the MCLP aim to allocate a set of facilities in a network to maximize coverage and the LSCP aim to find the minimum number of locations required, such that all demand nodes are covered. In facility location for emergency response problems the covered subject is often the population which can also be said about the coast guard. The coast guard problem has a lot of the modeling issues in common with emergency response problems. Hence, this report will address the problem of using covering problems for solving the coast guard fleet problem.

Objective

The objective for this thesis is to show how applied facility covering location models can be used to support decision makers in the optimization process of the coast guard problem. The thesis will focus on addressing the tactical planning problem, which for the coast guard problem are related to the deployment of resources. Establishing a deployment model for evaluating fleet effectiveness can contribute to further work on the operational coast guard problem. It shall be noted that this report presents a solution to the simplified problem, emphasizing on a theoretical study, and not a solution to the real problem.

Scope of work

The candidate should seek to cover the following points:



1. Perform a literature study, scoping on the application of facility location problems, in particular covering models.
2. Describe and present different mythologies for facility location problems that can be related to the coast guard deployment problem.
3. Present a thorough problem description, including the challenges concerning assumptions and data generation.
4. Develop a maximum covering location model for the coast guard planning problem, discuss the results and recommend further work

General

In the thesis the candidate shall present his personal contribution to the resolution of a problem within the scope of the thesis work.

Theories and conclusions should be based on a relevant methodological foundation that through mathematical derivations and/or logical reasoning identify the various steps in the deduction.

The candidate should utilize the existing possibilities for obtaining relevant literature.

The thesis should be organized in a rational manner to give a clear statement of assumptions, data, results, assessments, and conclusions. The text should be brief and to the point, with a clear language. Telegraphic language should be avoided.

The thesis shall contain the following elements: A text defining the scope, preface, list of contents, summary, main body of thesis, conclusions with recommendations for further work, list of symbols and acronyms, reference and (optional) appendices. All figures, tables and equations shall be numerated.

The supervisor may require that the candidate, in an early stage of the work, present a written plan for the completion of the work. The original contribution of the candidate and material taken from other sources shall be clearly defined. Work from other sources shall be properly referenced using an acknowledged referencing system.

The work shall follow the guidelines given by NTNU for the MSc Thesis work. The work load shall be in accordance with 30 ECTS, corresponding to 100% of one semester.

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- Computer code, input files, videos and other electronic appendages can be uploaded in a zip-file in DAIM. Any electronic appendages shall be listed in the thesis.

Supervision:

Main supervisor: Prof. Bjørn Egil Asbjørnslett

Co-supervisor: Ph.D Sigurd Solheim Pettersen

Deadline: 24.06.2018

Preface

This thesis is a part of my Master of Science degree in Marine Technology with specialization in Marine system design and logistics. The work has been written during the spring of 2018 and corresponds to 30 ECTS.

The work in this thesis has given an insight in the world of facility location within operations research, and how a maximum covering location approach can be applied to the coast guard planning problem. The focus in this thesis has been on the tactical coast guard deployment problem, which has had limited previous research. The result in this thesis does not provide a recommended solution to the deployment problem. However, it shows the potential in a maximum covering approach if better data can be developed.

Trondheim, June 21, 2018



Emil Benthien Høie

Acknowledgments

I would like to thank several people for their help and guidance through the process of writing this thesis. First, I would like to thank my main supervisor Bjørn Egil Asbjørnslett for constructive conversations, for guiding me around the complex problem that is coast guard planning, and for helping me to structure the report. Second, I would like to thank my co-supervisor Ph.D.Candidate Sigurd Solheim Pettersen for interesting discussions about the maximum covering location problem and providing me with central literature about the subject. I will also like to thank him for the excellent guidance with modeling issues in Xpress IVE, and for taking his time to read through some vital parts of the thesis

Finally, I would like to thank my office comrades for constructive conversations and for keeping up the spirits all the way to the end.

Thank you,

Emil Benthien Høie

Abstract

The purpose of this thesis is to see how a maximum covering location approach can be used for solving the coast guard deployment problem. The maximum covering location method has proven to be an effective method in facility location, especially within emergency response location, which is the reason for the interest in testing the applicability to the coast guard deployment problem.

The coast guard deployment problem is a complex problem. The problem involves determining the location of a coast guard fleet with different vessel capabilities to a specific operation. Measuring effectiveness on a coast guard deployment problem suffers from missions being of non-monetary value, and operation performance is often difficult to estimate due to the lack of operational data and mission requirements.

Through a performed literature review it became clear that one of the weaknesses with the classical maximal covering location problem is the notion of "all or nothing." A demand point is either covered or not. Therefore, a modified MCLP was constructed with weighted demand nodes and weighted neighbouring nodes to simulate the value of gradual coverage. In addition, a system for diminishing marginal return on demand nodes was created to simulate the effect of coverage by multiple vessels.

A computational study is presented with two main scenarios, one with a maximum response distance for each test vessel and two where maximum response time was defined for typical coast guard missions. The model is constrained within geographical boundaries of the waters under Norwegian jurisdiction. The results show that the model is able to locate feasible solutions for vessel deployment. In addition, show how testing a great variety of fleets, with global design constraints such as maximum cost or minimum utility, can support decisions makers in search of a suitable fleet within a design space area. The result also shows that computational time increases significantly with increased coverage radius or longer response time conditions.

The coast guard MCLP are faced with limitations and simplifications related to model and data discrepancies which can compromise the accuracy of the results. A further coast guard risk analysis is therefor advised. Also alternative objective functions such as the minimizing response time could be valuable for providing further insight into the coast guard planning problem.

Sammendrag

Formålet med denne oppgaven er å undersøke hvordan et maximum covering problem kan brukes for å løse et planleggingsproblem for kystvakten. Maximum covering problemer har vist seg å være en effektiv metode innen lokasjonsteori og operasjonsanalyse, og spesielt innen forskjellige sikkerhet og redningstjenester. Dette er årsaken til at man har fått interesse for hvordan dette også kan brukes for å se på det taktiske planlegging problemet rundt kystvakten. Taktisk planlegging er ofte forbundet med å bestemme hvilke operasjonelle lokasjoner som vil løse en bestemt målsetning best.

Å planlegge flåtedisponering for kystvakten er et komplekst problem. Problemet innebærer å bestemme lokasjonen til en flåte med fartøy som alle har forskjellige operasjonelle egenskaper og spesifikasjoner. Et av de vanskeligste problemene med flåte disponering er å evaluere hvor god løsningen er, fordi i motsetning til kommersielle flåtedisponeringsproblemer, som gjerne måler flåtens evne til å generere profitt, så er dette vanskelig for en kystvakt som har funksjon i å utføre oppdrag og ikke generere profitt. Mangelen på operasjonelle data om kystvakten er av naturlige årsaker også vanskelig å oppdrive, som bidrar ytterligere til kompleksiteten.

I denne oppgaven har det blitt utført et litteratur studium, hvor det ble klart at det originale maximum covering problemet hadde visse svakheter. Hvor en av de mest fremtredende var forenklingen om at enten så klarer man å dekke et oppdrag, ellers klarer man ikke å dekke et oppdrag. Dette førte til at et modifisert maximum covering problem ble generert, hvor operasjonelle områder som ligger i nærheten av et oppdragsområde blir gitt en verdi som gjør det mulig å vise verdien av å være i nærheten av et oppdrag men ikke klarer å dekke det. I tillegg så ble det generert et system som reduserer viktigheten av å dekke et oppdrag dersom det allerede er dekket av en enhet.

På bakgrunn av dette så ble det gjennomført et studie som fokuserte på 2 scenarier. I det ene så fikk hvert fartøy i flåten en bestemt maks distanse de kan dekke, og i det andre så fikk hvert oppdrag et krav til makstid innen det blir respondert til. For å enklere kunne visualisere problemet så ble problemet begrenset til norske farvann som i dag opereres av kystvakten. Resultatene viser

at å bruke en maximum covering modell gjør det mulig å finne mulige alternativer for hvordan man kan disponere en kystvakt flåte som tilfredsstillende visse krav. I tillegg, ved å teste modellen for et stort antall flåtekombinasjoner så kan man med enkle krav som budsjettbegrensinger eller krav til egenskaper finne et sett med flåter som kan brukes i videre planlegging og analyse. Resultatene viste også at kjøretiden til modellen ble nødt til å økes markant for å finne lovlig løsnings ved økt maksimal dekningsdistans for fartøyene.

Et av de største problemene med å bruke et maximum covering problem for et kystvakt planlegging problem er begrensinger og forenklinger gjort i forhold til data og datagenerering. Usikkerheten rundt denne dataen gjør at man ikke kan komme med en anbefaling til flåtedisponering. For å kunne bruke disse resultatene fra modellen krever det en grundig risiko analyse om oppdrag og oppdragstyper for kystvaken, for å kunne gi et bedre bilde av virkeligheten. I tillegg vil det være smart å se på alternative målfunksjoner som for eksempel å minimere responstiden til fartøyene. Dette vil kunne gi større innsikt og sammenlikningsgrunnlag for kystvakt problemet.

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Acronyms

DSS	=	Decision Support System
GA	=	Generic Algorithm
GIS	=	Geographic Information System
LSCP	=	Location Set Covering Problem
MCLP	=	Maximum Covering Location Problem
USD	=	United States Dollar
VRPSD	=	Vehicle Routing Problem with Stochastic Demand
LOA	=	Length Over All
CGV	=	Coast Guard Vessel

Chapter 1

Introduction

The coast guard fleet deployment problem is a complex problem. The problem involves determining the location of a coast guard fleet with different vessel capabilities to specific operations. Measuring effectiveness on a coast guard deployment problem suffers from missions being of non-monetary value, and operation performance is often difficult to estimate due to the lack of operational data and mission requirements.

Facility location problems have been popular within operations research, in particular, the location of fire stations, police stations, and hospitals. One issue with facility locations is that it can be hard to measure the effectiveness of the performance. Although problems like these are often budget restricted which can be a measure of effectiveness, they are still somewhat suffering from being of non-monetary value, because evaluating the safety and welfare of the public can be challenging. A typical class of facility problems that have been used in solving these problems is covering problems. The covering location problem are often categorized as Maximum covering location problems (MCLP) and location set covering problem(LSCP), where the MCLP aim to allocate a set of facilities in a network to maximize coverage and the LSCP aim to find the minimum number of locations required, such that all demand nodes are covered. In facility location for emergency response problems the covered subject is often the population which can also be said about the coast guard. The coast guard problem has a lot of the modeling issues in common with emergency response problems. Hence, this report will address the problem of using covering problems for solving the coast guard fleet deployment problem.

1.1 Objective

The objective for this thesis is to show how applied facility covering location models can be used to support decision makers in the optimization process of the coast guard problem. The thesis will focus on addressing the tactical planning problem, which for the coast guard problem are related to the deployment of resources. Establishing a deployment model for evaluating fleet effectiveness can contribute to further work on the operational coast guard problem. It shall be noted that this report presents a solution to the simplified problem, emphasizing on a theoretical and conceptual study, and not a solution to the real problem.

To assess the objectives, the following will be addressed:

1. Perform a literature study, scoping on the application of facility location problems, in particular covering models.
2. Describe and present different mythologies for facility location problems that can be related to the coast guard deployment problem.
3. Present a thorough problem description, including the challenges concerning assumptions and data generation.
4. Develop a maximum covering location model for the coast guard planning problem

This thesis has been developed in cooperation with [Pettersen et al. \(2017\)](#) which is working on an unpublished paper on the MCLP, and it is also a continuation of the work done by [Buland \(2017\)](#) about the coast guard fleet size and mix problem.

1.2 Background

The coast guard planning problem can be divided into three levels of planning: strategical-, tactical- and operational level of planning. Prior to this master thesis, a project thesis and a literature survey on the coast guard problem was conducted. Here we discovered that the research on the coast guard problem was limited and old. A paper by paper by [Darby-Dowman](#)

[et al. \(1995\)](#) lead to the path of researching the application of coast guard facility location problems for solving the tactical planning problem.

The strategic planning problem is concerned with long-term decisions that put massive pressure on the decision makers. The strategic planning problems are defined as problems that are affected in a period of 1 to 20 years. Fleet size and mix problems are typical strategic planning problems, where acquiring vessels is a substantial investment and can have a huge financial impact, making this an essential stage for decision makers. Concerning the coast guard, the master thesis of [Buland \(2017\)](#) he introduced a model with tradespace exploration and epoch-era analysis for developing fleet combinations and then use a value-centric perspective for choosing a fleet composition that would provide most satisfaction for the stakeholders. One of his primary issues was how the value of a fleet changed from different stakeholders in a Coast Guard fleet. He thoroughly performed a bottom-up design approach by contacting members of the Royal Norwegian Navy and did extensive research with a strategic level of planning. His results will serve as a basis for this research in solving the tactical planning problem.

Tactical level of planning is characterized by a planning period of weeks to months and years and involves decisions on how to utilize a network or area effectively to achieve the strategic objective. The problems include the planning of equipment management, resource allocation and utilization, and facility location. Facility location problems is a good example of how to solve the tactical planning problem, and for a coast guard, this can be reflected as a vessel deployment model. As mentioned above the focus of this thesis will be on the tactical planning problem, more specific, the maximum covering location problem.

Finally, we have the operational planning. Planning problems involving day to day operations and decisions are subject under this category. Dispatching response units and personnel are typical operational decisions, and operational planning is often required the time span is short, generally set to hours, days and minutes regarding response time. However, because of the time planning is short, operational level requires a lot of real data to provide significant results. In [Figure 1.1](#) is an illustrating the planning horizons for the different planning levels.



Figure 1.1: Pyramid showing the time scope of planning

1.3 Limitations

There are several limitations to this thesis, and one of the main limitations is the availability of relevant data. Because the Norwegian coast guard is a part of the Norwegian royal navy the access to data is restricted to open source material obtained mainly from [Kystverket](#), [Lovdata](#) and partly from the thesis of [Buland \(2017\)](#) and will be further described in chapter 2. This has resulted in the author making some design decisions regarding vessel capabilities, mission demand, and other data. However, it should be noted that the data selected is for experimental purposes only and is not related to the desirability of the Norwegian coast guard. The data used in this thesis is generated by the author, which is also one of the reasons why the aim of this thesis is not to come up with a definitive solution to the deployment problem, but rather present a conceptual design that shows the application of MCLP can be used for solving the coast guard problem.

1.3.1 Structure of report

This thesis will be structured as follows.

- Chapter 2 presents an introduction to the Norwegian coast guard. The chapter includes a description of the roles and tasks expected by the coast guard, the oceans of which the coast guard have jurisdiction and a description of the vessels operating today. This Chapter also gives a description of the marine traffic in operational waters to establish areas of interests when determining mission requirements.
- Chapter 3 provides a literature survey on facility location problems and other location problems of interest. The majority of the research papers addresses the covering problems, which is the key attention in this thesis. However, alternative papers are also reviewed. The chapter ends with some concluding remarks regarding the literature.
- Chapter 4 describes a brief introduction to the four most common facility location problems that are frequently mentioned in the literature survey, and that will be used in the thesis. Concluding remarks will also be addressed here with a short comparison of the selected methods.
- Chapter 5 presents a detailed problem description. This includes addressing the assumptions made to produce the data and the assumptions made in this thesis
- Chapter 6 present the model formulation and also, a short description of how data has been generated and preprocessed.
- Chapter 7 presents the result from the computational study, focusing on showing how MCLP has been used in the model.
- Chapter 8 provides a discussion of the results from the computational study. Furthermore, discrepancies within data and model formulation has been addressed with suggestions on how the study could be improved.
- Chapter 9 presents a conclusion of the work done in this thesis and a recommendation to further work

Chapter 2

The Norwegian coast guard

The purpose of this chapter is to give the reader a brief introduction to the Norwegian Coast Guard and the tasks and challenges that the Coast Guard must be able to handle. This chapter will also give an introduction to how the Norwegian Coast Guard fleet is structured today and a description of the different vessels and their operational capabilities.

2.1 Roles and tasks

The Norwegian Coast Guard is one of four departments within the Royal Norwegian navy including the Navy, the Coastal Ranger commando and the Norwegian Home Guard. The Coast Guards main purpose is to perform naval operations in Norwegian waters on behalf of the Norwegian Government.

The roles and tasks for the Norwegian Coast Guard are normally structured around peace-time scenarios where its primary objective is to maintain the Norwegian sovereignty in inland water, international water and in the exclusive economic zone (EEZ). These areas can be seen in Figure 2.1. As mentioned, the Norwegian Coast Guard operates under the Norwegian government and the specific jurisdiction of the Norwegian Coast Guard is listed in (Lovdata). The purpose of these laws is to make some general rules of facilitation that will make it easier for the Coast Guard to operate and make them able to contribute to state agency surveillance of the surrounding oceans. The sea areas that are covered under the Norwegian jurisdiction is the territorial waters, the EEZ, the area around Jan Mayen and the protected zones around Svalbard.



Figure 2.1: Map showing the oceans with Norwegian jurisdiction ([Lovdata](#))

As a department under the Norwegian naval defense, the coast guard is an important organ to guarantee that the Norwegian sovereignty and rights to the ocean, and also an organization that can assist the navy with resources and capacities that it possesses. One of the main areas of importance over the last years has been controlling the fisheries and the Norwegian government interests in the north. As a representative for civil preparedness and enforcement authorities, the coast guard is also able to respond to operations of non-military class such as search and rescue missions, tugging operations and oil spill recovery that may be a threat to the community. The Coast Guard should be able to handle a crisis in an operation area of low intensity and should be able to work independently and in cooperation with the military. In a situation of war the coast guard will maintain the task of monitoring and patrol Norwegian waters, and their presence is meant to act as an enforcement of the Norwegian sovereignty. ([Forsvarsstaben](#)

(2015)

The tasks and roles of the Norwegian Coast Guard are many, and in this report, the focus will concentrate on operations during peace-time. The complete list of roles and task concerning the Norwegian coast guard are listed in (Lovdata). A list of operational tasks is listed below.

- Oil recovery operations
- Tugging operations
- Fire fighting operations
- Ice Breaking
- Medical assistance and transportation
- Mechanical assistance
- Navigational assistance
- Diving assistance
- Participation in exercises
- Military crises

2.2 Vessels and Fleet Structure

The coast guard fleet structure is costumed to the specter of tasks mentioned above with a primary focus on good sea-keeping capabilities to operate in the exposed waters off the Norwegian coast. Today the coast guard fleet consists of 16 patrol vessels divided into two classes, inner- and outer Coast Guard. The inner coast guard consists of 6 vessels with a length of around 50 metes. These vessels primarily operate in the coastal areas up to 12 nautical miles off shore with controlling fisheries being one of the main tasks. In addition, patrolling, coastal monitoring, and support police and customs is also important missions. The inner coast guard is again divided into two classes the Nornen class and the Reine class. The vessels in the two classes have more or less the same capabilities. However, the Nornen class is outfitted with a small patrol

boat capable of operating independently for up to two days giving the Nornen class a larger operating range. Details about the inner coast guard can be seen in Table 2.1. In Figure 2.2 fleet an illustration of the Norwegian Coast Guard is displayed

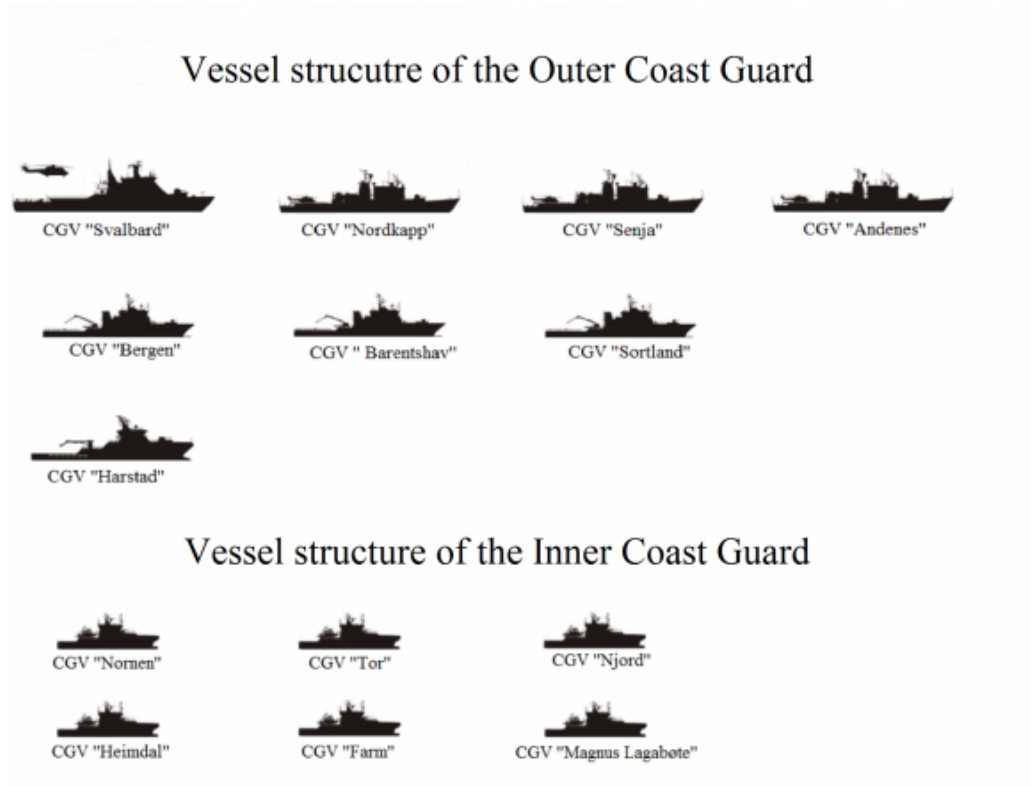


Figure 2.2: Illustration of the Norwegian coast guard fleet structure obtained from the master thesis of [Buland \(2017\)](#). The figures is collected from [Nilsen](#)

Table 2.1: Overview of Inner Coats Guard ([Forsvaret \(2017\)](#))

Vessels	Vessel data	Equipment and capabilities
<u>Nornen Class:</u>		
CGV Nornen	Displacement: 810 [tonnes]	Bollard pull: 32: [tonnes]
CGV Tor	LOA: 47,2 [m]	1 small patrol boat
CGV Tor	Beam: 10,3 [m]	1 small boats
CGV Heimdal	Drought: 4,2 [m]	oil recovery
CGV Farm	Speed: 16 [kn]	
<u>Reine Class:</u>		
CGV Magnus Lagabøte	Displacement: 790 [tonnes]	Bollard pull: 32 [tonnes]
	LOA: 49,6 [m]	2 small boats
	Beam: 10,3 [m]	Oil recovery
	Drought: 4,2 [m]	
	Speed: 16 [kn]	

The outer coast guard consists of 8 patrol vessels ranging from 83 to 105 meters in length, some have oil recovery capabilities, and some are outfitted with helicopters. As with the Nornen class, the capability to operate helicopters gives the vessels an ability to patrol vast areas of waters within a short time, making it highly effective in patrolling different fishing areas. In addition to being preventive regarding illegal fishing and activities, the helicopters are a tremendous asset regarding search and rescue missions. As we can see from [Table 2.2](#) some of the vessels are outfitted with oil recovery and tugging capabilities making the vessels able to react to marine accidents and casualties.

Table 2.2: Overview of Outer Coats Guard (Forsvaret (2017))

Vessels	Vessel data	Equipment and capabilities
<u>Nornen Class:</u>		
CGV Nordkapp	Displacement: 3 300 [tonnes]	Bollard pull: 70 [tonnes]
CGV Andenes	LOA: 105 [m]	2 small boats
CGV Senja	Beam: 14,3 [m] Draught: 5,6 [m] Speed: 22 [kn]	Helicopter
<hr/>		
CGV Svalbard	Displacement: 6 375 [tonnes] LOA: 103,7 [m] Beam: 19,1 [m] Draught: 6,5 [m] Speed: 17 [kn]	Bollard pull: 100 [tonnes] 2 small boats Helicopter Ice braking
<hr/>		
<u>Barentshav Class:</u>		
CGV Nordkapp	Displacement: 4 000 [tonnes]	Bollard pull: 150 [tonnes]
CGV Andenes	LOA: 93,2 [m]	2 small boats
CGV Senja	Beam: 16,6 [m] Draught: 6 [m] Speed: 20 [kn]	Oil recovery
<hr/>		
CGV Harstad	Displacement: 3 132 [tonnes] LOA: 83 [m] Beam: 15,5 [m] Draught: 6 [m] Speed: 18 [kn]	Bollard pull: 110 [tonnes] 2 small boats Oil recovery
<hr/>		

The multifunctional abilities of these vessels mean that the fleet can respond to almost any assignment. However, the challenge is to decide where it could be useful to allocate a vessel to respond to a potential accident or handle different operations. This is one of the main challenges that will be reviewed in this thesis and will be further explained in Chapter 5

2.3 Marine traffic characteristics

In Section 2.1 a list of potential operations that are important to consider when designing fleet deployment model for the coast guard is listed. In addition to being aware of the potential operations that may occur, it is also wise to get an overview of the marine traffic characteristics in the operational waters. The main reason for doing this is that with the Norwegian coast guard being a part the royal navy, they do not transmit their position because that would ruin the element of surprise when doing patrol and fishery monitoring. Hence looking at marine traffic can indicate waters of interest, and where there would be a demand for vessels.

Concentrating on the areas defined in [Lovdata](#) which can be seen on Figure 2.1, various vessel types are operating along the Norwegian coastline. Passenger vessels, bulk carriers, offshore vessels and fishing vessels are the most domination vessels off shore if we do not include pleasure crafts. Most pleasure crafts sails close to shore, protected from the open waters and although these crafts stand for the majority of the accidents reported they will typically be responded to by smaller vessels and will therefore not be a priority in this report.

Ships and vessels sailing in open waters are usually never stationary, so determining where it would be desirable to have a coast guard positioned can be challenging. Accidents are most often related to the ships and vessels position and the concentration of ships. Hence it could be useful to look at density maps to try and pinpoint areas that could be of interest.

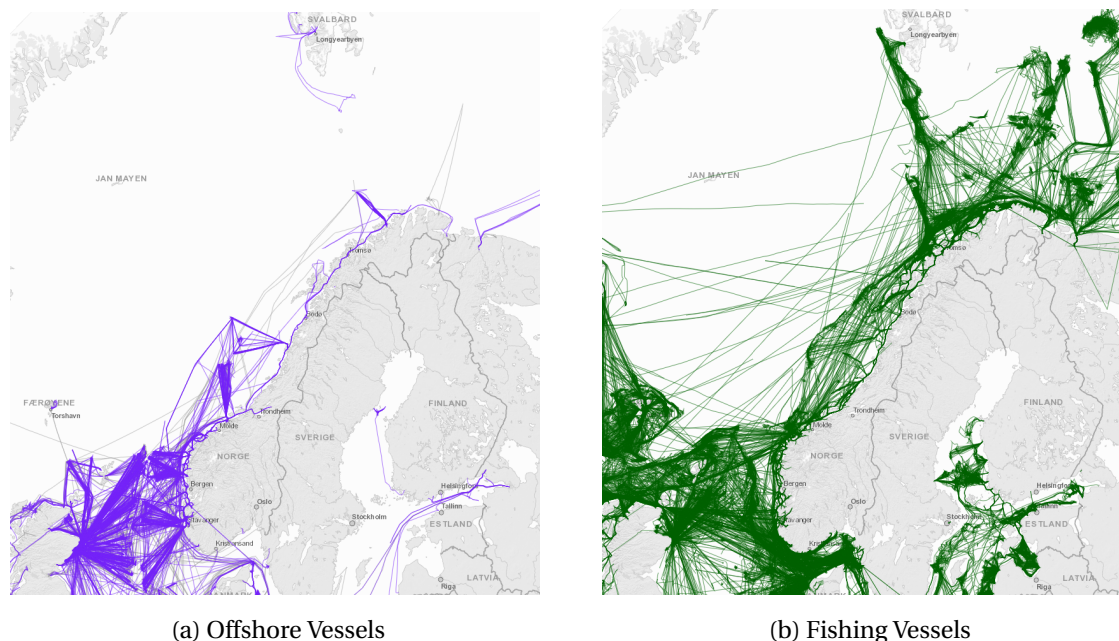


Figure 2.3: Density of fishing vessels and offshore vessels along the Norwegian Coastline [Kystverket](#)

From the figures above we see the density of traffic over the last five years from fishing- and offshore vessels. Norway has been a major actor in global oil production and as we can see from Figure 2.3a above the traffic related to the oil production stretches from the north sea and upwards along the coastline. Luckily the number of accidents related to oil production and the offshore industry is marginal. However, in the case of an accident, the environmental footprint could potentially be enormous which means that this is an area where it could be desirable to have operating coast guard vessels with oil-spill recovery capabilities. The ability for towing could also potentially be desirable because drifting vessels with maneuverability could cause severe damage. It should be mentioned that a lot of the offshore vessels operating in this area do have these capabilities and will be able to respond, but it is still a critical area to consider when evaluating potential deployment locations.

Norway is also a big fishing nation. From Figure 2.3b which represents fishing vessel traffic, we can see that fishing takes place all along the coast as well as in the Barents Sea. The Barents Sea is a rather shallow shelf popular for fishing and in search of hydrocarbons (oil and gas). The Barents Sea is located in the north of Norway and is split between Russia and Norway. The rich

fisheries and valuable ground have been a cause for boundary dispute between Norway and Russia for decades. In the boundary conflict, Norway favors a medial line established in the Geneva convention in 1958, while the Russians favor a meridian sector line, based on a Soviet decision in 1927 ([Wikipedia](#)). This led to the area being "Gray zone" up until now recently where the zone was divided between Russia and Norway. However, with this area being one of the most productive oceans in the world means that this is also an area of concern for the Coast Guard and an area that needs patrolling to maintain Norwegian jurisdiction and ownership.

Chapter 3

Literature Survey

The aim for this section is to present the reader for relevant literature concerning location science. This instance, the majority of the reviewed literature is connected to emergency response problems, server location problems and facility location problems. The purpose of the literature review is to give the reader an insight into the world of covering problems which this report will be based on. Also, the goal is to look at the different applications of the covering problems and see how and why covering location problems might be related to the problem that will be described further in Chapter 5

The different literature is structured and centered around covering models, but literature related to the coast guard problem has also been reviewed.

3.1 Facility location problems

The assignment in this thesis is to see how covering models can be used in deployment and scheduling problems for the Norwegian Coast Guard. Most of the data used in this thesis are generated by the author, and there is a significant level of uncertainty related to the data. Hence introducing stochastic elements to already self-generated data will not result in a more realistic representation of the maximum covering location problem (MCLP). Therefore the attention of this thesis and the following literature study will be on static and discrete models. This is a common approach for general covering problems.

3.1.1 Coast Guard scheduling and location

A typical problem for a Coast Guard is scheduling the fleet of vessels to perform a set of reports where each report and specific requirements regarding the number of ships needed and duration time. [Darby-Dowman et al. \(1995\)](#) has developed a software tool for scheduling Coast Guard cutters that address this problem. The model is a set partitioning model where being late for a report is allowed, but penalized. The objective of the model is to select a schedule that meets the most of the requirements in a given period. [Darby-Dowman et al. \(1995\)](#) emphasize the importance of a robust system in case of changes in the real world for the model to hold. The scheduling model shall support the tactical aspects of the problem and the corresponding operational requirements. Shortcomings of the model is that it is not fully automated and requires human decision making in processes where schedules overlap. The model cannot distinguish between operations, meaning that higher priority schedules need to be picked manually. Similarly, scheduling problem has been assessed by [Brown et al. \(1996\)](#), where they sought to: minimize the number of required patrol weeks missed, minimize the transit time to patrol areas and equitably distribute the patrol weeks among the cutters. As with [Darby-Dowman et al. \(1995\)](#), [Brown et al. \(1996\)](#) have also implemented penalties in the model to satisfy the requirements for weekly patrols and maintenance. The scheduling model resulted in only three missed patrol weeks which is a vast improvement from the manual scheduling done previously.

[Cline et al. \(1992\)](#) introduces a best-scheduling heuristic for scheduling coast guard buoy tenders for solving sizeable real-world routing problems. Their method reduced the real-world problem to a traveling salesman problem which has multiple known solution methods. The objective function to be minimized considers not only distance traveled but also arbitrary penalty factors so long as the penalties can be measured in units convertible into equivalent miles. Their investigation showed that the new method “the best-scheduling” could be applied to numerous problems and produce results. As mentioned in Chapter 2, helicopters are becoming a more and more critical part of the operations of the coast guard. [Hahn and Newman \(2008\)](#) has developed a mixed integer problem for deployment and maintenance of coast guard helicopters. The result is a scheduling plan for the weeks between maintenance that can consider multiple deployment sites.

[Radovilsky and Wagner \(2014\)](#) introduces a paper that investigates the effects that the “boat allocation tool,” which they had generated previously, on the US coast guard. The model where a deterministic model with the goal of finding the optimal deployment of the entire US coast guard fleet. In the model, they implement an inequality constraint which was used to analyze the effects of uncertainty in demand at the coast guard stations. The objective was to minimize the mismatch between the demand at specific stations operational hours and the supply of active boat hours. The paper shows that by implementing the model, they could reduce the number of stations significantly.

[Bhorgava \(1991\)](#) examines the challenges considering decision support systems (DSS) for a fleet mix planning problem for the US Coast Guard. In his paper, he describes in detail how a Coast Guard fleet mix problems can be a benchmark for a decision support system and how long-term planning horizon, necessity and uncertainty of fleet demand and mission objectives makes a Coast Guard mix a complex planning problem. He also evaluates what can be described as the "best mix" or a "good mix" and how to measure effectiveness on a fleet. [Bhorgava \(1991\)](#) concludes that the objective function must either be a function of minimizing the costs with vessel performances as constraints or if one should maximize vessel performance by having expenditure constraints and the vessel performance is best computed by performing simulations with suitable distributions.

3.1.2 Covering Location Problems

[Church and ReVelle \(1973\)](#) investigates the application of the maximum covering problem when adding mandatory closeness to the generic problem. The reason for this is that the general maximum covering tends to try and cover all the demand nodes, resulting in a population overestimation. The reason for this is that the only way to cover all nodes in a system is to add servers which will result in the servers covering the same nodes. By adding closeness constrain they allow for the model for not to completely cover all demand nodes, but allow the population to be nearby the servers/facilities. They conclude that MCLP is a powerful technique for decision making on location problems.

[Alexandris and Giannikos \(2010\)](#) has presented a paper based on a maximal covering location problem to study the effects of change in demand on maximal covering problems. They introduce an integer programming model to study how a change in demand space and coverage gaps change the results. By also introducing geographic information system they can show that the new model is more robust and requires fewer facility servers to have the same coverage as with conventional coverage models and at the same time reduce the coverage gaps. By using GIS (Geographic information system), they can introduce partial covering such as areas instead of discrete nodes common in the generic MCLP. In the paper, they state that their modified MCLP is more realistic and provides a more applicable notion of coverage.

An extension of the MCLP has been purposed by [Berman and Krass \(2002\)](#) where they introduce a model based on the MCLP with a generalization which they called GMCLP, such that it allows for partial covering of customers. The degree of coverage is determined by a non-increasing step function of the distance to the nearest facility. The study shows that in essence, their modified MCLP becomes an equivalent to the uncapacitated facility location problem. In more recent years [Berman et al. \(2010\)](#) have introduced a paper where the goal was to get an overview of the classes of models. The three classes of focus were gradual covering models, cooperative covering models, and variable radius models. The gradual covering models seek to ease the "all or nothing" constraint allowing facilities to be placed within a specific range from the demand. The cooperative models assume that the facilities can contribute to full coverage and that demand at a node is met when coverage is at a certain threshold. Variable radius models where instead of having a preset number of facilities to place the problem is denoted a budget that can be looked at as facilities with different range directly transferable to facility costs giving the problem more flexibility.

Within emergency response, the maximum covering problem has become a popular tool for determining locations for distributing emergency facilities. [Adenso-Díaz and Rodríguez \(1997\)](#) have formulated an MCLP problem to determine the location of ambulances in Leon, Spain. They present a tabu-search heuristic to solve a problem where 25 ambulances shall cover about 500.000 clients. The result showed that with 25 ambulances they were able to cover 99,5% of the population within a responsive of 25 minutes. It shall be noted that to cover the additional 0.5%

the required amount of ambulances had to be 36, hence again showing that complete coverage is not always favorable in cost terms.

In more recent years [Blanquero et al. \(2015\)](#) discusses a max covering location problem by a finite set of users and facilities. The objective is to maximize the expected demand covered where locations of facilities are sought along the edges in a network, making this a mixed integer non-linear program. Due to the non-discrete network the method applied to emergency response analysis and health care analysis. However, it is shown that the method only is applied to a finite problem with a relatively small set of candidates. [Blanquero et al. \(2015\)](#) have also developed a branch and bound strategy for solving the max covering problem.

[R.Paul et al. \(2016\)](#) has also reviewed a maximal condition covering problem. The objective for this paper is to analyze the existing and optimal locations for responding to large-scale emergencies. In essence, they seek to maximize cover population response with minimal changes to the existing response plan. The result was a grid showing the response "nodes" and the covering radius. The analysis showed that the improvement was 98% by only changing 30% of the original emergency structure.

In some situations, the number of clients at a facility might affect how the model will behave. To account for this [Marianov and ReVelle \(1996\)](#) has purposed a queuing model with stochastic arrivals at the facility to try to create a more realistic problem. In the model they define a minimum time for service, based either on the number of clients in the queue or a maximum weighting time. This resulted in an extension to the MCLP, the probabilistic maximum covering location-allocation problem (PMCLAP). One issue is that due to the complexity of real-world problems, researchers often prefer to use heuristics instead. [Pereira et al. \(2015\)](#) took the PMCLAP a step further and introduced a large neighborhood search heuristic to determine the location decisions of the problem. By solving the allocations as subproblems they were able to locate optimum in 95% of the cases.

In contrast to maximizing the covered population that was a basis in the MCLP, in some cases it could be preferable to minimize the use of facilities. [Hakimi \(1964\)](#) was one of the first to present concept of Location set covering problem (LSCP), where he wanted to minimize the number of police officers that could be distributed onto a highway network with a minimal distance from

any person. This problem was later remodeled into an integer programming problem by [Toregas et al. \(1971\)](#). [Toregas et al. \(1971\)](#) reviewed an emergency response problem as a set covering problem where the facilities had equal costs. The goal was to minimize the facility costs while covering all demand nodes.

A common denominator of the problems above is that they are a part of a problem class within discrete optimization problems. A maximal covering problem can be a challenging problem, but as long as restricted to a low number of possible nodes, the problem can quite easily be solved by exhaustive search of the solution space. [Fazel Zarandi et al. \(2011\)](#) introduces a solution method for MCLP that exceeds 900 nodes up to 2500 nodes. By introducing a customized generic algorithm (GA) [Fazel Zarandi et al. \(2011\)](#) can apply the MCLP into a more real-life application. The results show that the GA performs well and returns a solution that is far superior to the exact method regarding run-time. However, [Fazel Zarandi et al. \(2011\)](#) states that the to get even better performance from the GA heuristics should be added, and the effect of partial covering should also be addressed.

In a general covering problem, it is generally assumed that facilities are of equal cost and capabilities and objective often comes down to minimizing the number of facilities or maximizing the cover. However, in some cases, one might require to have facilities with different capabilities. [Colombo et al. \(2016\)](#) introduces a multi-mode covering location problem based on the MCLP which involves locating a given number of facilities and a constraint that limits the facility capability covering the same sites. He introduces a combination of heuristics which resulted in shorter computational time, and it also proved effective for solving large maximum covering location problems.

3.2 Routing problems

In Section 1.2 a pyramid illustrating the time frame of a planning process is shown, and although the attention in this thesis is on the tactical planning problem there is a close relationship between the processes. Hence some literature involving the operational level of planning is also reviewed.

Another take on solving the maximum covering problem is to combine it with vehicle routing problems. [Hachicha et al. \(2000\)](#) has created a multi-vehicle tour problem where a set of vertices must be covered by a finite number of vehicles. Their goal is to determine the minimum length of routes required to cover all vertices or at least be close to covering all. In order to solve the problem, they present three different heuristics; a modified savings heuristic, a modified sweep heuristic and a route-first/cluster-second heuristic. This kind of problem can typically be related to the delivery of healthcare by mobile units in developing countries. Another application for this type of problem is the post box location problem. In the paper [Hachicha et al. \(2000\)](#) can show that the heuristics can solve a realistic size problem within a reasonable time frame.

Combining the maximal covering location problem with routing problem is also assessed by [Megiddo et al. \(1983\)](#). The main characteristics with this problem are that the nodes representing potential location are located somewhere on the vertices in a network.

[Noava et al. \(2006\)](#) presents a vehicle routing problem with stochastic demands (VRPSD). The main contribution of the paper has been to show the application of set partitioning problems and how the general problem can easily be manipulated to satisfy routing problems with randomness applied. [Noava et al. \(2006\)](#) also discovered the disadvantage of using set partitioning due to the large generation of routes to obtain a good solution. For solving the VRPSD she introduced a heuristic producing competitive solutions.

3.3 Operational level of planning papers

[Bailey \(1994\)](#) has investigated the use of simulation-based dynamic optimization. His paper unfolds a scheduling problem and a methodology based on generating a sequence of a dynamic program that changes each step according to the way smuggler and cutters interact. In this paper he focuses on the smuggler in the way that the program tries to maximize the profit regarding contraband by looking at the cutters patrol pattern and their opportunity of diverting from the original pattern. He also looks at the flexibility that the smugglers have when information about the cutters are obtained. By using Monte Carlo methods, he was able to create a scheme that could predict the smugglers' performance during a particular cutter voyage.

[Maisiuk and Gribkovskaia \(2014\)](#) present a discrete event simulation model to evaluate different fleet configurations of supply vessels to the oil and gas industry. The supply vessels provide the offshore installations with supplies from a week to week basis, and if the vessel cannot complete the scheduled voyage a vessel off the spot market is hired. The objective is to determine a fleet configuration that considers the weather conditions and future vessel spot rates. The results are compared with real-world data for verification. Based on the simulation the conclusion was that chartering four vessels on long-term contracts are indeed the most effective configuration.

3.4 Concluding remarks on the literature

This chapter has presented literature on the coast guard problem and different facility location problems, emphasizing on maximum covering location problem. Facility location problems have been used to support decision makers in the allocation of equipment and determining location sites such as police stations, fire stations, and hospitals. The coast guard problem shares similar challenges that are experienced within emergency response, hence reviewing these papers has provided a good insight into how a the MCLP can be used in a coast guard problem. The MCLP can be a challenging problem. Thus the attention has been on static and discrete models.

Chapter 4

Facility location problems

Facility location problems have been a field of study in operations research for a century and have become a critical element in strategic planning for a wide range of problems in both private and public sector ([Zarandi et al. \(2012\)](#)). For example, for state governments, the location of emergency services such as police stations, hospitals and fire stations has utilized the facility locations diligently. In all cases, poor choice of location can increase the likelihood of damage to property or loss of life. In private sector facility locations problems is often related to where a facility should be located to minimize costs, where to place retail stores, etc. In recent, years cell services have become very important for the consumers, and poor judgment in location can lead to a decrease in competitiveness and an increase in costs. Correct use of facility location problems is vital for both private and public sector, and the success or failure depends on the locations chosen for establishing these facilities. In essence, all facility location problems can be related to a degree of costs, whether it costs regarding the loss of life or property, the cost of poor execution or poor performance ([Daskin \(1995\)](#)).

Facility location problems are today a desirable tool in operations research, and despite the different objectives the location problems seek to answer the same questions. ([Daskin \(1995\)](#)):

- How many facilities should be located?
- Where should each facility be located?
- How should demand for the facility services be allocated to the facilities?

The answer to these questions will depend on the objective of the underlying location problem and generating a robust model can be challenging. In the following sections, we will go through some common location problems which are frequently used in facility location and look at their applications.

4.1 Covering models

In many location problems, service to customers is dependent on the distance between the customers and facilities. In most cases, this means that the customer is assigned to the nearest facility and the goal is always to make sure that a facility serves all demand nodes or clients. This leads to the notion of coverage.

Classical covering models are viewed as static and deterministic models, meaning that the solution space is restricted to discrete nodes. For each demand node in a problem there is a subset N_i which represents the candidate facility nodes j that can serve or cover demand node i within a specified distance or time. In addition to showing candidate nodes that are within a certain distance or time constraint it can also be customized to be applied to mission capabilities for individual facilities (Colombo et al. (2016)). One can identify two main classes of demand covering problems, as proposed by Daskin (1995): mandatory covering problems, where all the demand area must be covered using the minimum number of servers and maximal covering models where the largest possible part of the demand area must be covered using a given number of servers (Daskin (1995)).

A common application of the covering models is within allocation of emergency facilities and equipment. Problems concerning fire station locations, emergency medical services(EMS) and patrol routing are examples of problems that have utilized covering models. In these cases covering models focus on covering specific demand nodes or maximizing the covered population within a specific time or distance (Toregas et al. (1971)).

This section is devoted to the primary types of covering location problems namely location set covering problems (LSCP) and maximum covering location problems (MCLP). There will also be introduced some extensions of the problems.

4.1.1 Location Set Covering Problem

The location set covering problem (LSCP) was first introduced by Hakimi (1964) and later re-modeled by Toregas et al. (1971) into an integer problem. The generalized LSCP is modeled as

a discrete deterministic model meaning that the chosen locations and demand points are pre-defined and the solution is also restricted to these points. The location set covering problem aims to minimize the number of facilities required while still covering all demand nodes within a certain service distance. [Toregas et al. \(1971\)](#) and [Church and ReVelle \(1973\)](#) formulated the problem as this:

Sets and Parameters:

- I = The sets of demand nodes i
- J = The sets of demand nodes j
- S = The distance beyond which a demand point is considered uncovered
- N_i = $\{j \in J \mid d_{ij} \leq S_i\}$. The set of facilities eligible to provide coverage to point i . S_i is the maximal service distance and d_{ij} is the shortest distance

Variables:

$$x_j = \begin{cases} 1 & \text{if a facility is located at site } j \\ 0 & \text{Otherwise} \end{cases}$$

Model:

$$\min z = \sum_{j \in J} x_j \quad (4.1)$$

$$s.t \sum_{j \in N_i} x_j \geq 1 \quad \forall i \in I \quad (4.2)$$

$$x_j = (0, 1) \quad \forall j \in J \quad (4.3)$$

The objective function (4.1) minimizes the number of service facilities that will be located. The constraints (4.2) make sure that all demand nodes are covered. By solving the problem above as an integer linear programming problem, we can ensure a feasible solution. The use of a branch and bound algorithm ensures that all integer solutions are found.

4.1.2 Maximal covering location problem

In contrast to the LSCP above where there are no restrictions in the number of facilities that can be built to cover all nodes, it is likely that this would not be the case in the real world. In reality, decision makers could be restricted by the amount facilities that can be built which can be reviewed as a budget constraint. Also, the LSCP treats all node by identically hence it does not distinguish between the importance of covering one specific node. The maximal covering location problem seeks to maximize the number of demand nodes. This problem leads to the fixating the number of facilities being built, and the demand nodes are given different weights depending on their level of importance. The goal is to maximize the number of covered nodes. Church and ReVelle (1973) originally proposed the following formulation.

Sets and Parameters:

- I = The sets of demand nodes i
- J = The sets of facility sites j
- S = The distance beyond which a demand point is considered uncovered
- a_i = The weighted demand at node i
- d_{ij} = The shortest distance from node i to facility site j
- N_i = $\{j \in J \mid d_{ij} \leq S_i\}$. The set of facilities eligible to provide coverage to point i . S_i is the maximal service distance and d_{ij} is the shortest distance
- P = Number of available facilities

Variables:

- x_j = $\begin{cases} 1 & \text{if a facility is located at site } j \\ 0 & \text{Otherwise} \end{cases}$
- y_i = $\begin{cases} 1 & \text{if a node is covered in the set } N_i \\ 0 & \text{Otherwise} \end{cases}$

Model:

$$\text{Max } z = \sum_{i \in I} y_i a_i \quad (4.4)$$

$$\text{s.t } \sum_{j \in N_i} x_j \geq y_i \quad \forall i \in I \quad (4.5)$$

$$\sum_{j \in J} x_j \leq P \quad (4.6)$$

$$y_i = (0, 1) \quad \forall i \in I \quad (4.7)$$

$$x_j = (0, 1) \quad \forall j \in J \quad (4.8)$$

N_i is the set of potential sites where a facility can be placed to cover demand point i . A demand node is covered when it is within a desired distance S from the facility. The objective is to maximize the covered population. Constraint 1 indicates that y_i is 1 only when one or more facilities are located at sites in the set N_i . The objective function (4.4) maximizes the number of covered demands, while constraint (4.5) requires that coverage at node i cannot be accounted for unless we locate at least one facility at one of the candidate facility sites j that can cover node i . This is defined in the subset N_i . Constraint (4.3) limit the number of facilities to be located to P . The last two constraints are standard integrity constraints.

4.2 Centering Problems

In the location set covering problem described above the objective is to determine the minimum number of facilities required for covering all demand nodes by using an exogenously specified distance between facilities and demand nodes. As examined by numerous authors such as [Daskin \(1983\)](#), [Toregas et al. \(1971\)](#), [Adenso-Díaz and Rodríguez \(1997\)](#) the required number of facilities needed to cover all nodes would be far more than the trade-off, and in most cases, it would not be possible to build that amount of facilities. This problem became the reason for the development of the MCLP which considers a predetermined set of facilities or resources available and determines the maximum possible coverage. Another approach for avoiding the problem with an unrealistic number of facilities is by the class called P-center problem. In the

P-center problem, the requirements are also to cover all nodes by locating a given number of facilities by minimizing the coverage distance. Instead of having a predetermined maximum distance “ S ” (which have been used in the previous problems), the model determines the minimal covering distance endogenously by locating P facilities. This Problem is known as a mini-max problem because it seeks to minimize the maximum distance between demand nodes and facilities.

The center covering problem is divided into two main classes, the vertex- and the absolute center problems. Whereas the absolute allows facilities to be located anywhere on the network the vertex is restricted to nodes. This thesis focuses on deterministic models therefore the absolute model will not be prioritized, and the focus will be on the vertex problem.

Sets and Parameters:

- I = The sets of demand nodes i
- J = The sets of facility sites j
- P = Number of facilities to be located
- d_{ij} = The shortest distance from node i to facility site j
- D = The maximum distance between a demand node and the nearest facility

Variables:

$$\begin{array}{l}
 x_j = \left\{ \begin{array}{ll} 1 & \text{if a facility is located at site } j \\ 0 & \text{Otherwise} \end{array} \right. \\
 y_{ij} = \left\{ \begin{array}{ll} 1 & \text{if a node } i \text{ is served by facility site } j \\ 0 & \text{Otherwise} \end{array} \right.
 \end{array}$$

Model:

$$\min z = D \quad (4.9)$$

$$s.t \sum_{j \in J} x_j = P \quad (4.10)$$

$$\sum_{j \in J} y_{ij} = 1 \quad \forall i \in I \quad (4.11)$$

$$y_{ij} - x_j \leq 0 \quad \forall i \in I, j \in J \quad (4.12)$$

$$D \geq \sum_{j \in J} d_{ij} y_{ij} \quad \forall i \in I \quad (4.13)$$

$$y_{ij} = (0, 1) \quad \forall i \in I, j \in J \quad (4.14)$$

$$x_j = (0, 1) \quad \forall j \in J \quad (4.15)$$

The objective in this model is to minimize the maximum distance between all demand nodes and facilities. Constraint (4.10) requires that P facilities are to be located and constraint (4.11) makes sure that every demand node is assigned to a facility. Constraint (4.12) makes sure that a demand node can only be assigned to a node where there is a facility, and (4.13) defines the maximum distance between a demand node i and a facility site j .

4.3 Median problem

Originally developed by [Hakimi \(1964\)](#), the p -median problem seeks to minimize the total weighted average distance between demand node and facility. The key decision is to determine where the P facilities should be located and which demand nodes should be located to which facility. [Charles S ReVelle \(1970\)](#) formulated the problem as follows:

Sets and Parameters:

- I = The sets of demand nodes i
 J = The sets of facility sites j
 h_i = demand at node i
 P = Number of facilities to be located
 d_{ij} = The shortest distance from node i to facility site j

Variables:

$$\begin{aligned}
 x_j &= \begin{cases} 1 & \text{if a facility is located at site } j \\ 0 & \text{Otherwise} \end{cases} \\
 y_{ij} &= \begin{cases} 1 & \text{if a node } i \text{ is served by facility site } j \\ 0 & \text{Otherwise} \end{cases}
 \end{aligned}$$

$$\min z = \sum_{j \in J} \sum_{i \in I} h_i d_{ij} y_{ij} \quad (4.16)$$

$$s.t. \sum_{j \in J} x_j = P, \quad (4.17)$$

$$\sum_{j \in J} y_{ij} = 1 \quad \forall i \in I \quad (4.18)$$

$$y_{ij} - x_j \leq 0 \quad \forall i \in I, j \in J \quad (4.19)$$

$$y_{ij} = (0, 1) \quad \forall i \in I, j \in J \quad (4.20)$$

$$x_j = (0, 1) \quad \forall j \in J \quad (4.21)$$

The objective function is to minimize the total demand-weighted distance between the demand nodes and the facilities. Constraint (4.17) makes sure that the required number of facility P is met, and constraint (4.18) makes sure that all demand points is connected to a facility. Constraint (4.19) allows for the demand node only to be connected to where there is a facility node j . Finally, the last two constraints are binary. It shall be noted that the formulation above only allows for the facilities to be located at a finite selected set of potential sites. These nodes represent the set of nodes J in the network, and [Hakimi \(1964\)](#) showed that for a given number of facilities P , it exists an optimal solution on the nodes of the network.

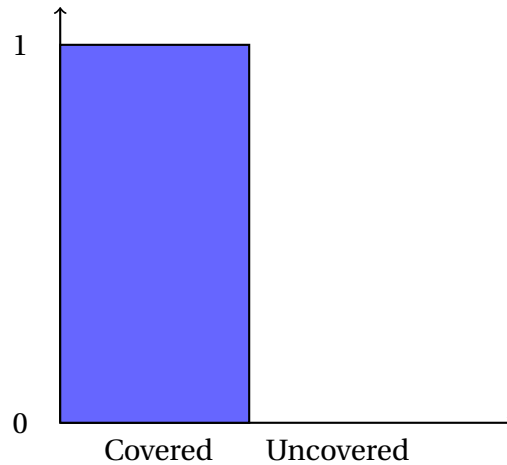


Figure 4.1: Illustrating the benefit a demand node is provided from facilities in covering problems

In covering and centering problems we assume that if a demand node is covered by a facility the demand node will get the full benefit of being covered. However, in many cases the benefit would have a gradual decrease with regards to an increase in distance between customer and facility. Median problems open up for the demand to be split between facilities.

4.4 Gradual coverage

In the methodologies above about covering problems, a key assumption is that a demand node is covered if the distance to the facility is less than R , and a demand node is uncovered if the distance to the facility is greater than a set distance R . In many situations, especially in a competitive environment it is unrealistic to assume that a customer A, located 4,9 kilometer from a facility X would be covered by facility X, while a customer B, at a distance 5,1 from facility would not be covered. Hence it makes more sense to introduce a model where coverage is gradually declining such that customer A would for example get 50% of the benefit from facility A, and customer B get 40%, instead of 100% and nothing. This has resulted in [Drezner et al. \(2004\)](#) developing a "gradual covering model".

The problem is formulated as a non-covering model instead of a maximum covering problem. A cost function is generated with a minimum distance l , a maximum distance u and weights w

between l and u . For a given distance d the cost function becomes:

$$c(d) = \begin{cases} 0, & d \leq l, \\ w(d-l), & l \leq d \leq u, \\ w(u-l), & d \geq u \end{cases}$$

Assuming that the cost of allocation a facility at node i is equal for all facility sites j we can formulate the problem below.

Sets and Parameters:

- I = The sets of demand nodes i
- J = The sets of facility sites j
- $C_i(d)$ = cost of placing a facility at node i
- P = Number of facilities to be located
- l = Inner covering radius
- u = Outer covering radius

Variables:

- $x_j = \begin{cases} 1 & \text{if a facility is located at site } j \\ 0 & \text{Otherwise} \end{cases}$
- $y_{ij} = \begin{cases} 1 & \text{if a node } i \text{ is served by facility site } j \\ 0 & \text{Otherwise} \end{cases}$

$$\min z = \sum_{j \in J} \sum_{i \in I} C_i(d) y_{ij} \quad (4.22)$$

$$s.t. \sum_{j \in J} x_j = P, \quad (4.23)$$

$$\sum_{j \in J} y_{ij} = 1 \quad \forall i \in I \quad (4.24)$$

$$y_{ij} - x_j \leq 0 \quad \forall i \in I, j \in J \quad (4.25)$$

$$y_{ij} = (0, 1) \quad \forall i \in I, j \in J \quad (4.26)$$

$$x_j = (0, 1) \quad \forall j \in J \quad (4.27)$$

As we can see, gradual covering problem is relatively similar to the p-median problem where the only difference is that instead of maximizing the total demanded weight we minimize the cost of locating a facility at a facility site j . The key difference here is the cost function which handles the change in distance between nodes and denotes a weighted cost to the location node.

4.5 Evaluation of location problems

The purpose of this chapter has been to look at different classes of facility location problems and to look at different applications for the classes. Knowing strengths and weaknesses of the models will contribute in the modeling of the future problem, and based on the research in this chapter it has become clear that further in this thesis the attention will be on formulating an MCLP for this coast guard problem.

Chapter 5

Problem formulation

In this chapter, a problem description of the coast guard deployment model is presented, before a detailed model formulation of the maximum covering problem is given in Chapter 6. The attention in this chapter will be on why the maximum covering model might be suitable for a coast guard location problem without going into detailed mathematics. Furthermore, a list of assumptions made in this thesis is presented and explanations on why the different decisions are made.

5.1 Problem structure

Facility location has been a fundamental area of research for over a century, and it has become a decisive role in the success of supply chain with applications within allocation facilities and equipment ([Fazel Zarandi et al. \(2011\)](#)). Facility location problems have been a common practice in private and public sector, in particular within emergency response planning. Covering location problems have, for a long time, been a prominent class of facility location where one seeks a solution that that covers a subset of costumers meeting a set of requirements.

As reviewed in Chapter 4, the maximum covering location problem (MCLP) has proven to be one of the most effective models in facility locations from a practical and theoretical point of view. As mentioned in Section 4.1.2, the premise for the maximum covering location problem is to establish the location to a set of facilities to maximize the weighted covered costumers of clients. The maximum covering problem has become particularly popular in the public sector for problems such as emergency response, the location of schools and hospitals, police stations and fire stations. The location problems share a lot of the desirable attributes with the coast

guard problem which is the reason for inventive the effects a maximum covering can have on a coast guard deployment problem.

The goal for this thesis is to see how MCLP can be used in deployment of coast guard fleet and show the trade-off between coverage and cost of multiple fleets with different fleet alternatives. First of all, we need to establish an initial fleet that can be implemented into the model. In Section 2.2 an introduction of the Norwegian coast guard fleet where presented. The vessels have a wide spread of applications and capabilities, and it would be a big challenge to include this fleet in the model. Hence the initial fleet in this thesis is the fleet presented by [Buland \(2017\)](#) in his thesis. The fleet consists of 8 vessels with different capabilities. The model will be limited to sea-going vessels only, meaning that all though some of the vessels have helicopter capabilities, effectively making the vessel able to respond faster to a mission and cover a greater area.

The second decision is to determine the type of missions and objectives to analyze. As reviewed in Section 2.1 there are multiple missions and operations of interest, but to limit the scope we have narrowed the analyze to 4 types for missions; search and rescue, tugging operations, patrolling and oils spill recovery. One of the reasons for limiting the operations to 4 is that some of the operations would require similar capabilities from the vessels, hence by reducing the number of mission types it becomes easier to distinguish between the missions, reducing the possibility of double counting. Also, making it easier to decide vessel capabilities for each of the eight vessels. Meaning that a vessel that does not have oil recovery capabilities cannot respond to an oil recovery mission, and so on. In [Lovdata](#) they mention controlling fisheries as one of the main priorities for the coast guard, and due to this, patrol areas gets denoted a vessel coverage requirement. Hence, these nodes must be covered to have a feasible solution.

Next is determining the areas and locations for the coast guard problem. Although this problem could be solved in a common network of nodes, the operational area is restricted to the waters under the Norwegian jurisdiction, this also helps readers visualize the scope of the problem. It has been difficult to obtain good data from the Norwegian coast guard on operations that can aid the job of forecasting the locations of operations that could be of interest. Collecting AIS data from the coast guard is, of course, the ideal approach, but for obvious reasons challenging.

This will discuss further in Chapter 8. To get around this, the locations chosen as a result of identifying marine traffic and political interests for the Norwegian government and coast guard. This assumption is a result of the availability of realistic data, making it hard to present any deployment recommendations to the coast guard problem. Below is an illustration showing the marine traffic over a year period and an illustration showing the chosen mission locations chosen for this study.

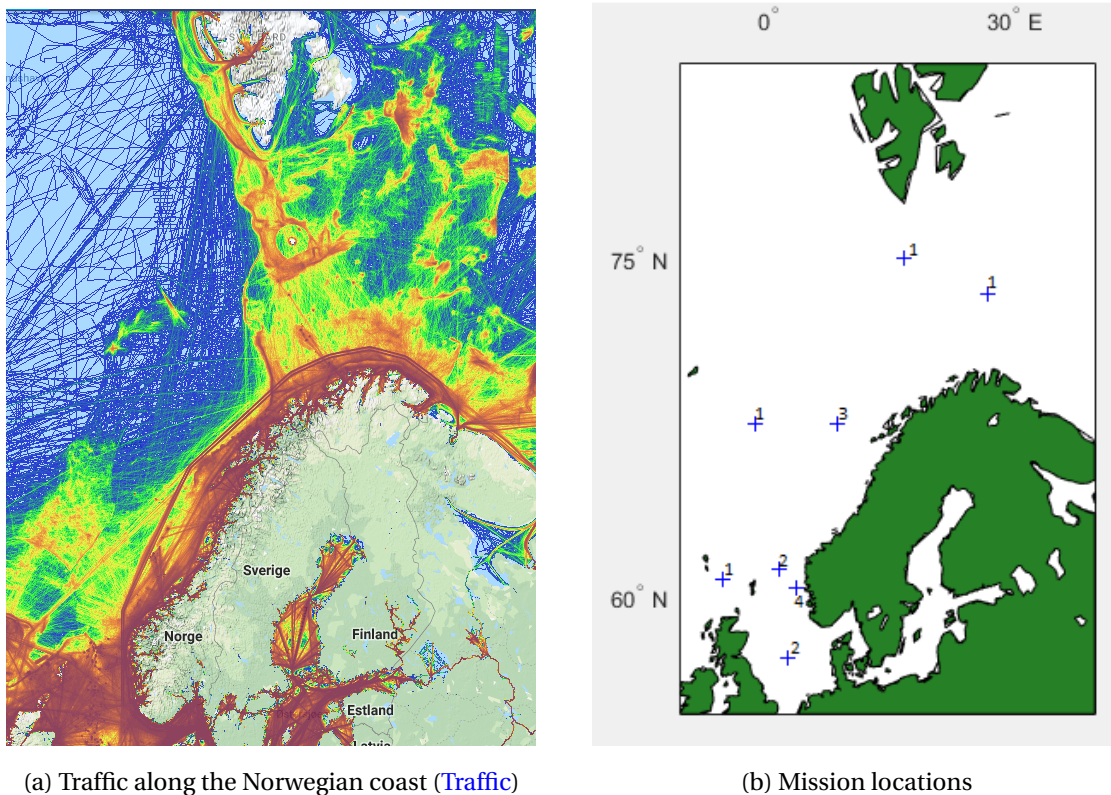


Figure 5.1: Showing traffic along the Norwegian coastline and the locations for the coast guard missions

The nodes marked with 1 is the patrol nodes which has patrol requirement of at least one vessel. In Section 2.3 we talked about the political interests and fishery activities. Hence we have allocated a set of demand nodes to the Barents Sea ([Wikipedia](#)) as well as to the left in the map where there is known commercial fisheries and shipping. The nodes marked with the number 2 represents demand nodes with oil recovery operation. This location is one of the biggest areas in the world for oil and gas production, hence having a vessel with oil recovery capabilities is

imperative. The node marked with 3 represents tugging operations. This location an important area, being a popular fishing ground all year around. Having tugging capabilities is also important at the oil and rescue nodes. However, the vessel with oil recovery capabilities are also able to do tugging operations, so this type of operation is covered. Lastly, the number 4 represents search and rescue operations. This area is a highly trafficked area along the Norwegian coastline and over the years accidents of different severity has developed here (NRK). It shall be noted that although it seems unrealistic that only one node is defined as a mission node, the neighboring areas is also important to cover. This will further be discussed in Section 5.2.

5.2 Demand nodes and weights

In this thesis, we will evaluate the system as a discrete system of nodes. A discrete MCLP implies that the set of potential facility locations have to be determined in advance (Berman et al. (2010)). The nodes are structured in the areas shown in Figure 2.1 with a 51 by 51 grid of nodes, adding up to 2601 nodes. Nodes that are positioned on shore are filtered away, and we are left with 1474 nodes. The problem is assumed to be an integer programming problem with no fractional solutions. ReVelle (1993) describes this problem as “integer friendly” because the solution of the LP – relaxation is often all integers making us able to solve the MCLP for large instances with conventional IP-solvers.

One fundamental assumption for the MCLP and made in this thesis is the notion of “all or nothing”. When a node is covered by a facility, it will have the full benefit of the applications from the facility (Daskin (1995)). This assumption can be unrealistic since that would mean if a facility had a covering radius of 5 kilometers, a node at 4,9 kilometer would be covered while a node at 5,1 would not be. However, it simplifies the problem, leaving us with a non-fractional solution. This effect is illustrated in Figure 4.1. In this model, we will also assume that the facilities in this problem have a fixed covering radius meaning that a node beyond this radius is not covered, while a node within is covered. With this information a subset containing all possible locations for the facilities related to each node is created, relaxing the number of possibilities to be checked in the model. In Figure 5.2a we can see the network of candidate nodes, where

nodes on shore are filtered away (Greene, 2014).

To evaluate the covering performance of the coverage from the facilities a weighted demand is denoted each node in the network corresponding to the importance of a particular mission. The weight applied to these nodes is decided by the author, and it is purely experimental. Based on the recognized locations in Figure 5.1a a source node gets denoted a weighted value which serves as a reward if covered by a facility. However, it is recognized that covering neighboring nodes should also be rewarded because that would imply that being close to a task gives a shorter response time in the event of an occurring assignment. This can be looked as a take on a “gradual covering model” addressed by Berman et al. (2010), where the neighboring nodes are denoted by a diminishing value according to the distance to the source node. In the event of nearby tasks occurring the nodes will get the accumulated score which would give these nodes a higher rating. This is a reasonable assumption, as with missions located close by each other it would imply that the probability of an event happening in this area is higher and desirable to have a facility located in this area. This effect can be seen from the contour plot below. It shall be noted that the weighted demand has been scaled up to give a better visual representation (Figure 5.2b).

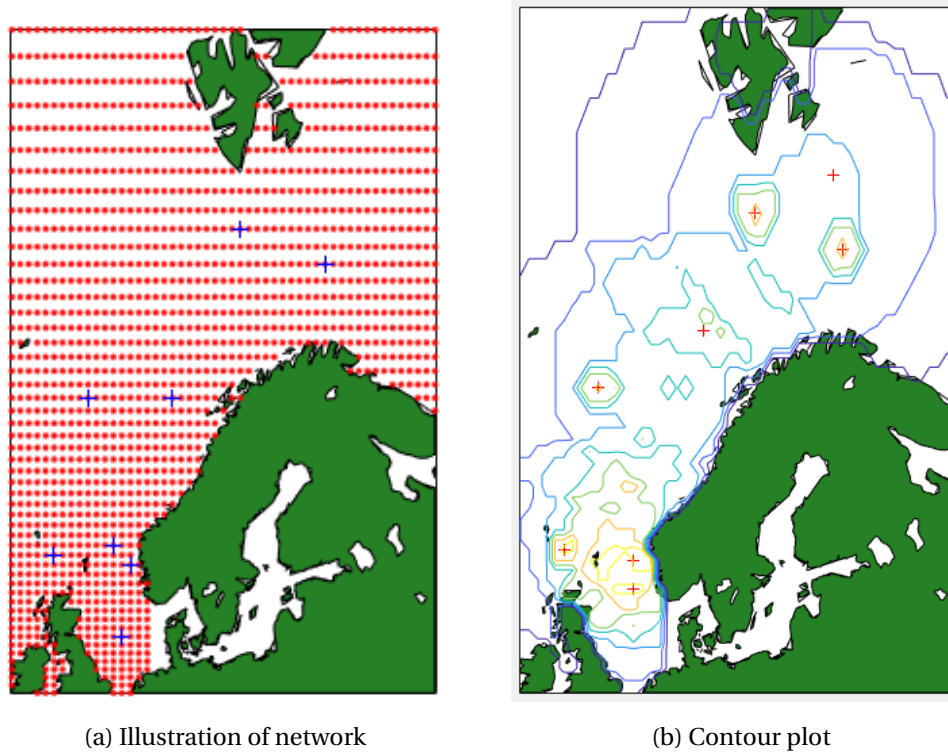


Figure 5.2: Displaying the network of demand nodes and a resulting contour plot of the weighted demand at mission nodes

5.3 Coverage by multiple vessels

The distribution of weighted nodes means that there will be some areas will be more desirable to cover than others. As established by [Church and ReVelle \(1973\)](#) and [Alexandris and Giannikos \(2010\)](#) the general MCLP will uncritically maximize the objective function, which results in that the vessels will be deployed very close to each other or even on top of each other to maximize the score. One way of handling this is to constrain the problem by saying that nodes can only be covered by one vessel resulting in a constant as shown below. This constraint states that a demand node can be covered by no more than one vessel, hence giving no reward for additional coverage.

$$\sum_{i \in I} y_{i,v} \leq 1 \quad (5.1)$$

This however, can be unrealistic in an accident scenario. For example in the event of a severe oil spill accident, having more than one vessel responding to the mission would most likely limit the spread of oil and an environmental crises. In Figure 5.3 below, an illustration of how the node weight reduces as a node is covered by more than one vessel. The illustration can be related to what in economics is known as diminishing marginal returns, where in this case a node covered by 1 vessel gets awarded a full weighted score. If a second vessel covers the same node, the second vessel gets awarded half of the original value of the weight. In case of a third vessel covering, it gets awarded half of the value from the second vessel. The mathematical formulation will be described in Chapter 7.

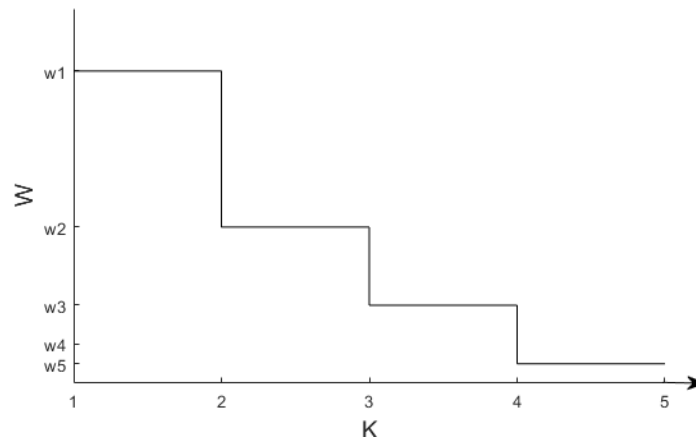


Figure 5.3: Graph showing the weight reduction during multiple node coverage

It shall be noticed that this assumption has been made by the author to try and simulate the effect of having coverage by multiple vessels. [Pantuso et al. \(2016\)](#) uses a similar assumption in their approach to estimate fleet renewal in different market conditions. [Colombo et al. \(2016\)](#) had an approach where he made sure that the different modes (his equivalent to facilities), gets can have different requirements. In the case of [Colombo et al. \(2016\)](#) the problem involved the number of required facilities to cover an individual demand node.

5.4 Summary of assumptions

Following is a short list of the primary assumptions made in this chapter and report.

- The problem is assumed to be a static and discrete planning problem. Static planning is characterized by that the input variables are the same and does not change during the planning process. By keeping the problem discrete means that locations sites feasible solutions are restricted to a network of nodes.
- The problem is assumed to be a mixed integer problem which means that no fractional solutions are permitted. This makes sense since dividing a vessel between two locations would not be possible. This assumption also results in the "all or nothing" assumption where for example a person at 4.9 kilometers would be fully covered while a person at 5.1 would not be if the coverage distance were 5 kilometers.
- A demand node that is selected as a mission will be denoted a weighted value, representing the importance of covering the node. Also, to respond to the gradual covering problem, neighbouring nodes gets denoted a reduced value depending on a fixed distance. If demand nodes are located next to each other then the scores will be added together.
- We have assumed that the facilities or vessels have constant predetermined coverage radius.

Chapter 6

Model Formulation

In this chapter, a mathematical model for coast guard maximum covering location problem is presented. Also a description of the data generation and preprocessing is described.

6.1 Model description

The coast guard problem in this thesis is formulated as a maximum covering location problem, where we seek to investigate if applied MCLP to a coast guard deployment problem can contribute to solving the tactical planning problem. The developed model has two main applications. Firstly it is designed to solve the tactical deployment problem concerning a set of objectives located on the map in Figure 5.1b For a given fleet the model will return the deployment solution that produces the highest weighted reward. Secondly, the model can be tested for other fleet configurations, letting decision makers able to evaluate if other combinations of vessels are more desirable regarding providing higher effectiveness.

Subset:

- d_{ij} Shortest distance from demand node i to candidate node j
- R_m Maximum response time to mission m
- VR_v Maximum range for vessel v
- A_{mv} 1 if vessel v is capable of responding to mission m

Sets:

- I The sets of demand nodes i
- J The sets of demand nodes j
- V The sets of vessel types v
- D_i Set of nodes that have vessel demand at node i
- M The sets of mission types m
- K Set where k represents the number of vessels
- N_{ivm} $N_{ivm} = \{j \mid d_{ij} \leq VR \cap A_{mv} = 1\}$, a subset of nodes j that is satisfies the condition inside the bracket.

Parameters:

- W_{imk} The weight donated to a node i for mission type m . k is 1 of the vessel responding is the first to respond, 2 if second, etc.
- D_i Vessel demand at node i
- P_v Available vessels type of v

Variables:

- x_{jv} Integer variable describing the number of vessels denoted to node j
- y_{imk} Integer variable describing if node i is covered by k number of ships

Model:

$$\max z = \sum_{i \in I} \sum_{m \in M} \sum_{k \in K} W_{imk} y_{imk} \quad (6.1)$$

$$s.t. \sum_{k \in K} y_{imk} - \sum_{v \in V} \sum_{j \in N_{ivm}} x_{jv} \leq 0 \quad \forall i \in I, m \in M \quad (6.2)$$

$$\sum_{v \in V} x_{iv} \geq D_i \quad \forall i \in I \quad (6.3)$$

$$\sum_{j \in J} x_{jv} \geq P_v \quad \forall v \in V \quad (6.4)$$

$$y_{imk} \leq 1 \quad \forall i \in I, m \in M, k \in K \quad (6.5)$$

$$x_{jv} \in \mathbb{Z}^+ \quad (6.6)$$

$$y_{imk} \in \mathbb{Z}^+ \quad (6.7)$$

The objective function 6.1 maximizes the covered nodes. The indices k represents a reduction in weighted value if node i is already covered. When $K = 1$ we have the original weight however when $k = 2$ it means that two vessels cover the same node and the value of the node gets halved (5.2). 6.2 makes sure that a capable vessel v only covers a node with mission requirement m at node j from the subset N_{ivm} . The subset also makes sure that the node is covered by a vessel that is within the time limit. The Subset can be described as follows.

$N_{ivm} = \{j \mid d_{ij} \leq VR \cap A_{mv} = 1\}$. VR is the individual vessel response range. $A_{mv} = 1$ means that vessel v is capable of performing mission m . Restriction 6.3 describes that all nodes with vessel demand must be serviced with one or more vessels. Constraint 6.4 tells that there cannot be deployed more vessels than there are in the fleet. Constraint 6.5 makes sure that a node can only be covered of one type of k , in essence this means that the model must decide how many vessels it wants to cover a node. This will depend on the objective function and what type of deployment that generates the highest cover or in this case score.

6.2 Data generation and preprocessing

The model formulation above will be solved with Xpress IVE software tool. However, although Xpress IVE is an excellent tool for solving optimization problems with numerous built-in strategies for solving to optima, working with a large set of variables internally in Xpress IVE is challenging. Hence the preprocessing of data will be performed in Matlab, where a .txt file will be generated as input to Xpress IVE. In the following sections, the preprocessing of data will be described.

6.2.1 Distance matrix

As mentioned in section 5.2 the problem consists of a network of 2601 nodes in a 51 by 51 grid of nodes. However, a distance matrix must be generated showing the distance between each node. For the distance between two geographical positions on the map the built-in Matlab function "*distance*" (MathWorks) has been used. The function calculates the distance between points on

a sphere with the radius of the earth as a reference and with nautical miles as distance unit. With all the points being on the ocean there difference in topography would have little effect on the resulting distance. An alternative method would have been using Pythagoras, and although this would be fine for nodes nearby each other there would be a significant deviation in distance as the nodes get further apart and the curvature of the earth will affect the result.

In Figure 5.2a we can see that nodes located on shore are filtered away, with the assistance of the "*Landmask*" function developed by Greene (2014). The function return 1 if a point is on land and 0 if the point is not. This reduces the number of calculations needed for developing the distance matrix. One attribute of the "*landmask*" function is that it enables for choosing the quality of the filtration, and from Figure 5.2a we can see that it does a fairly good job. In Section 5.1 a description of the operational area of the coast guard was described, which means that nodes located in the Baltic Sea and the Gulf of Bothnia are also removed. Reducing the total number of nodes to be addressed by the "*distance*" function by 43%. These calculations can be seen in the Matlab script "Distancematrix.m" in Appendix A.2

6.2.2 Node weights

In Section 5.1 the location of the different operations where determined, and in Section 5.2 an overall description of how the weight demand nodes where determined. In this thesis, the author has manually decided the node weight as well as how the weight of the neighbouring nodes is diminishing with respect to the distance from the root node. This can be seen from the script "MoselfileGen.m" in Appendix A.1. The script identifies the mission type and denotes a weighted value to the root node. In this script, the covering radius for each vessel is defined, which will be important for generating the text file that includes all the variables into IVE Xpress software.

The parameter WPC_{imk} in the model above represents the awarded weight of covering mission m at node i . In Section 5.3 we explained the effect of a node being covered by more than one vessel. For example, a patrol mission is in this thesis denoted 15 points. Hence if a vessel covers that very node it gets rewarded with 15 points. However, if a second vessel also covers that node,

it will be rewarded 7 points, and a third vessel will be rewarded 3.5 points. The reduction of awarded points is defined by equation 6.8. Here $WP(i)$ is the accumulated amount of points rewarded at each node i and k is the vessel number.

$$WPC_{imk} = \frac{WP(i)}{\frac{2^k}{2}} \quad (6.8)$$

The method of diminishing reward is recognized in economics as "Marginal Rate of Technical Substitution" ([Investopedia](#)), which shows how much the production must increase when reducing resources. The diminishing model used in this thesis is a modification to the "Marginal Rate of Technical Substitution." This approach is also applied in the paper of [Pantuso et al. \(2016\)](#) where they show the fleet price change as demand increases. This is illustrated in Figure 5.3.

6.2.3 Subset

In this thesis, two types of subsets will be generated. The first will be generated by a fixed vessel response distance and the second will be generated by a maximum mission response time.

As mentioned in Section 4.1.2 one of the key aspects of the MCLP is that we normally generate a subset that has predefined the potential locations that a facility or in this case a vessel can be located, and in this coast guard deployment case it also show what that are covered by placing a vessel at a location site. Above the subset $N_{ivm} = \{j \mid d_{ij} \leq VR \cap A_{mv} = 1\}$ was introduced. The subset says that j is a node in the subset N_{ivm} for node i , vessel v and mission m if the distance from j to node i is less than equal to the response distance of vessel v , and that vessel v has the operational capabilities of mission m . A_{mv} is the mission capability matrix that explains if a vessel is capable of responding to mission m . The second subset is based on the assumption that depending on the mission, there will be a individual maximum response time. The specific response times will be displayed in chapter 7. The formulation of the subset for the second scenario is defined as:

$N_{ivm} = \{j \mid \frac{d_{ij}}{VS_v} \leq RT_m \cap A_{mv} = 1\}$, where VS is the vessel speed and RT_m is the required response

time for mission m and VS_v is the maximum speed for vessel v .

The subsets are generated from a Matlab script which can be seen in [Appendix A.3](#).

Chapter 7

Computational Study and Result

In this chapter three different case scenarios of the coast guard deployment problem will be introduced, including scenario results. The problems will be solved using Matlab and Xpress optimization software on a laptop with an Intel(R) Core(TM) i7-8550U CPU with 8 GB of RAM. To limit the computational time there is included a maximum run time for each fleet. The developed optimization Xpress code can be seen in Appendix B.1.

In the following presented results, the notion of coverage has been normalized to a utility. The reason for this is that coverage in this instance is defined by the accumulated score that a fleet is covering and not actual area covered due to the weighted nodes discussed in Section 5.2. Hence it makes more sense to look at normalized results as a utility, where the highest objective value is assumed to be 1. The utility values are then defined as:

$$Utility = \frac{Objective\ value\ of\ fleet\ i}{Max\ Objective\ value} \quad (7.1)$$

7.1 Input data

All though we will test three different cases in this chapter the input data below is the same for all cases. The maximum response distance used in Table 7.1 is chosen by the author and does not an official requirement set by the Norwegian coast guard or any other organization for that matter. However, the distances are chosen on the ground that the vessel should be able to respond within a reasonable response time. For example, if we look at the data for the first vessel, that with a speed of 20 knots and a response distance of 50 nautical miles, effectively, the vessel should be deployed on a location site such that it can respond to a mission within

two and a half hours. In the case in Section 7.4 we will introduce a scenario where the fleet can respond to individual response time requirements. The CAPEX data for the vessels is borrowed from Buland (2017) where he determined a price estimate for the individual vessels based on the formulas obtained in Amdahl et al. (2011). Table 7.2 illustrates the capability matrix, which is referred to in Chapter 6 as the A matrix, an important parameter for generating the subsets. Table 7.3 illustrates the max response requirements for specific missions.

Table 7.1: Vessel data

Ship Types	1	2	3	4	5	6	7	8
Max velocity [kn]	20	21	23	25	18	28	22	16
Max Response Distance [nm]	50	40	70	90	60	60	45	50
CAPEX [mUSD]	27.2	36.5	34.3	38.38	36.92	77.22	55.22	51.93

Table 7.2: Capability matrix

Vessel Types	1	2	3	4	5	6	7	8
Patrol	0	1	1	1	1	0	1	1
Oil rec.	0	1	0	0	1	0	0	1
Tugging	1	1	1	1	1	1	1	1
SAR	0	0	1	1	0	1	1	1

Table 7.3: Response time Requirement

Patrol	24 hours
Oil Rec.	16 hours
Tugging	24 hours
SAR	12 hours

7.2 Case 1

In this first case, we shall see how the deployment of the initial vessel fleet consisting of eight different vessels, one of each. These vessel attributes and capabilities can be seen in Table 7.1 and Table 7.2. The goal with this test is to see if the model performs as expected before experimenting with multiple fleets and fleet sizes.

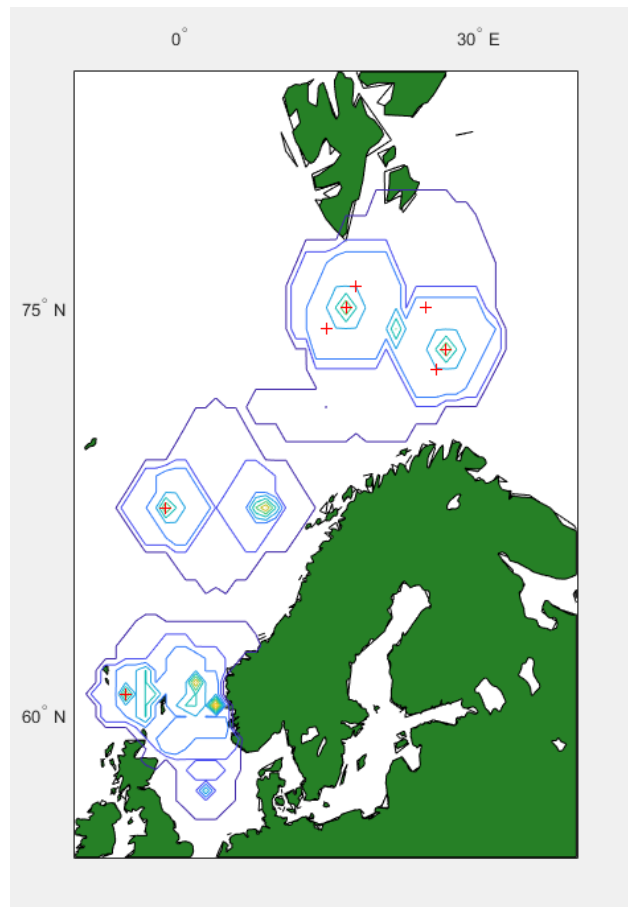


Figure 7.1: Deployment of single fleet with one vessel of each vessel type

The figure above illustrates the deployment of the initial fleet with one of each vessel type. From the generation of weighted nodes, we know that patrol nodes are awarded the most points so as expected some of the vessels have been allocated directly at these nodes and it is clear that constraint 6.3 is satisfied. Also, we can see that the effect of diminishing return on already covered nodes discussed in Section 6.8 has affected the deployment, and hence the vessels are some-

what spaced out between the contours. This effect will come become clearer when addressing the case in Section 7.3 which consists of multiple vessels.

7.3 Case 2

With the same input-data as for Case 1, we will now see how the coverage changes when testing for a number of fleets with a different number of vessels in each fleet. Before running the model a set of fleets is generated. The fleet generation is dependant on two parameters; the max number of vessels that can be generated for a curtain fleet and the number of fleets to be generated. The fleet generation can be seen in Appendix A.4. For this test scenario we have created 144 different fleets of different sizes, yet still with the same eight vessels used in Case 1. An outtake of the constructed fleet can be seen in Table 7.4.

Table 7.4: Example of developed fleet

Fleet number	Vessel Types								Number of vessels
	1	2	3	4	5	6	7	8	
39	3	0	4	3	3	4	1	1	19
40	4	3	1	4	2	0	4	3	21
41	1	1	2	2	4	4	3	4	21
42	3	4	3	1	1	1	2	1	16
43	4	3	0	3	0	3	0	0	13
44	0	1	4	1	1	1	4	2	14
45	3	2	3	0	2	2	2	4	18
46	2	2	2	1	1	3	3	2	16

Figure 7.3 shows a scatter plot where the blue dots represent all feasible solutions for a set of fleets. The number of vessels is plotted on the x-axis, and the objective function is plotted on the y-axis. Although the main objective of this thesis was to see how MCLP can assist decisions makers to address the tactical problem, the model also allows for addressing the strategical fleet size and mix problem. This plot gives a good visualization of how a large set of fleets performs

regarding coverage, and one can easily select a point and see the attributes of the fleet. As seen in Figure 7.3 the increase in weighted coverage grows significantly up to the point of 20 vessels and up until this point the different fleets are tightly grouped. However, after this point, we see that the dispersion of the weighted coverage increases. This is due to that the high rewarding nodes are covered more than once, providing less points for covering by the next vessel. This forces vessels to spread out and cover less profitable nodes. Another reason for the dispersion that occurs at around 20 vessels is that that the run-time for each fleet type is restricted to 20 seconds. There are two reasons for this; the first being that the model will try and solve to optimality for as long as it takes. Hence the model will not have time to fully locate optimality. Still, as we can see from Figure 7.2, the optimality gap shows that even though runs are time restricted, the solutions are close to optimality.

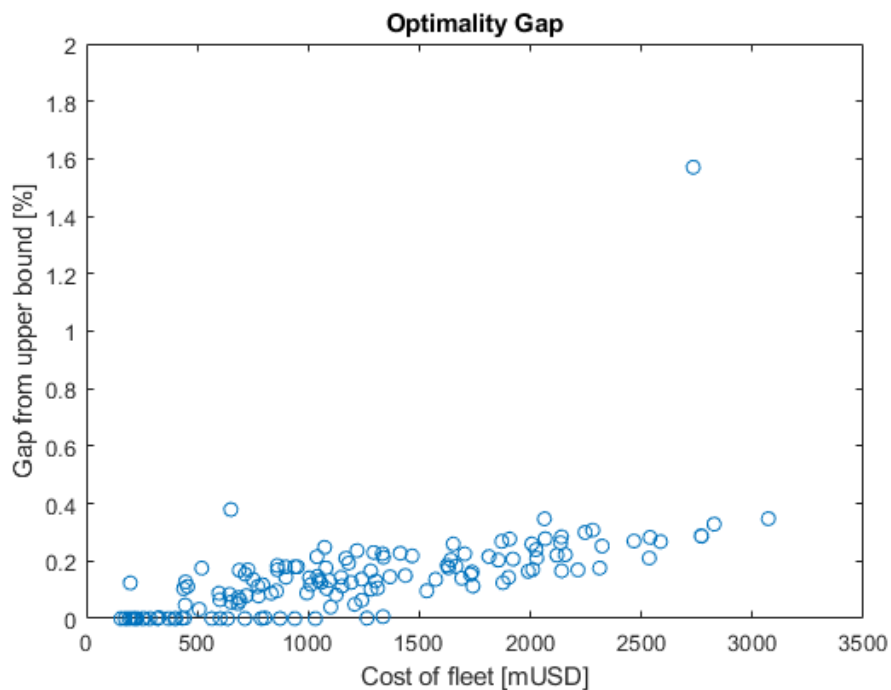


Figure 7.2: Optimality gap gap

The figure above shows that the located solutions are not far from the upper bound, and it shall be noted that this also means that the solution could potentially be closer to optimality than the figure illustrates. There are some points that strikes out, and this should be taken into consideration when evaluating fleet design if based on the solution.

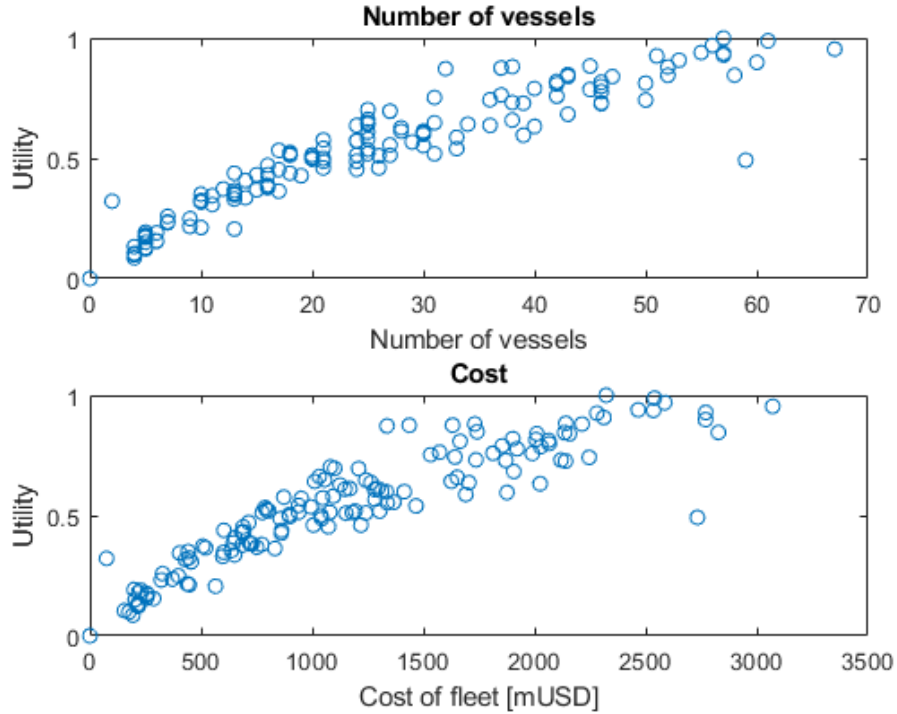


Figure 7.3: Scatter plot of number of vessels and cost of fleet

The scatter plots above shows that there the fleets are pretty gathered together. However, some points stick out. The red circle represents a point that has a high cost but does not provide any additional weighted coverage as expected regarding cost. The red point represents fleet number 135, and it consists of 59 vessels and costs \$2733M. First and foremost, considering that the Norwegian coast guard today has a fleet of 16 vessels, it is highly unlikely that a fleet of 59 vessels will become a reality, but we can learn from the results. As we can see the three most expensive vessels are vessel 6,7 and 8, and these are also the vessels that are in a high quantity in the fleet. This indicates that when decision makers shall determine an initial fleet, it could be wise to restrict the amount of these vessels to keep the costs down.

Table 7.5: Example of fleet

Vessel Type	1	2	3	4	5	6	7	8
Number of vessels	4	11	3	4	11	7	10	9
Vessel price [mUSD]	27	36	34	38	36	77	55	51

When exploring the different fleet alternatives, locating the Pareto frontier can help to visualize the change of utility for a given cost. The utility, in this case, is defined as the portion of coverage a fleet can serve in comparison to the maximum coverage for the complete scenario. Figure 7.4 shows the Pareto frontier for case 2 highlighted as red points. Points below the highlighted points are known as sub-optimal points, and in general, there should exist a solution on the path of the Pareto frontier. The highlighted points represents fleet 7, fleet 104 and fleet 142 and consists of 2, 32 and 57 vessels. If we look closely into the first red dot, we can get an indication as to why this fleet is providing such a high utility at a considerably lower cost than fleets at the same cost level. The fleet consists of two vessels, one of vessel 4 and one of vessel 5. As we can see vessel 4 and 5 both have patrol capabilities and due that the patrol nodes are the only nodes that are required to cover and the location of these nodes means that the fleet can meet the coverage constraints. However, only having two vessels provides little flexibility and such a fleet would be vulnerable to change in the location of potential patrol nodes. Although it would probably never be a situation where the Norwegian coast guard would acquire more than 20 vessels considering they have 16 today, the MCLP can be used to identify affordable fleet solutions or fleet solutions that can serve as an initial fleet for study in a further investigation under the consideration of some utility and cost constraint. In Figure 7.4 we can also see how one can identify a design space by introducing some cost and utility constraints and can narrow down the search for a good fleet mix.

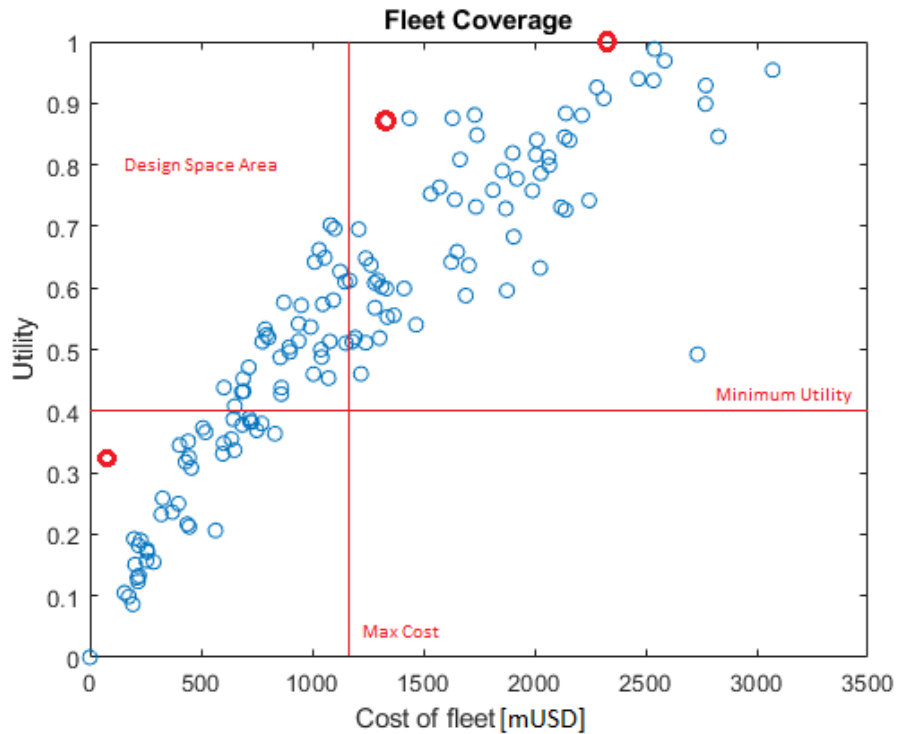


Figure 7.4: Pareto frontier for case 2 indicated by the red dots

Below is a figure showing the deployment of the result that gave the highest objective function. Of course it would never be an option to acquire 60 coast guard vessels, but it's interesting to see how the vessels would have been deployed. We can clearly see that the vessels will still be allocated to the areas that rewards the most points. With the coverage radius of the vessels being relatively small there are still a lot of pints rewarded for additional vessels to be deployed.

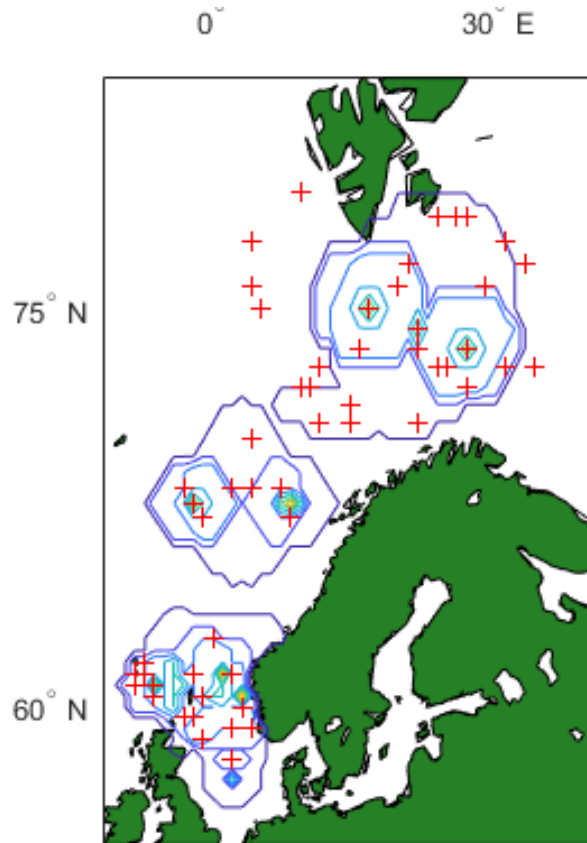


Figure 7.5: Deployment of vessels in case 2

7.4 Case 3

In the last two cases, we have assumed that some distance determined by the author determines the coverage distance. However, in reality, it would make more sense to look at the maximum response time for the individual and as a response, the maximum response time for the assessed operations is shown in Table 7.3. The result is a change in the subset from the subset described in Section 6.2.3 to the following subset: $N_{ivm} = \{j \mid \frac{d_{ij}}{VS} \leq RT_m \cap A_{mv} = 1\}$, where VS is the vessel speed and RT_m is the max response time for mission m . In the last two cases, the distances chosen meant that the vessels could respond to a mission with roughly two to three hours. Hence this effectively means that the covering distance for each vessel is increased significantly. This also means that the vessels have a lot more options to be placed on the network than before,

putting more stress on the model. This became clear when model returned no feasible solutions when time restricted to 20 seconds, so the time was increased to 240 seconds, resulting in feasible solutions shown in Figure 7.6.

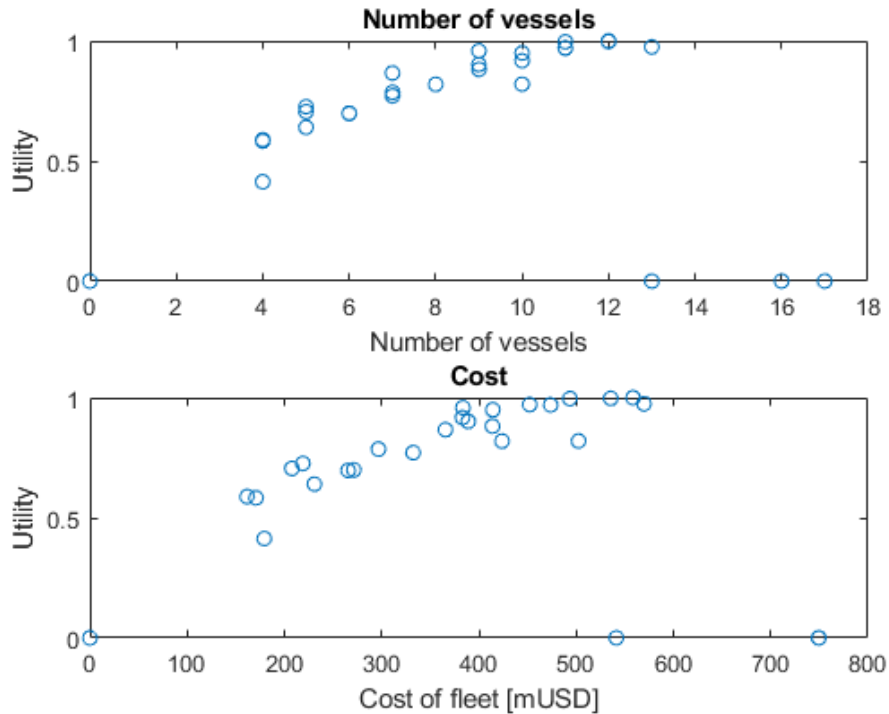


Figure 7.6: Scatter plot of fleet solutions

The results show that the change in subset conditions which led to an increase in coverage distance means that more demand nodes will be covered from a single vessel and that the coverage starts converging to a maximum level of coverage at around 7 vessels. In Section 5.2 we mentioned that all nodes are denoted a value of 1 or higher if the node is close to a mission site, and by looking at Figure 7.7 we see that there is room for placing vessels up in the left corner. However, as these nodes are only rewarded the score of 1 coverage, the added cost of additional vessels is not justified in additional coverage. From Figure 7.6 we also see that the model struggles to find feasible solutions for more than 14 vessels. Hence there is no incentive for solving for bigger fleets. These results do also give proof that the weighted diminishing marginal return presented in Section 5.3 is active, which is also one of the reasons why not more vessels are allocated the contoured areas where there is more award for covering. Notice from figure 7.7

that when the vessels have a high coverage radius, which translates to a longer response time requirement, the vessels do not have to be located at the epicenters of the contours. Instead they can be located nearby, and still offer full coverage support for the demand nodes.

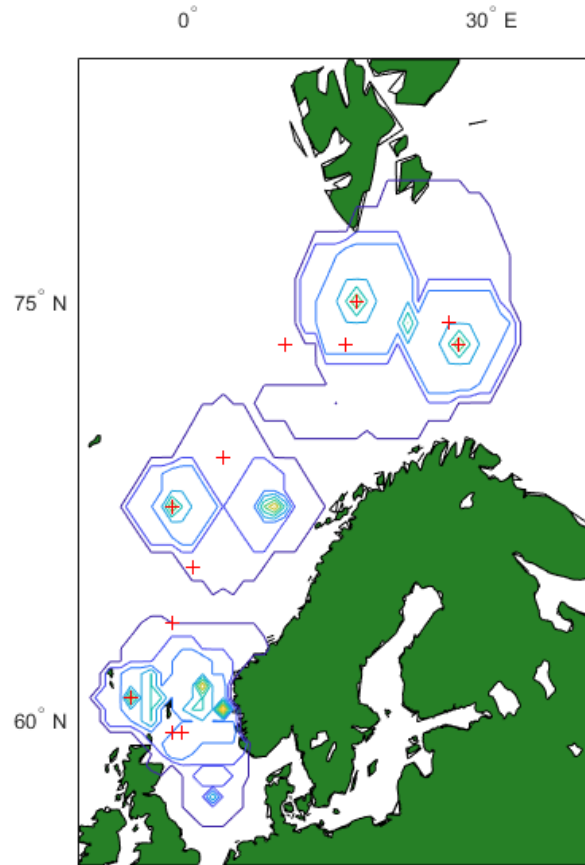


Figure 7.7: Deployment of fleet with maximum coverage

Chapter 8

Discussion

The objective of this thesis has been to see how facility covering location problems and in particular maximum covering location problems can be used to support decision makers in an optimization process of the coast guard problem. The focus has been on how MCLP can be used in solving the tactical deployment problem, but have also looked at how MCLP can contribute to solving the strategical fleet size and mix problem. The results in this thesis are not meant to reflect a real-world problem, but rather a simplified problem due to the lack of realistic data. In this chapter, the findings from the computational study are discussed. Also, discrepancies in model and data generation are discussed.

8.1 Computational studies

In chapter 7 MCLP was used for three case scenarios. The results showed how coverage with different fleets and fleet size. Compared to a classic MCLP the demand nodes are normally all given a value of one and the model cannot distinguish between facility locations, mainly because facilities are assumed homogeneous. In this thesis, we have denoted different values to the demand nodes depending on the mission to emphasize the importance of covering the specific nodes. From Figure 7.7 we see that the deployment is focused on the areas where there are most points to be awarded. This indicated that changing values on demand nodes can be used to show importance of covering the node or area. As mentioned in Chapter 7, by adding more vessels to the fleets the result starts converging to a maximum level of coverage, and we see that there is little added coverage from adding more vessels to the problem. These results are mostly expected and are also recognized in [Church and ReVelle \(1973\)](#) and other literature reviewed in Chapter 3. In the second and third case the effect of having a maximum covering distance to

having an individual maximum response time for the missions. Comparing the results we see that although there are far less vessels in the fleet in case 3 the overall coverage is ten times as great. The reason for this is that each vessel effectively has a higher coverage radius as discussed in Section 7.4 resulting in more nodes covered giving more points for fewer vessels. We can also see that the effect of the marginal return on the covered nodes in Case 2, where the covering radius is smaller. Which results in a reduction in awarded points faster. While in case 3, the large covering radius means that there is a higher reward per vessel, and more vessels get to "enjoy" the reward from mission nodes before the reward diminishes due to coverage by multiple vessels. All this goes to show how big the effect the coverage radius have on the model and that getting realistic data on this issue could be valuable for future design.

One challenge when using optimization software is that in many occasions locating an optimal solution can be time-consuming and near impossible, and therefore, the run-time of the model has been restricted to reduce computational time. In Chapter 7 the computational time was mentioned in that the run-time per fleet was restricted to 20 seconds for case 1 and 2 and 240 seconds for case 3, and the challenge then becomes how can we determine that we have an acceptable solution. An acceptable solution and the accuracy on an optimization problem will vary with the type of problem, and in general, there will a trade-off between having a lower computational time and accurate solutions. In this the lack of realistic data has resulted in that the author has created most of the data, and by Figure 7.2 we have shown that although having a maximum run-time the solutions are quite close to optimal, and Lundgren et al. (2012) has that near-optimal solutions are acceptable in instances where there is considerable uncertainty in input data which this thesis has. This has led to that in contrast to case 2 that tested 144 different fleets within a reasonable time; Case 3 has only managed 30 fleets.

A method for evaluating the fleets has been to look at the Pareto frontier showing the trade-off between cost and utility, and as explained in Section 7.3, by implementing some design restrictions such as a maximum cost or minimum utility as seen in in Figure 7.4 a design space can be established as a source for further fleet development. In Section 7.3 we discussed that the MCLP could serve as a support to solve the strategic fleet size and mix, and also, the design space area could be a good starting point when evaluating the operational problem. It is important to re-

alize that even though in this thesis seek to address the tactical problem, the close relationships between tactical, strategic and operational level of planning means that it is hard to single out a fleet in one stage and then move on to the next. In reality, there is an iterative process working through the stages back and forth to a definite solution is established. Hence, coming with decisive recommendations. All in all by looking at the results and especially Figure 7.7 there is a strong indication that MCLP could be a handy tool for fleet deployment.

8.1.1 Fleet generation

In Section 7.3 we discussed one of the points that deviated from the dominated area of feasible solutions, and we saw that the reason for this was that the fleet consisted of a high level of expensive multipurpose vessels. The fleets were generated randomly such that a fleet may select as much as 12 vessels of each vessel types, which gave the results given in Figure 7.4. However, by identifying the negative trade-off in utility by choosing a high number of expensive multipurpose vessels, constraining the number of vessels being made of this type could eliminate such a deviation and further improve the solution.

8.2 Model Discrepancies

Modeling an exact real-world process is very difficult in operations research, and in order model and understand this coast guard problem simplifications have been made. This section will identify some of the discrepancies to the model and evaluate the significance. Model discrepancies involve first and foremost the simplifications made in the model formulations. These simplifications are discussed in Chapter 5. One of the biggest discrepancies in the model is the assumption of the problem being static planning and discrete network. Static planning is characterized by that the input variables are the same and does not change during the planning process. This assumption is one of the more common simplifications made, and almost unavoidable in many planning processes due to the increase in complexity associated with dynamic planning. In addition, its assumed that the network of the problem is discrete, in this case

meaning that feasible solutions are constrained to a network of nodes also a common simplification made for MCLP (Daskin, 1995). However, with the large set of nodes used in this thesis, we still get a prominent result. In this report, we have assumed that the model is mixed integer programming problem meaning that no fractional solutions are permitted. This makes sense since dividing a vessel between two locations would not be possible. In a future problem, it could be desirable to change the perspective of the problem and develop a case where demand nodes are allowed to be partially covered by vessels. This would be particularly interesting if stochastic elements were added and adequate coverage could be justified by the probability of a mission or an accident.

8.2.1 Heuristics

Heuristics have not been implemented in the model formulation in this report, apart from heuristics used by Xpress IVE when solving the optimization problem. In case 2 the model solved for 144 fleets within a reasonable time. However, case 3 had a far greater computational time for only 30 fleets where it did not even manage to find a feasible solution within the time limit. Adding sufficient heuristics to this case could help to find a solution within a reasonable time hence should maybe be considered in the future. However, it is important to know that adding heuristics may provide a feasible solution, but the quality of the solution would still have to be addressed.

8.3 Data Discrepancies

One of the main issues with this thesis is the lack of realistic data related to the coast guard problem which has lead to that the author has had to generate a lot of the data. However, as mentioned in the introduction this thesis is a continuation on the work done by Buland (2017), hence to limit the scope of data collection, some input variables and vessel data have been borrowed from his work. The goal of this thesis has not been to present a solution as a design recommendation, but rather present result that shows how MCLP can be used to support decision

makers in the future. Therefore, borrowing the data does not harm the results as this project is to explore MCLP as a conceptual design. This is also the reason for the model has been kept strictly static and deterministic. In most cases and model formulations, adding uncertainty is imperative to try and recreate a real-world problem. However, since there already is a degree of uncertainty to the data used, adding stochastic elements will not improve the model, but make it harder to solve.

8.3.1 Demand zones and weights

Demand zones and weights are essential parameters to establish to make the MCLP work. The locations of these demand zones were extensively discussed in Chapter 5. However, again we need to emphasize that the chosen locations is purely experimental and has no relation to any demand of the Norwegian coast guard or other government agency. In general, each node in the network represent a location for potential mission or demand, and depending on the mission the node gets denoted a weighted demand reflecting the importance of the mission, and again, this weight is purely experimental and does not reflect the desirability expressed by the Norwegian coast guard. Additionally, the nearby nodes to the center node of a mission get denoted a value, illustration an type of gradual covering. These are all simplifications that affect the results. However, in this report, it illustrates how one could model a problem if more realistic data where obtained. Uncertainty related to this will always be an issue, and as seen from Case 2 and 3 the uncertainty normally increases with the complexity as reviewed by [Gaspar et al. \(2012\)](#). We see that the inaccuracies increases as the covering distance increases, hence making it harder to allocate the vessels, but the model still manages to locate feasible solutions.

8.3.2 Distance matrix

Another thing to take into consideration is the generation of the distance matrix. There are several ways to calculate the distance between two points, where the Pythagorean theorem might be one of the simpler calculations, and for a flat surface or if points are close together this will provide a fairly accurate result. However, when working with coordinates and the distance be-

tween the nodes become large, we need to be aware of how the distance is generated. As described in Section 6.2.1 we used the built distance function to generate. The function returns the Haversine distance function which calculates the arc length between two coordinates on an ellipsoid. This potential discrepancy has not been an issue in this report as we have used Haversine method. However, we have in this report stated that the lack of realistic data has led to a significant degree of uncertainty in results, and therefore for future studies on this field, where more realistic data is obtained its important be aware of the uncertainty tied to calculating distances with the Pythagorean theorem (Veness).

8.4 Alternative objective function

In most cases of MCLP the objective is to maximize the covered area subject to a set of constraints. Case 3 presented a solution where the objective was to cover specific missions within a maximum response time. However, the objective function was still to maximize coverage. Another perspective to the objective function could have been to look at minimizing response time, which could have given interesting comparable results to the cases tested in this thesis. An attempt to this was conducted. However, this resulted in enormous stress on the generated text file with the subset where the time for a vessel had to be generated for all nodes and all vessels, resulting in 54 million lines of text, which turned out to be too demanding for the computer used in this thesis. By developing a P-median approach, as described by Charles S ReVelle (1970), for the coast guard problem might resolve the problem in the future.

Chapter 9

Conclusion and Further Work

This chapter concludes the research done in this thesis and present insights towards relevant future work.

9.1 Conclusion

The purpose of this thesis is to see how a maximum covering location approach can be used for solving the coast guard deployment problem. The maximum covering location method has proven to be an effective method in facility location, especially within emergency response location, which is the reason for the interest in testing the applicability to the coast guard problem. One challenge for the MCLP in this thesis was how to measure the effectiveness of covering specific mission nodes. To resolve this issue mission demand nodes were denoted a weighted score, where a higher score indicates higher mission importance. In the literature, it became clear that one of the weaknesses of the classical MCLP is the notion of "all or nothing" coverage (5.2). This is resulting in denoting a reduced score to neighbouring nodes to illustrate the value of being almost covered. Included in the model were also a method for diminishing marginal return for demand nodes being covered by more than one vessel.

The computational study presented in this thesis presents two scenarios, one with a short individual response distance for each vessel type, and one where a maximum response time related to each specific mission type. This could be compared to the first scenario by multiplying the vessel speed, effectively resulting in a far greater coverage radius than the first scenario. The results showed, as can be expected that far less vessels were required before the model converged towards a maximum coverage. One of the primary objectives was to see how the applied method could support decision makers in solving the tactical planning problem, often related

to vessel deployment. By looking the highlighted points in Figure 7.4, a common denominator for the fleets is that these fleets consists of multipurpose vessels at low cost, which in this case gives a best result. In a real-world scenario however, a multipurpose vessel with capabilities such as ice class and helicopter could be preferable. These types of capabilities is not considered i this report, hence the model much rather select the less expensive vessels. The model also showed that MCLP could indeed be used as a support tool in vessel deployment. However, due to the lack of relevant data and uncertainty in the generated data providing, any definitive recommendations towards coast guard fleet deployment would be challenging. In addition to MCLP showing the ability to reflect on the tactical planning problem, it also showed applicability in the strategical planning problem. The model was able to test a vast number of fleets, and by adding some constraints to the plotted results, a design space including a set of fleets could be defined. This result could provide a better insight to the strategical fleet size and mix problem. However, this has not been optimized, nor been the objective of research in this thesis. The coast guard MCLP are faced with many limitations. In a design process, there will always have to be made model simplifications to address the real world problem. However, the primary limitation is the availability of relevant data, hence most of the data has been generated by the author. The goal for this study was to illustrate how MCLP could be adapted to a coast guard problem, which has been achieved and documented.

9.2 Further work

This thesis has conducted a study on how a maximum covering location approach could be used for addressing a coast guard problem. Although this thesis managed to show that MCLP has potential as a support for decision makers in a coast guard planning problem, some areas need to be addressed further. First and foremost, better and more relevant data needs to be generated or obtained from the Norwegian coast guard. A thorough mission and risk evaluation would be recommended could provide a more realistic result. Probability should be added to decide the weight of demand nodes. Another future study is to experiment with different objective function formulation, like for example minimizing response time, which could provide further

insight into the problem. Also vessel capabilities are in this study limited to patrol, tugging, SAR(search and rescue) and oil recovery. Also, including helicopter capabilities and ice class operability could be of interest as this would change how the fleet structure entirely. Furthermore, combining this study with routing is also an interesting study for the future. By including routing to the model it might be possible to get a more dynamic measure of effectiveness, since vessels are then able to be located on the paths between the nodes in the network.

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Appendix A

Matlab

A.1 Matlab - Weighted demand nodes generation

This script generates the mission locations and denoted weight to demand nodes

```
%% This file generates a text file %1

%clc; clear all;
%% Create distance matrix
%S = load('distMatrix.mat');
%Coord = S.Coord;

S = load('DistanceMatrixNY3.mat');
Coord = S.Coordny;

%run Distancematrix.m
%load vesseldata
vesseldata = xlsread('shipparameters.xlsx','Ark1','C4:J10');
%vessel Speed
VS = vesseldata(2,:);
%vessel capabilities
A = transpose(xlsread('shipparameters.xlsx','Ark1','C13:J16'));
% Mission response time
RT = xlsread('shipparameters.xlsx','Ark1','C18:C21');

%% Generate mission Demand points
% 1 = required poatrol node
% 2 = oil recovery operation
% 3 = Tugging operation
% 4 = Search and rescue

MP = zeros(length(Coord),1);

MP(671) = 2;
MP(630) = 2;
MP(1927) = 1;
MP(1419) = 1;
MP(490) = 1;
MP(272) = 1;
MP(730) = 4;
MP(1000) = 3;

Missiontype = MP;
Missiontype( ~any(Missiontype,2), : ) = [];
```

```

%% Create response time vector

RT = zeros(1,length(Missiontype));
for i = 1:length(Missiontype)
    if Missiontype(i) == 1
        RT(i) = 24;
    elseif Missiontype(i) == 2
        RT(i) = 16;
    elseif Missiontype(i) == 3
        RT(i) = 24;
    elseif Missiontype(i) == 4
        RT(i) = 12;
    end
end

%% Generating weights on the nodes
WP = ones(length(Coord),1); %weighted point matrix

for i = 1:length(Coord)
    if MP(i) == 1
        for j = 1:length(Coord)
            if S.dist(j,i) == 0
                WP(j) = WP(j)+6;
            elseif S.dist(j,i) <= 25 && S.dist(j,i) > 0
                WP(j) = WP(j) + 4;
            elseif S.dist(j,i) <= 100 && S.dist(j,i) > 25
                WP(j) = WP(j) + 3;
            elseif S.dist(j,i) <= 250 && S.dist(j,i) > 100
                WP(j) = WP(j) + 1;
            end
        end
    elseif MP(i) == 2
        for j = 1:length(Coord)
            if S.dist(j,i) == 0
                WP(j) = WP(j)+5;
            elseif S.dist(j,i) <= 25 && S.dist(j,i) > 0
                WP(j) = WP(j) + 3;
            elseif S.dist(j,i) <= 100 && S.dist(j,i) > 25
                WP(j) = WP(j) + 2;
            elseif S.dist(j,i) <= 250 && S.dist(j,i) > 100
                WP(j) = WP(j) + 1;
            end
        end
    elseif MP(i) == 3
        for j = 1:length(Coord)
            if S.dist(j,i) == 0
                WP(j) = WP(j)+7;
            elseif S.dist(j,i) <= 25 && S.dist(j,i) > 0
                WP(j) = WP(j) + 5;
            elseif S.dist(j,i) <= 100 && S.dist(j,i) > 25
                WP(j) = WP(j) + 2;
            elseif S.dist(j,i) <= 250 && S.dist(j,i) > 100
                WP(j) = WP(j) + 1;
            end
        end
    elseif MP(i) == 4
        for j = 1:length(Coord)
            if S.dist(j,i) == 0

```

```
        WP(j) = WP(j)+7;
    elseif S.dist(j,i) <= 25 && S.dist(j,i) > 0
        WP(j) = WP(j) + 5;
    elseif S.dist(j,i) <= 100 && S.dist(j,i) > 25
        WP(j) = WP(j) + 2;
    elseif S.dist(j,i) <= 250 && S.dist(j,i) > 100
        WP(j) = WP(j) + 1;

    end
end
end

%% Determining vessel range in nautical miles
%VR = [190 250 200 180 170 150 220 230];
VR = [50 40 70 90 60 60 45 50];

%% Inputdata to mosel file
%number of vessels
numVessels = length(VS);
%number of nodes
numNodes = length(Coord);
%number of missions
numMissions = 0; %pre-defining numMissions
for i = 1:length(MP)
    if MP(i) >= 1
        numMissions = numMissions + 1;

    end
end
```

A.2 Matlab - Distance Matrix generation

This script generates the distance matrix. The function "landmask" is used to filter away nodes that are on land ([Greene, 2014](#))

```

%% This script will generate the distances from one node to another
% with the hevasine method
tic
clc; clear all;
%load vesseldata
vesseldata = xlsread('shipparametrics.xlsx','Ark1','C4:J10');

%% Gereate number of nodes in the field
numnodes = 50; %

Lon = (-10:50/numnodes:40);
Lat = (52:28/numnodes:80);

coordinates = zeros(length(Lat),2);
coordinates(:,1) = Lat;
coordinates(:,2) = Lon;

kordinater = {Lat,Lon};
[a, b] = ndgrid(kordinater{:});
Coord = [a(:) b(:)];
[u,t] = size(Coord);

% This filters away coordinated that are in the water but are not of
% interest

for i = 1:u
    if landmask(Coord(i,1),Coord(i,2)) == 0
        Coord(i,:) = Coord(i,:);
    else
        Coord(i,:) = [NaN,NaN];
    end
end

for i = 1:u
    if Coord(i,2) >= 7 && Coord(i,1) <= 63
        Coord(i,:) = [NaN,NaN];
    elseif Coord(i,2) >= 13 && Coord(i,1) <= 67
        Coord(i,:) = [NaN,NaN];
    else
        Coord(i,:) = Coord(i,:);
    end
end

Coordny = Coord;

%create zero matrix for the distance matrix
dist = zeros(length(Coordny));

%% Calculate distances based on coordinates
for i = 1:length(Coordny(:,1)) %i = lat
    for j = 1:length(Coordny(:,2)) %j = lon
        dist(i,j) = distance(Coordny(i,1),Coordny(i,2),Coordny(j,1),...
```

```
        Coordny(j,2),earthRadius('nm')*1.1;  
    end  
end  
%  
toc
```

A.3 Matlab - Generating mosel text file

This script generates the text that is used in the optimization problem i Xpress. This script also generates the subsets used in Chapter 7.

```

%% Create Mosel text file
%% MoselfileGen nedd to be runned before running this
fid = fopen('DataInput_NY_case2.txt','wt');

%Sets:
fprintf(fid,'!Sets \n');
fprintf(fid,'nNODES: ');
    fprintf(fid,'%1.0f \n',numNodes);
fprintf(fid,'nVESSELS: ');
    fprintf(fid,'%1.0f \n',numVessels);
fprintf(fid,'nMISSIONS: ');
    fprintf(fid,'%1.0f \n',numMissions);
fprintf(fid,'nK_vessels: ');
    fprintf(fid,'%1.0f \n',20);%numVessels
    fprintf(fid,'\n');

%Subsets:
M = length(MP);

%% subset N_inv range dependant --> Case 2
%
fprintf(fid,'!Subsets \n');
%Writes the NI(i,v,m): such that j will become a node in the subset if
%the distance between node j and i is less than the max resonance distance
%for vessel v, and that vessel v can perform mission m.
fprintf(fid,'N: [ \n');
for i = 1:length(Coord)
    for v = 1:length(VS)
        for m = 1:numMissions
            nodesI = [];
            for j = 1:length(Coord)
                if S.dist(i,j)<= VR(v) && VR(v) >= 0 &&...
                    A(v,Missiontype(m)) == 1
                    nodesI = [nodesI, j];
                end
            end
            fprintf(fid, '(');
            fprintf(fid,'%1.0f %2.0f %2.0f',i,v,m);
            fprintf(fid, ') [');
            fprintf(fid,'%2.0f %2.0f %2.0f %2.0f %2.0f %2.0f %2.0f %2.0f %2.0f %2.0f ', nodesI);
            fprintf(fid,'] \n');
        end
    end
end
end
fprintf(fid,'] \n');

%% Subset where N_inv = (j|(d_ij/VS) <= TM) --> Case #

% fprintf(fid,'!Subsets \n');
% %Writes the NI(i,v,m): such that j will become a node in the subset if
% %the distance between node j and i is less than the max resonance distance
% %for vessel v, and that vessel v can perform mission m.
% fprintf(fid,'N: [ \n');
% for i = 1:length(Coord)

```

```

%   for v = 1:length(VS)
%       for m = 1:numMissions
%           nodesI = [];
%           for j = 1:length(Coord)
%               if S.dist(i,j)/VS(v) <= RT(m) && A(v,Missiontype(m)) == 1
%                   nodesI = [nodesI, j];
%               end
%           end
%       fprintf(fid, '(');
%       fprintf(fid, '%1.0f %2.0f %2.0f', i, v, m);
%       fprintf(fid, ') [');
%       fprintf(fid, '%2.0f %2.0f %2.0f %2.0f %2.0f %2.0f %2.0f %2.0f %2.0f %2.0f ', nodesI);
%       fprintf(fid, '] \n');
%   end
% end
% end
% fprintf(fid, '] \n');

%Generates the subset V of vessels v
fprintf(fid, 'VI: [ \n');
for i = 1:length(Coord)
    vessels = [];
    for v = 1:length(VS)
        vessels = [vessels, v];
    end
    fprintf(fid, '(');
    fprintf(fid, '%1.0f', i);
    fprintf(fid, ') [');
    fprintf(fid, '%2.0f %2.0f ', vessels);
    fprintf(fid, '] \n');
end
fprintf(fid, '] \n');

%Parameters:
fprintf(fid, '!Parameters \n');
%Writes the WCP(i,m,k): "Points rewarded for covering node i"

fprintf(fid, 'W: [ \n');
for i = 1:length(Coord)
    for k = 1:20:numVessels
        for m = 1:numMissions
            fprintf(fid, '(');
            fprintf(fid, '%1.0f %2.0f %2.0f', i, m, k);
            fprintf(fid, ') ');
            fprintf(fid, '%2.3f ', WP(i)/((2^k)/2));

            fprintf(fid, ' \n');
        end
    end
end
end
fprintf(fid, '] \n');

%Number of demanded vessels at node i
%VD = Vessel Demand
fprintf(fid, 'VD: [ \n');
for i = 1:length(Coord)
    fprintf(fid, '(');
    fprintf(fid, '%1.0f', i);
    fprintf(fid, ') ');
    if MP(i) == 1
        fprintf(fid, '%2.0f ', 1);
    end
    fprintf(fid, '\n');
end

```



```
    else
        fprintf(fid, '%2.0f ', 0);
        fprintf(fid, '\n');
    end

end

fprintf(fid, 'J\n');
fclose(fid);
```

A.4 Matlab - Run Model

This script runs Xpress through matlab, which enables us to solve for multiple vessels without doing manual inputs in Xpress IVE. In this script its possible to adjust the maximum time for longer solves by changing the value MAXTIME.

```

%% Plot results script.
%clear all
%% Create fleets of vessels
% Eight vessel-types in each fleet

FULL_ENUM = 0; %Fully enumerates design space
LATIN_HYP = 1; %Latin hypercube sampling

MaxObj = 0;
cost = [27.2000    36.5000    34.3000    38.3800    36.9200    77.2200    55.2200    51.9300];

numfleetype = 12; %Number of fleets for each numfleet --> 5 * 6 = 30
numfleet = 12; %Maximun number of vessels of each vesseltype

fleet = zeros(1,8);

for i = 1:numfleet
    if FULL_ENUM == 1
        Vs1 = 0:1:numfleetype;
        Vs2 = Vs1;
        Vs3 = Vs1;
        Vs4 = Vs1;
        Vs5 = Vs1;
        Vs6 = Vs1;
        Vs7 = Vs1;
        Vs8 = Vs1;
        Fleet_Comb = combvec(Vs1,Vs2,Vs3,Vs4,Vs5,Vs5,Vs6,Vs7);
    elseif LATIN_HYP == 1
        Fleet_Comb = round(i*lhsdesign(numfleetype,8,'smooth','off'));
    end

    r_fleet = length(fleet(:,1));

    for j = 1:length(Fleet_Comb(:,1))
        fleet(r_fleet + j,:) = Fleet_Comb(j,:);
    end
end

fleet(1,:) = [];

%% Fleets being run through Xpress

for i = 1:length(fleet(:,1))
    Vesselfleet = fleet(i,:);

    fid = fopen('fleet1.txt','wt');
    fprintf(fid,'P: [ \n');
    for j = 1:length(Vesselfleet)
        fprintf(fid, '(');
        fprintf(fid, '%1.0f',j);
        fprintf(fid, ')');
        fprintf(fid, '%2.0f %2.0f ',Vesselfleet(j));
    end
    fprintf(fid, '\n');
end

```

```

end
fprintf(fid, 'J\n');

% Initialize for Xpress Solver.
DEPLOY = 0; %zeros(I,V);
COVERI = 0; %zeros(I,M);
BINARY = 0; %zeros(I,M);
MAXTIME = -20; %Set maximum search time per instance.

[retcode, exitcode] = moselexec('maxcoveragesimple_Case3.mos')

if objval == 0
    i = i+1;
else
    FinalDeploy(i,:) = sum(DEPLOY);
    FinalObVal(i,1) = objval ;
    OptGap(i,1) = GAP;
    if objval >= MaxObj
        MaxObj = objval;
        finDeploy = DEPLOY;
    end
end
end

%% Determine coverage
covernodes = zeros(length(Coord),1);

for x = 1:length(Coord)
    for y = 1:numMissions
        for z = 1:numVessels
            if COVERI(x,y,z) >= 1
                covernodes(x) = 1;
            end
        end
    end
end
end
co = Coord(:,1);
aktuellenoder = isfinite(Coord(:,1));

cn = sum(covernodes);
an = sum(aktuellenoder);
schnitt = cn/an;

nodecoverage(i) = schnitt;

end

%% Locate optimality cap

for i = 1:length(OptGap)
    Optimality_gap(i) = abs(FinalObVal(i)-OptGap(i))/FinalObVal(i);
end
end

```

A.5 Matlab - Plot results

This script plot the results.

```

%% Plot results

load DistanceMatrixNY4.mat

%% Plot the weighted contours

clf('reset')

cost = [27.2000    36.5000    34.3000    38.3800    36.9200    77.2200    55.2200    51.9300];

%% Code finding deployd vessels for Case 1.

% Dep2 = [];
% for i = 1:length(finDeploy)
%     for j = 1:8
%         if finDeploy(i,j) >= 0.95 && finDeploy(i,j) <= 1.05
%             Dep2(i,:) = Coord(i,:);
%         end
%     end
% end
% Dep2( ~any(Dep2,2), : ) = [];

%% Code finding deployd vessels for case 2 and 3

Dep2 = [];
for i = 1:length(finDeploy)
    for j = 1:8
        if finDeploy(i,j) >= 0.95
            Dep2(i,:) = Coord(i,:);
        end
    end
end
Dep2( ~any(Dep2,2), : ) = [];

for i = 1:length(FinalDeploy(:,1))
    deploy(i,1) = sum(FinalDeploy(i,:));
    CostFleet(i,1) = cost*transpose(FinalDeploy(i,:));
end

%% plot on map

workingFolder = tempdir;
files = gunzip('gshhs_c.b.gz', workingFolder);
filename = files{1};
indexfile = gshhs(filename, 'createindex');

latlim = [52 80];

```

```

lonlim = [-10 40];
S = gshhs(filename, latlim, lonlim);

%This section makes sure that we wish to plot the coastline.
levels = [S.Level];
unique(levels);
L1 = S(levels == 1);

figure(2)
axesm('mercator', 'MapLatLimit', latlim, 'MapLonLimit', lonlim)
%gridm;
mlabel;
plabel;
grid off
geoshow([L1.Lat], [L1.Lon], 'Color', 'black')
geoshow('landareas.shp', 'FaceColor', [0.15 0.5 0.15]);

%% Display contours on the map
kontur = transpose(vec2mat(WP,51));
contourm(a,b,kontur)

%geoshow(Coord(:,1), Coord(:,2),'DisplayType','point', 'Marker','.', 'Color','red','MarkerSize',2)
%geoshow(Dep2(:,1), Dep2(:,2),'DisplayType','point', 'Marker','*', 'Color','blue','MarkerSize',2)
geoshow(Dep2(:,1), Dep2(:,2),'DisplayType','point', 'MarkerEdgeColor','red') %display deployment

%% display accident sights
% Isolate the accidents
acc = [];
for i = 1:length(MP)
    if MP(i) >= 1
        acc(i,:) = Coord(i,:);
    end
end
acc(~any(acc,2), :) = [];

%% Plot missions on map

%geoshow(Coord(:,1),Coord(:,2),'DisplayType','point', 'Marker','.', 'MarkerEdgeColor','red') %Plot coordinates
%geoshow(acc(:,1), acc(:,2),'DisplayType','point', 'MarkerEdgeColor','blue') % plot missions

tightmap

%% Utility plots and optimality gaps

utilityvector = FinalObVal/MaxObj;

figure(1)
subplot(2,1,1) % add first plot in 2 x 1 grid
plot(deploy,utilityvector,'o')
title('Number of vessels')
xlabel('Number of vessels')
ylabel('Utility')

subplot(2,1,2) % add second plot in 2 x 1 grid
plot(CostFleet,utilityvector,'o')
title('Cost')
xlabel('Cost of fleet [mUSD]')

```

```
ylabel('Utility')

figure(3)
plot(deploy,utilityvector,'o')

figure(5)
plot(CostFleet,utilityvector,'o')
title('Fleet Coverage')
xlabel('Cost of fleet [mUSD]')
ylabel('Utility')

% figure(4)
% plot(deploy,Optimality_gap,'o')

% figure(6)
% plot(CostFleet,Optimality_gap,'o')
```

Appendix B

Xpress IVE

B.1 Xpress IVE - Mosel file

This script runs and solves the MCLP and returns output results to Matlab

```
!@encoding CP1252
model SimpleMaxCovering
uses "moxprs"; !gain access to the Xpress-Optimizer solver

options explterm
options noimplicit

parameters
    DATAFILE = 'DataInput_NY_case2.txt';
    DATAFILE2 = 'fleet1.txt';
end-parameters

!Sets
declarations
    NODES: set of integer;           !Nodes
    VESSELS: set of integer;        !Vessel types
    MISSIONS: set of integer;       !Mission types
    K_vessels: set of integer;      !Number of vessels
    nNODES: integer;
    nVESSELS: integer;
    nMISSIONS: integer;
    nK_vessels: integer;
end-declarations

initializations from DATAFILE
    nNODES;
    nVESSELS;
    nMISSIONS;
    nK_vessels;
end-initializations

NODES := 1..nNODES;
VESSELS := 1..nVESSELS;
MISSIONS := 1..nMISSIONS;
K_vessels := 1..nK_vessels;

finalize(NODES);
finalize(VESSELS);
finalize(MISSIONS);
finalize(K_vessels);
```

```

declarations
    N: dynamic array(NODES,VESSELS,MISSIONS)    of set of integer;           !Nodes j to which a vessel v at node j can respond to mission m (Inner barrier)
    VI: dynamic array(NODES)                    of set of integer;           !Vessel types fit for nodes i.
end-declarations

initializations from DATAFILE
    N;
    VI;
end-initializations

forall(i in NODES, v in VESSELS, m in MISSIONS) finalize(N(i,v,m));
forall(i in NODES) finalize(VI(i));

!Parameters
declarations
    W: array(NODES,MISSIONS,K_vessels)        of real;
    VD: array(NODES)                          of real;
    P: array(VESSELS)                         of integer;
end-declarations

initializations from DATAFILE2
    P;
end-initializations

initializations from DATAFILE
    W;
    VD;
end-initializations

!For matlab:
declarations
    gap: mpvar;
    mipObjVal: mpvar;
    bestBound: mpvar;
end-declarations

declarations
    simplexiter: integer;
    DEPLOY: dynamic array(NODES,VESSELS) of integer;
    COVERI: dynamic array(NODES,MISSIONS) of integer;
    POINTS: dynamic array(NODES,VESSELS) of integer;
    MAXTIME: integer;
end-declarations

initializations from "matlab.mws:"
    DEPLOY;
    COVERI;
    POINTS;
    MAXTIME;
end-initializations

!Variables:
declarations
    x: dynamic array(NODES,VESSELS)          of mpvar;
    yI: dynamic array(NODES,MISSIONS,K_vessels) of mpvar;
end-declarations

!Creating variables

```



```

forall (i in NODES,v in VI(i)) do
    create (x(i,v));
    x(i,v) is_integer;
end-do

forall (i in NODES,m in MISSIONS,k in K_vessels) do
    create (yI(i,m,k));
    yI(i,m,k) is_integer;
end-do

declarations
    Objective:linctr;
    Constraint1: array(NODES,MISSIONS) of linctr;
    Constraint2: array(NODES) of linctr;
    Constraint3: array(VESSELS) of linctr;
    Constraint4: array(K_vessels,MISSIONS) of linctr;
    Constraint5: array(NODES) of linctr;
    Constraint6: array(NODES,MISSIONS,K_vessels) of linctr;
end-declarations

!Maximizes demand covered:

Objective:= sum(i in NODES)sum(m in MISSIONS)sum(k in K_vessels) W(i,m,k)*yI(i,m,k);

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!Ensures covering by vessel that are within range.
forall(i in NODES, m in MISSIONS) do
    Constraint1(i,m):=
        sum(k in K_vessels) yI(i,m,k) - sum(v in VI(i))sum(j in N(i,v,m)) x(j,v) <= 0;
end-do

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!Ensures sufficient patrol vessels at node i:
forall(i in NODES) do
    Constraint2(i):=
        sum(v in VI(i)) x(i,v) >= VD(i);
end-do

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

!Ensures that the number of vessels assigned cannot exceed the number of vessels in the fleet.
!forall(i in NODES, v in VI(i)) do
forall(v in VESSELS) do
    Constraint3(v):=
        sum(j in NODES) x(j,v) = P(v);
end-do

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

! This constraint ensures that inly one vessel of vessel k can cover one node.
! This leads to the reduction in score for vessel 2
forall(m in MISSIONS,k in K_vessels, i in NODES) do
    Constraint6(i,m,k) :=
        yI(i,m,k) <= 1;
end-do

declarations
    StopS: real;
end-declarations

setparam("XPRS_VERBOSE",true);
setparam("XPRS_MAXTIME",MAXTIME);

```

```
maximise(Objective);
simplexiter:=getparam("XPRS_simplexiter");
writeln(Objective, "%: ", getobjval);

! writes results to matlab
initializations to "matlab.mws:"
    simplexiter;
    evaluation of getparam("XPRS_BESTBOUND") as "GAP";
    evaluation of getobjval as "objval";
    evaluation of array(i in NODES,v in VESSELS) x(i,v).sol as "DEPLOY";
    evaluation of array(i in NODES,m in MISSIONS,k in K_vessels) yI(i,m,k).sol as "COVERI";

    end-initializations
!exit(getprobstat)

end-model
```