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Slope Stability Assessment with Bayesian Updating

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Abstract:
This thesis aims to explore the potential of implementing Bayesian updating in the probabilistic analysis of a geotechnical slope stability assessment. Bayesian updating is a well-defined method for combining supplementary information with already existing knowledge. In geotechnical slope stability assessment the previously existing knowledge would typically be a model of the slope, with material descriptions based on knowledge of the area and ground investigations. The supplementary information would typically be additional ground investigations, or observations of the performance of the slope. Including more information in the calculation model is expected to improve the accuracy of the predictions of slope behaviour. The model in this thesis is updated based on the observation that the slope is stable.

The Bayesian updating is implemented in two different analyses, where one models the spatial variability within each soil layer, while the other models each soil layer spatially homogeneous. The probabilistic analysis is implemented using Monte Carlo Simulations, and the slope stability assessment is conducted with an finite element model constructed in Plaxis 2D 2017.1. The shear strength of each material in the slope is modeled as a random variable. In the Single Random Variable (SRV) analysis these distributions are used to sample shear strength values that are applied to each layer. In the Conditional Random Field (CRF) analysis, these distributions form the basis for generating a random field where the shear strength of every element is modelled with a separate random variable. Each of the analyses are implemented in three different simulations with different variance of the input shear strength. This is done to examine the effect of increased uncertainty in the input parameters on the estimated behaviour of the slope, and to investigate whether increasing the uncertainty leads to estimates that are more conservative. Each simulation provides distributions of both the Factor of Safety of the slope and the shear strength of the materials both prior and posterior to updating. These are compared to examine the effects of the updating, and of using various input and calculation models.

The results show that Bayesian updating is an efficient method for incorporating observations of the performance of slopes into a slope stability assessment. The probability of failure is reduced significantly, and the shear strength distributions are made more accurate by the updating. In the random field, the effects of the updating are mostly concentrated to zones close to the failure mechanisms, in layers with an inaccurate initial estimate of the shear strength. The SRV analysis predicts a response that deviates significantly from the response of the CRF analysis. This indicates that neglecting the spatial variability entails a simplification that reduces the quality of the results significantly. Lastly, the results show that increasing the Coefficient of Variation in the input parameters completely changes the estimated behaviour of the slope, rather than altering it slightly in a conservative direction. This is not a suitable method for obtaining conservative estimates.

Keywords:

1. Bayesian updating
2. Reliability analysis
3. Slope stability
4. Random field

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Preface

This Master's thesis is written as the final part of the Master's programme in Geotechnical engineering at the Norwegian University of Science and Technology (NTNU, Trondheim). The work with this thesis was carried out over a course of 20 weeks during the spring semester of 2018, at the Geotechnical Engineering group at the institute of Civil and Environmental Engineering. The idea for this project was brought up by Ivan Depina, PhD at SINTEF.

The reader should have knowledge about basic geotechnical principles, and a basic understanding of probability theory.

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Astrid Hult

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A.H.

Abstract

This thesis aims to explore the potential of implementing Bayesian updating in the probabilistic analysis of a geotechnical slope stability assessment. Bayesian updating is a well-defined method for combining supplementary information with already existing knowledge. In geotechnical slope stability assessment the previously existing knowledge would typically be a model of the slope, with material descriptions based on ground investigations and knowledge of the area. The supplementary information would typically be additional ground investigations, or observations of the performance of the slope. Including more information in the calculation model is expected to improve the accuracy of the predictions of slope behaviour. The model in this thesis is updated based on the observation that the slope is stable.

The Bayesian updating is implemented in two different analyses, where one models the spatial variability within each soil layer, while the other models each soil layer spatially homogeneous. The probabilistic analysis is implemented using Monte Carlo Simulations, and the slope stability assessment is conducted with a finite element model constructed in Plaxis 2D 2017.1. The shear strength of each material in the slope is modeled as a random variable. In the Single Random Variable (SRV) analysis these distributions are used to sample shear strength values that are applied to each layer. In the Conditional Random Field (CRF) analysis these distributions are used as a basis for generating a random field where the shear strength of every element is modeled with a separate random variable.

Each of the analyses are implemented in three different simulations with different variance of the input shear strength. This is done to examine the effect of increased uncertainty in the input parameters, on the estimated behaviour of the slope, and to investigate whether increasing the uncertainty leads to more conservative estimates. Each simulation provides distributions of both the Factor of Safety of the slope and the shear strength of the materials, both prior and posterior to updating. These are compared to examine the effects of the updating, and of using various input parameters and calculation models.

The results show that Bayesian updating is an efficient method for incorporating observations of the performance of slopes into a slope stability assessment. The probability of failure is reduced significantly, and the shear strength distributions are made more accurate by the updating.In the random field, the effects of the updating are mostly concentrated to zones close to the failure mechanisms, in layers with an inaccurate initial estimate of the shear strength. The SRV analysis predicts a response that deviates significantly from the response of the CRF analysis. This indicates that neglecting the spatial

variability entails a simplification that reduces the quality of the results significantly. Lastly, the results show that increasing the Coefficient of Variation (COV) in the input parameters completely changes the estimated behaviour of the slope, rather than altering it slightly in a conservative direction. This is not a suitable method for obtaining conservative estimates.

Sammendrag

Denne oppgaven tar sikte på å undersøke potensialet for å implementere Bayesisk oppdatering i den probabilistiske analysen av en geoteknisk vurdering av skråningsstabilitet. Bayesisk oppdatering er en veldefinert metode for å kombinere tilleggsinformasjon med allerede eksisterende kunnskap. I en vurdering av skråningsstabilitet vil den tidligere eksisterende kunnskaper typisk være en modell av skråningen, med materialbeskrivelser basert på grunnundersøkelser og kunnskap om området. Den supplerende informasjonen vil typisk være ytterligere grunnundersøkelser, eller observasjoner av skråningens ytelse. Å inkludere mer informasjon i beregningsmodellen er forventet for å forbedre nøyaktigheten av prognosene for skråningens adferd. Modellen i denne oppgaven er oppdatert basert på observasjonen av at skråningen er stabil.

Bayesisk oppdatering er implementert i to forskjellige analyser. En av modellene modellerer den romlige variasjonen i hvert jordlag, mens den andre modellerer hver jordlag som homogent. Den probabilistiske analysen er implementert ved hjelp av Monte Carlo-simulering, og skråningsstabiliteten vurderes med en elementmetodemodell konstruert i Plaxis 2D 2017.1. Skjærstyrken til hvert materiale i skråningen modelleres som stokastiske variabler. I Single Random Variable (SRV)-analysen brukes fordelingene av disse variablene til å generere skjærstyrkeverdier som brukes på hvert lag. I Conditional Random Field (CRF)-analysen brukes disse fordelingene som grunnlag for generering av et stokastisk felt, hvor skjærestyrken til hvert element er modellert med en egen stokastisk variabel.

Hver av analysene er implementert i tre forskjellige simuleringer med forskjellig varians i input-skjærstyrken. Dette er gjort for å undersøke effekten av økt usikkerhet i input-parameterne på den estimerte skråningen skråningsstabiliteten, og for å undersøke om økt usikkerhet fører til mer konservative estimater. Hver simulering gir fordelinger av både sikkerhetsfaktoren til skråningen og av skjærstyrken til materialene både før og etter oppdatering. Disse sammenlignes for å undersøke virkningen av oppdateringen og virkningen av å bruk av ulike input-parametere og beregningsmodeller.

Resultatene viser at Bayesisk oppdatering er en effektiv metode for å inkorporere observasjoner av ytelsen til en skråning i en vurdering av skråningsstabilitet. Sannsynligheten for brudd reduseres betydelig, og estimatene av skjærstyrke blir mer nøyaktige ved oppdateringen. I det stokastiske feltet er virkningen av oppdateringen hovedsakelig konsentrert til soner nær bruddmekanismene, i jordlaglag med en unøyaktig første estimat av skjærstyrken. SRV-analysen estimerer en respons som avviker tydelig fra prognosen til CRF-analysen. Dette indikerer at neglisjering av romlig variabilitet innebærer en forenkling som reduserer kvaliteten på resultatene betydelig. Resultatene av denne

studien viser også at økning av variasjonskoeffisienten (COV) i inputparameterne medfører en stor endring i skråningens estimerte oppførsel, i stedet for å forandre oppførselen litt i en konservativ retning. Dette er ikke en egnet metode for å oppnå konservative estimater.

Contents

Nomenclature

Latin Symbols

a	Normalizing constant
A	The Cholesky decompostiton of C
c	Cohesion
C	Covariance matrix
E	Expected value operator
$f'(\mathbf{x})$	Prior probability distribution of $\mathbf{X}$
$f''(\mathbf{x})$	Posterior probability distribution of $\mathbf{X}$
F	Event "loss of stability"
F_s	Factor of Safety
$\mathbf{H}$	Linear function for applying observations to random field
H	Slope height
H_t	Tension crack depth
H_w	Submergence depth
k	Number of elements in the random field
ln	Natural logarithm
L	Vector of standard normal distributed random values
$L(\mathbf{x})$	Likelihood that $\mathbf{X} = \mathbf{x}$
m	Mean of log-normally distributed random variable X
n	Total number of realizations
n_{fail}	Number of failed realizations
N_0	Stability number
N_{kt}	Empirical cone penetration resistance factor
N_q	Bearing capacity ratio
$\mathbf{o}$	Observation
p_f	Probability of failure
$Pr(Z)$	Probability of event Z happening
$Pr(F\|Z)$	Probability of event F happening, given that Z happened

q_t	Cone tip resistance
q_t^*	Cone tip resistance adjusted for measurement uncertainty
R	Random field
s	Standard deviation of log-normally distributed random variable X
S_u	Undrained shear strength
x	Realization of random variable
X	Random variable
X_d	Design parameter
Y	Random variable
Y^*	Random variable adjusted for model uncertainty
Z	Observation

Greek Symbols

γ	Unit weight
δ	Separation length
ϵ_M	Model error
ϵ_q	Measurement error
ϵ_T	Transformation error
θ	Correlation length
μ_i	Mean value of random variable i
μ_q	Surcharge factor
μ_t	Tension crack factor
μ_w	Submergence factor
ρ	Pearson's correlation coefficient
ϕ	Internal friction angle
σ_i	Standard deviation of random variable i
σ_{vo}	In-situ total overburden pressure
σ'_{vo}	Effective vertical stress
τ_f	Average shear strength along slip surface
τ_{mob}	Mobilized shear stress along slip surface

Acronyms

CDF Cumulative Distribution Function

COV Coefficient of Variation

CPT Cone Penetration Test

CPTU Piezocone Penetration Test

CRF Conditional Random Field

FEM Finite Element Method

ISSMGE International Society for Soil Mechanics and Geotechnical Engineering

MCS Monte Carlo Simulation

NGU Geological Survey of Norway

NGI Norwegian Geotechnical Institute

NTNU Norwegian University of Science and Technology

NNRA Norwegian National Railways Administration

NPRA Norwegian Public Roads Administration

NVE Norwegian Water Resources and Energy Directorate

PDF Probability Density Function

PMF Probability Mass Function

RFEM Random Finite Element Method

SRV Single Random Variable

Chapter 1

Introduction

1.1 Background

An important task of geotechnical engineers in Norway is estimating the stability of both natural and built slopes. Especially slopes made up of sensitive clay materials are prone to slope failures and land slides, making it extra important to to be able to predict the behaviour of these slopes. Currently these calculations are usually done in a deterministic framework, where the properties of the soil are described with one single number for each parameter. These soil descriptions are applied to a model of the slope, and stresses and strains developed in the slope are calculated. The model is used to determine how well the slope will be able to withstand the load it is or will be exposed to.

However, the soil materials are not homogeneous. The properties of a soil material varies in space, also within a defined layer of for example quick clay or sand. This introduces a significant portion of uncertainty to the calculation, and makes it difficult to represent a large soil body with one single value. Deterministic methods usually deal with this uncertainty by applying a global or partial safety factor, to reduce the estimated soil performance, and make conservative estimates. This is an inaccurate way of assessing the uncertainty, and can lead to inefficient and expensive designs.

An alternative to deterministic design is to implement a probabilistic analysis. This approach represents soil parameters as random variables, described with probability distributions. Rather than determining one single value, one describes the probability of the parameter taking a range of different values. This gives an explicit estimate of the uncertainty of the input parameters, which can give more precise designs. The probabilistic calculation model takes in probability distributions of the input, and gives out probability distributions of the output, which in the case of slope stability will be an estimate of the slope's tendency to fail. The probability of failure is calculated based on the output distribution. The results of a probabilistic analysis gives a much better understanding of how the slope is likely to behave, than the result of a deterministic calculation does.

Probabilistic analysis is not widely used in the geotechnical community, but the potential of it is being explored in research and is gaining interest in the field. This thesis aims to explore the potential of expanding the probabilistic analysis further, by implementing Bayesian updating. Bayesian updating is a method of including new information into an already existing model, with the purpose of reducing uncertainties. An example is a model of a slope, where the soils are described with initial estimates, based on ground investigations. At a later point more information about the slope is obtained, either by supplementary tests, or by observing the behaviour of the slope. For example, the behaviour can be quantified in terms of settlements or in terms of whether the slope is stable or failed. This is thought to be an inexpensive way to improve a probabilistic design.

1.2 Problem Formulation

This master thesis will apply Bayesian updating to the probabilistic analysis of a slope stability problem. This is an effort to explore the potential of Bayesian updating in reducing the uncertainties in soil property estimation. Two models with different complexity will be used to model the slope. It is of interest to investigate both the effect Bayesian updating has on the analysis, and to compare the performance of the two different models. Furthermore, various shear strength distributions will be used as input, in order to see how the analysis responds to this.

Bayesian updating is thought to be a useful contribution to probabilistic analysis in geotechnical engineering because of the potential in improving information and reducing uncertainty at a low cost. The updating can potentially provide an efficient method of combining existing knowledge about the ground with either supplementary ground investigations or engineering judgment. The results of this thesis could be of interest for governmental agencies planning geotechnical designs in infrastructure projects, like Norwegian Public Roads Administration (NPRA), Norwegian National Railways Administration (NNRA) or Norwegian Water Resources and Energy Directorate (NVE). A calculation method that can make designs more efficient without need for additional ground investigations can potentially save large amounts of money.

1.3 Related Work

As mentioned previously, probabilistic analysis is a topic in growth in the field of geotechnical engineering. Several efforts have been made to investigate the topic in the academic community around the Norwegian University of Science and Technology (NTNU) and SINTEF during the last few years.
Bæverfjord (2015) did a study on adapting reliability analysis to slope stability evaluation using finite elements. A large effort was made in making this methodology available to the industry. She employed commonly used software and used data sets which are representative for the data sets that are commonly available to geotechnical engineers.
Depina et al. (2017) implements Bayesian updating on a slope stability evaluation conducted with Janbu's direct method (Janbu, 1954). They found results demonstrating that that the Bayesian framework can efficiently integrate information on observed slope performance in the safety assessment of a slope.
Wolebo (2016) developed a calculation model that implements probabilistic analysis using a conditional random field to model the spatial variability of the slope. This tool has been used in this study to conduct the probabilistic analysis.
The most relevant parts of the mentioned publications are presented throughout chapter 2, 3, 4 and 5 of this report.

1.4 What Remains to be Done?

Both reliability analysis with Bayesian updating and reliability analysis with conditional random field has been conducted before. This thesis will contribute to the field by performing a Bayesian updating on a conditional random field. The results from the random field analysis will be compared with a single random variable analysis where the spatial variability is not modeled.

1.5 Objectives

The main objectives of this Master's project are

1. Explore the effect of Bayesian updating on the probability of failure
2. Investigate the effect of Bayesian updating on the random field
3. Evaluate the difference between Single Random Variable analysis and Conditional Random Field analysis
4. Investigate whether varying the variability of the input distributions is an efficient approach to obtaining conservative estimates

1.6 Approach

The Bayesian updating will be implemented on a model of a real slope, located at Rissa in Trøndelag county, in Norway. This slope is well documented due to previous projects in the area, and a probabilistic model of the material properties is available. The available distributions for soil strength was made based on Piezocone Penetration Test (CPTU) measurements. However some of the shear strength values that the materials can take, according to the distributions, would lead to a failed slope. With these shear strength values, the slope would not be strong enough to carry its own weight. The plan for this project is to incorporate into the model the information that the slope is still standing. This will lead to an updating of the distribution of both the input distributions and the output distributions. The effects of Bayesian updating will be investigated by examining the differences in the distributions before and after updating.

The Bayesian updating will be implemented on a probabilistic analysis conducted with a Monte Carlo Simulation (MCS). The stability of the slope is evaluated using two different finite element models of the slope, with different complexity. The first one describes every layer with one single random variable for each of the soil parameter, such as the undrained shear strength or internal friction angle. The spatial variability is neglected in this approach. The second model estimates the shear strength properties of the soil using a random field. In the random field the shear strength is modeled separately in each element, to capture the spatial variability of the soil. This is a more complicated, but also more accurate representation of reality. The difference between these two models will be investigated, in terms of their interaction with the Bayesian updating, and their reaction to various input.

1.7 Limitations

The results in this study are likely affected by the limited computational capacity that was available for running the calculations. Each realization required approximately 10 minutes of calculation time, which made it impossible to aquire a sample size large enough to ensure converged results. The computational power was also a limitation on the possible mesh refinement in the finite element models. The element size could not be made small enough to properly model the spatial variability of the soil. Furthermore, there was no external uncertainty in the model, like an uncertain load, or a varying ground water level, that could influence the behaviour of the slope. This means that the estimated probability of failure after updating is only applicable as long as all conditions stay constant. The reliability of the slope under various external conditions is still unknown. Another limitation is the estimation of the model error, which has not been done explicitly for this analysis. The model error has a rather large impact on the results, and makes the estimation of the posterior probability imprecise.

1.8 Outline

The rest of the report is structured as follows:

Chapter 2 introduces relevant geotechnical background knowledge, focusing on slope stability evaluation and description of soil materials.

Chapter 3 introduces relevant statistical theories and techniques.

Chapter 4 describes the case study of the Rissa slope.

Chapter 5 introduces the methodology used in this thesis. This chapter also contains a presentation of previous work on the topic that has been influential on the choice of methods in this study.

Chapter 6 presents the results obtained in the Single Random Variable analysis.

Chapter 7 presents the results obtained in the Conditional Random Field analysis.

Chapter 8 is a discussion of the results.

Chapter 9 states the conclusions of the work, and provides suggestions for further work.

Chapter 2

Geotechnical Background

The topic of this thesis is a combination of statistics and soil mechanics. This chapter will introduce the relevant geotechnical models and soil characteristics that are used in this thesis. The the relevant statistical theories and techniques will be presented in the next chapter.

2.1 Slope Stability

When simulating a slope one wants to find out how stable it is. How can we expect the slope to behave? How much load can it bear? How large alterations can we make while preserving stability? Or do we need to take measures to make sure that the slope does not fail under the current conditions? To answer these questions, it is necessary to quantify how far away the slope is from failure. For this, we use the factor of safety.

2.1.1 Factor of Safety

Slope stability calculations are done in order to check whether a slope is safe or not. This can be an existing slope, or a slope that is planned to be constructed. The measurement of safety is called the Factor of Safety. It is defined as the ratio between the average shear strength of the soil along the slip surface, and and the mobilized shear stress developed along the slip surface.

$$F_s = \frac{\tau_f}{\tau_{mob}} \tag{2.1}$$

τ_f is the shear strength of the soil, and τ_{mob} is the mobilized shear stress. A slope is said to fail when the Factor of Safety is smaller than one, $F_s < 1$ (Bromhead (2006)). This is when the mobilized stress is larger than the available resistance in the soil. Design criteria for slopes are often defined on the Factor of Safety. National standards usually state the safety criterion for a slope. In Norway the required value of the factor of safety depends on the plausible failure mechanism and the consequence level of a potential failure.

2.1.2 Deterministic Slope Stability Calculations

Deterministic models for calculating the stability of slopes take in information about the slope, such as soil properties and geometry, and return the factor of safety. The input is deterministic, meaning that they are fixed. Two approaches with different levels of complexity are presented here.

Janbu's Direct Method

In 1954 Nilmar Janbu developed a method for calculating the factor of safety for slope stability, with the intention of eliminating the tedious calculations necessary at the time. It is a general solution, meant to find the most critical sliding surface without computing F_s for each possible sliding surface. This makes the work with finding the factor of safety of a slope a lot more efficient. However, the method is a simplification, based on several assumptions presented in Janbu (1954), and summarized here:

- The critical sliding surface has a cylindrical shape
- The slope is modeled with a two dimensional cross section assuming plane strain conditions
- At failure, the shear strength is fully mobilized along the sliding surface, except from in dry cracks
- The factor of safety is defined as the ratio between the shear strength and the average shear stress needed for equilibrium

Janbu provided a way of both identifying the location of the most critical slip surface, and calculating the Factor of Safety along this surface. The method is developed for both drained and undrained conditions. Only the the undrained approach is relevant to the results presented in this thesis, and thus the drained approach will not be presented.

Figure 2.1 shows a schematic of a slope that is simplified in order to fit the limitations of Janbu's direct method. It is a slope with height H and angle β, which is subjected to an external load q. The slip surface is assumed to be a toe circle. The stability charts and equations of the method are presented in Janbu (1954). The most central equation in the undrained case is also included here. Because neither external water level nor tension cracks are considered, some factors fall out of the equations:

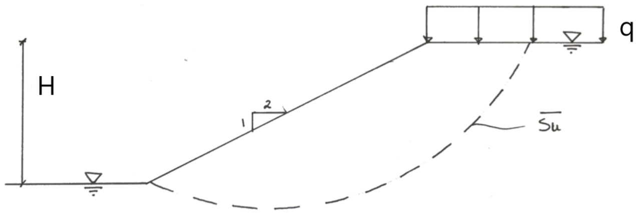

Figure 2.1: A schematic of an idealized slope used in Janbu's direct method.

$$H_t = 0 \Rightarrow \mu_t = 1 \tag{2.2}$$

$$H_w = 0 \Rightarrow \mu_w = 1 \tag{2.3}$$

H_w is the height of water submerging the slope, and H_t is the depth of tension cracks at the slope crest. μ_w and μ_t are the submergence factor and tension crack factor respectively, extracted from charts presented in Janbu (1954). This leads to a simplified version of the formula for the Factor of Safety:

$$FS = N_0 \cdot \frac{c}{\gamma H + q} \cdot \mu_q \tag{2.4}$$

N_0 is the stability number, c is the average shear strength along the slip surface, γ is the unit weight of the soil, H is the height of the slope, q is the external load and μ_q is the surcharge factor.

Janbu's method can be used to calculate the stability of slopes with simple geometries, where the failure surface is expected to have a cylindrical shape. The method is only reliable in cases where a plane strain failure mechanism is likely, such as in long embankments. The direct method does not cater for modeling of the spatial variability of the slope and it constrains the possible failure surfaces that can be identified. For that reason other more complicated models for predicting slope behaviour are used in more demanding cases.

Finite Element Method

In soil mechanics there are many different constitutive models available to describe the relation between stresses and strains in a material. However, the constitutive equations cannot be used directly on a continuous medium. The finite element method allows us to implement constitutive models on larger soil bodies. The approach is to discretize a slope into a finite number of elements, and use the constitutive equations on each element. Equilibrium is ensured in every element by redistributing stresses between the elements. If the stresses exceed the yield limit in an area, and cannot be redistributed to neighbouring elements, the slope will fail. The failure will start at the most critical point, and propagate until the shear surface has cut through the entire slope. In this way, the model finds the most critical slip surface by itself. It is not necessary to make assumptions about the shape or location of the failure surface. This is an advantage the finite element method has over many other approaches.

Furthermore, the finite element method allows more complex models than what limit equilibrium methods do. The geometry can be complex, and it is possible to include drains, geosynthetics and sheet pile walls, among other installations. Many different soil models are available, with various constitutive relations and failure criteria. Finite element models can be made in both two and three dimensions, allowing simulation of most mechanisms. The model used in this thesis is implemented in Plaxis 2D 2017.1, and the Mohr Coulomb constitutive model is used to model the soil. This model simulates the soil as linearly elastic until the yield stress is reached. After yielding, the soil is assumed to be perfectly plastic, meaning that the stresses do not change anymore, while the strains keep increasing. Yielding neither strengthens nor weakens the material. The model is a cross section of the slope, and the calculations assume plane strain conditions. It is a limitation that the model does not provide any knowledge about how the slope behaves after a potential failure. When the model slope fails the calculation breaks down. However, this is not a problem in most design applications, as the goal is to make sure the slope does not fail in the first place.

2.2 Soil Variability

One of the great difficulties in modeling geotechnical problems is to determine the soil characteristics. This is largely due to the spatial variability of soil materials. The deposition process that forms soils is subjected to randomness. This causes soil properties to fluctuate in space, also within a body of soil that has been classified as one material. Most testing and sampling of soils provides parameter values in a single point. Mapping a large area in this way is time consuming and expensive. Usually the number of tests one can take at one site is limited. In deterministic analysis one tries to perform representative tests on the soil, and estimate constant values for the soil parameters. These are applied throughout the material. One of the goals of this study is to evaluate the effect of neglecting the spatial variability, and look at how this interacts with the Bayesian updating. This is done by comparing results from slope stability assessments with and without modeling the spatial variability of soil.

2.2.1 Formation of Soil

Soil consists of small grains of rock and minerals, with water and air in the voids between the particles. Large deposits of soil are usually formed by particles that are transported by means of water, wind, gravity or glaciers, and deposited when the flow loses momentum (Knappett and Craig, 2012, chapter 1). For example, soil grains can be picked up by high velocity water at a source high up in the mountains where rivers are steep and fast. When the river reaches river planes it widens out and the water loses velocity. The capacity for transporting sediments decreases, and particles that were suspended in the water fall down to the bottom. The large particles are deposited first, and the smallest last, when the river reaches open water. Over millions of years soil deposits are formed. This deposition process is subject to randomness. There will be variation in the mineral composition in the grains, the velocity of the water, the course of the river, and many other elements. All of this leads to both layering of different types of soil, and variation within each soil layer. When the soil is deposited, the soil is still subjected to change. Loading conditions and flow of water are some processes that affects the properties of soil. These effects contribute to soil being a spatially variable material.

2.3 Uncertainty

International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE) defines uncertainty as a description of any situation or event that lacks certainty (ISSMGE (2004)). Soil properties are difficult to describe due to uncertainty from various sources. According to Der Kiureghian and Ditlevsen (2009) it is useful to characterize contributions to uncertainty according to origin. According to Phoon and Kulhawy (1999) the various sources of uncertainty include inherent soil variability, measurement uncertainty and transformation uncertainty. This section will also cover model uncertainty, which is occurs due to difficulties modeling the behaviour of a geotechnical system.

2.3.1 Aleatory and Epistemic Uncertainty

In geotechnical engineering there are many sources of uncertainties. Uncertainties are commonly categorized as aleatory or epistemic in character. Aleatory uncertainties describe uncertainties that originate from the natural variability of a property or an event (e.g. wave loads). A random variable subjected to aleatory uncertainty can be described with a distribution, but the variance caused by the aleatory uncertainty cannot be reduced. Uncertainties that originate from a lack of knowledge are called epistemic uncertainties (e.g. simplified geotechnical model). The variance of epistemic uncertainties can be reduced by obtaining more information and incorporating it in the calculation model.

As an example, consider measurements of inherently variable wave loads. The signal itself is affected by an aleatory uncertainty. However, the measuring device is assumed to introduce an epistemic uncertainty to the measurements. This uncertainty could be reduced by using a more precise measuring device. The final wave load measurement will be affected by both aleatory and epistemic uncertainty.

It can be discussed whether the spatial variability of soil represents an aleatory or epistemic uncertainty. The natural process of soil deposition is subjected to aleatory uncertainty. Making measurements in one location does not reduce the uncertainty in the process of soil formation. This would be much like taking one measurement of a wave load. However, in geotechnical problems it is often the soil in a limited area that is of interest. If the soil is available for measurements, the spatial variability can be known, with large confidence. The reason it is often unknown is the lack of measurements, making this an epistemic uncertainty. Investigating one limited soil volume would be analog to investigating wave loads at one point in a limited time period. It can be measured and analyzed. If the soil constituting the design structure is not available for testing, like in a planned embankment, the spatial variability is an aleatory uncertainty (Der Kiureghian and Ditlevsen, 2009).

Variance in one random variable can originate from many different sources. Some of the contributions can be reduced, and some cannot. It is useful to characterize the different uncertainties as aleatory or epistemic, based on their origin. This structuring provides a framework to decide which approach to take to reduce uncertainties in calculations.

2.3.2 Measurement Uncertainty

When collecting and handling soil samples, one will always risk disturbing the samples, and thus reducing the quality of the information that can be obtained from them. Measurement errors originate from equipment, operators and random testing effects (Phoon and Kulhawy, 1999). The errors arising from the way the samples are handled can be random, without any pattern from sample to sample, or they can be systematic. A systematic error affects all samples in the same way. An example of this is the systematic disturbance of clays that are sampled in pipes with diameter 54 mm or 73 mm. The material is reshaped, and its properties deviates from the in situ properties of the soil. More about this can be found in Amundsen et al. (2016). In the model employed in this thesis the measurement error of the Piezocone Penetration Test (CPTU) measurement ϵ_q is modeled as a log-normal distribution with mean $\mu_{\epsilon_q} = 1$ and standard deviation

σ_{ϵ_q}, and is applied as follows:

$$q_t^* = q_t \times \epsilon_q, \tag{2.5}$$

where q_t is the measured cone tip resistance, and q_t^* is the cone tip resistance adjusted for measurement error, which is used to estimate shear strength properties.

2.3.3 Transformation Uncertainty

Parameters measured in a soil test (e.g. cone tip resistance) often cannot be used in a geotechnical design. They must be transformed to design parameters (e.g. undrained shear strength), before they are useful in the simulation of the behaviour of a system. The transformation model is based on fitting empirical data or using certain theoretical relations to derive a relation between the measured parameters and design parameters (e.g. Lunne et al. (1997)). Uncertainties are introduced when fitting on empirical data or when making assumptions in theoretical deductions. These are categorized as transformation uncertainties. In this thesis the transformation uncertainty is denoted as ϵ_T and included in the calculations as follows:

$$X_d = X + \epsilon_T. \tag{2.6}$$

X_d is the design parameter, X is the output from the transformation model and ϵ_T is a zero-mean normally distributed random variable with standard deviation σ_{ϵ_T}, modeling the transformation uncertainty.

2.3.4 Model Uncertainty

The next step is to incorporate the design parameters into a calculation model, and simulate the behaviour of the system. Also this process introduces uncertainty. The model builds upon assumptions and simplifications of reality. Examples of such simplifications are the discretization of a continuous medium into a finite number of elements, and modeling a three dimensional slope using a two dimensional cross section. These simplifications makes it more likely that the output of the model is not an exact representation of reality. To compensate for this we add the model error to the output. The model error is the difference between the true behaviour of the system and the estimated behaviour. The model error is denoted with ϵ_M and modeled as a normal distribution with mean value $\mu_{\epsilon_M} = 0$ and standard deviation $\sigma_{\epsilon_M} = 0.05$, inspired by estimates of model errors in Wu (2009). It is incorporated in the model with the following equation

$$Y^* = Y + \epsilon_M \tag{2.7}$$

Y is the output from the calculation model, ϵ_M is the model error and Y^* is an estimate of the true system behaviour.

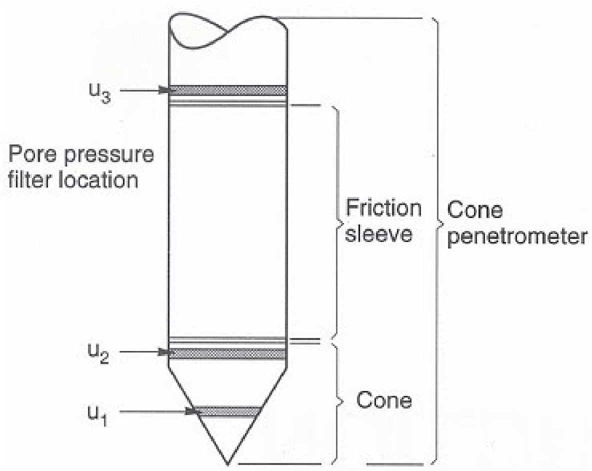

Figure 2.2: Schematic of a Piezocone (CPTU). (Lunne et al., 1997)

2.4 Cone Penetration Test - CPT

Cone penetration testing is an effective way of acquiring data on the stratification and physical and mechanical properties of a soil in-situ. The test is performed by pushing a series of rods with a cone at the bottom down through the soil at a constant rate, and measure the soil's resistance force against the cone. The device also measures the sleeve friction through a force sensor mounted along the bottom of the rod. Some cones can measure pore water pressure. These are called piezocones, and are used in CPTU tests. A schematic of such a device is shown in figure 2.2. Forces and pore pressures are measured continuously or intermittent as the cone is pushed through the soil. This provides a continuous or nearly continuous profile of soil, which makes this kind of testing cost-effective. Large amounts of data can be obtained while spending little time and resources. The continuous profile can be used to determine the spatial variability of the soil. CPT and CPTU tests are done in-situ, and investigate the soil in its natural environment. This is an advantage in-situ tests have over laboratory tests. Still, laboratory tests have other advantages, and a good site investigation program should include both in-situ tests and laboratory tests. Because CPT testing is cost-effective, it can be used to determine where to collect samples for laboratory tests (Lunne et al., 1997). CPT and CPTU tests are repeatable and reliable penetration tests, that provide useful data for use in geotechnical design.

Chapter 3

Statistical Background

To properly discuss the uncertainty present in soil characteristics this thesis will use several concepts from statistics. This chapter explains the relevant statistics without any further reading being required from the reader.

3.1 Basics of Probability

This section aims to introduce the basic principles of probability that are relevant to this thesis. The definitions given here will be used and built upon throughout this chapter.

3.1.1 Random Variable

The International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE) technical committee defines a random variable as *"a quantity, the magnitude of which is not exactly fixed but rather the quantity may assume any of the number of values described by a probability distribution"* (ISSMGE (2004)). A random variable is not like a traditional variable describing one quantity. It does not have a fixed value. A random variable rather describes the outcome of a random process. An example of such a process is a coin toss. The random variable X describes the result of the coin toss, and can take the values "heads" or "tails". The value of X varies for each time the coin is tossed, and it is impossible to know the result of the next toss. However, it is possible to know the characteristics of the random variable, saying how likely it is that the variable takes each possible value.

Discrete Random Variable

A discrete random variable has a countable number of possible values. The result of the coin toss mentioned earlier is an example of a discrete random variable. Other examples are the result of the roll of a die, which can take the values 1 through 6, or the sum of two die rolls. The characteristics of a discrete random variable are often described by a Probability Mass Function (PMF). This function distributes the probability mass over the possible outcomes. The sum of all the probability masses has to add up to 1. In both the coin toss and the roll of a die the probability mass is uniformly distributed over all the possible values. In other distributions, some values are assigned a higher probability of occurring than others.

Continuous Random Variable

A continuous random variable X is a variable that can take any value on a continuous spectrum. Such a variable is described by a Probability Density Function (PDF). The PDF distributes a probability density over the spectrum of possible outcomes. The integral over a PDF has to sum up to one, just like the summation over a PMF. To obtain a probability mass, which is the measure we commonly use, one needs to integrate the density over a range of X values. An example of a continuous random variable is the strength of a soil. The shear strength of a soil can be seen as the outcome of a random process that occurred when the soil was deposited. This process is affected by the composition of the available minerals, the grain size, the deposition conditions and the subsequent loading, among other conditions. This leads to a soil that can be described by a shear strength that theoretically can take any positive value. However, by knowing something about the soil type, and performing tests on a soil, we gather information about what values of the shear strength are more likely. The PDF of the shear strength of the soil describes this, by concentrating the probability density around more probable values.

3.1.2 Correlation

Correlation is a measure that describes the dependence between two random variables. The most commonly used quantification of correlation is the Pearson correlation coefficient. This coefficient is only sensitive to linear relationships between variables. It is used to describe how close two random variables are to having a linear relationship. The coefficient is commonly denoted $\rho_{X,Y}$, describing the dependence between variables X and Y. It is defined by

$$\rho_{X,Y} = \frac{C(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y} \tag{3.1}$$

where $C(X,Y)$ is the covariance between X and Y, and σ_i and μ_i are respectively the standard deviation and mean of variable i. E is the expected value operator. $\rho_{X,Y}$ can take values between -1 and 1. If $\rho_{X,Y} = -1$, X and Y have a perfect negative correla-

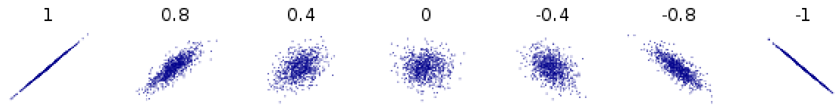

Figure 3.1: This figure shows seven plots of data from two random variables with varying correlation between them. Image: DenisBoigelot, https://commons.wikimedia.org/w/index.php?curid=15165296

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tion. $\rho_{X,Y} = 1$ means that X and Y are perfectly positively correlated. If $\rho_{X,Y} = 0$ the two variables are not correlated at all, and they are independent of each other. These cases, along with some intermediate ones, are shown in figure 3.1. How well two parameters are correlated does only say something about how noisy the plot is, not about the slope of the relationship. The correlation also does not say anything about causality.

An example of correlation in a geotechnical setting is the positive correlation between the Factor of Safety of a slope and the shear strength of the soil in the slope. Another example is the negative correlation between the stiffness of a soil and the settlements in the soil following loading.

3.2 Continuous Probability Distributions

There are many models that describe the distribution of random variables. Because soil parameters take values on a continuous spectrum we use continuous distributions to model them. The distributions used in this thesis will be described briefly.

3.2.1 Normal Distribution

The normal distribution is one of the most important continuous probability distributions. It is defined completely by its mean and standard deviation. The PDF of a normal distribution is symmetrical around its mean. The distribution is defined on all real numbers, but 99.7% of the probability mass is concentrated within three standard deviations of the mean. The probability density function of a normal distribution is described by

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}; \qquad -\infty < x < \infty \tag{3.2}$$

μ is the mean and σ is the standard deviation of the random variable X.

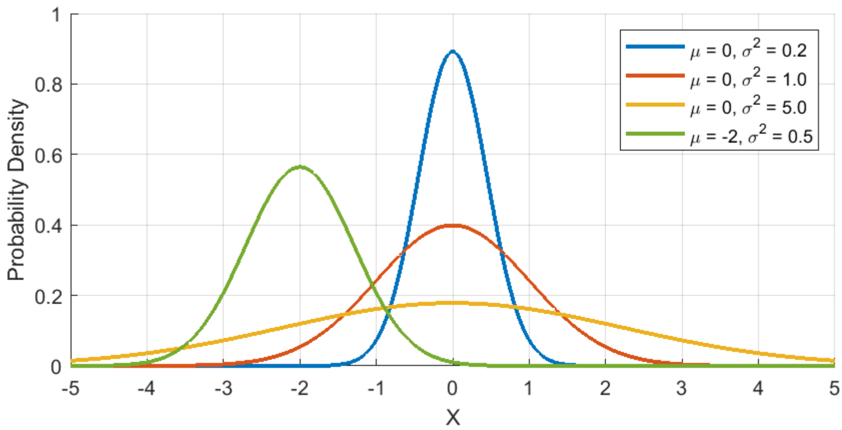


Figure 3.2: Four different normal distributions, showing the effect of varying mean and standard deviation.

Central Limit Theorem

One of the reasons that the normal distribution is so important is that many natural processes can be described by it. The reason for this can be described by the central limit theorem. The theorem proves that the normalized sum of several independent, random variables will tend towards a normal distribution (Billingsley, 1995, page 357). This also applies to random variables that are not normally distributed. This can be illustrated by taking a sample consisting of several observations. From this sample one can calculate the average value. If one performs this procedure many times, the mean values from each of the samples will form a distribution. According to the central limit theorem this distribution will tend towards a normal distribution. When the sample size grows, the distribution of means converges towards a normal distribution. A simple example is to observe 100 sets of 20 coin tosses. The number of heads in those 20 tosses will tend towards a normal distribution with mean 10.

Many processes behave similarly to this coin. For example, when performing a cone penetration test, each measurement does not return the shear strength of one single soil grain. A volume of soil within a radius of 10-20 cm around the cone is involved in a measurement (Fenton and Griffiths (2008)). The CPT cone averages the soil resistance over this volume. When the measurements are collated to one distribution, they are expected to tend towards a normal distribution. Because of this effect, which is present in most testing procedures, the normal distribution is applicable and useful when describing soil parameters. The next three subsections will describe variants of the normal distribution that will be used in this thesis.

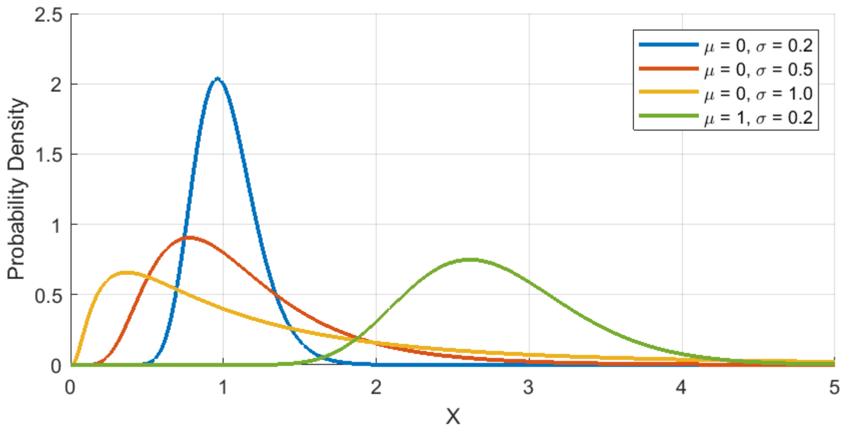


Figure 3.3: Four different log-normal distributions, showing the effect of varying mean and standard deviation.

3.2.2 Log-Normal Distribution

Another useful probability distribution is the log-normal distribution. It is well suited to fit soil parameters because it can only take positive values of X. This is because a log normally distributed random variable X can always be expressed as the exponential function of a random variable with a normal distribution. If Y has a normal distribution, then $X = e^Y$ has a log-normal distribution. The domain of a normal distribution is $[-\infty, \infty]$ and the domain of a log-normal distribution is $[0, \infty]$. The PDF of a log-normally distributed random variable is defined as

$$f(x) = \frac{1}{x\sqrt{2\pi\sigma^2}} e^{-\frac{(ln(x)-\mu)^2}{2\sigma^2}}; \qquad 0 < x < \infty \tag{3.3}$$

μ and σ describe the mean and standard deviation of the underlying normal distributed random variable, $ln(X)$, not the log-normally distributed X. The mean and standard deviation of X, m and s respectively, can be related to μ and σ through the following equations

$$\mu = \ln\left(\frac{m}{\sqrt{1+\frac{s^2}{m^2}}}\right) \quad ; \quad \sigma = \sqrt{\ln\left[1+\frac{s^2}{m^2}\right]}. \tag{3.4}$$

3.2.3 Bivariate Normal Distribution

A bivariate normal distribution describes two random variables and the relation between them. The two random variables can be represented by the vector $\mathbf{X} = [X_1 X_2]^T$. The distribution is described by a mean vector, $\mu = [\mu_1 \mu_2]$, and a covariance matrix.

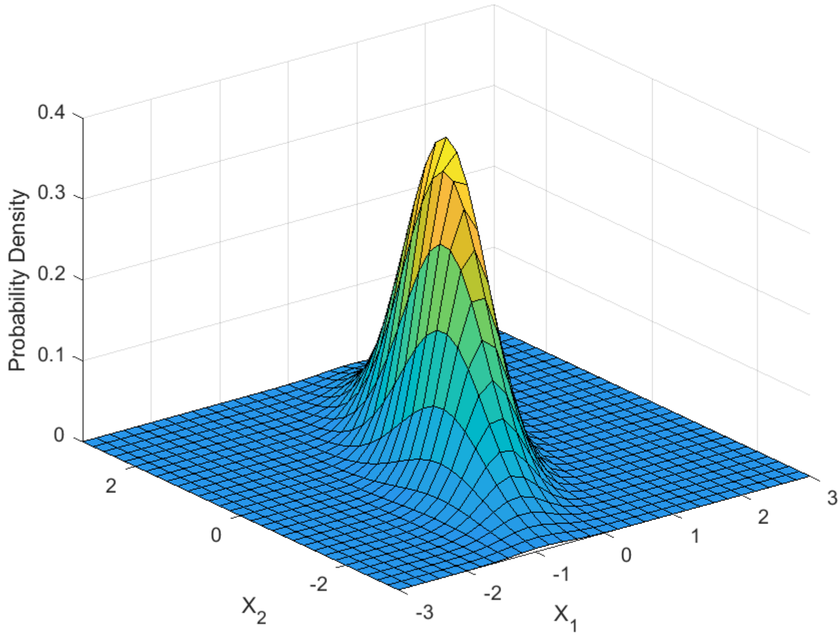


Figure 3.4: This surface shows the bivariate normal distribution of two random variables X_1 and X_2.

$$C = \begin{bmatrix} C(X_1,X_1) & C(X_1,X_2) \\ C(X_2,X_1) & C(X_2,X_2) \end{bmatrix} = \begin{bmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 \\ \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \tag{3.5}$$

μ_i and σ_i denotes the mean and standard deviation of X_i and are defined in the same way as for a single random variable. ρ_{12} is the pearson correlation coefficient between variable X_1 and X_2. The correlation coefficient can take values between -1 and 1. $\rho_{12} = 1$ means perfect positive correlation, and $\rho_{12} = -1$ means perfect negative correlation between the two variables. If the correlation coefficient is equal to zero, the bivariate distribution is a set of two independent random variables.

3.2.4 Multivariate Normal Distribution

A multivariate distribution is a generalization of the bivariate distribution into more dimensions. The multivariate normal distribution represents several random variables that are normally distributed. Again, each distribution is described by a mean and a standard deviation. Just like in the bivariate distribution, a covariance matrix containing the correlation coefficients relates these distributions to each other.

In geotechnical engineering, multivariate normal distributions are used to interpret multivariate data, which is obtained through measurements. When different tests are performed within close proximity of each other, the results will be correlated with each other. An example of this can be to take up undisturbed samples for oedometer and triaxial tests and performing Cone Penetration tests in locations close to each other. The soil parameters estimated from these tests will be correlated. The definition of what is close is related to the spatial variability of the soil on the site. This notion of closeness will be further formalized in section 3.4 about random fields. Multivariate data from different sources are correlated to a soil parameter, like the undrained shear strength. Using several correlated data sources can contribute to increase the reliability of our estimate of the soil parameter.

3.3 Probabilistic Analysis

3.3.1 Motivation

Current practice is to use deterministic analysis to analyze and predict the behaviour of geotechnical systems. This entails describing each soil material with one set of constant soil parameters, based on available results from tests and investigations. A characteristic value is assigned to each parameter. However, there is a lot of uncertainty associated with these values. Due to the inherent spatial variability of soil, it is difficult to make a good representation of the properties of a soil body using constant values.

To account for the uncertainty in the estimation, it is common to use a partial or global safety factor. The partial safety factor is a factor that is applied to the estimated input parameters to make them more conservative. The value is either reduced or increased according to what is conservative for the given parameter. The estimates of the system behaviour is made conservative by adjusting the input parameters towards conservative values. The global safety factor is related to the performance of the system in total, which is quantified in the Factory of Safety F_s in slope stability evaluations. The global safety factor is stated as a requirement of how high F_s needs to be, which varies according to the specifics of a given case. However, this is an inaccurate way of taking the uncertainty into account, and it completely neglects the spatial variability within each layer. This can lead to overly conservative estimates of soil behaviour and inefficient designs. In the other end of the spectrum, it can cause us to neglect weak zones that are critical to the behaviour of the modeled system.

The output of a deterministic analysis is a single estimate of the response, which is heavily reliant on the estimate of the input parameters. Examples of these outputs can be a deformation, or a Factor of Safety. This is where probabilistic analysis comes in.

In probabilistic analysis one tries to quantify the uncertainty, and describe it explicitly. Both soil parameters and external conditions, like loads or water levels, can be represented as random variables. This allows us to capture spatial and temporal variability. The characteristics of the distributions of the parameters are determined based on in-situ tests and laboratory tests. When using probability distributions to represent the input, one also gets a distribution of the output. This gives an estimate of how the system is likely to behave, which is more descriptive than one single estimate. When performing a probabilistic analysis of a system, there are many methods to choose from. The next subsection will describe the one used in this thesis.

3.3.2 Monte Carlo Simulation

This thesis employs a Monte Carlo Simulation to conduct the probabilistic analysis. This is an uncomplicated and robust numerical method that provides a good way of assessing the probability of failure of the slope. Numerical methods cannot use the information in probability distributions directly. They must sample values from the distributions, and use these in calculations. The principle of the Monte Carlo Simulation is to run a large number of deterministic simulations, each with input parameters sampled from probability distributions. This simulates many possible realizations of the slope to include uncertainty in estimates, and try to find the performance that corresponds to the input distributions. Each realization is run through the deterministic slope stability model of choice. This can be a simple Janbu's direct model or a more complex Finite Element analysis, or any other model. This returns a Factor of Safety F_s for each realization. Based on the distribution of the F_s we can calculate the probability that the slope looses stability, and fails. This is calculated as the number of failed realizations divided by the total number of realizations.

$$p_f = \frac{n_{fail}}{n} \tag{3.6}$$

p_f is the probability of failure, n_{fail} is the number of failed realizations and n is the total number of realizations. A realization is considered failed if it has a Factor of Safety smaller than one. In order to obtain accurate results, the number of realizations must be large enough, so that the distribution of the samples converges towards the distributions of the random variables they simulate.

3.4 Random Field

Materials rarely have completely constant properties throughout their volume. A random field is a statistical tool to describe the spatial variability of a property. The exact values of a property are difficult to quantify because one can only take so many samples from a material. However, based on the measurements that are available, one can try to discern a pattern that describes how the parameter varies in space. By capturing this pattern in a random field, one can incorporate spatial variability in an engineering model. The principle of a random field is to divide a volume or area into a field of discrete elements. Each element is assigned a random variable which describes the parameter that varies in space. All the random variables are correlated to each other, in a way that imitates the pattern found from the measurements. In this way, we model the material that has not been observed, based on what we did observe.

Random Field A random field R(x) defined in $\mathbb{R}^n$ is a function such that for every fixed $x \in \mathbb{R}^n$, $R(x)$ is a random variable in the probability space.

3.4.1 Gaussian Random Field

In a Gaussian Random Field the random variables are described by a multivariate normal distribution. This is a convenient distribution, because it makes it possible to describe the full random field completely with a vector holding the mean values of all the random variables, and a covariance matrix relating them. The multivariate distribution is described by the equation

$$f_{X_1 X_2 \ldots X_k}(x_1 x_2 \ldots x_k) = \frac{1}{(2\pi)^{k/2}} \frac{1}{|C|^{1/2}} \exp\left\{-\frac{1}{2}(X-\mu)^T C^{-1}(X-\mu)\right\}. \tag{3.7}$$

k is the number of elements in the random field, $\mathbf{X} = [X_1 X_2 \ldots X_k]$ is the vector of random variables and $\mu = E[\mathbf{X}]$ is the vector with the means of $\mathbf{X}$. C is the covariance matrix relating the variables in $\mathbf{X}$. Superscript T indicates that we use the transpose of the vector, and $|C|$ denotes the determinant of the covariance matrix.

Generating a random field that has unique means and standard deviations for each location requires a large amount of measurements. In a typical situation there are only measurements available in a few locations. Therefore, the random field is initiated with the same mean and standard deviation everywhere, leaving out a potential depth trend. Such a random field is called a stationary random field. It only serves to describe the relation between neighbouring elements i.e. based on their relative position, not their absolute position. This is considered a reasonable assumption, because the there is only a weak depth trend present in the data sets that have been used.

A Gaussian random field is said to be stationary if the probability function is dependent only on the distance in space between elements, rather than their absolute location. This assumption implies that the mean is $E[R(x_i)] = \mu$ for all i, and the covariance between x_i and x_j depends only on the distance between them, not the absolute location. Because of this, $C[R(x_i), R(x_j)] = C_R(x_i - x_j)$.

In a material there will typically be a dependency between neighboring elements. This dependency is stronger the closer two elements are located. The correlation between the elements in the random field is called autocorrelation.

3.4.2 Autocorrelation

Autocorrelation is the correlation of a variable with itself. A variable can for example be correlated with itself in either space or time. In geotechnical engineering this can be used to describe how dependent the soil properties in one point are on the properties of the surrounding soil. The autocorrelation of any function is always 1 if the signal is compared with an exact copy of itself. When one of the copies is shifted the correlation will start to change, and it becomes a measure of how variable the parameter is along its shifted dimension. In the case of soil properties, the autocorrelation is a function of the distance between the two points in space. The autocorrelation of a soil parameter is important to model spatial variability. If there are several measurements made in close proximity, an estimate of the autocorrelation can be made. More measurements lead to a more accurate estimation of the autocorrelation. However, these measurements should be sufficiently close to each other. The required closeness is known as the correlation length.

Correlation Length

Correlation length is defined as the distance in space beyond which autocorrelation is negligible. Points that are close to each other are usually strongly correlated. However, the further apart two points are, the weaker the correlation is between the two points. Beyond a certain distance, the two points are no longer correlated to each other. This is the correlation distance. It is used as a constant in the autocorrelation function. An example of an autocorrelation function is an exponential model. This model is used to generate the covariance matrix between the random variables describing the random field in this thesis. The function of the autocorrelation is given by

$$\rho(\delta) = exp\Big(-2\frac{\delta}{\theta}\Big) \tag{3.8}$$

ρ is the Pearson correlation coefficient between two points in space separated by the distance δ. θ is the correlation length describing the soil material. The function sees the separation distance between two points in relation to the correlation length.

3.4.3 Conditional Gaussian Random Field

After establishing a random field, it is useful to be able to incorporate further observations of the soil into the model. For example, if a parameter has been measured at a certain location, it needs not be estimated by the random field anymore. One can rather apply the measured values in the corresponding locations, and use a random field to simulate the surrounding soil. This is what a conditional random field allows us to do. It starts out with a random field, based on initial estimates of the soil properties. In this case this is a stationary random field. The joint probability density function of the random field R corresponds to a multivariate normal distribution, described by the relation

$$R \sim f(r) = N(r; \mu, C). \tag{3.9}$$

r is a realization of the random field, μ is the initial estimate of the mean and C is the covariance matrix. Additional observations $\mathbf{o} = \{o_1, \dots, o_n\}$ are obtained on n locations. C_o holds the covariance between the observed values. The observations can be incorporated into the random field, in such a way that they form a new and updated random field. Not only the elements that are observed get a changed distribution. Due to correlation between the distributions in each element, a change in one element also affects the the distributions describing the surrounding elements. The random field conditional on the observations o is defined by the following equations:

$$\mu_{r|o} = \mu + C\mathbf{H}^T \left[\mathbf{H}C\mathbf{H}^T + C_o\right]^{-1} (\mathbf{o} - \mathbf{H}\mu) \tag{3.10}$$

$$C_{r|o} = C - C\mathbf{H}^T \left[\mathbf{H}C\mathbf{H}^T + C_o\right]^{-1} \mathbf{H}C \tag{3.11}$$

$\mu_{r|o}$ and $C_{r|o}$ are the mean and covariance matrix respectively, describing the random field R, given that **o** has been observed. **H** is a linear function specified by an $n \times m$ matrix. n is the number of observations and m is the number of elements in the random field. The matrix is defined by $H(1, o_1) = 1$, $H(2, o_2) = 1$, $\dots$, $H(n, o_n) = 1$, and $H = 0$ everywhere else. **H** lets us combine the initial random field and the observations, as shown in equations 3.10 and 3.11. This is described more in detail in Depina and Wolebo (2017). However, **H** also restricts us to only incorporate observations that can be related to specific elements in the random field. Conditional random fields make it possible to update a random field when measurements of a parameter are available. The measurements are applied to the corresponding elements as random variables with very small variance. At all unobserved locations, the values are estimated by distributions that are updated to take the newly observed information into account.The random field will mostly be affected by the new information in a region close to the observations themselves, due to a limited correlation length.

3.4.4 Generating a Realization of a Random Field

To generate a random field one needs to know the mean μ and covariance C of the random field. μ is an $(m \times 1)$ vector and C is an $(m \times m)$ matrix, where m is the number of elements in the discrete random field. This can be an initial estimate where the mean is constant over all the elements, or a conditional random field, where both the mean and covariance are adjusted to take measurements into account. The covariance matrix is decomposed to give a lower triangle matrix A and an upper triangle matrix A^T, using a Cholesky decomposition.

$$C = AA^T \tag{3.12}$$

The random element comes from generating an $(m \times 1)$ vector of standard normal distributed random values, $L \sim N(\mu_L = 0, \sigma_L = 1)$. A realization of the random field can then be generated through the following equation:

$$R = \mu + AL \tag{3.13}$$

3.4.5 Probabilistic Slope Stability Analysis

A probabilistic analysis of the stability of a slope will give the probability of failure as output. This is more descriptive of the safety of the slope than the Factor of Safety obtained from a single deterministic analysis. The principle of the analysis is to generate many models of the slope, and assign soil properties sampled from probability distributions. There are two different approaches to this:

Single Random Variable An analysis using a single random variable to describe a soil parameter does not take spatial variability into account. It simply describes the whole material with one random variable per soil parameter. Each sample that is taken is applied to the entire layer of that material. The uncertainty is simplified, by only modeling the average strength in the soil. This can be done using a Finite Element Method (FEM), or a simpler model, like Janbu's direct method.

Random Finite Element Method In a Random Finite Element Method (RFEM) one simulates the spatial variability of the soil. The usual FEM mesh is replaced by a random field, where every element is described by its own probability distribution.

3.5 Bayesian Analysis

This study aims to explore the potential of incorporating Bayesian updating in probabilistic analysis of slope stability. It is thought that it can make calculations more efficient, and reduce uncertainties in estimates of soil parameters. This section will explain the basic principles of Bayesian updating, and how it can be implemented in this study.

3.5.1 Bayesian Updating on Binomial Distribution

Bayesian updating provides a way to consistently and effectively combine existing models and knowledge with new information. The concept starts out with some information about the distribution of a random variable. This is called the prior distribution. Then we obtain some new information about this random variable. The new information does not say exactly the same as the prior distribution. Now we could discard the new information, arguing that we have higher confidence in our prior distribution, or we could discard the prior, arguing that the new information is more reliable. A third and maybe better option, is to use Bayesian updating, which allows us to combine the information from both the prior distribution and the new information in a new distribution of our random variable. This new distribution is called the posterior distribution. We call the new information evidence upon which we update the prior distribution, and obtain the posterior distribution.

This will be illustrated with an example. We are interested in knowing the probability that a person has the flu. Out of 100 people, 20 people has the flu, and 80 do not have the flu. If you meet one of these 100 people you would assume that there is a 20% chance that this person has the flu. There is a test that can be taken to check if a person has the flu or not, but like every test, the result could be wrong. A positive result signifies that a person has the flu. Out of the 20 people with the flu, 90% (18 people) tests positive. The test also gives a positive result for 30% of the people that does not have the flu (24 people). These are false positive results. In total 42 out of 100 people test positive. This means that if a person tests positive, there is a 18/42= 0.429 chance that the person has the flu.

In terms of Bayesian updating, one would call the initial estimate of 20% probability of flu the prior distribution of X. X is the random variable holding information about whether the person has the flu or not. The test result provides extra information on whether the person has the flu or not. This is used as evidence to update the distribution of X. The updated distribution is called the posterior distribution of X, and estimates a 43% chance of the person having the flu. The posterior distribution contains more information on the persons health, than the prior.

3.5.2 Equations

One of the big advantages of Bayesian analysis is that it is a consistent and well defined way of combining different sources of information. Engineering judgment can be taken into account in a structured way, described by equations. Bayesian updating is a tool that can be used when performing a probabilistic analysis of a geotechnical problem. One starts out with initial estimates for a set of soil parameters. These are stored as random variables in the vector **X**. A geotechnical model is used to express events of interest as a function of the variables **X**. An example of such an event can be F = "loss of stability". A prior estimate of the probability of such an event happening is calculated based on the initial soil parameters. In Bayesian analysis we are interested in knowing the probability of an event, given that we have measured or observed some behaviour. This measurement or observation can be denoted as event Z. So, we are interested in the conditional probability of event F, given that event Z has been observed. This conditional probability can be written as

$$Pr(F|Z) = \frac{Pr(F \cap Z)}{Pr(Z)}. \tag{3.14}$$

According to Bayes' rule this conditional probability can be calculated as

$$Pr(F|Z) = \frac{Pr(Z|F)Pr(F)}{Pr(Z)}. \tag{3.15}$$

In Bayesian analysis the terms have the following meanings (Phoon and Ching, 2014):

- Pr(F): prior probability of F
- Pr(F|Z): posterior probability of F, given that Z has been observed
- Pr(Z|F): Likelihood of observing Z, given that F happens
- Pr(Z): Probability of making the observation Z

Bayes' rule tells us how to update a prior probability of event F to a posterior probability, after having made observation Z. However, we are not only interested in the probability of event F. We want to know how the distributions of all the variables in **X** are affected by the updating. Bayes' rule can be written in terms of **X**:

$$f''_X(\mathbf{x}) = aL(\mathbf{x})f'_X(\mathbf{x}). \tag{3.16}$$

Here $f'_X(x)$ denotes the prior distribution of X, $f''_X(x)$ denotes the posterior distribution and L(X) is the likelihood function, which is analog to $P(Z|F)$ in equation 3.15. The likelihood function is defined by $L(x) \propto Pr(Z|X = x)$. a is a normalizing constant. It is there to ensure that $f''_X(x)$ integrates to one. It corresponds to $\frac{1}{Pr(Z)}$ in equation 3.15. The function for a is

$$a = \frac{1}{\int_{-\infty}^{\infty} L(\mathbf{x}) f'_{\mathbf{X}}(\mathbf{x})\, d\mathbf{x}} \tag{3.17}$$

The integral in the denominator should be interpreted as $\int_{\mathbf{X}} d\mathbf{x} = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} dx_1 \dots dx_n$. If the function of the prior distribution of X, the likelihood function and the integral of X is available, one can calculate the posterior distribution of X using Bayes' rule.

3.5.3 Bayesian Updating in Geotechnical Engineering

It is often difficult to implement Bayes rule on continuous random variables in an analytical way. This is due to the integral in the denominator of equation 3.17. This integral only has an analytical solution if the prior and posterior distribution are of the same kind, for example normal distributions (Phoon and Ching, 2014). In most cases, the posterior distribution will not be the same kind of distribution as the prior, and there is no analytical solution to the equations. When this is the case, one can use numerical methods to approximate a solution to the equations. This thesis applies a Monte Carlo Simulation for this purpose.

Soils are inherently variable and difficult to characterize. This often leads to overly conservative estimates of soil parameters, and inefficient designs. Bayesian updating can be used in geotechnical engineering to include more available information and reduce uncertainty in the distributions of soil parameters (Depina et al., 2017). When using Bayesian updating in geotechnical engineering there are mainly two approaches. The first is to supplement the available information about the soil parameters with further soil tests and measurements. This approach gives direct information about the parameters. The parameters themselves are observed.

The second implementation is to use the prior distributions of the soil parameters as input to a calculation model. We carry out a Monte Carlo Simulation of the slope, to simulate the behaviour of a system that we can observe. This gives a prediction of the behaviour of the slope, based on our prior estimates of the soil parameters. Then, we observe the actual behaviour of the slope, out in the field. This can be an observatio of whether the slope has failed or not, or a measurement of deformations. Next step is to compare the output with the observed behaviour of the system. The realizations that do not correspond to the observed behaviour are deleted from the model, with both input and output. For example, if the slope is still standing, all the realizations predicting failure must be wrong, and should be removed. This leads to new and updated distributions of the soil parameters, where also the information we can observe is included.

Chapter 4

Case study: Rissa Slope

The aim of this thesis is to investigate the potential of using Bayesian updating to reduce uncertainties in slope stability calculations. To increase the relevance of the results, data from a real slope has been used as a basis for the calculations.

4.1 Background

The slope that has been modeled is located at Rissa in the municipality Indre Fosen, in Trøndelag county, Norway. The name Rissa is well known in the geotechnical community due to the quick clay landslide that took place there in april 1978. The slope used in this thesis is located in the western end of lake Botn, between Rein church and the shoreline, only 1.1 km away from the where the Rissa landslide initiated. The area is shown in figure 4.1. This slope is a suitable case study for a probabilistic analysis, because it is well documented, with a large amount of measurements done on several occasions. Norwegian Geotechnical Institute (NGI) proposed a stratigraphy of the slope when making a design for a road project. The Norwegian Public Roads Administration (NPRA) planned to build a new road across the slope in 2009, but the project was stopped due to challenges with slope stability.

In 2012 H. Kornbrekke (2012) wrote her master thesis on the stability of this specific slope. She compiled available field and laboratory test results, and analyzed the result of supplementary Piezocone Penetration Test (CPTU) measurements and block samples. As a result she presented a new stratigraphy of the slope, and new soil properties for the different materials that constitute the slope.

Amanuel Wolebo also used the same slope as a case study for his thesis in 2016. He presented an advanced probabilistic Slope stability analysis, modeling the spatial

Figure 4.1: Rissa - Overview of the studied area. Image: Kartverket (2018)

variability of the slope with conditional random fields. He described the soil in a probabilistic framework based on available CPTU measurements. The stratigraphy he used is based on the original estimates of NGI for the project of NPRA, due to Korsbrekkes work being published in Norwegian.

4.2 Soil Description

Rissa is located in a coastal area covered by marine deposits. These materials were deposited directly on bedrock when the glacier of the last ice age retracted from the area between 12500 and 9000 years ago. Since then the terrain at Rissa has heaved approximately 160 meters above the former sea level (NGU, 1987). The terrain below Rein church slopes gently down towards lake Botn. Large amounts of sensitive clay has been found in the slope. The slope consists of a thick layer of clay on bedrock, covered by a layer of sand on the upper half of the slope. The land heave has exposed the marine clay to flow of fresh water that has washed out the salts, and made the clay sensitive, and some places quick (Kornbrekke, 2012).

4.2.1 Site Investigations

The slope has been investigated at several occasions. NGI performed ground investigations in 2007, when making the first detail design for NPRA's road project, and in 2009. Several laboratory tests, total sounding and CPTU were performed. Geological

Survey of Norway (NGU) performed electrical resistivity measurements (R-CPTU) in cooperation with a master student from Norwegian University of Science and Technology (NTNU) in 2009 and 2010. NGU continued the geophysical investigations in 2011 and 2012 by making a seismic refraction measurement around Lake Botn and Rein church. Supplementary investigations were done in 2011, including CPTU, cylinder samples and block samples.

4.2.2 Layering

The slope studied in this thesis is the same slope that was studied by both Kornbrekke (2012) and Wolebo (2016). The soil descriptions used are based on his interpretations of data available from NPRA. The stratigraphy of the slope and location of the CPTU measurements are shown in figure 4.2. In NGI's results the clay body is separated into four layers of different clay materials. From top to bottom, there is quick clay, clay, sensitive clay and clay. A more detailed description of these materials will follow in the chapter 5. The slope is approximately 210 meters long in the horizontal direction. The soil layers are approximately 28-37 meters thick, and the surface of the slope is at +28 meters at the highest point and at +2 meters at the lowest point. CPTU measurements have been done in locations C4, C2, C5, KK3 and C3, as shown in figure 4.2.

4.2.3 Limitations

Information of the stratigraphy and material properties of the slope is only available on land. No soil investigations have been done on the bottom of the lake. This means that the model of the slope stops before the shoreline, and thus excludes factors that could be important to the stability of the slope. The support provided by the soils on the bottom of the lake is unknown. A slip surface extending into the lake bottom can potentially be the most critical one. This cannot be known based on available data, and contributed to the termination of the project of NPRA.

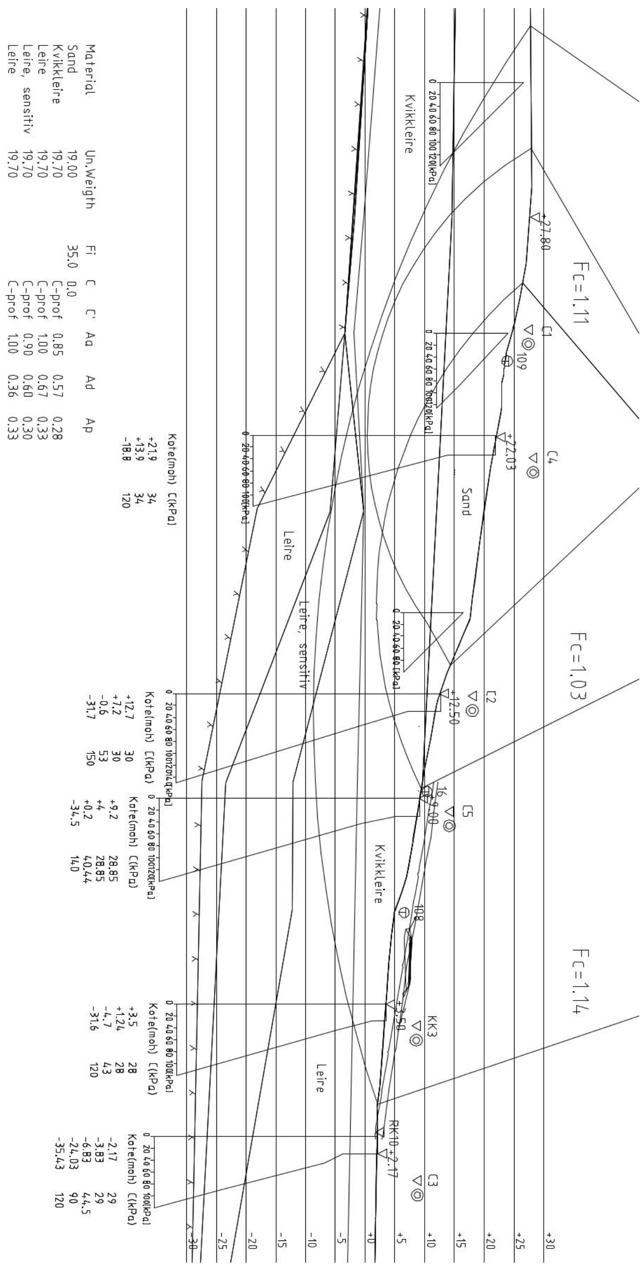


Figure 4.2: Stratigraphy of Rissa slope, suggested by NGI in 2009 (Kornbrekke, 2012)

Chapter 5

Method

This Chapter describes the methods used to implement Bayesian updating on a finite element analysis. Section 5.1 and 5.2 describe previous efforts on the topic that has been influential on the choice of method for this thesis. Section 5.3 explains how the behaviour of the slope has been simulated in a probabilistic framework, and how Bayesian Analysis has been implemented.

5.1 Bayesian Updating with Janbus Method

5.1.1 Scope

As a preparation for this Master's thesis a preliminary study was conducted as a project thesis during the fall semester of 2017, on the topic of Bayesian analysis of slope stability assessment. The scope was to investigate the potential of using Bayesian updating on slope stability calculations, and on the Percentual Improvement method especially. The Percentual Improvement method refers to a method of meeting safety requirements of slope stability in geotechnical engineering. For natural slopes where overall stability of larger areas is the issue the method can be used in cases where it is difficult to reach the target factor of safety, stated in current regulations. For slopes that fall under this category, it is sufficient to achieve a specified percentual improvement of the initial Factor of Safety. This is usually achieved by changing the slope geometry. The Percentual Improvement method is reviewed in detail in Oset et al. (2014). The aim of the project thesis was to investigate whether Bayesian updating could reduce the need for percentual improvement, by reducing uncertainties in the slope stability calculation. The work was a continuation of the work of Ulmke (2016).

5.1.2 Approach

In order to implement Bayesian updating it is necessary to run a probabilistic analysis. A Monte Carlo Simulation (MCS) was chosen for this purpose. The deterministic slope stability calculation in each realizationwas carried out using Janbu's direct method. An idealized slope was used in the calculations, and analyzed with an undrained approach. The soil consisted of one uniform material, where the undrained shear strength was modeled as a random variable. The slope was exposed to an accidental load of uncertain magnitude. Also this external load was modeled as a random variable. Because of the inherent variable nature of the load, it is classified as an aleatory uncertainty that cannot be reduced. The uncertainty in the shear strength is of epistemic character, and can be reduced.

The Percentual Improvement method was implemented by step wise reducing the slope height. For each geometry the of the slope, one MCS was carried out to obtain the probability of failure. Following this, a Bayesian updating was carried out, based on the observation that the slope was still standing under an observed load. This whole calculation was implemented for different values of the Coefficient of Variation (COV) of the input shear strength. The COV was varied in order to investigate whether this was a good way of making conservative estimates of the slope stability. This was implemented by keeping the mean constant and increasing the standard deviation.

5.1.3 Findings

The results from the analysis showed that both Percentual Improvement and Bayesian updating contribute to reducing the the probability of failure significantly. Reducing the uncertainty in the undrained shear strength by removing unlikely low values from the distribution do increase the reliability of the slope's stability. Another important finding is that increasing the COV seems to be non-conservative in combination with Bayesian updating. It was thought that increasing the uncertainty might be conservative. However, this move leads to an increased amount of both artificially high and artificially low values of the undrained shear strength of the soil. By removing a large part of the low values upon updating, one is left with a distribution with stronger values than what one started out with. The combination of the shape of the prior distribution of F_s and the application of the external caused effects that lead to the high COV simulation to cause the opposite of conservative estimates.

To compensate for assumptions and simplifications in the calculation model, a model error was applied. The results of the simulation indicated that the application of the model error had a significant effect on the posterior probability of failure. This was not investigated explicitly, but will be investigated in this master thesis. Also investigations of the effect of Bayesian updating on slope stability and the effect of varying the COV of the input parameters will be continued in this master thesis.

5.2 Advanced Probabilistic Slope Stability Analysis

Current practice in geotechnical engineering is to use deterministic analyses for estimating the stability of a slope. However, probabilistic analysis is gaining more interest, and is being explored as a way of approaching geotechnical problems.Wolebo (2016) wrote his master thesis about advanced probabilistice slope stability analysis.

5.2.1 Scope

A big part of Wolebo's work was constructing a tool that allows implementing a probabilistic analysis with conditional random fields in the finite element program Plaxis 2D. This entailed interpreting Piezocone Penetration Test (CPTU) profiles to represent soil parameters as random variables, and building a Python script that generated a random field based on these estimates and measurements. The script needed to interface with plaxis to pass the soil parameters to the model of the slope. The model was then used to run simulations investigating the effects of varying the input COV and the correlation length.

5.2.2 Approach

Wolebo used CPTU measurements as a basis for generating distributions of the undrained shear strength S_u of the clay materials and of the internal friction angle ϕ of the sand layer. A CPTU provides a vertical profile of the cone tip resistance q_t of a soil material. From this profile Wolebo extracted a Log-Normal distribution of q_t and the correlation length θ of each material using maximum likelihood estimation. The correlation length is assumed to be the same for shear strength as for the cone tip resistance in the respective layers. The undrained shear strength of the clay materials are estimated using the formula

$$S_u = \frac{\epsilon_q q_t - \sigma_{vo}}{N_{kt}}. \tag{5.1}$$

ϵ_q is the measurement error related to the CPTU equipment, σ_{vo} is the in-situ total overburden pressure and N_{kt} is the empirical cone penetration resistance factor. σ_{vo} is estimated with a deterministic value, and ϵ_q and N_{kt} are assigned probability distributions. The internal friction angle ϕ of the sand layer is derived from the cone tip resistance using the formulas

$$q_t = (N_q - 1)\sigma'_{vo} \tag{5.2}$$

and

$$N_q = \tan^2(\frac{\pi}{4} + \frac{\phi}{2})\exp(\pi \tan\phi). \tag{5.3}$$

σ'_{vo} is the effective vertical stress at the depth of penetration and N_q is the bearing capacity ratio. Instead of solving the equations analytically, ϕ is estimated numerically with a regression model. Measurement, regression and transformation errors are included in the estimate of ϕ. More information about this process can be found in Wolebo (2016).

Based on the estimates of the shear strength parameters a random field is generated. Then measurements from the CPTU profiles are applied at respective locations, and the random field is made conditional on these. The shear strength of every element in the random field is described by a unique probability distribution. For each realization of the random field new values are drawn for each element, and the random field is passed to Plaxis that completes the safety factor calculation. The calculation process will be explained more in detail in section 5.3.

5.2.3 Findings

The results show that varying the COV has a bigger impact on the behaviour of the system than varying the horizontal correlation length. This has confirmed that further investigations of the effect of changing the COV can yield interesting results. Otherwise, the most important results from thos study are the calculation model that can be used as a tool by others, and the probabilistic characterization of the materials in the Rissa slope. These have both been used in this thesis to expand the probabilistic analysis by including Bayesian updating. The next section will describe the model more in detail, and explain how it has been used in the work with this thesis.

5.3 Bayesian Updating with FEM

The model described in section 5.2 is the basis for the work with thesis. It has been used to generate prior distributions of factors of safety, and expanded to include Bayesian updating. Slope stability calculations are done in two different analyses. The Conditional Random Field (CRF) approach models the spatial variability within each layer. The Single Random Variable (SRV) approach does not, and applies one constant value of the shear strength to all elements within one material. This is to investigate whether a SRV approach is a valid simplification. There are differences in how the shear strength is modeled in the two analyses, but they both follow the same progression as shown in

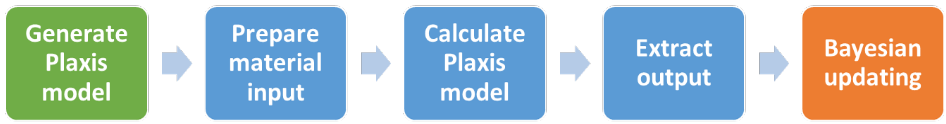


Figure 5.1: The progression of the calculation model.

figure 5.1.

The first step is to generate a plaxis model of the slope. In this thesis the all calculations are done on the Rissa slope. The mesh is divided according to soil layers. The element size used in this calculation is approximately 1 m. The sand layer was modeled with drained behaviour and the clay layers were modeled as undrained. No external load was added, as this was a safety assessment of a natural slope in its original constitution. All deterministically defined material properties are applied. In this case that is all but the shear strength parameters. Only the shear strength is modeled as a random variable, in order to isolate effects of the shear strength estimates.

Next step is to generate the shear strength values for the model. This step is different in the SRV analysis and the CRF analysis, and will be explained in subsections 5.3.1 and 5.3.2 respectively. The shear strength parameters are than passed to the Plaxis model, and the behaviour of the system is estimated in a safety calculation. This produces a factor of safety. When the calculation is finished the output, which is the factor of safety and the shear strength values, is extracted and stored.

These three steps, which are marked in blue, are repeated until the desired sample size is reached. This constitutes a Monte Carlo Simulation. In this work the sample size is set to 2000 realizations per simulation, due to limited computational capacity. A larger sample size would likely have yielded more stable results, but it was not feasible with the available computational power. The last step in the analysis is the Bayesian updating. This updates the prior distributions of the factor of safety and shear strength and gives posterior distributions. The Bayesian updating is equal for the two analyses, and will be explained in subsection 5.3.3.

5.3.1 Single Random Variable

The single random variable approach employed in this thesis is similar to the analysis described in section 5.1, where Janbu's method was employed. They both describe each material with one single random variable, and neglects any spatial variability within each layer. The difference is that the deterministic analysis in this thesis is a Finite Ele-

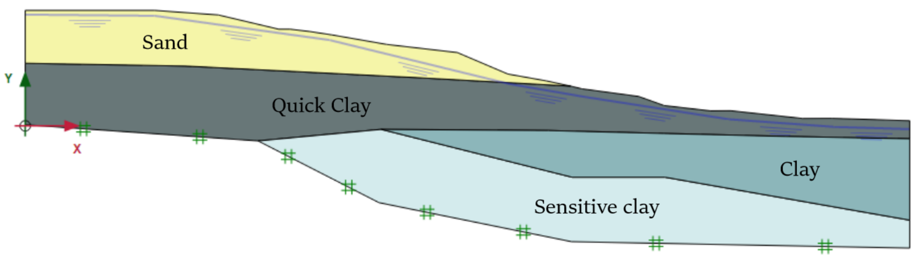


Figure 5.2: Finite element model used in SRV analysis.

Table 5.1: Input parameters used in the SRV analysis.

	Mean 1	CoV 1
layer	$\mu_\phi (°)$	CoV_ϕ
sand	35.4	0.15
	$\mu_{S_u} (kN/m^2)$	CoV_{S_u}
Quick Clay	54.52	0.20
Clay	62.25	0.25
Sensitive Clay	94.4	0.25

ment analysis, in stead of a limit equilibrium analysis. The finite element model of the slope is shown in figure 5.2.

The materials in the SRV analysis is described directly by the shear strength distributions derived by Wolebo (2016). These are presented in table 5.1. In the SRV analysis the lower Clay layer, right above bedrock is described as part of the Sensitive Clay layer. None of the CPTU measurements reach deep enough to measure the lower Clay layer, so there is no data from that layer to base a probabilistic description of the material on. In each of the realizations one value is drawn from each of the distributions, and applied to the entire layer it describes. This is equivalent to the limit state of a random field, where the correlation length is set to infinity in both vertical and horizontal direction. This gives full correlation between all elements in each material.

Three simulations of the SRV analysis have been completed, with varying COV. These are listed in table 5.2. It was of interest to see how varying the COV would affect behaviour of the system. To increase the COV the mean was kept constant and the standard deviation was increased. This is an attempt to increase the uncertainty of the estimates, and in that way make them more conservative.

Table 5.2: Simulations conducted in the SRV analysis.

Simulation	Mean	CoV
Simulation 1	Mean 1	CoV 1
Simulation 2	Mean 1	1.5 × CoV 1
Simulation 3	Mean 1	2.0 × CoV1

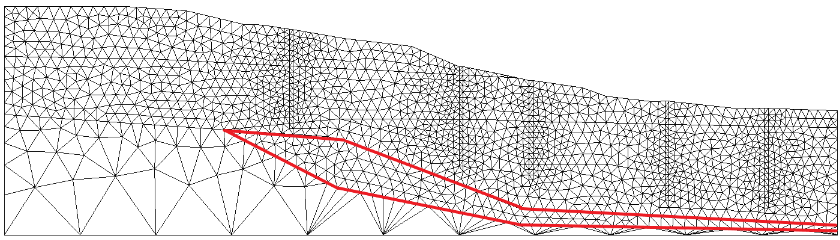


Figure 5.3: Mesh used in the CRF analysis. The stratigraphy is identical to the one used in 5.2, except the area marked with red borders, which is modeled as a separate clay layer.

5.3.2 Conditional Random Field

The Conditional Random field (CRF) analysis estimates the spatial variability within each layer. This is done by applying different shear strength properties in every element. Each element is described by one random variable, with its own distribution of shear strength. To make this random field, a mesh was first made to simulate the slope geometry. This was done in the software *gmsh*, which allows customizing of the mesh to model the CPTU measurements, and refine the mesh around these. The mesh is shown in figure 5.3. The stratigraphy is identical to the one used in 5.2, except the area marked with red borders. This is modelled as a separate clay layer, named Clay 2. Then the random field properties are determined. A Gaussian random field is completely described by a vector of the mean values of every element and a covariance matrix describing the relation between the elements. Only the elements within the same layer are correlated with each other. The following process is repeated for every material, to generate on random field for each layer.

First a stationary random field is made. This is based on the mean μ and standard deviation σ of the shear strength of the material, the correlation length θ of the material, and a chosen autocorrelation function. The distributions of the shear strength of each material is presented in table 5.3. In the CRF analysis the lower clay layer is modeled as a separate layer, with material properties extrapolated from the trend of the CPTU measurements. The upper clay layer is named Clay 1, and the lower clay Layer is named Clay 2. The autocorrelation function describes how correlated two elements are as a

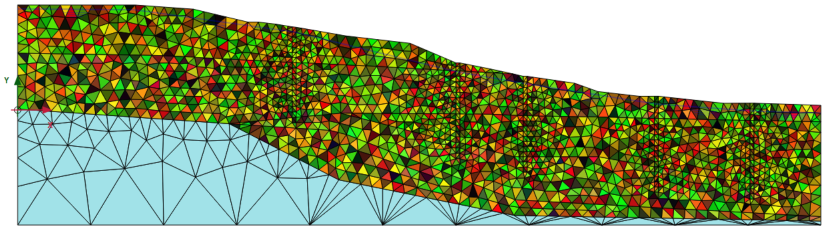


Figure 5.4: Finite element model used in CRF analysis.

function of the distance between them. An exponantial correlation function is used, which is defined as:

$$\rho(\delta) = \exp\left\{-2\frac{|\delta|}{\theta}\right\} \tag{3.8}$$

ρ is the correlation coefficient between two elements, δ is the distance in space between two elements, and θ is the correlation length of the material. From the equation it can be observed that when two locations are close to each other in space, $\delta \to 0$, the correlation coefficient approaches 1, $\rho \to 1$. If the locations are far apart, $\delta \to \infty$ the correlation approaches zero, $\rho \to 0$. The opposite relations apply to the correlation length. A large correlation length $\theta \to \infty$ leads to strong correlation $\rho \to 1$, and a small correlation length $\theta \to 0$ leads to weak correlation $\rho \to 0$.

The correlation between all elements are calculated using the autocorrelation function, and a covariance matrix is generated:

$$C = \begin{bmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \dots & \rho_{1n}\sigma_1\sigma_n \\ \rho_{21}\sigma_1\sigma_2 & \sigma_2^2 & \dots & \rho_{2n}\sigma_2\sigma_n \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1}\sigma_1\sigma_n & \rho_{n2}\sigma_2\sigma_n & \dots & \sigma_n^2 \end{bmatrix}. \tag{5.4}$$

n is the number of elements in the material and σ_i is the standard deviation of the shear strength in element i. In the stationary random field the mean and standard deviation are the same for all elements, and equal to the distributions in table 5.3. Then the CPTU measurements are applied to the elements corresponding to the locations where the measurements were taken. Next step is to make the random field conditional on these measurements, by updating the mean and covariance matrix using equations 3.10 and 3.11. This gives every element a unique distribution of its shear strength. In locations where measurements of the shear strength are available, these are applied. In unobserved locations the shear strength is estimated, based on the available measurements from other locations. The Conditional Random Field is now defined, and

Table 5.3: Input parameters used in the CRF analysis

	Mean 1	CoV 1	θ_V 1	θ_H 1
Layer	$\mu_\phi(°)$	CoV_ϕ	θ_V	θ_H
Sand	35.4	0.15	0.6	50
	$\mu_{S_u}(kN/m^2)$	CoV_{S_u}		
Quick Clay	54.52	0.2	0.3	50
Clay 1	62.25	0.25	0.3	50
Sensitive Clay	94.4	0.25	0.3	50
Clay 2	100.0	0.25	0.3	50

Table 5.4: Simulations conducted in the CRF analysis

Simulation	Mean	CoV	θ_V	θ_H
Simulation 4	Mean 1	CoV 1	θ_V 1	θ_H 1
Simulation 5	Mean 1	1.5 × CoV 1	θ_V 1	θ_H 1
Simulation 6	Mean 1	2.0 × CoV 1	θ_V 1	θ_H 1

realizations of the field are generated with the equation:

$$R = \mu + AL \tag{3.13}$$

where R is a vector with the shear strength values, μ is the mean values of the conditional random field and A is a lower triangular matrix such that $C = AA^T$. L is an $n \times 1$ vector of standard normal distributed random values. As mentioned, this process is carried out for all materials and the complete random field is assembled and passed to Plaxis. The calculation is completed and output is extracted. These steps correspond to the blue blocks in figure 5.1, which are repeated until the desired number of realizations are obtained, and the Monte Carlo Simulation is completed. This analysis has been done three times, with three different COV. These three simulations are listed in table 5.4. In order to vary the COV, the standard deviation changes in the three different simulations. Everything else is constant. The finite element model used in the CRF analysis is shown in figure 5.4. Every element in the mesh has a different colour to signify that the material properties are unique in each element.

5.3.3 Bayesian Updating

When the Monte Carlo Simulation is finished the output consists of the Factor of Safety of every realization, and the corresponding shear strength values, in both the SRV analysis and in the CRF analysis. The difference is that the CRF returns one shear strength value per element, while the SRV returns one per material. The Bayesian updating works in exactly the same way for both of these analyses.

Some of the realizations have $F_s >= 1$ and some have $F_s < 1$. However, the slope has been observed, and it is still standing. This means that the true Factor of Safety must be larger than 1. Because of assumptions and simplifications in the calculation model, there is uncertainty in whether the estimated F_s is equal to the true F_s. This uncertainty is quantified in the model error, ϵ_M, which is the difference between the true and the estimated value of F_s. This error is implemented as follows:

$$F_s^* = F_s + \epsilon_M \tag{5.5}$$

F_s^* is an estimate of the true Factor of Safety, and F_s is the Factor of Safety outputted from Plaxis. The model error is unknown, because the true Factor of Safety is unknown. ϵ_M is estimated with a normal distribution with mean 0 and standard deviation 0.05, based on findings in Wu (2009). The updating is then based on F_s^*. All realizations with $F_S^* <= 1$ are deemed to be impossible, and are removed from the collection of realizations. The constellation of shear strength values in those realizations would have lead to a failed slope. The remaining realizations form the posterior distributions of the F_s and the shear strength parameters.

The probability of failure, both before and after updating is based on on the factor of safety estimated by Plaxis, F_s. This because the aim of this project is to investigate the effect of Bayesian updating, and of using different inputs. Adding the model error on all results makes the results more blurred, and makes it more difficult to distinguish differences in the various simulations. The probability of failure is defined as

$$p_f = Pr(F_s <= 1) \tag{5.6}$$

Because some realizations will have $F_s^* > 1$, and survive the updating, yet have $F_s < 1$, the posterior probability of failure will be larger than zero.

5.3.4 Post Analysis Processing

After the Bayesian Updating the results consist of two sets of data for each simulation; the prior and posterior sets of realizations. The posterior realizations are a subset of the prior realizations. The further processing of the data is different in the SRV analysis and the CRF analysis. Because all of the materials have different distributions of the shear strength, all shear strength results was evaluated per material.

Single Random Variable

The results from the SRV simulations consist of one value of F_s and one shear strength value for each material, per realization. This is the basis for a prior and posterior distribution of theF_s, and a prior and posterior distribution of each of the materials' shear strength. Prior and posterior distributions of one random variable can be compared to look at the effect of Bayesian Updating. Either the the prior or the posterior distribution of the same random variable from different simulations can be compared to look at the effect of varying the input COV. By comparing the prior and posterior of different random variables one can investigate the interaction between Bayesian updating and variations in the input data. It can be interesting to investigate whether the different materials are affected differently by the updating. It is a goal to examine the reactions in the shear strength distributions, when updating based on F_S.

Conditional Random Field

The results from the CRF simulations consists of one value of F_s and one shear strength value for each element, per realization. Both the F_s and the shear strength of every single element is described by a prior and a posterior distribution. This is a much bigger amount of data, and it is more challenging to visualize effects of Bayesian Updating and of varying input data on the shear strength per material. Looking the mean value of the shear strength in each element over all realizations in a simulation is one option. Another is to look at the average value of the shear strength over all elements in a material, in each realization. These are ways to look at the average change in the whole random field. Another interesting approach is to identify elements and zones in the random field that experience larger effects of the Bayesian updating. This can identify which zones that have a big impact on the Factor of Safety of the slope, and can indicate that the estimation of parameters are inaccurate in these areas.

Chapter 6

Bayesian Updating with Single Random Variable Analysis

In the single random variable analysis the spatial variability within each material is not modelled. The shear strength of each material is described by one single random variable. For each realization of the slope, one value is drawn from the distribution of this variable, and applied to all elements within the material. The result is one prior and one posterior distribution of the shear strength parameters of each material, and of the Factor of Safety.

6.1 Updating of Factor of Safety

Figure 6.1 shows the prior distribution of the Factor of Safety F_s for three different simulations of the Single Random Variable (SRV) analysis. The histograms are fitted with Log-Normal distributions, as these seem to give a good fit. The three different simulations have different values of the Coefficient of Variation (COV) of the input shear strength. It is obvious that increasing the COV of the input leads to an increase of the COV of the output. Increasing the COV of the shear strength through increasing the standard deviation gives a larger range of probable values. A larger range of probable input values is expected to give a larger range of probable output values. Because the COV of shear strength is different in the different materials, it does not make sense to compare the value of input and output COV.

Another effect that can be seen in figure 6.1 is that when the input COV is increased, the mean of the output reduces. The yellow curve has a significantly lower mean than the blue curve. This is likely because the lower values of shear strength has more impact on the behaviour of the slope than the high values. If a realization has one weak layer, and other strong layers, a failure can initiate in the weak layer and propagate into strong layers. This means that when increasing the number of strong and weak realizations of the layers, the weak layers lead to more failures than the strong layers can prevent. This emphasis on the low values of S_u shifts the distribution of F_s to lower values.

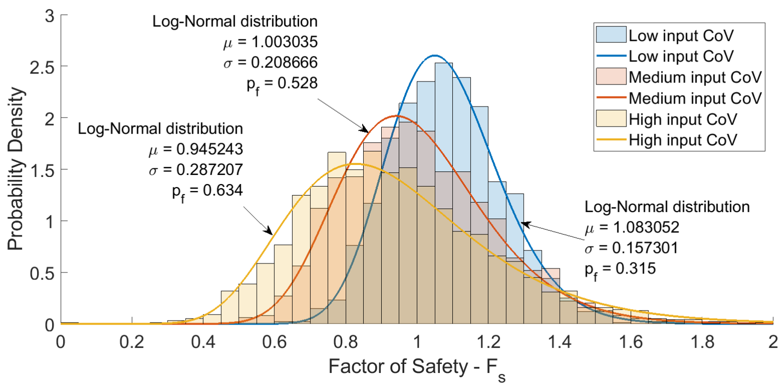


Figure 6.1: Estimated prior distributions of the Factor of Safety in the three Single Random Variable simulations.

Figure 6.2 shows the posterior distributions of F_s for the three different values of the COV of shear strength. The prior distributions are updated by removing most of the values of F_s that are lower than 1, to obtain the posterior distribution. Some values are still kept, because of the model error described in chapter 5. The probabilities of failure in the posterior distributions are still unacceptably high. However, this posterior probability of failure, p_f is only caused by the model error, which is there because we cannot have full confidence in the calculation model.

The simulation with the highest COV still have the highest standard deviation σ_{F_s} and mean μ_{F_s}. This is due to the longer right tail of the distribution, that remains after updating. This means that the simulation that was meant to be the most conservative, might not be so conservative after updating.

Another interesting effect is that the simulation with medium COV has the highest probability of failure after updating. The reason for this is not obvious. It might be due to statistical noise, because the simulations consist of a relatively small number of realizations. Another possible explanation is that the effect of the model error is sensitive to the shape of the prior distribution of F_s. The posterior probability of failure is defined as

$$p_{f,post} = \frac{n_{fail,post}}{n_{post}} \tag{6.1}$$

where $n_{fail,post}$ is defined as the number of failing realizations posterior to updating, and n_{post} is the total number of realizations posterior to updating. As mentioned in chapter 5, all of the failing realizations after updating are kept through the updating due to the model error. In the updating process the model error is added to the prior values of F_s. The model error adds more variability to the prior distribution of F_s. In a critical range around $F_s = 1$ the addition of the model error can cause the realizations in the prior distribution to shift from $F_s < 1$ to $F_s > 1$, or opposite. This can can cause realizations to be discarded, even though the slope was estimated to fail. After updating, the model error is removed again, and some realizations have $F_s < 1$ again. This means that it is the probability mass concentrated within the critical range that determines how large $n_{fail,post}$ is. The size of n_{post} is mainly determined by the probability mass on the upper side of $F_s = 1$ in the prior distribution. In all, this means that it is the ratio of probability mass in the critical range to the probability mass above $F_s = 1$ in the prior distribution of F_s that determines how large $p_f, post$ is, for a certain size of the model error.

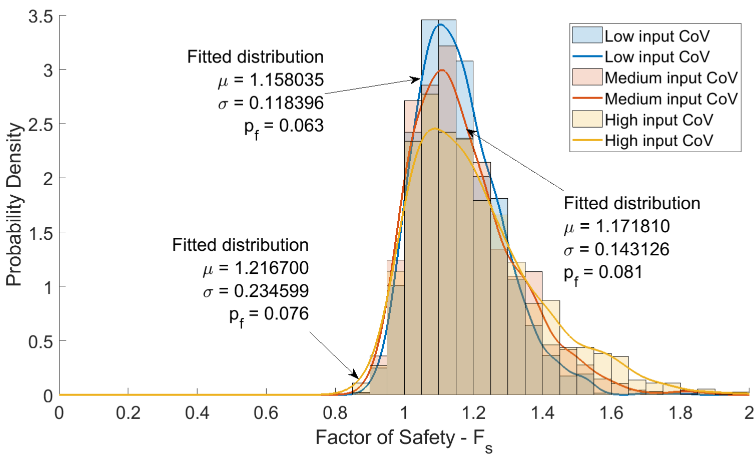


Figure 6.2: Estimated posterior distributions of the Factor of Safety in the three Single Random Variable simulations.

Figures 6.3, 6.4 and 6.5 show the prior and posterior distributions of F_s in the simulations with low, medium and high COV respectively. The updating has a significant effect on all of the three simulations. Because the simulation with the high COV has a lower mean and higher standard deviation in the prior distribution of F_s, it also has more realizations with F_s<1. This leads to a more extensive updating where more of the realizations of the slope are removed. How much the number of realizations and probability of failure changes over updating is listed in table 6.1. The higher the input COV is, the more realizations are removed. One could think that this means the updating is less efficient for the low COV simulation than for the high COV simulation. However, removing 62.8% of the realizations in the high COV is not necessarily a good thing. This implies that the prior distribution was very unrealistic, and poorly estimated. When so much of the previously gathered information is discarded, it might be a clue to go back and look at the methods used to determine the soil parameters and estimate the system behaviour. If we have so little confidence in the the low values, how reliable are the high values that are left? The high COV simulation in this project has an artificially high COV, and there might be good reasons for not having confidence in the results.

Table 6.1: Relative change of number of realizations and probability of failure due Bayesian to updating.

	Variable	Prior	Posterior	Relative change
Low CoV	$n_{realizations}$	2000	1363	31.85%
	p_f	0.315	0.062	80.31%
Medium CoV	$n_{realizations}$	2003	945	52.82%
	p_f	0.528	0.089	83.14%
High CoV	$n_{realizations}$	2008	747	62.80%
	p_f	0.634	0.081	87.22%

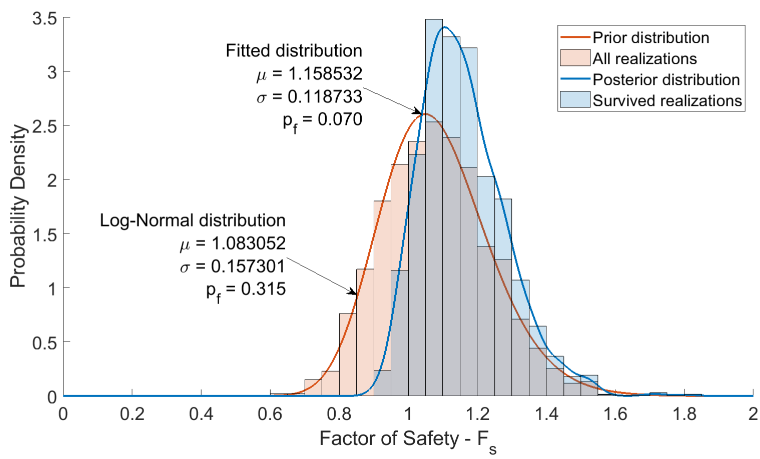


Figure 6.3: Estimated prior and posterior distribution of the Factor of Safety in the Low CoV Single Random Variable analysis.

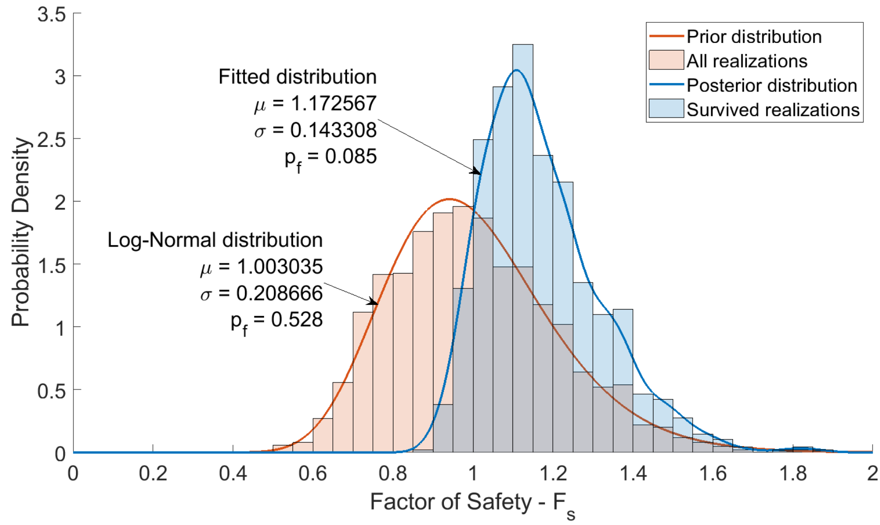


Figure 6.4: Estimated prior and posterior distribution of the Factor of Safety in the Medium CoV Single Random Variable analysis.

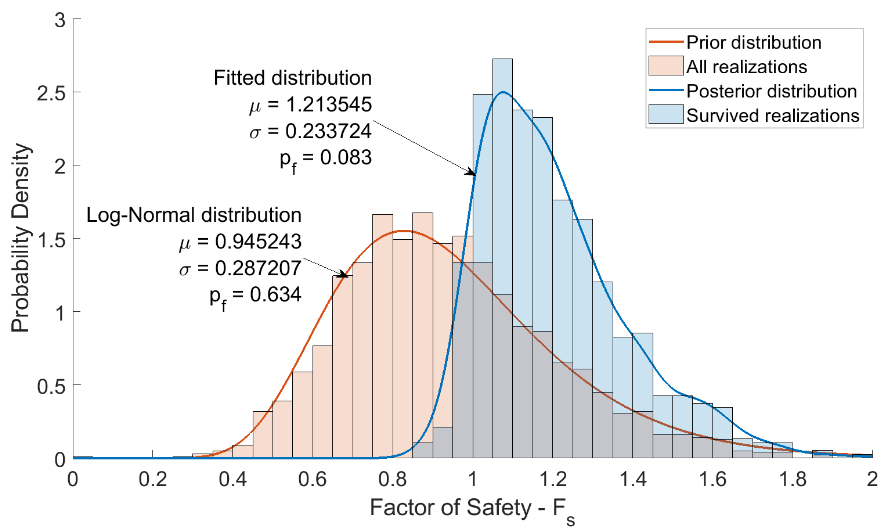


Figure 6.5: Estimated prior and posterior distributions of the factor of safety in the High CoV Single Random Variable analysis.

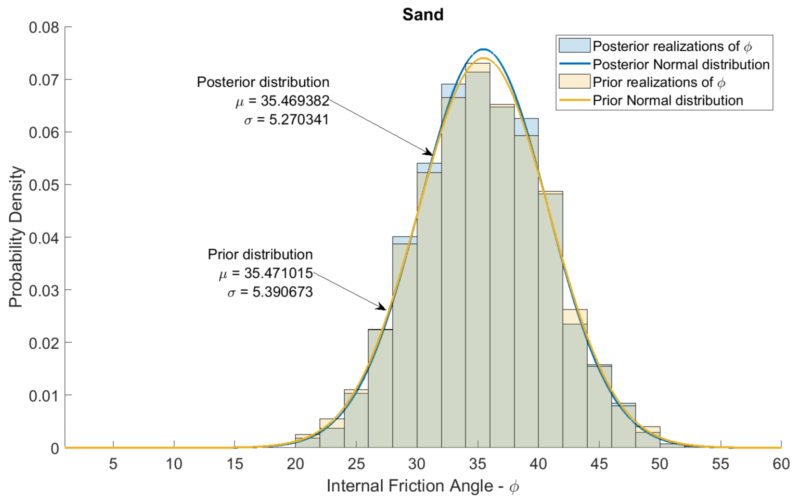


Figure 6.6: Estimated prior and posterior distribution of the internal friction angle of the Sand material, in the Single Random Variable analysis.

6.2 Updating of Shear Strength Distributions

The effect of updating on the distributions of the shear strength parameters are visible in all three simulations. The same trends are visible in all three results, so only one simulation will be presented. Because the simulation with the lowest COV is the most realistic one, the results of this simulation will be used to illustrate the effect that the Bayesian updating has on the shear strength distributions.

6.2.1 Sand

The prior and posterior distributions of the internal friction angle of sand are shown in figure 6.6. The difference between the two distributions is very small, making it difficult to distinguish them. Even though a large part of the realizations of the sand layers are deleted in the posterior distribution, it does not affect the shape of it. This indicates that the shear strength value of the sand has very little effect on the Factor of Safety of the slope. The realizations that were deleted had values spread out over the entire range of values of the internal friction angle.

6.2.2 Quick Clay

The prior and posterior distribution of the undrained shear strength S_u of the Quick Clay layer are shown in figure 6.7. The mean value of S_u of the quick clay increases with 5% when updating the distribution. This indicates that there is a relation between low

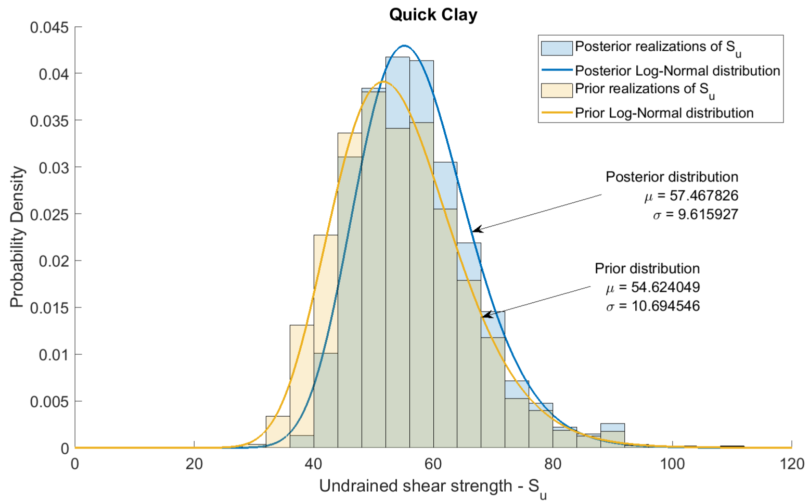


Figure 6.7: Estimated prior and posterior distribution of the undrained shear strength of the Quick Clay material, in the Single Random Variable analysis.

values of the shear strength of quick clay and low values of the Factor of safety. This is also confirmed by the reduction in the standard deviation, σ, which indicates that the deleted realization mainly had low values of S_u. Because there are several layers of different materials affecting the total shear strength of the slope, there is not a linear relation between the shear strength of the quick clay and the Factor of Safety. Because of this the change in the quick clay shear strength distribution is not as strong as the change in the F_s distribution in figure 6.3.

6.2.3 Clay

The prior and posterior distribution of the undrained Clay layer are shown in figure 6.8. The updating seems to have a similar effect on the Clay layer as it did on the Quick Clay layer. The mean value is increased, and the standard deviation is decreased, due to removal of values on the lower side.

6.2.4 Sensitive Clay

Figure 6.9 shows the prior and posterior distribution of the undrained shear strength of the Sensitive Clay layer. The distribution of this layer seems to have a similar response to the updating as the two other clay materials. This can be an indication that the materials that behave undrained have more effect on the Factor of safety than the Sand layer that is modeled with a drained response.

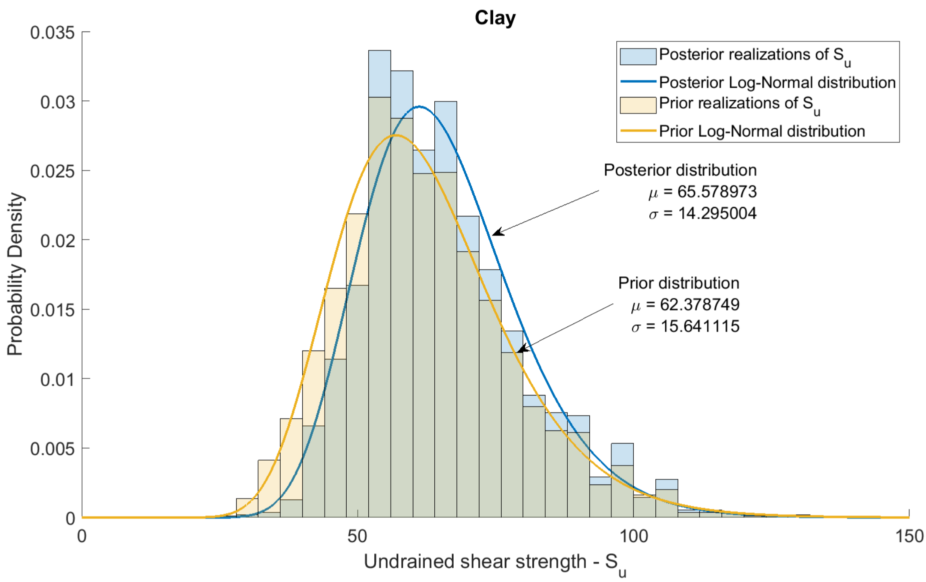


Figure 6.8: Estimated prior and posterior distribution of the undrained shear strength of the Clay material, in the Single Random Variable analysis.

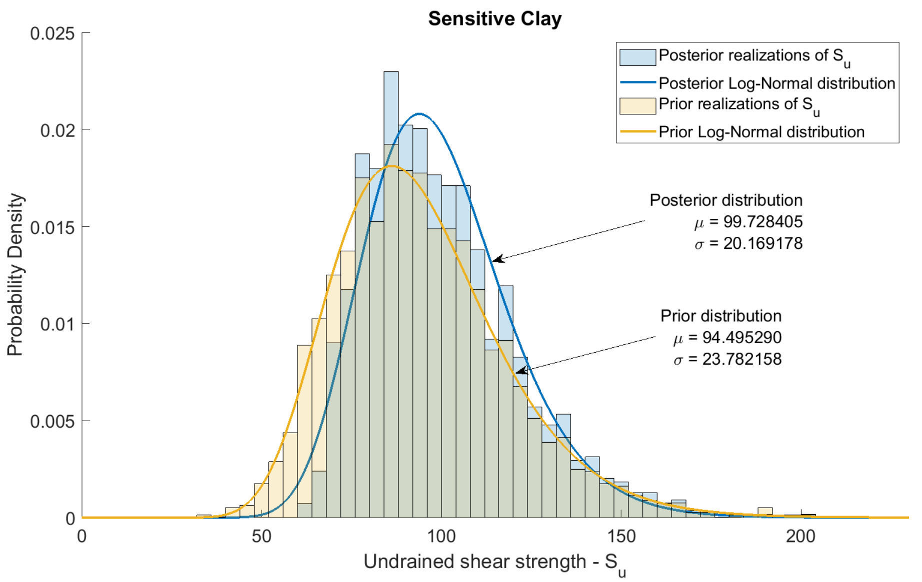


Figure 6.9: Estimated prior and posterior distribution of the undrained shear strength of the Sensitive Clay material, in the Single Random Variable analysis.

6.3 Effect of Model Error

The model error is applied in the process of updating to to avoid cutting a sharp line at $F_s = 1$. This is because it is not certain that the estimated F_s is entirely correct, and it would not be justifiable to remove all realizations of $F_s < 1$, and thereby reduce the probability of failure to zero. The model error is added in order to compensate for uncertainty that is introduced to the calculation model by assumptions and simplifications. The true model error is of course unknown, and a subjective estimate must be made. In this thesis the model error has been estimated as a normal distribution with mean $\mu_{\epsilon_M} = 0$ and standard deviation $\sigma_{\epsilon_M} = 0.05$. In subsection 6.1 it was discussed that the shape of the prior distribution has impact on the effect of the model error.

Figure 6.10 and 6.11 show that also the way that the model error is estimated has a influence on its effect on the posterior probability of failure. Figure 6.10 shows the prior and posterior distribution of the Factor of Safety from the simulation with low COV, and no model error applied during updating. The posterior distribution is cut off at $F_s = 1$. Figure 6.11 shows the same distribution, but with a small model error $\epsilon_M = N(0, 0.025)$. By comparing the figures 6.3, 6.10 and 6.11 it becomes clear that the model error blends out the results from the probabilistic analysis. The smaller the model error is, the stronger is the updating. On the other hand, the higher the confidence in the calculation model is, the smaller is the estimated model error. It is important to try to make an accurate estimate of the model error, in order to get an accurate representation of reality.

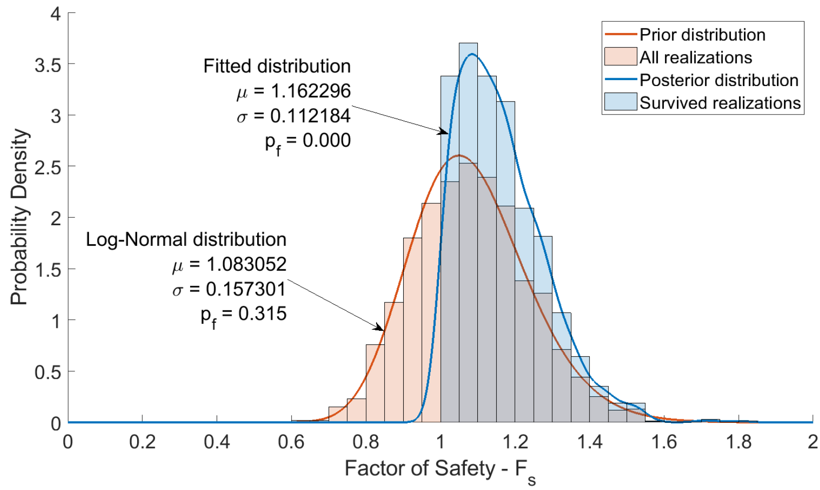


Figure 6.10: Estimated prior and posterior distribution of the factor of safety in the low CoV simulation, in the SRV analysis. No model error is applied during updating.

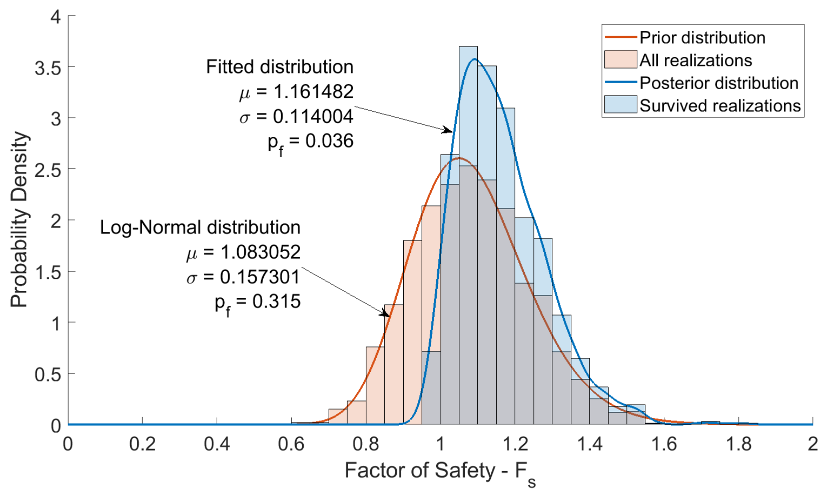


Figure 6.11: Estimated prior and posterior distribution of the factor of safety in the low CoV simulation, in the SRV analysis. A reduced model error is applied during updating.

Chapter 7

Bayesian Updating with Conditional Random Field

In the Conditional Random Field (CRF) analysis the shear strength of every single element is described by a separate random variable. This means that there is one prior and one posterior probability distribution of the shear strength in each element. It can be difficult to find trends in the results when the amount of data is so large. This chapter attempts to extract and visualize interesting effects Bayesian of updating on the distributions of shear strength and of Factor of Safety when modelling a slope with a random field.

7.1 Updating of Factor of Safety

Three Monte Carlo simulations has been carried out in the Bayesian analysis with a conditional random field, each with a different Coefficient of Variation (COV) of the input shear strength. The prior and posterior distributions of the Factors of Safety from these simulations are shown in figure 7.1. All the histograms are fitted with normal distributions.

The prior distributions show that the behaviour of the system changes completely when the COV is increased. The blue distributions correspond to the initial estimate of the soil parameters. In the other two distributions the standard deviation of the input is increased artificially in order to increase the COV. A higher COV of the input leads to a higher COV of the output, both because the standard deviation is increased, and because the mean is reduced. It is expected that the variance of the F_s increases when the amount of higher and lower values of the shear strength is increased. However, the large decrease in the mean value was not foreseen in the same way. This is the same effect as the one seen in the results of the Single Random Variable (SRV) analysis, but to a greater extent. In the random field there will be both high and low shear strength values throughout the mesh. The failure surface seeks out and connects weak elements and zones. The shifting of the distribution of F_s indicates that the weak elements have a larger effect on the F_s than the high elements. Strong elements cannot prevent failure as efficiently as low values can cause it.

The second subplot in figure 7.1 shows the posterior distributions of F_s. The blue and yellow distributions are almost identical to the prior distributions. In those simulations very few realizations are removed due to low values of F_s during the updating. The updating seems to only have a significant effect on the simulation with the high input COV. The change in the green distribution is not drastic from the prior to the posterior, but it is visible. The shifting of the mean values causes the distributions to barely overlap, which is the reason that the green distribution has a large portion of realization with $F_S < 1$, while the other two has close to none. This difference would likely be smaller if it was only the standard deviation that separated the distributions.

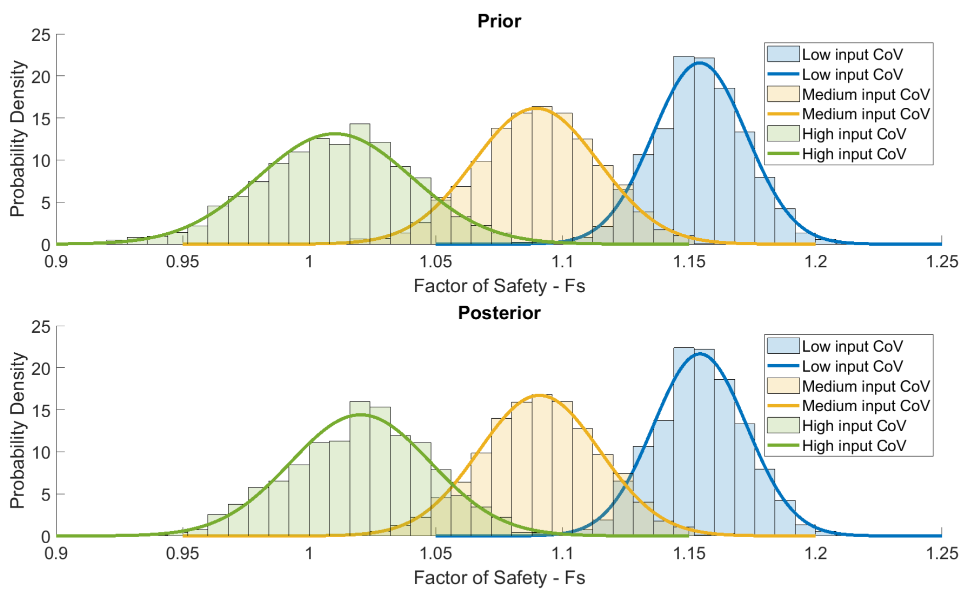


Figure 7.1: Prior and posterior distributions of the factor of safety, for three values of input COV, in the CRF analysis.

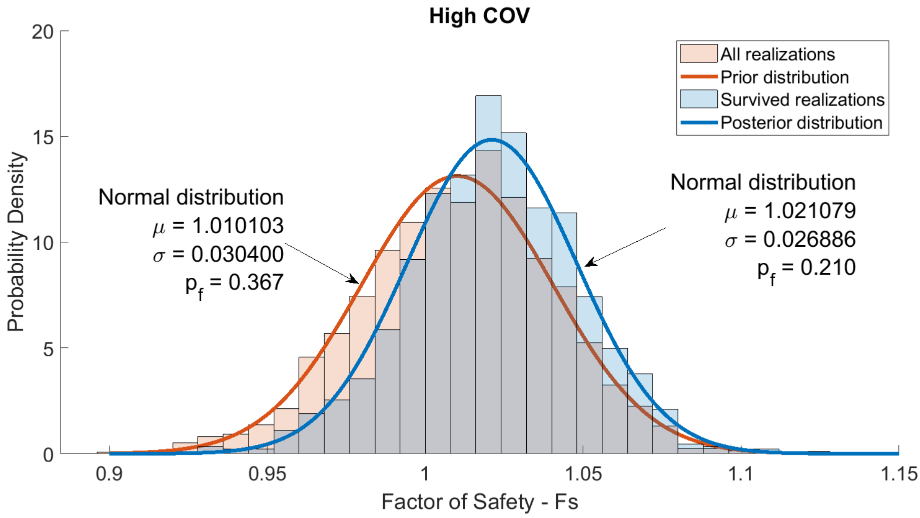


Figure 7.2: Prior and posterior distributions of the factor of safety for a high value of input COV.

A comparison of the prior and posterior distribution of F_s is shown in figure 7.2. Even though many of the realizations in the high COV simulation have $F_s < 1$, the Bayesian updating is not changing the distribution of the F_s drastically. This is due to the effect of the model error. The model error is included in the updating, and causes realizations with $F_s < 1$ to be kept, and realizations with $F_s > 1$ to be discarded, because of uncertainties in the estimate of F_s. The model error seems to have a very large effect on the updating in the CRF simulation. This is likely because the variance that is introduced with the model error is large compared to the variance that is already present in the prior distribution of F_s. The effect of the model error will be discussed further in section 7.3. The following section will present the effect of Bayesian updating on the shear strength distributions of the materials. This will be illustrated with results from the high COV simulation, as this is the only simulation where the updating gives a noticeable effect.

7.2 Updating of Shear Strength Distributions

Every simulation yields one prior and one posterior distribution of the shear strength of every single element in the Finite Element model. Because every distribution reacts differently to the updating, it is challenging to display the results. There are two main approaches; either to try to identify effects when looking at the average response of the random field, or to try to identify which zones are most sensitive to the updating, and investigate these. An attempt has been made on both of these approaches. In the shear strength distributions it is most interesting to look for effects of the Bayesian Updating.

7.2.1 Mean Shear Strength per Element

This subsection presents figures of the distributions of the average shear strength value of each element, through all simulations. There is one distribution for each material, including all elements in that layer. In these plots there is no reduction in the number of data points in each histogram due to updating, as both the prior and posterior distribution contain all elements. It is rather a change in the realizations that make up the mean value of the elements' shear strength.

Figure 7.3 shows the distribution of the average value of the shear strength per element in the sand layer. Because the random field of the sand layer is conditional on the CPTU measurements, and not stationary, the random variables describing the shear strength of each element are dependent on both each other and on the Piezocone Penetration Test (CPTU) measurements. Because of the non-stationarity of the conditional random field the distribution of the mean values per element cannot be expected to follow the Central Limit Theorem, and it can not be expected to form a normal distribution. This is confirmed by the distributions in figure 7.3. The effect of the updating is shown by the difference between the prior (red) and posterior (blue) distribution. The biggest difference between the two is that the peak is cut off during the updating, and the standard deviation of the posterior distribution is slightly higher than that of the prior.

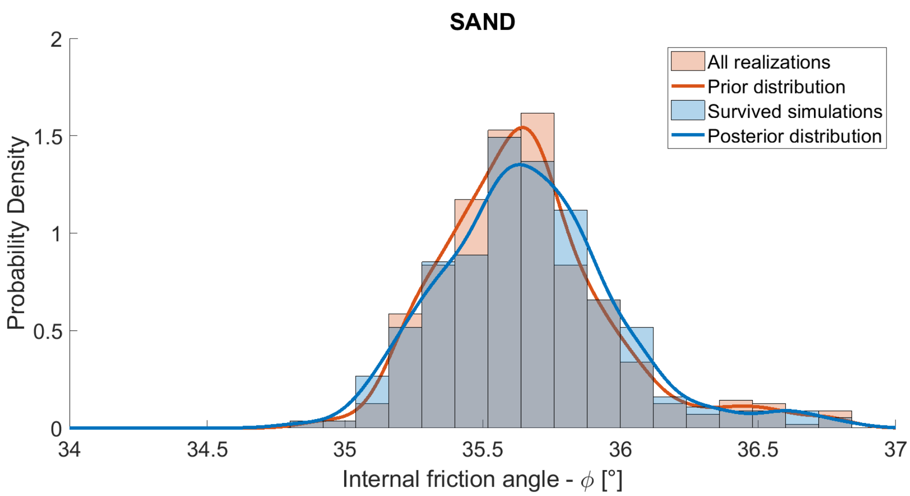


Figure 7.3: The distribution of average shear strength per Sand element, over all realizations.

Figure 7.4 shows the distributions of the mean value per element in the Quick Clay layer. Both distributions have odd shapes, likely due to the non-stationarity. There is one very clear peak, and two long tails of high and low values. Also here the biggest difference between the prior and posterior distribution is a reduction of the peak. The change is small, indicating that it is not low values in the Quick Clay layer that triggers failure in the slope. This can indicate that the estimation of the shear strength values of Quick Clay is accurate enough to not indicate slope failure.

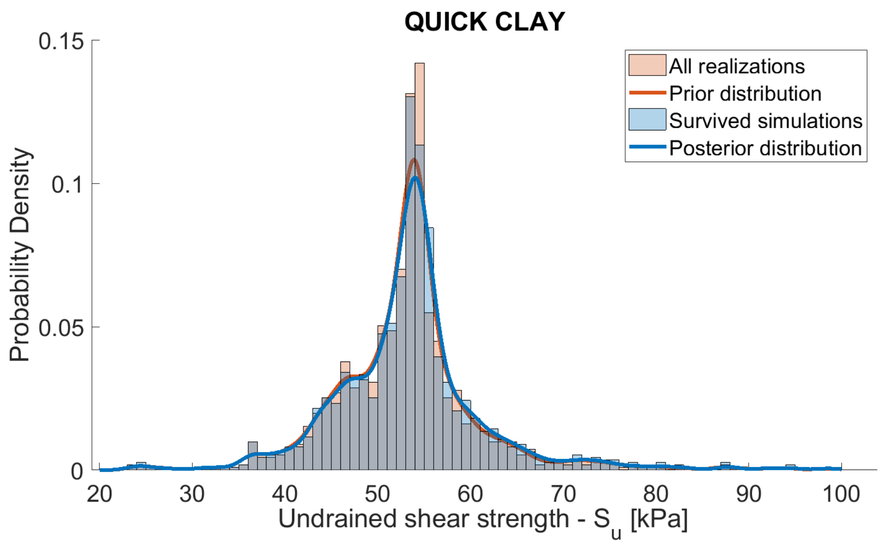


Figure 7.4: The distribution of average shear strength per Quick Clay element, over all realizations.

Figure 7.5 shows the prior and posterior distributions of the mean shear strength of the Clay 1 elements. The two distributions are very similar. The only visible difference is that the the peak is slightly reduced. The plot of the distributions indicate that low Clay 1 values does not cause realizations of the slope to fail. The realizations that failed, and were removed during updating, seem to have shear strength values from all parts of the distribution in the Clay 1 layer. This is perhaps due to the high density of CPTU measurements in this layer, leading to more accurate estimates of the shear strength.

Like in the previously mentioned materials, the biggest effect of the updating on the shear strength of the Sensitive Clay is a reduction of the peak, and thus increase of the standard deviation. The long and thin tail of low values does not seem to be affected by the updating. These elements keep a constant average shear strength value. This consistency is suspicious, and might indicate that these values stem from the CPTU

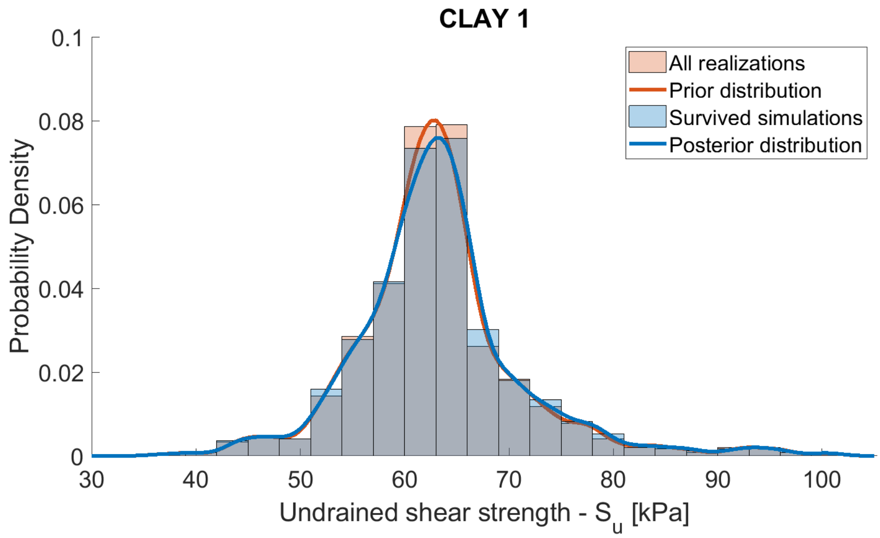


Figure 7.5: The distribution of average shear strength per Clay 1 element, over all realizations.

measurements. The shear strength distributions in the measured elements have a relatively small standard deviation, and will vary within a relatively small range of values throughout the realizations. The updating will not change the mean value of these elements.

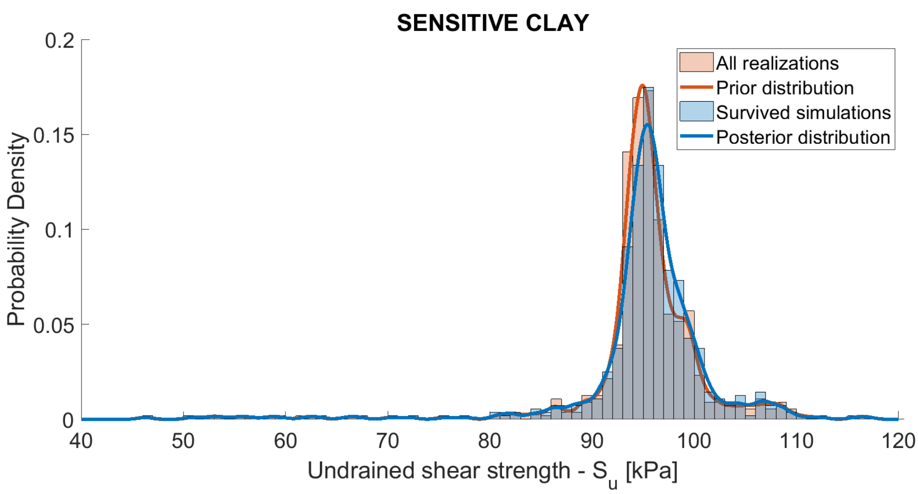


Figure 7.6: The distribution of average shear strength per Sensitive Clay element, over all realizations.

Because none of the CPTU profiles enter the Clay 2 layer, the random field of this layer is stationary. This means that the distributions of the shear strength in this layer is expected to form a more normal Probability Density Function (PDF). This is confirmed in figure 7.7. The updating gives a clear difference between the prior and posterior distribution. The mean is shifted towards higher values, because some elements get higher mean values of the shear strength, when weak realizations are removed. However, not all elements are affected in this way. Some elements do not influence the Factor of Safety. The slope fails or stands, regardless of the shear strength of those elements. These elements do not experience a change of the mean value. This is causing the standard deviation to increase from the prior to the posterior distribution. The elements become more spread out, because some elements change to higher values, and others stay at the same low value.

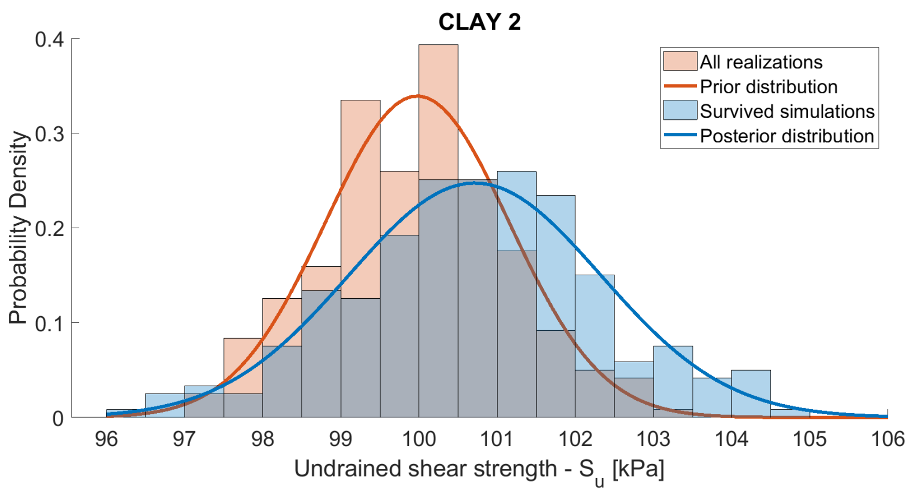


Figure 7.7: The distribution of average shear strength per Clay 2 element, over all realizations.

The bottom line after looking at the change in the mean values of each element over all realizations, seems to be that the estimated shear strength properties are more accurate in the layers where CPTU measurements are available. Making the random field conditional on these observations seem to remove the values of shear strength that would cause failure of the slope, from the distribution.

7.2.2 Mean Shear Strength per Simulation

The figures in the following subsection also show distributions of the shear strength, and are because of that split according to material. The plots show prior and posterior distributions of the average shear strength over all the elements in each realization.

That is, every data point in a histogram represents the average shear strength of all the elements(of a certain material) in one realization. There will be more data points in the prior distribution than in the posterior distribution, due to removing realizations during updating. The histograms are normalized, to form PDFs.

Figure 7.8 shows the prior and posterior distributions of the average shear strength of all Sand elements per realization. The two distributions are very similar, which confirms that the strength of the sand layer on top of the slope does not affect the F_s of the slope much. Removing the failed realizations does not alter the ratio of high to low values of the sand shear strength.

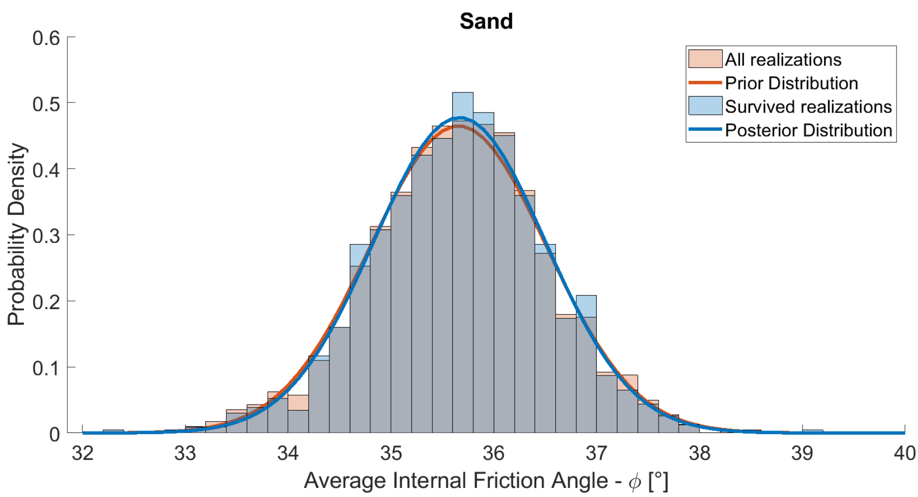


Figure 7.8: The distribution of the average shear strength per realization, over all Sand elements.

Figure 7.9, 7.10, 7.11 and 7.12 show the corresponding distributions of the Quick Clay, Clay 1, Sensitive Clay and Clay 2 respectively. The response to the updating is similar in all four clay layers. The posterior distribution is shifted a little towards higher values compared to the prior distribution. This means that the cohesive layers of the survived realizations on average have a slightly higher shear strength than the discarded realizations. Subsection 7.2.1 showed that there does not seem to be many specific elements in the Quick Clay, Clay 1 and Sensitive Clay layer that affect the F_s of the slope. It seems like it is rather the average value of S_u that is higher in the surviving realizations, and which elements that draw the mean up can differ from realization to realization. This was different in the Clay 2 material, where there seemed to be some elements that was clearly underestimated by the prior shear strength distribution. Low values of these key elements seemed to be tightly connected with a low F_s for the slope.

Back to the figures in this subsection, it is clear that there is only a small change in the average shear strength value from before to after updating. This is likely because the failure of the slope is only dependant on some of the elements in the model. Elements that are located far away from the failure surface do not affect whether the slope fails or not. The updating is independent of the fluctuation of the shear strength in these elements. When looking at the average value of all of the elements in a realization, these elements can blur out the effect that the updating has on the elements that are located close to the failure surface.

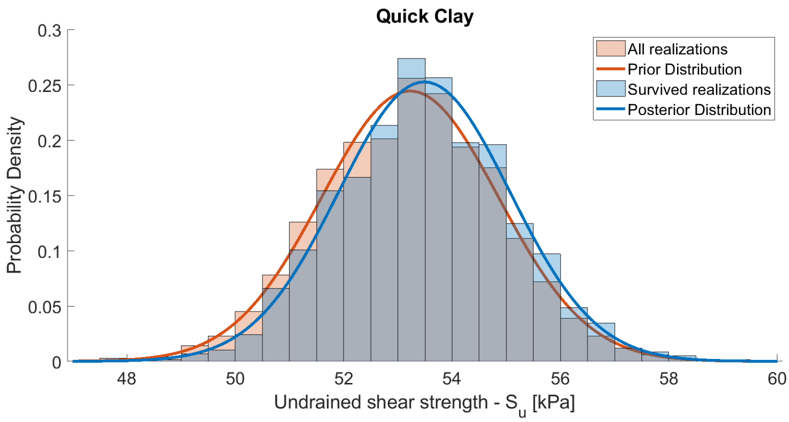


Figure 7.9: The distribution of the average shear strength per realization, over all Quick Clay elements.

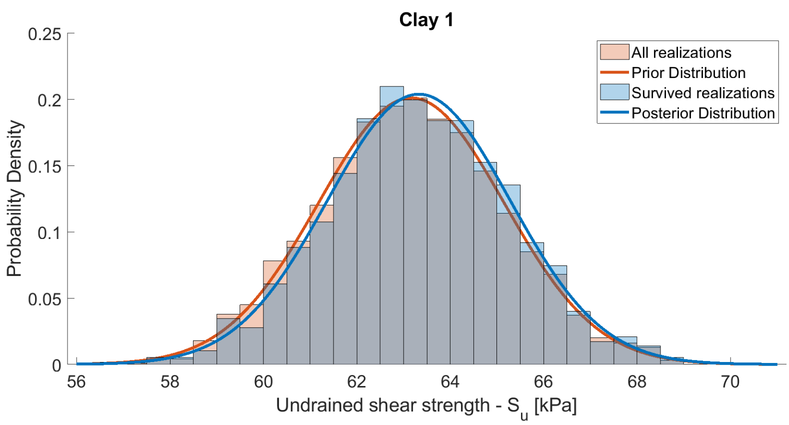


Figure 7.10: The distribution of the average shear strength per realization, over all Clay 1 elements.

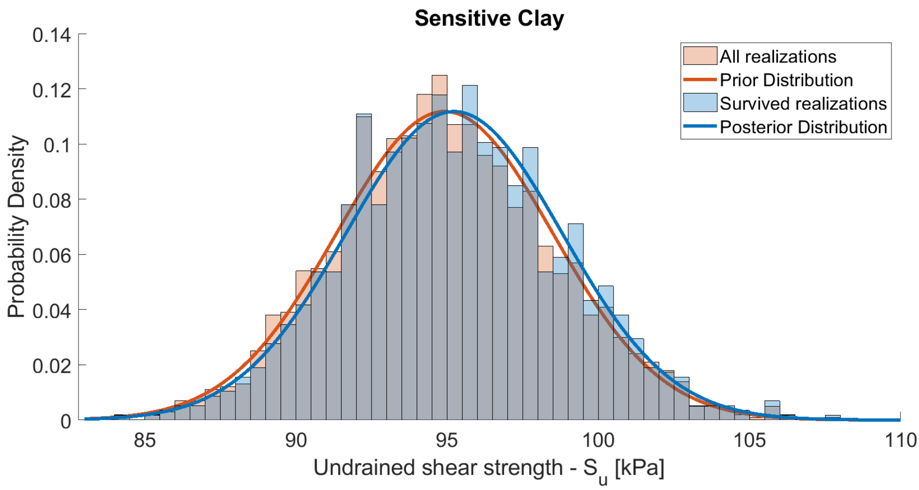


Figure 7.11: The distribution of the average shear strength per realization, over all Sensitive Clay elements.

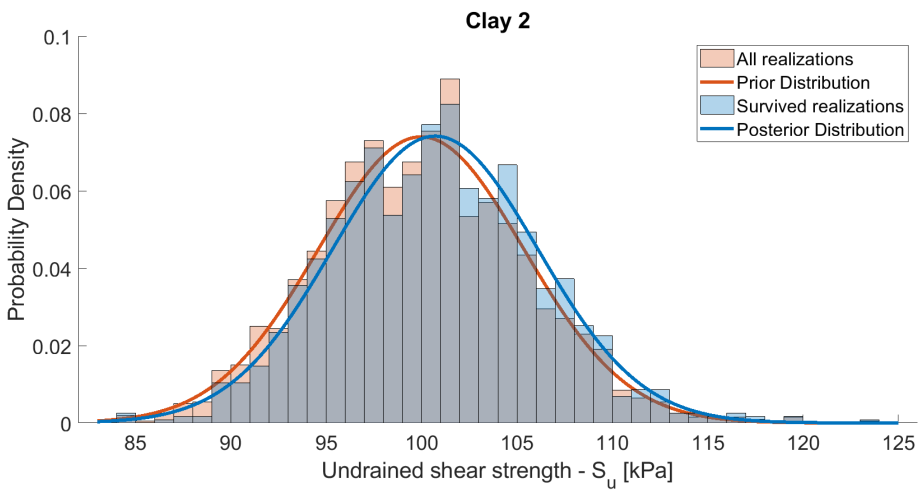


Figure 7.12: The distribution of the average shear strength per realization, over all Clay 2 elements.

7.2.3 Sensitive Zones

It is expected that some zones in the random field are more strongly related to the F_s of the slope than others. These zones will the also show a larger effect when updating based on F_s. This can be difficult to identify by looking at the average response of the entire slope. Figure 7.13 shows a figure of the random field, with the absolute value of the difference between the prior and posterior average shear strength indicated with colour in each element. This is the same entity that was plotted in the figures in subsection 7.2.1. The blue area in the lower left corner is bedrock, and the top layer is sand, which is defined with zero cohesion.

The plot is meant to show which elements that change the most when updating the random field. It is difficult to find clear patterns of failure surfaces. However, it is possible to identify that there are more elements with bright colours close to the bedrock than there are between the CPTU measurements. Especially the lowest layer, that consists of Clay 2, seems to have a higher concentration of red and yellow elements close to the steepest slope of the bedrock. This indicates that this zone is estimated to have lower shear strength than what is possible, as long as the slope is still standing. This is the layer where no CPTU measurements are available. In the layers where CPTU measurements are available, there seems to be more blue and turquoise elements, which indicate smaller change upon updating. This indicates that the CPTU measurements do contribute to give a more accurate representation of the shear strength of the soil. A limitation of this study is the relatively coarse random field discretization. The changes might have been easier to detect if the discretization was finer.

This simulation contains 2000 realizations. This is not enough to to give a stable output, independent of the sample size. It is likely that the results in shown in figure 7.13 also contain some statistical noise because of this. Some elements that are not expected to affect the F_s much, show a fairly large change in the shear strength upon updating. This can be seen in the lower right corner of the model. Some of the elements even experience a decrease of the average shear strength when removing failing realizations from the analysis. Perhaps these effects would disappear if the sample size was increased. This is left for other studies to find out.

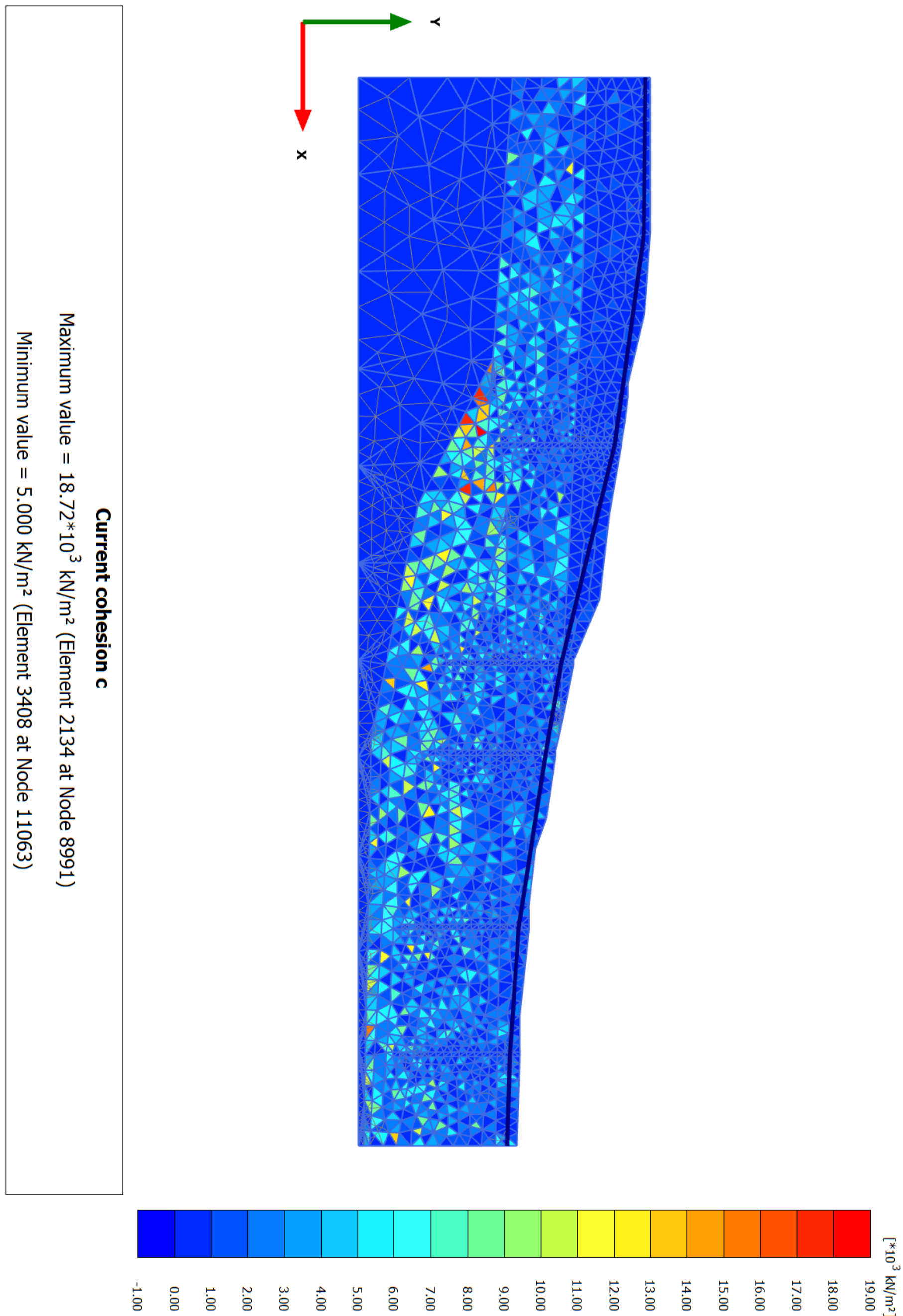
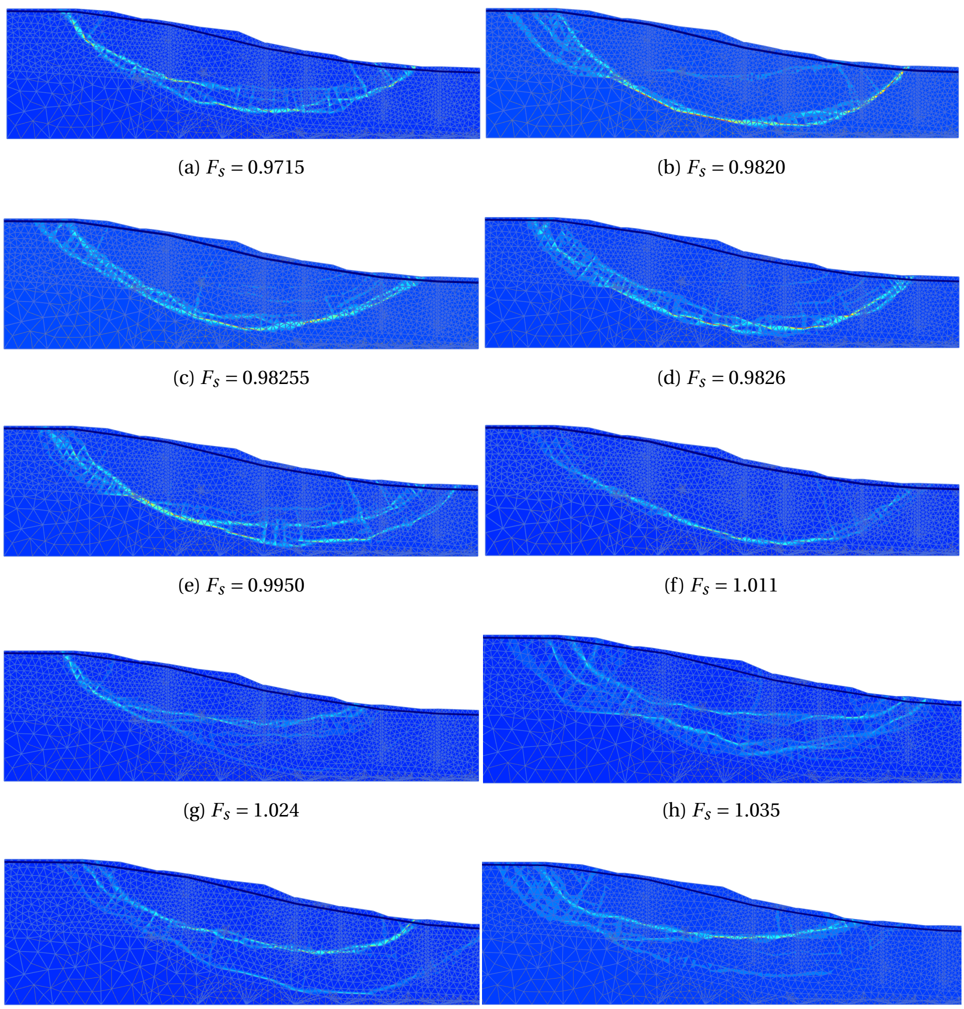


Figure 7.13: The absolute value of the difference in the average cohesion (S_u) of each element before and after updating. The values are multiplied with a factor of 1000.

Failure Mechanisms

Failure mechanisms from 10 of the realizations are shown in figure 7.14. They are sorted according to F_s, in increasing order. Half of the realizations failed, and half survived. This is a too small sample to draw any conclusions, but some patterns can be identified. The failure mechanisms seems to go deeper, towards the Clay 2 layer, when the $F_s < 1$. When more shallow failure mechanisms are critical F_s seems to be above 1. Some of the plots seem to develop two failure surfaces, in which the shallow one turns out to be the most critical one.



(a) $F_s = 0.9715$ (b) $F_s = 0.9820$

(c) $F_s = 0.98255$ (d) $F_s = 0.9826$

(e) $F_s = 0.9950$ (f) $F_s = 1.011$

(g) $F_s = 1.024$ (h) $F_s = 1.035$

(i) $F_s = 1.045$ (j) $F_s = 1.055$

Figure 7.14: Failure mechanisms from 10 realizations in the high COV simulation, in the CRF analysis.

7.3 Effect of Model Error

As seen in both in figure 7.1 and in the results from the SRV analysis the application and magnitude of the model error strongly affects the results of the updating. The model error is the reason that the posterior probability of failure is not zero, and it has a big impact on how many realizations are discarded in the updating process. The model error has been estimated with a Normal distribution with mean $\mu_{\epsilon_M} = 0$ and standard deviation $\sigma_{\epsilon_M} = 0.05$ in this thesis, N(0, 0.05). Figure 7.15 and 7.16 shows the prior and posterior distribution of the Factor of Safety with $\sigma_{\epsilon_M} = 0.00$ and $\sigma_{\epsilon_M} = 0.025$ respectively. These can be compared with figure 7.2 in sub section 7.1. The more the variation of the model error increases the more it blurs out the line between failing and surviving realizations. This has a very large effect on the results of the CRF analysis, because the variation in the distribution of F_s is quite small compared to the variation added by the model error. The model error that was used in this thesis is so large that it removes a large part of the effect of the Bayesian Updating. This indicates that the confidence in the calculation model is low, and a lot of uncertainty is added in order to compensate for this. In order to get reliable results it is important to take care when estimating the model error.

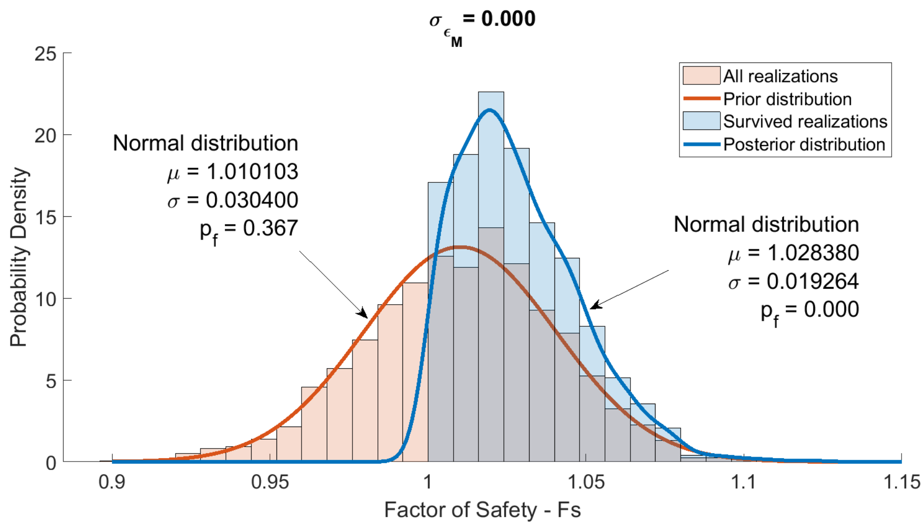


Figure 7.15: Prior and posterior distribution of the Factor of safety in the high COV simulation, updated with no model error.

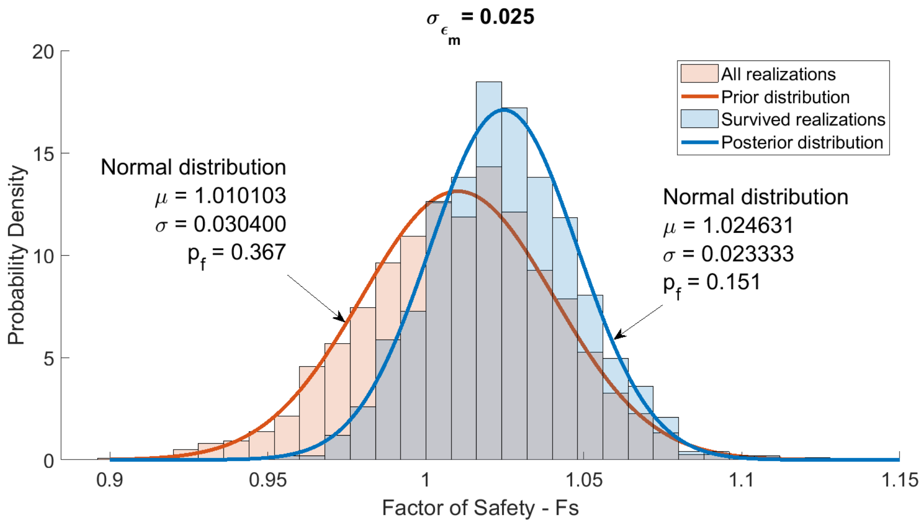


Figure 7.16: Prior and posterior distribution of the Factor of safety in the high COV simulation, updated with reduced model error.

Chapter 8

Discussion

This chapter is a discussion of the the implications of the results in chapter 6 and 7. The first section compares the results from the Single Random Variable (SRV) analysis and the Conditional Random Field (CRF) analysis. The discussion is focused on the effects of Bayesian updating, varying input Coefficient of Variation (COV) and the model error. The second and third sections highlight particularly interesting results from the SRV analysis and CRF analysis respectively, before a last discussion of the limitations of the work done in this thesis.

8.1 Comparison of SRV and CRF

The results of the SRV analysis and CRF analysis has been presented separately. This section will discuss similarities and differences between these two methods, and try to assess the applicability of the SRV analysis as a simplification of the slope material.

8.1.1 Updating the Factor of Safety

Figures 6.1 and 7.1 show the plots of the prior distributions of the Factor of Safety F_s in the SRV analysis and CRF analysis respectively. A clear difference between the two plots is the difference in the variance of the Factor of Safety. The COV is significantly higher in the results from the SRV analysis, than in the results from the CRF analysis. As seen earlier, the variance of the Factor of Safety F_s is dependent on the variance of the materials' shear strength. In the SRV the failure surface goes through a maximum of four different shear strength values, one for each material. Each of these can make a rather big impact on the slope's total strength. A single high or low value can change the average shear strength up or down significantly, and thus impact the Factor of Safety. In the CRF analysis the failure surface will always pass through a large number of elements, all with a different shear strength. The strength of the slope will always be an average of many high and low values, where each value has a smaller impact than in the SRV analysis. The realizations will be much more similar to each other than in the SRV analysis. This leads to a much wider range of responses in the SRV analysis than in the CRF analysis, and a much more spread out distribution of the F_s.

Furthermore, it was unexpected that the COV of the shear strength had a big impact on the system behaviour. Not only did the increased standard deviation in the input lead to increased standard deviation of the output; it also led to a significant reduction of the mean of the output. This is assumed to be due to the effect of increasing the amount of low shear strength values in the realizations. This effect was more pronounced in the CRF analysis than in the SRV analysis, but noteworthy in both. A probable reason for this difference is also found in the representation of the spatial variability. In the random field there will likely be some low shear strength values among the approximately 3000 elements. This means that close to all realizations will be affected by the increase of the COV of the input. In the SRV analysis there are only four shear strength values, and fewer realizations will contain and be dominated by low values. Fewer of the SRV realizations will be affected by increasing the COV, and the distribution changes less.

These two differences between the estimated response of the slope indicates that the SRV analysis is not an accurate simplification of the random field. The SRV is the limiting case of a random field, when the correlation length tends to infinity. In most of the cases it is used because it is on the conservative side with larger failure probabilities. The SRV analysis estimates a much higher probability of failure in all three simulations, and has a smaller effect of varying the input distributions.

8.1.2 Updating Shear Strength

When looking at the average shear strength value per element, over all realizations, the SRV analysis and the CRF analysis show different effects after updating. In the SRV analysis the Bayesian updating caused an increase in the average values of both the Quick Clay layer, Clay 1 layer, Sensitive Clay layer and the Clay 2 layer. This indicates that low values in any of the four layers are related to low values of the Factor of Safety. Only the Clay 2 layer showed a similar increase in the average value of shear strength per element, in the CRF analysis. The other three clay layers did not show the same effect. Piezocone Penetration Test (CPTU) measurements are incorporated directly into the random fields of these three layers. This is not done in the Clay 2 layer of the CRF analysis, or in any of the layers in the SRV analysis. It can seem like the direct application of measurements from CPTU profiles makes the soil parameter estimation more accurate, and makes it less likely that the estimated material parameters cause the slope to fail.

When looking at the average shear strength per simulation, over all elements in the random field each layer is represented by only one shear strength value. This is comparable to looking at the shear strength distribution of the materials of the SRV analysis. The change is similar in the two analyses. The general effect of the updating is that the shear strength distribution shifts towards higher values, in all materials except sand. However, the effect is bigger in the SRV analysis than in the CRF analysis. There is a stronger link between the shear strength values of the layers in the SRV analysis and F_s than there is between the average value of the random field and F_s. The small change in the CRF results indicate that it is not only the average shear strength of a random field that determines F_s, but also the arrangement of weak and strong values in space. This contribution is lost in the SRV analysis, because the spatial variability is not modelled. Here it is important to remember that not all elements in the random field are related to the Factor of Safety. Some elements vary independently of F_s, and will reduce the visible effect of updating when they are included in the average.

8.1.3 Model Error

A model error was added in the updating process in both of the two analyses. This model error was not estimated specifically for this calculation model, but rather taken from another model with similar assumptions of plane strain behaviour presented in Wu (2009). The same model error is applied in the SRV analysis and the CRF analysis. Because of the big difference in COV of F_s between the two analyses, the model error has quite different impact on the two updating processes. The variance added by the model error affects the CRF results more, because the relative increase of variance is significantly larger. This makes it valid to question both whether it is sensible to add the same model error to the two analyses, and whether the model error should be adjusted to the variance of the results it is applied to. The model error is there to compensate for assumptions and simplifications in the calculation model. The SRV analysis is more simplified than the CRF analysis, because of the neglect of the spatial variability within each layer. This can potentially justify applying a higher model error to the SRV results than to the CRF results.

The model error is so large that it blurs out a large part of the updating in the CRF analysis. This indicates that the confidence in the calculation model is so low that only a little bit of emphasis can be put on the results it produced. In this project this was not a conscious choice, but rather something that was discovered later on, when seeing the effect that the model error had on the results. However, if the confidence in the calculation model was so small that most of the results should be blurred out by the model error, it might be necessary to go back and have a second look at the model itself. If the results cannot be trusted, the model does not provide great value in the estimation of the slope stability. The required model error according to Wu (2009) could also reflect the low confidence in the input parameters. However, the Bayesian updating is actually an attempt to reduce uncertainty in the input parameters of the original model. The model error might have to be treated differently when using Bayesian updating. That is for another thesis to consider.

In order to investigate how the model itself behaves, and how it relates input to output, it would be interesting to perform an updating procedure with a minimal model error. This would not necessarily give reliable estimates of the posterior probability of failure, and posterior distributions of shear strength, but it could provide better insight on the model itself. It could give better answers about the conceptual differences between the SRV and the CRF analysis. The updating would be stronger, and perhaps clearer effects of Bayesian updating could be identified.

8.1.4 Varying COV

Every materials' shear strength properties has been described with probability distributions. These were used as basis for simulations to estimate the expected behaviour of the slope. In addition, several simulations with increased COV were carried out, in an effort to make the estimates more conservative. The thought was that more uncertainty in the estimates would be conservative. The results show that rather than making the results conservative, this approach completely changed the behaviour of the system. The low values that were introduced dominated the response of the slope. This could be viewed as conservative, but it takes focus away from the original shear strength estimates. This effect is especially pronounced in the CRF analysis.

Increasing the COV introduces more of both low and high shear strength values. The updating process removes a lot of the realizations that are dominated by low shear strength values, and leaves the high ones. Especially in the SRV analysis this has an unconservative effect. In this analysis, some realizations are dominated by high values, and will be left in the posterior distributions. The three simulations have similar posterior Probabilities of Failure, but the simulation where the COV is high has significantly more realizations with high Factor of Safety in the posterior distribution. The artificially high values, that were added when increasing the COV, are given more emphasis when many of the low values are removed. These unforeseen effects leads to the conclusion that increasing the COV is not a good method for making conservative estimates.

Another approach to make conservative estimates of the shear strength could be to shift the shear strength distribution towards lower values. This is more similar to the approach used in deterministic methods, when applying partial safety factors. However, when implementing Bayesian updating with Monte Carlo Simulation, this would lead to removing more realizations. Too strong updating can also be problematic, and will be discussed in section 8.2. The findings seems to point in the direction that the best way to obtain conservative estimates is to perform sufficient ground investigations. This is backed up with the finding that the random fields where CPTU measurements were applied within the conditional random field framework, seemed to give more accurate presentations of the soil.

8.2 Single Random Variable

As mentioned in section 8.1 the SRV analysis yields a distribution of Factor of safety, F_s with a high COV. Combined with rather low mean values of F_s this leads to a significant amount of realizations that fail. This means that a large proportion of the realizations are removed during updating. In the high COV simulation especially, 62.8% of the realizations were removed. This means that a lot of the information that the shear strength parameters are based on is removed from the analysis, and the basis on which we form the posterior distributions as quite small. Discarding all this information means that the confidence in the prior estimates are quite small. This is perhaps an invitation to go back and look at what made the prior estimates so unreliable, rather than continuing on with the prior distributions. If the methods used to generate the prior distributions cannot be trusted, they might benefit from a revision.

8.3 Conditional Random Field

8.3.1 Limitations of FEM Model with Random Field

The random field is implemented in order to model the spatial variability of the shear strength within each layer. An important parameter to describe this is the correlation length. It describes how quickly the shear strength fluctuates in space. The correlation length is the length in space beyond which correlation is negligible. In practical terms the correlation length is often defined separately in horizontal and vertical direction, and the horizontal correlation length is often much higher than the vertical correlation length. In this model the vertical correlation length is estimated to be 0.6 meters for the Sand layer, and 0.3 meters for all of the clay layers. However, the element size in the mesh is set to approximately 1 meter. Smaller element sizes makes the model too heavy to run. This makes it impossible to properly model the spatial variability of each layer. The random field does make the representation of the soil more accurate, but not as detailed a desired. This is a weakness of this random field model, that should be addressed with more detailed approaches.

8.3.2 Relation between Shear Strength and Factor of Safety

Both figure 7.7 and 7.13 indicates that the shear strength estimation in the Clay 2 layer is inaccurate. This seems to be extra important in the left end of this layer, which seems to be a zone that triggers failures if it is estimated with low shear strength. This knowledge can be used in recommendations for where further ground investigations should be located. Further ground investigations would likely give much more valuable information if it was focused on the Clay 2 layer, than if it provided more information about the other layers.

It should also be discussed that in the CRF analysis the Bayesian updating only gave proper effects on the high COV simulation. The material description in that simulation is artificially high, and not very realistic. This shows that Bayesian updating does not have impact on all cases. However, it is not unthinkable that Bayesian Updating is applicable on other case studies. Because of that, it is still interesting to use the high COV simulation to investigate the method in itself, and how it responds to varying input and model error.

8.4 Bayesian Updating

Because there is no external and aleatory uncertainty in in the calculation model, like a variable load or ground water level, most of the uncertainty is epistemic and due to the lack of knowledge about the soil conditions. This epistemic uncertainty can be reduced with the Bayesian updating, and the probability of failure would be reduced to zero, if it were not for the model error. The posterior probability of failure says more about the estimate of the model error than about the Bayesian updating itself. However, in reality the slope is affected by aleatory uncertainties. A fill could be placed on the slope crest, or a flood could cause the groundwater level to increase. The model and calculated posterior probability of failure say nothing about these cases.

Because of this, it is not so interesting to look at the effect of Bayesian updating in terms of probability of failure. It is more interesting to look at how the Bayesian changes the distributions of the shear strength parameters. This is knowledge that could be transferred into other calculations, by using the posterior distributions as input for new calculations. Bayesian updating was implemented in order to reduce the uncertainty in the input parameters, and in that way obtain a more accurate estimate of the probability of failure. The results from the SRV analysis show that the standard deviation of the shear strength parameters reduces during updating. When including the information on observed performance of the slope, low values are discarded, and the estimate of the shear strength parameters used in the SRV analysis becomes more accurate.

In the results of the CRF analysis the distributions used to sample shear strength values for each elements has not been examined directly. It is more interesting to look at the behaviour of the slope, than of single elements. Average values are used to look at trends throughout the random field. Because of this, the change in the shear strength distributions cannot be evaluated in the same way as for the SRV analysis. The elements will react differently to the updating. This is shown for example in figure 7.7, where the standard deviation of the distribution increases upon updating, because some elements experience an increase in average shear strength, while other elements do not. Other distribution shift towards higher values while keeping the same shape, and some distributions see very little change. Overall the results show that Bayesian updating is an efficient way of incorporating observations of the performance of a slope into slope stability assessment. This give new distributions of the input parameters, that are more accurate than the initial estimates.

8.5 Limitations

The results presented in this report have provided interesting insights in the workings of Bayesian updating, in combination with both the SRV analysis and with the CRF analysis. However, the calculation model has some limitations that should be highlighted. Firstly, the Monte Carlo Simulation (Monte Carlo Simulation (MCS)) consisted of only 2000 realizations per simulation. This is not enough to get a result that is independent of the sample size, and an accurate estimate of the behaviour of the system. The results are likely to be affected by statistical noise. The samples are not sufficiently converged towards the distributions they represent, and single realizations are given too much emphasis.

An example of this is the occurrence of elements that reduce in average shear strength when failing realizations are discarded during updating. There is no obvious reason that high shear strength of a zone or element would be related to low values of F_s. There are regions where there is no strong correlation between the shear strength and the F_s of the slope, and these elements will sometimes have high shear strength values in discarded realizations. However, it is equally likely that these unimportant elements have low values in discarded realizations. It is expected that influential elements would experience an increase in average shear strength over all realizations when updating, and that the average shear strength of the non-influential elements would not change, if the sample size was large enough. This is an assumption, that should be tested by running simulations with larger sample sizes. Unexpected effects could be discovered, and these would be more reliable if they could not be confused with statistical noise.

Another issue is the boundaries of the Finite Element Method (FEM) model. The vertical boundary on the bottom of the slope stops a short distance away from lake Botn. This boundary is locked in the horizontal direction, meaning that the soil cannot move across it. In this lies the assumption that the soil materials at the bottom of the lake are strong enough to support the slope. This is an uncertain claim, because no soil investigations are available from the materials at the bottom of the lake. This uncertainty was one of the reasons that the road project of the Norwegian Public Roads Administration (NPRA) was stopped back in 2009. The safety against slope failure could not be guaranteed based on the available information, because it was impossible to know whether the critical failure surface would extend into the seabed. This is not a problem when investigating the performance of a calculation model, rather than investigating the slope in itself. The goal of this thesis was to investigate the effects of combining Bayesian updating with a random field analysis, and comparing this to a simplified Single Random Variable analysis of the same slope. Estimating the reliability of the slope itself was not an objective, and this makes the boundaries of the model acceptable.

Chapter 9

Conclusions and Recommendations for Further Work

9.1 Summary and Conclusions

This thesis has explored the potential of implementing Bayesian updating as part of a probabilistic slope stability evaluation. The goal has been to investigate whether Bayesian updating can contribute to reduce uncertainty in soil parameters by including information about observed performance of the slope. The updating was implemented in two different probabilistic analyses with different levels of complexity, to investigate the effect the complexity level has on the estimated behaviour the modelled system. Furthermore, three different simulations were conducted in each analysis, with different Coefficient of Variation (COV) of the input shear strength parameters. Increasing the COV was suggested as a mean to make estimates more conservative.

The probabilistic analysis was conducted using a Monte Carlo Simulation (MCS), where the stability of every realization of the slope was assessed with a finite element model. The high complexity model employed a Conditional Random Field (CRF), modeling the spatial variability of the soil and incorporating measured values from cone penetration tests. The low complexity model conducted a Single Random Variable (SRV) analysis, where every layer was described with one random variable and the spatial variability within each layer was neglected.

Bayesian updating proves to be an efficient method of including information about observed slope performance, in a slope stability assessment. The probability of failure is reduced significantly in all simulations where the prior distribution predicts failing realizations. The updating removes inaccurate information from the prior distributions of soil properties. The confidence in the posterior distributions are higher than in the prior distributions.

The results from the CRF analysis show that the updating has more effect in some parts of the slope than others. Zones in the random field where the shear strength properties are inaccurate, and the inaccuracy also leads to estimates of a failed slope, experience a larger effect of the updating than the rest of the slope. This shows that Bayesian updating combined with a random field can be used as a basis when planning supplementary ground investigations. The value of the additional data can be increased if the testing is focused on zones where current estimates of soil properties cause erroneous predictions of slope behaviour.

Comparison of results from two analyses shows that the SRV analysis is not an accurate simplification of the CRF analysis. The simplified analysis predicts a much larger variance in the estimated values of the Factor of Safety F_s. These results are more conservative estimates of the slope stability, because they predict a higher probability of failure. However, the probability of failure is so overestimated compared to the results from the more detailed CRF analysis, that using the SRV analysis can lead to inefficient designs. The model used to conduct the CRF analysis is a research tool, and not suited for use in design processes. It could be useful to develop a calculation model that employs a hybrid of the two analyses, where both the simplicity of the SRV analysis and the accuracy of the CRF analysis were conserved.

Intuitively, it seems that increasing the uncertainty of the input parameters would give conservative estimates of the slope performance. However, the results show that increasing the COV of the shear strength parameters, through increasing the standard deviation, completely changes the predicted behaviour of the slope. The increased variance of the input shear strength assigns a higher probability density to both low and high values. The low values dominate the the behaviour of the slope in the prior estimates, and reduce the predicted performance more than intended. During updating, many of the weak realizations of the slope are rejected, and the high values of F_s are left to form the posterior distribution. This is especially evident in the results from the SRV analysis, where it leads to the opposite of conservative estimates of the posterior distribution of F_s. Varying the COV of the input shear strength does not seem to be a suitable approach to obtaining conservative slope stability estimates. The results of this thesis indicates that the most secure way of making conservative estimates is to perform thorough ground investigations, to obtain a detailed description of the soil stratification, and the spatial variability of properties within each layer.

9.2 Recommendations for Further Work

The potential of Bayesian updating has been examined in two different probabilistic slope stability analyses, where the shear strength of each material has been modeled as random variables. The effect of the COV of the input shear strength has been investigated. The first recommendation for further work on this topic is to implement the analysis on a computational cluster to speed up the calculations. This would make it possible to increase the sample size, and retrieve more accurate results. The effects of implementing Bayesian updating in slope stability assessments can be examined further by using the posterior distributions calculated in this thesis as input for a new calculation. This would allow investigation of whether other failure mechanisms become more critical after updating.

Another relevant expansion of the model would be to include external uncertainties, like uncertain groundwater levels and loads, to give a more realistic analysis of the investigated slope. This would also increase the possibilities of updating slope reliability with other types of observations (e.g., groundwater measurements, displacements). Also other soil parameters than the shear strength of the soil can be modeled as random variables. This should be done step wise, in order to isolate the effects of the different variables. Furthermore, it would be interesting to evaluate the effects of correlation lengths on the posterior distribution of an updated random field.

On a longer term it is thought that Bayesian updating can be used to calibrate partial factors of safety in deterministic design approaches. Eventually it should be a goal to develop a calculation tool that is a hybrid between SRV and CRF, which makes probabilistic analysis and Bayesian updating more available as a design tool.

List of Tables

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