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Methods for Cost Allocation Among Prosumers and Consumers Using Cooperative Game Theory

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Preface

This Thesis concludes my Master of Science (M.Sc) degree in Energy and Environmental Engineering with the Department of Electric Power Engineering at the Norwegian University of Science and Technology (NTNU). The thesis was written under the supervision of Professor Hossein Farahmand and PhD Candidate Markus Löschenbrand with the Department of Electric Power Engineering at NTNU.

It is with my deepest gratitude that I would like to thank my supervisors for your guidance and support throughout the project, and for your willingness to share your knowledge and answering my questions. I have always left our meetings strongly motivated and inspired for further work. I would also like to extend my gratitude to Research Assistant Sigurd Nikolai Bjarghov, for your valuable support and guidance whenever needed. Further, I am grateful for the help of Simon Indrøy Risanger for both providing me your game theoretical code, and for the support to realize the game theoretical framework in this thesis.

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Trondheim, June 25, 2018

Elise Tveita

Abstract

The decarbonization of the power sector is at the core of the transition to a sustainable energy future. In parallel with the growth in renewables, the costs of solar photovoltaic (PV) and electrical energy storage (EES) are decreasing, which facilitates installation of these devices at the residential level. With a higher penetration of distributed generation (DG) and electrical energy storage (EES), private end-users are taking a more active role in the power grid. Currently, independent operation of these devices is most common. With an increased amount of DG and EES available, opportunities for cooperation through power exchanges arise. In cooperative game theory, all players cooperate under joint benefits. Preliminary studies conducted by the author, show that such cooperation among prosumers and consumers yields reduced annual electricity cost compared to independent operation.

Focusing on cost allocation among end-users equipped with rooftop PV and batteries, the objective of this thesis is to analyze two possible solution concepts; the nucleolus and the Shapley value. An energy community consisting of private end-users is modeled as a cooperative game. By changing parameters that increase the value of the battery system in terms of reduced cost, this thesis aims to examine whether the deviation between the cost allocation methods increases as the value of the battery system is changed. The simulated energy community is based on data from private residences in Norway, provided by Trønderenergi Nett, the local distribution system operator (DSO).

Results show that both nucleolus and the Shapley value provide stable cost allocations under minor deviations, depending on the case. Results also show that the deviation between the methods increases, as the value of the battery system increases. The highest deviation between the methods is of 3 %. In this scenario, the value of the battery system is 8.84 %, which also represents the highest value of batteries within the considered scenarios.

Sammendrag

Dekarboniseringen av energisektoren er kjernen i overgangen mot en bærekraftig energifremtid. Kostnadene for solcellesystemer og elektrisk energilagring synker parallelt med økningen av fornybar energiproduksjon, dette gir insentiver for installasjon av disse enhetene blant sluttbrukere. Økt integrasjon av distribuert produksjon og elektrisk energilagring fører til at sluttbrukerne tar en mer aktiv rolle i kraftsystemet. I dag er selvstendig drift av disse enhetene mest utbredt, men med et økt antall produksjons- og lagringsenheter tilgjengelig blant sluttbrukerne, øker også samarbeidsmulighetene i form av kraftutveksling. I samarbeidende spillteori, samarbeider spillerne under delte fordeler. Tidligere studier gjennomført av forfatter, viser at samarbeid mellom sluttbrukere i distribusjonsnettet fører til reduserte årlige elektrisitetskostnader sammenlignet med selvstendig drift.

Ved å fokusere på kostnadsallokering blant sluttbrukere utstyrt med solcellepaneler og batterier, er målet med denne oppgaven å studere to løsningskonsepter fra spillteori: nucleolus og Shapleys metode. Et nabolag bestående av private sluttbrukere modelleres som et samarbeidende spill. Ved å endre ulike parametre som øker verdien av batterisystemet i form av redusert elektrisitetskostnad, er det ønskelig å undersøke hvorvidt forskjellen i kostnadsallokering fra de aktuelle løsningskonseptene øker, dersom verdien av batterisystemet endres. Det simulerte nabolaget er basert på data fra private sluttbrukere i Norge, levert av nettselskapet Trønderenergi Nett.

Resultater fra denne oppgaven viser at både nucleolus og Shapleys metode resulterer i stabile kostnadsallokeringer, videre avviker metodene minimalt i de simulerte scenarier. Resultater viser også at forskjellen i kostnadsallokeringene mellom metodene øker, når verdien av batterisystemet øker. Den største forskjellen mellom nucelolus og Shapleys metode er i denne oppgaven 3 %. I det aktuelle scenariet er verdien av batterisystemet 8.84 %, noe som også representerer den totalt høyeste batterisystemverdien blant simulerte scenarier.

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Abbreviations

DG	Distributed generation
DP	Dynamic programming
DSO	Distribution system operator
EES	Electrical energy storage
EPEX	European power exchange
EV	Electrical vehicle
GC	Green certificate
GHG	Greenhouse gas
LP	Linear programming
NOCT	Nominal operating cell temperature
PV	Photovoltaic
SCH	Supply chain management
STC	Standard test conditions
TSO	Transmission system operator
TSP	Traveling salesman problem
TU	Transferable utility
VAT	Value added tax

Chapter 1

Introduction

1.1 Motivation

By 2050, the European Commision target to cut the green house gas (GHG) emissions by 80-95 % compared to the 1990 levels. From 2017 to 2050, the share of of renewable energy in the power sector will increase from 25 % to 85 %, and a decarbonized power sector is at the core of the transition to a sustainable energy future. The increase in renewables is primarily due to the growth in solar and wind power generation. In parallel with the growth in renewables, the costs of solar photovoltaic (PV) is continuously decreasing. By 2020, the costs of solar PV are expected to halve relative to the 2015 levels, facilitating installation at residential scales [35]. With a higher share of unpredictable distributed generation (DG), new challenges arise. A higher share of DG leads to issues regarding bi-directional power flow in the distribution grid, which may cause problems regarding voltage stability- and frequency. To support the increase of renewables, electrical energy storage (EES) will play a crucial role. The EES can reduce constraints in the distribution network, irrespective of the constraints caused by the increasing share of renewables or change in demand pattern. As storage costs are decreasing, it is becoming an economic solution for private end-users, where it can can increase the share of renewable energy in the system to as high as 100 % [34].

When a higher amount of end-users are equipped with DG and EES, possibilities for cooperating operation rises. By exchanging power between end-users within a neighbourhood, a higher share of local produced power can be utilized. In addition, less power is required bought from the main grid. The possibility of cooperation through power exchange rises the question; Are there any economical benefits from such cooperation among the end-users?

Results from preliminary studies [37], show that cooperating in bidding at a power exchange provides reduced annual electricity cost compared to independent operation. However, as a prerequisite for the end-users to cooperate, the profitability after cost allocation for all end-users must exist.

1.2 Scope

Focusing on cost allocation among end-users equipped with rooftop PV and a battery system, the objective of this thesis is to evaluate two solution concepts from game theory; the nucleolus and the Shapley value. The thesis models an energy community as a cooperative game, where the central issue is how to allocate the annual electricity cost among a number of end-users. Furthermore, this thesis aims to study whether the deviation between the cost allocations proposed by nucleolus and the Shapley value, is related to the value of the battery system. The study is conducted by changing parameters that increase the value of the battery system in terms of reduced cost, which result in four different scenarios. The objectives of this thesis are the following:

- Apply the game theoretical solution concepts; the nucleolus and the Shapley value to a cooperative game, consisting of single end-users within an energy community.
- Evaluate whether the deviation in cost allocations provided by the two methods, is related to the battery system value within the community.

Furthermore, are the following model limitations and assumptions:

- The thesis is limited to study annual electricity cost for the end-users, thus investment- and maintenance cost of PV- and battery systems are neglected.

- The input data utilized for the simulations are deterministic, thus perfect foresight is available.
- The simulations are performed on a conceptual level, thus not directly applicable for a real-case study.

This thesis is based on its pre-work, conducted by the same author in the fall of 2017. The basis of case-studies of the resulting report is included, and further developed in this thesis.

1.3 Structure

This thesis is organized according to the following scheme:

Chapter 1, *Introduction*, introduces the reader to the motivation behind the field of cooperative game theory applied in power systems, as well as the scope and the objectives of this thesis.

Chapter 2, *Theory*, provides the relevant theory regarding the chosen optimization method. Therefrom, a literature review on cooperative game theory is carried out, followed by the theory behind the relevant game theoretical solution concepts.

Chapter 3, *Methodology*, presents the methodology for building the cooperative game, consisting of three layers; *System Layer*, *Optimization Layer* and the *Game Theoretical Layer*. Further, the overall simulation procedure is provided.

Chapter 4, *Data Input*, provides the input data used for obtaining the cooperative games, followed by the resulting games, which are used as input for the analyses of the game theoretical solution concepts.

Chapter 5, *Results and Discussion*, presents the results, along with the discussion of the results.

Chapter 6, *Conclusion and Further Work*, draws the main conclusion of the results, and suggests further work.

In addition, a scientific paper is provided in the appendix:

Appendix B - Comparison of cost allocation strategies among prosumers and consumers in a cooperative game

Chapter 2

Theory

The following chapter presents the reader the relevant background theory. Chapter 2.1 presents the relevant optimization theory, obtained from the pre-work of this thesis [37]. Therefrom, Chapter 2.2 provides the game theoretical solution concepts, including a literature review within cooperative games.

2.1 Dynamic Programming Optimization

The following chapter is based on theory from [9] and [13].

Dynamic programming (DP) is a solution strategy used to solve optimization problems, and it serves as a well-performing method for solving sequential decision problems over a broad time horizon. As a prerequisite to using DP, the problem in question must have a dynamic structure, where it can be divided into a discrete number of sequential steps. A DP problem may be interpreted as a sequential decision making, where one decision is made in each step.

The main idea of DP is to break the problem into smaller subproblems by dividing the problem into discrete sets of *states* and *stages*. Each stage usually represents one time step. Further, a sub-problem is defined as all possible states in one stage. For each state in a given stage, a decision has to be made. Each decision comes with a cost. A given decision in stage t leads to a new state in stage $t + 1$. The state reached in stage $t + 1$, given a state and a decision in stage t, is called *transition*. One has to obtain the cost for all the states in the given stage. Thus, there is a connection between the stages, and the objective is to find the optimal state in each stage that gives the global optimal solution.

The optimal solution for each state can be solved through the principle of recursion. Within DP optimization, the recursion principle can be divided into two solution procedures; *forward* and *backward*. In the former, the algorithm begins in the first stage, therefrom calculates how to reach the last stage at the lowest cost. In contrast, backward recursion is based on the algorithm begins in the last stage of the optimization horizon and calculates backward how to reach the first stage at the lowest cost. Both forward and backward recursion guarantee that the optimal solution for the global problem is obtained, as all possible combinations of states in all stages are calculated.

The DP algorithm generates a network of nodes, where all possible paths from first to the last stage are calculated to obtain the optimal global solution. This node network is illustrated in Figure 2.1.

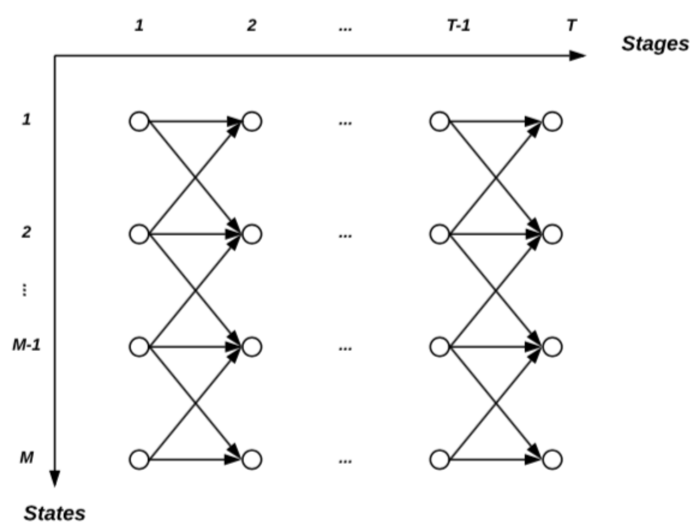

Figure 2.1: DP-node network with T stages and M states.

A drawback of DP is the increase in resource consumption, as the problem size increases. However, unlike greedy algorithms[1], DP algorithms guarantee that the optimal solution is obtained [13].

2.2 Coooperative Game Theory

2.2.1 Literature Review

The fundamentals of the modern-day game theory have its origin from the publication *Theory of Games and Economic Behaviour* [17]. Game theory constitutes a mathematical framework for analyzing the structure and resolution of conflict, and it is divided into two disciplines; *non-cooperative*, and *cooperative* or *coalition*. This thesis will focus on the latter.

Cooperative game theory studies cooperation among a group of players acting strategically. Methods within cooperative game theory have for decades been applied to a wide range of fields, such as in politics and economics. It proposes a well-performing tool for analyzing problems regarding the division of available resources among a group of players, and it has shown to be convenient for analyzing issues within operational research, such as in logistics and supply chain management (SCH). In [36], the authors apply the concept of the Shapley value for profit allocation, and conclude that there is great potential for applying cooperative game theory to explore cooperation within SCH. In [10], an algorithm based on the concept of the core is applied for horizontal cooperation of the traveling salesman problem (TSP). The proposed algorithm was shown to provide cost savings compared to non-cooperative operation.

Within power systems, cooperative game theory has historically been a tool for investment analysis, mainly focusing on power generation and transmission facilities. In [7], the cooperative game theoretical concept of the Kernel is applied for decentralized allocation of transmission costs, and

[1]A greedy algorithm is a simple algorithm which aims to find the best local solution. In the majority of problems, greedy algorithms do not provide the global optimal solution [13].

results show that a unique solution can be obtained. The considered problem in [7] is an extension of the work carried out in [6], which studies different solution concepts for allocating total transmission expansion costs among the players. In [11], the authors propose a generic framework for flexibility analysis in transmission expansion planning, by using the concept of the Shapley value.

In recent year, the interest of applying cooperative game theory in the distribution system is increased as it can serve as a well-performing tool for optimizing electricity costs and available resources. In [4], the authors analyze the value of sharing storage among consumers in a cooperative manner, and conclude that all players in a community would benefit from such cooperation. Ref. [14] studies cooperation among energy communities using cooperative game theory, and proves that each grid increases their individual profit by cooperating with the other energy communities. Furthermore, [5] studies how cooperative game theory can be applied for cost minimization within an energy community. Here, the authors propose the Shapley value for cost allocation, and conclude that both prosumers and consumers will obtain reduced cost when participating in the cooperative game.

2.2.2 An Introduction to Cooperative Games

In decision analysis using game theory, the presence of conflicting interests among players is central. However, cooperation among the players might also exist. To describe an individual or a group of individuals, the terms *player*, *decision-maker* and *agent* are in the literature used interchangeably. Concepts within game theory are based on two fundamental assumptions; *Rational behaviour* and *strategic reasoning*. These assumptions imply that players will make the strategic decisions that are best to achieve their goals, and then do the corresponding actions. A *game* is a description of a strategic interaction including the players' interest and the possible actions the players might take. A *solution* is a systematic description of an outcome that might emerge based on actions taken by the players.

Non-cooperative games study situations where the players are regarded as independent individuals without the possibility to directly communicate. If cooperation among the players is present, this is self-motivated without neither communication nor coordination of strategic decisions among them. In contrast, cooperative games focus on actions taken by groups of players, usually denoted by *coalitions*. Details of the internal interactions of players within a coalition are not considered, hence the focus is on what coalitions can achieve, furthermore the interaction among the coalitions. In cooperative games, it is of interest to examine which coalitions that may form and the possible payoffs resulting from different coalitions. An essential question is how to define *fairness*, and how to *fairly* allocate the total payoff for all players in the coalition.

The following chapters focus on cooperative game theory, its properties and solution concepts. In order to obtain a broad overview, [23] is used, whereas [24] and [31] are used for mathematical focus.

2.2.3 Definitions and Properties

Cooperative game theory is the study of the interactions among coalitions of players. Let $N = \{1,..., n\}$ be a finite set of in total n players. With n players, there exist 2^n possible coalitions. The *grand coalition* is the set consisting of all players, denoted by N. The *empty coalition* is the set consisting of none of the players, denoted by $\varnothing$. For every sub coalition $S \subseteq N$, there exists a value called the *characteristic function*, denoted by $\upsilon(S)$. The $\upsilon(S)$ represents the worth of coalition S, which is the maximum utility or payoff available for division among the players in S. Players that are not in a sub coalition, $N \backslash S$ are assumed to be unable to prevent S from achieving $\upsilon(S)$. In theory, focus on the utility $\upsilon(S)$ as *transferable*. A cooperative game with transferable utility (TU) implies that the utility obtained by the coalitions are measured in the same units. The transferable utility might for example be a joint currency which can be transferred among the players without losses. A cooperative TU game can be represented by (N,υ).

In cooperative game theory, for all N players in a game cooperate under joint benefits is assumed. This implies that the utility obtained by the players when forming a coalition, is guaranteed to be at least equal to, or greater than the utility obtained by operating individually. This property is expressed through the characteristic function, which must fulfill Eq. (2.1).

$$v(N) \geq \sum_{i \in N} v(i) \quad \text{and} \quad v(\varnothing) = 0 \tag{2.1}$$

The highest utility is usually obtained in the grand coalition. Due to the concept of *superadditivity*, the grand coalition will form. Superadditivity implies that if two disjoint coalitions respectively S_1 and S_2, decide to form one joint coalition, the utility resulted from the new coalition consisting of S_1 and S_2 is guaranteed to be equal to, or greater than the value obtained by S_1 and S_2 separately. This property is mathematically expressed through Eq. (2.2):

$$\begin{aligned} v(S_1 \cup S_2) \geq v(S_1) + v(S_1) \quad \forall S_1, S_2 \subset N \\ \text{s. t.} \quad S_1 \cap S_2 = \varnothing \end{aligned} \tag{2.2}$$

The essential issue within a cooperative TU game after forming the grand coalition, is how to divide the surplus utility among the players. There are two main approaches for solving this issue:

- Positive or descriptive approach: One tries to predict the outcome with highest probability of the interaction, hence the resulting utility should be the natural allocation when the coalition forms.

- Normative or prescriptive approach: The utility resulting from a coalition should be divided according to normative goals, usually expressed through axioms. Examples of this approach are the concept of nucleolus and the Shapley Value.

The allocation of utility is expressed through a payoff vector $\boldsymbol{x}$. For a game consisting of *n* players, a payoff vector is expressed as $\boldsymbol{x} = [x_1, x_2, ..., x_n]$, where x_i represents the payoff obtained by player i. When focusing on division of total utility among players, one seeks to obtain payoff vector with certain properties. The two properties of interest are; *group rationality* and *individual rationality*. Group rationality implies that the total utility should be divided among all the players within the coalition, expressed through Eq. (2.3). Individual rationality implies that a player will only join a coalition if this leads to at least the utility achieved if operating individually, expressed through Eq. (2.4).

$$\sum_{i \in N} x_i = v(N) \tag{2.3}$$

$$x_i \geq v(\{i\}) \quad \forall i \in N \tag{2.4}$$

A payoff vector $\boldsymbol{x}$ that fulfills both the requirements of group rationality and individual rationality, is called an *imputation*. Furthermore, an imputation $\boldsymbol{x}$ is stable if no alternative coalition will provide a higher payoff for any of its players. A stable imputation is said to be in the *core* of the game.

2.2.4 The Core

Due to the concept of superadditivity, the grand coalition provides the lowest electricity cost for the players. Despite this, the players will only join the grand coalition if the proposed allocation provides the players the highest payoff. To satisfy the equilibrium state, it must be ensured that none of the players want to upset the grand coalition N for some other sub coalition S. Due to the concept of rationality, the players seek to form the coalition where they expect to obtain the highest payoff. Let imputation $\boldsymbol{x}$ be a proposed division of the total utility $v(N)$. If there exists a sub coalition S where a higher payoff can be obtained, coalition S will form. Thus, the grand coalition is not stable. An imputation $\boldsymbol{x}$ is stable if no alternative coalition will provide a higher payoff to any of its players. Thus, the core denoted C of a TU game, is the set consisting of all stable imputations, mathematically expressed through Eq. (2.5):

$$C = \Big\{ x_i \mid x_i \in \{x_1, ..., x_n\}, \sum_{i \in N} x_i = v(N), \text{ and } \sum_{i \in S} x_i \geq v(S), \ \forall S \subset N \Big\} \tag{2.5}$$

The core may consist of several imputations, but it may also be empty, $C = \{\varnothing\}$. A TU game with an empty core will provide none stable payoff vectors, thus there exists none imputation where all players are satisfied with their received payoff. The size of the core can be interpreted as a measurement of stability. As a solution concept, the core provides a set of stable imputations without distinguishing any imputation as favourable to another. The core of a game consisting of four players is illustrated in Figure 2.2[2]. The points within the tetrahedron provide all possible allocations. Each player creates constraints regarding tolerable coalitions, resulting in a limited region. This region including its boundaries, is the core of the game, represented by the red polyhedron in Figure 2.2. The allocations within the core are stable, thus no other coalitions can provide a higher payoff to all the players.

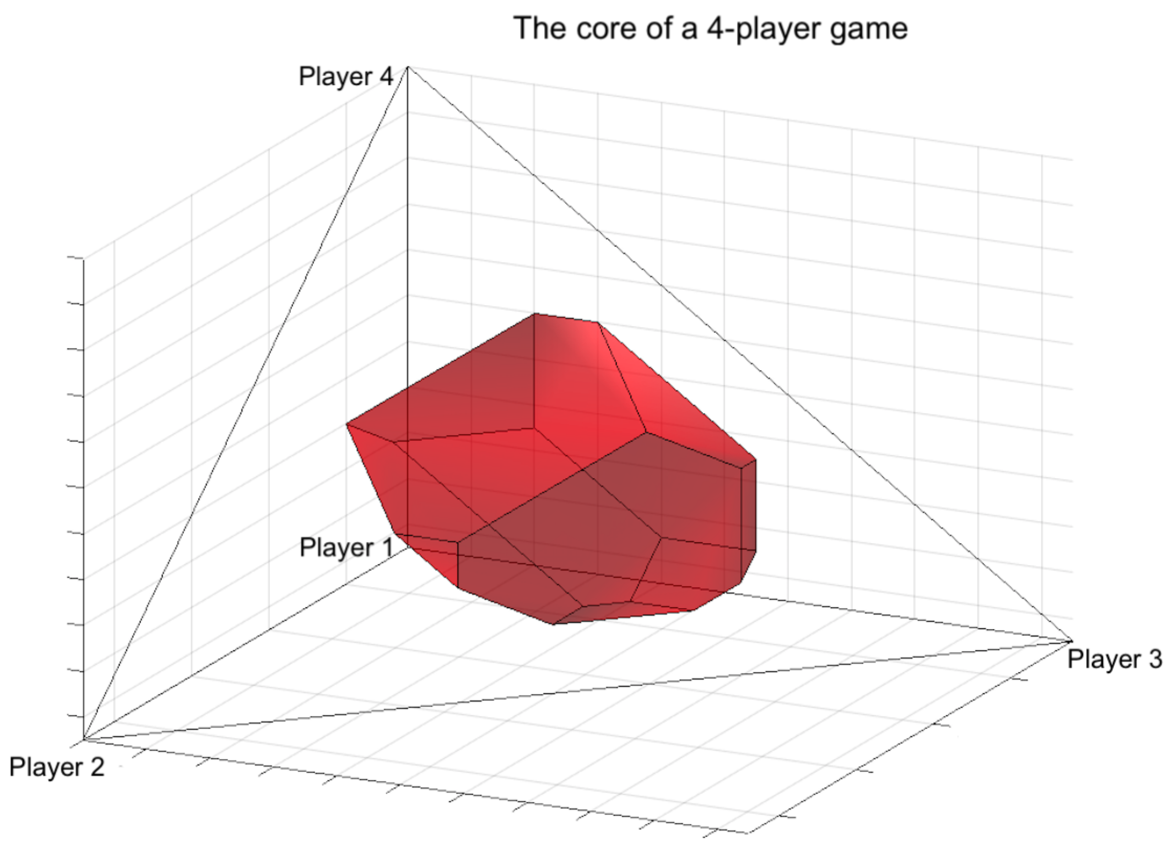

Figure 2.2: The core of a game consisting of 4 players. Stable allocations are obtained within the polyhedron.

2.2.5 The Shapley Value

Within cooperative games, the interpretation of fairness is a major question that distinguishes the different approaches for allocating total utility. While the core provides a set of solutions based on stability, different value concepts have been established to assign a player in a game a unique value.

[2]Figure created in MATLAB® using [39].

In 1953, Lloyd Shapley proposed a solution concept, whose interpretation of fairness is in terms of each player's individual contribution to a coalition [30]. The method assigns a unique value to each player based on four simple axioms. The axioms proposed by Shapley are as follows:

1. ***Efficiency***: All utility obtained by any player should be allocated. The total value of the players is the value of the grand coalition, hence $\upsilon(N) = \sum_{i \in N} \upsilon(i)$.
2. ***Symmetry***: Two players i and j that contribute the same to each coalition are substitutes, hence they should be treated equally. Player i and j are symmetric if $\upsilon(S \cup i) = \upsilon(S \cup j)$.
3. ***Null player***: A player i that contributes nothing, should receive nothing. Such a player is referred to as a null or a zero player. A player is a null if $\upsilon(S) = \upsilon(S \cup i)$.
4. ***Additivity***: The sum of two independent TU games, u and v must be the sum of the value of each game, hence $\phi(u + v) = \phi(u) + \phi(v)$.

For each player, there exists a unique value satisfying these axioms. This unique value is the Shapley value, denoted $\phi(\upsilon)$. For a TU game (N,υ) the Shapley value for player i is calculated by Eq. (2.6):

$$\phi_i(\upsilon) = \sum_{i \in S} \frac{(|S|-1)!(n-|S|)!}{n!}[\upsilon(S) - S\backslash\{i\})]. \tag{2.6}$$

The term $\frac{(|S|-1)!(n-|S|)!}{n!}$ expresses all possible orders the players can join the grand coalition, where each order has equal probability. The quantity $[\upsilon(S) - S/\{i\})]$ is the marginal contribution of player i to the coalition S. The summation is over all coalitions $S \subseteq N$, that consist of i. Thus, $\phi_i(\upsilon)$ is equal to the average marginal contribution of player i to the grand coalition, if the players sequentially form this coalition in a random order. For a game with $i = 1,..., n$ players, the Shapley value is given by the vector $\boldsymbol{\phi} = [\phi_i,...,\phi_n]$. If the considered game has an empty core, the Shapley value will necessarily be outside the core. Despite a game with a non-empty core where superadditivity is fulfilled, the Shapley values are not guaranteed to be within the core. Hence, the method does not ensure that the allocation proposed by Shapley is stable. Once the Shapley values for a cooperative game are calculated, additional examination of the core is required to check stability. The returned

Shapley value allocation, ϕ must be in the core of the game. Thus, Eq. (2.7) must be fulfilled.

$$\phi \in C \tag{2.7}$$

2.2.6 The nucleolus

The concept of the nucleolus was first introduced in [27]. While Shapley focuses on fairness in terms of individual contribution, the nucleolus is based on minimizing the players' dissatisfaction with their payoff. The idea behind the nucleolus is to minimize the maximum dissatisfaction the players in coalition *S* experience from a proposed imputation $\boldsymbol{x}$. Dissatisfaction is measured through an *excess function e(S,*$\boldsymbol{x}$*)*, expressed in Eq. (2.8).

$$e(S, \boldsymbol{x}) = v(S) - \sum_{i \in S} x_i = v(S) - \boldsymbol{x}(S) \tag{2.8}$$

If the excess *e(*$\boldsymbol{x}$*,S)* is positive, it represents the dissatisfaction resulted from the proposed imputation $\boldsymbol{x}$. In contrast, a negative *e(S,*$\boldsymbol{x}$*)* represents the additional payoff coalition *S* obtains from $\boldsymbol{x}$. The core is defined as the set of imputations, such that $\sum_{i \in S} x_i \geq v(S)$ for all coalitions S. Thus an imputation $\boldsymbol{x}$ is in the core only if all its excesses are negative or equal to zero. This condition expressed through Eq. (2.9).

$$C(N, v) = \{\boldsymbol{x} \in X \mid e(S, \boldsymbol{x}) \leq 0 \quad \forall S \subset N\} \tag{2.9}$$

Let $\boldsymbol{x}$ and $\boldsymbol{y}$ be two different payoff vectors from a coalition *S*. If max$\{e(S,\boldsymbol{x}) \mid S \subset N\}$> max$\{e(S, \boldsymbol{y}) \mid S \subset N\}$, the payoff vector $\boldsymbol{x}$ leads to a higher dissatisfaction among the players in *S*. If the two maxima are equal, the next greatest excesses with respect to $\boldsymbol{x}$ and $\boldsymbol{y}$ are compared. A payoff vector $\boldsymbol{x}$ will give an excess vector $\theta(\boldsymbol{x})$ for all sub coalitions of $N \backslash S = \varnothing$. This vector is denoted $\theta(\boldsymbol{x}) = [e(\boldsymbol{x}, S_1),...,e(\boldsymbol{x}, S_{2^n-2})]$. The excess vectors are ordered decreasing and *lexicographically*[3]. Thus, there exists a unique allocation that provides the lexicographically smallest vector. That is, the imputation that gives the least excess for all the subcoalitions. This unique payoff vector

[3] Lexicographic ordering implies that excesses are ordered in the same way as words are ordered in the dictionary.

is the nucleolus. The nucleolus of a cooperative game can be obtained by the concept of *least core*. Without comparing excesses from different payoff vectors, the concept of least core can be utilized for obtaining the minimal excess vector $\theta_{min}(\boldsymbol{x})$. The concept of the least core leads to the introduction of the ε-core, $C_\varepsilon(N,v)$, expressed through Eq. (2.10).

$$C_\varepsilon(N,v) = \boldsymbol{x} \in X \mid e(S,\boldsymbol{x}) \leq \varepsilon \quad \forall S \subset N\} \tag{2.10}$$

By letting $\varepsilon < 0$, the ε-core in Eq. (2.10) becomes more restrictive than the core represented in Eq. (2.5). If the ε-core is applied to a non-empty core, an ε_0 can be obtained which is the most constrained. If the ε_0 is further reduced, the ε-core becomes empty. The ε_0 represents the smallest maximum excess, and it is obtained by solving Eq. (2.11).

$$\varepsilon_0 = \min_{\boldsymbol{x} \in X} \max_{S \subseteq N \setminus \{\varnothing\}} e(S,\boldsymbol{x}) \tag{2.11}$$

There might exist multiple solutions to the problem in Eq. (2.11). Hence, multiple allocations can be a solution for ε_0. The nucleolus of a game is obtained by calculating the least core, further note which coalitions which are binding ε_0 from reducing further. Thus, the maximum smallest excess for a coalition is obtained. This excess represents the first element of the excess vector. Secondly, the second least core, ε_0^2 is obtained. The ε_0^2 is the least core when the excess expression of the binding coalition of the ε_0, $e(S_{\varepsilon_0},\boldsymbol{x})$ is removed, while the new resulting allocations are constrained to be equal to ε_0^2. The second element of the maximum excess vector then becomes ε_0^2. This process iterative continues until all coalitions have been binding to a least core. As a result, the lexicographical minimal vector $\theta_{min}(\boldsymbol{x}) = \{\varepsilon_0,\ \varepsilon_0^1,\ldots,\varepsilon_0^{2n-2}\}$ is obtained. All imputations are required to satisfy previous least cores, thus resulting in all possible imputations to a single payoff vector, that is the nucleolus.

Chapter 3

Methodology

This Chapter presents the methodology for modeling the energy community as a cooperative game. The modeling structure consists of three layers, illustrated in Figure 3.1[1]. The system layer is provided in Chapter 3.1, followed by the optimization layer in Chapter 3.2, both based on the pre-work of this thesis [37]. In Chapter 3.3, the game theoretical layer is presented.

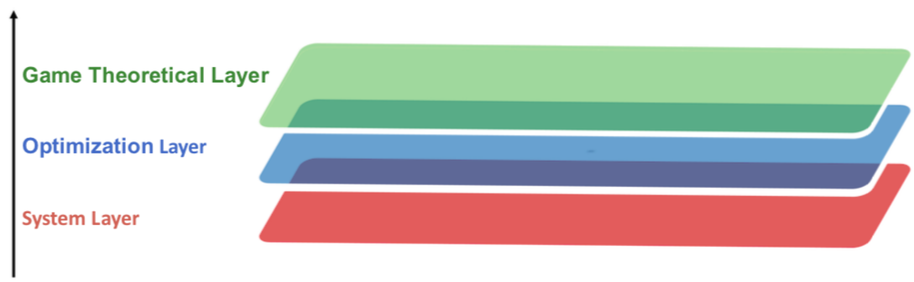

Figure 3.1: The energy community built as a cooperative game, consisting of three modeling layers.

3.1 System Layer

In this thesis, the term *end-user* corresponds to *player*. Thus, the author will throughout this thesis use the term player for denoting a single end-user within the energy community. The considered

[1]All figures in Chapter 3 are created by using Microsoft® PowerPoint for Mac.

energy community consists of both consumers and prosumers, in total 4 players. A consumer is regarded as an single end-user with a corresponding load that needs to be covered, whereas a prosumer is equipped with additional resources. From January 1st 2017, The Norwegian Water Resources and Energy Directorate (NVE)[2] revised the definition of a prosumer as:

"End-customer with consumption and production behind the point of connection, where feed in power into the point of connection never will exceed 100 kW. A prosumer is not allowed to have neither a production plant that is required to have a concession behind the point of connection, nor a turnover business that requires a turnover concession." [21]

The different players and their corresponding resources are described in Table 3.1.

Table 3.1: The end-users within the energy community and their corresponding resources.

Player	**Type of player**	**Resources**
1	Prosumer	Load, PV system, Battery 1
2	Prosumer	Load, PV system, Battery 2
3	Consumer	Load
4	Consumer	Load

Further, the following discrete sets are declared:

n: number of players	$n \in N$,	where N = {1,2,3,4}
p: number of PV systems	$p \in P$,	where P = {1,2}
z: number of batteries	$z \in Z$,	where Z = {1,2}
t: number of discrete time steps	$t \in T$,	where T = {1,.....,8760}

3.1.1 Load

The loads of the players will be modeled as a loss-less bus bar, as illustrated in Figure 3.2. As it can be seen from the figure, both the total loads of the players, as well as the PV production of player 1 and 2 are summarized, thus considered as one unit each. These assumptions are mathematically

[2]NVE: Norges vassdrags- og energidirektorat.

expressed in Eq. (3.1) and Eq. (3.2), respectively. Each time step t represents one hour.

$$P_{load}(t) = \sum_{n \in N} P_{load,n}(t) \tag{3.1}$$

$$P_{PV}(t) = \sum_{p \in P} P_{PV,p}(t) \tag{3.2}$$

Furthermore, from Figure 3.2 it can be seen that the batteries are modeled as positive loads when being charged, and negative loads when being discharged. Due to different battery specifications of battery 1 and 2, these are not modeled as a single unit. The power balance equation is given in Eq. (3.3), where $P_{grid}(t)$ represents the amount of grid imported energy in time step t. Furthermore, the loads and the PV production are considered inflexible, meaning that total load needs to be covered at all times.

$$P_{grid}(t) = \sum_{n \in N} P_{load,n}(t) + \sum_{z \in Z} P_{bat,z}(t) - \sum_{p \in P} P_{PV,p}(t) \tag{3.3}$$

$P_{grid}(t)$ is the slack variable in the load demand constraint in Eq. (3.3), meaning that it varies throughout the year, to ensure that demand is covered at all times. Load data will be provided in kWh, which will be assumed as covered by flat kW-power over that given hour. Thus, 1 kWh in hour t is equal to $P_{load}(t)$ = 1 kW. For the sake of simplicity, the term *load* will be used analogues to the term *power*.

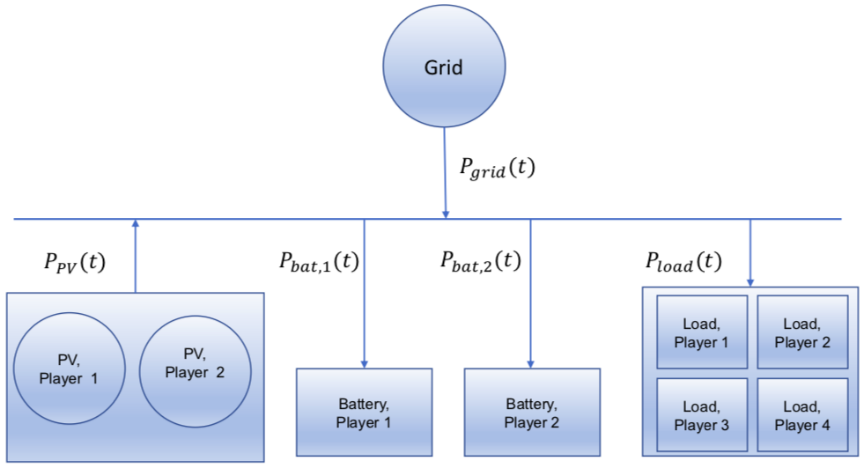

Figure 3.2: Power balance within the energy community, including arrows illustrating the power directions. Batteries shown as positive loads.

As the total load is modeled as one loss-less bus bar, power losses within the energy community are neglected. Similar, one does not consider transfer losses due to power exchange with the grid. Due to the scope of this thesis, these simplifications are regarded reasonable, but not applicable for a real case-study. In [12], the authors study transmitted power and relative losses for distribution networks, and suggests varying efficiencies to account for power losses due to power transfer within the distribution system. For developing a more realistic model, data from the [12] can be implemented in the considered case study.

3.1.2 PV Production

The PV production of player 1 and 2 are calculated by using a PV production model based on [25]. The PV production is modeled by the following parameters and variables:

P_{nom}	Nominal installed power	[kW]
η_{sys}	Total system efficiency	[%]
η_{inc}	Efficiency due to inclination angle of the panel	[%]
S	Solar irradiation	[kW/m^2]
S_{STC}	Solar irradiation under STC	[1 kW/m^2]
α_T	Temperature coefficient	[% /°C]
T_{cell}	Cell temperature	[°C]
$T_{cell,STC}$	Cell temperature under STC	[25 °C]
T_{amb}	Ambient temperature	[°C]
$NOCT$	Expected cell temperature in the module	[°C]

S_{STC} and $T_{cell,STC}$ are parameters under the standard test conditions (STC)[3], defined in order to facilitate comparison between PV systems from different providers, and for accounting for the change in performance as the temperature changes. The nominal operating cell temperature (NOCT) represents the expected pv cell temperature when the ambient temperature is 20°C, the solar irradiation is 0.8 kW/m^2, and the wind speed is 1 m/s. The NOCT is obtained from the data sheet for the given PV module. The η_{inc}[4] is the inclination angle efficiency factor, representing how much more irradiation is being absorbed due to inclination [3]. The total power produced by the PV systems of player 1 and 2 in time step t, $P_{PV}(t)$ is given by Eq. (3.4).

$$P_{PV}(t) = P_{nom}\eta_{sys}\eta_{inc}\frac{S}{S_{STC}}[1 + \alpha_T(T_{cell} - T_{cell,STC})] \tag{3.4}$$

In order to obtain T_{cell} for conditions different from $T_{cell,STC}$, Eq. (3.5) is used.

$$T_{cell} = T_{amb} + (\frac{NOCT - 20}{0.8})S \tag{3.5}$$

3.1.3 Battery Modeling

As mentioned in Chapter 3.1.1, the batteries of player 1 and 2 are modeled as positive loads when being charged and negative loads when being discharged. The net power, $P_{bat,z}(t)$ which is applied to battery z is given by Eq. (3.6). $P_{injected,z}(t)$ is the power which is being applied to battery z in

[3]Standard test conditions: S_{STC} = 1 kW/m^2, $T_{cell,STC}$ = 25 $°C$, $Airmass$ AM= 1.5.

[4]For roof mounted panels, the inclination angle is typically 20-40°, if placed on a roof [3].

time step t, while $\eta_{bat,z}$ is the efficiency of battery z.

$$P_{bat,z}(t) = \eta_{bat,z} P_{injected,z}(t),\ z \in Z \tag{3.6}$$

The $\eta_{bat,z}$ for battery z is dependent on whether the battery is being charged or discharged. The $\eta_{bat,z}$ for battery z is given in Eq. (3.7).

$$\eta_{bat,z} = \begin{cases} \eta_{ch,z}, & P_{bat,z}(t) > 0,\ z \in Z \\ \frac{1}{\eta_{disch,z}}, & P_{bat,z}(t) < 0,\ z \in Z \end{cases} \tag{3.7}$$

The state-of-charge (SOC)

The battery *State of Charge* (SOC) corresponds to the amount of which battery z is being charged or discharged in time step t. The SOC is given in percentage, expressed by Eq. (3.8).

$$SOC_z(t) \approx \frac{E_{bat,z}(t)}{E_{bat,z}^{max}} \qquad SOC_z(t) \in [0,1],\ z \in Z \tag{3.8}$$

where

$E_{bat,z}(t)$	Amount of energy stored in the battery z in time step t	[kWh]
$E_{bat,z}^{max}$	Nominal energy capacity of battery z	[kWh]

Degradation Factors

The battery efficiency is affected by factors such as the SOC, the charging power, aging and battery temperature. Since the studies in this thesis are performed on a conceptual level, degradation factors are not considered. In consequence, one assumes that maximum energy capacity, terminal power and efficiency are kept constant throughout the modeling time horizon.

3.1.4 The Energy Cost

Due to the liberalization of the energy market, Norwegian end-users are free to choose which energy company they want to provide their electric energy. In general, the energy supply companies offer three types of contract [22], among others a spot price contract. A spot price contract usually consists of a monthly fixed cost, the Nord Pool electricity spot price and a small term per kWh bought that goes to the energy supply company to ensure their incomes. In addition, the latter term includes the fixed cost for green certificates (GC)[5]. The annual energy cost $C_{ann,energy}$ whose the player is being charged, is expressed through Eq. (3.9).

$$C_{ann,energy} = 12C_{monthly,fixed} + \sum_{t \in T} P_{grid}(t)(C_{spotprice}(t) + C_{rev} + C_{CG}) \tag{3.9}$$

where

$P_{grid}(t)$	Energy consumption in time step t	[kWh/h]
$C_{monthly,fixed}$	Monthly fixed fee	[NOK/month]
$C_{spotprice}(t)$	Electricity spot price in time step t	[NOK/kWh]
C_{rev}	Marginal revenue to the energy supply company	[NOK/kWh]
C_{CG}	Fixed fee for the green certificate	[NOK/kWh]

As this thesis studies two different set of electricity spot prices, the $C_{spotprice}(t)$ varies the scenarios consider different spot prices. The remaining cost elements in Eq. (3.9) are kept consistent through all simulations. The original electricity spot price profile is equal to the profile used in the pre-work of this thesis [37]. Additionally, the price profile of the German electricity spot profile is obtained. This profile is chosen do to its high price fluctuations throughout the year.

3.1.5 Grid Tariff

In order to capture the total cost whose the players are being charged, the grid tariff from the distribution system operator (DSO) in the considered European Power Exchange (EPEX) area, is used.

[5]GC: Support instrument for renewable power production. GCs are given to companies with renewable power production. All power suppliers are obligated to buy a certain amount of power that has GC. The cost for the CGs is paid by the end-users through their electricity bill [20].

The current grid utility tariff structure in Norway is energy based, thus this structure will be used throughout this thesis. The energy based grid tariff implies that the players are being charged for the amount of kWh consumed, regardless on when it is delivered or at how high a power. The energy based structure has a fixed cost plus an extra fee per kWh.

As described in Chapter 3.1.1, all loads are summarized and considered as one unit. Thus, the players are not being charged grid tariffs for power exchanged within the energy community.

In 2010, NVE approved the *Prosumer agreement*, which implies that the prosumers are exempted to sign a balance agreement with Statnett, the transmission system operator (TSO). With the prosumer agreement, the prosumers are not regarded as market players, and they are not obliged to pay the the same grid tariffs as larger producers. The purpose of the prosumer agreement is to facilitate prosumers to feed their surplus power into the distribution grid [21]. The new definition of a prosumer, presented in Chapter 3.1, implies that the prosumers are not obliged to pay grid tariff costs for their sold energy. This, in turn facilitates for implementation of small-scale power production. The DSO is not obliged to buy the surplus power provided by the prosumers, yet it is common to do so. The prosumer is only paid the electricity spot price for selling energy to the grid. In contrast, one is being charged the electricity spot price, grid tariff cost and taxes when energy is bought from the grid. The annual grid tariff cost $C_{ann,grid}$, is expressed through Eq. (3.10).

$$C_{ann,grid} = C_{grid,fixed} + \sum_{t \in T} P_{grid}(t)(C_{grid,energy} + C_{consumertax}) \tag{3.10}$$

where

$P_{grid}(t)$	Energy consumption in time step t	[kWh/h]
$C_{grid,fixed}$	Annual fixed grid cost	[NOK/year]
$C_{grid,energy}$	Grid cost for consumed energy	[NOK/kWh]
$C_{consumertax}$	Consumer tax	[NOK/kWh]

The total electricity cost including both cost for energy and grid tariff is expressed in Eq. (3.11), where $C_{ann,energy}$ and $C_{ann,grid}$ is expressed through Eq. (3.9) and Eq. (3.10), respectively. The

VAT represents additional taxes, whose the end-users have to pay. The $C_{consumertax}$ is included along with the cost of grid tariff, although it is not a part of the actual grid tariff cost.

$$\sum_{t \in T} C_{el}(t) = C_{annual} = (C_{ann,energy} + C_{ann,grid})(1 + VAT) \tag{3.11}$$

3.1.6 The Value of the Battery System

The objective of this thesis is to study the deviation between the nucleolus and the Shapley as the battery system value changes. The term *battery system* represents both batteries within the energy community. To measure the value of the battery system, the term λ_{EES} is introduced. The λ_{EES} represents the relative reduced cost in for the grand coalition, provided by the battery system. Thus, there exists a λ_{EES} for each scenario. In order to obtain λ_{EES}, the annual electricity cost for the energy community without the battery system is calculated. Thus, $P_{grid}(t)$ is expressed through the following:

$$P_{grid}(t) = \sum_{n \in N} P_{load,n}(t) - \sum_{p \in P} P_{PV,p}(t) \tag{3.12}$$

$\sum_{n \in N} P_{load,n}(t)$ is the total load for the energy community in time step t, whereas $\sum_{p \in P} P_{PV,p}(t)$ is the total PV production within the same time step. As $\sum_{n \in N} P_{load,n}(t)$ and $\sum_{p \in P} P_{PV,p}(t)$ are inflexible, these parameters are not changed when the electricity cost without the battery system is calculated. To obtain the yearly electricity cost, Eq. (3.11) is used, with $P_{grid}(t)$ as expressed in Eq. (3.12).

3.2 Optimization Layer

3.2.1 Dynamic Programming Modeling

In preliminary studies [37], a dynamic programming (DP) algorithm was developed in order to obtain the minimized cost for a number of cooperating players. The code for the DP algorithm is implemented in MATLAB [16][6].

The objective function aims to minimize the annual cost of a certain number of players, by minimizing the cost of grid imported energy. In this way, dynamic programming optimizes operation of two batteries in parallel over a year, thus minimizing the annual cost for the players. By calculating the cost of every possible charge and discharge decision in every time step t within the optimization time horizon, a set of nodes is derived which results in lowest possible costs.

The objective function is given in Eq. (3.13). $C_{el}(t)$ represents the total electricity cost in time step t, including electricity spot price, grid tariff cost and taxes. Eq. (3.14a) shows the energy balance, which includes all players in the coalition and their respective PV system and batteries. Eq. (3.14b) and Eq. (3.14c) show the maximum and minimum battery state of charge, whereas Eq. (3.14d) and Eq. (3.14e) reflect the maximum charge and discharge power[7]. (3.14f) shows the stored energy in a given time step. Lastly, Eq. (3.14g) shows the battery state of charge equation.

$$\min f(P_{bat}) = \sum_{t \in T} C_{el}(t) P_{grid}(t) \tag{3.13}$$

[6]The optimization simulations are conducted on a DELL stationary computer with Intel® Core™ i7 CPU of 3.20 GHz and 16 GB RAM. The computation time for running the algorithm is approximately 12 seconds.

[7]The rated power is considered to be the continuous rated power.

s.t.

$$P_{grid}(t) = \sum_{n \in N} P_{load,n}(t) + \sum_{z \in Z} P_{bat,z}(t) - \sum_{p \in P} P_{PV,p}(t) \tag{3.14a}$$

$$SOC_z(t+1) \leq \boldsymbol{SOC_z^{max}} \quad z \in Z \tag{3.14b}$$

$$SOC_z(t+1) \geq \boldsymbol{SOC_z^{min}} \quad z \in Z \tag{3.14c}$$

$$P_{bat,z}(t) \leq P_{bat,z}^{max} \quad z \in Z \tag{3.14d}$$

$$P_{bat,z}(t) \geq -P_{bat,z}^{max} \quad z \in Z \tag{3.14e}$$

$$E_{bat,z}(t+1) \geq E_{bat,z}(t) + \eta_{bat,z} P_{bat,z}(t) \Delta t \quad z \in Z \tag{3.14f}$$

$$SOC_z(t+1) = \frac{E_{bat,z}(t+1)}{E_{bat,z}^{max}} \quad z \in Z \tag{3.14g}$$

Note that

$$\eta_{bat,z} = \eta_{ch,z}, \quad P_{bat,z}(t) \geq 0 \quad z \in Z$$

$$\eta_{bat,z} = \eta_{dis,z}, \quad P_{bat,z}(t) < 0 \quad z \in Z$$

$P_{bat,z}^{max}$ is the maximum amount of power which each battery can be charged or discharged, and it depends on the individual battery specifications. Due to the constraint of $P_{bat,z}^{max}$, there is for each SOC in stage t a limited range of valid SOCs, which battery z is allowed to reach in stage $t+1$. With M possible SOCs, the maximum change in SOC from stage t to stage $t+1$, for battery z is given by the following:

$$\Delta SOC_{max,z} = \frac{\eta_{bat,z} P_{bat,z}^{max}}{E_{bat,z}^{max}} M \tag{3.15}$$

The SOCs outside the $\Delta SOC_{max,z}$ are illegal. In the DP algorithm, these SOCs are set equal to infinity. The algorithm will loop through all legal SOCs for both batteries and calculate the related electricity cost for each transition. All the paths and their corresponding costs are stored in a matrix, denoted the *transition cost matrix*. When the costs for all paths are calculated, the algorithm will obtain the optimal path for both batteries by checking the cost for all possible combinations within the optimization time horizon. The transition cost matrix is illustrated in Figure 3.3, showing two arbitrary searching paths for battery 1 and 2[8]. When the algorithm has searched through all paths, the global optimal solution can be obtained. This global optimal solution corresponds to the charging strategy of battery 1 and 2 that provides the lowest electricity cost.

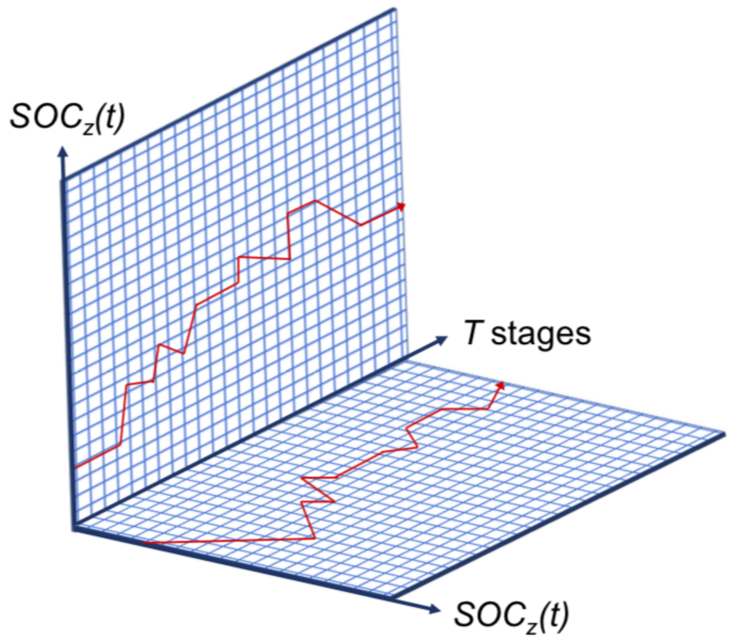

Figure 3.3: Searching strategy through the *transition cost matrix*. Arrows illustrate arbitrary paths for battery 1 and battery 2.

3.3 Game Theoretical Layer

The following chapter presents the reader the methodology for obtaining the Shapley value and nucleolus for the considered game. The methodology described in Chapter 3.3.4 is based on [26].

[8]Battery 1 and 2 correspond to player 1 and 2, respectively.

3.3.1 Prerequisites of Cooperation

The considered energy community can be modeled as a cooperative game. Each coalition formed by the players will lead to an outcome in form of the annual electricity cost. Further, the outcome of each coalition depends on the interaction among the players. This implies that the energy community satisfies the definition of a cooperative game. Further, the players act rational, with self-interest. Thus, the objective of each player is to minimize their individual electricity cost. Although the players act with self-interest, the grand coalition will form due to the concept of superadditivity.

As results from preliminary studies [37] show, the considered energy community including prosumers and consumers, fulfills the concept of superadditivity. Hence, the grand coalition will form. Despite this, the players will only join the grand coalition if the proposed cost allocation provides the players the highest payoff. As described in Chapter 3.1.1, $P_{grid}(t)$ operates as a slack variable, in order to ensure that load demand is fulfilled at all times. Thus, for sub coalitions that solely consist of consumers, demand is fulfilled by exclusively buying power from the grid. This, in turn, implies that all sub coalitions $S \subseteq N$ provide feasible solutions.

The considered energy community consists of n = 4 players, resulting in total 2^n = 16 possible coalitions. For each sub coalition $S \subseteq N$, there is a corresponding electricity cost. The electricity costs belonging to the coalitions are obtained by minimizing the annual cost for all possible coalitions $S \subseteq N$. As a result, a cooperative TU game *(N,v)* can be obtained. As this thesis analyzes four different scenarios, there exists a cooperative TU game *(N,v)* for each scenario. This game is used as input when the solution concepts nucleolus and the Shapley value are applied for cost allocation among the players.

The cooperating energy community cooperating as the grand coalition, is visualized through Figure 3.4. The arrows within the energy community indicate the direction of the power flow. Player 1 and 2 exchange power with each other, as well as sending power to player 3 and 4. As the arrows show, player 3 and 4 are only able to receive power. The energy community is connected to the main grid, to ensure that total load demand is met at all times.

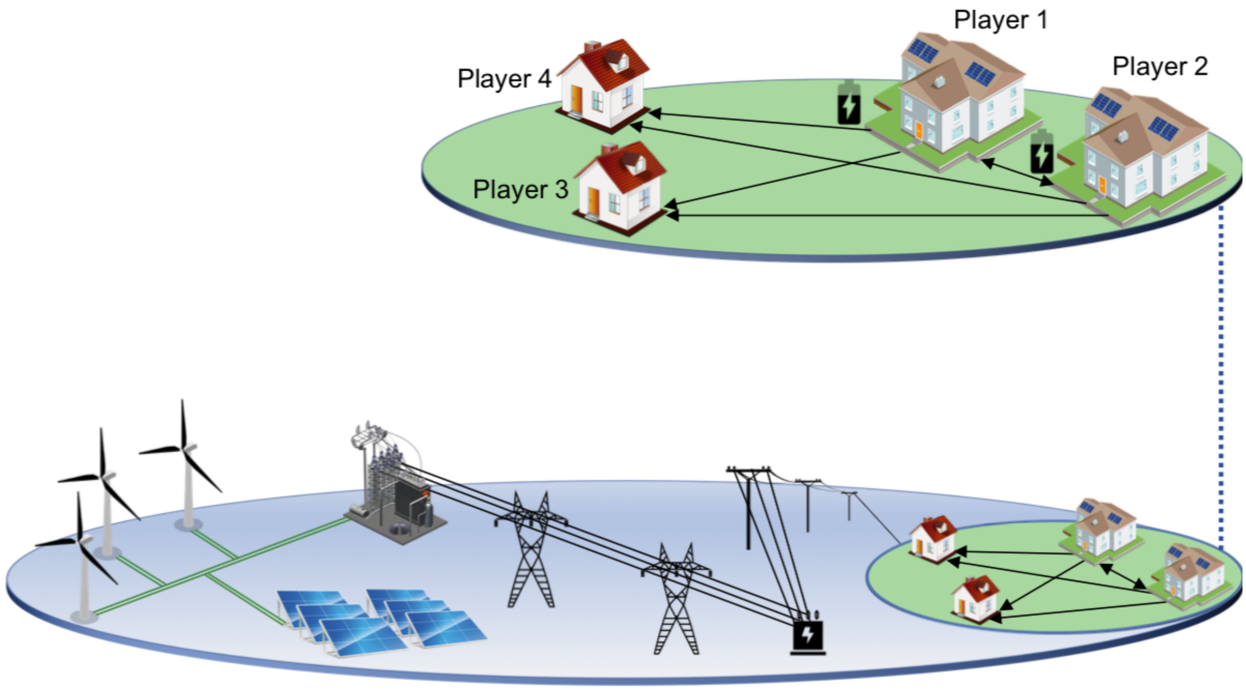

Figure 3.4: The energy community including 4 players cooperating as the grand coalition, as a part of a larger energy system.

3.3.2 The Core

To ensure that all end-users in a cooperative game want to form the grand coalition, the proposed value allocation needs to be in the core of the game. If there exists a non-empty core, the nucleolus is guaranteed to exist. Hence, additional examination is not required. In contrast, the Shapley method does not guarantee to provide stable cost allocations. Once the Shapley values for the game are obtained, these values must be checked for stability. The Shapley values for the game in each scenario are checked by Algorithm 1.

Algorithm 1 Check if allocation is in the core

Input: Cooperative TU game *(N,υ)* and allocation $\boldsymbol{x}$
Output: Statement of whether allocation is in the core

for all $S \subseteq N$ **do**
  if $\sum_{i \in S} x_i < \upsilon(S)$ **then**
    return Allocation is not in the core
  end if
end for
return Allocation is in the core

3.3.3 The Shapley Value

The Shapley value $\phi_i(\upsilon)$ for each player i is calculated based on each players' individual contribution to a coalition. Algorithm 2 provides the method for how the Shapley values for each game are obtained. When the Shapley value allocation vector, $\boldsymbol{\phi}$ for the n players are obtained, these values are used as input to Algorithm 1 to check their stability. The Shapley values, and subsequently their stability are calculated and checked using Microsoft® Excel version 16.10 for Mac.

Algorithm 2 Calculate value allocation using Shapley's method

Input: Cooperative TU game *(N,v)*
Output: The Shapley value allocation, $\boldsymbol{\phi}$

for all $i \in N$ **do**
  for all $S \subseteq N$ **do**
    if $i \in S$ **then**
      $\phi_i = \phi_i + \frac{(|S|-1)!(n-|S|)!}{n!}[\upsilon(S) - S\backslash\{i\})]$
    end if
  end for
end for
return $\boldsymbol{\phi}$

3.3.4 The Nucleolus

In this thesis, the nucleolus is obtained by the concept of finding the minimal excess vector $\theta_{min}(\boldsymbol{x})$ from the *least cores*. This method is based on an algorithm from [8], which is based on [2]. The $\theta_{min}(\boldsymbol{x})$ is found by iteratively solving a linear problem (LP). For each iteration, constraints based on previous maximum excesses are added to the problem. These constraints iteratively add cut to the LP problem, hence reduce the feasible region until one single point is obtained. This unique point is the nucleolus. For each iteration, constraints based on previous maximum excesses are added to the problem. These constraints add cut to the LP problem, hence reduce the feasible region until one unique nucleolus is obtained. The LP problem is given in Eq. (3.16)-Eq. (3.16c).

$$\min \varepsilon_k \tag{3.16}$$

$$\text{s.t.} \quad v(S) - \sum_{i \in S} x_i \leq \varepsilon_k \quad \forall S \subset N \text{ and } S \notin F_k \tag{3.16a}$$

$$\sum_{i \in N} x_i = v(N) \tag{3.16b}$$

$$\begin{aligned} &v(S) - \sum_{i \in S} x_i = \varepsilon_j, \\ &\forall S \subset F_j, j \in \{1, ..., k-1\} \end{aligned} \tag{3.16c}$$

$$\varepsilon_k \in \mathbb{R},\ x_i \in \mathbb{R} \quad \forall i \in N$$

The index k represents the number of iterations. The objective is to minimize ε_k for each stage. Eq. (3.16) ensures that ε_k does not exceed the maximum excess for imputation x. Group rationality is fulfilled by Eq. (3.16b), while Eq. (3.16c) ensures that the previous minimized maximum excess is still maintained. F_j represents the set including all coalitions which the excess constraint (3.16a) was binding at previous stages. Thus, F_k is the union of previously binding coalitions and $F_1 = \varnothing$. These coalitions are obtained by utilizing dual variables. For each coalition there is one constraint. If the corresponding dual variable is non-zero, the constraint, and hence the coalition is binding.

The iterative procedure continues until the unique solution is obtained. This results in maximal 2^n-2 iterations, which is the number of sub coalitions, except $S = N$ and $S = \varnothing$. Each LP consists of $n + 1$ variables and 2^n-2 constraints. As a result, the size of the problem will significantly increase as the number of players increases. The nucleolus of each game is obtained by using Algorithm 3.

The code including the algorithm for calculating the nucleolus, was provided by the the author of [26], and it is directly used for calculating the nucleolus for the considered games in this thesis. The algorithm is implemented in Python, and performed using Spyder through the Anaconda solver.

Algorithm 3 Calculate the nucleolus

Input: Cooperative TU game *(N,v)*
Output: The nucleolus, $\boldsymbol{x}_{nu}$

Initialize Previously binding coalitions $F_1 = \varnothing$
Initialize Vector of previous maximum excesses, $\boldsymbol{\varepsilon}$
Initialize Iteration $k = 1$

while $\boldsymbol{x}_{nu}$ not unique **do**
  Solve LP problem (3.16) for iteration k
  Append objective ε_k to vector of previous maximum excesses, $\boldsymbol{\varepsilon}$
  if (3.16b) and (3.16c) consist of n linearly independent constraints **then**
    Solution is unique and $\boldsymbol{x}_{nu} = \boldsymbol{x}_k$
  else
    for all Dual variables of constraints (3.16c) from the solution of iteration k **do**
      if Dual variable not equal to zero **then**
        Corresponding S is added to F_{k+1}
      end if
    end for
    $k = k + 1$
  end if
end while
return $\boldsymbol{x}_{nu}$

3.4 Scenarios

Using two different sets of load profiles and two different sets of electricity spot prices, four scenarios are developed. The two considered load profiles are denoted load P1 and load P2. The two sets of electricity spot prices are taken from Norway and Germany, referred to as NO3[9] and GER, respectively. Each scenario considers a different combination of load and price scenarios. The simulated scenarios including their parameters, are presented in Table 3.2. By simulating these scenarios, different values of the battery system can be obtained. Thus, a basis for analyses of the game theoretical methods with different battery system values, is developed. The Shapley value and nucleolus are applied to all scenarios, to study their performance under different conditions.

Table 3.2: The four simulated scenarios with different load profiles (Load P1/P2) and electricity spot prices (NO3/GER).

	Scenario			
Parameter	**#1**	**#2**	**#3**	**#4**
Load	Load P1	Load P1	Load P2	Load P2
Electricity spot price	NO3	GER	NO3	GER

The annual electricity cost of the energy community cooperating as the grand coalition without the battery system, is calculated. In addition, the electricity cost including the battery system is minimized by using the optimization algorithm described in Chapter 3.2.

3.5 Simulation Procedure

For each scenario, the value of the battery system is calculated for the grand coalition. In addition, both the nucleolus and the Shapley value are applied to the game. The total simulation procedure is represented through the flowchart in Figure 3.5.

[9]The considered area os located in Nordpool area 3, thus denoted NO3.

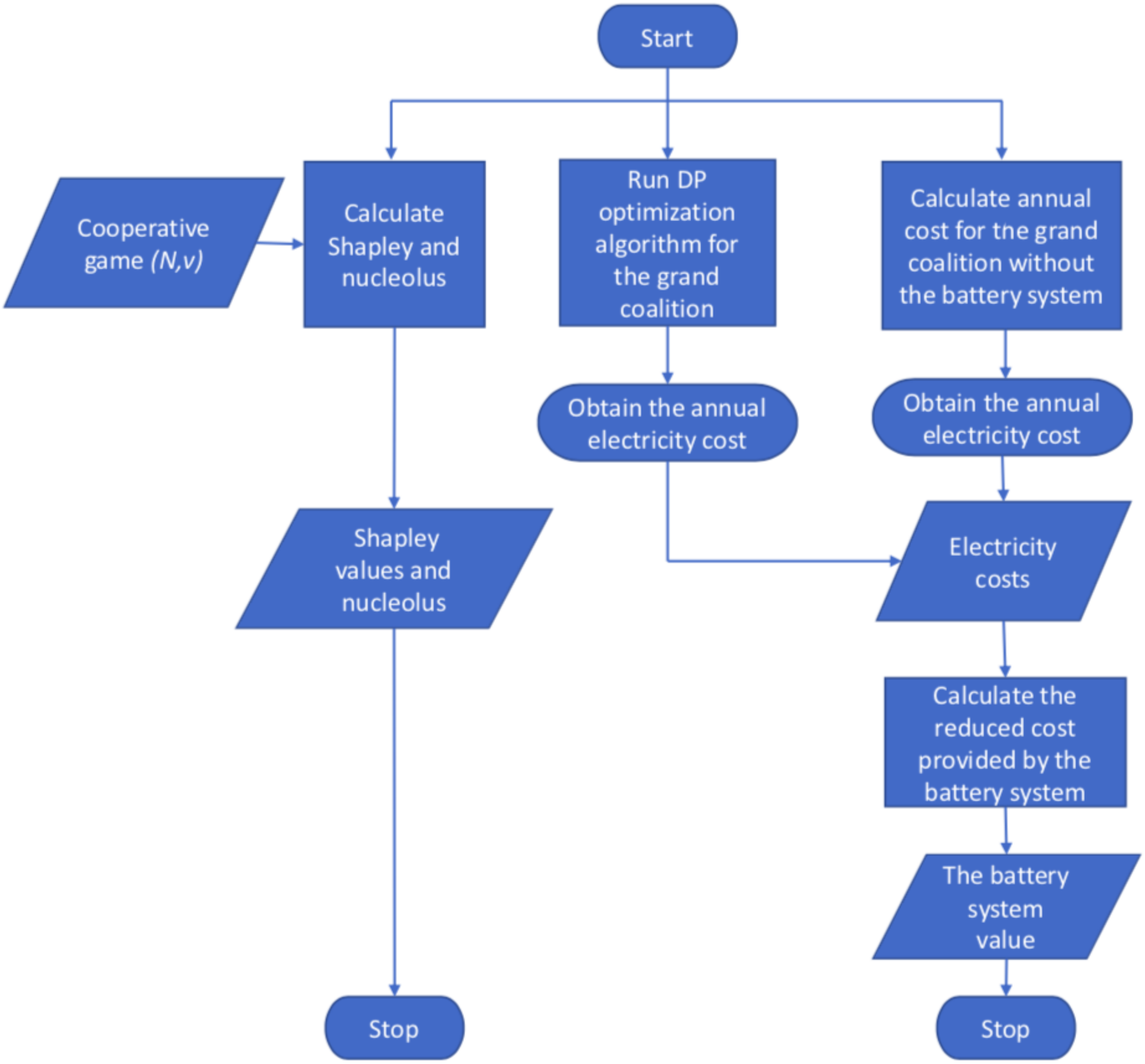

Figure 3.5: Total simulation procedure for each scenario. The Shapley value and nucleolus are obtained, along with the value of the battery system of the corresponding grand coalition.

Chapter 4

Input Data

The following chapter presents the input data used for simulating the different scenarios. Data presented in Chapter 4.1-4.5, are obtained from the pre-work of this thesis [37]. By utilization these data, the cooperative games *(N,υ)* for each scenario, can be obtained. The resulted games are provided in Chapter 4.6.

4.1 Load Profiles

The load profiles (load P1/load P2) are obtained from load data of two private end-users in the city of Trondheim, Norway. The load data of the two end-users were originally provided to the author of [3], by Trønderenergi Nett, the local DSO in the considered area[1]. The load data are re-used for the writing of this thesis. Both load data profiles are given in hourly resolution.

The load data are used to create two different load profiles, consisting of four players each[2]. Key data for load profile 1 and 2 are given in Table 4.1. As shown in the table, the key data differ to a great extent between the two profiles. Further, Table 4.2 and 4.3 provide key data for the different players within load profile 1 and 2, respectively. As the tables show, there is a significant difference

[1] Region NO3 in the Nord Pool spot market.
[2] Total load in each scenario is modeled as one unit: $\sum_{n \in N} P_{load,n}(t)$.

Table 4.1: Key data for the two load profiles.

Value	Load profile 1	Load profile 2
Annual consumption	114 270 kWh	28 868 kWh
Average consumption	13.04 kWh/h	3.295 kWh
Maximum consumption	35.40 kWh/h	10.00 kWh/h

between the annual, average and maximum consumption in profile 1 and 2. From Table 4.2, it can be seen that the largest energy consumption in profile 1 is of 43 303 kWh, while profile 2 provides an annual consumption of 7 217 kWh, as seen from Table 4.3.

In comparison, a private Norwegian household typically has an annual energy consumption of 20 000 kWh [29], whereas the annual energy consumption of a German household is approximately 3 500 kWh [38].

Table 4.2: Key data for the players in load profile 1.

	Players (Load profile 1)			
	1	**2**	**3**	**4**
Annual consumption [kWh]	43 303	43 303	13 830	13 830
Average consumption [kWh/h]	4.94	4.94	1.58	1.58
Maximum consumption [kWh/h]	15.0	15.0	4.94	4.94

Table 4.3: Key data for the players in load profile 2.

	Players (Load profile 2)			
	1	**2**	**3**	**4**
Annual consumption [kWh]	7 217	7 217	7 217	7 217
Average consumption [kWh/h]	0.824	0.824	0.824	0.824
Maximum consumption [kWh/h]	2.5	2.5	2.5	2.5

4.2 PV Production

For calculating the total PV power for each scenario, irradiation and temperature data are required. Irradiation and temperature data are taken from the Norwegian weather service [18]. The annual PV production within the energy community is 10 648 kWh. Figure 4.1 illustrates the total PV production, along with the mean temperature throughout the year. As Figure 4.1 shows, days with the highest PV production do not correlate with days with highest temperature, as PV production decreases with increased temperature [15].

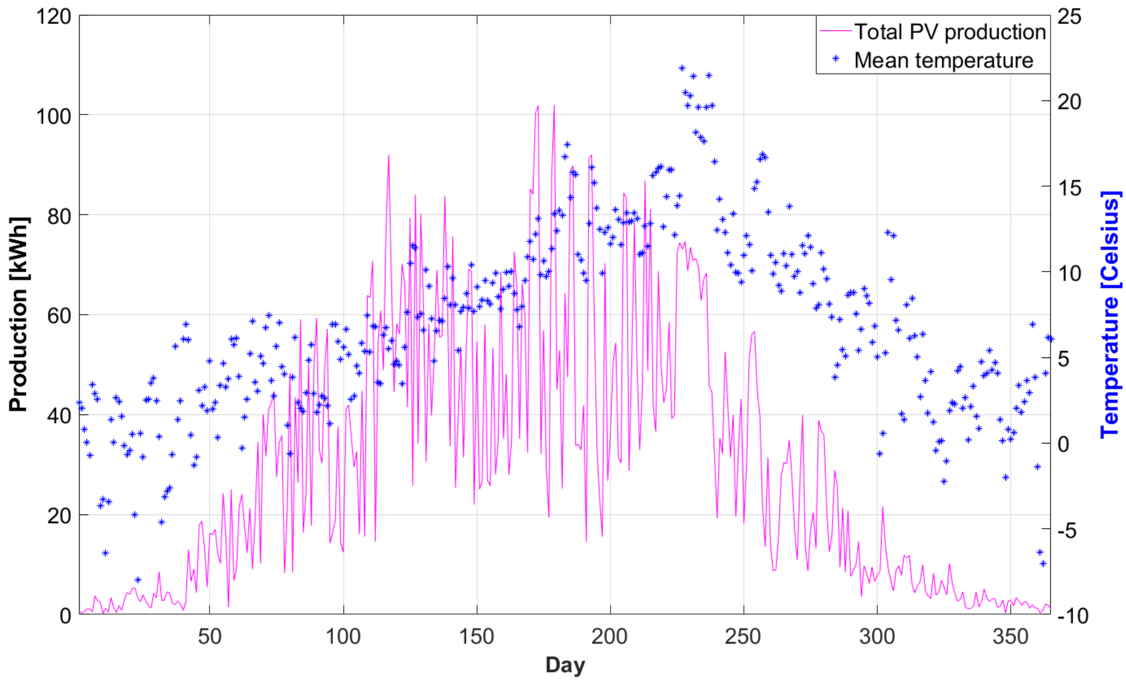

Figure 4.1: Total PV production within the energy community in 2015, along with the mean temperature.

The PV system is simulated by using the PV panel Sanyo HIT-240HDE4. The PV systems for player 1 and 2 require a rooftop area of 36.84 m^2 each. Data for the relevant PV panel is provided in Table 4.4.

Table 4.4: Key data for PV panel.

P_{nom}	α_T	$NOCT$	S	η_{inc}	η_{sys}
7 kW	-0.3 %/°C	44°C	0.190 kW/m^2	1.206	0.77

Figure 4.2 shows the daily load for load profile 1 and 2, along with the total PV production provided by player 1 and 2, throughout the year. As shown in Figure 4.2, the total PV production does only exceed the demand of load profile 2. For scenario 1, the battery system will not be utilized for storing power produced by the PV system, as produced power will solely be used for covering the load within the energy community.

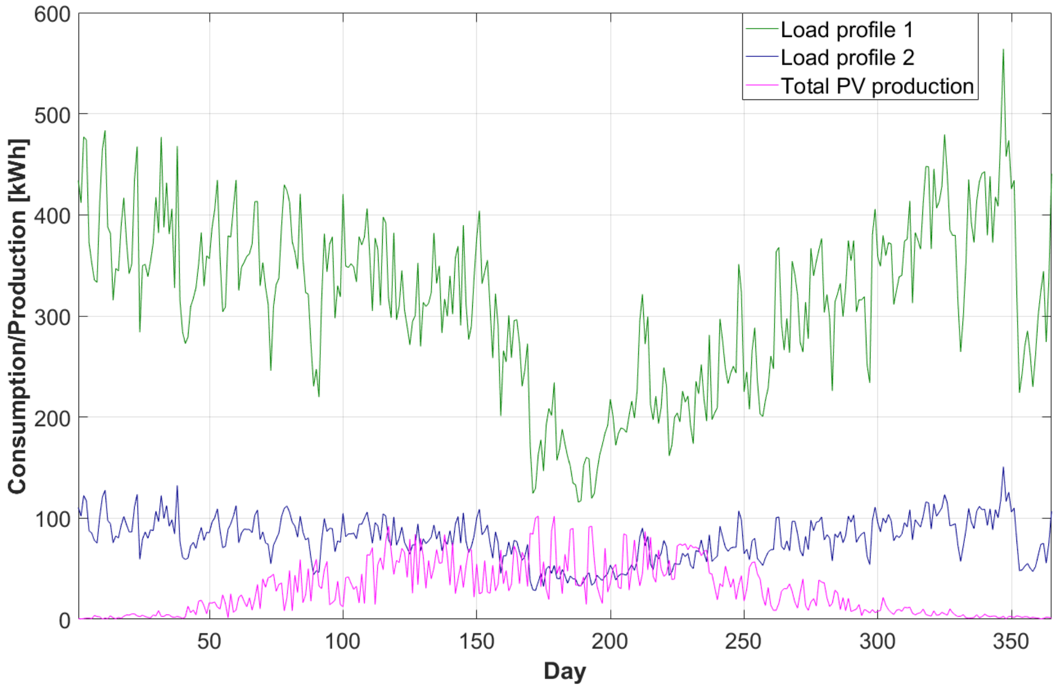

Figure 4.2: Total PV production throughout the year, along with the energy consumption of the two load profiles.

4.3 Battery Specifications

Two different batteries are used for the simulations. Battery specifications are given in Table 4.5. Battery 1 is based on data from a Tesla Powerwall [32], whereas battery 2 is based on data from a LG house battery [1].

Table 4.5: Battery specifications for the two batteries in the energy community.

	$P_{bat,z}^{max}$	$E_{bat,z}^{max}$	SOC_z^{max}	SOC_z^{min}	$\eta_{bat,z}$
Battery 1	7 kW	13.5 kWh	100 %	0 %	0.95
Battery 2	7 kW	9.8 kWh	100 %	0 %	0.90

4.4 Electricity Spot Price Profiles

The prices used for the simulated scenarios are taken from Nordpool [19] and EPEX spot markets [33] from 2015, both with hourly resolution. The spot price within the respective area (NO3) provides an average of 0.1898 NOK/kWh with minor fluctuations, whereas the German spot price (GER) provides an average price of 0.2831 NOK/kWh, and higher fluctuations. The two electricity spot price profiles are shown in Figure 4.3. Key data for both spot price profiles are given in Table 4.6.

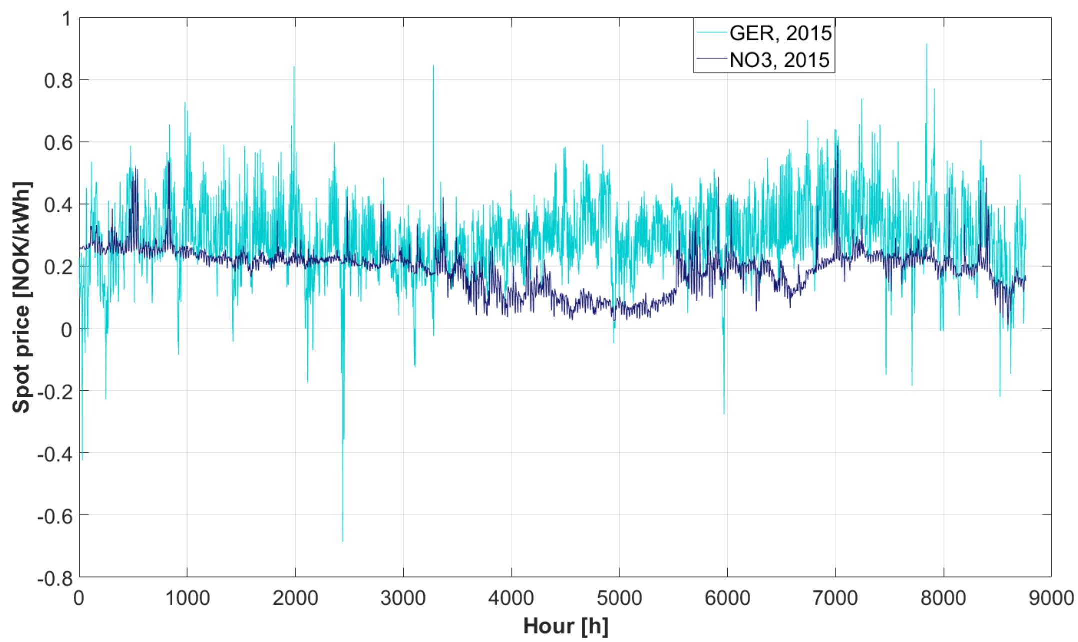

Figure 4.3: Electricity spot price of the GER- and NO3- profiles in 2015.

Table 4.6: Key data for electricity spot price profiles in 2015. All values given in [NOK/kWh].

	Average	Variance	Max	Min
GER	0.2831	0.0132	0.9150	-0.6860
NO3	0.1898	0.0047	0.5880	0.0110

The most remarkable observation from Table 4.6, is the minimum value of the GER-profile of -0.6860 NOK/kWh. Due to the rising share of high and inflexible renewable generation in the German power sector, the phenomenon of negative electricity spot prices on the EPEX is on the rise in Germany. The negative prices imply that power suppliers have to pay their customers to buy electric energy. In consequence, the German electricity spot price is highly volatile, as seen from Figure 4.3. For visualizing the difference in price fluctuations within each spot price profile, the profiles are plotted in heat diagrams. Figure 4.4 and 4.5 illustrate the average spot price for NO3 and GER, respectively. The prices are plotted for a specific hour at the specific weekday. The reader should note that the range of the NOK/kWh varies between Figure 4.4 and 4.5.

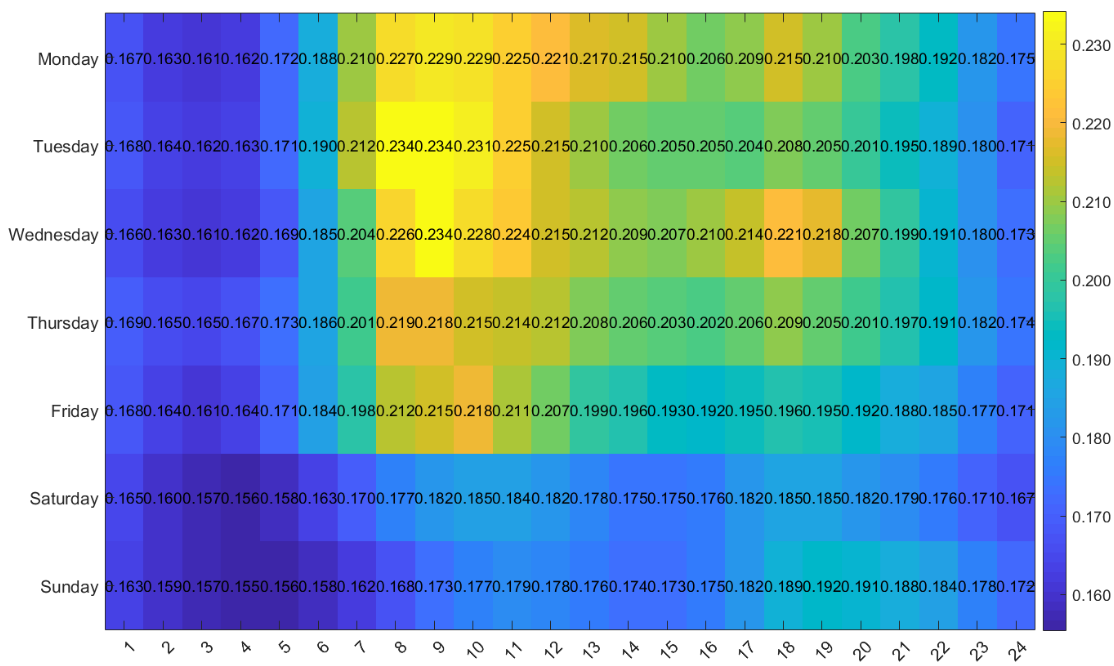

Figure 4.4: NO3-profile: Average hourly spot price in [NOK/kWh], for the specific hour at the specific weekday in 2015.

From these figures, one clearly sees the fluctuating prices within each profile. The spot prices in both Figure 4.4 and 4.5 are low at night, followed by high price-period between 07-10 am, from Monday to Friday. This increase is caused by the large number of required electricity appliances in the morning hours for both industrial, public and private end-users.

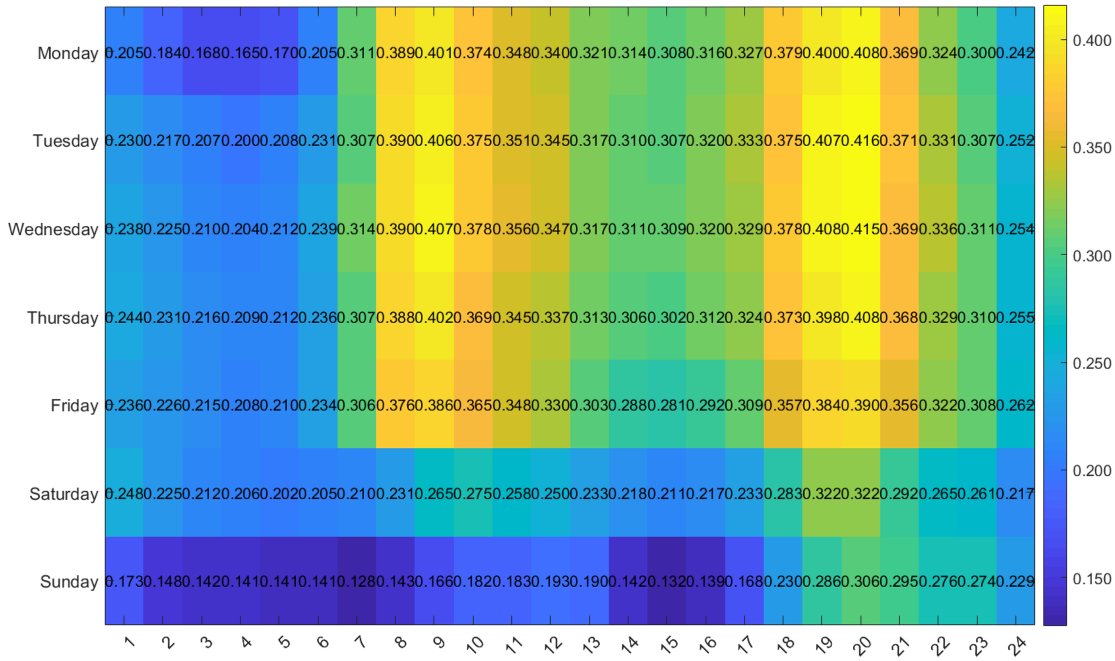

Figure 4.5: GER-profile: Average hourly spot price in [NOK/kWh], for the specific hour at the specific weekday in 2015.

The prices in both Figure 4.4 and 4.5 fall during the day, before a second high price-period is reached around 06-08 pm. This second high price-period is caused by end-users arriving from work, who start to use their electrical appliances at home. This tendency is in particular strong for the GER-profile in Figure 4.5. Despite the electricity spot price fluctuations during the week, it should be noted that the average electricity spot prices for private Norwegian households were 13.1 % lower in 2015 than the previous year 2014, and the lowest electricity spot price since 2005 [28]. The remaining cost terms related to the cost of electric energy are provided in Table 4.7, along with the grid tariff costs.

4.5 Grid Tariff

The energy-based grid tariff consists of a fixed term, in addition to a marginal cost for every kWh consumed. As the load data used in the simulations are taken from the city of Trondheim, the grid tariff cost is taken from the DSO in the same area, Trønderenergi Nett AS. All cost elements, except the electricity spot price are given in Table 4.7.

Table 4.7: Grid, energy, tax and VAT costs for end-users. The costs shown in the table are not including VAT.

Cost element	Cost	
$C_{monthly,fixed}$	37.5	[NOK/Month]
$C_{consumertax}$	0.124	[NOK/kWh]
C_{CG}	0.0369	[NOK/kWh]
C_{rev}	0.025	[NOK/kWh]
$C_{grid,fixed}$	1 340	[NOK]
$C_{grid,energy}$	0.22	[NOK/kWh]
VAT	25	[%]

4.6 The Cooperative TU games

The cooperative game *(N,v)* is used as input for analyzing the solution concepts of the nucleolus and the Shapley value. By utilizing the data input presented in Chapter 4, the cooperative games *(N,v)* can be obtained. The TU games used as input for the cost allocation methods of nucleolus and the Shapley value, are given in Table 4.8a-4.8d.

Table 4.8: The simulated TU games *(N,υ)* for all scenarios.

(a) TU game *(N,υ)* for scenario 1.

Scenario #1	
Coalition $S \subseteq N$	**Value** υ**(S) [NOK]**
{∅}	0
{1}	31 793
{2}	31 893
{3}	12 998
{4}	12 998
{1,2}	63 630
{1,3}	44 725
{1,4}	44 725
{2,3}	44 812
{2,4}	44 812
{3,4}	25 996
{1,2,3}	76 549
{1,2,4}	76 549
{1,3,4}	57 698
{2,3,4}	57 784
{1,2,3,4}	89 507

(b) TU game *(N,υ)* for scenario 2.

Scenario #2	
Coalition $S \subseteq N$	**Value** υ**(S) [NOK]**
{∅}	0
{1}	35 118
{2}	35 694
{3}	14 398
{4}	14 398
{1,2}	70 721
{1,3}	49 401
{1,4}	49 401
{2,3}	49 999
{2,4}	49 999
{3,4}	28 796
{1,2,3}	85 010
{1,2,4}	85 010
{1,3,4}	63 754
{2,3,4}	64 366
{1,2,3,4}	99 349

(c) TU game *(N,υ)* for scenario 3.

Scenario #3	
Coalition $S \subseteq N$	**Value** υ**(S) [NOK]**
{∅}	0
{1}	5 091
{2}	5 131
{3}	7 828
{4}	7 828
{1,2}	9 850
{1,3}	12 212
{1,4}	12 212
{2,3}	12 346
{2,4}	12 346
{3,4}	15 657
{1,2,3}	16 943
{1,2,4}	16 943
{1,3,4}	19 768
{2,3,4}	19 906
{1,2,3,4}	24 346

(d) TU game *(N,υ)* for scenario 4.

Scenario #4	
Coalition $S \subseteq N$	**Value** υ**(S) [NOK]**
{∅}	0
{1}	4 976
{2}	5 124
{3}	8 639
{4}	8 639
{1,2}	9 582
{1,3}	12 656
{1,4}	12 656
{2,3}	13 019
{2,4}	13 019
{3,4}	17 278
{1,2,3}	17 263
{1,2,4}	17 263
{1,3,4}	20 854
{2,3,4}	21 326
{1,2,3,4}	25 326

Chapter 5

Results and Discussion

This chapter presents to the reader the results from the game theoretical modeling, along with a discussion. Chapter 5.1- 5.3 present and discuss the findings from the considered scenarios, while a broader discussion is provided in Chapter 5.4.

5.1 Value of the Battery System

The problem in question is to evaluate the deviation between the Shapley value and nucleolus, and its relation to the battery system value. Consequently, it is of interest to study how the battery system value is changed within the different scenarios. The λ_{EES} for each scenario is shown in Table 5.1. By changing the parameters load and electricity spot price, results from Table 5.1 show the fluctuating contribution in cost reduction provided by the battery system. The corresponding values are plotted in Figure 5.1.

Table 5.1: Relative reduced cost in [%] provided by the battery system in each scenario.

	Scenario			
	#1 (Load P1/NO3)	**#2** (Load P1/GER)	**#3** (Load P2/NO3)	**#4** (Load P2/GER)
$\boldsymbol{\lambda_{EES}}$ [%]	**0.3917**	**1.4454**	**5.5221**	**8.8435**

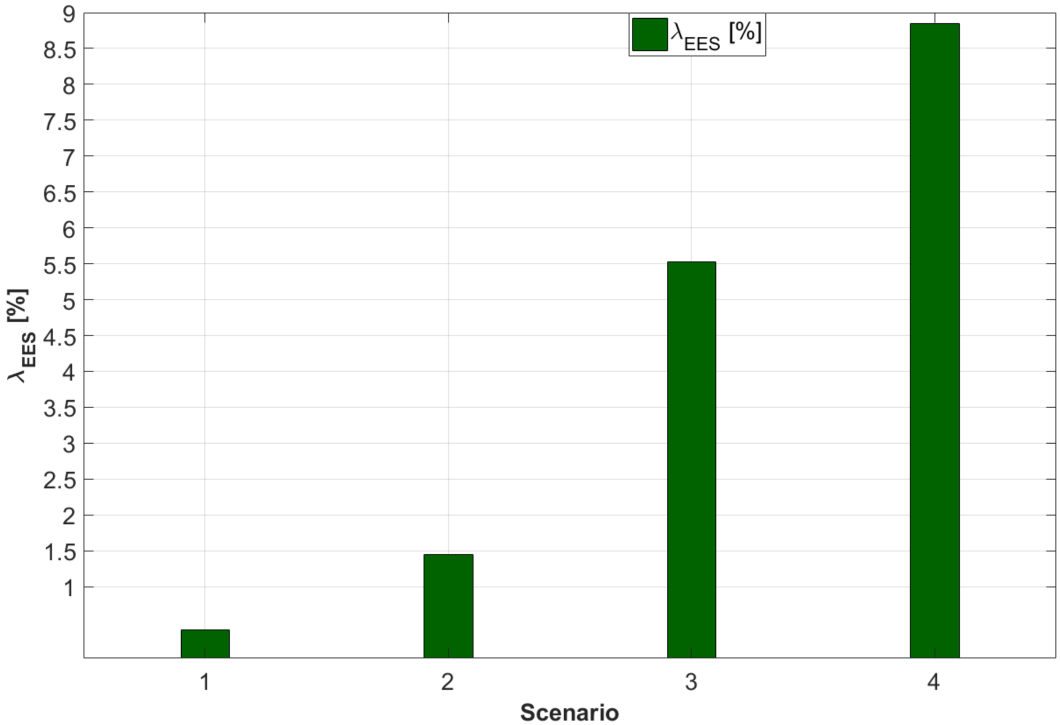

Figure 5.1: Relative reduced cost provided by the battery system in each scenario [%].

In scenario 1, the total load demand is high and the fluctuations in electricity spot prices are minor. The battery system is able to lower the cost by 0.39 %. The value of the battery system increases in scenario 2 and 3, whereas the highest cost reduction provided by the battery system, is obtained in scenario 4. Here, the PV production exceeds demand (Load profile 2) throughout several days in the year, as seen from Figure 4.2. Consequently, the batteries are utilized for storing the exceeded energy after load is covered. In addition, the electricity spot price in scenario 4 is the most fluctuating (GER), which implies that the batteries are utilized for both storing power produced by the PV system, and for buying power when prices are low, in order to store for periods with higher price peaks. The high utilization of the battery system is reflected through a cost reduction of 8.84 %. With a varying λ_{EES}, it is of interest to examine whether the deviation between cost allocations provided by nucleolus and the Shapley value is affected.

5.2 Cost Allocations

5.2.1 Deviation Between the Nucleolus and the Shapley Value

In a cooperative game, the initial question is whether the core is non-empty. The nucleolus can be obtained for the game of each scenario, thus there exists a non-empty core. Furthermore, the Shapley value is shown to be in the core of the game, for every scenario. Consequently, both methods provide stable cost allocations and are suitable for comparison.

Table 5.2 presents the relative deviation in cost allocations provided by nucleolus and the Shapley value. In addition, the preferred method of each player is expressed in parentheses. Cells marked with '(**Nu**)' indicates that nucleolus is the preferred method, whereas '(**Sh**)' corresponds to the Shapley value. Cells marked with '($\approx$)' indicate that the deviation is less than 0.1 %. Deviation less than this quantity can be considered negligible, consequently both methods provide almost identical cost allocations. The cost allocations in actual costs for each scenario, provided by nucleolus and the Shapley value, are given in Appendix A.

As the results in Table 5.2 show, the deviation between the methods is minor in all scenarios. Further, it can be seen that player 3 and 4 experience the same deviation. Due to equal demand, the same cost allocation method is preferred within each scenario. The relative deviation between

Table 5.2: Relative deviation in [%] between nucleolus and Shapley for each player in each scenario, along with their preferred method.

	Scenario			
Player	**#1**	**#2**	**#3**	**#4**
1	0.02 ($\approx$)	0.02 ($\approx$)	0.01 ($\approx$)	1.01 **(Sh)**
2	0.00 ($\approx$)	0.04 ($\approx$)	1.37 **(Nu)**	3.01 **(Nu)**
3	0.02 ($\approx$)	0.03 ($\approx$)	0.43 **(Sh)**	0.57 **(Sh)**
4	0.02 ($\approx$)	0.03 ($\approx$)	0.43 **(Sh)**	0.57 **(Sh)**

the methods is illustrated in Figure 5.2. Although the deviations between the methods are small, a significant increase can be obtained in scenario 3 and 4. There are in particular three cases worth highlighting. That is, cases where deviation is greater than 1 %. Deviation greater than this value occurs for player 1 in scenario 4, and for player 2 in scenario 3 and 4. These players are the prosumers within the energy community. From Table 5.2, it can be seen that player 1 prefers the Shapley value in scenario 4, which provides a cost 1 % lower than the nucleolus. In contrast, player 2 prefers the nucleolus in both scenario 3 and 4. These mentioned cases are illuminated when the deviation and its relation to the battery system value are to be analyzed.

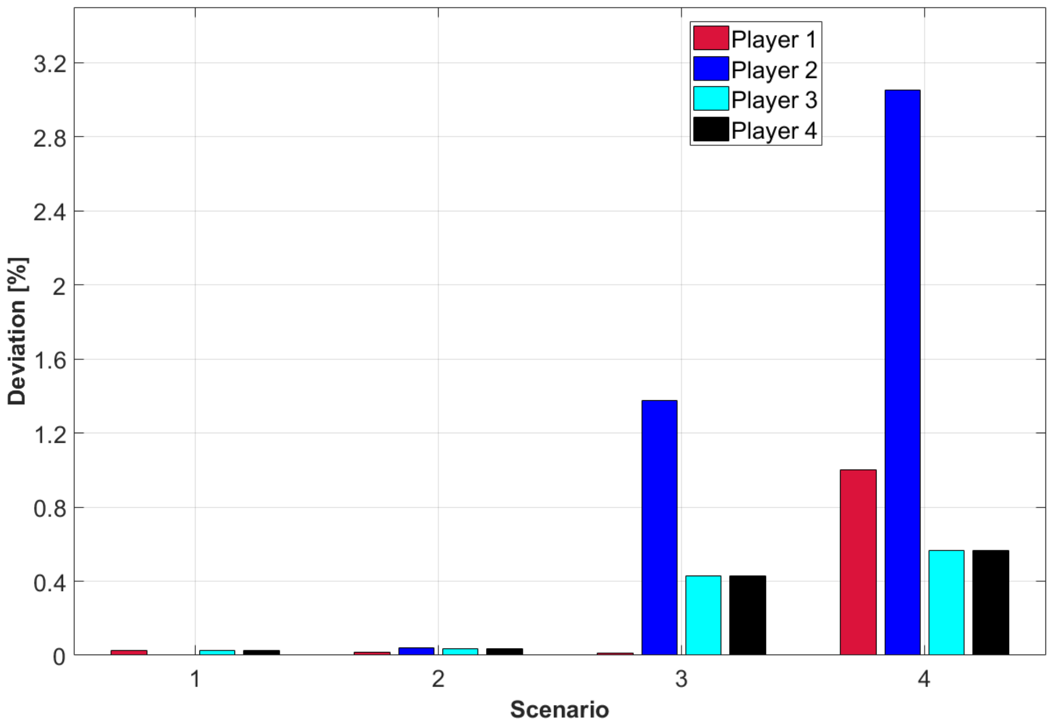

Figure 5.2: Relative deviation [%], between nucleolus and the Shapley value, for each player in each scenario.

5.3 Deviation Related to the Battery System

Figure 5.3 illustrates the deviation between the Shapley value and nucleolus for each player in all scenarios, plotted along with the relative cost reduction provided by the battery system, λ_{EES}. As Figure 5.3 shows, the deviation between Shapley and nucleolus tends to increase when the battery system value increases. For the interpretation of this finding, the conduct of the consumers and prosumers within the energy community is analyzed.

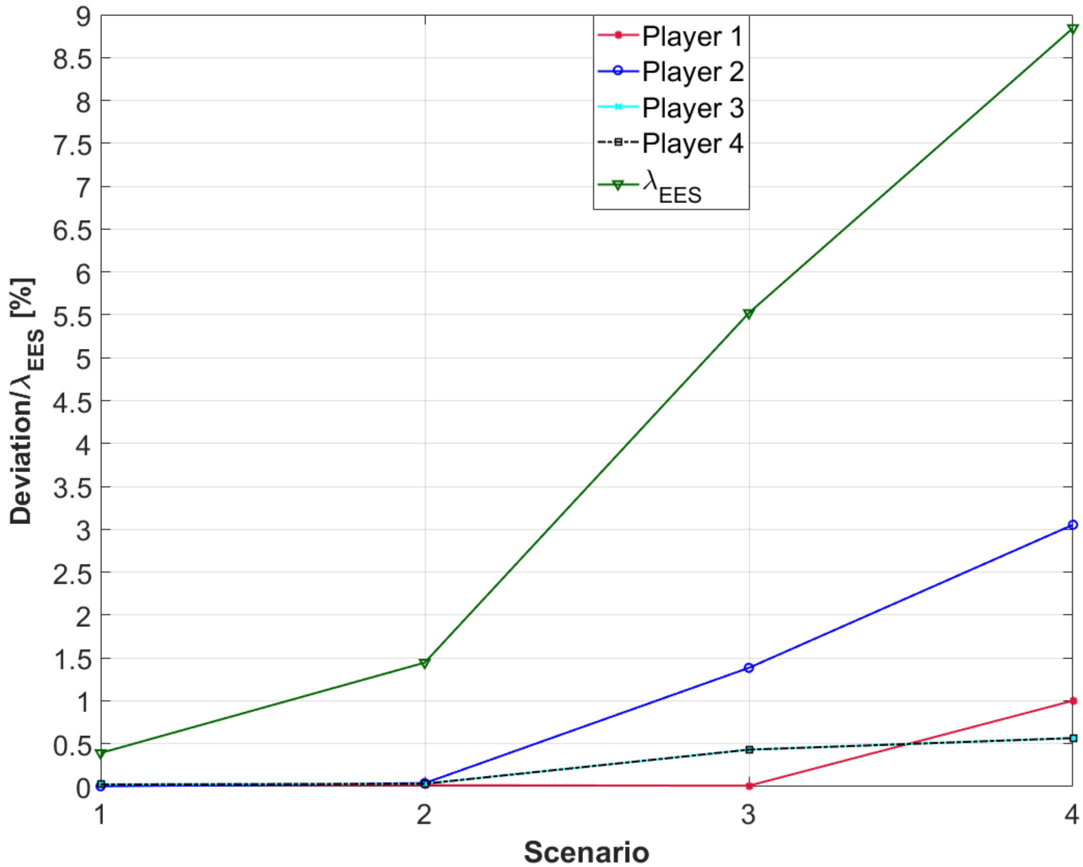

Figure 5.3: Relative deviation between nucleolus and Shapley for each player, along with the value of the battery system, in each scenario, [%].

The Consumers

As Figure 5.3 shows, the overall tendency for player 3 and 4 is a slight increase in deviation from 0.02 % in scenario 1, to 0.57 % in scenario 4. Despite this increase in relative deviation, this difference represents a marginal value in actual cost. If the Shapley value is applied to the system, player 3 and 4 would have been charged by 8 209 NOK each, whereas the nucleolus provides a cost of 8 256 NOK, as shown in Table A.1, Appendix A. These cost allocations correspond to a cost difference of 47 NOK, which is considered negligible in the context of an annual electricity bill. The results might indicate that the consumers are not that concerned regarding the chosen cost allocation method, irrespective of the value of the battery system.

The Prosumers

As shown in Table 5.2 and illustrated in Figure 5.3, the deviation experienced by player 1 and 2 is marginal in scenario 1 and 2. Here, the value of the battery system is minor, respectively 0.39 % and 1.45 %. In scenario 3, player 2 experiences a deviation of 1.37 %. In the same scenario, the value of the battery system is 5.52 %. In scenario 4, both the value of the battery system and the deviation experienced by player 2, increase. Yet, the deviation is 3.01 %, which corresponds to an actual cost of 136.8 NOK. Here, the battery system value is at its highest, that is 8.84 %. Despite this increase in deviation, the difference in actual cost is still minor. In scenario 4, player 1 experiences a deviation of 1.01 %, which corresponds to an actual cost of 43.4 NOK. Even though these deviations experienced by the prosumers can be considered marginal, the deviation increases, as the value of the battery system increases. Consequently, the results shows that the prosumers are more concerned regarding their preferred method, when the value of the battery system is high. A high λ_{EES} can be interpreted as a higher contribution to the overall cost reduction from player 1 and 2.

Despite the marginal deviation, it is of interest to examine what are the preferred methods by player 1 and 2. The overall highest deviation in the considered scenarios is obtained in scenario 4. Here, the battery system value is at its highest. Within this scenario, player 1 prefers the Shapley value, whereas player 2 prefers the nucleolus. In order to interpret the conflicting preferences, the conduct of the batteries in scenario 4, is analyzed. The reader should note that player 1 is equipped with the most efficient battery with largest energy capacity, whereas the battery of player 2 is less efficient and with lower energy capacity[1]. Figure 5.4 and 5.5 illustrate the annual conduct of battery 1 and 2, respectively.

[1]For battery specifications, see Table 4.5.

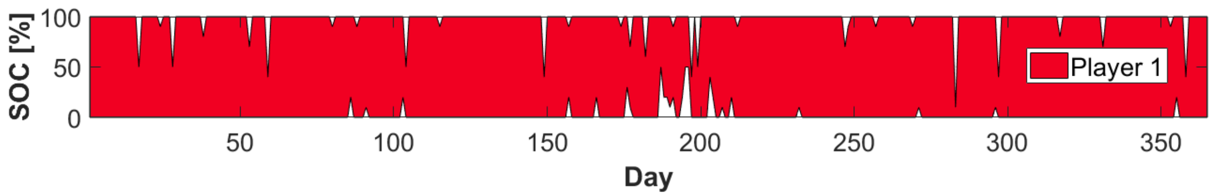

Figure 5.4: SOC of battery 1 (player 1) in [%] in scenario 4, throughout the year.

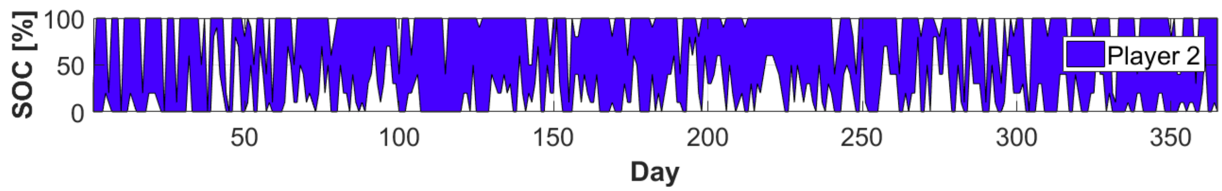

Figure 5.5: SOC of battery 2 (player 2) in [%] in scenario 4, throughout the year.

From these figures it can be seen that battery 1 is utilized to a greater extent than battery 2. Both batteries are utilized for storing energy in periods where PV production exceeds demand, and when the electricity spot price is high, such that it is economical profitable to buy power from the grid and store for later usage. Nevertheless, there is a significant difference between the utilization of the two batteries. The concept of the Shapley value allocates cost based on individual contribution to the grand coalition. This is the preferred method by player 1, whereas player 2 prefers the nucleolus. These results indicate that player 1 contributes more to the cost reduction than player 2. Figure 5.4 and 5.5 confirm this finding. Nevertheless, as mentioned above, player 1 is only being charged an extra cost of 43.4 NOK if nucleolus is applied to the game. In contrast, player 2 is more concerned regarding the selected method, as nucleolus provides an electricity cost which is 136.8 NOK lower than the Shapley value. Consequently, this high deviation between the methods is reflected through the difference in individual contribution between the players. The Shapley value captures this difference, consequently player 2 is being "punished".

Although the Shapley value interprets fairness based on individual contribution, an major issue when selecting cost allocation method, is how to define fairness. Despite the fact that it can be reasonable to interpret fairness in terms of individual contribution, this definition can also be interpreted as if the Shapley value rewards the most active player. In the considered scenario, this corresponds to player 1. Even if the battery of player 2 is utilized throughout the whole year, as illustrated in Figure 5.5, it could have been utilized even more if the efficiency had been just as high as of battery 1[2]. Due to battery 1, battery 2 has second priority and is less utilized. Despite the fact that battery 2 does contribute to the total cost reduction, player 1 has a great impact on the utilization of battery 2.

Consider a scenario where a prosumer invests in a battery with marginal improvements in terms of battery characteristics, compared to an already existing battery within the energy community. Due to minor improvements in performance, the new battery will have the first priority, in all time periods where only one battery needs to operate. Before the new battery was implemented in the energy community, the existing battery would have been operating, and it would have provided a value almost as high as the new battery. However, due to minor improvements, the introduction of the new battery will be favored, consequently it will reduce the operation of the existing battery. This, in turns, implies that the contribution of the existing battery will be limited. As the Shapley value favors players that contribute, it is of importance to be the most active player in the game, as this player is rewarded the most. In contrast, the less active player is being punished, due to its limited contribution. This issue might be a weakness of the Shapley value. Thus, the problem in question is whether fairness should be interpreted as actual contribution, or if less active players could be rewarded based on their potential contribution, irrespective of more active players in the game. In the considered scenario, the nucleolus might be the most suitable method. Although there are conflicting interests between player 1 and 2, player 1 experience a lower cost reduction by selecting the Shapley value, than player 2 would experience, if the nucleolus was applied.

5.3.1 Parameters Influencing the Value of the Battery System

As it can be seen from the results in Table 5.1, the simulated scenarios provide varying battery system values. Thus, the combination of the parameters; load demand, PV production and electricity spot price influence the utilization of the battery system. Despite marginal deviations between the

[2]Table 4.5: $\eta_{bat,1}$ = 0.95, $\eta_{bat,2}$= 0.90.

cost allocations in the considered scenarios, the deviation increases as the value of the battery system increases. Thus, the deviation in cost allocation method depends on parameters that increase the battery system value. The results in this thesis, show that the deviation depends on parameters that increase the battery system value, such as renewable sources and electricity spot price.

A deviation of 3 % corresponds to a minor cost difference of 136.8 NOK in the evaluated scenarios. Despite this, it is reason to believe that the deviation will increase as parameters such as electricity spot price and load fluctuates more, so that the battery system value increases. In addition, the end-users might expect different the grid tariff structures in the future. The combination of higher electricity spot prices and implementation of new grid tariff structures will facilitate for storage appliances, consequently the battery system value is expected to increase. A deviation of 3 % will represent a greater value in actual cost in an energy community where the overall costs are higher. In such scenarios the players are most likely to be more concerned regarding the selected method.

5.4 Incentives for Cooperation

In an energy community consisting of solely consumers without neither PV production nor battery systems, there is no incentive for cooperation, as there are no resources to operate in a cooperative manner. Thus, for an energy community to operate cooperative, a central question is how to facilitate the prosumers to join the cooperating operation. A rising issue is to analyze which player(s) are the most valuable for the cooperation.

Imagine a scenario where players have conflicting preferences regarding allocation method, as the deviation in actual costs is high. The majority of the players prefer nucleolus, whereas one single player prefers the Shapley value. A central issue is then which method to choose, as the chosen method may have large impacts whether the players want to join the grand coalition. Should the method preferred by the majority of the players be chosen, even though the single player who objects this method is the most valuable to the cooperation? Economic support or compensation provided by a regulator might solve this issue. If the Shapley values are outside the core of the game, this value can be stable if a regulator facilitate for cooperation in terms of economical support, to ensure that the proposed allocation is within the core.

Contrary, the considered issue can be studied the opposite way. Consider a regulator that has decided a cost allocation method for a given energy community. If the Shapley value is selected, the players are facilitated to contribute, as active players are rewarded at the expense of players with less resources. If nucleolus is applied to the system, it may be of less importance to be the most active player, as this method is based on minimizing the maximal dissatisfaction experienced by the players.

The Society and Environmental Aspects

In the considered games, the DSO is not regarded as a player, but operating as a slack variable to ensure that demand is met at all times. Despite this, the DSO might also have interest in the formation of the grand coalition. The cooperating energy community with internal power production leads to less power required bought from the main grid. In areas where there is risk of lack of adequate capacity, a cooperating energy community can prevent the DSO from invest in new lines to extend the capacity. In addition, a cooperating energy community might also have positive impacts on the environment, as it increases the share of renewables in the energy consumption. This, in turns, have positive impact on the overall society.

5.5 Model Simplifications, Assumptions and Consequences

Sources of error caused by assumptions and simplifications are provided in the following chapter. Chapter 5.5.1 is obtained from the pre-work of this thesis [37].

5.5.1 Optimization Modeling

Due to the available resources in the research community when the pre-work of this thesis was conducted, dynamic programming (DP) was utilized as optimization method. In contrast to greedy programming, DP guarantees that the optimal solution is obtained. However, when the problem in question rises, DP becomes both time and space consuming. With this in mind, it might be cumbersome to use the dynamic programming model developed in this thesis if additional batteries are to be implemented in the model. An alternative approach could be utilization of linear programming (LP). LP could be of particularly interest if additional batteries are implemented in the model, as it provides shorter computational time, with less required memory. Despite this mentioned advantage, LP is not as suitable as DP for solving problems spanned over a longer time horizon. If the problem is solved with LP, multiple LP-problems need to be solved due to limitations regarding the planning horizon. If the time horizon is kept to one year and the planning horizon is 24 hours, 365 LP problems must be performed. Another issue is the start- and terminal conditions required for the SOCs, as the SOCs at the start and the end of each planning horizon must be equal. This constraint represents a restriction in the LP problem, which is not required implemented in the DP problem. Although DP might be cumbersome with an increased amount of possible states in each stage, it has the ability to find the optimal solution for a longer time horizon. Hence, DP is well-performing method for the considered problems in this thesis.

5.5.2 System Modeling

Data Input

The load data utilized in this thesis offer an hourly resolution, meaning that peaks which occur within one hour are neglected. These neglected peaks caused by appliances such as water heaters are invisible to the model. For modeling the total load consisting of the four players within the energy community, two load profiles are scaled. Thus, the fluctuations caused by the high peaks are neglected for all the players. If the peaks had been accounted for, the battery system might have been utilized to a greater extent, which in consequence may have lowered the total electricity cost for the energy community. Whether this would influence the deviation in cost allocation method is uncertain. As described in Chapter 1.2, all simulations are modeled on a conceptual level, using deterministic input. Thus, the results obtained from this thesis can be considered as ideal, and utilized as a benchmark e.g., if studying a more realistic, stochastic model. Despite simplifications in load data resolution and assumed perfect forecast, the deviation between the cost allocation provided by nucleolus and the Shapley value is not affected, because the data used as input for the solution concepts are equal. Hence, the author believes that the given data are suitable for the considered study in this thesis.

Investment Cost

As this thesis solely focuses on allocation of the annual cost of electricity, the investment and maintenance cost of the PV system and batteries are not considered. To capture the whole picture regarding such investments, these costs could have been implemented in the cooperative game, to represent the what the prosumers have to pay. If the investment costs are to be implemented, the prosumers should receive some economical compensation for buying these devices, as the PV system and the batteries are valuable for all players within the game.

The DSO

The players in the considered scenarios are solely single end-users, thus the local DSO is not included as a player. However, for enabling the transfer of power within the energy community, a physical grid is required. Thus, the issue of whom is in charge of the grid constitution and operation is relevant. For a real case-study, the DSO could have been included as a player in the cooperative game. However, as the objective was to examine cooperation among the players, these mentioned approaches were outside the scope of this thesis.

Chapter 6

Conclusion and Further Work

6.1 Conclusion

The research conducted in this thesis, regarded analyses of the two game theoretical solution concepts nucleolus and the Shapley value, for cost allocation among single end-users within a cooperating energy community. By changing the input parameters, namely load and electricity spot price, four scenarios were obtained. Furthermore, the thesis aimed to examine whether the value of the battery system within the energy community has an impact on the deviation between the allocation methods. Results from this thesis show that nucleolus and the Shapley value propose approximately similar cost allocations in the considered scenarios. Within each scenario, the methods have shown to provide cost allocations within the core of the cooperative game. Thus, both nucleolus and the Shapley value can be considered stable. The results show that the deviation between the methods increases, as the value of the battery system increases. The overall highest deviation obtained in this thesis is of 3 %, representing an actual cost difference of 136.8 NOK. In the relevant scenario, the value of the battery system is 8.84 %, which represents the highest battery system value obtained from the considered scenarios.

Furthermore, the author believes that the interpretation of game fairness is a significant issue. Results from this thesis reflect that the Shapley value is based on individual contribution to the grand coalition. Consequently, this method rewards the most active player, whereas the less active player is punished due to its limited contribution. Nevertheless, this limited contribution is caused by the player holding the most resources which also operates most frequently. The fact that the Shapley value punishes the player who contributes less, although this limited contribution is caused by the most active player, may based on the results in this thesis be considered as a weakness of the allocation method. From the work of this thesis, the author wants to illuminate the following findings:

- Based on the considered case-studies, the author believes that both nucleolus and the Shapley value are well-suited for cost allocation for the set-up of the considered energy community.
- The deviation between nucleolus and the Shapley value increases as the battery system value increases.
- The interpretation of fairness is a central issue. Results reflect that the Shapley value is based on individual contribution; thus the most active player is favored at the expense of the player with lesser resources. In energy communities where the players have conflicting interest regarding the chosen methods, a regulator can ensure that a stable cost allocation can be obtained by providing economic support to players outside the core of the game.

6.2 Further Work

Going forward, continued research is needed within the topic of cooperative game theory applied within energy communities. Particularly because, to the author's knowledge, there is a limited number of research activities on this topic. During the work of this thesis, multiple ideas for additional analyses were discovered.

Complex Energy Communities

The author believes it is of high interest to study larger energy communities including more players. Summarized, the following aspects are proposed:

- Implementation of additional players with a diverse range of flexibility, such as wind- and PV production, electrical vehicles (EV) and battery systems. In addition, there exists potential for analysis of possibilities for including the DSO as a player in the cooperative game.
- Sensitivity analysis of battery systems with the goal to examine whether the deviation between the methods continues to increase, or if there exists a saturation point. These input parameters may be more fluctuating load demand, production, and electricity spot prices. Additionally, other grid tariff structures can be implemented.

Stability of the Shapley Value

In the light of more complex energy communities, the robustness of the Shapley value regarding its stability, is of particular relevance. That is, analyzing which set-up of players and input parameters which provide a Shapley value outside the core of the game. Furthermore, to study which parameters that have greatest impact on the stability of the Shapley value. Another aspect would be to study what happens to the energy community if central resources are unavailable.

Investment cost, Technical- and Environmental Aspects

Include investment costs of PV and battery systems, to study whether this would affect the deviation between the methods. As both the PV and the battery systems will degrade over time, these factors could be included. Additionally, the reduction in grid imported energy may reflect a reduction in GHG emission. By obtaining the reduction in GHG emissions, this value could be implemented as a cost in the cooperative game. Hence, a more realistic model can be developed.

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Appendices

Appendix A

The Nucleolus and the Shapley Value for all Scenarios

Table A.1: Cost allocation methods. All values given in [NOK].

(a) Scenario 1: Cost allocations resulted from nucleolus and the Shapley value. All values given in [NOK].

	Scenario #1	
Player	**Shapley**	**nucleolus**
1	31 742.9	31 735.3
2	31 833.9	31 835.3
3	12 965.1	12 968.3
4	12 965.1	12 968.3

(b) Scenario 2: Cost allocation methods. All values given in [NOK].

	Scenario #2	
Player	**Shapley**	**nucleolus**
1	35 026.3	35 031.7
2	35 621.7	35 607.7
3	14 350.5	14 354.8
4	14 350.5	14 354.8

(c) Scenario 3: Cost allocation methods. All values given in [NOK].

	Scenario #3	
Player	**Shapley**	**nucleolus**
1	4 615.4	4 615.0
2	4 719.4	4 655.0
3	7 505.6	7 538.1
4	7 505.6	7 538.1

(d) Scenario 4: Cost allocation methods. All values given in [NOK].

	Scenario #4	
Player	**Shapley**	**nucleolus**
1	4 290.4	4 333.8
2	4 617.8	4 481.0
3	8 208.9	8 255.6
4	8 208.9	8 255.6

Appendix B

Scientific Paper

Based on the knowledge and results achieved in this master thesis, the author has submitted a scientific paper to the *"International Conference on Smart Energy Systems and Technologies - SEST 2018"*. The full paper is attached in the following pages.

Comparison of cost allocation strategies among prosumers and consumers in a cooperative game

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***Abstract*— As the higher penetration of distributed generation (DG) and electrical energy storage (EES) is emerging, end-users are taking a more active role in the power grid. Yet, independent operation of these devices is most common. With an increased amount of DG and EES available, opportunities for cooperation in the operation of power exchange arises. In cooperative game theory, for all players in a game cooperate under joint benefits. Preliminary studies show such cooperation among prosumers and consumers provides reduced annual electricity cost compared to independent operation. Focusing on cost allocation among end-users equipped with rooftop PV and batteries, the objective of this paper is to evaluate two solution concepts from game theory; the nucleolus and the Shapley value. By changing parameters that increase the value of the battery system in terms of reduced cost, we aim to examine whether the deviation between the methods increases as the battery system value is changed. The simulated case is based on data from private residences in Norway. Results from our case show that both nucleolus and Shapley provide stable cost allocations with a marginal deviation. However, results also imply that the deviation between the methods tends to increase when the value of the battery system increases.**

Keywords—Cooperative game theory, Shapley, Nucleolus, Flexibility, Batteries, Distributed energy systems

Nomenclature

$\eta_{bat,z}$	Charg./discharg. efficiency of battery z [%]
λ_{EES}	Relative reduced cost provided by the battery system [%]
$\theta_{min}(\boldsymbol{x})$	Lexicographic smallest excess vector for payoff vector $\boldsymbol{x}$ [NOK]
$\upsilon(N)$	Worth of coalition set N [NOK]
$C_{el}(t)$	Electricity cost in time step t [NOK]
$E_{bat,z}(t)$	Energy capacity of battery z in time step t [kWh]
$E^{max}_{bat,z}$	Max. energy capacity of battery z [kWh]
$P_{bat,z}(t)$	Charg./discharg. power of battery z in time step t [kW]
$P^{max}_{bat,z}$, $P^{min}_{bat,z}$	Max. and min. charge rate of battery z [kW]
$P_{grid}(t)$	Power supplied or delivered to the grid in time step t [kWh/h]
$P_{load,i}(t)$	Load demand for player i in time step t [kWh/h]
$P_{PV}(t)$	Total photovoltaic power production in time step t [kWh/h]
C, $C_{\varepsilon}(N,\upsilon)$	The core and the ε-core of a cooperative game
$e(S,\boldsymbol{x})$	Excess experienced by players in S from payoff vector $\boldsymbol{x}$ [NOK]
F_k	Union of previously binding coalitions in k
k	Number of iterations in *least cores*
N, n, i	Set of all players, total number of players, and their index
$SOC_z(t)$	State of charge of battery z in time step t [%]
SOC^{max}_z	Max. battery state of charge of battery z [%]
SOC^{min}_z	Min. battery state of charge of battery z [%]
S	Subset of N
T, t, Δt	Total number of discrete time intervals, their index and time step
Z, z	Total number of batteries, and their index
$\phi(\upsilon)$	Shapley value [NOK]
$\phi_i(\upsilon)$	Payoff assigned to player i by Shapley [NOK]
ε_k	Max. excess vector in k [NOK]

I. Introduction

By 2050, solar photovoltaic (PV) and wind power might account for 52 % of the world's total electricity generation [1]. To support the increase of renewables, electrical energy storage (EES) will play a crucial role. As the costs of rooftop PV and batteries become more competitive economical, these applications are becoming more attractive for private end-users in the distribution grid. With an increased amount of DG and EES available among the end-users in the distribution system, possibilities for cooperating operation arise. In cooperative game theory, joint benefits for all players in a game to cooperate is assumed. Results from preliminary studies show that cooperating in bidding at a power exchange provides reduced electricity cost compared to independent operation. However, as a prerequisite for rational players to cooperate, the profitability after cost allocation for all players must exist. This, in turn, depends on the cost allocation among them.

Historically, cooperative game theory has been a tool for investment analysis, mainly focusing on power generation and transmission facilities. In [2], the authors propose a generic framework for flexibility analysis in transmission expansion planning using the concept of Shapley value. In recent year, the interest of applying cooperative game theory in the distribution system is increased, as it can serve as a well-performing tool for optimizing electricity costs and available resources. In [3], the authors analyze the value of sharing storage among consumers in a cooperative manner, and conclude that all players in a community would benefit from such cooperation. Ref. [4] studies cooperation among energy communities using cooperative game theory, and proves that each grid increases their individual profit by cooperating with the other energy communities. Furthermore, [5] studies how cooperative game theory can be applied for cost minimization within an energy community. Here, the authors propose the Shapley value for cost allocation, and conclude that both prosumers and consumers will obtain reduced cost when participating in the cooperative game.

In contrast to previous studies, this paper evaluates and compares the two game theoretical methods nucleolus and the Shapley value for cost allocation among single end-users within a single energy community. In addition, this paper aims to examine whether there is a relation between the cost allocations provided by the two game theoretical methods, and the value of the battery system.

We analyze a case study consisting of four end-users equipped with rooftop PV and a battery system as a cooperative game. Among the end-users, there are two prosumers equipped with one rooftop PV system and one battery each, whereas the two remaining end-users are consumers. We consider both the prosumers and consumers as players in the cooperative game. By changing the parameters: 1) demand, and 2) electricity spot price, four different scenarios are obtained. The parameters are changed in order to examine the deviation between the nucleolus and Shapley value as the battery system value changes. The simulations are performed by applying a dynamic programming optimization algorithm which calculates the annual electricity cost for each scenario, based on deterministic input. As the objective of the paper is to evaluate the game theoretical methods on a conceptual level, perfect foresight is used for simplicity, but not applicable to real case studies.

This paper is structured as follows. Section II presents the dynamic programming algorithm. Section III presents the game theoretical concepts used for cost allocation. Section IV presents the scenarios and the input data used. Results are presented in section V, followed by a discussion in section VI. Conclusion and further work are presented in chapter VII.

II. Dynamic programming algorithm

In order to obtain the annual electricity cost for a number of cooperating end-users, a dynamic programming (DP) optimization algorithm is utilized. The objective function aims to minimize the annual cost of a certain number of end-users, by minimizing the cost of grid imported energy. In this way, dynamic programming optimizes operation of two batteries in parallel over a year, thus minimizing the annual cost for the end-users. By calculating the cost of every possible charge and discharge decision in every time step within the optimization time horizon, a set of nodes is derived which results in lowest possible costs. In this case study, the time horizon is one year, thus T = 8760 hours. The objective function is shown in (1). Eq. (2a) shows the energy balance, which includes all players in the coalition and their respective PV and batteries. Eq. (2b) and (2c) show the maximum and minimum battery state of charge, whereas (2d) and (2e) reflect the maximum charge and discharge power[1]. Eq. (2f) shows the stored energy in a given time step. Finally, (2g) shows the battery state of charge equation.

[1]The rated power is considered to be the continuous rated power.

$$\min f(P_{bat}) = \sum_{t \in T} C_{el}(t) P_{grid}(t) \tag{1}$$

s.t.

$$P_{grid}(t) = \sum_{i \in N} P_{load,i}(t) + \sum_{z \in Z} P_{bat,z}(t) - P_{PV}(t) \tag{2a}$$

$$SOC_z(t+1) \leq SOC_z^{max} \quad z \in Z \tag{2b}$$

$$SOC_z(t+1) \geq SOC_z^{min} \quad z \in Z \tag{2c}$$

$$P_{bat,z}(t) \leq P_{bat,z}^{max} \quad z \in Z \tag{2d}$$

$$P_{bat,z}(t) \geq -P_{bat,z}^{max} \quad z \in Z \tag{2e}$$

$$E_{bat,z}(t+1) \geq E_{bat,z}(t) + \eta_{bat,z} P_{bat,z}(t) \Delta t \quad z \in Z \tag{2f}$$

$$SOC_z(t+1) = \frac{E_{bat,z(t+1)}}{E_{bat,z}^{max}} \quad z \in Z \tag{2g}$$

Note that

$$\eta_{bat,z} = \eta_{ch,z}, \quad P_{bat,z}(t) \geq 0 \quad z \in Z$$
$$\eta_{bat,z} = \eta_{dis,z}, \quad P_{bat,z}(t) < 0 \quad z \in Z$$

III. Game theoretical modelling

Cooperative game theory constitutes a mathematical framework for evaluating cooperation among a group of players. A cooperative game with *transferable utility* (TU) is represented as a pair (N,υ) [6]. N = $\{1,\ldots,$ n$\}$ is the finite set of n players that leads to 2^n possible coalitions. Further, υ denotes the characteristic function, representing the value of a coalition. For every coalition S, $S \subset N$, there exists a value $\upsilon(S)$, representing the worth of the coalition S. The value of the empty set, $\upsilon(\emptyset)$ = 0. The coalition consisting of all players is termed the *grand coalition*. Due to the concept of *superadditivity*[2], the $\upsilon(N)$ provides the highest payoff.

In this paper, each coalition formed by the players will lead to an outcome in form annual electricity cost. The outcome of each coalition depends on the interaction among the players. Thus, each simulated scenario satisfies the definition of a cooperative game. Secondly, the players are assumed rational and to act in their self-interest. Due to the concept of superadditivity, the grand coalition provides the lowest electricity cost for the players. Despite this, the players will only join the grand coalition if the proposed allocation provides the players the highest payoff.

To satisfy the equilibrium state, it must be ensured that none of the players want to leave the grand coalition N in order to join another sub coalition S. Due to rationality, the players seek to form the coalition where they expect to obtain the highest payoff. Let $\boldsymbol{x}$ = $\{x_1,x_2,...,x_n\}$ be a proposed cost allocation of the total payoff $\upsilon(N)$. If $\boldsymbol{x}$ fulfills the requirements of both individual and group rationality[3], it is denoted and *imputation*. Furthermore, an imputation $\boldsymbol{x}$ is stable if no alternative coalition will provide a higher payoff for any of its players. Hence, a stable imputation is said to be in the *core* of the game.

[2]*Superadditivity*: The value of a union of two disjoint coalitions is equal to, or greater than the sum of the coalitions' separate values.

[3]*Individual rationality*: A player will only join a coalition if this leads to at least the utility obtained by operating individually.
Group rationality: The total utility from a coalition should be divided among all the players within the coalition.

In order to fairly allocate the total payoff $\upsilon(N)$ for the players, the concepts of nucleolus and the Shapley value are introduced. These value concepts propose a unique allocation $\boldsymbol{x}$ based on some fairness principles. Nucleolus and Shapley differ in their interpretation of fairness, thus they do not necessary provide equal cost allocations. Before presenting these methods, we introduce the concept of the core.

A. The core

The core C of a TU game (N,υ) is the set consisting of all stable imputations, mathematically expressed through (3) [7].

$$C = \Big\{x_i \mid x_i \in \{x_1, ..., x_n\}, \sum_{i \in N} x_i = \upsilon(N), \sum_{i \in S} x_i \geq \upsilon(S),\ \forall S \subset N\Big\} \tag{3}$$

To ensure that all end-users in a cooperative game want to form the grand coalition, the proposed value allocation needs to be in the core of the game.

B. The Shapley value

Lloyd Shapley proposed a solution concept whose interpretation of fairness is in terms of each player's individual contribution to a coalition [8]. Shapley provides a simple method for cost allocation for all the players in the game based on four axioms. These axioms are as follows:

1) *Efficiency*: All utility obtained by any player should be allocated. The total value of the players is the value of the grand coalition, hence $\upsilon(N) = \sum_{i \in N} \upsilon(i)$.
2) *Symmetry*: Two players i and j that contribute the same to each coalition are substitutes, hence they should be treated equally. Player i and j are symmetric if $\upsilon(S \cup i) = \upsilon(S \cup j)$.
3) *Null player*: A player i that contributes nothing, should receive nothing. Such a player is referred to as a null or a zero player. A player is a null if $\upsilon(S) = \upsilon(S \cup i)$.
4) *Additivity*: The sum of two independent TU games, u and v must be the sum of the value of each game, hence $\phi(u + v) = \phi(u) + \phi(v)$.

For each player, there exists a unique value satisfying these axioms. This unique value is the Shapley value, denoted $\phi(\upsilon)$. For a TU game (N,υ) the Shapley value for each player i is calculated by the following:

$$\phi_i(\upsilon) = \sum_{i \in S} \frac{(|S| - 1)!(n - |S|)!}{n!}[\upsilon(S) - S\backslash\{i\})]. \tag{4}$$

Once the Shapley value for each player is calculated, the value allocation $\boldsymbol{\phi} = (\phi_i, ..., \phi_n)$ can be obtained. However, the method does not ensure that the allocation is stable. Once the Shapley values for the game are calculated, additional examination of the core is required to verify stability. Thus, $\boldsymbol{\phi}$ must fulfill the following condition:

$$\boldsymbol{\phi} \in C \tag{5}$$

C. The nucleolus

The concept of the nucleolus was first introduced in [9]. While Shapley focuses on fairness in terms of individual contribution, the nucleolus is based on minimizing the players' dissatisfaction with their payoff. The idea behind the nucleolus is to minimize the maximum dissatisfaction the players in coalition S experience from a proposed imputation $\boldsymbol{x}$. Dissatisfaction is measured through an *excess function e(S,$\boldsymbol{x}$)*, expressed in (6).

$$e(S, \boldsymbol{x}) = \upsilon(S) - \sum_{i \in S} x_i = \upsilon(S) - \boldsymbol{x}(S) \tag{6}$$

A negative *e(S,$\boldsymbol{x}$)* represents the additional payoff coalition S obtains from $\boldsymbol{x}$. Thus, an imputation $\boldsymbol{x}$ is in the core if and only if all excesses are negative or equal to zero.

$$C(N, \upsilon) = \{\boldsymbol{x} \in X \mid e(S, \boldsymbol{x}) \leq 0 \quad \forall S \subset N\} \tag{7}$$

A payoff vector $\boldsymbol{x}$ provides an excess vector $\theta(\boldsymbol{x}) = \{e(\boldsymbol{S}_1,\boldsymbol{x}),..., e(S_{2^n-2},\boldsymbol{x}\}$ for $S \subset N \backslash S = \emptyset$. Different allocations provide different excess vectors. The excess vectors are ordered *lexicographically*[4], thus there exists an allocation which corresponds to the lexicographic smallest excess vector $\theta_{min}(\boldsymbol{x})$. This unique payoff vector is the nucleolus. If the game has a non-empty core, a nucleolus for the game exists. In this paper, the nucleolus is obtained by finding $\theta_{min}(\boldsymbol{x})$ from *least cores*, a method based on [10], [11]. The least core concept leads to the introduction of the ε-core $C_\varepsilon(N,\upsilon)$, expressed through (8). By letting $\varepsilon < 0$, the ε-core becomes more restrictive than the core represented in (3).

$$C_\varepsilon(N, \upsilon) = \boldsymbol{x} \in X \mid e(S, \boldsymbol{x}) \leq \varepsilon \quad \forall S \subset N\} \tag{8}$$

By iteratively solving a linear programming (LP), the least core is obtained. The LP problem representing the least core is expressed in (9)-(9c). For each iteration k, constraints based on previous maximum excesses are added, thus reducing the feasible region. The feasible region is reduced until the nucleolus is obtained. Eq. (9a) ensures that ε_k does not exceed the maximum excess for imputation $\boldsymbol{x}$. Group rationality is fulfilled by (9b), while (9c) ensures that the previous minimized maximum excess is still maintained. F_j represents the set including all coalitions which the excess constraint (9a) was binding at previous stages. Thus, F_k is the union of previously binding coalitions and $F_1 = \emptyset$.

$$\min \varepsilon_k \tag{9}$$

$$\text{s.t.} \quad \upsilon(S) - \sum_{i \in S} x_i \leq \varepsilon_k \quad \forall S \subset N \text{ and } S \notin F_k \tag{9a}$$

$$\sum_{i \in N} x_i = \upsilon(N) \tag{9b}$$

$$\upsilon(S) - \sum_{i \in S} x_i = \varepsilon_j, \quad \forall S \subset F_j, j \in \{1, ..., k-1\} \tag{9c}$$

$$\varepsilon_k \in R,\ x_i \in R \quad \forall i \in N$$

[4]Lexicographic ordering means that the excesses are ordered in the same way as words are ordered in the dictionary.

IV. THE CASE STUDY

A. The Scenarios

Four scenarios are analyzed in total. Within each scenario, there are four players. Player 1 and 2 are prosumers with one rooftop PV system and one battery each, whereas player 3 and 4 are consumers. Using two different sets of load profiles and two different sets of electricity spot prices, nucleolus and Shapley method are applied to all scenarios in order to study their performance under different conditions. The load profiles are denoted Load P1 and Load P2. The two sets of spot prices are taken from Norway and Germany, denoted NO3 and GER respectively. The simulated scenarios are presented in tab. I.

TABLE I: The four simulated scenarios with different load profiles (Load P1/P2) and electricity spot prices (NO3/GER).

	Scenario			
Parameter	**#1**	**#2**	**#3**	**#4**
Load	Load P1	Load P1	Load P2	Load P2
Electricity spot price	NO3	GER	NO3	GER

B. Load Demand and PV production

Load data are based on Norwegian end-users located in Trondheim, Norway. Key data for the total load in each load scenario can be found in tab. II. The PV production for player 1 and 2 are calculated by using a PV production model based on [12]. Irradiation and temperature data are taken from the Norwegian weather service [13]. Fig. 1 shows the daily load in each load scenario along with the total PV production provided by player 1 and 2, during the year. As shown in 1, the total PV production does only exceed demand of load profile 2.

TABLE II: Key data for the two load profiles.

Value	Load profile 1	Load profile 2
Annual consumption	114 270 kWh	28 868 kWh
Average consumption	13.04 kWh/h	3.30 kWh
Maximum consumption	35.40 kWh/h	10.00 kWh/h

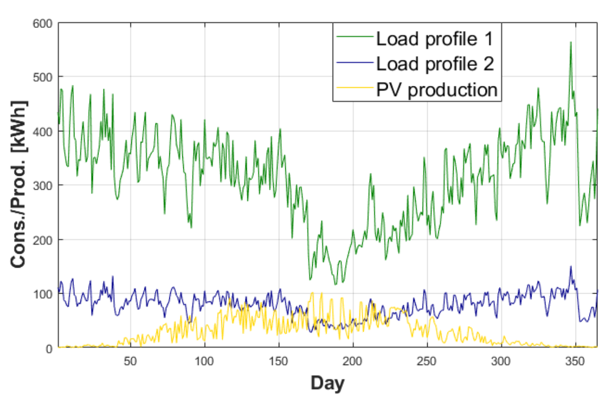

Fig. 1: Daily consumption for the two load profiles along with the daily PV production in 2015.

C. Battery Specifications

Two different batteries are used for the simulations. Battery specifications are presented in tab. III. Battery 1 is based on data from a Tesla Powerwall [14], while battery 2 is based on data from a LG house battery [15].

As we want to obtain the value of the battery system, the term λ_{EES} is introduced. The λ_{EES} represents the relative reduced cost for the grand coalition, provided by the battery system consisting of both batteries. Thus, there exists a λ_{EES} for each scenario.

TABLE III: Battery specifications.

	$P_{bat,z}^{max}$	$E_{bat,z}^{max}$	SOC_z^{max}	SOC_z^{min}	$\eta_{bat,z}$
Battery 1	7 kW	13.5 kWh	100 %	0 %	0.95
Battery 2	7 kW	9.8 kWh	100 %	0 %	0.90

D. Electricity Spot Prices

The prices used for the case studies are taken from Nordpool [16] and EPEX spot markets [17] from 2015, both with hourly resolution. The prices are shown in fig. 2. The spot price in the respective EPEX area[5] had an average of 0.1898 NOK/kWh with very low fluctuations, whereas the German spot price had an average price of 0.2831 NOK/kWh and much higher fluctuations. In addition to the electricity spot price, an obligatory green certificate cost of 0.0369 NOK/kWh and a retailers revenue margin of 0.025 NOK/kWh is added to every purchased kWh.

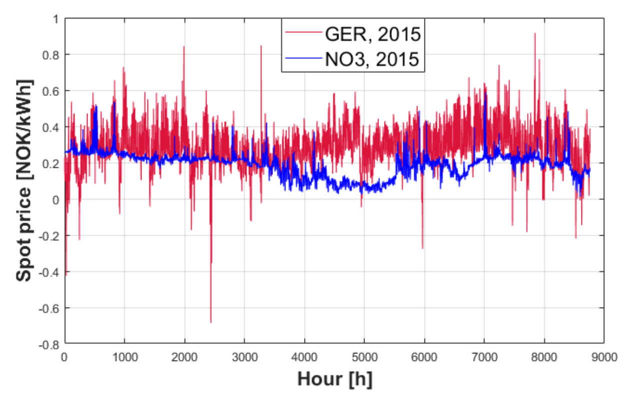

Fig. 2: Electricity spot price from Norway (NO3) and Germany (GER) in 2015.

E. Grid tariffs

In order to capture the annual cost that the end-user actually pays, the grid tariff from the relevant DSO is used. The current grid utility tariff structure in Norway is energy based, and has a fixed monthly cost plus an extra fee per kWh. All cost elements in addition to the spot price is shown in tab. IV.

[5]Region NO3 in the Nord Pool spot market.

TABLE IV: Grid, energy, tax and VAT costs for end-users. Note that all costs shown in the table are not including VAT.

Cost element	Cost
Fixed monthly cost [NOK/Month]	37.6
Grid energy cost [NOK/kWh]	0.22
Energy tax [NOK/kWh]	0.124
Green certificate fee [NOK/kWh]	0.0369
Retailer margin [NOK/kWh]	0.025
VAT [%]	25

V. Results

The initial question is if the core is non-empty. For each scenario, the nucleolus can be obtained, hence there exists a non-empty core. Furthermore, the Shapley value is shown to be in the core for each scenario. Thus both methods provide stable cost allocations.

Tab. V shows the relative deviation in cost allocations provided by nucleolus and the Shapley value. As the table content shows, the deviation between the methods is modest in all scenarios. Further, it can be seen that player 3 and 4 experience the same deviation. Due to equal demand, they prefer the same cost allocation method within each scenario.

TABLE V: Relative deviation in [%] between nucleolus and Shapley for each player in each scenario.

	Scenario			
Player	**#1**	**#2**	**#3**	**#4**
1	0.02	0.02	0.01	1.01
2	0.00	0.04	1.37	3.01
3	0.02	0.03	0.43	0.57
4	0.02	0.03	0.43	0.57

Tab. VI shows the preferred method for each player. Cells marked with '≈' indicate that the deviation is less than 0.1 %. Deviation less than this quantity can be considered negligible, thus both methods propose almost identical cost allocations.

TABLE VI: Preferred cost allocation method for each player in each scenario.

	Scenario			
Player	**#1**	**#2**	**#3**	**#4**
1	≈	≈	≈	Shapley
2	≈	≈	Nucleolus	Nucleolus
3	≈	≈	Shapley	Shapley
4	≈	≈	Shapley	Shapley

VI. Discussion

Fig. 3 shows the deviation between the nucleolus and Shapley, for each player plotted along with the cost reduction provided by the battery system, λ_{EES}. As fig. 3 shows, there is an increase in the value of the battery system for each scenario. By changing the parameters load and electricity spot price, the batteries' contribution to cost reduction varies for each scenario. In scenario 1, the total load demand is high (Load profile 1) and the fluctuations in electricity spot prices are modest. The batteries are only able to lower the cost with 0.39 %. The value of the battery system increases in scenario 2 and 3, whereas the highest cost reduction provided by the batteries is obtained in scenario 4. In this scenario, there are periods where PV production exceeds the demand (Load profile 2), as illustrated in fig. 1. In addition, the electricity spot price in scenario 4 is the most fluctuating (GER), which implies that the batteries are utilized for both storing power produced by the PV system, and for buying power when prices are low in order to store the power for high peak-periods. The high utilization of the batteries is reflected through a cost reduction of of 8.84 %. With a varying λ_{EES}, we aim to examine whether the deviation between nucleolus and Shapley is affected.

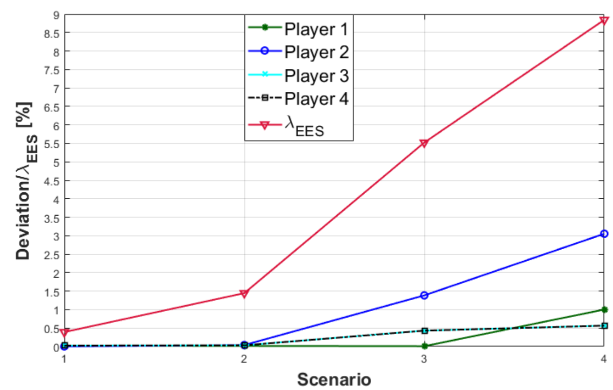

Fig. 3: Relative deviation between nucleolus and Shapley for each player along with the value of batteries, in each scenario.

As fig. 3 shows, player 3 and 4 experience exactly the same deviation within each scenario. Although the overall tendency for these players is a slight increase in deviation from 0.02 % in scenario 1 to 0.57 % in scenario 4, nucleolus and Shapley provide approximately similar cost allocations irrespective of the value of the battery system.

In scenario 1, player 1 experiences a deviation of 0.02 %, similar to player 3 and 4. However, in scenario 4, the deviation between the methods increases to 1.01 %. In the same scenario, player 2 experiences a deviation of 3.01 %. Even though these values can be considered marginal, there is a weak tendency that the prosumers experience a higher deviation in allocation method as the value of the battery system increases. Thus, the results might imply that the prosumers are more concerned regarding their preferred method when the value of the battery system is high. A high λ_{EES} can be interpreted as a higher contribution from player 1 and 2 to the energy community.

Despite this, player 1 and 2 do not necessary prefer the same allocation method. The overall highest deviation in method is found in scenario 4. For player 2, the cost proposed by nucleolus is over 3 % lower than the Shapley value. In contrast, player 1 prefers the Shapley value within the same scenario. Player 1 is equipped with the most efficient battery with largest energy capacity, as shown in tab. III. Thus, battery 1 is utilized more than battery 2. In other words, player 1 contributes more to the overall cost reduction. This difference in individual contribution is reflected through the preferred methods. While Shapley is preferred by player 1, player 2 prefers nucleolus within the same scenario.

In this paper, nucleolus and the Shapley value propose approximately similar cost allocations. Despite this, there is a tendency that the deviation in cost allocation increases as the value of the battery system increases. The value of the batteries is dependent on parameters such as renewable power production, price fluctuations and load demand. Thus, it can be interpreted as if the deviation in cost allocation method depends on these parameters. For an energy community where the players' available resources lead to marginal cost reduction, nucleolus and the Shapley propose almost similar cost allocations. Hence, for the case study presented in this paper, the players will not be concerned regarding their preferred method. In contrast, the deviation between the methods increases in scenarios where the available resources play a greater role in cost reduction within the energy community. Although the deviation between the methods is shown to be small in the presented scenarios, it might increase in larger energy communities consisting of more diversity among the players.

In an energy community consisting of solely consumers without neither PV production nor battery systems, there is no incentive for cooperation, as there are no resources to operate in a cooperative manner. Thus, for an energy community to operate cooperative, it is essential to facilitate the prosumers to join the cooperating operation. Although the consumers prefer the Shapley method in all presented scenarios, they never experience a deviation higher than 0.57 %. In contrast, player 2 experiences a deviation of 3 % in the same scenario. We believe that both nucleolus and the Shapley value are well-suited for cost allocation for the set-up of our case. Further we believe that it is of high importance to evaluate what is the aim of the cooperation. In scenarios where there are large deviations between the proposed methods, we believe that it is of high interest to study which players that are attractive for the cooperative operation to be beneficial. Both methods show similar fitness to solve the problem setting, whereas several aspects worth considering prior to implementation. The study shows that both methods provide solid cost allocations for local energy communities.

VII. Conclusion

In this paper we have evaluated nucleolus and the Shapley value for cost allocation among a set of cooperating end-users within an energy community. As the presented results show, both methods provide stable cost allocations. The deviation between the methods is small, and can be even considered negligible in some of the simulated scenarios. However, there is a tendency of a slight increase in deviation in scenarios where the battery system is able to contribute with a certain cost reduction. Thus, we interpret this as if the deviation in cost allocation method is affected by externalities that are able to increase the value of the battery system, such as renewable power production and fluctuation in electricity spot price.

Based on the presented results, both nucleolus and Shapley value serve the intended purpose. However, we believe that it is of importance to study how to encourage valuable players to join the cooperation. The results also imply that in larger systems or systems with price volatility or high use of flexibility, the methods deviate slightly and should be compared.

We believe that the definition of fairness in the context of game theoretical method in the considered case, has to be extended to consider other externalities such as renewable generation from other sources, different grid tariff structures and the size of the energy community. For future work, it is of interest to study how Shapley and nucleolus perform in larger energy communities with higher deviation between the players' individual resources. Another interesting aspect would be to include the grid operator as a player in the cooperative game.

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