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Time-domain Roll Motion Analysis of a Barge for Transportation of an Offshore Jacket Structure

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Marine Technology

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Background:

Large offshore foundations such as jackets are normally transported in dry condition by a transportation vessel (for example a barge) from the fabrication yard to the offshore site. The transportation route might be quite long, so that fatigue damage in jacket foundations and at joints to the barge deck during the sea voyage could be significant and has to be considered in design. The inertia loads on jacket due to wave-induced motions of the transportation vessel is the main cause of the fatigue loads. In particular roll motions of the vessel in oblique waves could be large due to resonance and small roll damping. Ship-shape floating structures typically have a small natural period in roll, in the order of 12-18s, and in some wave conditions, roll resonant motions might be inevitably excited. Large roll motions will induce large translational acceleration and inertia loads at the sea fastening point of the jacket. Wave radiation damping in roll for a barge is small and therefore viscous damping plays an important role in motion assessment.

In the industry, fatigue analysis is typically performed using frequency-domain hydrodynamic and motion analysis of transportation barges since it is efficient when a large number of sea states have to be considered for fatigue calculation. However, quadratic roll damping due to viscous effect has a non-linear dependency relative to roll motion response. In such frequency-domain methods the quadratic viscous damping has to be linearized with respect to the motion amplitude using an iterative procedure. In particular for irregular waves, a stochastic linearization should be considered. However, such linearization should be checked with time-domain roll motion simulations so that they are reasonable when considering the fatigue loads in jacket structures during transportation.

The purpose of this thesis is to establish a time-domain model in SIMA for roll motion analysis of the given transportation barge, to perform a sensitivity study of the roll damping (including quadratic and linearized viscous damping) on the effect of roll motions, and to compare the roll motion results with the frequency domain methods.

This thesis topic is proposed by DNV-GL. The barge concept and the basic hydrodynamic input model will be provided. In a parallel thesis work, the frequency-domain methods will be studied and the results will be compared with the time-domain results from this thesis.

Assignment:

The following tasks should be addressed in the thesis work:

1. Carry out a literature review on sea transportation, wave-induced ship roll motion analysis and in particular on roll damping modelling, time-domain hydrodynamic loads and motion analysis.
2. Study how to use the software SIMA. Based on the hydrodynamic data from the HydroD analysis, establish a time-domain model of the transportation barge. Design a reasonable equivalent stiffness and damping for surge, sway and yaw motions to avoid free rigid-body motions and to result into natural periods in the order of 40-80s.
3. Perform a decay test in all 6 DOFs of motions to check the time-domain model. Compare the natural periods for heave, roll and pitch with the frequency-domain results.

4. Consider the wave direction of 45 degrees and 90 degrees, regular wave cases with wave height of 1m and 6m using quadratic roll damping and linearized roll damping, perform time-domain simulations and obtain the roll motion RAO (roll motion amplitude divided by wave elevation amplitude). Compare these RAO results with the frequency-domain methods with stochastic linearization.

5. Consider irregular wave cases ($H_s=1.5, 2.5, 3.5, 4.5, 5.5, 6.5$ m and corresponding $T_p=7.5, 8.5, 9.5, 10.5, 11.5, 12.5$ s), obtain roll motion spectra and compare them with the frequency-domain results.

6. Conclude the work and give recommendations for future work.

7. Write the MSc thesis report.

In the thesis the candidate shall present his personal contribution to the resolution of problem within the scope of the thesis work.

Theories and conclusions should be based on mathematical derivations and/or logic reasoning identifying the various steps in the deduction.

The candidate should utilize the existing possibilities for obtaining relevant literature.

The thesis should be organized in a rational manner to give a clear exposition of results, assessments, and conclusions. The text should be brief and to the point, with a clear language. Telegraphic language should be avoided.

The thesis shall contain the following elements: A text defining the scope, preface, list of contents, summary, main body of thesis, conclusions with recommendations for further work, list of symbols and acronyms, reference and (optional) appendices. All figures, tables and equations shall be numerated.

The supervisor may require that the candidate, in an early stage of the work, present a written plan for the completion of the work. The plan should include a budget for the use of computer and laboratory resources that will be charged to the department. Overruns shall be reported to the supervisor.

The original contribution of the candidate and material taken from other sources shall be clearly defined. Work from other sources shall be properly referenced using an acknowledged referencing system.

The thesis shall be submitted electronically (pdf) in DAIM:

- Signed by the candidate
- The text defining the scope (this text) (signed by the supervisor) included
- Computer code, input files, videos and other electronic appendages can be uploaded in a zip-file in DAIM. Any electronic appendages shall be listed in the main thesis.

The candidate will receive a printed copy of the thesis.

Supervisor: NTNU: Zhen Gao
Co-supervisor: DNV-GL: Limin Yang

Deadline for thesis report: 11.06.2018

Preface

This thesis is written by Ingrid Mehn-Andersen and is the final part of the Master in Science in Marine Technology degree specializing in marine structures. The work has been carried out during the spring of 2018 at the Department of Marine Technology (IMT) at the Norwegian University of Science and Technology (NTNU) in Trondheim. In this course, the following report accounts for 100 % of the final grade, and the workload corresponds to 30 ECTS.

The project was proposed and initiated by DNV GL and Limin Yang. I would like to thank him for his work as co-supervisor. His advice has been helpful. I would also sincerely like to thank my supervisor in this project, Professor Zhen Gao for good guidance throughout the work. He gladly shared his knowledge and answered all questions throughout the spring.

During the autumn of 2017, I worked on a project thesis that was a pre-study to this master thesis. The primary focus of this project thesis was a literature study on time domain transportation analysis of roll motions. Some of this work is included in the theory in this thesis.

This report is written for readers with a basic knowledge of marine technology on the field of hydrodynamics and marine structures.

Trondheim, 11.06.2018



Ingrid Mehn-Andersen

Executive Summary

This paper presents a literature study on roll motions for barges, and time domain and frequency domain simulations. In addition, results from time and frequency domains have been compared in regular and irregular waves to verify the stochastic linearization process performed to linearize roll motion.

Most vessel responses can be calculated with acceptable accuracy in the frequency domain, whereas it is more difficult for roll response. This is due to roll motion being highly non-linear compared to other degrees of freedom. The non-linearity arises from flow separation. Estimating roll damping is therefore associated with difficulties and uncertainty. Consequently, empirical formulas have been developed for classical ship shapes (Ikeda et al. (1977)). In addition, roll natural period generally falls within the frequency range of a typical wave energy spectrum. This causes dynamic amplification of roll motion. Hence it is of utmost importance that good estimation of roll damping is made for such structures.

The barge has zero forward speed in the time domain simulations. To achieve realistic results for a barge with forward speed, stiffness was added in surge, sway and yaw. A proper damping was also needed to avoid resonant motions in these DOFs. Due to coupled motions, the level of damping in sway had great impact on the roll motion. In order to achieve acceptable horizontal offset and steady state roll motion, a damping ratio of 1 % was found to be optimal.

The natural periods obtained in SIMA and HydroD are presented in Table 1. The stiffness in surge, sway and yaw were zero in HydroD, the system is therefore free to oscillate without restrictions in these DOFs, and no natural frequencies are found. The difference in the heave, roll and pitch may be due to coupled motions, where the damping in other degrees of freedom affects the damping in roll. Another reason can be differences in calculation processes between SIMA and HydroD. Two concrete examples are that SIMA uses the damped natural frequency instead of the natural frequency, and the added mass for infinity frequency instead of added mass corresponding to the natural frequency.

Table 1: Comparison of Natural Periods between HydroD and SIMA.

	Obtained T_d SIMA	Obtained T_0 HydroD	Difference
Surge	53.50	-	-
Sway	54.375	-	-
Heave	11.7	12.1	3.3 %
Roll	14.8	15.6	3.3 %
Pitch	10.7	11.5	6.9 %
Yaw	54.33	-	-

For regular waves, an iteration process in HydroD provided the linearized damping for regular waves with 45 degrees wave heading. In the comparison of the linearized and quadratic damping RAO's in SIMA and HydroD, similarities were observed for most frequencies. However, around the natural frequencies, differences were observed, as presen-

ted in Figure 1. All in all there is good consistency between the linearized and quadratic damping. This may be due to coupled effects or conservative results in the frequency domain analyzes.

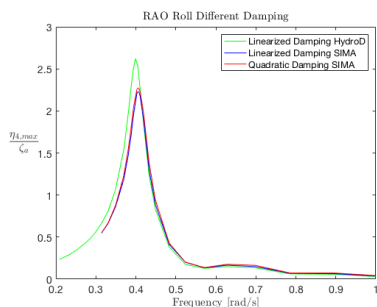


Figure 1: Comparison of Linearized and Quadratic Damping in HydroD and SIMA.

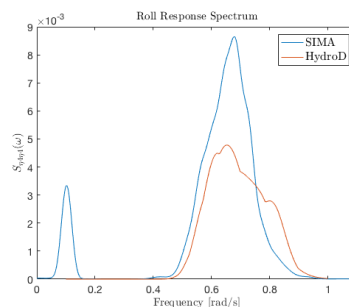


Figure 2: Response Spectra for Condition Number 1 with 90 degrees wave heading.

In Figure 2, the response spectra for roll motion in SIMA are compared with HydroD. One can see that the wave spectra from SIMA contain a peak at 0.1 [rad/s]. This coincides with the natural period in sway and is most likely due to the coupled motions. The form of the spectrum from HydroD is different from SIMA, the total energy from the methods was therefore compared using the standard deviation. In roll motion, the amount of energy appeared to be similar, however the energy contributes at different frequencies.

In SIMA different wave seeds were used, with the same environmental conditions. They turned out looking completely different. Because of large differences between analyses with equal conditions, it is advised to test more wave seeds. This is the only way one can be certain that a wide range of scenarios are tested, and the correct behaviour of the barge is recorded.

Sammendrag

Denne oppgaven inneholder en litteraturstudie på rullebevegelser for lektere, sammen med teori om simuleringer i tids- og frekvensdomene. I tillegg er simuleringer mellom tidsdomene og frekvensdomene blitt sammenlignet i vanlige og uregelmessige bølger for å verifisere den stokastiske lineariseringsprosessen som er presentert for å linearisere rullbevegelsen.

De fleste bevegelser i ett skip kan beregnes med akseptabel nøyaktighet i frekvensdomene, dette er vanskeligere for rullerespons. Rullebevegelsen er nemlig svært ikke-lineær sammenlignet med andre grader av frihet. Denne ikke-lineariteten oppstår fra strømningsseparasjon. Å beregne rulldemping er derfor forbundet med vanskeligheter og usikkerhet. Empiriske formler har derfor blitt utviklet for klassiske skipformer (Ikeda et al. (1977)). Den naturlige perioden i rull faller vanligvis innenfor frekvensområdet for et typisk bølgeenergispektrum. Dette medfører dynamisk forsterkning av rullebevegelse. Det er derfor viktig å estimere rulledemping med god nøyaktighet.

Lekteren ligger i ro i tidsdomensimuleringene. For å oppnå realistiske resultater for en lekter med fremdriftshastighet, ble stivhet tilsatt i jag, svai og gir. Det var da også nødvendig å legge til demping for å unngå resonansbevegelser i disse frihetsgradene. På grunn av koblede bevegelser hadde nivået av demping i svai stor påvirkning på rullens bevegelse. For å oppnå akseptabel horisontal offset- og stabil rullebevegelse, ble et dempningsforhold på 1 % funnet å være optimalt.

De naturlige periodene funnet i SIMA og HydroD er presentert i Tabell 2. Stivheten i jag, svai og gir var null i HydroD, systemet er derfor fritt til å svinge uten restriksjoner. Den naturlige frekvenser er derfor ikke funnet for disse frihetsgradene. Forskjellen i hiv, rull og stamp kan skyldes koblede bevegelser, hvor demping i andre frihetsgrader påvirker dempingen i rull. En annen grunn kan være forskjeller i beregningsprosessene i SIMA og HydroD. To konkrete eksempler er at SIMA bruker den dempede naturlige frekvensen i stedet for den naturlige frekvensen, og tillegsmassen for uendelig frekvens i stedet for tillegsmassen for den naturlige frekvensen.

Table 2: Sammenligning av Egenperioder mellom SIMA og HydroD.

	T_d funnet i SIMA	T_0 funnet i HydroD	Forskjeller
Surge	53.50	-	-
Sway	54.375	-	-
Heave	11.7	12.1	3.3 %
Roll	14.8	15.6	3.3 %
Pitch	10.7	11.5	6.9 %
Yaw	54.33	-	-

For vanlige bølger ble en iterasjonsprosess gjennomført i HydroD for å finne den lineærisede dempingen for vanlige bølger med 45 grader. RAO ble laget for å sammenligne den lineære og kvadratiske dempingen i SIMA og HydroD. De var forholdsvis like for de

fleste frekvenser. Det var imidlertid forskjeller rundt den naturlige frekvensen i rull, som vist i Figur 3. Alt i alt stemmer den lineære og kvadratiske dempingen godt overrens. De forskjellene som er, kan skyldes koblede effekter eller konservative resultater i frekvensdomene analysene.

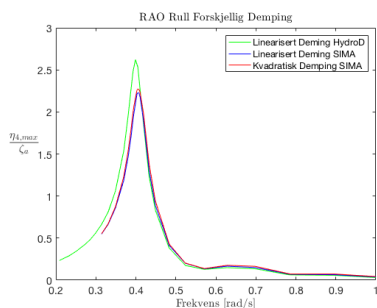


Figure 3: Bevegelseskarakteristikk i Rull med Sammenligning av Linearisert og Kvadratisk Demping i SIMA og HydroD.

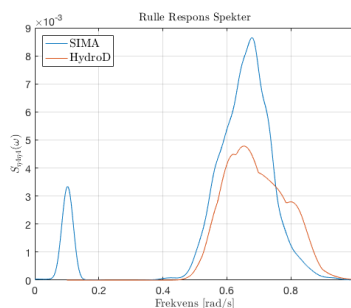


Figure 4: Rulle Respons Spekter ved Kondisjon 1 og Bølger med Angrepsvinkel på 90 Grader.

I Figur 4 sammenlignes responsspektrene for rullebevegelse i SIMA med HydroD. Man kan se at bølgespektrene fra SIMA inneholder en topp på 0,1 [rad/s]. Dette sammenfaller med den naturlige perioden i svai og er mest sannsynlig på grunn av de koblede effektene. Spekteret fra HydroD er forskjellig fra SIMA, den totale energien fra metodene ble derfor sammenlignet ved bruk av standardavviket. Den totale mengden energi i rullebevegelsen i de to metodene er forholdsvis like, energien bidrar derimot ved forskjellige frekvenser.

I SIMA ble det brukt forskjellige bølge "seed" ved samme sjøtilstand. De viste seg å gi helt forskjellige resultater. På grunn av store forskjeller mellom de forskjellige analysene med like forhold, anbefales det å teste flere bølge seeds. Dette er den eneste måten man kan være sikker på at et bredt spekter av scenarier blir testet, og riktig oppførsel av lekteren registrert.

Acronyms

Abbreviation	Explanation
CoG	Center of gravity
DOF	Degree of freedom
FFT	Fast Fourier Transform
GM	Metacentric height
IFFT	Inverse fast Fourier transform
JONSWAP	Joint North Sea Wave Project
PM	Pierson-Moskowitz
RAO	Response amplitude operator
SIMA	Simulation Workbench for Marine Applications
SIMO	Simulation of Marine operations
StD	Standard deviation
VAR	Variance
VCG	Vertical center of gravity
VIV	Vortex induced vibration
WAFO	Wave Analysis for Fatigue and Oceanography
WADAM	Wave Analysis by Diffraction and Morison Theory

List of Symbols

$\ddot{\eta}_j$	Body acceleration
$\dot{\eta}_j$	Body velocity
ϵ	Random phase angle
η_1	Surge motion
η_2	Sway motion
η_3	Heave motion
η_{4a}	Roll motion amplitude
η_4	Roll motion
η_5	Pitch motion
η_6	Yaw motion
γ	Peakedness parameter
λ	Wave length
ω	Angular frequency
ω_d	Damped angular frequency
ω_n	Natural angular frequency
\overline{GM}_L	Longitudinal metacentric height
\overline{GM}_T	Transverse metacentric height
ϕ	Velocity potential
ϕ_0	Incident wave potential
ϕ_D	Diffraction potential
ϕ_R	Radiation potential
ρ	Density of the fluid
σ	Standard deviation
σ^2	Variance
θ	Roll angle for forced roll
θ_0	Roll amplitude

ξ	Damping ratio
ζ	Wave elevation
$\zeta_a = A$	Wave amplitude
$\zeta_w = H$	Wave Height
A_{WP}	Waterplane area
B_{BK}	Bilge keel damping
B_e	Eddy damping
B_f	Frictional damping
B_L	Lift damping
B_w	Wave damping
c_w	Phase velocity
E	Total amount of energy in a sea state
F_{exc}	Excitation force
F_{rad}	Radiation force
g	Acceleration due to gravity
$H(\omega)$	Transfer function
$h(t)$	Impulse response function
H_{m0}	Estimate of H_s if the significant wave height is calculated from wave spectrum
H_s	Significant wave height
I_A	Hydrodynamic added mass in roll
I_M	Moment on inertia of barge
k	Wave number
$k(\tau)$	Retardation function
p	Pressure
p_a	Atmospheric pressure
$R(\tau)$	Autocorrelation function
$S(\omega)$	Spectrum
S_{0B}	Mean wetted hull surface
T	Wave period
T_0	Natural Period
T_d	Damped Period
T_p	Spectral period or Peak period
T_z	Zero up-crossing wave period
z	Elevation of a point, with reference to mean free-surface level
A	Added-mass matrix
B	Damping matrix
C	Restoring matrix
F	External force matrix
M	Mass matrix
n	Unit perpendicular
V	Velocity vector

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Chapter 1

Introduction

1.1 Background

Barges are often used in ocean transport of large cargoes, even complete platforms like jackets. They are commonly built at construction yards far away from the installation site and then transported to the location. Hence the transportation barges have a considerable role in setting up an offshore field as these large beam box shape structures move heavy items from the fabrication yard to the actual field of deployment. These non propelled barges are towed by one or more tugs at a normal tow speed of 5.0-6.0 knots (Negi and Dhavalikar (2011)). The transportation is thus often an important part of the installation and decommissioning. During transportation there will be motions on the barge that again will affect the structure, hence there will be fatigue damage and shortening of the fatigue life during transportation. An example model of a jacket on a transportation barge can be seen in Figure 1.1 and main dimensions can be found in Section 4.1.1.

Roll motion is in many cases the dominant of forces in sea fastening systems, due to resonance and small roll damping. Ship-shape floating structures typically have a small natural period in roll, in the order of 12-18s, and in some wave conditions roll resonant motions might be inevitably excited. The barge will therefore be exposed to large roll motions. This can be critical as prediction of roll damping has been a challenging task for naval architects. Most vessel responses can be calculated with acceptable accuracy in the frequency domain, whereas it is more difficult for roll response due to the nonlinear behavior of roll damping. In case of heave, pitch, sway and yaw the principal source of damping is creation of waves. But this source and others are small for damping in roll. Thus the total roll damping coefficient is relatively small and is therefore the most difficult damping to calculate, among all degrees of freedom.

In addition roll motions of heavily loaded barge-type hull forms in ocean transit, either self-propelled or under tow, can be problematic in heavy seaways because of the ves-

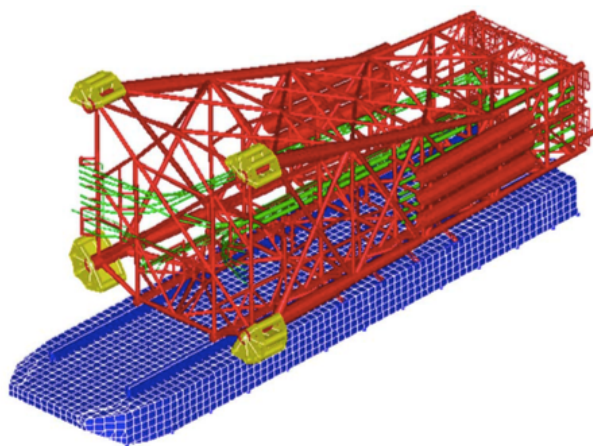


Figure 1.1: Structural Model of Jacket on Barge (Bøe et al. (2017)).

sels' high vertical center of gravity (VCG), low freeboard, and shallow draft (Magnuson (2010)).

The sea fastening system secures the cargo to the deck of the barge during transportation. This may be done by welded connections or chain lashings (Natskår and Moan (2010)). Large roll motions will induce large translational acceleration and inertia loads at the sea fastening point of the jacket. It is therefore of high importance to calculate the roll motion correctly. Hence the challenge is to develop a reliable method for calculating the equivalent linearized roll damping which enables the required response statistics to be calculated in the frequency domain for operational strength and fatigue analysis. As a result of the non linearities it can be difficult to calculate the roll response in the frequency domain. However time domain analysis can be time consuming when a large number of sea states have to be considered for eg. fatigue calculation. Frequency domain calculations have therefore become the norm in the industry for fatigue analysis (Hajiarab et al. (2011)). The stochastic nature of the ocean environment is the main source of the fatigue demand for welded structural elements (Negi and Dhavalikar (2011)). During a voyage this is often dominated by inertia loads due to wave induced motions of the transportation vessel and/or deformation loads caused by wave induced bending and torsion of the transportation vessel (Bøe et al. (2017)).

This thesis topic is proposed by DNV-GL as a preparation for new guidelines and recommendation for transportation fatigue analyzes. Today's standard can be found in DVL GL Marine Operations standard DNVGL-ST-N001 (DNV GL (2016)), but they does not give a complete guidance for transportation fatigue calculations (Bøe et al. (2017)). One of the topics with few guidelines is how to deal with non-linear viscous roll damping when doing transportation analyses. The first step for solving this problem is to verify the linearization technique, which will be done in this thesis.

1.2 Objectives

The objectives in the report are:

1. Do a literature review on time-domain analysis of floating structures (ships) in waves, with focus on sea transportation, wave-induced ship roll motion analysis, time-domain hydrodynamic loads and motion analysis.
2. To establish a time-domain model in SIMA with the given hydrodynamic analysis results from HydroD.
3. To perform a decay test in all six degrees of freedom to check the time-domain model.
4. To perform dynamic response analysis considering regular and irregular waves.
5. To compare the frequency-domain results obtained by Anders Juul Weiby with the time-domain results and conclude on the accuracy of the linearization methods for roll damping.

1.3 Structure of Report

The rest of this paper is divided into 7 chapters. An explanation of the content in each chapter is as follows.

Chapter 2 gives a brief insight in the hydrodynamic loads in time and frequency domain. Some pages in this chapter were written during project work autumn 2017.

Chapter 3 explains how a damped system can be modeled. In addition it goes deeper into roll damping and why it has a nonlinear behavior. Some pages from this chapter were also written during project work autumn 2017.

Chapter 4 describes the modelling done in SIMA and the theory behind other program used in this thesis.

Chapter 5 presents and discusses all results obtained from SIMA.

Chapter 6 compares the results obtained in SIMA to the results obtained by Anders Juul Weiby in HydroD (Weiby (2018)).

Chapter 7 concludes the project and describes the findings throughout the report.

Chapter 8 gives a description of further work that can be addressed.

1.4 Scope and Limitations

The work done in this thesis will mainly focus on roll motion. This is due to the objectives listed earlier. The motion and spectrum of the five other degrees of freedom will in most cases also be plotted but not discussed at the same depth as roll.

The modification in the model is done to create steady state motions for the roll motions. The adjustments have also been done to minimize the difference between the time and frequency model in roll. The other motions are then only discussed in order to look at the influence on roll.

In the model analyzed, the mass is modeled as one mass with one centre of gravity. This means that the forces between the structures are not taken into account.

Chapter 2

Hydrodynamics

Generally, scientists and engineers are interested in the kinematics and dynamics of waves. This is regarded as the foundation for the design and construction of fixed or floating structures in the marine environment as the response of installations and ships is mainly governed by wave loads. Consequently, this chapter shall provide an insight into hydrodynamical loads and how structures are affected. Regular and Irregular waves will be defined in the first section. The other sections will focus on the linear and second-order hydrodynamic loads in frequency and time domain.

2.1 Sea Environment

As ocean waves vary in shape, height, length and velocity, it is important to distinguish between regular and irregular waves. Additionally, the principles of an irregular sea state shall be described and eventually shall lead to the realization of the random water surface under realistic conditions.

2.1.1 Potential Theory

Potential flow can describe the velocity field as the gradient of a scalar function, the velocity potential. Assuming that the fluid under study is incompressible, inviscid and irrotational, the potential theory can be used to calculate wave loads on fixed structures and motions of a body floating in the fluid. Most of the equations used in this section can be found in Faltinsen (1993). The fluid in the potential theory is described by a velocity potential ϕ from which the fluid characteristics such as fluid velocity, fluid acceleration, fluid pressure and surface elevation can be found. As fluid is assumed to be ideal in the potential theory, calculation of phenomena such as viscous damping, slamming pressure and forces on slender structures from this theory is not reliable. This is due to the fact that

these phenomena are directly related to fluid behaviours such as viscosity, vortex shedding and compressibility. However these nonlinear phenomena can be considered in theory by adding viscous terms.

The first assumption is that the fluid is irrotational, $\nabla \times \mathbf{V} = 0$. This means that the velocity vector, \mathbf{V} , can be written as the gradient of a scalar variable as shown in Equation 2.1.

$$\mathbf{V} = \nabla\phi = \frac{\partial\phi}{\partial x}\mathbf{i} + \frac{\partial\phi}{\partial y}\mathbf{j} + \frac{\partial\phi}{\partial z}\mathbf{k} \quad (2.1)$$

The pressure can be obtained from the Bernoulli equation, found in Equation 2.2.

$$p - p_a = -\rho gz - \rho \frac{\partial\phi}{\partial t} - \frac{1}{2}\rho(\nabla\phi)^2 \quad (2.2)$$

The fluid field is defined by physical boundary conditions that should be considered in the calculations. These include kinematic boundary condition and dynamic free-surface condition.

The kinematic boundary condition includes the boundary at the sea bottom and at the body, and can be found in Equation 2.3 and 2.4 respectively. These equations restrict the fluid motion through the seabed and the body.

$$\left. \frac{\partial\phi}{\partial n} \right|_{z=-h} = 0 \quad (2.3) \quad \frac{\partial\phi}{\partial n} = \underbrace{\mathbf{V}_B}_{\text{body velocity}} \cdot \mathbf{n} \quad (2.4)$$

For the free-surface there are both dynamic and kinematic boundary conditions that must be combined. Since the pressure at the free surface should be equal to the atmospheric pressure, the dynamic boundary condition can be defined as in Equation 2.5.

$$g\zeta + \left. \frac{\partial\phi}{\partial t} \right|_{z=0} = 0 \quad (2.5)$$

The kinematic free-surface condition can be found by defining the surface elevation as $z = \zeta(x, y, t)$ and the function $F, F(x, y, z, t) = z - \zeta(x, y, t)$. A fluid particle on the free-surface is assumed to stay on the free-surface, this means that $DF/Dt = 0$. By removing the higher order terms, the kinematic free-surface condition can be found by Equation 2.6 (Faltinsen (1993)).

$$\left. \frac{\partial\zeta}{\partial t} = \frac{\partial\phi}{\partial z} \right|_{z=0} \quad (2.6)$$

By combining Equation 2.5 and 2.6, a new Equation 2.7 can be found.

$$\left. \frac{\partial^2\phi}{\partial t^2} + g \frac{\partial\phi}{\partial z} \right|_{z=0} = 0 \quad (2.7)$$

This can be illustrated as shown in Figure 2.1.

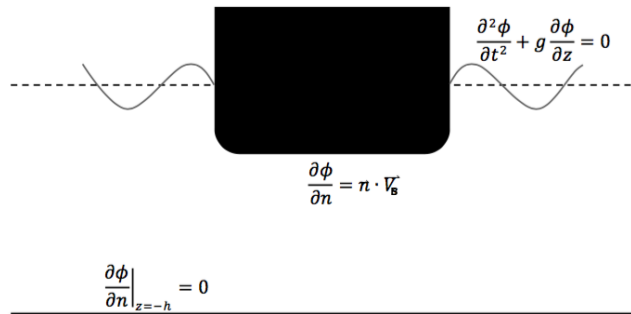


Figure 2.1: Boundary Conditions of Object in Fluid, used in Potential Theory.

Wave Potential

From the dynamic free surface potential it can be shown that the wave potential needs to be at the form $\phi = f(z)\sin(kx - \omega t)$. By inserting this in Equation 2.7, it can be found that the wave potential will be as shown, $\phi = (A_1 e^{kz} + B_1 e^{-kz})\sin(kx - \omega t)$. If the two other boundary conditions are used the wave potential will be as shown in Equation 2.8 (Pettersen (2007)).

$$\phi = \frac{g\zeta_A}{\omega} \frac{\cosh k(h+z)}{\cosh kh} \sin(kx - \omega t) \quad (2.8)$$

This can be used in shallow water, but at greater depths it can be simplified as shown in Equation 2.9.

$$\phi = \frac{g\zeta_A}{\omega} e^{kz} \sin(kx - \omega t) \quad (2.9)$$

2.1.2 Regular Waves

Regular waves oscillate with period $T = \frac{2\pi}{\omega}$ and with wave length $\lambda = \frac{2\pi}{k}$, where k is the wave number. The wave potential for finite water depths is shown in Equation 2.8 and for infinite water depths in Equation 2.9. Linear dispersion relationship can be found by implementing the general potential velocity in the combined free surface boundary conditions, expressed in Equation 2.7. The dispersion relation for finite water depths is shown in Equation 2.10 and for infinite water depths in Equation 2.11.

$$\omega^2 = kg \tanh kh \quad (2.10)$$

$$\omega^2 = kg \quad (2.11)$$

The phase velocity is different from the fluid velocity, and depends on the wave length, $c_W = \frac{\lambda}{T} = \frac{\omega}{k}$.

Regular incident waves are far from similar to how ocean waves appear in reality. However they can be useful to describe more general waves under the assumption of linear conditions due to super position principle (Greco (2012)).

Phase Angle

The phase angle can be defined as the difference between the wave and the movement, if the wave elevation amid-ship is measured at the same time as the movement at the same place. This can be seen in Figure 2.2. If the green line is the wave elevation and the blue line the movement, the phase difference can be measured as shown.

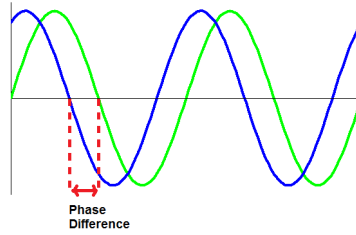


Figure 2.2: Phase Difference Between Wave Elevation and Movement.

If the movement is roll, this can be described as seen in Equation 2.12 and 2.13. The phase angle is ϵ_4 , and a positive value means that the roll motions are in front of the wave elevations.

$$\zeta = \zeta_a \cos \omega t \quad (2.12)$$

$$\eta_4 = \eta_{4a} \cos \omega t + \epsilon_4 \quad (2.13)$$

2.1.3 Irregular Waves

In Section 2.1.2 regular waves were considered, where wave crests and troughs alternate consistently with the same amplitude and period. However, the actual waves have an irregularity and randomness, mainly due to the inconsistency in wind speed and direction. Thus, a more realistic capture of the sea surface demands a stochastic consideration of the wave pattern. The main concept of reproducing the surface elevation and wave kinematics of an irregular sea state propagating in the positive x-direction involves the superposition of a large number of regular wave components. This can be done as shown in Equation 2.14.

$$\zeta = \sum_{j=1}^N A_j \sin \omega_j t - k_j x + \epsilon_j \quad (2.14)$$

Where A_j , ω_j , k_j and ϵ_j mean respectively the wave amplitude, circular frequency, wave number and random phase angle of wave component number j . In Myrhaug and Lian (2009) it is assumed that:

1. The wave process is stationary within a short term interval, between 20 minutes and 3 hours, where the mean value and variance of the process will be constant.
2. The wave elevation is normally distributed with zero mean.
3. The wave process is ergodic, which means that one time series is representative for the wave process.

Wave Spectrum

The total energy in a sea state can be described as seen in Equation 2.15, as a sum of N linear wave components.

$$\frac{E}{\rho g} = \sum_{n=1}^N \frac{1}{2} \zeta_{An}^2(\omega_n) \quad (2.15)$$

The wave spectrum can therefore be introduced as $S(\omega)$, where the area in a narrow frequency interval is proportional to the energy for all wave components within the interval, as seen in Equation 2.16 and Figure 2.3. Most energy of an irregular sea state is limited to a relatively narrow and discrete frequency band as seen in the figure.

$$\frac{1}{2} \zeta_{An}^2 = S(\omega_n) \Delta\omega \quad (2.16)$$

When $N \rightarrow \infty$, this means that $\Delta\omega \rightarrow 0$. The total energy may then be integrated. $S(\omega)$ will contain all information about the statistical properties for the waves. As the wave elevation is assumed to be normally distributed, the mean value is zero. The variance may then be given as seen in Equation 2.17.

$$\frac{E}{\rho g} = \sigma^2 = \int_0^{\infty} S(\omega) d\omega \quad (2.17)$$

Pierson-Moskowitz (PM) Spectrum

The Pierson-Moskowitz spectrum are described by Equation 2.18. These spectra have one peak and a steep front at low frequencies.

$$S(\omega) = \frac{A}{\omega^5} \exp\left[-\frac{B}{\omega^4}\right] \quad (2.18)$$

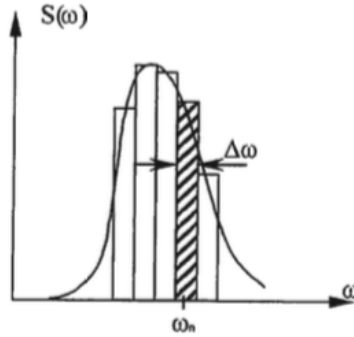


Figure 2.3: Definition of Wave Spectrum (Myrhaug and Lian (2009)).

JONSWAP Spectrum

In this thesis the JONSWAP spectrum (Joint North Sea Wave Project) will be used. This is a result of a multinational measuring project in the south-east part of the North Sea. The specter may given as i Equation 2.19 (Myrhaug and Lian (2009)).

$$S(f) = \alpha g^2 (2\pi)^{-4} f^{-5} \exp\left[-\frac{5}{4}(T_p f)^{-4}\right] \gamma \exp\left[-\frac{(T_p f - 1)^2}{2\sigma^2}\right] \quad (2.19)$$

JONSWAP spectrum is equal to the PM spectrum if $\gamma = 1$. The difference between the two spectra is how the energy is distributed along the frequency axis. In the JONSWAP spectrum there will be more energy concentrated near the peak frequency, and less energy on frequencies further away from the peak frequency, compared to the PM spectrum. This is illustrated in Figure 2.4. The JONSWAP spectrum is assumed to be a good model when $3.6\sqrt{H_{m0}} \leq T_p \leq 5\sqrt{H_{m0}}$.

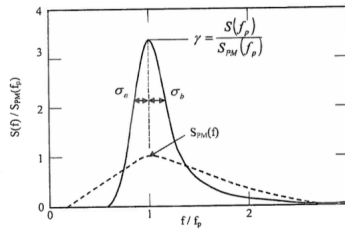


Figure 2.4: Comparison of JONSWAP and PM Spectra for same Sea State (Myrhaug and Lian (2009)).

Wave seed is a random number where every seed number will give different maximum wave rise, maximum wave fall and also the time windows when the peak is occurs in the 3hrs simulations (Gunawan Suwarno and Choon Hua Lee (2016)). The form of irregular waves depend on the wave spectrum formulation, gamma, seed and the wave data (height

and period). This means that irregular wave analyses will require wave realization which relies on wave seeds selection.

Scatter Diagram

Another common approach for modeling irregular waves are through directional long term scatter diagrams. They are used to show how the data points within a set are distributed with regard to different parameters. In this case, intervals for the significant wave heights, H_s , are defined along the first column, while intervals for the spectral period, T_p , are specified along the first row. By assigning each position in the diagram the number of observed sea states that fit the specified combination, a scatter diagram for the data sample is generated.

This can be used to find the statistical properties of the individual parameters as well as the connection between them. By dividing the route into different legs where the wave climate and vessel heading can be assumed to be constant, several scatter diagrams are collected for different wave directions for each leg. Each scatter diagram is then weighted in accordance with expected duration for each leg. An example of scatter diagrams can be seen in Figure 2.5.

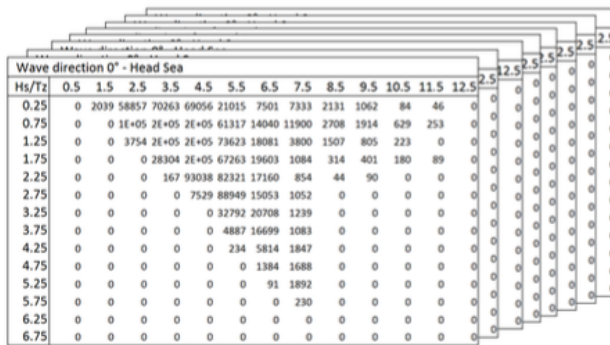


Figure 2.5: Directional Long-Term Scatter Diagrams (Bøe et al. (2017)).

Through scatter diagrams the mean values for H_s and T_p during a short-term period are used. As the duration of the transport is relatively short compared to the long-term statistics of the scatter diagrams, the deviation from the expected weather could be considerable. It is therefore recommended to use conservative scatter diagrams including storm conditions (Bøe et al. (2017)).

2.2 Linear Wave-Induced Motions and Loads

To obtain results in irregular seas it is possible to linearly superpose results from regular wave components (Faltinsen (1993)). Steady state will be assumed, this implies that the linear dynamic motions and loads oscillates harmonically with the same frequency as the wave loads. The hydrodynamic problem in regular waves is normally dealt with as

two sub-problems: the wave excitation loads and the hydrodynamic loads, also known as diffraction and radiation problem, as seen in Figure 2.6.

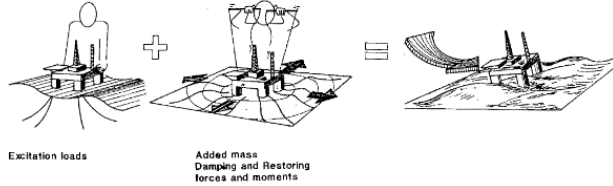


Figure 2.6: Superposition of Wave Excitation, Added Mass, Damping and Restoring Loads (Faltinsen (1993)).

A barge can move in six degrees of freedom (DOF) where the first three represent translation motions and the last three represent rotations. A right-handed coordinate system (x,y,z) fixed with respect to the mean position of the body is used, with positive z vertically upwards through the center of gravity. This can be seen in Figure 2.7.

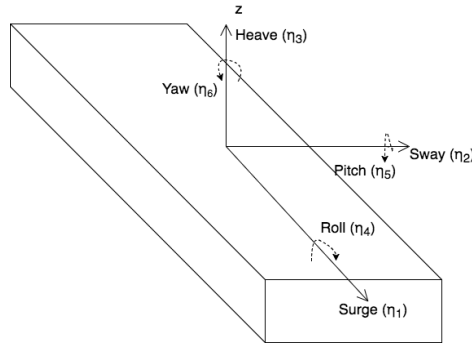


Figure 2.7: Six Degrees of Freedom for a Vessel.

2.2.1 Diffraction Problem

In the diffraction problem the body is fixed (restrained from oscillation) and interacts with incident waves. The wave potential can be used as shown in Equation 2.9. This problem can be split into two, as shown in Figure 2.8.

The figure at the left side illustrates that the flow due to ϕ_0 penetrates the body with normal velocity, as the body was not there, $\partial\phi_0/\partial n$. This causes hydrodynamic loads on the body called Froude-Kriloff loads. To recover the body impermeability, the body presence causes a flow associated with ϕ_D . The sum of these two loads gives the wave excitation force. The contributions are obtained by integrating the incident wave dynamic pressure and the diffraction pressure along the mean wetted hull.

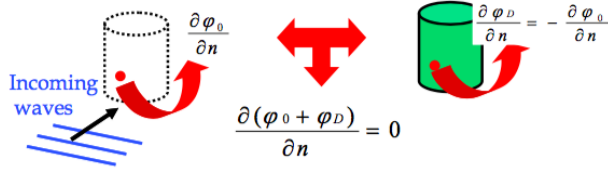


Figure 2.8: Splitting Wave Potential for the Diffraction Problem (Greco (2012)).

$$F_{exc,k}(t) = - \underbrace{\int_{S_{OB}} \rho \frac{\partial \phi_0}{\partial t} n_k dS}_{\text{Froude-Kriloff loads}} - \underbrace{\int_{S_{OB}} \rho \frac{\partial \phi_D}{\partial t} n_k dS}_{\text{Diffraction loads}} \quad (2.20)$$

2.2.2 Radiation Problem

In the radiation problem the body is forced to oscillate in its six degrees of freedom with frequency ω . The moving body generates waves associated with the radiation velocity potential and is subjected to hydrodynamic loads identified as added-mass, damping and restoring terms. The restoring term is connected with the variation of the buoyancy due to body motions. The added-mass and damping terms are connected with the dynamic pressure caused by body motions. The force can then be found as in Equation 2.21.

$$F_{rad,k}(t) = - \int_{S_{OB}} \rho \frac{\partial \phi_R}{\partial t} n_k dS = \sum_{j=1}^6 (-A_{kj} \ddot{\eta}_j - B_{kj} \dot{\eta}_j) \quad (2.21)$$

2.3 Equation of Motion

A floating barge may have independent degrees of motions, which may be described as a mass-damper-spring system as described in Equation 2.22.

$$(\mathbf{M} + \mathbf{A}(\omega)) \ddot{\boldsymbol{\eta}} + \mathbf{B}_1(\omega) \dot{\boldsymbol{\eta}} + \mathbf{B}_2(\omega) |\dot{\boldsymbol{\eta}}| \dot{\boldsymbol{\eta}} + \mathbf{C}\boldsymbol{\eta} = \mathbf{F}_0(\omega) e^{i\omega t} \quad (2.22)$$

Here $(\mathbf{M} + \mathbf{A}(\omega)) \ddot{\boldsymbol{\eta}}$ is the mass forces consisting of a mass term and an added mass term. ω is the angular frequency of the wave in rad/s . The second term $\mathbf{B}_1(\omega) \dot{\boldsymbol{\eta}}$ is the linear damping force from potential damping (wave radiation), while $\mathbf{B}_2(\omega) |\dot{\boldsymbol{\eta}}| \dot{\boldsymbol{\eta}}$ is the quadratic damping force. This represents the viscous damping. The last term $\mathbf{C}\boldsymbol{\eta}$ represents the restoring forces.

In roll motion Equation 2.22 can be described as Equation 2.23. To simplify and limit the problem of nonlinear damping, the equation is formulated as an equation of first degree (Faltinsen (1993)). Where $B_{44}(\dot{\eta}_4) = B_1 \dot{\eta}_4 + B_2 \dot{\eta}_4 |\dot{\eta}_4| + \dots$ is the nonlinear damping coefficient in roll.

$$(I_{44} + A_{44})\ddot{\eta}_4 + B_{44}\dot{\eta}_4 + C_{44}\eta_4 = F_4 \quad (2.23)$$

2.3.1 Parameter Dependence

The added mass and damping coefficients have a strong frequency dependence. In addition they depend on the motion mode, and the added mass in heave is therefore not the same as in sway. In Figure 2.9, the frequency dependence is illustrated.

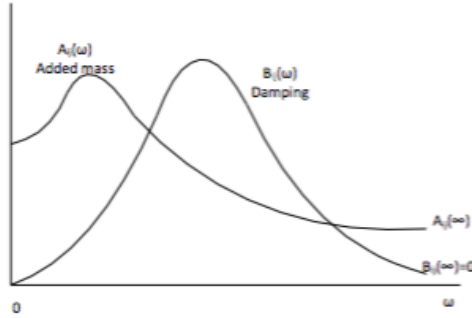


Figure 2.9: Added Mass and Damping Plotted against Frequency (Gao (2017)).

This implies that the damping coefficient goes to zero when $\omega \rightarrow 0$ or $\omega \rightarrow \infty$. This means that $\mathbf{B}(w) = \mathbf{b}(w) + \mathbf{B}_\infty = \mathbf{b}(w)$. The added mass converges against \mathbf{A}_∞ and can be written as $\mathbf{A}(w) = \mathbf{a}(w) + \mathbf{A}_\infty$.

2.3.2 Restoring Forces and Moments

The restoring forces and moments are due to a difference between the buoyancy and its weight. In equilibrium there will be balance between these two forces. The restoring force is due to buoyancy change when the barge is brought out of equilibrium. If the body has a x-z plane as a symmetry plane, the only non-zero coefficients are C_{33} , C_{35} , C_{44} , C_{55} and C_{53} (Faltinsen (1993)).

$$C_{33} = \rho g A_{WP} \quad (2.24) \quad C_{35} = C_{53} = -\rho g \int \int_{A_{WP}} x ds \quad (2.25)$$

$$C_{44} = \rho g V (z_B - z_G + \rho g \int \int_{A_{WP}} y^2 ds = \rho g V \overline{GM}_T) \quad (2.26)$$

$$C_{55} = \rho g V (z_B - z_G + \rho g \int \int_{A_{WP}} x^2 ds = \rho g V \overline{GM}_L) \quad (2.27)$$

2.3.3 Natural Periods

The natural or resonance periods, damping level and wave excitation level are important parameters in assessing the amplitudes of motion of the barge. Relatively large motions are likely to occur if the structures are excited with oscillation periods close to the resonance period. However, if the damping is high or the excitation level is relatively low due to cancellation effects, the response may be similar to the response at other periods. The uncoupled and undamped periods can be calculated as seen in Equation 2.28.

$$T_{ni} = 2\pi \sqrt{\frac{M_{ii} + A_{ii}(\omega)}{C_{ii}}} \quad (2.28)$$

For an unmoored structure there are no uncoupled resonance periods in surge, sway and yaw.

A simplified equation for the uncoupled natural period in roll can be seen in Equation 2.29 (Faltinsen (1993)).

$$T_{n4} = 2\pi \sqrt{\frac{Mr_{44}^2 + A_{44}(\omega)}{C_{44}}} = 2\pi \sqrt{\frac{Mr_{44}^2 + A_{44}(\omega)}{\rho g V \overline{GM}_T}} \quad (2.29)$$

Where M is the mass of the ship, r_{44} is the radius of gyration, A_{44} is the added mass due to roll motion and \overline{GM}_T is the transverse metacentric height. As seen in the last section, added mass has a strong frequency dependence. An iteration process must therefore be used to find the correct natural period.

The parameter which has the strongest influence on the natural period is the metacentric height. This should therefore be adjusted in such a way that the natural period in roll is not in the middle of the wave frequency, even though it can not be completely avoided. T_{n4} is typically 8-12 s for conventional merchant vessels, while the wave period is about 5-10 s.

This means that ships without roll stabilization equipment are exposed to strong resonance effects in roll, and the response may be substantial. The roll wave damping has a tendency to be small due to cancellation effects for normal midship sections. Cancellation means that the roll moment caused by the pressure forces on the ship sides tends to counteract the roll moment caused by the pressure forces on the ship bottom. For a circular cross-section the roll damping will be zero if the moment axis coincides with the cylinder axis. If the cross sectional beam is small or large, the cancellation effect will be smaller. This can be seen in Figure 2.10 where the two dimensional roll wave damping B_{44}^{2D} is a function of the beam-draught ratio for rectangular cross sections. The amplitude at resonance depends on the damping level, and correct damping is therefore important in order to estimate the correct excitation forces (Faltinsen (1993)).

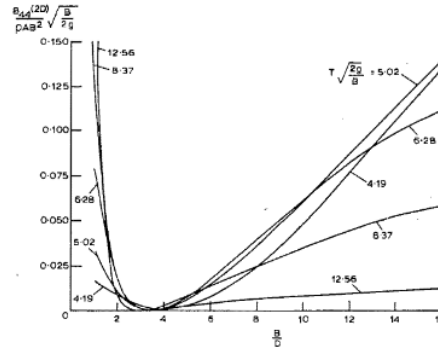


Figure 2.10: Two-Dimensional Roll Wave Damping (Faltinsen (1993)).

Linearized Roll Restoring Moment

In Equation 2.26 the restoring coefficient in roll was shown. When the roll angles are small this is a valid assumption. However, for large roll angles the restoring coefficient will exceed the linear part of the GZ-curve as seen in Figure 2.11 (Wawrzynski and Krata (2016)).

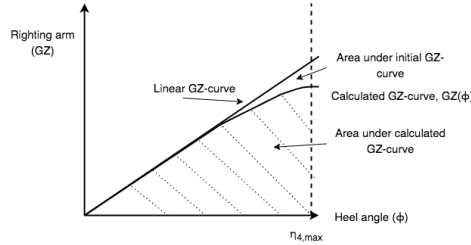


Figure 2.11: Linearized Roll Restoring Moment.

Equivalent restoring stiffness for different angles has been found in Natskär and Moan (2010) by forced roll tests. The equation of motion is then written as in Equation 2.30.

$$(I_M + I_A)\ddot{\theta} + B_{eq}\dot{\theta} + C_{eq}\theta = M(t) \quad (2.30)$$

The equivalent restoring, C_{eq} is found by assuming that the work performed when rolling the model, statically is the same as when using C_{eq} . This can be seen in Equation 2.31. The result from model tests can be seen in Figure 2.12. From this figure it can be seen that the assumption of expressing $C_{44} = \rho g V \overline{GM}_T$ is valid only for roll angles smaller than 10-15 degrees.

$$\frac{1}{2}C_{eq}\theta_0^2 = \int_0^{\theta_0} M(\theta)d\theta \Rightarrow C_{eq} = \frac{2}{\theta_0^2} \int_0^{\theta_0} M\theta d\theta \quad (2.31)$$

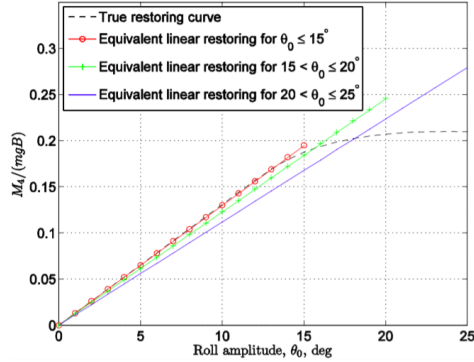


Figure 2.12: Actual Restoring Curve from Forced Roll Tests (Natskår and Moan (2010)).

2.3.4 Coupled Motions

The equation of motion shown in Section 2.3 does not take into account coupled effects. As there will be coupled effects between the motions, the true mathematical model describing the movements will be a combination of six coupled second order differential equations. In Equation 2.32 the cross-coupled coefficients are taken into account, where there is a force in direction j because of a motion in direction k .

$$(\mathbf{M}_{jk} + \mathbf{A}_{jk}(\omega))\ddot{\boldsymbol{\eta}}_k + \mathbf{B}_{jk}(\omega)\dot{\boldsymbol{\eta}}_k + \mathbf{C}_{jk}\boldsymbol{\eta}_k = \mathbf{F}_j e^{i\omega t} \quad (2.32)$$

For a structure with no forward speed and no current it can be shown by using Green's second identity that the added mass and damping coefficients satisfy the symmetry relations $A_{jk} = A_{kj}$ and $B_{jk} = B_{kj}$.

For a floating body with lateral symmetry in shape and weight distribution, the six coupled equations of motion can be reduced to two sets of equations, where the first set consisting of surge, heave, and pitch can be decoupled from the second set consisting of sway, roll, and yaw (S. N. Das and S. K. Das (2003)). This is due to the linear theory. If the body is forced in heave the pressure is symmetrical about the z -axis and the resulting horizontal forces are therefore zero. The case is similar for pitch. If forces sway, roll and yaw motion are considered, the pressure distribution will be asymmetric about the x - z plane. This implies that A_{1k} , A_{3k} , A_{5k} , B_{1k} , B_{3k} and B_{5k} equals zero for $k=2,4$ and 6 (Faltinsen (1993)). It can then be concluded from the equation of motion that the vertical and longitudinal motions (heave, pitch, surge) are uncoupled from the lateral motions (sway, yaw, roll). In this thesis, the second set will be the focus as roll motion is of greatest interest.

2.3.5 Acceleration

When analyzing rotations in roll, an important consideration is to determine the derivatives of the motion, because fatigue damage is highly dependent on the acceleration. The motion

and speed can be derived to find the acceleration. This is seen in Equation 2.33, where $\ddot{\eta}_k$ is seen as the acceleration.

$$\begin{aligned}\eta_k &= \bar{\eta}_0 e^{i\omega t} \\ \dot{\eta}_k &= i\omega \bar{\eta}_0 e^{i\omega t} \\ \ddot{\eta}_k &= -\omega^2 \bar{\eta}_0 e^{i\omega t}\end{aligned}\tag{2.33}$$

In order to find the maximum accelerations on the sea-fastening, the coordinate system has to be changed from an earth fixed coordinate system (x,y,z) to a body fixed coordinate system (X,Y,Z). This will give the accelerations in X and Y-direction in the deck plane, as well as Z-acceleration normal to the deck plane. Estimation of the forces on the sea-fastening equipment can then be done.

The acceleration in the different directions can be described as seen in Equation 2.34.

$$\begin{aligned}a_x &= \ddot{\eta}_1 + z\ddot{\eta}_5 - y\ddot{\eta}_6 \\ a_y &= \ddot{\eta}_2 + z\ddot{\eta}_4 + x\ddot{\eta}_6 \\ a_z &= \ddot{\eta}_3 + y\ddot{\eta}_4 - x\ddot{\eta}_5\end{aligned}\tag{2.34}$$

2.4 Frequency Domain Analysis

The equation of motion is seen as a combination of time and frequency domain. Therefore, some modification must be done to solve the equation in frequency domain. First, $\boldsymbol{\eta}$ is a matrix, $\boldsymbol{\eta} = [\eta_1 + \eta_2 + \eta_3 + \eta_4 + \eta_5 + \eta_6]^T$, where the indexes 1-6 are the six degrees of freedom. The derivatives can be seen Equation 2.33, which describes the movement, speed and acceleration. This can be inserted into Equation 2.22 and the time dependent parts can be removed from all parts except the quadratic one.

To account for the non linearity in roll, $B_{44}(\dot{\eta}_4) = B_1\dot{\eta}_4 + B_2\dot{\eta}_4|\dot{\eta}_4| + \dots$, the quadratic damping term must be linearized. For regular waves, the equivalent linear damping is found by demanding the same amount of energy dissipated from the linear system as from the non-linear system. The equivalent damping in roll is then as shown in Equation 2.35 (Pettersen (2007)).

$$\begin{aligned}B_{44,2} &= \int_0^T B_F \dot{\eta}_4 \frac{\partial \eta_4}{\partial t} dt = 4 \int_0^{\frac{T}{4}} B_2 |\dot{\eta}_4| \dot{\eta}_4 \frac{\partial \eta_4}{\partial t} dt \\ &\implies B_{44,eq} = B_{44,1} + \frac{8}{3\pi} B_{44,2} \theta_0 \omega\end{aligned}\tag{2.35}$$

For an irregular sea state, the equivalent stochastic linearization is as shown in Equation 2.36 (Natskår and Steen (2013)).

$$B_{44,eq} = B_{44,1} + 2\sqrt{\frac{2}{\pi}} \sigma_{\dot{\eta}_4} B_{44,2}\tag{2.36}$$

In Equation 2.36 σ_{η_4} is the standard deviation of the roll velocity and can be determined from the roll RAO and the sea spectrum. This is shown in Equation 2.37 (Drobyshevski and Whelan (2010)).

$$\sigma_{\eta_4} = \sqrt{\int_0^{\infty} \omega^2 \theta_0^2(\omega) S_0(\omega) d\omega} \quad (2.37)$$

Here $\theta_0(\omega)$ is the roll response amplitude operator (RAO), which depends on the equivalent damping coefficient $B_{44,eq}$. $S_0(\omega)$ is the wave spectrum. An iterative procedure is applied, where repetitive computations of RAO are undertaken by solving equations of motions until convergence is obtained (Drobyshevski and Whelan (2010)).

The equation of motion in the frequency domain for roll can then be expressed as shown in Equation 2.38.

$$[\mathbf{C} - \omega^2(\mathbf{M} + \mathbf{A}(\omega)) + i\omega\mathbf{B}_{eq}(\omega)]\boldsymbol{\eta}_0 = \mathbf{F}_0(\omega) \quad (2.38)$$

2.4.1 Transfer Function

Having achieved the equation of motion in the frequency domain (Equation 2.38) there are different ways of solving the problem for a forced vibration. The frequency response method, with complex numbers, is the most common. By taking the inverse of the expression within the brackets in Equation 2.38, we obtain Equation 2.39 (Newland (1993)).

$$\mathbf{X}(\omega) = \underbrace{[\mathbf{C} - \omega^2(\mathbf{M} + \mathbf{A}(\omega)) + i\omega\mathbf{B}_{eq}(\omega)]^{-1}}_{\text{Transfer function} - \mathbf{H}(\omega)} \mathbf{F}_0(\omega) \quad (2.39)$$

The transfer function states how the system reacts to a harmonic load with a given frequency, where the magnitude gives the ratio between the amplitude of the harmonic response to the amplitude of the harmonic force. There are many types of transfer functions. The one shown here is the force to motion transfer function, $\mathbf{H}(\omega)_{FX}$. In linear theory another transfer function is the linear correlation between wave amplitude and force, $\mathbf{H}(\omega)_{ZF}$. If these transfer functions are multiplied, the wave to motion transfer function can be found, $\mathbf{H}_{ZX}(\omega) = \mathbf{H}(\omega)_{FX}\mathbf{H}(\omega)_{ZF}$. This can also be obtained as shown in Equation 2.40 (Gao (2017)).

$$|\mathbf{H}(\omega)_{ZX}| = \left| \frac{\eta}{\zeta_a} \right| \quad (2.40)$$

In sea keeping the special name response amplitude operator, RAO, is often used and denotes the ratio between the response amplitude of any studied variable to the wave amplitude. The word operator is used because RAO operates on the frequency spectrum of a sea state to produce a spectrum of motion, moment, stress and so on (Bergdahl (2009)).

2.4.2 Spectral Analysis

Analyzing model test results, it will often be convenient to transform the measured time history signal to the frequency domain. This can be obtained by a spectral analysis in which the spectral density function of the signal is found. The spectral analysis is usually done using the Fast Fourier Transform (FFT) technique.

Assuming that the process takes place over the time interval ($0 < t < T$, $T \rightarrow \infty$). The power spectrum of the wave, $S_{xx}(\omega)$ is defined as the Fourier transform of the autocorrelation function $R_{xx}(\tau)$ (Equation 2.41). In the same way the cross spectrum between the wave and motion can be found as shown in Equation 2.42 (Steen (2014)).

$$S_{xx} = \int_{-\infty}^{+\infty} R_{xx}(\tau) e^{-i\omega\tau} d\tau \quad (2.41) \quad S_{xy}(\omega) = \int_{-\infty}^{+\infty} R_{xy}(\tau) e^{-i\omega\tau} d\tau \quad (2.42)$$

The linear transfer function between the response and wave can be seen in Equation 2.43, expressed by the cross-spectrum. The transfer function can also be obtained from the wave input spectrum and measured response spectrum directly, which is shown in Equation 2.44 (Steen (2014)).

$$H(\omega) = \frac{S_{xy}(\omega)}{S_{xx}(\omega)} \quad (2.43) \quad |H(\omega)|^2 = \frac{S_{yy}(\omega)}{S_{xx}(\omega)} \quad (2.44)$$

2.5 Time Domain Analysis

An alternative to solving the equation of motion in the frequency domain by equivalent linearization of the damping, is to solve the equation of motion in time domain. The equation of motion can then be rewritten as shown in Equation 2.45 using the parameter dependence from Section 2.3.1.

$$-\omega^2(\mathbf{M} + \mathbf{A}_\infty)\mathbf{X}(w) + (i\omega\mathbf{a}(w) + \mathbf{b}(w))i\omega\mathbf{X}(w) + \mathbf{C}\mathbf{X}(w) = \mathbf{F}_0(w) \quad (2.45)$$

Since time and frequency should not be mixed in ordinary differential equations (ODE), the equation of motion has to be transformed to Cummins (1962) equation. First the Fourier and Inverse Fourier transforms are shown in Equation 2.46 and 2.47 respectively.

$$\mathbf{X}(w) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathbf{x}(t) e^{-i\omega t} dt = \mathfrak{F}[\mathbf{x}(t)] \quad \mathbf{x}(t) = \int_{-\infty}^{+\infty} \mathbf{X}(w) e^{i\omega t} dt = \mathfrak{F}^{-1}[\mathbf{X}(w)] \quad (2.46) \quad (2.47)$$

Equation 2.48 is then obtained by using the Inverse Fourier transform. The second term represents the wave radiation force vector, excluding the added mass force vector for infinite frequency (Gao (2017)).

$$(\mathbf{M} + \mathbf{A}_\infty)\ddot{\mathbf{x}}(t) + \int_{-\infty}^{+\infty} [i\omega\mathbf{a}(w) + \mathbf{b}(w)]i\omega\mathbf{X}(w)e^{i\omega t}dw + \mathbf{C}\mathbf{x}(t) = \mathbf{f}_0(t) \quad (2.48)$$

The integral is a convolution of the Fourier transforms and may be solved using Parseval's theorem, $2\pi \int_{-\infty}^{+\infty} F(\omega) \cdot \overline{G(\omega)}d\omega = \int_{-\infty}^{+\infty} f(\tau) \cdot \overline{g(\tau)}d\tau$. This is seen in Equation 2.49.

$$\int_{-\infty}^{+\infty} \underbrace{(i\omega a_{ij}(\omega) + b_{ij}(\omega))}_{F(\omega)} e^{i\omega t} \underbrace{i\omega X_j(\omega)}_{\overline{G(\omega)}} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(\tau) \cdot \overline{g(\tau)}d\tau \quad (2.49)$$

Two new functions are then defined, as seen in Equation 2.50 and 2.51. This leads to a new connection, $f(\tau) = 2\pi k_{ij}(t + \tau)$.

$$f(\tau) = \int_{-\infty}^{+\infty} (i\omega a_{ij}(\omega) + b_{ij}(\omega))e^{i\omega(t+\tau)}d\omega k_{ij}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} (i\omega a_{ij}(\omega) + b_{ij}(\omega))e^{i\omega\tau}d\omega \quad (2.50)$$

$$(2.51)$$

If this is inserted, the following relation can be shown as in Equation 2.52.

$$\overline{g(\tau)} = \int_{-\infty}^{+\infty} \overline{G(\omega)e^{i\omega\tau}}d\omega = \int_{-\infty}^{+\infty} \overline{G(\omega)e^{-i\omega\tau}}d\omega = \int_{-\infty}^{+\infty} i\omega X_j(\omega)e^{-i\omega\tau}d\omega = \dot{x}_j(-\tau) \quad (2.52)$$

The second term component is $\frac{1}{2\pi} \int_{-\infty}^{+\infty} 2\pi k_{ij}(t + \tau)\dot{x}_j(-\tau)d\tau$. By setting $\tau' = -\tau$ and collecting the components in the matrix form, it can be shown that $\int_{-\infty}^{+\infty} \mathbf{k}(t - \tau')\dot{\mathbf{x}}(\tau')d\tau'$. The Fourier transform pair can then be seen in Equations 2.53 and 2.54.

$$\mathbf{k}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathbf{K}(\omega)e^{i\omega\tau}d\omega = \frac{1}{2\pi} \mathfrak{F}^{-1}[\mathbf{K}(\omega)] \quad (2.53)$$

$$\mathbf{K}(\omega) = \int_{-\infty}^{+\infty} \mathbf{k}(\tau)e^{i\omega\tau}d\tau = 2\pi \mathfrak{F}[\mathbf{k}(\tau)] \quad (2.54)$$

The equation of motion can therefore be expressed as in Equation 2.55, which is described as the Cummins' equation which was discovered by (Ogilvie (1964)). This equation is an integro differential equation that contains a convolution term representing fluid memory effects associated with the dynamics of the radiation forces (Perez and Fossen (2008)).

$$(\mathbf{M} + \mathbf{A}_\infty)\ddot{\mathbf{x}}(t) + \int_{-\infty}^{+\infty} \mathbf{k}(t - \tau)\dot{\mathbf{x}}(\tau)d\tau + \mathbf{C}\mathbf{x} = \mathbf{f}(t) \quad (2.55)$$

The second term is the convolution between the retardation function and the body velocity. It represents the wave radiation force vector in the time domain, this is the potential roll

damping as explained in Section 3.2.1. Other parts of the damping must be added to include for non-linearity in roll.

This includes a convolution term representing fluid memory effects associated with the dynamics of the radiation forces. This is done by incorporate the energy dissipation due the radiated waves consequence of the motion of the structure (Tristan Pérez and Thor I. Fossen (2008)).

2.5.1 Retardation Function

The retardation function can be calculated either by the frequency dependent added mass or potential damping coefficient.

$$\mathbf{k}(\tau) = \frac{1}{\pi} \int_0^{+\infty} (\mathbf{b}(\omega)\cos\omega\tau - \omega\mathbf{a}(\omega)\sin\omega\tau)d\omega \quad (2.56)$$

The radiation force has a memory effect, represented by the convolution term, meaning that the force will not disappear immediately after the body stops the movement. However $\mathbf{k}(\tau) = 0$ for $\tau < 0$, the radiation force will therefore be zero before the body starts to oscillate (Gao (2017)).

Ogilvie (1964) used Fourier transform for sinusoidal excitation forces, and the representation was found for the added mass and damping terms as seen in Equation 2.57 and 2.58. This is called Kramers-Kronig relations.

$$\mathbf{a}(\omega) = -\frac{1}{\omega} \int_0^{+\infty} \mathbf{k}(\tau)\sin\omega\tau d\tau \quad (2.57) \quad \mathbf{b}(\omega) = \int_0^{+\infty} \mathbf{k}(\tau)\cos\omega\tau d\tau \quad (2.58)$$

Inverse Fourier transform can also be used to rewrite the time-and frequency retardation as seen in Equation 2.59 and 2.60.

$$\mathbf{k}(\tau) = \frac{2}{\pi} \int_0^{+\infty} \mathbf{b}(\omega)\cos\omega\tau d\omega \quad (2.59) \quad \mathbf{k}(\tau) = -\frac{2}{\pi} \int_0^{+\infty} \omega\mathbf{a}(\omega)\sin\omega\tau d\omega \quad (2.60)$$

2.5.2 Impulse Response Method

$H(\omega)$ gives the steady state response of a system to a sine wave input. By measuring H for all frequencies the dynamic characteristics of a system can be completely defined. An alternative approach is to calculate the response motion caused by a transient excitation force by use of the impulse response function. The response to a unit impulse at $t=0$ is represented by the unit impulse response function, $h(t)$.

As complete information about either the frequency response function or the impulse response function fully defines the dynamic characteristic of a system, it follows that it is possible to derive one from the other, and vice versa. This can be done using Fourier transform as seen in Equations 2.61 and 2.62 (Newland (1993)).

$$H(\omega) = \int_{-\infty}^{+\infty} h(t)e^{-i\omega t} dt \quad (2.61) \quad h(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} H(\omega)e^{i\omega t} d\omega \quad (2.62)$$

This pair of Fourier transform is similar to the pair of the retardation function and its frequency-domain form, $\mathbf{k}(\tau)$ and $\mathbf{K}(\omega)$. The input is the velocity, $\dot{\mathbf{X}}(\omega) = i\omega\mathbf{X}(\omega)$ and the output is the radiation force, $(i\omega\mathbf{a}(\omega) + \mathbf{b}(\omega))i\omega\mathbf{X}(\omega)$ without the contribution from \mathbf{A}_∞ (Gao (2017)).

Using this the time dependent load can be calculated using the impulse response function and the wave elevation (Natsk ar and Steen (2013)), where the impulse function can be calculated as shown in Equation 2.63 using the transfer function for wave loads as calculated by potential theory.

$$f(t) = \frac{1}{2\pi} \int_0^\infty \mathbf{h}_{ZF}(\tau)\zeta(t - \tau)d\tau \quad (2.63)$$

2.6 Relation Between Frequency and Time Domain

If the loads, or response, are defined in frequency domain or time domain it is possible to interact between the different domains. The different methods for calculating response of a body subjected to pseudo-random irregular waves characterized by a wave spectra is shown in Figure 2.13. For a linear system the response will be the same going from the wave spectrum over to the time domain integration and back to the response spectrum as going directly down on the frequency-domain side. Fast Fourier transformation is an efficient numerical technique of transforming a time signal to a frequency-domain representation in the form of a Fourier transform, similar to a Fourier series. Inverse Fourier transformation can be used from the frequency domain to the time domain (Bergdahl (2009)).

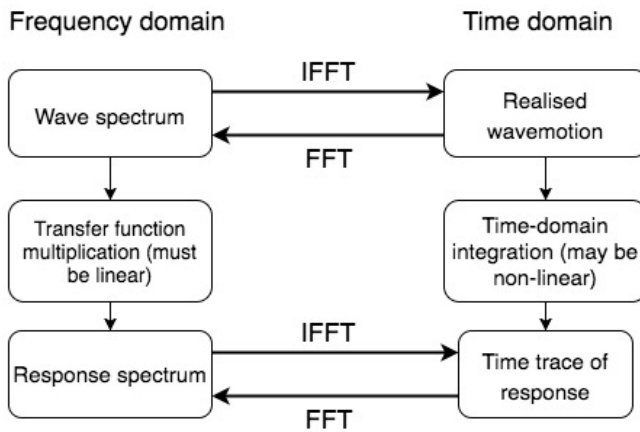


Figure 2.13: Frequency Domain vs. Time Domain (Bergdahl (2009)).

2.7 Second-Order Non-linear Hydrodynamic Loads

In the linear solution as described in Chapter 2.2, both the free-surface condition and the body boundary conditions are satisfied on respectively the mean position of the free-surface and the submerged hull surface. The fluid pressure and the velocity of fluid particles on the free-surface are also linearized. However, in second order theory the non-linearities in the velocity of fluid particles on the free surface are accounted for. This is done by keeping all terms in the velocity potential, fluid pressure and wave loads linear with either the wave amplitude or proportional to the square of the wave amplitude. The solution of the second order problem results in mean forces, and forces oscillating with difference-frequency and sum-frequencies in addition to the linear solution. The second-order forces are often neglected because they might be small compared to the first-order force, but they are important for structures with low natural frequencies (Faltinsen (1993)).

2.7.1 Mean Wave Drift Forces and Moments

In potential-flow theory, the mean wave drift forces originate from the diffraction and radiation of waves from the body. This implies that these loads are greater for large structures compared to the wave length, because of their capability of wave making. For a surface piercing body a major contribution to the horizontal mean wave force is due to the relative vertical motion between the structure and the waves. This causes some of the body surface to be part of the time in water and part of the time out of water. This results in a non-zero mean pressure even in regular harmonically oscillating waves. If the relative vertical motion differs around the waterline, the result is a non-zero mean force.

Unlike for the slowly-varying loads, it can be shown that it is only the first-order velocity potential that contributes to the mean drift force. Hence, the mean drift force can be obtained from a linear potential-flow analysis. There are several calculation methods which can be used like Maruo's formula and direct pressure integration. When the results of mean wave loads in regular waves are known, it is easy to obtain results in an irregular sea as the mean force contribution from each wave component can be added together using super-position.

2.7.2 Slow Drift Loads

Slow drift motions are resonance oscillations excited by non-linear interaction effects between the waves and the body motion. The slow drift motions are caused by a sea state consisting of waves with at least two different frequencies, where the difference between them leads to resonance, $\omega_i - \omega_j$. Slow-drift motions are of equal importance as the linear first-order motions in design of mooring systems for large volume structures. The slow-drift resonance oscillations occur in the surge, sway and yaw motions for moored structures, and can occur in heave, pitch and roll for free floating structures.

A general formula for slow-drift excitation loads can be expressed as seen in Equation 2.64 for irregular waves. Where A_i is the wave amplitudes and the coefficients T_{jk}^{ic} and T_{jk}^{is} can

be interpreted as second-order transfer functions for the difference frequency loads.

$$F_i^{SV} = \sum_{j=1}^N \sum_{k=1}^N A_j A_k [T_{jk}^{ic} \cos(\omega_k - \omega_j)t + (\epsilon_k - \epsilon_j) + T_{jk}^{is} \sin(\omega_k - \omega_j)t + (\epsilon_k - \epsilon_j)] \quad (2.64)$$

2.7.3 Sum-Frequency Effects

There are non-linear effects which have higher frequencies than the dominant frequency components in the wave spectrum. This is due to terms oscillating with frequencies $2\omega_1$, $2\omega_2$ and $\omega_1 + \omega_2$. These may be important for exciting the resonance oscillations in the heave, pitch and roll of a tension leg platform.

In this thesis only linear hydrodynamic loads will be considered. But the mean drift force was calculated from WADAM and imported into HydroD. This is neglected in this work, but it is the most important second-order effect for this problem (Greco (2012)).

Chapter 3

Damping

Damping influences an oscillating system by reducing, restricting or preventing its oscillations. In damped vibrations, external resisting forces act on the vibrating object. The object loses energy due to resistance and as a result, the amplitude of vibrations decreases exponentially. This chapter will give further insight into how such a system can be modeled. The first section focuses on general damped system, while the second and third section focuses on roll damping.

3.1 Damped System with one Degree of Freedom

The equation of the motion is $m\ddot{u} + b\dot{u} + cu = 0$. In the absence of a damping term, $b=0$, the natural frequency of the system would be $\omega_0 = \sqrt{c/m}$. In this case a system with damping will be investigated.

This equation of motion can be solved by assuming $u = Ce^{st}$. The characteristic equation of non-trivial solutions can be seen in Equation 3.1 (Larsen (2009)).

$$s = \omega_0 \left(-\frac{b}{2m\omega_0} \pm \sqrt{\left(\frac{b}{2m\omega_0}\right)^2 - 1} \right) \quad (3.1)$$

The damping factor b can give 3 different types of solutions: critical damping, over damped and under damped. The amplitudes for each case can be seen as a time series in Figure 3.1.

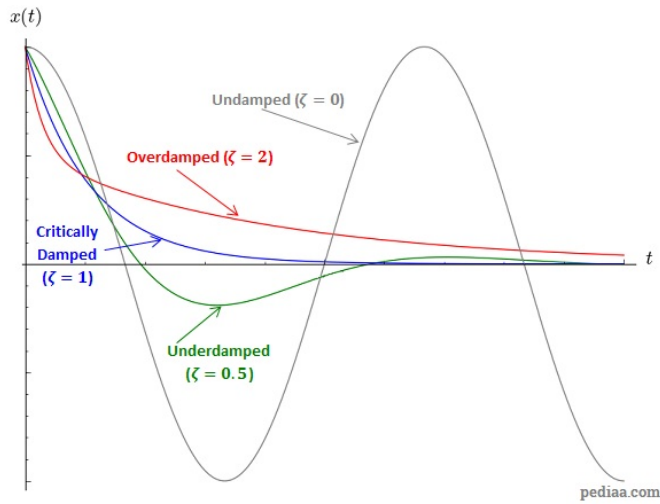


Figure 3.1: Difference between Damped and Undamped Systems (Nipun (2017)).

3.1.1 Critical Damping

For critical damping Equation 3.2 is used, where factor b only depends on the stiffness and the mass. When the real damping is equal to the critical one, there will not be any fluctuations, merely an asymptotic approach towards the static equilibrium.

$$b = b_c = 2m\omega_0 = 2m\sqrt{\frac{k}{m}} = 2\sqrt{mk} \quad (3.2)$$

In marine structures, damping is never as large as the critical, but it is useful as a reference for the real damping. The damping ratio is a parameter that characterizes the frequency response of a second order ordinary differential equation. This can be seen in Equation 3.3. If $\xi = 1$, the system has critical damping. Under this condition, the oscillating object returns to its equilibrium position as soon as possible without completing any more oscillations.

$$\xi = \frac{\text{actual damping}}{\text{critical damping}} = \frac{b}{b_c} = \frac{b}{2m\omega_0} \quad (3.3)$$

3.1.2 Over-Damped

If s in Equation 3.1 is a real number the solution is simply decaying exponential with no oscillation. This case occurs for $\xi > 1$.

3.1.3 Under-Damped

This is the most normal situation for most marine structures, where the object continues to oscillate, but with an amplitude decreasing exponentially as $e^{i\omega_n\sqrt{1-\xi^2}t}$. This case occurs for $0 < \xi < 1$. If the system is under-damped, s in Equation 3.1 looks like Equation 3.4.

$$s = \omega_0(-\xi \pm \sqrt{\xi^2 - 1}) = -\xi\omega_0 \pm i\omega_0\sqrt{1 - \xi^2} \quad (3.4)$$

Introduction of a new term, damped natural frequency, can be useful. This is seen in Equation 3.5, the natural period can thereafter be found as in Equation 3.6. In most practical cases this will be similar to the natural frequency.

$$\omega_d = \omega_0\sqrt{1 - \xi^2} \quad (3.5) \quad T_d = \frac{2\pi}{\omega_d} \quad (3.6)$$

3.2 Roll Damping

In the 1970's, strip methods for predicting ship motions in 5-degree of freedoms in waves were established. The methods are based on potential flow theories, Ursell-Tasai method, source distribution method etc., and can predict pitch, heave, sway and yaw motions of ships in waves with fairly good accuracy (Kawahara et al. (2012)). Strip methods, however, do not work well on roll motion. The total roll damping of a floating vessel can be divided into potential and viscous components. The potential component can be predicted accurately since it has a linear characteristic, however the viscous component is non-linear and prediction of this is more problematic. Therefore, some empirical formulas or experimental data are used to predict the roll damping.

Several papers which both summarize the work of others and present a fundamentally new practical estimation technique for roll damping were presented in the late 1970's (Falzarano et al. (2015)). This technique separates the roll damping into components and ignores their interactions and the coupling of roll to other degrees of freedom (Ikeda et al. (1978)). The components are shown in Equation 3.7 and consist of the frictional (B_f), the wave (B_w), the eddy (B_e) and the bilge keel (B_{BK}) components at zero forward speed. At forward speed, the lift (B_L) is added. It should be noted that this method summarizes the roll-damping coefficients. This may not be justifiable hydro-dynamically, but the subdivision is a convenience which allows computation of the individual components (Chakrabarti (2000)). The $B_{44,eq}$ is therefore equal to the equivalent damping coefficient shown in Section 2.4.

$$B_{44,eq} = B_f + B_e + B_w + B_L + B_{BK} \quad (3.7)$$

This method is used for general cargo ship hulls and may not be applicable to shallow draft, high beam to draft ratio, transport barges and similar vessels (Falzarano et al. (2015)).

3.2.1 Potential Roll Damping

Wave Damping

Wave damping is caused by free surface waves and is thus a function of the wave parameters. It is computed quite accurately from the linear diffraction/radiation theory (ref Section 2.2.2). This is therefore the only damping used in other degrees of freedom with reasonable success in providing satisfactory results.

In the presence of forward speed, empirical formulas specified by Ikeda et al. (1978) should be avoided. Instead a panel method theory developed by Salvesen et al. (1970) should be used to predict the forward speed added mass and radiation damping, as this is theoretically accurate and provides more correct results (Falzarano et al. (2015)).

For the present case, a rectangular barge, there is no forward speed or current. In addition, the barge is symmetric about the x-y and the y-z axis as shown in Section 4.1.2. Hence the linear damping will be, $B_{jk} = B_{kj}$ for coupled effects (Faltinsen (1993)).

3.2.2 Viscous Roll Damping

The radiation potential can not include viscous effects, which are non-linear with the roll amplitude and velocity. The viscous part of the roll motion increases the damping significantly. Hence the motion and forces in sea fastening will be highly overestimated (Natskår and Steen (2013)).

Skin Friction

Friction damping is caused by viscous skin friction stress on the hull surface of the ship form as the ship rolls. The empirical expression for ship friction damping coefficient for laminar flow and zero speed is shown in Equation 3.8, and for turbulent flow in Equation 3.9 (Kato (1957)).

$$B_{f0} = \frac{4}{3\pi} \rho S r_e^3 R_0 \omega C_f \quad (3.8)$$

$$B_{f0} = 0.787 \rho S r_e^2 \sqrt{\omega \nu} \left(1 + 0.00814 \left(\frac{r_e^2 R_0^2 \omega}{\nu} \right)^{0.386} \right) \quad (3.9)$$

Where S is the wetted surface area, r_e is the effective bilge radius, R_0 is the roll amplitude and C_f is the friction coefficient.

Eddy Making Damping

The viscous eddy damping considered for a naked hull is caused by pressure variation on the naked hull due to flow separation. This is caused by sharp corners and the shedding of

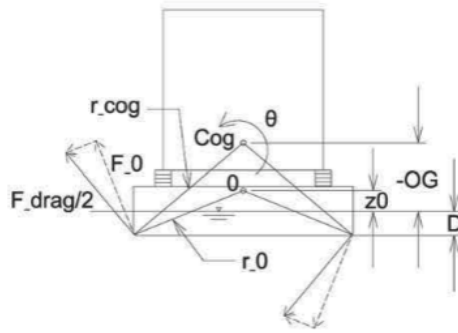


Figure 3.2: Cross Section and Definition of Drag Forces (Natskår and Steen (2013)).

vortices around the bottom of the ship. For slender ships, the vortices are shed from the forward and the aft regions, near the stem and stern and at the bilge circle near midship. For a vessel with fuller shape, as a barge-like structure, the mid ship region contributes significantly to the phenomenon. The empirical formula for estimating the eddy damping is similar to the estimation of drag force on a cylinder using a drag coefficient.

This method is shown by Natskår and Steen (2013) and Chakrabarti (2000) based on Tanaka (1961). The force from the viscous damping can be written as the drag part in Morrisons equation, expressed as in Equation 3.10.

$$F_{drag} = \frac{1}{2} \rho C_d A_{wet} r_{cog}^2 |\dot{\theta}| \dot{\theta} \quad (3.10)$$

The force acts on the submerged part of the barge. However as it is mainly due to vortex shedding it is assumed to act as a concentrated force at the barge bilges, $F_{drag}/2$ at each side. This can be seen in Figure 3.2. The roll moment about point 0 is expressed as $M_4 = B_{44,2} |\dot{\theta}| \dot{\theta} \approx F_{drag} r_0$.

The damping can then be expressed as in Equation 3.11, where $r_{cog} = \sqrt{(B/2)^2 + (D - OG)^2}$ and $OG = D - KG$.

$$B_{44,2} = \frac{1}{2} \rho C_d A_{wet} r_{cog}^2 r_0 = \rho C_d L D^4 \left(H_0^2 + \left(1 - \frac{OG}{D} \right)^2 \right) (H_0 + 1) \sqrt{H_0^2 + \left(1 + \frac{z_0}{D} \right)^2} \quad (3.11)$$

This equation is more applicable for smaller roll angles, and tends to overestimate the experimental damping (Chakrabarti (2000)). This should be taken into account and values should be treated with care as vortex shedding is the dominant roll damping component for a box shaped floating vessel (Hajjarab et al. (2011)).

3.2.3 Bilge-Keel Damping

The bilge keel is an effective and efficient way to increase the overall damping and thus stabilize excessive roll motion of a ship or a barge. When the bilge keel is present, it produces additional damping due to normal force on the keel plus pressure variation in the hull surface caused by the presence of the bilge keel. These contributions vary with the amplitude of roll and the wave frequency. The total bilge keel damping may be separated into two components: the damping of the bilge keels themselves, normal pressure damping, and the interaction between the bilge keels, hull and waves, which gives the hull damping. This can be presented as $B_{BK} = B_{BKN} + B_{BKH}$. In Equation 3.12 the normal force component is shown and in Equation 3.13 the pressure component of damping is shown, both per unit length.

$$B_{BKN} = \frac{8}{3\pi} \rho r_{cb}^3 b_{BK} \omega R_0 f^2 C_D \quad (3.12)$$

$$B_{BKH} = \frac{4}{3\pi} \rho r_{cb}^3 D^2 \omega R_0 f^2 \left(- \left(-22.5 \frac{b_{BK}}{\pi r f R_0} - 1.2 \right) A_2 + 1.2 B_2 \right) \quad (3.13)$$

As the barge for this task will be modeled without bilge-keel this will be zero.

3.2.4 Lift Damping

The lift damping in roll occurs in the form of a lift moment similar to the lift force caused by a ship moving forward with the sway motion. This can be calculated by Equation 3.14 (Ikeda et al. (1978)).

$$B_L = 0.075 \rho U L D^3 k_N \left(1 - 2.8 \frac{OG}{D} + 4.667 \left(\frac{OG}{D} \right)^2 \right) \quad (3.14)$$

The lift damping varies linearly with the speed U and is independent of the frequency of roll motion. This will therefore be negligible in the case of a barge with zero forward speed.

3.2.5 Effect of Forward Speed

In SIMO it is not possible to run an analysis with forward speed, this would however make the case more complex. In this thesis, it will be neglected, which will result in some accuracy compared to the reality. If the forward speed had been included, two of the roll damping coefficients could be affected by the forward speed. The eddy damping coefficient will be significantly reduced with forward speed. Whereas in this task with zero forward speed it may be the dominant damping coefficient. Lift damping is however negligible for zero speed but varies linearly with the speed and becomes dominant. The bilge keel component is assumed to be unaffected by forward speed (Falzarano et al. (2015)).

3.3 Overview of Roll Damping Systems

Roll motion of vessels can be one of the most important motions to damp, thus several systems have been developed for the sole purpose of roll damping. The following section will consist of a brief discussion of bilge keel and anti-roll tanks. There are several other systems to damp the roll motion as well, but these two are widely used in addition to active fins.

3.3.1 Bilge Keel

Bilge keels are flat plates protruding from the ship side. The longitudinal lengths of the keels are about 25 to 50 % of the length of the vessel, with depths of about 1-3 % of the width. In Section 3.2.3 an explanation of how it effects the roll motion is given.

3.3.2 Anti-roll Tanks

Anti-roll tanks provide damping by containing tanks of water that move with the same velocity as the ship's rolling but with another phase, creating a counteracting moment that reduces the net excitation moment. The tanks are tuned by the water height of the tanks, and are often tuned to have maximum contribution at the natural roll period, for obvious reasons. Passive tanks have been measured to reduce the resonant roll motion by up to 90 % and significant roll motion by up to 50-60 % (Pettersen (2007)).

3.4 Experimental Methods for Analyzing Roll Damping

Several methods and techniques of analyzing roll damping experimentally and numerically exist. The methods used for analyzing the barge in this thesis are decay test, regular waves and irregular waves.

3.4.1 Decay Test

Performing a decay test is easy, fast and cheap compared to other methods. The decay test is an identification test to obtain information on natural periods and damping of the model. It is essential to know these parameters to avoid resonance and excessively heavy loads that could lead to damage and fatigue. The test is done by applying a constant force or moment to the structure causing it to deflect, followed by a release of the force or moment. When released, the system will react by seeking back to its equilibrium position, showing harmonical oscillation of motions before it goes back to rest. By analyzing the motion, the damping and natural period of the system are revealed.

3.4.2 Simulations in Regular and Irregular Waves

Regular waves are in principal harmonic sinusoidal waves. The results of regular waves can not describe how a vessel will behave in a typical environment, these waves can nonetheless produce valuable information on how the vessel will respond to different wave frequencies. In this thesis the barge will be analyzed in regular waves to validate the results between time and frequency domain. In order to obtain more realistic simulation scenarios, irregular waves are carried out.

It is not well established or used as an experimental method. A challenge to this method is to measure or estimate the work applied on the model in roll by the waves, which will be needed to calculate the roll damping. It is an advantage that it is a realistic way of testing since the coupling between sway, roll and yaw is included, thus the centre of rotation will vary. Also, this method will, if any, capture slow drift damping in the system.

Chapter 4

Procedure

In this chapter an explanation of how the modelling is done will be presented, in addition to an explanation of the different analyses carried out. A summary of the programs used in this thesis will also be presented.

4.1 Model

4.1.1 Main Dimensions

The model consists of a barge with a jacket on top, an example of which was seen in Figure 1.1 in Section 1.1. Here they are modelled as one mass with one centre of gravity. The forces between the structures are therefore not taken into account. The main dimensions of the model can be seen in Table 4.1.

Table 4.1: Main Dimensions of the Model.

Draft APP [m]	7.8
Weight [t]	119662
Length of bilge keel [m]	102.5
Width of bilge keel [m]	0.8
System LCG [m]	133.79 (from APP)
System TCG [m]	-0.001
System VCG [m]	30.81
Radius of Gyration of Roll	33.58
Radius of Gyration of Pitch	59.81
Radius of Gyration of Yaw	59.11

4.1.2 Coordinate System

The coordinate system of the barge alone can be seen in Figure 4.1. The x -axis is positive in the forward direction, the y -axis is positive in the port side direction and the z -axis is positive in the vertical upwards direction. It has origo in the projection of the CoG at the waterline, meaning the CoG is in most cases shifted just slightly forward of midship and up from the keel to the waterline (mean draught) depending on the loading condition.

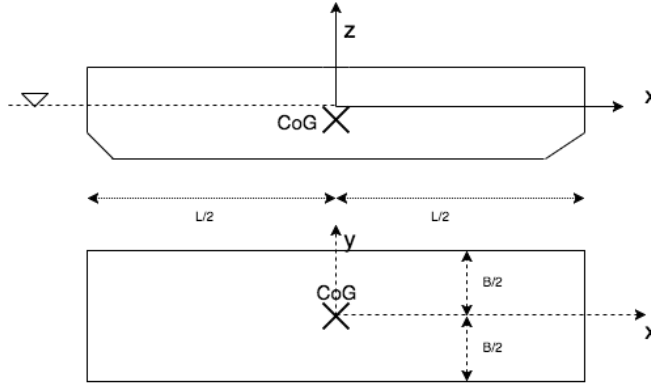


Figure 4.1: Coordinate System for the Barge.

A barge will oscillate with certain amplitudes in time within a certain range of frequencies, and in space within a certain range of wavelengths. The movements of an arbitrary point can be written as in Equation 4.1

$$\mathbf{s} = (\eta_1 + z\eta_5 - y\eta_6)\mathbf{i} + (\eta_2 - z\eta_4 + x\eta_6)\mathbf{j} + (\eta_3 + y\eta_4 - x\eta_5)\mathbf{k} \quad (4.1)$$

4.2 Model from HydroD

The panel model was made in HydroD with dimension as described in Section 4.4.1 and then imported into SIMA. After some adjustment several analyses were done in both regular and irregular waves. The imported model contained values from the potential theory only. Running time domain simulations in SIMA the barge will have zero forward speed, to achieve realistic results for a barge with forward speed some of the parameters had to be corrected.

4.2.1 Restoring Forces and Moments

The stiffness matrix obtained from HydroD can be seen in Table 4.2. To avoid large offset in the horizontal motions when waves are added from different directions, stiffness

is added in surge, sway and yaw. Before this was done SIMA displayed an error message *Heading of body outside of range*, meaning that the barge had too much yaw rotation.

Table 4.2: Stiffness Matrix from HydroD.

	x	y	z	rx	ry	rz
x	0	0	0	0	0	0
y	0	0	0	0	0	0
z	0	0	1.7163e+08	0	1.3724e+09	0
rx	0	0	0	3.18e+10	0	-6.522e+08
ry	0	0	1.3724e+09	0	8.6682e+11	-1.1735e+06
rz	0	0	0	0	0	0

The stiffness was added using the equation from Section 2.3.3. The natural period is set as a constant at 60 seconds in all three directions. When assuming that the periods are uncoupled, although this is not the case in reality, the stiffens can be calculated as seen in Equation 4.2 for surge and sway. Equation 4.3 is used to calculate the stiffness in yaw.

$$C_{11} = C_{22} = \frac{M_{ii} + A_{ii}(\omega)(2\pi)^2}{60^2} \quad (4.2)$$

$$C_{66} = \frac{Mr_{66}^2 + A_{66}(\omega)(2\pi)^2}{60^2} \quad (4.3)$$

As stated in Section 2.3.1 the added mass and damping coefficients have a strong frequency dependence. This means that the added mass has to be found at 60 seconds. In Figure 4.2, 4.3 and 4.4 the frequency dependent added mass can be seen. The added mass is read to be 4.44e06 in surge 3.69e7 in sway and 1.35e11 in yaw.

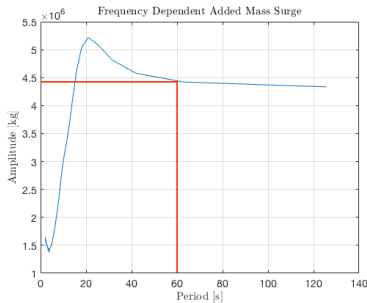


Figure 4.2: Frequency Dependent Added Mass in Surge.

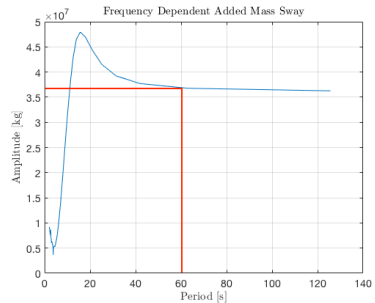


Figure 4.3: Frequency Dependent Added Mass in Sway.

The final stiffness matrix ended up as seen in Table 4.3.

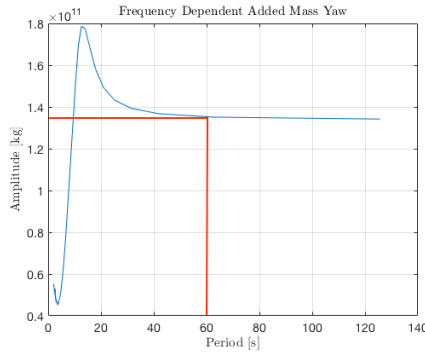


Figure 4.4: Frequency Dependent Added Mass in Yaw.

Table 4.3: Final Stiffness Matrix Used in SIMA.

	x	y	z	rx	ry	rz
x	1.36e+06	0	0	0	0	0
y	0	1.716e+06	0	0	0	0
z	0	0	1.7163e+08	0	1.3724e+09	0
rx	0	0	0	3.18e+10	0	-6.522e+08
ry	0	0	1.3724e+09	0	8.6682e+11	-1.1735e+06
rz	0	0	0	0	0	6.068e+09

4.2.2 Linear Damping

The linear damping matrix obtained from HydroD can be seen in Table 4.2. This is the damping due to potential theory only. As seen in Section 3.2, the only roll damping component from the potential theory is the wave damping. This is seen in Equation 4.4.

$$B_{44,eq} = \underbrace{B_w}_{\text{Roll damping from potential theory}} + B_f + B_e + B_L + B_{BK} \quad (4.4)$$

DNV GL has found the total linear damping, 3.92e+9 [Nms], using StarCCM+. The potential damping from HydroD was 4.87e+05 [Nms]. The linear damping due to other sources than the potential damping can then be found as the difference between the total linear damping and the potential damping, 3.92e+9 [Nms] - 4.87e+05 [Nms]. This damping is added in the linear damping matrix in SIMA, 3.9195e+9.

In Section 5.3.1 some results will be presented, where it will be shown that the roll motion is interrupted by coupled effects between sway, roll and yaw. In principle, we should not have any resonant motions in these degrees of motions for a ship with forward speed. This is corrected by adding damping in surge, sway and yaw. Total damping ratio as shown in Equation 3.3, should be about 15-20 % in surge, sway and yaw for a ship-type structure (DNV GL (2010)). The linear damping terms are calculated as seen in Equation 4.5 for

Table 4.4: Linear Damping Matrix from HydroD.

	x	y	z	rx	ry	rz
x	1298.1	0	0	0	0	0
y	0	12492	0	0	0	0
z	0	0	0	0	0	0
rx	0	0	0	4.87e+05	0	0
ry	0	0	0	0	0	0
rz	0	0	0	0	0	1.6056e+07

surge and sway and as seen in Equation 4.6 for yaw.

$$B_{11} = B_{22} = \xi 2(M_{ii} + A_{ii}(\omega)) \frac{2\pi}{60} \quad (4.5) \quad B_{66} = \xi 2(Mr_{66}^2 + A_{66}(\omega)) \frac{2\pi}{60} \quad (4.6)$$

The final linear damping matrix ended up as seen in Table 4.5.

Table 4.5: Final Linear Damping Matrix Used in SIMA.

	x	y	z	rx	ry	rz
x	3.899e+06	0	0	0	0	0
y	0	4.919e+06	0	0	0	0
z	0	0	0	0	0	0
rx	0	0	0	3.9195e+9	0	0
ry	0	0	0	0	0	0
rz	0	0	0	0	0	1.738e+10

4.2.3 Wave Drift Force

The wave drift forces can be calculated in WADAM using momentum conservation of pressure integration (Det Norske Veritas (2005)). However, the wave drift force is a second order hydrodynamic load, as described in Section 2.7.1, and was deleted in SIMA. The calculations done in HydroD and SIMA will only focus on the linear hydrodynamic loads.

4.3 Analysis in SIMA

4.3.1 Decay Tests

For a decay test the wave input must be low enough to be neglected. First a ramp force or moment are added, secondly a constant force or moment are added. The values found in Table 4.6 are used in the decay test.

Table 4.6: Characteristic Values Used in Decay Tests.

Motion	Force [kN] or Moment [kNm]	Simulation Length [s]	Ramp Duration [s]	Constant force duration [s]
Sway	2000	2000	50	100
Surge	2500	2000	50	100
Heave	2.5e+06	2000	50	100
Roll	2.5e+08	2000	50	100
Pitch	2.5e+08	2000	50	100
Yaw	25000	2000	50	100

As presented in Section 4.2.2, the only damping included in the model for heave and pitch is potential damping. It is therefore hard to find the natural period in the same manner as the other degrees of freedom. Instead of a normal decay test, response spectrum analysis of the time series are performed. The frequency with most energy is then the natural period in that degree of motion.

4.3.2 Analysis in Regular Waves

To compare results between the frequency domain and time domain analysis are carried out in regular waves. This is mainly done with two different wave directions, 45 and 90 degrees, some simulations are however carried out for 0 and 15 degrees as well. In addition, several wave amplitudes and periods are used. The interval for periods were chosen to be 5-20 seconds. However, as the natural period of roll is about 16 seconds some extra periods were added close to the natural period to capture the true behavior of the barge. Several analyses were done in regular waves using these parameters.

Time domain analyses in SIMA are analyzed using Matlab. Amplitudes in different motions can then be found. From the frequency domain solution the results are given as a response amplitude operator, as explained in Section 2.4.1, which is the ratio between the response amplitude of any studied variable to the wave amplitude. The RAO from the frequency domain solution is then compared to the RAO from the time domain solution.

Linear Damping

The first step is to check whether the two programs and analysis types give the same results. This is done by comparing the results with the linear damping only, indicating if the components in the equation of motion are similar or not.

Linearized Damping

Secondly the linear and quadratic damping are added. Unfortunately it is impossible to add linear and quadratic damping in HydroD using regular waves. The damping must therefore be linearized in frequency domain. To get accurate results, the linearization should be done for each sea state. Unfortunately, this is impossible in HydroD as the results are presented as a RAO for one wave heading. To get the linearization, Anders Juul Weiby has performed an iteration process in HydroD for regular waves with 45 degrees. This iteration process is presented in Section 2.4. The main equation can be seen in Equation 4.7. To compare results done in SIMA with Hydro D, the linearized damping was added to the linear damping matrix.

$$B_{44,eq} = B_{44,1} + \frac{8}{3\pi} B_{44,2} \eta_{4,max} \omega_n = 6.99e + 09 \quad (4.7)$$

As the iteration process is time consuming, this is done only for 45 degrees. The foundation for comparison results is therefore limited. For further work on verifying the linearization method, more wave headings should be used.

Quadratic Damping

The last step in regular waves is to add the quadratic damping. In SIMA, this is done by adding a new matrix *Quadratic Damping*. The value added in the matrix was found by DNV GL using StarCCM+ as seen in Section 4.4.4. They found the quadratic damping in roll to be 2.17e11 [Nms²].

4.3.3 Analysis in Irregular Waves

To analyze the barge for irregular waves a JONSWAP spectrum is used. Several combinations of environmental data are analyzed, where Hs, Tp and wave seed are changed for each simulation. For each condition, six three-hour simulations (after transient condition) are carried out. In Table 4.7 several condition numbers are defined. For the case of simplicity, these numbers are referred to in the next chapters to define a sea state.

Six wave seeds are chosen to compare the results between them and get a good basis for comparison. As seen in Section 2.1.3, every wave seed will give different wave realization and therefore different roll motion. The effect of different wave seeds will therefore be studied. When all the analyses are completed the standard deviation for roll motion for each simulation is found. The roll spectra between the different periods, wave heights and wave seeds will also be compared.

Table 4.7: Environmental Data for Analysis in Irregular Waves.

Condition number	Hs [m]	Tp [s]	Wave Directions [deg]	Wave Seeds
1	1.5	7.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
2	1.5	8.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
3	1.5	9.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
4	1.5	10.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
5	1.5	11.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
6	1.5	12.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
7	2.5	7.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
8	2.5	8.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
9	2.5	9.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
10	2.5	10.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
11	2.5	11.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
12	2.5	12.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
13	3.5	7.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
14	3.5	8.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
15	3.5	9.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
16	3.5	10.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
17	3.5	11.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
18	3.5	12.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
19	4.5	7.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
20	4.5	8.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
21	4.5	9.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
22	4.5	10.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
23	4.5	11.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
24	4.5	12.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
25	5.5	7.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
26	5.5	8.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
27	5.5	9.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
28	5.5	10.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
29	5.5	11.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
30	5.5	12.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
31	6.5	7.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
32	6.5	8.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
33	6.5	9.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
34	6.5	10.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
35	6.5	11.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202
36	6.5	12.5	0, 15, 45, 90	100, 101, 102, 200, 201, 202

4.4 Computational Tools

4.4.1 HydroD

HydroD is a platform combining the different SESAM programs for hydrostatic/stability analysis and hydrodynamic analysis. It is an interactive application for computation of hydrodynamics and stability, wave loads and motion response for ships and offshore structures. The wave loads and motions are computed by Wadam or Wasim (Det Norske Veritas (2009)). The loads calculated can be transferred to a structural mode, before structural analyses with respect to strength or fatigue are performed.

To determine the floating position, trim and draught, either actual mass and buoyancy can be used or the desired floating position can be defined. Compartments are then automatically filled. The hydrodynamic analysis may be performed using the actual floating position independently of the panel model to determine worst loading conditions to be used in structural strength analysis. These analyses are normally performed in the frequency domain.

In HydroD six different types of finite element models may be built (Det Norske Veritas (2009)). Panel FEM is the one used in this master thesis to calculate the hydrodynamic loads and responses from potential theory as described in Section 2.1.1. The roll damping in the potential theory is caused by free surface waves as seen in Section 3.2.1.

WADAM

For a vessel with forward speed, WASIM is used to calculate the wave loads and motions, while WADAM is used for a vessel without forward speed. Therefore the program used for this task will be WADAM as forward speed is neglected.

WADAM (Wave Analysis by Diffraction And Morison theory) calculates loads using Morison's equation for slender structures and first and second order 3D potential theory for large volume structures. For structures which consist of both slender and large volume parts, Morison's equation and potential theory are used (Det Norske Veritas (2005)).

Wave loads are calculated as described in Newman (1993), where potential theory is used, to calculate first order radiation and diffraction effects on large volume structures. The actual implementations are based on Wamit, which uses a 3D panel method to evaluate velocity potentials and hydrodynamic coefficients (Det Norske Veritas (2005)).

4.4.2 SIMA

SIMA is a workbench developed by MARINTEK, and is now included in the SESAM package offered by DNV GL (2018). It offers a complete solution for the simulation and analysis of marine operations and floating systems. It includes the entire process from modelling, definition of the simulation and its execution, to interpretation, post-processing and documentation of the results. 2D and 3D graphical presentations of the system, give a

quick understanding of the results (Sintef (2018)).

The dynamic analyses in SIMA are based on time integration, where the results are calculated from equilibrium equations step by step in time domain. SIMA workbench consists of several numerical softwares such as SIMO, RIFLEX, VIVANA and so on. For coupled analysis, the workflow can be seen in Figure 4.5. For the simulation of the barge, SIMO will be the main numerical software.

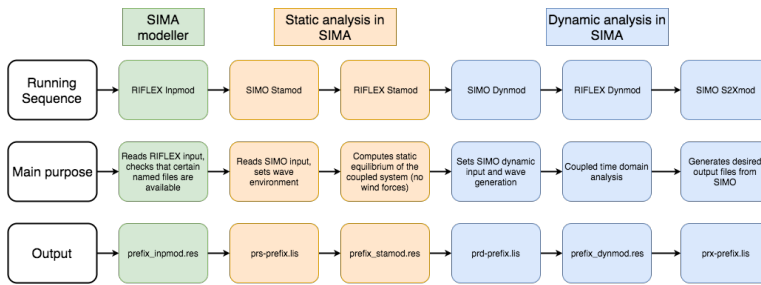


Figure 4.5: SIMA Program Flow Chart.

SIMO

SIMO is a computer program for time-domain simulation of motions and station-keeping behaviour of complex systems of floating vessels and suspended loads. Typical scenarios which can be simulated by the program are offshore crane operations, offshore loading and TLP tether installation (SIMO project team (2012a)).

Time domain simulation is used for solving the equation of motion. This means that non-linearities are allowed for all terms in the equation, and any excitation forces may be applied and transient responses are calculated.

The barge is assumed to be stationary and a stiffness matrix for the horizontal direction will be used for station-keeping. When the body is assumed to have the same position during the simulations, FFT can be used to obtain time series of wave excitation forces.

Hydrodynamic coefficients, like added-mass and radiation damping, first order wave force and second order mean drift forces, are usually obtained from a diffraction solver like WADAM. The results from WADAM are imported into SIMO.

Two main types of bodies can be created in SIMA, where the loading is defined as frequency-dependent hydrodynamic coefficients or position-dependent hydrodynamic coefficients. The first one corresponds to large bodies and includes 6 dofs, while the second one is used for small bodies and includes 3 dofs only. Consequently the first will be used to model the barge (SIMO project team (2012b)).

To calculate the natural periods and eigen vectors, mass and stiffness matrices are calculated, and the system is solved by a standard Jacobian solver. The mass matrix is built from the virtual mass matrix, including both structural and hydrodynamic mass. The frequency dependency of the latter is not accounted for in the analysis as asymptotic values are used.

This means that added mass at infinite frequency is used (SIMO project team (2007)).

The results are presented as plots of time series, spectral analysis and extreme value estimates. This can be done for the position, velocity, acceleration, wave elevation or wave particle motion.

4.4.3 Matlab

Matlab is a high-performance language for technical computing. It combines numerical analysis, matrix computation, signal processing, and graphics in an easy-to-use environment where problems and solutions are expressed just as they are written mathematically, without the need of traditional programming (The MathWorks (2004)).

WAFO

A toolbox made for Matlab is called WAFO (wave analysis for fatigue and oceanography). This can be used to analyze the data from SIMO by running statistical analysis in Matlab (Brodtkorb et al. (2000)). WAFO allows treatment of more complicated problems for example spatial waves with time dynamics can be handled thus extending the analysis to random fields.

In the WAFO toolbox the function *dat2spec* is used to perform spectral analysis of the roll motion in irregular waves. The default option is frequency smoothing using a parzen window function on the estimated autocovariance function. One of the parameters that can be changed is *L*, maximum lag size of the window function. As *L* decreases the estimate becomes smoother and the bandwidth increases.

Spectrum bias is caused by the fact that an infinite length sequence is reduced to a finite length sequence. This can be seen as a windowing of an infinite sequence of data (Lindgren et al. (2013)). The bias of the peaks is caused by the main lobe width and the bias of the peaks decreases with increasing *n*.

4.4.4 StarCCM+

STAR-CCM+ software is an all-in-one solution for multidisciplinary engineering simulation including computational fluid dynamics (CFD), computational solid mechanics (CSM), heat transfer, multiphase flow, particle dynamics, reacting flow, electrochemistry, acoustics and rheology (Siemens PLM Software (2016)).

Prior to this thesis, DNV GL has used StarCCM+ to do a CFD analysis of the model. The results obtained can be seen in 4.8. STARCCM+ offers both a finite-volume and finite-element method. This is used to get comparable results of the damping and period in roll.

Table 4.8: Quantities Extracted from the StarCCM+ Simulation.

Quantity	Value	Unit
Ratio of linear Damping to critical damping	0.024	[-]
Ratio of quadratic damping to critical damping per motion amplitude	0.0091	[1/deg]
Roll period	16.0	[s]
Stiffness	3.21e10	[Nm]
Estimated total roll inertia (including added mass)	2.08e11	[kgm ²]
Estimated linear damping	3.92e9	[Nms]
Estimated quadratic damping	2.17e11	[Nms ²]

Chapter 5

Results from SIMA

In this chapter all results obtained in SIMA will be presented and discussed. The barge is analyzed in regular and irregular waves, and the results will be compared to time domain simulation results in the next chapter.

5.1 Decay Test

In Figure 5.1 the behaviour of the barge in surge can be seen for the decay test. The ramp force is applied from 50 [s] to 100 [s] and the constant force is applied from 100 [s] to 200 [s]. The offset of the barge in surge motion is in meters and after the force is released a decreasing motion can be seen. In MATLAB, a fitting to the SIMA curve (red dotted line) was performed and the dimensionless linear and quadratic damping are found as b_1 and b_2 . Multiplied with the mass, the true linear and quadratic damping coefficients are found.

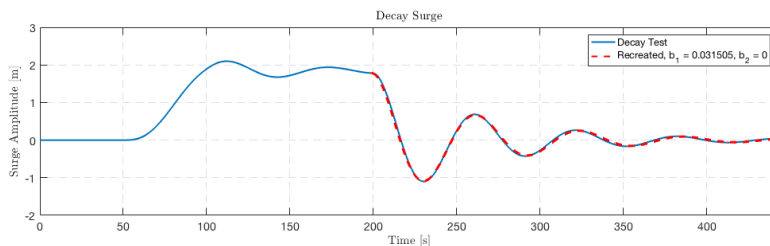


Figure 5.1: Results for Decay Test in Surge.

Figure 5.2 represents the motion of the barge in sway. The damping coefficient for this motion is retrieved in the same manner as for surge.

The same procedure is followed for the remaining motions roll and yaw, however they have

5.1. DECAY TEST

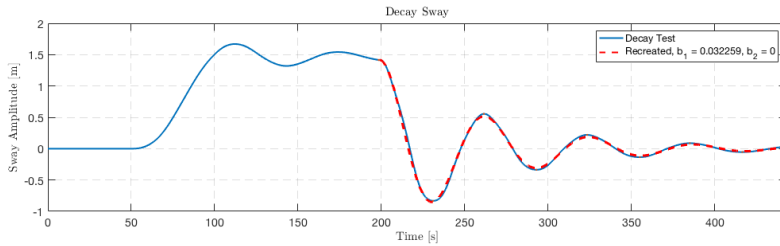


Figure 5.2: Results for Decay Test in Sway.

offsets in degrees, not meters, as they are rotations and not translations. The results can be seen in Figures 5.3 and 5.4, respectively. To find the true linear and quadratic damping coefficients, b_1 and b_2 have to be multiplied with the moment of inertia.

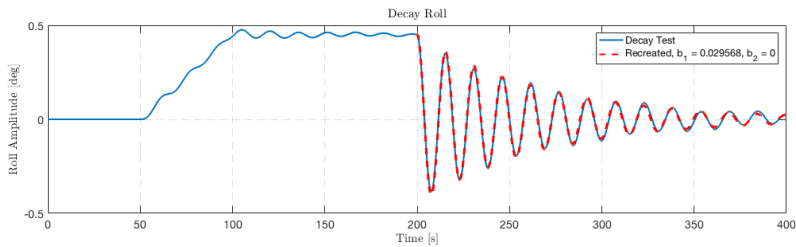


Figure 5.3: Results for the decay test in Roll.

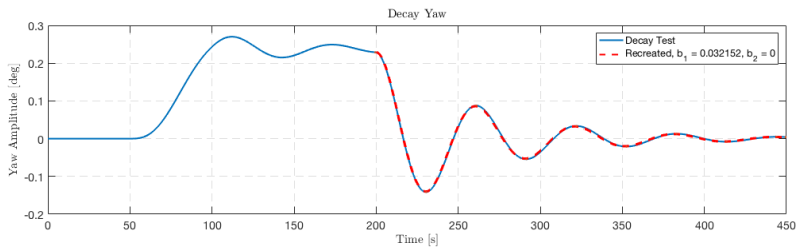


Figure 5.4: Results for the decay test in Yaw.

After the system is released it is evident that the motions decrease. This is because of the damping. Graphs of the descending speed in the system can be seen in Appendix A.

The time series for heave and yaw are seen in Figures 5.3 and 5.4, respectively. In these degrees of freedom, it was impossible to do the curve fitting for finding b_1 and b_2 , as these degrees of motion are zero in the matrix in SIMA.

The damping coefficient results from SIMA can be seen in Table 5.1. They coincide well with the damping added in SIMA. Hence, the Matlab script works well and provides correct values that are trustworthy. The coefficients b_1 and b_2 are dimensionless damping coefficients. They are therefore multiplied with the mass, $\mathbf{M} + \mathbf{A}$, for the translation

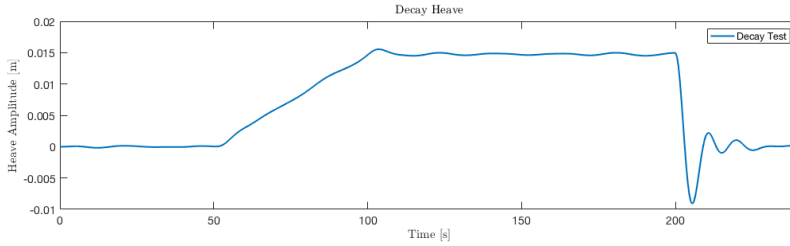


Figure 5.5: Results for Decay Test in Heave.

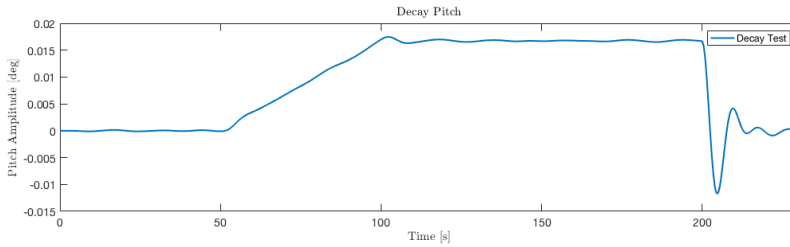


Figure 5.6: Results for Decay Test in Pitch.

and moment of inertia, $\mathbf{M}r^2 + \mathbf{A}$, for the rotations, in order to get the correct damping coefficient (\mathbf{B}).

Table 5.1: Damping Coefficients in Different Degrees of Motion.

	b_1	B_1 from Decay test	B_1 from SIMA	Difference
Surge	0.031505	3.91e+06	3.899e+06	0.28 %
Sway	0.032259	5.05e+06	4.919e+06	2.59 %
Roll (Quadratic)	0.029568	6.99e+09	6.787e+09 (linearized)	3.6 %
Yaw	0.0321525	1.78e+10	1.738e+10	2.36 %

In the decay test for roll, quadratic damping is used. However the Matlab program only calculates b_1 while b_2 is found as zero. The result is, however, similar to the linearized roll which results in the same motion as the quadratic damping. This will be presented in the coming sections. Table 5.1 indicates consistency between estimated and actual damping coefficients.

5.1.1 Natural Periods

The barge is designed to have natural periods at 60 seconds for surge, sway and yaw. The natural periods are obtained from the decay test, and can be seen in Table 5.2. From this, it is shown that the natural period is a bit shorter than expected, this may be due to damped natural period. SIMA uses the damped natural frequency instead of the natural frequency

5.1. DECAY TEST

in all calculations. This can be seen in Section 3.1.3. The damped natural period and frequency can be calculated. In surge, sway and yaw, 15 % of the critical damping is added which means $\xi = 0.15$. For roll, the percentage of critical damping is calculated to be 14.2 %. However, one may see that the natural period still has a difference of around 8 %. This may be due to coupled motions and that the damping in other degrees of motion effect the damping in the motion we are looking at.

Table 5.2: Natural Periods from Decay Test

	Obtained T_0	Obtained ω_0	Calculated T_d	Difference
Surge	53.50	0.0187	59.32	9.8 %
Sway	54.375	0.0184	59.32	8.33 %
Roll	14.85	0.0674	15.84	6.25 %
Yaw	54.33	0.0184	59.32	8.41 %

Even though the damping in heave and pitch are zero, the stiffness in these directions will result in a natural period. A spectral analysis of the motions after the force is released in the decay test is therefore carried out. The response spectrum can be seen in Figures 5.7 and 5.8.

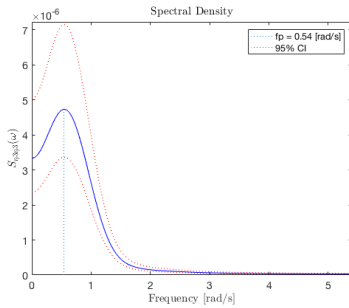


Figure 5.7: Heave Response Spectrum after Decay Test.

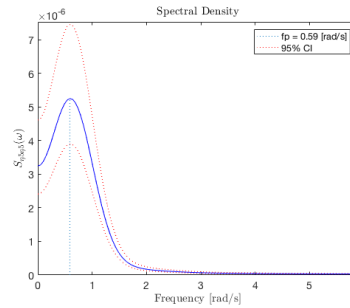


Figure 5.8: Pitch Response Spectrum after Decay Test.

The natural periods obtained from the spectral analysis can be seen in Table 5.3.

Table 5.3: Natural Periods from Spectral Analysis.

	Obtained T_0	Obtained ω_0	Calculated T_0	Difference
Heave	11.636	0.0859	10.933	6.04 %
Pitch	10.649	0.0939	10.516	1.25 %

5.2 Wave Force

In Section 4.2.3, it was explained that the wave drift force was removed from the model as HydroD does not take second order effects into account. In Figure 5.9, roll motion without wave drift force can be seen. And in Figure 5.10, roll motion with wave drift force is presented. It is seen from the figures that the roll motion with wave drift force is altered and oscillates around a non zero value. The mean wave force is therefore removed to avoid this problem.

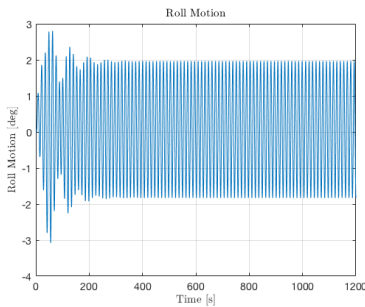


Figure 5.9: Roll Motion Without Wave Drift Force.

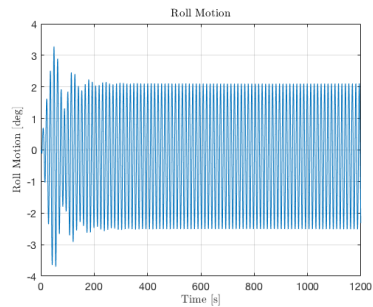


Figure 5.10: Roll Motion With Wave Drift Force.

5.3 Different Damping in Regular Waves

5.3.1 Damping from HydroD

In the first analysis the only change made in the damping matrix was for roll motion. This resulted in large fluctuations in surge, sway, roll and yaw. When linear and quadratic damping is added in roll, the motions for wave amplitude 6 meters and peak period 13 seconds can be seen in Figures 5.11, 5.12, 5.13 and 5.14. The same motions for wave amplitude 1 meters and peak period 13 can be seen in Appendix B, as well as for linear roll damping only for the same parameters.

5.3. DIFFERENT DAMPING IN REGULAR WAVES

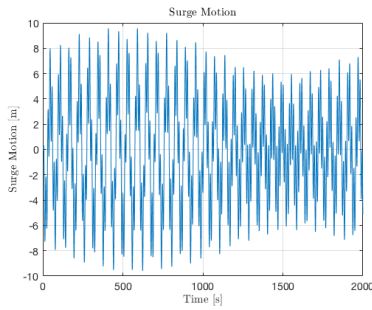


Figure 5.11: Surge Motion for Initial Damping from HydroD. Wave Amplitude 6 [m], Peak Period 13 [s] and Wave Heading 45 Degrees.

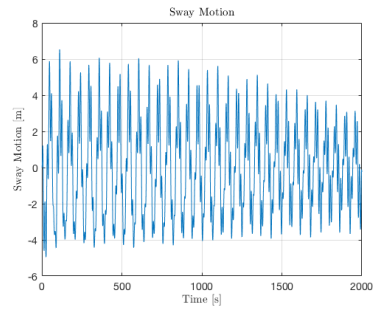


Figure 5.12: Sway Motion for Initial Damping from HydroD. Wave Amplitude 6 [m], Peak Period 13 [s] and Wave Heading 45 Degrees.

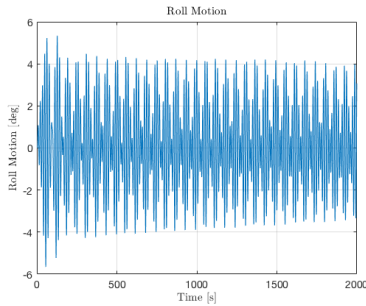


Figure 5.13: Roll Motion for Initial Damping from HydroD. Wave Amplitude 6 [m], Peak Period 13 [s] and Wave Heading 45 Degrees.

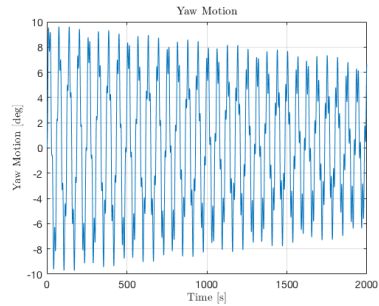


Figure 5.14: Yaw Motion for Initial Damping from HydroD. Wave Amplitude 6 [m], Peak Period 13 [s] and Wave Heading 45 Degrees.

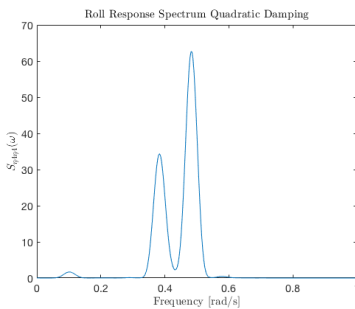


Figure 5.15: Response Spectrum for Roll with Initial Damping from HydroD. Wave Amplitude 6 [m], Peak Period 13 [s] and Wave Heading 45 Degrees.

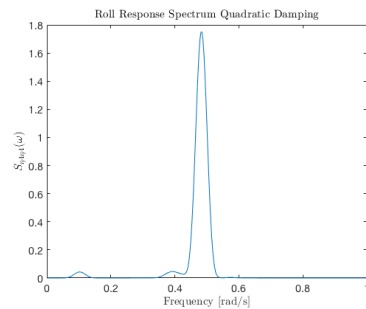


Figure 5.16: Response Spectrum for Roll with Initial Damping from HydroD. Wave Amplitude 1 [m], Peak Period 13 [s] and Wave Heading 45 Degrees.

It can also be seen that the horizontal offsets are large in surge, sway and yaw. To obtain more stable roll motion, and control the offsets, damping is added as seen in Section 4.2.2. A spectrum analysis of the roll motion, created by wave amplitude 6m and peak period 13, can be seen in Figure 5.15. Three peaks are seen in the figure. The largest at 0.48 [rad/s], which is due to the wave period as the regular waves had peak period at 13. The second peak is at 0.38 [rad/s], this is due the natural period in roll at 16 seconds. A small peak can also be seen at 0.1 [rad/s], the natural period for sway and yaw is at 60 seconds and due to coupled effects, roll motion is effected by the natural period. The roll response spectrum for wave amplitude 1m and peak period 13 can be seen in Figure 5.16, the same peaks are present in this spectrum but it is clear that the peak for roll motion is much smaller, this is due to the wave amplitude which again creates lower roll amplitudes. The response spectrum for linear roll damping only with the same parameters can be seen in Appendix B.

5.3.2 15 % Total Damping Ratio

Linear Roll Damping

In the first analysis only linear damping is added in roll. The wave elevation and resulting roll motion in 45 degrees regular waves for 15 % total damping ratio, 6 meter wave amplitude and 13 seconds peak period can be seen in Figures 5.17 and 5.18. One can see that the roll motion has an irregular response the first 200 seconds before it becomes steady state. The roll amplitude can then be found at steady state. Tables of the roll amplitude results from time domain analyses with linear damping and different wave directions can be seen in Appendix C.1. In addition to time series of sway, yaw and the roll force for the same parameters and different wave directions.

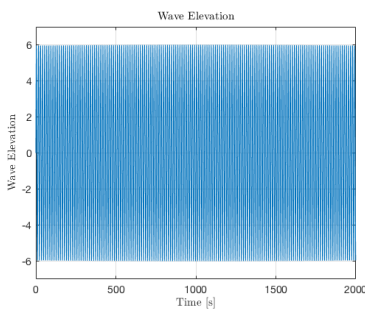


Figure 5.17: Wave Elevation for 6 [m] Wave Amplitude and 13 [s] Peak Period for Linear Damping.

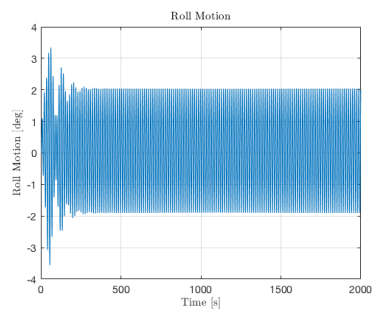


Figure 5.18: Roll Motion for 6 [m] Wave Amplitude and 13 [s] Peak Period for Linear Damping.

Linearized Roll Damping

To compare the results between the frequency and time domain solutions with quadratic damping, linearized damping is added in both programs as calculated in Section 4.3.2.

The wave elevation was shown in the last section, and the resulting roll motion in 45 degrees regular waves for 15 % total damping ratio, 6 meter wave amplitude and 13 seconds peak period can be seen in Figure 5.19. The same time series and tables as for linear roll can be found in Appendix C.2.

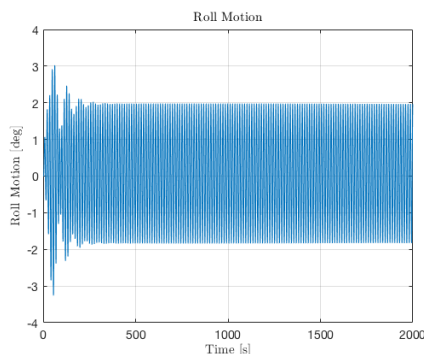


Figure 5.19: Roll Motion for 6 [m] Wave Amplitude and 13 [s] Peak Period for Linearized Damping.

It can be seen that the difference in roll amplitude for linear and linearized roll in regular waves, as presented in Figure 5.18 and 5.19, is quite small for this sea state. A comparison of all sea states is done in Section 5.5.2.

Quadratic Roll Damping

The quadratic damping was given in the task, calculated from StarCCM+. This is added in the quadratic damping matrix in order to compare the results with the linearized damping. The wave elevation is the same as earlier, and the resulting roll motion in 45 degrees regular waves for 15 % total damping ratio, 6 meter wave amplitude and 13 seconds peak period can be seen in Figure 5.19. Appendix C.3, contains the same time series as earlier, as well as tables showing the roll amplitude with quadratic damping for different wave heights and wave heading.

The difference in roll amplitude for linear and quadratic roll in regular waves as seen in Figure 5.18 and 5.19 is quite small for this sea state. A comparison of all sea states is done in Section 5.5.2.

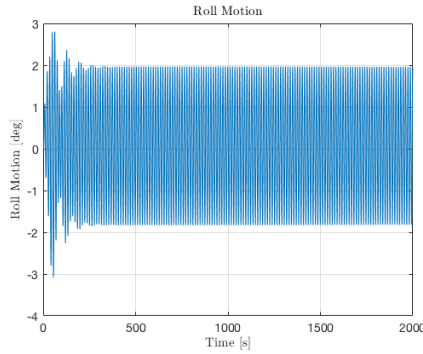


Figure 5.20: Roll Motion for 6 [m] Wave Amplitude and 13 [s] Peak Period for Quadratic Damping.

5.3.3 Sway Damping

Sway is coupled with roll, as seen in Section 2.3.4, and the level of damping in these rotations will therefore have an impact on the other. In Section 4.2.2 the total damping ratio was set to be 15 %, however it turned out that the coupling between the motions is strong and such a high level in damping also damped the roll motion. In Figure 5.21 and 5.22 a comparison of different RAO in sway with different levels of damping can be seen.

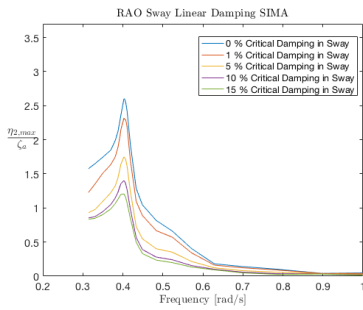


Figure 5.21: RAO Sway with Different Damping in Sway.

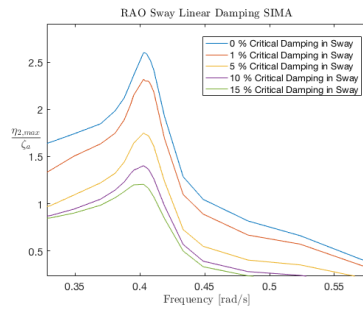


Figure 5.22: RAO Sway with Different Damping in Sway, Focused on the Peak.

As seen in Figure 5.22 and 5.24 the RAO for sway is similar to that for roll, indicating that the sway motion is due to the roll of the barge. When only the percentage of critical damping in sway is changed this has a significant impact on the RAO in roll. The damping should therefore be chosen with great caution. Consequently, time series of roll motion with different types of sway damping is studied. Peak period is chosen at 16 seconds, the natural period in roll, to look at the resonance effects. Figures 5.25, 5.26, 5.27 and 5.28 show that the roll motion is stable for very low percentage of the total damping ratio in sway. To make sure that the motion is stable and avoid offsets in horizontal direction, the

5.3. DIFFERENT DAMPING IN REGULAR WAVES

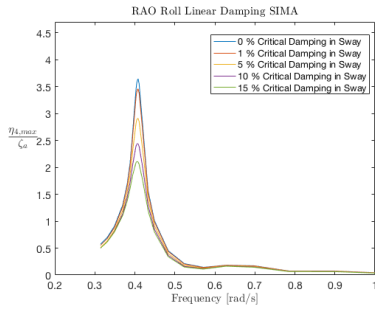


Figure 5.23: RAO Roll With Different Damping in Sway.

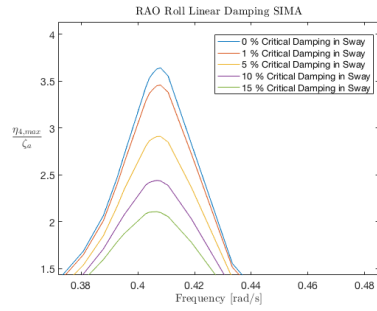


Figure 5.24: RAO Roll with Different Damping in Sway, Focused on the Peak.

sway damping is set at 1 %.

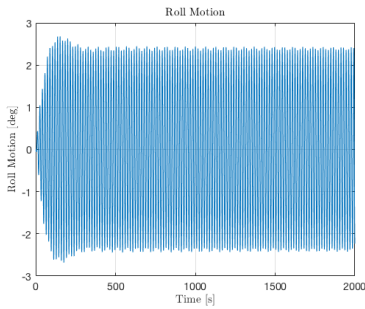


Figure 5.25: Roll Motion for 6 [m] wave amplitude and 16 [s] peak period with a 0% Damping Ratio.

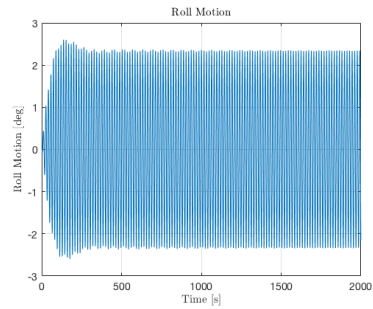


Figure 5.26: Roll Motion for 6 [m] wave amplitude and 16 [s] peak period with a 1% Damping Ratio.

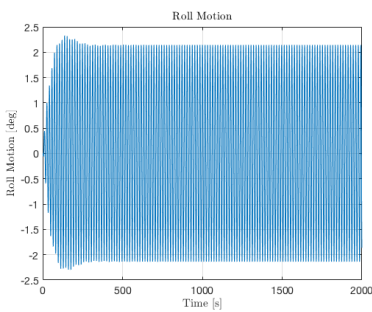


Figure 5.27: Roll Motion for 6 [m] Wave Amplitude and 16 [s] Peak Period with a 5% Damping Ratio.

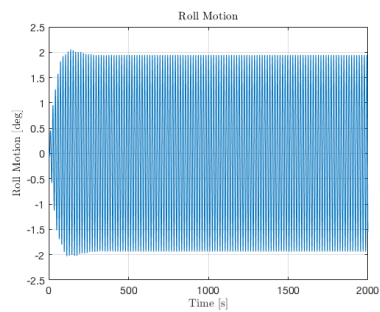


Figure 5.28: Roll Motion for 6 [m] Wave Amplitude and 16 [s] Peak Period with a 10% Damping Ratio.

5.3.4 Yaw Damping

Yaw is coupled with roll as seen in Section 2.3.4. Now, the effect of damping in yaw on the roll motion is investigated. In Section 4.2.2 the total damping ratio was set to be 15 %. In Figures 5.21 and 5.22 a comparison of different RAO in yaw with different levels of damping is presented. There is however no correlation between the total damping ratio in yaw and the roll motion. The damping is therefore set to be 15 %.

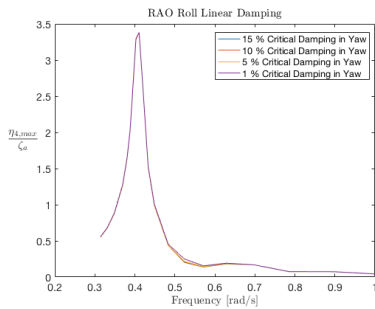


Figure 5.29: RAO Roll With Different Damping in Yaw.

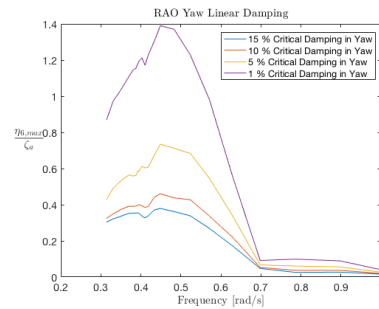


Figure 5.30: RAO Yaw With Different Damping in Yaw.

5.4 Regular Waves

The main analysis in this thesis will use a total damping ratio of 1 % in sway and 15 % in yaw. This is done to ensure reasonable offsets in the horizontal motions. The results will also be compared with the time domain results, where no restoring effect and additional damping were considered for surge, sway and yaw motions. To avoid having too large differences, the smallest possible damping in sway is added.

5.4.1 Linear Roll Damping

The analysis including linear damping only in roll, is redone because of the change in damping. The resulting roll motion in 0, 15, 45 and 90 degrees regular waves, 6 meter wave amplitude and 16 seconds peak period can be seen in Figures 5.31, 5.32, 5.33 and 5.34.

The roll motion has a longer period with irregular response, compared to the case with 15 % total damping ratio in sway. The motion does, however, not fluctuate more than approximately half a degree, before the fluctuations diminish more and more and becomes steady state. The roll amplitude can then be found at steady state. Time series of the other five degrees of motion and roll force for the same parameters and different wave direction can be seen in Appendix D.1. Tables for the amplitudes in the five other degrees of motion

5.4. REGULAR WAVES

and roll force for the same parameters and different wave direction, can be seen in the same Appendix.

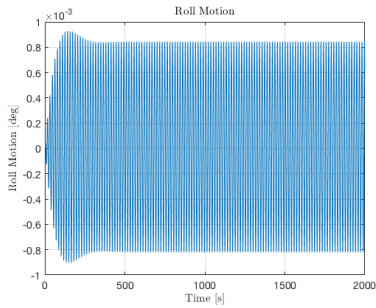


Figure 5.31: Roll Motion with Linear Damping, for 6 [m] Wave Amplitude and 16 [s] Peak Period with 0 Degrees Wave Heading.

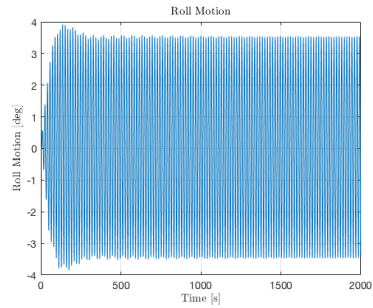


Figure 5.32: Roll Motion with Linear Damping, for 6 [m] Wave Amplitude and 16 [s] Peak Period with 15 Degrees Wave Heading.

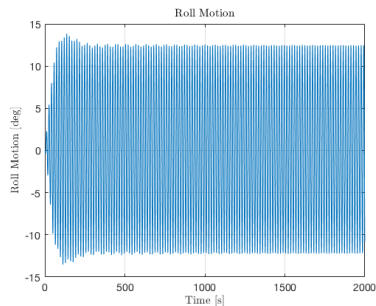


Figure 5.33: Roll Motion with Linear Damping, for 6 [m] Wave Amplitude and 16 [s] Peak Period with 45 Degrees Wave Heading.

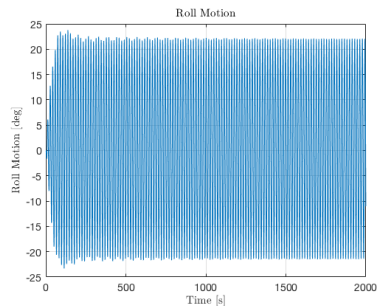


Figure 5.34: Roll Motion with Linear Damping, for 6 [m] Wave Amplitude and 16 [s] Peak Period with 90 Degrees Wave Heading.

It can be seen that the roll motion strongly depends on the wave direction. For 0 degrees, the roll amplitude is very small and became steady state fast. When the wave heading increases, the wave amplitude increases as well. But in addition, the motion becomes more fluctuating for a longer period. This will be discussed in Section 5.5.1.

Table 5.4: Roll Amplitude with Linear Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0131	0.0262	0.0394	0.0526	0.0658	0.0789
6	0.0250	0.0501	0.0751	0.1002	0.1253	0.1504
7	0.0644	0.1286	0.1926	0.2564	0.3200	0.3833
8	0.0691	0.1381	0.2073	0.2765	0.3456	0.4146
9	0.1503	0.3002	0.4496	0.5984	0.7467	0.8942
10	0.1659	0.3310	0.4957	0.6599	0.8240	0.9877
11	0.1349	0.2677	0.3995	0.5307	0.6609	0.7903
12	0.2001	0.3969	0.5929	0.7858	0.9741	1.1557
13	0.4123	0.8245	1.2355	1.6490	2.0654	2.4822
14	0.8876	1.7734	2.6573	3.5399	4.4211	5.3032
14.5	1.2936	2.5843	3.8707	5.1520	6.4276	7.6980
15	1.9042	3.7907	5.6431	7.4467	9.1893	10.8612
15.3	2.1765	4.3140	6.3808	8.3516	10.2097	11.9468
15.4	2.2204	4.3964	6.4907	8.4753	10.3354	12.0623
15.45	2.2286	4.4107	6.5073	8.4901	10.3412	12.0531
15.5	2.2264	4.4047	6.4952	8.4688	10.3115	12.0210
15.55	2.2256	4.4035	6.4952	8.4662	10.3049	12.0041
15.6	2.2098	4.3727	6.4494	8.4105	10.2382	11.9253
15.9	1.9860	3.9347	5.8138	7.6001	9.2788	10.8396
16	1.8968	3.7601	5.5613	7.6001	8.8927	10.3968
16.2	1.7002	3.3774	5.0110	6.5834	8.0806	9.4947
16.5	1.4664	2.9177	4.3408	5.7237	7.0588	8.3358
17	1.1826	2.3571	3.5170	4.6558	5.7674	6.8461
18	0.8580	1.7132	2.5640	3.4083	4.2442	5.0697
19	0.6625	1.3236	1.9829	2.6396	3.2933	3.9432
20	0.5470	1.0925	1.6364	2.1786	2.7192	3.2582

5.4.2 Linearized Roll Damping

The resulting roll motion for analysis including linearized damping in 45 degrees regular waves, 6 meter wave amplitude and 16 seconds peak period can be seen in Figures 5.35, 5.36, 5.37 and 5.38.

The roll amplitude results from time domain analysis with linearized damping and 45 degrees wave direction can be seen in Table 5.5. The same time series and tables as presented for linear damping is also found for linearized damping, and presented in Appendix D.2.

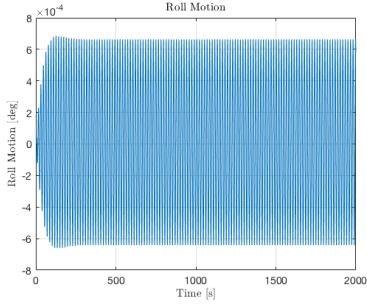


Figure 5.35: Roll Motion with Linearized Damping, for 6 [m] Wave Amplitude and 16 [s] Peak Period with 0 Degrees Wave Heading.

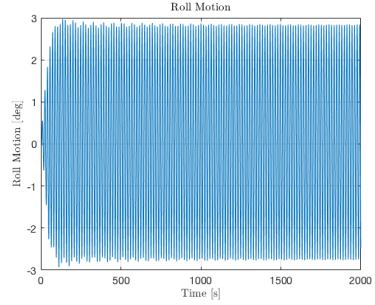


Figure 5.36: Roll Motion with Linearized Damping, for 6 [m] Wave Amplitude and 16 [s] Peak Period with 15 Degrees Wave Heading.

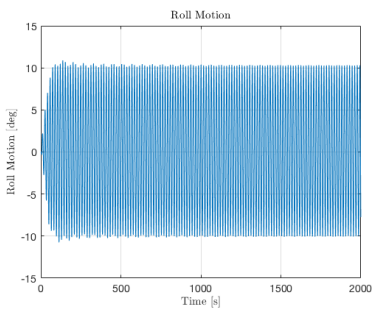


Figure 5.37: Roll Motion with Linearized Damping, for 6 [m] Wave Amplitude and 16 [s] Peak Period with 45 Degrees Wave Heading.

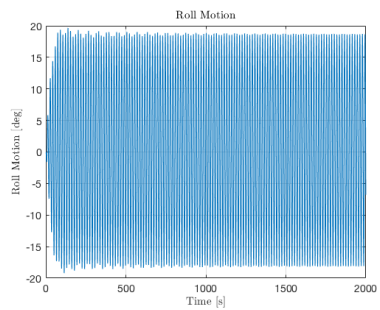


Figure 5.38: Roll Motion with Linearized Damping, for 6 [m] Wave Amplitude and 16 [s] Peak Period with 90 Degrees Wave Heading.

Table 5.5: Roll Amplitude with Linearized Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0131	0.0262	0.0394	0.0526	0.0658	0.0789
6	0.0250	0.0501	0.0751	0.1002	0.1253	0.1504
7	0.0644	0.1286	0.1926	0.2564	0.3200	0.3833
8	0.0691	0.1381	0.2073	0.2765	0.3456	0.4146
9	0.1503	0.3002	0.4496	0.5984	0.7467	0.8942
10	0.1659	0.3310	0.4957	0.6599	0.8240	0.9877
11	0.1349	0.2677	0.3995	0.5307	0.6609	0.7903
12	0.2001	0.3969	0.5929	0.7858	0.9741	1.1557
13	0.4123	0.8245	1.2355	1.6490	2.0654	2.4822
14	0.8876	1.7734	2.6573	3.5399	4.4211	5.3032
14.5	1.2936	2.5843	3.8707	5.1520	6.4276	7.6980
15	1.9042	3.7907	5.6431	7.4467	9.1893	10.8612
15.3	2.1765	4.3140	6.3808	8.3516	10.2097	11.9468
15.4	2.2204	4.3964	6.4907	8.4753	10.3354	12.0623
15.45	2.2286	4.4107	6.5073	8.4901	10.3412	12.0531
15.5	2.2264	4.4047	6.4952	8.4688	10.3115	12.0210
15.55	2.2256	4.4035	6.4952	8.4662	10.3049	12.0041
15.6	2.2098	4.3727	6.4494	8.4105	10.2382	11.9253
15.9	1.9860	3.9347	5.8138	7.6001	9.2788	10.8396
16	1.8968	3.7601	5.5613	7.6001	8.8927	10.3968
16.2	1.7002	3.3774	5.0110	6.5834	8.0806	9.4947
16.5	1.4664	2.9177	4.3408	5.7237	7.0588	8.3358
17	1.1826	2.3571	3.5170	4.6558	5.7674	6.8461
18	0.8580	1.7132	2.5640	3.4083	4.2442	5.0697
19	0.6625	1.3236	1.9829	2.6396	3.2933	3.9432
20	0.5470	1.0925	1.6364	2.1786	2.7192	3.2582

5.4.3 Quadratic Roll Damping

The resulting roll motion for quadratic damping analysis in 45 degrees regular waves, 6 meter wave amplitude and 16 seconds peak period are seen in Figure 5.18.

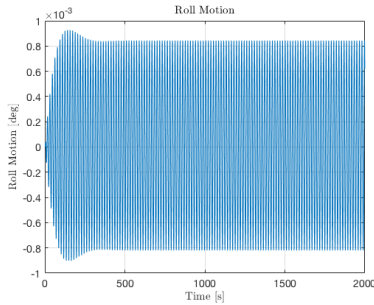


Figure 5.39: Roll Motion with Quadratic Damping, for 6 [m] Wave Amplitude and 16 [s] Peak Period with 0 Degrees Wave Heading.

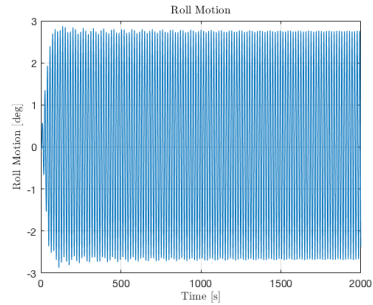


Figure 5.40: Roll Motion with Quadratic Damping, for 6 [m] Wave Amplitude and 16 [s] Peak Period with 15 Degrees wave Heading.

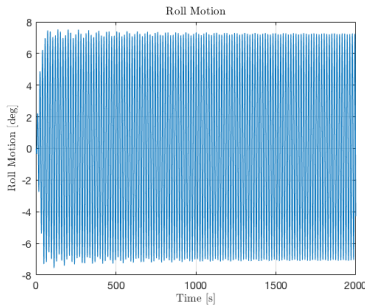


Figure 5.41: Roll Motion with Quadratic Damping, for 6 [m] Wave Amplitude and 16 [s] Peak Period with 45 Degrees Wave Heading.

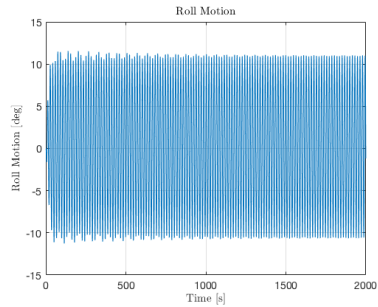


Figure 5.42: Roll Motion with Quadratic Damping, for 6 [m] Wave Amplitude and 16 [s] Peak Period with 90 Degrees Wave Heading.

The roll amplitude results from time domain analyses with quadratic damping and 45 degrees wave direction can be seen in Table 5.6. The same time series and tables as presented for linear damping is also found for linearized damping, and presented in Appendix D.3.

Table 5.6: Roll Amplitude with Quadratic Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0131	0.0262	0.0394	0.0526	0.0658	0.0789
6	0.0250	0.0501	0.0751	0.1002	0.1253	0.1504
7	0.0644	0.1286	0.1926	0.2564	0.3200	0.3833
8	0.0691	0.1381	0.2073	0.2765	0.3456	0.4146
9	0.1503	0.3002	0.4496	0.5984	0.7467	0.8942
10	0.1659	0.3310	0.4957	0.6599	0.8240	0.9877
11	0.1349	0.2677	0.3995	0.5307	0.6609	0.7903
12	0.2001	0.3969	0.5929	0.7858	0.9741	1.1557
13	0.4123	0.8245	1.2355	1.6490	2.0654	2.4822
14	0.8876	1.7734	2.6573	3.5399	4.4211	5.3032
14.5	1.2936	2.5843	3.8707	5.1520	6.4276	7.6980
15	1.9042	3.7907	5.6431	7.4467	9.1893	10.8612
15.3	2.1765	4.3140	6.3808	8.3516	10.2097	11.9468
15.4	2.2204	4.3964	6.4907	8.4753	10.3354	12.0623
15.45	2.2286	4.4107	6.5073	8.4901	10.3412	12.0531
15.5	2.2264	4.4047	6.4952	8.4688	10.3115	12.0210
15.55	2.2256	4.4035	6.4952	8.4662	10.3049	12.0041
15.6	2.2098	4.3727	6.4494	8.4105	10.2382	11.9253
15.9	1.9860	3.9347	5.8138	7.6001	9.2788	10.8396
16	1.8968	3.7601	5.5613	7.6001	8.8927	10.3968
16.2	1.7002	3.3774	5.0110	6.5834	8.0806	9.4947
16.5	1.4664	2.9177	4.3408	5.7237	7.0588	8.3358
17	1.1826	2.3571	3.5170	4.6558	5.7674	6.8461
18	0.8580	1.7132	2.5640	3.4083	4.2442	5.0697
19	0.6625	1.3236	1.9829	2.6396	3.2933	3.9432
20	0.5470	1.0925	1.6364	2.1786	2.7192	3.2582

5.5 Comparison of Results in Regular Waves

5.5.1 Comparison of Wave Direction

The wave heading will affect the roll amplitude. This can be seen in Figures 5.43 and 5.44. At zero degrees wave direction, sway, roll and yaw motions will be close to zero as they cannot be caused by wave excitation. But when the wave direction is changed to 15, 45 and 90 the roll amplitude will increase proportionally. Beam waves yield highest amplitudes as these waves contribute the most to roll motion.

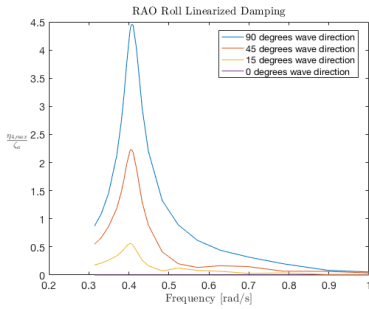


Figure 5.43: RAO Roll with Linearized Damping Comparison of Wave Heading.

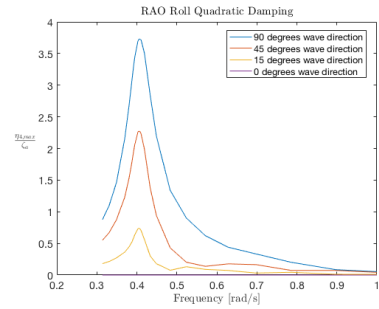


Figure 5.44: RAO Roll with Linear Damping Comparison of Wave Heading.

5.5.2 Comparison of Linear, Linearized and Quadratic Damping

In Figure 5.45 a comparison between linear, linearized and quadratic damping in regular waves are shown. The amplitude in roll is similar for all three damping methods for all frequencies, except around the natural frequencies. If the wave frequency equals the natural frequency, the amplitude of the oscillations will grow (and grow). This effect is known as resonance. The quadratic and linearized damping will be affected most in this area and avoid resonance effects. It can also be seen that the linearized and quadratic damping is similar for regular waves at 45 degrees, this means that the linearization manages to match the quadratic damping in a good way.

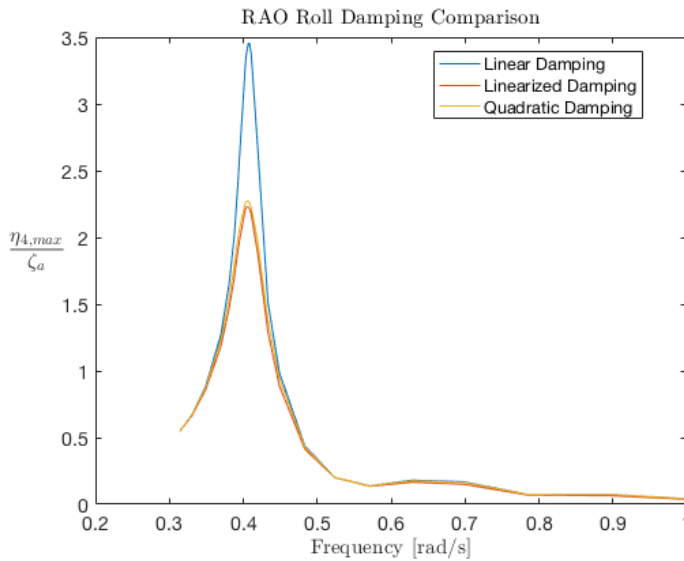


Figure 5.45: RAO Roll Comparison of Damping Techniques, 45 Degrees Wave Heading.

It is also interesting to look at the percentage differences between linear and quadratic damping in Table 5.7. Not surprisingly, the largest difference is found at 15.4 seconds, just around the natural period. In addition it is shown that there is a substantial correlation with the wave heading. When the wave heading is changed the amplitude increases, the percentage difference is therefore largest where the amplitude is largest.

Table 5.7: Percentage Differences Between Linear and Quadratic Damping.

Tp	Percentage difference		
	15 deg	45 deg	90 deg
5	0 %	0 %	0 %
6	0 %	1 %	0 %
7	0 %	2 %	0 %
8	1 %	1 %	0 %
9	0 %	3 %	2 %
10	1 %	2 %	0 %
11	1 %	0 %	1 %
12	1 %	0 %	1 %
13	0 %	2 %	4 %
14	1 %	5 %	11 %
14.5	2 %	10 %	22 %
15	10 %	37 %	64 %
15.3	17 %	52 %	83 %
15.4	17 %	53 %	84 %
15.45	17 %	52 %	82 %
15.5	17 %	50 %	80 %
15.55	16 %	48 %	77 %
15.6	15 %	45 %	72 %
15.9	8 %	27 %	45 %
16	6 %	21 %	36 %
16.2	4 %	12 %	23 %
16.5	2 %	7 %	13 %
17	1 %	3 %	6 %
18	0 %	1 %	2 %
19	0 %	1 %	1 %
20	0 %	0 %	0 %

5.5.3 Comparison of Wave Amplitude

In Figures 5.46 and 5.47, the results for different wave amplitudes are shown. The y-axis denomination is $\frac{\eta_{4,max}}{\zeta_a}$. To find the roll amplitude for wave amplitudes other than 1 meter for a normal RAO, the value can be multiplied with the desired wave amplitude. This is correct for linear damping results outside the natural frequency range. For frequencies around 15-16 seconds, significant offsets are observed.

The range of offsets for quadratic damping has the highest effect on motions close to the natural frequency. Motions in quadratic damping are therefore most affected. This is also why the percentage differences between linear and quadratic damping are largest around the natural frequency. The theory fits the results from Table 5.7: when the percentage difference is larger than a few percentages, it gives different results if the RAO is multiplied with the wave amplitude.

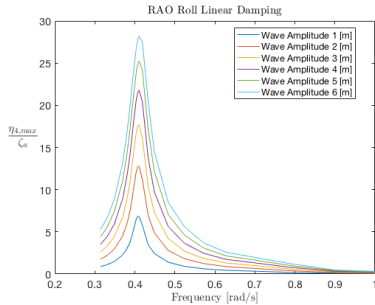


Figure 5.46: RAO Roll with Linear Damping. Comparison of Wave Amplitude.

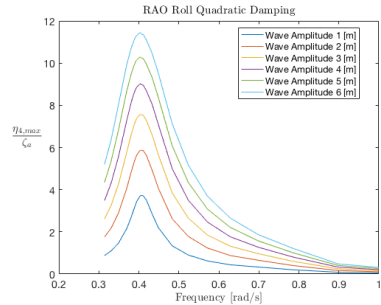


Figure 5.47: RAO Roll with Quadratic Damping. Comparison of Wave Amplitude.

5.6 Irregular Waves

Several environmental combinations are analyzed for irregular waves. Time series for conditions number 9 and 12 are seen in Figures 5.48 and 5.49 respectively, both with 45 degree wave direction. The wave elevations for the same conditions are seen in Figures 5.50 and 5.51.

Irregular waves are analyzed by spectral analysis as presented in Section 2.4.2. The spectra corresponding to the motion seen in Figures 5.48 and 5.49, are presented in Figures 5.52 and 5.53, respectively. In the spectra with significant wave height 2.5 [m] and peak period 9.5 [s], the largest peak is found at 0.42 [rad/s], which is close to the natural period in roll. The second peak is around the peak period for the wave elevation, 0.66 [rad/s]. It seems like some resonant motions are present in this area, based on the long rounded shape of this peak. In Figure 5.53, roll response spectra for significant wave height 2.5 [m] and peak period 12.5 [s] are shown. In this spectra there is only one peak, but the peak is 7.5 times higher for peak period 12.5 than it was for peak period 9.5. This is because the peak period comes closer to the natural period in roll, and resonant motions occur.

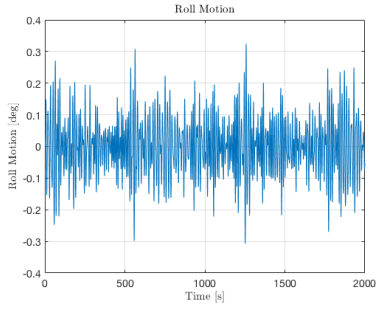


Figure 5.48: Roll Motion for Irregular Waves with Significant Wave Height 2.5 [m] and Peak Period 9.5 [s].

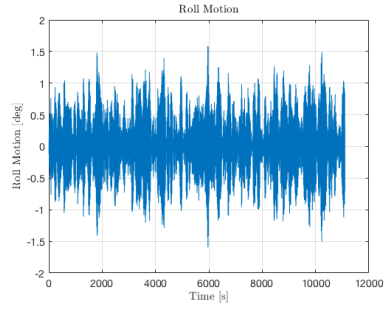


Figure 5.49: Roll Motion for Irregular Waves with Significant Wave Height 2.5 [m] and Peak Period 12.5 [s].

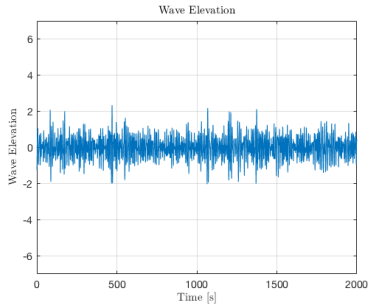


Figure 5.50: Wave Elevation for Irregular Waves with Significant Wave Height 2.5 [m] and Peak Period 9.5 [s].

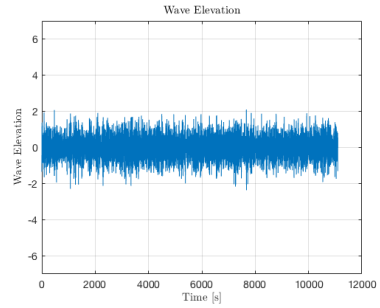


Figure 5.51: Wave Elevation for Irregular Waves with Significant Wave Height 2.5 [m] and Peak Period 12.5 [s].

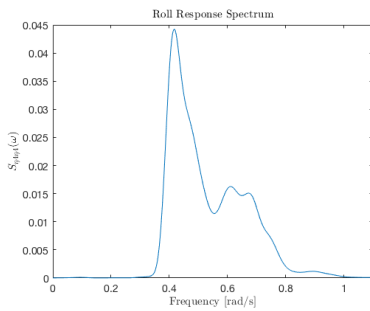


Figure 5.52: Roll Response Spectrum for Irregular Waves with Significant Wave Height 2.5 [m] and Peak Period 9.5 [s].

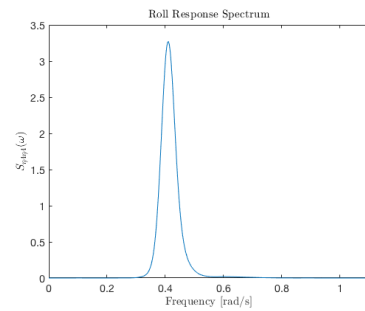


Figure 5.53: Roll Response Spectrum for Irregular Waves with Significant Wave Height 2.5 [m] and Peak Period 12.5 [s].

5.6.1 Comparison of Wave Seeds

The different wave seeds will generate different wave elevations as seen from the wave realizations. This will also result in different responses in roll and therefore different response spectrum. The response spectrum will be used to compare the results of frequency and time domain analyzes. The differences will therefore be investigated to anticipate the uncertainty level of the results. The standard deviation for all conditions with wave heading 45 degrees can be seen in Table 5.8. The standard deviation of the standard deviations presented in the table varies between 0.0014 and 0.0780, the uncertainty will therefore depend on the condition number.

When looking at the results for incoming waves at 90 degrees, the standard deviation presented in Appendix E, appears to be significantly higher than the ones presented in Table 5.8. Beam sea would however cause larger motions. Standard deviations of the standard deviations vary between 0.0022 and 0.0708, and vary greatly with condition as before.

Table 5.8: Standard Deviation for Different Wave Seeds in 45 deg Wave Heading.

Condition number	Wave Seed 1	Wave Seed 2	Wave Seed 3	Wave Seed 4	Wave Seed 5	Wave Seed 6	Standard Deviation
1	0.02305	0.02464	0.02305	0.02441	0.02138	0.02136	0.0014
2	0.03248	0.03551	0.03255	0.03453	0.02963	0.03081	0.0022
3	0.05382	0.05927	0.05377	0.05772	0.05160	0.05452	0.0028
4	0.10102	0.11711	0.11025	0.11243	0.09956	0.10955	0.0068
5	0.20022	0.20622	0.17083	0.18679	0.18679	0.16918	0.0155
6	0.28427	0.28234	0.26687	0.28388	0.22142	0.24766	0.0051
7	0.03816	0.04069	0.03832	0.04046	0.03578	0.03542	0.0022
8	0.05407	0.05915	0.05418	0.05750	0.04933	0.05132	0.0037
9	0.08909	0.09831	0.08918	0.09558	0.08555	0.09036	0.0047
10	0.16639	0.19238	0.18095	0.18438	0.16384	0.17958	0.0109
11	0.32292	0.33326	0.27932	0.30338	0.27467	0.28732	0.0240
12	0.45551	0.45291	0.42772	0.45219	0.35868	0.39802	0.0076
13	0.03816	0.04069	0.03832	0.04046	0.03578	0.03542	0.0024
14	0.05407	0.05915	0.05418	0.05750	0.04933	0.05132	0.0050
15	0.08909	0.09831	0.08918	0.09558	0.08555	0.09036	0.0066
16	0.16639	0.19238	0.18095	0.18438	0.16384	0.17958	0.0148
17	0.32292	0.33326	0.27932	0.30338	0.27467	0.28732	0.0314
18	0.45551	0.45291	0.42772	0.45219	0.35868	0.39802	0.0506
19	0.06552	0.06752	0.06767	0.06962	0.06684	0.06124	0.0029
20	0.10087	0.10935	0.10134	0.10695	0.09417	0.09461	0.0062
21	0.15692	0.17436	0.15848	0.16797	0.15040	0.15813	0.0086
22	0.29091	0.33504	0.31450	0.32000	0.28618	0.31120	0.0183
23	0.54872	0.56825	0.48535	0.52143	0.47218	0.49334	0.0379
24	0.76679	0.76348	0.72078	0.75505	0.61405	0.67351	0.0610
25	0.08003	0.08250	0.08265	0.08505	0.08161	0.07483	0.0035
26	0.12519	0.13471	0.12528	0.13237	0.11787	0.11641	0.0074
27	0.18974	0.21205	0.19321	0.20250	0.18114	0.18924	0.0110
28	0.33737	0.38853	0.36401	0.37025	0.33149	0.35874	0.0212
29	0.64745	0.67169	0.57813	0.61833	0.55997	0.58463	0.0434
30	0.91012	0.90670	0.85626	0.89360	0.73382	0.80132	0.0702
31	0.09452	0.09747	0.09762	0.10047	0.09635	0.08841	0.0041
32	0.14776	0.15908	0.14789	0.15627	0.13911	0.13750	0.0087
33	0.22232	0.24980	0.22800	0.23685	0.21178	0.21993	0.0135
34	0.37467	0.43225	0.40405	0.41081	0.36781	0.39594	0.0238
35	0.71845	0.74725	0.64741	0.69000	0.62414	0.65144	0.0471
36	1.03967	1.03617	0.97935	1.01846	0.84339	0.91733	0.0780

Different wave seeds will generate different roll response spectra. This is investigated by plotting the six different response spectra from six different wave seeds in the same figure. This can be seen for condition number 11 in Figure 5.54, and for condition number 12 in Figure 5.55. For condition number 11 the standard deviation was 0.0240, while it was 0.0076 for condition number 12. Hence, one may see that if the standard deviation in Table 5.8 is large the response spectra will vary more than if the standard deviation is low.

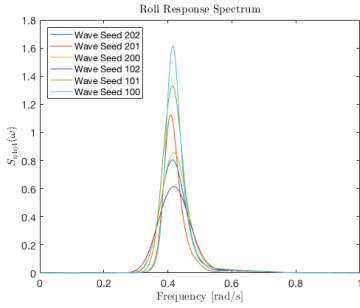


Figure 5.54: Comparison of Wave Seeds. Condition Number 11.

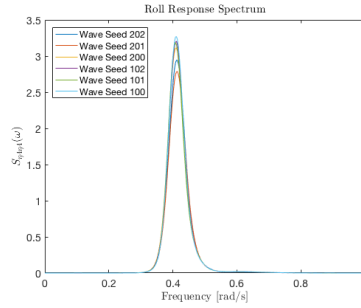


Figure 5.55: Comparison of Wave Seeds. Condition Number 12.

5.6.2 Comparison of Environmental Input

The spectra will also vary with the significant wave height and peak period, this can be seen in Figures 5.56 and 5.57, respectively. As seen in Section 2.1.3 and Equation 2.17 the energy roll motion is proportional to the area under the graph, $\frac{E}{\rho g} = \sigma^2 = \int_0^\infty S(\omega)d\omega$. Figure 5.56 shows that the area under the graphs decreases less, depending on the significant wave height, than it does for Figure 5.57 depending on the peak period. This means that the significant wave height contributes less to the the energy in the roll motion than the peak period, as the area decreases faster when the peak period decreases compared to when the significant wave height decreases.

The factor that contributes most to roll motion is the wave heading. This is seen in Figure 5.58. Roll motion is negligible for 0 degrees waves, and very small for 15 degrees. It is however quite large for 45 degrees and again twice as large for 90 degrees. 90 degrees of incoming waves should therefore definitely be avoided by all means due to the significantly high responses. In addition the Table with standard deviations for the different wave seeds presented in Appendix E for 90 degrees of incoming waves, shows that there is large variation between the standard deviations. This results in a large uncertainty

5.6. IRREGULAR WAVES

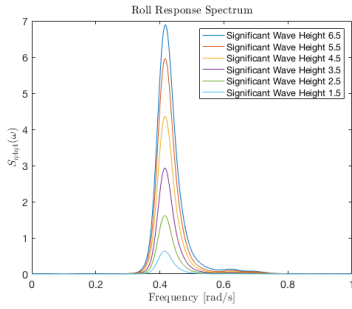


Figure 5.56: Roll Response Spectrum. Comparison of Different Significant Wave Heights.

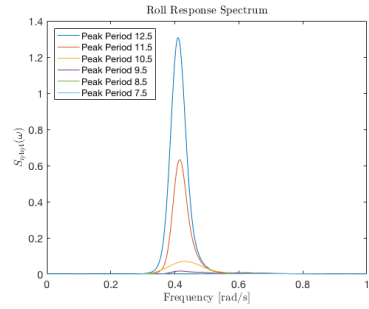


Figure 5.57: Roll Response Spectrum. Comparison of Different Peak Periods.

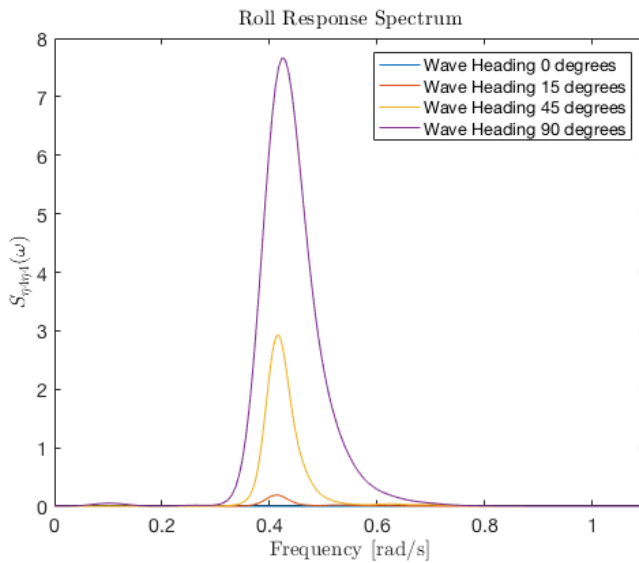


Figure 5.58: Roll Response Spectrum. Comparison of Different Wave Heading.

Chapter 6

Comparison of Results in Time and Frequency-Domain

In this section results obtained from SIMA will be compared to results done in HydroD by Anders Juul Weiby (Weiby (2018)). At first the natural periods found from decay tests in SIMA are compared with the natural periods from HydroD. The results compared in regular waves are shown in Section 5.4 with 45 and 90 degrees heading with linear, linearized and quadratic damping. For irregular waves the results obtained in SIMA are shown in Section 5.6.

6.1 Natural Periods

The natural periods found in SIMA and HydroD can be seen in Table 6.1. In surge, sway and yaw stiffness was added to avoid large horizontal movement. This was not done in HydroD and the stiffness in these directions is zero. It is therefore not possible to obtain natural period in these degrees of motions in HydroD. The natural periods in HydroD is found in waves with period of 16 seconds.

Table 6.1: Comparison of Natural Periods in SIMA and HydroD.

	Obtained T_d SIMA	Obtained T_0 HydroD	Difference
Heave	11.7	12.1	3.3 %
Roll	14.8	15.6	3.3 %
Pitch	10.7	11.5	6.9 %

The natural periods are similar in the compared directions. In surge, sway and yaw additional stiffness is added in SIMA and they can therefore not be compared. One of the

reasons for the differences may be the calculation method. SIMA uses the natural damped frequency, while HydroD uses the natural frequencies, as discussed in Section 4.4.2. This should however only result in about 0.15 % deviation, since the damping ratio in surge, sway, roll and yaw is 0.15. Another difference in the calculation process is the use of additional damping, which was explained in Section 2.3.1. HydroD uses the added mass corresponding to the natural frequency, while SIMA on the other hand will be using the added mass for infinity frequency, as explained in Section 4.4.2. Some deviation in the results is therefore natural and expected.

6.2 Regular Waves

6.2.1 Linear Damping

45 Degrees Wave Heading

To compare the models, only linear damping is included in addition to the potential damping. The RAO between the roll amplitude and wave amplitude is then compared between the time domain solution and frequency domain solution. The comparison of frequency and time domain in 45 degrees regular waves is seen in Figure 6.1. As shown, the RAO is almost equal for frequencies exceeding 0.41 [rad/s]. The peak is however 0.7 degrees higher for frequency domain, in addition to an offset of approximately 0.1 [rad/s]. This is due to different natural frequency in frequencies domain compared to time domain. This again leads to a constant difference between the graphs from 0.3 [rad/s] to the peak.

To investigate the difference in amplitude for the peaks, RAO for the other motions is plotted to check if it may be due to coupled effects or other effects which are calculated differently. The RAO for surge, sway, heave, pitch and yaw can be seen in Figures 6.2, 6.3, 6.4, 6.5 and 6.6. In addition the roll moments are compared to check all parts of the equation of motion, this is seen in Figure 6.7.

All motions fit almost perfectly except for the two coupled motions, sway and yaw. It is however seen in Section 5.3.4 how the damping in yaw has small or no influence on the roll motion. One of the reasons for offsets in roll may, on the other hand, be the sway motion. As seen in Section 5.3.3, the damping in sway had a large impact on the roll motion. The roll motion with zero damping in sway is seen in Figure 6.8, one can see that the peak increases in RAO in roll.

Discretization may also be reason for the peak's offset. In Figure 6.8 all the data points can be seen as circles. For the simulations done in SIMA, several points are added around the peak to find the maximum value and the true peak. This is however not the case for the data set obtained from HydroD simulated by Weiby (2018). The peak in frequency

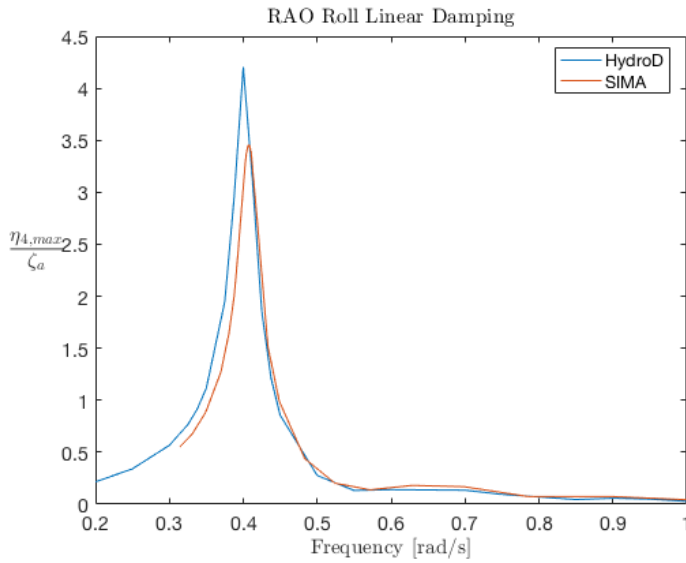


Figure 6.1: RAO Roll Comparison of SIMA and HydroD with Linear Damping in 45 Degrees Wave Heading.

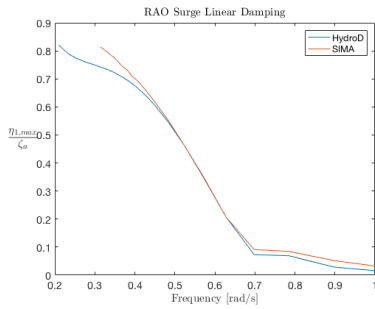


Figure 6.2: RAO Surge Comparison of SIMA and HydroD with Linear Damping in 45 Degrees Wave Heading.

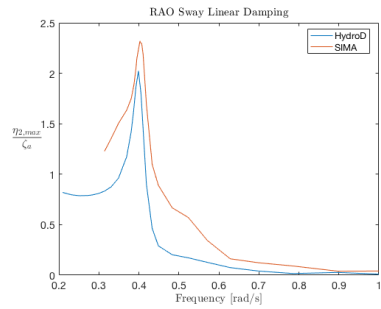


Figure 6.3: RAO Sway Comparison of SIMA and HydroD with Linear Damping in 45 Degrees Wave Heading.

domain may therefore be misplaced as several data points should have been added close to the peak. This leads to an uncertainty of the true behavior.

6.2. REGULAR WAVES

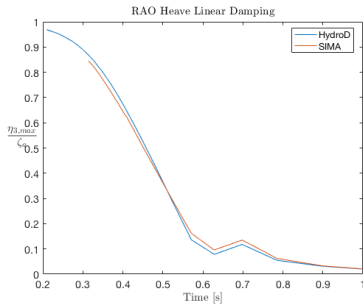


Figure 6.4: RAO Heave Comparison of SIMA and HydroD with Linear Damping in 45 Degrees Wave Heading.

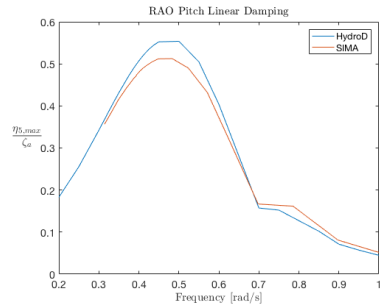


Figure 6.5: RAO Pitch Comparison of SIMA and HydroD with Linear Damping in 45 Degrees Wave Heading.

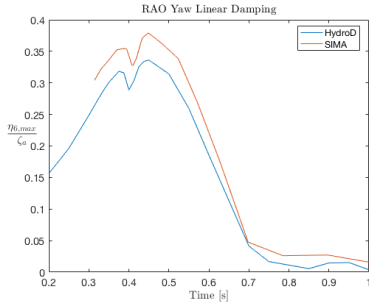


Figure 6.6: RAO Yaw Comparison of SIMA and HydroD with Linear Damping in 45 Degrees Wave Heading.

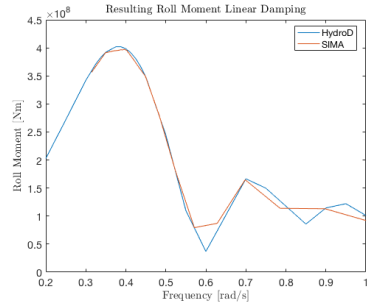


Figure 6.7: Roll Moment, Comparison of SIMA and HydroD with Linear Damping in 45 Degrees Wave Heading.

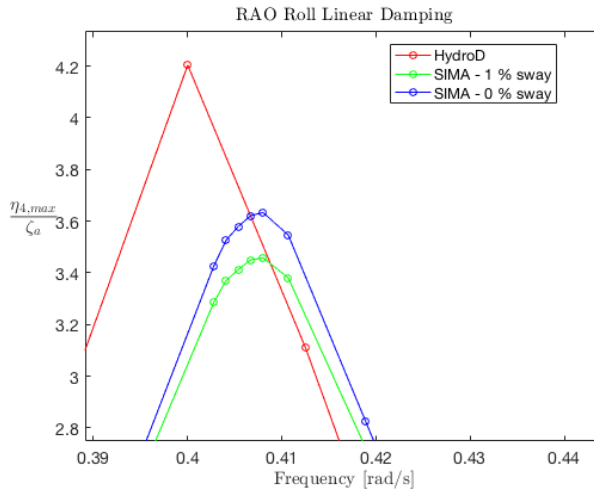


Figure 6.8: Discretization of RAO in Roll with Linear Damping. Focusing on the Peak.

6.2.2 90 Degrees Wave Heading

For 90 degrees wave heading, the comparison between frequency and time domain is seen in Figure 6.9.

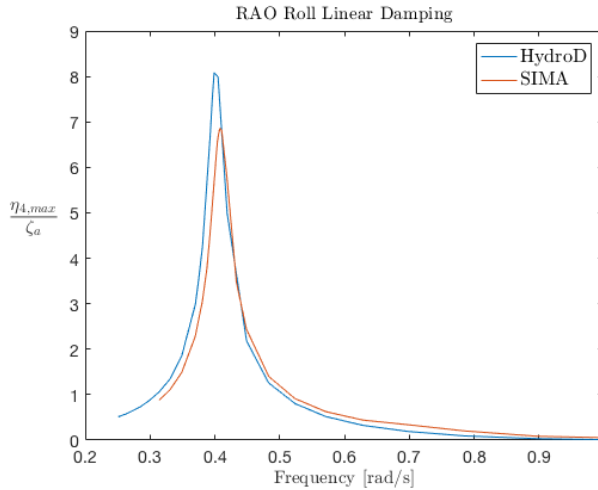


Figure 6.9: RAO Roll Comparison of SIMA and HydroD with Linear Damping in 90 degrees Wave Heading.

Some of the other degrees of motions are plotted for 90 degrees as well. The RAO for surge, sway and yaw can be seen in Figures 6.10, 6.11 and 6.12. In addition, the roll moments are compared to check all parts of the equation of motion, this is seen in Figure 6.13. The RAO for heave and pitch in 90 degrees is similar to the RAO in 45 degrees, which can be seen in Appendix F.

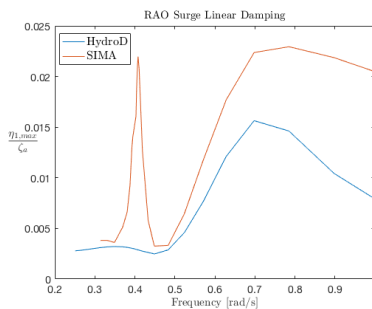


Figure 6.10: RAO Surge Comparison of SIMA and HydroD with Linear Damping in 90 degrees Wave Heading.

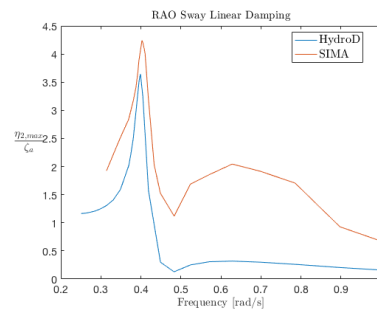


Figure 6.11: RAO Sway Comparison of SIMA and HydroD with Linear Damping in 90 degrees Wave Heading.

For the RAO in surge for time domain a peak is seen at the natural period in roll. This may affect the roll motions although the value and offsets in surge are small. For comparison the amplitude at around 0.41 [rad/s] is at 0.022, while the amplitude for roll is at 8.5 for the time domain. In addition surge is not connected to sway through coupled motions. Hence, the impact from surge is expected to be quite small.

The RAO in sway is also different in time domain versus frequency domain. In the same way as for 45 degrees the motions in sway will affect the damping in roll. When the wave direction is changed from 45 to 90 degrees the effect may be even larger than for 45 degrees.

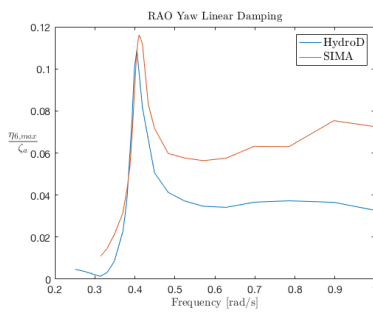


Figure 6.12: RAO Yaw Comparison of SIMA and HydroD with Linear Damping in 90 Degrees Wave Heading.

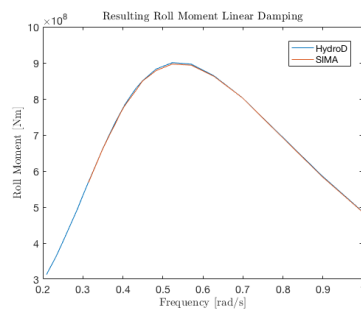


Figure 6.13: Roll Moment, Comparison of SIMA and HydroD with Linear Damping in 90 Degrees Wave Heading.

The resulting roll moment seen in Figure 6.13 is however almost equal. The calculation method should therefore give the same results in the different degrees of motion as long as the input is the same.

6.2.3 Linearized Damping

As the results between time and frequency domain was quite similar for linear damping, the linearized damping is added. In Figures 6.14 the comparison with linearized damping can be seen.

As shown in the figure, the form is the same as for linear damping with 45 and 90 degrees wave heading. But the peak is lower, as expected, when the damping is increased. Other RAO's are also compared. RAO in sway and yaw can be seen in Figure 6.15 and 6.16. The other motions are presented in in Appendix F.

The reasons for differences in the RAO's with linearized damping are the same as explained for linear damping. In Figure 6.17 the discretization is shown for linearized damping. This shows that there are added data points in the linearized discretization compared

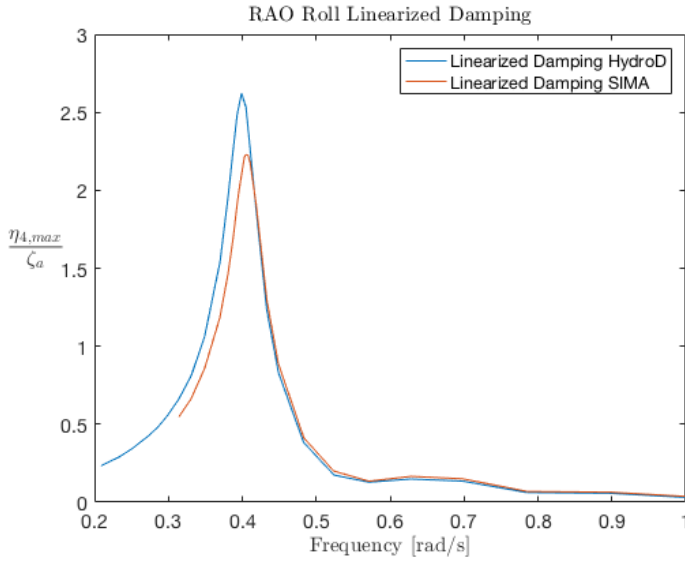


Figure 6.14: RAO Roll Comparison of SIMA and HydroD with Linearized Damping in 45 Degrees Wave Heading.

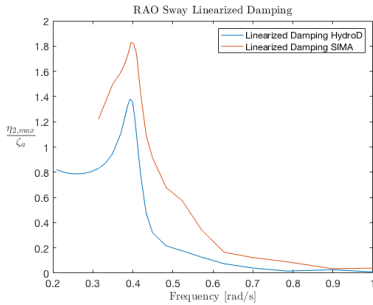


Figure 6.15: RAO Sway Comparison of SIMA and HydroD with Linearized Damping in 45 Degrees Wave Heading.

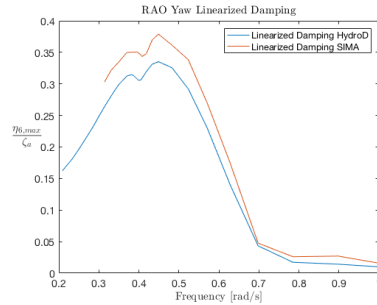


Figure 6.16: RAO Yaw Comparison of SIMA and HydroD with Linearized Damping in 45 Degrees Wave Heading.

to the linear. Hence, the results have a higher accuracy. In the figure, the RAO for roll with 0 % damping in sway is shown. This peak is close to the results for frequency domain.

The iteration process to find the linearized damping for 90 degrees wave heading is not performed in time domain. These results are therefore not presented. It can however be assumed that the form of RAO in all degrees of motions is similar to the form for linear damping.

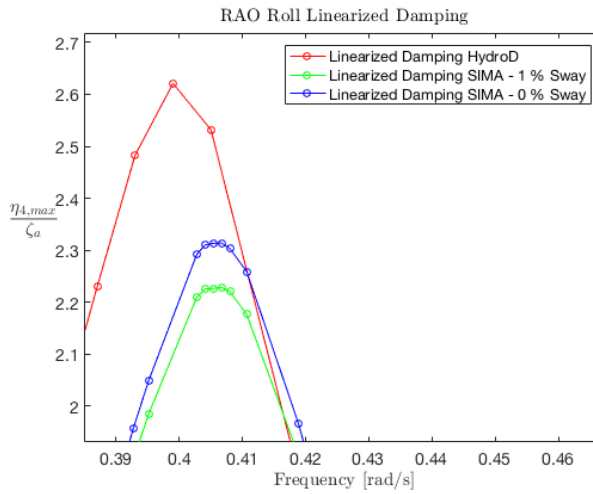


Figure 6.17: Discretization of RAO in Roll with Linearized Damping. Focusing on the Peak.

6.2.4 Quadratic Damping

As seen in Section C.3 where the results for linear, linearized and quadratic damping were compared, the amplitude for quadratic damping was higher than the linearized. The results for the quadratic damping will therefore be more similar to the frequency domain linearized results. Figure 6.18 shows that the quadratic damping with zero damping in sway gives the most similar results compared to the frequency domain.

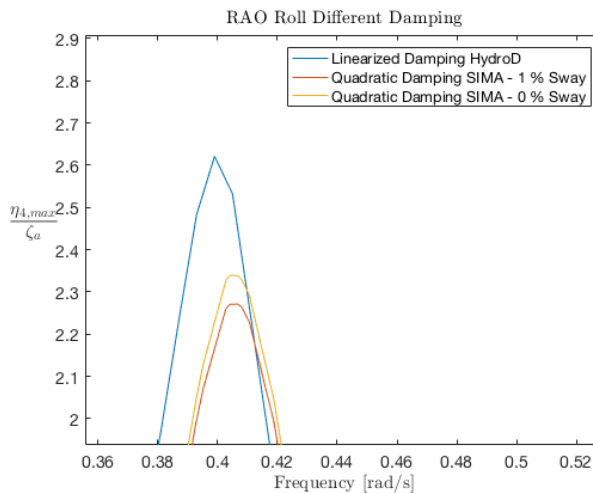


Figure 6.18: RAO Roll Comparison of SIMA and HydroD with Linearized and Quadratic Damping in 45 Degrees Wave Heading.

6.3 Irregular Waves

Several analyses are done in irregular waves. In this chapter the mean value of the standard deviation from the six wave seeds obtained for each condition is used. Quadratic damping is used in all analyses in both SIMA and HydroD.

6.3.1 45 Degrees Wave Heading

To compare the results between frequency and time domain in irregular waves the response spectra are plotted against each other. This can be seen for significant wave height 1.5 meters and different peak periods in Figures 6.19, 6.20, 6.21, 6.22, 6.23 and 6.24. For wave height 2.5, 3.5, 4.5, 5.5 and 6.5 meters, the spectra are presented in Appendix G.1.

From the wave spectra it can be seen that the energy is present for different frequencies in time and frequency domain. If the input in HydroD and SIMA are different, there would be no point in comparing the output. The wave spectra were therefore compared. The wave spectra coincide perfectly and excluded as an error source when comparing the roll motion spectra.

For condition number 1 it can be seen that there is a small amplitude at the natural period in roll, while there is no response in this area for frequency domain.

In both condition number 1 and 3 the responses are more jagged in frequency domain than for time domain. This may be due to the calculation method for the spectrum. As seen in Section 4.4.3, WAFO toolbox is used to calculate the roll response spectra in time domain. The maximum lag size of the window function and bandwidth given in this calculation may effect the form of the spectra.

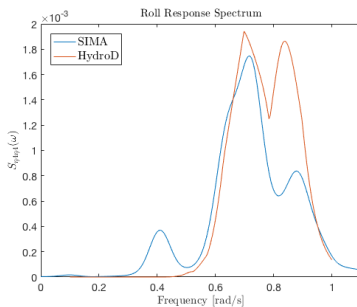


Figure 6.19: Response Spectra Condition Number 1.

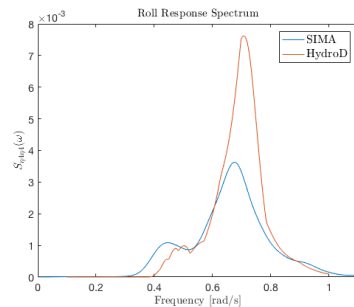


Figure 6.20: Response Spectra Condition Number 2.

For condition number 5 and 6, the peak is placed at the same amplitude around the natural period in roll. Nevertheless, there are some differences in the width of the graph. Espe-

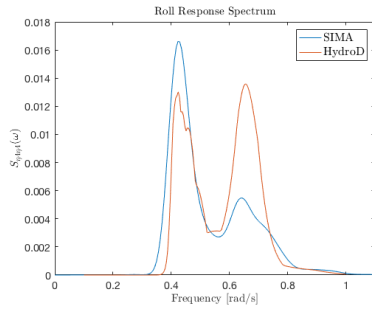


Figure 6.21: Response Spectra Condition Number 3.

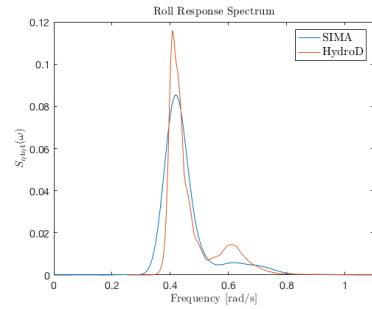


Figure 6.22: Response Spectra Condition Number 4.

cially in condition number 12, it can be seen that the response from HydroD has a higher and narrower peak. This indicates that the response is more concentrated around one frequency. The response in time domain has a wider range of frequencies in the response, which means that the variance in the frequencies is higher.

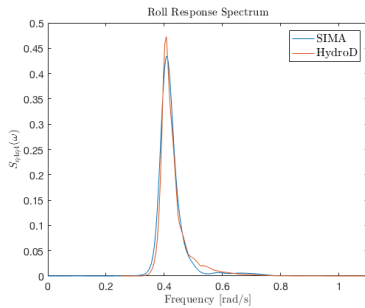


Figure 6.23: Response Spectra Condition Number 5.

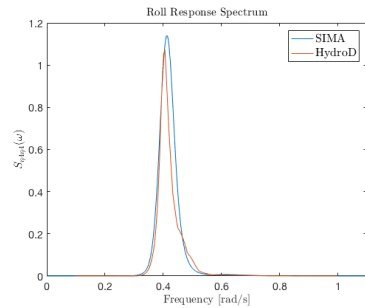


Figure 6.24: Response Spectra Condition Number 6.

Even though some of the plots are different, the energy in the spectra is more similar. As seen in Section 2.1.3, the standard deviation can be found by integrating the area under the graph. The standard deviation from SIMA and HydroD can be seen in Table 6.2. The comparison of the standard deviation shows that the energy in the roll motion is similar, it just contributes at different frequencies. In this table the mean value for each wave seed is used. The wave seeds are also an uncertainty. As seen in Section 5.6.1, six different spectra from six different wave seeds with the same environmental conditions may look completely different. The standard deviation of the standard deviations from the wave seeds was also large for each condition.

From Table 6.2 it can also be seen that the standard deviation in high periods is almost always larger for time domain, while the difference in small periods is larger in frequency

Table 6.2: Comparison of Standard Deviation between HydroD and SIMA.

Condition number	Mean Std SIMA	Std HydroD	Difference	Percentage Difference
1	0.02298	0.0227	2.8e-4	1.22 %
2	0.03259	0.0344	-1.81e-3	5.26 %
3	0.05511	0.0539	1.21e-3	2.19 %
4	0.10832	0.0953	0.01302	12.02 %
5	0.18504	0.1645	0.02054	11.10 %
6	0.28305	0.2416	0.04145	14.64 %
7	0.03814	0.0386	-4.6e-4	1.19 %
8	0.05426	0.0574	-3.14e-3	5.47 %
9	0.09134	0.0892	2.14e-3	2.34 %
10	0.17792	0.1584	0.01952	10.97 %
11	0.30014	0.2685	0.03164	10.54 %
12	0.45310	0.3936	0.0595	13.13 %
13	0.05254	0.0540	-1.46e-3	2.70 %
14	0.07684	0.0804	-5.56e-3	4.43%
15	0.12719	0.1248	2.39e-3	1.88%
16	0.24565	0.2194	0.02625	10.68 %
17	0.40984	0.3685	0.04134	10.08%
18	0.57399	0.5386	0.03539	6.17%
19	0.06640	0.0694	-3e-3	4.32 %
20	0.10121	0.1033	-2.09e-3	2.02 %
21	0.16105	0.1598	1.25e-3	0.78 %
22	0.30964	0.2793	0.03034	9.80 %
23	0.51488	0.4652	0.04968	9.65 %
24	0.71561	0.6753	0.04031	5.63 %
25	0.08111	0.0849	3.79e-3	4.46 %
26	0.12530	0.1263	-1e-3	0.79 %
27	0.19465	0.1948	-1.5e-4	0.08 %
28	0.35840	0.3382	0.0202	5.64 %
29	0.61003	0.5586	0.05143	8.43 %
30	0.85030	0.8064	0.0439	5.16 %
31	0.09581	0.1003	-4.49e-3	4.48 %
32	0.14794	0.1492	-1.26e-3	0.84 %
33	0.22811	0.2297	-1.59e-3	0.69 %
34	0.39759	0.3960	1.59e-3	0.40 %
35	0.67978	0.6493	0.03048	4.48 %
36	0.97240	0.9325	0.0399	4.10 %

domain.

In Table 6.3, the percentage difference between the standard deviation is shown for different significant wave heights and peak periods. The table shows that the difference is largest for condition number 6, with the lowest wave height and highest peak. However, the two spectra in Figure 6.24, look similar. For the Figures presented in this section, the spectres from condition number 1, 2 and 3 appear to be more different, but the percentage difference is small. The standard deviation should therefore always be compared as this gives a more correct perception of the differences.

6.3.2 90 Degrees Wave Heading

The same conditions as presented in the last section are analyzed for 90 degrees wave heading. Response spectra are plotted against each other. This can be seen for significant wave height 1.5 meters and different peak periods in Figures 6.25, 6.26, 6.27, 6.28, 6.29 and 6.30. For wave height 2.5, 3.5, 4.5, 5.5 and 6.5 meters, the spectra are presented in

Table 6.3: Percentage Differences between Standard Deviation in 45 Degrees.

Tp \ Hs	1.5	2.5	3.5	4.5	5.5	6.5
7.5	1.22 %	1.19 %	2.70 %	4.32 %	4.46 %	4.48 %
8.5	5.26 %	5.47 %	4.43 %	2.02 %	0.79 %	0.84 %
9.5	2.19 %	2.34 %	1.88%	0.78 %	0.08 %	0.69 %
10.5	12.02 %	10.97 %	10.68 %	9.80 %	5.64 %	0.40 %
11.5	11.10 %	10.54 %	10.08 %	9.65 %	8.43 %	4.48 %
12.5	14.64 %	13.13 %	6.17 %	5.63 %	5.16 %	4.10 %

Appendix G.2.

One can see that the wave spectra from SIMA for condition number 1 has a peak at 0.1 [rad/s] for 90 degrees. This was not present for 45 degrees. This is most likely due to the coupling with sway, which becomes more dominant at 90 than 45 degrees. This is also present for condition number 2. There is, however, no sign of the same coupled frequencies in HydroD.

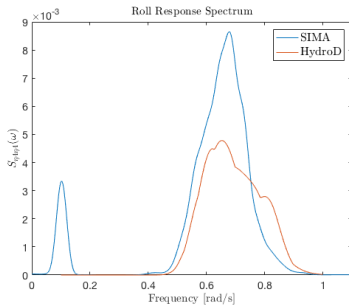


Figure 6.25: Response Spectra Condition Number 1.

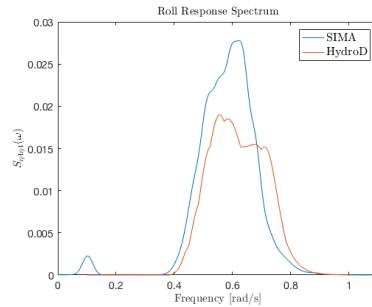


Figure 6.26: Response Spectra Condition Number 2.

For condition number 4, 5 and 6 the form of the spectres are similar to the spectres for 45 degrees, but with higher peaks. The same reasons for differences, as discussed for 45 degrees, are applicable for 90 degrees as well.

The standard deviation from SIMA and HydroD for 90 degrees can be seen in Table 6.4. In Table 6.3, the percentage difference between the standard deviation is showed for different significant wave heights and peak periods.

The table shows that the largest difference appears for condition number 3. The percentage

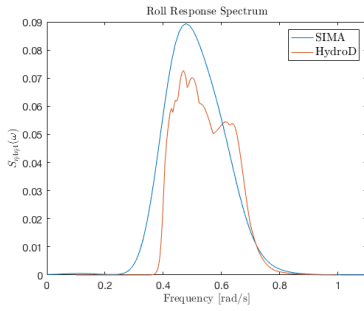


Figure 6.27: Response Spectra Condition Number 3.

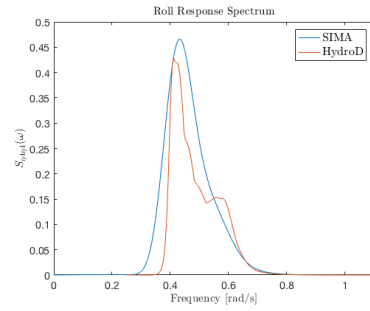


Figure 6.28: Response Spectra Condition Number 4.

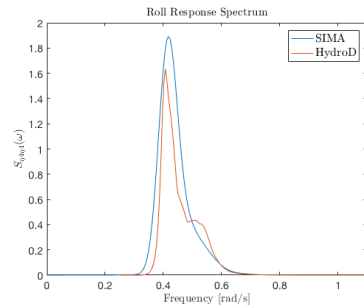


Figure 6.29: Response Spectra Condition Number 5.

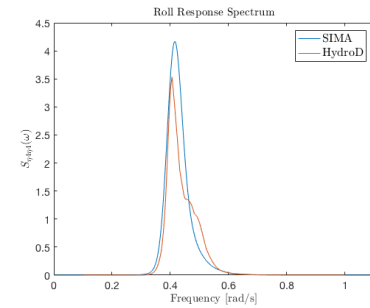


Figure 6.30: Response Spectra Condition Number 6.

difference is very different for 90 degrees compared to 45 degrees. Some of the values are much smaller, while others is much higher, which means larger variations for 90 degrees.

It seems like the worst cases are 3.5 and 4.5 [m] significant wave height and small peak periods. For higher periods, the standard deviations are almost the same.

Table 6.4: Comparison of Standard Deviation for Conditions.

Condition number	Mean Std SIMA	Std HydroD	Difference	Percentage Difference
1	0.03736	0.0342	3.16e-3	8.45 %
2	0.07083	0.0695	1.33e-3	1.88 %
3	0.13814	0.1313	6.84e-3	4.95 %
4	0.25339	0.2316	0.02179	8.60 %
5	0.39843	0.3716	0.02683	6.73 %
6	0.57334	0.5175	0.05584	9.74 %
7	0.06131	0.0588	2.51e-3	4.09%
8	0.11790	0.1157	2.2e-3	1.87%
9	0.22823	0.2155	0.01273	5.58%
10	0.41205	0.3821	0.02995	7.27%
11	0.63503	0.5991	0.3593	5.66%
12	0.89716	0.8263	0.07086	7.90 %
13	0.07958	0.1057	-0.02612	24.7%
14	0.16100	0.2080	-0.047	22.60 %
15	0.31693	0.3843	-0.6737	17.53 %
16	0.56419	0.5265	0.03769	6.68 %
17	0.85555	0.8146	0.04095	4.79 %
18	1.12810	1.1160	0.0121	1.07%
19	0.09645	0.0822	0.01425	14.77 %
20	0.19789	0.1619	0.03599	18.19 %
21	0.39142	0.3022	0.08922	22.79 %
22	0.70738	0.6670	0.04038	5.71 %
23	1.06347	1.0202	0.04327	4.07 %
24	1.38927	1.3866	2.67e-3	0.19%
25	0.11771	0.1292	-0.01149	8.89 %
26	0.23169	0.2541	-0.02241	8.82%
27	0.45628	0.4677	-0.01142	2.44 %
28	0.82320	0.8043	0.0189	2.30 %
29	1.25409	1.2181	0.03599	2.87 %
30	1.63470	1.6427	-8e-3	0.49 %
31	0.13891	0.1527	-0.01379	9.03 %
32	0.27365	0.3001	-0.02645	8.81 %
33	0.51637	0.5504	-0.03403	6.18 %
34	0.92310	0.9384	-0.0153	1.63 %
35	1.41221	1.4086	3.61e-3	0.25 %
36	1.86236	1.8872	-0.02484	1.32 %

Table 6.5: Percentage Differences between Standard Deviation in 90 Degrees.

Tp\ Hs	1.5	2.5	3.5	4.5	5.5	6.5
7.5	8.45 %	4.09 %	24.7 %	14.77 %	8.89 %	9.03 %
8.5	1.88 %	1.87 %	22.60 %	18.19 %	8.82 %	8.81 %
9.5	4.95 %	5.58 %	17.53 %	22.79 %	2.44 %	6.18 %
10.5	8.60 %	7.27%	6.68 %	5.71 %	2.30 %	1.63 %
11.5	6.73 %	5.66%	4.79 %	4.07 %	2.87 %	0.25 %
12.5	9.74 %	7.90 %	1.07 %	0.19 %	0.49 %	1.32 %

Chapter 7

Conclusion

This paper has focused on investigating the differences in damping for roll motions in time domain and frequency domain simulations. It has been seen that roll motion is one of the most dominating motions, contributing with significant force to the sea fastening systems. The barge will therefore be exposed to large roll motions and correct calculation of roll motion is important.

The total roll damping of a floating vessel may be divided into potential and viscous components. Component one, the potential component can be predicted accurately because of its linear characteristic. The viscous component is however non-linear and its prediction is more problematic. This means that it has to be linearized in frequency domain. Stochastic linearization as seen in Section 2.4 is the most common, which may lead to some inaccuracy in the frequency domain. However time domain analysis can be time consuming.

The barge will have zero forward speed in the time domain simulations, as SIMA cannot handle analyses with forward speed. To achieve realistic results for a barge with forward speed some values had to be corrected. As we do not have any hydrostatic restoring effect in surge, sway and yaw, stiffness is added to avoid free rigid-body motions. Otherwise, the SIMA simulations would result in large offsets in the horizontal motions. A proper damping is then also needed so that the resonant motions in these DOFs are reasonable. It has been seen that the level of damping in sway has a great impact on the roll motion. The percentage of sway damping was chosen at the lowest level where the horizontal offset was acceptable and the roll motion reached steady state without fluctuations due to coupled effects. This happened at 1 % of critical damping in sway.

Stiffness was added to avoid large horizontal movement and resulted in natural periods for surge, sway and yaw. The natural periods from the decay test in these degrees of motions were 53.5, 54.8 and 54.3, respectively. The stiffness in these directions was zero in HydroD, therefore the natural period is zero. There is accordingly no point in comparing periods in these degrees of freedom. When comparing results from HydroD and SIMA, natural periods in heave, roll and pitch only differed 3-6 % from each other. In SIMA

they were 11.7, 14.8 and 10.7, respectively. The difference may be due to coupled motions, where the damping in other degrees of freedom affects the damping in roll. Another reason can be differences in calculation processes between SIMA and HydroD. Two concrete examples are that SIMA uses the damped natural frequency instead of the natural frequency, and the added mass for infinity frequency instead of added mass corresponding to the natural frequency.

For regular waves, an iteration process in HydroD provided the linearized damping for regular waves with 45 degrees wave heading. Comparing the linear, linearized and quadratic damping RAO's, it was seen that they were similar for all frequencies, except around the natural frequencies. In this area resonance may occur and the quadratic and linearized damping will damp the roll motion mostly in this area to avoid resonance effects. It was also seen that there was good consistency between the linearized and quadratic damping. For both linear and linearized damping the difference between time and frequency domain analysis is about 10-15 %. This may be due to coupled effects or conservative results in the frequency domain analyzes.

For irregular waves, six wave seeds have been used for each sea state. As seen in Section 5.6.1, six different spectra from six different wave seeds with the same environmental conditions, may look completely different. The standard deviation of the standard deviations from the wave seeds was also large for each condition. To get an applicable mean value of the roll motions, several wave seeds should be used.

Another reason for the different form of the spectres is that the energy is present for different frequencies in time and frequency domain. The total energy may however be more similar. The comparison of the standard deviations showed that the energy in the roll motion is similar, it just contributes at different frequencies.

Both in regular and irregular waves the factor contributing most to the roll motion was the wave heading. When the barge is exposed to beam sea the amplitude in roll increases over 2.5 times the amplitude in 45 degrees. 90 degrees wave heading should therefore be avoided, and it is rare that the barge will experience beam waves. A given scenario may however be in case of tug engine failure or tow line breakage. The barge will then drift and typically be exposed to beam sea (Natskår and Steen (2013)). It should therefore be in the calculations to take precaution.

Chapter 8

Recommendations for Further Work

Recommendations for later work will first of all be to find the linearization factor for regular waves in several degrees to verify the linearization technique for other degrees than 45. This will lead to a better foundation for a conclusion of the linearization method.

As presented earlier, DNV GL has done extensive computational fluid dynamics (CFD) analyses of the system analyzed in this thesis. A comparison of results obtained with linear, linearized and quadratic damping to these results, could give even better insight into the validity of these models, and more knowledge as well. CFD is known to demand a lot of computer power, but when carried out properly, also produces accurate results.

For irregular waves it was seen that the roll response spectres from SIMA and HydroD had a good fitting for some sea states. Others were however quite different and the linearization from HydroD had a bad fitting. One of the reasons for the differences may be the wave seeds. Wave spectra from the different wave seeds were compared and found very different. More wave seeds should therefore be introduced in order to verify how the linearization works for irregular waves.

In these calculations only linear effects have been taken into account. This theory requires a linearization of the free surface boundary condition by simply neglecting all higher order terms. As seen in Section 2.7 some of the second order hydrodynamic loads should be investigated to find the effects of second order loads on roll motion. Analyses with non linearities in the waves should also be investigated, this leads to higher crests and smaller trough, which would influence the roll motion.

Further investigations can be performed using nonlinear effects at the model. The model has one mass with one centre of gravity. This means that the forces between the structures are not taken into account. In addition vortex induced vibration (VIV) due to wind, water impact loads on overhanging cargo and vibration caused by slamming on transportation

vessel will effect the fatigue life.

Even though non linear effects are ignored, another wave spectrum may still be introduced. In this thesis JONSWAP spectra is used. This is designed for the North Sea, and may therefore not fit the wave elevation during the transportation. Which wave spectra that give the most correct behavior should be investigated and used in further analyzes.

The main reason for estimation of roll motion is to later find the acceleration and use this in fatigue damage calculation. Further work should therefore be done to use the spectra obtained in frequency and time domain in order to find the acceleration. The fatigue damage for time and frequency domain can then be compared to look at the differences and in the end conclude on the accuracy of the linearization technique. It can be assumed that the time domain model results, as long as the model is modelled correctly, results in more realistic results, true to the natural behavior.

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Appendix A: Results from Decay Test

Figure A.1, A.2, A.3 and A.4 shows the descending speed in the system for all four considered motions.

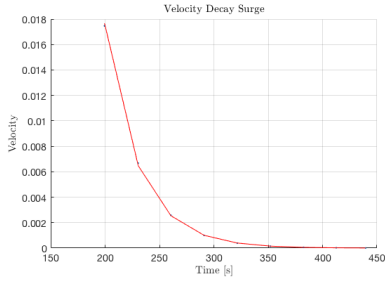


Figure A.1: Velocity for a Decay Test in Surge.

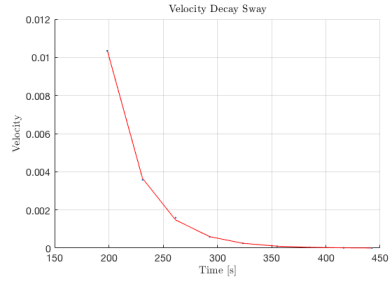


Figure A.2: Velocity for a Decay Test in Sway.

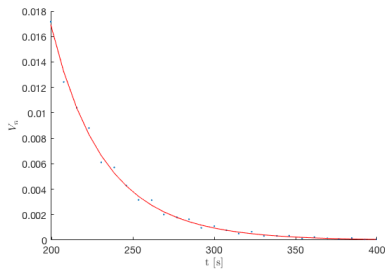


Figure A.3: Velocity for a Decay test in Roll.

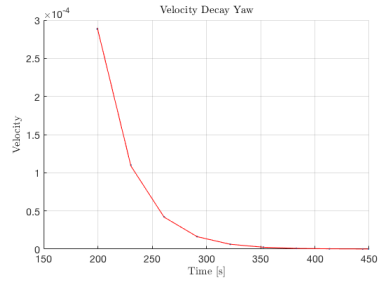


Figure A.4: Velocity for a Decay Test in Yaw.

Appendix B: Damping from HydroD

B.1 Quadratic Damping

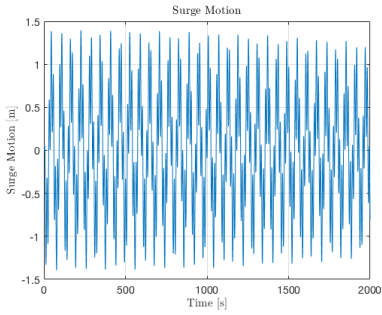


Figure B.1: Surge Motion Wave Period 13 [s] Significant Wave Height 1 [m].

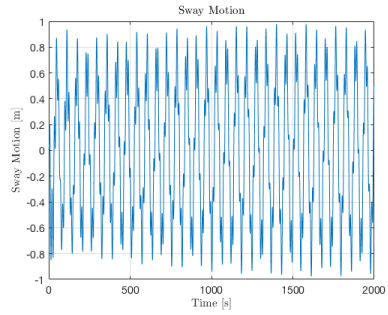


Figure B.2: Sway Motion Wave Period 13 [s] Significant Wave Height 1 [m].

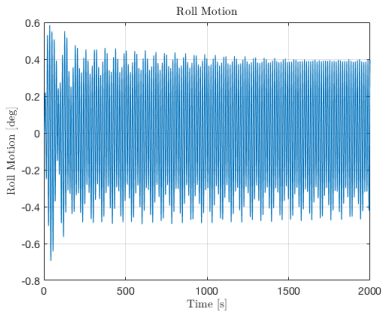


Figure B.3: Roll Motion Wave Period 13 [s] Significant Wave Height 1 [m].

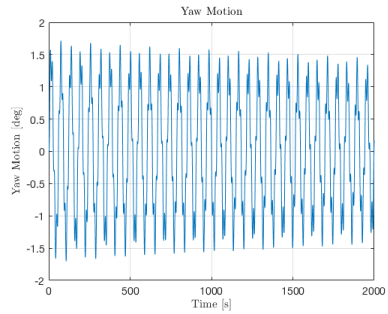


Figure B.4: Yaw Motion Wave Period 13 [s] Significant Wave Height 1 [m].

B.2. LINEAR DAMPING

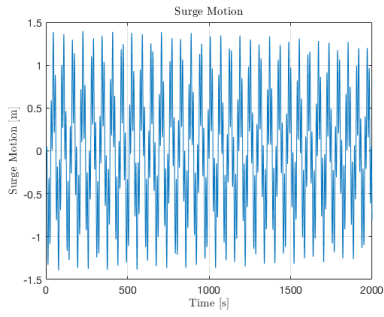


Figure B.5: Surge Motion Wave Period 13 [s] Significant Wave Height 1 [m].

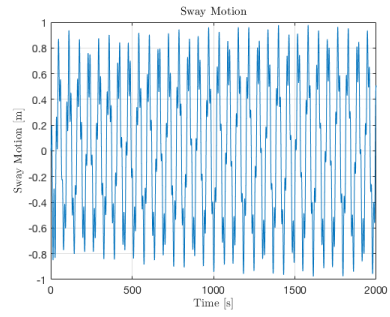


Figure B.6: Sway Motion Wave Period 13 [s] Significant Wave Height 1 [m].

B.2 Linear Damping

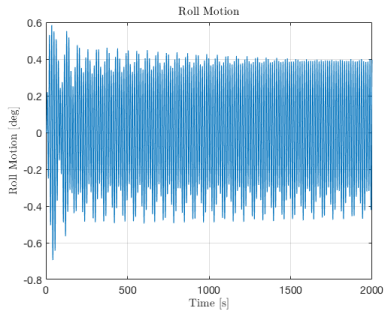


Figure B.7: Roll Motion Wave Period 13 [s] Significant Wave Height 1 [m].

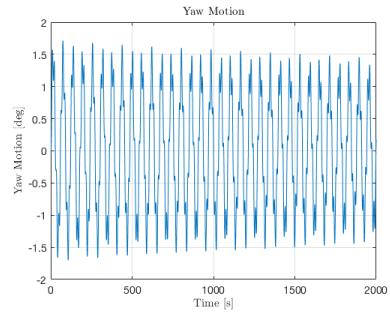


Figure B.8: Yaw Motion Wave Period 13 [s] Significant Wave Height 1 [m].

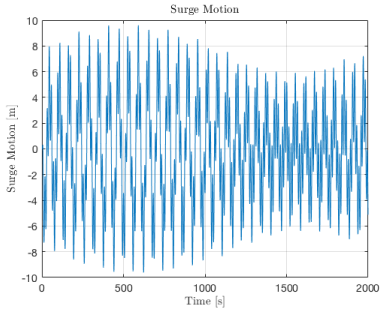


Figure B.9: Surge Motion Wave Period 13 [s] Significant Wave Height 6 [m].

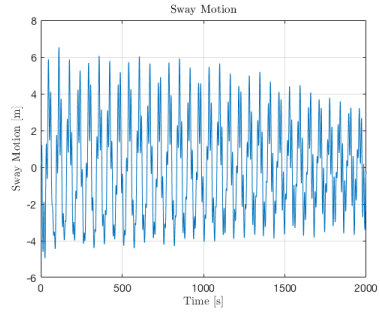


Figure B.10: Sway Motion Wave Period 13 [s] Significant Wave Height 6 [m].

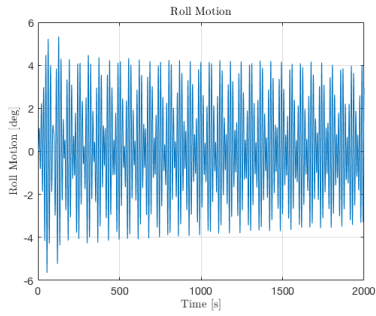


Figure B.11: Roll Motion Wave Period 13 [s] Significant Wave Height 6 [m].

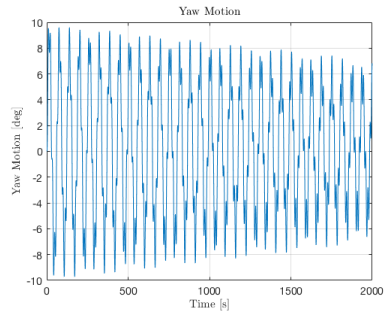


Figure B.12: Yaw Motion Wave Period 13 [s] Significant Wave Height 6 [m].

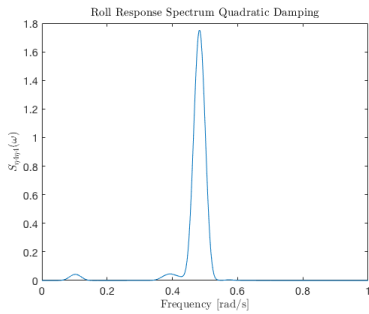


Figure B.13: Response Spectrum for Roll in Regular Waves Wave Period 13 [s] Significant Wave Height 6 [m].

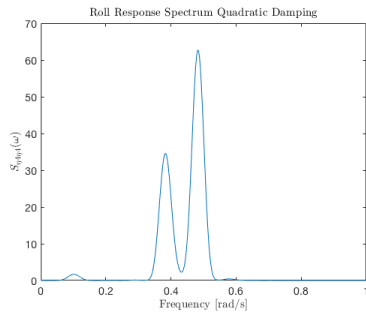


Figure B.14: Response Spectrum for Roll in Regular Waves Wave Period 13 [s] Significant Wave Height 6 [m].

Appendix C: 15 % total damping ratio

C.1 Linear Roll Damping

C.1.1 45 Degrees

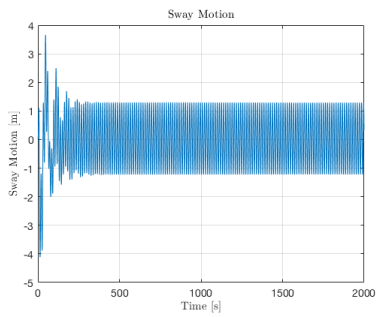


Figure C.1: Sway Motion Wave Period 13 [s] Significant Wave Height 6 [m].

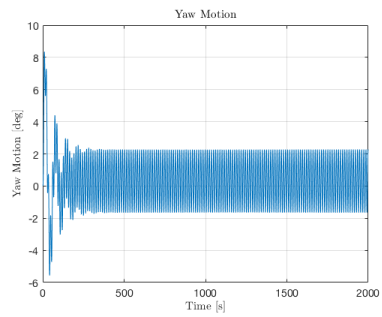


Figure C.2: Yaw Motion Wave Period 13 [s] Significant Wave Height 6 [m].

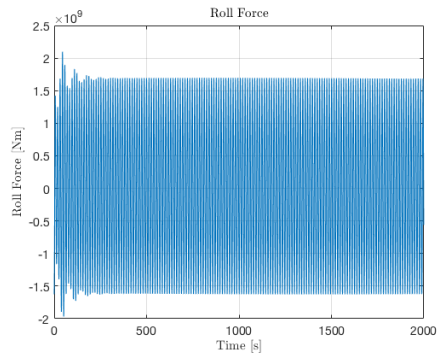


Figure C.3: Roll Force Wave Period 13 [s] Significant Wave Height 6 [m].

Table C.1: Roll Amplitude with Linear Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0114	0.0229	0.0344	0.0459	0.0575	0.0691
6	0.0229	0.0459	0.0687	0.0916	0.11445	0.1373
7	0.0641	0.1280	0.1917	0.2551	0.3182	0.3811
8	0.0644	0.1287	0.1928	0.2571	0.3212	0.3850
9	0.1445	0.2886	0.4324	0.5755	0.7180	0.8599
10	0.1643	0.3282	0.4918	0.6552	0.8185	0.9816
11	0.1105	0.2197	0.3285	0.4382	0.5476	0.6564
12	0.1482	0.2994	0.4525	0.6033	0.7484	0.8858
13	0.3405	0.6818	1.0274	1.3815	1.7453	2.1182
14	0.8009	1.6015	2.4028	3.2071	4.0122	4.8250
14.5	1.1894	2.3797	3.5717	4.7664	5.9663	7.1730
15	1.7809	3.5509	5.2975	7.0099	8.6802	10.3066
15.3	2.0554	4.0802	6.0472	7.9350	9.7309	11.4265
15.6	2.0861	4.1326	6.1058	7.9825	9.7476	11.3942
15.9	1.8728	3.7154	5.5004	7.2059	8.8192	10.3303
16	1.7865	3.5474	5.2592	6.9017	8.4602	9.9264
16.2	1.5975	3.1775	4.7233	6.2209	7.6577	9.0269
16.5	1.3741	2.7380	4.0811	5.3934	6.6656	7.8906
17	1.1050	2.2052	3.2954	4.3718	5.4294	6.4636
18	0.8001	1.5989	2.3956	3.1889	3.9777	4.7611
19	0.6116	1.2231	1.8343	2.4450	3.0550	3.6644
20	0.5008	1.0020	1.5034	2.0051	2.5080	3.0122

Table C.2: Sway Amplitude with Linear Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0185	0.0371	0.0558	0.0748	0.0942	0.1135
6	0.0104	0.0211	0.0319	0.0424	0.0525	0.0618
7	0.0270	0.0548	0.0837	0.1118	0.1380	0.1631
8	0.0259	0.0516	0.0771	0.1030	0.1278	0.1503
9	0.0483	0.0962	0.1442	0.1914	0.2373	0.2809
10	0.1740	0.3473	0.5184	0.6913	0.8612	1.0279
11	0.2692	0.5399	0.8091	1.0761	1.3411	1.5951
12	0.3374	0.6682	0.9823	1.2905	1.6078	1.9123
13	0.3577	0.7187	1.0808	1.4528	1.8201	2.1958
14	0.3772	0.7424	1.1111	1.4849	1.8385	2.1922
14.5	0.3690	0.7355	1.1030	1.4556	1.8031	2.1727
15	0.3473	0.7000	1.0563	1.4055	1.7583	2.0973
15.3	0.3411	0.6770	1.0198	1.3754	1.7262	2.0674
15.6	0.3413	0.6811	1.0161	1.3518	1.7032	2.0602
15.9	0.3464	0.6912	1.0329	1.3736	1.7043	2.0387
16	0.3470	0.6939	1.0352	1.3761	1.7115	2.0406
16.2	0.3455	0.6942	1.0414	1.3824	1.7202	2.0564
16.5	0.3438	0.6855	1.0317	1.3783	1.7198	2.0524
17	0.3428	0.6819	1.0174	1.3489	1.6833	2.0273
18	0.3275	0.6524	0.9728	1.2907	1.6070	1.9290
19	0.3134	0.6236	0.9310	1.2428	1.5578	1.8728
20	0.2949	0.5876	0.8798	1.1735	1.4674	1.7613

Table C.3: Yaw Amplitude with Linear Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0185	0.0371	0.0558	0.0748	0.0942	0.1135
6	0.0104	0.0211	0.0319	0.0424	0.0525	0.0618
7	0.0270	0.0548	0.0837	0.1118	0.1380	0.1631
8	0.0259	0.0516	0.0771	0.1030	0.1278	0.1503
9	0.0483	0.0962	0.1442	0.1914	0.2373	0.2809
10	0.1740	0.3473	0.5184	0.6913	0.8612	1.0279
11	0.2692	0.5399	0.8091	1.0761	1.3411	1.5951
12	0.3374	0.6682	0.9823	1.2905	1.6078	1.9123
13	0.3577	0.7187	1.0808	1.4528	1.8201	2.1958
14	0.3772	0.7424	1.1111	1.4849	1.8385	2.1922
14.5	0.3690	0.7355	1.1030	1.4556	1.8031	2.1727
15	0.3473	0.7000	1.0563	1.4055	1.7583	2.0973
15.3	0.3411	0.6770	1.0198	1.3754	1.7262	2.0674
15.6	0.3413	0.6811	1.0161	1.3518	1.7032	2.0602
15.9	0.3464	0.6912	1.0329	1.3736	1.7043	2.0387
16	0.3470	0.6939	1.0352	1.3761	1.7115	2.0406
16.2	0.3455	0.6942	1.0414	1.3824	1.7202	2.0564
16.5	0.3438	0.6855	1.0317	1.3783	1.7198	2.0524
17	0.3428	0.6819	1.0174	1.3489	1.6833	2.0273
18	0.3275	0.6524	0.9728	1.2907	1.6070	1.9290
19	0.3134	0.6236	0.9310	1.2428	1.5578	1.8728
20	0.2949	0.5876	0.8798	1.1735	1.4674	1.7613

APPENDIX C. 15 % TOTAL DAMPING RATIO

Table C.4: Force Amplitude with Linear Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	60637970	121180125	181626445	241976921	302231375	362389578
6	82471324	164859734	247163429	329380109	411506515	493537765
7	113106445	225921109	338420453	450571281	562326656	673621750
8	113896406	227800664	341695031	455622640	569580968	683531593
9	164775250	329453453	493857656	657855781	821394875	984410062
10	87052988	174319468	261934062	350011078	438651609	527997250
11	79375402	159169992	238547460	315568296	388275468	454437765
12	181498484	362980687	543161656	721527187	896070625	1064136031
13	279714406	560022062	840310781	1119157687	1395627625	1668319375
14	349385656	699909093	1051512218	1405532312	1762655750	2123893750
14.5	364485484	729675937	1095947687	1463706625	1835403500	2214640750
15	379452968	758954343	1138579625	1520417125	1903706687	2286687250
15.3	387501125	774740843	1161066875	1546718437	1930610250	2311472875
15.6	395504203	789708500	1181352187	1570405875	1955785562	2337019750
15.9	397619265	793930156	1187537625	1576823875	1960192812	2336727875
16	396279640	791265906	1183851375	1572982500	1957259437	2335503625
16.2	396735437	792452531	1185627687	1574558312	1957752437	2335289750
16.5	395470765	789617250	1181192937	1570368062	1954857750	2333904625
17	394569375	788385593	1180893937	1571045500	1957669187	2340271625
18	391199781	782015343	1172043937	1560950000	1948642125	2334134625
19	372786843	745375812	1117614250	1489547312	1860781625	2230984250
20	356280593	712459437	1068437812	1424128000	1779581125	2134500000

C.1.2 15 Degrees

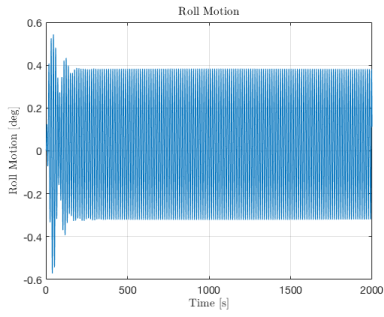


Figure C.4: Roll Motion Wave Period 13 [s] Significant Wave Height 6 [m].

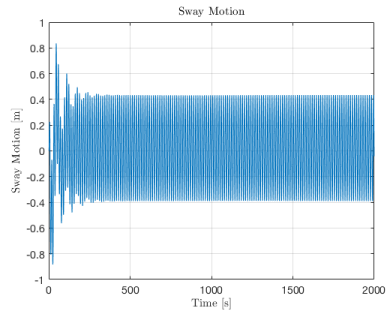


Figure C.5: Sway Motion Wave Period 13 [s] Significant Wave Height 6 [m].

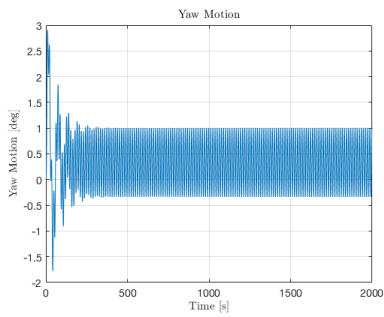


Figure C.6: Yaw Motion Wave Period 13 [s] Significant Wave Height 6 [m].

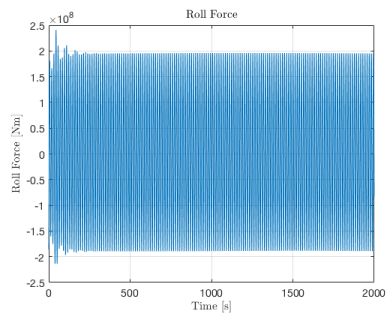


Figure C.7: Roll Force Wave Period 13 [s] Significant Wave Height 6 [m].

Table C.5: Roll Amplitude with Linear Damping at 15 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0079	0.0159	0.0238	0.0317	0.0397	0.0476
6	0.0085	0.0170	0.0255	0.0340	0.0425	0.0510
7	0.0108	0.0216	0.0323	0.0431	0.0538	0.0646
8	0.0384	0.0769	0.1154	0.1539	0.1925	0.2312
9	0.0273	0.0546	0.0819	0.1092	0.1365	0.1638
10	0.0657	0.1314	0.1971	0.2627	0.3282	0.3936
11	0.0833	0.1663	0.2488	0.3307	0.4123	0.4931
12	0.1230	0.2447	0.3648	0.4833	0.6004	0.7158
13	0.0603	0.1203	0.1799	0.2390	0.2974	0.3551
14	0.1307	0.2610	0.3901	0.5172	0.6415	0.7622
14.5	0.2223	0.4438	0.6628	0.8781	1.0881	1.2916
15	0.3820	0.7621	1.1373	1.5047	1.8620	2.2066
15.3	0.4706	0.9378	1.3975	1.8460	2.2800	2.6967
15.6	0.5081	1.0120	1.5067	1.9881	2.4526	2.8968
15.9	0.4742	0.9446	1.4069	1.8571	2.2917	2.7077
16	0.4563	0.9092	1.3547	1.7890	2.2089	2.6113
16.2	0.4162	0.8293	1.2362	1.6338	2.0191	2.3899
16.5	0.3677	0.7330	1.0934	1.4464	1.7897	2.1212
17	0.3085	0.6154	0.9188	1.2171	1.5085	1.7915
18	0.2404	0.4799	0.7175	0.9521	1.1828	1.4088
19	0.1899	0.3793	0.5677	0.7546	0.9393	1.1213
20	0.1604	0.3205	0.4802	0.6391	0.7969	0.9534

Table C.6: Yaw Amplitude with Linear Damping at 15 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0032	0.0063	0.0095	0.0126	0.0157	0.0188
6	0.0044	0.0087	0.0130	0.0174	0.0216	0.0259
7	0.0076	0.0154	0.0234	0.0315	0.0397	0.0479
8	0.0068	0.0135	0.0202	0.0269	0.0335	0.0400
9	0.0126	0.0252	0.0378	0.0504	0.0631	0.0758
10	0.0153	0.0306	0.0459	0.0614	0.0768	0.0921
11	0.0551	0.1105	0.1659	0.2206	0.2743	0.3264
12	0.0970	0.1927	0.2865	0.3806	0.4731	0.5636
13	0.1252	0.2502	0.3748	0.4987	0.6214	0.7422
14	0.1478	0.2951	0.4413	0.5859	0.7285	0.8683
14.5	0.1485	0.2966	0.4436	0.5890	0.7320	0.8718
15	0.1463	0.2921	0.4368	0.5799	0.7206	0.8597
15.3	0.1485	0.2959	0.4417	0.5856	0.7276	0.8668
15.6	0.1515	0.3019	0.4506	0.5972	0.7411	0.8819
15.9	0.1532	0.3059	0.4569	0.6054	0.7504	0.8908
16	0.1533	0.3060	0.4571	0.6055	0.7502	0.8898
16.2	0.1527	0.3045	0.4547	0.6025	0.7469	0.8873
16.5	0.1530	0.3049	0.4549	0.6028	0.7479	0.8899
17	0.1536	0.3064	0.4577	0.6069	0.7535	0.8971
18	0.1509	0.3012	0.4504	0.5979	0.7435	0.8868
19	0.1458	0.2908	0.4348	0.5773	0.7181	0.8569
20	0.1390	0.2775	0.4152	0.5520	0.6877	0.8222

Table C.7: Sway Amplitude with Linear Damping at 15 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0042	0.0083	0.0125	0.0166	0.0207	0.0248
6	0.0034	0.0068	0.0102	0.0137	0.0171	0.0205
7	0.0045	0.0089	0.0133	0.0178	0.0222	0.0267
8	0.0200	0.0399	0.0600	0.0801	0.1002	0.1203
9	0.0175	0.0351	0.0526	0.0701	0.0876	0.1051
10	0.0225	0.0450	0.0675	0.0899	0.1124	0.1348
11	0.0304	0.0609	0.0914	0.1220	0.1526	0.1832
12	0.0705	0.1408	0.2109	0.2807	0.3504	0.4198
13	0.0734	0.1468	0.2203	0.2941	0.3681	0.4425
14	0.0990	0.1979	0.2968	0.3959	0.4952	0.5946
14.5	0.1314	0.2624	0.3929	0.5225	0.6508	0.7777
15	0.2137	0.4264	0.6372	0.8447	1.0481	1.2463
15.3	0.2765	0.5516	0.8235	1.0908	1.3521	1.6063
15.6	0.3212	0.6407	0.9565	1.2670	1.5706	1.8660
15.9	0.3282	0.6548	0.9784	1.2974	1.6105	1.9167
16	0.3249	0.6484	0.9690	1.2855	1.5966	1.9013
16.2	0.3148	0.6285	0.9400	1.2482	1.5522	1.8513
16.5	0.3025	0.6042	0.9044	1.2022	1.4970	1.7881
17	0.2891	0.5778	0.8655	1.1517	1.4359	1.7178
18	0.2814	0.5625	0.8433	1.1235	1.4029	1.6814
19	0.2728	0.5457	0.8184	1.0910	1.3633	1.6353
20	0.2754	0.5509	0.8265	1.1022	1.3780	1.6539

C.1. LINEAR ROLL DAMPING

Table C.8: Force Amplitude with Linear Damping at 15 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	33095398	66184023	99265910	132341156	165409625	198471164
6	27095707	54185027	81267949	108344496	135414664	162478500
7	27979423	55958240	83936128	111912730	139887656	167859343
8	47830662	95661019	143490171	191317140	239140851	286960390
9	29609125	59249179	88924726	118634992	148380468	178161531
10	59200160	118388652	177557218	236698093	295803843	354867593
11	47544060	94996027	142323632	189497242	236489710	283275968
12	24908526	49766869	74571734	99109417	123383746	147551148
13	32882959	65699380	98287328	130720074	162691968	193993304
14	69459375	138820320	207711835	275772515	342652562	408461234
14.5	81236082	162297414	242932507	322737812	401283250	478362640
15	92462082	184607218	275975687	366126328	454869437	541764125
15.3	98664066	196855632	294018359	389817765	483716593	575303968
15.6	104857480	209109757	312135281	413449078	512555203	608914687
15.9	108418582	216191656	322742921	427449046	529668578	628917437
16	109120964	217644382	324864468	430206171	533027718	632766343
16.2	110612011	220599367	329399250	436445734	541368187	643468781
16.5	112524050	224469757	335306281	444445906	551459000	655700062
17	116103269	231732125	346313515	459344359	570277093	678608312
18	121894980	243374218	364067796	483601859	601534562	717706625
19	119394632	238478023	356928359	474436171	590683718	705611406
20	117152648	234069554	350510343	466235890	581096187	694898500

C.1.3 0 Degrees

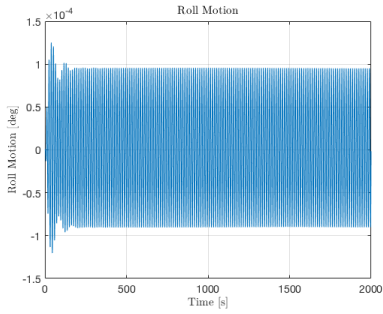


Figure C.8: Roll Motion Wave Period 13 [s] Significant Wave Height 6 [m].

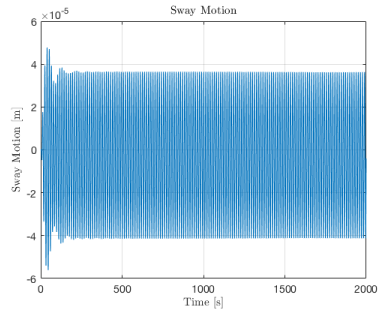


Figure C.9: Sway Motion Wave Period 13 [s] Significant Wave Height 6 [m].

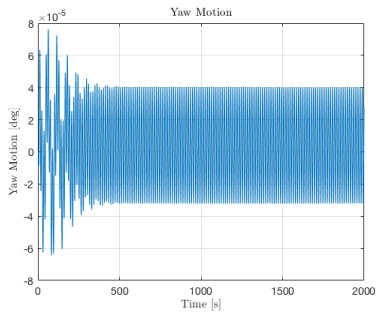


Figure C.10: Yaw Motion Wave Period 13 [s] Significant Wave Height 6 [m].

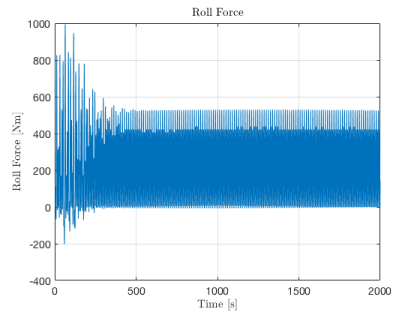


Figure C.11: Roll Force Wave Period 13 [s] Significant Wave Height 6 [m].

Table C.9: Roll Amplitude with Linear Damping at 0 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	3.1758e-07	6.3519e-07	9.5199e-07	1.2635e-06	1.5995e-06	1.91400e-06
6	6.9426e-07	1.3917e-06	2.0810e-06	2.7749e-06	3.4675e-06	4.16893e-06
7	1.4633e-06	2.9253e-06	4.3861e-06	5.8482e-06	7.3078e-06	8.75697e-06
8	3.1047e-06	6.2039e-06	9.3025e-06	1.2386e-05	1.5483e-05	1.85600e-05
9	5.3071e-06	1.0620e-05	1.5940e-05	2.1263e-05	2.6589e-05	3.19205e-05
10	9.3766e-06	1.8744e-05	2.8119e-05	3.7504e-05	4.6900e-05	5.62983e-05
11	1.1878e-05	2.3744e-05	3.5608e-05	4.7488e-05	5.9378e-05	7.12750e-05
12	1.1490e-05	2.2999e-05	3.4532e-05	4.6091e-05	5.7675e-05	6.93466e-05
13	1.5613e-05	3.1195e-05	4.6769e-05	6.2339e-05	7.7923e-05	9.34853e-05
14	3.6280e-05	7.2498e-05	1.0860e-04	1.4458e-04	1.8045e-04	2.16154e-04
14.5	6.3619e-05	1.2709e-04	1.9025e-04	2.5296e-04	3.1511e-04	3.76524e-04
15	1.0232e-04	2.0430e-04	3.0555e-04	4.0576e-04	5.0461e-04	6.01807e-04
15.3	1.2058e-04	2.4071e-04	3.5997e-04	4.7795e-04	5.9423e-04	7.08543e-04
15.6	1.2351e-04	2.4671e-04	3.6932e-04	4.9103e-04	6.1161e-04	7.30849e-04
15.9	1.1468e-04	2.2922e-04	3.4351e-04	4.5737e-04	5.7068e-04	6.83384e-04
16	1.1054e-04	2.2099e-04	3.3130e-04	4.4138e-04	5.5115e-04	6.60627e-04
16.2	1.0140e-04	2.0278e-04	3.0411e-04	4.0539e-04	5.0659e-04	6.07774e-04
16.5	8.9604e-05	1.7926e-04	2.6896e-04	3.5876e-04	4.4872e-04	5.38875e-04
17	7.4326e-05	1.4863e-04	2.2299e-04	2.9742e-04	3.7193e-04	4.46667e-04
18	5.4972e-05	1.1000e-04	1.6512e-04	2.2035e-04	2.7573e-04	3.31302e-04
19	4.3952e-05	8.7966e-05	1.3208e-04	1.7629e-04	2.2062e-04	2.65118e-04
20	3.6489e-05	7.2993e-05	1.0956e-04	1.4635e-04	1.8336e-04	2.20663e-04

APPENDIX C. 15 % TOTAL DAMPING RATIO

Table C.10: Yaw Amplitude with Linear Damping at 0 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	2.7836e-07	5.5730e-07	8.3737e-07	1.1100e-06	1.3875e-06	1.67348e-06
6	1.1750e-06	2.3499e-06	3.5200e-06	4.6987e-06	5.8735e-06	7.06147e-06
7	2.2006e-06	4.3978e-06	6.5835e-06	8.7698e-06	1.0957e-05	1.31352e-05
8	3.2307e-06	6.4657e-06	9.7039e-06	1.2949e-05	1.6194e-05	1.94486e-05
9	3.7543e-06	7.5004e-06	1.1250e-05	1.4975e-05	1.8708e-05	2.24228e-05
10	5.6397e-06	1.1283e-05	1.6911e-05	2.2547e-05	2.8181e-05	3.38257e-05
11	3.8080e-06	7.6285e-06	1.1426e-05	1.5195e-05	1.9072e-05	2.29185e-05
12	4.6760e-06	9.3744e-06	1.4093e-05	1.8779e-05	2.3363e-05	2.78991e-05
13	8.0093e-06	1.6006e-05	2.3962e-05	3.1927e-05	3.9792e-05	4.79122e-05
14	1.1164e-05	2.2328e-05	3.3607e-05	4.4834e-05	5.6136e-05	6.74546e-05
14.5	1.1332e-05	2.2737e-05	3.4185e-05	4.5728e-05	5.7323e-05	6.90448e-05
15	1.1912e-05	2.3890e-05	3.6072e-05	4.8565e-05	6.1260e-05	7.42196e-05
15.3	1.1942e-05	2.3931e-05	3.6073e-05	4.8605e-05	6.1439e-05	7.45488e-05
15.6	1.2332e-05	2.4638e-05	3.7020e-05	4.9505e-05	6.2128e-05	7.47999e-05
15.9	1.2303e-05	2.4666e-05	3.7190e-05	4.9718e-05	6.2451e-05	7.52296e-05
16	1.2491e-05	2.4997e-05	3.7564e-05	5.0137e-05	6.2813e-05	7.53776e-05
16.2	1.2843e-05	2.5701e-05	3.8587e-05	5.1678e-05	6.4902e-05	7.83128e-05
16.5	1.3549e-05	2.7177e-05	4.0908e-05	5.4883e-05	6.9069e-05	8.35591e-05
17	1.4644e-05	2.9303e-05	4.3966e-05	5.8683e-05	7.3461e-05	8.81423e-05
18	1.6432e-05	3.2900e-05	4.9430e-05	6.6008e-05	8.2793e-05	9.95764e-05
19	1.6582e-05	3.3158e-05	4.9852e-05	6.6677e-05	8.3635e-05	1.00813e-04
20	1.6715e-05	3.3459e-05	5.0275e-05	6.7192e-05	8.4253e-05	1.01528e-04

C.1. LINEAR ROLL DAMPING

Table C.11: Sway Amplitude with Linear Damping at 0 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	1.2767e-07	2.5586e-07	3.8366e-07	5.0475e-07	6.3982e-07	7.6463e-07
6	2.8574e-07	5.6984e-07	8.5054e-07	1.1391e-06	1.4162e-06	1.6990e-06
7	5.6645e-07	1.1344e-06	1.6923e-06	2.2623e-06	2.8231e-06	3.3868e-06
8	1.1652e-06	2.3246e-06	3.4791e-06	4.6332e-06	5.7803e-06	6.9292e-06
9	2.0146e-06	4.0375e-06	6.0490e-06	8.0664e-06	1.0100e-05	1.2134e-05
10	3.7321e-06	7.4759e-06	1.1221e-05	1.4966e-05	1.8702e-05	2.2455e-05
11	4.7792e-06	9.5543e-06	1.4324e-05	1.9096e-05	2.3862e-05	2.8629e-05
12	4.7168e-06	9.4402e-06	1.4167e-05	1.8903e-05	2.3633e-05	2.8373e-05
13	6.5986e-06	1.3196e-05	1.9791e-05	2.6333e-05	3.2875e-05	3.9418e-05
14	1.5603e-05	3.1195e-05	4.6753e-05	6.2158e-05	7.7533e-05	9.2749e-05
14.5	2.7607e-05	5.5157e-05	8.2571e-05	1.0980e-04	1.3679e-04	1.6356e-04
15	4.4879e-05	8.9629e-05	1.3407e-04	1.7814e-04	2.2168e-04	2.6475e-04
15.3	5.3028e-05	1.0590e-04	1.5851e-04	2.1067e-04	2.6226e-04	3.1321e-04
15.6	5.4597e-05	1.0908e-04	1.6339e-04	2.1735e-04	2.7112e-04	3.2444e-04
15.9	5.0813e-05	1.0160e-04	1.5228e-04	2.0300e-04	2.5361e-04	3.0414e-04
16	4.9169e-05	9.8336e-05	1.4743e-04	1.9655e-04	2.4565e-04	2.9471e-04
16.2	4.5187e-05	9.0397e-05	1.3563e-04	1.8092e-04	2.2620e-04	2.7163e-04
16.5	4.0121e-05	8.0278e-05	1.2039e-04	1.6066e-04	2.0099e-04	2.4155e-04
17	3.3414e-05	6.6833e-05	1.0027e-04	1.3371e-04	1.6727e-04	2.0113e-04
18	2.4978e-05	5.0004e-05	7.5072e-05	1.0017e-04	1.2541e-04	1.5074e-04
19	2.0234e-05	4.0489e-05	6.0818e-05	8.1201e-05	1.0166e-04	1.2221e-04
20	1.7019e-05	3.4020e-05	5.1102e-05	6.8277e-05	8.5568e-05	1.0296e-04

Table C.12: Force Amplitude with Linear Damping at 0 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.5491	1.7077	4.8828	8.7891	19.5312	17.5781
6	2.4414	8.0042	14.4091	27.3438	42.4173	62.5000
7	2.9297	13.6719	28.0457	49.3896	70.3125	100.3433
8	7.9368	33.9416	74.2188	134.3394	199.4566	307.4966
9	5.8594	21.1506	44.6229	81.5370	128.9100	185.1161
10	21.4844	60.4269	160.6314	239.8932	423.3104	608.6728
11	9.7656	42.9688	89.8438	151.0655	242.5916	357.6594
12	5.2424	24.6928	50.9858	104.0757	166.7885	223.1864
13	14.2037	57.2426	134.4873	232.7048	314.8696	532.4606
14	42.4264	134.7937	332.0312	552.8972	980.8546	1429.7502
14.5	52.7344	176.8796	417.9688	945.3125	1554.6875	2281.2500
15	62.5000	281.2500	531.2500	1054.6875	2046.8750	3500.0000
15.3	62.5000	304.6875	632.8125	1273.4375	2593.7500	3828.1250
15.6	76.1719	316.4062	671.8750	1375.0000	2640.6250	4546.8750
15.9	56.6406	269.5312	640.6250	1328.1250	2890.6250	3531.2500
16	74.2188	312.5000	750.0000	1359.3750	2453.1250	4343.7500
16.2	66.9034	261.6520	703.1250	1656.2500	2765.6250	3281.2500
16.5	73.9288	267.5781	640.6250	1585.9375	2453.1250	3234.3750
17	82.0312	328.1250	593.7500	1476.5625	2328.1250	3062.5000
18	82.5060	429.6875	670.9847	1439.4792	2165.8103	3264.2257
19	88.4473	350.4865	914.0625	1363.5125	2168.5553	3125.7043
20	77.3106	312.6273	704.7601	1333.3377	2334.9731	3219.9930

C.2 Linearized Roll Damping

C.2.1 45Degrees

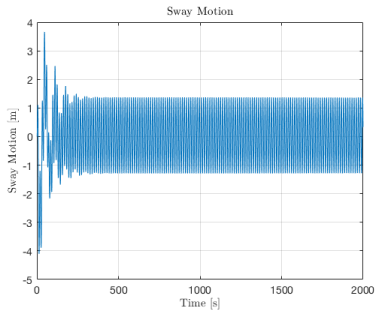


Figure C.12: Sway Motion Wave Period 13 [s] Significant Wave Height 6 [m].

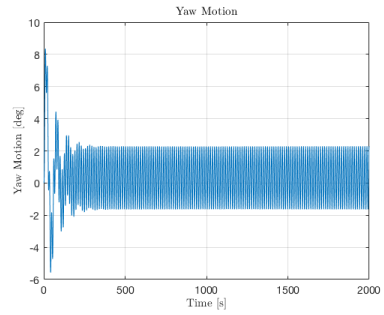


Figure C.13: Yaw Motion Wave Period 13 [s] Significant Wave Height 6 [m].

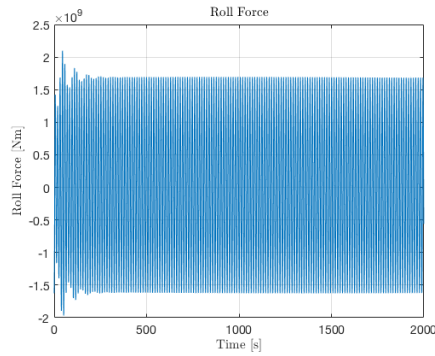


Figure C.14: Roll Force Wave Period 13 [s] Significant Wave Height 6 [m].

Table C.13: Roll Amplitude with Linearized Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0105	0.0210	0.0316	0.0422	0.0528	0.0635
6	0.0211	0.0421	0.0631	0.0841	0.1051	0.1261
7	0.0604	0.1206	0.1807	0.2405	0.3000	0.3594
8	0.0628	0.1256	0.1884	0.2511	0.3138	0.3763
9	0.0628	0.2773	0.4155	0.5531	0.6903	0.8268
10	0.1599	0.3194	0.4785	0.6372	0.7957	0.9538
11	0.1088	0.2167	0.3248	0.4325	0.5398	0.6473
12	0.1455	0.2926	0.4402	0.5850	0.7266	0.8615
13	0.3268	0.6537	0.9834	1.3149	1.6506	1.9876
14	0.7309	1.4623	2.1945	2.9295	3.6663	4.4042
14.5	1.0305	2.0607	3.0910	4.1228	5.1539	6.1903
15	1.4033	2.8015	4.1887	5.5590	6.9072	8.2302
15.3	1.5610	3.1105	4.6372	6.1304	7.5810	8.9821
15.6	1.6103	3.2048	4.7684	6.2876	7.7519	9.1536
15.9	1.5199	3.0252	4.5029	5.9405	7.3278	8.6569
16	1.4726	2.9321	4.3665	5.7645	7.1167	8.4172
16.2	1.3711	2.7316	4.0711	5.3800	6.6496	7.8735
16.5	1.2254	2.4440	3.6486	4.8324	5.9886	7.1113
17	1.0279	2.0521	3.0689	4.0750	5.0666	6.0396
18	0.7722	1.5435	2.3128	3.0795	3.8423	4.6004
19	0.5994	1.1986	1.7976	2.3962	2.9944	3.5920
20	0.4939	0.9879	1.4819	1.9763	2.4713	2.9669

Table C.14: Sway Amplitude with Linearized Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0063	0.0130	0.0200	0.0274	0.0353	0.0433
6	0.0084	0.0168	0.0253	0.0338	0.0423	0.0505
7	0.0292	0.0590	0.0892	0.1195	0.1495	0.1802
8	0.0258	0.0512	0.0769	0.1017	0.1253	0.1490
9	0.0258	0.0998	0.1494	0.1992	0.2491	0.2992
10	0.0932	0.1861	0.2789	0.3715	0.4639	0.5561
11	0.1345	0.2678	0.4009	0.5346	0.6696	0.8068
12	0.2020	0.4039	0.6070	0.8122	1.0199	1.2307
13	0.2469	0.4923	0.7372	0.9824	1.2282	1.4745
14	0.3538	0.7057	1.0555	1.4030	1.7480	2.0899
14.5	0.4931	0.9846	1.4744	1.9622	2.4474	2.9301
15	0.7284	1.4530	2.1719	2.8845	3.5882	4.2808
15.3	0.8798	1.7530	2.6148	3.4603	4.2855	5.0874
15.6	0.9901	1.9711	2.9361	3.8792	4.7951	5.6805
15.9	1.0247	2.0410	3.0418	4.0213	4.9746	5.8982
16	1.0239	2.0399	3.0416	4.0231	4.9799	5.9086
16.2	1.0101	2.0137	3.0054	3.9802	4.9339	5.8631
16.5	0.9819	1.9591	2.9276	3.8838	4.8244	5.7467
17	0.9381	1.8733	2.8034	3.7264	4.6402	5.5432
18	0.8839	1.7667	2.6479	3.5267	4.4028	5.2756
19	0.8412	1.6820	2.5223	3.3622	4.2013	5.0394
20	0.8304	1.6605	2.4906	3.3205	4.1503	4.9801

Table C.15: Yaw Amplitude with Linearized Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0185	0.0372	0.0558	0.0748	0.0942	0.1135
6	0.0104	0.0211	0.0318	0.0424	0.0524	0.0618
7	0.0270	0.0547	0.0836	0.1117	0.1378	0.1629
8	0.0258	0.0516	0.0771	0.1030	0.1278	0.1503
9	0.0258	0.0963	0.1445	0.1918	0.2379	0.2816
10	0.1741	0.3475	0.5187	0.6918	0.8618	1.0286
11	0.2693	0.5400	0.8093	1.0764	1.3417	1.5961
12	0.3374	0.6686	0.9837	1.2929	1.6118	1.9196
13	0.3574	0.7176	1.0792	1.4485	1.8141	2.1809
14	0.3770	0.7419	1.1092	1.4816	1.8333	2.1840
14.5	0.3695	0.7364	1.1037	1.4556	1.8008	2.1677
15	0.3520	0.7094	1.0701	1.4232	1.7793	2.1218
15.3	0.3481	0.6904	1.0386	1.3988	1.7532	2.0960
15.6	0.3472	0.6917	1.0311	1.3712	1.7265	2.0856
15.9	0.3483	0.6950	1.0379	1.3796	1.7097	2.0494
16	0.3478	0.6954	1.0374	1.3796	1.7150	2.0479
16.2	0.3452	0.6936	1.0408	1.3813	1.7203	2.0547
16.5	0.3429	0.6835	1.0295	1.3749	1.7154	2.0480
17	0.3416	0.6796	1.0140	1.3446	1.6777	2.0212
18	0.3266	0.6508	0.9704	1.2875	1.6025	1.9240
19	0.3129	0.6226	0.9295	1.2401	1.5540	1.8677
20	0.2944	0.5869	0.8795	1.1729	1.4662	1.7588

C.2. LINEARIZED ROLL DAMPING

Table C.16: Force Amplitude with Linearized Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	6.0638e+07	1.2118e+08	1.8163e+08	2.4198e+08	3.0223e+08	3.6239e+08
6	8.2471e+07	1.6486e+08	2.4716e+08	3.2938e+08	4.1151e+08	4.9354e+08
7	1.1311e+08	2.2592e+08	3.3842e+08	4.5057e+08	5.6233e+08	6.7363e+08
8	1.1390e+08	2.2780e+08	3.4170e+08	4.5563e+08	5.6959e+08	6.8356e+08
9	1.1390e+08	3.2945e+08	4.9386e+08	6.5786e+08	8.2141e+08	9.8443e+08
10	8.7053e+07	1.7432e+08	2.6193e+08	3.5001e+08	4.3865e+08	5.2799e+08
11	7.9376e+07	1.5918e+08	2.3857e+08	3.1563e+08	3.8840e+08	4.5466e+08
12	1.8150e+08	3.6299e+08	5.4318e+08	7.2164e+08	8.9635e+08	1.0645e+09
13	2.7971e+08	5.5998e+08	8.4012e+08	1.1187e+09	1.3944e+09	1.6663e+09
14	3.4937e+08	6.9977e+08	1.0510e+09	1.4042e+09	1.7594e+09	2.1179e+09
14.5	3.6446e+08	7.2945e+08	1.0951e+09	1.4617e+09	1.8307e+09	2.2061e+09
15	3.7945e+08	7.5891e+08	1.1384e+09	1.5199e+09	1.9027e+09	2.2849e+09
15.3	3.8755e+08	7.7510e+08	1.1623e+09	1.5496e+09	1.9361e+09	2.3206e+09
15.6	3.9559e+08	7.9041e+08	1.1837e+09	1.5757e+09	1.9660e+09	2.3540e+09
15.9	3.9770e+08	7.9460e+08	1.1898e+09	1.5820e+09	1.9701e+09	2.3537e+09
16	3.9635e+08	7.9187e+08	1.1859e+09	1.5778e+09	1.9664e+09	2.3514e+09
16.2	3.9679e+08	7.9292e+08	1.1872e+09	1.5783e+09	1.9651e+09	2.3481e+09
16.5	3.9551e+08	7.8993e+08	1.1823e+09	1.5729e+09	1.9600e+09	2.3427e+09
17	3.9459e+08	7.8854e+08	1.1814e+09	1.5724e+09	1.9603e+09	2.3450e+09
18	3.9121e+08	7.8207e+08	1.1722e+09	1.5614e+09	1.9495e+09	2.3357e+09
19	3.7279e+08	7.4540e+08	1.1177e+09	1.4897e+09	1.8611e+09	2.2316e+09
20	3.5628e+08	7.1247e+08	1.0685e+09	1.4242e+09	1.7798e+09	2.1348e+09

C.3 Quadratic Roll Damping

C.3.1 45 Degrees

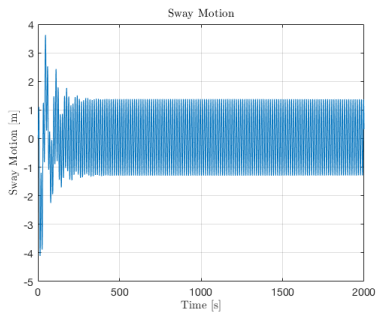


Figure C.15: Sway Motion Wave Period 13 [s] Significant Wave Height 6 [m].

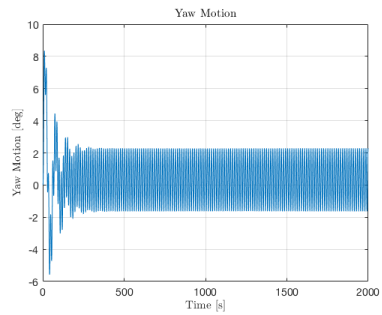


Figure C.16: Yaw Motion Wave Period 13 [s] Significant Wave Height 6 [m].

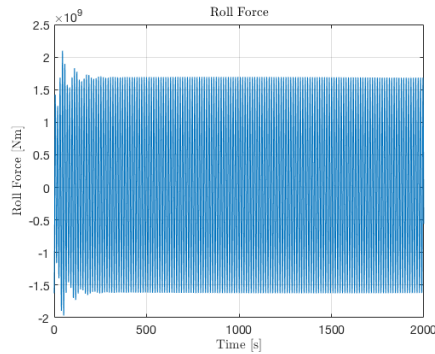


Figure C.17: Roll Force Wave Period 13 [s] Significant Wave Height 6 [m].

C.3. QUADRATIC ROLL DAMPING

Table C.17: Roll Amplitude with Quadratic Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0115	0.0230	0.0344	0.0457	0.0570	0.0684
6	0.0231	0.0460	0.0686	0.0911	0.1133	0.1355
7	0.0635	0.1259	0.1874	0.2479	0.3077	0.3667
8	0.0644	0.1283	0.1918	0.2550	0.3180	0.3807
9	0.1441	0.2855	0.4247	0.5621	0.6979	0.8323
10	0.1638	0.3256	0.4858	0.6447	0.8027	0.9596
11	0.1112	0.2209	0.3308	0.4404	0.5499	0.6586
12	0.1503	0.3033	0.4563	0.6047	0.7487	0.8854
13	0.3449	0.6845	1.0205	1.3516	1.6772	1.9603
14	0.7953	1.5281	2.2184	2.8679	3.4786	4.0508
14.5	1.0948	2.0390	2.8772	3.6327	4.3213	4.9605
15	1.5106	2.6476	3.5940	4.4194	5.1587	5.8358
15.3	1.6328	2.8293	3.8175	4.6767	5.4457	6.1463
15.6	1.6780	2.9187	3.9414	4.8283	5.6220	6.3483
15.9	1.5908	2.8292	3.8655	4.7728	5.5831	6.3278
16	1.5027	2.7085	3.7494	4.6592	5.4699	6.2035
16.2	1.4391	2.6351	3.6670	4.5773	5.4014	6.1583
16.5	1.2844	2.4126	3.4154	4.3206	5.1443	5.9100
17	1.0312	1.9970	2.9089	3.7587	4.5496	5.2930
18	0.7691	1.5200	2.2516	2.9703	3.6706	4.3502
19	0.5952	1.1852	1.7687	2.3448	2.9162	3.4837
20	0.4898	0.9776	1.4629	1.9456	2.4253	2.9036

Table C.18: Sway Amplitude with Quadratic Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0051	0.0100	0.0159	0.0229	0.0363	0.0444
6	0.0084	0.0168	0.0253	0.0338	0.0422	0.0504
7	0.0303	0.0609	0.0916	0.1222	0.1524	0.1829
8	0.0256	0.0517	0.0775	0.1024	0.1259	0.1498
9	0.0508	0.1010	0.1512	0.2015	0.2521	0.3029
10	0.0930	0.1861	0.2794	0.3727	0.4659	0.5589
11	0.1351	0.2685	0.4013	0.5355	0.6711	0.8091
12	0.1996	0.4002	0.6029	0.8079	1.0156	1.2260
13	0.2403	0.4838	0.7312	0.9831	1.2395	1.4844
14	0.3544	0.7223	1.0962	1.4708	1.8473	2.2293
14.5	0.4966	0.9831	1.4595	1.9293	2.3851	2.8307
15	0.7825	1.4493	2.0506	2.6138	3.1635	3.6910
15.3	0.9066	1.6435	2.3021	2.9136	3.4891	4.0445
15.6	1.0199	1.8434	2.5738	3.2466	3.8792	4.4814
15.9	1.0591	1.9431	2.7293	3.4518	4.1290	4.7719
16	1.0487	1.9505	2.7598	3.5057	4.2053	4.8694
16.2	1.0457	1.9613	2.7885	3.5530	4.2707	4.9521
16.5	1.0144	1.9394	2.7962	3.5989	4.3571	5.0788
17	0.9487	1.8582	2.7284	3.5614	4.3606	5.1402
18	0.8857	1.7606	2.6231	3.4722	4.3072	5.1278
19	0.8407	1.6779	2.5112	3.3400	4.1638	4.9823
20	0.8294	1.6572	2.4833	3.3077	4.1301	4.9504

C.3. QUADRATIC ROLL DAMPING

Table C.19: Yaw Amplitude with Quadratic Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0185	0.0372	0.0559	0.0750	0.0944	0.1137
6	0.0104	0.0211	0.0319	0.0425	0.0525	0.0618
7	0.0271	0.0549	0.0838	0.1119	0.1380	0.1630
8	0.0259	0.0517	0.0772	0.1031	0.1279	0.1503
9	0.0483	0.0962	0.1444	0.1917	0.2379	0.2817
10	0.1741	0.3476	0.5184	0.6917	0.8620	1.0292
11	0.2697	0.5407	0.8101	1.0790	1.3447	1.5995
12	0.3376	0.6682	0.9821	1.2917	1.6112	1.9180
13	0.3577	0.7186	1.0812	1.4500	1.8178	2.1774
14	0.3766	0.7404	1.1088	1.4787	1.8266	2.1777
14.5	0.3692	0.7364	1.1047	1.4576	1.8024	2.1685
15	0.3504	0.7117	1.0777	1.4367	1.7998	2.1474
15.3	0.3471	0.6943	1.0494	1.4173	1.7792	2.1297
15.6	0.3463	0.6954	1.0407	1.3884	1.7508	2.1159
15.9	0.3478	0.6964	1.0427	1.3884	1.7229	2.0709
16	0.3490	0.6984	1.0427	1.3891	1.7274	2.0662
16.2	0.3451	0.6938	1.0418	1.3839	1.7249	2.0605
16.5	0.3431	0.6834	1.0291	1.3745	1.7142	2.0468
17	0.3427	0.6806	1.0147	1.3442	1.6776	2.0201
18	0.3274	0.6516	0.9707	1.2874	1.6017	1.9215
19	0.3133	0.6230	0.9301	1.2409	1.5537	1.8656
20	0.2947	0.5873	0.8801	1.1735	1.4664	1.7581

APPENDIX C. 15 % TOTAL DAMPING RATIO

Table C.20: Force Amplitude with Quadratic Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	5.9940e+07	1.1979e+08	1.7955e+08	2.3921e+08	2.9879e+08	3.5828e+08
6	8.1900e+07	1.6370e+08	2.4540e+08	3.2699e+08	4.0848e+08	4.8984e+08
7	1.1280e+08	2.2531e+08	3.3750e+08	4.4934e+08	5.6076e+08	6.7170e+08
8	1.1390e+08	2.2780e+08	3.4169e+08	4.5563e+08	5.6959e+08	6.8355e+08
9	1.6478e+08	3.2945e+08	4.9385e+08	6.5785e+08	8.2138e+08	9.8439e+08
10	8.7053e+07	1.7432e+08	2.6194e+08	3.5002e+08	4.3866e+08	5.2800e+08
11	7.9250e+07	1.5921e+08	2.3863e+08	3.1571e+08	3.8849e+08	4.5474e+08
12	1.8145e+08	3.6291e+08	5.4311e+08	7.2158e+08	8.9617e+08	1.0644e+09
13	2.7971e+08	5.6002e+08	8.4017e+08	1.1187e+09	1.3941e+09	1.6658e+09
14	3.4903e+08	6.9919e+08	1.0501e+09	1.4027e+09	1.7569e+09	2.1119e+09
14.5	3.6447e+08	7.2944e+08	1.0949e+09	1.4606e+09	1.8280e+09	2.1995e+09
15	3.7846e+08	7.5760e+08	1.1376e+09	1.5195e+09	1.9021e+09	2.2844e+09
15.3	3.8754e+08	7.7521e+08	1.1631e+09	1.5520e+09	1.9417e+09	2.3316e+09
15.6	3.9558e+08	7.9062e+08	1.1851e+09	1.5800e+09	1.9761e+09	2.3734e+09
15.9	3.9768e+08	7.9478e+08	1.1912e+09	1.5865e+09	1.9806e+09	2.3747e+09
16	3.9634e+08	7.9200e+08	1.1870e+09	1.5817e+09	1.9758e+09	2.3704e+09
16.2	3.9678e+08	7.9302e+08	1.1882e+09	1.5818e+09	1.9737e+09	2.3659e+09
16.5	3.9549e+08	7.8997e+08	1.1829e+09	1.5753e+09	1.9662e+09	2.3559e+09
17	3.9451e+08	7.8834e+08	1.1812e+09	1.5725e+09	1.9618e+09	2.3497e+09
18	3.9099e+08	7.8158e+08	1.1714e+09	1.5602e+09	1.9485e+09	2.3360e+09
19	3.7278e+08	7.4536e+08	1.1176e+09	1.4896e+09	1.8609e+09	2.2313e+09
20	3.5617e+08	7.1221e+08	1.0680e+09	1.4234e+09	1.7785e+09	2.1330e+09

C.3.2 15 Degrees

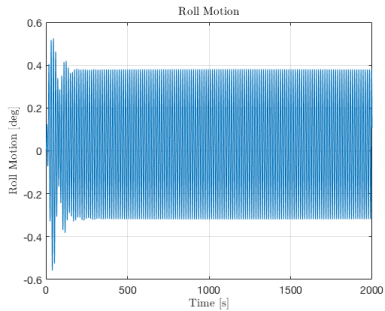


Figure C.18: Roll Motion Wave Period 13 [s] Significant Wave Height 6 [m].

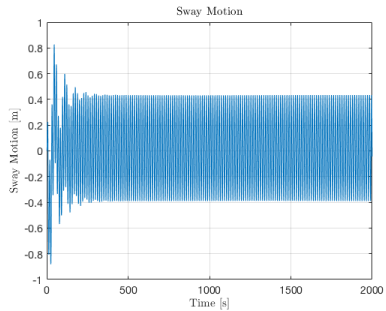


Figure C.19: Sway Motion Wave Period 13 [s] Significant Wave Height 6 [m].

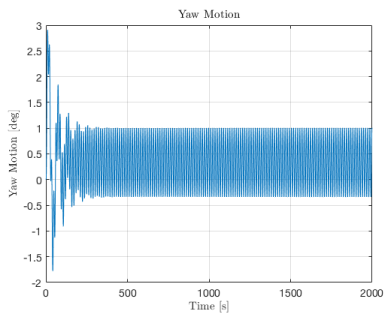


Figure C.20: Yaw Motion Wave Period 13 [s] Significant Wave Height 6 [m].

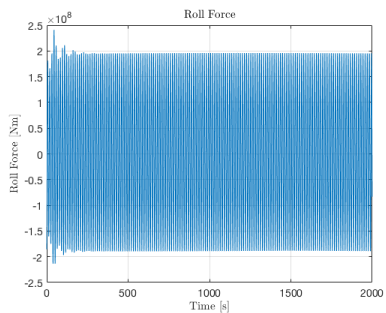


Figure C.21: Roll Force Wave Period 13 [s] Significant Wave Height 6 [m].

Table C.21: Roll Amplitude with Quadratic Damping at 15 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0081	0.0161	0.0241	0.0321	0.0401	0.0480
6	0.0085	0.0171	0.0256	0.0340	0.0425	0.0509
7	0.0108	0.0215	0.0322	0.0429	0.0535	0.0641
8	0.0385	0.0768	0.1148	0.1527	0.1903	0.2278
9	0.0273	0.0546	0.0818	0.1090	0.1360	0.1631
10	0.0663	0.1321	0.1976	0.2627	0.3274	0.3918
11	0.0841	0.1674	0.2500	0.3316	0.4123	0.4920
12	0.1240	0.2460	0.3665	0.4851	0.6030	0.7199
13	0.0615	0.1226	0.1831	0.2431	0.3021	0.3599
14	0.1349	0.2676	0.3975	0.5240	0.6461	0.7633
14.5	0.2186	0.4294	0.6315	0.8244	1.0076	1.1805
15	0.3740	0.7119	1.0198	1.3021	1.5614	1.7999
15.3	0.4398	0.8287	1.1781	1.4948	1.7834	2.0474
15.6	0.4753	0.8955	1.2728	1.6147	1.9267	2.2120
15.9	0.4514	0.8618	1.2376	1.5823	1.8991	2.1908
16	0.4207	0.8113	1.1728	1.5067	1.8143	2.0980
16.2	0.4040	0.7833	1.1386	1.4705	1.7797	2.0674
16.5	0.3607	0.7067	1.0363	1.3484	1.6446	1.9243
17	0.2945	0.5822	0.8614	1.1324	1.3950	1.6467
18	0.2327	0.4630	0.6898	0.9127	1.1318	1.3455
19	0.1852	0.3694	0.5521	0.7329	0.9115	1.0874
20	0.1570	0.3136	0.4694	0.6241	0.7776	0.9299

C.3. QUADRATIC ROLL DAMPING

Table C.22: Yaw Amplitude with Quadratic Damping at 15 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0032	0.0063	0.0095	0.0126	0.0157	0.0188
6	0.0044	0.0087	0.0130	0.0174	0.0216	0.0259
7	0.0076	0.0154	0.0234	0.0315	0.0397	0.0479
8	0.0068	0.0135	0.0202	0.0269	0.0335	0.0400
9	0.0126	0.0252	0.0379	0.0505	0.0632	0.0760
10	0.0153	0.0306	0.0460	0.0615	0.0769	0.0922
11	0.0551	0.1106	0.1660	0.2209	0.2748	0.3271
12	0.0971	0.1931	0.2872	0.3813	0.4742	0.5652
13	0.1252	0.2501	0.3747	0.4986	0.6213	0.7422
14	0.1478	0.2951	0.4413	0.5860	0.7287	0.8685
14.5	0.1486	0.2967	0.4439	0.5893	0.7323	0.8720
15	0.1465	0.2930	0.4386	0.5828	0.7248	0.8639
15.3	0.1489	0.2975	0.4450	0.5909	0.7353	0.8773
15.6	0.1518	0.3030	0.4530	0.6014	0.7478	0.8918
15.9	0.1532	0.3060	0.4575	0.6068	0.7534	0.8964
16	0.1537	0.3069	0.4584	0.6077	0.7538	0.8959
16.2	0.1526	0.3043	0.4544	0.6023	0.7472	0.8888
16.5	0.1529	0.3046	0.4543	0.6019	0.7469	0.8891
17	0.1537	0.3064	0.4576	0.6068	0.7534	0.8971
18	0.1510	0.3012	0.4503	0.5978	0.7434	0.8868
19	0.1458	0.2908	0.4346	0.5771	0.7180	0.8570
20	0.1390	0.2773	0.4149	0.5514	0.6868	0.8208

Table C.23: Sway Amplitude with Quadratic Damping at 15 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0042	0.0084	0.0126	0.0167	0.0208	0.0249
6	0.0034	0.0068	0.0102	0.0136	0.0170	0.0204
7	0.0045	0.0089	0.0134	0.0178	0.0223	0.0267
8	0.0201	0.0400	0.0600	0.0798	0.0996	0.1194
9	0.0177	0.0354	0.0531	0.0708	0.0884	0.1060
10	0.0226	0.0453	0.0681	0.0910	0.1139	0.1369
11	0.0306	0.0612	0.0917	0.1221	0.1524	0.1828
12	0.0711	0.1418	0.2121	0.2820	0.3515	0.4206
13	0.0741	0.1481	0.2223	0.2967	0.3714	0.4464
14	0.1013	0.2030	0.3051	0.4076	0.5105	0.6138
14.5	0.1315	0.2627	0.3932	0.5235	0.6526	0.7804
15	0.2175	0.4240	0.6209	0.8092	0.9898	1.1632
15.3	0.2643	0.5089	0.7381	0.9545	1.1603	1.3567
15.6	0.3063	0.5876	0.8499	1.0969	1.3313	1.5548
15.9	0.3173	0.6149	0.8959	1.1626	1.4169	1.6602
16	0.3100	0.6054	0.8873	1.1570	1.4155	1.6640
16.2	0.3086	0.6048	0.8891	1.1623	1.4255	1.6792
16.5	0.2991	0.5910	0.8754	1.1521	1.4212	1.6829
17	0.2828	0.5630	0.8401	1.1137	1.3835	1.6492
18	0.2778	0.5548	0.8309	1.1059	1.3795	1.6516
19	0.2706	0.5410	0.8111	1.0810	1.3505	1.6195
20	0.2739	0.5477	0.8215	1.0953	1.3690	1.6427

C.3. QUADRATIC ROLL DAMPING

Table C.24: Force Amplitude with Quadratic Damping at 15 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	3.3095e+07	6.6184e+07	9.9266e+07	1.3234e+08	1.6541e+08	1.9847e+08
6	2.7096e+07	5.4185e+07	8.1268e+07	1.0834e+08	1.3541e+08	1.6248e+08
7	2.7775e+07	5.5546e+07	8.3317e+07	1.1109e+08	1.3887e+08	1.6664e+08
8	4.7831e+07	9.5661e+07	1.4349e+08	1.9132e+08	2.3914e+08	2.8696e+08
9	2.9608e+07	5.9250e+07	8.8926e+07	1.1864e+08	1.4838e+08	1.7817e+08
10	5.9170e+07	1.1833e+08	1.7747e+08	2.3658e+08	2.9565e+08	3.5469e+08
11	4.7422e+07	9.4761e+07	1.4198e+08	1.8906e+08	2.3595e+08	2.8265e+08
12	2.4841e+07	4.9615e+07	7.4282e+07	9.8808e+07	1.2316e+08	1.4732e+08
13	3.2870e+07	6.5652e+07	9.8270e+07	1.3071e+08	1.6268e+08	1.9397e+08
14	6.9459e+07	1.3882e+08	2.0770e+08	2.7574e+08	3.4259e+08	4.0836e+08
14.5	8.1236e+07	1.6229e+08	2.4292e+08	3.2269e+08	4.0118e+08	4.7818e+08
15	9.2453e+07	1.8459e+08	2.7594e+08	3.6605e+08	4.5469e+08	5.4140e+08
15.3	9.8667e+07	1.9689e+08	2.9417e+08	3.9022e+08	4.8459e+08	5.7692e+08
15.6	1.0486e+08	2.0917e+08	3.1241e+08	4.1422e+08	5.1424e+08	6.1202e+08
15.9	1.0842e+08	2.1625e+08	3.2299e+08	4.2816e+08	5.3127e+08	6.3194e+08
16	1.0913e+08	2.1770e+08	3.2509e+08	4.3084e+08	5.3443e+08	6.3544e+08
16.2	1.1061e+08	2.2063e+08	3.2956e+08	4.3691e+08	5.4243e+08	6.4554e+08
16.5	1.1253e+08	2.2449e+08	3.3540e+08	4.4473e+08	5.5213e+08	6.5703e+08
17	1.1611e+08	2.3175e+08	3.4639e+08	4.5954e+08	5.7070e+08	6.7942e+08
18	1.2169e+08	2.4314e+08	3.6390e+08	4.8352e+08	6.0167e+08	7.1805e+08
19	1.1929e+08	2.3825e+08	3.5673e+08	4.7434e+08	5.9075e+08	7.0578e+08
20	1.1697e+08	2.3383e+08	3.5032e+08	4.6616e+08	5.8110e+08	6.9495e+08

C.3.3 0 Degrees

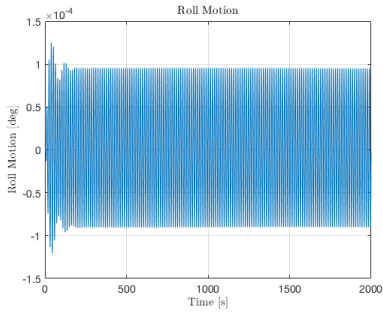


Figure C.22: Roll Motion Wave Period 13 [s] Significant Wave Height 6 [m].

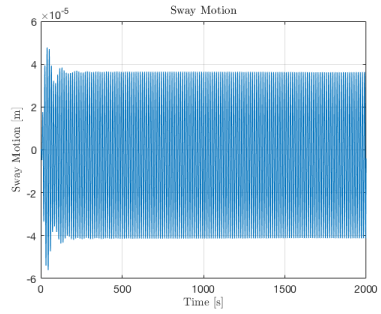


Figure C.23: Sway Motion Wave Period 13 [s] Significant Wave Height 6 [m].

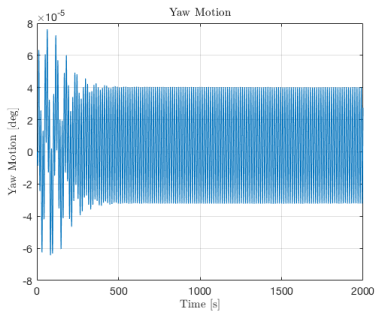


Figure C.24: Yaw Motion Wave Period 13 [s] Significant Wave Height 6 [m].

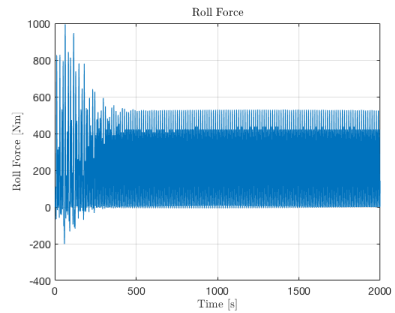


Figure C.25: Roll Force Wave Period 13 [s] Significant Wave Height 6 [m].

C.3. QUADRATIC ROLL DAMPING

Table C.25: Roll Amplitude with Quadratic Damping at 0 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	3.1718e-07	6.3442e-07	9.5088e-07	1.2619e-06	1.5972e-06	1.9115e-06
6	6.9155e-07	1.3863e-06	2.0728e-06	2.7640e-06	3.4541e-06	4.1531e-06
7	1.4654e-06	2.9293e-06	4.3920e-06	5.8562e-06	7.3173e-06	8.7697e-06
8	3.1219e-06	6.2389e-06	9.3553e-06	1.2456e-05	1.5571e-05	1.8668e-05
9	5.3375e-06	1.0676e-05	1.6019e-05	2.1362e-05	2.6712e-05	3.2069e-05
10	9.4469e-06	1.8885e-05	2.8317e-05	3.7735e-05	4.7144e-05	5.6569e-05
11	1.1978e-05	2.3943e-05	3.5899e-05	4.7872e-05	5.9853e-05	7.1877e-05
12	1.1640e-05	2.3306e-05	3.5001e-05	4.6727e-05	5.8477e-05	7.0274e-05
13	1.5948e-05	3.1881e-05	4.7793e-05	6.3690e-05	7.9557e-05	9.3485e-05
14	3.7488e-05	7.4946e-05	1.1234e-04	1.4962e-04	1.8672e-04	2.2364e-04
14.5	6.3619e-05	1.2708e-04	1.9025e-04	2.5296e-04	3.1510e-04	3.7651e-04
15	1.0591e-04	2.1146e-04	3.1625e-04	4.2002e-04	5.2234e-04	6.2295e-04
15.3	1.2058e-04	2.4071e-04	3.5995e-04	4.7791e-04	5.9418e-04	7.0847e-04
15.6	1.2351e-04	2.4670e-04	3.6930e-04	4.9100e-04	6.1156e-04	7.3077e-04
15.9	1.1468e-04	2.2921e-04	3.4350e-04	4.5735e-04	5.7064e-04	6.8333e-04
16	1.0579e-04	2.1151e-04	3.1713e-04	4.2258e-04	5.2782e-04	6.3286e-04
16.2	1.0140e-04	2.0278e-04	3.0410e-04	4.0537e-04	5.0657e-04	6.0775e-04
16.5	8.9604e-05	1.7925e-04	2.6896e-04	3.5875e-04	4.4871e-04	5.3886e-04
17	7.1363e-05	1.4273e-04	2.1415e-04	2.8565e-04	3.5730e-04	4.2914e-04
18	5.3319e-05	1.0667e-04	1.6009e-04	2.1358e-04	2.6727e-04	3.2116e-04
19	4.2855e-05	8.5776e-05	1.2881e-04	1.7196e-04	2.1521e-04	2.5865e-04
20	3.5711e-05	7.1457e-05	1.0737e-04	1.4349e-04	1.7984e-04	2.1649e-04

Table C.26: Yaw Amplitude with Quadratic Damping at 0 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	2.7846e-07	5.5752e-07	8.3769e-07	1.1103e-06	1.3877e-06	1.6738e-06
6	1.1779e-06	2.3558e-06	3.5287e-06	4.7102e-06	5.8884e-06	7.0797e-06
7	2.2041e-06	4.4050e-06	6.5938e-06	8.7844e-06	1.0974e-05	1.3157e-05
8	3.2374e-06	6.4791e-06	9.7241e-06	1.2976e-05	1.6228e-05	1.9489e-05
9	3.7573e-06	7.5064e-06	1.1258e-05	1.4986e-05	1.8723e-05	2.2441e-05
10	5.6517e-06	1.1307e-05	1.6946e-05	2.2593e-05	2.8240e-05	3.3896e-05
11	3.8089e-06	7.6294e-06	1.1425e-05	1.5199e-05	1.9079e-05	2.2926e-05
12	4.6766e-06	9.3781e-06	1.4096e-05	1.8783e-05	2.3373e-05	2.7906e-05
13	8.0208e-06	1.6027e-05	2.3998e-05	3.1977e-05	3.9803e-05	4.7912e-05
14	1.1159e-05	2.2316e-05	3.3588e-05	4.4832e-05	5.6156e-05	6.7503e-05
14.5	1.1332e-05	2.2737e-05	3.4184e-05	4.5728e-05	5.7323e-05	6.9045e-05
15	1.1804e-05	2.3690e-05	3.5785e-05	4.8185e-05	6.0800e-05	7.3738e-05
15.3	1.1942e-05	2.3931e-05	3.6073e-05	4.8606e-05	6.1437e-05	7.4549e-05
15.6	1.2332e-05	2.4638e-05	3.7020e-05	4.9506e-05	6.2129e-05	7.4797e-05
15.9	1.2303e-05	2.4666e-05	3.7190e-05	4.9718e-05	6.2451e-05	7.5231e-05
16	1.2602e-05	2.5218e-05	3.7904e-05	5.0555e-05	6.3351e-05	7.5997e-05
16.2	1.2843e-05	2.5701e-05	3.8587e-05	5.1678e-05	6.4902e-05	7.8313e-05
16.5	1.3549e-05	2.7177e-05	4.0908e-05	5.4883e-05	6.9069e-05	8.3560e-05
17	1.4699e-05	2.9409e-05	4.4128e-05	5.8887e-05	7.3727e-05	8.8476e-05
18	1.6459e-05	3.2949e-05	4.9501e-05	6.6098e-05	8.2907e-05	9.9714e-05
19	1.6586e-05	3.3167e-05	4.9868e-05	6.6704e-05	8.3676e-05	1.0088e-04
20	1.6731e-05	3.3504e-05	5.0365e-05	6.7344e-05	8.4502e-05	1.0189e-04

C.3. QUADRATIC ROLL DAMPING

Table C.27: Sway Amplitude with Quadratic Damping at 0 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	1.2798e-07	2.5650e-07	3.8463e-07	5.0600e-07	6.4132e-07	7.6652e-07
6	2.8630e-07	5.7098e-07	8.5215e-07	1.1413e-06	1.4192e-06	1.7024e-06
7	5.6489e-07	1.1312e-06	1.6877e-06	2.2559e-06	2.8152e-06	3.3783e-06
8	1.1718e-06	2.3379e-06	3.4998e-06	4.6617e-06	5.8165e-06	6.9704e-06
9	2.0308e-06	4.0681e-06	6.0950e-06	8.1279e-06	1.0178e-05	1.2226e-05
10	3.7547e-06	7.5205e-06	1.1289e-05	1.5055e-05	1.8815e-05	2.2593e-05
11	4.8221e-06	9.6394e-06	1.4451e-05	1.9270e-05	2.4079e-05	2.8893e-05
12	4.7782e-06	9.5650e-06	1.4357e-05	1.9161e-05	2.3962e-05	2.8774e-05
13	6.7262e-06	1.3451e-05	2.0175e-05	2.6849e-05	3.3524e-05	3.9418e-05
14	1.6130e-05	3.2255e-05	4.8334e-05	6.4258e-05	8.0131e-05	9.5916e-05
14.5	2.7606e-05	5.5156e-05	8.2570e-05	1.0980e-04	1.3679e-04	1.6356e-04
15	4.6390e-05	9.2641e-05	1.3857e-04	1.8408e-04	2.2906e-04	2.7349e-04
15.3	5.3027e-05	1.0590e-04	1.5850e-04	2.1065e-04	2.6223e-04	3.1318e-04
15.6	5.4596e-05	1.0907e-04	1.6338e-04	2.1733e-04	2.7110e-04	3.2441e-04
15.9	5.0812e-05	1.0159e-04	1.5228e-04	2.0299e-04	2.5359e-04	3.0412e-04
16	4.6938e-05	9.3910e-05	1.4081e-04	1.8769e-04	2.3457e-04	2.8149e-04
16.2	4.5186e-05	9.0396e-05	1.3562e-04	1.8092e-04	2.2619e-04	2.7162e-04
16.5	4.0121e-05	8.0277e-05	1.2039e-04	1.6066e-04	2.0098e-04	2.4154e-04
17	3.2062e-05	6.4142e-05	9.6248e-05	1.2835e-04	1.6066e-04	1.9316e-04
18	2.4188e-05	4.8437e-05	7.2743e-05	9.7087e-05	1.2156e-04	1.4609e-04
19	1.9730e-05	3.9460e-05	5.9271e-05	7.9142e-05	9.9050e-05	1.1909e-04
20	1.6662e-05	3.3322e-05	5.0065e-05	6.6894e-05	8.3844e-05	1.0091e-04

Appendix D: Regular Waves

D.1 Linear Roll Damping

D.1.1 45 Degrees

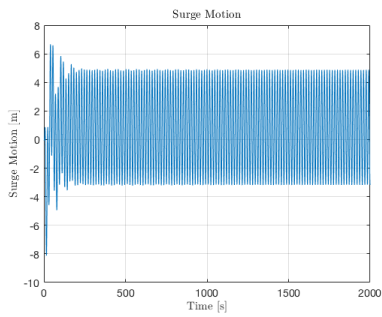


Figure D.1: Surge Motion Wave Period 16 [s] Significant Wave Height 6 [m].

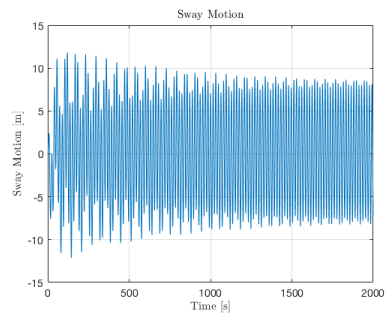


Figure D.2: Sway Motion Wave Period 16 [s] Significant Wave Height 6 [m].

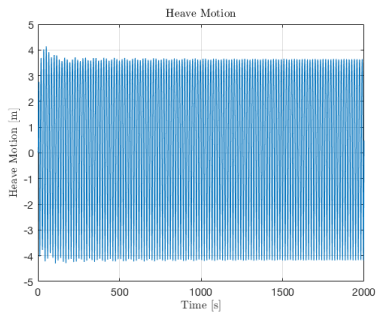


Figure D.3: Heave Motion Wave Period 16 [s] Significant Wave Height 6 [m].

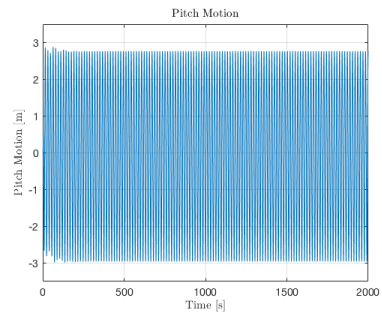


Figure D.4: Pitch Motion Wave Period 16 [s] Significant Wave Height 6 [m].

D.1. LINEAR ROLL DAMPING

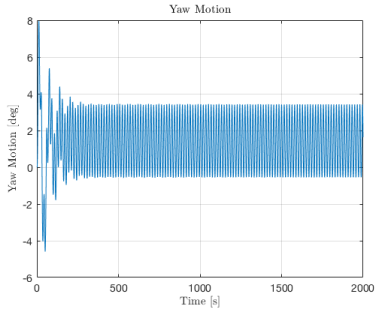


Figure D.5: Yaw Motion Wave Period 16 [s] Significant Wave Height 6 [m].

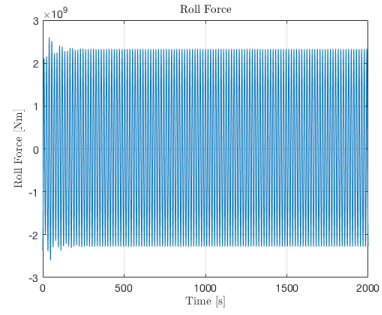


Figure D.6: Roll Force Wave Period 16 [s] Significant Wave Height 6 [m].

Table D.1: Surge Amplitude with Linear Damping at 45 Degrees.

Tp \ Hs	1	2	3	4	5	6
5	0.0140	0.0286	0.0436	0.0589	0.0742	0.0901
6	0.0237	0.0479	0.0725	0.0974	0.1222	0.1466
7	0.0520	0.1035	0.1548	0.2042	0.2507	0.2942
8	0.0845	0.1676	0.2496	0.3299	0.4095	0.4906
9	0.0915	0.1834	0.2752	0.3663	0.4565	0.5460
10	0.2066	0.4115	0.6141	0.8134	1.0079	1.1975
11	0.3488	0.6944	1.0354	1.3700	1.6967	2.0136
12	0.4625	0.9215	1.3738	1.8173	2.2537	2.6875
13	0.5527	1.1038	1.6513	2.1931	2.7388	3.2814
14	0.6199	1.2374	1.8506	2.4575	3.0567	3.6475
14.5	0.6486	1.2936	1.9331	2.5659	3.1952	3.8181
15	0.6734	1.3433	2.0071	2.6628	3.3098	3.9486
15.3	0.6865	1.3681	2.0399	2.6979	3.3416	3.9961
15.4	0.6919	1.3769	2.0488	2.7028	3.3390	3.9887
15.45	0.6923	1.3770	2.0476	2.6994	3.3397	3.9872
15.5	0.6943	1.3808	2.0526	2.7047	3.3419	3.9890
15.55	0.6959	1.3831	2.0540	2.7036	3.3450	3.9921
15.6	0.6978	1.3863	2.0576	2.7063	3.3491	3.9967
15.9	0.7072	1.4038	2.0828	2.7451	3.4068	4.0569
16	0.7102	1.4104	2.0943	2.7601	3.4234	4.0817
16.2	0.7199	1.4325	2.1321	2.8196	3.4935	4.1551
16.5	0.7313	1.4577	2.1745	2.8773	3.5669	4.2484
17	0.7435	1.4835	2.2174	2.9423	3.6574	4.3807
18	0.7751	1.5484	2.3185	3.0840	3.8439	4.6007
19	0.7956	1.5900	2.3822	3.1716	3.9580	4.7404
20	0.8139	1.6257	2.4351	3.2436	4.0539	4.8664

Table D.2: Sway Amplitude with Linear Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0275	0.0527	0.0802	0.1033	0.1239	0.1437
6	0.0420	0.0843	0.1267	0.1690	0.2110	0.2529
7	0.0392	0.0794	0.1206	0.1623	0.2042	0.2461
8	0.0907	0.1816	0.2715	0.3597	0.4457	0.5289
9	0.1233	0.2450	0.3658	0.4866	0.6077	0.7292
10	0.1622	0.3220	0.4812	0.6404	0.8003	0.9600
11	0.3437	0.6766	1.0021	1.3239	1.6459	1.9719
12	0.5713	1.1310	1.6807	2.2217	2.7558	3.2854
13	0.6676	1.3180	1.9538	2.5783	3.1918	3.7930
14	0.8897	1.7572	2.6043	3.4325	4.2425	5.0341
14.5	1.0967	2.1715	3.2256	4.2597	5.2738	6.2678
15	1.6923	3.3497	4.9576	6.5034	7.9769	9.3713
15.3	2.1852	4.2878	6.2616	8.0828	9.7465	11.2600
15.4	2.2690	4.4455	6.4773	8.3407	10.0381	11.5826
15.45	2.2985	4.5011	6.5540	8.4338	10.1412	11.6900
15.5	2.3000	4.5038	6.5576	8.4385	10.1477	11.6991
15.55	2.3003	4.5013	6.5503	8.4238	10.1261	11.6714
15.6	2.3199	4.5371	6.5974	8.4799	10.1878	11.7359
15.9	2.1508	4.2342	6.2135	8.0693	9.7959	11.3967
16	2.0489	4.0417	5.9482	7.7503	9.4471	11.0338
16.2	1.8900	3.7418	5.5371	7.2620	8.9114	10.4803
16.5	1.7476	3.4647	5.1413	6.7691	8.3419	9.8554
17	1.6349	3.2479	4.8317	6.3800	7.8885	9.3539
18	1.5041	2.9924	4.4617	5.9094	7.3336	8.7329
19	1.3535	2.6963	4.0267	5.3435	6.6452	7.9308
20	1.2279	2.4447	3.6520	4.8510	6.0430	7.2292

Table D.3: Heave Amplitude with Linear Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0055	0.0111	0.0166	0.0221	0.0276	0.0331
6	0.0147	0.0295	0.0442	0.0589	0.0736	0.0883
7	0.0333	0.0665	0.0998	0.1329	0.1660	0.1990
8	0.0623	0.1247	0.1870	0.2494	0.3117	0.3740
9	0.1352	0.2702	0.4051	0.5400	0.6748	0.8095
10	0.0959	0.1916	0.2883	0.3863	0.4862	0.5880
11	0.1616	0.3218	0.4798	0.6347	0.7845	0.9279
12	0.2946	0.5896	0.8839	1.1750	1.4627	1.7434
13	0.4094	0.8203	1.2331	1.6465	2.0605	2.4733
14	0.5079	1.0180	1.5361	2.0613	2.5935	3.1320
14.5	0.5541	1.1153	1.6842	2.2619	2.8500	3.4484
15	0.5955	1.1993	1.8135	2.4388	3.0744	3.7175
15.3	0.6201	1.2590	1.9224	2.6043	3.2920	3.9796
15.4	0.6265	1.2735	1.9481	2.6402	3.3394	4.0445
15.45	0.6299	1.2802	1.9579	2.6540	3.3614	4.0644
15.5	0.6313	1.2834	1.9628	2.6529	3.3579	4.0609
15.55	0.6349	1.2874	1.9627	2.6579	3.3553	4.0478
15.6	0.6386	1.2890	1.9600	2.6462	3.3338	4.0324
15.9	0.6579	1.3176	1.9764	2.6322	3.2825	3.9262
16	0.6642	1.3316	1.9991	2.6633	3.3215	3.9716
16.2	0.6781	1.3600	2.0448	2.7294	3.4136	4.0927
16.5	0.6943	1.3928	2.0968	2.8063	3.5165	4.2246
17	0.7239	1.4507	2.1836	2.9200	3.6584	4.3971
18	0.7737	1.5495	2.3268	3.1078	3.8939	4.6825
19	0.8151	1.6320	2.4504	3.2707	4.0927	4.9193
20	0.8459	1.6929	2.5413	3.3913	4.2429	5.0963

Table D.4: Pitch Amplitude with Linear Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0160	0.0319	0.0478	0.0637	0.0796	0.0954
6	0.0382	0.0764	0.1146	0.1527	0.1907	0.2287
7	0.0811	0.1620	0.2428	0.3234	0.4040	0.4843
8	0.1616	0.3231	0.4845	0.6458	0.8069	0.9679
9	0.1668	0.3334	0.4997	0.6656	0.8310	0.9959
10	0.3137	0.6270	0.9398	1.2504	1.5576	1.8607
11	0.4319	0.8637	1.2954	1.7255	2.1529	2.5765
12	0.4902	0.9797	1.4687	1.9564	2.4421	2.9265
13	0.5130	1.0251	1.5364	2.0468	2.5561	3.0642
14	0.5121	1.0234	1.5338	2.0440	2.5535	3.0618
14.5	0.5054	1.0105	1.5156	2.0197	2.5217	3.0225
15	0.4959	0.9929	1.4920	1.9933	2.4966	3.0034
15.3	0.4905	0.9842	1.4810	1.9807	2.4847	2.9940
15.4	0.4883	0.9783	1.4719	1.9733	2.4802	2.9918
15.45	0.4868	0.9756	1.4686	1.9688	2.4756	2.9875
15.5	0.4859	0.9748	1.4673	1.9645	2.4672	2.9781
15.55	0.4849	0.9731	1.4653	1.9621	2.4635	2.9694
15.6	0.4835	0.9701	1.4611	1.9568	2.4575	2.9629
15.9	0.4752	0.9519	1.4310	1.9171	2.4087	2.9056
16	0.4713	0.9455	1.4238	1.9067	2.3945	2.8873
16.2	0.4671	0.9356	1.4062	1.8797	2.3560	2.8363
16.5	0.4581	0.9176	1.3791	1.8431	2.3105	2.7813
17	0.4434	0.8867	1.3304	1.7750	2.2229	2.6738
18	0.4149	0.8302	1.2467	1.6648	2.0848	2.5079
19	0.3839	0.7680	1.1525	1.5375	1.9237	2.3110
20	0.3564	0.7127	1.0692	1.4260	1.7830	2.1406

Table D.5: Yaw Amplitude with Linear Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0185	0.0371	0.0558	0.0748	0.0942	0.1135
6	0.0107	0.0217	0.0328	0.0436	0.0541	0.0635
7	0.0272	0.0551	0.0839	0.1121	0.1386	0.1639
8	0.0260	0.0515	0.0772	0.1019	0.1268	0.1494
9	0.0476	0.0951	0.1422	0.1888	0.2339	0.2792
10	0.1741	0.3473	0.5196	0.6928	0.8630	1.0301
11	0.2692	0.5398	0.8091	1.0746	1.3357	1.5884
12	0.3385	0.6701	0.9847	1.2850	1.5901	1.8879
13	0.3620	0.7185	1.0828	1.4589	1.8365	2.2265
14	0.3793	0.7473	1.1094	1.4739	1.8303	2.1628
14.5	0.3711	0.7389	1.1040	1.4633	1.8003	2.1468
15	0.3388	0.6759	1.0189	1.3609	1.6904	2.0175
15.3	0.3280	0.6514	0.9737	1.3099	1.6568	1.9980
15.4	0.3278	0.6545	0.9755	1.2999	1.6457	1.9942
15.45	0.3305	0.6570	0.9800	1.3019	1.6419	1.9905
15.5	0.3322	0.6594	0.9852	1.3057	1.6370	1.9833
15.55	0.3351	0.6642	0.9945	1.3193	1.6468	1.9805
15.6	0.3382	0.6709	1.0028	1.3334	1.6617	1.9921
15.9	0.3526	0.7049	1.0527	1.3930	1.7382	2.0825
16	0.3545	0.7103	1.0624	1.4070	1.7453	2.0874
16.2	0.3541	0.7063	1.0604	1.4106	1.7538	2.0919
16.5	0.3537	0.7028	1.0463	1.3904	1.7368	2.0799
17	0.3523	0.7014	1.0444	1.3786	1.7091	2.0340
18	0.3355	0.6690	0.9987	1.3224	1.6380	1.9522
19	0.3221	0.6410	0.9556	1.2645	1.5678	1.8857
20	0.3042	0.6078	0.9115	1.2152	1.5188	1.8217

Table D.6: Force Amplitude with Linear Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	6.0638e+07	1.2118e+08	1.8163e+08	2.4198e+08	3.0223e+08	3.6239e+08
6	8.2472e+07	1.6486e+08	2.4717e+08	3.2939e+08	4.1152e+08	4.9355e+08
7	1.1311e+08	2.2592e+08	3.3842e+08	4.5057e+08	5.6232e+08	6.7360e+08
8	1.1390e+08	2.2780e+08	3.4170e+08	4.5561e+08	5.6957e+08	6.8352e+08
9	1.6478e+08	3.2947e+08	4.9388e+08	6.5788e+08	8.2144e+08	9.8446e+08
10	8.7054e+07	1.7432e+08	2.6194e+08	3.5002e+08	4.3867e+08	5.2803e+08
11	7.9387e+07	1.5910e+08	2.3840e+08	3.1530e+08	3.8784e+08	4.5378e+08
12	1.8153e+08	3.6305e+08	5.4315e+08	7.2108e+08	8.9487e+08	1.0620e+09
13	2.7976e+08	5.6013e+08	8.4068e+08	1.1201e+09	1.3977e+09	1.6718e+09
14	3.4942e+08	7.0011e+08	1.0520e+09	1.4062e+09	1.7643e+09	2.1270e+09
14.5	3.6450e+08	7.2997e+08	1.0970e+09	1.4657e+09	1.8399e+09	2.2211e+09
15	3.7946e+08	7.5894e+08	1.1383e+09	1.5180e+09	1.8990e+09	2.2779e+09
15.3	3.8736e+08	7.7327e+08	1.1564e+09	1.5353e+09	1.9095e+09	2.2770e+09
15.4	3.9002e+08	7.7792e+08	1.1615e+09	1.5393e+09	1.9101e+09	2.2729e+09
15.45	3.9144e+08	7.8063e+08	1.1657e+09	1.5446e+09	1.9157e+09	2.2776e+09
15.5	3.9196e+08	7.8116e+08	1.1652e+09	1.5433e+09	1.9140e+09	2.2757e+09
15.55	3.9377e+08	7.8492e+08	1.1709e+09	1.5504e+09	1.9216e+09	2.2832e+09
15.6	3.9529e+08	7.8780e+08	1.1749e+09	1.5551e+09	1.9269e+09	2.2887e+09
15.9	3.9750e+08	7.9261e+08	1.1831e+09	1.5667e+09	1.9414e+09	2.3057e+09
16	3.9619e+08	7.9034e+08	1.1803e+09	1.5646e+09	1.9411e+09	2.3079e+09
16.2	3.9667e+08	7.9190e+08	1.1837e+09	1.5699e+09	1.9483e+09	2.3172e+09
16.5	3.9546e+08	7.8938e+08	1.1802e+09	1.5673e+09	1.9489e+09	2.3228e+09
17	3.9459e+08	7.8841e+08	1.1806e+09	1.5704e+09	1.9561e+09	2.3366e+09
18	3.9122e+08	7.8210e+08	1.1722e+09	1.5612e+09	1.9485e+09	2.3341e+09
19	3.7283e+08	7.4554e+08	1.1179e+09	1.4899e+09	1.8613e+09	2.2319e+09
20	3.5630e+08	7.1256e+08	1.0687e+09	1.4247e+09	1.7806e+09	2.1363e+09

D.1.2 15 Degrees

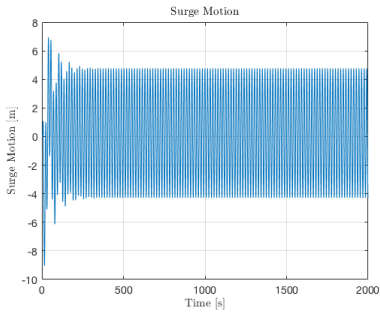


Figure D.7: Surge Motion Wave Period 16 [s] Significant Wave Height 6 [m].

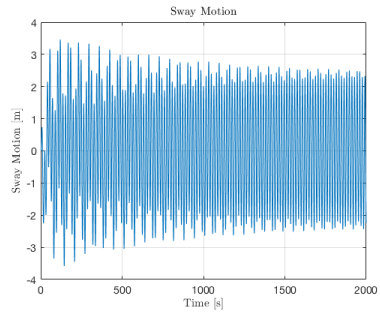


Figure D.8: Sway Motion Wave Period 16 [s] Significant Wave Height 6 [m].

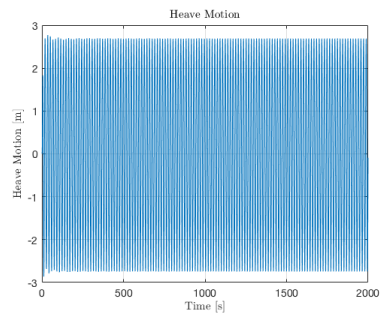


Figure D.9: Heave Motion Wave Period 16 [s] Significant Wave Height 6 [m].

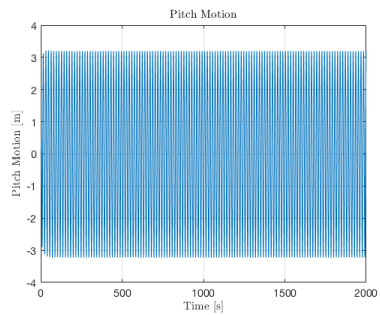


Figure D.10: Pitch Motion Wave Period 16 [s] Significant Wave Height 6 [m].

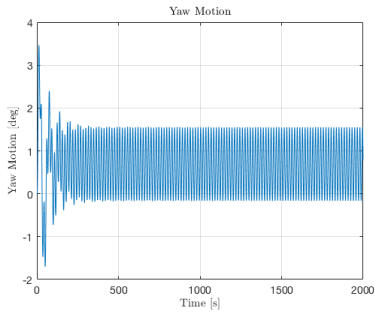


Figure D.11: Yaw Motion Wave Period 16 [s] Significant Wave Height 6 [m].

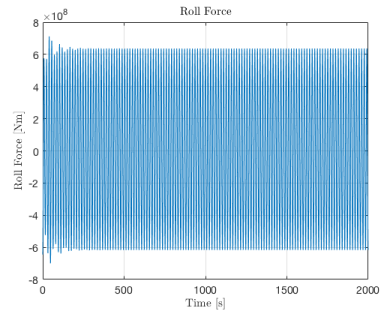


Figure D.12: Roll Force Wave Period 16 [s] Significant Wave Height 6 [m].

Table D.7: Surge Amplitude with Linear Damping at 15 Degrees.

Tp \ Hs	1	2	3	4	5	6
5	0.0115	0.0231	0.0347	0.0463	0.0581	0.0699
6	0.0258	0.0517	0.0776	0.1037	0.1298	0.1560
7	0.0511	0.1022	0.1533	0.2044	0.2555	0.3065
8	0.0922	0.1844	0.2767	0.3690	0.4612	0.5535
9	0.1439	0.2878	0.4314	0.5750	0.7184	0.8616
10	0.1630	0.3259	0.4889	0.6519	0.8150	0.9782
11	0.1593	0.3198	0.4818	0.6451	0.8099	0.9759
12	0.2556	0.5121	0.7693	1.0267	1.2840	1.5407
13	0.4004	0.8001	1.1990	1.5968	1.9932	2.3881
14	0.5495	1.0987	1.6481	2.1972	2.7458	3.2936
14.5	0.6121	1.2242	1.8361	2.4478	3.0590	3.6694
15	0.6732	1.3462	2.0187	2.6904	3.3611	4.0305
15.3	0.7054	1.4098	2.1126	2.8131	3.5109	4.2057
15.4	0.7168	1.4323	2.1455	2.8555	3.5616	4.2636
15.45	0.7226	1.4436	2.1622	2.8773	3.5885	4.2953
15.5	0.7272	1.4530	2.1761	2.8956	3.6108	4.3215
15.55	0.7320	1.4623	2.1898	2.9134	3.6324	4.3466
15.6	0.7377	1.4736	2.2065	2.9353	3.6594	4.3782
15.9	0.7649	1.5285	2.2897	3.0475	3.8011	4.5500
16	0.7724	1.5436	2.3125	3.0781	3.8398	4.5972
16.2	0.7901	1.5791	2.3662	3.1505	3.9313	4.7083
16.5	0.8146	1.6284	2.4407	3.2510	4.0588	4.8635
17	0.8493	1.6979	2.5451	3.3904	4.2335	5.0740
18	0.9250	1.8494	2.5451	3.6951	4.6158	5.5346
19	0.9705	1.9406	2.9099	3.8784	4.8457	5.8116
20	1.0156	2.0306	3.0450	4.0584	5.0708	6.0819

Table D.8: Sway Amplitude with Linear Damping at 15 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0075	0.0149	0.0223	0.0297	0.0371	0.0444
6	0.0106	0.0212	0.0318	0.0424	0.0530	0.0637
7	0.0160	0.0318	0.0476	0.0634	0.0790	0.0947
8	0.0344	0.0688	0.1033	0.1379	0.1725	0.2074
9	0.0477	0.0954	0.1432	0.1910	0.2390	0.2870
10	0.0579	0.1157	0.1734	0.2310	0.2885	0.3458
11	0.0476	0.0954	0.1434	0.1915	0.2395	0.2873
12	0.1087	0.2165	0.3231	0.4284	0.5322	0.6343
13	0.1534	0.3049	0.4547	0.6025	0.7482	0.8912
14	0.2305	0.4567	0.6789	0.8974	1.1127	1.3246
14.5	0.2818	0.5590	0.8314	1.0986	1.3604	1.6168
15	0.4378	0.8694	1.2926	1.7052	2.1055	2.4920
15.3	0.5735	1.1378	1.6884	2.2212	2.7331	3.2210
15.4	0.6033	1.1970	1.7762	2.3365	2.8742	3.3866
15.45	0.6156	1.2214	1.8124	2.3839	2.9323	3.4547
15.5	0.6193	1.2289	1.8237	2.3990	2.9513	3.4777
15.55	0.6229	1.2362	1.8349	2.4142	2.9708	3.5017
15.6	0.6259	1.2411	1.8400	2.4179	2.9708	3.5023
15.9	0.5980	1.1873	1.7641	2.3248	2.8668	3.3883
16	0.5736	1.1395	1.6946	2.2359	2.7613	3.2690
16.2	0.5352	1.0635	1.5825	2.0902	2.5848	3.0649
16.5	0.5028	0.9993	1.4883	1.9678	2.4366	2.8934
17	0.4889	0.9726	1.4501	1.9202	2.3818	2.8343
18	0.4727	0.9409	1.4501	1.8607	2.3110	2.7540
19	0.4374	0.8720	1.3038	1.7325	2.1580	2.5803
20	0.4088	0.8148	1.2186	1.6209	2.0226	2.4244

Table D.9: Heave Amplitude with Linear Damping at 15 Degrees.

Tp \ Hs	1	2	3	4	5	6
5	0.0068	0.0135	0.0203	0.0270	0.0338	0.0406
6	0.0148	0.0296	0.0445	0.0593	0.0741	0.0889
7	0.0309	0.0617	0.0926	0.1234	0.1543	0.1851
8	0.0637	0.1275	0.1913	0.2551	0.3189	0.3828
9	0.0993	0.1986	0.2979	0.3972	0.4965	0.5958
10	0.1599	0.3199	0.4798	0.6398	0.7998	0.9597
11	0.1579	0.3158	0.4737	0.6317	0.7897	0.9479
12	0.1249	0.2497	0.3745	0.4994	0.6243	0.7495
13	0.1551	0.3102	0.4652	0.6200	0.7745	0.9287
14	0.2351	0.4703	0.7053	0.9399	1.1739	1.4071
14.5	0.2991	0.5984	0.8978	1.1969	1.4955	1.7936
15	0.3557	0.7116	1.0676	1.4235	1.7791	2.1340
15.3	0.3849	0.7700	1.1549	1.5394	1.9232	2.3061
15.4	0.3945	0.7889	1.1830	1.5765	1.9692	2.3608
15.45	0.3999	0.7998	1.1994	1.5982	1.9961	2.3928
15.5	0.4037	0.8075	1.2110	1.6137	2.0154	2.4159
15.55	0.4080	0.8161	1.2239	1.6308	2.0368	2.4415
15.6	0.4135	0.8271	1.2404	1.6530	2.0646	2.4750
15.9	0.4444	0.8891	1.3341	1.7788	2.2228	2.6660
16	0.4526	0.9062	1.3605	1.8151	2.2696	2.7238
16.2	0.4752	0.9509	1.4271	1.9032	2.3790	2.8542
16.5	0.5033	1.0072	1.5114	2.0160	2.5205	3.0248
17	0.5483	1.0971	1.6464	2.1959	2.7455	3.2951
18	0.6265	1.2533	1.6464	2.5085	3.1369	3.7657
19	0.6920	1.3845	2.0775	2.7711	3.4653	4.1599
20	0.7435	1.4873	2.2316	2.9765	3.7219	4.4679

Table D.10: Roll Amplitude with Linear Damping at 15 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0092	0.0183	0.0275	0.0367	0.0459	0.0551
6	0.0104	0.0207	0.0311	0.0415	0.0519	0.0623
7	0.0126	0.0253	0.0379	0.0505	0.0631	0.0757
8	0.0438	0.0877	0.1318	0.1760	0.2204	0.2649
9	0.0314	0.0628	0.0943	0.1257	0.1571	0.1885
10	0.0733	0.1466	0.2197	0.2926	0.3654	0.4379
11	0.0905	0.1803	0.2695	0.3579	0.4452	0.5315
12	0.1330	0.2643	0.3932	0.5192	0.6418	0.7606
13	0.0757	0.1512	0.2262	0.3004	0.3737	0.4456
14	0.1817	0.3627	0.5418	0.7182	0.8905	1.0578
14.5	0.3086	0.6155	0.9188	1.2165	1.5067	1.7877
15	0.6193	1.2335	1.8366	2.4230	2.9879	3.5270
15.3	0.8184	1.6259	2.4117	3.1663	3.8820	4.5533
15.4	0.8537	1.6948	2.5109	3.2914	4.0281	4.7158
15.45	0.8607	1.7081	2.5294	3.3142	4.0548	4.7452
15.5	0.8588	1.7044	2.5240	3.3065	4.0434	4.7288
15.55	0.8533	1.6936	2.5082	3.2861	4.0193	4.7019
15.6	0.8401	1.6677	2.4707	3.2384	3.9627	4.6373
15.9	0.6944	1.3804	2.0495	2.6938	3.3069	3.8837
16	0.6388	1.2702	1.8873	2.4838	3.0540	3.5931
16.2	0.5443	1.0833	1.6118	2.1251	2.6187	3.0890
16.5	0.4570	0.9100	1.3556	1.7902	2.2105	2.6138
17	0.3651	0.7278	1.0858	1.4365	1.7780	2.1087
18	0.2726	0.5439	1.0858	1.0777	1.3379	1.5920
19	0.2143	0.4278	0.6399	0.8499	1.0572	1.2611
20	0.1806	0.3605	0.5395	0.7172	0.8934	1.0679

Table D.11: Pitch Amplitude with Linear Damping at 15 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0169	0.0339	0.0508	0.0678	0.0847	0.1016
6	0.0416	0.0832	0.1248	0.1664	0.2080	0.2496
7	0.0881	0.1762	0.2643	0.3524	0.4405	0.5286
8	0.1512	0.3024	0.4535	0.6047	0.7559	0.9071
9	0.2180	0.4360	0.6539	0.8719	1.0899	1.3078
10	0.2106	0.4213	0.6319	0.8425	1.0531	1.2637
11	0.2299	0.4599	0.6899	0.9199	1.1498	1.3796
12	0.3186	0.6372	0.9558	1.2741	1.5922	1.9098
13	0.4162	0.8324	1.2485	1.6644	2.0798	2.4948
14	0.4917	0.9834	1.4750	1.9664	2.4579	2.9493
14.5	0.5103	1.0205	1.5307	2.0409	2.5510	3.0609
15	0.5252	1.0504	1.5757	2.1010	2.6264	3.1518
15.3	0.5316	1.0631	1.5946	2.1261	2.6575	3.1888
15.4	0.5337	1.0675	1.6013	2.1352	2.6691	3.2029
15.45	0.5346	1.0692	1.6038	2.1384	2.6732	3.2082
15.5	0.5348	1.0697	1.6048	2.1400	2.6754	3.2108
15.55	0.5357	1.0714	1.6070	2.1428	2.6787	3.2146
15.6	0.5365	1.0730	1.6095	2.1460	2.6825	3.2189
15.9	0.5369	1.0737	1.6106	2.1475	2.6844	3.2213
16	0.5354	1.0708	1.6062	2.1417	2.6771	3.2125
16.2	0.5346	1.0693	1.6041	2.1389	2.6738	3.2088
16.5	0.5314	1.0629	1.5945	2.1262	2.6581	3.1900
17	0.5255	1.0509	1.5764	2.1020	2.6277	3.1536
18	0.5099	1.0197	1.5764	2.0396	2.5497	3.0600
19	0.4802	0.9604	1.4406	1.9208	2.4012	2.8817
20	0.4537	0.9073	1.3609	1.8145	2.2682	2.7221

Table D.12: Yaw Amplitude with Linear Damping at 15 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0032	0.0063	0.0095	0.0126	0.0157	0.0188
6	0.0044	0.0089	0.0132	0.0176	0.0220	0.0262
7	0.0077	0.0155	0.0235	0.0317	0.0399	0.0482
8	0.0068	0.0136	0.0204	0.0271	0.0337	0.0403
9	0.0126	0.0252	0.0378	0.0505	0.0632	0.0760
10	0.0150	0.0300	0.0449	0.0600	0.0751	0.0901
11	0.0548	0.1101	0.1652	0.2197	0.2732	0.3252
12	0.0968	0.1922	0.2859	0.3794	0.4713	0.5611
13	0.1257	0.2512	0.3762	0.5005	0.6235	0.7447
14	0.1479	0.2951	0.4413	0.5859	0.7285	0.8683
14.5	0.1484	0.2965	0.4437	0.5895	0.7334	0.8743
15	0.1446	0.2882	0.4308	0.5730	0.7145	0.8553
15.3	0.1454	0.2895	0.4317	0.5728	0.7118	0.8478
15.4	0.1471	0.2926	0.4358	0.5767	0.7152	0.8512
15.45	0.1484	0.2951	0.4397	0.5816	0.7209	0.8571
15.5	0.1493	0.2967	0.4415	0.5835	0.7232	0.8594
15.55	0.1503	0.2991	0.4456	0.5893	0.7298	0.8663
15.6	0.1515	0.3015	0.4494	0.5943	0.7357	0.8725
15.9	0.1554	0.3101	0.4630	0.6127	0.7579	0.8967
16	0.1557	0.3109	0.4643	0.6146	0.7603	0.8996
16.2	0.1557	0.3102	0.4632	0.6137	0.7607	0.9031
16.5	0.1560	0.3109	0.4641	0.6151	0.7633	0.9085
17	0.1564	0.3121	0.4664	0.6189	0.7691	0.9165
18	0.1534	0.3063	0.4664	0.6093	0.7588	0.9069
19	0.1486	0.2965	0.4433	0.5891	0.7335	0.8764
20	0.1422	0.2845	0.4271	0.5702	0.7141	0.8592

Table D.13: Roll Force Amplitude with Linear Damping at 0 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	3.3095e+07	6.6184e+07	9.9266e+07	1.3234e+08	1.6541e+08	1.9847e+08
6	2.7096e+07	5.4185e+07	8.1269e+07	1.0835e+08	1.3542e+08	1.6248e+08
7	2.7979e+07	5.5958e+07	8.3936e+07	1.1191e+08	1.3989e+08	1.6786e+08
8	4.7830e+07	9.5660e+07	1.4349e+08	1.9132e+08	2.3914e+08	2.8696e+08
9	2.9609e+07	5.9248e+07	8.8922e+07	1.1863e+08	1.4837e+08	1.7815e+08
10	5.9201e+07	1.1839e+08	1.7756e+08	2.3671e+08	2.9582e+08	3.5489e+08
11	4.7544e+07	9.4996e+07	1.4232e+08	1.8950e+08	2.3649e+08	2.8328e+08
12	2.4909e+07	4.9770e+07	7.4573e+07	9.9111e+07	1.2339e+08	1.4754e+08
13	3.2884e+07	6.5703e+07	9.8294e+07	1.3074e+08	1.6273e+08	1.9404e+08
14	6.9464e+07	1.3884e+08	2.0774e+08	2.7583e+08	3.4273e+08	4.0858e+08
14.5	8.1238e+07	1.6231e+08	2.4295e+08	3.2280e+08	4.0142e+08	4.7863e+08
15	9.2457e+07	1.8456e+08	2.7579e+08	3.6578e+08	4.5420e+08	5.4064e+08
15.3	9.8641e+07	1.9663e+08	2.9332e+08	3.8816e+08	4.8055e+08	5.7003e+08
15.4	1.0068e+08	2.0058e+08	2.9897e+08	3.9521e+08	4.8862e+08	5.7874e+08
15.45	1.0187e+08	2.0291e+08	3.0241e+08	3.9961e+08	4.9384e+08	5.8485e+08
15.5	1.0259e+08	2.0427e+08	3.0445e+08	4.0237e+08	4.9731e+08	5.8869e+08
15.55	1.0363e+08	2.0647e+08	3.0773e+08	4.0665e+08	5.0250e+08	5.9471e+08
15.6	1.0482e+08	2.0875e+08	3.1104e+08	4.1087e+08	5.0768e+08	6.0086e+08
15.9	1.0840e+08	2.1600e+08	3.2207e+08	4.2579e+08	5.2646e+08	6.2357e+08
16	1.0911e+08	2.1748e+08	3.2430e+08	4.2885e+08	5.3040e+08	6.2831e+08
16.2	1.1061e+08	2.2052e+08	3.2912e+08	4.3582e+08	5.4009e+08	6.4124e+08
16.5	1.1252e+08	2.2441e+08	3.3512e+08	4.4408e+08	5.5072e+08	6.5441e+08
17	1.1611e+08	2.3175e+08	3.4633e+08	4.5935e+08	5.7026e+08	6.7856e+08
18	1.2190e+08	2.4339e+08	3.4633e+08	4.8372e+08	6.0187e+08	7.1835e+08
19	1.1940e+08	2.3852e+08	3.5702e+08	4.7461e+08	5.9118e+08	7.0641e+08
20	1.1716e+08	2.3409e+08	3.5058e+08	4.6659e+08	5.8184e+08	6.9622e+08

D.1.3 0 Degrees

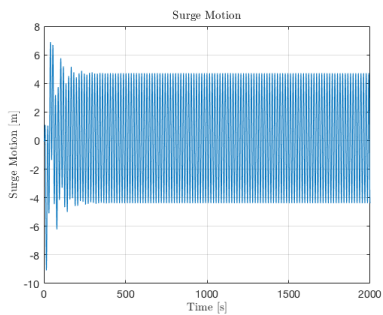


Figure D.13: Surge Motion Wave Period 16 [s] Significant Wave Height 6 [m].

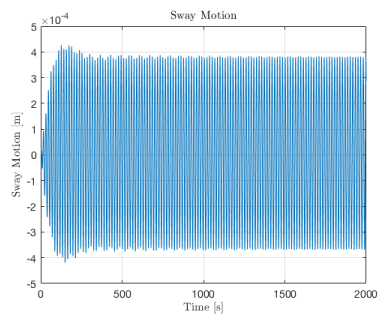


Figure D.14: Sway Motion Wave Period 16 [s] Significant Wave Height 6 [m].

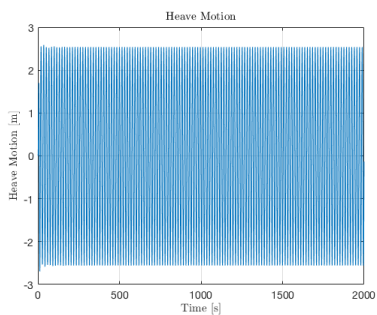


Figure D.15: Heave Motion Wave Period 16 [s] Significant Wave Height 6 [m].

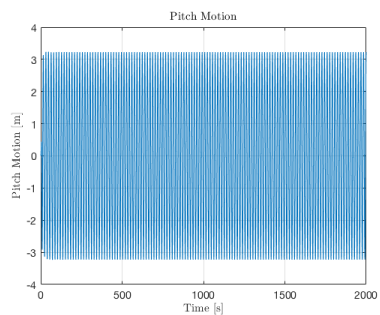


Figure D.16: Pitch Motion Wave Period 16 [s] Significant Wave Height 6 [m].

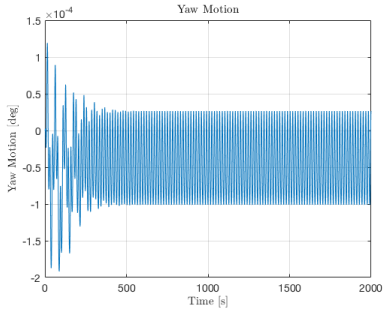


Figure D.17: Yaw Motion Wave Period 16 [s] Significant Wave Height 6 [m].

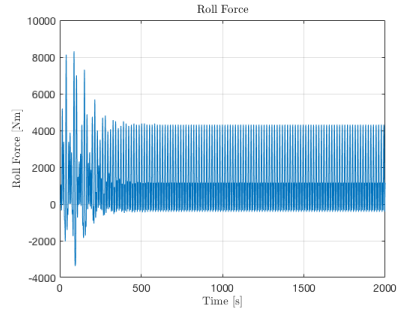


Figure D.18: Roll Force Wave Period 16 [s] Significant Wave Height 6 [m].

Table D.14: Surge Amplitude with Linear Damping at 0 Degrees.

Tp \ Hs	1	2	3	4	5	6
5	0.0065	0.0130	0.0195	0.0260	0.0325	0.0390
6	0.0237	0.0474	0.0712	0.0949	0.1186	0.1424
7	0.0533	0.1067	0.1600	0.2133	0.2665	0.3198
8	0.0949	0.1898	0.2848	0.3799	0.4751	0.5704
9	0.1384	0.2767	0.4150	0.5531	0.6911	0.8290
10	0.1751	0.3501	0.5249	0.6997	0.8744	1.0489
11	0.1578	0.3156	0.4734	0.6311	0.7888	0.9465
12	0.2264	0.4528	0.6794	0.9060	1.1327	1.3596
13	0.3711	0.7421	1.1133	0.9060	1.8556	2.2268
14	0.5275	1.0551	1.5827	2.1104	2.6382	3.1661
14.5	0.5950	1.1902	1.7856	2.3811	2.9768	3.5727
15	0.6617	1.3236	1.9856	2.6477	3.3100	3.9724
15.3	0.6970	1.3941	2.0912	2.7884	3.4856	4.1829
15.4	0.7092	1.4184	2.1276	2.8369	3.5461	4.2554
15.45	0.7157	1.4315	2.1472	2.8628	3.5785	4.2941
15.5	0.7208	1.4417	2.1625	2.8833	3.6040	4.3248
15.55	0.7260	1.4520	2.1779	2.9038	3.6296	4.3554
15.6	0.7323	1.4645	2.1966	2.9287	3.6607	4.3926
15.9	0.7621	1.5243	2.2866	3.0489	3.8114	4.5740
16	0.7704	1.5408	2.3114	3.0820	3.8527	4.6234
16.2	0.7894	1.5789	2.3683	3.1578	3.9473	4.7369
16.5	0.8160	1.6320	2.4481	3.2643	4.0805	4.8968
17	0.8545	1.7089	2.5632	3.4175	4.2716	5.1256
18	0.9372	1.8744	2.8116	3.7487	4.6857	5.6227
19	0.9870	1.9740	2.9610	3.9479	4.9348	5.9217
20	1.0366	2.0732	3.1098	4.1464	5.1830	6.2195

D.1. LINEAR ROLL DAMPING

Table D.15: Sway Amplitude with Linear Damping at 0 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	1.4591e-07	2.9283e-07	4.3961e-07	5.7846e-07	7.3298e-07	8.7663e-07
6	3.3194e-07	6.5974e-07	9.8250e-07	1.3188e-06	1.6312e-06	1.9619e-06
7	6.5832e-07	1.3157e-06	1.9549e-06	2.6093e-06	3.2365e-06	3.8871e-06
8	1.2667e-06	2.5134e-06	3.7629e-06	5.0161e-06	6.2641e-06	7.5110e-06
9	2.1054e-06	4.2447e-06	6.3852e-06	8.5648e-06	1.0832e-05	1.3120e-05
10	4.0275e-06	8.1110e-06	1.2234e-05	1.6386e-05	2.0575e-05	2.4818e-05
11	5.0567e-06	1.0117e-05	1.5173e-05	2.0256e-05	2.5328e-05	3.0409e-05
12	4.9639e-06	9.9271e-06	1.4886e-05	1.9842e-05	2.4795e-05	2.9743e-05
13	7.1374e-06	1.4232e-05	2.1271e-05	1.9842e-05	3.5244e-05	4.2261e-05
14	1.7537e-05	3.5049e-05	5.2773e-05	7.0628e-05	8.8626e-05	1.0657e-04
14.5	3.2753e-05	6.5451e-05	9.8142e-05	1.3075e-04	1.6330e-04	1.9550e-04
15	6.4596e-05	1.2903e-04	1.9288e-04	2.5596e-04	3.1825e-04	3.7929e-04
15.3	8.3448e-05	1.6648e-04	2.4892e-04	3.3002e-04	4.0992e-04	4.8816e-04
15.4	8.6201e-05	1.7212e-04	2.5733e-04	3.4181e-04	4.2485e-04	5.0652e-04
15.45	8.6238e-05	1.7223e-04	2.5788e-04	3.4279e-04	4.2611e-04	5.0892e-04
15.5	8.5422e-05	1.7082e-04	2.5587e-04	3.4014e-04	4.2377e-04	5.0602e-04
15.55	8.4474e-05	1.6906e-04	2.5314e-04	3.3702e-04	4.2001e-04	5.0247e-04
15.6	8.2435e-05	1.6485e-04	2.4713e-04	3.2959e-04	4.1130e-04	4.9275e-04
15.9	6.8915e-05	1.3797e-04	2.0749e-04	2.7762e-04	3.4832e-04	4.1953e-04
16	6.3791e-05	1.2783e-04	1.9214e-04	2.5710e-04	3.2288e-04	3.8919e-04
16.2	5.4470e-05	1.0951e-04	1.6494e-04	2.2111e-04	2.7779e-04	3.3521e-04
16.5	4.6185e-05	9.2864e-05	1.4016e-04	1.8836e-04	2.3740e-04	2.8713e-04
17	3.6919e-05	7.3988e-05	1.1166e-04	1.5040e-04	1.8988e-04	2.3042e-04
18	2.6811e-05	5.4373e-05	8.2835e-05	1.1233e-04	1.4301e-04	1.7412e-04
19	2.1525e-05	4.4080e-05	6.7612e-05	9.2075e-05	1.1785e-04	1.4462e-04
20	1.8316e-05	3.7776e-05	5.8556e-05	8.1037e-05	1.0515e-04	1.3480e-04

Table D.16: Heave Amplitude with Linear Damping at 0 Degrees.

Tp \ Hs	1	2	3	4	5	6
5	0.0070	0.0140	0.0210	0.0280	0.0350	0.0420
6	0.0153	0.0305	0.0458	0.0611	0.0763	0.0916
7	0.0309	0.0617	0.0926	0.1234	0.1543	0.1852
8	0.0632	0.1265	0.1897	0.2531	0.3164	0.3798
9	0.0996	0.1992	0.2989	0.3985	0.4981	0.5978
10	0.1550	0.3101	0.4652	0.6204	0.7756	0.9309
11	0.1677	0.3353	0.5030	0.6706	0.8383	1.0060
12	0.1327	0.2654	0.3982	0.5309	0.6637	0.7964
13	0.1379	0.2758	0.4138	0.5309	0.6901	0.8284
14	0.2027	0.4054	0.6082	0.8110	1.0138	1.2167
14.5	0.2683	0.5366	0.8049	1.0733	1.3418	1.6102
15	0.3250	0.6501	0.9752	1.3003	1.6255	1.9507
15.3	0.3545	0.7090	1.0635	1.4182	1.7729	2.1277
15.4	0.3641	0.7282	1.0923	1.4566	1.8209	2.1854
15.45	0.3695	0.7390	1.1086	1.4783	1.8480	2.2180
15.5	0.3731	0.7460	1.1187	1.4913	1.8638	2.2369
15.55	0.3776	0.7552	1.1329	1.5108	1.8888	2.2670
15.6	0.3830	0.7659	1.1491	1.5323	1.9157	2.2992
15.9	0.4149	0.8298	1.2448	1.6601	2.0755	2.4912
16	0.4234	0.8473	1.2718	1.6968	2.1224	2.5486
16.2	0.4471	0.8943	1.3416	1.7891	2.2370	2.6852
16.5	0.4765	0.9533	1.4303	1.9077	2.3854	2.8635
17	0.5234	1.0469	1.5707	2.0948	2.6193	3.1443
18	0.6050	1.2101	1.8155	2.4214	3.0278	3.6349
19	0.6739	1.3479	2.0223	2.6971	3.3724	4.0484
20	0.7283	1.4566	2.1852	2.9143	3.6438	4.3737

D.1. LINEAR ROLL DAMPING

Table D.17: Roll Amplitude with Linear Damping at 0 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	3.5561e-07	7.1125e-07	1.0659e-06	1.4149e-06	1.7928e-06	2.1446e-06
6	7.8578e-07	1.5755e-06	2.3553e-06	3.1421e-06	3.9261e-06	4.7217e-06
7	1.5990e-06	3.1951e-06	4.7896e-06	6.3855e-06	7.9781e-06	9.5553e-06
8	3.2865e-06	6.5621e-06	9.8363e-06	1.3084e-05	1.6343e-05	1.9588e-05
9	5.4614e-06	1.0930e-05	1.6410e-05	2.1892e-05	2.7376e-05	3.2869e-05
10	9.7124e-06	1.9414e-05	2.9110e-05	3.8789e-05	4.8461e-05	5.8099e-05
11	1.2245e-05	2.4474e-05	3.6698e-05	4.8929e-05	6.1136e-05	7.3358e-05
12	1.1934e-05	2.3878e-05	3.5840e-05	4.7814e-05	5.9795e-05	7.1783e-05
131	1.6526e-05	3.3019e-05	4.9467e-05	4.7814e-05	8.2203e-05	9.8480e-05
141	3.9853e-05	7.9622e-05	1.1926e-04	1.5867e-04	1.9780e-04	2.3663e-04
14.5	7.3857e-05	1.4748e-04	2.2066e-04	2.9318e-04	3.6490e-04	4.3551e-04
15	1.4568e-04	2.9047e-04	4.3350e-04	5.7400e-04	7.1116e-04	8.4437e-04
15.3	1.8700e-04	3.7270e-04	5.5585e-04	7.3530e-04	9.1003e-04	1.0791e-03
15.4	1.9293e-04	3.8467e-04	5.7406e-04	7.6006e-04	9.4175e-04	1.1184e-03
15.45	1.9287e-04	3.8470e-04	5.7441e-04	7.6114e-04	9.4400e-04	1.1224e-03
15.5	1.9104e-04	3.8112e-04	5.6932e-04	7.5483e-04	9.3697e-04	1.1151e-03
15.55	1.8851e-04	3.7620e-04	5.6228e-04	7.4604e-04	9.2698e-04	1.1045e-03
15.6	1.8389e-04	3.6712e-04	5.4907e-04	7.2910e-04	9.0675e-04	1.0817e-03
15.9	1.5373e-04	3.0730e-04	4.6062e-04	6.1347e-04	7.6579e-04	9.1817e-04
16	1.4251e-04	2.8499e-04	4.2745e-04	5.6992e-04	7.1233e-04	8.5487e-04
16.2	1.2021e-04	2.4050e-04	3.6091e-04	4.8153e-04	6.0249e-04	7.2385e-04
16.5	1.0131e-04	2.0270e-04	3.0426e-04	4.0607e-04	5.0823e-04	6.1088e-04
17	8.0818e-05	1.6164e-04	2.4263e-04	3.2371e-04	4.0501e-04	4.8664e-04
18	5.7345e-05	1.1471e-04	1.7218e-04	2.2967e-04	2.8731e-04	3.4512e-04
19	4.5164e-05	9.0357e-05	1.3559e-04	1.8088e-04	2.2633e-04	2.7198e-04
20	3.7238e-05	7.4418e-05	1.1157e-04	1.4882e-04	1.8623e-04	2.2370e-04

Table D.18: Pitch Amplitude with Linear Damping at 0 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0178	0.0356	0.0534	0.0711	0.0889	0.1067
6	0.0426	0.0852	0.1278	0.1704	0.2130	0.2556
7	0.0883	0.1767	0.2650	0.3533	0.4416	0.5300
8	0.1555	0.3110	0.4665	0.6220	0.7776	0.9331
9	0.2133	0.4265	0.6398	0.8530	1.0663	1.2796
10	0.2192	0.4383	0.6575	0.8767	1.0958	1.3150
11	0.2162	0.4324	0.6486	0.8648	1.0811	1.2973
12	0.2931	0.5862	0.8793	1.1725	1.4656	1.7587
13	0.3936	0.7873	1.1809	1.1725	1.9682	2.3619
14	0.4795	0.9590	1.4385	1.9180	2.3975	2.8770
14.5	0.5026	1.0052	1.5079	2.0105	2.5131	3.0158
15	0.5216	1.0431	1.5647	2.0862	2.6078	3.1294
15.3	0.5301	1.0601	1.5902	2.1202	2.6503	3.1804
15.4	0.5329	1.0659	1.5988	2.1317	2.6647	3.1976
15.45	0.5342	1.0684	1.6026	2.1368	2.6710	3.2053
15.5	0.5346	1.0693	1.6039	2.1386	2.6732	3.2078
15.55	0.5359	1.0719	1.6078	2.1437	2.6797	3.2156
15.6	0.5372	1.0744	1.6117	2.1489	2.6861	3.2233
15.9	0.5391	1.0782	1.6173	2.1564	2.6955	3.2346
16	0.5382	1.0765	1.6147	2.1530	2.6912	3.2295
16.2	0.5380	1.0761	1.6141	2.1521	2.6901	3.2282
16.5	0.5360	1.0720	1.6080	2.1440	2.6800	3.2160
17	0.5319	1.0637	1.5956	2.1275	2.6594	3.1913
18	0.5191	1.0383	1.5574	2.0766	2.5957	3.1149
19	0.4904	0.9808	1.4712	1.9616	2.4521	2.9425
20	0.4646	0.9292	1.3937	1.8583	2.3229	2.7875

D.1. LINEAR ROLL DAMPING

Table D.19: Yaw Amplitude with Linear Damping at 0 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	2.7842e-07	5.5742e-07	8.3750e-07	1.1102e-06	1.3875e-06	1.6739e-06
6	1.1757e-06	2.3511e-06	3.5221e-06	4.7013e-06	5.8768e-06	7.0658e-06
7	2.2010e-06	4.3985e-06	6.5843e-06	8.7711e-06	1.0958e-05	1.3136e-05
8	3.2304e-06	6.4650e-06	9.7027e-06	1.2948e-05	1.6192e-05	1.9446e-05
9	3.7574e-06	7.5070e-06	1.1260e-05	1.4990e-05	1.8728e-05	2.2449e-05
10	5.6422e-06	1.1287e-05	1.6919e-05	2.2557e-05	2.8195e-05	3.3843e-05
11	3.8135e-06	7.6391e-06	1.1441e-05	1.5217e-05	1.9103e-05	2.2961e-05
12	4.6830e-06	9.3901e-06	1.4111e-05	1.8803e-05	2.3394e-05	2.7928e-05
13	8.0429e-06	1.6084e-05	2.4099e-05	1.8803e-05	4.0050e-05	4.8225e-05
14	1.1252e-05	2.2502e-05	3.3877e-05	4.5231e-05	5.6688e-05	6.8166e-05
14.5	1.1552e-05	2.3188e-05	3.4878e-05	4.6675e-05	5.8527e-05	7.0552e-05
15	1.1875e-05	2.3914e-05	3.6198e-05	4.8757e-05	6.1827e-05	7.5297e-05
15.3	1.1251e-05	2.2682e-05	3.4403e-05	4.6731e-05	5.9725e-05	7.3325e-05
15.4	1.1269e-05	2.2676e-05	3.4259e-05	4.6423e-05	5.8994e-05	7.2561e-05
15.45	1.1306e-05	2.2664e-05	3.4258e-05	4.6231e-05	5.8674e-05	7.1873e-05
15.5	1.1318e-05	2.2661e-05	3.4178e-05	4.6028e-05	5.8155e-05	7.1129e-05
15.55	1.1357e-05	2.2789e-05	3.4361e-05	4.6113e-05	5.8338e-05	7.0761e-05
15.6	1.1480e-05	2.2988e-05	3.4464e-05	4.6215e-05	5.8086e-05	7.0281e-05
15.9	1.2043e-05	2.4116e-05	3.6394e-05	4.8722e-05	6.1202e-05	7.3813e-05
16	1.2341e-05	2.4702e-05	3.7132e-05	4.9563e-05	6.2153e-05	7.4623e-05
16.2	1.2848e-05	2.5671e-05	3.8440e-05	5.1441e-05	6.4542e-05	7.7924e-05
16.5	1.3605e-05	2.7284e-05	4.1088e-05	5.5125e-05	6.9443e-05	8.4003e-05
17	1.4748e-05	2.9498e-05	4.4234e-05	5.9018e-05	7.3827e-05	8.8632e-05
18	1.6500e-05	3.3036e-05	4.9647e-05	6.6283e-05	8.3113e-05	9.9950e-05
19	1.6650e-05	3.3341e-05	5.0194e-05	6.7205e-05	8.4380e-05	1.0174e-04
20	1.6749e-05	3.3584e-05	5.0970e-05	6.8843e-05	8.7398e-05	1.0679e-04

Table D.20: Roll Force Amplitude with Linear Damping at 0 Degrees.

Tp \ Hs	1	2	3	4	5	6
5	0.5487	1.7061	4.8828	8.7891	19.5312	17.5781
6	2.4414	8.0091	14.4127	27.3438	42.4417	62.5000
7	2.9297	13.6719	28.0327	49.3654	70.3125	108.1524
8	7.9372	33.9656	74.2188	134.3323	199.4394	307.4612
9	5.8594	21.1539	44.6293	81.5257	128.9974	185.2345
10	21.4844	60.4355	160.5355	239.8583	422.9555	608.1034
11	9.7656	41.1935	89.8438	151.6080	243.5577	359.1967
12	5.2429	24.6969	56.8331	103.9112	166.3459	244.8191
13	14.1985	57.1671	116.3543	103.9112	314.1177	529.5300
14	42.8894	135.7463	332.0312	561.5233	866.6585	1233.7543
14.5	52.7344	160.8071	421.8750	968.7500	1570.3125	1960.9375
15	54.6875	253.9062	789.0625	1406.2500	1984.3750	3687.5000
15.3	70.3125	296.8750	796.8750	1703.1250	3125.0000	5000.0000
15.4	76.1719	296.8750	804.6875	1765.6250	2812.5000	4906.2500
15.45	70.3125	359.3750	859.3750	1796.8750	3171.8750	5125.0000
15.5	66.4062	328.1250	875.0000	1781.2500	3375.0000	4781.2500
15.55	78.1250	335.9375	882.8125	1656.2500	3187.5000	4843.7500
15.6	82.0312	281.2500	859.3750	1500.0000	3375.0000	5031.2500
15.9	76.1719	242.1875	703.1250	1312.5000	2328.1250	4062.5000
16	57.4509	304.6875	640.6250	1718.7500	2171.8750	3765.6250
16.2	69.6380	359.3750	590.9128	1593.7500	2078.1250	3203.1250
16.5	83.9844	293.7562	682.0071	1414.0625	1803.4306	2655.9470
17	81.0547	322.2656	714.8438	1250.0000	1879.3616	3125.0000
18	73.9567	429.6875	844.9301	1489.3735	2391.1359	4015.6250
19	77.8670	343.5361	793.1845	1399.1951	2018.7924	2972.6515
20	77.3566	355.4688	886.3999	1282.8542	2170.5382	3816.4985

D.1.4 90 Degrees

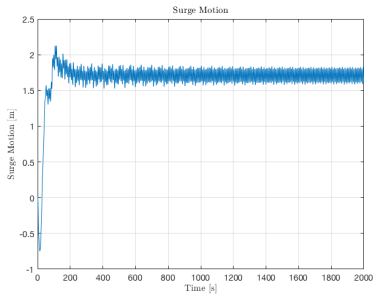


Figure D.19: Surge Motion Wave Period 16 [s] Significant Wave Height 6 [m].

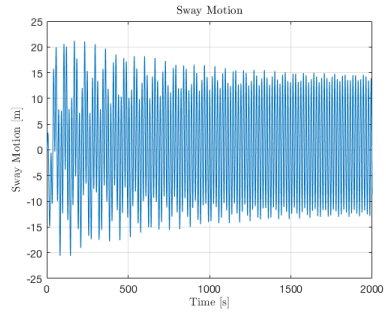


Figure D.20: Sway Motion Wave Period 16 [s] Significant Wave Height 6 [m].

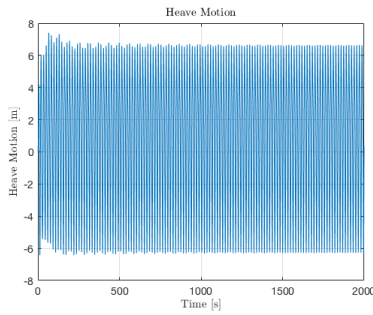


Figure D.21: Heave Motion Wave Period 16 [s] Significant Wave Height 6 [m].

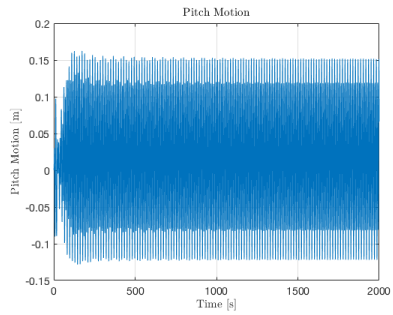


Figure D.22: Pitch Motion Wave Period 16 [s] Significant Wave Height 6 [m].

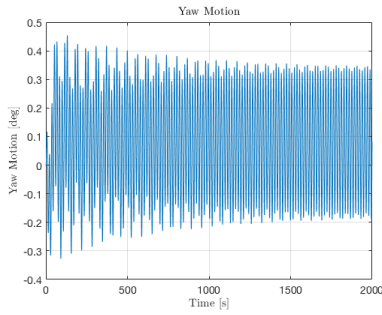


Figure D.23: Yaw Motion Wave Period 16 [s] Significant Wave Height 6 [m].

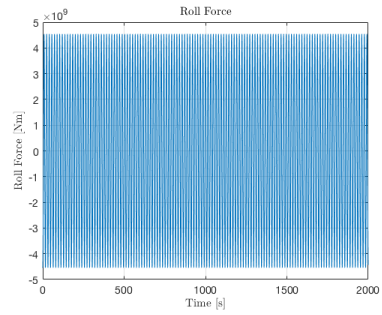


Figure D.24: Roll Force Wave Period 16 [s] Significant Wave Height 6 [m].

Table D.21: Surge Amplitude with Linear Damping at 90 Degrees.

Tp \ Hs	1	2	3	4	5	6
5	0.0072	0.0149	0.0221	0.0273	0.0310	0.0375
6	0.0198	0.0408	0.0598	0.0793	0.0938	0.1126
7	0.0219	0.0459	0.0669	0.0822	0.0895	0.0855
8	0.0229	0.0487	0.0707	0.0846	0.1069	0.1451
9	0.0224	0.0457	0.0667	0.0842	0.1070	0.1418
10	0.0177	0.0359	0.0531	0.0675	0.0804	0.1019
11	0.0118	0.0237	0.0346	0.0385	0.0456	0.0546
12	0.0064	0.0130	0.0201	0.0271	0.0308	0.0351
13	0.0033	0.0085	0.0130	0.0198	0.0339	0.0496
14	0.0033	0.0093	0.0273	0.0438	0.0639	0.1143
14.5	0.0058	0.0185	0.0539	0.1018	0.1623	0.2143
15	0.0127	0.0398	0.0768	0.1190	0.1672	0.2202
15.3	0.0208	0.0602	0.1013	0.1336	0.1701	0.2085
15.4	0.0220	0.0633	0.1036	0.1361	0.1676	0.2039
15.45	0.0212	0.0615	0.0996	0.1311	0.1650	0.2012
15.5	0.0199	0.0587	0.0946	0.1289	0.1630	0.2014
15.55	0.0183	0.0552	0.0907	0.1245	0.1614	0.2018
15.6	0.0161	0.0501	0.0839	0.1205	0.1621	0.2047
15.9	0.0141	0.0536	0.0964	0.1381	0.1750	0.2112
16	0.0131	0.0541	0.0959	0.1355	0.1707	0.2041
16.2	0.0092	0.0448	0.0848	0.1199	0.1502	0.1774
16.5	0.0066	0.0254	0.0538	0.0753	0.1084	0.1478
17	0.0051	0.0179	0.0339	0.0580	0.0845	0.1147
18	0.0036	0.0094	0.0207	0.0371	0.0573	0.0833
19	0.0038	0.0089	0.0160	0.0269	0.0421	0.0621
20	0.0038	0.0084	0.0150	0.0243	0.0368	0.0570

Table D.22: Sway Amplitude with Linear Damping at 90 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.4568	0.9302	1.3865	1.7956	2.1607	2.5311
6	0.5585	1.1115	1.6396	2.1891	2.7639	3.3407
7	0.9287	1.8403	2.7503	3.6651	4.5700	5.4437
8	1.7065	3.3782	5.0514	6.7514	8.4549	10.1704
9	1.9167	3.7999	5.6917	7.6185	9.5425	11.4422
10	2.0441	4.0537	6.0528	8.0635	10.0954	12.1519
11	1.8562	3.6645	5.4423	7.2070	8.9791	10.7752
12	1.6854	3.3094	4.8673	6.3614	7.8003	9.1843
13	1.1181	2.1986	3.2349	4.2253	5.1751	6.0948
14	1.5254	3.0265	4.4929	5.9164	7.2921	8.6183
14.5	2.0291	4.0285	5.9805	7.8687	9.6774	11.4025
15	3.2023	6.2591	9.0689	11.6246	13.9298	15.9881
15.3	4.0021	7.6678	10.8675	13.6523	16.1621	18.3994
15.4	4.1132	7.8509	11.1131	13.9433	16.4347	18.6950
15.45	4.1440	7.9025	11.1585	13.9758	16.4799	18.7299
15.5	4.1942	7.9879	11.2831	14.1423	16.6643	18.9268
15.55	4.2162	8.0491	11.3894	14.3022	16.8836	19.2086
15.6	4.2395	8.1047	11.4848	14.4449	17.0781	19.4570
15.9	3.8850	7.5617	10.9343	14.0037	16.8047	19.3796
16	3.6868	7.2445	10.5856	13.6792	16.5295	19.1690
16.2	3.4131	6.7724	10.0198	13.1125	16.0280	18.7689
16.5	3.1618	6.3050	9.4006	12.4263	15.3655	18.1999
17	2.8400	5.6751	8.4933	11.2852	14.0455	16.7751
18	2.5188	5.0472	7.5839	10.1274	12.6760	15.2274
19	2.2084	4.4265	6.6545	8.8927	11.1410	13.3991
20	1.9242	3.8541	5.7931	7.7414	9.6993	11.6670

Table D.23: Heave Amplitude with Linear Damping at 90 Degrees.

Tp \ Hs	1	2	3	4	5	6
5	0.0299	0.0598	0.0897	0.1195	0.1491	0.1790
6	0.0795	0.1588	0.2372	0.3146	0.3907	0.4660
7	0.1932	0.3857	0.5765	0.7651	0.9510	1.1360
8	0.4044	0.8093	1.2147	1.6198	2.0192	2.4130
9	0.6916	1.3833	2.0754	2.7671	3.4582	4.1464
10	0.8985	1.7984	2.7017	3.6083	4.5184	5.4291
11	0.9653	1.9346	2.9081	3.8857	4.8676	5.8537
12	0.9739	1.9551	2.9436	3.9390	4.9411	5.9494
13	0.9608	1.9300	2.9104	3.9013	4.9022	5.9126
14	0.9664	1.9517	2.9556	3.9772	5.0153	6.0753
14.5	0.9651	1.9625	3.0033	4.0830	5.1995	6.3482
15	0.9786	2.0135	3.0995	4.2157	5.3371	6.4715
15.3	0.9799	2.0365	3.1865	4.4072	5.6585	6.9171
15.4	0.9888	2.0871	3.2880	4.5479	5.8229	7.0986
15.45	0.9918	2.1071	3.3270	4.5881	5.8623	7.1361
15.5	0.9896	2.1149	3.3475	4.6222	5.9062	7.1896
15.55	0.9943	2.1209	3.3514	4.6257	5.9158	7.2045
15.6	0.9933	2.1188	3.3459	4.6127	5.8948	7.1818
15.9	0.9652	1.9897	3.1063	4.3021	5.5369	6.7943
16	0.9603	1.9556	3.0276	4.1785	5.3783	6.6061
16.2	0.9594	1.9316	2.9249	3.9707	5.0900	6.2571
16.5	0.9598	1.9297	2.9093	3.8979	4.8951	5.9053
17	0.9607	1.9307	2.9129	3.9047	4.9039	5.9079
18	0.9639	1.9330	2.9095	3.8975	4.8943	5.8995
19	0.9717	1.9469	2.9258	3.9084	4.8977	5.8944
20	0.9739	1.9498	2.9284	3.9101	4.8974	5.8895

Table D.24: Roll Amplitude with Linear Damping at 90 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0528	0.1063	0.1570	0.2059	0.2531	0.2988
6	0.0397	0.0767	0.1103	0.1488	0.1930	0.2364
7	0.0849	0.1685	0.2504	0.3311	0.4104	0.4878
8	0.2022	0.4036	0.6065	0.8147	1.0287	1.2576
9	0.3383	0.6738	1.0064	1.3374	1.6676	1.9916
10	0.4410	0.8767	1.3074	1.7328	2.1483	2.5733
11	0.6269	1.2469	1.8604	2.4769	3.0942	3.7068
12	0.9140	1.8209	2.7216	3.6641	4.6369	5.6316
13	1.3981	2.8023	4.2139	5.6327	7.0592	8.4938
14	2.4385	4.8719	7.2972	9.7123	12.1155	14.5064
14.5	3.4741	6.9282	10.3529	13.7495	17.1000	20.3878
15	5.7580	11.1797	16.0748	20.3706	24.1048	27.5016
15.3	6.8082	12.7761	17.7035	21.7934	25.2389	28.1915
15.4	6.8617	12.7715	17.6065	21.6039	24.9754	27.8716
15.45	6.7909	12.6404	17.4314	21.3679	24.6799	27.5404
15.5	6.6895	12.4707	17.2143	21.1306	24.4231	27.2579
15.55	6.5697	12.2746	16.9719	20.8630	24.1458	26.9833
15.6	6.3649	11.9343	16.5298	20.3683	23.6362	26.4758
15.9	4.9957	9.6027	13.6952	17.3019	20.4724	23.2868
16	4.5532	8.8358	12.7223	16.2246	19.3892	22.2104
16.2	3.7786	7.4433	10.9208	14.1481	17.0984	19.8231
16.5	3.0499	6.0529	8.9669	11.7585	14.4099	16.9455
17	2.2714	4.5269	6.7509	8.9428	11.0882	13.1784
18	1.4917	2.9826	4.4709	5.9548	7.4328	8.9054
19	1.1051	2.2111	3.3181	4.4262	5.5353	6.6457
20	0.8801	1.7606	2.6417	3.5239	4.4075	5.2928

Table D.25: Pitch Amplitude with Linear Damping at 90 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0136	0.0272	0.0408	0.0543	0.0677	0.0811
6	0.0263	0.0527	0.0792	0.1058	0.1324	0.1591
7	0.0421	0.0846	0.1276	0.1702	0.2109	0.2482
8	0.0582	0.1160	0.1711	0.2215	0.2654	0.3041
9	0.0622	0.1240	0.1831	0.2377	0.2855	0.3298
10	0.0492	0.0984	0.1468	0.1933	0.2377	0.2786
11	0.0326	0.0656	0.0986	0.1311	0.1629	0.1942
12	0.0217	0.0439	0.0665	0.0892	0.1123	0.1357
13	0.0140	0.0287	0.0439	0.0597	0.0764	0.0938
14	0.0101	0.0211	0.0330	0.0460	0.0602	0.0754
14.5	0.0088	0.0193	0.0316	0.0457	0.0618	0.0882
15	0.0095	0.0262	0.0480	0.0723	0.0976	0.1236
15.3	0.0114	0.0334	0.0603	0.0886	0.1168	0.1448
15.4	0.0116	0.0346	0.0622	0.0910	0.1200	0.1488
15.45	0.0115	0.0345	0.0624	0.0916	0.1208	0.1516
15.5	0.0112	0.0341	0.0621	0.0916	0.1225	0.1538
15.55	0.0110	0.0336	0.0614	0.0920	0.1236	0.1554
15.6	0.0106	0.0330	0.0614	0.0921	0.1236	0.1559
15.9	0.0092	0.0289	0.0555	0.0858	0.1176	0.1497
16	0.0088	0.0274	0.0527	0.0821	0.1133	0.1453
16.2	0.0077	0.0237	0.0467	0.0743	0.1046	0.1362
16.5	0.0067	0.0203	0.0401	0.0647	0.0926	0.1225
17	0.0052	0.0154	0.0305	0.0497	0.0719	0.0963
18	0.0029	0.0080	0.0155	0.0252	0.0369	0.0505
19	0.0018	0.0047	0.0088	0.0143	0.0209	0.0288
20	0.0014	0.0036	0.0068	0.0110	0.0161	0.0221

Table D.26: Yaw Amplitude with Linear Damping at 90 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0244	0.0468	0.0703	0.0902	0.1080	0.1277
6	0.0712	0.1384	0.2036	0.2657	0.3269	0.3921
7	0.0754	0.1504	0.2219	0.2933	0.3565	0.3998
8	0.0630	0.1249	0.1859	0.2341	0.2680	0.2887
9	0.0632	0.1261	0.1826	0.2295	0.2746	0.2865
10	0.0575	0.1151	0.1695	0.2232	0.2663	0.2969
11	0.0564	0.1132	0.1703	0.2283	0.2882	0.3361
12	0.0576	0.1160	0.1762	0.2398	0.3062	0.3730
13	0.0598	0.1196	0.1789	0.2367	0.2912	0.3406
14	0.0718	0.1439	0.2162	0.2884	0.3596	0.4279
14.5	0.0834	0.1667	0.2495	0.3332	0.4163	0.4972
15	0.1120	0.2189	0.3175	0.4070	0.4883	0.5623
15.3	0.1162	0.2204	0.3105	0.3882	0.4564	0.5172
15.4	0.1136	0.2142	0.3000	0.3732	0.4370	0.4940
15.45	0.1111	0.2095	0.2933	0.3647	0.4279	0.4850
15.5	0.1086	0.2056	0.2878	0.3582	0.4203	0.4761
15.55	0.1061	0.2009	0.2813	0.3506	0.4118	0.4677
15.6	0.1019	0.1937	0.2725	0.3404	0.3997	0.4537
15.9	0.0754	0.1459	0.2104	0.2694	0.3231	0.3738
16	0.0675	0.1323	0.1932	0.2505	0.3029	0.3516
16.2	0.0540	0.1074	0.1597	0.2097	0.2567	0.3022
16.5	0.0433	0.0863	0.1287	0.1701	0.2104	0.2504
17	0.0313	0.0627	0.0944	0.1262	0.1587	0.1913
18	0.0214	0.0431	0.0652	0.0879	0.1111	0.1350
19	0.0146	0.0305	0.0461	0.0597	0.0757	0.0923
20	0.0108	0.0232	0.0353	0.0479	0.0610	0.0747

Table D.27: Roll Force Amplitude with Linear Damping at 90 Degrees.

Tp \ Hs	1	2	3	4	5	6
5	0.5487	1.7061	4.8828	8.7891	19.5312	17.5781
6	2.4414	8.0091	14.4127	27.3438	42.4417	62.5000
7	2.9297	13.6719	28.0327	49.3654	70.3125	108.1524
8	7.9372	33.9656	74.2188	134.3323	199.4394	307.4612
9	5.8594	21.1539	44.6293	81.5257	128.9974	185.2345
10	21.4844	60.4355	160.5355	239.8583	422.9555	608.1034
11	9.7656	41.1935	89.8438	151.6080	243.5577	359.1967
12	5.2429	24.6969	56.8331	103.9112	166.3459	244.8191
13	14.1985	57.1671	116.3543	103.9112	314.1177	529.5300
14	42.8894	135.7463	332.0312	561.5233	866.6585	1233.7543
14.5	52.7344	160.8071	421.8750	968.7500	1570.3125	1960.9375
15	54.6875	253.9062	789.0625	1406.2500	1984.3750	3687.5000
15.3	70.3125	296.8750	796.8750	1703.1250	3125.0000	5000.0000
15.4	76.1719	296.8750	804.6875	1765.6250	2812.5000	4906.2500
15.45	70.3125	359.3750	859.3750	1796.8750	3171.8750	5125.0000
15.5	66.4062	328.1250	875.0000	1781.2500	3375.0000	4781.2500
15.55	78.1250	335.9375	882.8125	1656.2500	3187.5000	4843.7500
15.6	82.0312	281.2500	859.3750	1500.0000	3375.0000	5031.2500
15.9	76.1719	242.1875	703.1250	1312.5000	2328.1250	4062.5000
16	57.4509	304.6875	640.6250	1718.7500	2171.8750	3765.6250
16.2	69.6380	359.3750	590.9128	1593.7500	2078.1250	3203.1250
16.5	83.9844	293.7562	682.0071	1414.0625	1803.4306	2655.9470
17	81.0547	322.2656	714.8438	1250.0000	1879.3616	3125.0000
18	73.9567	429.6875	844.9301	1489.3735	2391.1359	4015.6250
19	77.8670	343.5361	793.1845	1399.1951	2018.7924	2972.6515
20	77.3566	355.4688	886.3999	1282.8542	2170.5382	3816.4985

D.2 Linearized Roll Damping

D.2.1 45 Degrees

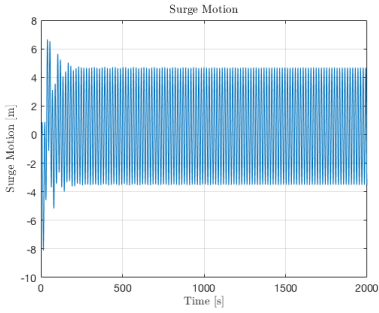


Figure D.25: Surge Motion Wave Period 16 [s] Significant Wave Height 6 [m].

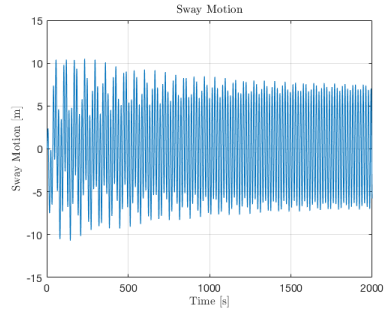


Figure D.26: Sway Motion Wave Period 16 [s] Significant Wave Height 6 [m].

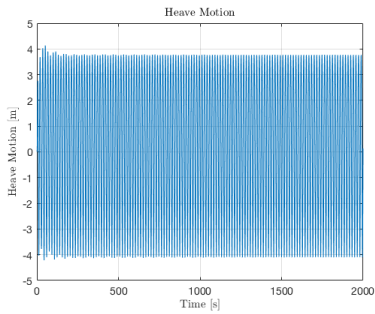


Figure D.27: Heave Motion Wave Period 16 [s] Significant Wave Height 6 [m]

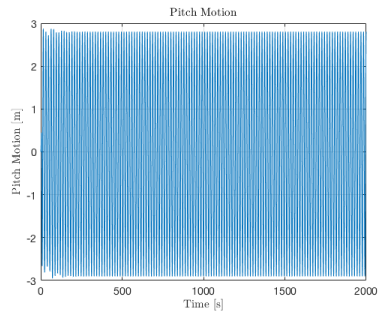


Figure D.28: Pitch Motion Wave Period 16 [s] Significant Wave Height 6 [m].

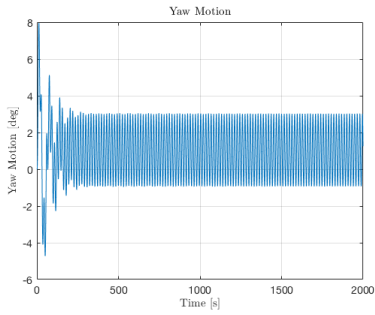


Figure D.29: Yaw Motion Wave Period 16 [s] Significant Wave Height 6 [m].

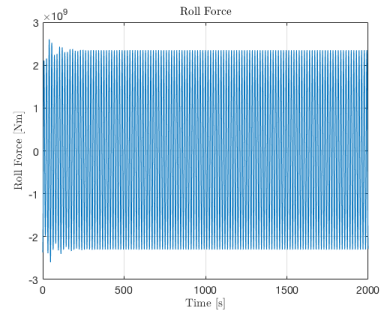


Figure D.30: Roll Force Wave Period 16 [s] Significant Wave Height 6 [m].

Table D.28: Surge Amplitude with Linearized Damping at 45 Degrees.

Tp \ Hs	1	2	3	4	5	6
5	0.0140	0.0286	0.0436	0.0589	0.0742	0.0901
6	0.0237	0.0479	0.0725	0.0974	0.1222	0.1466
7	0.0520	0.1035	0.1547	0.2042	0.2507	0.2942
8	0.0845	0.1675	0.2496	0.3299	0.4095	0.4905
9	0.0915	0.1834	0.2752	0.3664	0.4566	0.5461
10	0.2065	0.4115	0.6141	0.8134	1.0079	1.1972
11	0.3488	0.6944	1.0355	1.3704	1.6975	2.0152
12	0.4626	0.9223	1.3763	1.8227	2.2600	2.6883
13	0.5526	1.1033	1.6501	2.1910	2.7280	3.2643
14	0.6200	1.2376	1.8509	2.4577	3.0567	3.6472
14.5	0.6486	1.2934	1.9326	2.5653	3.1954	3.8186
15	0.6733	1.3433	2.0077	2.6649	3.3138	3.9546
15.3	0.6869	1.3707	2.0480	2.7156	3.3706	4.0116
15.4	0.6926	1.3816	2.0631	2.7334	3.3893	4.0304
15.45	0.6932	1.3827	2.0643	2.7343	3.3900	4.0394
15.5	0.6951	1.3868	2.0707	2.7430	3.4004	4.0477
15.55	0.6970	1.3901	2.0707	2.7474	3.4044	4.0598
15.6	0.6988	1.3937	2.0801	2.7539	3.4117	4.0715
15.9	0.7083	1.4106	2.1025	2.7804	3.4554	4.1276
16	0.7112	1.4165	2.1116	2.7804	3.4696	4.1412
16.2	0.7204	1.4356	2.1413	2.8336	3.5201	4.1946
16.5	0.7312	1.4579	2.1762	2.8828	3.5764	4.2665
17	0.7437	1.4846	2.2202	2.9478	3.6647	4.3868
18	0.7750	1.5480	2.3177	3.0825	3.8414	4.6010
19	0.7954	1.5893	2.3806	3.1686	3.9529	4.7355
20	0.8142	1.6270	2.4376	3.2457	4.0527	4.8623

Table D.29: Sway Amplitude with Linearized Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0264	0.0520	0.0776	0.1030	0.1188	0.1383
6	0.0418	0.0837	0.1258	0.1678	0.2096	0.2511
7	0.0359	0.0730	0.1111	0.1496	0.1883	0.2269
8	0.0902	0.1805	0.2699	0.3576	0.4430	0.5257
9	0.1243	0.2469	0.3687	0.4905	0.6127	0.7352
10	0.1657	0.3291	0.4922	0.6555	0.8197	0.9841
11	0.3448	0.6785	1.0043	1.3258	1.6469	1.9712
12	0.5738	1.1348	1.6843	2.2236	2.7541	3.2780
13	0.6785	1.3396	1.9863	2.6198	3.2397	3.8476
14	0.9173	1.8106	2.6812	3.5302	4.3579	5.1654
14.5	1.0923	2.1620	3.2101	4.2362	5.2393	6.2180
15	1.4371	2.8469	4.2227	5.5584	6.8509	8.0963
15.3	1.6971	3.3558	4.9612	6.5017	7.9694	9.3600
15.4	1.7549	3.4693	5.1256	6.7108	8.2167	9.6397
15.45	1.7752	3.5095	5.1840	6.7852	8.3050	9.7401
15.5	1.7812	3.5215	5.2019	6.8088	8.3341	9.7748
15.55	1.7830	3.5255	5.2019	6.8151	8.3401	9.7798
15.6	1.8173	3.5888	5.2940	6.9186	8.4545	9.8991
15.9	1.8298	3.6191	5.3500	7.0094	8.5894	10.0866
16	1.7887	3.5408	5.2398	7.0094	8.4373	9.9240
16.2	1.7357	3.4388	5.0964	6.6986	8.2398	9.7160
16.5	1.6657	3.3032	4.9037	6.4605	7.9680	9.4226
17	1.5902	3.1595	4.7019	6.2124	7.6870	9.1229
18	1.4951	2.9738	4.4335	5.8716	7.2864	8.6766
19	1.3501	2.6883	4.0135	5.3245	6.6205	7.9008
20	1.2209	2.4317	3.6330	4.8258	6.0111	7.1892

Table D.30: Heave Amplitude with Linearized Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0055	0.0111	0.0166	0.0221	0.0276	0.0331
6	0.0147	0.0295	0.0442	0.0589	0.0736	0.0883
7	0.0333	0.0665	0.0997	0.1329	0.1660	0.1990
8	0.0623	0.1247	0.1870	0.2493	0.3117	0.3740
9	0.1352	0.2702	0.4051	0.5400	0.6748	0.8095
10	0.0959	0.1916	0.2884	0.3865	0.4862	0.5877
11	0.1616	0.3218	0.4798	0.6347	0.7846	0.9281
12	0.2945	0.5896	0.8838	1.1751	1.4628	1.7439
13	0.4094	0.8201	1.2325	1.6451	2.0576	2.4671
14	0.5078	1.0173	1.5319	2.0531	2.5796	3.1103
14.5	0.5533	1.1117	1.6758	2.2465	2.8237	3.4070
15	0.5937	1.1920	1.7952	2.4068	3.0248	3.6476
15.3	0.6157	1.2363	1.8642	2.4994	3.1410	3.7853
15.4	0.6228	1.2505	1.8834	2.5252	3.1737	3.8236
15.45	0.6265	1.2578	1.8961	2.5417	3.1912	3.8414
15.5	0.6292	1.2620	1.8974	2.5428	3.1993	3.8568
15.55	0.6323	1.2682	1.8974	2.5594	3.2098	3.8616
15.6	0.6364	1.2760	1.9201	2.5688	3.2185	3.8744
15.9	0.6571	1.3155	1.9746	2.6318	3.2851	3.9333
16	0.6631	1.3279	1.9922	2.6318	3.3110	3.9621
16.2	0.6770	1.3559	2.0347	2.7125	3.3868	4.0555
16.5	0.6937	1.3897	2.0869	2.7843	3.4842	4.1814
17	0.7236	1.4495	2.1785	2.9115	3.6456	4.3794
18	0.7736	1.5490	2.3258	3.1055	3.8903	4.6771
19	0.8151	1.6319	2.4502	3.2702	4.0915	4.9173
20	0.8459	1.6927	2.5410	3.3907	4.2422	5.0953

Table D.31: Pitch Amplitude with Linearized Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0160	0.0319	0.0478	0.0637	0.0796	0.0954
6	0.0382	0.0764	0.1146	0.1527	0.1907	0.2287
7	0.0811	0.1620	0.2428	0.3234	0.4040	0.4843
8	0.1616	0.3231	0.4845	0.6458	0.8069	0.9679
9	0.1668	0.3334	0.4997	0.6656	0.8310	0.9959
10	0.3137	0.6270	0.9398	1.2504	1.5576	1.8607
11	0.4319	0.8637	1.2954	1.7256	2.1530	2.5768
12	0.4902	0.9797	1.4687	1.9565	2.4425	2.9273
13	0.5130	1.0252	1.5366	2.0471	2.5566	3.0650
14	0.5121	1.0236	1.5341	2.0438	2.5536	3.0626
14.5	0.5055	1.0109	1.5166	2.0220	2.5264	3.0304
15	0.4959	0.9915	1.4888	1.9892	2.4920	2.9964
15.3	0.4901	0.9812	1.4749	1.9705	2.4673	2.9682
15.4	0.4880	0.9767	1.4662	1.9604	2.4601	2.9629
15.45	0.4866	0.9738	1.4630	1.9555	2.4547	2.9576
15.5	0.4854	0.9727	1.4630	1.9563	2.4526	2.9517
15.55	0.4847	0.9714	1.4630	1.9542	2.4504	2.9498
15.6	0.4833	0.9685	1.4570	1.9490	2.4445	2.9435
15.9	0.4753	0.9521	1.4308	1.9118	2.3988	2.8914
16	0.4712	0.9447	1.4217	1.9118	2.3881	2.8781
16.2	0.4669	0.9347	1.4043	1.8769	2.3528	2.8324
16.5	0.4579	0.9170	1.3777	1.8406	2.3063	2.7766
17	0.4434	0.8867	1.3311	1.7773	2.2255	2.6775
18	0.4149	0.8298	1.2454	1.6623	2.0809	2.5022
19	0.3839	0.7678	1.1520	1.5367	1.9221	2.3084
20	0.3565	0.7128	1.0691	1.4257	1.7828	2.1403

Table D.32: Yaw Amplitude with Linearized Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0185	0.0372	0.0559	0.0748	0.0942	0.1135
6	0.0107	0.0217	0.0327	0.0435	0.0537	0.0632
7	0.0271	0.0549	0.0838	0.1120	0.1382	0.1634
8	0.0260	0.0515	0.0771	0.1021	0.1271	0.1497
9	0.0476	0.0949	0.1426	0.1900	0.2354	0.2793
10	0.1743	0.3479	0.5200	0.6935	0.8638	1.0310
11	0.2692	0.5399	0.8092	1.0749	1.3361	1.5893
12	0.3383	0.6704	0.9868	1.2886	1.5958	1.8992
13	0.3612	0.7152	1.0742	1.4437	1.8059	2.1720
14	0.3790	0.7471	1.1068	1.4694	1.8240	2.1501
14.5	0.3696	0.7350	1.0961	1.4498	1.7798	2.1254
15	0.3481	0.6935	1.0479	1.3979	1.7340	2.0677
15.3	0.3443	0.6825	1.0157	1.3592	1.7107	2.0515
15.4	0.3434	0.6840	1.0157	1.3458	1.6975	2.0447
15.45	0.3444	0.6837	1.0167	1.3441	1.6909	2.0412
15.5	0.3454	0.6857	1.0210	1.3471	1.6893	2.0402
15.55	0.3463	0.6859	1.0210	1.3506	1.6827	2.0370
15.6	0.3473	0.6879	1.0260	1.3570	1.6844	2.0323
15.9	0.3507	0.7011	1.0461	1.3814	1.7181	2.0475
16	0.3505	0.7022	1.0500	1.3814	1.7199	2.0536
16.2	0.3503	0.6987	1.0493	1.3964	1.7354	2.0647
16.5	0.3504	0.6956	1.0363	1.3784	1.7248	2.0646
17	0.3497	0.6965	1.0374	1.3698	1.6939	2.0158
18	0.3342	0.6668	0.9959	1.3194	1.6353	1.9435
19	0.3215	0.6404	0.9557	1.2664	1.5714	1.8759
20	0.3031	0.6042	0.9035	1.2011	1.4969	1.7906

D.2. LINEARIZED ROLL DAMPING

Table D.33: Roll Force Amplitude with Linearized Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	6.0638e+07	1.2118e+08	1.8163e+08	2.4198e+08	3.0223e+08	3.6239e+08
6	8.2472e+07	1.6486e+08	2.4717e+08	3.2939e+08	4.1152e+08	4.9355e+08
7	1.1311e+08	2.2592e+08	3.3842e+08	4.5057e+08	5.6232e+08	6.7361e+08
8	1.1390e+08	2.2780e+08	3.4171e+08	4.5562e+08	5.6958e+08	6.8354e+08
9	1.6478e+08	3.2946e+08	4.9388e+08	6.5789e+08	8.2145e+08	9.8449e+08
10	8.7054e+07	1.7432e+08	2.6194e+08	3.5002e+08	4.3866e+08	5.2802e+08
11	7.9388e+07	1.5911e+08	2.3842e+08	3.1537e+08	3.8798e+08	4.5404e+08
12	1.8153e+08	3.6306e+08	5.4317e+08	7.2135e+08	8.9525e+08	1.0624e+09
13	2.7975e+08	5.6003e+08	8.4021e+08	1.1187e+09	1.3947e+09	1.6657e+09
14	3.4940e+08	6.9990e+08	1.0512e+09	1.4042e+09	1.7592e+09	2.1172e+09
14.5	3.6445e+08	7.2950e+08	1.0953e+09	1.4614e+09	1.8305e+09	2.2043e+09
15	3.7943e+08	7.5866e+08	1.1373e+09	1.5164e+09	1.8963e+09	2.2744e+09
15.3	3.8748e+08	7.7424e+08	1.1595e+09	1.5423e+09	1.9226e+09	2.2985e+09
15.4	3.9020e+08	7.7934e+08	1.1658e+09	1.5490e+09	1.9279e+09	2.3018e+09
15.45	3.9165e+08	7.8215e+08	1.1705e+09	1.5553e+09	1.9351e+09	2.3092e+09
15.5	3.9216e+08	7.8274e+08	1.1705e+09	1.5546e+09	1.9344e+09	2.3087e+09
15.55	3.9399e+08	7.8660e+08	1.1705e+09	1.5623e+09	1.9432e+09	2.3188e+09
15.6	3.9551e+08	7.8955e+08	1.1804e+09	1.5672e+09	1.9490e+09	2.3246e+09
15.9	3.9764e+08	7.9378e+08	1.1868e+09	1.5751e+09	1.9568e+09	2.3303e+09
16	3.9630e+08	7.9124e+08	1.1833e+09	1.5751e+09	1.9536e+09	2.3282e+09
16.2	3.9673e+08	7.9239e+08	1.1854e+09	1.5738e+09	1.9560e+09	2.3304e+09
16.5	3.9549e+08	7.8966e+08	1.1811e+09	1.5696e+09	1.9535e+09	2.3308e+09
17	3.9460e+08	7.8849e+08	1.1809e+09	1.5710e+09	1.9574e+09	2.3391e+09
18	3.9123e+08	7.8211e+08	1.1723e+09	1.5613e+09	1.9487e+09	2.3342e+09
19	3.7283e+08	7.4554e+08	1.1179e+09	1.4899e+09	1.8612e+09	2.2318e+09
20	3.5630e+08	7.1256e+08	1.0687e+09	1.4246e+09	1.7804e+09	2.1359e+09

D.3 Quadratic Roll Damping

D.3.1 45 Degrees

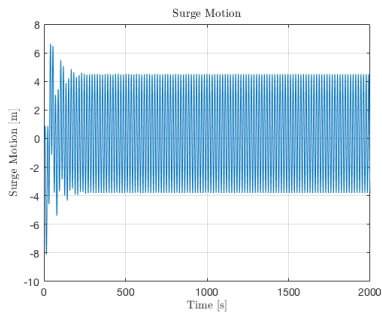


Figure D.31: Surge Motion Wave Period 16 [s] Significant Wave Height 6 [m].

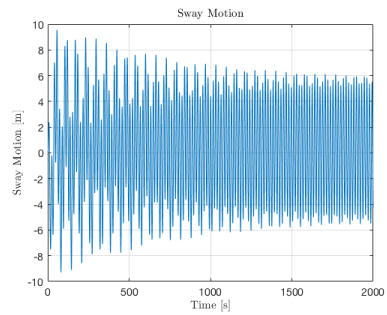


Figure D.32: Sway Motion Wave Period 16 [s] Significant Wave Height 6 [m].

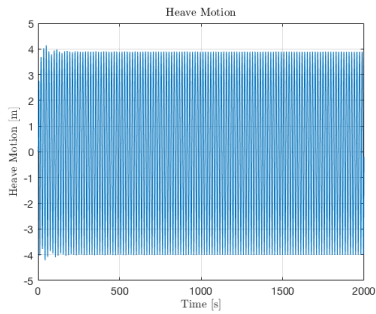


Figure D.33: Heave Motion Wave Period 16 [s] Significant Wave Height 6 [m].

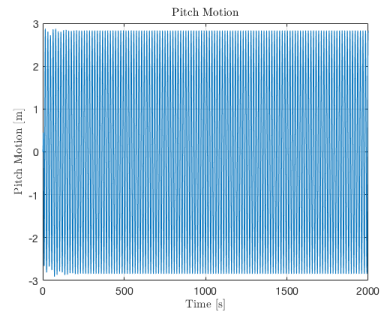


Figure D.34: Pitch Motion Wave Period 16 [s] Significant Wave Height 6 [m].

D.3. QUADRATIC ROLL DAMPING

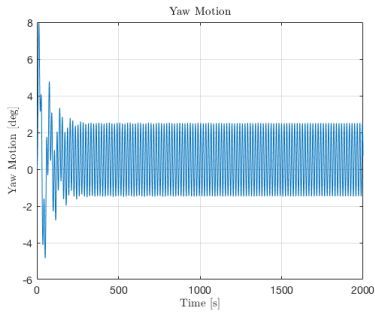


Figure D.35: Yaw Motion Wave Period 16 [s] Significant Wave Height 6 [m].

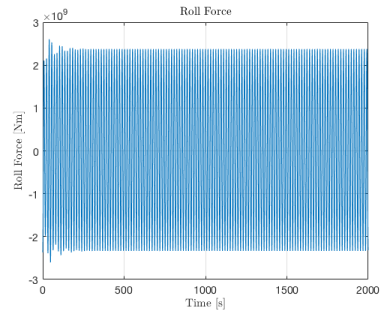


Figure D.36: Roll Force Wave Period 16 [s] Significant Wave Height 6 [m].

Table D.34: Surge Amplitude with Quadratic Damping at 45 Degrees.

Tp \ Hs	1	2	3	4	5	6
5	0.0140	0.0286	0.0436	0.0589	0.0742	0.0901
6	0.0237	0.0479	0.0725	0.0974	0.1222	0.1466
7	0.0520	0.1035	0.1548	0.2042	0.2507	0.2942
8	0.0845	0.1676	0.2496	0.3299	0.4095	0.4905
9	0.0915	0.1834	0.2752	0.3664	0.4566	0.5461
10	0.2065	0.4115	0.6141	0.8133	1.0079	1.1972
11	0.3488	0.6944	1.0354	1.3701	1.6971	2.0146
12	0.4626	0.9218	1.3751	1.8209	2.2579	2.6875
13	0.5527	0.9218	1.6502	2.1905	2.7237	3.2538
14	0.6200	1.2376	1.8509	2.4575	3.0560	3.6458
14.5	0.6486	1.2934	1.9325	2.5651	3.1952	3.8182
15	0.6733	1.3433	2.0081	2.6667	3.3184	3.9635
15.3	0.6869	1.3712	2.0512	2.7253	3.3926	4.0523
15.4	0.6925	1.3825	2.0684	2.7487	3.4224	4.0887
15.45	0.6932	1.3839	2.0706	2.7521	3.4272	4.0950
15.5	0.6951	1.3880	2.0770	2.7610	3.4389	4.1098
15.55	0.6970	1.3915	2.0822	2.7681	3.4480	4.1212
15.6	0.6988	1.3953	2.0882	2.7765	3.4591	4.1354
15.9	0.7083	1.4120	2.1100	2.8019	3.4874	4.1754
16	0.7111	1.4177	2.1186	2.8132	3.5013	4.1929
16.2	0.7203	1.4363	2.1464	2.8499	3.5470	4.2450
16.5	0.7312	1.4579	2.1782	2.8910	3.5959	4.2998
17	0.7437	1.4847	2.2210	2.9511	3.6740	4.3959
18	0.7751	1.5480	2.3177	3.0828	3.8424	4.6007
19	0.7955	1.5893	2.3806	3.1687	3.9531	4.7352
20	0.8141	1.6266	2.4373	3.2457	4.0530	4.8616

Table D.35: Sway Amplitude with Quadratic Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0275	0.0526	0.0802	0.1033	0.1241	0.1435
6	0.0420	0.0842	0.1222	0.1628	0.2033	0.2435
7	0.0388	0.0779	0.1174	0.1570	0.1961	0.2350
8	0.0906	0.1812	0.2708	0.3586	0.4441	0.5268
9	0.1235	0.2457	0.3677	0.4902	0.6134	0.7374
10	0.1631	0.3252	0.4874	0.6502	0.8153	0.9818
11	0.3439	0.6771	1.0029	1.3247	1.6464	1.9714
12	0.5714	1.1313	1.6806	2.2197	2.7502	3.2737
13	0.6693	1.1313	1.9710	2.6078	3.2355	3.8606
14	0.9011	1.7990	2.6886	3.5613	4.4220	5.2608
14.5	1.0978	2.1577	3.1807	4.1531	5.0969	6.0116
15	1.4625	2.7048	3.8404	4.9077	5.9281	6.9028
15.3	1.7156	3.0821	4.3054	5.4375	6.5021	7.5220
15.4	1.7729	3.1655	4.4140	5.5770	6.6740	7.7185
15.45	1.7936	3.1972	4.4503	5.6189	6.7215	7.7721
15.5	1.7998	3.2125	4.4754	5.6464	6.7511	7.8039
15.55	1.8033	3.2133	4.4711	5.6378	6.7443	7.8019
15.6	1.8395	3.2827	4.5647	5.7464	6.8547	7.9050
15.9	1.8712	3.4150	4.7971	6.0740	7.2719	8.4077
16	1.8345	3.3810	4.7668	6.0442	7.2425	8.3792
16.2	1.7833	3.3560	4.7922	6.1189	7.3610	8.5352
16.5	1.7006	3.2812	4.7490	6.1252	7.4187	8.6417
17	1.6157	3.1677	4.6557	6.0748	7.4273	8.7196
18	1.5033	2.9839	4.4350	5.8559	7.2437	8.5965
19	1.3534	2.6934	4.0164	5.3204	6.6042	7.8672
20	1.2267	2.4400	3.6414	4.8315	6.0106	7.1805

D.3. QUADRATIC ROLL DAMPING

Table D.36: Heave Amplitude with Quadratic Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0055	0.0111	0.0166	0.0221	0.0276	0.0331
6	0.0147	0.0295	0.0442	0.0589	0.0736	0.0883
7	0.0333	0.0665	0.0997	0.1329	0.1660	0.1990
8	0.0623	0.1247	0.1870	0.2494	0.3117	0.3740
9	0.1352	0.2702	0.4052	0.5400	0.6748	0.8095
10	0.0959	0.1916	0.2883	0.3864	0.4862	0.5877
11	0.1616	0.3218	0.4798	0.6347	0.7846	0.9280
12	0.2945	0.5896	0.8839	1.1751	1.4629	1.7440
13	0.4094	0.5896	1.2327	1.6453	2.0575	2.4659
14	0.5078	1.0174	1.5311	2.0494	2.5700	3.0910
14.5	0.5536	1.1115	1.6725	2.2359	2.8018	3.3685
15	0.5938	1.1905	1.7889	2.3889	2.9905	3.5929
15.3	0.6158	1.2334	1.8521	2.4710	3.0890	3.7051
15.4	0.6228	1.2478	1.8735	2.4995	3.1249	3.7497
15.45	0.6265	1.2548	1.8839	2.5129	3.1419	3.7694
15.5	0.6292	1.2604	1.8926	2.5254	3.1583	3.7903
15.55	0.6324	1.2658	1.8997	2.5335	3.1665	3.7994
15.6	0.6365	1.2737	1.9108	2.5468	3.1831	3.8199
15.9	0.6572	1.3146	1.9713	2.6260	3.2781	3.9292
16	0.6632	1.3270	1.9893	2.6496	3.3074	3.9626
16.2	0.6773	1.3551	2.0316	2.7059	3.3775	4.0469
16.5	0.6938	1.3893	2.0839	2.7761	3.4657	4.1537
17	0.7237	1.4496	2.1771	2.9059	3.6336	4.3591
18	0.7737	1.5492	2.3258	3.1050	3.8882	4.6725
19	0.8151	1.6319	2.4503	3.2703	4.0914	4.9166
20	0.8459	1.6928	2.5410	3.3908	4.2422	5.0952

Table D.37: Pitch Amplitude with Quadratic Damping at 45 Degrees.

Tp \ Hs	1	2	3	4	5	6
5	0.0160	0.0319	0.0478	0.0637	0.0796	0.0954
6	0.0382	0.0764	0.1146	0.1527	0.1907	0.2287
7	0.0811	0.1620	0.2428	0.3234	0.4040	0.4843
8	0.1616	0.3231	0.4845	0.6458	0.8069	0.9679
9	0.1668	0.3334	0.4997	0.6656	0.8310	0.9959
10	0.3137	0.6270	0.9398	1.2504	1.5576	1.8606
11	0.4319	0.8637	1.2954	1.7255	2.1530	2.5767
12	0.4902	0.9797	1.4687	1.9565	2.4424	2.9272
13	0.5130	0.9797	1.5365	2.0471	2.5567	3.0652
14	0.5121	1.0236	1.5342	2.0436	2.5535	3.0630
14.5	0.5055	1.0110	1.5168	2.0225	2.5279	3.0330
15	0.4959	0.9912	1.4874	1.9859	2.4851	2.9843
15.3	0.4901	0.9804	1.4718	1.9635	2.4562	2.9492
15.4	0.4880	0.9762	1.4643	1.9542	2.4469	2.9406
15.45	0.4866	0.9735	1.4609	1.9485	2.4381	2.9300
15.5	0.4854	0.9722	1.4603	1.9493	2.4386	2.9312
15.55	0.4847	0.9709	1.4586	1.9473	2.4363	2.9278
15.6	0.4834	0.9679	1.4544	1.9420	2.4302	2.9185
15.9	0.4753	0.9520	1.4298	1.9085	2.3874	2.8677
16	0.4712	0.9445	1.4200	1.8979	2.3777	2.8589
16.2	0.4669	0.9346	1.4039	1.8753	2.3485	2.8234
16.5	0.4579	0.9168	1.3770	1.8390	2.3038	2.7710
17	0.4434	0.8867	1.3315	1.7780	2.2265	2.6781
18	0.4149	0.8298	1.2452	1.6617	2.0795	2.4994
19	0.3839	0.7679	1.1520	1.5365	1.9217	2.3075
20	0.3565	0.7128	1.0691	1.4257	1.7828	2.1405

D.3. QUADRATIC ROLL DAMPING

Table D.38: Yaw Amplitude with Quadratic Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0185	0.0371	0.0558	0.0748	0.0942	0.1135
6	0.0107	0.0217	0.0328	0.0436	0.0540	0.0634
7	0.0272	0.0550	0.0839	0.1121	0.1384	0.1636
8	0.0260	0.0515	0.0771	0.1020	0.1269	0.1496
9	0.0476	0.0950	0.1424	0.1896	0.2351	0.2791
10	0.1741	0.3476	0.5198	0.6932	0.8635	1.0308
11	0.2692	0.5398	0.8091	1.0747	1.3358	1.5889
12	0.3384	0.6702	0.9857	1.2873	1.5944	1.8978
13	0.3617	0.6702	1.0752	1.4430	1.8000	2.1590
14	0.3792	0.7473	1.1061	1.4663	1.8168	2.1381
14.5	0.3695	0.7349	1.0966	1.4509	1.7813	2.1232
15	0.3470	0.6992	1.0638	1.4247	1.7717	2.1158
15.3	0.3436	0.6919	1.0374	1.3930	1.7555	2.1047
15.4	0.3428	0.6930	1.0368	1.3799	1.7420	2.0968
15.45	0.3438	0.6922	1.0367	1.3749	1.7338	2.0916
15.5	0.3449	0.6942	1.0411	1.3790	1.7336	2.0935
15.55	0.3458	0.6932	1.0409	1.3780	1.7216	2.0832
15.6	0.3468	0.6942	1.0413	1.3810	1.7181	2.0779
15.9	0.3505	0.7019	1.0493	1.3872	1.7240	2.0488
16	0.3507	0.7017	1.0497	1.3895	1.7221	2.0518
16.2	0.3509	0.6977	1.0465	1.3920	1.7287	2.0530
16.5	0.3515	0.6950	1.0326	1.3718	1.7137	2.0480
17	0.3509	0.6969	1.0358	1.3657	1.6864	2.0057
18	0.3351	0.6677	0.9962	1.3187	1.6336	1.9399
19	0.3220	0.6411	0.9564	1.2670	1.5723	1.8758
20	0.3038	0.6056	0.9050	1.2017	1.4956	1.7863

Table D.39: Roll Force Amplitude with Quadratic Damping at 45 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	6.0638e+07	1.2118e+08	1.8163e+08	2.4198e+08	3.0223e+08	3.6239e+08
6	8.2472e+07	1.6486e+08	2.4717e+08	3.2939e+08	4.1152e+08	4.9355e+08
7	1.1311e+08	2.2592e+08	3.3842e+08	4.5057e+08	5.6232e+08	6.7361e+08
8	1.1390e+08	2.2780e+08	3.4170e+08	4.5561e+08	5.6957e+08	6.8352e+08
9	1.6478e+08	3.2947e+08	4.9388e+08	6.5789e+08	8.2144e+08	9.8448e+08
10	8.7054e+07	1.7432e+08	2.6194e+08	3.5002e+08	4.3867e+08	5.2802e+08
11	7.9387e+07	1.5910e+08	2.3840e+08	3.1532e+08	3.8789e+08	4.5390e+08
12	1.8153e+08	3.6306e+08	5.4315e+08	7.2124e+08	8.9512e+08	1.0623e+09
13	2.7975e+08	3.6306e+08	8.4028e+08	1.1187e+09	1.3942e+09	1.6642e+09
14	3.4941e+08	6.9994e+08	1.0511e+09	1.4033e+09	1.7563e+09	2.1096e+09
14.5	3.6446e+08	7.2946e+08	1.0948e+09	1.4594e+09	1.8258e+09	2.1934e+09
15	3.7943e+08	7.5867e+08	1.1375e+09	1.5171e+09	1.8981e+09	2.2783e+09
15.3	3.8747e+08	7.7455e+08	1.1612e+09	1.5476e+09	1.9343e+09	2.3202e+09
15.4	3.9020e+08	7.7976e+08	1.1681e+09	1.5557e+09	1.9424e+09	2.3293e+09
15.45	3.9164e+08	7.8262e+08	1.1730e+09	1.5626e+09	1.9509e+09	2.3393e+09
15.5	3.9215e+08	7.8321e+08	1.1731e+09	1.5621e+09	1.9510e+09	2.3396e+09
15.55	3.9398e+08	7.8711e+08	1.1790e+09	1.5704e+09	1.9609e+09	2.3519e+09
15.6	3.9550e+08	7.9008e+08	1.1833e+09	1.5756e+09	1.9674e+09	2.3593e+09
15.9	3.9762e+08	7.9416e+08	1.1892e+09	1.5824e+09	1.9736e+09	2.3624e+09
16	3.9628e+08	7.9154e+08	1.1854e+09	1.5780e+09	1.9688e+09	2.3575e+09
16.2	3.9671e+08	7.9254e+08	1.1867e+09	1.5785e+09	1.9674e+09	2.3540e+09
16.5	3.9548e+08	7.8969e+08	1.1818e+09	1.5723e+09	1.9604e+09	2.3455e+09
17	3.9459e+08	7.8847e+08	1.1810e+09	1.5718e+09	1.9599e+09	2.3451e+09
18	3.9122e+08	7.8211e+08	1.1723e+09	1.5614e+09	1.9490e+09	2.3348e+09
19	3.7283e+08	7.4554e+08	1.1179e+09	1.4898e+09	1.8612e+09	2.2317e+09
20	3.5630e+08	7.1256e+08	1.0687e+09	1.4246e+09	1.7803e+09	2.1357e+09

D.3.2 15 Degrees

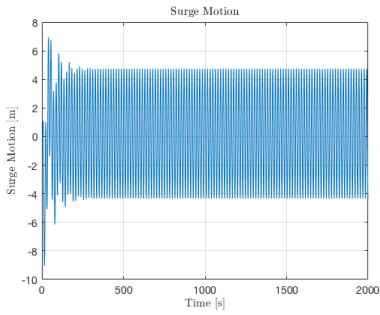


Figure D.37: Surge Motion Wave Period 16 [s] Significant Wave Height 6 [m].

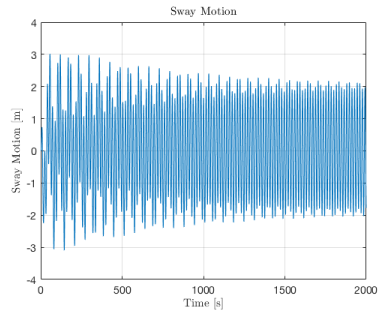


Figure D.38: Sway Motion Wave Period 16 [s] Significant Wave Height 6 [m].

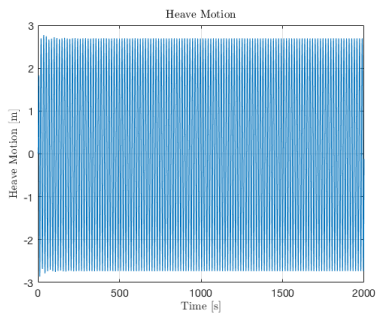


Figure D.39: Heave Motion Wave Period 16 [s] Significant Wave Height 6 [m].

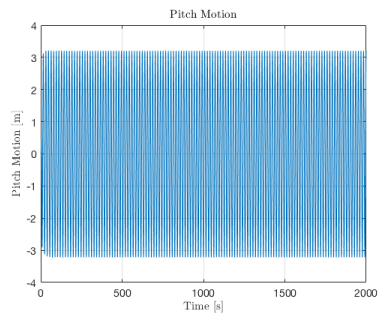


Figure D.40: Pitch Motion Wave Period 16 [s] Significant Wave Height 6 [m].

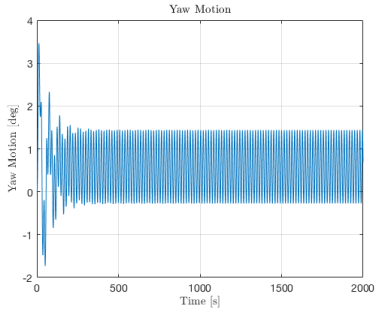


Figure D.41: Yaw Motion Wave Period 16 [s] Significant Wave Height 6 [m].

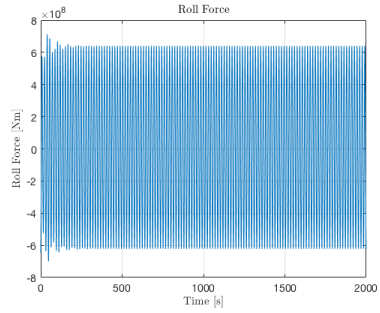


Figure D.42: Roll Force Wave Period 16 [s] Significant Wave Height 6 [m].

Table D.40: Surge Amplitude with Quadratic Damping at 15 Degrees.

Tp \ Hs	1	2	3	4	5	6
5	0.0115	0.0231	0.0347	0.0463	0.0581	0.0699
6	0.0258	0.0517	0.0776	0.0463	0.1298	0.1560
7	0.0511	0.1022	0.1533	0.2044	0.2555	0.3065
8	0.0922	0.1844	0.2767	0.3690	0.4612	0.5535
9	0.1439	0.2878	0.4314	0.5750	0.7184	0.8616
10	0.1630	0.3259	0.4889	0.6519	0.8150	0.9782
11	0.1593	0.3198	0.4818	0.6451	0.8099	0.9759
12	0.2556	0.5121	0.7692	1.0266	1.2837	1.5401
13	0.4004	0.8001	1.1991	1.5969	1.9934	2.3885
14	0.5495	1.0987	1.6481	2.1971	2.7456	3.2932
14.5	0.6121	1.2242	1.8361	2.4476	3.0585	3.6685
15	0.6732	1.3462	2.0186	2.6902	3.3607	4.0299
15.3	0.7054	1.4100	2.1135	2.8154	3.5156	4.2138
15.4	0.7169	1.4328	2.1472	2.8597	3.5701	4.2781
15.45	0.7226	1.4441	2.1641	2.8821	3.5980	4.3114
15.5	0.7273	1.4535	2.1781	2.9008	3.6212	4.3391
15.55	0.7321	1.4629	2.1921	2.9191	3.6438	4.3659
15.6	0.7377	1.4742	2.2089	2.9414	3.6715	4.3988
15.9	0.7650	1.5290	2.2914	3.0518	3.8100	4.5658
16	0.7725	1.5440	2.3139	3.0817	3.8471	4.6100
16.2	0.7901	1.5793	2.3667	3.1520	3.9346	4.7144
16.5	0.8146	1.6283	2.4407	3.2511	4.0593	4.8649
17	0.8493	1.6979	2.5452	3.3908	4.2342	5.0751
18	0.9250	1.8493	2.7728	3.6949	4.6154	5.5340
19	0.9705	1.9405	2.9098	3.8781	4.8452	5.8108
20	1.0156	2.0307	3.0451	4.0588	5.0715	6.0831

D.3. QUADRATIC ROLL DAMPING

Table D.41: Sway Amplitude with Quadratic Damping at 15 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0075	0.0149	0.0223	0.0296	0.0368	0.0441
6	0.0106	0.0211	0.0317	0.0296	0.0528	0.0634
7	0.0160	0.0318	0.0476	0.0634	0.0791	0.0947
8	0.0342	0.0682	0.1022	0.1362	0.1702	0.2041
9	0.0477	0.0954	0.1432	0.1911	0.2391	0.2872
10	0.0580	0.1159	0.1739	0.2317	0.2895	0.3472
11	0.0475	0.0948	0.1420	0.1891	0.2359	0.2823
12	0.1085	0.2159	0.3219	0.4264	0.5292	0.6302
13	0.1533	0.3045	0.4537	0.6009	0.7457	0.8878
14	0.2307	0.4575	0.6811	0.9018	1.1199	1.3349
14.5	0.2821	0.5600	0.8341	1.1031	1.3664	1.6236
15	0.4193	0.8057	1.1701	1.5142	1.8419	2.1572
15.3	0.5275	0.9880	1.4059	1.7962	2.1630	2.5093
15.4	0.5530	1.0345	1.4718	1.8771	2.2572	2.6161
15.45	0.5640	1.0545	1.4995	1.9117	2.2982	2.6634
15.5	0.5675	1.0612	1.5094	1.9249	2.3149	2.6838
15.55	0.5717	1.0701	1.5226	1.9419	2.3354	2.7075
15.6	0.5764	1.0798	1.5360	1.9572	2.3502	2.7195
15.9	0.5724	1.0967	1.5837	2.0415	2.4754	2.8864
16	0.5559	1.0737	1.5614	2.0210	2.4559	2.8689
16.2	0.5279	1.0335	1.5177	1.9802	2.4214	2.8426
16.5	0.5004	0.9887	1.4624	1.9200	2.3608	2.7848
17	0.4877	0.9676	1.4383	1.8987	2.3494	2.7899
18	0.4729	0.9416	1.4048	1.8617	2.3115	2.7539
19	0.4373	0.8716	1.3026	1.7297	2.1532	2.5731
20	0.4087	0.8144	1.2176	1.6191	2.0193	2.4187

Table D.42: Heave Amplitude with Quadratic Damping at 15 Degrees.

Tp \ Hs	1	2	3	4	5	6
5	0.0068	0.0135	0.0203	0.0270	0.0338	0.0406
6	0.0148	0.0296	0.0445	0.0270	0.0741	0.0889
7	0.0309	0.0617	0.0926	0.1234	0.1543	0.1851
8	0.0637	0.1275	0.1913	0.2551	0.3189	0.3827
9	0.0993	0.1986	0.2979	0.3972	0.4965	0.5958
10	0.1599	0.3199	0.4798	0.6398	0.7998	0.9597
11	0.1579	0.3158	0.4737	0.6317	0.7897	0.9479
12	0.1249	0.2497	0.3745	0.4993	0.6243	0.7495
13	0.1551	0.3102	0.4652	0.6200	0.7745	0.9287
14	0.2351	0.4703	0.7053	0.9399	1.1738	1.4070
14.5	0.2991	0.5984	0.8976	1.1966	1.4950	1.7927
15	0.3556	0.7115	1.0672	1.4226	1.7773	2.1313
15.3	0.3849	0.7699	1.1546	1.5387	1.9222	2.3047
15.4	0.3945	0.7889	1.1830	1.5766	1.9695	2.3614
15.45	0.3999	0.7998	1.1995	1.5985	1.9968	2.3942
15.5	0.4037	0.8073	1.2104	1.6129	2.0144	2.4149
15.55	0.4080	0.8161	1.2238	1.6309	2.0373	2.4428
15.6	0.4135	0.8270	1.2402	1.6528	2.0647	2.4757
15.9	0.4444	0.8888	1.3333	1.7773	2.2207	2.6633
16	0.4526	0.9057	1.3593	1.8127	2.2659	2.7185
16.2	0.4752	0.9508	1.4265	1.9021	2.3772	2.8516
16.5	0.5033	1.0072	1.5113	2.0154	2.5195	3.0231
17	0.5483	1.0971	1.6463	2.1958	2.7452	3.2946
18	0.6265	1.2533	1.8807	2.5085	3.1368	3.7655
19	0.6920	1.3845	2.0775	2.7711	3.4653	4.1599
20	0.7435	1.4873	2.2316	2.9764	3.7218	4.4678

D.3. QUADRATIC ROLL DAMPING

Table D.43: Roll Amplitude with Quadratic Damping at 15 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0091	0.0183	0.0273	0.0364	0.0454	0.0543
6	0.0103	0.0206	0.0309	0.0364	0.0512	0.0613
7	0.0126	0.0251	0.0375	0.0498	0.0621	0.0742
8	0.0434	0.0861	0.1284	0.1700	0.2112	0.2520
9	0.0313	0.0625	0.0936	0.1245	0.1553	0.1860
10	0.0726	0.1439	0.2140	0.2830	0.3510	0.4182
11	0.0895	0.1769	0.2622	0.3457	0.4274	0.5074
12	0.1315	0.2587	0.3820	0.5029	0.6214	0.7371
13	0.0754	0.1502	0.2242	0.2973	0.3692	0.4394
14	0.1796	0.3547	0.5248	0.6890	0.8471	0.9994
14.5	0.3017	0.5904	0.8662	1.1269	1.3720	1.6017
15	0.5622	1.0374	1.4511	1.8176	2.1458	2.4415
15.3	0.7018	1.2564	1.7235	2.1297	2.4890	2.8100
15.4	0.7279	1.2984	1.7786	2.1954	2.5636	2.8925
15.45	0.7351	1.3124	1.7984	2.2201	2.5924	2.9243
15.5	0.7358	1.3170	1.8071	2.2318	2.6063	2.9397
15.55	0.7362	1.3208	1.8139	2.2430	2.6219	2.9596
15.6	0.7323	1.3169	1.8136	2.2471	2.6306	2.9733
15.9	0.6408	1.1947	1.6847	2.1204	2.5101	2.8633
16	0.6010	1.1347	1.6170	2.0495	2.4371	2.7846
16.2	0.5255	1.0142	1.4675	1.8831	2.2634	2.6117
16.5	0.4477	0.8747	1.2816	1.6654	2.0238	2.3568
17	0.3614	0.7146	1.0573	1.3874	1.7030	2.0024
18	0.2718	0.5408	0.8060	1.0662	1.3204	1.5673
19	0.2140	0.4268	0.6377	0.8460	1.0513	1.2535
20	0.1805	0.3603	0.5390	0.7161	0.8914	1.0643

Table D.44: Pitch Amplitude with Quadratic Damping at 15 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0169	0.0339	0.0508	0.0678	0.0847	0.1016
6	0.0416	0.0832	0.1248	0.0678	0.2080	0.2496
7	0.0881	0.1762	0.2643	0.3524	0.4405	0.5286
8	0.1512	0.3024	0.4535	0.6047	0.7559	0.9071
9	0.2180	0.4360	0.6539	0.8719	1.0899	1.3078
10	0.2106	0.4213	0.6319	0.8425	1.0531	1.2637
11	0.2299	0.4599	0.6899	0.9199	1.1498	1.3796
12	0.3186	0.6372	0.9558	1.2741	1.5922	1.9098
13	0.4162	0.8324	1.2485	1.6644	2.0798	2.4948
14	0.4917	0.9834	1.4750	1.9665	2.4580	2.9494
14.5	0.5103	1.0205	1.5307	2.0408	2.5508	3.0607
15	0.5252	1.0504	1.5755	2.1006	2.6258	3.1509
15.3	0.5316	1.0631	1.5945	2.1259	2.6573	3.1887
15.4	0.5337	1.0675	1.6013	2.1351	2.6689	3.2028
15.45	0.5346	1.0692	1.6038	2.1385	2.6732	3.2080
15.5	0.5348	1.0697	1.6047	2.1398	2.6750	3.2102
15.55	0.5357	1.0714	1.6072	2.1429	2.6788	3.2146
15.6	0.5365	1.0730	1.6095	2.1460	2.6826	3.2193
15.9	0.5369	1.0737	1.6106	2.1475	2.6845	3.2215
16	0.5354	1.0709	1.6064	2.1420	2.6776	3.2134
16.2	0.5346	1.0693	1.6040	2.1387	2.6736	3.2084
16.5	0.5314	1.0629	1.5944	2.1261	2.6578	3.1896
17	0.5255	1.0509	1.5764	2.1020	2.6277	3.1536
18	0.5099	1.0197	1.5296	2.0396	2.5497	3.0601
19	0.4802	0.9604	1.4406	1.9208	2.4012	2.8817
20	0.4537	0.9073	1.3609	1.8145	2.2682	2.7221

D.3. QUADRATIC ROLL DAMPING

Table D.45: Yaw Amplitude with Quadratic Damping at 15 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0032	0.0063	0.0095	0.0126	0.0157	0.0188
6	0.0044	0.0089	0.0132	0.0126	0.0220	0.0262
7	0.0077	0.0155	0.0235	0.0316	0.0399	0.0481
8	0.0068	0.0136	0.0204	0.0270	0.0336	0.0401
9	0.0126	0.0252	0.0378	0.0505	0.0633	0.0760
10	0.0150	0.0300	0.0450	0.0600	0.0751	0.0901
11	0.0548	0.1101	0.1653	0.2199	0.2734	0.3254
12	0.0968	0.1923	0.2863	0.3803	0.4728	0.5633
13	0.1257	0.2512	0.3763	0.5007	0.6239	0.7452
14	0.1479	0.2953	0.4417	0.5867	0.7297	0.8699
14.5	0.1483	0.2963	0.4432	0.5886	0.7317	0.8716
15	0.1454	0.2907	0.4355	0.5799	0.7234	0.8659
15.3	0.1468	0.2944	0.4415	0.5873	0.7318	0.8738
15.4	0.1484	0.2969	0.4446	0.5909	0.7357	0.8788
15.45	0.1495	0.2990	0.4476	0.5948	0.7402	0.8838
15.5	0.1503	0.3002	0.4490	0.5967	0.7426	0.8865
15.55	0.1511	0.3020	0.4518	0.6000	0.7462	0.8903
15.6	0.1520	0.3037	0.4542	0.6029	0.7496	0.8937
15.9	0.1549	0.3086	0.4605	0.6100	0.7564	0.8986
16	0.1551	0.3088	0.4606	0.6098	0.7553	0.8963
16.2	0.1553	0.3088	0.4605	0.6096	0.7556	0.8977
16.5	0.1558	0.3100	0.4622	0.6119	0.7589	0.9029
17	0.1563	0.3116	0.4655	0.6173	0.7668	0.9135
18	0.1534	0.3062	0.4582	0.6090	0.7585	0.9065
19	0.1486	0.2965	0.4436	0.5898	0.7350	0.8791
20	0.1421	0.2841	0.4260	0.5676	0.7092	0.8507

Table D.46: Roll Force Amplitude with Quadratic Damping at 15 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	3.3095e+07	6.6184e+07	9.9266e+07	1.3234e+08	1.6541e+08	1.9847e+08
6	2.7096e+07	5.4185e+07	8.1269e+07	1.3234e+08	1.3542e+08	1.6248e+08
7	2.7979e+07	5.5958e+07	8.3936e+07	1.1191e+08	1.3989e+08	1.6786e+08
8	4.7830e+07	9.5660e+07	1.4349e+08	1.9132e+08	2.3914e+08	2.8696e+08
9	2.9609e+07	5.9248e+07	8.8922e+07	1.1863e+08	1.4837e+08	1.7815e+08
10	5.9201e+07	1.1839e+08	1.7756e+08	2.3671e+08	2.9582e+08	3.5490e+08
11	4.7544e+07	9.4996e+07	1.4232e+08	1.8950e+08	2.3649e+08	2.8328e+08
12	2.4909e+07	4.9770e+07	7.4573e+07	9.9112e+07	1.2339e+08	1.4756e+08
13	3.2884e+07	6.5703e+07	9.8295e+07	1.3075e+08	1.6273e+08	1.9405e+08
14	6.9464e+07	1.3884e+08	2.0774e+08	2.7582e+08	3.4271e+08	4.0853e+08
14.5	8.1238e+07	1.6230e+08	2.4290e+08	3.2264e+08	4.0105e+08	4.7802e+08
15	9.2457e+07	1.8456e+08	2.7580e+08	3.6578e+08	4.5424e+08	5.4074e+08
15.3	9.8650e+07	1.9676e+08	2.9379e+08	3.8941e+08	4.8316e+08	5.7466e+08
15.4	1.0070e+08	2.0076e+08	2.9965e+08	3.9700e+08	4.9231e+08	5.8520e+08
15.45	1.0188e+08	2.0311e+08	3.0315e+08	4.0155e+08	4.9785e+08	5.9180e+08
15.5	1.0260e+08	2.0447e+08	3.0527e+08	4.0442e+08	5.0157e+08	5.9627e+08
15.55	1.0364e+08	2.0667e+08	3.0852e+08	4.0876e+08	5.0690e+08	6.0252e+08
15.6	1.0483e+08	2.0895e+08	3.1184e+08	4.1302e+08	5.1214e+08	6.0876e+08
15.9	1.0841e+08	2.1611e+08	3.2252e+08	4.2707e+08	5.2926e+08	6.2876e+08
16	1.0911e+08	2.1755e+08	3.2462e+08	4.2979e+08	5.3253e+08	6.3237e+08
16.2	1.1061e+08	2.2055e+08	3.2925e+08	4.3619e+08	5.4102e+08	6.4315e+08
16.5	1.1252e+08	2.2442e+08	3.3519e+08	4.4427e+08	5.5119e+08	6.5538e+08
17	1.1611e+08	2.3175e+08	3.4632e+08	4.5933e+08	5.7025e+08	6.7858e+08
18	1.2190e+08	2.4339e+08	3.6411e+08	4.8368e+08	6.0181e+08	7.1824e+08
19	1.1940e+08	2.3851e+08	3.5702e+08	4.7460e+08	5.9117e+08	7.0638e+08
20	1.1716e+08	2.3409e+08	3.5058e+08	4.6654e+08	5.8172e+08	6.9590e+08

D.3.3 0 Degrees

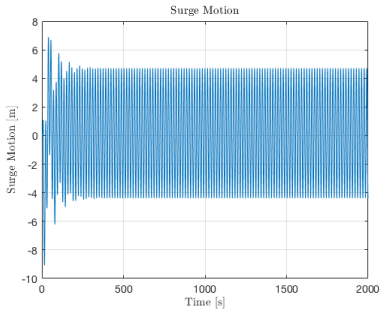


Figure D.43: Surge Motion Wave Period 16 [s] Significant Wave Height 6 [m].

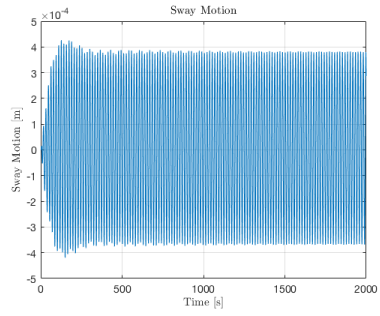


Figure D.44: Sway Motion Wave Period 16 [s] Significant Wave Height 6 [m].

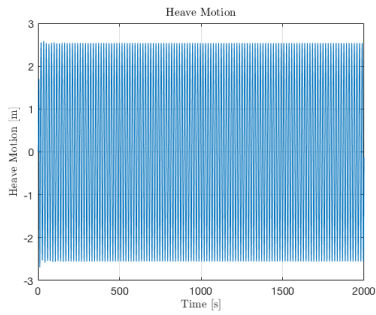


Figure D.45: Heave Motion Wave Period 16 [s] Significant Wave Height 6 [m].

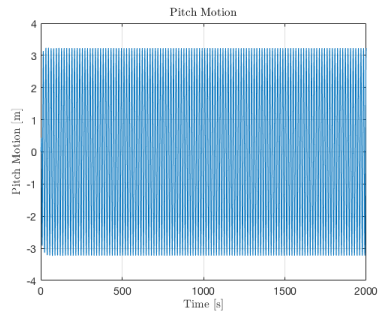


Figure D.46: Pitch Motion Wave Period 16 [s] Significant Wave Height 6 [m].

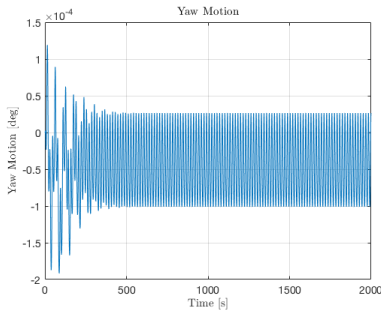


Figure D.47: Yaw Motion Wave Period 16 [s] Significant Wave Height 6 [m].

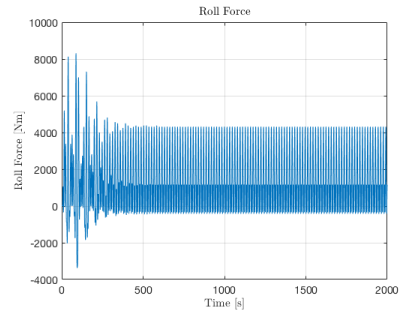


Figure D.48: Roll Force Wave Period 16 [s] Significant Wave Height 6 [m].

Table D.47: Surge Amplitude with Quadratic Damping at 0 Degrees.

Tp \ Hs	1	2	3	4	5	6
5	0.0065	0.0130	0.0195	0.0260	0.0325	0.0390
6	0.0237	0.0474	0.0712	0.0949	0.1186	0.1424
7	0.0533	0.1067	0.1600	0.2133	0.2665	0.3198
8	0.0949	0.1898	0.2848	0.3799	0.4751	0.5704
9	0.1384	0.2767	0.4150	0.5531	0.6911	0.8290
10	0.1751	0.3501	0.5249	0.6997	0.8744	1.0489
11	0.1578	0.3156	0.4734	0.6311	0.7888	0.9465
12	0.2264	0.4528	0.6794	0.9060	1.1327	1.3596
13	0.3711	0.7421	1.1133	1.4844	1.8556	2.2268
14	0.5275	1.0551	1.5827	2.1104	1.8556	3.1661
14.5	0.5950	1.1902	1.7856	2.3811	2.9768	3.5727
15	0.6617	1.3236	1.9856	2.6477	3.3100	3.9724
15.3	0.6970	1.3941	2.0912	2.7884	3.4856	4.1829
15.4	0.7092	1.4184	2.1276	2.8369	3.5461	4.2554
15.45	0.7157	1.4315	2.1472	2.8628	3.5785	4.2941
15.5	0.7208	1.4417	2.1625	2.8833	3.6040	4.3248
15.55	0.7260	1.4520	2.1779	2.9038	3.6296	4.3554
15.6	0.7323	1.4520	2.1966	2.9287	3.6607	4.3926
15.9	0.7621	1.5243	2.2866	3.0489	3.8114	4.5740
16	0.7704	1.5408	2.3114	3.0820	3.8527	4.6234
16.2	0.7894	1.5789	2.3683	3.1578	3.9473	4.7369
16.5	0.8160	1.6320	2.4481	3.2643	4.0805	4.8968
17	0.8545	1.7089	2.5632	3.4175	4.2716	5.1256
18	0.9372	1.8744	2.8116	3.7487	4.6857	5.6227
19	0.9870	1.9740	2.9610	3.9479	4.9348	5.9217
20	1.0366	2.0732	3.1098	4.1464	5.1830	6.2195

D.3. QUADRATIC ROLL DAMPING

Table D.48: Sway Amplitude with Quadratic Damping at 0 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	1.4591e-07	2.9283e-07	4.3961e-07	5.7846e-07	7.3298e-07	8.7663e-07
6	3.3194e-07	6.5974e-07	9.8250e-07	1.3188e-06	1.6312e-06	1.9619e-06
7	6.5832e-07	1.3157e-06	1.9549e-06	2.6093e-06	3.2365e-06	3.8871e-06
8	1.2667e-06	2.5134e-06	3.7629e-06	5.0161e-06	6.2641e-06	7.5110e-06
9	2.1054e-06	4.2447e-06	6.3852e-06	8.5647e-06	1.0832e-05	1.3120e-05
10	4.0275e-06	8.1110e-06	1.2234e-05	1.6386e-05	2.0575e-05	2.4818e-05
11	5.0567e-06	1.0117e-05	1.5173e-05	2.0256e-05	2.5328e-05	3.0409e-05
12	4.9639e-06	9.9271e-06	1.4886e-05	1.9842e-05	2.4795e-05	2.9743e-05
13	7.1374e-06	1.4231e-05	2.1271e-05	2.8254e-05	3.5244e-05	4.2261e-05
14	1.7537e-05	3.5049e-05	5.2772e-05	7.0627e-05	3.5244e-05	1.0657e-04
14.5	3.2752e-05	6.5450e-05	9.8140e-05	1.3075e-04	1.6330e-04	1.9550e-04
15	6.4594e-05	1.2902e-04	1.9286e-04	2.5593e-04	3.1821e-04	3.7923e-04
15.3	8.3444e-05	1.6646e-04	2.4889e-04	3.2996e-04	4.0984e-04	4.8804e-04
15.4	8.6196e-05	1.7210e-04	2.5730e-04	3.4175e-04	4.2475e-04	5.0637e-04
15.45	8.6234e-05	1.7221e-04	2.5785e-04	3.4273e-04	4.2602e-04	5.0879e-04
15.5	8.5418e-05	1.7081e-04	2.5583e-04	3.4009e-04	4.2367e-04	5.0589e-04
15.55	8.4470e-05	1.6904e-04	2.5311e-04	3.3697e-04	4.1992e-04	5.0236e-04
15.6	8.2431e-05	1.6904e-04	2.4710e-04	3.2954e-04	4.1123e-04	4.9265e-04
15.9	6.8914e-05	1.3796e-04	2.0748e-04	2.7760e-04	3.4828e-04	4.1947e-04
16	6.3790e-05	1.2783e-04	1.9213e-04	2.5708e-04	3.2286e-04	3.8915e-04
16.2	5.4470e-05	1.0950e-04	1.6494e-04	2.2111e-04	2.7779e-04	3.3519e-04
16.5	4.6184e-05	9.2863e-05	1.4016e-04	1.8835e-04	2.3740e-04	2.8712e-04
17	3.6919e-05	7.3988e-05	1.1166e-04	1.5040e-04	1.8988e-04	2.3041e-04
18	2.6811e-05	5.4373e-05	8.2835e-05	1.1233e-04	1.4300e-04	1.7412e-04
19	2.1525e-05	4.4080e-05	6.7612e-05	9.2074e-05	1.1785e-04	1.4461e-04
20	1.8316e-05	3.7776e-05	5.8556e-05	8.1036e-05	1.0515e-04	1.3480e-04

Table D.49: Heave Amplitude with Quadratic Damping at 0 Degrees.

Tp \ Hs	1	2	3	4	5	6
5	0.0070	0.0140	0.0210	0.0280	0.0350	0.0420
6	0.0153	0.0305	0.0458	0.0611	0.0763	0.0916
7	0.0309	0.0617	0.0926	0.1234	0.1543	0.1852
8	0.0632	0.1265	0.1897	0.2531	0.3164	0.3798
9	0.0996	0.1992	0.2989	0.3985	0.4981	0.5978
10	0.1550	0.3101	0.4652	0.6204	0.7756	0.9309
11	0.1677	0.3353	0.5030	0.6706	0.8383	1.0060
12	0.1327	0.2654	0.3982	0.5309	0.6637	0.7964
13	0.1379	0.2758	0.4138	0.5519	0.6901	0.8284
14	0.2027	0.4054	0.6082	0.8110	0.6901	1.2167
14.5	0.2683	0.5366	0.8049	1.0733	1.3418	1.6102
15	0.3250	0.6501	0.9752	1.3003	1.6255	1.9507
15.3	0.3545	0.7090	1.0635	1.4182	1.7729	2.1277
15.4	0.3641	0.7282	1.0923	1.4566	1.8209	2.1854
15.45	0.3695	0.7390	1.1086	1.4783	1.8480	2.2180
15.5	0.3731	0.7460	1.1187	1.4913	1.8638	2.2369
15.55	0.3776	0.7552	1.1329	1.5108	1.8888	2.2670
15.6	0.3830	0.7552	1.1491	1.5323	1.9157	2.2992
15.9	0.4149	0.8298	1.2448	1.6601	2.0755	2.4912
16	0.4234	0.8473	1.2718	1.6968	2.1224	2.5486
16.2	0.4471	0.8943	1.3416	1.7891	2.2370	2.6852
16.5	0.4765	0.9533	1.4303	1.9077	2.3854	2.8635
17	0.5234	1.0469	1.5707	2.0948	2.6193	3.1443
18	0.6050	1.2101	1.8155	2.4214	3.0278	3.6349
19	0.6739	1.3479	2.0223	2.6971	3.3724	4.0484
20	0.7283	1.4566	2.1852	2.9143	3.6438	4.3737

D.3. QUADRATIC ROLL DAMPING

Table D.50: Roll Amplitude with Quadratic Damping at 0 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	3.5561e-07	7.1125e-07	1.0659e-06	1.4149e-06	1.7928e-06	2.1446e-06
6	7.8578e-07	1.5755e-06	2.3553e-06	3.1421e-06	3.9261e-06	4.7217e-06
7	1.5990e-06	3.1951e-06	4.7896e-06	6.3855e-06	7.9780e-06	9.5552e-06
8	3.2865e-06	6.5621e-06	9.8363e-06	1.3084e-05	1.6343e-05	1.9588e-05
9	5.4614e-06	1.0930e-05	1.6410e-05	2.1892e-05	2.7376e-05	3.2869e-05
10	9.7124e-06	1.9414e-05	2.9110e-05	3.8789e-05	4.8461e-05	5.8099e-05
11	1.2245e-05	2.4474e-05	3.6698e-05	4.8929e-05	6.1136e-05	7.3358e-05
12	1.1934e-05	2.3878e-05	3.5840e-05	4.7814e-05	5.9795e-05	7.1783e-05
131	1.6526e-05	3.3018e-05	4.9467e-05	6.5868e-05	8.2202e-05	9.8480e-05
14	3.9853e-05	7.9621e-05	1.1926e-04	1.5867e-04	8.2202e-05	2.3663e-04
14.5	7.3856e-05	1.4748e-04	2.2066e-04	2.9318e-04	3.6489e-04	4.3550e-04
15	1.4568e-04	2.9046e-04	4.3347e-04	5.7394e-04	7.1107e-04	8.4424e-04
15.3	1.8699e-04	3.7267e-04	5.5578e-04	7.3517e-04	9.0983e-04	1.0788e-03
15.4	1.9292e-04	3.8463e-04	5.7398e-04	7.5992e-04	9.4153e-04	1.1181e-03
15.45	1.9286e-04	3.8466e-04	5.7433e-04	7.6100e-04	9.4379e-04	1.1221e-03
15.5	1.9103e-04	3.8108e-04	5.6924e-04	7.5470e-04	9.3677e-04	1.1148e-03
15.55	1.8850e-04	3.7617e-04	5.6221e-04	7.4592e-04	9.2679e-04	1.1043e-03
15.6	1.8388e-04	3.7617e-04	5.4900e-04	7.2899e-04	9.0658e-04	1.0814e-03
15.9	1.5373e-04	3.0728e-04	4.6059e-04	6.1342e-04	7.6571e-04	9.1804e-04
16	1.4250e-04	2.8498e-04	4.2742e-04	5.6988e-04	7.1226e-04	8.5478e-04
16.2	1.2021e-04	2.4050e-04	3.6090e-04	4.8151e-04	6.0246e-04	7.2381e-04
16.5	1.0131e-04	2.0270e-04	3.0425e-04	4.0606e-04	5.0822e-04	6.1085e-04
17	8.0818e-05	1.6164e-04	2.4262e-04	3.2370e-04	4.0500e-04	4.8663e-04
18	5.7345e-05	1.1471e-04	1.7218e-04	2.2966e-04	2.8731e-04	3.4512e-04
19	4.5164e-05	9.0356e-05	1.3559e-04	1.8088e-04	2.2633e-04	2.7197e-04
20	3.7238e-05	7.4418e-05	1.1157e-04	1.4882e-04	1.8623e-04	2.2370e-04

Table D.51: Pitch Amplitude with Quadratic Damping at 0 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0178	0.0356	0.0534	0.0711	0.0889	0.1067
6	0.0426	0.0852	0.1278	0.1704	0.2130	0.2556
7	0.0883	0.1767	0.2650	0.3533	0.4416	0.5300
8	0.1555	0.3110	0.4665	0.6220	0.7776	0.9331
9	0.2133	0.4265	0.6398	0.8530	1.0663	1.2796
10	0.2192	0.4383	0.6575	0.8767	1.0958	1.3150
11	0.2162	0.4324	0.6486	0.8648	1.0811	1.2973
12	0.2931	0.5862	0.8793	1.1725	1.4656	1.7587
13	0.3936	0.7873	1.1809	1.5746	1.9682	2.3619
14	0.4795	0.9590	1.4385	1.9180	1.9682	2.8770
14.5	0.5026	1.0052	1.5079	2.0105	2.5131	3.0158
15	0.5216	1.0431	1.5647	2.0862	2.6078	3.1294
15.3	0.5301	1.0601	1.5902	2.1202	2.6503	3.1804
15.4	0.5329	1.0659	1.5988	2.1317	2.6647	3.1976
15.45	0.5342	1.0684	1.6026	2.1368	2.6710	3.2053
15.5	0.5346	1.0693	1.6039	2.1386	2.6732	3.2078
15.55	0.5359	1.0719	1.6078	2.1437	2.6797	3.2156
15.6	0.5372	1.0719	1.6117	2.1489	2.6861	3.2233
15.9	0.5391	1.0782	1.6173	2.1564	2.6955	3.2346
16	0.5382	1.0765	1.6147	2.1530	2.6912	3.2295
16.2	0.5380	1.0761	1.6141	2.1521	2.6901	3.2282
16.5	0.5360	1.0720	1.6080	2.1440	2.6800	3.2160
17	0.5319	1.0637	1.5956	2.1275	2.6594	3.1913
18	0.5191	1.0383	1.5574	2.0766	2.5957	3.1149
19	0.4904	0.9808	1.4712	1.9616	2.4521	2.9425
20	0.4646	0.9292	1.3937	1.8583	2.3229	2.7875

D.3. QUADRATIC ROLL DAMPING

Table D.52: Yaw Amplitude with Quadratic Damping at 0 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	2.7842e-07	5.5742e-07	8.3750e-07	1.1102e-06	1.3875e-06	1.6739e-06
6	1.1757e-06	2.3511e-06	3.5221e-06	4.7013e-06	5.8768e-06	7.0658e-06
7	2.2010e-06	4.3985e-06	6.5843e-06	8.7711e-06	1.0958e-05	1.3136e-05
8	3.2304e-06	6.4650e-06	9.7027e-06	1.2948e-05	1.6192e-05	1.9446e-05
9	3.7574e-06	7.5070e-06	1.1260e-05	1.4990e-05	1.8728e-05	2.2449e-05
10	5.6422e-06	1.1287e-05	1.6919e-05	2.2557e-05	2.8195e-05	3.3843e-05
11	3.8135e-06	7.6391e-06	1.1441e-05	1.5217e-05	1.9103e-05	2.2961e-05
12	4.6830e-06	9.3901e-06	1.4111e-05	1.8803e-05	2.3394e-05	2.7928e-05
13	8.0429e-06	1.6084e-05	2.4099e-05	3.2141e-05	4.0050e-05	4.8225e-05
14	1.1252e-05	2.2502e-05	3.3877e-05	4.5231e-05	4.0050e-05	6.8166e-05
14.5	1.1552e-05	2.3188e-05	3.4878e-05	4.6675e-05	5.8527e-05	7.0551e-05
15	1.1875e-05	2.3914e-05	3.6198e-05	4.8760e-05	6.1832e-05	7.5304e-05
15.3	1.1251e-05	2.2682e-05	3.4402e-05	4.6730e-05	5.9725e-05	7.3319e-05
15.4	1.1269e-05	2.2677e-05	3.4260e-05	4.6424e-05	5.8998e-05	7.2557e-05
15.45	1.1306e-05	2.2665e-05	3.4259e-05	4.6231e-05	5.8673e-05	7.1871e-05
15.5	1.1319e-05	2.2662e-05	3.4179e-05	4.6029e-05	5.8162e-05	7.1127e-05
15.55	1.1357e-05	2.2790e-05	3.4362e-05	4.6115e-05	5.8338e-05	7.0770e-05
15.6	1.1480e-05	2.2790e-05	3.4465e-05	4.6217e-05	5.8090e-05	7.0286e-05
15.9	1.2043e-05	2.4116e-05	3.6393e-05	4.8722e-05	6.1202e-05	7.3812e-05
16	1.2341e-05	2.4702e-05	3.7132e-05	4.9563e-05	6.2153e-05	7.4623e-05
16.2	1.2848e-05	2.5671e-05	3.8440e-05	5.1441e-05	6.4542e-05	7.7923e-05
16.5	1.3605e-05	2.7284e-05	4.1088e-05	5.5126e-05	6.9443e-05	8.4004e-05
17	1.4748e-05	2.9498e-05	4.4234e-05	5.9018e-05	7.3827e-05	8.8632e-05
18	1.6500e-05	3.3036e-05	4.9647e-05	6.6283e-05	8.3113e-05	9.9949e-05
19	1.6650e-05	3.3341e-05	5.0194e-05	6.7205e-05	8.4380e-05	1.0174e-04
20	1.6749e-05	3.3584e-05	5.0969e-05	6.8843e-05	8.7397e-05	1.0679e-04

Table D.53: Roll Force Amplitude with Quadratic Damping at 0 Degrees.

Tp \ Hs	1	2	3	4	5	6
5	0.5487	1.7061	4.8828	8.7891	19.5312	17.5781
6	2.4414	8.0091	14.4127	27.3438	42.4417	62.5000
7	2.9297	13.6719	28.0327	49.3654	70.3125	108.1524
8	7.9372	33.9656	74.2188	134.3323	199.4394	307.4612
9	5.8594	21.1539	44.6293	81.5257	128.9974	185.2346
10	21.4844	60.4355	160.5355	239.8583	422.9556	608.1035
11	9.7656	41.1935	89.8438	151.6080	243.5576	359.1967
12	5.2429	24.6969	56.8331	103.9112	166.3460	244.8192
13	14.1985	57.1671	116.3544	231.9154	314.1177	529.5302
14	42.8894	135.7463	332.0312	561.5226	314.1177	1233.7565
14.5	52.7344	160.8071	421.8750	968.7500	1570.3125	1960.9375
15	54.6875	253.9062	789.0625	1406.2500	1984.3750	3687.5000
15.3	70.3125	296.8750	796.8750	1703.1250	3125.0000	5000.0000
15.4	76.1719	296.8750	804.6875	1765.6250	2812.5000	4906.2500
15.45	70.3125	359.3750	859.3750	1796.8750	3171.8750	5125.0000
15.5	66.4062	328.1250	875.0000	1781.2500	3375.0000	4781.2500
15.55	78.1250	335.9375	882.8125	1656.2500	3187.5000	4843.7500
15.6	82.0312	335.9375	859.3750	1500.0000	3375.0000	5031.2500
15.9	76.1719	242.1875	703.1250	1312.5000	2328.1250	4062.5000
16	57.4509	304.6875	640.6250	1718.7500	2171.8750	3765.6250
16.2	69.6379	359.3750	590.9045	1593.7500	2078.1250	3203.1250
16.5	83.9844	293.7549	681.9946	1414.0625	1803.3586	2655.7753
17	81.0547	322.2656	714.8438	1250.0000	1879.3399	3125.0000
18	73.9567	429.6875	844.9259	1489.3593	2391.1011	4015.6250
19	77.8670	343.5370	793.1882	1399.2089	2018.7919	2972.7161
20	77.3566	355.4688	886.3996	1282.8510	2170.5238	3816.4648

D.3.4 90 Degrees

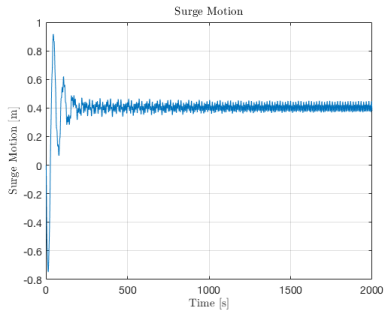


Figure D.49: Surge Motion Wave Period 16 [s] Significant Wave Height 6 [m].

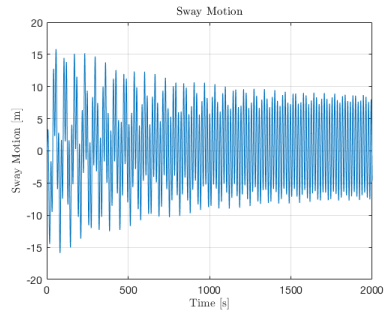


Figure D.50: Sway Motion Wave Period 16 [s] Significant Wave Height 6 [m].

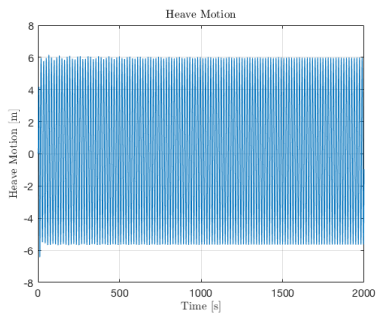


Figure D.51: Heave Motion Wave Period 16 [s] Significant Wave Height 6 [m].

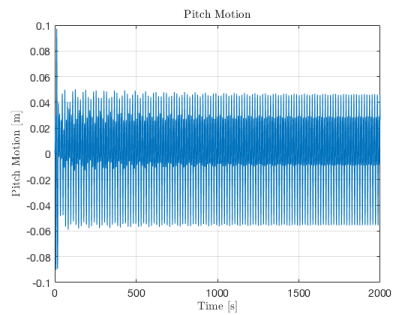


Figure D.52: Pitch Motion Wave Period 16 [s] Significant Wave Height 6 [m].

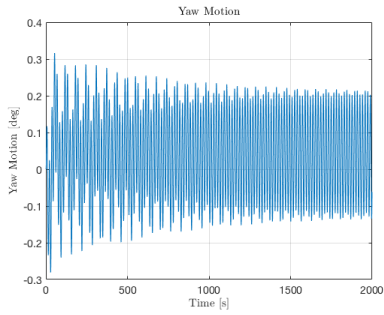


Figure D.53: Yaw Motion Wave Period 16 [s] Significant Wave Height 6 [m].

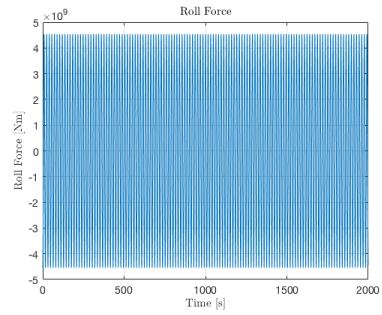


Figure D.54: Roll Force Wave Period 16 [s] Significant Wave Height 6 [m].

Table D.54: Surge Amplitude with Quadratic Damping at 90 Degrees.

$T_p \backslash H_s$	1	2	3	4	5	6
5	0.0072	0.0149	0.0221	0.0273	0.0310	0.0375
6	0.0198	0.0408	0.0598	0.0793	0.0938	0.1126
7	0.0219	0.0459	0.0669	0.0822	0.0895	0.0855
8	0.0229	0.0487	0.0707	0.0845	0.1068	0.1448
9	0.0224	0.0457	0.0667	0.0842	0.1068	0.1414
10	0.0177	0.0359	0.0530	0.0672	0.0800	0.1009
11	0.0118	0.0237	0.0348	0.0387	0.0463	0.0556
12	0.0064	0.0126	0.0182	0.0233	0.0279	0.0326
13	0.0031	0.0068	0.0107	0.0150	0.0232	0.0290
14	0.0031	0.0080	0.0140	0.0207	0.0283	0.0367
14.5	0.0033	0.0114	0.0190	0.0269	0.0347	0.0478
15	0.0060	0.0165	0.0267	0.0373	0.0478	0.0576
15.3	0.0075	0.0165	0.0260	0.0355	0.0446	0.0527
15.4	0.0073	0.0160	0.0250	0.0343	0.0439	0.0553
15.45	0.0071	0.0159	0.0253	0.0350	0.0451	0.0568
15.5	0.0072	0.0160	0.0254	0.0351	0.0460	0.0579
15.55	0.0072	0.0161	0.0256	0.0356	0.0470	0.0589
15.6	0.0072	0.0183	0.0290	0.0370	0.0527	0.0611
15.9	0.0073	0.0174	0.0286	0.0404	0.0526	0.0651
16	0.0073	0.0174	0.0287	0.0406	0.0533	0.0673
16.2	0.0069	0.0166	0.0281	0.0410	0.0547	0.0689
16.5	0.0050	0.0161	0.0278	0.0400	0.0531	0.0692
17	0.0049	0.0135	0.0234	0.0376	0.0529	0.0701
18	0.0036	0.0104	0.0187	0.0291	0.0432	0.0606
19	0.0038	0.0087	0.0154	0.0249	0.0369	0.0516
20	0.0038	0.0083	0.0139	0.0215	0.0311	0.0427

D.3. QUADRATIC ROLL DAMPING

Table D.55: Sway Amplitude with Quadratic Damping at 90 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.4567	0.9297	1.3855	1.7953	2.1601	2.5292
6	0.5585	1.1115	1.6394	2.1888	2.7633	3.3398
7	0.9287	1.8401	2.7498	3.6642	4.5686	5.4417
8	1.7057	3.3750	5.0494	6.7433	8.4334	10.1378
9	1.9140	3.7898	5.6704	7.5829	9.4959	11.3927
10	2.0387	4.0347	6.0137	7.9979	9.9971	12.0198
11	1.8509	3.6462	5.4052	7.1460	8.8891	10.6503
12	1.6875	3.3273	4.9238	6.4855	8.0299	9.5720
13	1.1375	2.3222	3.9105	5.2154	7.0236	9.0617
14	1.6618	3.3703	5.1326	6.9066	8.6784	10.4399
14.5	1.9734	3.8461	5.6750	7.4819	9.2633	11.0216
15	2.4156	4.4053	6.2802	8.1047	9.8641	11.6239
15.3	2.7775	4.9471	6.9617	8.8862	10.7516	12.5770
15.4	2.8260	5.0069	7.0336	8.9673	10.8283	12.6448
15.45	2.8336	5.0187	7.0280	8.9474	10.7977	12.6145
15.5	2.8750	5.0445	7.0337	8.9193	10.7520	12.5579
15.55	2.9258	5.1550	7.1755	9.0919	10.9370	12.7394
15.6	2.9845	5.2633	7.3386	9.3059	11.1937	13.0358
15.9	3.0513	5.4515	7.6195	9.6587	11.6226	13.5298
16	3.0463	5.4777	7.6690	9.7261	11.6930	13.5929
16.2	3.0156	5.5263	7.7901	9.9106	11.9350	13.8892
16.5	2.9588	5.5820	7.9789	10.2249	12.3657	14.4297
17	2.7523	5.3484	7.8030	10.1310	12.3556	14.4965
18	2.5070	4.9848	7.4200	9.8053	12.1378	14.4278
19	2.2031	4.3964	6.5734	8.7306	10.8655	12.9761
20	1.9199	3.8343	5.7418	7.6415	9.5356	11.4218

Table D.56: Heave Amplitude with Quadratic Damping at 90 Degrees.

Tp \ Hs	1	2	3	4	5	6
5	0.0299	0.0598	0.0897	0.1195	0.1491	0.1790
6	0.0795	0.1588	0.2372	0.3146	0.3907	0.4660
7	0.1932	0.3857	0.5765	0.7651	0.9510	1.1361
8	0.4044	0.8093	1.2146	1.6196	2.0186	2.4126
9	0.6916	1.3833	2.0755	2.7672	3.4574	4.1455
10	0.8985	1.7985	2.7015	3.6080	4.5179	5.4293
11	0.9653	1.9350	2.9095	3.8893	4.8745	5.8651
12	0.9738	1.9551	2.9441	3.9408	4.9452	5.9572
13	0.9608	1.9307	2.9128	3.9053	4.9070	5.9169
14	0.9660	1.9485	2.9436	3.9483	4.9608	5.9797
14.5	0.9634	1.9435	2.9351	3.9353	4.9425	5.9557
15	0.9605	1.9369	2.9245	3.9203	4.9229	5.9314
15.3	0.9629	1.9374	2.9213	3.9130	4.9108	5.9138
15.4	0.9630	1.9386	2.9226	3.9132	4.9097	5.9114
15.45	0.9625	1.9368	2.9189	3.9075	4.9016	5.9020
15.5	0.9598	1.9346	2.9187	3.9101	4.9077	5.9109
15.55	0.9621	1.9365	2.9188	3.9076	4.9019	5.9033
15.6	0.9609	1.9350	2.9184	3.9094	4.9082	5.9128
15.9	0.9585	1.9287	2.9090	3.8980	4.8934	5.8944
16	0.9552	1.9231	2.9000	3.8839	4.8757	5.8742
16.2	0.9572	1.9217	2.8970	3.8800	4.8713	5.8681
16.5	0.9586	1.9212	2.8855	3.8578	4.8409	5.8301
17	0.9599	1.9223	2.8862	3.8551	4.8280	5.8026
18	0.9636	1.9313	2.9020	3.8754	4.8519	5.8284
19	0.9717	1.9467	2.9246	3.9051	4.8879	5.8728
20	0.9738	1.9496	2.9277	3.9086	4.8951	5.8852

D.3. QUADRATIC ROLL DAMPING

Table D.57: Roll Amplitude with Quadratic Damping at 90 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0526	0.1056	0.1554	0.2025	0.2478	0.2914
6	0.0397	0.0766	0.1100	0.1483	0.1921	0.2351
7	0.0849	0.1683	0.2501	0.3303	0.4082	0.4829
8	0.2020	0.4024	0.6027	0.8053	1.0084	1.2171
9	0.3331	0.6546	0.9671	1.2735	1.5754	1.8700
10	0.4396	0.8785	1.3210	1.7681	2.2126	2.6546
11	0.6219	1.2358	1.8505	2.4693	3.0853	3.6984
12	0.9014	1.7821	2.6496	3.4997	4.3384	5.1602
13	1.3388	2.6215	3.8443	4.9962	6.0791	7.1315
14	2.1961	4.0654	5.6464	7.0855	8.3648	9.5173
14.5	2.8427	4.9165	6.5875	8.0372	9.3091	10.4522
15	3.5084	5.6286	7.2807	8.6869	9.9515	11.0949
15.3	3.7146	5.8564	7.5356	8.9626	10.2238	11.3651
15.4	3.7250	5.8614	7.5356	8.9577	10.2143	11.3503
15.45	3.7293	5.8724	7.5493	8.9774	10.2386	11.3785
15.5	3.7240	5.8744	7.5656	9.0007	10.2673	11.4122
15.55	3.7167	5.8737	7.5697	9.0108	10.2820	11.4302
15.6	3.6963	5.8558	7.5659	9.0167	10.2956	11.4502
15.9	3.4562	5.6514	7.3969	8.8658	10.1513	11.3146
16	3.3462	5.5534	7.3005	8.7972	10.1028	11.2697
16.2	3.0839	5.2852	7.0563	8.5669	9.8836	11.0739
16.5	2.6934	4.8326	6.6015	8.1330	9.4765	10.7042
17	2.1501	4.0584	5.7662	7.2684	8.6672	9.9211
18	1.4631	2.8906	4.2613	5.5641	6.8274	8.0276
19	1.0969	2.1778	3.2492	4.3092	5.3490	6.3640
20	0.8782	1.7529	2.6216	3.4830	4.3429	5.1960

Table D.58: Pitch Amplitude with Quadratic Damping at 90 Degrees.

Tp \ Hs	1	2	3	4	5	6
5	0.0136	0.0272	0.0408	0.0543	0.0677	0.0811
6	0.0263	0.0527	0.0792	0.1057	0.1324	0.1591
7	0.0421	0.0846	0.1276	0.1702	0.2110	0.2482
8	0.0582	0.1160	0.1711	0.2215	0.2655	0.3041
9	0.0622	0.1240	0.1831	0.2377	0.2858	0.3301
10	0.0492	0.0985	0.1469	0.1934	0.2378	0.2787
11	0.0326	0.0656	0.0987	0.1315	0.1634	0.1951
12	0.0217	0.0439	0.0665	0.0894	0.1127	0.1363
13	0.0140	0.0286	0.0436	0.0591	0.0750	0.0913
14	0.0100	0.0203	0.0311	0.0421	0.0534	0.0651
14.5	0.0083	0.0172	0.0263	0.0356	0.0451	0.0546
15	0.0071	0.0148	0.0227	0.0322	0.0409	0.0498
15.3	0.0066	0.0139	0.0223	0.0307	0.0392	0.0486
15.4	0.0064	0.0139	0.0219	0.0303	0.0392	0.0494
15.45	0.0063	0.0137	0.0218	0.0303	0.0391	0.0499
15.5	0.0062	0.0135	0.0216	0.0301	0.0396	0.0505
15.55	0.0061	0.0134	0.0214	0.0299	0.0398	0.0506
15.6	0.0059	0.0132	0.0213	0.0302	0.0404	0.0515
15.9	0.0059	0.0135	0.0223	0.0320	0.0426	0.0539
16	0.0059	0.0136	0.0225	0.0324	0.0431	0.0546
16.2	0.0057	0.0137	0.0228	0.0330	0.0440	0.0558
16.5	0.0054	0.0133	0.0225	0.0329	0.0441	0.0561
17	0.0046	0.0116	0.0204	0.0303	0.0414	0.0538
18	0.0028	0.0074	0.0137	0.0214	0.0305	0.0411
19	0.0018	0.0046	0.0086	0.0138	0.0201	0.0275
20	0.0013	0.0032	0.0058	0.0090	0.0131	0.0180

D.3. QUADRATIC ROLL DAMPING

Table D.59: Yaw Amplitude with Quadratic Damping at 90 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	0.0244	0.0468	0.0703	0.0901	0.1080	0.1276
6	0.0712	0.1384	0.2036	0.2657	0.3269	0.3921
7	0.0754	0.1504	0.2219	0.2933	0.3565	0.3997
8	0.0630	0.1250	0.1859	0.2338	0.2679	0.2891
9	0.0632	0.1261	0.1828	0.2294	0.2745	0.2848
10	0.0574	0.1149	0.1694	0.2229	0.2670	0.2982
11	0.0561	0.1123	0.1684	0.2251	0.2807	0.3260
12	0.0574	0.1144	0.1712	0.2271	0.2795	0.3254
13	0.0588	0.1163	0.1720	0.2251	0.2746	0.3218
14	0.0677	0.1294	0.1862	0.2388	0.2870	0.3306
14.5	0.0727	0.1317	0.1839	0.2316	0.2754	0.3154
15	0.0765	0.1317	0.1802	0.2243	0.2647	0.3012
15.3	0.0728	0.1237	0.1683	0.2091	0.2478	0.2841
15.4	0.0718	0.1223	0.1665	0.2073	0.2453	0.2807
15.45	0.0710	0.1210	0.1651	0.2055	0.2430	0.2774
15.5	0.0703	0.1199	0.1638	0.2039	0.2407	0.2748
15.55	0.0693	0.1188	0.1621	0.2015	0.2383	0.2720
15.6	0.0680	0.1166	0.1598	0.1986	0.2340	0.2672
15.9	0.0589	0.1048	0.1459	0.1842	0.2199	0.2533
16	0.0560	0.1008	0.1412	0.1784	0.2130	0.2448
16.2	0.0490	0.0910	0.1294	0.1654	0.1988	0.2303
16.5	0.0408	0.0783	0.1141	0.1480	0.1811	0.2140
17	0.0317	0.0637	0.0956	0.1269	0.1572	0.1864
18	0.0220	0.0453	0.0695	0.0946	0.1201	0.1459
19	0.0149	0.0303	0.0467	0.0639	0.0875	0.1081
20	0.0109	0.0220	0.0337	0.0461	0.0593	0.0734

Table D.60: Roll Force Amplitude with Quadratic Damping at 90 Degrees.

Tp\ Hs	1	2	3	4	5	6
5	2.8006e+08	5.5972e+08	8.3882e+08	1.1168e+09	1.3941e+09	1.6708e+09
6	4.3963e+08	8.7771e+08	1.3104e+09	1.7370e+09	2.1559e+09	2.5659e+09
7	5.8500e+08	1.1677e+09	1.7517e+09	2.3285e+09	2.8992e+09	3.4619e+09
8	7.0788e+08	1.4150e+09	2.1191e+09	2.8167e+09	3.5070e+09	4.1888e+09
9	8.0354e+08	1.6060e+09	2.4061e+09	3.2023e+09	3.9942e+09	4.7814e+09
10	8.6214e+08	1.7236e+09	2.5842e+09	3.4424e+09	4.2989e+09	5.1529e+09
11	8.9345e+08	1.7865e+09	2.6790e+09	3.5707e+09	4.4616e+09	5.3512e+09
12	8.9665e+08	1.7929e+09	2.6889e+09	3.5845e+09	4.4797e+09	5.3744e+09
13	8.7872e+08	1.7571e+09	2.6353e+09	3.5131e+09	4.3906e+09	5.2677e+09
14	8.4918e+08	1.6979e+09	2.5462e+09	3.3942e+09	4.2418e+09	5.0891e+09
14.5	8.2449e+08	1.6487e+09	2.4726e+09	3.2964e+09	4.1199e+09	4.9433e+09
15	8.0281e+08	1.6054e+09	2.4077e+09	3.2098e+09	4.0118e+09	4.8135e+09
15.3	7.9042e+08	1.5807e+09	2.3709e+09	3.1610e+09	3.9510e+09	4.7408e+09
15.4	7.8627e+08	1.5725e+09	2.3586e+09	3.1446e+09	3.9304e+09	4.7162e+09
15.45	7.8392e+08	1.5678e+09	2.3515e+09	3.1352e+09	3.9187e+09	4.7021e+09
15.5	7.8141e+08	1.5627e+09	2.3439e+09	3.1249e+09	3.9059e+09	4.6867e+09
15.55	7.8034e+08	1.5606e+09	2.3407e+09	3.1208e+09	3.9007e+09	4.6805e+09
15.6	7.7797e+08	1.5558e+09	2.3335e+09	3.1112e+09	3.8888e+09	4.6663e+09
15.9	7.6271e+08	1.5254e+09	2.2879e+09	3.0504e+09	3.8127e+09	4.5750e+09
16	7.5740e+08	1.5148e+09	2.2722e+09	3.0295e+09	3.7868e+09	4.5439e+09
16.2	7.4646e+08	1.4929e+09	2.2394e+09	2.9858e+09	3.7321e+09	4.4782e+09
16.5	7.3000e+08	1.4600e+09	2.1900e+09	2.9200e+09	3.6501e+09	4.3802e+09
17	7.0628e+08	1.4126e+09	2.1188e+09	2.8250e+09	3.5312e+09	4.2375e+09
18	6.5973e+08	1.3195e+09	1.9792e+09	2.6389e+09	3.2986e+09	3.9583e+09
19	6.1151e+08	1.2230e+09	1.8345e+09	2.4460e+09	3.0575e+09	3.6690e+09
20	5.6831e+08	1.1366e+09	1.7049e+09	2.2732e+09	2.8415e+09	3.4098e+09

Appendix E: Standard Deviation for Wave Seeds in 90 Degrees

Table E.1: Standard Deviation for Different Wave Seeds in 90 deg.

Condition number	Wave Seed 1	Wave Seed 2	Wave Seed 3	Wave Seed 4	Wave Seed 5	Wave Seed 6	Standard Deviation
1	0.03754	0.04120	0.03651	0.04221	0.03272	0.03395	0.0038
2	0.07379	0.07727	0.06847	0.07636	0.06275	0.06631	0.0059
3	0.14117	0.14931	0.13210	0.14733	0.12595	0.13299	0.0093
4	0.25396	0.27392	0.25232	0.26350	0.22807	0.24858	0.0154
5	0.25396	0.27392	0.25232	0.26350	0.22807	0.24858	0.0351
6	0.58007	0.56993	0.57950	0.57672	0.56315	0.57065	0.0066
7	0.06142	0.06786	0.05983	0.06932	0.05375	0.05567	0.0063
8	0.12278	0.12865	0.11398	0.12711	0.10448	0.11038	0.0098
9	0.23352	0.24664	0.21850	0.24309	0.20817	0.21946	0.0152
10	0.41477	0.44567	0.40948	0.42737	0.37170	0.40328	0.0248
11	0.69203	0.69795	0.60300	0.64814	0.56740	0.60170	0.0531
12	0.90625	0.89141	0.90601	0.90121	0.88311	0.89500	0.0091
13	0.07859	0.08988	0.07689	0.08998	0.07036	0.07177	0.0086
14	0.16704	0.17599	0.15495	0.17433	0.14351	0.15022	0.0134
15	0.32466	0.34245	0.30372	0.33715	0.28916	0.30442	0.0211
16	0.57002	0.61044	0.55968	0.58406	0.50984	0.55111	0.0338
17	0.92754	0.93693	0.81879	0.87385	0.76499	0.81123	0.0688
18	1.21669	1.19870	1.13082	1.17648	0.98632	1.05957	0.0894
19	0.09433	0.11076	0.09248	0.10863	0.08601	0.08651	0.0108
20	0.20386	0.21697	0.18867	0.21596	0.17889	0.18297	0.0167
21	0.40077	0.42392	0.37537	0.41751	0.35652	0.37441	0.0268
22	0.71702	0.76583	0.70049	0.73156	0.63974	0.68964	0.0425
23	1.14854	1.16133	1.02378	1.08676	0.95156	1.00887	0.0830
24	1.49659	1.47632	1.39155	1.44324	1.22229	1.30564	0.1067
25	0.11510	0.13517	0.11297	0.13253	0.10492	0.10559	0.0131
26	0.23733	0.25466	0.21909	0.25433	0.21261	0.21213	0.0199
27	0.46617	0.49572	0.43819	0.48885	0.41487	0.43389	0.0324
28	0.83689	0.89379	0.81322	0.85303	0.74224	0.80000	0.0515
29	1.34980	1.36685	1.21251	1.28321	1.12204	1.19016	0.0959
30	1.75919	1.73731	1.63656	1.69333	1.44507	1.53672	0.1225
31	0.13578	0.15947	0.13342	0.15636	0.12382	0.12459	0.0155
32	0.28026	0.30072	0.25893	0.30024	0.25113	0.25061	0.0234
33	0.52603	0.56247	0.49692	0.55554	0.46912	0.48814	0.0378
34	0.93957	1.00656	0.91030	0.96013	0.82837	0.89365	0.0610
35	1.51303	1.53889	1.36885	1.45152	1.25989	1.34107	0.1076
36	2.00203	1.98039	1.86332	1.92595	1.65152	1.75094	0.1374

Appendix F: Additional Results, RAO in Time- and Frequency Domain

F.1 Linear Comparison

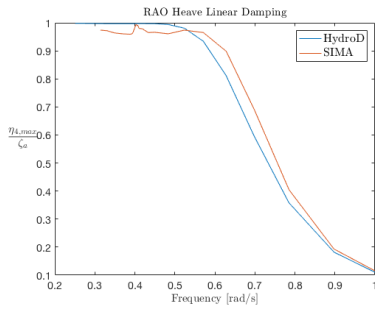


Figure F.1: RAO Heave Linear Damping.

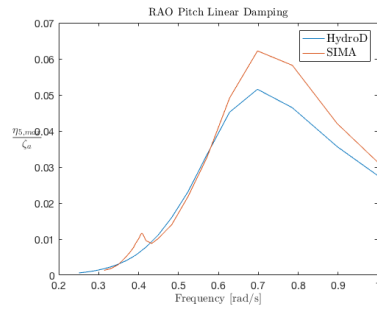


Figure F.2: RAO Pitch Linear Damping.

F.2 Linearized Comparison

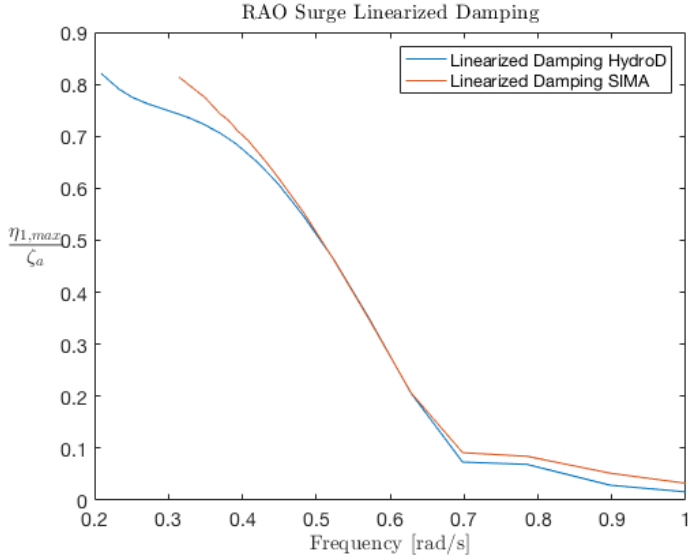


Figure F.3: Comparison of RAO in Roll between Time- and Frequency Domain Solution.

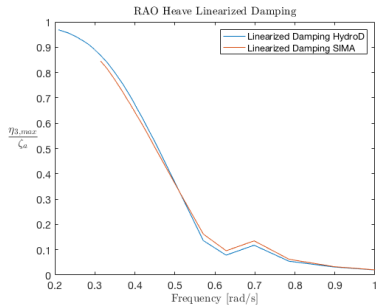


Figure F.4: RAO Heave Linearized Damping.

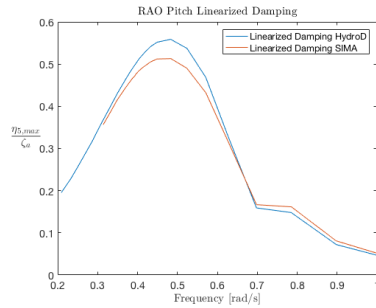


Figure F.5: RAO Pitch Linearized Damping.

Appendix G: Additional Results, Response Spectra for Irregular Waves

G.1 45 Degrees

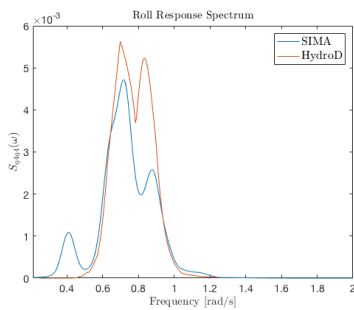


Figure G.1: Response Spectra Condition Number 7. 45 Degrees.

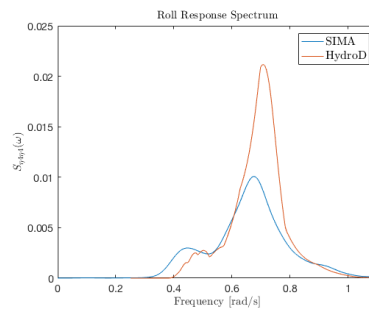


Figure G.2: Response Spectra Condition Number 8. 45 Degrees.

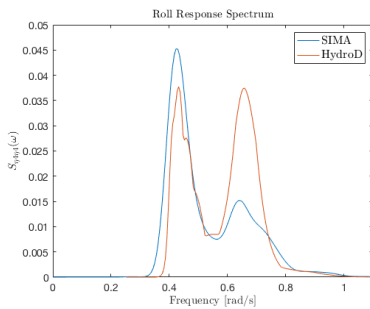


Figure G.3: Response Spectra Condition Number 9. 45 Degrees.

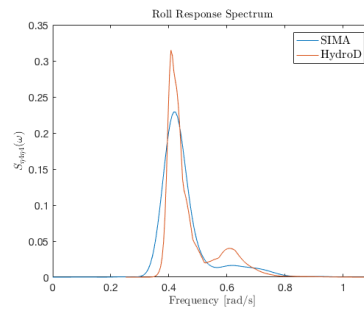


Figure G.4: Response Spectra Condition Number 10. 45 Degrees.

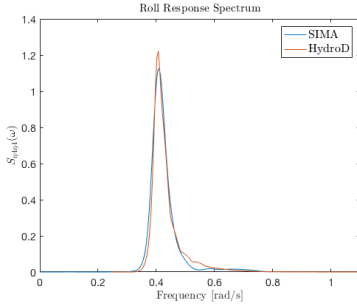


Figure G.5: Response Spectra Condition Number 11. 45 Degrees.

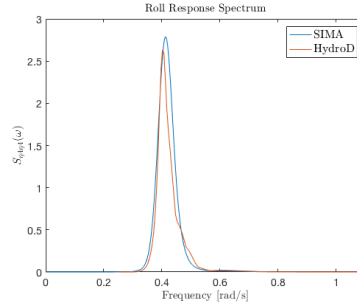


Figure G.6: Response Spectra Condition Number 12. 45 Degrees.

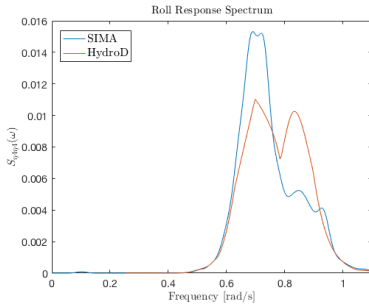


Figure G.7: Response Spectra Condition Number 13. 45 Degrees.

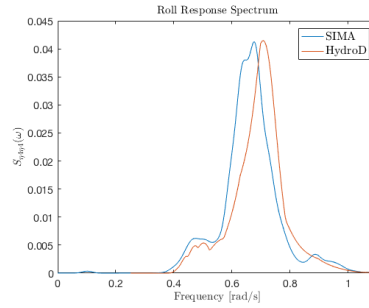


Figure G.8: Response Spectra Condition Number 14. 45 Degrees.

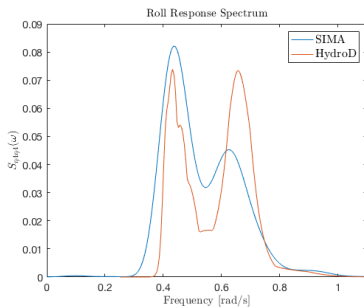


Figure G.9: Response Spectra Condition Number 15. 45 Degrees.

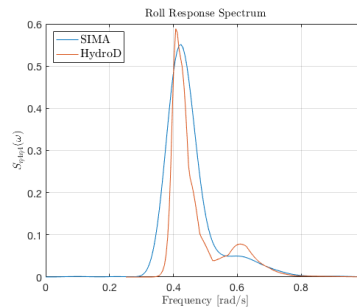


Figure G.10: Response Spectra Condition Number 16. 45 Degrees.

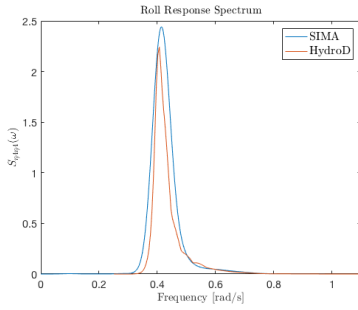


Figure G.11: Response Spectra Condition Number 17. 45 Degrees.

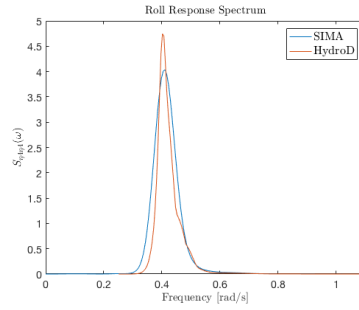


Figure G.12: Response Spectra Condition Number 18. 45 Degrees.

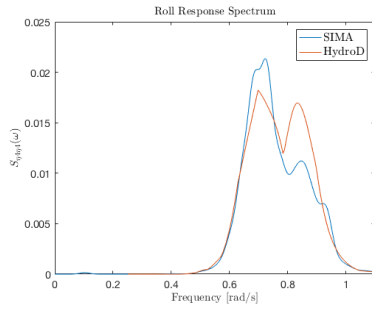


Figure G.13: Response Spectra Condition Number 19. 45 Degrees.

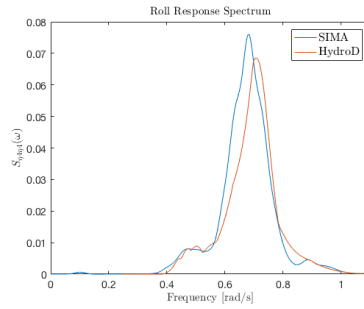


Figure G.14: Response Spectra Condition Number 20. 45 Degrees.

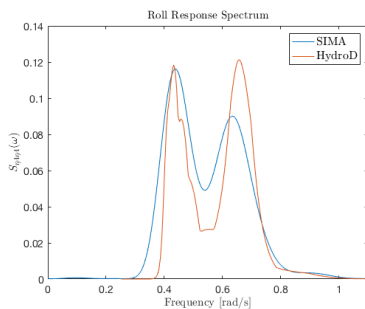


Figure G.15: Response Spectra Condition Number 21. 45 Degrees.

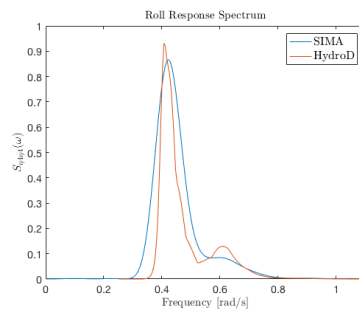


Figure G.16: Response Spectra Condition Number 22. 45 Degrees.

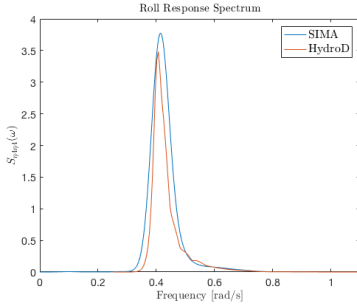


Figure G.17: Response Spectra Condition Number 23. 45 Degrees.

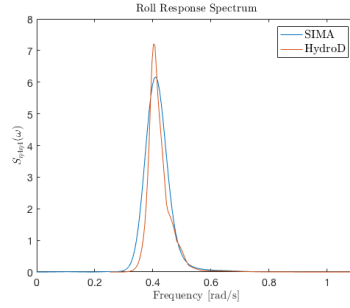


Figure G.18: Response Spectra Condition Number 24. 45 Degrees.

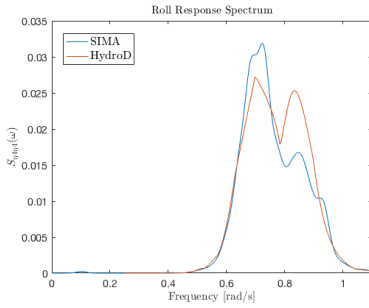


Figure G.19: Response Spectra Condition Number 25. 45 Degrees.

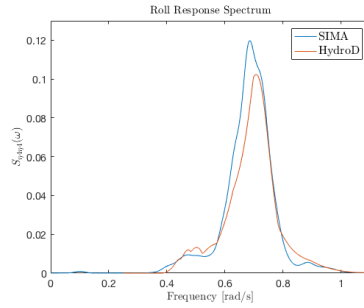


Figure G.20: Response Spectra Condition Number 26. 45 Degrees.

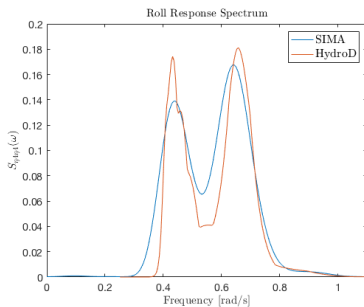


Figure G.21: Response Spectra Condition Number 27. 45 Degrees.

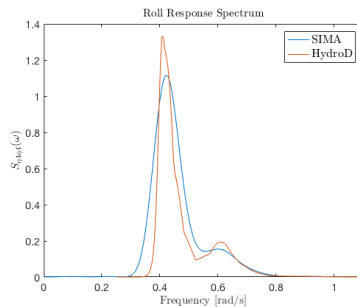


Figure G.22: Response Spectra Condition Number 28. 45 Degrees.

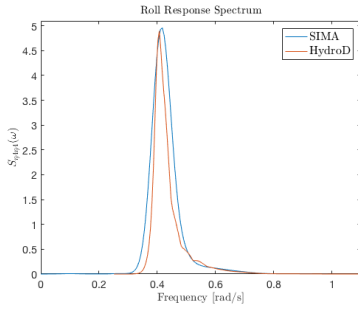


Figure G.23: Response Spectra Condition Number 29. 45 Degrees.

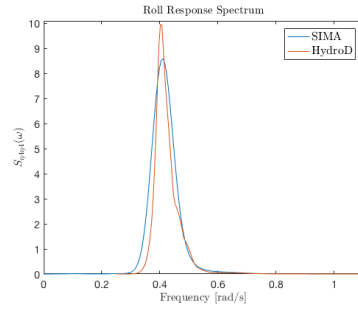


Figure G.24: Response Spectra Condition Number 30. 45 Degrees.

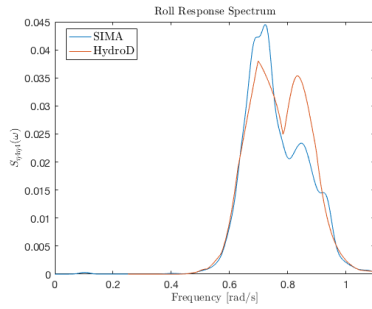


Figure G.25: Response Spectra Condition Number 31. 45 Degrees.

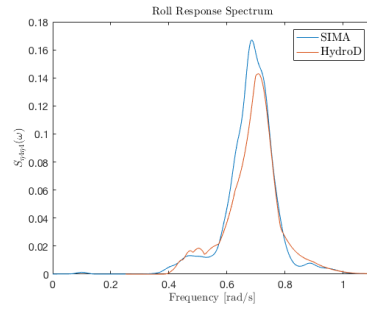


Figure G.26: Response Spectra Condition Number 32. 45 Degrees.

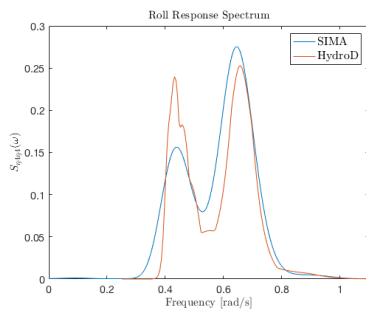


Figure G.27: Response Spectra Condition Number 33. 45 Degrees.

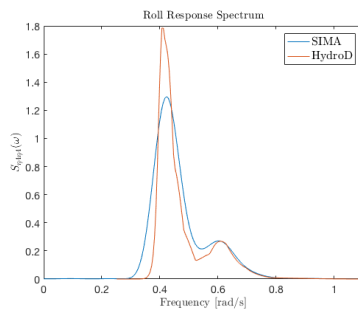


Figure G.28: Response Spectra Condition Number 34. 45 Degrees.

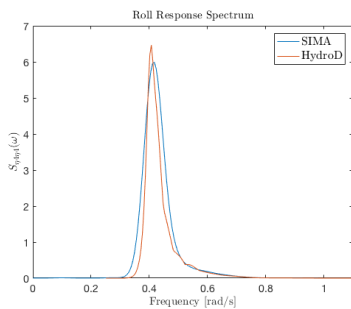


Figure G.29: Response Spectra Condition Number 35. 45 Degrees.

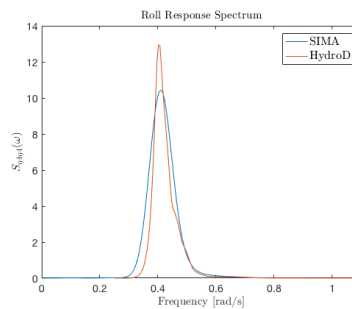


Figure G.30: Response Spectra Condition Number 36. 45 Degrees.

G.2 90 Degrees

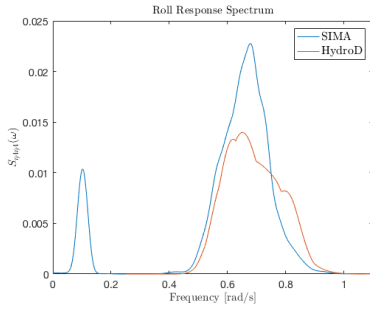


Figure G.31: Response Spectra Condition Number 7. 90 Degrees.

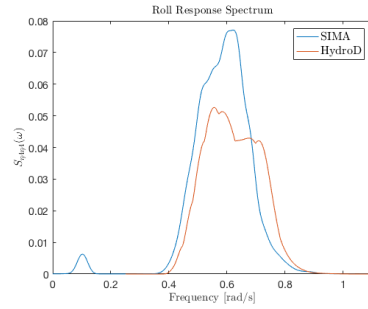


Figure G.32: Response Spectra Condition Number 8. 90 Degrees.

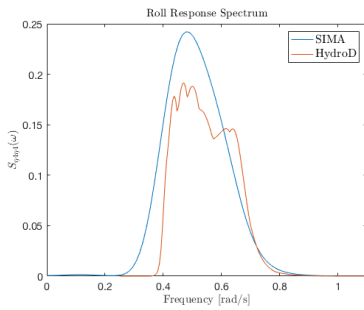


Figure G.33: Response Spectra Condition Number 9. 90 Degrees.

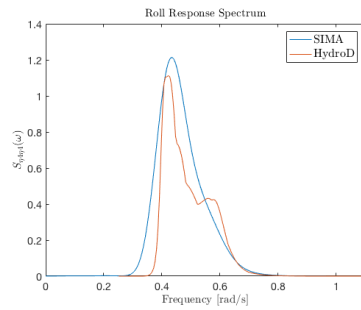


Figure G.34: Response Spectra Condition Number 10. 90 Degrees

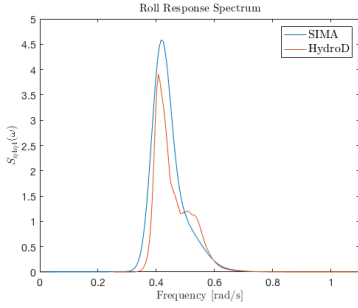


Figure G.35: Response Spectra Condition Number 11. 90 Degrees

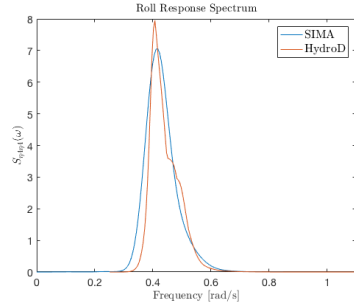


Figure G.36: Response Spectra Condition Number 12. 90 Degrees

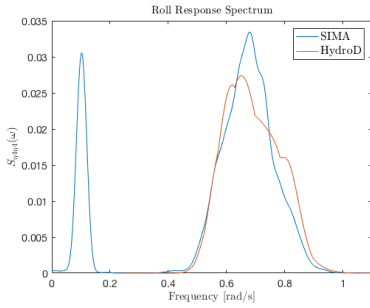


Figure G.37: Response Spectra Condition Number 13. 90 Degrees

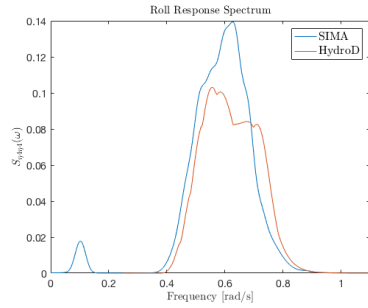


Figure G.38: Response Spectra Condition Number 14. 90 Degrees

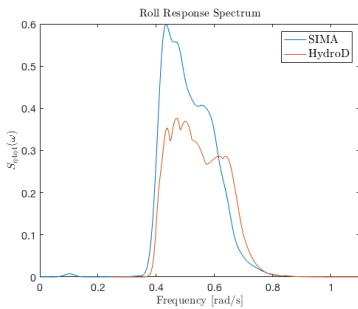


Figure G.39: Response Spectra Condition Number 15. 90 Degrees

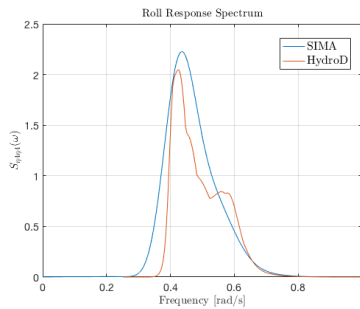


Figure G.40: Response Spectra Condition Number 16. 90 Degrees

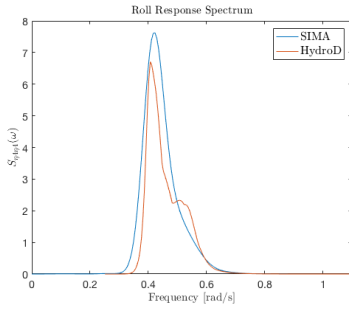


Figure G.41: Response Spectra Condition Number 17. 90 Degrees

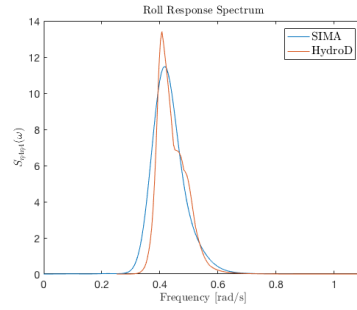


Figure G.42: Response Spectra Condition Number 18. 90 Degrees

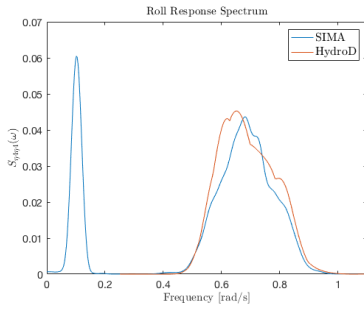


Figure G.43: Response Spectra Condition Number 19. 90 Degrees

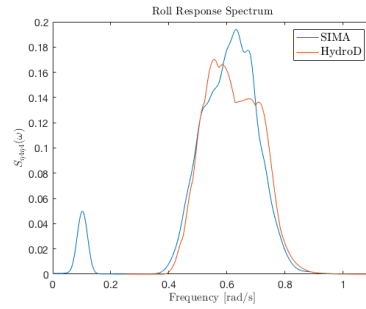


Figure G.44: Response Spectra Condition Number 20. 90 Degrees

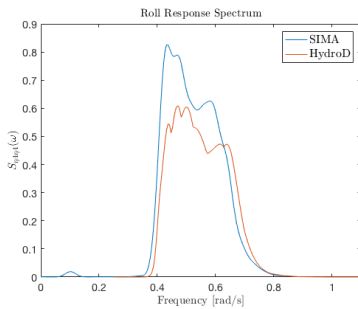


Figure G.45: Response Spectra Condition Number 21. 90 Degrees

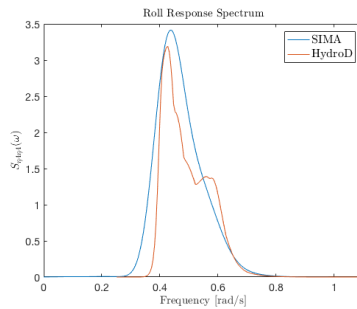


Figure G.46: Response Spectra Condition Number 22. 90 Degrees

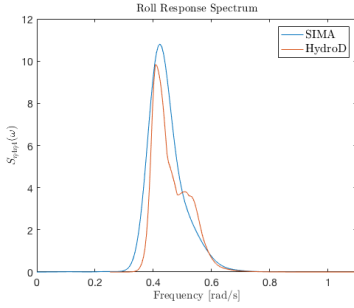


Figure G.47: Response Spectra Condition Number 23. 90 Degrees

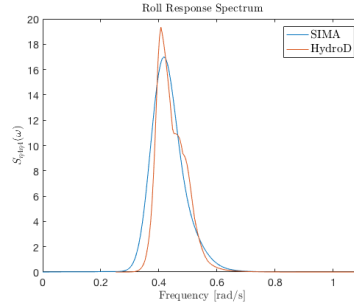


Figure G.48: Response Spectra Condition Number 24. 90 Degrees

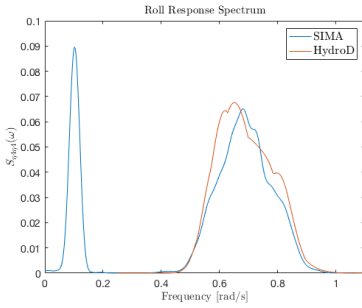


Figure G.49: Response Spectra Condition Number 25. 90 Degrees

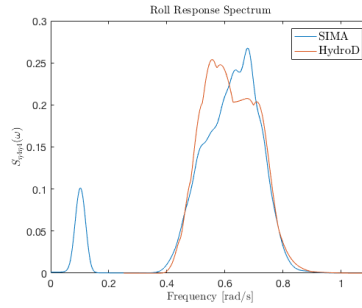


Figure G.50: Response Spectra Condition Number 26. 90 Degrees

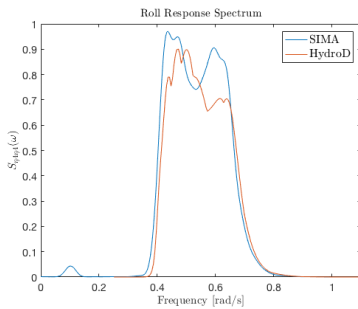


Figure G.51: Response Spectra Condition Number 27. 90 Degrees

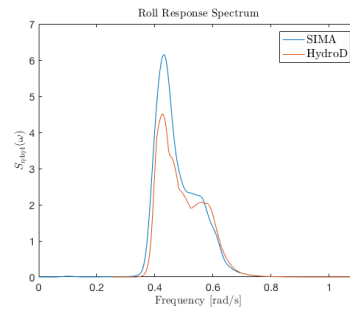


Figure G.52: Response Spectra Condition Number 28. 90 Degrees

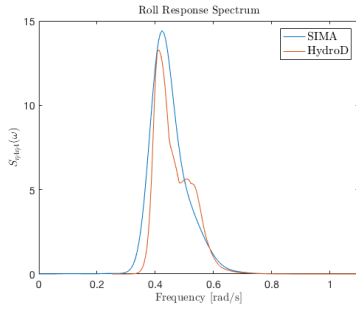


Figure G.53: Response Spectra Condition Number 29. 90 Degrees

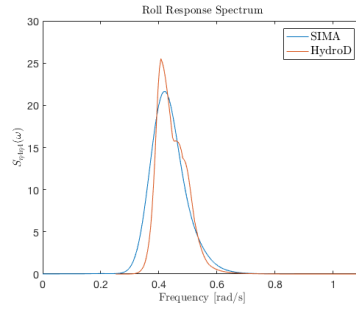


Figure G.54: Response Spectra Condition Number 30. 90 Degrees

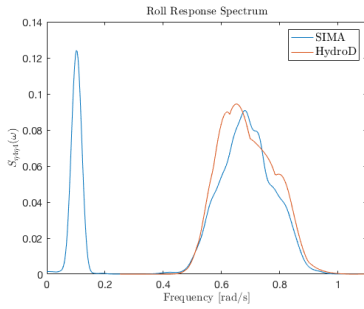


Figure G.55: Response Spectra Condition Number 31. 90 Degrees

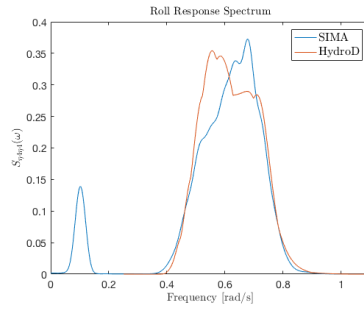


Figure G.56: Response Spectra Condition Number 32. 90 Degrees

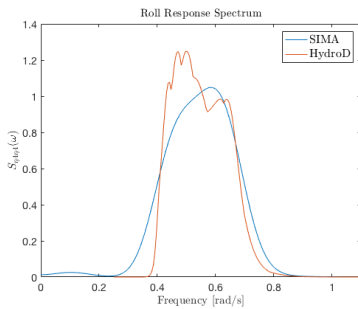


Figure G.57: Response Spectra Condition Number 33. 90 Degrees

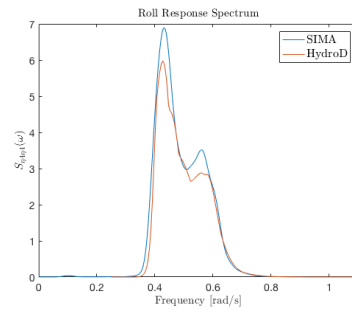


Figure G.58: Response Spectra Condition Number 34. 90 Degrees

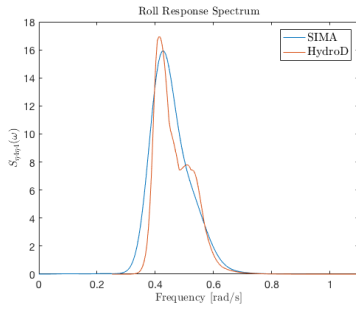


Figure G.59: Response Spectra Condition Number 35. 90 Degrees

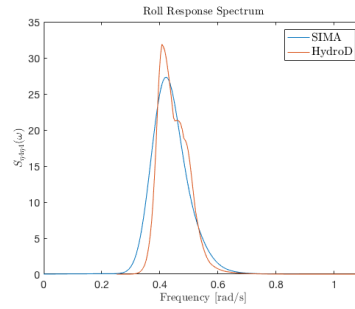


Figure G.60: Response Spectra Condition Number 36. 90 Degrees