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Alternate representation of butterfly robot

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1 Preface

I would like to thank my supervisor, Prof Anton Shiraev and PhD Candidate Christian Fredrik Sætre for guidance when I needed it. Also I am grateful for support from Bettina Grorud.

As this paper follows the work on the previous paper of the author "Control and motion generation of underactuated butterfly robot", the introduction and problem is similar and will contain similar arguments.

Abstract

This paper focus on the challenging tasks of controlling underactuated systems. A well known example is the butterfly robot, where the task is to have continuous rotation of the plates without losing contact between ball and plates. The first step is to reduce the degrees of freedom of the dynamical model to facilitate the choice of holonomic constraint. The next step is trajectory planning with motion generation, and lastly the stabilization of this motion is required to obtain the desired trajectory.

2 Abstrakt

Denne oppgaven fokuserer på utfordringene knyttet til kontroll av underaktuerte systemer. En velkjent problemstilling er "butterfly robot", der målet er en kontinuerlig rotasjon av et sett med plater uten å miste kontakt mellom ballen og platene. Det første målet er å redusere frihetsgraden til den dynamiske modellen for å gjøre det mulig å velge gode valg av "holonomic constraints" som er begrensende dynamikk. Det neste målet er å skape baneplanlegging med "motion generation", og til slutt skal systemet stabiliseres til å følge denne ønskede banen.

Contents

3 Introduction

When interacting with a certain object, in general, there will be some "limb" actuating a desired force upon the object in some degrees of freedom. If there are forces that "explicitly" affects every degree of freedom of the object, the action is called a prehensile manipulation. For humans this task is trivial, and is often referred to as grasping. When enclosing a hand around some adequately sized object, even infants can manipulate several objects in any desirable degree of freedom. In a general case for humans, these degrees of freedom often include the three cartesian coordinates and the three axis of rotation. This is a convenient choice of coordinates as humans can manipulate relate and manipulate objects in these DoF without any knowledge about motion planning or dynamical systems.

However, if there are degrees of freedom without any explicit manipulating forces from the actuators, the system is called underactuated. This will result in a system where it is not possible to satisfy every desired trajectory in some configuration space. Correlated to human interaction this could be an event where the object is sliding or rolling in an open hand. These systems are much more complicated for humans to control, and may take some practice to sufficiently manipulate the object in the restricted configuration space, e.g. tricks with football.

When manipulating objects with robotic limbs the difference in complexity regarding prehensile and non-prehensile manipulation is considerable. If there is a proper grasp around an object with some robotic tool, the manipulation of the object is reduced to manipulating the orientation of the robotic tool. However, there are situations where it is necessary to manipulate some object in an under-actuated system. This may be if the object is particularly delicate or or the robotic tool is not able to completely grasp the object in question. Then there would be some degrees of freedom without direct actuation.

In this case arbitrary manipulation of the object is restricted, and it is necessary to reduce the degrees of freedom such that the reduced system is completely actuated. This done by introducing some relationship between the general coordinates, called virtual holonomic constraint. The reduced system can then be controlled by some adequate controller, although the shape of the trajectories for the entire configuration space will depend on the choice of the virtual holonomic constraint.

4 Choice of coordinates

In several earlier papers regarding the butterfly robot, the choice of generalized coordinates have been mainly polar coordinates. These coordinates are advantageous for many reasons, especially when implementing virtual holonomic constraints and obtaining constraint forces . These coordinates are also intuitive and provide more information than the cartesian coordinate system.

However it is beneficial to obtain the equations of motion with other coordinate systems, as other solutions to trajectory planning, virtual holonomic constraints or constraint dynamics may present itself. Thus it is chosen to calculate the equations of motion with cartesian coordinates with respect to the ball, in both body- and the inertial frame related to the plates.

The butterfly robot consists of two parallel 8-shaped plates, with some electrical engine providing torque in the center of the two plates. There is placed a perfectly round spherical object on top of the two plates and can roll freely along the shape of the plates.

4.1 Excess coordinates

In the general case we have the generalized coordinates:

$$q = \begin{bmatrix} x_p & y_p & \theta_b & x_b & y_b & \theta_p \end{bmatrix}^T \tag{4.1}$$

Taken from the authors previous article, $x_b = x^2$, $y_b = y^2$, $\theta_b = \phi$ and $\theta_b = \theta^1$ as seen in figure 2.1.

The distance between the center of the plates to the edge of the plates is defined as:

$$\delta(\beta) = 0.1095 - 0.0405cos(2\beta), \quad \vec{\delta}(\beta) = \delta(\beta) \begin{bmatrix} sin(\beta) & cos(\beta) \end{bmatrix}^T \tag{4.2}$$

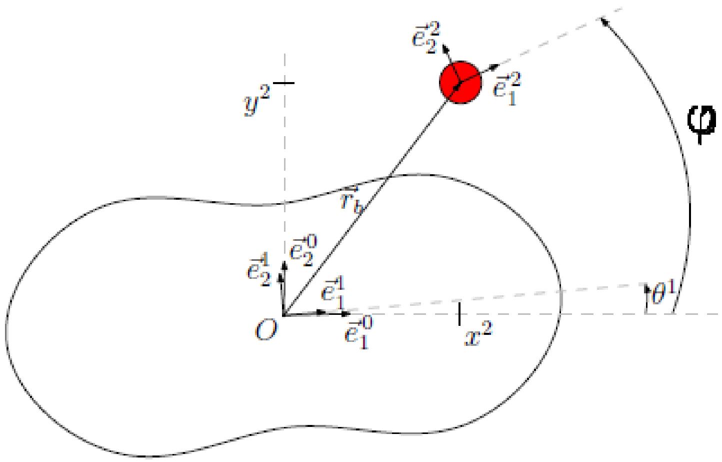
$\vec{e}_2^{\,2}$
$\vec{e}_1^{\,2}$
y^2
φ
$\vec{r}_b$
$\vec{e}_2^{\,0}$
$\vec{e}_2^{\,1}$
$\vec{e}_1^{\,1}$
θ^1
O
$\vec{e}_1^{\,0}$
x^2

Figure 4.1

5 Calculating equation of motion

To obtain the equation of motion of the system it is necessary to calculate the Lagrangian of the system. This calculation includes the kinetic and potential energy of the entire system. Therefore this will be the first step when obtaining the EoM.

5.1 Obtaining kinetic and potential energy

It is expected that the kinetic and potential energies of the system are trivial to obtain when using the aforementioned coordinate system.

5.1.1 Energies of butterfly plates

Firstly the orientational variables are stated, then derivated to obtain the velocities. Finally well known operations are done to obtain the kinetic energies of the system:

$$x_p = y_p = 0 \tag{5.1}$$

$$\tag{5.2}$$

It is clear that there is no linear velocities regarding the plates, as the center of mass coincides with the center of rotation. Thus only the angular velocity is considered. The angular velocity of the plates is simply $\dot{\theta}$

Finally the kinetic energy of the plates are calculated:

$$T_p = \frac{1}{2} J_p (\dot{\theta}_p)^2 \tag{5.3}$$

Since the center of mass of the plates do not move, the potential energy of the system stay constant.

5.1.2 Energies of ball

With a similar procedure we calculate the kinetic and potential energies of the ball.

The states regarding both the position and rotation of the ball is already conveniently expressed in the generalized coordinates.

Thus we can immediately obtain the linear and rotational velocities of the ball:

$$v_b = \sqrt{\dot{x}_b^2 + \dot{y}_b^2} \qquad \omega_b = \dot{\theta}_b \tag{5.4}$$

Then, calculating the kinetic energy of the ball is trivial:

$$T_b = \frac{1}{2} m_b v_b^2 + \frac{1}{2} J_b \omega_b^2 = \frac{1}{2} m_b (\dot{x}_b^2 + \dot{y}_b^2) + \frac{1}{2} J_b \dot{\theta}_b^2 \tag{5.5}$$

Finally, the potential energy of the ball is also conveniently described by the generalized coordinate:

$$V_b = m_b g h = m_b g y_b \tag{5.6}$$

Thus we obtained the Lagrangian:

$$\mathcal{L} = T_p + T_b - V_b = \frac{1}{2}(J_p \dot{\theta}_p^2 + m_b(\dot{x}_b^2 + \dot{y}_b^2) + J_b \dot{\theta}_b^2) - m_b g y_b \tag{5.7}$$

Then, the equation of motion can be obtained by:

$$\frac{d}{dt}(\frac{\partial \mathcal{L}}{\partial \dot{q}_j}) - \frac{\partial \mathcal{L}}{\partial q_j} = u_j \qquad ; \qquad (j = 1, ..., 4) \tag{5.8}$$

First we calculate the equation of motion with respect to the rotation of the plates, namely $q_1 = \theta_p$:

$$\frac{d}{dt}(\frac{\partial \mathcal{L}}{\partial \dot{q}_1}) - \frac{\partial \mathcal{L}}{\partial q_1} = u_1 \implies J_p \ddot{\theta}_p = u \tag{5.9}$$

Then we calculate the equation of motion with respect to both the linear velocities of the ball, $q_2 = x_b$ and $q_3 = y_b$:

$$\frac{d}{dt}(\frac{\partial \mathcal{L}}{\partial \dot{q}_2}) - \frac{\partial \mathcal{L}}{\partial q_2} = 0 \implies m_b \ddot{x}_b = 0 \tag{5.10}$$

$$\frac{d}{dt}(\frac{\partial \mathcal{L}}{\partial \dot{q}_3}) - \frac{\partial \mathcal{L}}{\partial q_3} = 0 \implies m_b \ddot{y}_b + m_b g = 0 \tag{5.11}$$

Finally we have the equation of motion related to the rotation of the ball, $q_4 = \theta_b$:

$$\frac{d}{dt}(\frac{\partial \mathcal{L}}{\partial \dot{q}_4}) - \frac{\partial \mathcal{L}}{\partial q_4} = 0 \implies J_b \ddot{\theta}_b = 0 \tag{5.12}$$

We can clearly see that the dynamics of this uncoupled system is trivial, until any constraint dynamics are introduced. The only external force influencing the system is the electric engine turning the plates.

5.2 System matrices

Since the equation of motion for the unconstrained system is simple, the resulting system matrices will also be trivial.

5.2.1 Inertia matrix

$$\mathbf{M}(q) = \begin{bmatrix} J_p & 0 & 0 & 0 \\ 0 & m_b & 0 & 0 \\ 0 & 0 & m_b & 0 \\ 0 & 0 & 0 & J_b \end{bmatrix} \tag{5.13}$$

The inertia matrix describes the forces or torques required to accelerate a body, linearly or by rotation, when purely focusing on the linear inertia and moment of inertia of the body. This matrix does not include the dynamics of friction, damping or constraining forces in the system.

5.2.2 Coriolis matrix

In this unconstrained case, there is neither any Coriolis nor centrifugal forces present in the system, thus
$\mathbf{C}(q, \dot{q}) = 0$

5.2.3 Gravity vector

Now, the gravitational influence on the system is considered. Intuitively, the gravity would only affect the vertical displacement of the ball. By simple inspection of the equation of motion, this is indeed the case, and the gravity vector is easily obtained.

$$\mathbf{G} = \begin{bmatrix} 0 & 0 & m_b g & 0 \end{bmatrix}^T \tag{5.14}$$

5.2.4 Unconstrained system equation

The resulting system can now be represented on the form:

$$\mathbf{M}(q)\ddot{q} + \mathbf{C}(q, \dot{q})\dot{q} + \mathbf{G}(q) = \mathbf{B}u \tag{5.15}$$

As discussed earlier, the Coriolis and centrifugal matrix, $\mathbf{C}(q, \dot{q})$, does not affect the system and is equal to zero. Furthermore, the actuation from the electric engine only directly affects the angle of the plates. Thus the vector describing the influence of the actuation on the system is easily obtained as: $\mathbf{B} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T$.

5.3 Constraint Dynamics

In this unconstrained case, the ball and the plates move independent of each other and results in a trivial dynamic system as shown in 5.2.4.

These dynamics are of little interest, as the thesis considers the case when the ball is in contact with the plates. Thus the next step is to include the constraining dynamics. This process will add important data regarding the constraining forces, throughout certain trajectories. The estimation of these forces will help confirm that the constraints will hold.

5.3.1 Offset constraint

The first constraint to consider is the fact that the ball does not depart from the frame. The dynamics present when the ball is compliant, and there is some compression leading to line contact between the ball and the frame are complex and will not be considered here. Therefore only point contact is considered. This constraint may be presented as follows:

$$h_1 = \omega = 0 \tag{5.16}$$

$$\omega = R_c - (\delta(\alpha) + r) \tag{5.17}$$

$$R_c = \sqrt{x_b^2 + y_b^2} \tag{5.18}$$

$$\alpha = \beta - \theta_p \tag{5.19}$$

$$\beta = atan2(\frac{y_b}{x_b}) \tag{5.20}$$

Here, it is assumed that $\alpha = \phi$ in order to describe the offset between the ball and the frame as the difference between the distance from the origin to the ball, and the distance given by the radius of the frame in addition to the radius of the ball. As shown in 5.17 the radius of the frame is given by the function $\delta(\alpha)$ where α is the angle between the ball and the horizontal line through the plates. This it then described by the difference between the angle between the ball and the horizontal plane, β, in the inertial frame and the angle of the plates, θ_p, which can easily be measured by some encoder in the electrical engine. Finally, the angle β is obtained in 5.20, where $atan2(\frac{y}{x})$ is used. This function takes two arguments, and returns a real value even when $x = 0$.

5.3.2 Non-slip constraint

The second constraint is the non-slip constraint. This constraint ensures that the relative velocity between the ball and the frame in the contact point is zero. The most practical description of this constraint is to ensure that the relative linear velocity of the ball is equal to the rotational velocity of the ball times its radius. What is meant by the relative linear velocity of the ball is the velocity of the ball relative to the plates.

There are several issues when obtaining any equation for this constraint. Firstly, it is difficult to obtain a velocity constraint dynamic that is integrable. There will now be presented two different suggestions as constraint dynamics.

5.3.2.1 First Non-slip constraint

The first constraint is derived from earlier articles such as, (BR ARTICLE). This constraint states that the distance traveled along the plates s is equal to the rotation of the ball times its radius, $\theta_b r$. The equation looks as follows:

$$h_2 = s - \theta_b r = const \tag{5.21}$$

$$\dot{h_2} = \dot{s} - \dot{\theta}_b r = 0 \implies \sqrt{\dot{x}_b^2 + \dot{y}_b^2} - \delta(\beta)\dot{\theta}_p - \dot{\beta}_b r = 0 \tag{5.22}$$

$$\tag{5.23}$$

With β as defined in 5.20. Here, the intuitive interpretation of the equation is that the linear velocity is equal to the total rotational velocity of the ball. However this equation is not integrable, and finding an analytic solution for the positional constraint is very difficult. This makes it impossible to simulate the system with the constraining dynamics. An analytic solution is also required to reduce the degrees of freedom of the dynamical system. Therefore another approach was attempted.

5.3.2.2 Second Non-slip constraint

In this section the idea was to present the constraint with two independent equations for each positional variable. This may present another way to find an analytic solution to the positional constraint. This might also help reducing the degrees of freedom of the system.

$$\dot{x}_b = \dot{\theta}_b r sin(\xi) - \delta(\beta)\dot{\theta}_p sin(\theta_p) \tag{5.24}$$

$$\dot{y}_b = \dot{\theta}_b r cos(\xi) - \delta(\beta)\dot{\theta}_p cos(\theta_p) \tag{5.25}$$

Here, the linear velocity in both x- and y-direction in the inertial frame is set equal to the total rotational velocity of the ball. The term including ξ describes the linear velocity in each respective direction, x and y, origin from the rotation of the ball. The rotation speed of the ball is multiplied by its radius and is scaled by a function of the angle of the surface the ball is in contact with.

The angle of the contact point between the ball and the plates, ξ, is calculated by the following equation:

$$tan(\xi_p) = \dot{\delta}(\theta_p) \implies arctan(\dot{\delta}(\theta_p)) = \xi_p \tag{5.26}$$

$$\xi = \xi_p + \beta + \theta_p \tag{5.27}$$

This relation is made when seeing that $\dot{\delta}(\theta_p)$ provides the curvature of the plates in the contact point. Now the angle of the tangent in the point contact is given by $\arctan(\dot{\delta}(\theta_p))$.

As seen in 5.27, the angle of the tangent to the curvature is added with the angle of both the balls placement on the plates and the angle of the plates. Thus ξ is the angle of the tangent on the contact point in the inertial frame, related to the vertical plane, e.g. $\xi = \frac{\pi}{2}$ is when the tangent points parallel to the x-direction.

5.3.3 Reduction of degrees of freedom

Now the degrees of freedom may be reduced by calculating the derivative of the constraints. $\ddot{x}_b$ and $\ddot{y}_b$. However these calculations are involved and is not represented in this paper.

5.4 Discussion

As there was made no analytic calculation of the excessive or the reduced equation, there was no possibility of making a simulation with any results. However it is possible to reduce the degrees of freedom with the constraints previously made.

With cartesian coordinates, it is difficult to properly obtain an analytic solution of the dynamic constraints, and therefore it is not possible to start the process of finding trajectory dynamics and stabilizing controllers.

Polar coordinates provides more easy ways to obtain the constrain dynamics and to reduce the degrees of freedom, further work should be done to obtain analytic solutions to the positional constraints.

5.5 Summary

Obviously there is little experimental results in this paper as the reduced equation of motion was not obtained. There is much theory and unexplored possibilities in this subject and many hours have been used to obtain relevant knowledge, even though little result is shown.

6 References

[1] Maksim Surov, Anton Shiriaev, Leonid Freidovich, Sergei Gusev, Leonid Paramonov *Case study in non-prehensile manipulation: Planning and orbital stabilization of one-directional rollings for the 'Butterfly' robot*

[2] Gusev, Anton S. Shiriaev & Leonid B. Freidovich (2016) *SDP-based approximation of stabilising solutions for periodic matrix Riccati differential equations*, International Journal of Control, 89:7, 1396-1405,

[3] A.S. Shiriaev, L.B. Freidovich, S.V. Gusev. *Transverse Linearization for Controlled Mechanical Systems With Several Passive Degrees of Freedom IEEE Transactions on Automatic Control* 893–906, 2010.

[4] H.M. Francke. Motion planning and analysis of the underactuated "Butter y" robot. University of Technology, Eindhoven, Netherlands, DC 2015.092, 2015.

[5] Stefan Johansson Bo Kågstrøm Anton Shiriaev Andras Varga. *COMPARING ONE-SHOT AND MULTI-SHOT METHODS FOR SOLVING PERIODIC RICCATI DIFFERENTIAL EQUATIONS*, Eindhoven, International

[6] Fahim Shakib. *MODELING AND CONTROL OF THE UNDERACTUATED BUTTERFLY ROBOT*, Eindhoven, 2016