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The Compliant Tower Concept for Offshore Wind Turbines

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Submission date: June 2018
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DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

Report Title: The Compliant Tower Concept for Offshore Wind Turbines	Date: 11/06/2018 Number of pages (incl. appendices): 75			
	Master Thesis	x	Project Work	
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Abstract:

Offshore wind turbines are getting larger and the water depths where they are located are getting deeper as well. This causes problems of eigen frequencies for rigid structures where a jacket structure could be too expensive in depths around 50 meters. At this depth, floating structures will not be efficient due to the hydrodynamic behaviour and the mooring system for this depth can be challenging. That is the reason why compliant structures are analysed for depths around 50 meters due to their low eigenfrequencies of the structure while having a load path into the seabed.

For the realization of the thesis, two main tasks have been done. First, the implementation of a code to calculate all the steps in the design of an offshore wind turbine. This method has covered from the determination of natural frequencies and displacements at the structure to code checks necessary to satisfy the guidelines and requirements. This methodology will be the base for a preliminary design of the structure. The main theory to develop this program has been the finite element method.

The second main task is the design of a compliant tower for a specified site location using the methodology for preliminary design create beforehand. Different tests have been done and different options for the design have been studied like the use of guy cables to control the displacements, the incorporation of a mass trap in the structure and changes in the slenderness of the tower to achieve a lower natural frequency. This program will be useful for many different cases of environmental loads. The rotor-nacelle assembly used has a rating of 5 MW and a rotor radius of 126 m. Possibility of a larger wind turbine has been studied to reduce even more the natural frequencies.

Another important factor like the soil-tower interaction has been also considered and the analysis of the behaviour of the soil has been done according to p-y curves. Differences in behaviour for different types of soil has been evaluated.

After arriving to a final design of the compliant tower satisfying all the limit states evaluated, the results of the program are shown and it can be seen that the representation of deflections are reliable while the data for bending moment reactions and shear forces is not completely exact due to the higher influence of bending strain energy than for the shear energy for high flexibility members.

Although further research is needed about compliant towers, this preliminary analysis is satisfactory to ensure that compliant towers could become much more common in the future.

Keywords:

1. Offshore
2. Compliant
3. Eigenfrequencies
4. Wind Turbine

MASTER THESIS

(TBA4920 Marine Civil Engineering, master thesis)

Spring 2018
for
Ruben Martinez Ochagavia

The compliant tower concept for offshore wind turbines

Compliant tower konsept for offshore vindmøller

OPPGAVETEKST (To fill in on DAIM)

The main task is to develop a methodology for performing a load analysis of compliant towers for offshore wind turbines. This analysis approach shall be used for preliminary design and concept evaluation, and can therefore be simplified. It is suggested to implement the method in e.g. MATLAB, based on a few eigenmodes. The equations shall be solved in the time-domain and simplified code checks shall be performed. Using this method, a compliant tower support structure shall be designed for a 50m reference site and compared with other support structure concepts developed for these site conditions. Different options for the design can be evaluated, and the robustness of the design with respect to changes in input parameters or site conditions shall be studied.

ADDITIONAL INFORMATION

BACKGROUND (On why and how)
Offshore wind turbines are getting larger and larger, and site water depths are getting deeper as well. This leads to problems with eigenfrequencies of the support structures, as the traditional monopole concept becomes too soft. Multi-member jacket structures are an alternative. Their main advantage are a stiffer system and less excitation from hydrodynamic loads. However, these structures can become costly for deep-water sites (50m and beyond). Floating platforms are another possibility, but their hydrodynamic behaviour is complex and moorings can be challenging at these depths. The compliant tower (sometimes also called semi-floater) is a hybrid concept that has very low global eigenfrequencies similar to a floater, while at the same having a load path into the seabed and a substructure similar to a fixed bottom turbine. The concept has not been studied in detail for earlier turbines, but for future very large wind turbines it might be attractive and should be re-evaluated.

TASK DESCRIPTION (Tentative work for the thesis)
Description of task
The thesis consists of two main tasks. The first task is to come up with a methodology for performing a simplified loads analysis of compliant towers that can be used for preliminary design. The second task is to design a compliant tower for a reference site.

Aims and purpose
The aim is to understand what the main considerations are for the design of compliant towers for offshore wind turbines, i.e., which are the critical issues and design drivers for such a concept, and how robust is it with respect to e.g. site conditions. A secondary aim is to obtain a reference design that can be compared with other support structure concepts in terms of weight and/or cost.

Subtasks and research questions
The first ask is to get familiar with the design of wind turbine support structures. This includes studying the environmental loads, the existing wind turbines and their operational parameters, the dynamical analysis of such a system, postprocessing techniques (rainflow counting, code checks).
The main task is to develop a simplified load analysis method. This could be based on a simple modal model using the first n=1,2,3 eigenmodes of the structure. These can be derived from finite element / wind turbine simulation software. Turbine loads can be obtained from a rotational spectrum or a modification of load time series from a fixed rotor. Wave loads can be computed from the Morison formula. The solution of the dynamical equations in the time domain can be done in MATLAB. This approach can be further refined if time permits (e.g. using an existing BEM code in MATLAB).
The design of the tower is based on time-domain simulations and should consider both fatigue and extreme loads. Code checks from the relevant standards shall be used in a simplified way. The resulting design shall be compared against other designs in the literature.
Parameter studies shall be performed to further understand how changes in the input parameters affect the design, e.g., changes in loads or uncertainty about system parameters. In addition, different design options could be evaluated, e.g. the use of guy cables or buoyancy elements.

Suggested schedule (total of 20 weeks)
1. Familiarization with the topic, literature study [4 weeks]
2. Developing and implementing a simplified load analysis method [6 weeks]
3. Design of a compliant tower for a 50m reference site [4 weeks]
4. Parameter studies [3 weeks]
8. Writing of thesis [3 weeks]

General about content, work and presentation
The text for the master thesis is meant as a framework for the work of the candidate. Adjustments might be done as the work progresses. Tentative changes must be done in cooperation and agreement with the professor in charge at the Department.

In the evaluation thoroughness in the work will be emphasized, as will be documentation of independence in assessments and conclusions. Furthermore the presentation (report) should be well organized and edited; providing clear, precise and orderly descriptions without being unnecessary voluminous.

The report shall include:

- Standard report front page (from DAIM, http://daim.idi.ntnu.no/)
- Title page with abstract and keywords
- Preface
- Summary and acknowledgement. The summary shall include the objectives of the work, explain how the work has been conducted, present the main results achieved and give the main conclusions of the work.
- Table of content including list of figures, tables, enclosures and appendices.
- A list explaining important terms and abbreviations should be included.
- List of symbols should be included
- The main text.
- Clear and complete references to material used, both in text and figures/tables. This also applies for personal and/or oral communication and information.
- Text of the Thesis (these pages) signed by professor in charge as Attachment 1.
- The report musts have a complete page numbering.

The thesis can as an alternative be made as a scientific article for international publication, when this is agreed upon by the Professor in charge. Such a report will include the main points as given above, but where the main text includes both the scientific article and a process report.

Advice and guidelines for writing of the report is given in: “Writing Reports” by Øivind Arntsen. Additional information on report writing is found in “Råd og retningslinjer for rapportskriving ved prosjekt og masteroppgave ved Institutt for bygg, anlegg og transport” (In Norwegian). Both are posted on It’s-learning.

Submission procedure
Procedures relating to the submission of the thesis are described in DAIM (http://daim.idi.ntnu.no/). Printing of the thesis is ordered through DAIM.

On submission of the thesis the candidate shall submit also to the professor in charge a CD/DVD with the paper in digital form in pdf and Word (editable) version, the underlying material (such as data collection, time series etc., if possible) in digital form.

Documentation collected during the work, with support from the Department, shall be handed in to the Department together with the report.

According to the current laws and regulations at NTNU, the report is the property of NTNU. The report and associated results can only be used following approval from NTNU (and external cooperation partner if applicable). The Department has the right to make use of the results from the work as if conducted by a Department employee, as long as other arrangements are not agreed upon beforehand.

Tentative agreement on external supervision, work outside NTNU, economic support etc.
Separate description is to be developed, if and when applicable.

Health, environment and safety (HSE) https://innsida.ntnu.no/hms-for-studenter
NTNU emphasizes the safety for the individual employee and student. The individual safety shall be in the forefront and no one shall take unnecessary chances in carrying out the work. In particular, if the student is to participate in field work, visits, field courses, excursions etc. during the Master Thesis work, he/she shall make himself/herself familiar with "Fieldwork HSE Guidelines". The document is found on the NTNU HMS-pages at
https://innsida.ntnu.no/wiki/-/wiki/English/Fieldwork+-+for+participants

The students do not have a full insurance coverage as a student at NTNU. If you as a student want the same insurance coverage as the employees at the university, you must take out individual travel and personal injury insurance.

Start and submission deadlines
The work on the Master Thesis starts on (date) 15.01.2018

The thesis report as described above shall be submitted digitally in DAIM at the latest (date:)__________________ at 3pm.

Professor in charge: Michael Muskulus

Other supervisors: Tu Ying

Trondheim, 15.01.2018. (revised: N/A)

Michael Muskulus

Professor in charge (sign)

Abstract

Offshore wind turbines are getting larger and the water depths where they are located are getting deeper as well. This causes problems of eigen frequencies for rigid structures where a jacket structure could be too expensive in depths around 50 meters. At this depth, floating structures will not be efficient due to the hydrodynamic behaviour and the mooring system for this depth can be challenging. That is the reason why compliant structures are analysed for depths around 50 meters due to their low eigenfrequencies of the structure while having a load path into the seabed.

For the realization of the thesis, two main tasks have been done. First, the implementation of a code to calculate all the steps in the design of an offshore wind turbine. This method has covered from the determination of natural frequencies of the structure and the solution of the dynamical equation of dynamics to obtain displacements depending on time for different environmental conditions in the tower, to the obtention of the force response of the structure and the code checks necessary to satisfy the guidelines and requirements for Ultimate Limit State, Fatigue Limit State and Serviceability Limit State. This methodology will be the base for a preliminary design of the structure. The main theory to develop this program has been the finite element method and the type of beam used for the calculation can be decided by the user between the Bernoulli beam and the Timoshenko beam.

The second main task is the design of a compliant tower for a specified site location using the methodology for preliminary design create beforehand. The location of the compliant tower is the Dutch North Sea and an analysis has been performed to try to locate the optimal natural frequencies for the structure based on the frequencies of the environmental loads acting on the tower. Different tests have been done and different options for the design have been studied like the use of guy cables to control the displacements, the incorporation of a mass trap in the structure and changes in geometrical aspects have been done to achieve a lower natural frequency. These changes are the length from the seabed level to the rotor-nacelle assembly and the slenderness of the monopile studied. All the elements introduced in the code can be changed easily and therefore, this program will be useful for many different cases.

The rotor-nacelle assembly used has a rating of 5 MW and a rotor radius of 126 m. Possibility of a larger wind turbine has been studied to reduce even more the natural frequencies.

Another important factor like the soil-tower interaction has been also considered and the analysis of the behaviour of the soil has been done according to p-y curves. Differences in behaviour for different types of soil has been evaluated.

After arriving to a final design of the compliant tower satisfying all the limit states evaluated, the results of the program are shown and it can be seen that the representation of deflections are reliable while the data for bending moment reactions and shear forces is not completely exact due to the higher influence of bending strain energy than for the shear energy for high flexibility members.

Although further research is needed about compliant towers, this preliminary analysis is satisfactory to ensure that compliant towers could become much more common in the future.

Acknowledgment

I would like to thank my supervisor Michael Muskulus and Tu Ying for their technical advice, providing me with the necessary information to continue when I have been stuck during the thesis work.

I would like to thank also to my parents and my sister, who have supported me not only during the master thesis, but also during all my life. You are an example to follow.

And last but not least, to Laura Lázaro for helping me when I have been more stressed in the realization of the thesis and for sharing with me unforgettable moments.

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Abbreviations

DOF	Degree of Freedom
FE	Finite Element
OWT	Offshore Wind Turbine
RNA	Rotor-Nacelle Assembly
MSL	Mean Sea Level
API	American Petroleum Institute
ULS	Ultimate Limit State
FLS	Fatigue Limit State
SLS	Serviceability Limit State
ALS	Accidental Limit State
DLC	Design Load Case
DAF	Dynamic Amplification Factor

1. Introduction

In this chapter the motivation for the master thesis and an introduction about offshore wind turbines and wind energy is described. Later, the main objectives of the thesis are explained.

1.1. Motivation

An expansion of renewable energies is happening in the last years and among them, wind energy is becoming an important source of energy for the energy industry.

The use of oil and other fossil fuels as energy source are in constant growing but these kinds of energies are not inexhaustible. The increase in their use is related with the growing in some sectors as automotive or industry. Developed countries are consumers, but they are not producers of energy, so it makes that these countries are susceptible to any change which can appear.

The main reason to the improvement in the use of renewable energies are the environmental impact. CO2 is the main responsible of the greenhouse effect, this phenom is a process in which thermal radiation from world surface is absorb by greenhouse gases which are in the high atmosphere. Then, these gases send this radiation in every direction, so part of the radiation goes again to the planet surface and to the low atmosphere causing an increase of surface temperature compare with the temperature without these gases.

If the energy demand it is satisfied with energy from fossil sources, it could cause an irreversible damage. Therefore, it is necessary the use of renewable energies to reduce the emissions and the harmful effects.

The European Union has kept, as a main goal to be achieved in 2020, to reduce 20% the greenhouse gases emission, increasing the percentage of the use of renewable source of energy at least 20%.

In Figure 1.1, the forecast of primary energy consumption is shown. As can be seen, even though the growing value of the renewable energies, it is necessary adopt measures to reduce CO2 emissions, which come from fossil fuels, to avoid the increasing of the emissions during the following years.

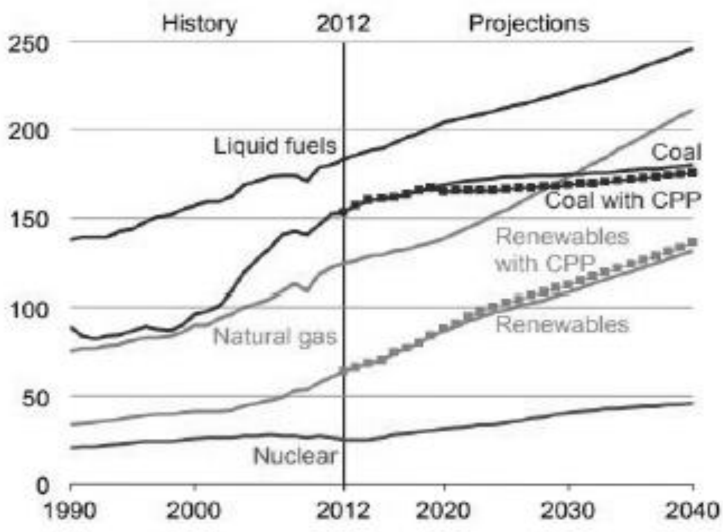


Figure 1.1. Forecast of primary energies consumption in the world. (Source:IEA, World Energy Outlook 2016).

Among renewable energy, wind energy is the most productive way of renewable energy and it comes from the wind power which generate electricity from the wind flows. Generally, the electrical energy is generated by turbines which can be located onshore or offshore. This kind of energy is considered ‘green’ since does not generate any pollution.

According to the International Energy Agency, wind energy onshore is the second biggest electricity generator source and offshore energy is starting to grow in the same way.

Currently, Europe is the leader in wind energy and this sector generates around 300,000 jobs and 72 billion of annual euros. So, in order to increase the use of wind energy around the world, wind farms are starting to be developed offshore during the last years. They are growing more because they have a reduce visual impact in comparison with onshore wind turbines and the acoustic pollution is almost null. The evolution of WT can be seen in Figure 1.2.

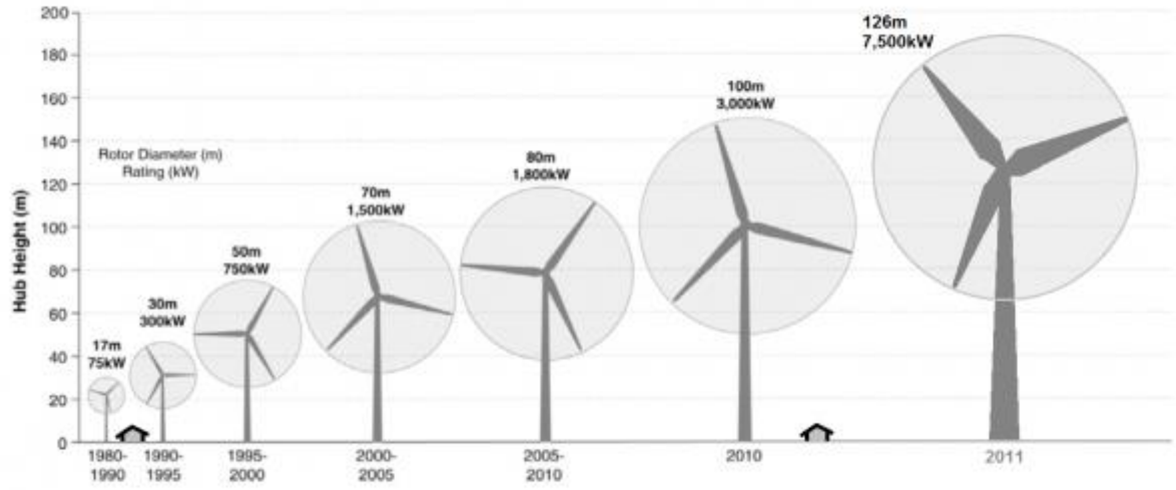


Figure 1.2. Evolution of Wind Turbines. (Source: National Renewable Energy Laboratory)

Having a special focus in offshore wind turbines (OWT), they are getting larger and site water depts are getting deeper as well. This leads to problems related to eigenfrequencies of the support structure for fixed structures. Jacket structures are designed to solve this problem and being stiffer, but they can be really expensive for deep water sites around 50 meters depth. Other possibility is the installation of floating platforms but the response of these systems to wave and wind loads is complex and mooring can be a problem for this depth. Therefore, an intermediate solution could be designed in order to have very low natural frequencies as floating structures but having at the same time a load path into the seabed and a substructure similar to a rigid structure.

1.2. Types of Offshore Wind Turbines

OWT can be classified in three different types of structures:

1.2.1. Fixed Bottom Wind Turbine

As explained before, this kind of structures are stiff and are fixed to the seabed to avoid displacements and rotations at the base. They can be classified in four groups:

- **Monopile:**

The WT is located above a tubular structure made of steel which is campled in the seabed (Figure 1.3). This structure is usually around 6 meters of diameter and they are used for depths below 35 meters.

This kind of anchor is reliable in any kind of soil, from soft clays to soft rock where is possible to drive the monopile in the seafloor. When driving piles is not possible due to hard soils, a monopile can be installed after the perforation of a hole beforehand. After the installation it would be ensured with grouting. However, this process is not recommended for the possible movement of sediments.

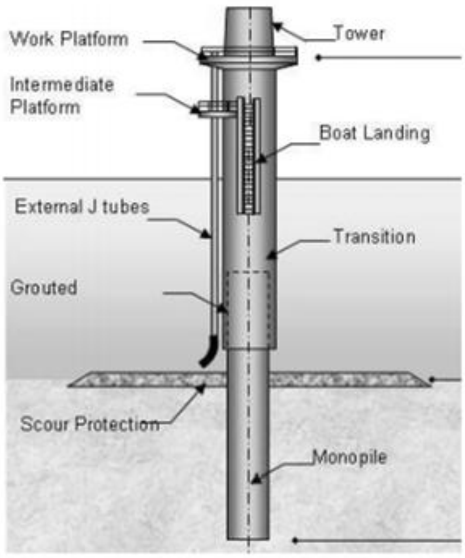


Figure 1.3. Monopile representation for fixed bottom structures.

- **Gravity based structures:**

This kind of structures are in equilibrium by its selfweight how can be seen in Figure 1.4.

This solution is used for water depths between 20 and 80 meters and they are mode adequate for firm ground.

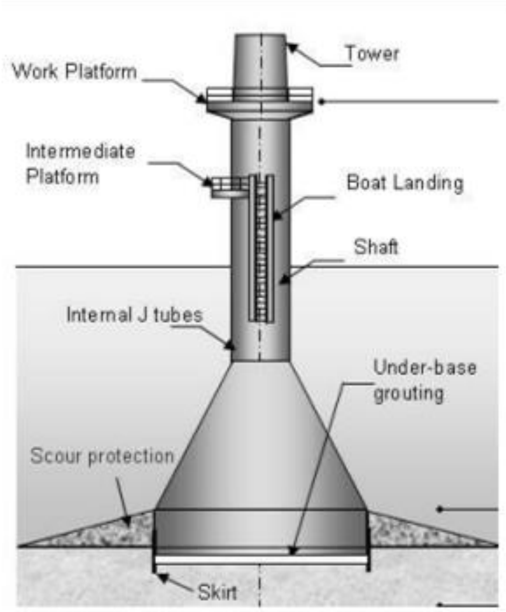


Figure 1.4. Example of a Gravity-based structure.

- **Jacket structure:**

The wind turbine is located above a truss structure of three or four supporting legs which are partially submerged into the seabed (Figure 1.5). They can be used until 80 meters of water depth although for more than 40-50 meters they start to be too expensive.

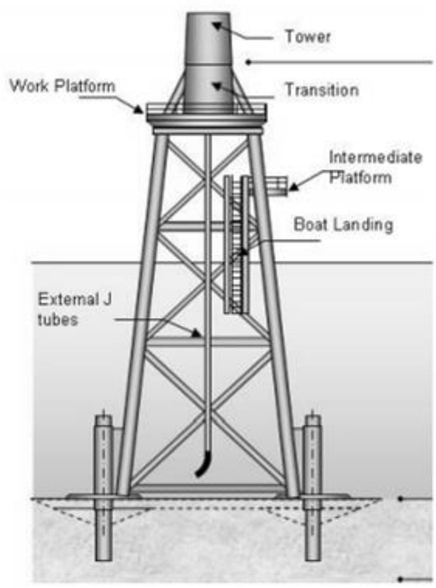


Figure 1.5. Jacket structure.

- **Tripod Structures:**

They are shown in Figure 1.6 and the RNA stands above three embedded legs in the surface of the seafloor thanks to inverted bucket structures.

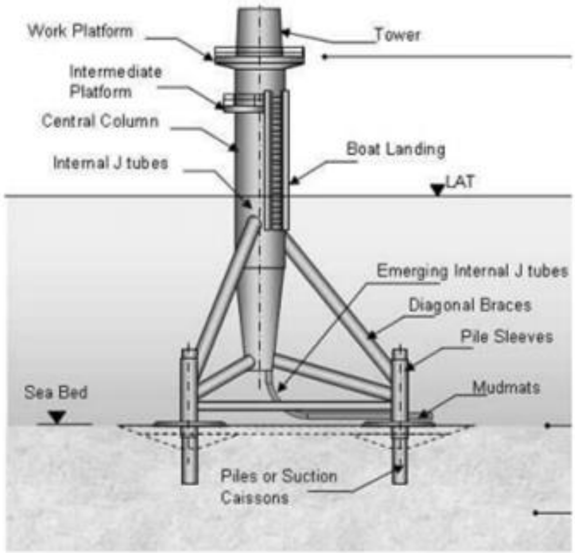


Figure 1.6. Tripod Structure.

1.2.2. Floating Wind Turbine

They are not in direct contact with the seafloor and are anchored thank to different methods like catenaries or pretension cables. They can appear floating or semi-submerged in the water. This kind of wind turbines are commonly used for large deep waters. The main problem will be the analysis of the stability of the structure due to the high wave and wind loads in these depths.

The different floating structures can be divided in three groups:

- **Structures stabilized by flotation:**

The structure will be floating in the water surface and they are anchored to the seabed with cables to contribute to stability of the system. It is used for zones with small wave loads. One example of this type would be a platform tri-floater in which the structure is semi-submerged. It is formed by three cylindrical floating columns joined with steel bars and it is anchored to the seafloor through cables (Figure 1.7).

Figure 1.7. Semi-submergible tri-floating structure. (Source:www.erwind.es)

- **TLP Structures:**

The RNA stands over a platform anchored with four high-resistance cables to the seabed. Cables are in tension and that is the tension which keeps floatability. The problem of this system is the complexity of the cable system which is not allowed for all kinds of soil and it is expensive. However, the WT can be located in a really high deep (See Figure 1.8).

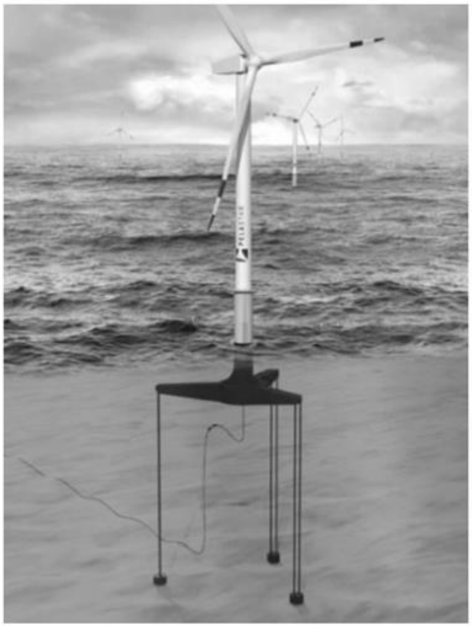


Figure 1.8. TLP Structure (Source: cimaterocks.com)

- **Spar Structures:**

The RNA stands in a cylindrical column of steel totally submerged which has a counterweight in the extreme to give stability to the system. It is anchored to the seabed through cables.

See Figure 1.9. for more details about the structure.

Figure 1.9. Spar Structure (Source: www.kirt-thomsen.com)

1.2.3. Compliant Wind Turbine

In earlier design, this solution was not viable because of the size of wind turbines, but now with the appearance of very large wind turbines it could be a good solution at depths around 50 meters depth where the other types of structures have more disadvantages.

The main idea of compliant towers is to have the main natural frequency below the frequency range of the wave loads. So, the location of natural frequencies will be: (see Figure 1.10)

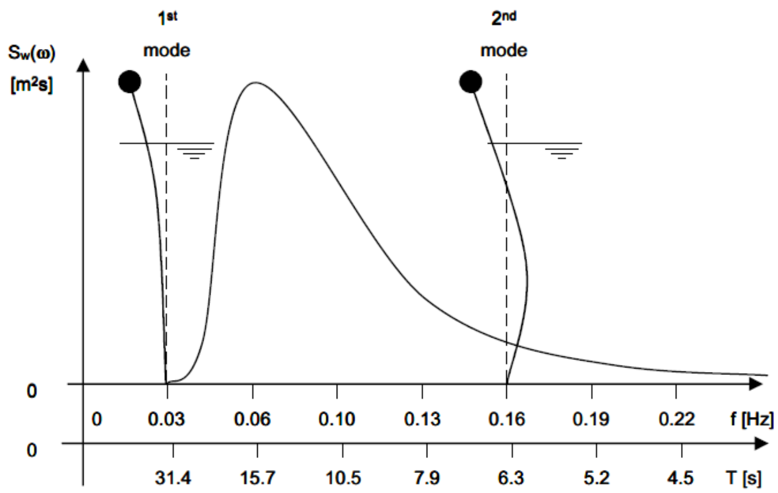


Figure 1.10. Location of natural frequencies of a compliant tower.

In order to achieve that, different configurations can be designed as shown in Figure 1.11.

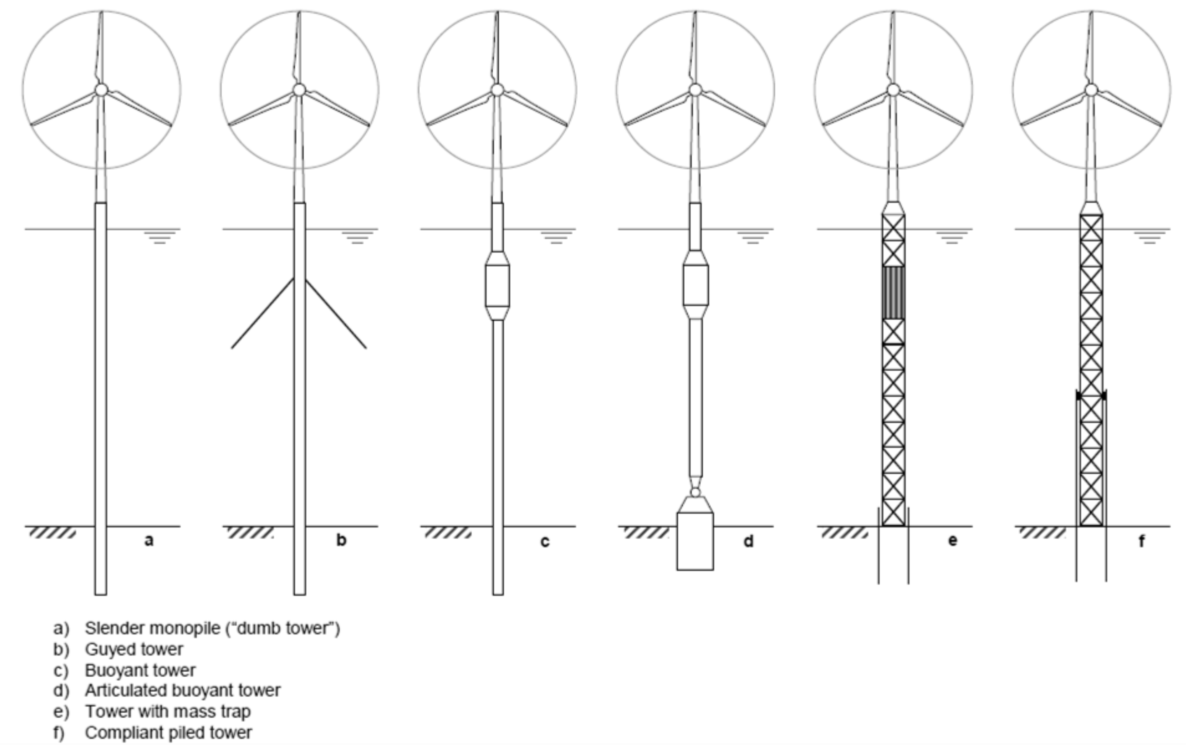


Figure 1.11. Different types of Compliant Towers.

The cases studied in the present thesis are compliant towers a), b) and c), it means, compliant towers which could be a simple extension of the monopile concept for fixed bottom structures.

In these cases, the diameter is reduced to reach the eigenfrequency searched and besides, more elements like mass traps, guy cables of buoyancy cans can be added to the system.

1.3. Objectives

The objectives of the thesis could be divide in two main tasks.

The first one would be the design of a methodology for the preliminary design of the structure. For this task, a Matlab code will be created to analyse the environmental loads, introduce them in a previous design selected for the compliant tower and after the calculation of the response forces acting along the structure, a code check will be developed to verify that the standards, guidelines and requirements are satisfied.

The second task will be the design of a compliant tower for a reference site with the code created previously.

Furthermore, there will be some subtasks that should be carried out like:

- Studying the different environmental loads acting in the structure, the existing wind turbines in wind energy industry and their operational parameters.
- A dynamic analysis of the structure and postprocessing techniques.
- Use of wind turbine simulations software: ASHES will be used for the determination of the forces acting on the rotor-nacelle assembly.
- Parameter study: Analysis of how a variation in input parameters can change the final result of the design.
- Finally, different design options will be evaluated like the use of guy cables, mass traps, slenderer cylinders, etc.

2. Methodology

In this chapter, all the theoretical background needed for the development of the thesis is shown. It involves from the modelling of the finite element model and calculation of natural frequencies, to the obtention of external loads to calculate the internal reactions and if they satisfy the requirements and guidelines.

This explanation will be the base for the understanding of the analysis and the following discussion and conclusion according to the results.

2.1. Finite Element Model

The main idea of this model is to consider the variations in the mechanical properties along the length of the structure. For that purpose, the structure has been discretized in different beams (with 2 nodes per beam) which represent the whole structure. Each element will have constant mechanical properties, so, the more elements in which the tower is divided, the more exact the solution is. The number of elements would be an input parameter in the final method to allow the user to decide the relation between approximation to the exact solution and speed of the program.

The structure is modelled with two degrees of freedom in each node, considering the displacement and rotation (transversal and rotational) as can be seen in Figure 2.1 and Figure 2.2.

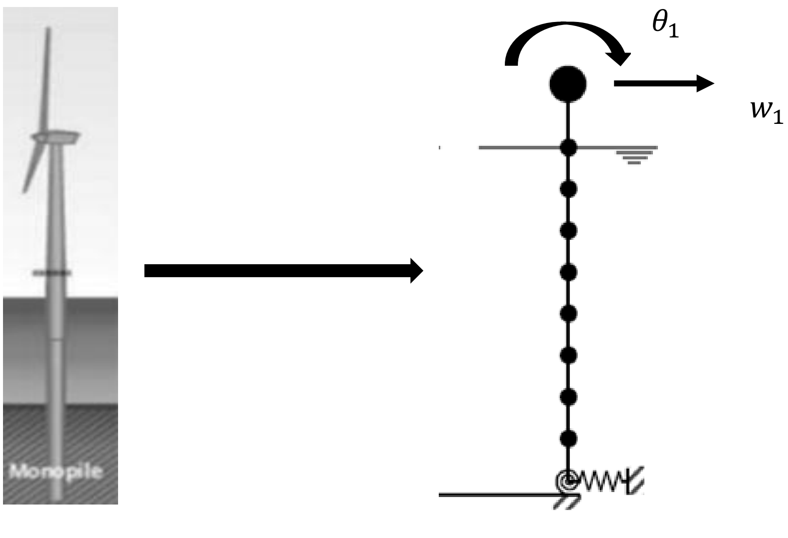
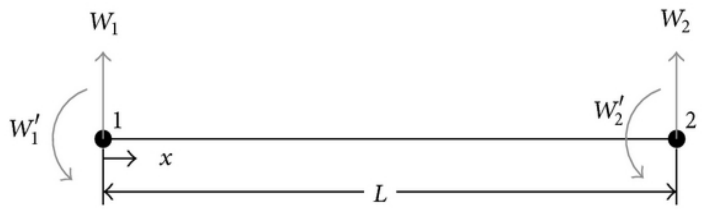


Figure 2.1. Discretization of the OWT and degrees of freedom of each node.

Each beam element of the structure will have 4 DOF (2 from each node) and therefore, each element will have four nodal values which will be the solution of the finite element problem as can be seen in Figure 2.2.



$$u^e = \begin{bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{bmatrix}$$

Figure 2.2. Degrees of freedom on a two nodes beam element and nodal values. θ = w'

2.1.1. Element Matrices

These matrices will represent the behaviour of the element when different loads are acting on its nodes and they are called the stiffness and mass matrix.

The formulation used for the determination of the element matrices has been obtained using Euler-Bernoulli beam elements and the equilibrium equation for this system is given by [1]:

$$EI\frac{\partial^4 w}{\partial x^4} = q$$

where E is the Young’s modulus of the material, I is the second moment of inertia about the axis perpendicular to the plane analysed (then, EI is the flexural rigidity) and q is the uniform transverse load per unit length (L).

The continuous variable w is approximated in terms of discrete nodal values and it can be written as:

$$w = [N_1 \;\; N_2 \;\; N_3 \;\; N_4] * (u^e)$$

Where N_i are the cubic shape functions which can be defined as: (see Figure 2.3)

$$N_1 = \frac{1}{L^3} * (L^3 - 3Lx^2 + 2x^3)$$

$$N_2 = \frac{1}{L^2} * (L^2x - 2Lx^2 + x^3)$$

$$N_3 = \frac{1}{L^3} * (3Lx^2 - 2x^3)$$

$$N_4 = \frac{1}{L^2} * (x^3 - Lx^2)$$

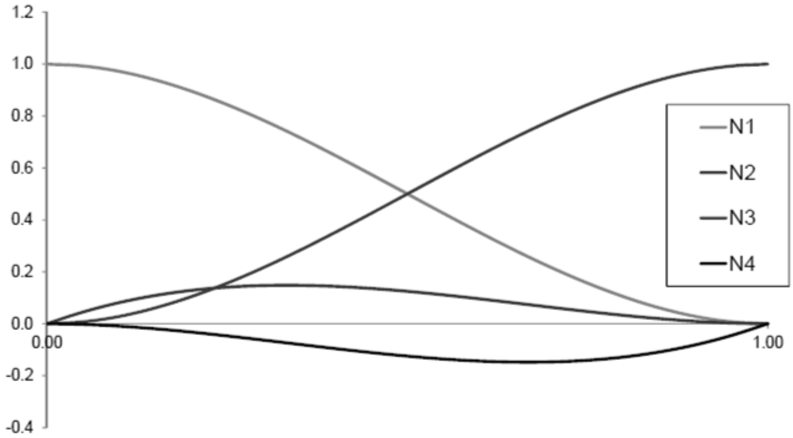


Figure 2.3. Cubic shape functions.

Substituting w in the equilibrium equation and applying Galerkin's method, the next integral is obtained.

$$\int_0^L \begin{Bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{Bmatrix} EI \frac{\partial^4}{\partial x^4} [N_1 \quad N_2 \quad N_3 \quad N_4] \begin{Bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{Bmatrix} \mathrm{d}x = \int_0^L \begin{Bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{Bmatrix} q \,\mathrm{d}x$$

At the beginning of this research, a way to try to solve this integral for the determination of the mechanical properties along the tower has been searched but the results were unsatisfactory due to the obtention of a stiffness matrix not symmetrical and the following problems trying to calculate the eigenvalues with this type of matrices.

However, assuming a constant beam element with no change in geometry and properties of the material used, a simplification can be done in order to avoid differentiating four times and it leads to the obtention of the element stiffness relationship. This procedure is followed for the most of the softwares using the finite element method thanks to its

$$EI \begin{bmatrix} \frac{12}{L^3} & \frac{6}{L^2} & -\frac{12}{L^3} & \frac{6}{L^2} \\ & \frac{4}{L} & -\frac{6}{L^2} & \frac{2}{L} \\ & & \frac{12}{L^3} & -\frac{6}{L^2} \\ \text{Symmetrical} & & & \frac{4}{L} \end{bmatrix} \begin{Bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{Bmatrix}$$

where F_i and M_i are the forces and moments at the nodes.

In matrix notation, it would be

$$K * u = F$$

where K is the stiffness matrix of the element.

Furthermore, when the beam element is vibrating transversely, it would be subjected to an additional restoring force depending on the inertia or mass matrix (M). It can be defined as

$$M = \int \rho * [N]^T * [N] * dV$$

and the solution to this integral will be

$$M = \frac{\rho A L}{420}\begin{pmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{pmatrix}$$

The way to arrive to these matrices (stiffness and mass) can be found in many engineering text books and following different paths. The method shown here is following the differential equation explained above, but another method can be based on a consideration of energy and it will lead to the same characterization of the element properties of the beam.

Another formulation related to the one used by Bernoulli is the Timoshenko beam element, which has the same DOF as in Figure 2.2. and the transversal displacement and the rotation are approximated independently as:

$$\mathrm{w(x)} = \mathrm{N_1(x)w_1} + \mathrm{N_2(x)w_2}$$

$$\theta(\mathrm{x}) = \mathrm{N_1(x)}\theta_1 + \mathrm{N_2(x)}\theta_2$$

where the shape functions in this case are:

$$N_1(x) = \frac{x_2 - x}{L} \qquad N_2(x) = \frac{x - x_1}{L}$$

Then, using the approximations of cinematic and constitutive equations for a Timoshenko beam, the exact stiffness matrix for a Timoshenko beam element can be described as:

$$\frac{EI}{(1+\Phi)}\begin{bmatrix} \frac{12}{L_e^3} & \frac{6}{L_e^2} & \frac{-12}{L_e^3} & \frac{6}{L_e^2} \\ \frac{6}{L_e^2} & \frac{(4+\Phi)}{L_e} & -\frac{6EI}{L_e^2} & \frac{(2-\Phi)}{L_e} \\ \frac{-12}{L_e^3} & \frac{-6}{L_e^2} & \frac{12}{L_e^3} & \frac{-6}{L_e^2} \\ \frac{6}{L_e^2} & \frac{(2-\Phi)}{L_e} & -\frac{6}{L_e^2} & \frac{(4+\Phi)}{L_e} \end{bmatrix}$$

where the factor $\boldsymbol{\Phi}$=12EI/(GA$_s$L^2) represents the shear slenderness of the beam and GA$_s$ would be the shear rigidity. Notice that the Timoshenko model is simplified into a Bernoulli model when this factor is equal to 0.

However, as the objective of the tower is to have a low first natural frequency (high flexibility), the bending strain energy along the structure becomes more significant than the shear energy [2]. So, to represent that issue, the following stiffness matrix has been used after the dynamic analysis, to transform displacements and rotations obtained into shear forces and bending moments.

$$K_s = \begin{bmatrix} \mu GA/L & \mu GA/2 & -\mu GA/L & \mu GA/2 \\ \mu GA/2 & \mu GAL/4 & -\mu GA/2 & \mu GAL/4 \\ -\mu GA/L & -\mu GA/2 & \mu GA/L & -\mu GA/2 \\ \mu GA/2 & \mu GAL/4 & -\mu GA/2 & \mu GAL/4 \end{bmatrix}$$

where μ is the correction factor for shear energy and it is generally taken 0.9 for circular cross sections [3].

Furthermore, a geometric stiffness matrix was considered during the first steps of the thesis to calculate the buckling load for the first buckling mode shape of the structure. It was done solving the eigenvalue problem $|K-\lambda K_G|=0$ but later, trying to be more conservative and following the guidelines, the buckling load proposed in NORSOK has been selected. Anyway, the geometric stiffness matrix selected was:

$$K_g^e = \frac{1}{30l}\begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix}$$

2.1.2. System Matrices

Once the element matrices can be defined, an assembly of all the element matrices has to be carried out to represent the whole structure. It can be done due to the fact that each element has a node in common with the next element, so, for instance, the stiffness matrix for 2 elements connected by one node would be

$$\mathbf{K} = \begin{bmatrix} k_{11}^{(1)} & k_{12}^{(1)} & k_{13}^{(1)} & k_{14}^{(1)} & 0 & 0 \\ k_{21}^{(1)} & k_{22}^{(1)} & k_{23}^{(1)} & k_{24}^{(1)} & 0 & 0 \\ k_{31}^{(1)} & k_{32}^{(1)} & (k_{33}^{(1)} + k_{11}^{(2)}) & (k_{34}^{(1)} + k_{12}^{(2)}) & k_{13}^{(2)} & k_{14}^{(2)} \\ k_{41}^{(1)} & k_{42}^{(1)} & (k_{43}^{(1)} + k_{21}^{(2)}) & (k_{44}^{(1)} + k_{22}^{(2)}) & k_{23}^{(2)} & k_{24}^{(2)} \\ 0 & 0 & k_{31}^{(2)} & k_{32}^{(2)} & k_{33}^{(2)} & k_{34}^{(2)} \\ 0 & 0 & k_{41}^{(2)} & k_{42}^{(2)} & k_{43}^{(2)} & k_{44}^{(2)} \end{bmatrix}$$

Following this procedure for all the elements we can arrive to the final stiffness matrix and mass matrix.

Besides, not only the properties from the tower must be included but also other elements should be introduced in the program. These elements are:

- **Rotor-nacelle assembly:**

It is clear that the RNA weight on the top of the tower will have a considerable importance in the behaviour of the whole system, so the mass and the inertia of the RNA should be added to the information of the top node. All the mass is lumped in the degree of freedom of the transverse displacement in the top node and the total inertia is included in the rotational degree of freedom as the mass moment of inertia about that node using basic dynamics as $I = m*L^2 / 3$.

- **Water inside the substructure of the tower:**

The water inside the tower from the seabed to the mean sea level (MSL) can be also considered like a added mass (weight and inertia) and it is introduced in the corresponding elements in the same way as explained above for the tower. Besides, if a mass trap element is used in the structure, it would be introduced in the system following the same procedure because the main characteristic of this element is to establish a region in which the mass of water inside that region would be considered also for the dynamic behaviour of the tower.

- **Ground inside the monopile in the foundation:**

If the foundation is taken into account, according to the explanation in the previous element, the ground inside to monopile should be also included in the program with an added mass effect.

- **Stiffness from the ground:**

Again, if the user of the program created selects the foundation as another variable in the system, it is very important to consider the stiffness transmitted by the ground depending on the displacement of the tower. It will be added as a lumped stiffness in the displacement DOF and the value of this stiffness will be discussed later.

- **Stiffness from the cable:**

If this element is chosen, the stiffness contribution of the cable will depend also on the displacement at that point of the structure, so the stiffness added in the displacement DOF will be discussed later in the section about acting loads on the structure.

2.2. Natural Frequencies

The natural frequencies are the frequencies at which the tower tends to vibrate in absence of any damping nor driving force acting.

Then, using the fundamental equation in structural dynamics, the natural frequencies of the tower can be obtained:

$$M\ddot{u} + C\dot{u} + Ku = P(t)$$

where C is the damping matrix and P is the external force which are considered zero for the determination of the natural frequencies.

Thus, the equation to solve would be:

$$M\ddot{u} + Ku = 0 \rightarrow [K - \omega^2 M]\varphi = 0$$

where ω is the natural frequency (eigenvalue) and φ is the mode shape for that natural frequency (eigenvector). So, there will be as many natural frequencies as number of degree of freedom (N) in the structure. However, the most important ones in the final behaviour of the tower are the first modes (lower frequencies) and the program will be based on the three first modes.

Then, once the system stiffness and mass matrix and the natural frequencies are defined, the fundamental equation of dynamics has to be solved to figure out the displacement and rotation along the tower depending on time.

Thanks to the theory found in [4], this equation can be transformed from a system of N equations with N unknows to a system of N independent equations.

The resultant equation will be:

$$\ddot{y} + 2\eta\omega_n\dot{y} + \omega_n^2 y = \frac{P^*}{M^*}$$

where,

u= $\varphi * y$;

η is equal to the damping as a percentage of the critical damping.

P^*= $\varphi^t *$ P;

M^*=$\varphi^t *$ M $* \varphi$;

n is going from 1 to N (one equation for each natural frequency).

This solution is valid for the stationary response of the structure, so the boundary conditions in the code written are the beginning of the simulation in idle state.

The other inputs in the equation are the damping ratio and the external load applied. The damping ratio is used as a fixed value for all the modes in most of the cases of this characteristics (1%) as in [5] for example.

Regarding the external loads, they will be explained and described in the next section.

2.3. Environmental Loads

Offshore wind turbines are exposed to many different loads during their design lifetime and putting all of them together in a unique model while keeping the model reliable can become really difficult. As explained before, the idea of this thesis is to create a model for preliminary design, so this simplified model will not have all the environmental forces included but it will have the most important ones to have an idea of the response of the tower and being able of introducing this design in advanced programs to have further knowledge about the behaviour of the structure with a more realistic model. It means, this model created tries to find a good relation between efficiency in time and reliability of the structure.

Knowing this, the most important loads which have been introduced in the model are:

- Wave loads
- Loads acting in the RNA (Wind Loads, 3P loading, 1P loading)
- Gravitational loads from the RNA
- Loads from the soil
- Guy cable reaction

2.3.1. Wave Loads

In order to determine the loads acting in the tower from the waves, the Morison equation is used [6]. This equation has two terms, the inertia force which come from the potential theory and oscillating flows and the drag force from real flows and constant currents. The total force is:

$$\boxed{F(t) = \frac{\pi}{4}\,\rho\, C_M D^2 \cdot \dot{u}(t) + \frac{1}{2}\,\rho\, C_D D\, \cdot u(t)\,|u(t)|}$$

where the first term is the inertia force per unit length and the second one is the drag force per unit length.

Besides, u(t) in this case is the velocity of the wave and $\dot{u}(t)$ is the acceleration.

The coefficients Cm and Cd were selected according to the API recommendations. It is a simple way to select them because they do not depend on the KC (Keulegan Carpenter) number but they can represent in a good way the behaviour of the wave loads. DNV and Clauss add the effect of the KC number. The suggested drag and inertia coefficient values from API and SNAME are:

Table 2.1. Morison coefficients suggested by API and SNAME.

	Smooth		**Rough**	
	C_D	C_M	C_D	C_M
API	**0.65**	**1.6**	1.05	1.2
SNAME	0.65	2.0	1.0	1.8

The horizontal velocity and acceleration of the wave in each time and depth from the water surface were calculated based on the theory for linear waves:

Horizontal Water Particle Velocity

$$u = \frac{agk}{\omega}\frac{\cosh(kz+kd)}{\cosh(kd)}\cos(kx-\omega t) = a\omega\frac{\cosh(kz+kd)}{\sinh(kd)}\cos(kx-\omega t)$$

Horizontal Water Particle Acceleration

$$a_x = agk\frac{\cosh(kz+kd)}{\cosh(kd)}\sin(kx-\omega t)$$

in which, a is the wave amplitude (m) (wave height divided by 2),

k is the wave number (2*pi/L) and L is the wave length (m),

ω is the wave frequency in radians/s (2*pi/T) and T is the wave period (s),

z is the depth of the analysed point from the still water line (MSL),

d is the water depth from the seabed to the still water line,

x will be zero due to the analysis at one exact point in space (at the tower),

t is time and g is the gravity (9.81m/s^2).

Thus, considering a wave displacement from the still water line of

$$\eta = a * \cos(kx - \omega t)$$

and using the dispersion relation

$$\omega^2 = g * k * \tanh(k * d)$$

it is possible to determine the velocity and acceleration of the wave at each depth and time just knowing three parameters the wave height, wave period and total depth for the still water line.

So, in most of the cases it would be enough to calculate the inertia and drag forces acting, but in the case of a compliant tower, the velocity and acceleration of the structure should be also included in the calculation. Thus, the Morison equation would be:

$$F = \underbrace{\rho V \dot{u}}_{a} + \underbrace{\rho C_a V (\dot{u} - \dot{v})}_{b} + \underbrace{\frac{1}{2} \rho C_d A (u - v) |u - v|}_{c}.$$

where a is the Froude-Krylov force, b is the hydrodynamic mass force, c is the total drag force and Ca is the added mass coefficient and it is related to the inertia coefficient as Cm=1+Ca.

v(t) would be the velocity of the tower, so an iteration process is needed to calculate the final velocity and acceleration at each node of the structure.

Finally, the total force per length was calculated at each node of the structure and it is multiplied by the length of tower which correspond for each node. It will be the length of the element for all the nodes (half the length of the element above and below) except for the node at the seabed (half of the length) and the highest node, where the force was calculated for the crest of the wave and it will be applied in the closest node multiplying by the length corresponding by that node affected by the wave (below the crest).

2.3.2. Rotor-Nacelle Assembly Loads

These loads are probably the most important loads when designing a compliant tower because the forces are acting far from the seabed and the deflections they can cause could lead to the failure of the structure or to exceed the operational values for the power production of the OWT.

The loads acting in the RNA could be summarized in 1P, 3P, 6P, … loads, wind loads, centrifugal forces from the blades, torsional moments on turbine due to an irregular loading of blades or sea ice and icing on turbine blades. The method used to introduce most of these forces has been to develop a simple reference model in the software ASHES considering a totally stiff, no mass and undamped tower. Afterwards, carrying out a batch analysis for the most important cases, the total force and moment acting at the top of the tower can be obtained along time.

However, this method will have some weak points, among them, the relative velocity of the tower will not be included in the resultant force and it could change the final behaviour of the tower due to the fact that this additional force will try to stabilize the tower. But, if we introduce that stabilizing force afterwards, the previous issue can be transformed in an advantage because that force from that simple model will be reliable for a huge number of cases in which the tower will move in a different way.

The non-linearity of the thrust acting on the rotor has been calculated following the next formula [7]:

$$F = \frac{1}{2}\rho\pi R^2 c_T v^2$$

in which ρ is the density of air, R is the rotor radius, c_T is the effective thrust coefficient and v is the velocity of the air. Actually, v is the relative velocity of the wind, but as the tower is considered infinitely stiff, the RNA will not have displacements at the top.

Then, including the velocity of the tower in the equation: $v = v_{wind} - v_{tower}$, the stabilizing force can be calculated as:

$$F_{stab} = F * (-2 * \frac{v_{tower}}{v_{wind}} + \frac{{v_{tower}}^2}{{v_{wind}}^2})$$

The term $\frac{{v_{tower}}^2}{{v_{wind}}^2}$ can be neglected and the resultant force can be considered as the aerodynamic damping exerted by the tower.

Another important factor when defining the wind load, is the turbulence of the wind, because in reality wind does not have a uniform speed along time and space, so, the turbulence value for each wind speed should be defined to introduce it into the software.

The turbulence selected has been based on [8] and it determines different models to represent the turbulence depending on the wind speed (see Figure 2.4.):

$$I(U) = \frac{(15 + aU)}{(1 + a)U} \cdot I_{15}$$

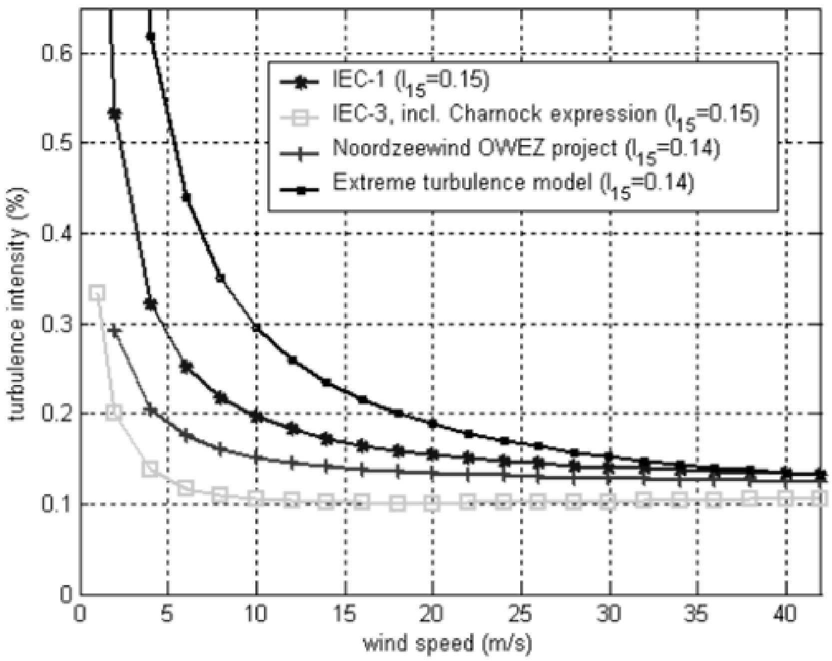


Figure 2.4. Turbulence intensity

The distribution used for this thesis has been the one from the Noordzeewind OWEZ project (I_{15}=0.14 and a=5) because IEC-1 is too conservative and IEC-3 is too much optimistic, so the chosen one would be nice for the consideration of wake effects without being too conservative. The model used for the extreme turbulence model is shown also in Figure 2.4.

2.3.3. Gravitational Load from the RNA

Another load introduced in the code is the bending moment produced by the displacement of the RNA from the vertical axis. The bending moments produced by all the elements are not incorporated to the program because the weight and the distance from the vertical axis is not as significant as in the case of the RNA. This bending moment will be an unfavourable load because it tends to destabilize the tower to higher displacements and stresses. A scheme of this load can be seen in Figure 2.5.

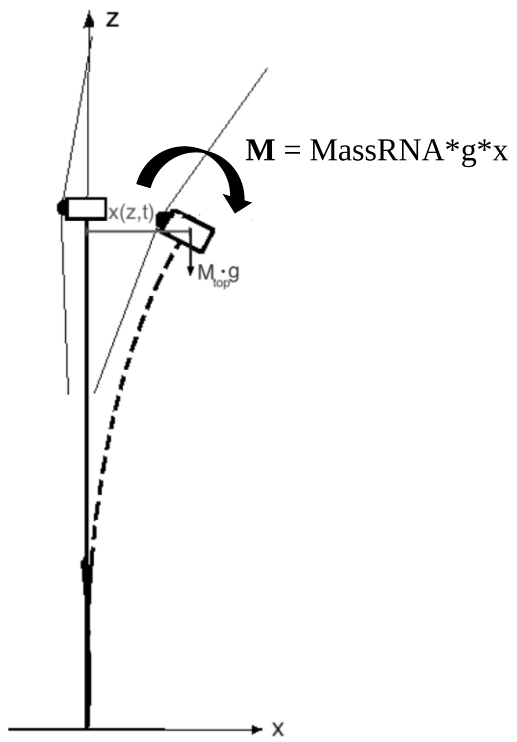


Figure 2.5. Bending moment produced by the RNA selfweight.

This load has been entered in the program considering that the center of gravity of the RNA would be in the top node. It has been considered in that way because the moments produced by the deviation of the center of gravity and other geometrical aspects of the WT has been already introduced by the ASHES software.

2.3.4. Soil-Tower interaction

The soil adds stiffness to the system and it is one of the most effective elements to absorb the forces and moments from the external loads acting on the tower. The soil will be one of the most difficult parameters to represent because it acts in a complex way, but some simplifications will be done to try to get an accurate model of the foundation.

In the code created, there are some inputs parameters to determine which type of representation of the soil the user wants to use. The different types of models are:

- **No foundation:**

It is the simplest way to represent the ground and it considers that the structure is campled and it is totally stiff from the seabed to lower depths. It is not a good way to represent compliant towers because it does not contribute to flexibility and getting low natural frequencies is really hard with this kind of solution. For this solution, the bending moment reaction and force reaction at the base is shown at the end of the execution of the code.

- **Uncoupled springs:**

This solution considers that the soil at the seabed level will have a displacement stiffness and rotational stiffness (see Figure 2.6). It is a good way to represent what is happening above that level but, on the other hand, to get these values is not an easy task.

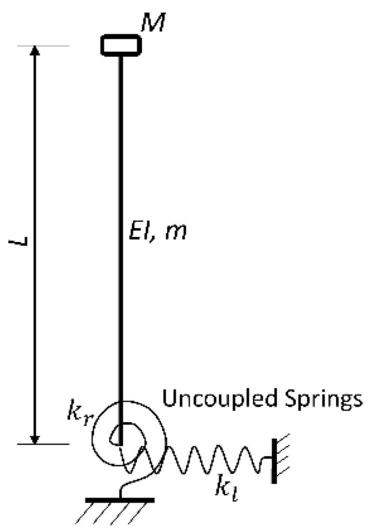


Figure 2.6. Representation of uncoupled springs in the system.

- **Foundation included:**

The stiffness will be introduced following a Winkler model which considers the interaction between soil and tower as a finite amount of springs. The spring stiffness will depend on many factors as characteristics of the soil and depth. One of the main problems of this method is that the shear capacity of the soil is neglected but, anyway, this can be a good way to represent the soil if a huge number of elements are introduced in the system.

Apart from these cases, the bottom of the structure can be selected as a free ending, supported, with uncoupled springs and campled for the cases where foundation is included. Uncoupled springs will not have much sense for this case with foundation, but the code allows to introduce that type of soil. The supported bottom is not really useful because it will not be close to reality. And finally, the doubt is to consider a free-ending or a campled bottom. In reality, it will be a free end but, as the moments and displacements in higher depths are getting lower and lower, it is usual to represent the bottom of the structure campled to ensure that there will not be displacements nor rotations in the lowest point of the structure. So, foundation and campled bottom has been the solution adopted but the user of the Matlab code will be able to change this solution in a straightforward way in the program.

Focusing on the foundation, the procedure to get the stiffness depending on the depth has been carried out following the guidelines of API-2A-WSD [9]. This method is based on load-deflection (p-y) curves and it distinguish between soft clay and sand.

- **Load-Deflection Curves for Soft Clay**

This relation is normally non-linear and it is represented in Table 2.2. This table are for the cases where equilibrium has been reached under cyclic loading, which is the case evaluated.

Table 2.2. p-y curve for soft clay.

$X > X_R$		$X < X_R$	
P/p_u	y/y_c	p/p_u	y/y_c
0.00	0.0	0.00	0.0
0.23	0.1	0.23	0.1
0.33	0.3	0.33	0.3
0.50	1.0	0.50	1.0
0.72	3.0	0.72	3.0
0.72	∞	0.72 X/X_R	15.0
		0.72 X/X_R	∞

where X is the depth below the soil surface,

X_R is the depth below soil surface to bottom of reduced resistance zone,

p is the actual lateral resistance,

p_u is the ultimate resistance,

y is the actual lateral deflection,

$y_c = 2.5 * \varepsilon * D$,

and ε is the strain which occurs at one-half the maximum stress on laboratory unconsolidated undrained compression tests of undisturbed soil samples.

Cyclic loading causes deterioration of lateral bearing capacity below that for static loads. In Figure 2.7. the difference between the static loading and the cyclic loading is shown for a random example.

In the absence of more definite criteria, the following selection of the ultimate lateral resistance is recommended varying from 3c to 9c as X increases from 0 to X_R:

$$p_u = 3c + \gamma X + J\,\frac{cX}{D}$$

$$p_u = 9c \text{ for } X \geq X_R$$

in which c is the undrained shear strength for undisturbed clay soil samples,

γ is the effective unit weight of soil and

J is the dimensionless empirical constant with values ranging from 0.25 to 0.5.

In general, minimum values of X_R should be about 2.5 pile diameters.

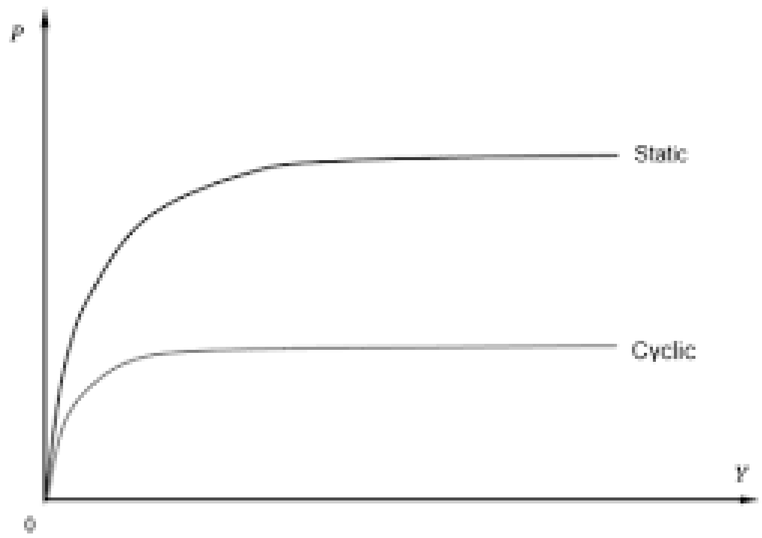


Figure 2.7. Example showing the difference between static and cyclic loading (p-y curves).

- **Load-Deflection Curves for Sand**

The load-deflection curve for sand are also non-linear and with no relevant data, the next expression will represent the behaviour of sand depending on depth.

$$P = A \times p_u \times \tanh\left[\frac{k \times H}{A \times p_u} \times y\right]$$

where A is equal to 0.9 for cyclic loading and

k is the initial modulus of subgrade reaction depending on the angle of internal friction (see Figure 2.8).

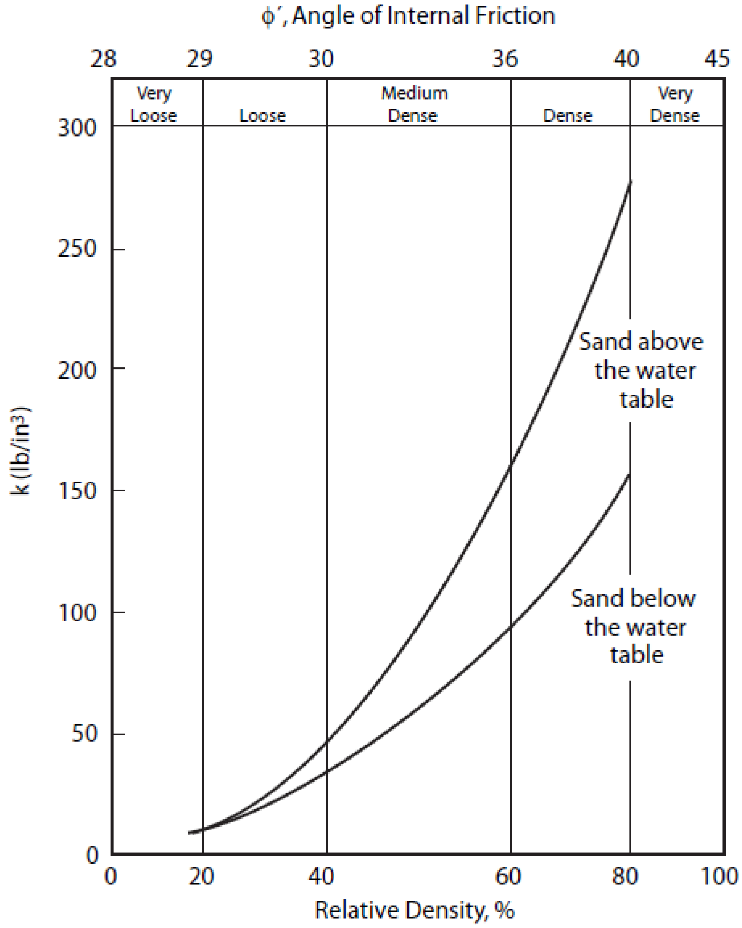


Figure 2.8.Value of k depending on the angle of internal friction.

In this case, the lateral bearing capacity for sand varies following different equations for shallow depths and deep depths, taking the smallest value of them for a given depth:

$$p_{us} = (C_1 \times H + C_2 \times D) \times \gamma \times H$$

$$p_{ud} = C_3 \times D \times \gamma \times H$$

where p_u is the ultimate resistance (force/unit length) and

C_1, C_2 and C_3 are the coefficients as function of the angle of internal friction (see Figure 2.9).

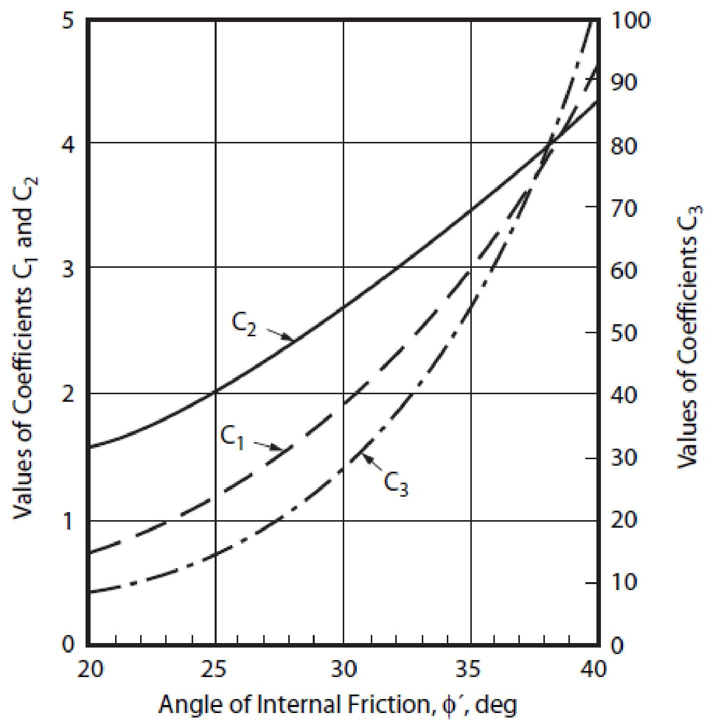


Figure 2.9. Coefficients depending on the angle of internal friction.

Finally, after having defined the p-y curve, the stiffness for the calculation of the natural frequencies must be decided before knowing the displacement produced.

Although the p-y curves describe a nonlinear soil behaviour, the monopile foundation of a real wind turbine is generally working in the linear regime [10]. Therefore, the initial stiffness of the foundation is used in this thesis.

The code will have a section to select the type of soil in the site location and there will be an option to select 2 layers of different properties along the depth of the monopile.

2.3.5. Guy Cable Load

It is also possible the introduction of guy cables in the structure. As in the case of the soil, the guy cable loads will depend on the displacement of the tower at the anchor with the cable. It means, that it will not have a linear relation neither and therefore the initial stiffness contribution will be the one selected for the determination of the natural frequency (as stated for the soil).

The determination of the horizontal force caused by the guy cable in each moment in time can be derived from the next expression [11] explained in Figure 2.10:

$$X = l - l_s + x$$

$$X = l - h\left(1 + 2\frac{T_H}{wh}\right)^{\frac{1}{2}} + \frac{T_H}{w}\cosh^{-1}\left(1 + \frac{wh}{T_H}\right)$$

in which T_H is the horizontal force, w is the weight per unit length of the cable, h is the height from the top of the cable to the seabed, l is the total length of the cable and l_s the suspended length.

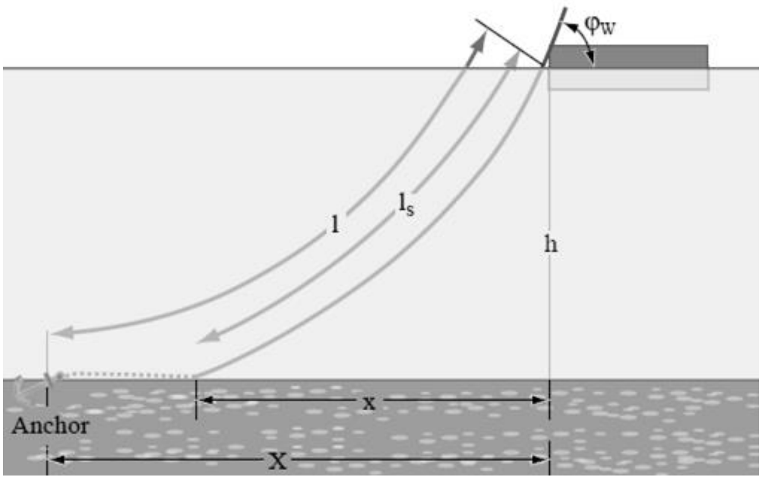


Figure 2.10. Representation of parameters for the calculation of horizontal forces at the cable.

So, the horizontal force at the cable will change depending on the value of X, which will be the initial one plus the displacement of the tower at that point. Furthermore, in reality, there will be more than one cable installed at the tower to act for all the possible direction of external forces. A usual solution is to use three guy cables spaced 120º in plant view. Therefore, the resultant force acting for each displacement of the tower will not depend only on one cable but also in the whole system of cables (because one displacement will cause a higher force acting in the cable opposite to the external loads and a lower force in the other cables in the direction of the displacement). Then, a solution has been developed geometrically for three cables and introduced in the code. The case studied is the one where one of the cables is in the direction of the external forces because it will cause the maximum possible tension at that cable.

External forces acting on the cable will be neglected because they will not be significant for the results and it will make the problem much more complex.

Besides, the election of the type of cable is important for the efficiency of the system and to reduce displacements in an appropriate way. The most common solution for water depths below 100 m are chains acting just by gravity, so this will be the type of cable used.

Chains are chosen for these depths because they are heavy, they have high breaking strength, high elasticity and no bending effect. These are the reason why a chain system is more efficient that wires and high-tech fibres for compliant towers.

2.4. Requirements and Guidelines

The code, after calculating the resulting forces and bending moments in each node, will check if the ultimate limit state (ULS), the fatigue limit state (FLS), the serviceability limit state (SLS) and the cable tension will be satisfied, and for that purpose, different guidelines will be followed like NORSOK, IEC and DNV.

2.4.1. Ultimate Limit State

The ULS has been studied according to NORSOK N-004 [12]. The firs requirements to fulfil are the thickness of tubular members above 6mm and a relation of the diameter divided by the thickness below 120.

In NORSOK, if there is no specification about the material factor, a factor of 1.15 will be selected.

Afterwards, there are several checks for different kinds of loads like compression forces, shear forces, bending moments and the combination between them.

- **Axial compression:**

Axial tensile loads will not be analysed because the weight from the rotor-nacelle assembly will cause that the predominant forces acting are compressive loads.

Then, tubular members subjected to axial compressive loads should satisfy:

$$N_{Sd} \leq N_{c,Rd} = \frac{Af_c}{\gamma_M}$$

where

N_{Sd} is the design axial force obtained from the program,

f_c is the characteristic axial compressive strength,

A is the cross sectional area and

γ_M is the safety factor defined before.

In order to define f_c, the following expressions are used:

$f_c = [1.0 - 0.28\lambda^2] * f_y$ for $\lambda \leq 1.34$

$f_c = \frac{0.9}{\lambda^2} f_y$ for $\lambda > 1.34$

$$\lambda = \sqrt{\frac{f_{cl}}{f_E}} = \frac{kl}{\pi i}\sqrt{\frac{f_{cl}}{E}}$$

in which f_{cl} is the characteristic local buckling strength,

λ is the column slenderness parameter,

f_y is the characteristic yield strength

f_E is the Euler bucking strength,

k is the effective length factor,

l is the unbraced length and

i is the radius of gyration

The effective length factor for campled-free beams is usually 2, but for being in the side of security against buckling, a factor of 2.1 has been selected.

The characteristic local buckling strength should be calculated from:

$$f_{cl} = f_y \qquad \text{for } \frac{f_y}{f_{cle}} \leq 0.170$$

$$f_{cl} = \left(1.047 - 0.274\frac{f_y}{f_{cle}}\right)f_y \qquad \text{for } 0.170 < \frac{f_y}{f_{cle}} \leq 1.911$$

$$f_{cl} = f_{cle} \qquad \text{for } \frac{f_y}{f_{cle}} > 1.911$$

and

$$f_{cle} = 2C_eE\frac{t}{D}$$

where f_{cle} is the characteristic elastic local buckling strength and

C_e is the critical elastic buckling strength = 0.3

- **Bending moment:**

Elements should be checked against bending loads following this condition:

$$M_{Sd} \leq M_{Rd} = \frac{f_m W}{\gamma_M}$$

where M_{Sd} is the bending moment,

f_m is the characteristic bending strength,

W is the elastic section modulus and

γ_M will be specified later.

The characteristic bending strength should be:

$$f_m = \frac{Z}{W}f_y \qquad \text{for} \qquad \frac{f_yD}{Et} \leq 0.0517$$

$$f_m = (1.13 - 2.58(\frac{f_yD}{Et}))(Z/W)f_y \qquad \text{for} \qquad 0.0517 < \frac{f_yD}{Et} \leq 0.1034$$

$$f_m = (0.94 - 0.76(\frac{f_yD}{Et}))(Z/W)f_y \qquad \text{for} \qquad 0.1034 < \frac{f_yD}{Et} \leq 120f_y/E$$

in which

$W = \frac{\pi}{32}\frac{(D^4-(D-2t)^4)}{D} = elastic\ section\ modulus$

$Z = \frac{1}{6}(D^3 - (D-2t)^3) = plastic\ section\ modulus$

The condition for the last case ($\frac{f_y D}{Et} \leq 120 f_y/E$) is due to the fact that the outer diameter divided by the thickness should be below 120 (as stated before), but this condition has not been introduced in the program to allow the calculation of the behaviour of the tower without fulfilling this requirement.

- **Shear:**

The elements of the simulation subjected to beam shear forces should satisfy:

$$V_{Sd} \leq V_{Rd} = \frac{Af_y}{2\sqrt{3}\gamma_M}$$

where V_{Sd} is the design shear force

and the material factor is equal to 1.15.

- **Axial compression and bending moment:**

The following conditions should be fulfil at all cross sections along the length of the tower:

$$\frac{N_{Sd}}{N_{c,Rd}} + \frac{1}{M_{Rd}}\left(\frac{C_m M_{Sd}}{1 - \frac{N_{Sd}}{N_E}}\right) \leq 1.0$$

and

$$\frac{N_{Sd}}{N_{cl,Rd}} + \frac{M_{Sd}}{M_{Rd}} \leq 1.0$$

where

C_m is the moment reduction factor considered 1 for this case,

N_E is the Euler buckling strength,

$N_{cl,Rd} = \frac{f_{cl}A}{\gamma_M}$ = design axial local buckling resistance and

$N_E = \frac{\pi^2 EA}{(kl/i)^2}$

- **Shear and bending moment:**

The interaction between shear forces and bending moments should satisfy that the direction of the shear and the moment vector are orthogonal within ±20º. It can be satisfied with the following formulas:

$\frac{M_{Sd}}{M_{Rd}} \leq \sqrt{1.4 - \frac{V_{Sd}}{V_{Rd}}}$ for $\frac{V_{Sd}}{V_{Rd}} \geq 0.4$

$\frac{M_{Sd}}{M_{Rd}} \leq 1.0$ for $\frac{V_{Sd}}{V_{Rd}} < 0.4$

- **Material factor:**

The material factor will be 1.15 for shear and it will be defined for the following formulas for other cases different from shear forces:

$\gamma_M = 1.15$ for $\lambda_s < 0.5$

$\gamma_M = 0.85 + 0.60\lambda_s$ for $0.5 \leq \lambda_s \leq 1.0$

$\gamma_M = 1.45$ for $\lambda_s > 1.0$

where

$\lambda_s = \frac{|\sigma_{c,Sd}|}{f_{cl}}\lambda_c$, considering no hydrostatic pressure,

$\lambda_c = \sqrt{\frac{f_y}{f_{cle}}}$ and

$\sigma_{c,Sd} = \frac{N_{Sd}}{A} + \frac{M_{Sd}}{W}$

2.4.2. Fatigue Limit State

In [12], basic information is given about fatigue analysis requirements, but more detailed information is given in [13] with respect to fatigue design.

The aim of fatigue design is to ensure that the structure has an adequate fatigue life. The design fatigue live for the structure components should be based on the structure service life specified by the operator, but if no structure service life is given by the operator, a fatigue service life of 15 years should be used.

The FLS should be analysed along all the elements of the structure because every joint or other type of stress concentration can be a source of fatigue cracking. This program created will make a basic fatigue analysis, but it will be enough for a preliminary design of the structure.

First of all, the number of load cycles shall be multiplied with the appropriate factor in Table 2.3 before the development of the fatigue analysis.

Table 2.3. Design Fatigue Factors.

Classification of structural components based on damage consequence	**Access for inspection and repair**		
	No access or in the splash zone	**Accessible**	
		Below splash zone	**Above splash zone**
Substantial consequences	10	3	2
Without substantial consequences	3	2	1

'Substantial consequences' means that the failure of the joint will entail danger of loss of human life, significant pollution or mayor financial consequences.

'Without substantial consequences' means that the structure satisfies the requirement to damaged condition according to the accidental limit state (ALS) with failure in the actual joints as the defined damage.

Welds in joints below 150 m water depth should be assumed inaccessible for in-service inspection.

For this thesis, substantial consequences are considered due to the mayor financial consequences of a failure by fatigue.

Then, the fatigue study should be based on S-N curves, which measure the number of cycles needed for the fatigue failure for a determined stress range.

The analysis based on fracture mechanics will be used in further analysis of the structure and it will not be covered in the program created for the analysis.

In order to get the fatigue life, the assumption of linear cumulative damage can be used (Palmgren-Miner rule).

So, the accumulated fatigue damage D would be:

$$D = \sum_{i=1}^{k} \frac{n_i}{N_i} = \frac{1}{\bar{a}} \sum_{i=1}^{k} n_i \cdot (\Delta\sigma_i)^m \leq \eta$$

where

$\bar{a}$ = intercept of the design S-N curve with the log N axis

m= negative inverse slope of the S-N curve

k = number of stress blocks

n_i = number of stress cycles in stress blocks

N_i = number of cycles to failure at constant stress range $\Delta\sigma_i$

η= usage factor = 1 / Design Fatigue Factor

So, the methodology followed to figure out the accumulated fatigue damage will be done in these steps:

1) Determining the stress range distribution expressed by a stress histogram, having at least 20 stress blocks to ensure a reasonable numerical accuracy. This stress distribution is obtained thanks to the *rainflow.m* function from the Matlab web. It calculates the number of cycles for each stress block and the associated stress amplitude and mean stresses.

2) Modifications have been done in the code to obtain the stress range and the number of cycles applying the factor in Table 2.3.

3) S-N curves are drawn for a mean stress of zero, but in reality, predominant forces of compliant towers will be bending moments varying among high values, causing a mean stress different from zero. So, corrections for the mean stress should be applied. Formulas for that correction are expressed for example by Soderberg, Goodman or Gerber [14] (see Figure 2.11).

4) Selection of the parameters $\bar{a}$ and m of the S-N curve which will define the fatigue design.

5) For each block of constant stress range, the fatigue damage was calculated with the formula shown before.

6) Once, the damage of each constant stress range, all the damages are added according to Palmgren-Miner.

7) The damage obtained will be the damage corresponding to the time analysed in the simulation, so this damage should be adapted to the time those external loads are acting on the structure and afterwards, it will be possible to determine how many years there will be until the failure of the structure by fatigue. These years should be higher than the fatigue service life selected (15 years).

8) Repeat this calculation for all the nodes of the tower.

The corrections for the mean stress are done according to the Haigh’s diagrams which is based on the Goodman equation as can be seen in the figure below.

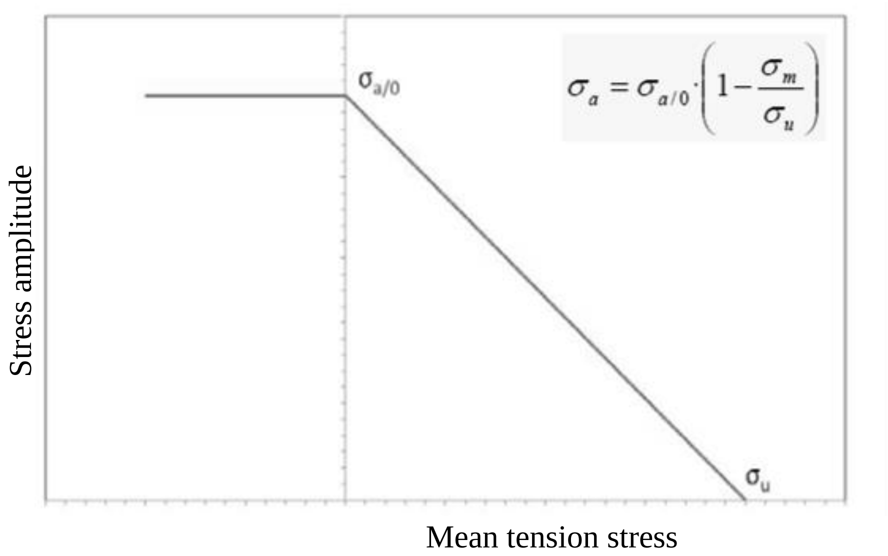


Figure 2.11. Effect of the mean stress on the fatigue design.

Thus, as can be seen in the previous figure, for tensile mean stresses, the real damage produced in the structure will be higher. However, for compressive mean stresses that difference in the amplitude selected could be the same than for zero mean stresses. Therefore, as there will be mean stresses acting in compression due to the compression force exerted by the weight of the RNA, the stress range selected for the calculation of the fatigue damage will be the same than for zero mean stresses.

2.4.3. Serviceability Limit State

For the serviceability limit state (SLS) for compliant towers for offshore wind turbines no relevant data has been found, so approximate values from previous experience from both floating and bottom fixed WT projects have been selected [15].

These values are related to the inclination of the tilt at the tower top with respect to the vertical axis (without including the initial geometric tilt of the nacelle) and to the maximum horizontal acceleration at the tower top.

- **Inclination of tilt:**
  - Operational cases: Limited to 5 deg for the mean value and 10 deg for the maximum value achieved.
  - Non-operational load cases: 15 deg as maximum value along the time analysed.

- **Maximum acceleration:**
  - Operational load cases: The limitation will be 0.3g for the maximum value obtained.
  - Non-operational load cases: It is limited to 0.6g for the maximum value obtained.

in which g is the gravity acceleration equals to 9.81 m/s^2.

However, could be possible that for a specific environmental condition, one or more of these limits would be exceeded. In those cases, there are two main options than can be carried out.

One of them is to stop the WT operations for those cases in which some limit is exceeded. Consequently, this case should be taken out of the analysis of the annual production of the wind turbine.

Another option is to assess the impact of that situations in the loads transferred to the structure in order to quantify them and conclude whether that overload is significant or not.

2.4.4. Design Load Cases

One of the most important parts of the design phase is to evaluate the environmental conditions to convert them into the significant load cases studied for the limit states.

The different DLCs that can be selected are found at Table 1 in NEK IEC [16].

The program developed has been designed to calculate the most critical FLS and ULS considered, it means, for DLC 1.2 and DLC 6.1a, respectively (see Table 2.4). But changing the inputs in the program and introducing other values for the external loads, the program offers the possibility to analyse other DLC.

Table 2.4. Design Load Cases selected according to IEC.

Design situation	DLC	Wind condition	Waves	Wind and wave directionality	Sea currents	Water level	Other conditions	Type of analysis	Partial safety factor
1) Power production	1.2	NTM $V_{in} < V_{hub} < V_{out}$	NSS Joint prob. distribution of H_s, T_p, V_{hub}	COD, MUL	No currents	NWLR or ≥ MSL		F	*
6) Parked (standing still or idling)	6.1a	EWM Turbulent wind model $V_{hub} = k_1 V_{ref}$	ESS $H_s = k_2 H_{s50}$	MIS, MUL	ECM	EWLR		U	N

2.4.5. Load Factors

The election of the load factors can be chosen from different guidelines and they will depend on the limit state studied. There are two different approaches when calculating the design load effect (Figure 2.12). The first approach consists on the multiplication of the load factor times the characteristic load effect calculated after the simulation and the second one multiplies the load factor times the characteristic load and the simulation is carried out afterwards. The approach selected for the program has been the second approach because it will represent in a better way the non-linear behaviour of some parts of the structure which is the case of the guy cables and the foundation.

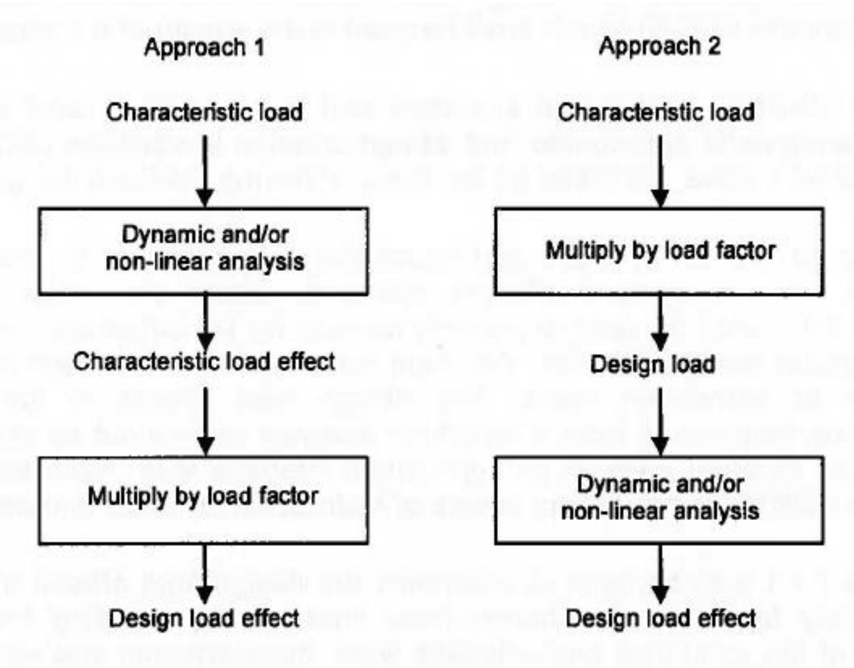


Figure 2.12. Types of approach for applying the load factor.

The load factors depending on the limit state will be:

- **Ultimate Limit State:**

For instance, the recommended values for the ULS at IEC are the following ones depending on the type of design situation for unfavourable loads or favourable loads (Table 2.5):

Table 2.5. Design load factor according to IEC.

Unfavourable loads			Favourable[12] loads
Type of design situation (see Tables 1 and 2)			All design situations
Normal (N)	Abnormal (A)	Transport and erection (T)	
1,35*	1,1	1,5	0,9

However, as it was explained before, the design load factors selected are defined in another guideline (DNV) [17]. Following this document, the load factors for ULS are in Table 2.6.

Table 2.6. Load factors for ULS.

Combination of design loads	Load categories			
	G	Q	E	D
a)	1.3	1.3	0.7	1.0
b)	1.0	1.0	1.3	1.0
Load categories are: G = permanent load Q = variable functional load E = environmental load D = deformation load For description of load categories see Sec.2. (at DNV)				

Combinations a) and b) should be considered in both operating and extreme conditions. In the program designed, only the combination b) is analysed because the environmental loads will be the most important load acting along this type of structure and therefore the higher coefficient for this kind of load will be decisive. If other coefficients are required for the analysis, they will be easy to change in the code.

Deformation loads are the loads caused by inflicted deformations like loads at the foundation and loads at the guy cable.

- **Fatigue Limit State:**

The load factor in the FLS is 1.0 for all load categories.

- **Serviceability Limit State:**

The load factor in the SLS is 1.0 for all load categories.

2.4.6. Cable Strength

For the calculation of the integrity of the guy cable the data from [11] is used. It says that the breaking strength for the chain is:

$$CBS = c * (44 - 0.08D)D^2 \qquad (N)$$

where D is the diameter in mm and c is obtained from Table 2.7.

Table 2.7. Values of c for the calculation of the Catalogue breaking strength.

Grade (specification)	c
ORQ	21.1
3	22.3
3S	24.9
4	27.4

Once the breaking strength has been calculated, the maximum admissible tension should be obtained applying a safety factor. The different safety factors depending on the regulations followed are (See Table 2.8):

Table 2.8. Safety factor coefficients for the guy cable.

IACS		**API RP 2SK**	
Condition	**Safety factor**	**Condition**	**Safety factor**
Intact	1.8	Intact	1.67
One line removed	1.25	One line removed	1.25
Transient	1.1	Transient	1.05

in which the safety factor is the breaking strength divided by the maximum tension allowed at the cable.

For IACS (International Association of Classification Societies) safety factors: for survival conditions for operating conditions, these safety factors are increased by about 50 %.

3. Analysis and Discussion

In this section, the analysis for the environmental conditions of a specific site location has been studied and a discussion about the different results obtained depending on the input parameters has been carried out.

3.1. Initial data

Before starting running the Matlab code, some initial parameters should be decided beforehand. These parameters correspond to the material used for the tower, the environmental characteristic loads and many other values related to the RNA, the water depth and geometry of the tower. Most of the data can be found in the NREL study [5], the Upwind Design Basis [8] and [18].

3.1.1. Tower Properties

The material used for the calculation has been steel with Young's modulus of 210 GPa and shear modulus of 84 GPa. The normal value of steel's density is 7,850 kg/m3 but, a density of 8,500 kg/m3 has been selected to take into account bolts, the paint, welds and flanges not included in the thickness of the tower.

Regarding to the characteristics selected for the WT, they can be seen in Table 3.1.

Table 3.1. Properties for the NREL 5-MW Wind Turbine.

Rating	5 MW
Rotor Orientation	Upwind
Configuration	Blades
Rotor Diameter	126 m
Shaft Tilt	5º
Rotor Mass	110,000 kg
Nacelle Mass	240,000 kg

So, the total added mass to the top node is 350,000 kg and the inertia has been calculated taking into account the geometry of the RNA observed in Figure 3.1.

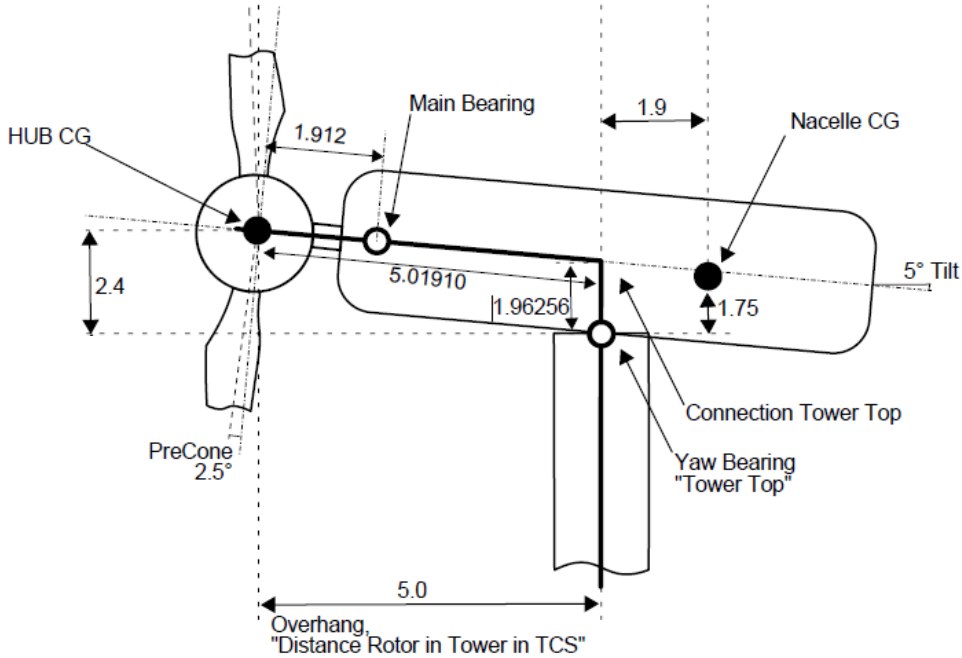


Figure 3.1. RNA used in this thesis based on [18].

The rotor frequency range (1P) lies between 0.115 Hz and 0.202 Hz, so the 3P frequency will be between 0.35 and 0.61 Hz. These frequencies come from the interaction of the rotation of the blades with the tower structure and the 6P, 9P, and so on frequency are also happening in the system but the impact of this frequencies in higher natural frequencies are not so significant in the results obtained, so just the 1P and 3P frequencies are studied.

The geometry of the tower will be one of the input parameters to introduce in the program, so the design of all the parameters may be modified to find the best solution to the site location selected. However, some logical restrictions must be considered like the distance from the RNA to the highest wave predicted at the site, which should be above the distance of the radius of the rotor plus a safety distance.

Besides, the different parts of the compliant tower introduced in the system are (Figure 3.2):

- **Tower structure:**

This will be higher part of the compliant tower, going from the RNA to the substructure. The diameter and thickness will not be constant along the height of the tower, so they will vary linearly after introducing the diameter and thickness at both the bottom and the top of the tower.

- **Substructure/Transition piece:**

The substructure will go from some meters above the MSL (to be decided by the user of the program) to the seabed. The diameter and thickness through the transition piece will be constant.

- **Monopile:**

It goes from the seabed level to the length that has been introduced for the monopile underground (foundation). The diameter and the thickness will be also constant along the length.

However, if the geometry of the structure needs to be different from the stated above, the code can be modified easily to change the vector of the final diameter and thickness and make calculations according to these values.

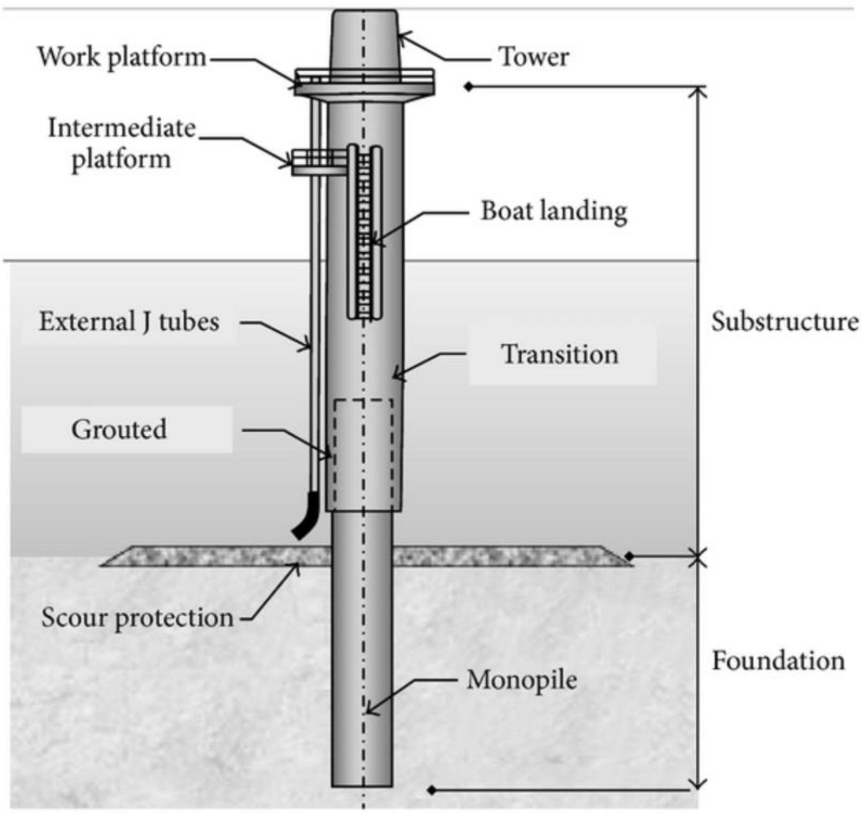


Figure 3.2. Parts of the OWT used for the simulation.

3.1.2. Site Location and Environmental Loads

With regard to the site location and the environmental loads, they are obtained from [8]. The offshore site is located in the Dutch North Sea (see Figure 3.3) and a water depth of 50 m has been assumed (which is in the range of depths where the compliant tower structure can be profitable). The name where the data was obtained from is 'K13-Alpha 3' (53º13'04'' north and 3º13'13'' east).

The water density considered for the seawater is 1,025kg/m^3.

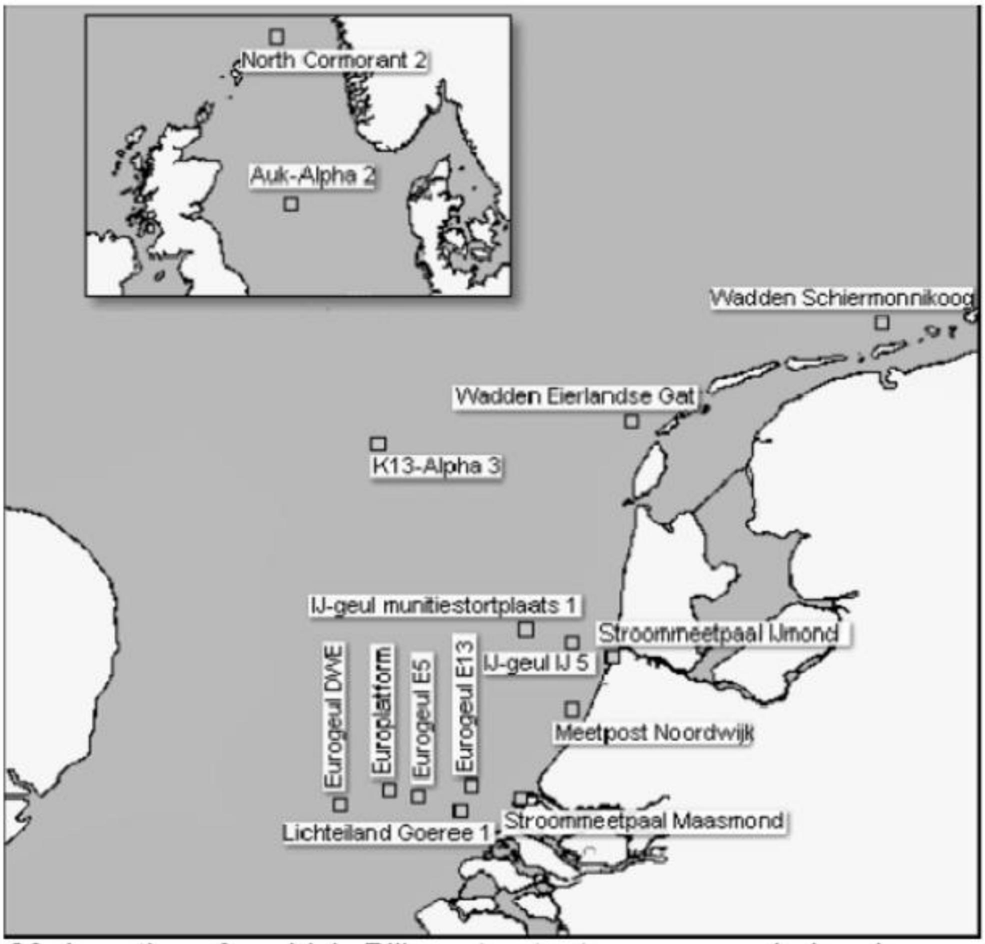


Figure 3.3. Locations where measurements of wind and wave data are obtained [8]

Afterwards the assessment of the loads at this site location is executed by the Upwind Design Basis document for the deep water site and the characteristic loads acting in the tower will be:

- **Wave Loads:**

The relation obtained between wave height and return period for extreme values was found to be:

$$H_{s,3hrs}(T_{return}) = 0.6127 * \ln(x) + 7.042$$

And the relation between the maximum wave height and significant wave height is:

$$H_{max} = 1.86H_s$$

Finally, the wave period associated with the maximum wave heights are in the range of:

$$11.1\sqrt{H_s/g} \leq T \leq 14.3\sqrt{H_s/g}$$

Assuming the lower limit to give the most severe loading condition, the following significant wave height and wave period has been obtained for a return period of 50 years (Table 3.2). If other period between these values is required, it is easy to change in the code before the simulation.

Table 3.2. Extreme wave heights for 50 years of return period.

T_{return} (years)	H_s (m)	H_{max} (m)	$T(H_{max})$ (s)
50	9.4	17.48	10.87

Apart from that, the effect of breaking waves will be neglected because the water depth is high, and therefore the wave height will be far from the breaking limit. Thus, linear wave theory has been selected for the calculation of velocities and accelerations of the wave (as stated in the methodology section).

- **Wind Loads:**

Extreme values for wind speeds will follow the next expression:

$$V_{hub,10min}(T_{return}) = 2.5536 * \ln(x) + 32.736$$

According to [16], the 10 minute wind speeds should be multiplied by a factor of 0.9 to get the value for 3 hours stationary situations (Table 3.3).

Table 3.3. Conversion between extreme wind speeds of different averaging periods.

Averaging period	10 min	1 h	3 h
Correction factor relative to extreme 10-min average wind speed	1,00	0,95	0,90

So, the final wind speed for 50 years return period is 42.73 m/s and this velocity will be the one introduced in the ASHES software for the calculation of loads transmitted to the tower.

The wind speeds used here are at hub height.

For mean values for the FLS, in the offshore industry wave climate data is generally expressed in a 2-dimensional scatter diagram giving the number of occurrences of each combination of significant wave height Hs and peak spectral period Tp. For offshore wind turbine design the 2-D scatter diagram must be expanded to include Vw as a third dimension. To derive the 3-D scatter diagram, the parameters Hs and Tp and Vw will be used. The wind and wave data is subsequently gathered in bins. The Vw bins cover 2 m/s.

Finally, the results obtained at [8] for the given offshore site for the different wind bins are (see Table 3.4):

Table 3.4.Lumped scatter diagram of the given offshore site for sorted wind bins.

V [ms]	TI[%]		Hs [m]	Tp [m]	f [%]	occ./year [hrs]
	normal	extreme				
2	29,2	99,3	1,07	6,03	0,06071	531,8
4	20,4	53,1	1,1	5,88	0,08911	780,6
6	17,5	37,1	1,18	5,76	0,14048	1230,6
8	16	30	1,31	5,67	0,13923	1219,7
10	15,2	25,4	1,48	5,74	0,14440	1264,9
12	14,6	22,3	1,7	5,88	0,12806	1121,8
14	14,2	20,1	1,91	6,07	0,10061	881,3
16	13,9	18,5	2,19	6,37	0,07554	661,7
18	13,6	17,2	2,47	6,71	0,04878	427,3
20	13,4	16,1	2,76	6,99	0,03151	276,1
22	13,3	15,3	3,09	7,4	0,01924	168,6
24	13,1	14,6	3,42	7,8	0,00977	85,6
26	12	14	3,76	8,14	0,00474	41,6
28	11,9	13,5	4,17	8,49	0,00243	21,3
30	11,8	13,1	4,46	8,86	0,00093	8,2
32	11,8	12,7	4,79	9,12	0,00053	4,6
34-42	11,7	12,3	4,9	9,43	0,00019	1,6

So, to make a preliminary design, not all the values for wind speed are introduced in ASHES but the most representative for the whole situation of the structure to try to make it damage equivalent to the previous Table. The results introduced are in Table 3.5.

Table 3.5. Wind speeds introduced in ASHES and their corresponding wave load cases and frequency of occurrence.

V [ms]	TI[%]		Hs [m]	Tp [m]	occ./year [hrs]
	normal	extreme			
8	16	35	1,31	5,67	5027.6
14	14,2	24	1,91	6,07	2664.8
20	13,4	19	2,76	6,99	1034.9

The extreme case will be also introduced in ASHES for a wind speed of 42.73 m/s and a turbulence intensity of 13%.

3.2. Natural Frequencies and Mode Shapes

One of the outputs of the program will be an image of the first three natural frequencies and mode shapes which will be considered for the design of the structure.

An example showing the results for a campled solution is shown below in Figure 3.4.

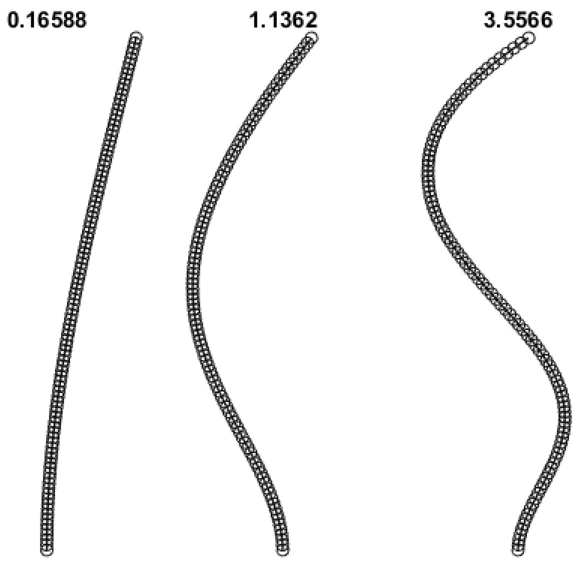


Figure 3.4. Natural frequencies (Hz) and mode shapes for a random case (first three mode shapes from left to right).

Then, to see how the structure response to dynamic loads depends on the natural frequencies, different cases with different load cases will be shown.

3.2.1. Static vs. Dynamic Loads

In order to analyse the difference between static and dynamic loads using the program, a load of 1 MN is applied at the top of the structure to see the deflections caused by it. The deformation of the tower can be seen in Figure 3.5. for the same case as shown in Section 3.2.

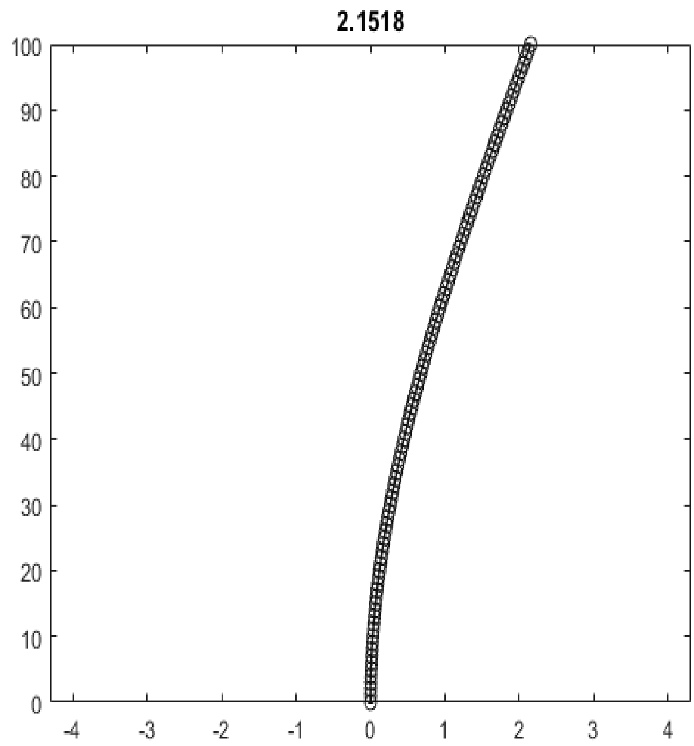


Figure 3.5. Displacements at the top for a static load in m.

So, the maximum displacement will occur at the top of the tower and the values will be of 2.15 m.

However, if a cyclic load of 1 MN is applied at the top of the structure, the displacement will depend on the dynamic amplification factor (DAF) and it will be higher when the frequency of the external force approximates to the natural frequency of the system (see Figure 3.6).

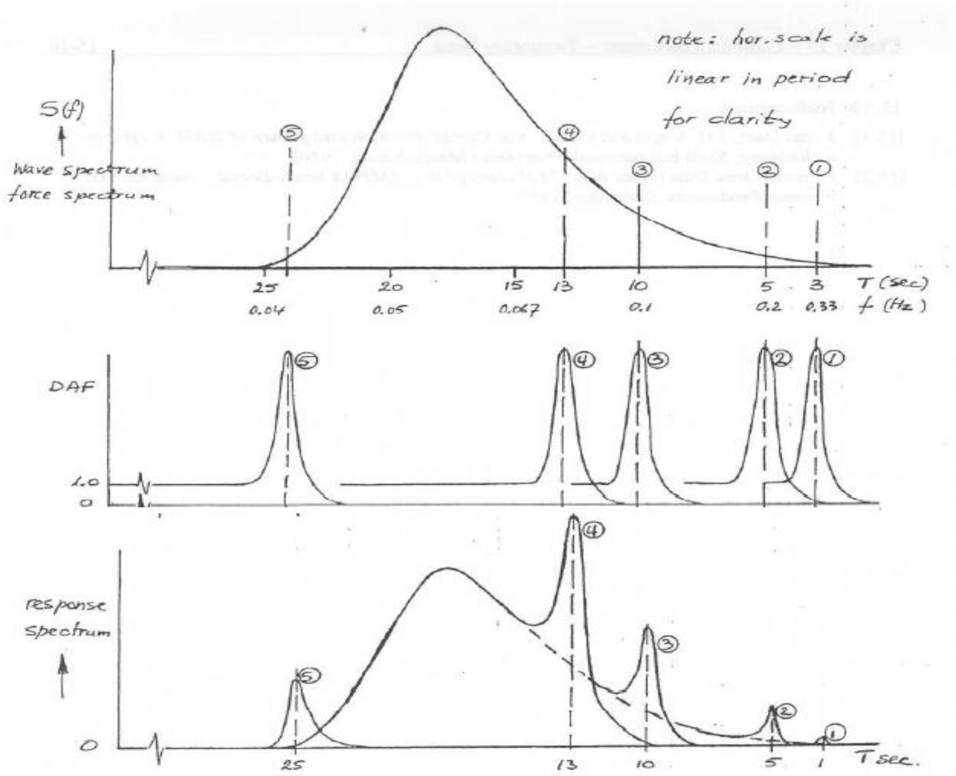


Figure 3.6. Effect of natural frequency on DAF.

So, knowing that the first natural frequency is 0.166 Hz and the second one is 1.136, a comparation of cases has been done when the external load frequency is close to the first natural frequency (0.17 Hz) and when it is between natural frequencies (0.55 Hz). The results of the displacement along time are in Figure 3.7 and Figure 3.8.

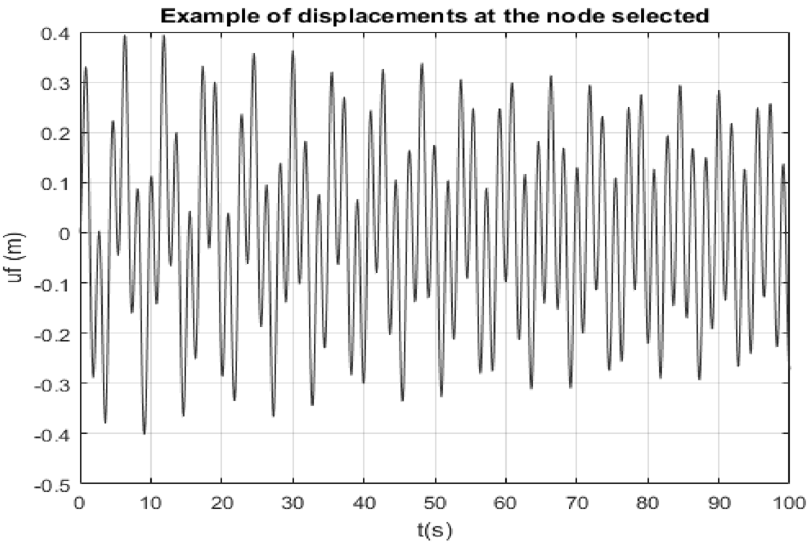


Figure 3.7. Displacements of the top of the tower for a load frequency of 0.55 Hz.

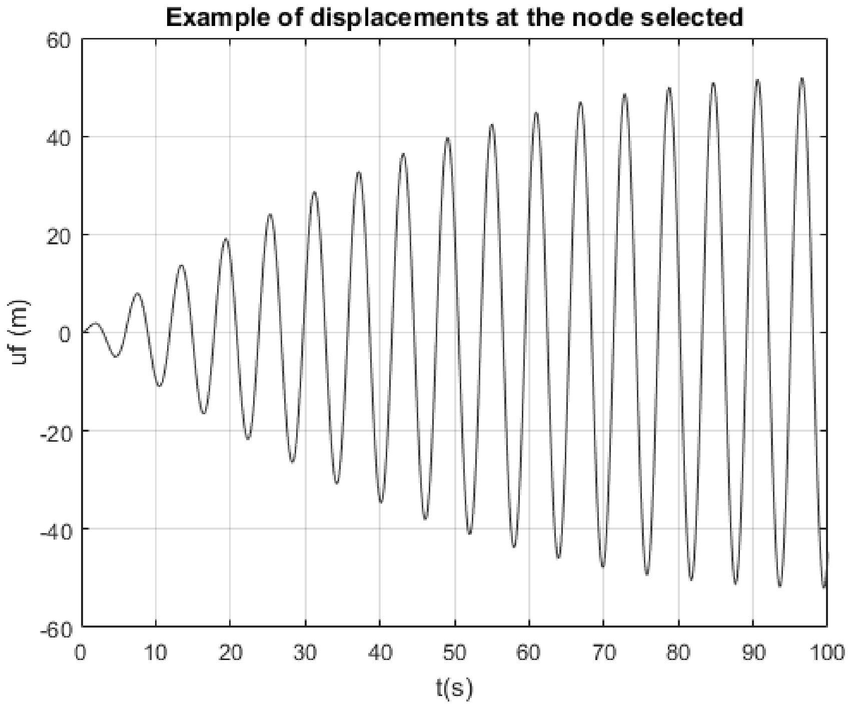


Figure 3.8.Displacements of the top of the tower for a load frequency of 0.17 Hz.

The difference between both cases is really high. In the case of 0.17 Hz of external force, the DAF is close to the maximum, so the displacement at the top will increase too much and the resultant forces acting on the structure will exceed the maximum allowed value for the integrity of the structure (just the damping is acting on the structure).

The maximum DAF can be calculated with the next formula:

$$DAF = \frac{1}{2 * \eta}$$

where the ratio of the damping with respect to the critical damping η is 1%, so the maximum DAF would be 50 for the case where the external load frequency and the natural one coincides.

For the case where the external load frequency is 0.55, the maximum displacement at the top is less than in the static case and it is due to the fact that the first natural energy is the one with more impact on the final behaviour of the structure and if the external load frequency is higher than the natural frequency, the mass will be dominant over the loads and the DAF will be lower than 1 (tends to 0). For cases with the external load frequency lower than the first natural frequency, the stiffness will be dominant over loads and the DAF will tend to go closer to 1.

Although the first natural frequency is the most important frequency for the final behaviour of the structure, forces acting close to other natural frequencies could also cause the failure of the structure. That is the reason why the three first natural frequencies are analysed in this program.

3.2.2. Frequency allowance for OWT

Knowing that the frequencies of the external loads should be as far from the natural frequencies as possible, the structure should be designed to avoid that coincidences.

Then, after calculating the frequencies of the external loads, the compliant tower should be designed to avoid that frequencies.

In the next figure (Figure 3.9), a representation of the main loads frequencies acting and the natural frequencies for the compliant tower searched is shown:

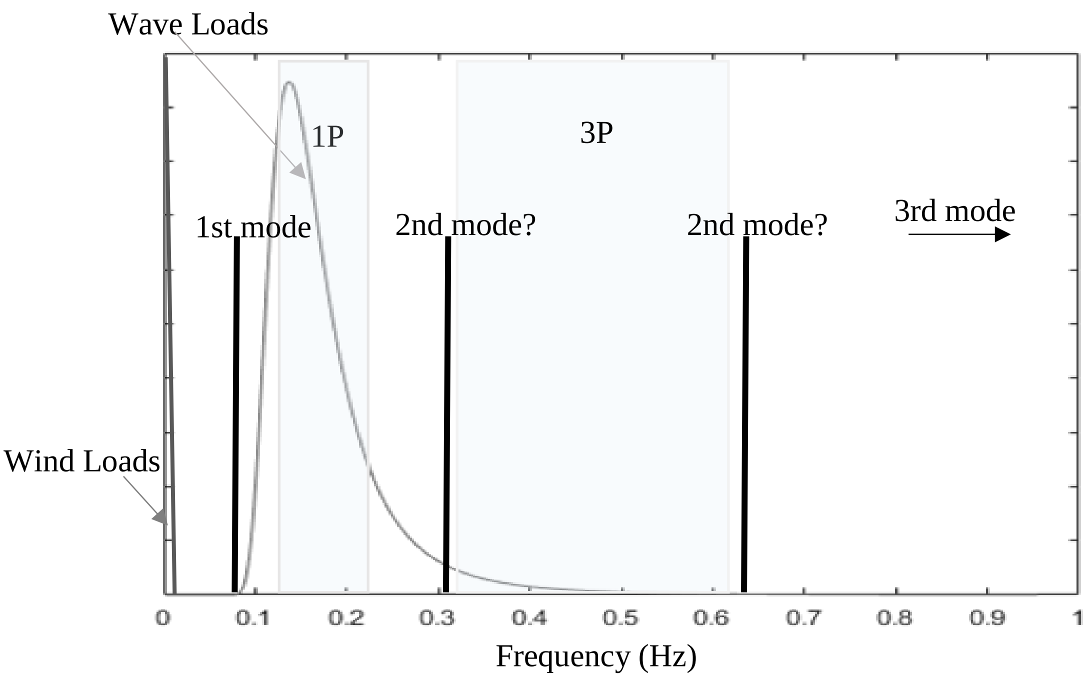


Figure 3.9. Frequencies acting on the structure and possible solutions for the natural frequencies of the compliant tower.

The final configuration of the natural frequencies for the compliant tower should be below the 1P loading and the wave loads frequencies for the first natural frequency. For the second natural frequency, it should be above or below the 3P loading (depending on the results) and the third natural frequency should be above the 3P loading.

3.2.3. Effect of Input Parameters in the Natural Frequencies

The combination of all the parameters should cause that the natural frequencies of the tower lies in the specified ranges. For that purpose, a previous knowledge of how the different parameters affect the natural frequency should be carried out (see Table 3.6).

Table 3.6.Sensitivity analysis of main parameters.

Parameter	Initial value (m)	% changed	1st natural freq (0,136 Hz)	Variation in 1st nat. freq. (%)	2nd natural frequency (0,896 Hz)	Variation in 2st nat. freq. (%)
Length	140	10%	0.11729	-13.8017	0.80131	-10.5910
D top tower	4	10%	0.1358	-0.1984	0.90754	1.2620
t top tower	0.02	10%	0.1355	-0.4189	0.89843	0.2455
D bottom tower	5.5	10%	0.13693	0.6320	0.91938	2.5830
t bottom tower	0.03	10%	0.13598	-0.0661	0.89819	0.2187
D substructure	5.5	10%	0.15405	13.2138	0.92998	3.7658
t substructure	0.03	10%	0.14176	4.1817	0.91366	1.9448
wdepth	50	10%	0.13539	-0.4997	0.8472	-5.4707

The main parameters are represented in the table above, although there are many factors apart from these parameters that will affect the natural frequencies of the material.

Besides, not only the natural frequencies should be considered on the design of the compliant tower, but also the resultant bending moments and stresses at the structure which will be the ones used for the checking of stability at the different limit states. So, maybe a reduction in thickness could improve the behaviour of the tower but that reduction will cause higher stress at specific nodes in the structure.

The previous table is calculated considering the structure campled at the seabed, so other parameters as the stiffness given by the soil will be also determinant when designing a compliant tower.

Therefore, the different types of soil solutions are analysed to see the differences in natural frequencies.

- **Campled solution:**

It is the solution used for the parameter study, so the main natural frequencies will be 0.136, 0.896 and 2.263 Hz.

- **Uncoupled springs:**

The stiffness used for the calculation are based on [19] and they are k11=24.5 MN/m and k22=4,130 MNm, corresponding to the transversal displacement DOF and rotational DOF, respectively. The characteristic of the soil used for the calculation and the monopile are in Table 3.7.

Table 3.7. Soil characteristics.

Upper Layer	**Soft clay**	2 m thick
	cu	20 kPa
	Effective unit weight	8 kN/m3
Sublayer	**Dense Sand**	-
	Angle of friction	38º
	Effective unit weight	11 kN/m3
Monopile	Length	26
	Diameter	3.2
	Thickness	0.035

The results obtained from this model are: 0.0805, 0.514 and 1.373 Hz, so they are considerably below the campled solution and they are close to the natural frequencies the compliant tower needs.

- **Foundation:**

Here, the difference between the different types of soil (sand and soft clay) and the campled solution is analysed. The selection of the characteristics of the monopile and the different soils are shown in Table 3.8.

Table 3.8. Characteristics selected for sand, soft clay and the initial values for the monopile.

Clay		**Sand**	
Parameter	**Value**	**Parameter**	**Value**
Saturated unit weight	19 kN/m3	Saturated unit weight	19 kN/m3
c_u	20 kPa	Angle of friction	38º
J	0.5	C1	3.8
X_R	2.5*D	C2	4
ε	0.01	C3	76
		ksand	120 lb/in3
Monopile			
Length	35		
Diameter	5		
Thickness	0.035		

The differences between both soils with regard to natural frequencies of the structure are in Figure 3.9.

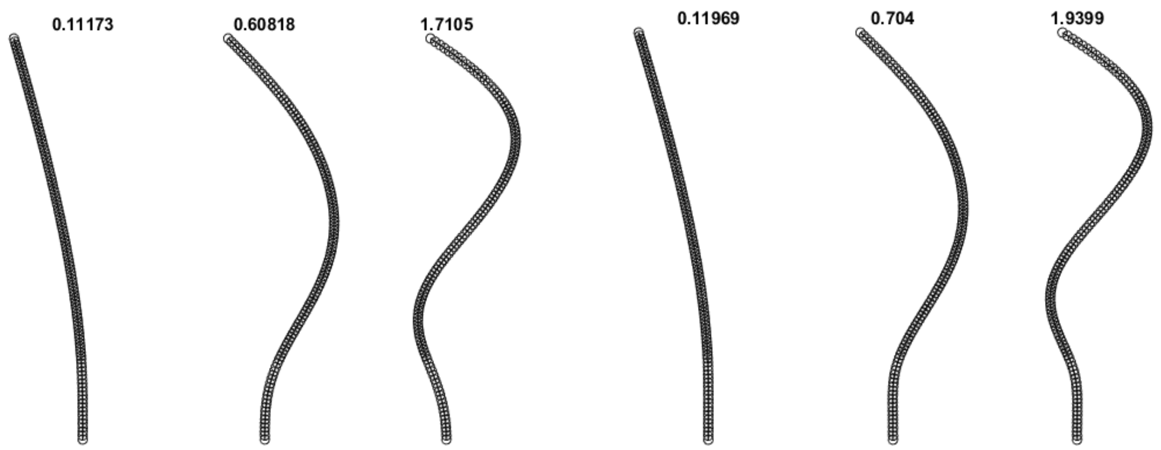


Table 3.9. Natural frequencies for soft clay (left) and sand (right).

As could be expected, the solution in clay foundations are less stiff and therefore, the natural frequencies will be a bit lower.

3.3. Load-Deflection Curves

As it was mentioned in the methodology, the first displacements of the p-y curve of the soil will determine the stiffness of the soil given to the system.

Analysing different cases (sand and soft clay) the results for the p-y curves obtained are shown in Figure 3.11). The parameters for both soils have tried to be realistic for a normal case and they are shown in Table 3.8.

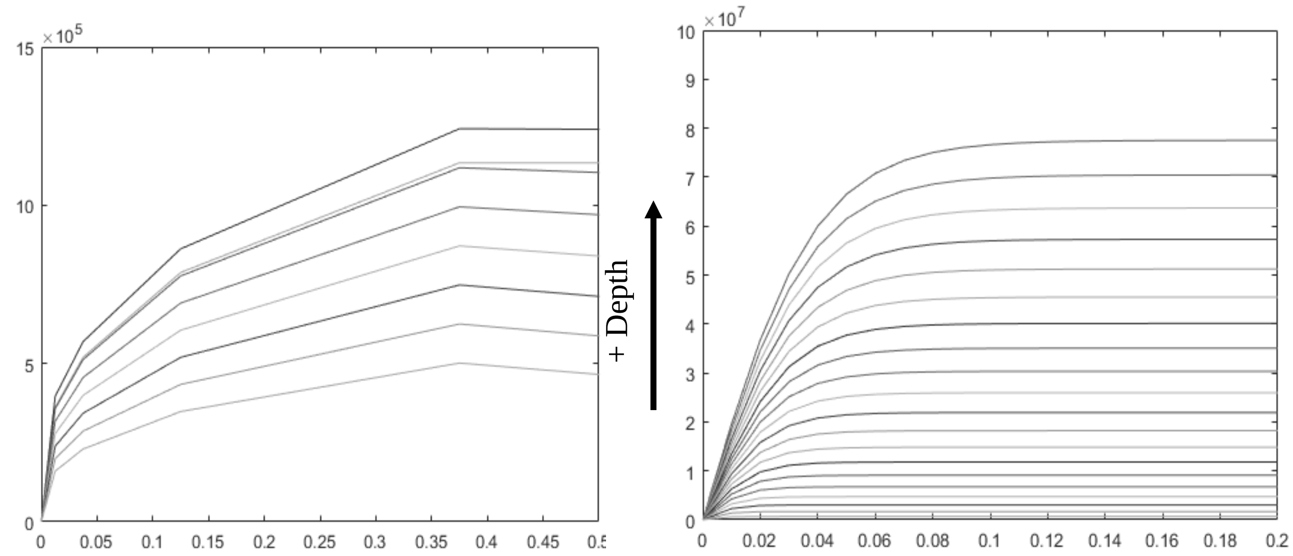


Figure 3.11. p-y curves for soft clay (left) and sand (right).

The analysis of both soils has been carried out with the same amount of elements at the monopile because it will determine the force exerted at each node. The differences between both types of soil are obvious due to the high difference to reach the highest pressure at the soil. Besides, the displacement needed to reach the highest pressure is larger in the case of clay and that is the main reason why the stiffness of the clay will be less (less slope at the beginning of the curve). However, this slope will not be so different at the beginning in both cases and the results obtained just show a small difference reflected in the natural frequencies obtained.

In the graph for the soft clay, there are less curves drawn, and it is because for a depth higher than X_R, the p-y curve will be constant. On the other hand, the curve for the sand will increase with depth according to the formula shown in the methodology.

In addition to the soil conditions, the guy cable will also give some stiffness to the system. In this case, the p-y curve will have a different shape (Figure 3.12) so the total horizontal restoring force acting in the tower from all the cables will increase more each time when the displacement is increased.

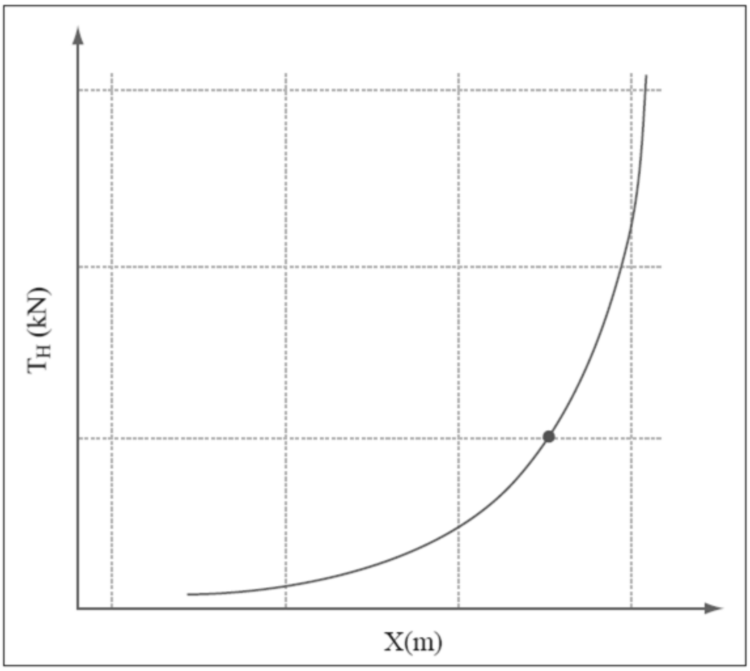


Figure 3.12. Force-Displacement curve for the guy cable.

3.4. Stabilization of the Compliant Tower

At the beginning of the computation (for instance an extreme case), the tower will take a short time to stabilize itself. It is due to the fact that extreme forces are acting from the idle position of the tower without forces to the position of equilibrium for that forces in a short period of time. It causes that the displacements, and consequently velocities and accelerations vary considerably before it reaches the equilibrium position (see Figure 3.13). For that reason, it is usual to take out the first 120 seconds of simulation. Besides, the total time of the simulation will be 720 seconds and after taking out the first 2 minutes, there will be 10 minutes left for the analysis which will be used to evaluate the ULS and the FLS. This time will increase considerably the running time of the program, but it will be needed to have accurate values related to Fatigue Limit States.

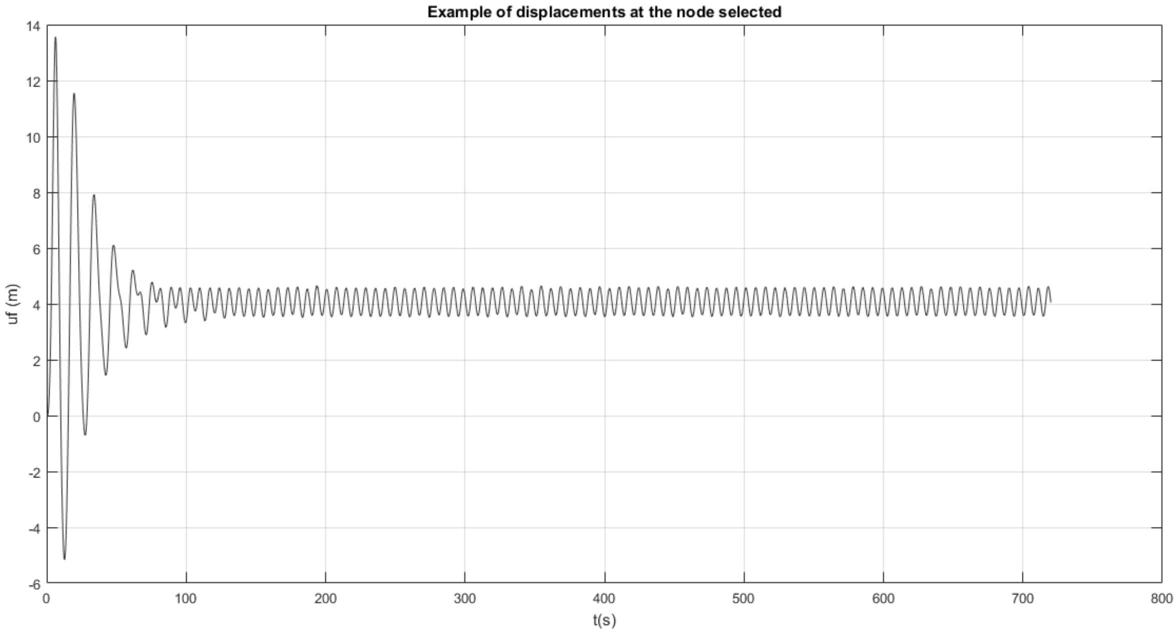


Figure 3.13. Displacements at the top considering the total time simulated.

3.5. Bending Moments and Shear Forces

Along 10 minutes of simulation, the bending moments and shear forces will be analysed for all the nodes at the tower and a FLS and ULS calculation will be performed.

An example of the reactions at the seabed will be shown in Figure 3.14 and 3.15 for a campled solution.

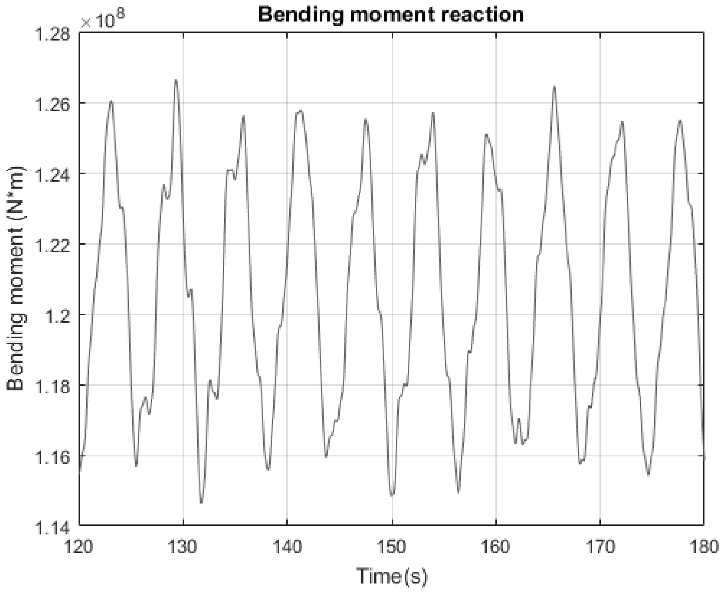


Figure 3.14. Bending moment reaction at the seabed level for a campled solution.

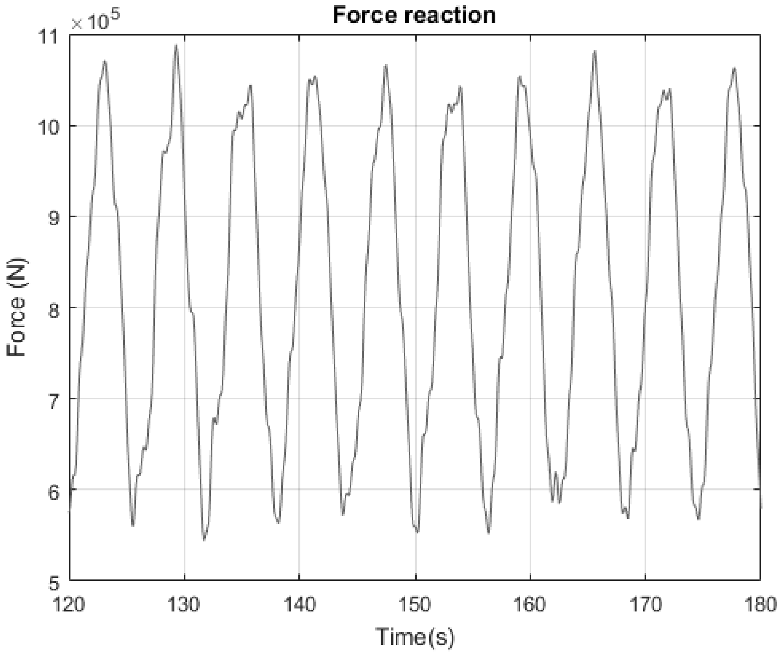


Figure 3.15. Force reaction for the campled solution.

For the ULS, the maximum reaction loads in each node for the whole period will be compared with the maximum allowed loads and the interaction between different kinds of loads.

For the FLS, the rainflow counting has been carried out to distribute the stress ranges in different blocks. The results from the rainflow counting at the base is shown in Figures 3.16, 3.17, 3.18 and 3.19. With these results, the FLS has been done in the program using a simple S-N curve with parameters m=4 and a=10^10 without endurance limit because of the marine environment.

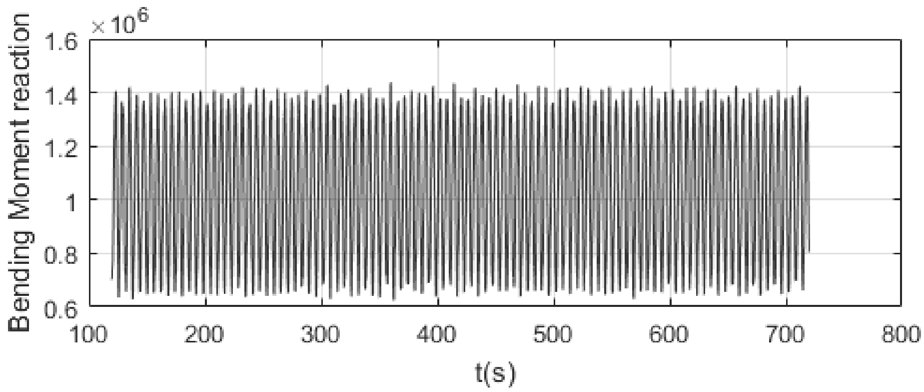


Figure 3.16. Bending moment load effect in a node close to the seabed for a case with foundation included.

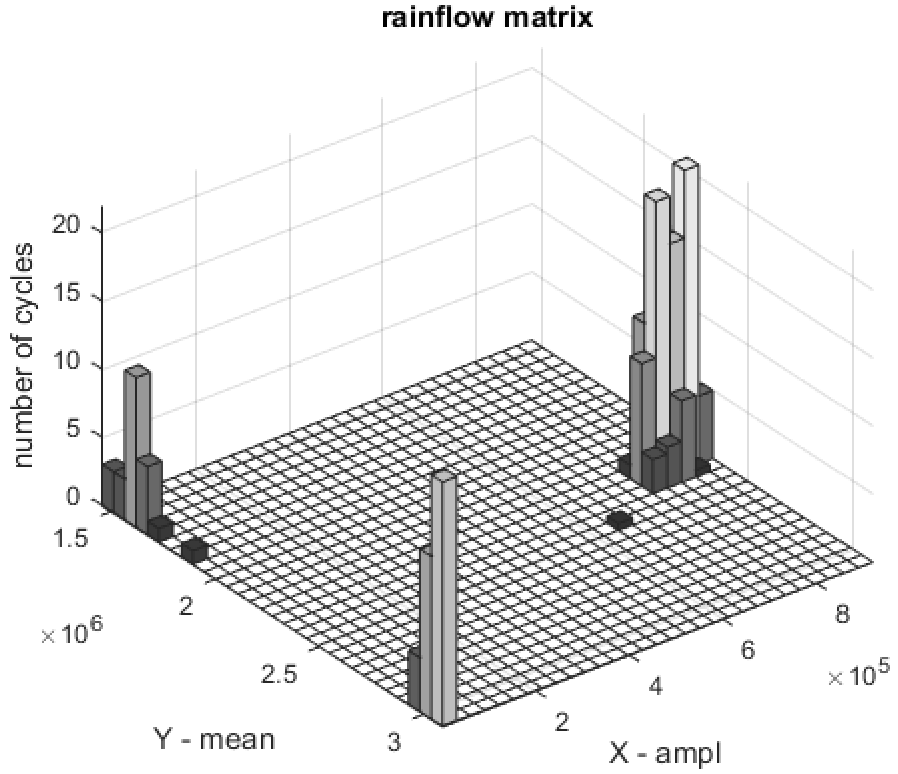


Figure 3.17. Rainflow matrix for the amplitud and mean of stresses and number of cycles.

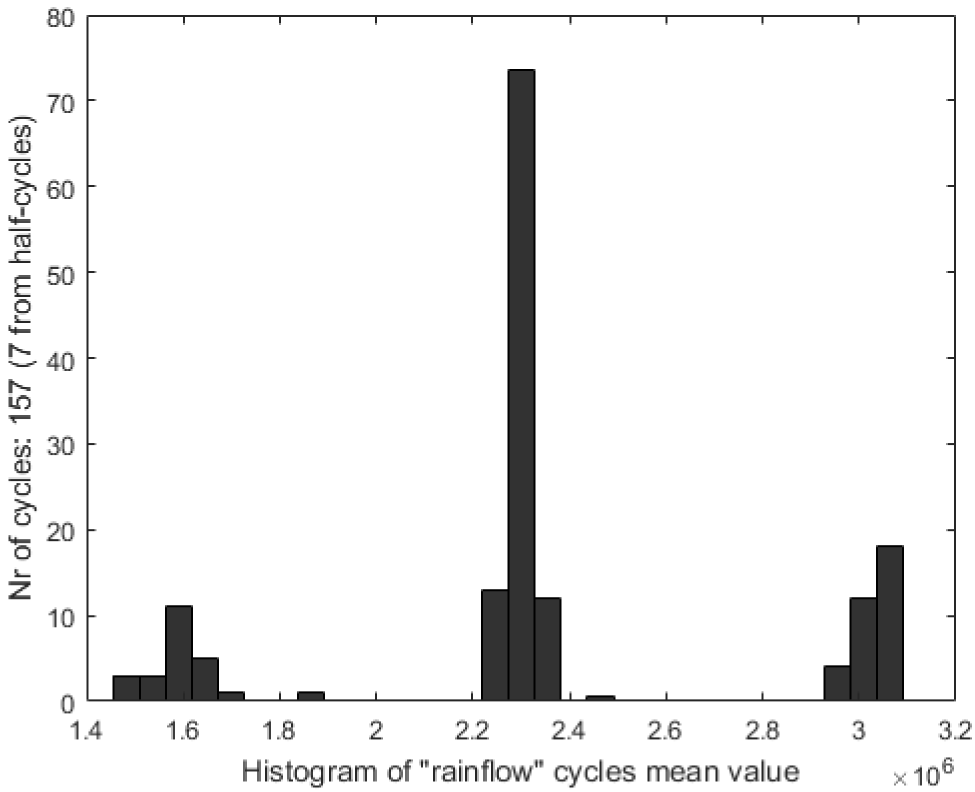


Figure 3.18. Mean value of stresses and number of cycles obtained from the rainflow counting.

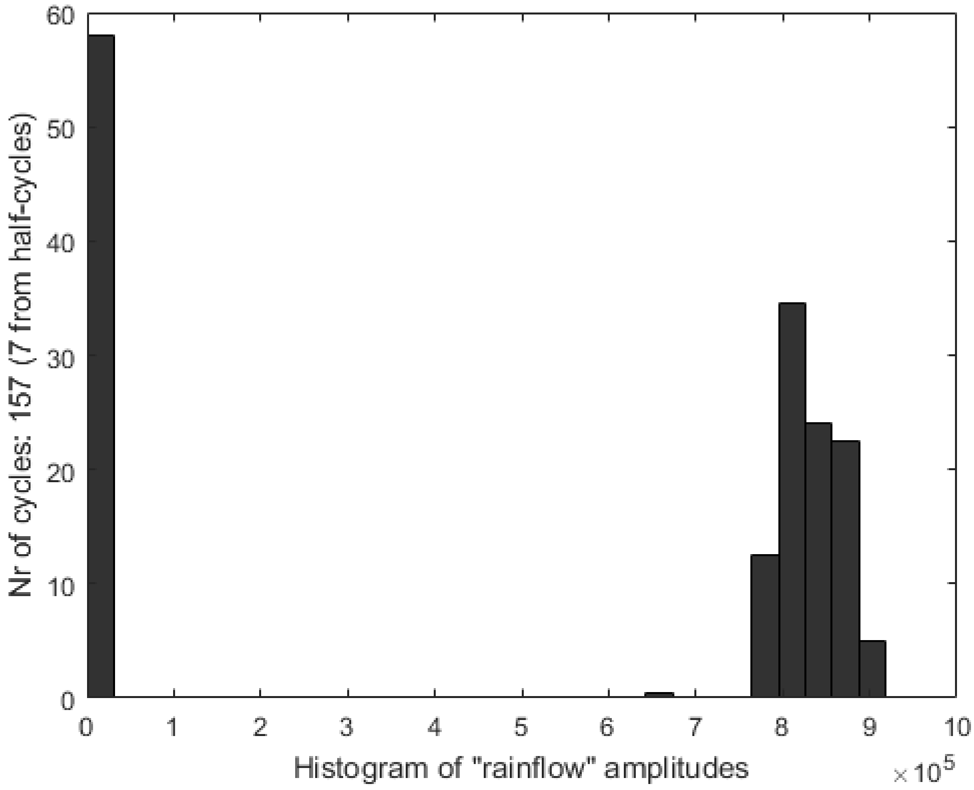


Figure 3.19. Stress amplitude and number of cycles for each stress amplitude block.

These data make sense because the bending moment reaction in Figure 3.16 are really similar, so the amplitude will be more or less the same for all the simulation. And the small variation of this bending moment reaction is represented for the cases of small stress amplitude ranges.

These are the reasons why the rainflow matrix has two different locations for amplitudes.

Once the FLS is done, the results will show if the FLS is satisfied for the specific case analysed, so, if more cases are carried out for the FLS analysis, the data from the damage should be exported to Excel (for example) and all the cases simulated should be added to determine whether the total sum of cases at each node is still below 1 or not.

3.6. Preliminary Calculations

Several tests have been done changing the input data to try to figure out the best solution of a compliant tower at water depth 50 m and with sand in the foundation.

Reducing the diameter of the structure to make it slenderer and increasing the length a bit above the minimum height possible (100 meters above the water depth), the first frequency stands in the position it should (around 0.08 Hz). Instead, the second frequency lies in the zone of 3P (around 0.5 Hz) and it makes that the structure rise above the recommended values for operational issues in some cases.

In order to try to reduce the second natural frequency, a mass trap has been considered a few meters below the splash zone. It will consist in a element added to the system to take into account the added mass around the mass trap also. It reduces the second frequency for high diameters of the mass trap but this increasement in diameter will produce higher loads acting because of the waves thanks to the higher surface that the waves hit when it goes through the tower. Thus, this element will be a possible input into the program, but it is not a recommended solution. It could be more useful for jacket structures where the jacket can just be closed in some parts to take into account the water inside.

Apart from that, the guy cable influence will be also analysed, and the results shows that the displacements are reduced efficiently if the relation between horizontal distance and total length of the cable is selected in a good way. It is caused because it can have almost null influence in the stiffness for the calculation of the natural frequencies but, at the same time, it can act with a high restoring force for a bigger displacement. One negative point is the influence in the acceleration because it will increase due to the fact that the cable will stop the movement of the structure sooner. Results of displacements at the top comparing 2 different configurations for the guy cable is shown in Figure 3.20.

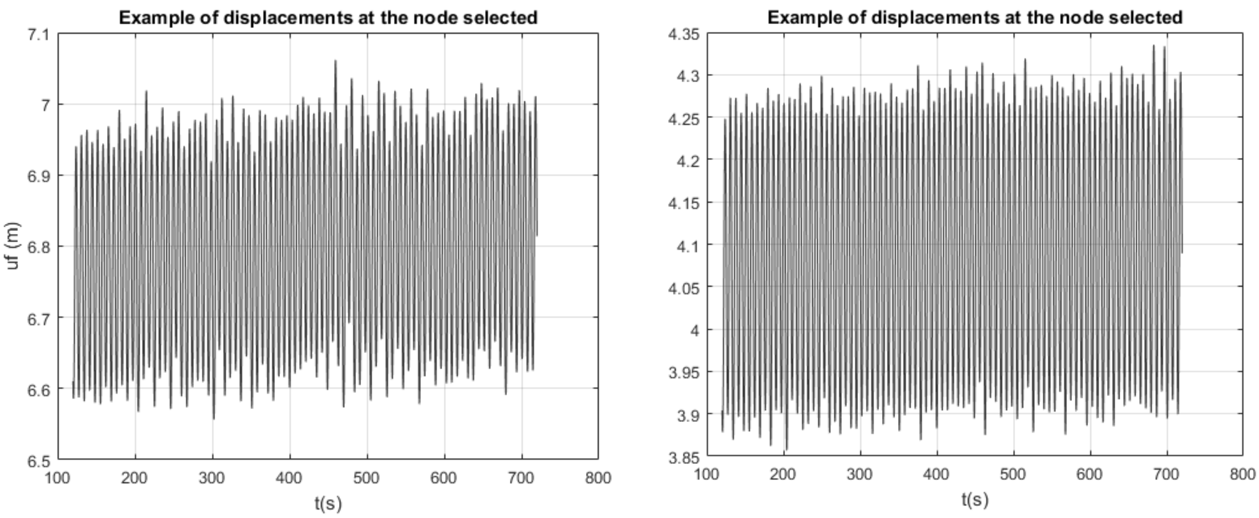


Figure 3.20.Displacements at the top for a horizontal distance of the cable of 470 m (left) and 490 m (right).

After having knowledge of all the parameters with significant impact on the results, the compliant tower has to be designed to satisfy all the checks.

4. Results

In this section the final design for a specific case is explained and the outputs from the software are shown.

4.1. Suggested Design

The specific site location is based on the assumption that the soil is formed by clay and the water depth is 50 m.

The design of the compliant tower in order to check ULS, FLS and SLS is shown in Table 4.1.

Table 4.1.Final design of the compliant tower.

Parameter	Value
Length from the seabed	150 m
D top tower	3.5 m
t top tower	0.020 m
D bottom tower	4.5 m
t bottom tower	0.030 m
D substructure	4 m
t substructure	0.030 m
Length monopile	50 m

Parameter	Value
D monopile	4 m
t monopile	0.034
Height of the transition piece (from the MSL to the tower)	20 m
Cable height	70 m
Cable horizontal distance	485 m
Cable total length	500 m
Diameter of the cable	100 mm
Specification of the cable	4

The number of elements selected for the tower and substructure are 100 elements and for the monopile 50 elements have been selected.

The restricted relation mentioned in the methodology between diameter and thickness is not fulfilled in this tower design, so further analysis should be done about buckling. However, this requirement was not fulfilled to achieve a higher slenderness and being able to reduce natural frequencies.

The cable height has been decided to be anchored above the MSL and avoiding the zone where waves could hit it directly. Locations above the MSL will enable the easy maintenance of this element. Although on the other hand, operation of ships close to the zone should take care of the cable.

4.2. Behaviour of the Compliant Tower

Entering the previous data for the simulation of the Matlab code the result obtained are the following:

- **Natural frequencies**:

The natural frequencies for the solution selected are (see Figure 4.1):

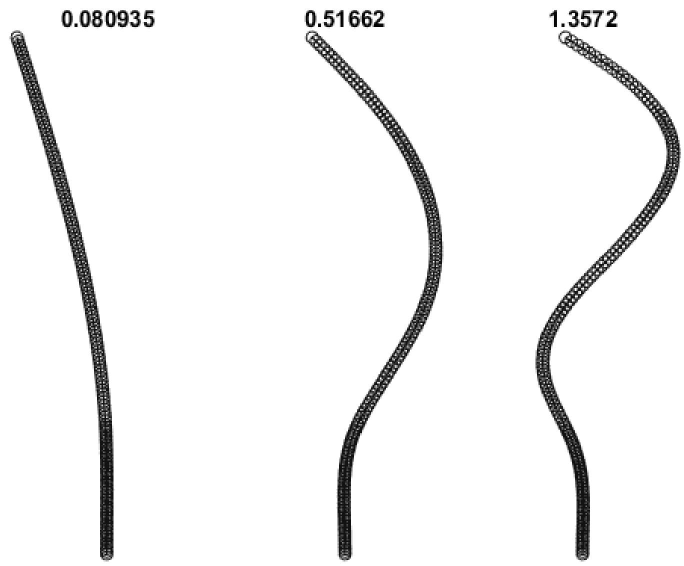


Figure 4.1. Natural frequencies for the design selected.

- **Displacements at the cable anchor:**

The displacements at the anchor of the guy cable is shown in the next graph (see Figure 4.2).

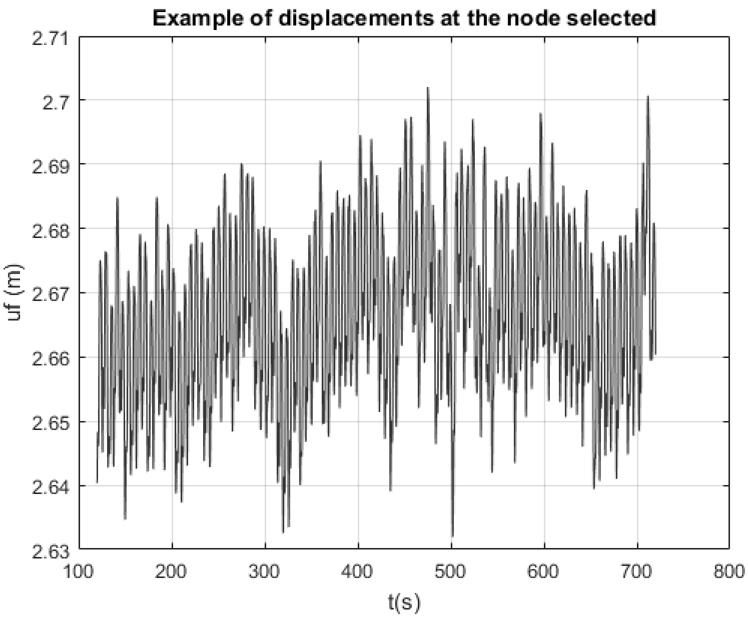


Figure 4.2. Displacements at the Cable for the design solution.

- **Displacements, speeds and accelerations at the top:**

The most important serviceability conditions are for the data at the RNA (top node). Then, the relation between displacements, speeds and accelerations along the first 10 minutes of simulation are (see Figure 4.3):

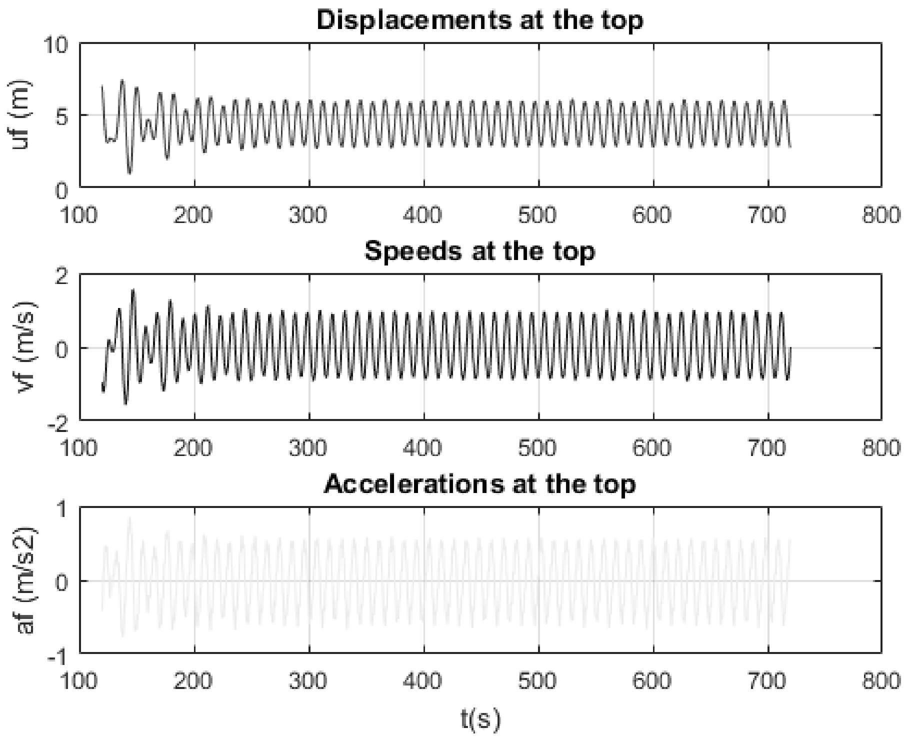


Figure 4.3. Displacements, speeds and accelerations at the top of the tower.

The results show how the variance of the values stands really constant along time.

- **Checks:**

The ULS is satisfied for the wind extreme case and the significant wave height for a return period of 50 years. All the nodes satisfy these checks, having the maximum moments around the seabed level.

The SLS is satisfied for all the cases thanks to the action of the guy cable. It reduces the maximum displacement, which reduces the maximum tilt in the RNA.

The FLS has been satisfied considering the three operational cases for the extreme turbulence model explained before and adding the damage from all the cases, the maximum damage will be 0.3082 at one node close to the seabed (node 51 is the seabed level) (See Table 4.2).

Table 4.2. Damage at the most critical node.

Node	Damage 8m/s	Damage 14m/s	Damage 20m/s	Total Damage
52	0.06496	0.1233	0.11989	0.30817

One of the main problems is that the second natural frequency is inside the 3P frequency selected, so, although for the analysed cases the conditions are fulfilled, a more detailed study should be done in more sophisticated software for the analysis of more wind bins. Another solution to reduce the natural frequencies is to increase the mass at the top, therefore, the analysis for a higher RNA could be performed to see if it is more viable for this case. Anothe issue is that the resultant loads obtained are not excessively high even applying the different matrix defined for Timoshenko explained in the methodology. So, the resultant loads will be underestimated and further analysis for more flexibles structures should be carried out.

Finally, after the execution of the code, a prompt will appear to ask the user if a video of the displacements along time should be shown. It will take more time for the computation if a video is required and that is the reason why it is asked after the simulation.

5. Conclusion

The conclusion about the findings and proposals about further work related to this master thesis is discussed in this section.

5.1. Conclusion

This thesis has developed a simplified model for preliminary design of a compliant tower modelling the most significant environmental loads acting on the tower. The response of the structure to the different loads shows a reliable solution to the dynamic fundamental equation. However, the effect loads at each node will vary significantly depending on the stiffness matrix used for the obtention of reactions. It is due to the higher bending strain energy exerted for flexible structures in comparison with the shear strain energy. This effect can be approximated for the high order Timoshenko beam to model static analysis by using this matrix after the solution of final displacements along time.

Although the flexibility of the tower is high, and deflections are higher than in jacket structures, the tilt and acceleration limits stablished are fulfilled for the operational and extreme cases. On the other hand, tilts angles are always close to the limit for operational conditions and thus, more data will be needed for the efficiency of offshore wind turbines with high tilt angles to see if that solution could be cost-effective in comparation with other kind of structures at similar depths. For instance, from 70 meters water depth floating offshore wind turbines are commonly used and it could be a limiting depth for the design of compliant towers. Also, for shallower waters, rigid structures are used. So, for intermediate depths between these type of structures, compliant tower could be a nice solution if environmental condition allows that the frequency of the design loads is not too low. Besides for shallower waters and around 50 meter water depth higher rating of wind turbines can be used for compliant towers. It would increase the mass of the top node and therefore, the natural frequencies will be reduced.

With regard to the code created, the running time will be about ten minutes for a time simulation of 720 seconds. It could be optimized but the iterations to calculate forces from the waves, from the wind and response forces like the soil and cable opposition to movement make it difficult to optimize for this simplified model.

One of the main advantages of the methodology is that the input data can be changed easily in the program to show the results for the conditions that the user requires. Besides, it makes not only the calculation of efforts at each node of the structure, but also uses this design loads to check if the different limit states are satisfied. Also, the design cases selected will depend on the environmental loads, so changing this input data, the different design load cases can be analysed easily.

Finally, another important aspect is that this method can be also used for the design of rigid structures with a monopile configuration because the main difference with compliant towers will be the natural frequency and the use of guy cables to control the displacement can be taken out of the program before the simulation.

5.2. Further Work

Further analysis related to compliant towers and its implementation in water depths around 50 meters depth could be carried out.

As compliant towers are flexible, small forces can cause considerable deflections at the top and then, further analysis about start up of the structure or response to high variations in wind should be analysed. Variation in directionality of environmental loads could be also significant and could be a good extension to this master thesis.

Also, as has been stated before, efficiency of power production should be carried out due to the high tilt angles obtained.

Regarding to the guy cable, more analysis could be done to determine more exact values about the behaviour of guy cables against wave forces and currents acting on them. For example, a dynamic analysis could be done just for the guy cable.

And obviously, more economical investigations about viability of this kind of structures should be performed due to extra elements needed in the tower like the use of scour protection.

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Appendix A

Matlab Code Summary

Time processing of the program will be around 10 minutes for 720 seconds of simulation and the data already introduced in the code is the solution with the environmental conditions for the extreme case. Analysis for ULS, FLS and SLS will be carried out in all the simulations, so the searched limit state should be the one in which the user has to focus.

All the input data is explained in the function where it is introduced in the code.

The Matlab Code can be found in the attached documents uploaded to DAIM.

```
%%%%%%%%%%%%% Compliant tower calculation %%%%%%%%%%%%%%%%%%%%

% Written by Ruben Martinez Ochagavia Spring 2018 (in Matlab R2016b)

%  This programme will calculate if a specific design for a
%compliant tower fulfil the requirements for ULS, FLS and SLS. It
%will calculate %from the stiffness and mass matrix to the different
%loads %acting on the structure and the reactions in the structure
%for cyclic loading.
%
%  The inputs needed for the calculation should be introduced in
these %functions:
%  --- CompliantTowerCalculation.m (main script)
%  --- CableT0.m (if a guy cable is use in the structure)
%  --- Checks.m
%  --- DynamicLoad.m
%  --- myode.m
%  --- StaticLoad.m (if a static load is introduced - not
needed)(from previous calculations)
%  --- StiffnessGround.m (if the foundation is included)
%
                        %%%%%%%
```