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Improved lumping of offshore wind turbine fatigue load cases

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Coastal and Marine Engineering and Management
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Master of Science in

COASTAL AND MARINE ENGINEERING AND MANAGEMENT

CoMEM

Improved lumping of offshore wind turbine fatigue load cases

Norwegian University of Science and Technology
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CoMEM Thesis

This thesis was completed by:
Alahyar Koochekali

Under supervision of:
Professor
Michael Muskulus

As a requirement to attend the degree of
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Name: Alahyar Koochekali				
Professor in charge/supervisor: Michael Muskulus				
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Abstract:

Stochastic environmental modelling, and structure response calculation are two major steps in determining fatigue damage for offshore wind turbines. However, there are challenges ahead of each step. First, site-specific or high-quality met-ocean data that contain joint measurements of wind and wave are not always available. To generate joint met-ocean data, copula was used. Copula is a function that can capture autocorrelation and dependence of two random variables. The copula was calculated for a specific location in the North Sea for pairs of random met-ocean variables (wave height, wind speed) and (wave height, wave period). This joint function was transferred to another location where only marginals were known. New data were generated by combining these data sets.
Second, even though integrated time-domain analysis of offshore wind turbines can capture the dynamic response of a structure, such simulations are time-consuming. Lumping is a way to reduce a full sea-state to some load cases by weighting environmental parameters by a certain criterion. A simple lumping method was applied to generate lumped sea-state representing pairs of random variables. This lumping method is applied on both real data and copula-generated data. In addition, the difference between fatigue damage caused by real data and copula generated-data were calculated using a formula based on the quasi-static response of a wind turbine. The results show using copula together with simple lumping method can provide a very good estimation of lumped wind and wave height, and fatigue damage. The average difference measured was less than 15%. The copula accuracy reduces in predicting lumped wave height and period, and fatigue damage due to less correlation of wave height and wave period.

Keywords:

1. Lumping
2. Copula
3. offshore Wind turbine fatigue
4. CoMEM

NTNU
Faculty of Engineering Science
Department of Civil and Environmental Engineering

MASTER THESIS
(TBA4920 Marine Civil Engineering, master thesis)

Spring 2018
for
Student: Alahyar Koochekali

Improved lumping of offshore wind turbine fatigue load cases

BACKGROUND

The combination of meteorological and met ocean characteristics of site location is essential in the fatigue damage calculations over lifetime of dynamically sensitive offshore wind turbines. A cost efficient and reliable design against fatigue lies in determining a method for combination of wind and wave climate which guarantees same fatigue damage over structure's lifetime. Considering time consuming and complicated structural analysis of OWTs with various interactions, it is not feasible in early design stages to consider all wind and wave combination in different load cases. To prevent that, lumping of wind and wave data are proposed. 2D and 3D scatter diagrams are normally used to represent long term statistical dependency of environmental conditions. Based on this representation of environmental data and structural response, lumping technics should be developed to predict fatigue damage. Lumping technics are lied on different concepts.

TASK DESCRIPTION

Description of task

The aim is to improve the lumping of load cases used in the preliminary design of offshore wind farms. The first objective is to study the dependence between wind speed and sea state parameters for different sites, using publicly available correlated data. It shall be investigated by how much the joint distribution can be predicted from the marginal alone. Then the response shall be included, and a method shall be developed for lumping the fatigue load cases that is more accurate than the existing approaches. Practical issues such as how to use the response simulation effort in the best possible way can be addressed as well.

Subtasks and research questions

Is it possible to use correlation of data and apply that at another location combining with the marginals of that site? Is this method capable of predicting fatigue damage.

Review of literature (3 weeks)
Review of literature on different combination of joint data and methods to investigate and produce joint data considering their correlation. Review on different lumping methods of sea state for determining fatigue damage considering the response of structure.

Gather environmental data (3 weeks)
Find different source of met-ocean data preferably open-source data which has enough joint measurements of wind speed, wave heigth and wave period. Check the availability of data.

Generate data using joint measurements from another location (5 Weeks)
Find a method to combine joint measurements from one location with marginals measured in another location. The goal is to use the marginal from one location where the joint measurements are missed.

Determine the structure's response (3 weeks)
Find a method to calculate the response of structure under environmental loading and determine the fatigue damage in the structure by this loading. Time-domain and frequency domain analysis can be an option.

Writing the thesis (4 weeks)
Summarize all the finding on the previous steps of the research and write them properly to reflect the result of this research.

General about content, work and presentation
The text for the master thesis is meant as a framework for the work of the candidate. Adjustments might be done as the work progresses. Tentative changes must be done in cooperation and agreement with the professor in charge at the Department. In the evaluation thoroughness in the work will be emphasized, as will be documentation of independence in assessments and conclusions. Furthermore, the presentation (report) should be well organized and edited; providing clear, precise and orderly descriptions without being unnecessary voluminous.

The report shall include:

- Standard report front page (from DAIM, http://daim.idi.ntnu.no/)
- Title page with abstract and keywords. (MScTitlePage[IBM]).
  CoMEM students must include CoMEM as one of the keywords.
- CoMEM page (Only CoMEM students) (CoMEM MSc title Page templateNTNU).
- Preface
- Summary and acknowledgement. The summary shall include the objectives of the work, explain how the work has been conducted, present the main results achieved and give the main conclusions of the work.
- Table of content including list of figures, tables, enclosures and appendices.
- A list explaining important terms and abbreviations should be included.
- List of symbols should be included
- The main text.
- Clear and complete references to material used, both in text and figures/tables. This also applies for personal and/or oral communication and information.
- Thesis task description (these pages) signed by professor in charge as Attachment 1.
- The report musts have a complete page numbering.

The thesis can as an alternative be made as a scientific article for international publication, when this is agreed upon by the Professor in charge. Such a report will include the main points as given above, but where the main text includes both the scientific article and a process report.

Submission procedure
Procedures relating to the submission of the thesis are described in IV faculty webpage
https://www.ntnu.edu/iv/master-thesis-regulation

On submission of the thesis the candidate shall submit to the professor in charge a CD/DVD('s) or a link to a net-cloud including the report in digital form as pdf and Word (or other editable form) versions and the underlying material (such as data collection, time series etc.).

Documentation collected during the work, with support from the Department, shall be handed in to the Department together with the report.

According to the current laws and regulations at NTNU, the report is the property of NTNU. The report and associated results can only be used following approval from NTNU (and external cooperation partner if applicable). The Department has the right to make use of the results from the work as if conducted by a Department employee, as long as other arrangements are not agreed upon beforehand.

Start and submission deadlines
The work on the Master Thesis starts on : 12 February 2018________________________

The thesis report as described above shall be submitted digitally in DAIM at the latest: 9 July 2018 at 3pm (revised: 16 July 2018 at 5pm).

Professor in charge: Michael Muskulus
Professor
Department of Civil and Environmental Engineering
NTNU

Other supervisors: ________________________

Trondheim, dd.mm.yyyy. (revised: dd.mm.yyyy)

____
Professor in charge (sign)

Acknowledgement

I would like to thank my parents for directing me to the path of education from my childhood and giving me the chance to follow my dreams. In addition, I wold like to thank them for giving me the most valuable gift a parent can give to their child, my brother, Amirabbas, and my dear sister Raana.

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1 Introduction

An adequate representation of the site-specific met-ocean data has several important applications in marine and coastal industry. Cost-efficient and reliable design of offshore wind turbines especially for fatigue damage are highly dependent on joint met-ocean data. In addition, reducing environmental data uncertainties can effectively improve the planning of installation and maintenance-operation of marine structures (Leontaris, Morales-Nápoles, & Wolfert, 2016).

Offshore wind turbines are ubiquitous. The main drawback of offshore wind power is its comparatively high capital cost. This high cost can be attributed to more expensive support structures, grid connections and offshore installations (Bilgili, Yasar, & Simsek, 2011). However, it is expected that the wind industry will reduce its cost by 40% until 2020. Application of large offshore wind farms and large-sized monopiles in water depths of [25-40 meters] may be a solution to reduce the costs. Design of these monopiles are typically governed by fatigue damage with significant contributions from wave excitation (Ziegler, 2015).

Integrated time-domain analysis that can capture dynamic behaviour of the structure, and model simultaneous stochastic aerodynamics and hydrodynamics, is an accurate tool for fatigue design of offshore wind turbines (Passon, 2015). However, to avoid excessive computation times of these simulations, input reduction is imperative (Walstra, Hoekstra, Tonnon, & Ruessink, 2013). Lumping is a way to reduce a full-sea-state to some load cases by weighting environmental parameters by a certain criterion. In terms of fatigue damage, this criterion can be the desire to achieve approximately the same damage for all load cases of met-ocean data.

However, joint measurement of site-specific met-ocean data like wind speed and wave height are not always available. The pairwise dependence between random variables like wave height and wind speed can be described using classical families of bivariate distributions such as the Normal, Wiebull, or Gumbel distribution. However, one drawback of using these joint distributions is that univariate distributions or marginals of random variables must be characterized by the same parametric family of distributions (Genest & Favre, 2007). Another way of modelling dependence between random variable is to use Copula. “A Copula is a multivariate probability distribution, whose variables have uniform marginals” (Vanem, 2016) and can capture the dependence behaviour of two or more random variables.

1.1 Research objectives and methodology

Accurate lumping of site-specific joint met-ocean data is necessary for fatigue damage estimation. While the lumping method for fatigue analysis is still under investigation, joint-measurement of data is also not always available. For example, wind speed measurement for a given offshore station may start some years after wave measurements.

This research has two main objectives:

- *Is it possible to use joint values measured at another location in the North Sea (recorded in Copula function) and combine them with the marginal of two random variables from another location to generate lumped joint data?*
- *Can this lumped data be used to estimate fatigue damage?*

In order to address these objectives, long-term met-ocean data were gathered from four locations in the North Sea. Corrupted data and uncorrelated measurements were observed in the data which were omitted from data. The marginals of wave height, and wind speed, and wave period were estimated. The Copula for all four sites were determined in a matrix. The marginal domain was discretised using an adaptive mesh. The mesh grid is transferred to the copula domain using the cumulative distribution function of marginals. A finite difference method was applied to determine copula difference or joint probability of occurrence in adapted copula domain. The above calculations were performed in MATLAB and the code is available upon request. To verify this method data from one location are regenerated using copula from the same location. In the next step the data were lumped using a simplified method. After that, a simplified method was used to compare fatigue damage from this method with the damage caused by real data. Even though these simplified methods do not capture the dynamic behaviour of structures, they are a simple indication of fatigue damage.

1.2 Outline of report

Chapter 1	General definition of the topic, small background and main research questions are explained in this chapter.
Chapter 2	A brief explanation of previous works on lumping of sea-state for fatigue damage, and application of Copula in marine engineering are reported.
Chapter 3	Statistical theories that are applied in this research are presented. Copula definition and step-by-step guide to generation of empirical copula is explained. Analytical distributions and copulas are defined. The lumping process and formulations are described. Methods to estimate fatigue damage are reported.
Chapter 4	Various steps of generating joint data from the marginals of one location together with a Copula from another location are explained. First, data gathering and raw data

for bivariate wave height and wind speed are presented. Then, a 6-step approach to transfer marginals to copula domain and calculate lumped joint data are presented.

Chapter 5 The results of above of calculation from chapter 4 are presented and discussed here. The generation of data using Copula is performed for pairs of (Wind Speed, Wave Height) and (Wave height, Wind Speed). A copula from each location is picked and data are generated for other locations. These data are plotted versus real data measures at each location and the average difference are plotted. Furthermore, fatigue damage estimated by these data are also compared with the fatigue damage estimation of real data. The results are discussed in this chapter.

Chapter 6 The conclusion of current study and future subjects for research are suggested in this chapter. Lumped data generated by copula models are compared to real lumped data for pairs of (Wave height, Wind speed) and (Wave height, Wave Period). The result show that Copula method can predict wind speed and wave height data with accuracy of 10% excluding extremely high or low values. The average difference of generated data using Copula and real data for wave period and wave height are less than 20%.

2 Review of literature

Wind turbines are getting larger and are being used in deeper waters. In this situation, fatigue damage becomes dominant in the design of large monopile substructures (Passon & Branner, 2014). The fatigue design is dominated by the dynamic response of structures under the joint met-ocean climate (Passon, 2015). Since integrated nonlinear calculation of structural response under simultaneous wind and wave loading is time consuming, the scattering of wind and wave parameters result in too many load cases for fatigue design (Kühn, 2001). Meanwhile the availability of site-specific joint met-ocean data is important in fatigue design, Association of lumping sea-state to wind speed is also detrimental in the fatigue damage calculations(Passon, 2015). Due to the importance of availability of joint met-ocean data and lumping of sea-state for fatigue damage, each subject is reviewed separately.

2.1 Joint distribution and Copula application

Stochastic descriptions and simulations of oceanographic variables are essential for coastal and marine engineering applications. (Jäger & Nápoles, 2017). Due to the dependency of met-ocean parameters, like wave height and wind speed, univariate assessment of random variables cannot provide a complete assessment of the probability of occurrence(Masina, Lamberti, & Archetti, 2015). A through statistical description requires the joint use of random variables marginals to obtain multivariate distributions (De Michele, Salvadori, Passoni, & Vezzoli, 2007). For instance the joint behaviour of wave heights and periods, together with the estimation of possible extreme conditions are used to determine design criteria for risk analyses of marine structures and coastal environments (Salvadori, Tomasicchio, & D'Alessandro, 2014).

The concept of joint distributions (Haver, 1987) is widely used in marine industry (Bitner-Gregersen, 2015). The use of Copula, however, has been increasingly popular in recent years (Vanem, 2016). The theory of Copula is introduced by Sklar (1959). Sklar (1959) stated that the joint cumulative distribution function $F(x, y)$ of any pair of continuous random variables (X, Y) is a function of its marginal cumulative distributions and a Copula that describes the dependence between random variables:

$$F(x, y) = C[F(x), G(y)] \tag{1}$$

where $F(x)$ and $G(y)$ are marginal cumulative distributions and C is the Copula. In other words, if $F(x, y)$ is known $C, F,$ and G can be uniquely determined. The Copula is defined in the domain of [0,1]. For more information see the section 3.1.3.

Copula has been previously applied in financial applications. The concept of Copula was elaborated later and applied in civil and hydrology engineering (Genest & Favre, 2007). Four different analytical bivariate Copulas were used to describe joint behaviour of extreme water level and significant wave height (Tao, Dong, Wang, & Guedes Soares, 2013). Bivariate and multi-variate Copula model were used for sea storm modelling considering wave height, storm duration, storm direction, and storm interval (De Michele, Salvadori, Passoni, & Vezzoli, 2007).

The original met-ocean data may not be independent and identically distributed (iid) random variables as they may contain ties and serial dependence (Vanem, 2016). To remove the serial dependence caused by seasonal effect or short-term dependencies, weekly data were subsampled from 3-hour data (Vanem, 2016). To model the joint distribution of significant wave height and zero-crossing wave period, different bivariate modelling technics including conditional models, bivariate log-normal model and several bivariate parametric Copulas were used (Vanem, 2016). It was concluded that while most common families of Copulas cannot capture the dependence structure data, as opposed to the conditional model, advanced Copula technics can be significant improvement in determining joint behaviour. The Archimedean family of Copula with a specific attention to the tail behaviour of Copulas were implemented to generate a large number of realistic time series of wind speed and wave height data to schedule marine operation and cable installation for offshore wind turbines. In the same study, three different sets of tests were used to examine goodness of fit for different Copulas (Leontaris, Morales-Nápoles, & Wolfert, 2016).

2.1.1 Knowledge gap

The joint behaviour of random variables at a certain location are not always known. However, it is possible to apply the joint behaviour from another location to simulate the joint behaviour. Especially, if the two locations have similarity in the relevant physical characteristics. Such a proposal has not yet been investigated and will be investigated in this research.

2.2 Lumping of sea-state for fatigue damage

To reduce the number of load cases for fatigue-damage design of an offshore wind turbine, a scatter plot of joint sea-state was lumped into a number of sea-states in such a manner that they preserve the damage that accumulates in structure (Kühn, 2001).

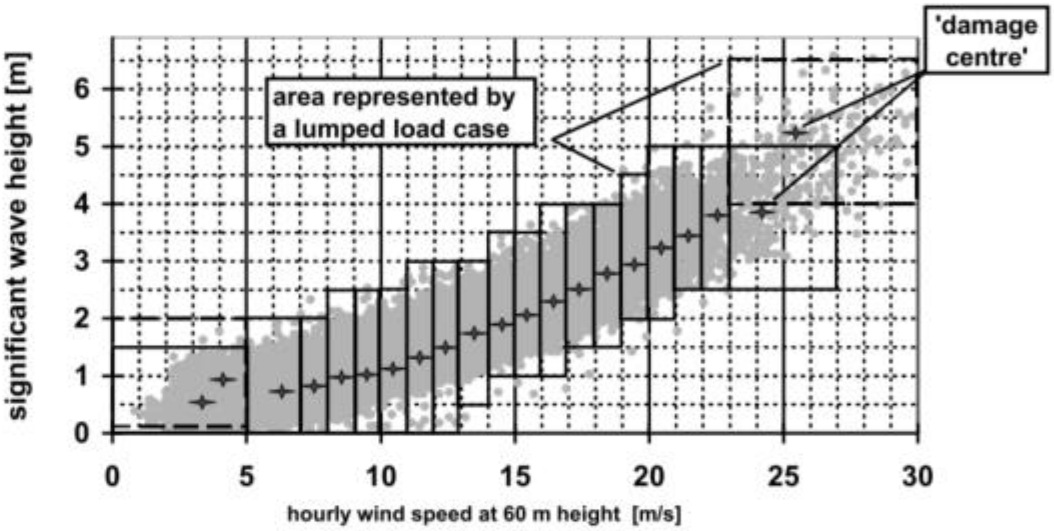


Figure 2-1. Lumping of a sea-state scatter plot for a certain wind speed (Kühn, 2001)

Four main random variables play a role in determining wind and wave load combination for fatigue design,: mean wind speed at hub height, wind direction, wave direction, number of seeds for turbulent wind and irregular wave realisations(Passon & Branner, 2014).

Most of the traditional lumping methods aim at lumping sea-states. Estimation of lumping loads were used based on a quasi-static behaviour of an offshore wind turbine. This lump loads refined through an iterative process by a correction factor (Kühn, 2001). The traditional lumping methods for lumping sea-state works in two ways: preserving the wave period distribution and averaging wave height over each bin of wave period(Mittendorf, 2009) or the other way around, preserving wave height distribution and averaging wave period over wave height bins (Tony Burton, 2012)(for further information see section 3.2.2). However, these methods cannot provide accurate lumped load cases for larger offshore wind turbines established on monopiles where dynamics of structures lead fatigue damage (Passon & Branner, 2014). Neither of these methods takes dynamics of structrues into account. Dynamics of structure can be captured using frequency domain analysis. By Assuming a narrow bounded response spectrum for a monopile, lumping of sea-state was performed based on square root of spectral wave energy at the structure's first natural frequency for each sea-state (Seidel, 2014). Damage equivalent wave height and wave periods were introduced to lump sea-state based on preserving fatigue damage neglecting lumping of wind speed (Passon & Branner, 2014). The structure's response was summarized in a damage matrix (for further information3.2.3). A recent wind-wave correlation method combined sea-state lumping with wind speed considering dynamic response of offshore wind turbines (Passon, 2015). The goal of this new method was establishment of wind-wave correlations in terms of a unique, damage equivalent $Hs - Tp$ combination per wind speed bin, wind direction, and wave direction.

2.2.1 Knowledge gap

Accurate wind-wave correlation methods for the fatigue design of OWTs considering the stochastic sea-state and structures' dynamic behaviour do not exist and guidance is also missing in all relevant design guidelines (Passon, 2015).

3 Concepts and theories

3.1 Statistics of marginals and joint variables

3.1.1 Basic statistics

The cumulative distribution function for a variable X is denoted by $F_X(x)$, and it is defined by:

$$F_{X(x)} = P[X \leq x] \tag{2}$$

The probability that the continuous variable is in the interval $[a, b]$ is obtained as

$$P[a \leq X \leq b] = F_X(b) - F_X(a) \tag{3}$$

If

$$\frac{\lim_{\Delta x \to 0} \left(\left(F_X(x + \Delta x)\right) - F_X(x)\right)}{\Delta x} \tag{4}$$

exist, the probability density function for variable the X is designated by

$$f_x(x) = \frac{dF_X(x)}{dx} \tag{5}$$

which represents the probability that X is located within the interval $[x, x + dx]$. In other words,

$$P[a < x \leq b] = \int_a^b \frac{dF_{X(x)}}{dx} \tag{6}$$

$$F_X(x) = \int_{-\infty}^{b} f_X(\zeta) d\zeta \tag{7}$$

In addition to the analytical probability distributions, some continuous distributions which are frequently applied are Normal, Lognormal, Exponential, Gamma, Weibull, and Rayleigh distributions.

Meanwhile normal distribution is described only based on mean and standard deviation, the other distributions are parametric distributions which require definition of one or more parameters.

Understanding which distribution better fits to real data is important. There are different ways to examine the goodness of fit. Using probability paper, Chi-square test, and the Kolmogorov test are three ways to examine goodness of fit but the first method is the easiest way to visualize goodness of fit.

Most of the events are described by two or more stochastic variables. Even though it is possible to consider multiple random variables to investigate the joint behaviour or general behaviour of a stochastic process, normally bivariate variables, or two random variables are considered. The joint cumulative distribution function for two variables is defined as

$$F_{XY}(x, y) = P[(X \leq x) \cap (Y \leq y)] \tag{8}$$

The joint probability density function is given as the following limiting value if it exists:

$$f_{XY} = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \frac{F(x+\Delta x, y+\Delta y) - F(x, y)}{\Delta x \Delta y} \tag{9}$$

$$f_{XY}(x, y) = \frac{\partial^2 F_{XY}(x, y)}{\partial x \partial y} \tag{10}$$

If $f_{XY}(x, y)$ is known, we can find $F_{XY}(x, y)$ by

$$F_{XY}(x, y) = \int_{-\infty}^{y} \int_{-\infty}^{x} f_{XY}(u, v) du dv \tag{11}$$

These properties and expressions can also be generalized directly to several variables.

3.1.2 Sea wave stationary random variables and scatter diagram

The long-term sea-state is customarily represented by several discrete short-term sea-states which are considered to be stationary Gaussian stochastic process and can be characterized by wave spectra,$S_{\zeta\zeta}$. The Jonswap spectrum is normally used to describe sea-state. From the long-term statistic, the occurrence frequency of the various sea-states can be represented by a continuous joint probability density function of wave height (H_S) and wind speed (W_s).
Applying conditional probability distribution, the joint distribution is defined as

$$f(H_s, T_p) = f(H_s) f(T_p | H_s) \tag{12}$$

where the wave height distribution and conditional probability can be calculated based on the wave spectrum(Haver, 1987).
Since both the structure's response and met-ocean data can be presented in terms of frequency, the frequencies of various sea wave types are presented in Figure 3-1. The green fonts represent the initiation source of waves.

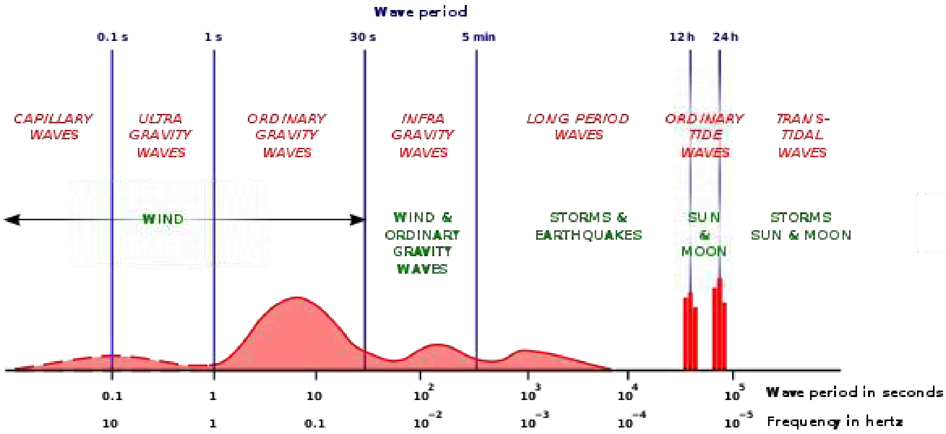


Figure 3-1. Wave frequency range

To compare these frequencies with a structure's frequency, the structure's natural frequency ranges are shown as bold line in Figure 3-2.

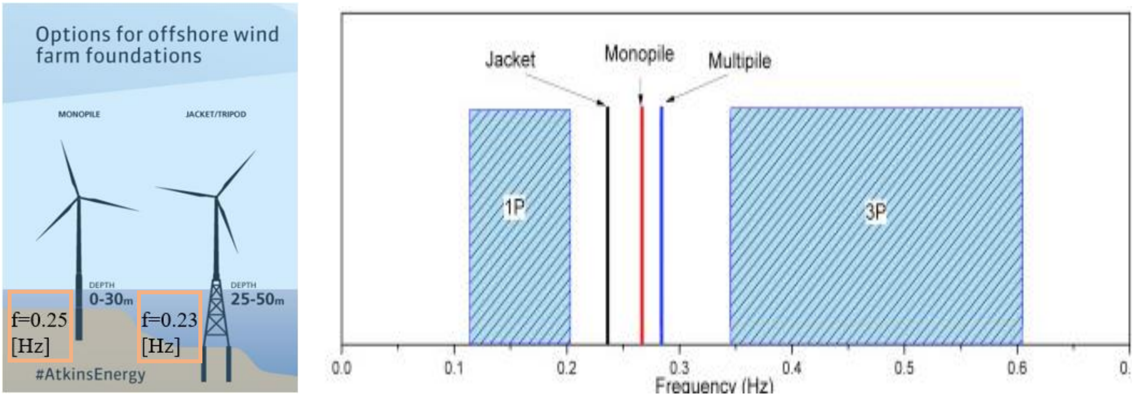


Figure 3-2. Natural frequency range (Shi et al., 2015)

3.1.3 Copula

3.1.3.1 Emprical copula

Copula isolate the marginal properties from the dependence structure of random variables. Combining Copula with marginal distribution leads to generation of joint distribution which can represent multivariate behaviours (Jäger & Nápoles, 2017).

The Copula of two or more random variables (X, Y) with marginals distribution functions $F(x) = P(X \leq x)$, is defined as the joint cumulative distribution of $\big(U = F_X(x), V = F_Y(y)\big)$

$$F_{X,Y} = C(u, v) = P\big[F_{X(x)} \leq u \cap F_Y(y) \leq v\big], \qquad u, v \in [0,1] \tag{13}$$

Where U and V are uniformly distributed.

Based on Sklar's theory there exists a unique Copula C for which above equation holds, and the joint dependency of X and Y is characterized fully and uniquely by C (Genest & Favre, 2007).

It can be mathematically proven that a Copula associated with a random pair of (X, Y) is "invariant by monotone increasing transformation of the marginals", $F(x), G(y)$. One of the functions that meets this requirement is the rank of random variables, therefore, pairs of rank of X, Y should have the same as copula as marginals (Genest & Favre, 2007)

$$(R_1, S_1), \ldots (R_n, S_n) \tag{14}$$

where R_i is the rank of X_i among $X_1, \ldots, X_n$ and $S_i$is the rank of Y_i among $Y_1, \ldots, Y_n$. By dividing R & S by $\frac{1}{n+1}$, where n is the number of measurements of X and Y, the rank domain is rescaled to $[0,1]^2$ domain which is the domain of *empirical copula* can be defined as*:*

$$C_n(u, v) = \frac{1}{n}\sum_{i=1}^{n} 1(\frac{R_i}{n+1} \leq u, \frac{S_i}{n+1} \leq v) \tag{15}$$

"For any given pair of (u, v), it may be shown that $C_n(u, v)$ is a rank-based estimation of the unknown quantity of $C(u, v)$ whose large -sample distribution is centered at C(u,v) and normal. Copula density $c(u, v)$ equals the derivative of $C(u, v)$ relative to its arguments, and based on Sklar's theory it can be written as"

$$f(x, y) = f(x)g(y)c(F(x), G(y))$$

If the copula density is 1 then the variables are independent(Vanem, 2016).

3.1.3.2 Analytical Copulas and tail dependency

Various types of parametric families of Copulas exist. Three main classes of Copulas are elliptical Copula, which can capture radially symmetric behaviour, Archimedean Copulas, which have the capability of taking into account lower and upper tail behaviour, and extreme value Copulas, "which arise in the limit of component-wise maxima, but could also be used to model general positive dependence structures (Vanem, 2016).

Three of the most common Copulas are the Gaussian, Gumbel, and Clyton Copulas, where the last two are the most used one-parameter Archimedean Copulas. These Copulas where used to generate wave height and wind period time series to model a schedule of cable installation operation (Leontaris, Morales-Nápoles, & Wolfert, 2016).

The Guassian Copula is given by

$$\mathrm{C(u, v)} = \Phi_\rho\,(\Phi^{-1}(u), \Phi^{-1}(v)) \tag{16}$$

Where Φ the standard normal distribution function and Φρ the standard bivariate normal distribution function with linear correlation coefficient ρ, see section 3.1.3.5.

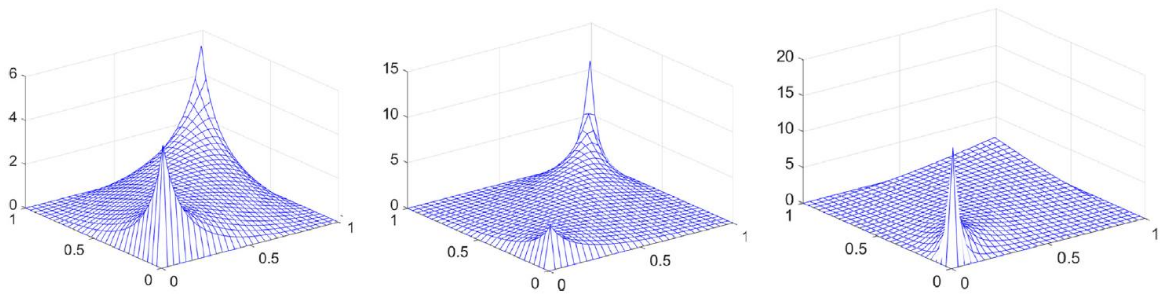
The Gumbel copula is defined as:

$$C(u, v; \theta) = \exp\left[-\left[(-\ln(u))^{\theta} + (-\ln(v))^{\theta}\right]^{\frac{1}{\theta}}\right] \tag{17}$$

And Clyton copula is defined as

$$C(u, v; \beta) = 9u^{-\beta} + v^{-\beta} - 1)\wedge(-1/\beta) \tag{18}$$

A general shape of density function of these copulas is shown in below Figure 3-3.These figures can be compared with the analytical density copulas extracted from the four locations for different pairs of wind and wave presented in Figure 3-3.



Gaussian copula ($\rho = 0.7$) Gumbel copula ($\theta = 2, r_s = 0.7$) Clayton copula ($\beta = 2.1, r_s = 0.7$)

Figure 3-3. Density of function of three main copulas and different tail dependency

3.1.3.3 Copula generation process

To use emprical or analytical copulas the following procedure can be followed:

1- Importing data
2- Transform observation into pseudo -observation; transformation to rank
3- Use either analytical or empirical formula to generate copula
4- In terms of analytical copula perform goodness of copula fit analysis(for further detail (Leontaris et al., 2016))

3.1.3.4 Copula versus joint distribution

One limitation of using joint distribution is that marginal behaviour must also be characterized by the same family of univariate distributions, while the copula avoids this restriction (Genest & Favre, 2007).

3.1.3.5 Dependence of random variables

One way to measure the dependence of two random variables is Pearson's correlation coefficient which is defined as

$$r_{X,Y} = \frac{COV(X, Y)}{\left(VAR(X).VAR(Y)\right)^{0.5}} \tag{19}$$

Where COV is the covariance of two random variables. $-1 \leq r_{X,Y} \leq 1$ and the correlation coefficient is an expression for the linear dependency between X and Y.

Another method of measuring the dependence is Spearman's rho where the correlation between pairs (R_i, S_i) of ranks is measured:

$$\rho_n = \frac{\sum_{i=1}^{n} (R_i - \bar{R})(S_i - \bar{S})}{\sqrt{\sum_{i=1}^{n} (R_i - \bar{R})^2 \sum_{i=1}^{n} (S_i - \bar{S})^2}} \in [-1, 1] \tag{20}$$

where

$$\bar{R} = \frac{1}{n}\sum_{i=1}^{n} R_i = \frac{n+1}{2} = \frac{1}{n}\sum_{i=1}^{n} S_i = \bar{S} \tag{21}$$

This can be written in an easier form of :

$$\rho_n = \frac{12}{n(n+1)(n-1)}\sum_{i=1}^{n} R_i S_i - 3\frac{n+1}{n-1} \tag{22}$$

The Spearman's rho is superior to Pearson's r in that ρ_n estimates a population parameter that is always well defined, whereas there are heavy-tailed distributions (such as the Cauchy, for example) for which a theoretical value of Pearson's correlation does not exist. (Genest & Favre, 2007).

P-values range from 0 to 1, where values close to 0 correspond to a significant correlation in R and a low probability of observing the null hypothesis.

3.2 Lumping

3.2.1 Definition and application

A simple lumping was done based on a quasi-static assumption of behaviour of different lumping processes as mentioned in 2.2.7. The wave scatter diagram is subdivided into several cells; within each cell is a single sea-state representing average value of all sea-states located in this cell. The probabilities of occurrence for all sea-states within the cell are summed up.

Real data distribution is shown in a 3D histogram using MATLAB function in Figure 5-7. To summarize the data for engineering application purposes the scatter diagram is averaged over different combinations of cells.

3.2.2 Lumping process

In reality, a range of $H_s - T_p$ combination is associated for each set of mean wind speed, wind direction and wave direction. However, for practical applications only one representative of $H_s - W_s$ is associated for each set of these basic parameters to limit the number of load simulations.

A full wave climate is described by the directional scatter diagram which is compromisd of i=1:n wave height classes $H_{s,i}$ and $j = 1{:}m$ wind speed-classes $T_{p,j}$. The lumping of this scatter diagram is aimed at the establishment of $H_s - T_p$ relations with one representative wave period associated to one significant wave height or vice versa.

Two approaches, outlined below, can be used to determine averaged wave height and wind speed.

3.2.2.1 Lumping for two random Variables(X,Y)

First, the general process for any two random variables is discussed, and then it is attributed to met-ocean parameters.

3.2.2.2 Preservation of wave period distribution and lumping wave height

To do this simple averaging the following steps are needed

1- Bin the both marginals (H_s, W_s) or(H_s, T_p)

2- Apply this formula to calculate averaged wave height for each bin speed

$$H_{s,j} = \frac{\sum_{i=1}^{n} P_{i,j} . H_{s_{i,j}}}{\sum_{i=1}^{n} P_{i,j}} \tag{23}$$

3- Plot the lumped sea-state

In this calculation the wind speed distribution is preserved while the wave height is averaged over each class of wind speed. This calculation is shown in Figure 3-5, where the blue box represents averaging over wave height for each wind bin.

			class i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
			Hs [m]	0.00	0.54	0.89	1.43	1.77	2.32	2.66	3.20	3.54	4.09	4.43	4.97	5.32	5.86	6.20	6.75	7.09	7.63	7.97	8.52	
				0.54	0.89	1.43	1.77	2.32	2.66	3.20	3.54	4.09	4.43	4.97	5.32	5.86	6.20	6.75	7.09	7.63	7.97	8.52	8.86	
			Hs,i[m]	0.27	0.71	1.16	1.60	2.04	2.49	2.93	3.37	3.82	4.26	4.70	5.14	5.59	6.03	6.47	6.92	7.36	7.80	8.25	8.69	
class j	Ws[m/s]		Ws,j																					
1	0.00	1.35	0.68	0.006	0.015	0.020	0.017	0.007	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.067
2	1.35	2.70	2.03	0.006	0.019	0.037	0.042	0.050	0.027	0.007	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.191
3	2.70	4.05	3.38	0.003	0.010	0.020	0.027	0.046	0.051	0.030	0.010	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.201
4	4.05	5.40	4.73	0.001	0.004	0.009	0.013	0.025	0.040	0.043	0.023	0.011	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000			0.000	0.174
5	5.40	6.75	6.08	0.001	0.002	0.004	0.005	0.011	0.019	0.028	0.026	0.019	0.008	0.003	0.001	0.000	0.000	0.000	0.000	0.000	$P_{i.i}$		0.000	0.124
6	6.75	8.10	7.43	0.000	0.001	0.001	0.002	0.004	0.008	0.013	0.018	0.019	0.011	0.005	0.002	0.000	0.000	0.000	0.000	0.000			0.000	0.085
7	8.10	9.45	8.78	0.000	0.000	0.000	0.001	0.002	0.003	0.006	0.008	0.014	0.014	0.009	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.062
8	9.45	10.80	10.13	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.004	0.008	0.009	0.007	0.005	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.040
9	10.80	12.15	11.48	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.003	0.005	0.005	0.004	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.024
10	12.15	13.50	12.83	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.003	0.003	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.014
11	13.50	14.85	14.18	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.002	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.009
12	14.85	16.20	15.53	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.005
13	16.20	17.55	16.88	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.003
14	17.55	18.90	18.23	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
15	18.90	20.25	19.58	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
16	20.25	21.60	20.93	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17	21.60	22.95	22.28	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
18	22.95	24.30	23.63	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
19	24.30	25.65	24.98	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	25.65	27.00	26.33	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
																								1

Figure 3-4. Scatter diagram of wave height and Wind speed for CN4.

3.2.2.3 Preservation of wave height distribution and lumping wave period

To do this simple averaging, the following steps need to be taken:

1- Bin both marginals (H_s, T_p)or (H_s, W_s)

2- Aapply this formula to calculate averaged wind speed for each wave height bins

$$W_{s,i} = \frac{\sum_{j=1}^{m} P_{i,j}.W_{s_{i,j}}}{\sum_{j=1}^{m} P_{i,j}} \tag{24}$$

3- Plot the lumped sea-state

In this calculation the wave height distribution is preserved meanwhile the wind speed is averaged over each class of wind speed. This calculation is shown in Figure 3-5 where the orange box represents averaging over wind speed for each wave height.

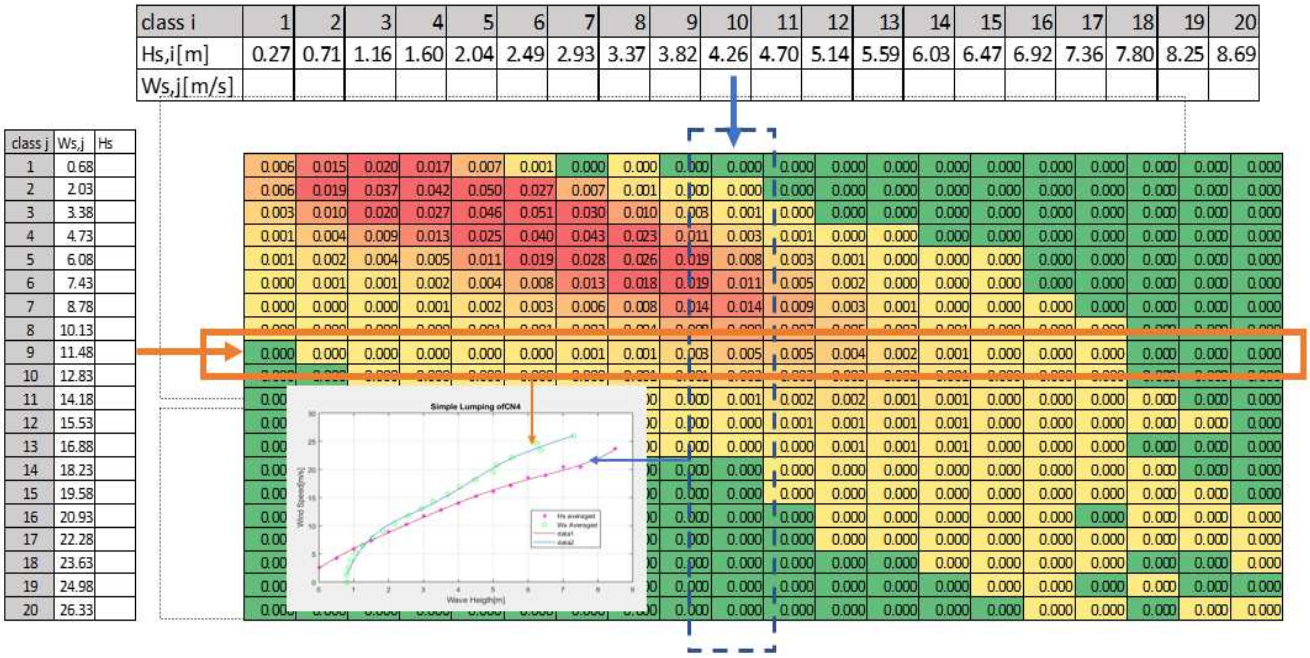


Figure 3-5. Lumping Process for Wave height averaged (Blue), and Wind Speed averaged (Orange)

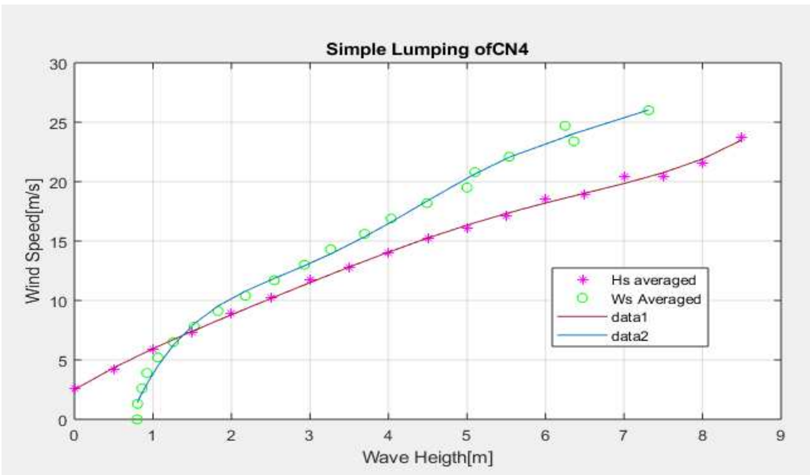


Figure 3-6. Summary of lumped scatter plot

3.2.3 Fatigue damage and damage equivalent lumping

Fatigue is the process of material degradation when exposed to continually changing stresses. Fatigue-damage calculation, in practice, has four main stages: stochastic environmental modelling, structural response calculation, establishment of stress range distribution and damage accumulation (Kühn, 2001).

Two-dimensional and 3-dimensional scatter diagrams and Lumping of scatter diagrams, in practice, are used for the first step.

Either frequency domain(Seidel, 2014) or integrated time domain analysis (Skau, Grimstad, Page, Eiksund, & Jostad, 2018) can be used to determine stress range in structures due to combined loading. Frequency domain is not as accurate as time domain analysis as it cannot capture nonlinearities in structure and environmental loading. However, it is a very efficient way to calculate response of structures to dynamic loading with good accuracy, which is advantageous for fatigue damage calculation(Seidel, 2014). Due to the dynamic nature of the loads on the wind turbine structure, the slenderness of the system, various non-linearities and interactions, and the simultaneous effect of environmental loading, integrated time domain simulation can bring more accurate calculation of structures response (Amar Bouzid, Bhattacharya, & Otsmane, 2018).

Applying Rain flow counting and linear damage accumulation rule of Palmgren-Miner together with standard S-N curves leads to fatigue damage calculations for the last two steps.

3.2.3.1 Cumulative fatigue damage and damage equivalent load

To calculate the cumulative fatigue damage on a structure the Palmgren-Miner rule can be used. This rule assumes that the fatigue damage produced by an individual stress cycle is constant:

$$D_i = \frac{1}{N_i} \tag{25}$$

Di is the fatigue damage produced by stress cycle i, and Ni is the number of stress cycles of the stress range Si to failure.

The rule also implies that the fatigue damage for a stress history with changing stress ranges can be calculated using linear accumulation of the partial fatigue damage produced by individual cycles. The accumulated fatigue damage(D) is given by:

$$D = \sum_{i=1}^{k} \frac{n_i}{N_i} \qquad (26)$$

where n_i is the total number of stress cycles of the stress range Si, and k is the number of different stress ranges S (Chen et al., 2011).

The damage equivalent load (DEL) is a measure which enables the comparison of different fatigue load spectrums. The slope of the S-N curve m and the number of cycles N_{eq} are getting fixed. From the damage calculated it can then be determined which constant amplitude would have generated the same amount of damage based on the fixed parameters, (Hendriks and Bulder, 1995).

$$DEL = [\sum_{i=1}^{k} \left(\frac{(N_i s)^m}{N_{eq}}\right)]^{\left(\frac{1}{m}\right)} \qquad (27)$$

3.2.3.2 Long-term fatigue damage based on Kuhn's simplified method

Kuhn (2001) proposed a simplified method for fatigue damage calculations following these steps:

1- Establish 3d scatter diagram $(H_{s,i}, T_{z,j}, V_k)$ where $(H_{s,i}, T_{z,j})$ are binned sea-state scatter diagram and V_k is the binned wind speed; i is wave height bin number, j is wave period bin number, and k is wind speed bin number.

2- Aerodynamic fatigue analysis for each Vk

3- Hydrodynamic fatigue analysis for each $(H_{s,i}, T_{z,j}, V_k)$

4- Superposition of short-term fatigue damage for each $(H_{s,i}, T_{z,j}, V_k)$

5- Long-term fatigue damage load calculations

$$\Delta\sigma_{eq,ah} = \sqrt[\mu]{\sum_i \sum_j \sum_k \Delta\sigma^{\mu}_{eq,ah,i,j,k}\Big|_{p=1} p_{i,j,k}} \qquad (28)$$

Here the $p_{i,j,k}$ is the probability of occurrence of each $(H_{s,i}, T_{z,j}, V_k)$

3.2.3.3 Damage equivalent lumping

To do integrated time domain simulation of wind turbines load cases need to be reduced. One Criteria for lumping is to do the weighing of environmental parameters based on their relative fatigue damage. Assuming linear and quasi-static response of a structure, the stress ranges are considered proportional to the significant wave height, standard deviation of wind speed and thus mean wind speed.(Kühn, 2001). The fatigue damage can be proportional to the environmental conditions in the below formulation (Kühn, 2001) :

$$D \propto \Delta\sigma \propto H_s^{\mu}$$
$$D \propto n_{total} \propto \frac{1}{T_z} \tag{29}$$
$$D \propto \Delta\sigma^{\mu} \propto V^{\mu}$$

Where D is the damage, V is the mean wind velocity at hub height, μ is the the slope of S-N curve, H_s Significant wave height and T_z is the mean zero crossing period.

In other words, the wave height and wave period lumping can be summarized as(Passon & Branner, 2014)

$$H_{s,m,j} = \left(\sum_{i=1}^{N(H_s)} \frac{(p_{i,j} H_{s,i}^{m})}{p_j} \right)^{\frac{1}{m}} \tag{30}$$

$$T_{z,n,i} = \left(\sum_{i=1}^{N(T_z)} \frac{(p_{i,j}/T_{z,j})}{p_j} \right)^{-1} \tag{31}$$

Another important frequency for fatigue damage of structure is the first natural frequency of the structure (Seidel, 2014):

$$D \propto (\sqrt{S_{\zeta\zeta}(\omega_0)}) \tag{32}$$

Where the $S_{\zeta\zeta}$ is wave energy spectrum and ω_0 is the first natural frequency of the structure.

4 Using copula to generate dependent data for another location

4.1 Data gathering

For this thesis four different environmental data sets have been used. Each data set contains average wind speed(m/s), that is averaged from the wind speeds observed in the time interval of interest, wind direction, significant wave height(m), which is the mean values over the upper third of the observed wave heights during the time interval, wave direction, and wave period. The datasets are collected from four different locations along the North Sea. These locations are shown by mark in the Figure 4-1.

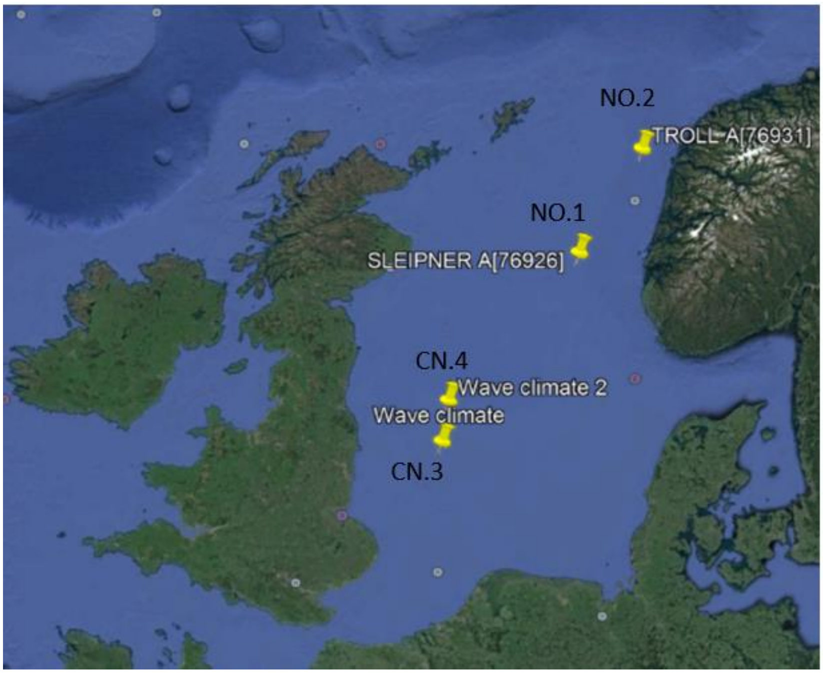


Figure 4-1. Data set locations

The first two data-sets conver periods of16 years (2002-2018) and 24 years (1994-2018) of data measured in ten- minute intervals from two offshore stations, 76931 and 76926, located in the northeastern part of the North Sea, available on Norwegian Meteorological Institute website (www.eklima.no). The third and fourth environmental data sets cover a period of 24 years (1992-2016) in three-hour intervals from two offshore locations in the centre of North Sea, available on (www.waveclimate.com). Three-hourly time series of wind and wave parameters are generated from each grid point of an in-house wave model covering the period 1979 to present day. The different locations are named based on the table below.

Table 1. Data source

Station Name	Source	Duration
NO.1	www.eklima.no	2002_2018
NO.2	www.eklima.no	1994_2018
CN.3	www.waveclimate.com	1992_2016
CN.4	www.waveclimate.com	1992_2016

4.2 Excluding unrealistic values

Missing data and defect measurements are available in all of the data sets. To keep data correlated, the missing values for all random variables measured at the same time are excluded from the data. The number of excluded data are relatively small as compared to the size of the remaining data set.

In this report it is assumed that measured met-ocean data are independent and identically distributed random variables(iid). However, the original met-ocean data may not be iid as they may contain ties and serial dependence (for further information see section 6). The wind speed, wave height and wave period are three random variables that are treated in this analysis. The pre-processed data for wave height and wind speed at four different locations are plotted in the Figure 4-2.

4.3 Using copula to generate lumped data

The goal is to use marginal data from one location ($location(B)$) combined with a Copula generated in another location ($location\ (A)$) and generate new joint data. These joint data are lumped in the next step. The original joint measurements for each location are also lumped. To examine to what extent this method works, a comparison is made between lumped data calculated by this method to the lumped measured joint data at the same location.

A numerical process is defined to perform the above calculations. The mesh grid generation at different steps and the mathematics behind the numeric calculations for each step are explained and graphically shown in this section

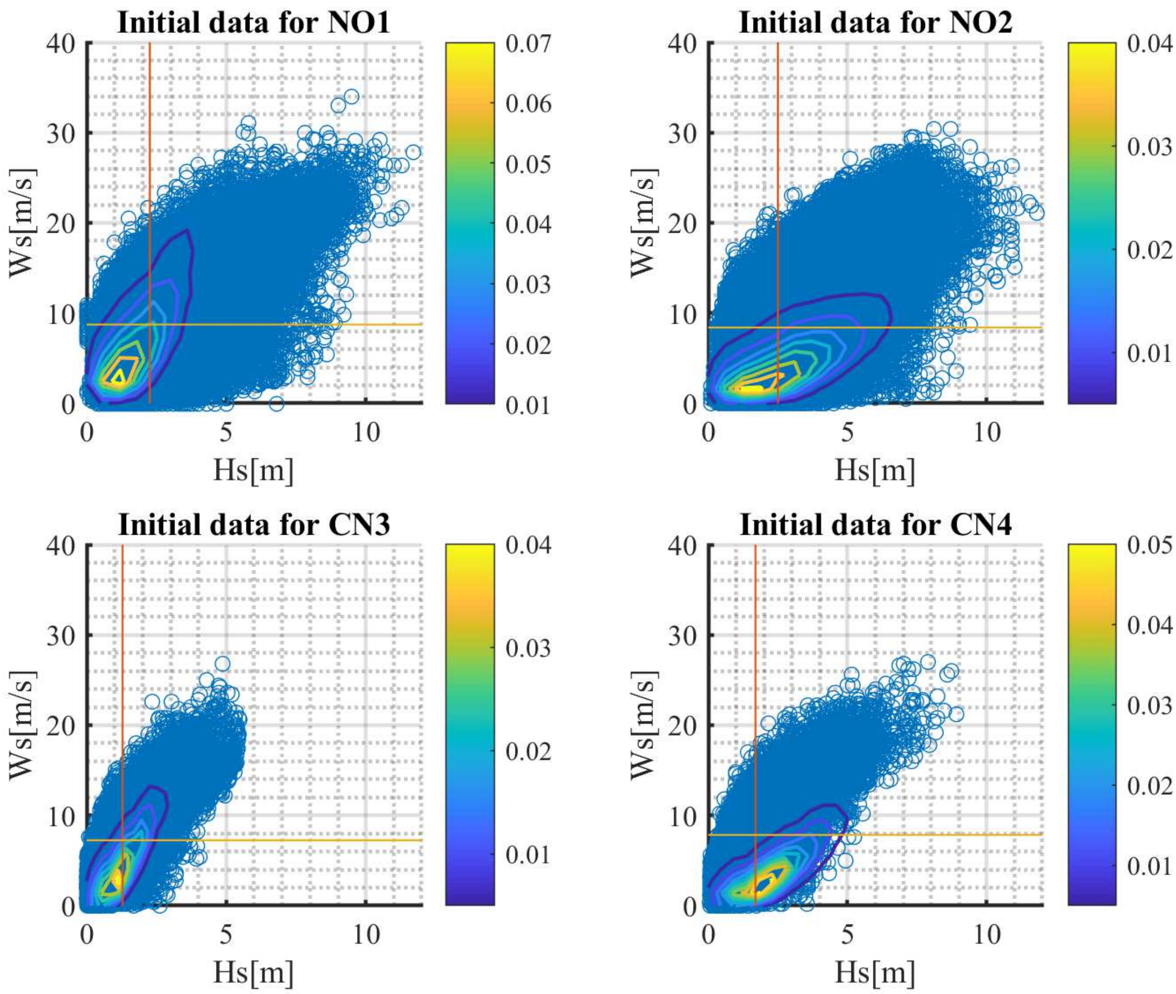


Figure 4-2. Scatter plot of measurements for Hs and Ws

4.3.1 Procedure to generate lumped data at location B using copula at location A

This approach has the following steps:

1- Generate Copula grid and calculate Copula from the joint measured variables measured at location A (pairs of (X, Y), where X and Y can be $(H_s, T_p) or (H_S, W_s)$.

2- Calculate copula density inside the cells of Copula grid

3- Adaptive bin generation for first marginal of location B (Variable X)

4- Transforming this bin to copula domain, $[0,1]^2$using the CDF of each marginal, and bin generation in [0,1] domain for location B (inside the copula grid)

5- Finding marginal bin's location relative to the closest copula cell grids

6- Calculate copula density and marginal representing each bin for one of the marginals (from the cells of copula grid)

7- Apply steps 2 to 6 for the other marginal (Variable Y)

8- Lumping new mesh grid of X over Y (Wave height over Wind speed)

4.3.1.1 Generating emprical Copula

The following steps were performed numerically using MATLAB to generate Copula for two random variable measurements X $= (x_1, x_2, \dots x_n)$ and $Y = (y_1, y_2, \dots, y_n)$:

1- Add a very small value between [1e-5,1e-9] to each measurement of x, y in order to make pairs of (x, y) distinguishable in case of repetitive numbers in each marginal.

2- Sort out two the marginals$(R_1 = rank(X), and\ S_1 = rank(Y))$

3- Assign rank of each of measurements of X (r_1) and Y (s_1) separately.

4- Choose one of the marginals that is sorted(R_1)

5- Find the corresponding value (y) that is measured at the same u_1 , to form the pairs of (x, y) and from that form the rank pairs (r_1, s_1).

6- Divide the pairs of (R_1, S_1) by the number of measurements to transfer data to [0,1] domain.

7- Transform each marginal to copula domain [0,1] by using the analytical Cumulative Distribution Function (CDF) of each random variable.

8- Generating copula mesh grid; assigning equally spaced mesh grid size of 0.01 in copula domain $[0,1]^2$ for each axis of the copula (U, V).

9- Calculate copula for each node in copula domain or for each pair of (U, V) by this formula

$$C_n(u, v) = \frac{1}{n}\sum_{i=1}^{n} 1(\frac{R_i}{n+1} \leq u, \frac{S_i}{n+1} \leq v) \tag{33}$$

$$U = CDF(X) \tag{34}$$

$$V = CDF(Y) \tag{35}$$

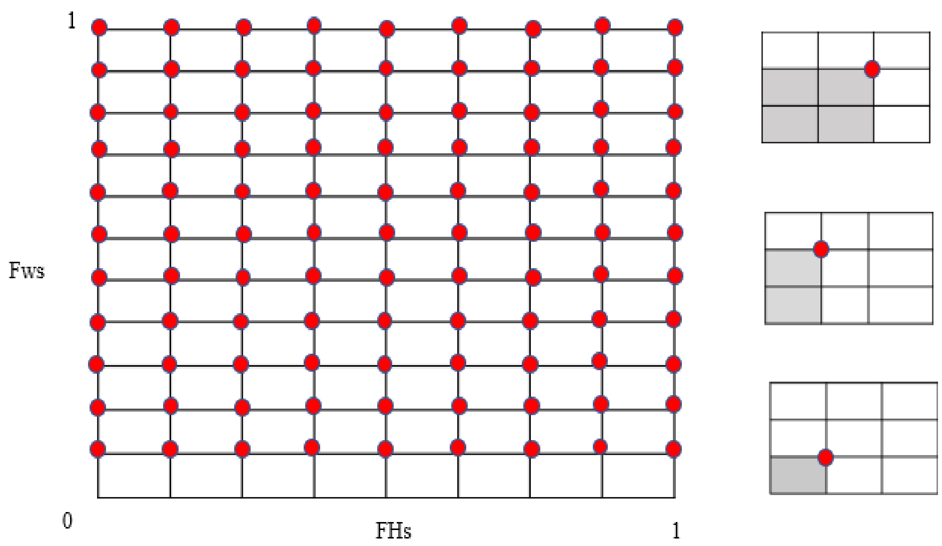


Figure 4-3. Copula mesh generation and copula calculation

An example of the uniform distribution of scaled ranked data at location CN4 versus real distribution of marginals are plotted in Figure 4-4.

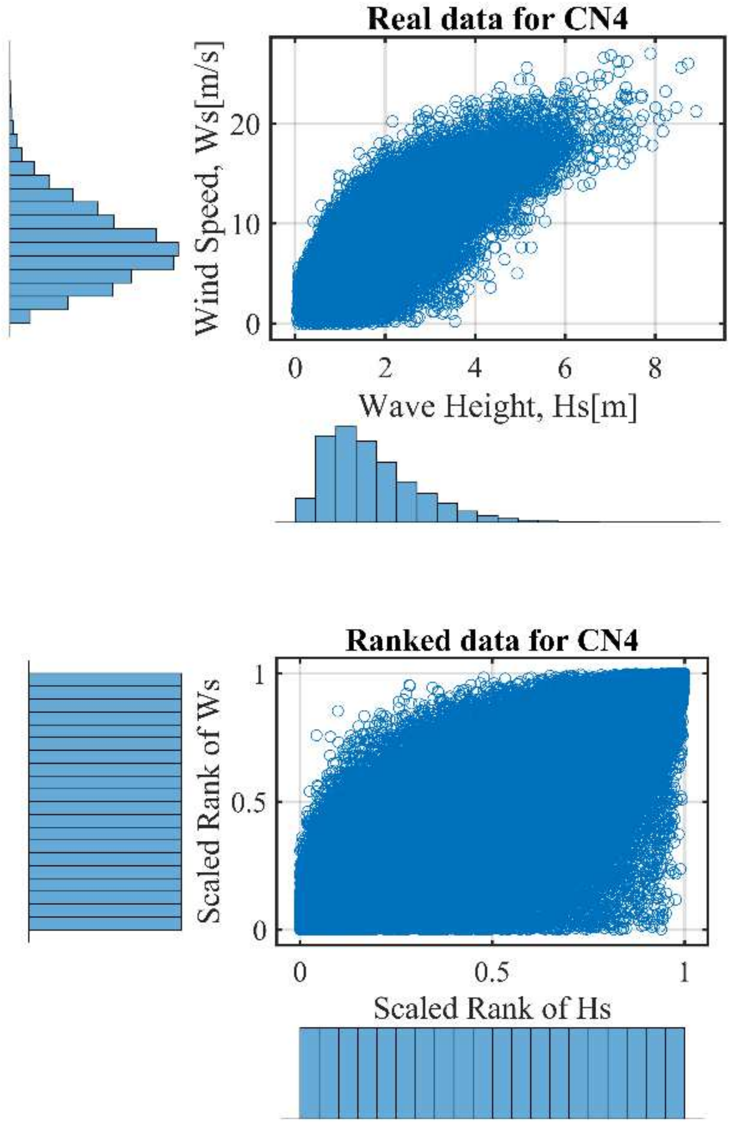


Figure 4-4. Real distribution of marginals versus uniform scaled ranked data at station CN4

4.3.1.2 Calculate copula density inside each cell of the Copula grid from location A

The copula density can be calculated by summarizing Copula in grid points inside the centre of each cell. The following numerical scheme represents this calculation, which is shown in the Figure 4-5.

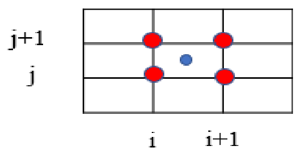


Figure 4-5. Numerical Stencil for copula density calculation

$$C(U,V) = F_{XY}(X,Y), Copula\ or\ rank\ and\ joint\ proability \quad (36)$$

$$c(U,V) = \frac{dC}{dUdV} = \frac{C_{i+1,j+1} - C_{i+1,j} - C_{i,j+1} + C_{i,j}}{dU.dV} = f(X,Y) \quad (37)$$

Where C is copula and c is the Copula density. Considering 100-point discretization for each axis in Copula grid, the copula density will be summarized in a 99 × 99 matrix.

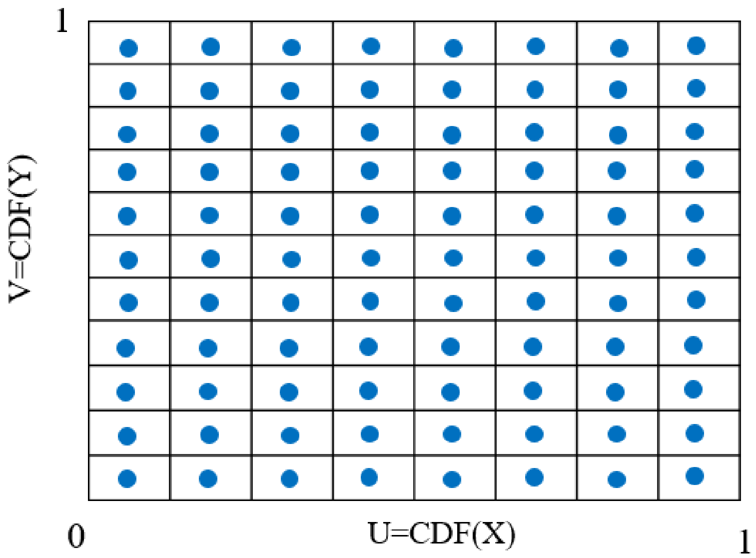


Figure 4-6. Transfer copula density to centre of every cell in Copula grid

4.3.1.3 Adaptive mesh grid generation for marginals from location B

It is assumed that marginal values for random variable Hs and W s are known. To generate the joint data from the marginals, different bin ranges from each marginal are required. In order to take this into account, the density of measurements from three different bin size along each marginal is chosen to cover the whole range of marginals. The formulation in each zone is shown in below Figure 4-7.

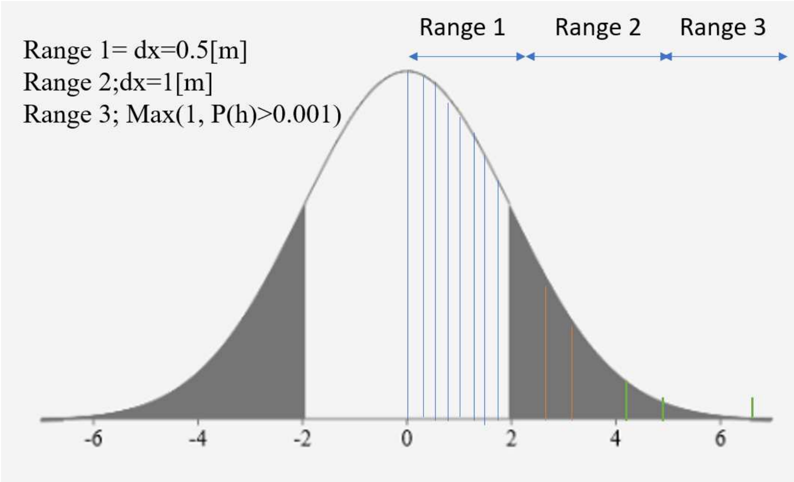


Figure 4-7. Adaptive mesh size

4.3.1.4 Turning this grid to [0,1] domain by taking CDF of marginals

The generated adaptive mesh in the previous step from the marginal will be transferred to copula domain by taking the emprical CDF of these values (step 2 of Figure 4-8). The discretization of another marginal is not changed in this step. The output of this section would be a new mesh grid for the imported marginals in the copula domain in one axis.

4.3.1.5 Finding marginal bin's location relative to the closest copula cell grids

The Copula is divided based on relative distance of Marginal grid to the copula grid by simple linear relation (step 3 in Figure 4-8):

$$C_{left} = \frac{dx_1}{dx} C_{cell} \ and \ C_{Right} = dx_2/dx \tag{38}$$

4.3.1.6 Calculate copula density and marginal representing each bin for one of the marginals

The Copula density in each cell of new grid is calculated using equation.39 (step 4 in Figure 4-8). In the next step the representant wave heigth for each column of mesh grid is summarized using equation.40 (step 5 in Figure 4-8). The same process is followed for the other marginal as shown in Figure 4-9.

$$H_{cell} = \frac{H_j + H_{j+1}}{2} \ ; where\ j\ is\ the\ copula\ grid\ number, here\ j = 1{:}\,100 \tag{39}$$

$$H_{bin} = \frac{\left(\sum_{i=1}^{m} P_{(cell,i)}\ H_{cell,i}\right)}{\sum_{i=1}^{m} P_{(cell,i)}} \tag{40}$$

$$H_{column} = \frac{\sum_{k=1}^{100} P_{(bin,k)}\, H_{bin,k}}{\sum_{k=1}^{100} P_{(bin,k)}} ; 100\ is\ copula\ grid\ of\ another\ marginal \tag{41}$$

$$P_{column} = \sum_{k=1}^{100} P_{(bin,k)} \tag{42}$$

$$\sigma_{copula}^2 = \sum_{k=1}^{100} P_{(bin,k)}\ (H_{bin,k} - H_{column})^\wedge 2 \tag{43}$$

$$\sigma_{tot}^2 = H_{ave} - \frac{H_{column}}{sqrt\left(\sigma_{avg}^2 + \sigma_{copula}^2\right)} \tag{44}$$

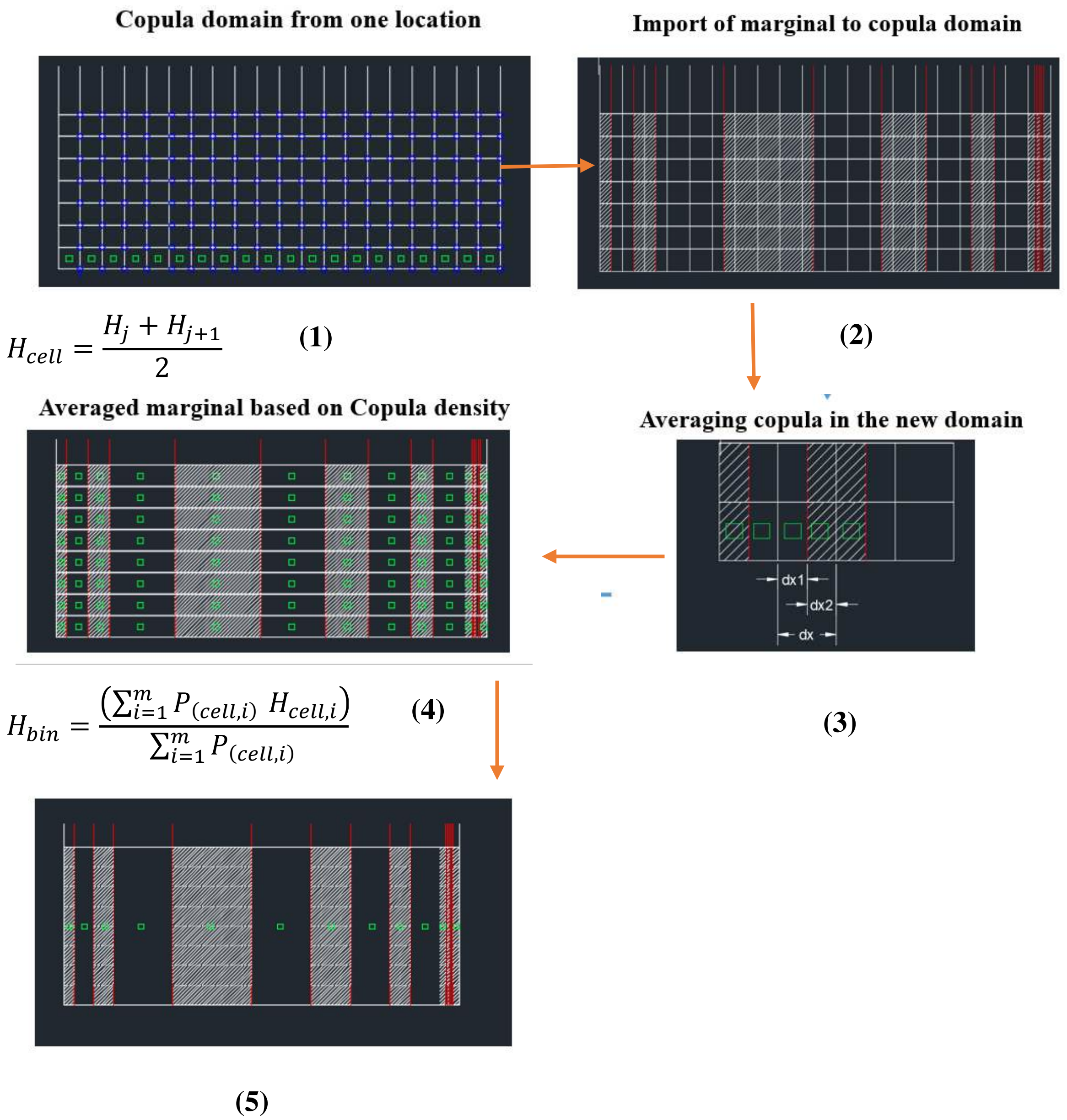


Figure 4-8. Import X marginal to Copula mesh grid

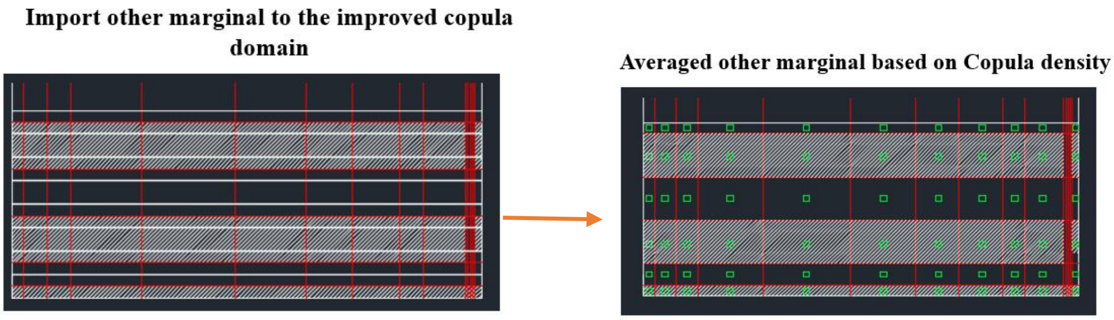


Figure 4-9. Import Y marginal to Copula mesh grid

4.3.1.7 Simplistic method to compare lumped load fatigue damage

The real structure's fatigue damage under dynamics loading was not performed due to time limitation regarding setting both time-domain and frequency domain simulation. A simple relationship between fatigue damage and met-ocean data based on section 3.2.3.3 is applied to calculate a relative damage between two different methods of lumping sea-states follows

$$\frac{D_2 - D_1}{D_2} \propto \frac{H_{s,2}^{\mu} - H_{s,1}^{\mu}}{H_{s,2}}$$
$$\frac{D_2 - D_1}{D_2} \propto \frac{V_2^{\mu} - V_1^{\mu}}{V_1} \tag{45}$$
$$\frac{D_2 - D_1}{D_2} \propto \frac{1/T_{z,2} - 1/T_{z,1}}{1/T_{z,2}}$$
$$Subscript\ 2 = Copula\ lumped\ data$$
$$Subscript\ 1 = Real\ lumped\ data$$

Where μ is the slope of S-N curve [3-5]. An estimate of $\mu = 4$ is assumed for the relative fatigue damage comparison above.

The simplistic formulation given above is used to calculate the fatigue damage difference between lumped real data and lumped data generated by copula. The lumped values that have a very low probability of occurrence are excluded from this calculation. The long-term fatigue damage is proportional to the probability of occurrence multiplied by met-ocean the damage and these values do not have big impact on fatigue damage. As a result, the lumped values at upper and lower tails where the probability of occurrence of marginals are less than 1%, have been excluded from the calculations.

Furthermore, a qualitative analysis is performed based on capability of lumped sea sates to capture important frequencies for fatigue damage of structures especially natural frequency of wind turbines. A natural frequency of $0.25[hz]$ corresponding to natural period of $T = \frac{1}{f[hz]} = 4\ [s]$ is assumed for a monopile to check the sensitivity of lumping of wave period around this value.

5 Results and Discussion

The results of calculations and data presentations are shown in this section. A discussion on the applicability of the copula method in determining lumped data and applicability of these data in simple estimation of fatigue damage are presented and discussed in this section.

5.1 Marginals of random variables

Bar chart of distribution for the three met-ocean random variables (H_S, W_s, T_p) are shown in the figures below. To make it easier to compare the density of the data, the amounty of data per bin is normalized with the total number of measurements. In order to compare the distribution of the random variables with some well-known distributions, different distributions are fitted to real measurements.

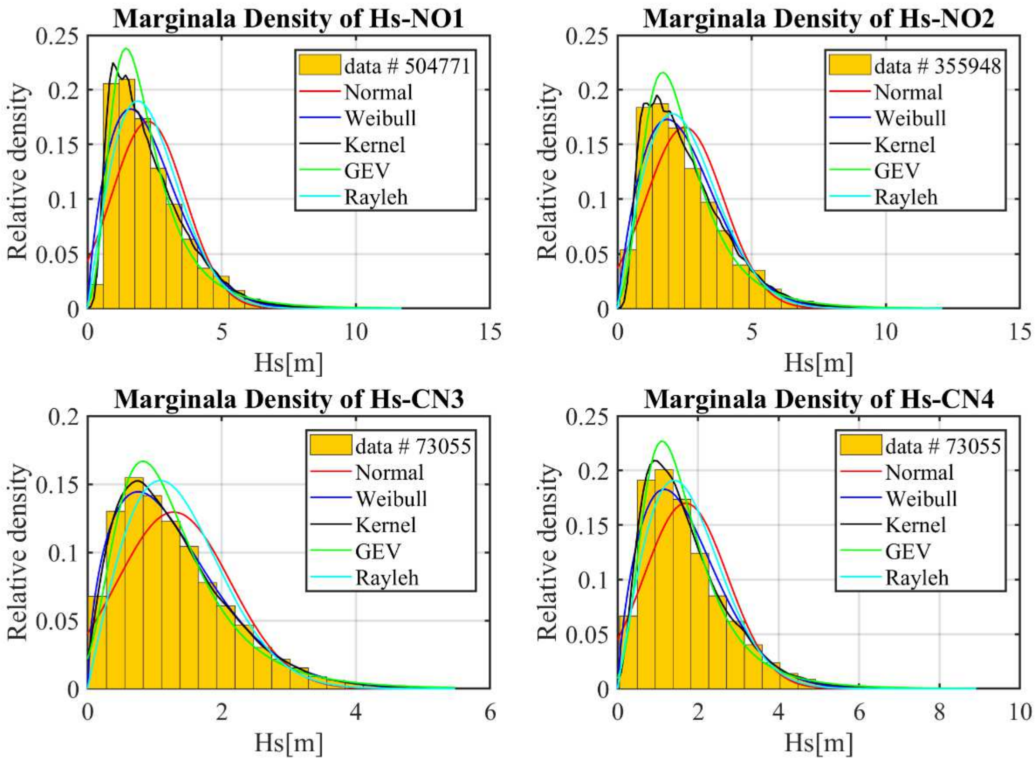


Figure 5-1. Wave height density

The yellow bars represent the empirical distribution of measurements and the lines represents different type of distributions fitted to data. It can be seen that none of the distributions fully fit the data. Kernel distribution is considered as a continuous emprical distribution of real data.

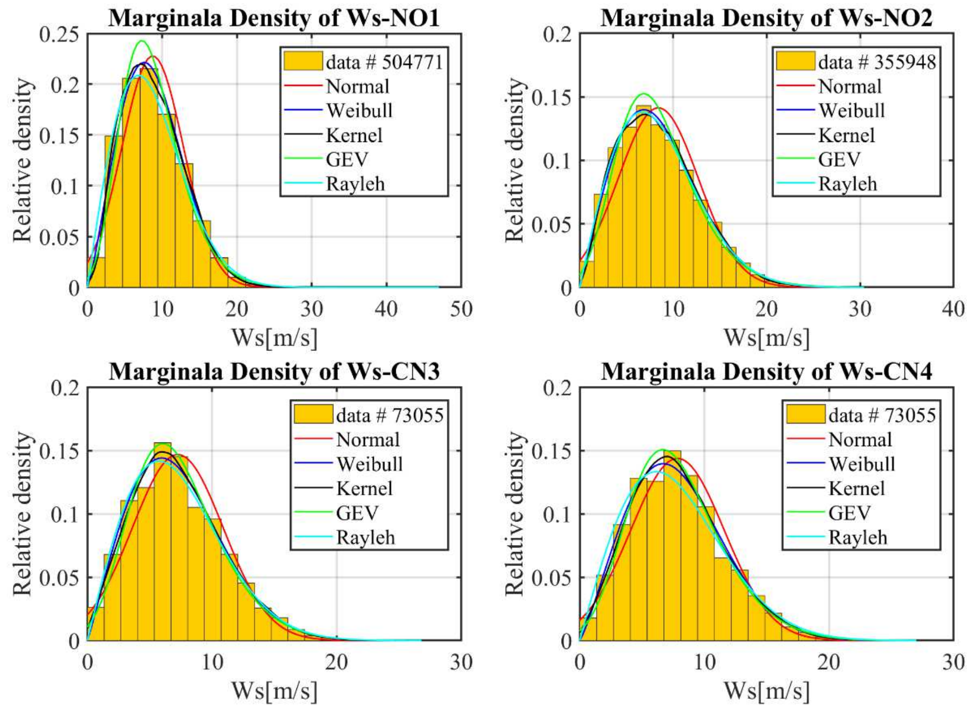


Figure 5-2. Wind Speed density

Like wave height, none of the distributions closely fit wind data. However, from visual observations, the Weibull distribution has a better fit to the data. It can be seen that extreme values have a very low probability of occurrence in comparison with mean values.

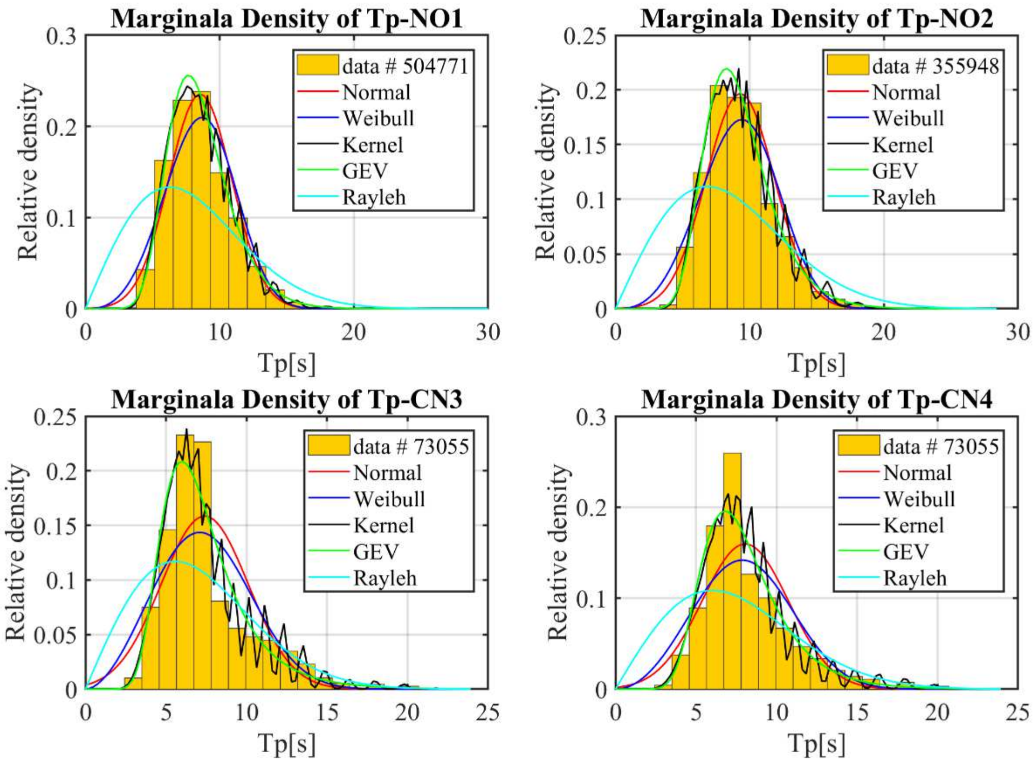


Figure 5-3. Wave period density

As can be seen above, none of the families of distributions fully fit the data, and each marginal has a different distribution than other marginals. This difference clarifies why families of joint distributions cannot fully capture the joint behaviour of these random variables. Comparing the above pictures, it can be seen that the mean values and maximum values are different for different locations, meanwhile the distribution of different random variables ‘shapes are similar.

5.1.1 Goodness of fit

To examine the goodness of analytical distributions to real data, the probability plot is used. Here a graph presenting the probability plots for wind speed at location “NO1” is shown. The probability plots for other parameters at different locations are shown in 1 Appendix .

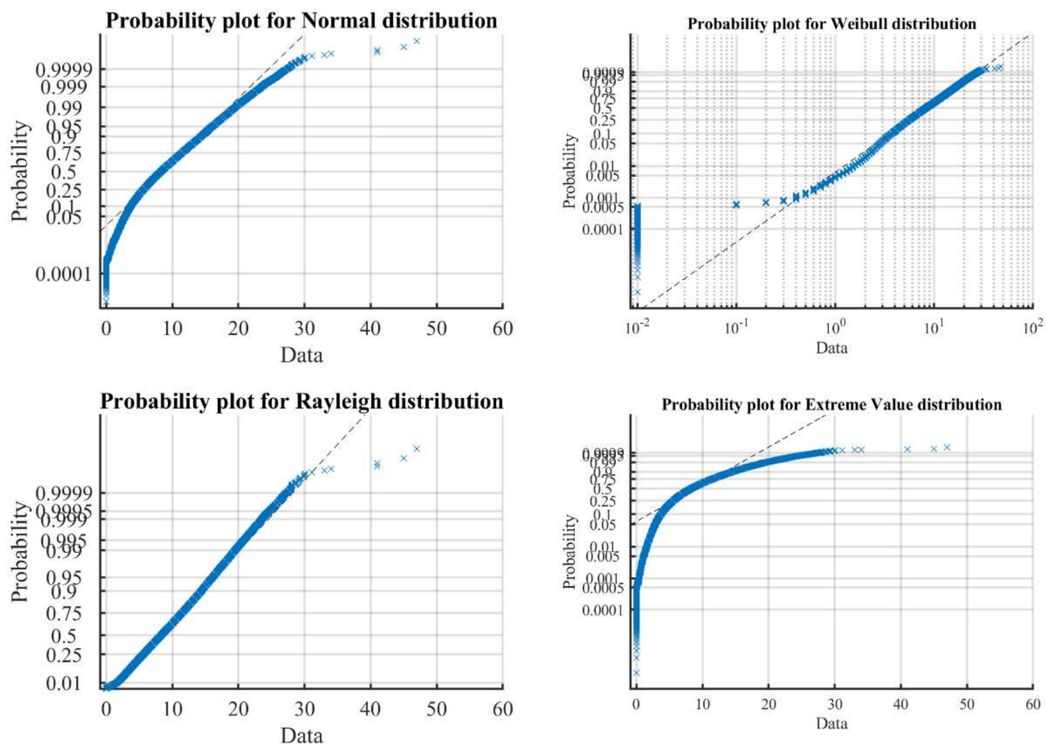


Figure 5-4. Probability plot for Ws at NO1

According to Figure 5-4, the fitted distributions do not fully fit the data. The incompatibility is obvious in the tail where the minimum and maximum values, with very low probability of occurrence, take place. It should be noted that the vertical axes represent the cumulative probability (section 3.1.1).

5.2 Bivariate presentation of data (wave height and wind speed)

The pre-processed data for these four locations are plotted in the Figure 5-5. Each measurement is plotted as a blue circle, and a cloud of these circles representing all of these measurements. Contour lines showing the density of measurements are also plotted inside the scatterplot. The horizontal and vertical lines represent the mean of each variable. The contour lines have a relatively similar shape to each other which shows the joint behaviour follows the same pattern in different locations.

A density plot of measurements is shown in Figure 5-6. Spearman's Rho as a measure of dependence calculated for bivariate data shows similar values for different locations which means the degree of linear dependency between wave height and wind speed at one location looks like the other sites.

A 3D-histogram of joint measurements are plotted in Figure 5-7. This figure has a better visualization of the joint measurement distribution. For high values of wind speed and wave height, the frequency of occurrence is much lower than small waves and wind speed. According to section 3.2.3, the long-term fatigue damage is proportional to probability of occurrence of sea-state.

In locations CN3 and CN4, the wave height and wind speed mean are lower than NO1 and NO2. This may be due to the fact that these locations are located in the shadow zone of the North Sea and the wind wave blowing over long fetch from the North-West do not touch them directly.

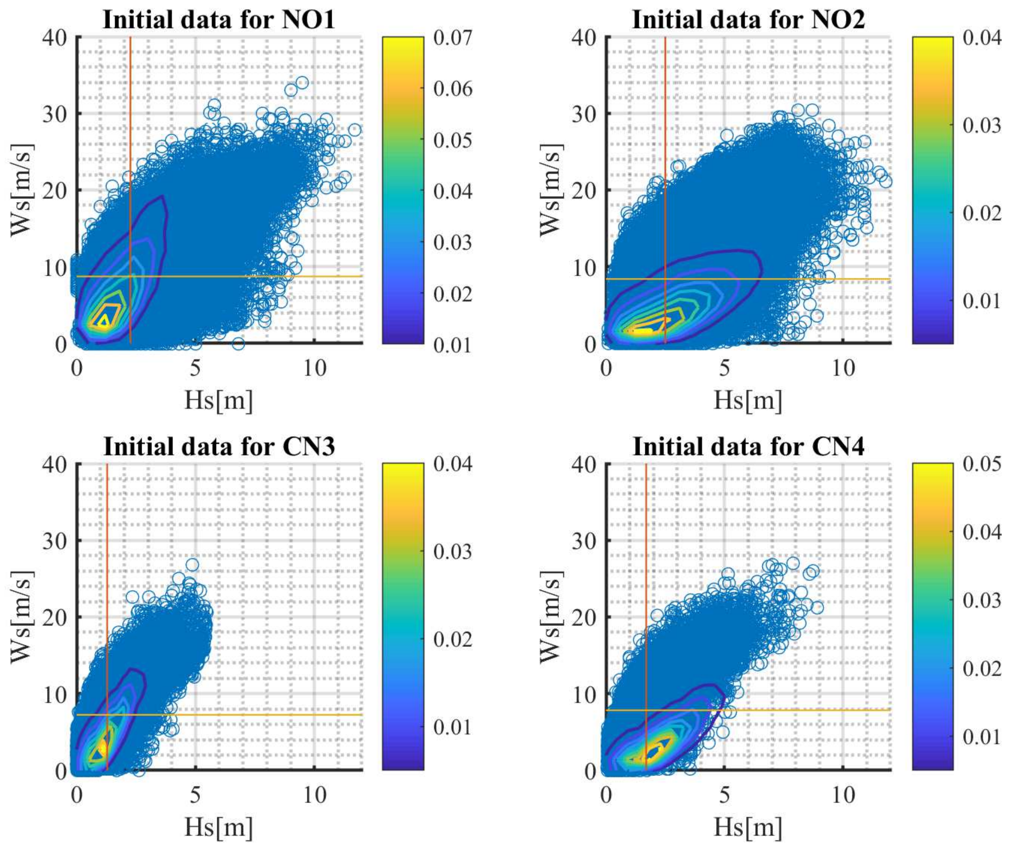


Figure 5-5. Scatter plot of measurements for Hs and Ws

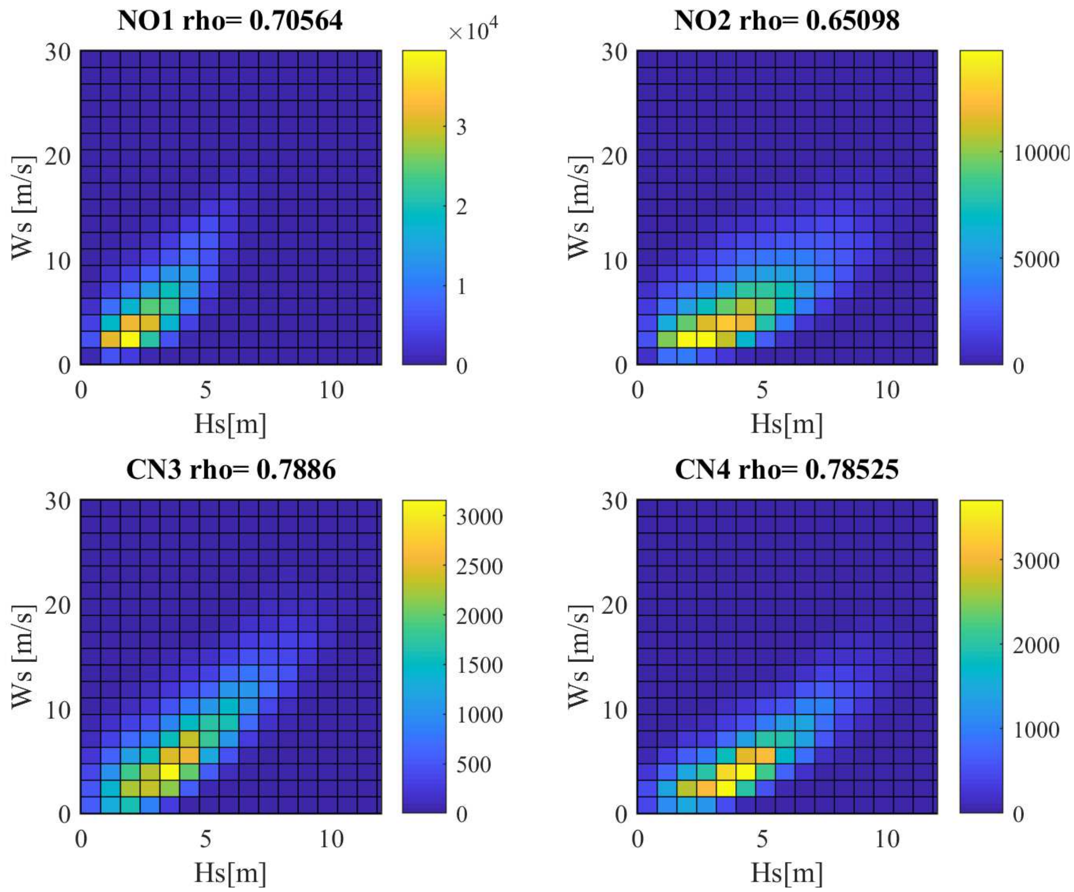


Figure 5-6. Density plot for all stations

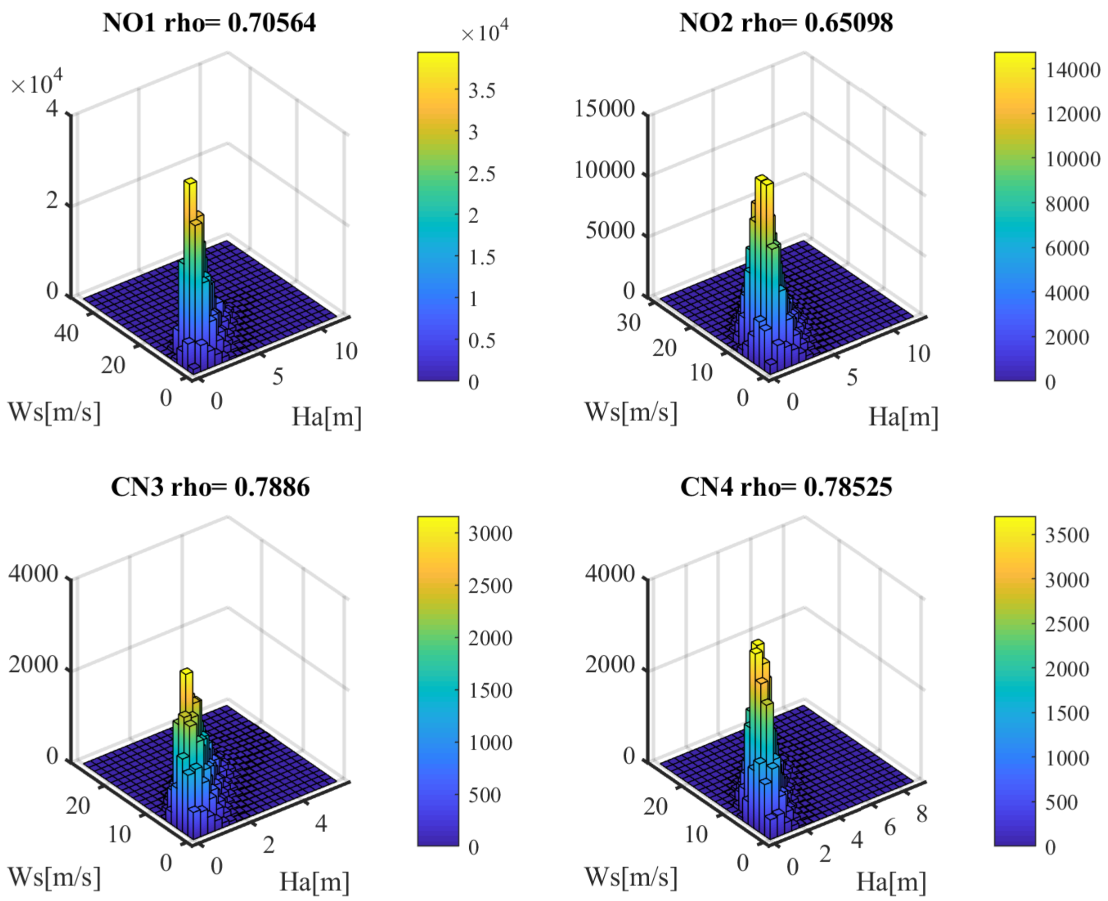


Figure 5-7. 3D histogram of joint measurement for Hs and Ws

5.2.1 Simplified lumped data for wind speed and wave height

To lump the joint values of wind and wave, a simple averaging method is performed by keeping the distribution of wind speed and averaging over wave height and the other way around as mentioned in section 3.2.2. The result of this lumping is shown in Figure 5-8.

If two random variables are linearly dependant($\rho = 1$), the two generated lines in this graph will be adjusted on each other. As is expected, there is a difference in the two graphs as the two variables are not completely dependent. The choice of each method should be based on the application of lumped data, which physical phenomenon are going to be described by lumped data and to the extent to which they are sensitive to each variable. A polynomial of the order of 5 is fitted to the data to represent the trend of lumping.

5.2.2 Copula calculations

5.2.2.1 Rank generation

Following the first 6 steps mentioned in 4.3.1.1, the measured data are transferred to copula domain, $[0,1]^2$ domain, with a uniform distribution for each marginal. While the original data is plotted in Figure 4-2 , the pairs of rank representing each pair of measurements are plotted in Figure 5-9 . The correlation of joint values is preserved in this transformation.

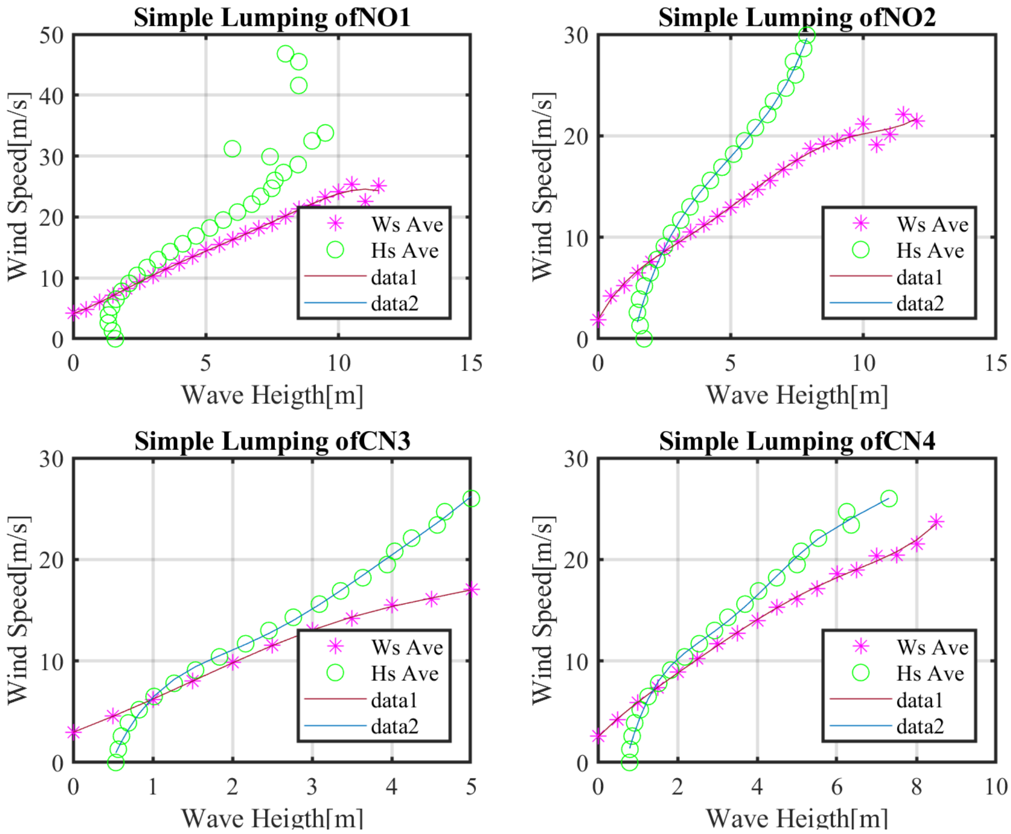


Figure 5-8. Comparison of lumping by wave height averaging (Green) and wind speed averaging (Red)

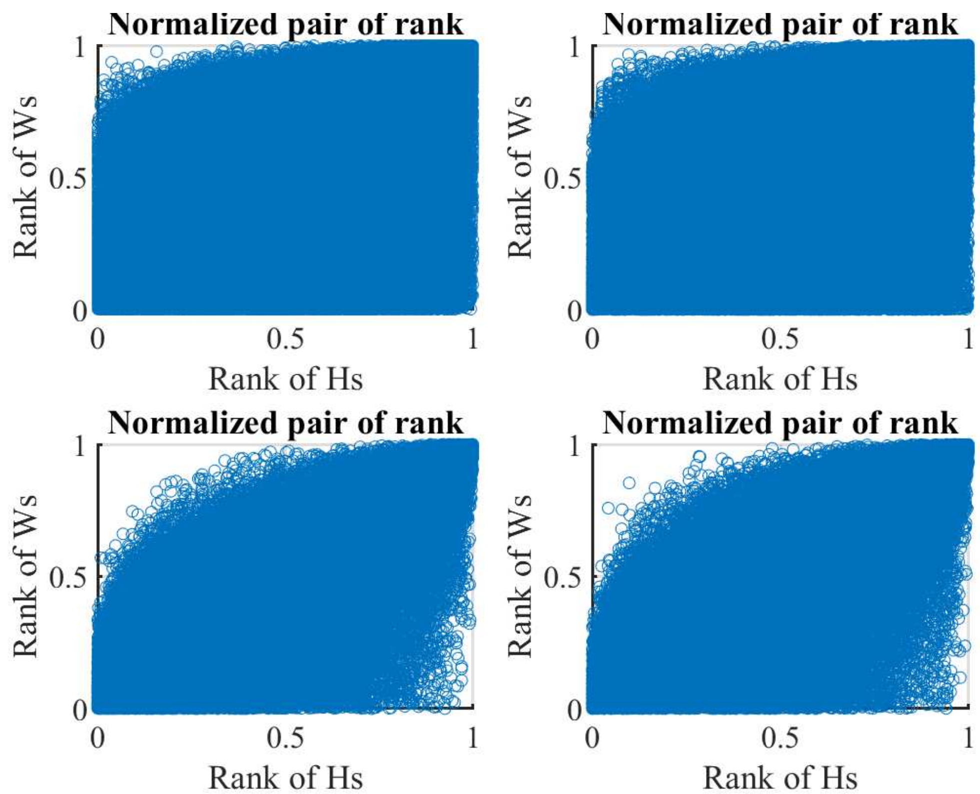


Figure 5-9. Generation of rank pairs(R,S) from measurement pairs(Hs,Ws)

5.2.2.2 Copula density

The 3D-scattering of the pairs of ranks are plotted in Figure 5-10 which represents the density of ranked correlation or the copula density.

As shown in the Figure 5-10 the wave height and wind speed are highly correlated on both tails. This can be interpreted as saying that high wind velocity and high wave height are highly correlated, while this correlation is significantly less for an average or normal sea-state. Comparing with Figure 3-3, it can be observed that this copula has non-symmetric tale.

5.2.2.3 Copula

Applying an interval of 0.01, the Cumulative distribution of Wave height and wind speed are discretised into a 100 × 100 mesh grid. The value in the grid points of each cell is calculated from section 4.1.2.1.

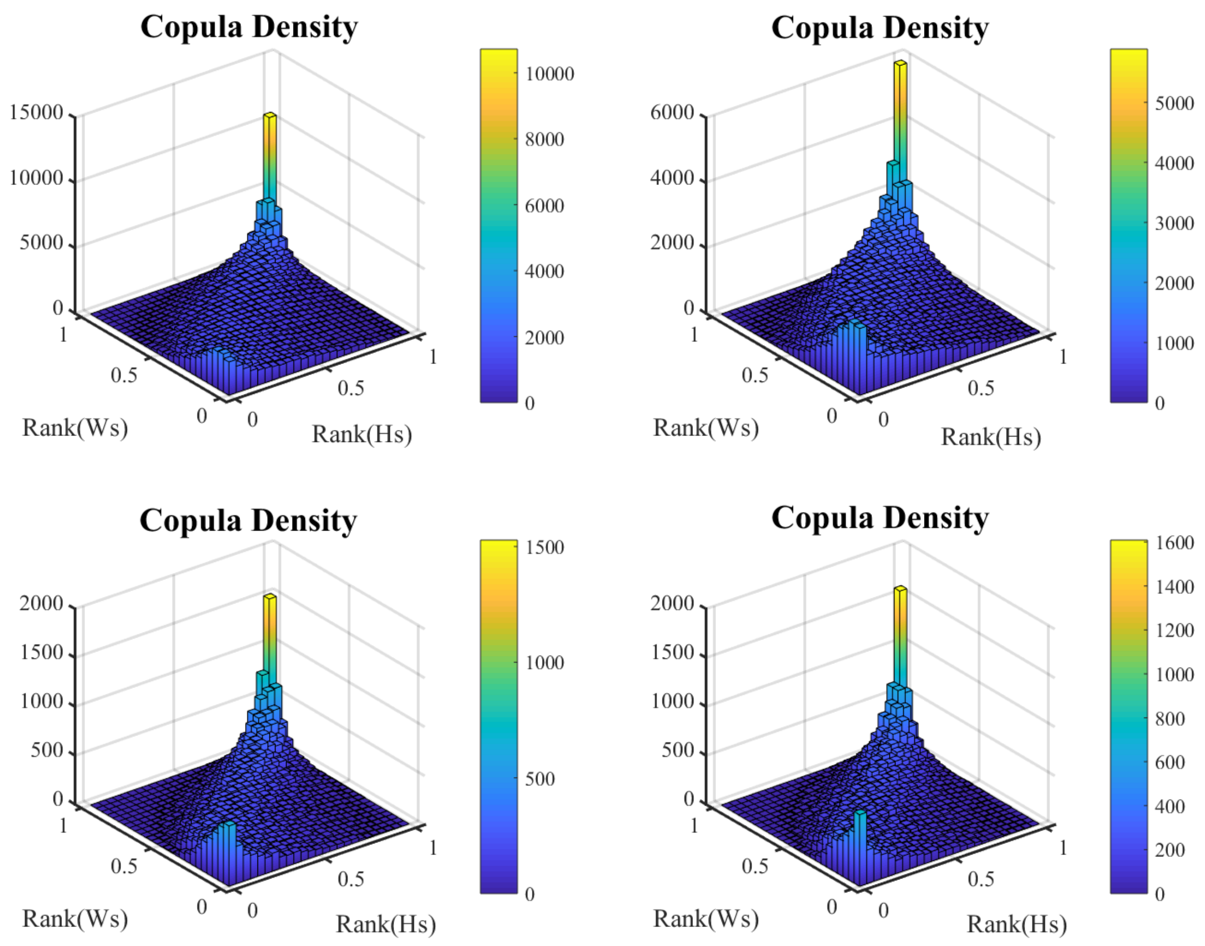


Figure 5-10. Copula Density

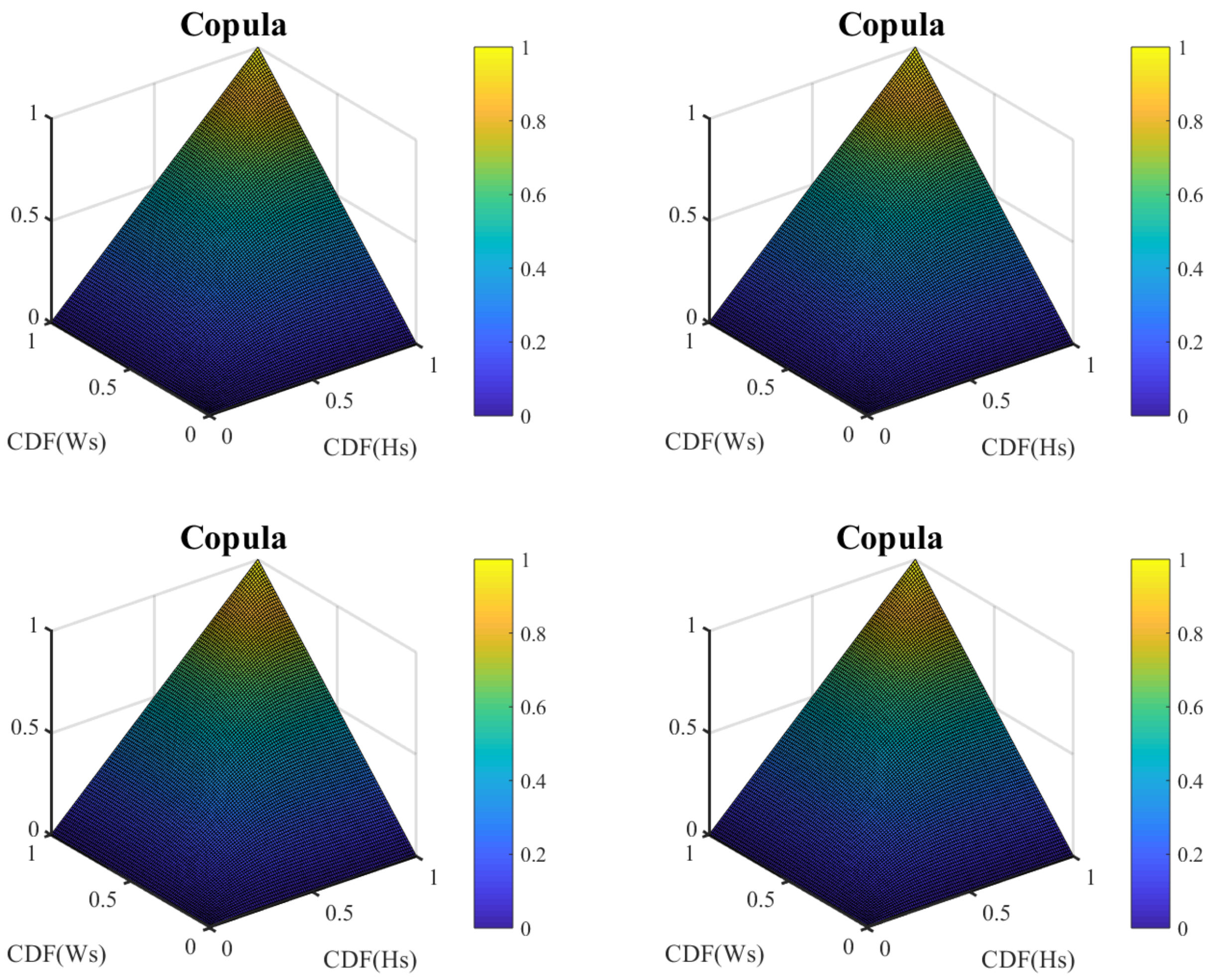


Figure 5-11. Copula

The figure above represents Copula which is a cumulative distribution of joint variables. As it is expected it keeps increasing to one. To interpret this figure in engineering applications, the results of this figure are applied in Figure 5-13.

5.2.3 Comparing copula

To see the possibility of attributing copula from one location to another location the copula from two different locations are subtracted. In the Figure 5-12, $CO_2 - CO_1$ represents subtraction of copula at location 2 from copula at location 1. The percentile difference of copula varies spatially for different cells. Even though the maximum difference at few locations reach 40 percent, the average copula difference is between 10-15 %. This similarity of copula can be interpreted as saying that the joint behaviour of two random variables follow the same pattern as another location. To examine the validity of this interpretation, copula from one location is applied to another location. Due to the close distance between NO1 and NO2 the copula difference of Co2-Co1 has very low values meaning the joint behaviour of wind and wave has a very similar behaviour since the source of wind and wave are similar in those locations. However, the difference of NO1 and CN3 or CN4 shows more difference in copula values. This can be attributed to the distance of these locations and the difference of wind and wave blown to that region. However, as can be seen the general difference is small which shows the general climate in the North Sea can be presented by Copula.

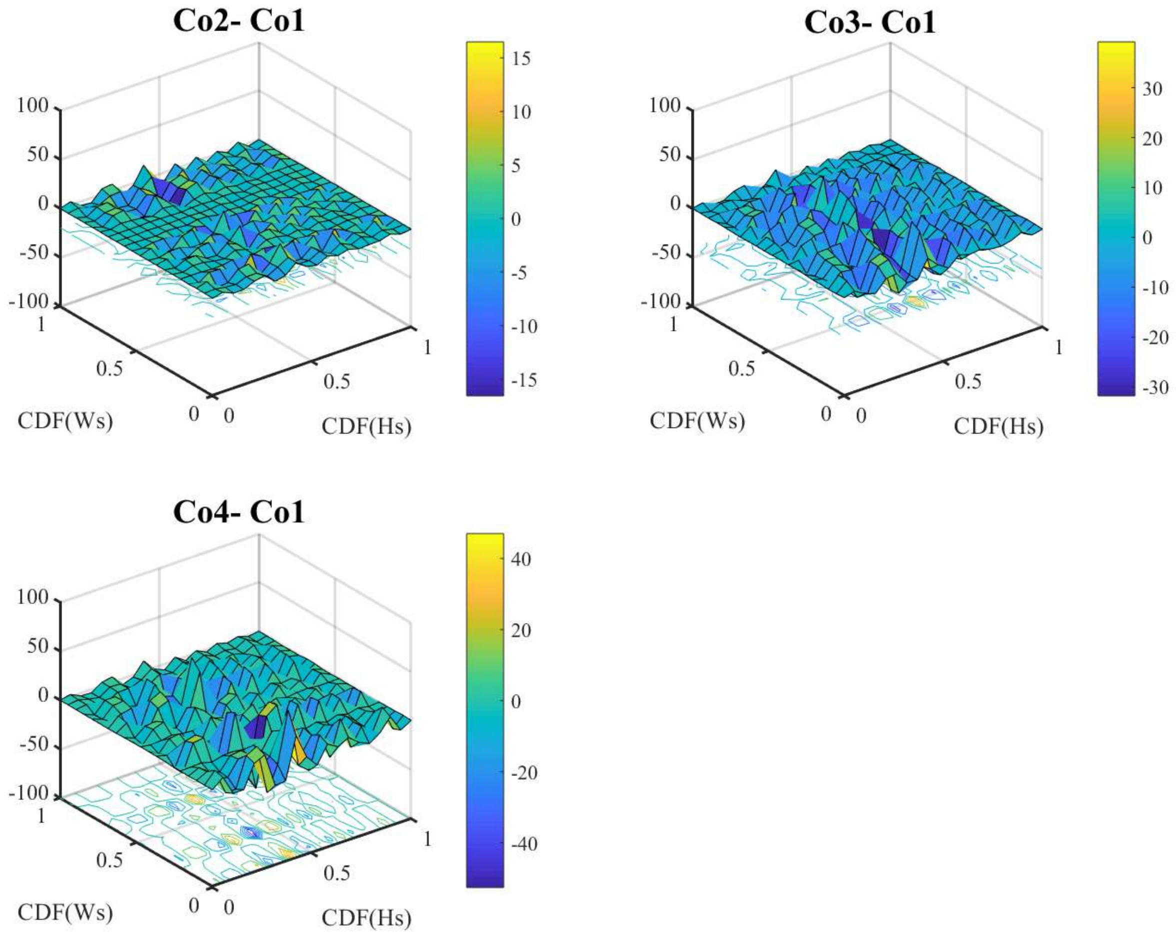


Figure 5-12. Copula difference

5.2.4 Applying copula for one location to another

The 8-step procedure mentioned in section 4.3 is followed. In Figure 5-13 the source copula is calculated from NO1 which means it is assumed the joint behaviour of wave height and wind speed at four locations are governed by copula at NO1. The marginals at each station are based on that station. However, it is assumed that the correlation between wave height and wind speed are unknow at NO2, CN3, CN4, and the correlation of them are imported from NO1.The generated joint data are lumped over wind speed (averaged over wind speed) and the results are plotted in Figure 5-13 as circles. The real data at each location are lumped over wind speed and are plotted as red stars. The difference between red dots and circles show how well copula from NO1 can predict the joint behaviour in other locations in the North Sea.

The blue line represents the wave height that 99% of measurements are smaller than that. In other words, it represents the upper tail of joint data. The reason that the number of points in this region is relatively big in comparison with the 1% probability of occurrence is that the interval in the adaptive mesh grid in this region is defined as the maximum of 1[m] wave height difference or probability of occurrence of 0.001. The measurement inside the CDF(Hs)=99% is not as accurate as other locations. This point represents the big tail in Figure 5-10. This is the domain with a very low probability of occurrence but with a very high correlation. Therefore, determination of copula in this cell is quite sensitive and a small deviation between copula of 0.997 and 0.998 can cause lumped data difference. However, as is shown in Figure 5-1, the density of these values is quite low in that region. The standard deviation of generated data during lumping for each bin is calculated. As shown in the graph this value is negligible.

The lumping process is done over wind speed while keeping the wave height distribution unchanged. The bin size for wave height is equal to 0.5[m] for lumping of real data. The bin size for lumping of generated data is based on adaptive mesh generation process.

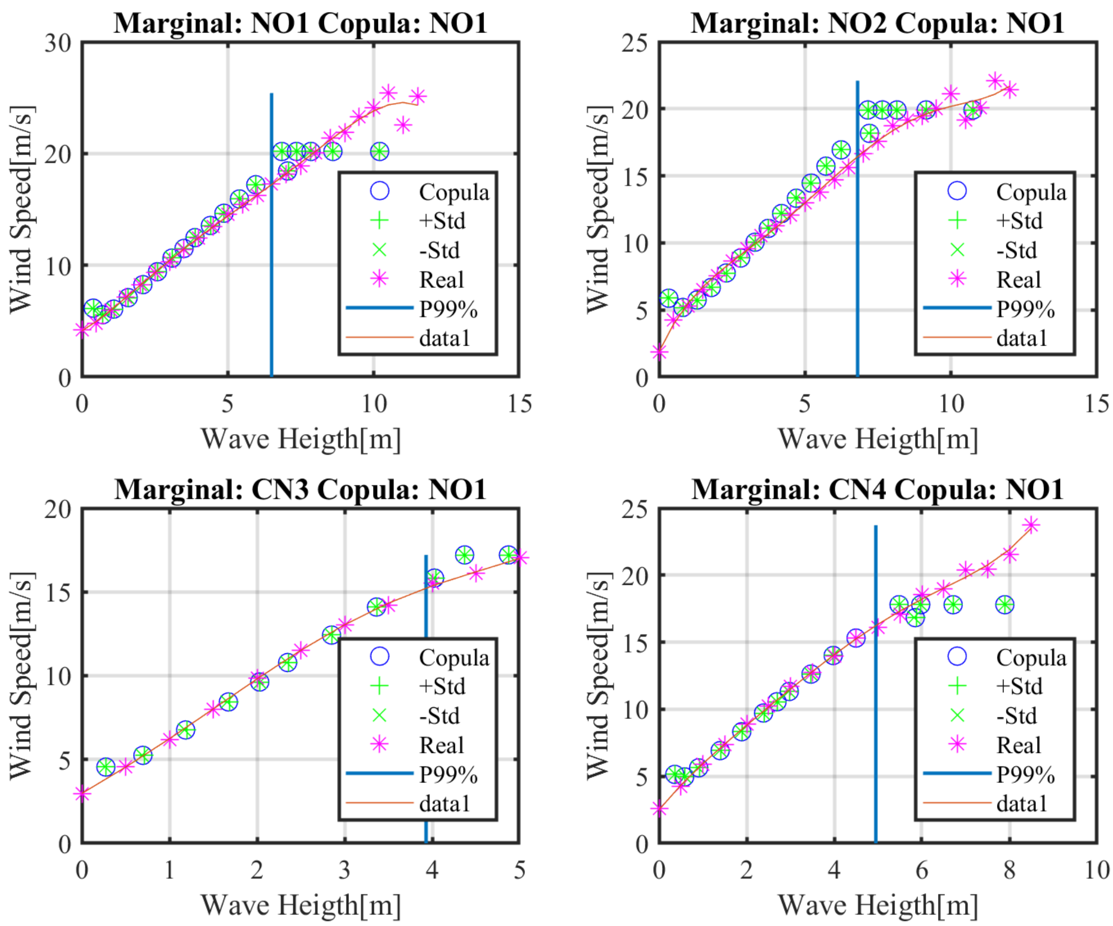


Figure 5-13. Generated data using copula from NO1

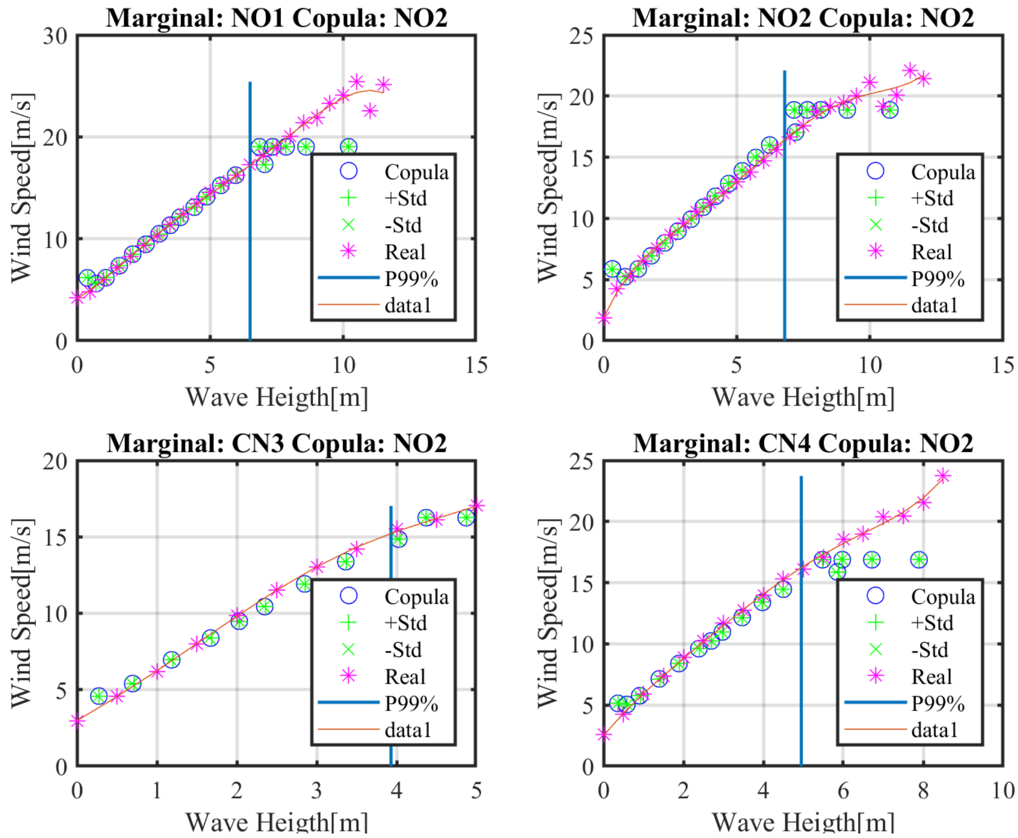


Figure 5-14. Generated data using copula from NO2

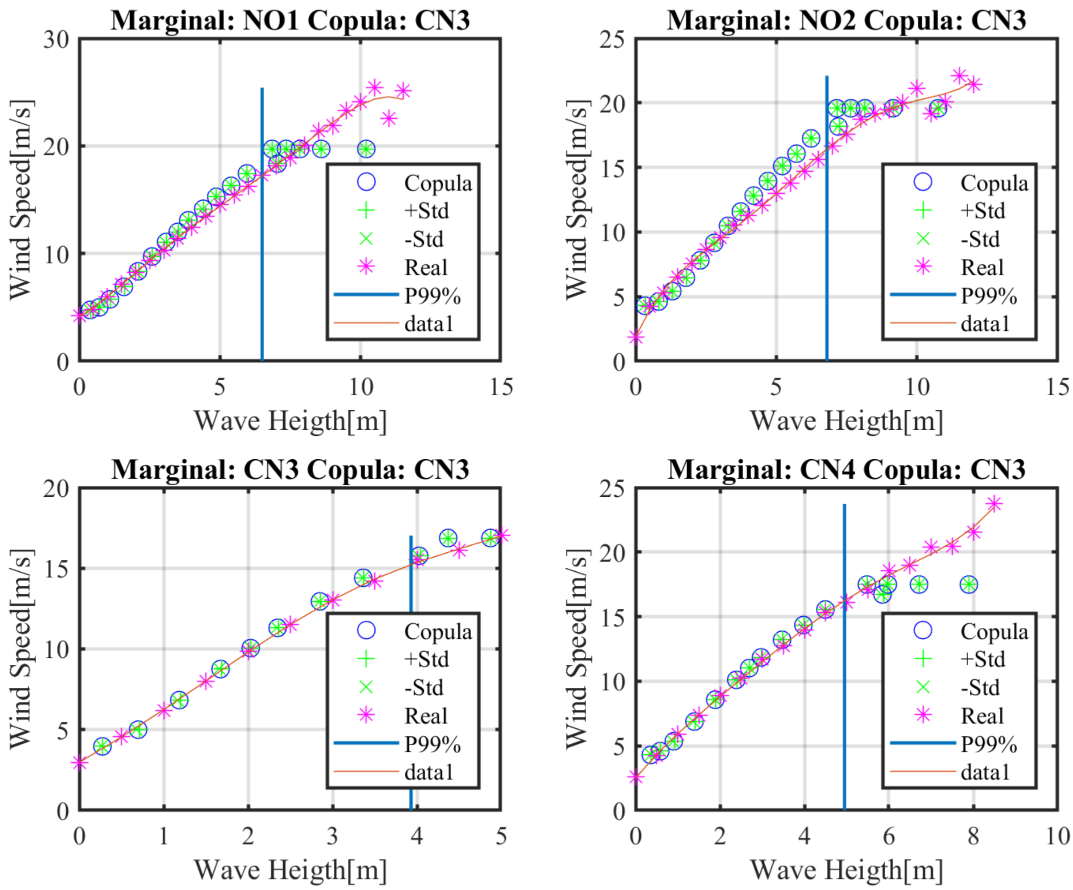


Figure 5-15. Generated data using copula from NO3

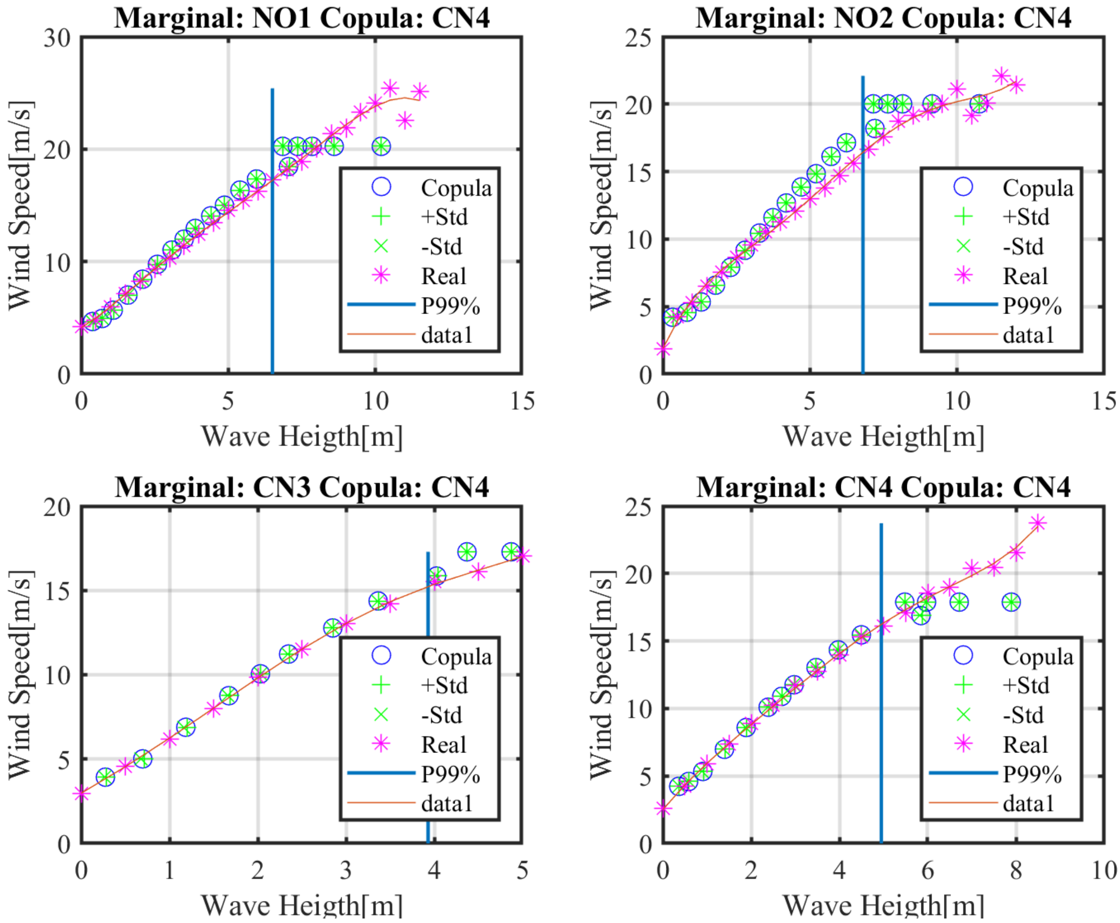


Figure 5-16. Generated data using copula from NO4

To examine the effectiveness of this method for another locations, the data are generated for different locations using copula from NO2, CN3, and CN4. Except the tails, the generated lumped data represent a very good estimation of real lumped data. The quantitative difference are plotted in Figure 5-14 - Figure 5-16.

5.3 Bivariate Modelling of wave height and wave period

The joint behaviour of raw data for wave height and wave period was investigated. First, a representation of raw data is shown and then the joint behaviour is examined by calculating copula. The use of copula to generate lumped data is also investigated. To make this section brief, the rest of figures are plotted in the Appendix.

5.3.1 Bivariate presentation of data (Wave height and wave period)

The scatter plot of $H_S$and $T_p$are shown in Figure 5-17. Scatter plot of Tp and Hs. In comparison with the wind speed and wave height, the joint behaviour is less linear. The relatively circular-contour lines representing the degree of dependence between the pairs of random valuables are less dispersed. The data at CN3 and CN4 are represented at period interval of 0.38[s]. The two lines show the mean values of each marginal.

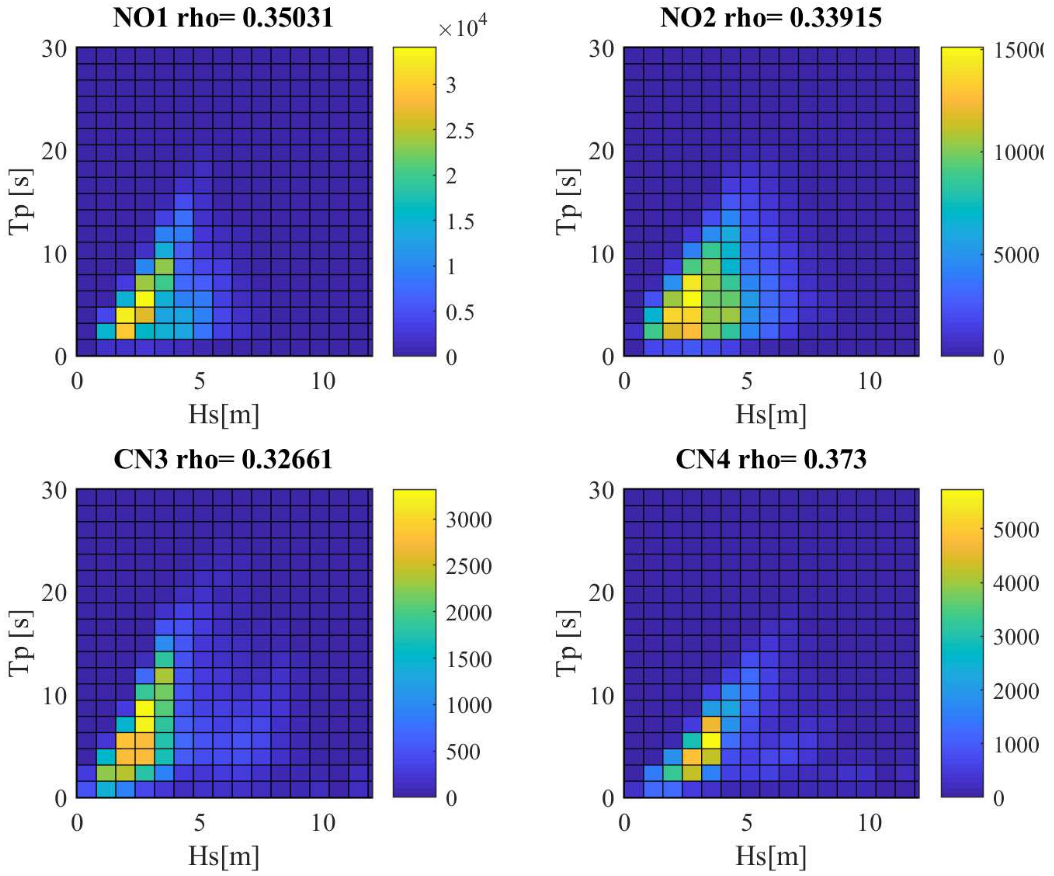


Figure 5-17. Scatter plot of Tp and Hs

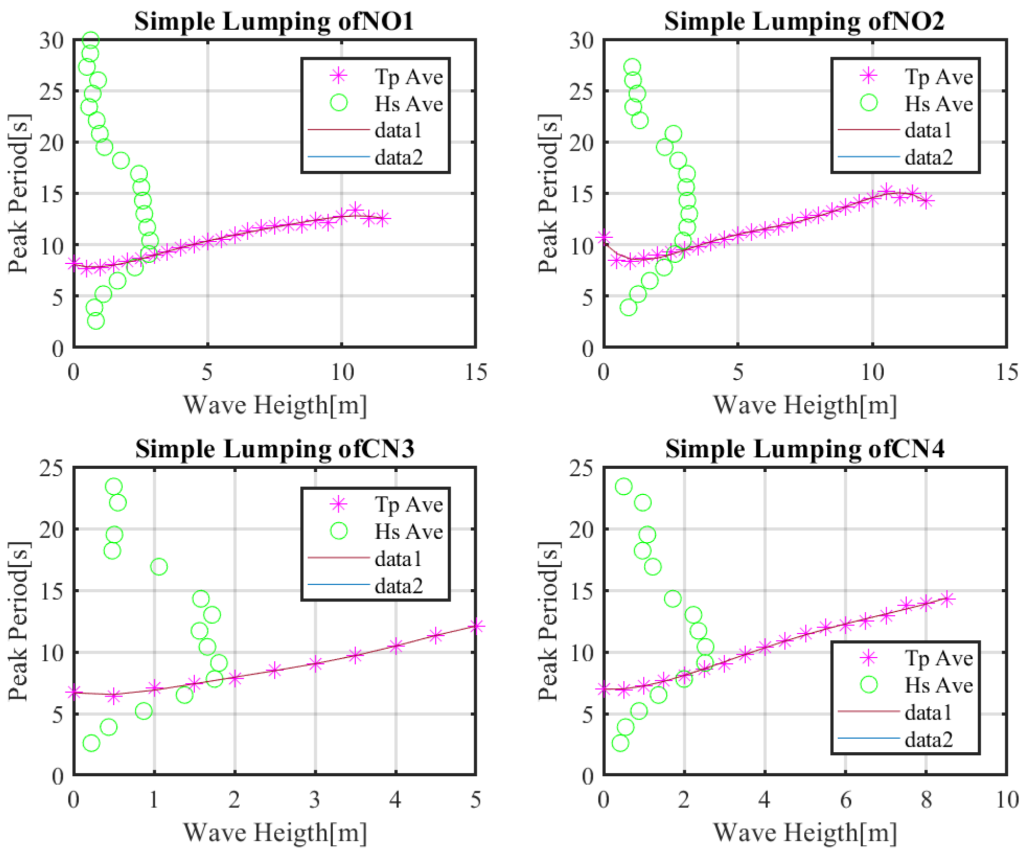


Figure 5-18.Comparison of Lumping data over Tp and Hs

The data are simply lumped by averaging over wave height and wave period which are shown in Figure 5-18. The difference between lumping over each variable is clear in this picture. This difference is justified by circular contour lines in comparison with the relatively linear dependence of variables as shown in Figure 5-5. The correlation coefficient is between [0.3-0.4] which also expresses that the data are not linearly dependant.

5.3.2 Copula calculation

After transferring pairs of (Hs, Tp) to pairs of rank (R, S), the copula density is shown in Figure 5-19. The tail dependency is obvious in this figure. In addition, the density of dependency is not only diagonal, and it is spread.

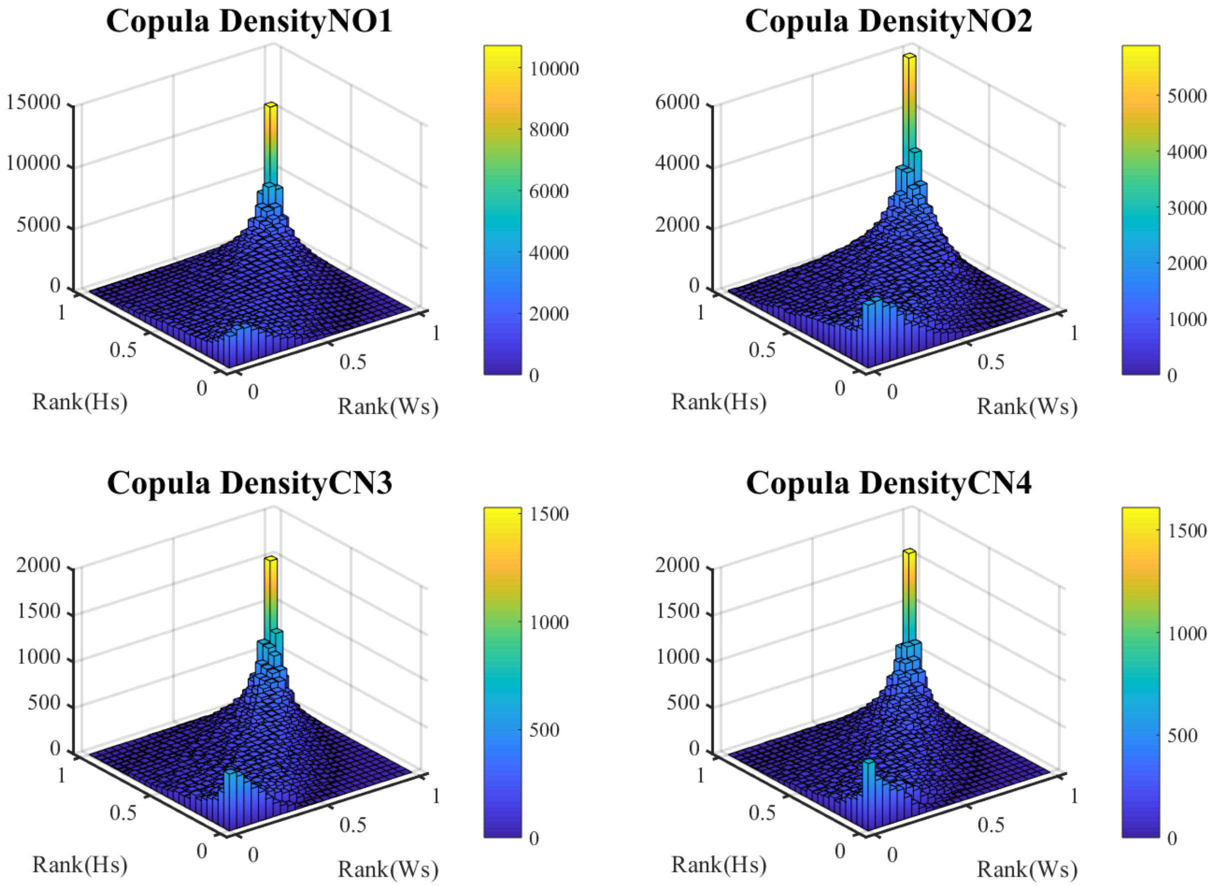


Figure 5-19. Copula density Tp and Hs

The copula is generated and shown in the appendix. The copula difference from copula at location NO1 is shown below.

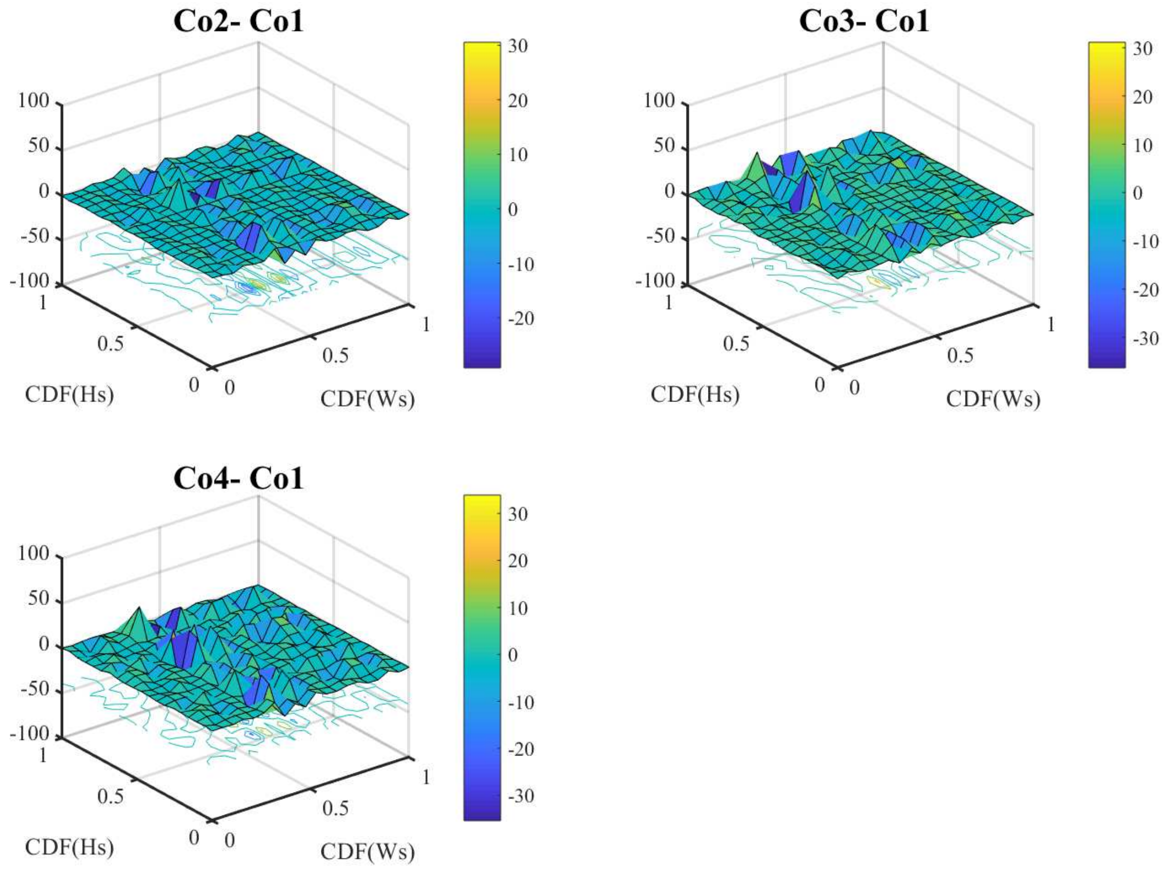


Figure 5-20. Copula difference measure at grid points of CDF(Hs) and CDF(Ws)

The maximum copula difference is less than 40 % while the average copula difference is around 15%. Like wave height and wind speed the copula from NO1 and NO2 shows more similarity relative to copula difference of NO1 and CN3 due to different wave climate that they experience.

5.3.3 Applying copula for one location to another

The 8-step procedure mentioned in chapter 4 is applied to generate lumped data using copula. The generated scatter data are lumped over waver periods (the wave height distribution remains constant). The results show that copula method does not predict the behaviour in the tail due to high correlation in that region. In addition, the copula method over estimates the lumped data. The blue line represents the tail of the data and the wave height with the probability of occurrence of 1%. In comparison to copula approach for wind speed and wave height this method has less accuracy in predicting the lumped data, especially as the wave height increases.

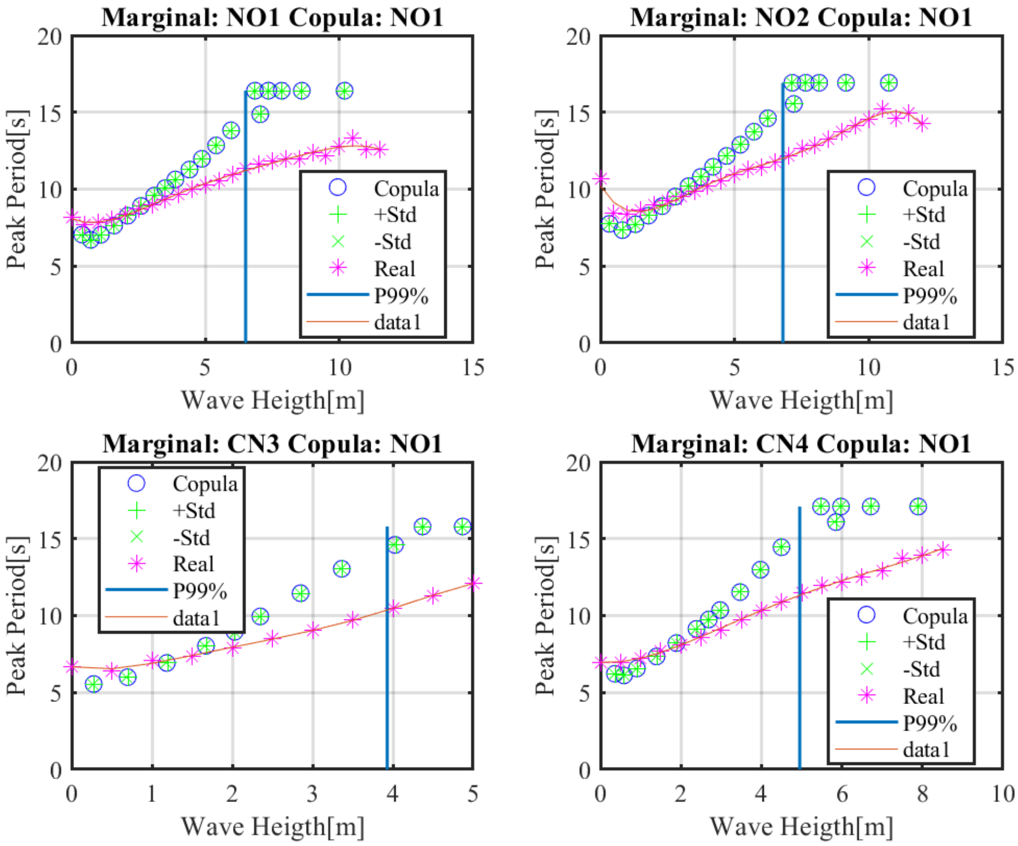


Figure 5-21. Data generated copula from NO1

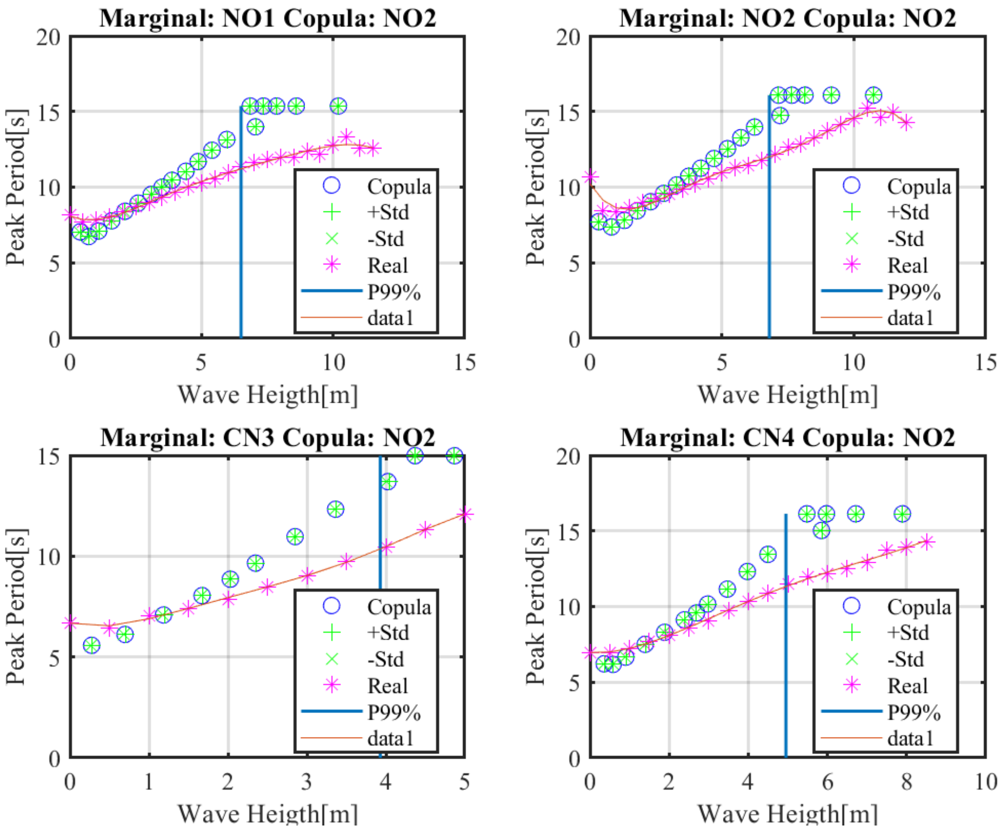


Figure 5-22. Data generated from NO2

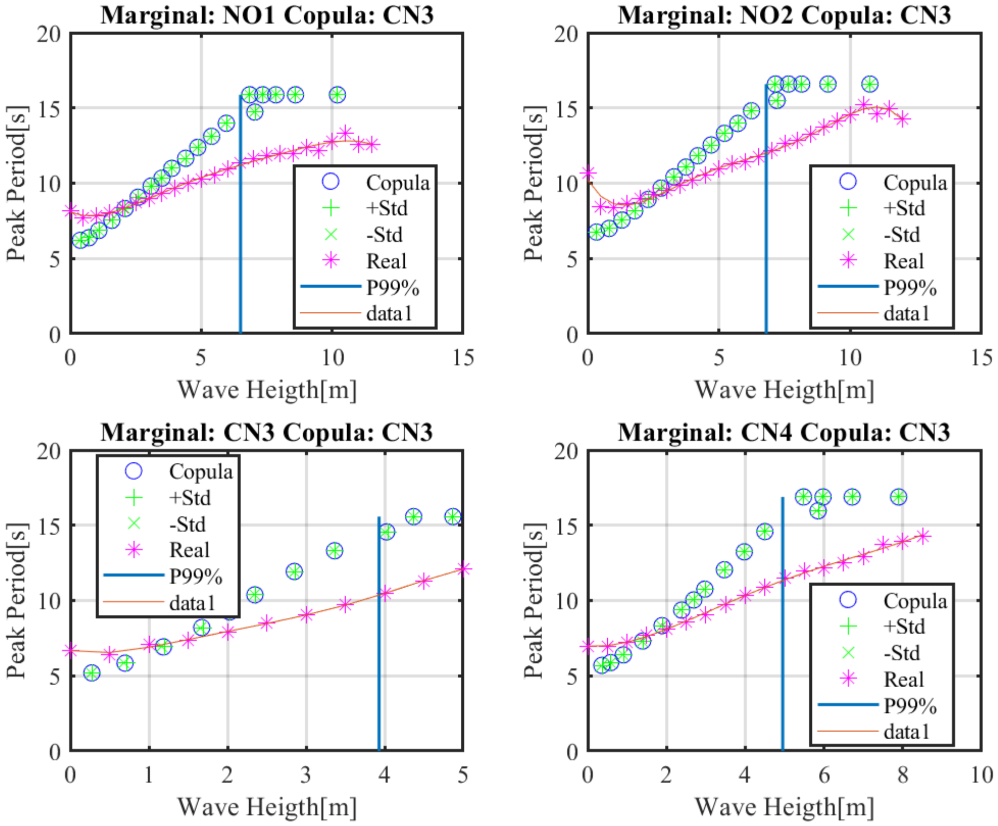


Figure 5-23. Generated data from CN3

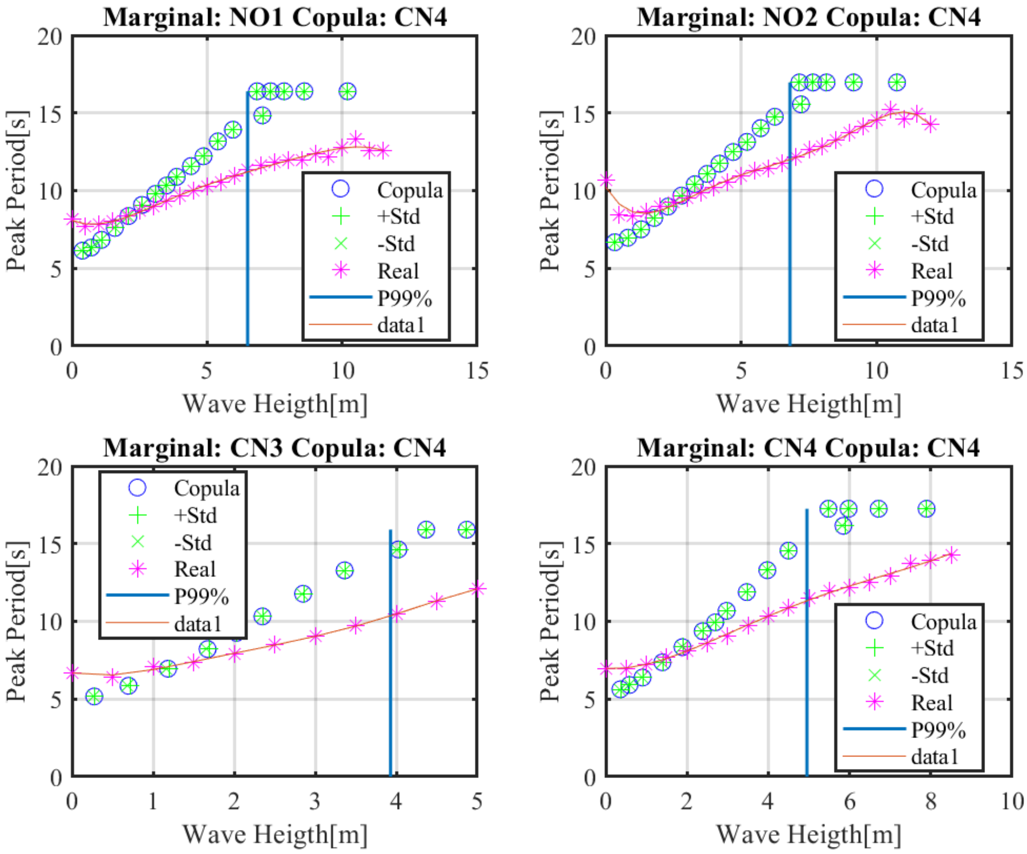


Figure 5-24. Generated data from CN4

5.4 Discussion of copula lumped data

Lumped data generated by copula are compared to the lumped real data. A linear interpolation between lumped copula data is performed to find the values (wind speed and wave period) that have similar values for wave height of real lumped data. Then the corresponding wind speed and wave period for both sources are compared using the relative difference. The lumped data are generated based on the copula at NO1.

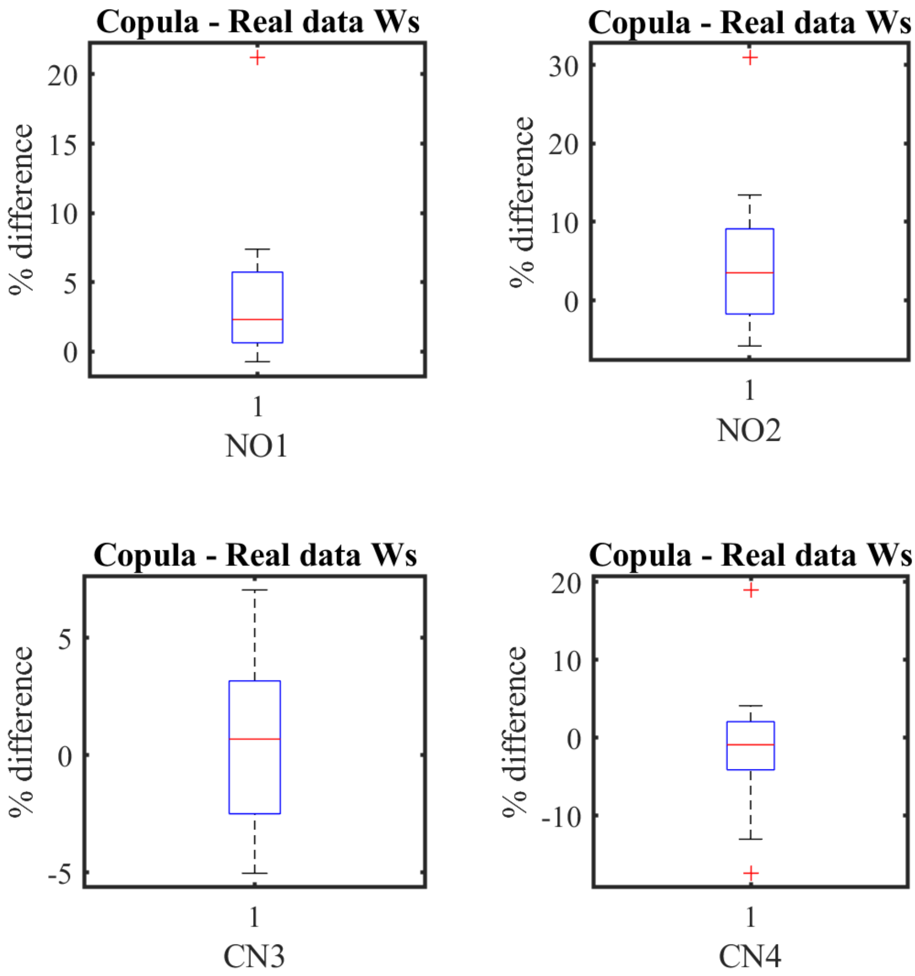
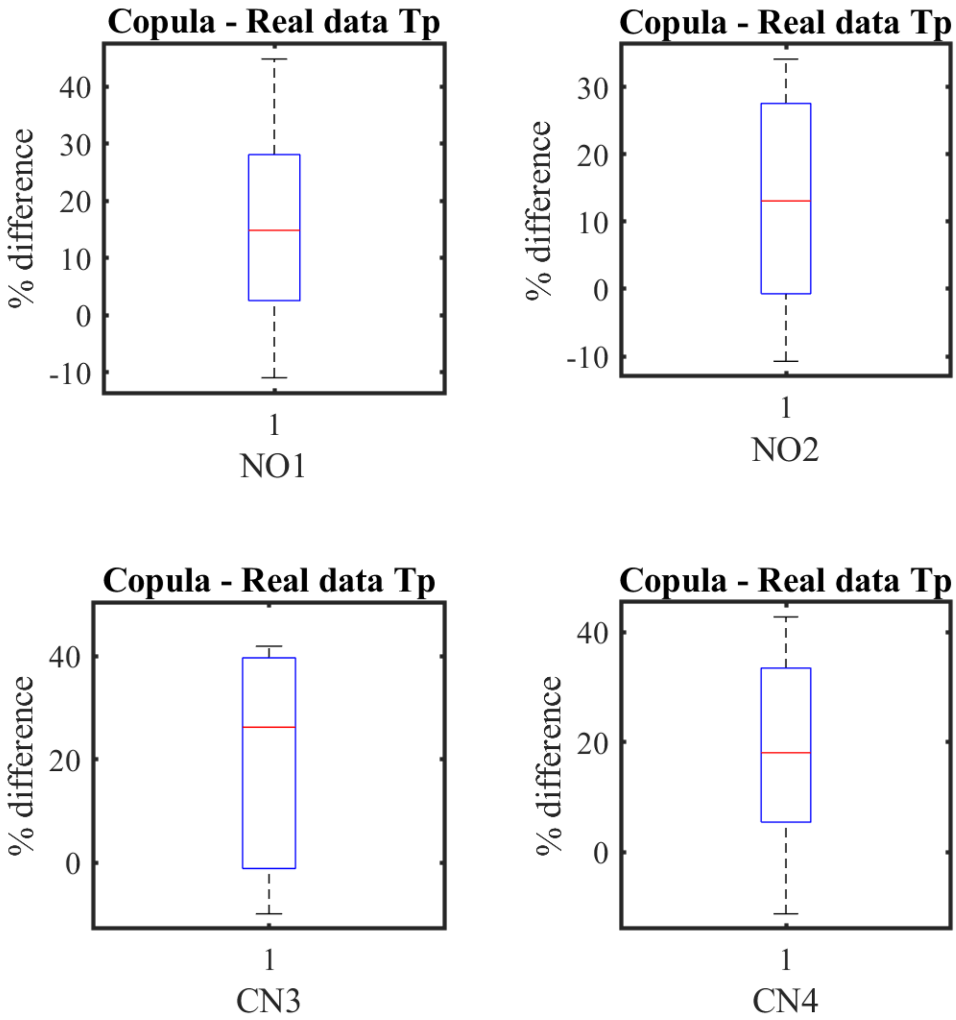


Figure 5-25. Difference of Ws from Copula and real data calculated by lumping; Copula from NO1

The relative difference between the lumped data generated by copula and real data is less than 10% for all locations. The maximum difference is also related to the tails where there is a low probability of occurrence and less importance for fatigue damage design.



DFigure 5-26. Difference of Ws from Copula and real data calculated by lumping; Copula from NO1

The lumped peak period calculated from copula cannot be estimated as accurately as wind speed. The average difference is less than 25% and the copula over estimates the value for lumped peak period. One reason for the difference between the application of Copula for wave period and wind speed can be attributed to the type of correlation between wave height and each of these variables. Wind speed and wave height are more linearly correlated as the Pearson's r is relatively close to one. This can be attributed to the fact that wave height is generated by wind and therefore there is a more direct correlation. However, in terms of peak period, the correlation is less and at the very least the correlation is not linear. This can be attributed to the fact that the wave train is made of of various components with different frequencies such as swell waves and wind generated waves. This diversity of components reduces the correlation of wave height and wave period relative to the correlation of wave height and wind speed.

5.5 Discussion on application of copula lumped data for fatigue damage estimation

The difference between fatigue damage from lumped load based on real data and copula generated data are compared in this section using the method described in 4.3.1.7. The lumped data that are presented in Figure 5-21and Figure 5-13 are combined with the damage formula mentioned below, which is based on the quasi-static behaviour of structure, to have a very simple estimation of fatigue damage difference between lumped data .

$$\frac{D_2 - D_1}{D_2} \propto \frac{V_2^{\mu} - V_1^{\mu}}{V_1}$$

$$\frac{D_2 - D_1}{D_2} \propto \frac{1/T_{z,2} - 1/T_{z,1}}{1/T_{z,2}}$$

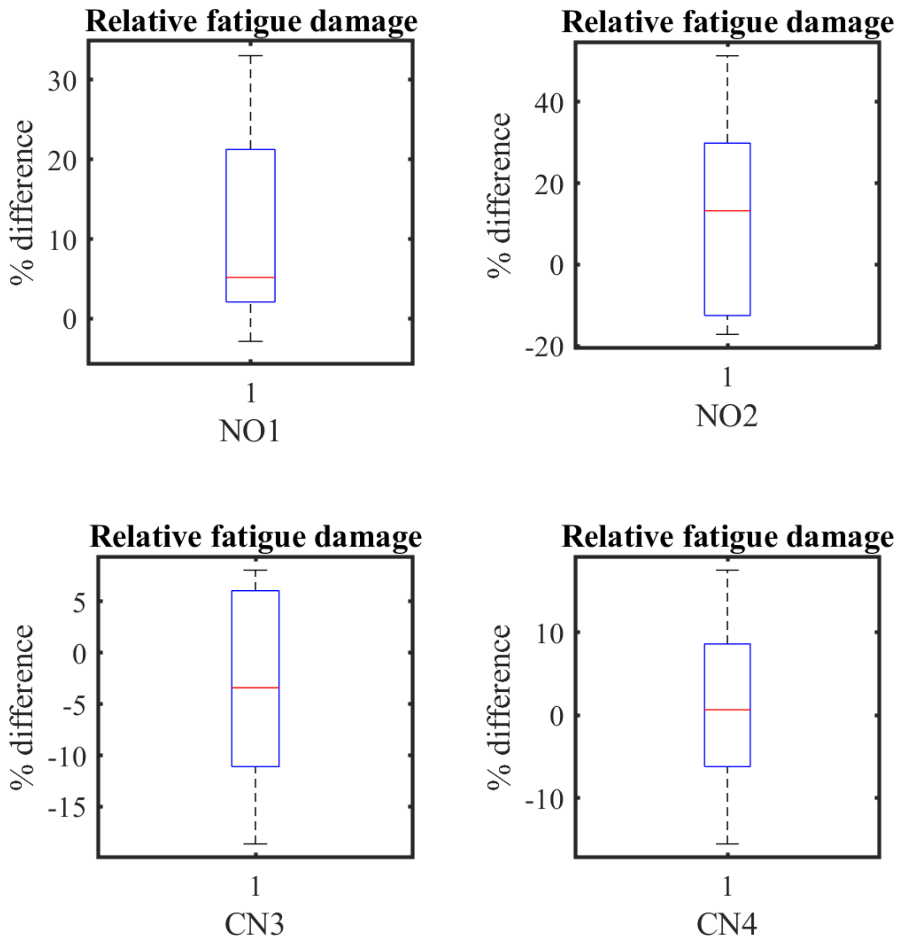


Figure 5-27. Relative fatigue damage difference from real data and copula data for Ws using NO1

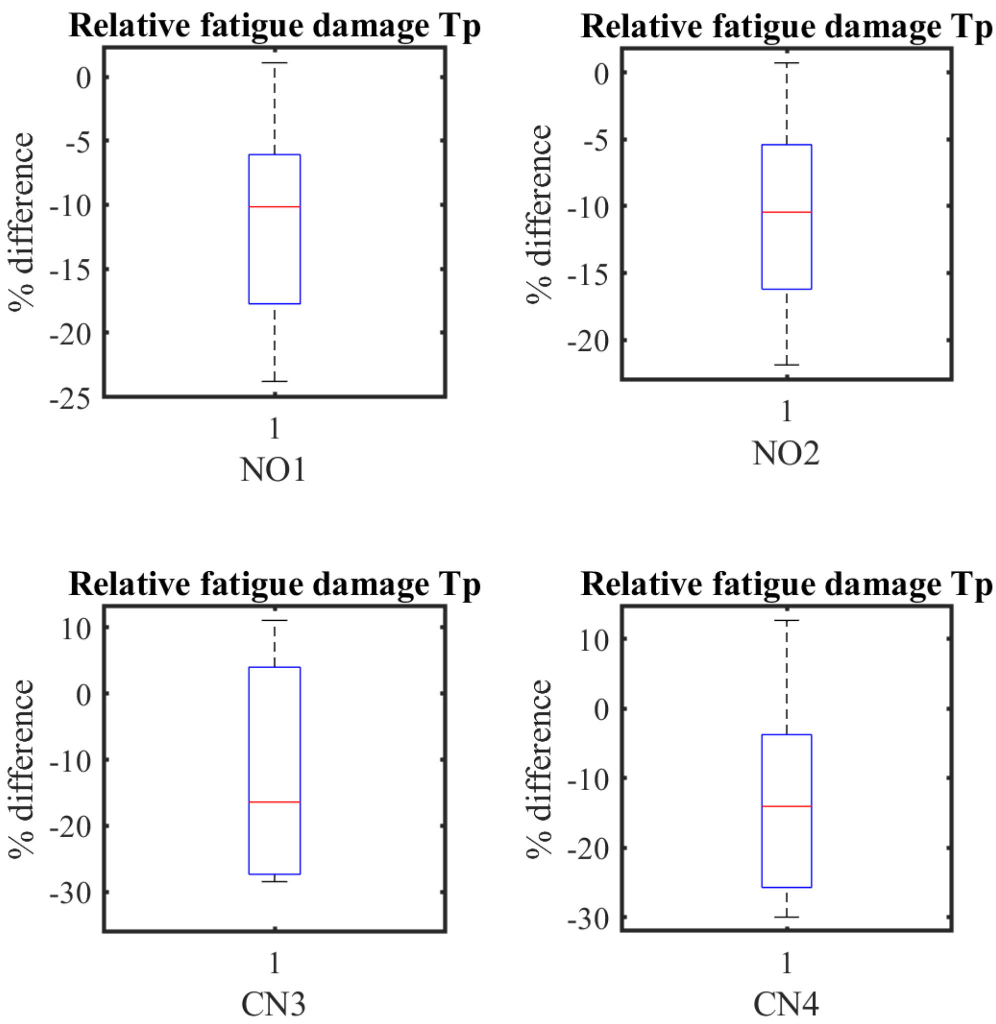


Figure 5-28. Relative fatigue damage difference for lumped Tp using NO1

As can be seen from Figure 5-27 and Figure 5-28, the average fatigue damage difference between lumped data caused by wave period and wind speed are less than 20 % which shows lumped data based on copula can have a good approximation for fatigue damage.

Another aspect of examining the applicability of copula generated data is to look at the important met-ocean frequencies near a structure's first natural frequency where the damage is significant. For a monopile, it is assumed that the natural frequency of the structure equals to 0.25[hz] or natural period of 4[s]. However, based on the peak period distribution, and joint distribution of Hs-Tp, it is less than 1%. (Figure 5-3, Figure 5-17). As can be seen from Figure 5-21 to Figure 5-24, this frequency has such a low probability that is not captured by the lumping of real data and copula generated data which show that copula method has a good application for structures in this range of frequency as well..

6 Conclusion and future work

This research has presented regenerated copulas from different bivariate random variables. The generated copula is combined with the marginal from three different locations to generate new data. The results are lumped and compared with lumped data measured at the same location. The fatigue damage difference between the real data and copula generated data is calculated for different locations.

6.1 Conclusion

- The copula density suggested that both pairs of (Ws, Hs) and(Hs, Tp) were unsymmetrically tail dependent. The upper tail represents extreme storm condition. This shape of tail dependency represents the analytical Gumbel copula.
- The stations that were located relatively close to each other had quite similar copula with average difference of less than 10%. An increase in the distance between locations was reflected as the copula difference between two locations increased up to 15%. The local maximum difference of 40% was observed in the results
- The copula method could estimate the pairs of (Hs, Ws) for another location with an average difference of less than 10 % compared to real data for the four locations. The average difference increased for pairs of (Hs, Tp) to 23% which is twice that of wind speed and wave height. This could be attributed to the correlation of data. When data were more linearly dependant, like (Hs, Ws) , the copula method provided more accurate results.
- To prevent underestimation of fatigue damage, lumping should be done over wave height, rather than wave period, as the wave frequencies connect to the structure's natural frequency. However, since the wave frequencies close to natural frequency range of monopiles had a very low probability of occurrence; (less than 1%), lumping over wave period did not affect fatigue damage calculations.
- The copula method could not provide a good estimation of both the upper and lower tails due to high correlation of values.
- The average differences for calculating fatigue damage using the copula method both for the lumped wind speed and lumped wave period, were between 8-20 %. Smaller values from the stations close to the copula source, and larger values from the distant stations. The fatigue damages that were related to both upper and lower tails were excluded in the calculation due to low probability of occurrence.

- The simple lumping approach over each random variable led to different lumped load cases. If variables are not linearly dependent, lumping can lead to very different values for lumped load cases. The results suggest that lumping over wave height differs from lumping over wave period significantly, as the linear dependence is less. However, if data are more linearly correlated, with a higher Pearson's r, like pairs of (Ws, Hs), the difference of lumping load cases are less.
- The results suggest that pairs of (Ws, Hs) are more linearly correlated than (Hs, Tp).
- The data sets were measured at time interval of10 min and 3-hours. However, in long-term analysis of data, the marginals and copulas had the same behaviour and distributions between all four locations.

6.2 Future research proposals

- The determination of copula and marginals in this report are done based on emprical copula. It is highly recommended to do these calculations based on analytical copulas and marginals and see to what extent they can estimate lumped data. Spearman's ρ and Kendall's τ are two methods for calculating goodness of fit that work well with ranked data. The application of Vine and Kachi copula in describing met-ocean data also needs to be investigated.
- Besides wind speed, wave height, and wave period, wind and wave direction are also random variables in sea-state data. A multi-dimensional analysis of met-ocean data using copula is recommended. (De Michele et al., 2007)
- The original met-ocean data may not be independent and identicall distributed random variables(iid) as they may contain ties and serial dependence. Subsampling data to remove seasonal effect can be a solution to remove serial dependence(Vanem, 2016). In addition, to break the ties in data, during the copula generation process, the rank of data can be assigned randomly (Kojadinovic & Yan, 2010).
- The lumping was performed using a simple approach in this thesis. The effect of lumping method on the accuracy of copula generated data need be investigated. In addition, more advanced lumping method is recommended to be applied so as to be capable of capturing structures dynamic behaviour and fatigue damage equivalency(Passon, 2015).
- Copula methods can be applied in the scheduling of marine operations to optimize the cost of operation of offshore wind farms. Marine operations are highly dependent on

the prediction of joint met-ocean data. The applications of copula in marine operation are recommended for investigation (Leontaris et al., 2016).

- Separation of wind generated waves from swell waves and calculating two separate copulas is an interesting topic to work on as it can distinguish the joint behaviour of random variables when there is a difference between the sources of wave generation.

▪

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Appendix

7.1 Goodness of fit for location "NO1"

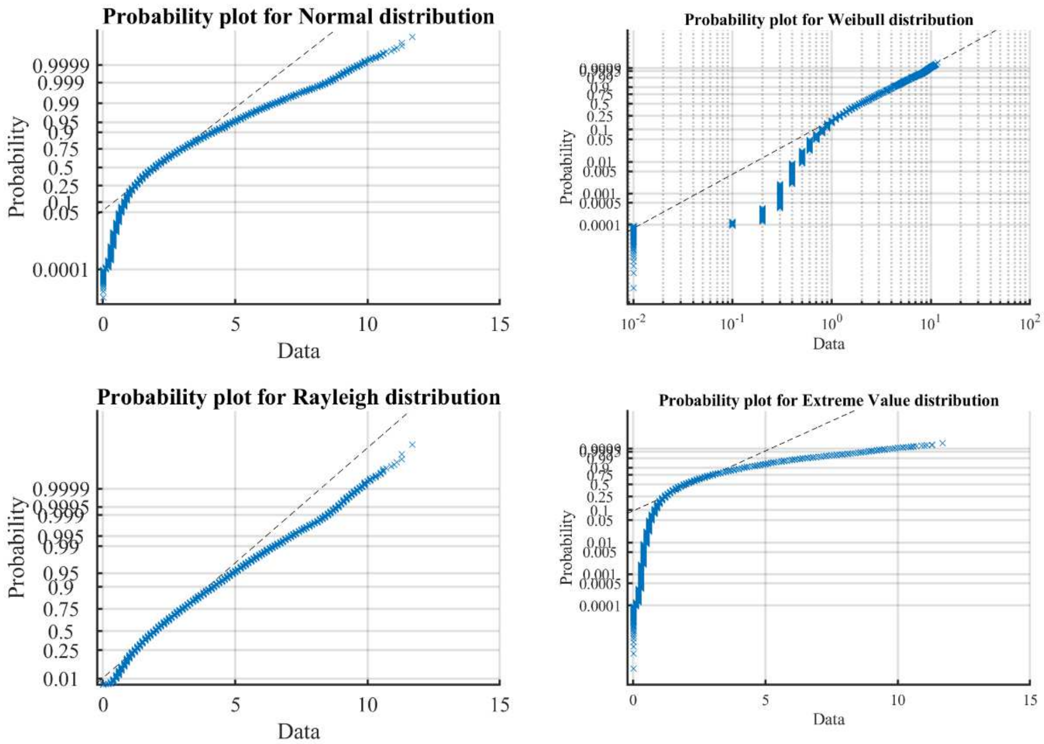

Figure 7-1. Probability plot for Hs at NO1

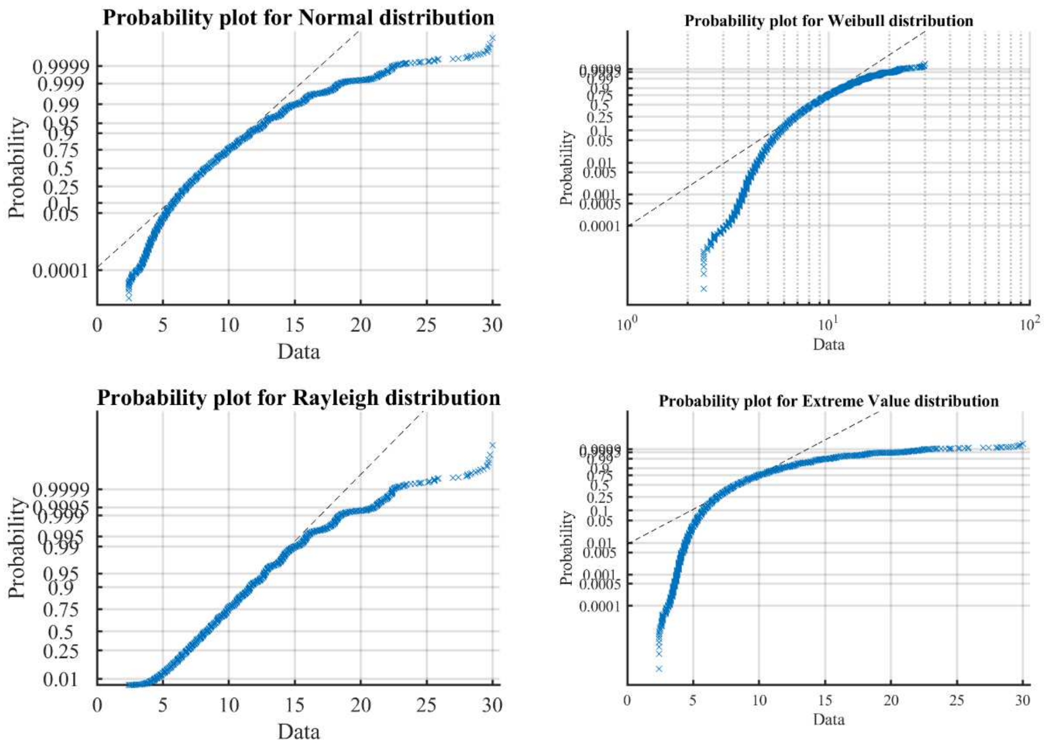

Figure 7-2. Probability paper for Tp at NO1

7.2 Joint values for wave period and wave height

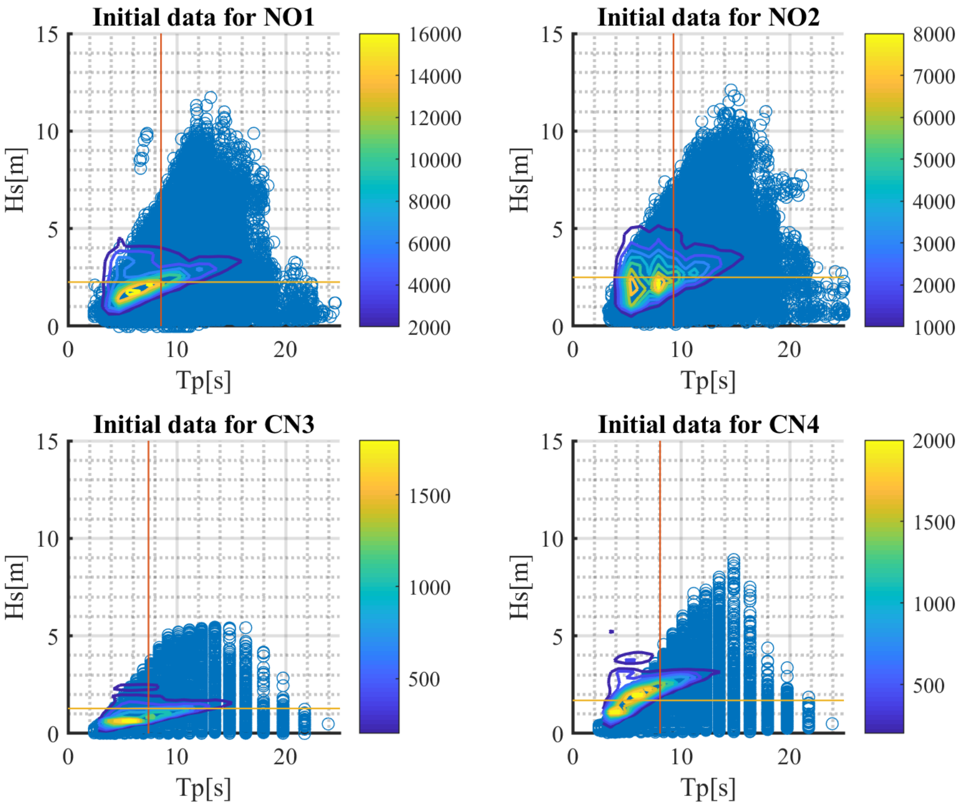

Figure 7-3. Density plot for all stations

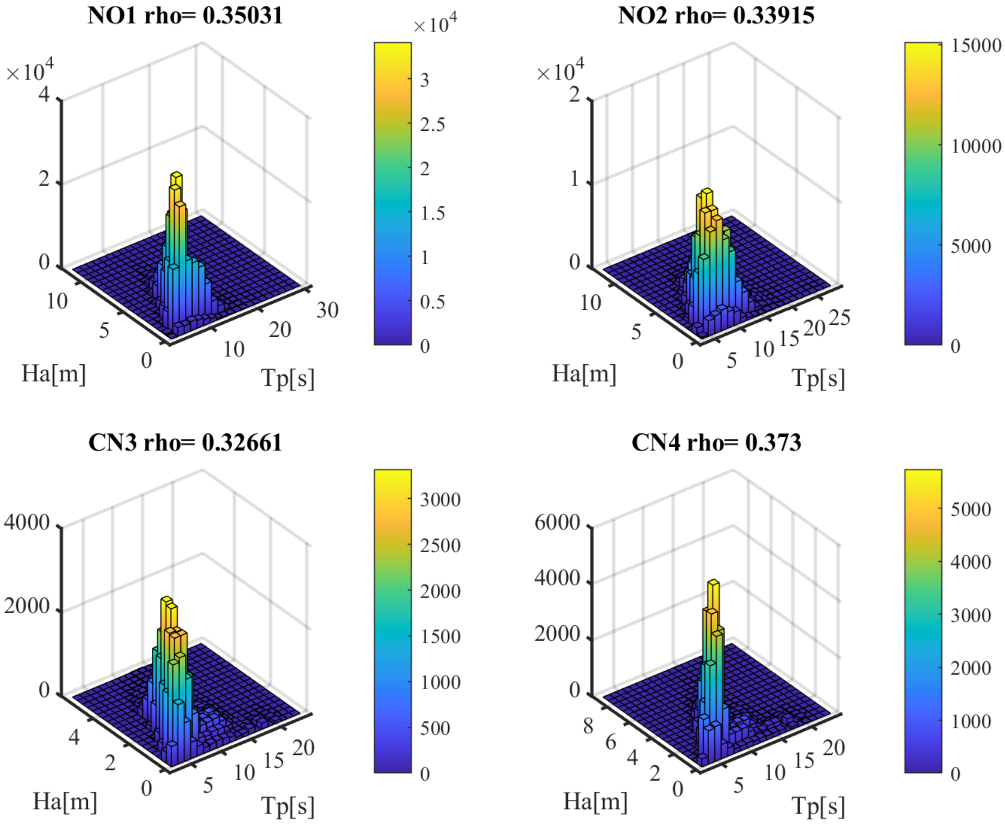

Figure 7-4. 3D histogram for all stations

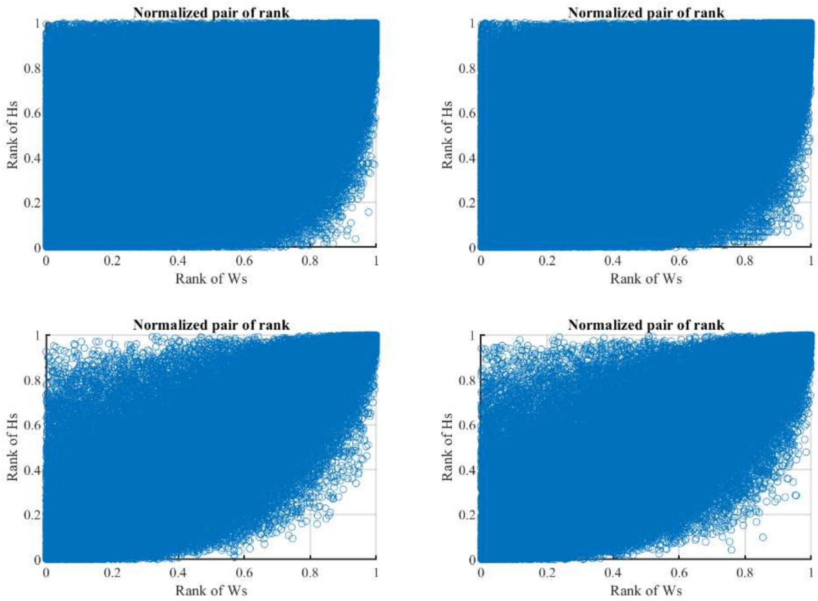

Figure 7-5. Rank values for pair of Hs and Tp

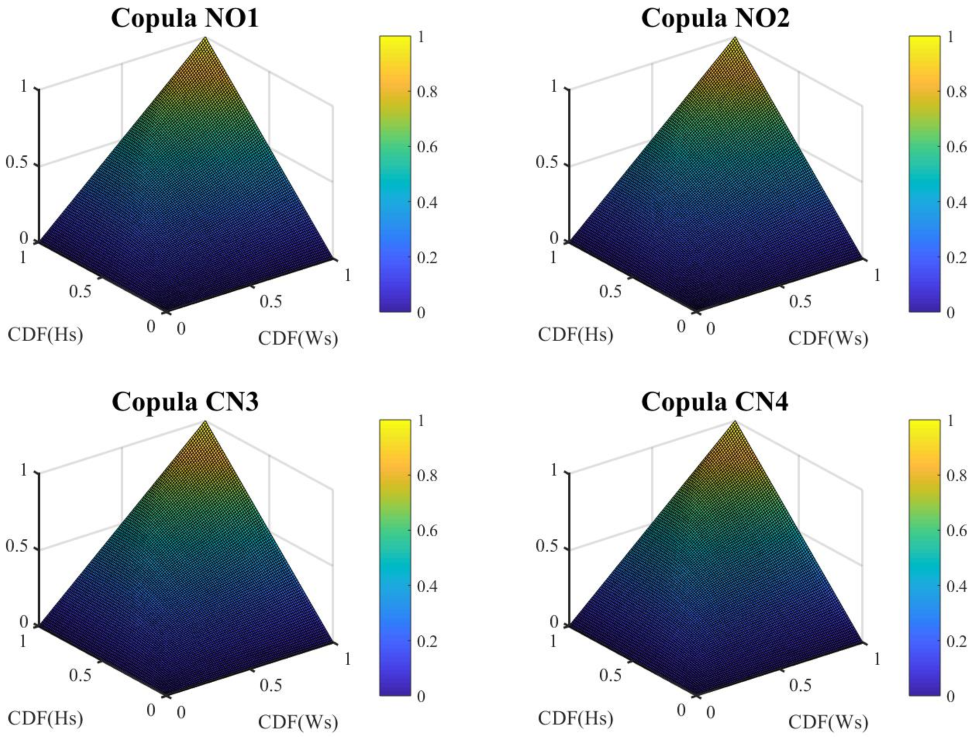

Figure 7-6. Copula for ranked vales