NTNU
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Tackling Variability in Renewable Energy Production and Electric Vehicle Consumption with Stochastic Optimization

The Benefits of Using the Stochastic Quasi-Gradient Method compared with Exact Methods and Machine Learning

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Preface

The work presented in this master thesis, is the culmination of the students work done at the Norwegian University of Science and Technology, NTNU Trondheim. It is done as part of the effort of obtaining a degree in Energy and Environmental Engineering from the Department of Electric Power Engineering .

As the student has already finished a master program in Industrial Engineering, this thesis builds upon that work by utilizing the fairly deep treatment of optimization methods done previously, in analyzing electrical power systems and the challenges they face. The student also has some background from financial subjects, which have lent another perspective on variability and volatility, dynamic and stochastic systems and methods to analyze this. Moreover, the student is also influenced by his exchanges to ETH, Zrich, and Tsinghua University, Beijing, which both proved fruitful for new academic impulses.

The main endeavour of this thesis is to address how energy storage will help facilitate the inclusion of more variable production from renewable energy sources (RES) in the power grid, as well as the unpredictability of Electric Vehicle (EV) charging. To do this, the student proposes that the Stochastic Quasi-Gradient (SQG) method might be an suitable tool for analysis. It is a method that, among other applications, has been deployed in finance - a field which has been studying volatile, dynamic and stochastic systems for a long time. Additionally, the field of Operations Research lends methods that might let one bridge even more advanced statistical methods from finance with problems faced in power systems with the rise of volatility from RES.

The methodical strength of the SQG method is that it is neither a fully exact or deterministic method, nor a full-blown heuristic. It uses an estimate of the stochastic gradient at each iteration step in the solution process. Hence, it should not get stuck in local minima and be able to deal with more complex problem, whilst still have a concept of how best to improve the solution with each iteration. Thus, it should be more suited and faster than the other type of methods mentioned. This is further discussed and analyzed throughout the thesis. Moreover, a few other examples are provided to highlight the added benefit and strengths of the SQG method in optimizing energy system storage with RES.

Oslo, April 9, 2018

Sondre Flinstad Harbo

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There are many to whom the author want to extend his gratefulness before diving deeper into the material.

First of all, my greatest regards needs to be given to professor Magnus Korpås whom has been my supervisor for this thesis. Each discussion with him always proves very rewarding by lending insights, ideas and ingenuity to the problem at hand. Also his Ph.D. student Salman Zaferanlouei, deserves thanks for many a good discussion and help with some of the AC-Optimal Power Flow code.

Another professor who deserves my sincere thanks, is Alexei Gaivoronski. Not only do I owe everything that has to do with the implementation of the Stochastic Quasi-Gradient (SQG) method to his excellent work on the topic, he also has developed the software that functions as the main optimization engine in this work. It is also always very rewarding, and also humbling, to get answers from Alexei, which conveys how much knowledge he has on the subject.

I also need to thank the academic faculty at the Norwegian University of Science and Technology (NTNU), and especially the Department of Electric Power Engineering for excellent courses and much valuable knowledge. Also the Managerial Economics, Finance and Operations Research group at the Department of Industrial Economics and Technology Management for all their teaching on optimization and stochastic methods.

Yet, this thesis is not only a product of teaching at NTNU, but has other impulses. The year of exchange spent at ETH, Zrich, in 2014/2015 was a great time of academic growth for the student. Moreover, the autumn semester spent at Tsinghua University, Beijing, during 2017 gave many new insights and impulses.

As such, the student is also very grateful for associate professor Zechun Hu's interest at having me as a guest researcher at his Smart Grid Operation and Optimization Lab. The student is also happy for the discussions and input he has provided. At Tshinghua, the student also had many interesting discussions with PhD student Hongcai Zhang, especially on Electric Vehicles.

I am also very appreciative for being able to use a picture from my previous employer Statkraft AS as cover illustration on this thesis.

Lastly, I need to thank my family for their continued and non-relenting support. I am not only amazed they always believe in me with all my projects and endeavours, but also put up with all my exclamations on how exciting a little maths can be.

.

Than you all for great guidance, support and direction.

S.F.H.

Summary

The work presented in thesis investigates different applications for implementing the Stochastic-Quasi Gradient (SQG) model to solve stochastic multistage AC-OPF problems, and compares it with a Stochastic-Dynamic Programming (SDP) approach and an Evolutionary algorithm.

Where the SDP quickly becomes too cumbersome to solve, the thesis also shows the other two as more appropriate tools, where the SQG method works better in larger cases, the Evolutionary algorithm in smaller.

Hence, to analyze how energy storage may optimally be used for incorporating variable renewable energy sources to bigger grid networks, the SQG method may be of academic and practical interest.

Sammendrag

Arbeidet presentert i denne oppgaven undersker forskjellige applikasjoner fo implementering en Stochastisk-Quasi Gradient (SGQ) model for å løse stochastisk multisteg AC-OPF problemer, og sammenligner denne med en Stochastisk-Dynamisk Programmerings (SDP) fremgangsmåte og en Evolusjonær algoritme.

Der SDP metoden fort blir for kronglete å løse, viser denne oppgaven ogs at det finnes to andre mer passende verktøy, der SQG metoden virker bedre for strre tilfeller, og den Evolusjonære for små.

Dermed, for å analysere hvordan energilagring optimalt kan bli benyttet for å inkludere mere variabele fornybare energikilder i strre netverk, kan SQG metoden være av akademisk og praktisk interesse.

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Chapter 1

Introduction

This chapter introduces the thesis. First it places the topic in a greater context, and highlights why the specific problems that are dealt with here are relevant. Thereafter, it is presented a brief description of the approach used and the methods deployed in the thesis. Then, the author also tries to gauge some for the contribution of the work, before lastly an outline for the rest of the thesis is given.

1.1 Centrality of the topic

Climate change might pose one of the greatest challenges humanity will face in the coming decades, many believe. In making the transition to a more sustainable society and economy, there are many issues to be addresses. One of these is the raising variability in the power system as one includes more and more renewable energy sources.

As more and more wind farms, photo-voltaic systems and electric vehicles are incorporated into the energy system, greater and greater variability is introduced to the power system. This because all the aforementioned technologies associated with a more greener society are in nature quite volatile in their energy production or consumption; you can not easily control when the wind blows, the sun shines or even when people want to charge their Plug-in Electric Vehicles. Hence, to strike the required physical balance between produced and consumed energy in the power system becomes an increasingly more difficult task for the controllable, dispatchable generators in the system, both on the short, medium and long term.

Furthermore, the increasing variability of production and demand is also problematic for the electrical component's in the power gird. This because many of these are developed for conditions where the power transferred, its voltage and current magnitudes, are fairly stable. For instance, the time delay of electromagnetic induction in a transformer means that momentary changes cannot occur but produces extremely high currents and voltages. Many other physical grid components are also sensitive to rapid change power values. And in this way, it is of importance to the whole system that voltage quality, reactive power and transmission frequency are all minimally effected by the increasing variability of production and consumption.

To deal with this variability, the solution is in general to include some way to store the excess energy that the intermittent energy sources produces at time of abundance. Similarly, when the RES production is lower, both in the short term - for instance hour to hour - or in the long term - summer to winter - the Energy System Storage (ESS) will feed the power back out to the grid.

Thus, an optimal operation of such ESS is then paramount, so that it stores and discharges the energy effectively and enhances the incorporation of RES.

1.2 Motivation

As was illustrated above, there is a great need to find methods that help analyze how to tackle the rising variability in the energy systems using energy storage.

In tackling this problem, optimization techniques are natural candidates as they are apt at finding efficient ways to solve mathematical problems. Yet, there are three main issues that sets the problem we want to analyze apart from more regular optimization problems.

The first complicating aspect of our problem is its dynamic character. This because to first store and later dispatch energy, is to effectively move energy from one period in time to the next. Hence, we need to consider the effects on the energy system of reallocating the energy. In short, we want to find out at which point it is most beneficial for the system to use additional energy from the energy storage or to add energy to the storage, considering both its past and future demand and consumption.

Secondly, the problem at hand also requires us to deal with complex mathematics and non-linearity. This is because the physical phenomena that occurs during power transmission, such as the presence of both active and reactive power, are relationships that are be distescribed with non-linear mathematical formulations. There are some methods that simplify this by either linearizing the mathematics, or at least make it conic. Yet, be able to fully analyze the effect on issues such as voltage stability and quality, we opt for a fully fledged Alternating Current Optimal Power Flow (AC-OPF) as the basis of our power system modelling. Yet, dealing with such non-linear systems is especially a challenge for optimization techniques, as it requires them not only to find an optimum, but the global optimum.

The third issue that makes our problem challenging is that it is in its nature a stochastic problem. That is, we want our analysis and optimal energy storage operation policy to take into account the probable production of the intermittent energy sources in the following time period. Thus, the solution we find to the problem should in some way be able to hedge against the volatility of the production from the RES, such that the effects of extreme events are dosed down. In addition, the probability functions of the RES might not easily formulated mathematically. Hence, our solution method would benefit from being able to tackle complex descriptions of the stochastic variables as well.

Fortunately, the methodology proposed in this thesis is able to deal with all the aforementioned issues. The Stochastic Quasi-Gradient (SQG) method, which will be treated more in depth in subsequent chapters, is a method with roots in approximate techniques and with elements of Stochastic - or heuristic - Optimization methods. Thus, it is able to tackle the issues of dynamics, non-linearity and stochastic described above. On the other hand, it also make use of analytically concepts, namely the gradient, in improving its approximate solution. Hence, it should also in theory be a faster solution method than random search based heuristics.

1.3 Researchers approach and methodology

In this thesis, the main objective of the author has been to apply and continue the Stochastic Quasi-Gradient implementation for a multistage AC-OPF with stochastic variables. The author has also compared the SQG performance with other solution methods for the same problem, namely an exact method and a heuristic. To further demonstrate the possible benefits of the SQG method, the author makes some illustrative examples analyzing the energy storage problem in a power grid using the proposed methodology.

The basic implementation of the SQG method for this specific problem was done in the previous Master thesis written by the student in Industrial Economics and Technology Management (see Harbo (2017b)). Most of the coding for the simulations have been done in Matlab®. This not only because it has an integrated optimization toolbox, but also because the SQG software developed by Professor A.A.Gaivoronski and the MATPOWER software developed by Ray D. Zimmerman are both implemented in Matlab®.

To compare the performance of the SQG method with corresponding exact and heuristic methods, two different solution techniques has been chosen and a few test cases has been developed and implemented. To represent the exact methods, an solution method using Stochastic Dynamic Programming has been deployed. Likewise, representing heuristic methods, the problem has also been solved using an Evolutionary Algorithm. Moreover, further cases have been developed for the SQG method, in order to show the further benefits of this solution method.

1.4 Contribution

This thesis has further implemented and applied a multistage, dynamic and stochastic AC-OPF model solved by the SQG method. A lot of time has been spend developing and implementing different cases on which to run the models, and also on testing and tuning the proposed methods. Moreover, another comparable solution method using Evolutionary programming has been developed.

The starting point for this thesis, has in many ways been the work done with the authors previous thesis Harbo (2017b). Thus many of the main contributions of this thesis is topics that was suggested as further work in that last thesis. The contribution of this thesis is:

- Development of a Stochastic Optimization heuristic to solve the same case for comparison using an Evolutionary Algorithm
- Implementation an even faster solver of the AC-OPF model
- Started implementation of using a discrete, dynamic AC-OPF for the whole period to calculate the gradient for the solution in the SQG process
- Inclusion of a time varying electricity price in the modelling

- Using the IEEE case grid data to develop a larger model for a test power system, with the 24 bus case fully implemented and the 118 bus case started
- Using the AMB simulation of EV together with the optimization models in analyzing the grid response

1.5 Outline of the thesis

The thesis is structured in 7 chapters, as well as its preface, acknowledgements, summary and appendices.

Firstly, an introduction is given in this first chapter, highlighting the relevance of the topic, the motivation for the work, the approach taken and the possible contribution of the work. The second chapter describes the problem further, whilst the third chapter reviews the related literature and presents some of the theoretical concepts useful for the further discussions and understanding of topics treated in the thesis. Chapter four presents the mathematical methodologies that are the backbone of the solution method utilized. The fifth chapter discusses the implementation of the models and methods, and presents the cases used for the application of the solution approaches. Chapter six presents the results from solving the problems with the different methods and for the different cases. Lastly, chapter seven concludes the thesis, and proposes further research to extend the topics and methods treated here.

Chapter 2

Problem description

In the following chapter, a more in depth introduction and explanation of the problem is given. A problem statement is explicitly formulated to highlight for the reader what the ultimate research question of this thesis is. Thereafter a further description of the problem, from both a Power System and a Optimization perspective is given. Lastly, there is a discussion of the scope of the thesis and what is left outside its reach.

One might notice that some of the content covered in of both this chapter, and the following chapter 3 on literature, might seem somewhat basic. Yet, the author has decided to include an introduction to the fundamentals underlying the thesis to help unfamiliar readers get to grips with the most essential concepts for understanding the following chapter, something I have gotten good feedback on from interested colleagues after sharing previous work with them. However, due to considerations of length, the contents are covered quite briefly. Non-the-less, references to the most prominent texts on each subject will be given, should the reader wish deeper indulgence in the topics and methodology treated here.

2.1 Problem statement

The main endeavour of this thesis, is to discover the benefits and possibilities of using the SQG method to analyze energy storage used in electric grids under stochastic and multi-period considerations. The research question that is sought answered by this thesis is then formulated as

Which benefits and possibilities does the Stochastic Quasi-Gradient method offer in analyzing and optimize multistage power system operation with energy storage under uncertainty?

2.2 Description of problem and scope

A more in-depth explanation of the problem is provided here, first from a power system analysis and then from optimization methods view point. This section also sets limits to what is considered to be in focus of - and equally important what is considered out of scope for - the thesis.

2.2.1 Power System Optimization

When dealing with power system optimization, one uses relations and phenomena from power system engineering and power system analysis to find the optimal way to produce energy for a system, given its characteristics and energy production and consumption.

The energy produced and transported is subject to physical laws and constrained by the physical limits of the electrical components of the power system, as well as rules set to enforce power security and stability.

To analyze this, several methods are available, such as the Economic Dispatch approach, regular Load Flow studies or Fast Decoupled Load Flow, and the standard Direct Current Optimal Power Flow (DC-OPF). Both the Economic Dispatch, but especially OPF methods, are readily incorporated with power market considerations as well, with power bids for both producers and consumers.

However, these approaches does not fully consider the impact of losses and reactive phase shift of the power in the system. This is critical for a realistic analysis with respect to the physical constraints and security of the system. The ability to tackle all of these desired properties are characteristics that are unique to Alternating Current (AC) power systems, is only fully consider in an approach called the Alternating Current Optimal Power Flow (AC-OPF) method. This is because elements such as coils, capacitors, and long transmission lines all experience electromagnetic temporal energy storage of power causing phase shift between the voltage and current profiles.

For DC this is not an issue, as the voltage and current are both a product of unidirectional flow of electric charge. For AC systems however, which is most common for power grids, the current and voltage alternates periodically between negative and positive values as the charge flow changes direction. Hence, the reactive electrical components cause the phase between current and voltage to shift its phase relative to each other. Since power is a product of the current and voltage at any given moment, a phase shift between the two means among other things that the active power delivered is less than the apparent power when looking at the voltage and current produced from the generator side. As a result, the overcompensation for the reduced active effect due to reactive power's seizing of apparent power, transformer stations and other grid components risk being overloaded. Hence, the effect of phase shifts throughout the system is critical information to consider for a optimal power flow analysis both with varying loads and generation, as well as for the physical constraints and stability constraints.

2.2.2 The variability challenge when transitioning to a sustainable energy system

As Europe strives to emit less and less CO_2, there will be a need to shift the energy production away from using fossil based resources to more sustainable sources. Hence, the electricity demand is expected to increase further as production switches from conventional fossil fuelsDe Vita et al.. Moreover, the introduction of a larger share of renewable energy sources (RES), will increase the variability of production. This is because the main sources of renewable energy are inherently rather volatile in its production.
Also the adoption of electrical vehicles (EV's) induces another, yet similar challenge to the demand side of the power system. The increased electricity demand from charging EVs is not necessarily easy to predict as EV owners often are free to charge whenever they want. Additionally, the basic charging that happens in the afternoon when EV owners return home from work might also yield greater imbalances in the grid.

2.2.2.1 Energy system impact of a growing share of renewable energy sources

The increased share of RES in the power system does not come without problems. Traditionally, power systems have covered most of their load by using thermal power plants, gas turbines and hydro power. Many such power stations of are to a large degree dispatchable. That is, they can be turned on and off, or at least be regulated up and down, to meet the fluctuating electricity need. This gives the grid operator flexibility to meet the load demanded at any time. However, with the increasing share of renewable energy sources fed into the grid, the relative amount of dispatchable source decreases. This, because you cannot turn the sun and wind on whenever electricity demand is high. This leads to unbalances. Over a 24 hour perspective, the sun will only shine during daytime, whereas energy might be needed in the evening when people cook their dinner. Likewise, on both a week, month and year horizon, there will be greater need to store energy

With deviations between the produced energy in the system and the energy consumed by system loads, the frequency in the grid drops before Frequency Containment Reserves, Frequency Restoration Reserves and Replacement Reserves are put in place. The grid is to operate within a set safety margin (eg. 50 Hz, +/- 0.5 Hz) for its components to be safe from damage, and the amount of frequency that drops for a certain drop in power is determined by the system response. The installed synchronous generators are an important factor of frequency dynamics and stability for the system. With their stored kinetic energy they add rotational inertia to the system. This inertia works as a natural spring, providing kinetic energy or absorbs it from the grid, to stabilize any small deviation in frequency. Generators in wind turbines are tiny in comparison, resulting in a significantly smaller amount of kinetic energy and rotational inertia to contribute in grid stability. PV systems contain no rotational energy as they convert solar energy to electricity. Even though it to some degree is possible to use power inverters to set up synthetic inertia, power systems with low rotational inertia are making frequency control and power system operation more challenging, as presented by Ulbig et al. (2014) or Mercier et al. (2009).

Therefore it is very beneficial to be able to transfer energy from a period of plenty to a period of deficit through the use of energy storage. To move demand around is known as *peak-shaving* and *valley filling*, which reduces the difference between peak and trough. See figure 2.1 for an illustration.

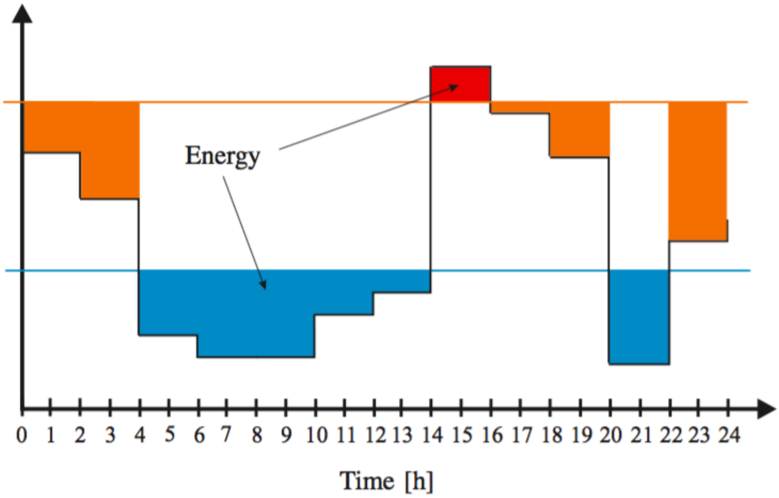

Figure 2.1: Peak-shaving and Valley-filling of High and Low Load Periods Galus et al. (2012)

Such flexible demand response might be quite possible in a future scenario where the grid, its components and constituencies, is connected with information flow allowing coordination. This state is termed *Smart Grids* (see figure 2.2), and is a promising development for stabilizing purposes, for instance with having EV deliver power to the grid in times of need as an Vehicle-to-Grid service.

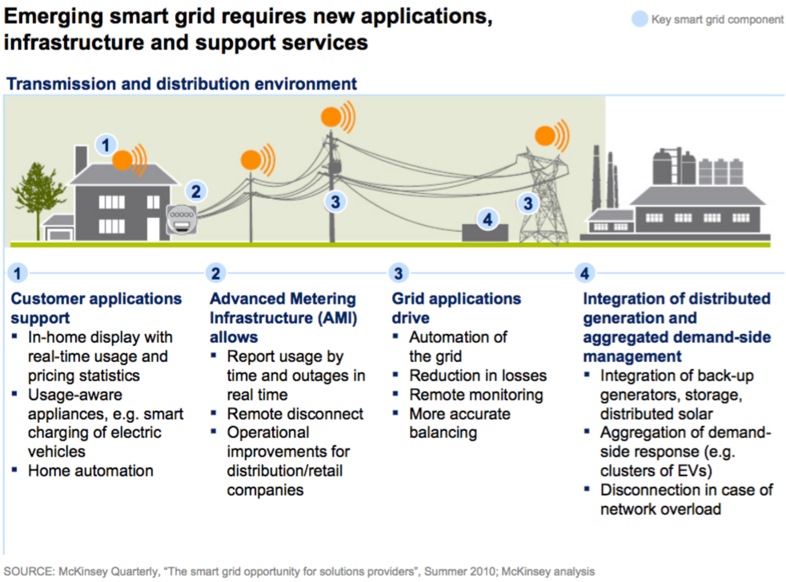

Figure 2.2: Smart Grid characteristics McKinsey & Company (2014)

2.2.2.2 Energy system impact of a rising EV fleet

As argued above, an increasing number of EVs might impose volatility from the demand side on the grid. This because the EV owners often are free to charge whenever, and it poses a big strain on the grid if enough cars is to charge at once.

For an example on how increasing volatility affects the grid, one might consider assessments of how EV adoption is likely to affect the Norwegian grid. Norway is by far the country where most EVs have been adopted, with battery EVs having over 20% market share of new sold cars in 2017. Publications done by The Norwegian Water Resources and Energy Directorate (NVE), Spilde and Skotland (2015) and Skotland et al. (2016), show that the electrification of the transportation system will pose challenges to the grids transformer stations and voltage quality. The findings of the reports are that the Norway's already installed grid in the near term will fairly easily tackle the charging demand of the EVs in terms of total capacity. The transmission lines are in generally not expected to experience problems. Yet quite a few transformers, especially in rural areas, are likely to become overloaded at times in a future with a larger EVs fleet. The extreme case here assume full adoption and a conscience factor of 75%. From figure 2.3, displaying responds on when people normally charge their EVs, such a high conscience factor seems unlikely for most time instance. Yet, after a vacation break, or a weekend, it might be the case that many EV owners want to charge their cars at the same time. Non-the-less, the Norwegian transmission system operators have quite some time to replace old and soon to be underdimensioned parts of the system before we see a full adoption of EVs.

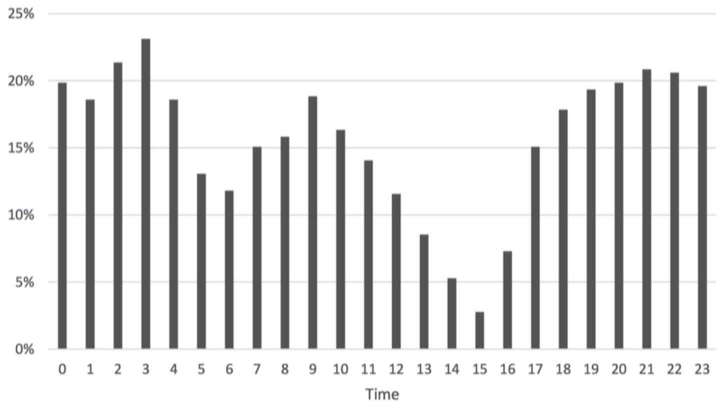

Figure 2.3: Answers from 398 respondents to ”Cross out those hours during the day when you normally charge your EV?” Tveter (2014)

In terms of voltage quality and stability however, the NVE reports show there is a somewhat greater concern. For instance, they present that measurements done by SINTEF has showed that even the connection of one 32 A single-phase charger may cause significant reduction in voltage quality. Hence, for one household, the connection and charging of an EV would double the demanded power from that house during time of charging. Thus, if all households in an area connect for charging at the same time, that will be very troublesome for their local transformer station and voltage quality.

2.2.3 Optimization Methods

In the following, some key aspects of the optimization methods deployed here are brought forth. Optimization problems may be classified in many ways; whether they are linear or non-linear, convex or non-convex, dynamic or static, stochastic or deterministic.

There are many well developed methods for solving linear programs, for instance the SIMPLEX method or duality methods. A non-linear problem on the other hand, is more difficult to solve, especially when the non-linearity give rise to a non-convexity. That is, there is a possibility of having both local and global optima. That makes it harder to develop algorithms or methods that guarantee the fully optimal solution.

In tackling non-convex and global optimization problems, there are many different approaches. These may be characterized as deterministic methods, stochastic optimization methods, or heuristics and meta-heuristics. The first tries to use exact, analytically calculations and methods to find the optimum. The second, stochastic optimization methods, are in simplified terms exact methods that takes heed of random variables in their solution process in order not to be trapped in a local optimum. The third, heuristics, may generally be described as an trial-and-error or random search approach. This category often overlaps with the previous one and share common features, such as including a random element in their search process. For instance, the Evolutionary Algorithm used in this thesis, employs a randomized mating process of the individuals according to their fitness. A goal for many of heuristic methods is that they are able to learn about the problem without being explicitly programmed, and hence quite a few of these are also considered machine learning methods.

In the field of operations research and optimization, the term stochastic programming is used differently. It there refers to mathematical decision making under uncertainty where some part of the problem is not fully known. One approach to solving such problems is to discretize the probability space and solve the corresponding deterministic simplification of the problem, which is deployed in this thesis using Stochastic Dynamic Programming.

The method of Stochastic Quasi-Gradient proposed in this thesis, might be characterized as a Stochastic Optimization method. It is not only aimed at solving optimization programs that are subject to uncertainty. It is also able to deal with non-linearity, non-convexity and complex functions to find a global solution. Being a stochastic optimization method, and neither a fully exact approach nor a full-blown heuristic, it should in principle be faster than both its counterparts. It should be able to get close to the global optimum in a way that exact methods might have some difficulties with. Moreover, it will be able to use some analytic information from the solution process - the Stochastic Quasi-Gradient - which should aid its search so it may be faster than a heuristic based one.

2.2.4 Scope of Thesis

When working on this thesis, some limitations had to be imposed on what not to do, and what was to be the main focus.

Since the topic of this thesis is similar to the work done for the Industrial Economics and Technology Management thesis, many of the concepts from literature, mathematical models and methods are the same. Whereas the previous work explored the feasibility of solving multi-period, stochastic AC-OPF using the SQG method, this work has focused on the further implementation, application and testing. Thus, the author has considered whether to leave out all information that has been included elsewhere, referencing the relevant parts of previous work. However, the author has decided to leave in leave abbreviated and more clear cut introductions and explanations on the most important concepts and models treated in this thesis. This is done to make the document work as a stand-alone text, even though some of the section here carry overlap with the previous thesis.

Moreover, as the mathematical optimization concepts has been the topic of another thesis, a detailed assessment of these aspects has been left out here. Instead, the author has brought in more information on the variability challenge and focused on the inclusion of electrical vehicles. It shall be noted that some ambitions where cut back on during the work, namely the full implementation of the 118 bus case and a machine learning implementation with the ABM of EVs.

Chapter 3

Review of related literature

This chapter starts with a presents and some of the most important literature sources for this thesis. A brief review of some of the fundamental theoretical concepts required to understand the thesis' models has also been included, to facilitate understanding for unfamiliar readers and enhance the further discussion of methodology, implementation and results.

3.1 Review of Literature

The work presented in this thesis is a culmination of the student's academic journey at the Norwegian University of Science and Technology. As such, it is heavily influenced by the students previous semester projects, Harbo (2016) and Harbo (2017a), and especially the master thesis Harbo (2017b) written at the Department of Industrial Engineering and Technology Management. Hence, these works will be pointed to at points where the reader may find further details of the models and implementation, that is left out of this report to keep it within a reasonable length. Also, some of the sections in this chapter naturally lies close to that what has been presented there.

This work draws from several different streams of literature, as it may be placed in the conjunction of two different fields; Power System Analysis and Operations Research. It also uses an Agent Based Model, a technique arisen from the field of Complexity Science.

The most important sources on optimization methods has come from Nocedal and Wright (2006) and Birge and Louveaux (2011). The first gives a comprehensive guide to numerical optimization and the second a valuable explanation of the fundamentals of

Stochastic Programming. Also the books by Kall and Wallace (1994), Pflug (1988) have been important. These books are all considered seminal works in the field.

To the SQG method, works such as Ermoliev (1983), Ermoliev and Wets (1988), Gaivoronski (1988), Gaivoronski (2005) and Becker and Gaivoronski (2014) have all been of inspiration. Where the paper, and following book of Ermoliev introduces the SQG method theoretically, the works of Gaivoronski specifies how this method is implemented and the many advantages it has. Additionally, the work of Peeta and Zhou (2006) together with the Becker and Gaivoronski paper, has provided ideas and examples for what to consider when implementing, approximating values and tuning the SQG model.

For the consideration of dynamic programming, the book of Bellman (1957) is an important contribution. However, also the works of Zaferanlouei et al. (2016) and Erdal (2017)has been of influence for the author.

On Evolutionary Algorithms, the books of Spall (2003) and Haupt and Haupt (2004) give solid introductions and examples. For the establishment of the field, the works of Rechenberg (1965), Rechenberg (1989) has been central, and the popular and widely used adoption to Genetic Algorithms presented by Haupt and Haupt (2004) has also been important. Moreover the thesis of Gundersen et al. (2017) has been of great inspiration for the author in how to use and implement this type of optimization heuristic. Lastlty, the works of Carr (2014) and Maringer (2005) has given a useful introduction on how to adopt these concepts onto problems with continuous variables.

For power systems, reference may be found with Grigsby (2012) and Crow (2012). The first provides an introduction to electric power engineering and power system analysis, and the second offers a more comprehensive treatment on computational techniques used to analyze power systems. Additionally, optimal power flow methods and their computational performance are presented further in the works of Sperstad and Marthinsen (2016) and Castillo and O'Neill (2013).

On variability, the paper of Graabak and Korpås (2016) offers an in depth review, and Jones (2017) gives a even more comprehensive treatment of the fundamentals of the topic. For more specifics on the volatility on the production side, works such as Aigner (2013) and Yordanov (2012) provides some insights on wind and solar PV power respectively.

On variability from the consumption side, Kremers (2013) provides some insight on the effect of more power demanding appliances. However, the charging of Electric Vehilces (EVs) are of main concern, as their individual demand for power might at times coincide to create consumption peaks posing serious challenges to the grid. Some empirical studies that collected data from existing populations of EVs are Smart and Schey (2012) and Smart et al. (2012), showing some of the behaviour and impact of a rising EV fleet. Publications from NVE on the other hand, Spilde and Skotland (2015) and Skotland et al. (2016), goes into detail of how Electric Vehicle (EV) adoption will impact the grid, in the specific case of Norway where the adoption is the furthest.

To gain further understanding of the energy demand from EVs, the author developed a bottom-up model using Agent Based Modeling (ABM) of this during a pre-thesis project. The whole report, with all modelling details and results are discussed in Harbo (2017a). The the works of Bustos-Turu et al. (2014) and Galus et al. (2012) have also been important in this regard, as they both consider how PEV may be modelled using ABM.

Many other papers have also been visited, that are not introduced to keep the length of this document somewhat contained.

3.2 The variability challenge of the energy system transition

This sections shows some examples of the problems with the volatility introduced to the energy system though the transition to a more sustainable economy nature induced and society.

As mentioned in the introduction, and also in a recent book on the topic Jones (2017), an increased share of renewable energy in the electricity mix means that the power system has to tackle higher variability, uncertainty and be more flexible. The general solution to this issue is to include Energy System Storage, as noted by Beaudin et al. (2010) for instance.

The variability is twofold; both on the consumption side, and on the production side. On the demand side, the growing fleet of EV's and home appliances such as induction stoves means that the instantaneous demand for electric power vary more. On the supply side, renewable energy sources like PV power and especially wind power are both means of production that have more volatile production profiles.

Historically, a typical household has one demand peak in the morning and one in the evening, making the power demand varying through the day. Industrial sites often have higher consumption throughout the work day, and lower at night. The is also a difference in energy demanded throughout the year, due to the varying temperature and the corresponding energy expenditure for heating and cooling. However, these patters are quite similar day to day and thus rather predictable, especially when aggregated for greater areas. Yet, as the energy system makes a transition to become more sustainable, the aforementioned issues turns the previous predictability on its head.

3.2.1 Integration of Intermittent, Renewable Energy Sources

As assessed by for instance Graabak and Korpås (2016), the inclusion of PV and especially wind adds variability to the energy system. Solar power is only able to be produced during the day naturally. Yet, the resulting power is not only determined by the irradiation at a given time of day, but also the amount of clouds present at the specific locations. In the same vein, wind power is only produced when the wind conditions are suitable; with too low or too high wind, the production is either not substantial or shut down. Even though aggregation of greater production ares makes the most rapid changes be smoothed out to some degree, these energy sources are non-the-less substantially less controllable than traditional energy sources.

3.2.1.1 PV power

Solar power capacity installed globally has have developed rapidly over the past years. As seen in figure 3.1, the installations have grown from a cumulative total of 6 GW in 2006 to 291 GW in 2016 according to IRENA, and with 100 GW to be added in 2017 alone according to BNEF.

This incredible growth is in part fuelled by the steadily declining prices of PV cells, with an reduction in costs by 80% since 2008 (Liebreich (2016)). Going forward, the costs are predicted to fall further. IRENA for instance expects a decrease by at least 30% by 2025 (Taylor et al. (2016)), hence enabling a future further growth of PV installations and makeing solar PV a feasible energy source, both for commercial and residential use.

However, a main drawback of PV power is that the system produces most of the energy at midday, and nothing during night, requiring that a isolated solar PV system employs storage of energy to even out the delivery of power.

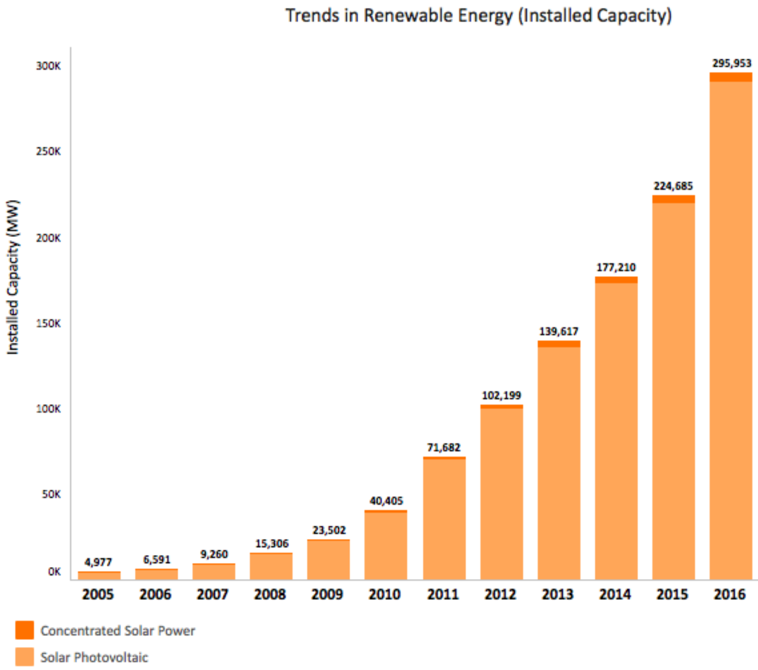

Figure 3.1: Cumulative installed solar PV capacity, Whiteman et al. (2017) IRENA

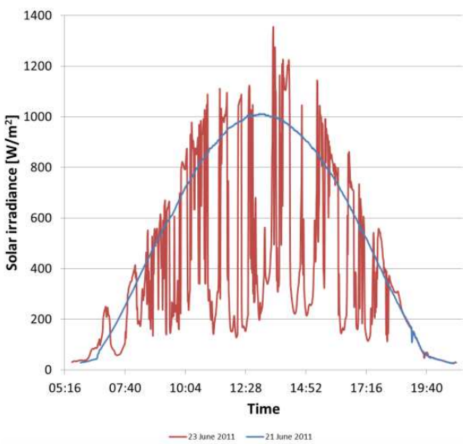

Figure 3.2: Measured irradiance, clear versus partially overcast summer day, Southern Norway Yordanov (2012)

Figure 3.2 shows the solar irrradiance during a day in southern Norway. The irradiance intensity reflects the time of day, while the fluctuation is due to partially shading. This variability in irradiance makes the power output less predictable.

3.2.1.2 Wind Power

Wind power have also developed immensely over the past years, growing from 73 GW in 2006 to 467 GW in 2016 according to IRENA and displayed in figure 3.3. Installed power capacity between 2005 and 2016 increased from 6% to 16.7% comparted with total production capacity irrespective of source. By 2030 it is predicted to be more than 114,000 wind turbines in Europe, having an total installed power of 392 GW where 294 GW is by onshore wind and 98 GW is offshore wind, equal to meeting 31% of EU electricity demand. Corbetta et al. (2015).

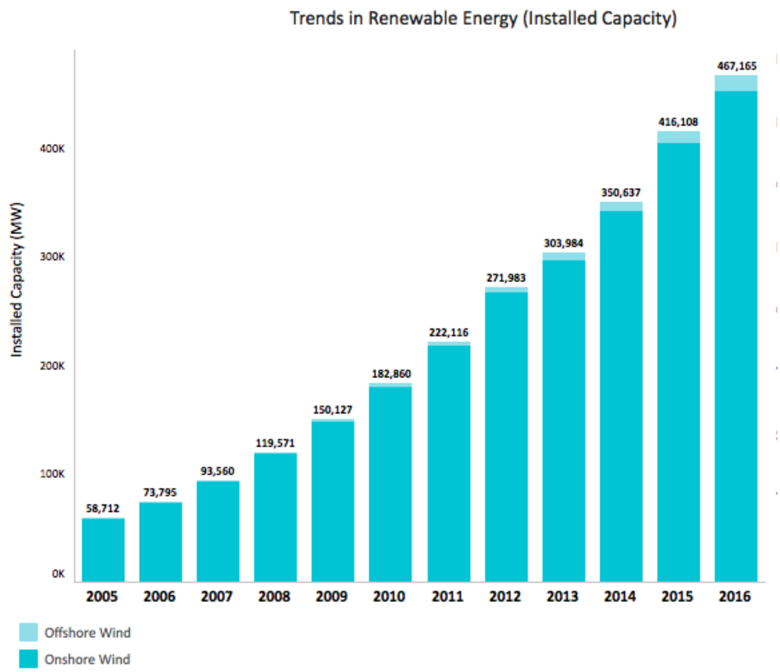

Figure 3.3: Cumulative installed wind power capacity, Whiteman et al. (2017) IRENA

Again this increase has partly been driven by cost reductions, down by 50% since 2009 (Liebreich (2016)). Because of the natural behavior of wind, power output from wind farms can have large fluctuations. The hour-to-hour power production have large fluctuations and may range from maximum output to nearly no production in minutes. This results in different challenges for the grid operator, such as voltage variations, the sudden need of primary and secondary reserves and cycling losses of thermal power plants.

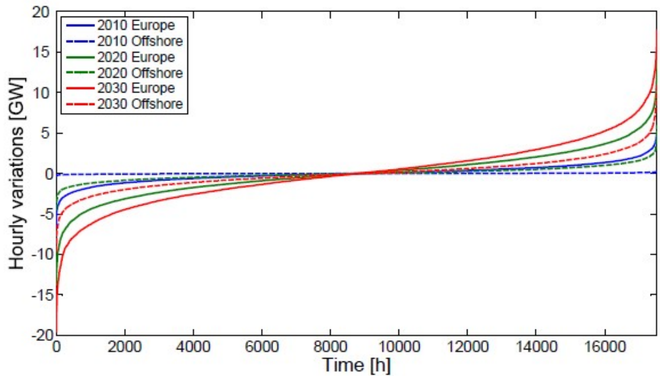

Figure 3.4: wind Hourly variation Aigner (2013)

Figure 3.4 shows an increase in wind power production variability which are possible to take place in future. While the hourly variations in 2010 do not have considerable change, it is expected to double in 2020 and triple in 2030.Aigner (2013).

Hence, the need if flexibility options such as Energy System Storage or even cross-border energy is a necessity. This thesis dives into the former.

3.2.2 Incorporating a larger EV fleet

A larger fleet of EVs is likely to introduce more variability on the demand side, in a similar, yet opposite and somewhat forceful manner as the incorporation of RES to the energy mix. The reason it is important to understand the impact of EV adoption, is because a event with many EV owner charging their vechicles at the same time substantially increases the demand from the grid and rises challenges for its components. For an assessments of how Electric Vehicle (EV) adoption is likely to affect the Norwegian grid specifically, publications from NVE Spilde and Skotland (2015) and Skotland et al. (2016), give valuable insights. The first paper show that the electrification of the Norwegian transportation system will reduce the consumed energy in that sector by 80%, but non-the-less still pose challenges to the transformer stations in the grid and the voltage quality. The second report also treats some of the same issues, but focuses even more on electric vehicles, the adoption outlook forward and charging behaviour.

To gain further understanding of the expected evolution of energy demand from EVs, one may study areas that have a few of them in use already, or try to build a model for simulation and analysis. There are some studies that have collected available data to do the former, and the author has in a previous work done the latter using insights from the former as input on what charging behaviour is common. As discussed in Harbo (2017a), one approach is to use simulation to capture the fundamentals characteristics of the system and yield desired insights in a bottom-up manner. In this thesis, we use the Agent Based Modelling framework built in as part of the work with Harbo (2017a) to simulate EV charging behavior. The reader is encouraged to visit this piece of work, or an abbreviated upcoming version soon to be published, if more details on the modelling is desired.

3.3 Introduction to Optimization

The following section gives a brief introduction to mathematical optimization, and the most relevant methodology and theory to understand when reading this thesis. For a more comprehensive treatment of the mathematical methodology underlying the Stochastic Quazi-Gradient method and optimization techniques specifically employed in this thesis, one is encouraged to visit Harbo (2017b), where the details are delved into in greater depth.

3.3.1 Basics of optimization

At its core, the field of optimization is about finding the best solution to a mathematically formulated problem. Here, one often use models and analytic methods to fund the best decision to make out of a set of feasible ones. In a general, formalized manner, an optimization problem may be formulated as in equation 3.1. Here, and objective function is defined, seeking to minimize the value of a function dependant on some decision variables, where the decision variables are given some constraints. This is expressed as

$$\begin{aligned} \min \quad & f(\mathbf{x}) \\ \text{s.t.} \quad & \mathbf{x} \in \mathbf{X} \end{aligned} \tag{3.1}$$

where $f(\mathbf{x})$ is the objective function and $\mathbf{X}$ is the set of possible solutions for the vector of control variables $\mathbf{x}$.

3.3.2 Non-linear Optimization Using Lagrangian Multipliers

An optimization problem is considered non-linear if either the objective function or any of the constraints contain non-linear terms. This is the case in many real-world problems.

Sometimes, it is quite possible to simplify the problem in such a manner that it is solvable by techniques used for linear, or even quadratic, problems. Yet at times, it is either not possible or not desirable to reduce the detail of the optimization problem to a level of non-linearity. An example of a non-linear problem of particular interest to this thesis is the power flow of the AC-OPF model. One method often used to analyze these problems are by using Lagrangian Multipliers.

In general, non-linear optimization problems may be expressed similar to linear ones, as in Lundgren et al. (2012) we can formulate the general problem as

$$\begin{aligned} \min \quad & f(\mathbf{x}) \\ \text{s.t.} \quad & g_i(\mathbf{x}) \leq b_i \quad \forall i \in 1, 2, \cdots, N \end{aligned} \tag{3.2}$$

where some terms in the constraints $g_i(\mathbf{x})$ or $f(\mathbf{x})$ are non-linear.

More importantly for this thesis is cases where $g_i(\mathbf{x})$ and $f(\mathbf{x})$ are differential functions. Then, the Karush-Kuhn-Tucker (KKT) conditions may provide general requirements for optimality for the non-linear problem at hand, which gives rise to the technique of Lagrangian multipliers and nodal price analysis. The conditions for the point **x** to be a local minimia is

$$\Delta f(\mathbf{x'}) = v_i \Delta g_i(\mathbf{x'}), \quad v_i \geq 0, \quad \forall i \in 1, 2, \cdots, N \tag{3.3}$$

$$g_i(\mathbf{x'}) \leq b_i \quad \forall i \in 1, 2, \cdots, N \tag{3.4}$$

$$v_i(b_i - g_i(\mathbf{x'})) = 0 \quad \forall i \in 1, 2, \cdots, N \tag{3.5}$$

where the constraints in the first equation 3.3 gives the dual feasibility, the second constraints 3.4 defines primal feasibility and the last constraints 3.5 ensures complementary. From this we formulate the Lagrangian function as follows

$$\mathcal{L}(\mathbf{x},\mathbf{v}) = f(\mathbf{x}) + \sum_{i=1}^{N} v_i(b_i - g_i(\mathbf{x})) \tag{3.6}$$

where $v_i \geq 0$ is now the Lagrangian multiplier for the ith constraint.

In applied cases, the Lagrangian multipliers are also know as the shadow price and it ties together the derivatives of the costs and the constraints imposed on the control variables. When the mathematical model analyzed is and power system, formulated as an

Optimal Power Flow problem as described in 3.7.2 and explained in more detail in for instance Wangensteen (2012), these give the nodal price of power at each particular bus (or node) in the power system. Then the Lagrangian multipliers for the system may be expressed as

$$v = \begin{bmatrix} c + \gamma \\ d \end{bmatrix} \tag{3.7}$$

where c is the marginal cost of generation at the bus, γ is shadow cost added to the cost to give the nodal price given that there is a constraint on generation capacity at the bus, and d is the marginal willingness to pay of the utility costumers at the bus if it is a load bus.

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3.3.3 Line Search Methods

When trying to find the best solution of an optimization problem, there are different ways or strategies one may utilize. The simplex method mentioned earlier seeks to find the optimum of a linear, convex, problem by checking its vertices, that is points of intersection between different constrains, based on the knowledge that the optimum should lie in one of these points. It also moves from one point to the next along the line given by two constraints' intersection, in a way that yields most improvement per move.

Another approach is that of line search methods. Generally, they compute a search direction d^s and a step length ρ^s for each iteration. That is finding the next approximation for the solution $xs + 1$ by

$$x^{s+1} = x^s - \rho^s d^s \tag{3.8}$$

One intuitive method is here is the gradient approach. It tries to find the the optimal solution by figuring out the direction of the rate of improvement at its starting point, and then move a step of a certain length in that direction. Another line search method is Newton's method or Newton-Raphson method known from numerical analysis, utilizing the Hessian matrix in making its steps. Further mathematical details of these methods are not discussed there, but can be found treated for the purpose of discussing the SQG method in Harbo (2017b), or more extensive in Jameson (1995) and Nocedal and Wright (2006).

3.3.4 Interior-point methods

Interior point - or barrier - methods are, together with active-set sequential quadratic programming methods, considered the most powerful way to solve large-scale non-linear problems. For understanding we may comparing it conceptually to the to the Simplex method, a long time standard approach to solving linear programs and the dominating solution method for more small scale linear problems. The Simplex method checks vertices along the feasible region in searching for the optimal solution, which generally requires a large number of inexpensive iterations. Yet, as the problem becomes bigger, e.g. increasing the number of constraints and variables, the solution time scales exponentially with size of problem.

The Interior point methods on the other hand use another strategy. It approaches the boundary in the limit, from interior (or exterior) of feasible region. The term "interior point" comes from the early development of the method that started at some initial point inside the feasible region - an interior point - and used barrier methods to make sure the iterations stayed within the feasible region. Hence, interior points never actually lie on the boundary, and thus is able to tackle non-linear cases as well. Indeed, the method has proven similarly efficient for linear as non-linear cases. In contrast to the Simplex method it uses a small amount of iterations, yet each iteration is more computationally expensive.

This thesis employs interior-point methods as part of its solution process, in two different variats. For one, the Matlab® `fmincon` solver utilized in this thesis solves the problem primarily by using the line search Newton method discussed in section 3.3.3. The MIPS solver, or Matpower Interior Point Solver Zimmerman and Wang (2016), used as comparison in this thesis also the primal-dual approach of the Newton approach.However, a further and full treatment of the interior point is not given here, as it would require the introduction of a lot more theoretical, mathematical concepts, that are out of scope of this thesis. For a more comprehensive treatment see for instance Nocedal and Wright (2006).

3.4 Stochastic Programming

A demanding issue for optimization problems is how to tackle uncertainty as part of the mathematical formulation and solution of its models. This is the focus for a group of optimization methods on stochastic optimization.

3.4.1 Basis of Stochastic Programming

These models have historically emerged from recourse problems and probability constrained models as Higle (2005) mentions in a brief and informal introduction to stochastic programming. These models arise when one of more variables of the model is best described by random variables. A thorough presentation of Stochastic Programming is given in the books of Kall and Wallace (1994) and Birge and Louveaux (2011). As stated in their works, a stochastic problem can generally be formulated as

$$\begin{aligned} \min_{x \in X} \quad & \mathbb{E}_\omega f_0(x, \omega) \\ \text{s.t.} \quad & \mathbb{E}_\omega f_i(x, \omega) \leq 0, \quad \forall i \in 1, 2, \cdots, M. \end{aligned} \tag{3.9}$$

where one seeks to minimize the expected value (the $\mathbb{E}$ operator) of the objective function f_0 by choosing the right values for the control variables x given some constrains f_i and the realization of the random variables ω.

3.4.2 Multistage Stochastic Programming

Multistage Stochastic problems arise in many real world cases, where one in to make several decisions over a period and where unforeseen events happen during the course of time. As before, Stochastic problems are characterized with uncertainty in the parameters describing them. Multistage problems, are problems that reach over several periods of time, in which one tried to find a optimal policy or strategy for solving the problem for all the desired stages. Hence they may also be called multi-period problems, yet for a multistage stochastic optimization problem is not only solved for a multiple of periods.The execution of the strategy or policy should allow for reactions to what previously has happened and react to the new situation, in a manner that optimizes the overall objective. The book by Pflug and Pichler (2010) give a comprehensive treatment of multistage stochastic optimization.

3.4.2.1 Stochastic Dynamic Programming

Stochastic Dynamic Programming, often abbreviated as SDP, is a method of stochastic optimization with roots in dynamic programming. It uses discretized probability distributions through Markow Chains - and specifically the Markow Decision Process - to calculate the optimal policy for a specific problem, which all might be found in described Hillier and

Lieberman (2010).

From dynamic programming, we may utilize its method for systematic evaluation of the optimal combination of some interrelated decisions. This approach discretizes the states and the time steps. Central to dynamic programming is the *Bellman Optimality Principle*, see for instance Bellman (1957) which says that for any given current state, an optimal policy for the following states is independent of policy decisions made in the previous states. Then, if we may somehow find the optimal policy and its value for the last stage, we may recursively calculate the optimal policy for each stage back to the beginning. For a further explanation of this method in the context of power systems and multistage stochastic problems, see Harbo (2017b).

3.5 Stochastic Quasi-Gradient Methods

A limiting issue most Stochastic Programming methods have in common that they require the problem have a certain degree of convexity. Equally critically, these other approaches often become computational demanding when introducing several time steps or stages. A method better suited to tackle these issues - issues that are key characteristics of the Optimal Power Flow models of interest that are non-convex and multistage - is the Stochastic Quasi-Gradient method that is an integral part of this thesis.

The Stochastic Quasi-Gradient (SQG) methods were introduced in Ermoliev (1976), developed in Ermoliev (1983) and Ermoliev and Wets (1988), and early implemented by for instance Gaivoronski (1988). They are characterized as both a Monte-Carlo approach and an iterative sampling algorithm, and are designed to tackle nonlinear stochastic programming problems with continuous random distributions. They are numerical techniques with background in stochastic approximation, gradient projection and mathematical, algorithmic programming. In essence, instead of using exact values, they employ asymptotically consistent estimates to evaluate the functions in question and their derivatives. They are aimed at solving stochastic problems where both objective function and constraints may be of complex nature, for instance non-linear and non-convex. When applied to deterministic cases, they may be considered random search techniques (Ermoliev (1983)).

Within the field of Stochastic programming, SQG methods are considered a complement to the large scale linear stochastic programming methods(Gaivoronski (1988)).

Such linear stochastic programs has been put quite an effort into and are often represented by discrete Scenario Trees and solved using Benders Decomposition. The linear approach solves the stochastic programming problem of **??** and **??** by its deterministic equivalent after discretizing the probability function of ω into a finite number of points, $\omega_i \; \forall \; i \in 1, ..., K$ where k is the number of discrete points, and replacing the spacial property of the original problem with sums, see equation **??**. This approach is well suited for linear problems with large dimension for which the random variables are well described by discretizations of only a few points and where high precision is required and possible from these. Yet, these deterministic methods often encounter computational difficulties when the number of stages or required points of dicretization becomes moderate. This since the solution time scales with K^T (Gaivoronski (2005)), where K is the number of discrete points of the approximated probability distribution and T is the number of stages.

The SQG approach on the other hand, is well suited for problems with a smaller dimension, but where the number of stages are greater. Hence problems of dynamic character, especially stochastic dynamic optimization problems, are suitable cases for the SQG approach which solution time only scales linearly with T. The SQG approach also is well suited when the probability distributions are either complex or best described by continuous distributions, problems with many stochastic variables or problems of non-linear nature. However, due to its iterative, approximate nature, the convergence to a final, super precise solution often takes many more iterations that just coming well within the vicinity of the solution. Non-the-less this approach is of particular interest when optimizing simulation models depending on some finite number of parameters which may be challenging to pose as a linear or even non-linear programming model, and where the estimation of the underlying stochastic functions does not have high precision them self.

In the Stochastic Quasi-Gradient methodology, one solves the problem of minimizing

$$\min_{x \in X} F^0(x) = \mathbb{E}_\omega[f(x, \omega)]$$

from the objective equation in 3.9. where $\omega \in \mathbb{R}^k$ is a vector of random parameters and the decision variables are $x \in X \subset \mathbb{R}^n$. Here, X defines the set of feasible solutions. This is usually convex and simple to allow more easy computation of projection onto the set, yet more complex inequalities may in principle be used. They may for instance given by upper and lower bounds or linear constrains defined by $Ax \leq b$ like in **??** and **??**. More generally the constraints may be expressed as

$$\text{s.t.} \quad F^i(x) = \mathbb{E}_\omega f_i(x, \omega) \leq 0, \quad \forall i \in 1, 2, \cdots, M.$$

as in 3.9.

As a general illustrative description, the iterative nature of the SQG approach start from an initial point x_0 and makes a step of size ρ_s in the direction opposite (in the case of a minimization problem) to the current estimate ξ^s of the gradient of $F^0(x)$ at point x_s. See Becker and Gaivoronski (2014) or Harbo (2017b) for more details on how to choose step direction, step length, and the projection onto a feasible set.

3.6 Stochastic, Heuristic Optimization

In the manner of Spall (2003), Stochastic Optimization techniques are views as methods which both generate and use randomly realized parameters in making its iterations towards an optimized solution. Hence, the problem they try to solve may not necessarily be of a stochastic character it self, as these techniques are frequently and powerfully employed on deterministic, yet difficult (eg. NP-hard), problems to generate a solution that is sufficiently good within reasonable time.

As such, many of these techniques are also known as heuristics, and termed heuristic optimization (Maringer (2005)). Examples are

- Random search
- Machine learning, especially reinforcement learning
- Recursive linear estimation
- Model selection
- Stochastic approximation
- Simulation-based optimization
- Simulated annealing
- Markov chain Monte Carlo
- Genetic and evolutionary algorithms
- Ant Systems and Ant Colony Optimization
- Optimal experimental design

See for instance the works mentioned above for further details. In this thesis we have chosen to implement a genetic, evolutionary algorithm, as a comparison to the SDP and SQG methods.

3.6.1 Evolutionary and Genetic Algorithms

Evolutionary algorithms are, as the name suggests, strategies which uses inspiration from nature and its process of evolution in order to directly search for an optimal solution. As seen in for instance Rechenberg (1965), Rechenberg (1989), such algorithms often use populations of vectors to represent possible solutions, and have them mutate and recombine in generations in order to iterate forth a better solution. Since this type of algorithm is able to learn about the problem and its environment during its iterations without being specifically programmed, they may also be considered machine learning algorithms.

These types of algorithms are especially associate with the genetic inspired versions. As seen in for example Holland (1975), genetic algorithms represents solutions by a *chromosome*, as illustrated in figure 3.5, which have a probability of being reproduced according to its fitness.

10	15	10	5	0	10	20

Figure 3.5: Chromosome vector

Hence, the individual solutions in the populations have their aptness evaluated by a *fitness function*. For optimization problems, this fitness function may simply just be the objective function. Likewise, the determination of which solutions should be used as parents, happen according to a *selection operator*, so that the more fit individuals are more likely to reproduce. Such a selection operator may be based on the evaluation of the individual's fitness alone, or be subject to other factors. For instance Gundersen et al. (2017) use both education of offspring and a selection of parents based on the relative diversity the respective individual add.

A basic example of an selection operator in the case where one has a maximization problem is to define the probability of choosing x_1 as a parent as

$$P(x_1 isselectedasparent) = \frac{f(x_1)}{\sum_i^n f(x_i)} \tag{3.10}$$

where f indicates the evaluation by the fitness function, n is the number of individuals in the population and i denotes each specific individual. Hence, the higher fitness value an individual has, the more likely it is to reproduce.

Another alternative is to use weighted rankings, based on the sorting of the individuals from highest to lowest, as for instance presented in Carr (2014). For an implementation in which we are only to keep k individuals (and naturally $k < n$), we may calculate the probability of choosing x_i as

$$P(x_i\ is\ selected\ as\ parent) = \frac{k - i + 1}{\sum_{j=1}^{k} j}. \tag{3.11}$$

To give an example, if x_1 is the individual with highest fitness value, x_2 the one with second highest etc., and we for instance is only to keep 5 individuals, then

$$P(x_1\ is\ selected\ as\ parent) = \frac{5 - 1 + 1}{1 + 2 + 3 + 4 + 5} = \frac{5}{15} = 0.33. \tag{3.12}$$

With this approach, the fitness values may assume both positive and negative values.

A new generation of solutions is generated by the combining the chromosomes of two parent solutions. The exact way the characteristics of the two parents are inherited, are determined by the *crossover operator*. One way to do this operation is to determine a crossover element in the vector, say the j^{th} element of some pair of chromosomes, at which the chromosomes are split before being recombined with its counterpart corresponding remnant after the cut off. This is illustrated in figure 3.6.

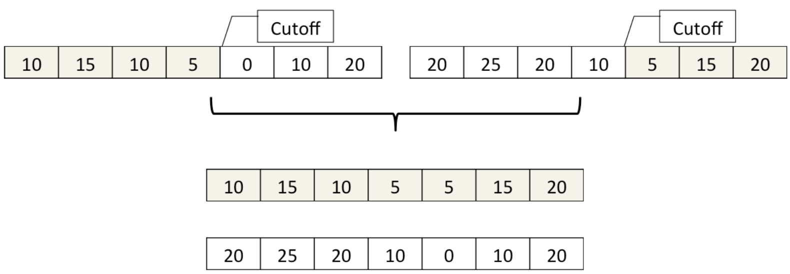


Figure 3.6: Recombination of parent chromosomes into a new individual based on cut off

Moreover, the child might also experience a mutation in its chromosome, determined by a *mutation operator*. This might be as simple as selecting a random element of the chromosome vector and replacing it with a random number within the bounds of the possible values it can undertake. Another approach would for instance be to perturbate the values of a chromosome with some percentage given by random realizations.

In general, evolutionary algorithms are useful in creating a group of good solutions where one is the best current approximation. Since it takes additional effort to manage the population, methods such as Simulated Annealing may be faster if the problem is not too rough. Yet, the evolutionary algorithms are better if the solution space have several local minima or maxima, as it is less likely to get ”trapped” in a local optimum.

3.6.1.1 Optimization of Continuous Variables

For our specific problem, the decision variables are continuous. However, Evolutionary algorithms are often employed for problems which require an integer solution. Thus, the relaxation of dealing with continuous variables allows the use of different cross over and mutation operators, since we are not forced to return the corresponding chromosome vector with integer values only.

One operator to be employed on continuous variable problems was proposed in Haupt and Haupt (2004), where one chooses two parents x_1 and x_2 and generate two offspring as a linear combination of the two chromosomes weighted by a factor β. That is

$$\begin{aligned} x_{1,new} &= (1-\beta)\cdot x_1 + \beta \cdot x_2 \\ x_{2,new} &= (1-\beta)\cdot x_2 + \beta \cdot x_1 \end{aligned} \tag{3.13}$$

where β is between 0 and 1. This cross over process is illustrated in figure 3.7 below.

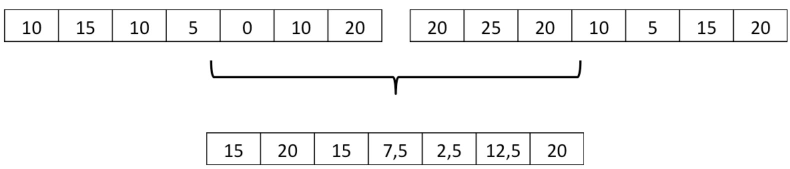


Figure 3.7: Recombination of parent chromosomes into a new individual

In this case β is set to 0.5. That is, the resulting chromosome has in each cell of its vector the average value of the identical cells from its parents.

3.7 Optimization of Power Systems

In this section, an basic description of Power System Analysis and Optimal Power Flow is given, as an background for the models and cases considered later in this thesis.

3.7.1 Power System Analysis

From the fundamentals of electrical engineering, we are well familiar with Ohm's and Kirchhoff's laws on voltage $U = I \cdot R$ and current $\sum I = 0$. Further more, we assume known that in alternating current (AC) systems for instance described in Grigsby (2012), complex power may be expressed as the sum of active and reactive power $\mathbf{S} = P + jQ =$ **VI*** and also as the the multiplication of the phasors of voltage and the conjugate of current.

For ease of analysis we often transform all values to per unit [p.u.] numbers and represent the system through one-line diagrams thus simplifying it to one phase instead of three, as seen in figure 3.8 from Grigsby (2012).

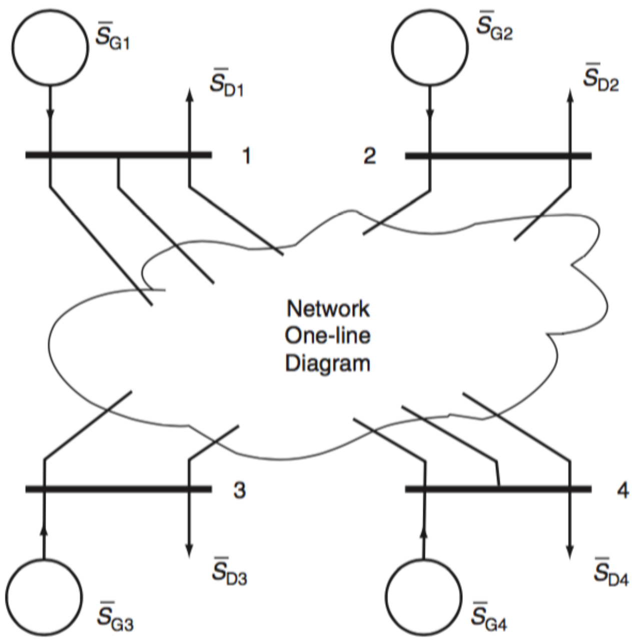


Figure 3.8: One-line system of four-bus network.

When developing the basic power flow equations for such a system, we need to consider the network topology and the characteristics (impedance, resistance) of the power lines and components. This is most often expressed in matrix notation, as with an n-bus system

$$\left[I_{Bus}\right] = \left[Y_{Bus}\right]\left[V_{Bus}\right] \tag{3.14}$$

$$\begin{bmatrix} I_1 \\ \vdots \\ I_i \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & \cdots & Y_{1i} & \cdots & Y_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Y_{i1} & \cdots & Y_{ii} & \cdots & Y_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Y_{n1} & \cdots & Y_{ni} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_i \\ \vdots \\ V_n \end{bmatrix} \tag{3.15}$$

where $Y_{ij} = G + \jmath B$ is the admittance matrix of the network. Thus, the expression for power flow becomes

$$S_i = V_i \Big(\sum_{i=1}^{n} Y_{ij}^* \cdot V_{ij}^* \Big) \tag{3.16}$$

and

$$P_i = \sum_{j=1}^{n} |V_i||V_j| \big(G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij} \big) = \sum_{j=1}^{n} |Y_{ij}||V_i||V_j| \cos(\delta_i - \delta_j - \theta_{ij}) \tag{3.17}$$

$$Q_i = \sum_{j=1}^{n} |V_i||V_j| \big(G_{ij} \sin \delta_{ij} + B_{ij} \cos \delta_{ij} \big) = \sum_{j=1}^{n} |Y_{ij}||V_i||V_j| \cos(\delta_i - \delta_j - \theta_{ij}) \tag{3.18}$$

where δ is the voltage angle and θ is the lag. As presented in Crow (2012) , one of the most common way to solve these problems are through iterations useing the NewtonRalphson Method and calculation the Jacobian

$$\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} = \begin{bmatrix} \frac{\partial \Delta P}{\partial \delta} & \frac{\partial \Delta P}{\partial V} \\ \frac{\partial \Delta Q}{\partial \delta} & \frac{\partial \Delta Q}{\partial V} \end{bmatrix} \tag{3.19}$$

for the power flow equations, solving for the know power injections P and Q by

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \tag{3.20}$$

and finding the converging values of δ and $|V|$ through iteration.

The power flows equations from load flow studies expressed here are used as base for calculating the Optimal Power Flow of a system. Moreover, these equations clearly expresses the non-linear, non-convex physical properties that are required of our optimization methods when solving them.

3.7.2 Optimal Power Flow

The technique of Optimal Power Flow (OPF) is in essence an optimization of how power shall flow through a grid with respect to some objective function and by choosing the best values for the control variables. In many cases the objective is to minimize some cost or loss function dependent on the state variables and control variables defined in the formulation of the problem.

As seen in the previous section 3.7.1, and in for instance Cain et al. (2012), one way of formulating the OPF problem in a general manner is the following:

$$\begin{aligned} \text{Maximize} \quad & f(P) \\ \text{subject to} \quad & g(x, P) = 0 \\ & h_{min} \leq h(x, P) \leq h_{max} \\ & P_{min} \leq P \leq P_{max}, \end{aligned} \tag{3.21}$$

where $f(P)$ is the objective function. The equality constraints, $g(x, P)$, of the system expresses the physical law of power balance, or the conservation of energy, as stated in the equations 3.17 and 3.18 for active and reactive power respectively. The inequality constraints, $h(x, P)$, represents the physical limitations of the system that often are operational rules set by the operator so that power system components are not over- (or under-) loaded. Examples for such constraints are voltage limits, current limits of the transmission lines. The input and output variables P, is typically the generator values for the slack generator, and possibly other generator or buses. The variables x are the state variables.

Many books in power system analysis will cover this topic extensively, for in the book *Power Systems* by Grigsby (2012).

In the simplest version of the Optimal Power Flow problem, one assumes direct current (DC) and sett all bus voltage to 1pu. Yet, the more realistic case is an alternating current (AC) power flow, in which we also have to treat the phases of the system voltages and currents. Further detail on how these parameters and variables may be expressed is found in section E.1 on the AC-OPF.

One of the most common ways for solving OPF problems is through the use of Lagrangian multiplier discussed in section 3.3.2, which readily give insight on the price at the different nodes in the system.

3.7.3 Multistage Optimization of Power Systems

Power systems are a natural application in which to consider several time steps, and develop a multistage model for. This is especially due to the rise of increasingly more Energy Storage Systems (ESS) to be able to store variably generated renewable energy. Hence, one is interested in using this stored energy in some optimal manner, which make the models take on a dynamic character.

Other applications is a deterministic, dynamic ACOPF, often labeled DOPF, in which one solves the power flow for all time steps simultaneously coupled together by the energy storage in the power system. A review of such models may be found in Sperstad and Marthinsen (2016), as well as a comparison of the different solution methods implemented for this and their performance.

One article develop the AC-OPF to a dynamic problem, the article of Zaferanlouei et al. (2016) who presents a formulation of this problem where AC-OPF power flow with constraints as given in 3.21, 3.17 and 3.18 over a T step time period coupling the periods thought the dynamic equations of the battery, that

$$E_{ST,i}(t) = E_{ST,i}(t-1) + \Delta t \cdot \eta_{chrg} \cdot P_{Ch}(t) - \Delta t \cdot \frac{P_{Dch}(t)}{\eta_{dischrg}} \tag{3.22}$$

where $E_{ST,i}(t)$ is the energy stored at bus i at the end of time step t, $P_{Ch,i}(t)$ and $P_{Dch,i}(t)$ are the active charging and discharging power of a certain energy storage at bus i during time step t, $\eta_{Ch,i}$ and $\eta_{Dch,i}$ is the charging and discharging efficiency of the energy storage at bus i, and Δt is the amount of time between the time step increments.

Additionally, the energy stored is also clearly subject to some capacity constraints, such as

$$0 \leq E_{ST,i}(t) \leq E_{ST,i}^{max}. \tag{3.23}$$

and so may the discharge and charging values be too

$$\begin{aligned} 0 \quad &\leq P_{Ch}(t) \quad &\leq P_{Ch}^{max} \\ 0 \quad &\leq P_{Dch}(t) \quad &\leq P_{Dch}^{max} \end{aligned} \tag{3.24}$$

setting bounds on the dynamics of the problem and transfer of energy between states.

3.7.4 Stochastic Programming of Power Systems

The inclusion of stochastic variables in the AC-OPF, lets one directly model uncertainty as part of the problem, to find some solution that hedges against unwanted realizations.

For instance we may want to add stochastic PV generation and house hold energy consumption, eg as fluctuations around a base production curve or load curve respectively. For instance as

$$P_L(t) = P_{avg} \pm P_{rnd}(\omega) \tag{3.25}$$

where P_{avg} is the average power for given time period (eg at noon) over the last week and $P_{rnd}(\omega)$ is some random realization for instance based on a normal distribution with $= P_{avg}$ and $\sigma(t)$ is calculated or assumed somehow. To make things relatively simple in this thesis, uniform distributions around some mean have been used for all the stochastic variables.

In this thesis, we seek to solve a multistage, stochastic program of a power system with ESS. In many regards this is a difficult problem to solve as it in principle quickly becomes rather heavy computationally. One common approach is the use of Scenario Trees and especially for the linear version of the problem - an economic dispatch problem. Another approach, is that of calculating water values using dynamic programming, or Stochastic Dynamic Programming.

However, many of these approaches scale exponentially in running time according to their level of discretization and the number of time steps, and soon become intractable. In this thesis we propose the use of the SQG method to solve the stochastic, multistage program.

Chapter 4

Methodology

This chapter presents the methods used to solve the problems presented. The solution methods will be given an overview, discussing some details of their methodology and presenting figures of their architecture or illustrative flowcharts.

The mathematical optimization models that is the fundamental for this to work, are discussed in length in Harbo (2017b), and the most necessary mathematical framework is presented in appendix E for the interested reader.

4.1 AC-OPF solution methods

This thesis uses a full AC-OPF model with non-linearity and non-convexity, and solve it using the Interior-point method. The thesis uses Matlab's built-in optimization solver `fmincon`, but also implements a new, faster solver called *MIPS* as well. Figure 4.1 and 4.2 illustrate how the architectural is laid out for these tow cases.

4.2 Stochastic Quasi Gradient Method

The SQG approach as discussed in 3.5 uses a privately implemented solver developed by Alexei Gaivoronski (2016) and for instance utilized in Gaivoronski (2005) and Becker and Gaivoronski (2014). An overview of the SQG approach, as presented in his 2005 paper, is displayed below in figure 4.3. As can be seen from the figure, both the optimized model,

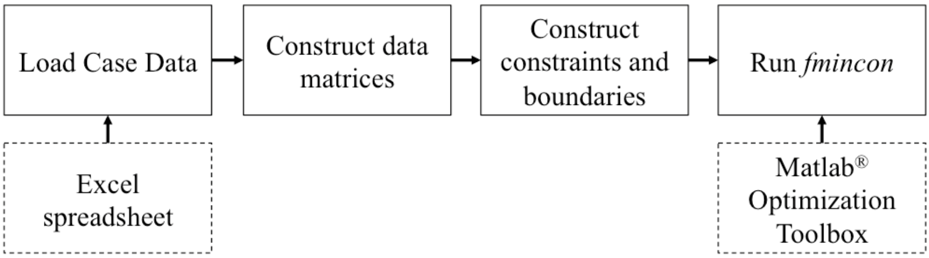


Figure 4.1: AC-OPF architectural flowchart using the *fmincon* solver

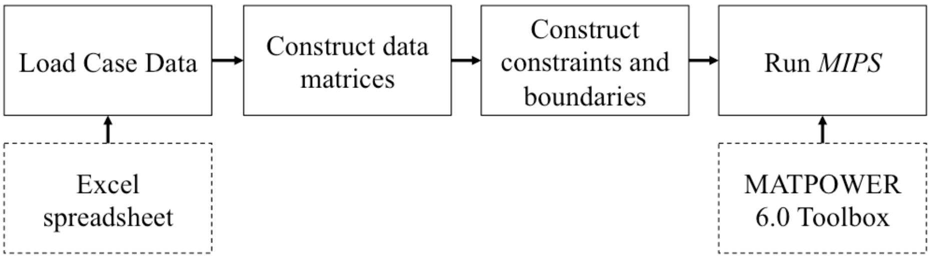


Figure 4.2: AC-OPF architectural flowchart using the *MIPS* solver

user interface and underlying AC-OPF simulation model is also developed to be run together with the SQG optimization engine.

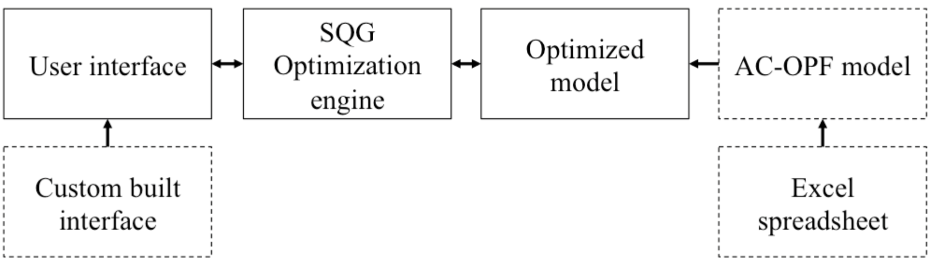


Figure 4.3: Overview of Stochastic Quasi-Gradient Architecture

A key question of the SQG method is how to retrieve the stochastic quasi-gradient, which is used to determine in which direction the iteration algorithm is to make its next step. A more detailed overview of how this process works is found in figure 5.1, divided into three main parts; initialization, the SQG iterations and the post-analysis. It may also be observed that for the SQG iteration, the AC-OPF is solved for all the time steps, before extracting the stochastic gradient.

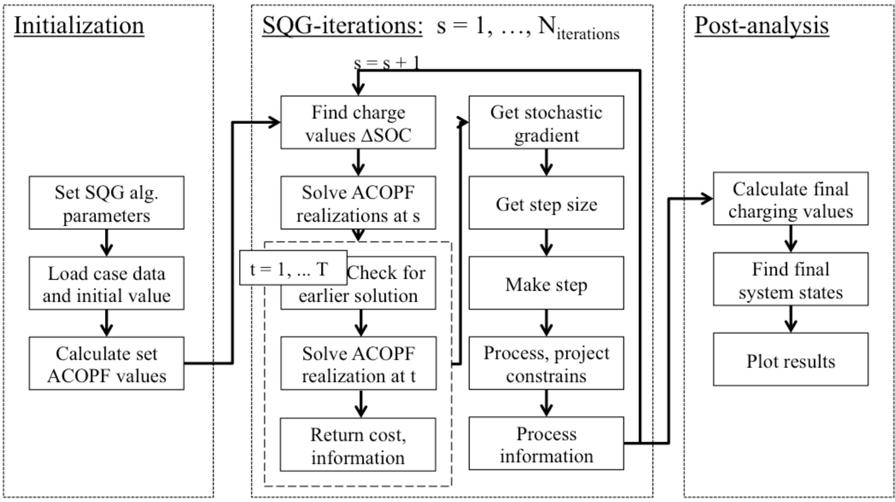


Figure 4.4: Iteration process of SQG algorithm for ACOPF

In order to find the gradient for the problem at each iteration, this thesis applies a method of calculating the gradient in from some analytic expression

The challenge when estimating the gradient for the battery, is that it would be beneficial to discharge at all the time steps considered in isolation Yet, for some time steps to discharge, other need to charge so that energy is available. In Harbo (2017b) the author presents a method for which to calculate the gradient directly in simple cases. It may be observed that one more unit of energy discharged by the battery at a time step, is equivalent to reducing the total demand of the loads in the power system by one unit. This may be be expressed formally as

$$\begin{aligned}\frac{dQ(t,P_{Battery},\xi)}{dP_{Battery}} &= \frac{dC_G\big(P_{Gen}(t,P_{Battery},\boldsymbol{\xi})\big)}{dP_{Battery}} \\ &= \frac{dC_G\big(P_{Gen}(t,P_{Battery},\boldsymbol{\xi})\big)}{dP_{Gen}} \cdot \frac{dP_{Gen}}{dP_{Battery}}\end{aligned} \tag{4.1}$$

and since

$$\frac{dP_{Gen}}{dP_{Battery}} = -1 \mid P_{Loss} \approx 0 \tag{4.2}$$

we get

$$\begin{aligned}\frac{dQ(t,P_{Battery},\xi)}{dP_{Battery}} &= \frac{dC_G\big(P_{Gen}(t,P_{Battery},\boldsymbol{\xi})\big)}{dP_{Gen}} \\ &= -\,C_G'\big(P_{Gen}(t,P_{Battery},\boldsymbol{\xi})\big) \mid t, P_{Battery}, \boldsymbol{\xi}.\end{aligned} \tag{4.3}$$

Despite having a good indication on how the objective function of the separate time steps might be improved by alternating the battery charging policy, this does not necessarily translate directly into gradients for the first stage decision parameters.

To achieve this, it would for instance be possible to shift the gradient values with respect to stored energy at each time step so that they are centered around the mean of the gradients of the related AC-OPF solutions.

4.3 Stochastic Dynamic Programming

To compare the SQG method with an exact method, the simplest case was also solved using Stochastic Dynamic Programming. An overview of this process is found in figure 4.5, where a backward recursion method is applied. For more details on the method and algorithm developed for this, see Harbo (2017b).

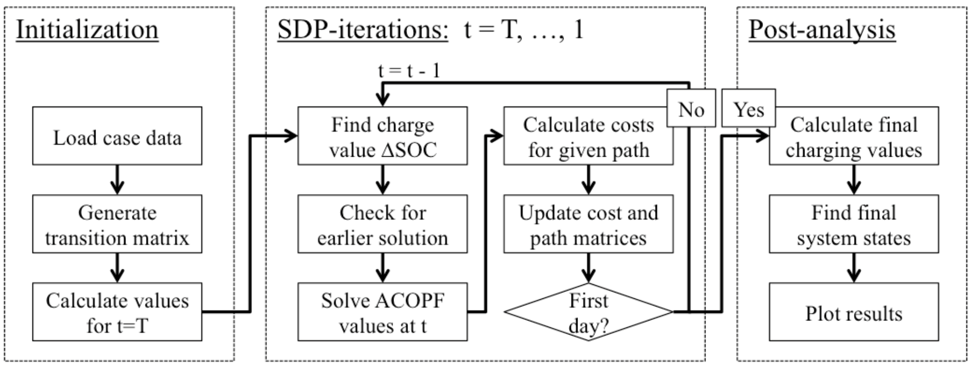


Figure 4.5: Illustration of SDP method for the AC-OPF.

4.4 Evolutionary Algorithm

This thesis also develops a new Evolutionary algorithm to solve the stochastic, multistage AC-OPF case. In figure 4.6 an architectural overview of the code is showed.

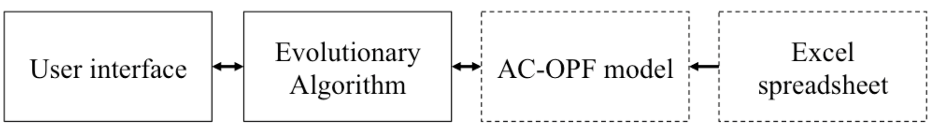


Figure 4.6: Overview of architecture for the AC-OPF using an Evolutionary Algorithm

Figure 4.7 shows a flowchart of the algorithm. First the process is initialized, before it iterates through evolutionary life-cycles, and finishes with some post-processing. During the main iterations, the algorithm chooses individuals to mate using a probability based on their fitness and creates an offspring using their properties. Thereafter the offspring may be inflicted a mutation of sort, before it is evaluated with the fitness function. This evaluation is done by solving the AC-OPF model. At the end of the iteration, a fraction of the best fit individuals is kept, before new individuals are selected to mate and start the process all over again.

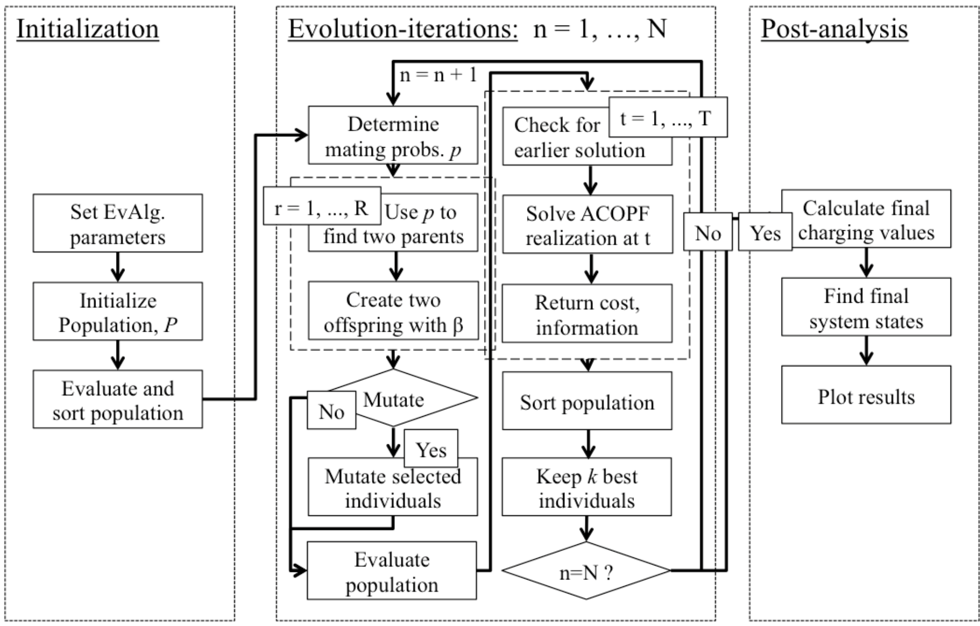


Figure 4.7: Illustration of solution method for the AC-OPF using an Evolutionary Algorithm

Further details on the specific algorithm from employed in this thesis, can be found in the section 5.4.1.

4.5 Modelling PEV power demand and charging behaviour with agents

A mathematical foundation for the Agent Based Models (ABM) used to simulate Plug-in Electric Vehicles (PEV) behaviour and energy demand is presented in this section. In particular, the focus will be on the charging strategies assigned to the agents in different runs of the simulation.

The models are primarily a bottom-up computational program where one defines a set of agents with certain characteristics and interaction rules, as well as properties of the environment. Based on this a computer simulation is done, in which a desired number of agents operate and interact for an specified length of computational time increments. By using ABM one seeks insights insights on complex problems, that is not possible to gain through explicit techniques. As a result, the mathematics here is not necessarily very advanced.

To simulate EV charging, we need to define some charging strategies or charging behaviour that the agents should adhere to. This is perhaps the most interesting and important part of the model. The strategies are what will have the most impact on the results, and an analysis of what aspect of a strategy caused the results to differ, will give valuable insights on how PEV agents may behave given certain conditions. Here, we may specify a few strategies to be implemented.

- One common charging strategy to consider is a ”dumb” charging strategy, meaning the agents are not concerned about the price of the electricity, etc. but charge whenever they have the need.

- The first strategy allow for minimal interaction between the agents, which limits the amount of Self-Organization and Emergence (see section Harbo (2017a) for details) to be observed. Another possibility is then to have a probabilistic charging strategy based on SOC and system price, which additionally will let us analyze if the system planner may be able to shift some of the load away from the peaks.

One may of course define even more accurate or different strategies, for instance based on behavioral economics, but this will be sufficient for our analysis here.

There is a possibility to let the agents achieve some kind of learning effects and posses memory, or have information about forecasts and other market data, which then could let them reach some kind of optimized results that are not explicitly solved by the modeller. Such a model could be interesting to compare to an optimization model based on some of the same data, to see if the agent based model with bottom-up optimization provide some similar results to a top-down mathematical one.

Some research was done by the student on this, and reinforcement learning was identified as a method well suited. However, due to the specifics of the case sought to implement, the method would have to be customized so much that it would be difficult to implement via off-the-shelf software. Hence, due to time consideration, a independent coding of reinforcement learning for the ABM is left to be done for another time.

Chapter 5

Implementation and cases

This chapter presents how the method, models and cases from previous chapters are implemented. All the optimization done in this thesis is implemented in the programming language Matlab®, where as the Agent Based Model used to generate EV charging values has been programmed in Java®.

This thesis has built upon the models built in Harbo (2017b), and used the same backbone for the SQG method, the SDP method and the AC-OPF simulation as earlier.

As part of this degrees work, a lot of time and effort was spent building the ABM of EV's that was part of the pre-master project reported in Harbo (2017a). This work is also to be published at the PSCC in 2018, and much time was also spent during this thesis period to get that paper published.

A new effort done specifically for this thesis, is the development and implementation of an Evolutionary algorithm to solve the stochastic multistage AC-OPF problem, and both the research and coding of this method has taken quite a bit of effort.

For the SQG method to be of sufficiently quick, the underlying model should not take too much time, as it is to be ran many times throughout the optimization iterations. Hence, this thesis also has implemented the *MIPS* solver for the AC-OPF simulations in SQG method. From there, it needed to be adjusted for the different cases to be tested on.

On top of that, a lot of time has been spent developing and testing many different cases for the SQG method. The implementation of the 24 bus case is all new, and has been implemented for several variations such as several batteries and EV charging.

Also some time was spent implementing the 118 bus case, but the time spent preparing the data input for the model and prospect of increased time to run models and testing them, made the student put this case to the side after a while.

5.1 AC-Optimal Power Flow

For the implementation of the basic AC-OPF the Interior Point method was chosen, on basis of research like Castillo and O'Neill (2013) and Sperstad and Marthinsen (2016) that indicate this to be a method that is able to deal with the non-linearity and size of the AC-OPF problem. It was implemented using both *fmincon* from the Matlab® *Optimization Toolbox* and the MATPOWER solver *MIPS*. Harbo (2017b) may be visited for details except on the MIPS method.

5.2 The SQG method

The implementation of the SQG method for AC-OPF was started briefly in Harbo (2016) and done fully fledged in Harbo (2017b). The optimization engine for the method was granted the student from his professor by Alexei Gaivoronski, who has over several years and publications (eg. Gaivoronski (2016)) developed and implemented the tool in developed in Matlab® . The solution process is illustrated in figure 5.1.

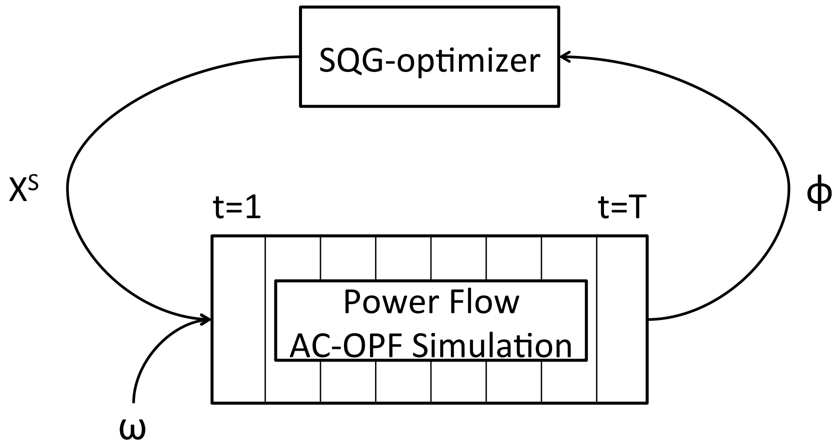


Figure 5.1: SQG overview

As the SQG iterates, it stars with some values for the variables X at iteration s, and runs the AC-OPF simulation with those and the random realizations ω for all T time steps. Then it returns the values ϕ, which the SQG optimization engine uses to calculate the new value for the variables X.

For more details on the SQG method, an algorithmic description of the SQG solution process, and information on how the solution parameters should be tuned, please see Harbo (2017b).

5.3 Solving stochastic, multistage AC-OPF with SDP

To have something to compare the SQG method to, a SDP solution was implemented for the problem at hand. Again, the reader is referred to Harbo (2017b) for the algorithmic details of this approach.

5.4 Solving stochastic, multistage AC-OPF with an Evolutionary Algorithm

To compare our solution method for modelling and optimize energy storage in and electric grid, we develop an Evolutionary Algorithm that uses an underlying AC-OPF simulation in evaluating the fitness of the individual solutions.

An algorithmic description for the Evolutionary algorithm employed is found below:

5.4.1 Evolutionary Algorithm for solving the AC-OPF problem

Algorithm 1 Evolutionary Algorithm for AC-OPF

```
Initialization:
    a) Set algorithm parameters.
    b) Initialize a fully random population, P.
    c) Evaluate all individuals in P according to the fitness function.
    d) Sort the population according to their fitness values.
for Current generation ¡ Max iterations do
    Determine the probability p_i of x_i being a parent according to equation 3.11.
    for Number of reproductions do
        Randomly select two parents form P, according to the probabilities of p.
        Create two offspring according Haupt's method as written out in equation 3.13.
    end for
    if Mutation criteria is met then
        Mutate the population by changing random elements to random numbers according to predefined algorithm parameters.
    end if
    Evaluate the fitness of the newly generated individuals.
    Sort the population according to their fitness values.
    Keep the first k individuals, discard the rest.
end for
```

5.5 Cases

This thesis has implemented the SQG method for the 24 bus test case from IEEE, and started implementing it for the 118 test case. Additionally, it used the 9 bus IEEE test case, and a constructed 4 bus case introduced in Harbo (2017b), and introduced some new methods to these as well.

Specific data for the cases is found in appendix B.1.

5.5.1 4 bus power system

In the 4 bus case - a network of four nodes - the problem is as simple as possible. As such, we let both the cost of wind energy, but also the battery cost be null. The case is presented in figure 5.2.

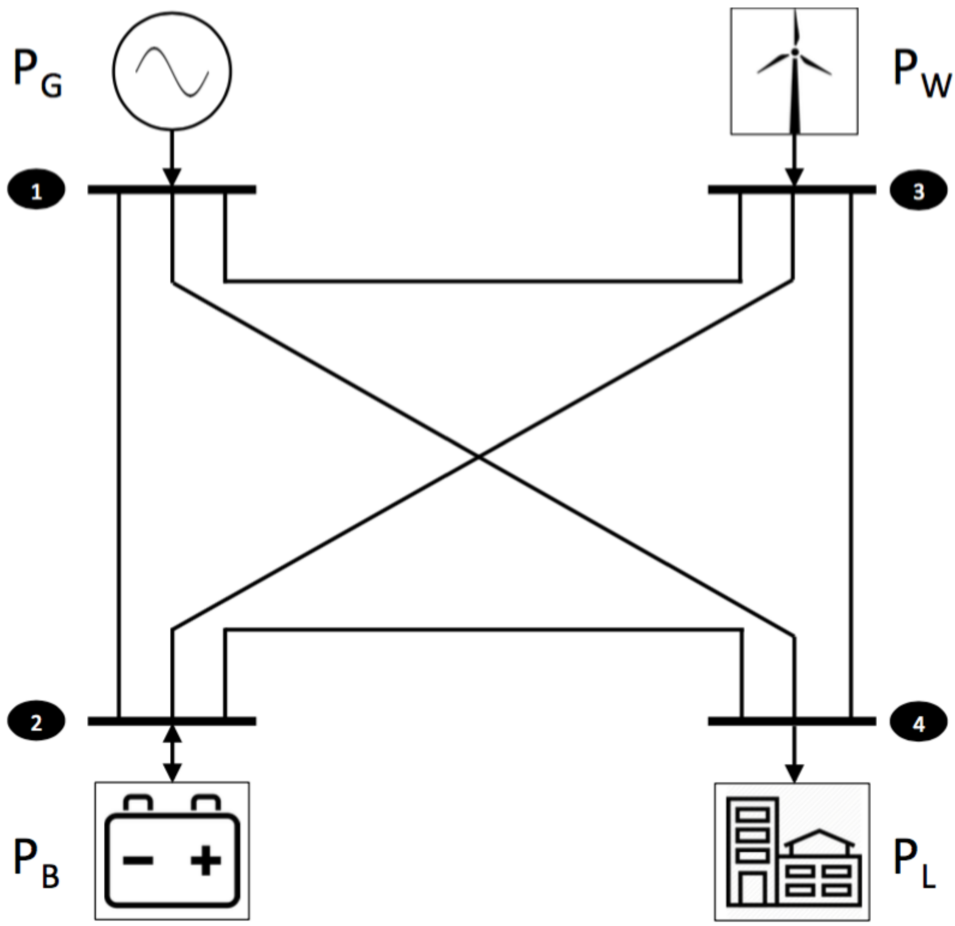


Figure 5.2: Illustration of 4 bus case.

Each power bus, or busbar, is in this case connected to a power generator, consumer or storage.

- **Bus 1** is connected to a generator (e.g. coal, gas or stiff grid), and is the slack bus for the model. The cost of this generator in this case is displayed in 5.3
- **Bus 2** is connected to the energy storage, and how much to charge or discharge is the decision to be made for the problems we consider.
- **Bus 3** is connected to a wind park. It produces wind energy, but the exact amount is uncertain. It is considered to be given as a random number in a uniform distribution.

- **Bus 4** is connected to households and industry consumers, which represents the power demand in the system.

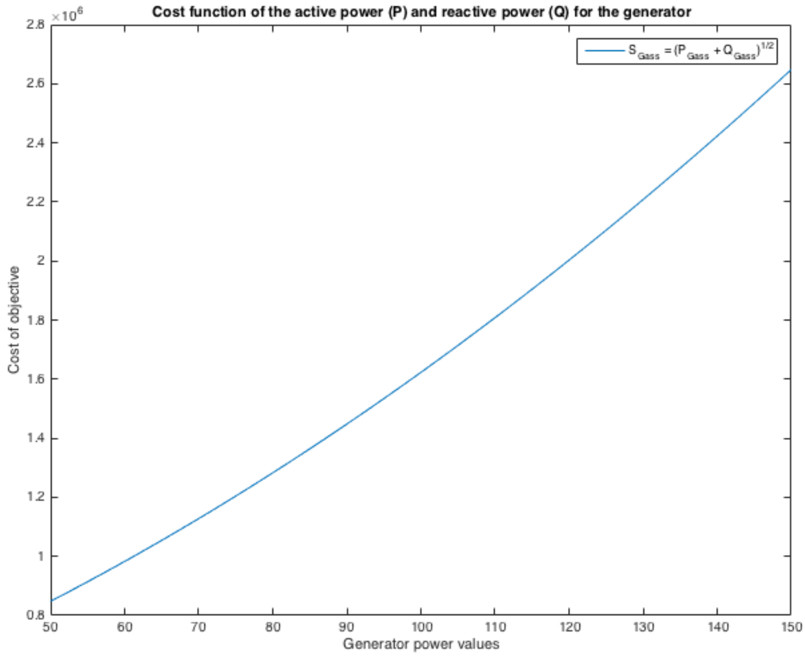


Figure 5.3: Cost function of the 4 bus case

The data for the specified case can be found in the tables **??**, B.2 and B.3 in appendix B.1.1.

5.5.2 9 bus power system

For the 9 bus case we let three of the buses be transformers like in the original case, three of them be power generating buses, two be load buses at the last be the energy storage E_{ST} that may discharge (or charge) power to the grid with $P_{Battery}$. Of the generating buses, one is a slack bus with a connected to an external grid P_{Grid} that will compensate for what ever energy demanded that is not met by the production at any given instance. The two others will be a wind generator park, P_{Wind} as before, and a PV plant P_{PV}, which both will have stochastic production. As for the loads, we let one represent households $P_{Household}$ and the other businesses or industry $P_{Business}$, both with stochastic demand.

The data for the 9 bus case can be found in the tables B.4 and B.5 in appendix section B.1.2.

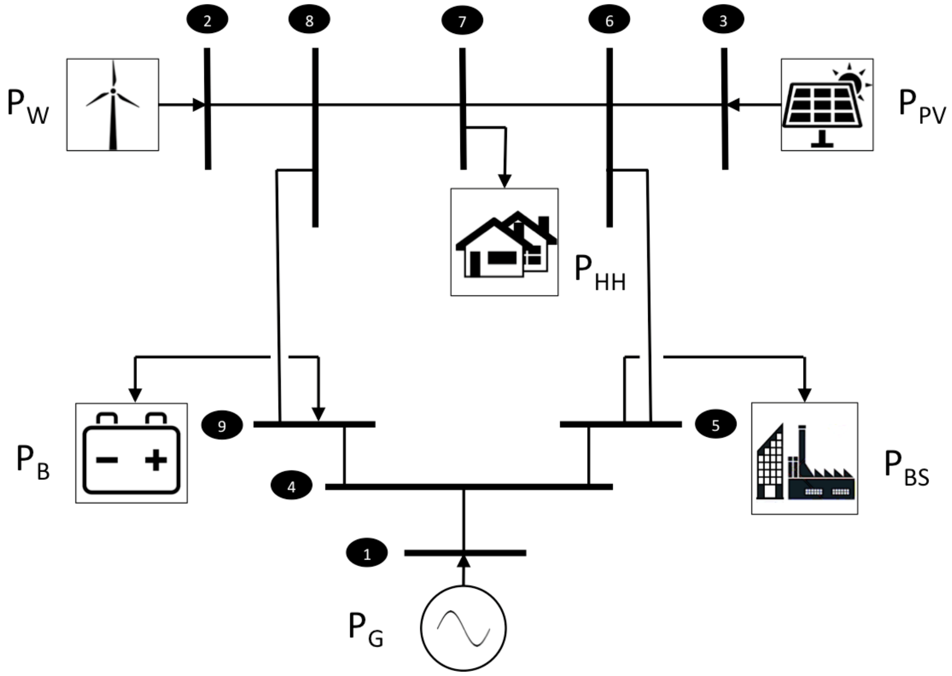


Figure 5.4: Modified IEEE nine-bus system.

For this case the mean wind and PV generation follows the curves presented in figure 5.6, and the mean power load from the Business and Household is presented in 5.6. For simplicity, the values they may realize to for each time step are all assumed to have a uniform distribution around their mean for that time step. The range of these distribution has for this case been chosen as ±25% for wind, ±10% for PV whenever the mean is more than null, ±5% Business and ±10% for Household.

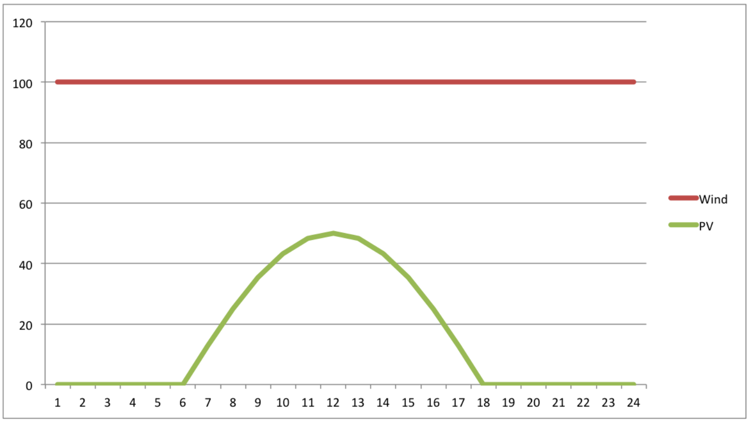


Figure 5.5: Mean Production Profiles for Stochastic Generation

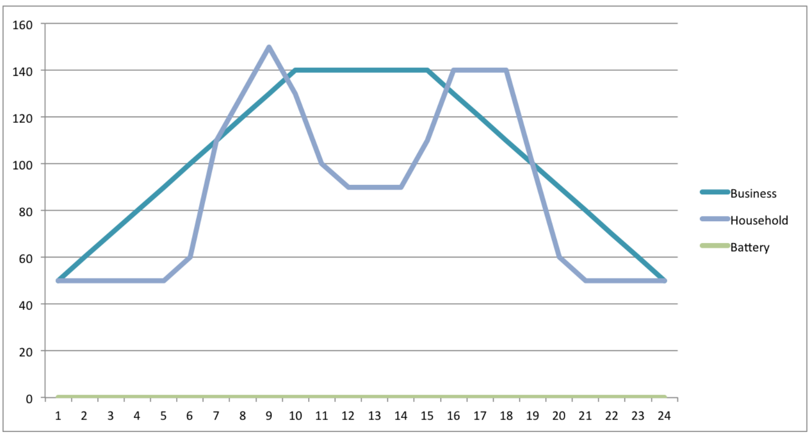


Figure 5.6: Mean Demand Profiles for Stochastic Loads

Note here, that neither the wind or the battery generates or consumes reactive power $Q_{L,4}(t)$. Thus the generator has only to supply reactive power $Q_{G,1}(t)$ according to the demanded reactive power and the phase changes across the network cables during the specific solution. The wind power generated is what is under stochastic influence in this case, and is given as by a uniform distribution with a specific range around a given mean wind energy for the time step.

5.5.3 24 bus power system

One goal for the SQG method, is for it to be able to solve the optimal storage of energy in bigger, complex power networks. Hence it is natural to test our method on an slightly larger electricity grid. The data was first published in Grigg et al. (1999), and can be readily found on the internet. The data for the 9 bus case can be found in the tables B.6 and B.7 in appendix section B.1.3.

Where the 9 bus system still is fairly simple, the 24 bus system as seen in figure 5.7 has a more diverse topography with different voltage zones and lines with diverse interconnections and transfer constraints. There are 9 generators dispersed throughout the system, serving 16 different loads.

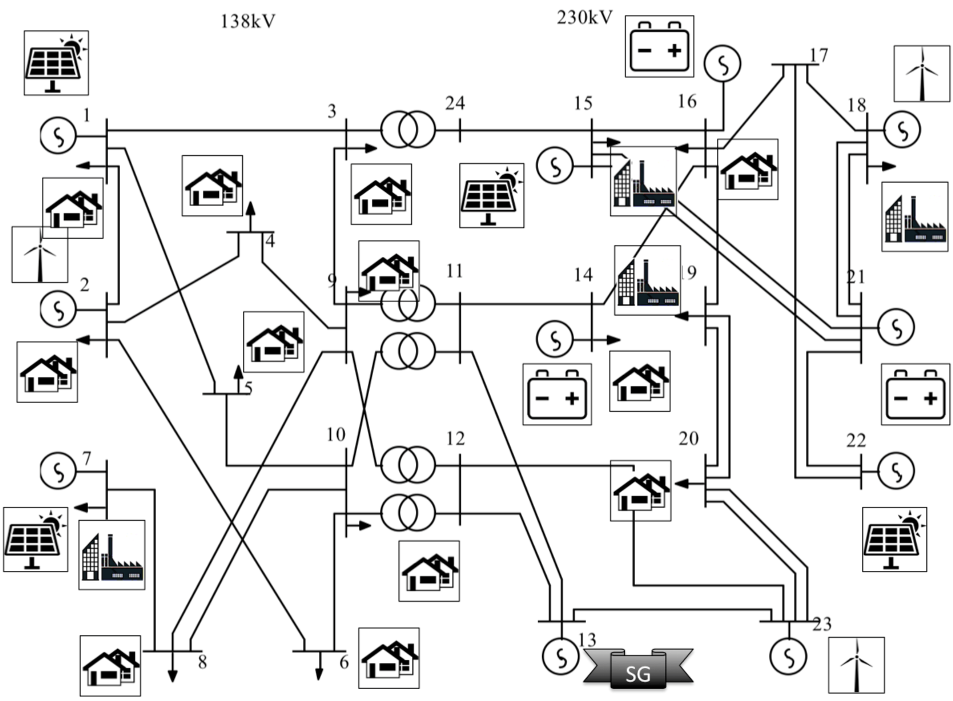


Figure 5.7: 24 bus system

In a similar way as with the 9 bus system, the IEEE system has been adapted to serve the needs for the tests we wish to execute. Each generator has been classified as either a PV plant or a wind farm, and every load as either a residential area or an business or industry consumer. Based on this, for each time step, we adjust the production and consumption according to their respective profiles as illustrated in figure 5.8. Additionally, the loads, and both the RES of wind and PV have their produced energy defied as stochastic variables produced within a uniform band of uncertainty around these profiles.

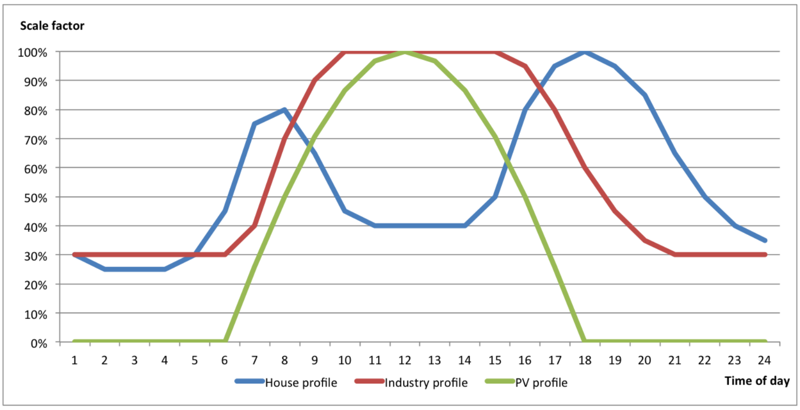


Figure 5.8: Scaling of the time dependent production and consumption

One of the generators has also been classified as a slack bus connecting to a stiff grid (indicated by the SG label), with which the power system can exchange energy. Hence, most of the time, the RES will produce an insufficient amount of energy, and the system has to import energy from the grid at the current electricity price. Hence, the goal of the optimization is to use the battery in such a manner that one is able to maximize the usage of the available renewable energy.

5.5.4 118 bus power system

As mentioned in appendix section B.1.2, the data for the 118 bus case has been left out of the thesis report as it is simply not worth the space on paper, but may be found cited in the aforementioned appendix section and also readily on the Internet. However, to illustrate the complexity of this system, the figure 5.9 is provided. As the system has over 50 generators and over 150 lines, one might easily understand that the amount of equations needed to model, simulate and solve this problem is quite substantial. see IEEE (1973),

Spent quite some time without produceing good res

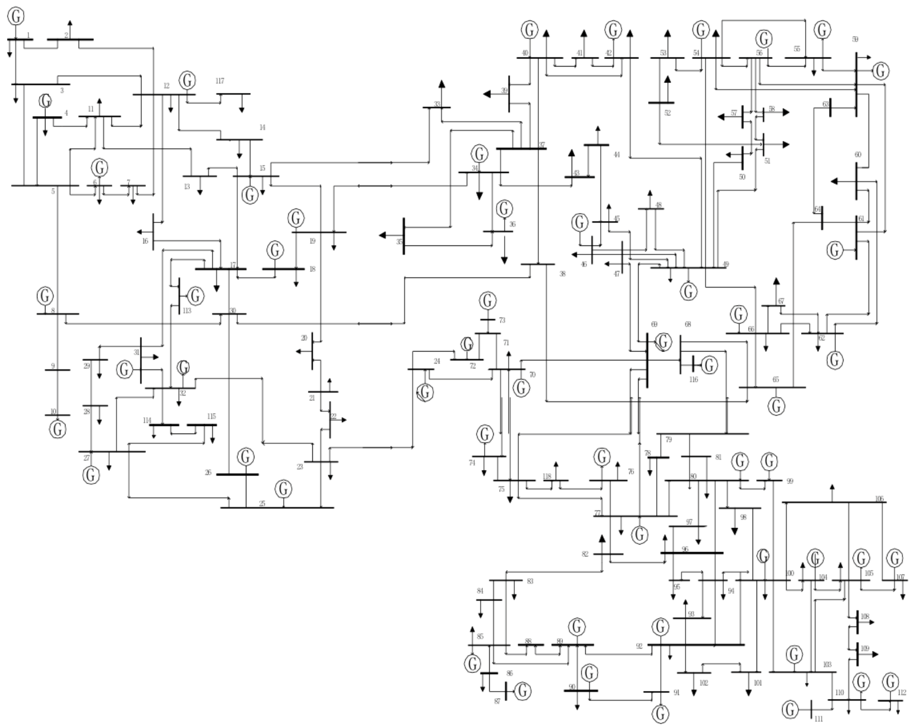


Figure 5.9: 118 bus system

5.5.5 EV charging simulation with ABM

To couple the AMB of EV to the multistage AC-OPF model, we use the data generated by the EV simulations as input to the grid model. We here include two of the charging strategies discussed in Harbo (2017a), dumb charging and charging based on SOC and price.

Figure 5.10 depicts the total demanded power on the y-axis from the grid during 10 days, with EVs using the dumb charging strategy. The x-axis shows the time of day, whilst the different series are the different days.

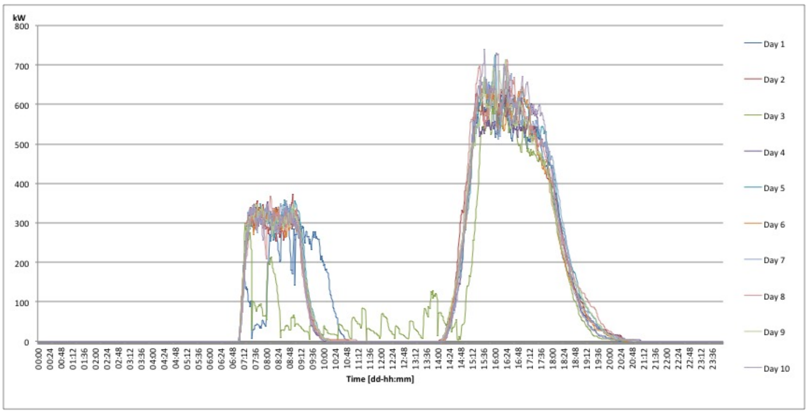


Figure 5.10: Total power demand from 1500 EVs during 10 days with dumb charging

The graph in figure 5.11 shows the total demanded power with the agents considering their SOC and the power price before deciding probabilistic whether or not to charge.

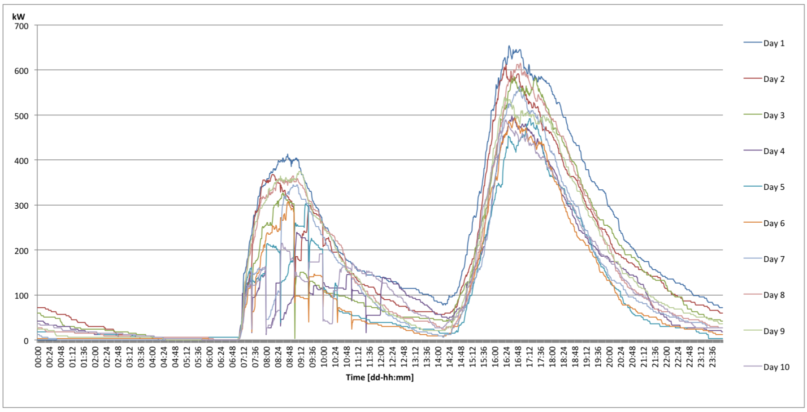


Figure 5.11: Total power demand from 1500 EVs during 10 days with charging strategy based on SOC and price and P_{max}=4500

5.5.6 Implementation of the cases of Stochastic, Multistage AC-OPF using SQG

In implementing the AC-OPF model, much of the foundation was layed in Harbo (2017b). Yet as a development of the work done here, the student implemented the *MIPS* solver for the basic model as well, spending a considerable amount of time configuring this.

Additionally, a stiff grid was included as the slack bus in the AC-OPF solution for all except the simplest 4 bus case, with a time varying spot price as presented in figure **?**.

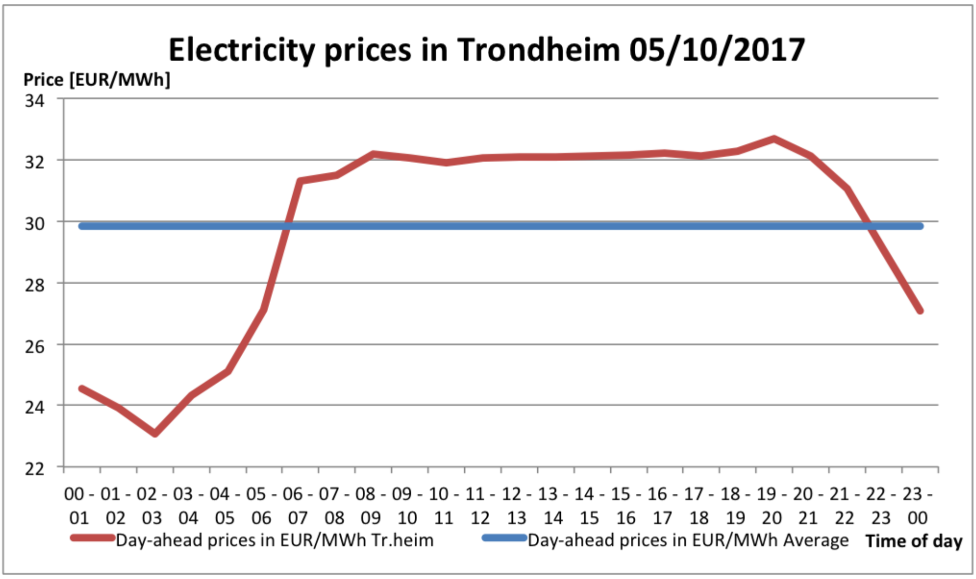


Figure 5.12: Electricity price determining cost of each time step, from Nordpool (2017)

The implementation of the 24 bus case has been done for several variations, such as more than one battery and EV charging. For the EV cases to be included as input to the simulations, the data presented in figure 5.10 and 5.11, we need to have the data aggregated from a minute level to an hour level. To do this, we add up all values within each hour across 10 days, and divide them by the corresponding number of minutes. The resulting input is displayed in figure 5.13 and 5.14.

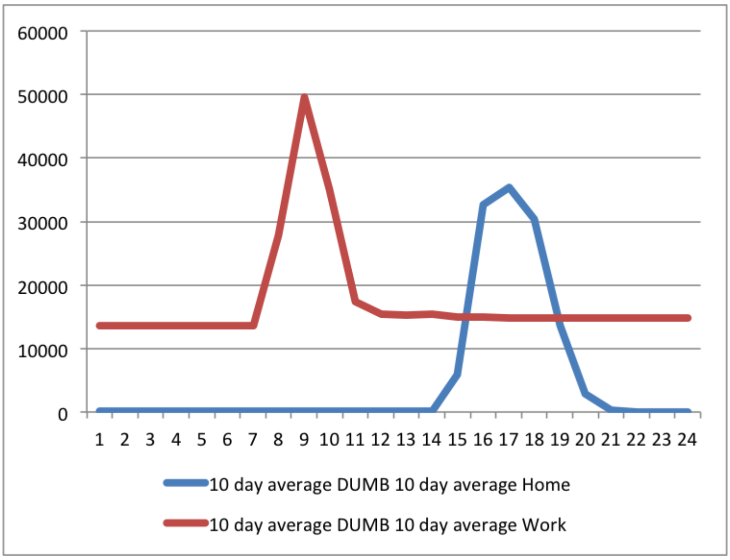


Figure 5.13

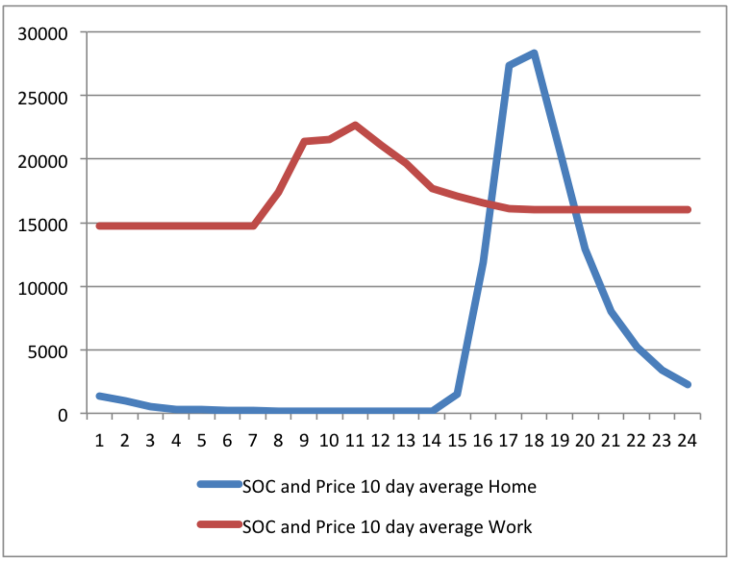


Figure 5.14

Time was also spent working on an implementation for the 118 bus case, but this effort was considered too time consuming after a while. However, the code that has been implemented for this will also be delivered with the thesis in its online hand-in.

Chapter 6

Results and discussion

The most important results from the thesis work is presented it this section. A main goal has been to compare the SQG method with an SDP and Evolutionary Algorithm for the same problem, to asses the strength and performance of the different methods. Another has been to illustrate futher valuable analysis empowered by the SQG method. As such, both these aspects will be provided. As will discussions not only on the stand alone observations from single results, but also on the comparative level.

For all simulations and computations a MacBook Air has been used. It dates from mid 2013 with a $1,3GHz$ Intel $i5$ core and $4GB1600MHz$ ram.

6.1 Comparative analysis

This section presents results from solving the stochastic, multistage AC-OPF using different methods, and compares their output and solution times. It starts with the 4 bus case, before moving on to the 9 bus and 24 bus case.

6.1.1 Solving the 4 bus case

Figure 6.1 shows a standard profile for the generator values. For different methods, the only thing differing is the values for the generator in blue, that comes as a result of how much the battery is charged. Additionally, for simulations with randomness, the value for wind in the yellow line also changes from iteration to iteration.

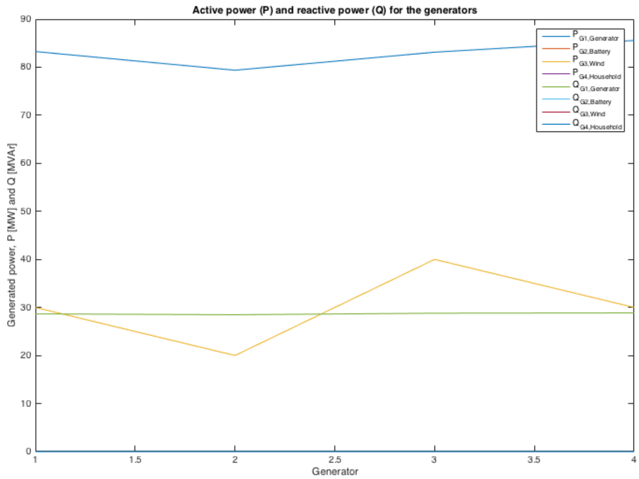


Figure 6.1: Typical generator profiles for 4 bus case

When using the different methodologies for solving this problem, their high-level performance is seen in table 6.1. As can be assessed from the table, the SQG method used here, does not reach the same solution as the other methods in this case. A reason for that may be that the gradient retrieving function used is not the best for the type of problem, and may be updated. In terms of time spent calculating the answer, we see that the SDP uses more time than the other two methods. In fairness, the SQG and Evolutionary algorithm solution times is determined by the number of iterations they run. However, for the Evolutionary algoritm at least, we see that it is able to come within 2% of the exact, yet discretizised, solution of the SDP metod, in about a third of the time.

6.1.1.1 SQG method

For the SQG method, figure 6.2 below presents the loads in the system and the battery charging profile for this case. The other figure presented here, figure 6.3 shows the development of the function value using this method. The development of the variables in this case may be found in appendix C.1 in figure C.1 and C.2.

Table 6.1: Solution time for 4 bus case with different methods

Method	SQG using *fmincon*	SDP	EvAlg
Solution value	16 865	12 000	12 273
Solution time [s]	445	1317	264

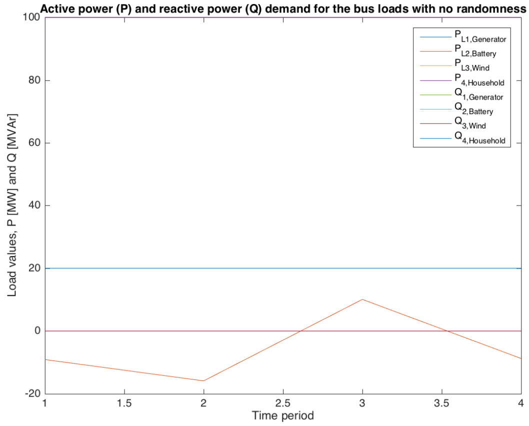


Figure 6.2: Battery charging with direct gradient for 4 bus case

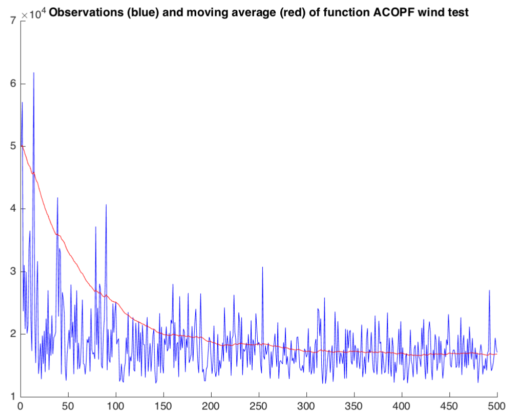


Figure 6.3: Function value development for SQG solution of 4 bus case

In the first figure, we see that the battery is discharged in the beginning of the simulation, but charges up in the third time step. The second figure shows the convergence to the approximated value of the objective function in red, with the different realizations in blue. This is produced using 500 iterations of the SQG process with the *fmincon* solver.

6.1.1.2 SDP method

Figure 6.4 displays the solution of the same problem tackled by the SDP method. For this graph, a discretization level of 51 has been used. It is fully possible to provide even more detailed results. A run was done with double level of discretization found in figure C.3 in the appedix C.1, but taking about 10 hours to solve. The transition probabilities, and corresponding transition matrix, or this case may be found in appendix B.1, in figure B.2.

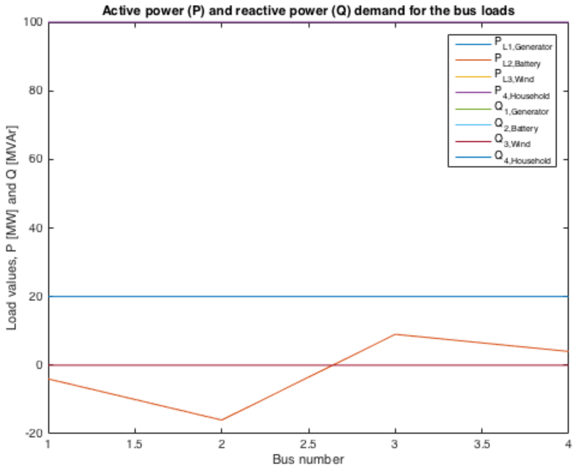


Figure 6.4: Battery charging with SDP for 4 bus case

The overall picture here is the same as for the SQG case, that the battery has its lowest charge in time step 2 and highest in timestep 3. Yet, the battery values at the end and beginning are different here.

6.1.1.3 Evolutionary algorithm method

As for the Evolutionary algorithm done for the 4 bus case, a population of 20 individual was used, and iterated though 30 cycles of mating and fitness evaluation. Figure 6.5 presents the corresponding generator values, 6.6 the corresponding load values and 6.7 the development of the population in this case. As explained earlier, the generator values are a direct result of the decided charging profile of the battery. Hence they are not included for all the different methods, but brought in here for the sake of a few comments below.

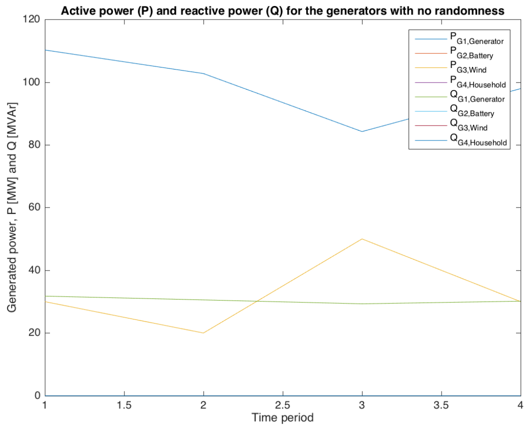


Figure 6.5: Generator values for EvAlg solution for 4 bus system

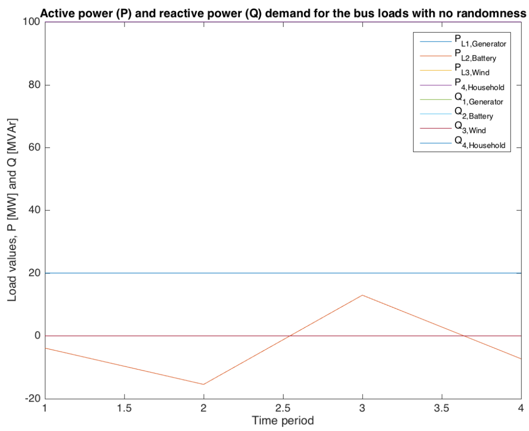


Figure 6.6: Load values for EvAlg solution for 4 bus system

In the first two figures on the load and generator results shows that battery overall profile is much the same as for the SQG and the SDP solution. It also worth noticing that the charging profiles in figure 6.6 is chosen to be similar to the average wind profiles in figure 6.5.

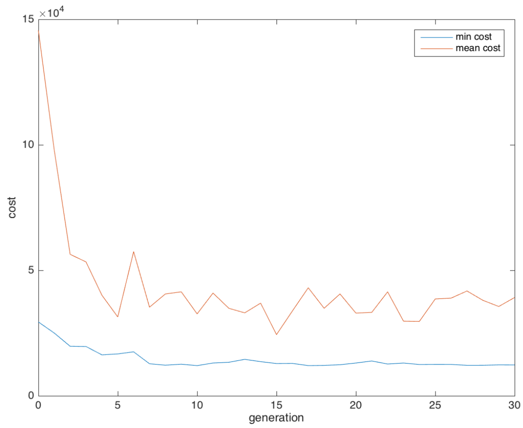


Figure 6.7: Population improvement during algorithm iterations

The development of the population shows that the Evolutionary algorithm relatively quickly finds a relatively good approximation to the solution. As the population mutates, the average function value of the population fluctuates, and proposes new candidates for possible solutions.

6.1.1.4 Comparison and discussion

In the end, we see that the charging profiles in all cases are similar in that they imitate the expected wind values for the case. We also see that the evolutionary algorithm was able to find a fairly good solution quite quickly, whereas the SQG solution might have had problems with the gradient estimation function for this case.

6.1.2 Solving the 9 bus case

Figure 6.8 presents a typical profile for the system generators with no randomness in the 9 bus system solution. The red line denotes the mean wind production, which during the simulations are random. The yellow line shows the mean PV production, also realized as a stochastic variable in the simulations. The blue and the purple lines represents the generated active and reactive power from the gas generator for the specific solution this graph is extracted from.

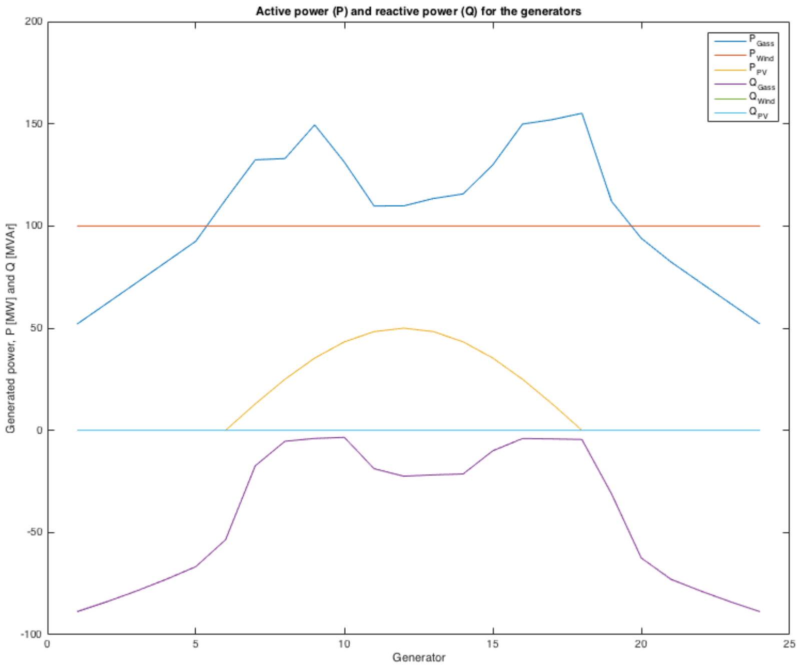


Figure 6.8: Generator values for SQG solution for 9 bus system

In table 6.2 shows the high level results of the results using the different techniques. It may be noted the authour did not go thorugh with a full simulation using the SDP approach even thouhg it was implemented. This was because the the number of iterations required for this exponentially more computing heavy method either proved to take too long (the run was cancelled after running for 20 hours), or simply draining too much memory from the computer used. Another thing to note here, is that the author also implemented the MIPS solver for the SQG approach.

From the table we see that the SQG solutions again does not get as low for their function value as the Evolutionary Algorithm, indicating that they again have problems with their gradient function. They also take quite a bit more time that previous implemented

Table 6.2: Solution time for 9 bus case with different methods

Method	SQG using *fmincon*	SQG using *MIPS*	SDP	EvAlg
Solution value	203 480	166 450	N/A	56 430
Solution time [s]	6 415	12 958	N/A	8 495

versions of a similar problem. In Harbo (2017b), the 9 bus case was implemented not using a varying spot price, but having generators with cost functions, which the code was able to solve for 250 SQG iterations in 450 seconds.

A reason that may explain both the excessive time spent and the poor results, is the particularity of the stiff-grid case. The use of a simple spot price function means that the derivatives of the generation at each time step becomes a constant, and the second derivatives in the hessian becomes 0. Hence, the solvers, and especially the MIPS method using the hessian operator actively, may not only have difficulty calculating the gradient, but also ha convergence issues because of the singularity in the Jacobian matrix, see Baker et al. (2012) and Baker et al. (2013) for more details. Hence, the cost of the objective function should either be changes in future work on this method so that the second order derivatives exist.

6.1.2.1 SQG method

In the following, more detailed results using the SQG method for the 9 bus case are presented.

Fmincon

The figure 6.9 shows the energy loads and charging profiles resulting from the SQG method using fmincon when forcing the system to use all RES that are available.

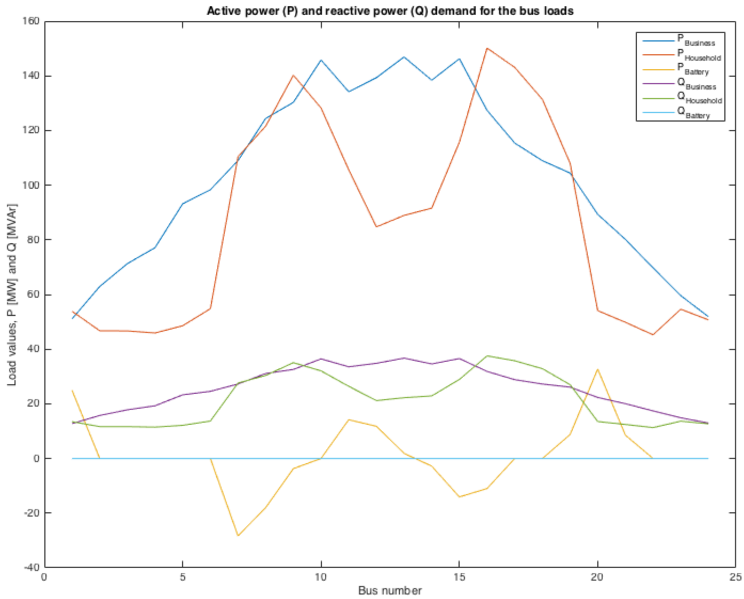


Figure 6.9: Generator values for Gradient SQG solution for 9 bus system

The results seem fairly logical, with the battery discharging during time of excessive demand, and using times of lower demand to recharge.

MIPS

Figure 6.10 displays the loads from using the MIPS solver for the SQG method. The results make some sense, charging at midnight when demand is low. Yet in the middel of the day, the charging and discaring seem somewhat random.

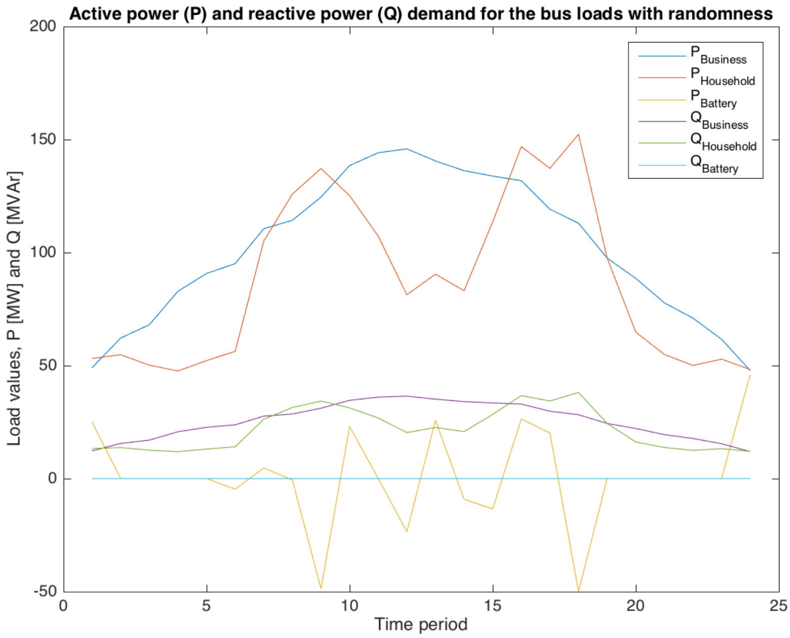


Figure 6.10: Generator values and available power for MIPS SQG solution for 9 bus system

Figure 6.11 and 6.12 shows more details behind the results. The first figure displays how much of the available renewable energy is take usage of, when the solution is not forced to use all. As we can see the solution technique here is not able to utilize all the renewable energy, which here is free and increases the cost of the solution. The second figure shows the imported power from the grid and the charging profile of the battery put together. Further figures from this case showcasing the convergence of the variables are found in C.1.

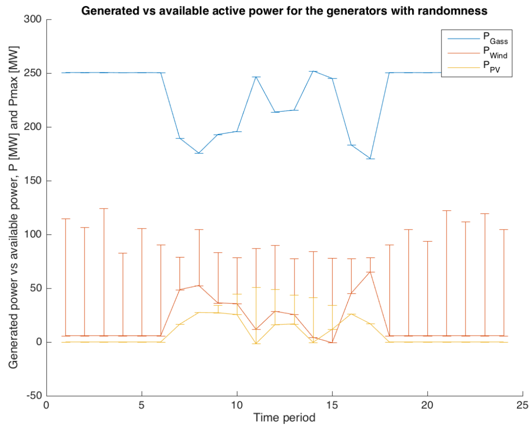


Figure 6.11: Generator values and available power for MIPS SQG solution for 9 bus system

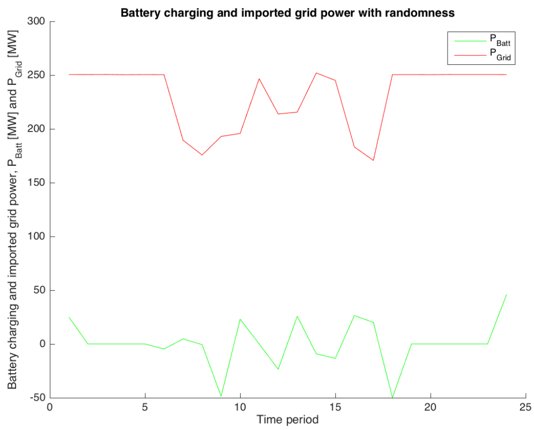


Figure 6.12: Battery and grid exchange for MIPS SQG solution for 9 bus system

6.1.2.2 Evolutionary algorithm method

The Evolutionary algorithm deployed here used a population of 40 individuals, 100 life-cycle iterations and was solved in 8 495 seconds. The figure 6.13 shows the corresponding load values that are given as a solution. Yet, we observe that the resulting charging profile does not seem very logical, but rather random. Figure 6.14 shows the development of the populations. This simulation shows little development in terms of convergence, with the mean just slightly creeping down, and the minimum actually increasing. Thus, it might also be the case that there is something wrong in how the objective function calculates individual fitness.

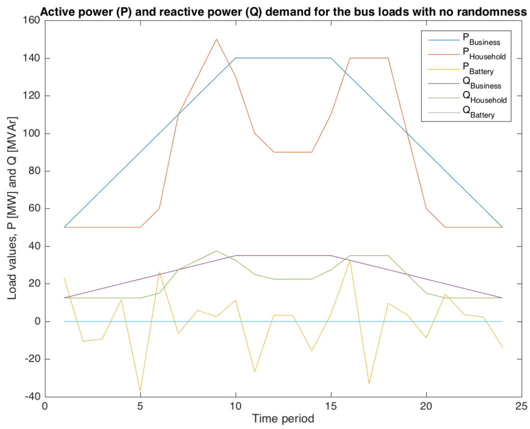


Figure 6.13: Load values for EvAlg solution for 9 bus system

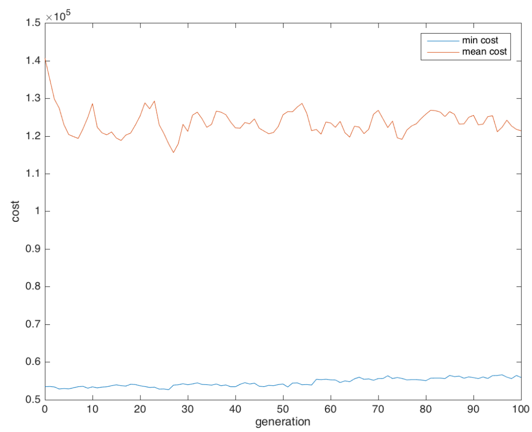


Figure 6.14: Population improvement during algorithm iterations

6.1.2.3 Comparison and discussion

When taking the results from the SQG method and the Evolutionary algorithm method together, it is evident that the stiff grid case implemented has not been the easiest to solve for the current set up with the chosen techniques, and more time would needed to be spent in understanding how to attack the formulated problem wih these techniques here.

6.1.3 Solving the 24 bus case

A typical solution of the 24 bus case is displayed below with the generator values under random influence in figure 6.15 and the load values also subject to randomness in 6.16. This case has thus far only been implemented for the SQG method, but for both the *fmincon* and the *MIPS* solver.

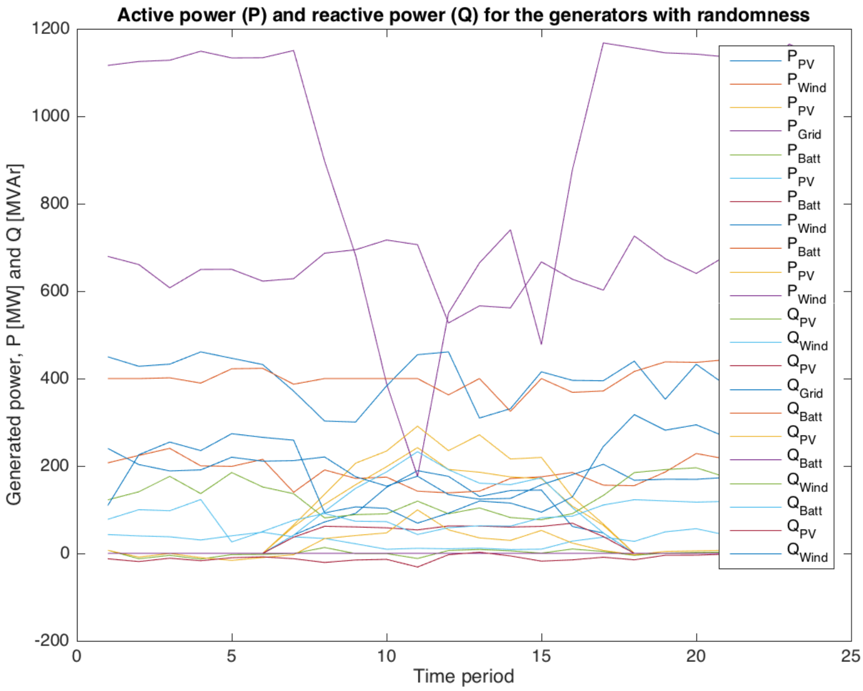


Figure 6.15

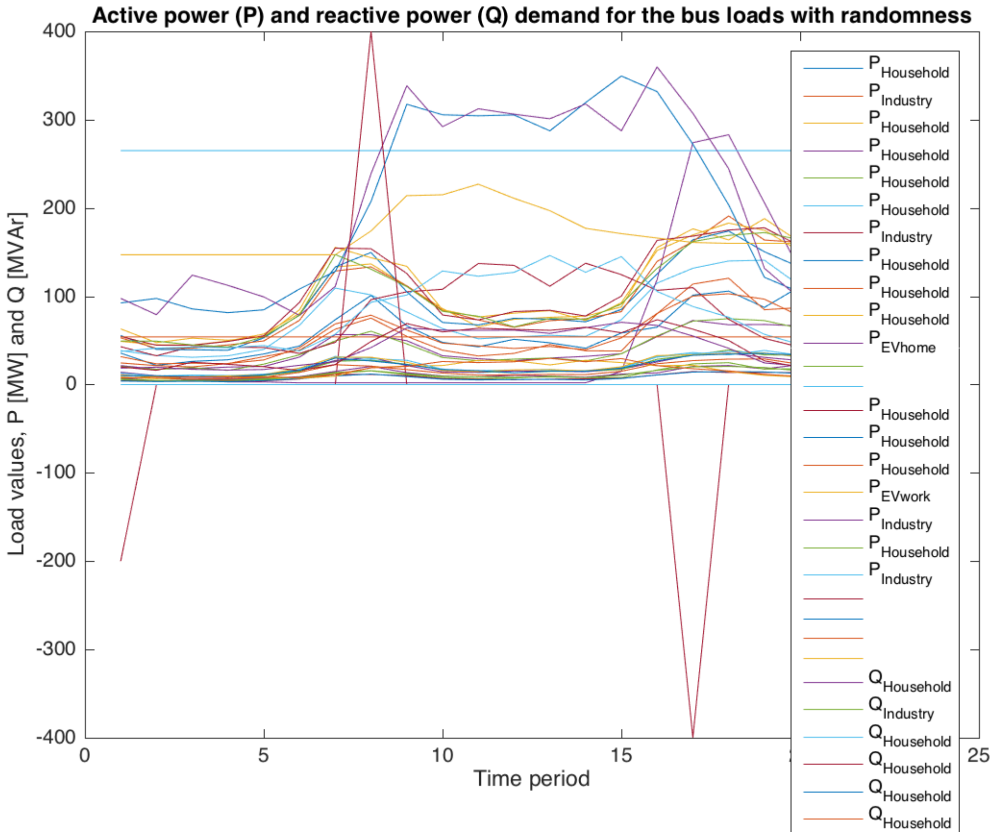


Figure 6.16

Table 6.3 shows the solution times and results for the SQG method for with the two different solvers.

Again, the log from running the models show that the MIPS implementation again has some problems with convergence, which may explain why it has a greater solution value. However, now it outperforms the *fmincon* solver with respect to time, which is what is expected. From small tests on simpler models, the *MIPS* solver should be substantially faster than the the *fmincon* solver when it is to solve bigger problems as it makes use of spare matrix mathematics among other techniques.

Table 6.3: Solution time for 24 bus case with different methods

Method	SQG using *fmincon*	SQG using *MIPS*
Solution value	518 410	676 510
Solution time [s]	9 283	3 209
Iterations	25	25

6.1.3.1 FMINCON

In figure 6.17, the results from the 24 bus solution with the *fmincon* solver is presented. The bars atop the lines show how much of the available power is utilized in the 24 bus case. We see that most of the power is used at many time steps, with some exceptions that might be caused by the randomness in the specific realization.

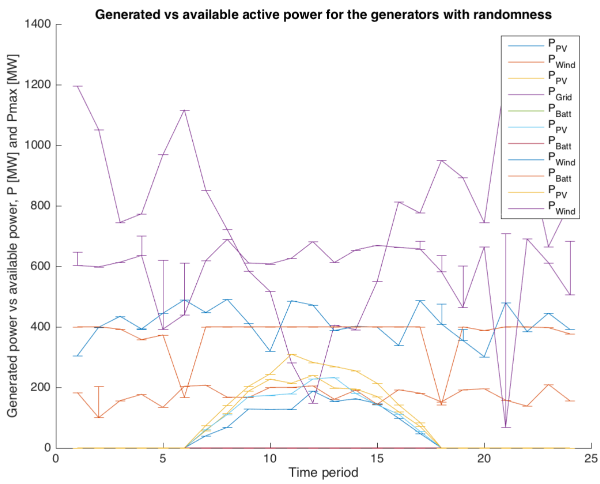


Figure 6.17

Figure 6.18 shows the corresponding loads also with randomness. Here the charging profile can also be spotted as the negative load when demand starts to rise. Figure 6.19 shows the charging profile juxtaposed towards the import from the stiff grid, where the charging and discharging to some small degree help even out the grid import.

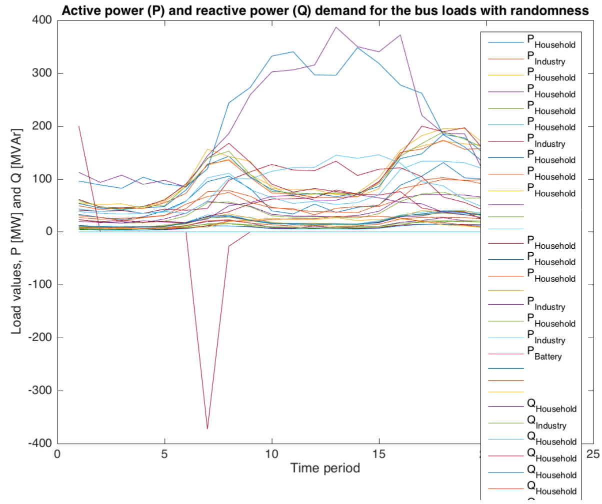


Figure 6.18

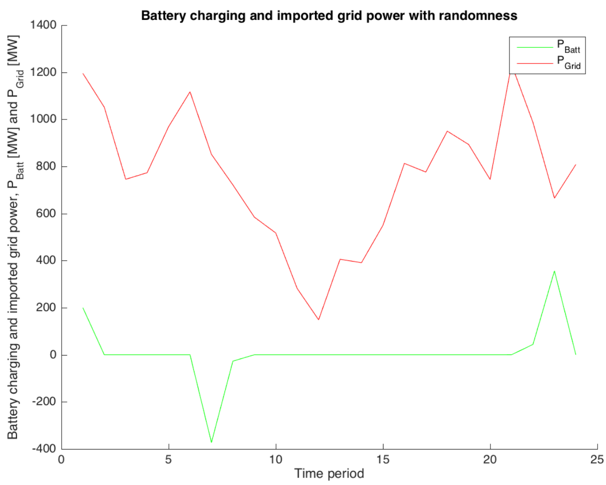


Figure 6.19

6.1.3.2 MIPS

In figure 6.20 we see the generation profiles for the 24 bus case with the *MIPS* solver implemented. We may notice that also here, much of the free renewable energy is use. Yet not as much seems to be utilized as in the *fmincon* solution, which may be how the latter has a better resulting function value. Figure 6.21 again show the loads, but how with no randomness in the demand functions. We again see the battery profile with its negative discharge.

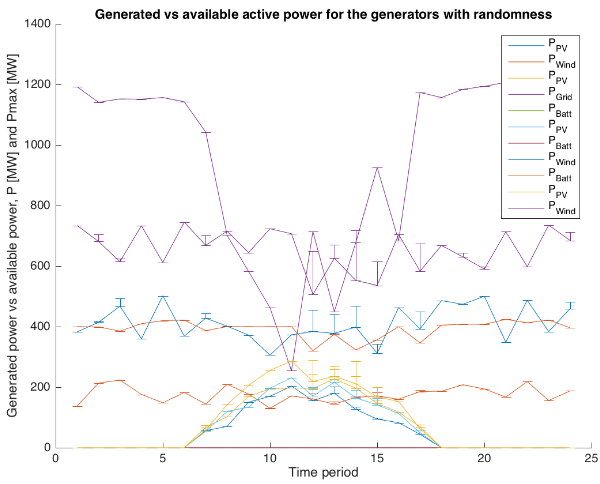


Figure 6.20

Figure 6.22 shows the spot price juxtaposed against the function value of each step (with the axis here should be reversed), which shows why the solutions charge around step 7 when the price increases.

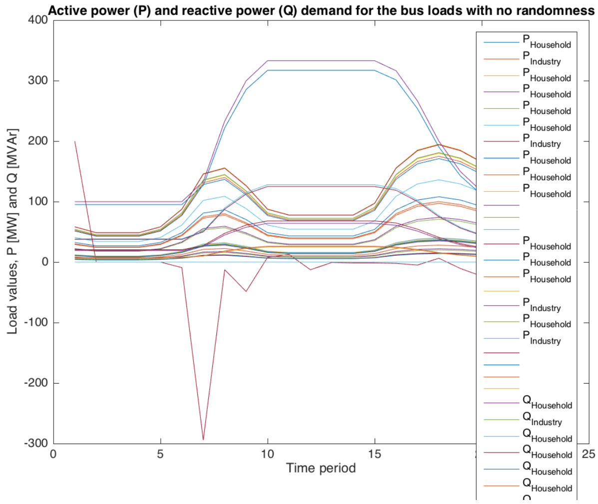


Figure 6.21

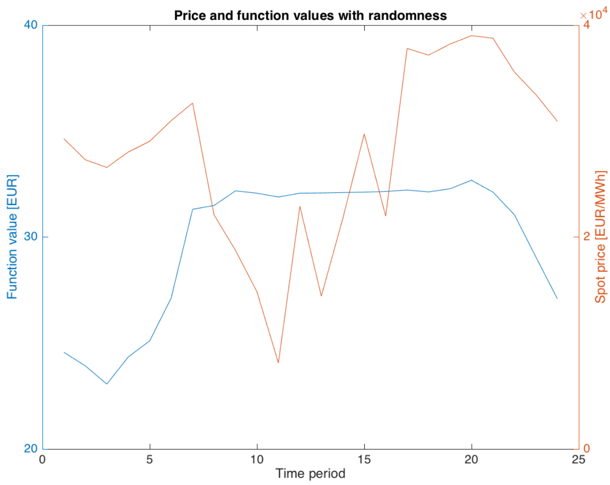


Figure 6.22

6.1.3.3 Comparison and discussion

From the results of the 24 bus case, it is clear that the solvers was better able to extract and utilize the available power from the renewable resources. It is also evident that the *MIPS* solver improves the solution times even more, but again have some convergence issues when the second order derivatives in the hessian are null and the Jacobian becomes singular.

Comparing these results to the ones from the 9 bus case, we also see that the Evolutionary algorithm may be a more powerful technique for smaller systems, whereas the SQG method is more adept at bigger systems. This may be supported by the intuitive fact that for a smaller solution space it should be easier to find a good solution with random search, but that for a bigger solution space it pays off to take time to calculate an analytical measure of some sort that gives guide for how to improve the solution with the iterations.

6.2 Extended analysis of 24 bus case with SQG

In Harbo (2017b), the author analysed how a different range for the random variables and different levels of battery storage affected the solution of the 4 and 9 bus cases using the *fmincon* solver. As such, this type of assesment is left out of this thesis.

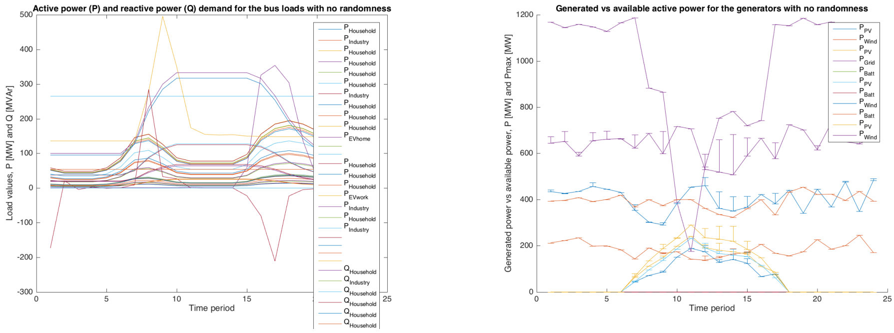
In this thesis we instead to to additional assessments, one on how the inclusion of electrical vehicles may be added to the model, and the other how the model may include several batteries in its solution method.

6.2.0.1 Valuating flexible EV load

Here we model how additional demand from EVs affects the grid. As a first step, this is only done by changing two of the demand loads to be an EV load specified from the ABM simulation. One for household charging one for city charging.

Dumb charging

Figure 6.23 presents the loads and generator values of the 24 bus case with EV profiles subject to dumb charging. We see that the EV demand spike at time step 8, means that the battery charges the step before.



(a) Load values

(b) Generated and available power

Figure 6.23: Results from 24 bus case solved with inclusion of dumb charged EVs

Figure 6.24 shows the objective function of the solution. As evident from this, the solution has yet to converge to a optimal solution, indicating that these simulations may need more time and tuning in order to produce better results.

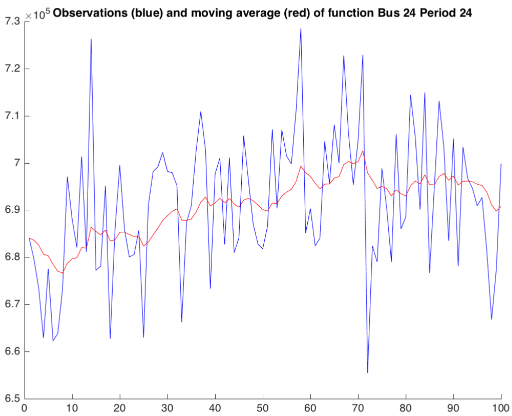
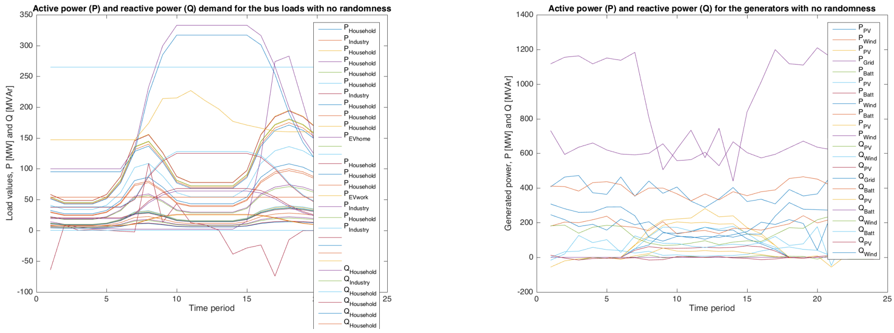


Figure 6.24: Objective function values with dumb charging

Charging regulated through SOC and Price

Figure 6.25 presents the loads and generator values of the 24 bus case with EV profiles that are affected by SOC of the vehicles and the power price in the system. This time the charging is less abrupt, and the battery does not have the same extreme movements.



(a) Load values

(b) Generated and available power

Figure 6.25: Results from 24 bus case solved with inclusion of EVs charged based on SOC and Price

Figure 6.26 shows the objection function values for the SOC and price case. Again, the function needs more time and tuning to converge. However, the interesting part here is that both cases hover around the same value range, indicating that there is not too much difference from a cost perspective for the grid of including EV charging, if it has other measures to distribute the energy over time such as Energy Syste Storage.

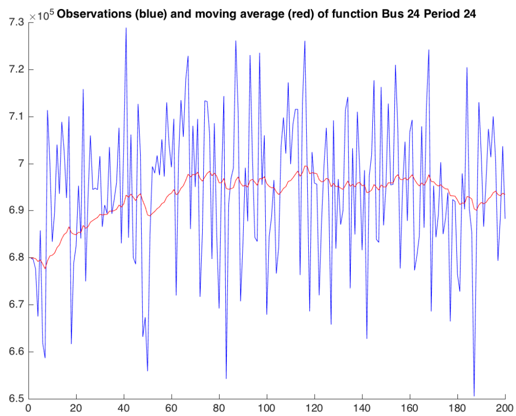


Figure 6.26: Objective function values with charging by impacted price and SOC

6.2.0.2 SQG method with several batteries

Another implementation was made for the SQG method with several batteries. It was ran for 100 iterations, and came up with a solution in 2600 seconds with a function value of

940 000. A selected charging value for each battery is displayed in the figures 6.27, 6.28 and 6.29 below. For this implementation, the lack of derivatives on the price function is quite crippling, with the current set up, as it is not possible to calculate the gradient directly using the method layed forth in section 4.2. Hence more time might be spent on investigating how a gradient can be calculated directly for this case with several batteries. An alternative is to use a finite differences approximation, as also utilized in Harbo (2017b), but this would take a much longer time to compute.

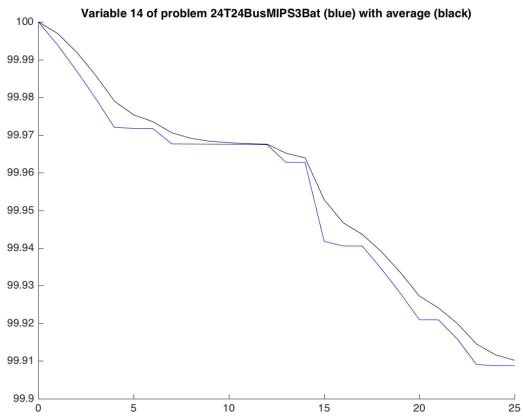


Figure 6.27: Objective function values with dumb charging

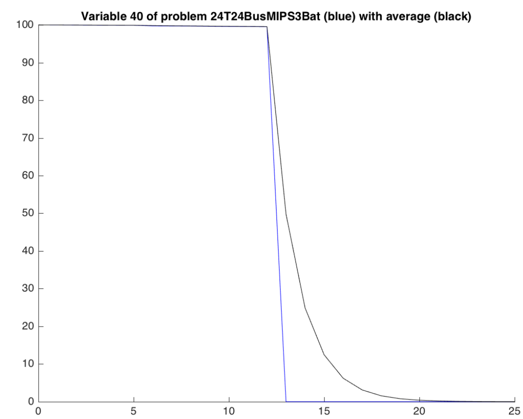


Figure 6.28: Objective function values with dumb charging

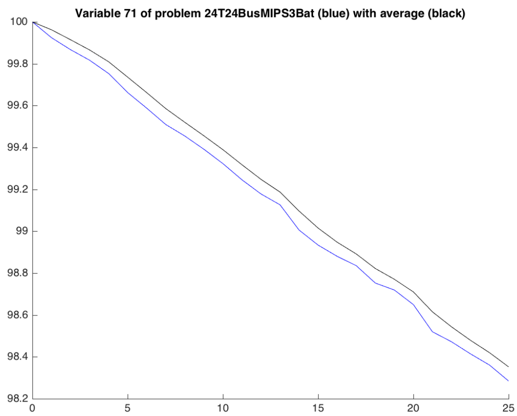


Figure 6.29: Objective function values with dumb charging

Chapter 7

Concluding remarks and future research

This chapter makes some concluding remarks about the work presented and its findings. It also suggests some areas of future research delving deeper or extending the issues and methods presented in this thesis.

7.1 Conclusion

This thesis shows that both the Stochastic-Quasi Gradient (SQG) method and the method of Evolutionary algorithms are applicable tools when analyzing the stochastic multistage AC-OPF problem presented in this thesis.

From the 4 bus case, we saw that both the SQG method and Evolutionary algorithm solved the optimization problem in less time than a more deterministic, or analytic, approach of Stochastic-Dynamic Programming (SDP). The Evolutionary approach came quicker to come to a good solution than the SQG for this smaller case. For the 9 bus case, we saw that the SDP method began to be too computationally heavy to use.

For the 24 bu case, we saw that the SQG method with the MIPS solver implementation especially, was able to be more competitive on solution time. It also alludes that the Evolutionary algorithm may be a more powerful technique for smaller systems, whereas the SQG method is more adept at bigger systems.

In chapter 2 we asked:

Which benefits and possibilities does the Stochastic Quasi-Gradient method offer in analyzing and optimize multistage power system operation with energy storage under uncertainty?

As an answer to this, we have found that the SQG method might be beneficial to use for bigger systems, whereas Evolutionary algorithms for smaller. We have also noted that the analytically information the solution method gives, may add additional information from its simulations. In conclusion, these methods may be used to analyze energy storage and help in bringing more variable renewable energy sources into in the electrical grid.

7.2 Further work

There are may ways this work may be continued forward, both in terms of methodology and in terms of application. Some of these are mentioned in Harbo (2017b) and has not been addressed here, but are included for the sake of completeness.

In terms of methodology improvements, some further work may consist of:

- Spend more time toning and testing the models presented here, especially the 9 bus case for the SQG method.
- Implement or develop an even faster solver of the AC-OPF model.
- Expand the AC-OPF model include more direct measures of stability, such as the N-1 criterion or continuation power flow.
- Expand the AC-OPF model to include a market coupling, for instance through bidding of power from the different sources to be solved by a market clearing algorithm.
- Do more research on how to calculate direct gradients for the SQG more appropriately,especially for the case with several batteries.
- Implement parts of the model using parallel programming, to speed up the solution process.
- Continue the implementation of a discrete, dynamic AC-OPF for the whole period to calculate the gradient for the solution in the SQG process.
- Implement a version where the dispatchable generation is the decision variable not charging, with more complex constraints for the SQG solver.
- Find new ways to implement the decision rule version of the model, to gain more insights from this approach.

In terms of application, ideas for futher work are

- Use the method in this thesis to valuate how much economic profit a energy grid operator can gain if it is able to exercise some control of when the Electric Vehicles in the grid are charging.
- Do a more in depth analysis of the connection between EVs and the grid, by letting the ABM and the optimization model here interact.
- Valuate control of household demand, through for instance price signals in a smart grid system.
- Include electricity price, and for instance water inflow to hydro power reservoirs, as a stochastic variables.
- Use real grid data to develop a full fledged model for an existing power system.
- Use real data to implement forecasts of the variable generation of variable generation and demand.
- Develop an online model that uses real time data to calculate the optimal level of a distributed battery in a micro grid for any time instance.
- Use the model to analyze how much wind can be included in a power system with and without batteries, given it has to satisfy the stability and safety limits of the current grid.
- Couple with the EV model

These suggestions are neither exhaustive nor exclusive, but may serve as inspiration the versatility of the proposed methods.

Bibliography

Aigner, T. (2013). *System Impacts from Large Scale Wind Power*. PhD thesis, NTNU Trondheim, Norway.

Baker, K., Hug, G., and Xin, L. (2012). Inclusion of inter-temporal constraints into a distributed newton-raphson method. *North American Power Symposium*.

Baker, K., Zhu, D., Hug, G., and Xin, L. (2013). Jacobian singularities in optimal power flow problems caused by intertemporal constraints. *North American Power Symposium*.

Beaudin, M., Zareipour, H., Schellenberglabe, A., and Rosehart, W. (2010). Energy storage for mitigating the variability of renewable electricity sources: An updated review. *Energy for Sustainable Development*, 14:302–314.

Becker, D. M. and Gaivoronski, A. A. (2014). Stochastic optimization on social networks with application to service pricing. *Computational Management Science*, 11:531562.

Bellman, R. E. (1957). *Dynamic Programming*. Princeton University Press, Princeton.

Birge, J. R. and Louveaux, F. (2011). *Introduction to Stochastic Programming*. Springer, New York, US, 2nd edition.

Bustos-Turu, G., van Dam, K. H., Acha, S., and Shah, N. (2014). Estimating plug-in electric vehicle demand flexibility through an agent-based simulation model. *IEEE Innovative Smart Grid Technologies Conference Europe (ISGT-Europe)*.

Cain, M. B., O'Neill, R. P., and Castillo, A. (2012). History of optimal power flow and formulations. Technical Report Optimal Power Flow Paper 1, Federal Energy Regulatory Commission, Washington DC, US.

Carr, J. (2014). An introduction to genetic algorithms.

Castillo, A. and O'Neill, R. P. (2013). Computational performance of solution techniques applied to the acopf. Technical Report Optimal Power Flow Paper 5, Federal Energy Regulatory Commission, Washington DC, US.

Corbetta, G., Ho, A., and Pineda, I. (2015). Wind Energy Scenarios for 2030. Technical report, European Wind Energy Association.

Crow, M. L. (2012). Computational methods for electric power systems. In Grigsby, L. L., editor, *Power Systems*, chapter 5. CRC Press, Florida, US, 3rd edition.

De Vita, A., Tasios, Evangelopoulou, Forsell, Fragiadakis, Fragkos, and Zampara. EU Reference Scenario 2016 Energy, Transport and GHG Emissions : Trends to 2050. Technical report, European-Commission, Luxembourg.

Erdal, J. S. (2017). Stochastic optimization of battery system operation strategy under different utility tariff structures. Master's thesis, NTNU Trondheim, Norway.

Ermoliev, Y. (1976). *Methods of Stochastic Programming*. (in Russian), Nauka,Moscow.

Ermoliev, Y. (1983). Stochastic quasigradient methods and their application to systems optimization. *Stochastics*, 9:1–36.

Ermoliev, Y. and Wets, R., editors (1988). Oxford University Press, Springer-Verlag.

Gaivoronski, A. A. (1988). Stochastic quasigradient methods and their implementation. In Ermoliev, Y. and Wets, R. J.-B., editors, *Numerical Techniques for Stochastic Optimization*, pages 313–351. Springer-Verlag.

Gaivoronski, A. A. (2005). Stochastic quasigradient methods and their implementation. In Wallace, S. W. and Ziemba, W. T., editors, *Applications of Stochastic Programming*, pages 37–60. MOS-SIAM.

Gaivoronski, A. A. (2016). SQG v.5 reference manual. Unpublished, Norwegian University of Science and Technology.

Galus, M. D., Wietor, F., and Andersson, G. (2012). Incorporating valley filling and peak shaving in a utility function based management of an electric vehicle aggregator. *IEEE PES Innovative Smart Grid Technologies Europe (ISGT Europe)*.

Graabak, I. and Korpås, M. (2016). Variability characteristics of european wind and solar power resources a review. *Energies*, 9.

Grigg, C., Wong, P., Albrecht, P., Allan, R., Bhavaraju, M., Billinton, R., Chen, Q., Fong, C., Haddad, S., and Kuruganty, S. (1999). The ieee reliability test system 1996 - a report prepared by the reliability test system task force of the application of probability methods subcommittee. *IEEE Transactions on Power Systems*, 14.3:1010–1020.

Grigsby, L., editor (2012). *Power Systems*. CRC Press, Florida, US, 3rd edition.

Gundersen, A., Johansen, M., and Benjamin, K. (2017). Exact and heuristic solution approaches for a routing problem found within snow plowing operations. Master's thesis, NTNU Trondheim, Norway.

Harbo, S. F. (2016). Agent based modelling, simulation and optimization of electric power flow with plug-in electric vehicles. Unpublished specialization project report in TIØ4500, Norwegian University of Science and Technology.

Harbo, S. F. (2017a). Agent based modelling and simulation of plug-in electric vehicles. Unpublished specialization project report in TET4520, Norwegian University of Science and Technology.

Harbo, S. F. (2017b). Tackling variability of renewable energy with stochastic optimization of energy system storage - solving a stochastic, multistage ac optimal power flow problem with the stochastic quasi-gradient method. Master's thesis, NTNU Trondheim, Norway.

Haupt, R. L. and Haupt, S. E. (2004). *Practical Genetic Algorithms*. John Wiley and Sons, New York, US, 2nd edition.

Higle, J. L. (2005). Stochastic programming: Optimization when uncertainty matters. *INFORMS - Tutorials in Operations Research*.

Hillier, F. S. and Lieberman, G. J. (2010). *Numerical Optimization*. McGraw-Hill International, New York, US, 9th edition.

Holland, J. H. (1975). *Adaptation in Natural and Artificial Systems: An Introductory Analysis with Applications to Biology, Control and Artificial Intelligence*. MIT Press, Cambridge, US, 1st edition.

IEEE, W. G. (1973). Common data format for the exchange of solved load flow data. *IEEE Transactions on Power Apparatus and Systems*, PAS-92, No. 6:1916–1925.

Jameson, A. (1995). Gradient based optimization methods. *MAE Technical Report No. 2057*, Princeton University.

Jones, L. (2017). *Renewable Energy Integration - Practical Management of Variability, Uncertainty, and Flexibility in Power Grids*. Elsevier, 2nd edition.

Kall, P. and Wallace, S. W. (1994). *Stochastic Programming*. John Wiley and Sons, Chichester, 2nd edition.

Kremers, E. (2013). *Modelling and Simulation of Electrical Energy Systems Through a Complex Systems Approach Using Agent-Based Models*. PhD thesis, UPV, Valencia, Spain.

Liebreich, M. (2016). In search of the miraculous. Technical report, Bloomberg, New Energy Finance.

Lundgren, J., Rnnqvist, M., and Vrbrand, P. (2012). *Optimization*. Studentlitteratur, Malm, Sweden, 1st edition.

Maringer, D. G. (2005). *Portfolio Management with Heuristic Optimization*. Springer, New York, US, 1st edition.

McKinsey & Company (2014). Electric vehicles in europe: gearing up for a new phase? Technical report, for the Amsterdam Roundtables Foundation, Amsterdam, Netherland.

Mercier, P., Cherkaoui, R., and Oudalov, A. (2009). Optimizing a battery energy storage system for frequency control application in an isolated power system. *IEEE Transactions on Power Systems*, 24.3:1469–1477.

Nocedal, J. and Wright, S. J. (2006). *Numerical Optimization*. Springer, New York, US, 2nd edition.

Nordpool (2017). Market data. `https://www.nordpoolgroup.com/Market-data1/Dayahead/Area-Prices/NO/Hourly/?view=table`. [Online; accessed 15-October-2017].

Peeta, S. and Zhou, C. (2006). Stochastic quasi-gradient algorithm for the off-line stochastic dynamic traffic assignment problem. *Transportation Research*, 40:179206.

Pflug, G. (1988). Stepsize rules, stopping times, and their implementation in stochastic quasigradient algorithms. In Ermoliev, Y. and Wets, R. J.-B., editors, *Numerical Techniques for Stochastic Optimization*, pages 353–372. Springer-Verlag.

Pflug, G. C. and Pichler, A. (2010). *Multistage Stochastic Optimization*. Springer, New York, US.

Rechenberg, I. (1965). Cybernetic solution path of an experimental problem. *Royal Aircraft Establishment Translation*.

Rechenberg, I. (1989). Evolution strategy: Natures way of optimization. In Bergmann, H. W., editor, *Optimization: Methods and Applications, Possibilities and Limitations*, pages 106–126. Springer.

Skotland, C. H., Eggum, E., and Spilde, D. (2016). Hva betyr elbiler for strmnettet? Technical report, The Norwegian Water Resources and Energy Directorate (NVE), Oslo, Norway.

Smart, J. and Schey, S. (2012). Battery electric vehicle driving and charging behavior observed early in the ev project. *SAE Int. J. Alt. Power*.

Smart, J., Scoffield, D., and Schey, S. (2012). A first look at the impact of electric vehicle charging on the electric grid in the ev project. *SAE Int. J. Alt. Power*.

Spall, J. C. (2003). *Introduction to Stochastic Search and Optimization - Estimation, Simulation, and Control*. John Wiley and Sons, New York, US, 1st edition.

Sperstad, I. B. and Marthinsen, H. (2016). Optimal power flow methods and their application to distribution systems with energy storage. Technical report, SINTEF Energy Research, Trondheim, Norway.

Spilde, D. and Skotland, C. (2015). Hvordan vil en omfattende elektrifisering av transportsektoren pvirke kraftsystemet? Technical report, The Norwegian Water Resources and Energy Directorate (NVE), Oslo, Norway.

Taylor, M., Ralon, P., and Ilas, A. (2016). The power to change: Solar and wind cost reduction potential to 2025. Technical report, IRENA.

Tveter, H. T. (2014). Large scale transition from conventional to electric vehicles and the consequences for the security of electricity supply - a demand side analysis of electricity consumption. Master's thesis, NHH, Norwegain School of Economics, Bergen, Norway.

Ulbig, A., Borsche, T. S., and Andersson, G. (2014). Impact of low rotational inertia on power system stability and operation. *IFAC Proceedings Volumes*, 47.3:7290–7297.

Wangensteen, I. (2012). *Power System Economics*. Fagbokforlaget, Bergen, Norway, 2nd edition.

Whiteman, A., Rinke, T., Esparrago, J., Arkhipova, I., and Elsayed, S. (2017). Renewable capacity statistics 2017. Technical report, IRENA.

Yordanov, G. H. (2012). *Characterization and Analysis of Photovoltaic Modules and the Solar Resource Based on In-Situ Measurements in Southern Norway*. PhD thesis, Norwegian University of Science and Technology, Trondheim, Norway.

Zaferanlouei, S., Ranaweera, I., Korpås, M., and Farahmand, H. (2016). Optimal scheduling of plug-in electric vehicles in distribution systems including pv, wind and hydropower generation.

Zimmerman, R. D. and Wang, H. (2016). Matpower interior point solver - users manual. Technical Report MIPS 1.2.2, Power Systems Engineering Research Center.

Appendix A

Acronyms and Definitions

A.1 Acronyms

ABM Agent Based Model

AC Alternating Current

BNEF Bloomberg New Energy Finance

ESS Energy System Storage

EV Electric Vehicles

IRENA The International Renewable Energy Agency

NVE The Norwegian Water Resources and Energy Directorate

OPF Optimal power flow

PEV Plug-in Electric Vehicle

RES Renewable Energy Sources

SDP Renewable Energy Sources

SOC State of Charge, eg. how much power there is available on a PEV's battery compared to its capacity.

SSB Statistics Norway, *Statistisk Sentralbyrå* in Norwegian.

A.2 Definitions

In this thesis, these important concepts are referenced to as the folloling:

Dispatchable generation Power production that is easy and quick to turn on and off.

Simulation The word simulation is in this thesis is often used for referencing the low-lever part of the models running the deterministic AC-OPF optimization given the realization of stochastic variables and charging policy. The results returned are used by the SQG-optimization engine to optimize the stochastic dynamic AC-OPF.

Stochastic Optimization The use of stochastic variation in an iterative optimization method in order to help find the global optimum.

Stochastic Programming Refers to decision making under uncertainty, by modeling and formulation of exact mathematical programs to optimize the outcome by manipulation of some decision variables.

Multi-Period Stochastic AC-OPF or MP-S-AC-OPF is an AC-OPF model with ESS and stochastic variables, solved by the SQG method.

Stochastic Dynamic AC-OPF or SD-AC-OPF, is an AC-OPF model with ESS (thus dynamic) and stochastic variables, solved by Stochstic Dynamic Programming (SDP).

Appendix B

Additional Information on the Models

B.1 Case data

The data for the 9 bus, 24 bus and 118 bus case are all from IEEE standard tests systems. It is handed in in excel files along with the thesis, and also readily found by searching the internet. The 4 bus case is an constructed example case by the author.

B.1.1 Case 1: the 4 bus power system

Table B.1 presents the given demand and generation (average in the stochastic case) of the different buses, that the AC-OPF takes as input to its solution enforcing the values by upper and lower boundaries. In table B.2 the corresponding constrains on the buses voltage maximum and minimum, as well as power bounds on active and reactive power. Table B.3 shows the line constraints for the network. Note that the values are the same for all three phases. The system is assumed to have base values of $100MVA$ for base power and $115kV$ as base voltage.

Table B.1: Given demand and generated power for the 4 bus case

Bus number	Pd	Qd	Pg	Qg
1	0	0	0	0
2	10	0	0	0
3	0	0	30	0
4	100	20	0	0

Table B.2: Constraints value for the buses for the 4 bus case

Bus	Vmax	Vmin	Qmax	Qmin	Pmax	Pmin
1	1,1	1,1	200	-200	200	0
2	1,1	0,9	0	0	0	0
3	1,1	0,9	0	0	30	30
4	1,1	0,9	0	0	0	0

Table B.3: Constraints value for the lines for the 4 bus case

$\mathbf{Line}_{from,to}$	$\mathbf{R}_{i,j}$	$\mathbf{X}_{i,j}$	$\mathbf{I}_{i,j}^{max}$
2,4	0,5	0,3	250
2,3	0,5	0,3	250
2,1	0,5	0,3	250
1,3	0,3	0,2	250
1,4	0,3	0,2	250
3,4	0,2	0,1	250

B.1.2 Case 2: the 9 bus power system

Table B.4 shows the constrains on the buses voltage maximum and minimum, as well as power bounds on active and reactive power. In table B.5 the line constraints for the network are presented. Note again that the values are the same for all three phases, and that the system is assumed to have base values of $100MVA$ for base power and $115kV$ as base voltage.

Table B.4: Constraints value for the buses for the 9 bus case

Bus number	Discription	Vmax	Vmin	Pmax	Pmin	Qmax	Qmin
1	Gas, slack	1,1	1,1	300	0	300	-300
2	Wind	1,1	1	300	0	0	0
3	PV	1,1	1	300	0	0	0
4	Transformator	1,1	0,9	-	-	-	-
5	Business	1,1	0,9	-	-	-	-
6	Transformator	1,1	0,9	-	-	-	-
7	House	1,1	0,9	-	-	-	-
8	Transformator	1,1	0,9	-	-	-	-
9	Battery	1,1	0,9	50	0	0	0

Table B.5: Constraints value for the lines for the 9 bus case

$\textbf{Line}_{from,to}$	$\mathbf{R}_{i,j}$	$\mathbf{X}_{i,j}$	$\mathbf{B}_{i,j}$	$\mathbf{I}_{i,j}^{max}$
1,4	0	0,0576	0	250
4,5	0,017	0,092	0,158	250
5,6	0,039	0,17	0,358	150
3,6	0	0,0586	0	300
6,7	0,0119	0,1008	0,209	150
7,8	0,0085	0,072	0,149	250
8,2	0	0,0625	0	250
8,9	0,032	0,161	0,306	250
9,4	0,01	0,085	0,176	250

B.1.3 Case 3: the 24 bus power system

Table B.6 shows the constrains on the buses voltage maximum and minimum, as well as power bounds on active and reactive power. In table B.7 the line constraints for the network are presented. Note again that the values are the same for all three phases, and that the system is assumed to have base values of $100MVA$ for base power and $115kV$ as base voltage.

Table B.6: Constraints value for the buses for the 24 bus case

Bus nr.	Discription	Vmax	Vmin	Pmax	Pmin	Qmax	Qmin
1	PV + House	1,05	0,95	192	0	80	-50
2	Wind + Industry	1,05	0,95	192	0	80	-50
3	House	1,05	0,95	0	0	0	0
4	House	1,05	0,95	0	0	0	0
5	House	1,05	0,95	0	0	0	0
6	House	1,05	0,95	0	0	0	0
7	PV + Industry	1,05	0,95	300	0	180	0
8	House	1,05	0,95	0	0	0	0
9	House	1,05	0,95	0	0	0	0
10	House	1,05	0,95	0	0	0	0
11	Only transmission	1,05	0,95	0	0	0	0
12	Only transmission	1,05	0,95	0	0	0	0
13	Connection stiff grid	1,05	0,95	1500	0	240	0
14	Battery + House	1,05	0,95	0	0	200	-50
15	PV + House	1,05	0,95	215	0	110	-50
16	Battery + House	1,05	0,95	155	0	80	-50
17	Only transmission	1,05	0,95	0	0	0	0
18	Wind + Industry	1,05	0,95	400	0	200	-50
19	House	1,05	0,95	0	0	0	0
20	Industry	1,05	0,95	0	0	0	0
21	Battery	1,05	0,95	400	0	200	-50
22	PV	1,05	0,95	300	0	96	-60
23	Wind	1,05	0,95	660	0	310	-125
24	Only transmission	1,05	0,95	0	0	0	0

Table B.7: Constraints value for the lines for the 24 bus case

$\textbf{Line}_{from,to}$	$\mathbf{R}_{i,j}$	$\mathbf{X}_{i,j}$	$\mathbf{B}_{i,j}$	$\mathbf{I}_{i,j}^{max}$
1,2	0,0026	0,0139	0,4611	175
1,3	0,0546	0,2112	0,0572	175
1,5	0,0218	0,0845	0,0229	175
2,4	0,0328	0,1267	0,0343	175
2,6	0,0497	0,192	0,052	175
3,9	0,0308	0,119	0,0322	175
3,24	0,0023	0,0839	0	400
4,9	0,0268	0,1037	0,0281	175
5,10	0,0228	0,0883	0,0239	175
6,10	0,0139	0,0605	2,459	175
7,8	0,0159	0,0614	0,0166	175
8,9	0,0427	0,1651	0,0447	175
8,10	0,0427	0,1651	0,0447	175
9,11	0,0023	0,0839	0	400
9,12	0,0023	0,0839	0	400
10,11	0,0023	0,0839	0	400
10,12	0,0023	0,0839	0	400
11,13	0,0061	0,0476	0,0999	500
11,14	0,0054	0,0418	0,0879	500
12,13	0,0061	0,0476	0,0999	500
12,23	0,0124	0,0966	0,203	500
13,23	0,0111	0,0865	0,1818	500
14,16	0,005	0,0389	0,0818	500
15,16	0,0022	0,0173	0,0364	500
15,21	0,0063	0,049	0,103	500
15,21	0,0063	0,049	0,103	500
15,24	0,0067	0,0519	0,1091	500
16,17	0,0033	0,0259	0,0545	500
16,19	0,003	0,0231	0,0485	500
17,18	0,0018	0,0144	0,0303	500
17,22	0,0135	0,1053	0,2212	500
18,21	0,0033	0,0259	0,0545	500
18,21	0,0033	0,0259	0,0545	500
19,20	0,0051	0,0396	0,0833	500
19,20	0,0051	0,0396	0,0833	500
20,23	0,0028	0,0216	0,0455	500
20,23	0,0028	0,0216	0,0455	500
21,22	0,0087	0,0678	0,1424	500

B.2 SDP data

Transition probability matrix with discterization level of 51.

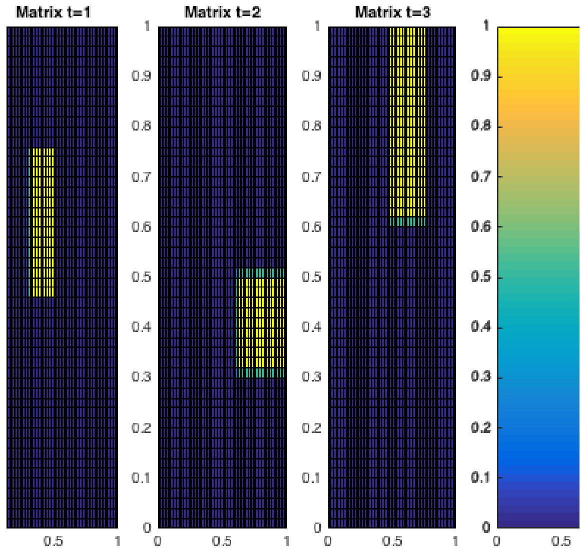


Figure B.1: Transition matrix with SOC and wind discretizised int 51 levels

Appendix C

Additional Results and Graphs from the Models

Due to concerns on length for this report, a number of figures may have been left out that might be of interest to the reader.

In this appendix, a few extra graphs and other relevant results will be presented.

C.1 Further figures form the simulations

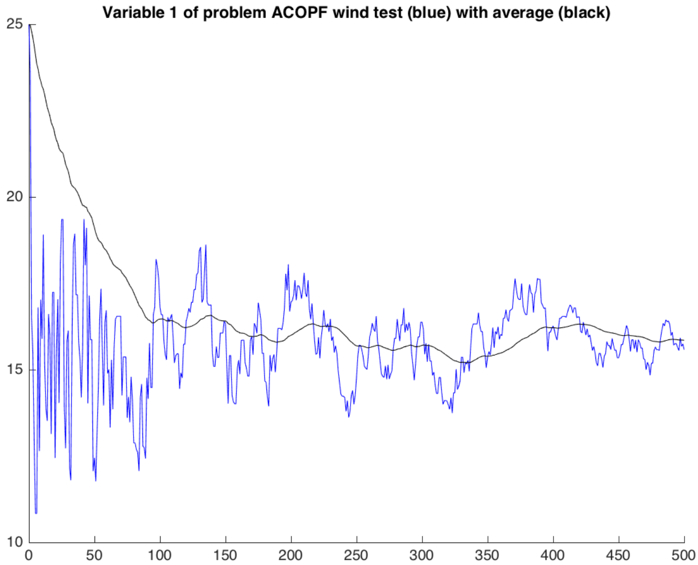


Figure C.1: Approximation of variable 1 for the 4 bus case with the SQG method

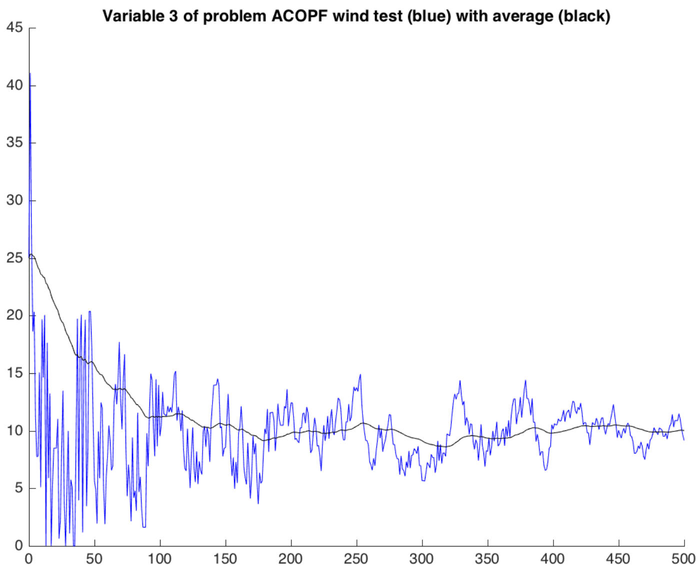


Figure C.2: Approximation of variable 3 for the 4 bus case with the SQG method

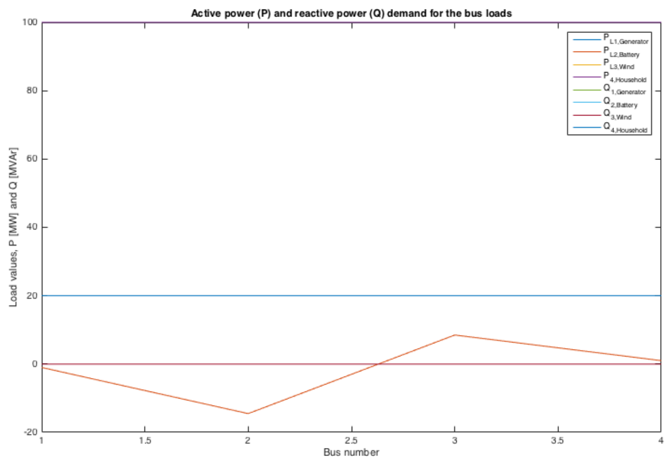
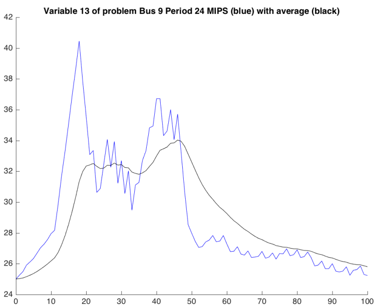
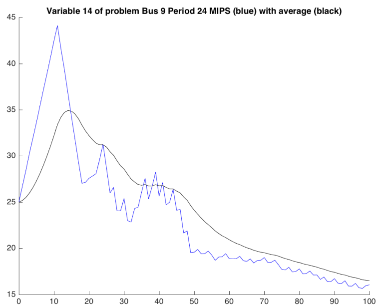


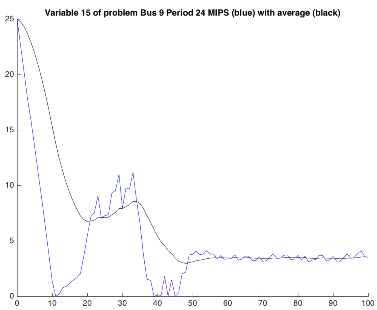
Figure C.3: SDP soultion of 4 bus case discretizised to 101 different levels of SOC



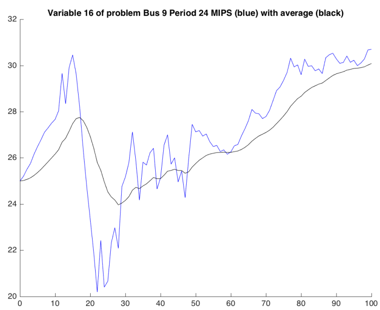
(a) t = 13



(b) t = 14



(c) t = 15



(d) t = 16

Figure C.4: Iterations of four of the variables for charging for the 9 bus case solved with MIPS

Appendix D

Information on the implemented code

Attached with the online hand-in of the thesis, the programmed code will be found. It contains the models developed and implemented throughout this work that has been presented in this document, as well as the data utilized and extensive and detailed results that was deemed to plentifully to include in the thesis.

Please feel free to contact the author using email `sondreharbo@gmail.com` if there are any questions regarding the code, or if a reader of this report wants to see and try running some of the code.

Appendix E

Mathematical Optimization Models

In the following are the mathematical models underling this thesis, as selected excerpts from Harbo (2017b). See the referred work for more details if desired.

E.1 Basic AC-Optimal Power Flow Models

The technique called an Alternating Current Optimal Power Flow (ACOPF) is used to calculate how generators best produce their power given cost and losses due to resistance in the transmission grid, accounting for both active and reactive power.

First we introduce a general deterministic model for AC-Optimal Power Flow.

E.1.1 Assumptions

As standard for most Power Flow studies we assume as earlier that:

1. A balanced loading of the three phases in the power system.
2. The power system operates in steady-state.
3. Constant system frequency.
4. Power demand and production except in the slack bus is assumed known.
5. No energy storage is allowed in the system.
6. Prices is assumed to be constant for the basic model, but may be dynamic in extended formulations, for instance a reaction to the shadow prices of the power bidding of the different producers.

The first assumption here, allows us to simplify the parts using π-diagrams and constructing a single-line model of the system. This makes it simpler when building the mathematical model of transmission lines with corresponding impedance, buses, generators and loads. The second means that we disregard transient and dynamic behavior of the system, for instance in the case of lost loads, generator tripping, short circuits or other contingencies. This is also a pre-requisite of the third assumption, which also lets us disregard generator drooping and frequency response here.

E.1.2 Notation

The following general notation is applied in the mathematical model:

Indicies:

i, j Bus, or node, indicies

Sets:

G Distributed generators in the network
L Distributed loads in the network
N Number of buses in the network

Parameters:

$I_{line,rated}$	Rated capacity for current in the lines.
$Q_{G,i}^{min}, Q_{G,i}^{max}$	Minimum and maximum reactive power capacity of the generators.
V_{min}, V_{max}	Minimum and maximum voltage amplitude.
$\delta_{min}, \delta_{max}$	Minimum and maximum voltage angle.
P_i^L, Q_i^L	Demand, or load, for active and reactive power at a given bus i.
$Y_{i,j}, \theta_{i,j}$	The admittance and angle for the line between bus i and bus j in the system.
C_i^G	Price of production with generator at bus i.
$V_{slack} = 1p.u., \delta_{slack} = 0$	Voltage amplitude and angle of a bus.

Variables:

P_{slack}^G, Q_{slack}^G	Active and reactive power production from the slack bus generator.
V_i, δ_i	Voltage amplitude and angle of a bus.

E.1.3 Model

A general AC-Optimal Power Flow model is presented below, minimize cost of all generator, whilst upholding energy balance and physical constraints for all buses.

$$
\begin{aligned}
\min \quad & \sum_{i \in G} C^G \cdot P_i^G \\
\text{s.t.} \quad & P_i^G - P_i^L = \sum_{j=1}^{N} |\mathbf{V_i}||\mathbf{V_j}||\mathbf{Y_{i,j}}| \cos(\delta_j - \delta_i + \theta_{i,j}) \\
& Q_i^G - Q_i^L = \sum_{j=1}^{N} |\mathbf{V_i}||\mathbf{V_j}||\mathbf{Y_{i,j}}| \sin(\delta_j - \delta_i + \theta_{i,j} \\
& Q_i^{min} \leq Q_i \leq Q_i^{max} \\
& V_{min} \leq V_i \leq V_{max} \\
& |\mathbf{I}_{line,(i,j)}| \leq I_{line,rated} \\
& \delta_{min} \leq \delta_i \leq \delta_{max} \\
& i, j \in N.
\end{aligned}
\tag{E.1}
$$

In this model, the objective function is to minimize the cost of running the generators for a given production cost. The first two equality constraints are the power flow equations in this Alternating Current Network. It is derived from Kirchhoff's current law, which says that current in and out of every bus has to be equal The inequality constraints that follow represents physical limits in the power network. The firs inequality, states the upper and lower limits of reactive power that is possible to produce or consume at the different distributed generators. The second inequality, are the limits on how high or low voltage can be. This is usually within 0.95 and 1.05 p.u. - that is, within 95-105%. The line constraints are the line current between bus i and j being derived from Ohm's law $U = RI$ and here expressed by

$$\mathbf{I}_{line,(i,j)}(t) = \frac{\mathbf{V}_i - \mathbf{V}_j}{\mathbf{Y}_{i,j}}. \tag{E.2}$$

This model may be solved by many approaches, such as Lagrangian Multipliers or an iterative Newton-Raphson method, or the Interior Point Method that we use here.

E.2 Dynamic AC-Optimal Power Flow with Energy Storage

When one is to consider energy storage in power system optimization, such as letting the models incorporate batteries or pump-hydro power plants, the mathematical formulation takes on a dynamic character. This is an effect of the energy storage equations needed to be introduced, as seen in 3.22. For this thesis, the time increments will almost always being 1 hour, and thus left out. Moreover, one needs to constraint the minimum and maximum power of the battery as seen in 3.23, and possibly also it upper and lower charging and discharging values.

Such equations will be introduced in the following dynamic models, to connect the AC-OPF solution of each time step though the storage dynamics.

In this section a model is formulated which includes the possibility to store some energy in for instance a battery or a hydro power dam, and hence the model undertakes a dynamic character.

E.2.1 Assumptions

First we make some new assumptions.

1. The power demand and production except in the slack bus is assumed known by its stochastic distribution functions.
2. Energy storage is allowed in the system, but only supplies active power.
3. Prices is assumed to be constant for a certain time period for this model, and are not dependent on the dynamics of the model.

E.2.2 Notation

We now update the general notation from the previous model to having the possibility to be different for different time periods, and include the following new notation:

New indicies:

t Time instance

New sets:

T Time intervals

New parameters:

$P_{Ch,i}^{max}, P_{Dch,i}^{max}$	Maximum active charging and discharging power of a certain energy storage at bus i.
$E_{ST,i}^{max}$	Rated energy capacity of the energy storage at bus i.
$\eta_{Ch,i}, \eta_{Dch,i}$	Charging and discharging efficiency of the energy storage at bus i.
Δt	Time step increment.

New variables:

$P_{Ch,i}(t), P_{Dch,i}(t)$	Active power charging and discharging from the energy storage at bus i for time instance t.
$E_{ST,i}(t)$	Energy stored at time step t, which is given by the charge and discharge during t.

E.2.3 Model

Incorporating these new assumptions and variables into the model, we have:

$$
\begin{aligned}
\min \quad & \sum_{t=1}^{T} \sum_{i \in G} C_i^G(t) \cdot P_i^G(t) \cdot \Delta t \\
\text{s.t.} \quad & P_{G,i}(t) - P_{L,i}(t) + P_{Dch,i}(t) - P_{Ch,i}(t) = ... \\
& \quad \sum_{j=1}^{N} |\mathbf{V_i}(t)||\mathbf{V_j}(t)||\mathbf{Y_{i,j}}(t)| \cos(\delta_j(t) - \delta_i(t) + \theta_{i,j}(t) \\
& Q_{G,i}(t) - Q_{L,i}(t) = \sum_{j=1}^{N} |\mathbf{V_i}(t)||\mathbf{V_j}(t)||\mathbf{Y_{i,j}}(t)| \sin(\delta_j(t) - \delta_i(t) + \theta_{i,j}(t) \\
& Q_{G,i}^{min} \leq Q_{G,i}(t) \leq Q_{G,i}^{max} \\
& V_{min} \leq V_i(t) \leq V_{max} \\
& |\mathbf{I}_{line,(i,j)}(t)| \leq I_{line,rated} \\
& 0 \leq P_{Ch}(t) \leq P_{Ch}^{max} \\
& 0 \leq P_{Dch}(t) \leq P_{Dch}^{max} \\
& E_{ST}(t) = E_{ST}(t-1) + \Delta t \cdot \eta_{chrg,i} \cdot P_{Ch,i}(t) - \Delta t \cdot \frac{P_{Dch,i}(t)}{\eta_{dischrg,i}} \\
& \forall t \in [0, t, 2t, ..., T] \quad \text{and} \quad i, j \in N.
\end{aligned}
\tag{E.3}
$$

In this model, the objective function is to minimize the cost of running the generators, for a given production cost and over all time periods.

E.2.4 Stochastic Multistage AC-Optimal Power Flow

In this section seeks to provide a somewhat through mathematical framework of the problem at the heart of the thesis; solving an AC-OPF for several time periods under the influence of stochastic variables. An introduction and general formulation to this type of problem might be found in **??**.

Note that a very important assumption for this model is that after deciding how much power to charge or discharge at the start of the period before the realized wind energy is known, this decision cannot be altered. Thus, $P_{Battery}(t)$ is fixed during the time step, in which the wind generation is realized and gas generator supplies the remanding energy demanded. Hence, it is not possible to suddenly decide to charge more if the wind power generated is realized to be in the upper range of its possible values. The power balance for each time step is assured by the gas generator, and not to be influenced by sudden change in charging policy during the time step.

E.2.5 Model

With power production and loads given by some stochastic realization, the problem may be formulated in the same manner as in the deterministic case, only where the power values are random and the method tries to minimize the expected value of the objection function. In the model below, the random values are denoted as depending on the stochastic realization $\boldsymbol{\xi}$. The decision variable for each singe time step is to determine how much power the dis-patchable gas generator has to supply in order to make up for the remaining demand are are not met by the other power sources as well as covering all physical losses P_{loss} of power in the system. We assume that using the generator c_G is much more costly than the other power sources. We therefore seek how to optimally charge the battery so that it charges when energy is in abundance, and discharges when it is in shortage.

A model for this may be formulated as follows:

$$\min_{P_B, P_G} \quad \mathbb{E}\sum_{t=1}^{T} C\Big(P_G(t,\boldsymbol{\xi}) \mid P_B(t)\Big)$$

$$\begin{aligned}
\text{s.t.} \quad & P_i(t) = \sum_{j=1}^{N} |\mathbf{V_i}(t)||\mathbf{V_j}(t)||\mathbf{Y_{i,j}}(t)| \cos\big(\delta_j(t) - \delta_i(t) + \theta_{i,j}(t)\big) \\
& Q_i(t) = \sum_{j=1}^{N} |\mathbf{V_i}(t)||\mathbf{V_j}(t)||\mathbf{Y_{i,j}}(t)| \sin\big(\delta_j(t) - \delta_i(t) + \theta_{i,j}(t)\big) \\
& P_i^{min} \le P_i \le P_i^{max} \\
& Q_i^{min} \le Q_i \le Q_i^{max} \\
& V_{min} \le V_i \le V_{max} \\
& \delta_{min} \le \delta_i \le \delta_{max} \\
& |\mathbf{I}_{line,(i,j)}| \le I_{line,rated} \\
& E_{ST}(t-1) - E_{ST,2}^{max} \le P_B(t) \le E_{ST}(t-1) \\
& E_{ST}(t) = E_{ST}(t-1) - P_B(t) \\
& \forall t \in [1,2,...,T] \quad \text{and} \quad \forall i \in [1,..,N].
\end{aligned} \tag{E.4}$$

E.2.6 Reformulation of the decision variables to SOC

Where the implementation of the energy storage constrains in the S-MS-AC-OPF formulation from E.4 is not as simply bounded as might be thought at first, another approach would be to reformulate the model in terms of SOC. That means, in stead of letting the decision parameter be how much to charge at a given time step, one lets it be the decision of what state of charge, or stored energy, one is to have at each time step.

In effect this becomes the same decision, as the charging or discharging of the battery for each time step is the difference of the energy levels of the battery at the current and the previous time steps. That is

$$P_{Battery}(t) = E_{ST}^{max} * \big(SOC(t-1) - SOC(t)\big) = E_{ST}(t-1) - E_{ST}(t) \tag{E.5}$$

where

$$SOC(t) = \frac{E_{ST}(t)}{E_{ST}^{max}}. \tag{E.6}$$

The benefit of this formulation is that the constrains on the decision variable becomes very simple, it reduces to only upper and lower bounds of no energy storage and maximum energy storage - the battery capacity E_{ST}^{max}. This it should be very easy for the SQG solver to project a step that is out of the feasible area, down onto the set X of feasible values for x as described in equations **??** and **??**. See Harbo (2017b) for further details.