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An equilibrium market power model for power markets and tradable green certificates, including Kirchhoff's Laws and Nash-Cournot competition

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ABSTRACT

We investigate the economic impacts of introducing tradable green certificates to promote electricity produced from renewable energy sources. We formulate a mixed complementarity, multi-region, partial equilibrium model, clearing both the electricity and green certificate markets under the assumption of Nash-Cournot market competition. We introduce a mixed complementarity formulation of the tradable green certificate policy scheme. The main contribution of this paper is to combine a public support scheme for electricity production with a power market model in which strategic generators compete and exercise market power in a capacitated transmission network with spatial energy exchange.

Any policy instrument interfering with the free market solution in a partial equilibrium model will reduce social welfare as a result of deadweight losses from the policy. These welfare losses may be substantial. We show that losses from tradable green certificates influence different market actors depending on the market conditions, but existing firms are likely to bear most of these losses.

In markets with Cournot competition, where producers act strategically, green certificates help to increase market competition if new firms are able to enter the market. Existing firms will not be motivated to compete with new generation capacity. The consumer surplus from introducing tradable green certificates under Cournot competition may increase, despite the deadweight losses the policy incurs.

1. Introduction

This paper models the use of tradable green certificates to support deployment of renewable electricity in the power market. During the last decade, many public support schemes have supported renewable electricity as a way forward to fight climate change, in addition to improving security of energy supply, promoting technological development and innovation, and providing opportunities for employment and regional development (EU, 2009; REN21, 2015). The power sector contributes more than any other sector to the reduction in the share of fossil fuels in the global energy mix (IEA, 2014). Global energy demand is rising, and electricity is the fastest-growing final form of energy. Existing support schemes for electricity include feed-in tariffs, feed-in premiums, tradable green certificates and investment subsidies, possibly combined with tenders/auctions in various forms. In this paper we focus on tradable green certificates (also called 'renewable energy certificates' or RECs). Our model also includes imperfect market competition, since many international electricity markets have been liberalized and redesigned during the last three decades and market competition and possible misuse of market power is a relevant issue in these markets (Borenstein and Bushnell, 1999; Borenstein et al., 1995; Bushnell, 2003).

The main contribution of this paper is to combine a public support scheme for electricity production with a power market model in which strategic generators compete and exercise market power in a capacitated transmission network with spatial energy exchange. Policy instruments constitute interference to the market, and will inevitably lead to deadweight losses. Since we address imperfect competition in this paper, we also get partial welfare gains from the instrument due to reduced market power and thus increased competition. We employ a deterministic partial equilibrium model to find the cost of reaching a target quota of renewable electricity production. Our work builds on previous research on Nash-Cournot equilibria in power markets (Hobbs, 2001; Hobbs et al., 2008; Metzler et al., 2003). We develop a mixed complementarity, multi-region, partial equilibrium model, with an underlying alternating current (AC) network represented by a linearized DC

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network approximation (Schweppe et al., 1988). Both the electricity and green certificate markets are cleared under the assumption of Nash-Cournot market competition. The shared constraints from the network imply that a solution to our model constitutes a Generalized Nash Equilibrium. To our knowledge this is the first model that combines an AC approximation of the transmission network with equilibrium modeling of support schemes and in addition allows imperfect competition.

A literature review follows in Section 2. Section 3 describes the tradable green certificate scheme in further detail. Section 4 describes our mathematical model. In Section 5 we illustrate through a small example how this modeling approach can be used to identify welfare distribution effects between producers, consumers and transmission system owner (TSO), in order to find winners and losers of the game. Section 6 concludes. An Appendix A shows detailed numerical results and supplementary figures from the example.

2. Literature review

The following literature review focuses on equilibrium models for power markets, the inclusion of economic policy instruments in power market models and on modeling physical aspects of the network.

2.1. Perfect versus imperfect competition

Although many policies have been implemented to promote competition, there is still evidence of market power in electricity markets (Bushnell et al., 2008; Dahlan and Kirschen, 2012; Mirza and Bergland, 2015; Pérez de Arce and Sauma, 2016). In order to model imperfect competition we must handle different players participating in the market through a game where they take into account other players' actions. The action of one agent influences the payoff of another agent. A Nash equilibrium describes an equilibrium between agents interacting through their payoffs (Nash, 1950).

In some instances a Nash-Cournot model with multiple players optimizing their own payoffs may be expressed as an optimization problem (Facchinei and Pang, 2003).[1] However, our policy problem includes a tax agent who constrains both primal and dual variables together. Consumers pay a tax on their electricity consumptions in order to finance green certificates that subsidize renewable producers. This restriction produces an equilibrium condition which cannot be expressed in an optimization problem. Thus we formulate an equilibrium model instead of an optimization model. The equilibrium formulation also allows introduction of more realistic demand functions, which would prevent us from expressing our equilibrium problem as an optimization problem (Hobbs et al., 2008).

A generalized Nash equilibrium involves agents that interact both at the level of their payoffs, but also through their strategy sets. The action of an agent can influence the payoff of another agent, but it can also change the set of actions that this agent can undertake. In our model it is easy to see that the strategy of a firm is constrained by production from other firms through the bounds on the energy flows in the network (Kirchhoff's laws). This dependence implies that the equilibrium is a Generalized Nash Equilibrium (Wei and Smeers, 1999).

Our approach is to solve an equilibrium problem consisting of each player's KKT conditions together with market clearing conditions, and solve this problem to obtain a generalized Nash equilibrium.

2.2. Economic models for policy instruments

Economic instruments for achieving environmental goals are classically categorized as either price-based or quantity-based, depending on which of these two variables is chosen by a regulator. By fixing one of the variables (for example price), the other (i.e. quantity) is determined by the market. Traditional price-based policy instruments are taxes and subsidies (Pigou, 1920). The most common policy instruments supporting renewable electricity are feed-in tariffs, feed-in premiums and tradable green certificates. In recent years these instruments are increasingly used in various hybrid policies, especially in combination with competitive bidding (tendering) (Couture et al., 2015; Held et al., 2014; REN21, 2015). Complementarity models are very suitable for policy analysis (Gabriel et al., 2013a). The price-driven versus quantity-driven policy instruments correspond to the duality between model constraints with primal variables representing real-world physical properties (quantities) and the accompanying dual variables (representing prices).

Other papers focus on the dynamics of certificate prices and build models for forecasting prices and volumes in the certificate markets. Such models take into account banking, borrowing and penalty options. The certificate price must equal the discounted expected value in the next time-step, and also the penalty price times the probability of a shortage of credits at the compliance date. Wolfgang et al. (2015) describe a methodology where they simulate climatic variables like wind, sun and reservoir inflow affecting electricity generation, and calculate strategies for the certificate inventory by stochastic dynamic programming using the EMPS model. They report case study forecasted certificate prices based on predefined capacities for production and transmission. Coulon et al. (2015) build a stochastic price model where they allow for dynamic endogenous investment in generation dependent on certificate prices. They demonstrate the important role of market design in determining price behaviour, and suggest a function for deciding the penalty of non-compliance with the certificate obligation. Boomsma and Linnerud (2015) use a somewhat related real options approach to compare market and policy risk under different renewable electricity support schemes. They treat the certificate price as stochastic and find that differences in market risk between support schemes are less than commonly believed due to price diversification. Neither of these models considers an underlying transmission network with its corresponding system effects or the effects of imperfect competition. We include in our model both of these perspectives, at the expense of treating the dynamic development of the support scheme from year to year. We may still include seasonal system dynamics by including time-periods in our model.

2.3. Modeling the economics and the physical aspects of the network

Many studies of electricity markets disregard transmission constraints entirely, or use a transhipment network that ignores Kirchhoff's voltage law (Hobbs et al., 2000). Most of these studies do not consider support schemes. As far as the authors are aware, no previous studies of policy schemes for renewable electricity have included the AC characteristics of transmission networks. We give a short overview of the most relevant models and their approach and scope.

Bushnell (2003) presents a mixed complementarity model to analyze competition between multiple firms possessing a mixture of hydroelectric and thermal generation resources. He studies how Cournot competitors may act strategically and increase profit by allocating more flexible hydro production to off-peak periods than they would under perfect competition. However, he does not consider network transmission constraints. Neither does the study by Linares et al. (2008), which includes regulatory support schemes like tradable green certificates. The authors develop an oligopolistic generation-expansion model for the electricity sector simulating regulatory instruments. They formulate a linear complementarity model which allows the optimization problem for each firm considering the power, carbon and green certificate markets to be solved simultaneously, but do not consider a transmission network or zonal electricity prices. Another study that considers the green certificate support scheme in an equilibrium model without considering transmission is provided by Marchenko (2008). The study evaluates how well a green certificate market mechanism is able to optimize the

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[1] If an equilibrium problem can be expressed as a variational inequality (VI) with symmetric Jacobian, then an equivalent optimization problem could be found.

total economic effect taking into account external costs compared with other policy instruments. Gabriel et al. (2013b) solve Nash-Cournot energy production games while restricting some variables to be discrete in a recent power market study. Their approach allows for more realistic modeling, for example regarding investments or operational start up decisions. The study neither considers transmission networks nor support scheme aspects.

Other approaches include transmission networks, but assume that electricity can be transported as in a transhipment network. Only Kirchhoff's current law is imposed in such models, while the voltage law (also called Kirchhoff's loop rule) that forces power to flow in parallel paths is disregarded. This approach simplifies the mathematical models, but the corresponding analytic results lead to propositions that are misleading. One example of a common but misleading proposition is that power only flows from nodes with lower prices to nodes with higher prices. This and more examples are thoroughly described by Wu et al. (1996).

Böhringer et al. (2007) investigate economic impacts from using feed-in tariffs or tradable green certificates to promote electricity from renewable energy sources within the EU. Producers compete in a Cournot oligopoly with iso-elastic demand. Their model covers transport between neighboring areas. However the model only handles transport costs between adjacent regions, and arbitrage opportunities through transit areas are not recognized. Kirchhoff's voltage law is not imposed, so transmission is modeled as a transhipment network.

Nagl (2013) looks at renewable support schemes under perfect competition, but extends the study with a time dimension spanning four decades and introduces alternative weather years to capture weather uncertainty. He investigates the effect of weather uncertainty on the financial risk of green electricity producers under feed-in tariffs and tradable green certificates. Electricity demand is assumed to be inelastic. The model relies on transport capacities between adjacent regions. Arbitrage opportunities through transit regions are not recognized, and Kirchhoff's voltage law is not imposed.

The electricity market study by Vespucci et al. (2010) does not include support schemes, but represents the market as a non-cooperative game and assumes that generation firms are Cournot players that decide their strategy in order to maximize their profit. The model operates on a network with five zones and four transmission links. It does not contain cycles, so Kirchhoff's voltage law is not relevant. The study assumes linear demand curves, and each producer solves a mathematical program with equilibrium constraints (MPEC) assuming quadratic production costs. The model does not include time periods.

A different electricity market model approach (without the inclusion of a support scheme) is provided by Vespucci et al. (2013). They use a mixed integer linear programming model of a zonal electricity market. They solve a two-stage model where a dominant producer exerts market power on a capacitated transmission network in order to maximize market share while guaranteeing an annual profit. The model includes hourly decision variables within a year. As the previous one, this model also operates on a network with five zones and four transmission links without cycles, so Kirchhoff's voltage law is not relevant.

Pérez de Arce and Sauma (2016) compare four different incentive policies for renewable energy in an oligopolistic market with price-responsive demand. They include a quota system among the incentive policies, and induce penalties to firms who fail to comply with the obligation instead of introducing a certificate market that provide subsidies. Their network consists of two nodes linked by one line, where Kirchhoff's voltage law is not relevant.

In order to calculate more realistic network electricity flows, the linearized "DC" load flow model (Schweppe et al., 1988) is frequently used. This is an approximation of an alternating current (AC) model, focusing on real power with linear approximations of the power flow equations. Losses are often disregarded, but several different DC approximations are discussed by Stott et al. (2009).

Several studies have combined a DC approximated transmission network with oligopolistic market models (Hobbs, 2001; Hobbs et al., 2008; Metzler et al., 2003; Neuhoff et al., 2005). None of these includes policy support schemes for renewable electricity. Hobbs (2001) uses constant power transmission distribution factors (PTDF) to describe the power flow, and shows that a model with bilateral power markets and arbitrage is equivalent to a POOLCO power market in which each producer sells power to the grid at their area price. The transmission model is extended with nonlinear losses, controllable DC lines and phase shifters in Hobbs et al. (2008). The PTDF-based formulation is not possible, since changes in line loadings with respect to changed injections will be nonlinear. Kirchhoff's voltage law is instead imposed by restricting the sum of potential differences (voltages) around any network loop to be zero. A similar version is used by Bjorndal and Jornsten (2007), who study benefits from congestion management using an optimization model that maximize social welfare assuming perfect competition, linear demand functions and affine production cost functions.

A recent modeling advance is given by Munoz et al. (2013), who study transmission investments and their cost and performance implications for renewable portfolio standards assuming that the market equilibrium is the solution that minimizes total system costs. They show that ignoring transmission constraints when considering investments in renewables will increase the total costs. Perez et al. (2016) proceed further to include trading of renewable energy certificates between regions with different renewable obligations and thus regional certificate prices. They find that most of the economic benefits are captured if approximately 25% of renewable energy credits are allowed to be acquired from out of state. They however assume perfect competition, inelastic demand and that renewable targets are met in the most cost-efficient manner.

Limpaitoon et al. (2011) combine an oligopolistic electricity market, a lossless DC-approximated transmission network and a cap-and-trade emissions permits market. They show that market structure and congestion can have significant impact on the market performance. Limpaitoon et al. (2014) proceed further to analyze market combinations in the permits market, and how initial levels of permit allocations influence the results. They show that a firm with more efficient technologies and high levels of initial permits can withhold permits, and that strategic permit trading may influence patterns of transmission congestion. Their model covers a cap-and-trade permits market instead of a green certificate market. The cap-and-trade scheme limits greenhouse gas emissions by creating a cost on emissions, instead of rewarding new renewable generation technologies which do not emit greenhouse gases. The permits lead to welfare redistribution between firms whereas the cost of certificates are transferred to consumers via a tax, thus creating different welfare redistribution outcomes. Their model does not consider arbitragers, who eliminate any non-cost based price differences between regions.

Our model combines inclusion of transmission constraints and investments in renewables, as recommended by Munoz et al. (2013). We develop one integrated complementarity model capable of representing regional power markets with imperfect competition among multiple players, system effects from a physical AC network affecting multiple regional markets and a tradable green certificate policy instrument to support renewable electricity. We also include arbitragers that are able to exploit non-technical price differences between regions. To our awareness no model combining these elements exists in the previous literature, and our results show that all of these aspects are of importance when studying the effects of a policy scheme.

3. The tradable green certificate scheme

The tradable green certificate system is a market based support scheme providing financial support to promote new electricity generation based on renewable energy sources. For each megawatt-hour

(MWh) of renewable energy produced, a tradable certificate is issued to the generator, who can then sell the certificate in the marketplace. The demand could be voluntary (based on preferences), or mandatory as a quota obligation on retailers or end-users (Nilsson and Sundqvist, 2007). It is common to design the system as technology neutral in order to promote competition between technologies eligible for certificates, for example by recognizing that all renewable energy sources in accordance with directive EU Directive 2009/28/EC (EU, 2009) qualifies for the right to certificates. The system can also be geared towards particular types of renewable energy (Coulon et al., 2015; El Kasmioui et al., 2015).

The authorities must ensure the following actions in a typical application:

- decide a mandatory quota obligation which is imposed on market participants
- issue certificates to producers of eligible electricity generation
- maintain a registry over certificates, keeping track of traded certificates
- cancel redeemed certificates according to the quota obligation
- impose penalties to parties who do not fulfill their quota obligation

Fig. 1 shows demand and supply of green certificates based on Morthorst (2000), who describes development of a green certificate market. The demand for electricity certificates is inflexible, and represented by a vertical demand curve. The supply curve is a mixture of short-run marginal cost (SRMC) for existing renewable generation and long-run marginal cost (LRMC) of new renewable generation. The figure shows a situation where the electricity price plus certificate price must cover the LRMC of new renewable generation. The certificate price will correspond to the needed markup to the electricity price such that the last renewable capacity can recover its capital costs (included in LRMC).

Fig. 2 shows a theoretical market solution, indicating how the certificate price adds on the electricity price such that the last produced unit covers its (long run) marginal cost.

The price p_0 is the equilibrium market price before the green certificate scheme is introduced. With introduction of certificates we see renewable electricity generation crowding out parts of the old generation. The electricity price then decreases to p_p, thus remaining old producers receives p_p-p_0 less than before for each sold unit. Renewable generation earns the producer price of electricity plus the green certificate price ($p_p + p_{gc}$). Consumers pay the consumer price p_c, which is equal to the producer price of electricity plus the tax rate that is necessary to finance the value of the green certificates (indicated by the blue rectangle). Thus the tax rectangle and the certificate rectangle should cover the same area.

It is evident that ceteris paribus, the market solution in Fig. 2 would imply a significant transfer from old producers (decreasing their profit) to consumers (increasing their consumer surplus), while new producers earn a profit from the combined electricity and certificate income.

Introducing a financial support scheme to remunerate expensive renewable power generation instead of cheaper (polluting) power generation implies that a welfare loss is imposed to society, unless the climate benefits of the scheme are quantified. Market regulations are justified when they can alleviate market imperfections such as externalities, which prevent the market from optimal resource allocation and maximized welfare. Burning fossil fuels that emit greenhouse gases today may for example have big future societal costs that are not reflected in today's prices and not taken into account by market participants.

Our partial equilibrium model does not capture the benefits of the policy scheme, so introducing a support scheme will inevitably result in a welfare loss. These losses are attributed to the policy instruments and are called deadweight losses. Fig. 2 indicates a deadweight loss from the green certificates, and an additional deadweight loss from the tax. In our analyses we investigate who bears these welfare losses. Regardless of this, we assume that the overall targets for the green certificate support scheme justify the welfare losses we find, but that is not within the scope of the model.

4. Mathematical model

We have r regions and f firms. There are i generation technologies available, and some of these are eligible for green certificates which are traded in a common market across the regions. A levelized production cost is associated with each technology. Each firm can be located in several regions, and operate several technologies. Regions are connected by links with limited capacity. If there are price differences between regions, the transmission system operator (TSO) earns the difference on the power flowing through the link. A transport cost could also be associated with each link as a fixed rate, generating additional income to the grid owner (TSO). For simplicity, we assume no such fixed rates in the model presented here.

We assume that electricity supply is characterized by the existing technologies' SRMC, and by the LRMC for technologies that require capacity investments. Renewable electricity generation receives tradable green certificates according to production volume. We assume that the certificate price is formed such that the combined income from electricity and green certificates covers the LRMC for the last capacity investment that fulfills the quota obligation. In the following we just refer to marginal costs of production.

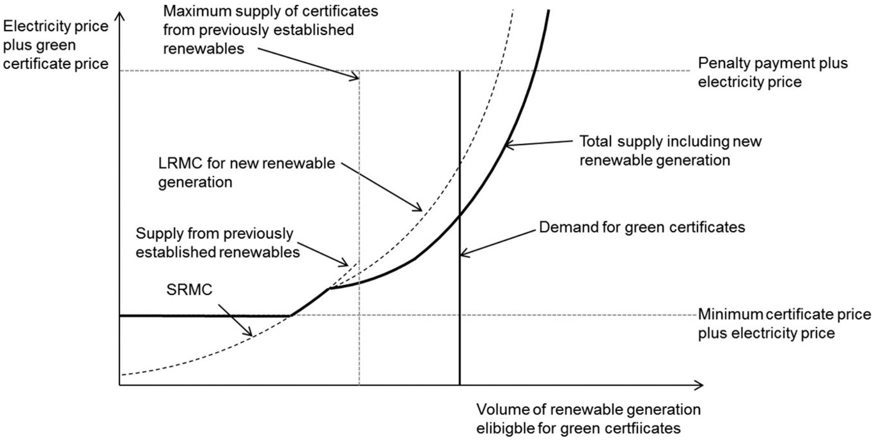

Fig. 1. Demand and supply at a green certificate market.

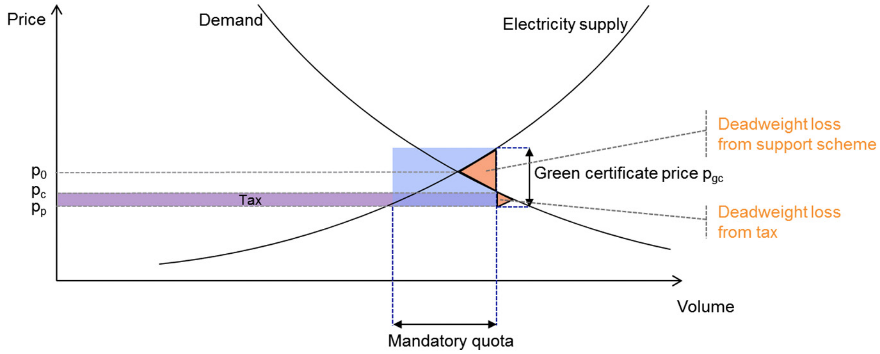

Fig. 2. Market solution in a tax financed certificate support scheme.

Furthermore we assume that electricity suppliers choose to comply with the quota requirement, instead of paying a quota obligation penalty fee. Our model is static in the sense that we consider a yearly quota and a certificate market in equilibrium. We do not consider banking or lending certificates. Although we consider a representative year, we could decompose this chosen period into time segments as in standard capacity expansion models. We do not include such a time segment dimension here in order to simplify notation, but in empirical applications it would be essential to recognize the diverse operation of the system in different time segments.

Furthermore we assume that the electricity and certificate markets are complete, such that there is a unique price for shared resources - and all relevant players share this price. In an incomplete market, players may price the shared constraints differently.

Let us define the following notation.

Sets

R	regions (we assume that each node represents a region), indexed by *r* and *k*
I	generation technologies, indexed by *i*
I_c	generation technologies eligible for electricity certificates, indexed by *i*
F	electricity producing firms, indexed by *f* or *g*
N	loops in electricity network, indexed by ν
K	lines in electricity network, indexed by (r, k)
K_ν	lines in loop ν, indexed by (r, k)

Parameters

c_i	marginal cost of production for technology *i*
$Q_{0,\,r}$	demand quantity intercept in region *r*
$P_{0,\,r}$	price intercept in region *r*
L_{rk}	transport capacity from region *r* to region *k*
R_{rk}	reactance on link from region *r* to region *k*
G_{ifr}	production capacity of technology *i* in firm *f* in region *r*
$k_{rk\nu}$	indicates if line from region r to region k is included in loopflow ν, takes values −*1*, *0* or *1*
V	green certificate volume

Variables

s_{fr}	supply by firm *f* to region *r*
x_{ifr}	production in firm *f* using technology *i*
z_{rk}	net flow from region *r* to region *k*
a_r	arbitrage flow into region *r*
p_r	supplier price of electricity in region *r*
w_r	(dual) transport cost from the grid into region *r*
κ_ν	(dual) grid transport cost to impose Kirchhoff's voltage law in loop *v*
τ_{rk}	(dual) price on grid transmission capacity from region *r* to region *k*
φ_{ifr}	(dual) price on production capacity by firm f and technology i
ω_f	(dual) marginal income by firm *f*
γ	(dual) marginal cost of restricting net arbitrage to zero
μ	price of green certificate
t_μ	consumption tax rate to finance the green certificate support scheme

4.1. Producer problem (Cournot competition)

The producers choose their generation and sales in order to maximize profit. They are aware that their production will influence the market price (Cournot competition). Producer *f* solves the following quadratic program:

$$\underset{s_{fr},x_{ifr}}{Max}\sum_{r\in R}\left[(p_r-w_r)s_{fr}-\sum\nolimits_{i\in I}(c_i-w_r)x_{ifr}+\sum\nolimits_{i\in I_c}\mu\, x_{ifr}\right]$$

$$=\sum\nolimits_{r\in R}\left[\begin{array}{c}\left(P_{0r}-\left(\dfrac{P_{0r}}{Q_{0r}}\right)\left(\sum\nolimits_{g\in F}s_{gr}+a_r\right)-t_\mu-w_r\right)s_{fr}\\ -\sum\nolimits_{i\in I}(c_i-w_r)x_{ifr}\ +\ \sum\nolimits_{i\in I_c}\mu\, x_{ifr}\end{array}\right]$$

Each producer maximizes its profit, which is comprised by three components: income, production cost and certificate income. The wheeling cost w_r is paid to the TSO for transporting power s_{fr} to region *r* from the transmission network. When the producer generates power x_{ifr} in region *r*, the TSO pays the regional wheeling fee to the producer for receiving power into the network. The producer also receives the certificate price μ for each MWh of renewable electricity x_{ifr} generated using a technology eligible for certificates $i \in I_c$.

$$Supply:\ \sum\nolimits_{r\in R}s_{fr}-\sum\nolimits_{i\in I}\sum\nolimits_{r\in R}x_{ifr}\le 0,\quad f\in F\quad(\omega_f)$$

The supply constraint inhibits the producer from selling more power s_{fr} than it produces x_{ifr}. It is possible to produce more power than supplied. This would imply that the marginal income ω_f is zero.

$$Prodlim:\ \ x_{ifr}\le G_{ifr},\, i\in I, f\in F,\quad r\in R\ \left(\varphi_{ifr}\right)$$

The prodlim constraint represents production limits. We assume that each production facility has an upper capacity bound G_{ifr}, with a shadow price of φ_{ifr}. At last we add nonnegativity constraints on the decision variables for supply and generation.

$$s_{fr}\ge 0, x_{ifr}\ge 0$$

The Karush-Kuhn-Tucker (KKT) conditions are found by formulating the Lagrangian function and taking partial derivatives with respect to the independent variables and to the Lagrange-multipliers (dual variables). We collect the complete set of KKT conditions in the end of this section.

4.2. Grid owner/TSO problem (Nash-Bertrand assumption)

We assume that the grid owner naively acts as a price taker, and chooses grid flows to maximize its profit while adhering to Kirchhoff's current and voltage laws and transmission capacities.

$$\underset{z_{rk},z_{kr}}{Max} \sum\nolimits_{r\in R}\left[w_r\left(\sum\nolimits_{f\in F}s_{fr}-\sum\nolimits_{f\in F}\sum\nolimits_{i\in I}x_{ifr}+a_r\right)\right]$$

The grid owner maximizes his income from the wheeling fee on power flowing to each region.

$$KCL: \sum\nolimits_{f\in F}s_{fr}-\sum\nolimits_{f\in F}\sum\nolimits_{i\in I}x_{ifr}+a_r-\sum\nolimits_{k\in R}z_{kr}+\sum\nolimits_{k\in R}z_{rk}=0, r\in R \quad (w_r)$$

The KCL constraint states Kirchhoff's current law: The sum of currents flowing into a node or region is equal to the sum of currents flowing out of that node, so the sum of all currents meeting in region r must be zero.

$$KVL: \sum\nolimits_{(r,k)\in K_v}R_{rk}(z_{kr}-z_{rk})=0, \quad v\in N \quad (\kappa_v)$$

Kirchhoff's voltage law (also called Kirchhoff's loop rule) is represented by the KVL constraint. The law says that the directed sum of the electrical potential differences (voltages) around any closed cycle in the network is zero. A potential difference over the cycle would create a current, and we cannot have a positive flow running through any cycle in the network. The sum of flows adjusted by the reactance R_{rk} of the line between region r and region k must be zero.

$$Flowlim: z_{rk}\leq L_{rk}, \quad (r,k)\in K \quad (\tau_{rk})$$

The flowlim constraint represents the capacity of the lines. This capacity depends on temperature, security limits and other parameters, but we assume a directed net transfer capacity L_{rk} for each line.

We also need nonnegativity constraints on the directed flow variables.

$$z_{rk}\geq 0$$

4.3. Arbitrager (Nash-Bertrand assumption)

If there are price differences between regions, arbitragers try to buy power at a lower price and sell at a higher price, exploiting these price differences.

We assume that the arbitrager is a price taker, and solve the following profit maximization problem:

$$\underset{a_r}{Max} \sum_{r\in R}[(p_r-w_r)a_r]$$

The arbitrager will buy power in region k and sell to region r if $p_r - w_r > p_k - w_k$.

$$Arbizero: \sum\nolimits_{r\in R}a_r=0 \quad (\gamma)$$

Since the arbitrager does not generate power, the sum of regional arbitrage quantities a_r must be zero. These variables can be both positive and negative.

$$a_r \text{ free}, \qquad r\in R$$

4.4. Tax agent

The tax agent minimizes the tax needed to finance the green certificates that are necessary to fulfill the renewable quota obligation.

$$\underset{t_\mu}{\text{Min}}\ t_\mu$$

The tax rate should be as low as possible, in order to minimize the socioeconomic deadweight loss the tax will incur. The tax on electricity must cover the value of the certificates.

$$Tax: \sum\nolimits_{r\in R}\sum\nolimits_{f\in F}s_{fr}t_\mu\geq\sum\nolimits_{r\in R}\sum\nolimits_{f\in F}\sum\nolimits_{i\in I_c}x_{ifr}\mu \quad (\lambda_1)$$

The tax rate is nonnegative.

$$t_\mu\geq 0 \quad (\lambda_2)$$

4.4.1. KKT conditions

$$\frac{\partial\mathcal{L}}{\partial t_\mu}=1-\lambda_1\left(\sum_{r\in R}\sum_{f\in F}s_{fr}\right)-\lambda_2=0$$

$$\frac{\partial\mathcal{L}}{\partial\lambda_1}=-\sum_{r\in R}\sum_{f\in F}s_{fr}t_\mu+\sum_{r\in R}\sum_{f\in F}\sum_{i\in I_c}x_{ifr}\mu\leq 0 \quad \perp(\lambda_1\geq 0)$$

$$\frac{\partial\mathcal{L}}{\partial\lambda_2}=-t_\mu\leq 0 \quad \perp(\lambda_2\geq 0)$$

We see from $\frac{\partial\mathcal{L}}{\partial t_\mu}$ that at least one of the dual variables must be strictly positive.

$$\lambda_1>0\Rightarrow\sum_{r\in R}\sum_{f\in F}s_{fr}t_\mu-\sum_{r\in R}\sum_{f\in F}\sum_{i\in I_c}x_{ifr}\mu=0$$

$$\lambda_2>0\Rightarrow t=0$$

Thus the tax financing condition can be written:

$$\left(0\leq\sum_{r\in R}\sum_{f\in F}s_{fr}t_\mu-\sum_{r\in R}\sum_{f\in F}\sum_{i\in I_c}x_{ifr}\mu\right)\perp(t_\mu\geq 0)$$

4.5. Consumer/market clearing

The representative consumer acts as a price taker. We assume a linear demand curve. Her willingness to pay for a quantity q_r is $w_r(q_r)=P_{0r}-\frac{Q_{0r}}{P_{0r}}q_r$. The consumer wants to maximize her consumer surplus:

$$\underset{Q_r^*}{Max}\int_0^{Q_r^*}(w_r(q_r)-p_r-t_\mu)\,dq_r \quad \text{where} \quad w_r(q_r)=P_{0r}-\frac{Q_{0r}}{P_{0r}}q_r$$

4.5.1. KKT condition

$$\frac{d}{dQ_r^*}\left[\int_0^{Q_r^*}w_r(q_r)\,dq_r-p_rQ_r^*-t_\mu Q_r^*\right]=0$$

$$P_{0r}-\frac{P_{0r}}{Q_{0r}}q_r=p_r+t_\mu \text{ where } q_r=\sum\nolimits_{f\in F}s_{fr}+a_r$$

Thus the market clearing condition which maximizes consumer surplus is:

$$P_{0r}-\frac{P_{0r}}{Q_{0r}}\left(\sum_{f\in F}s_{fr}+a_r\right)-\ p_r-t_\mu=0, p_r \text{ free}, r\in R$$

4.6. Certificate/quota constraint

Regulating authorities decide a volume V of new renewable electricity production.

$$Elcert:\ \sum_{i\in I_c}\sum_{f\in F}\sum_{r\in R}x_{ifr}\geq V \qquad \perp(\mu\geq 0)$$

The dual price μ of this constraint becomes the value of certificates. This is the lowest certificate value needed to achieve the target of renewable production. Producers could choose to generate more than the target, in which case the certificate value will be zero.

The combined KKT conditions and the additional quota constraint that constitute the full equilibrium model are as follows:

Producers:

$$\left(0\leq -p_r\ \ +\frac{P_{0r}}{Q_{0r}}s_{fr}+w_r+\omega_f\right)\perp(s_{fr}\geq 0), f\in F, r\in R$$

$$\left(0\leq c_i-w_r-\mu+\varphi_{ifr}-\omega_f\right)\perp(x_{ifr}\geq 0), f\in F, r\in R, i\in I_c$$

$$\left(0\leq c_i-w_r\qquad +\varphi_{ifr}-\omega_f\right)\perp(x_{ifr}\geq 0), f\in F, r\in R, i\in I\backslash I_c$$

$$\left(0\leq \sum_{i\in I}\sum_{r\in R}x_{ifr}-\sum_{r\in R}s_{fr}\right)\perp(\omega_f\geq 0),\ \ f\in F$$

$$(0\leq G_{ifr}-x_{ifr})\perp\left(\varphi_{ifr}\geq 0\right), i\in I, f\in F, r\in R$$

TSO:

$$\left(0\leq w_r-w_k+\sum_{v\in N}k_{rkv}\kappa_v+\tau_{rk}\right)\perp(z_{rk}\geq 0), (r,k)\in K$$

$$\left(\sum_{f\in F}s_{fr}-\sum_{f\in F}\sum_{i\in I}x_{ifr}+a_r-\sum_{k\in R}z_{kr}+\sum_{k\in R}z_{rk}=0\right)\ (w_r \text{ free}), r\in R$$

$$\left(\sum_{(r,k)\in K_v}R_{rk}(z_{kr}-z_{rk})=0\right)\ (\kappa_v \text{ free}), v\in N$$

$$(0\leq L_{rk}-z_{rk})\perp(\tau_{rk}\geq 0),\ \ (r,k)\in K$$

Arbitrager:

$$(-p_r+w_r+\gamma=0)\ (a_r \text{ free}), r\in R$$

$$\left(\sum_{r\in R}a_r=0\right)\ (\gamma \text{ free})$$

Tax agent:

$$\left(0\leq \sum_{r\in R}\sum_{f\in F}s_{fr}t_\mu-\sum_{r\in R}\sum_{f\in F}\sum_{i\in I_c}x_{ifr}\mu\right)\perp(t_\mu\geq 0)$$

Certificate/quota condition:

$$\left(0\leq \sum_{i\in I_c}\sum_{f\in F}\sum_{r\in R}x_{ifr}-V\right)\perp(\mu\geq 0)$$

Market clearing condition:

$$\left(P_{0r}-\frac{P_{0r}}{Q_{0r}}\left(\sum_{f\in F}s_{fr}+a_r\right)-\ p_r-t_\mu=0\right)\ (p_r \text{ free}), r\in R$$

5. Illustrative example

In our example, authorities want to subsidize electricity generated by renewable technologies at the expense of cheaper but polluting technologies using fossil fuels. The example is based on and expanded from Hobbs (2001). It illustrates the application of the suggested model, and is designed to permit verification by the reader. We have three price zones, $r = 1, 2, 3$ and each pair of zones is interconnected by a single transmission line (see Fig. 3). All three lines have equal impedances. Each zone has customers, and the demand functions are

$$w_r(q_r)=40-\frac{40}{500}q_r, \text{for } r=1,2 \text{ and } w_3(q_3)=32-\frac{32}{620.4}q_3.$$

There are two producers $f = 1, 2$, each with one generator. Firm 1's generator is sited at $r = 1$, while 2's is at $r = 2$. Both generators have unlimited capacity, and a constant marginal cost: \$15/MWh for firm 1, and \$20/MWh for firm 2. The only transmission cost arises from congestion.

We consider two different transmission systems. One with infinite transmission capacity and no congestion, and one with congestion on a capacitated transmission line between region 1 and 2. The flow capacity is 25 MW either direction. These two cases are solved for three types of competition: Perfect competition, Cournot competition without arbitrage and Cournot competition with arbitrage. The arbitrager eliminates price differences between regions, erasing any non-cost based differences in price. Such price differences do not appear under perfect competition.

We expand the example by introducing a quota obligation of 80 MWh renewable electricity. This represents a production increase between 8 and 15% compared to previous production in the different cases. We assume that all existing generation is based on fossil fuels, and that both existing firms may invest in renewable generators with a LRMC equal to \$24/MWh (located in the same region as the existing generator). We also introduce potential new firms in regions 1 and 2 with the opportunity to invest in the same generation technology with the same costs as the existing firms. A regulator issues a certificate for each MWh of renewable electricity, and electricity suppliers must buy its relative share of certificates. The certificate cost is allocated to consumers by a certificate tax on top of the electricity price.

Introducing new firms under Cournot competition has its own effects regardless of the support scheme. We want to separate these effects from the support scheme effects, so we define three variants for each case (in addition to the original case). The variants are summarized in Table 1.

The model has been programmed in GAMS,[2] and the mixed complementarity problem has been solved with the PATH solver (Dirkse and Ferris, 1995).

Numerical results from each case variant are reported in the Appendix A, see Tables 3 to 5. The tradable green certificates together with the consumer tax lead to large deadweight losses in our numerical example, amounting to >100% of the total certificate value in all cases.

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[2] General Algebraic Modeling System, see www.gams.com.

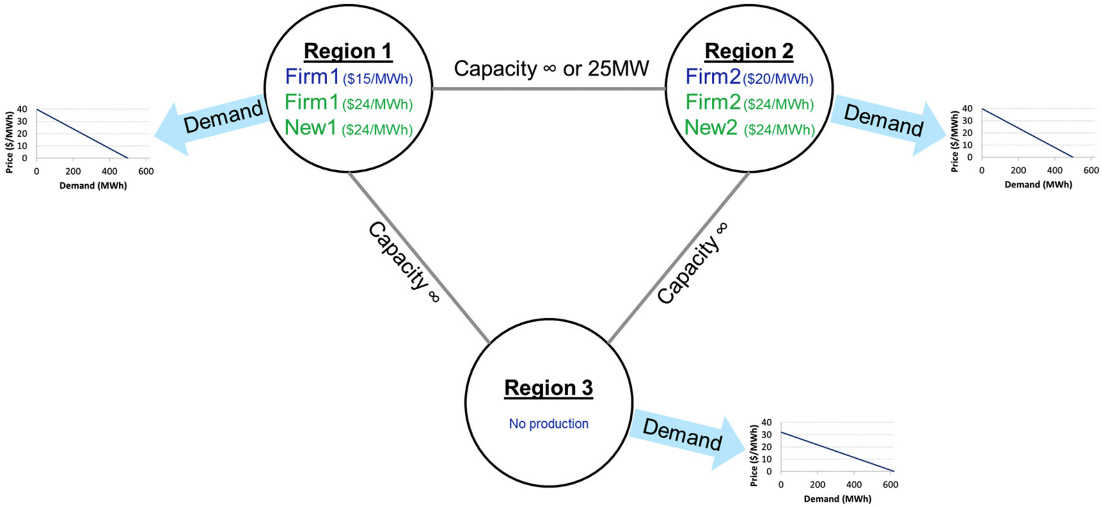

Fig. 3. Illustrative example.

The reason for this extreme result is that the certificates cover the increase in production costs, but provide no profit to producers because of constant marginal cost. Consumers pay for more expensive production, without getting any extra direct benefit (keep in mind that the reason for the scheme is not represented in the partial model, so these benefits are not captured). Thus the whole value of the certificates is lost, and the deadweight loss from the consumer tax results in a net loss.

Our analysis proceeds as follows: First we describe properties of each case equilibrium briefly, and then discuss a sensitivity analysis in one of the cases to get a sense of the equilibrium. Then we look at welfare effects of the different case variants, and summarize the aggregated social welfare effects for consumers and firms.

5.1. Perfect competition - uncapacitated network

In this case it does not matter where new production enters, since there is no lack of network capacity. The cheapest generation (wherever located) will enter the market. Since all renewable generators have identical cost, the model solution is indeterminate - any distribution of the mandatory quantity between the four firms is a valid solution. (In Table 5 the 80MWh is arbitrarily allocated to the existing firm $f = 2$.)

The certificate price must cover the difference between current marginal generation marginal renewable generation (equal to 24 minus 15). Producers get no profit from the new generation, since electricity price plus certificate price only cover production cost. Since we have only one renewable technology, the total certificate value balances the cost of certificate production and the scheme does not contribute to increased welfare at all. Then the tax has a negative welfare effect. The new and more expensive generation replaces some of the old and cheaper (but presumably dirtier) generation, and consumers face a higher total price when we include the certificate tax. The total certificate value is 720, but the deadweight loss of the certificates and the tax is higher: 733. The scheme decreases net social welfare more than the scheme costs. The welfare loss depends on the cost of renewable electricity - the higher the cost, the higher the deadweight loss.

5.2. Perfect competition - $LinkCapacity_{12} = 25$ MW

The capacitated network favors production in Region 2, since there is a network bottleneck towards the cheapest generation in Region 1. All renewable generation will be located in Region 2 under perfect competition. As long as the existing and the new firm in region 2 have the same marginal cost of renewable generation, it does not matter which of these firms is producing.

Region 2 has a higher electricity price, since the fossil generation in this region is more expensive than in Region 1. The certificate price needed to cover the marginal renewable generation cost will be correspondingly *lower* (equal to 24 minus 20) compared with the uncapacitated case. Producers earn zero profit, but the TSO earns a profit which does not change when the tradable certificate scheme is introduced. Consumers must pay a certificate tax, which expose them to increased electricity prices. Thus consumer surplus decreases in this case.

5.3. Cournot competition, no arbitrage - uncapacitated network

Under Cournot competition, both new firms will enter the market and generate equal amounts of renewable electricity (since they have identical costs). There is no lack of network capacity, thus the *location* of new firms does not matter. New firms will make positive profits, at the expense of existing firms. Consumer electricity prices after tax decrease, thanks to increased production – thus the consumer surplus increases. Since the electricity prices are high under Cournot competition, the resulting certificate price becomes lower than under perfect competition.

5.4. Cournot competition, no arbitrage - $LinkCapacity_{12} = 25$ MW

New firms still enter the market, but now the location matters. The capacitated network favors production in Region 2, but the new firm in region 1 also enters the market, generating 14 out of 80 MWh. This

Table 1
Case variants.

Variant	Firms able to invest in renewable generation	Support scheme	Comment
0)	Only existing firms	No mandatory quota	Original example
a)	Only existing firms	Mandatory quota with green certificates	Isolated support scheme effects without new firms
b)	New firms may enter	No mandatory quota	Isolated effects from introducing new firms
c)	New firms may enter	Mandatory quota with green certificates	Combined effects from new firms and support scheme

case has the highest regional electricity price, and correspondingly the lowest certificate price.

5.5. Cournot competition with arbitrage - uncapacitated network

Price differences are arbitraged away in this case, so the electricity price is equal in all regions – and lower than the marginal cost of renewable generation. This means that new firms will not enter the market without a mandatory quota with corresponding support. The certificate price and certificate tax remain the same in the arbitrage case as in the one without arbitrage. Both new firms enter and the location of the new firms does not matter. Since they have the same marginal production cost, they generate the same amounts of electricity. Consumers face a lower after tax electricity price than before the scheme, so consumer surplus increases.

5.6. Cournot competition with arbitrage - $LinkCapacity_{12} = 25$ MW

The capacitated network again favors the new firm in region 2, and when arbitrage is possible conditions are even harder for the new firm in region 1. It is able to run with a small profit, and produces 8 out of 80 MWh. The new firm in Region 2 generates the remaining 72 MWh. Electricity price in Region 2 before the support scheme is above the renewable marginal cost of production, so the new firm in Region 2 would enter the market even without a mandatory quota and corresponding support (but with a smaller production volume of 29 MWh).

Generation from each technology in each case is depicted in Fig. 8 in the Appendix A. In cases with Cournot competition there may be opportunities for new firms to enter the market and be able to earn a profit even without a mandatory renewable quota. When arbitrage is possible, competition is harder and new firms produce less compared to cases without arbitrage.

Regional production depends strongly on network transmission capacities. New firms produce equally when the network is uncapacitated. The capacitated network favors production in Region 2, but we see that the new firm in Region 1 still generates a small share of the renewable electricity in the capacitated Cournot cases with mandatory renewable generation quota.

5.7. Sensitivity analysis

Under Cournot competition new firms have higher incentives to employ new technology than existing firms. With equal marginal costs of production, profit gains are higher for new firms than existing ones. In this sensitivity analysis we investigate what happens if the existing Firm 2 has a superior renewable technology, allowing to generate renewable electricity with lower production cost than new firms. We assume a limited capacity of 100 MWh, due to limited natural resources.

We find that marginal production cost must be considerably lower for the existing firm to produce at the expense of new firms that want to enter the market. This is evident in Fig. 4, which shows electricity production by firm in the case of Cournot competition without arbitrage in a capacitated network. Remember that new firms (New 1 and New 2) have a renewable cost of \$24/MWh.

If old Firm 2 can generate renewable electricity at a lower cost than the original technology (\$20/MWh), it runs at full capacity. The certificate price is zero, since the renewable quota target is surpassed. The electricity price in region 2 is still high enough to allow the new firm "New2" to enter the market. Corresponding electricity prices and certificate prices are reported in Fig. 5. If the renewable generation cost of Firm 2 increases above \$20, the firm cuts its renewable generation and the certificate price must cover the difference between the renewable and the fossil generation cost. All regional prices decrease with increasing generation cost of Firm 2, but the certificate price increases more. At a generation cost of \$20.4/MWh firm "New1" is able to enter the market, since electricity price in Region 1 plus certificate price has risen above the production cost of \$24/MWh. If Firm 2's renewable generation cost is above \$21.45/MWh, the certificate price does no longer cover the price difference and Firm 2 chooses not to compete with new firms on renewables, despite having a lower production cost.

In this particular case the consumers gain (see Fig. 5) when new firms conquer market shares. Thanks to increased market competition they pay lower electricity prices even though they must finance the green certificates that support more expensive renewable generation. Thus the consumer surplus increases, despite the deadweight losses from the support scheme. Similar sensitivity analyses for each Cournot case are reported in the Appendix A.

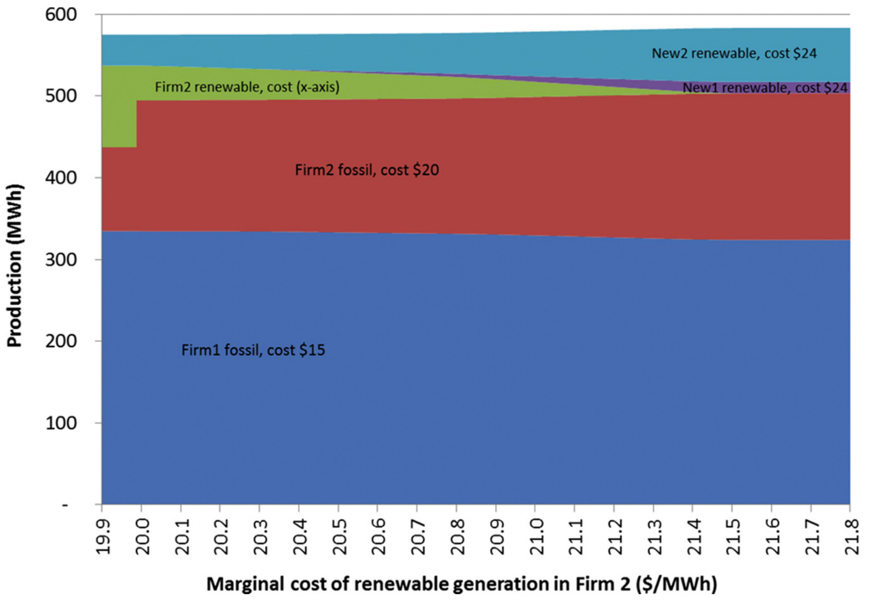

Fig. 4. Equilibrium under Cournot competition without arbitrage in capacitated network.

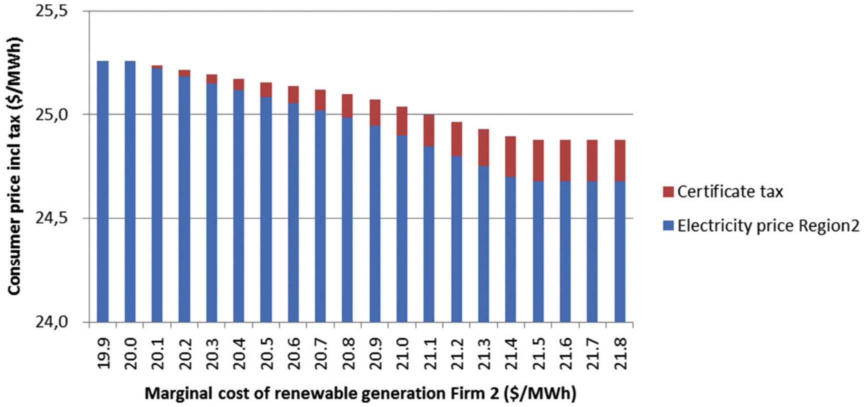

Fig. 5. Consumer price in region 2.

5.8. Social welfare effects in different cases

The sensitivity analysis of our selected case indicated that consumer surplus may increase. Fig. 6 shows the aggregated social welfare effects for consumers (measured by consumer surplus) and firms (measured by profit) for each of our six cases (looking at variant c) where new players may enter the market). All cases show reductions in net social welfare.

Perfect competition constitutes the strongest form of competition among suppliers, and firms earn no profit in our example (but the TSO earns a profit in the capacitated cases). Consequentially consumers must pay the full deadweight loss incurred by the certificate scheme.

In contrast, under Cournot assumption the competition among firms is weaker, and firms make high profits. The certificate scheme leads to increased consumer surplus under Cournot competition while total firm profit decrease thanks to increased competition among suppliers. New firms enter the market and increase the market competition since they increase their profits from renewable generation more than the existing Cournot players do. The existing firms choose not to generate renewable electricity, in order to hold back production and keep prices as high as possible. These results confirm our qualitative analysis in Section 3.

None of the cases provides increased profits to the producers (in total). New firms may still make positive profits, but these come at the expense of existing firms. As market competition gets more intense, firms lose profit and consumers gain surplus (regardless of the support scheme).

In order to assess deadweight losses from the support scheme, we need to decompose the social welfare changes into separate effects:

1) Both the tradable certificate scheme with mandatory renewable quota and the accompanying consumer tax have deadweight losses.
2) New producers entering the market will increase competition regardless of the green certificates, leading to welfare changes that are independent of the support scheme.

In Fig. 7 we have decomposed the net social welfare changes for each of the cases. The first column shows the original net social welfare. Column number two and three shows the change in consumer surplus and firms profit respectively, from allowing new firms to enter the market. Column 4 and 5 shows the combined scheme and tax deadweight losses for consumers and firms respectively, leading to the new net social welfare shown in column 6.

Allowing new firms under Cournot competition has its own welfare effects, which are negative for (existing) firms and positive for consumers. Notice that arbitrage makes it unprofitable for new firms to enter the market in the uncapacitated network Cournot case without the mandatory renewable quota.

Fig. 7 shows that welfare effects are diverse, even in this small example. Perfect competition provides the highest net social welfare, but also the highest deadweight loss from the certificate scheme, which in these cases hits the consumers. In the Cournot cases the deadweight losses hit the old firms, for which the entire losses are even higher than the

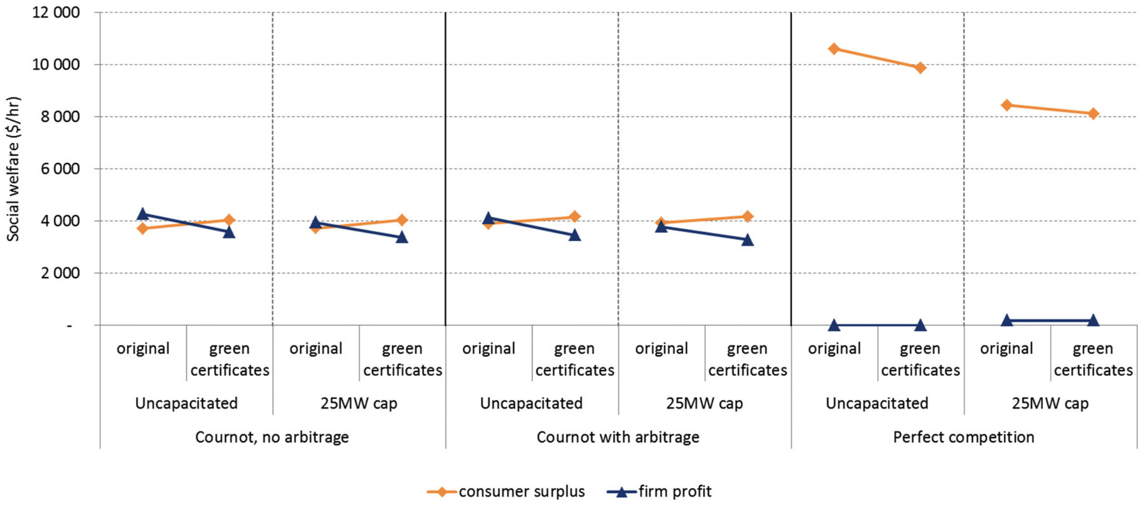

Fig. 6. Social welfare effects for consumers and firms by case with increasing market competition.

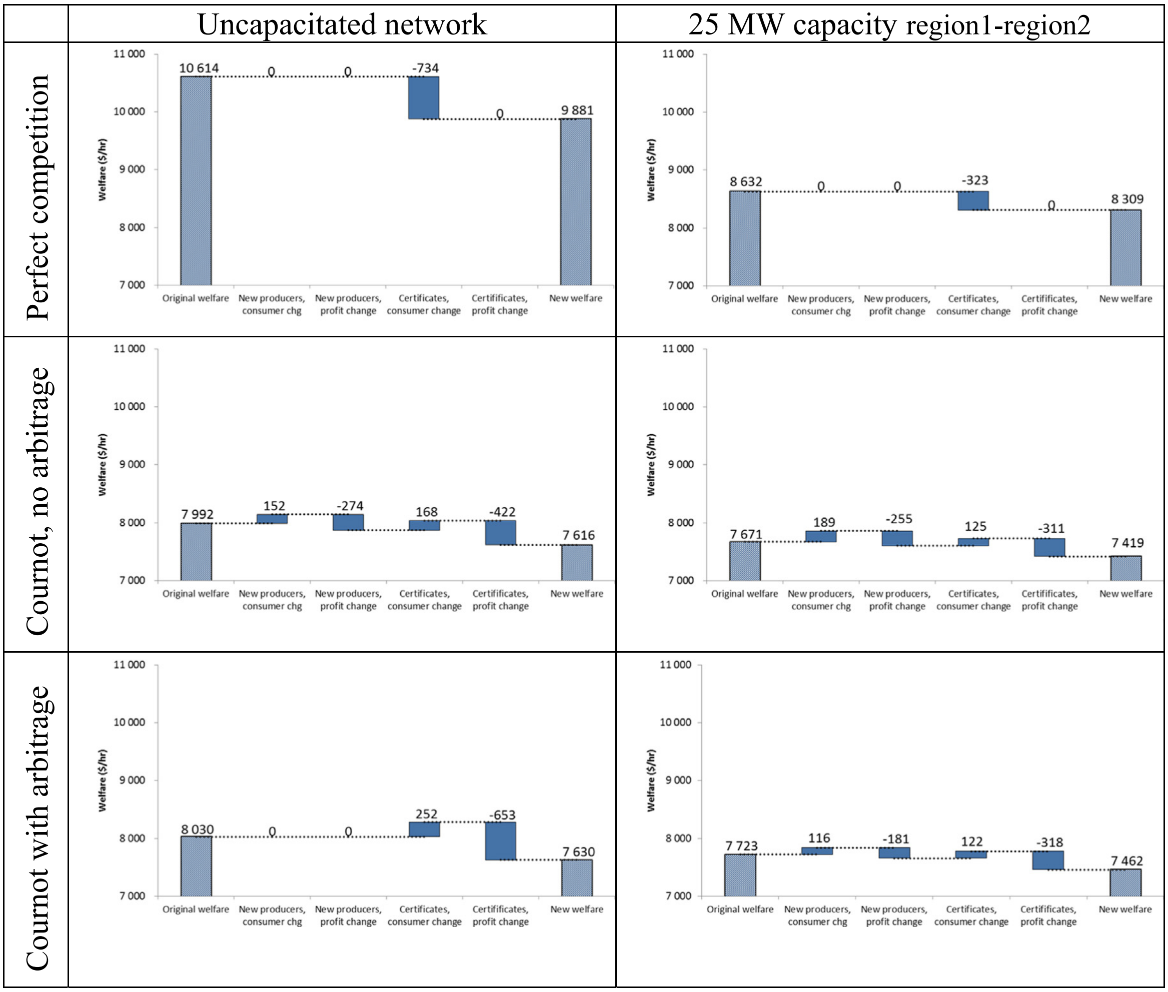

Fig. 7. Decomposed net social welfare effects.

deadweight losses of the support scheme. The capacitated network dampens the impacts compared to the uncapacitated network. Total welfare losses are similar in magnitude for each capacitated case, in contrast to the uncapacitated ones. In the next section we look at a variation where the deadweight losses are shared between consumers and firms.

5.9. Isolated deadweight losses from the support scheme without new players

To further demonstrate the diversity of impacts, we constrain renewable generation to existing firms only and compare the isolated deadweight losses to the situation where we allow new firms. Table 2 shows isolated deadweight losses compared to net social welfare and certificate values. The game outcome becomes the same in each case: Firm 2 should generate the mandatory renewable electricity (but some cases have additional alternative solutions). Thus the isolated social welfare effects from the tradable green certificates are similar in each new case.

In this constrained situation the market competition does not increase, and the deadweight losses are now divided equally between firms and consumers under Cournot competition. It is important to note that consumers do not gain from the support scheme unless new players enter the market (see Table 2). New players are essential in order to improve consumer surplus. Deadweight losses increase, both in absolute terms and relative to welfare. Deadweight losses relative to certificate value however decrease, because the support scheme is more expensive. The deadweight loss is still higher than the total value of certificates in all cases.

The small example shows that a diversity of effects may follow from a tradable green certificate scheme, depending on market competition and network bottlenecks.

6. Conclusions

We present a combined policy model and power market model including network properties from Kirchhoff's circuit laws. We know from previous work that the electric transmission network gives rise to important system effects, and that different forms of market competition have consequences for welfare distribution among market players. In a small example we have demonstrated that both network effects and different forms of market competition give rise to diverse effects of welfare redistribution from a tradable green certificate scheme.

The partial deadweight losses could be substantial. In our example the deadweight losses are higher than the whole value of certificates – each dollar spent on a green certificate would be a direct deadweight loss to society. This extreme result stems from the stylized example, but also realistic deadweight losses may become high, as we will report on in further studies.

The distribution of losses depends on the market power situation. Under perfect competition consumers bear the whole deadweight loss in our example, but this is because firms already have zero profit and thus have nothing to lose. The general picture in Fig. 2 shows that existing firms may lose substantial profits from a green certificate

Table 2
Social welfare losses relative to net social welfare and certificate value.

			Original welfare	certificate value	consumer surplus	profit change	deadweight loss	loss as % of welfare	loss as % of certificate value
Allowing new firms	Uncapacitated								
		Perfect competition	10,614	720	-734	-	-734	-6.9 %	-102%
		Cournot, no arbitrage	7,870	140	168	-422	-254	-3.2 %	-181%
		Cournot with arbitrage	8,030	140	252	-653	-400	-5.0 %	-287%
	25MW cap								
		Perfect competition	8,632	320	-323	-	-323	-3.7 %	-101%
		Cournot, no arbitrage	7,605	116	125	-311	-186	-2.4 %	-160%
		Cournot with arbitrage	7,723	140	122	-318	-196	-2.5 %	-140%
Only existing firms	Uncapacitated								
		Perfect competition	10,614	720	-734	-	-734	-6.9 %	-102%
		Cournot, no arbitrage	7,992	320	-217	-217	-433	-5.4 %	-135%
		Cournot with arbitrage	8,030	320	-217	-217	-433	-5.4 %	-135%
	25MW cap								
		Perfect competition	8,632	320	-323	-	-323	-3.7 %	-101%
		Cournot, no arbitrage	7,671	320	-217	-217	-433	-5.7 %	-135%
		Cournot with arbitrage	7,723	320	-217	-217	-433	-5.6 %	-135%

scheme. Cournot competition is a milder form of competition where firms are able to exploit market power. The support scheme may offer opportunities for new firms to enter the market. This would increase market competition, and consumer surplus may increase while existing firms lose profit. Even if existing firms are permitted to keep their market power under Cournot competition, they must bear half of the deadweight losses incurred by the support scheme.

The support scheme may also offer opportunitites for new firms to gain at the expense of old firms. The sensitivity analysis indicates that existing firms will not be motivated to compete with new generation capacity.

We draw the conclusion that existing firms will typically bear the biggest burdens from a green certificate scheme. The tradable green certificates may lead to substantial reallocations of welfare from existing firms to both consumers and new firms.

The transmission network has major importance for the localization of new production, and for social welfare redistribution effects from the support scheme. Therefore, the combined model of policy instruments in power markets including system effects from the network proves useful in assessing regionalized effects of the support scheme and evaluating alternative policy instruments promoting production of renewable energy.

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Appendix A. Numerical results and sensitivity analyses

Results from the expanded example are presented in the following tables.

Table 3
Profits and social welfare ($/hr).

			Consumer surplus			Profit					Consumer surplus	Profit	Net social welfare
			Region 1	Region 2	Region 3	Firm 1	Firm 2	New 1	New 2	TSO			
Uncapacitated	Perfect competition	0) Original	3,906	3,906	2,802	-	-	-	-	-	10,614	0	10,614
		a) Certif existing	3,665	3,665	2,550	-	-	-	-	-	9,881	0	9,881
		b) New producers	3,906	3,906	2,802	-	-	-	-	-	10,614	0	10,614
		c) Certif new	3,665	3,665	2,550	-	-	-	-	-	9,881	0	9,881
	Cournot, no arbitrage	0) Original	1,406	1,406	906	3,543	731	-	-	-	3,718	4,273	7,992
		a) Certif existing	1,334	1,334	834	3,391	666	-	-	-	3,502	4,056	7,558
		b) New producers	1,482	1,482	906	3,347	635	9	9	-	3,870	3,999	7,870
		c) Certif new	1,581	1,581	876	2,980	470	64	64	-	4,039	3,577	7,616
	Cournot with arbitrage	0) Original	1,633	1,633	646	3,465	653	-	-	-	3,912	4,118	8,030
		a) Certif existing	1,555	1,555	586	3,313	588	-	-	-	3,696	3,901	7,597
		b) New producers	1,633	1,633	646	3,465	653	-	-	-	3,912	4,118	8,030
		c) Certif new	1,723	1,723	718	2,952	442	36	36	-	4,165	3,465	7,630
25MW cap$_{1,2}$	Perfect competition	0) Original	3,906	2,500	2,038	-	-	-	-	188	8,444	188	8,632
		a) Certif existing	3,786	2,404	1,931	-	-	-	-	188	8,121	188	8,309
		b) New producers	3,906	2,500	2,038	-	-	-	-	188	8,444	188	8,632
		c) Certif new	3,786	2,404	1,931	-	-	-	-	188	8,121	188	8,309
	Cournot, no arbitrage	0) Original	1,586	1,237	906	2,541	1,297	-	-	105	3,729	3,942	7,671
		a) Certif existing	1,509	1,170	834	2,414	1,207	-	-	105	3,512	3,726	7,238
		b) New producers	1,653	1,358	906	2,574	978	-	59	76	3,918	3,687	7,605
		c) Certif new	1,705	1,429	909	2,402	763	9	137	65	4,043	3,376	7,419
	Cournot with arbitrage	0) Original	1,905	1,383	646	2,526	1,167	-	-	97	3,933	3,790	7,723
		a) Certif existing	1,820	1,311	586	2,397	1,079	-	-	97	3,717	3,573	7,289
		b) New producers	1,896	1,472	681	2,551	961	-	19	78	4,050	3,609	7,659
		c) Certif new	1,877	1,576	718	2,450	668	1	117	54	4,171	3,291	7,462

Table 4
Prices ($/MWh).

			Electricity price			Transmission price			Capacity dual	Certificate price	Certificate tax
			Region 1	Region 2	Region 3	Region 1	Region 2	Region 3	R1-R2		
Uncapacitated	Perfect competition	0) Original	15.0	15.0	15.0	-	-	-	-	-	-
		a) Certif existing	15.0	15.0	15.0	-	-	-	-	9.0	0.8
		b) New producers	15.0	15.0	15.0	-	-	-	-	-	-
		c) Certif new	15.0	15.0	15.0	-	-	-	-	9.0	0.8
	Cournot, no arbitrage	0) Original	25.0	25.0	22.3	-	-	-	-	-	-
		a) Certif existing	24.8	24.8	22.1	-	-	-	-	4.0	0.6
		b) New producers	24.6	24.6	22.3	-	-	-	-	-	-
		c) Certif new	23.9	23.9	22.3	-	-	-	-	1.7	0.2
	Cournot with arbitrage	0) Original	23.8	23.8	23.8	-	-	-	-	-	-
		a) Certif existing	23.6	23.6	23.6	-	-	-	-	4.0	0.6
		b) New producers	23.8	23.8	23.8	-	-	-	-	-	-
		c) Certif new	23.2	23.2	23.2	-	-	-	-	1.7	0.2
25MW cap$_{1,2}$	Perfect competition	0) Original	15.0	20.0	17.5	-2.5	2.5	-	7.5	-	-
		a) Certif existing	15.0	20.0	17.5	-2.5	2.5	-	7.5	4.0	0.4
		b) New producers	15.0	20.0	17.5	-2.5	2.5	-	7.5	-	-
		c) Certif new	15.0	20.0	17.5	-2.5	2.5	-	7.5	4.0	0.4
	Cournot, no arbitrage	0) Original	24.1	25.9	22.3	-1.4	1.4	-	4.2	-	-
		a) Certif existing	23.9	25.7	22.1	-1.4	1.4	-	4.2	4.0	0.6
		b) New producers	23.7	25.3	22.3	-1.0	1.0	-	3.0	-	-
		c) Certif new	23.3	24.7	22.1	-0.9	0.9	-	2.6	1.4	0.2
	Cournot with arbitrage	0) Original	22.5	25.1	23.8	-1.3	1.3	-	3.9	-	-
		a) Certif existing	22.3	24.9	23.6	-1.3	1.3	-	3.9	4.0	0.6
		b) New producers	22.6	24.7	23.6	-1.0	1.0	-	3.1	-	-
		c) Certif new	22.4	23.9	23.2	-0.7	0.7	-	2.2	1.7	0.2

Table 5
Demand, generation and transmission (MWh).

			Demand			Fossil production		Renewable production			Flow		
			Region 1	Region 2	Region 3	Firm 1	Firm 2	Firm 2	New 1	New 2	R1-R2	R1-R3	R2-R3
Uncapacitated	Perfect competition	0) Original	312.5	312.5	329.6	954.6	-	-	-	-	318.2	323.9	5.7
		a) Certif existing	302.7	302.7	314.4	839.9	-	*80.0*	-	-	253.3	283.9	30.6
		b) New producers	312.5	312.5	329.6	954.6	-	-	-	-	318.2	323.9	5.7
		c) Certif new prod	302.7	302.7	314.4	839.9	-	*80.0*	x	x	253.3	283.9	30.6
	Cournot, no arbitrage	0) Original	187.5	187.5	187.4	392.2	170.2	-	-	-	74.0	130.7	56.7
		a) Certif existing	182.6	182.6	179.8	383.5	81.6	80.0	-	-	74.0	126.9	52.9
		b) New producers	192.5	192.5	187.4	382.2	160.2	-	15.0	15.0	74.0	130.7	56.7
		c) Certif new prod	198.8	198.8	184.3	362.0	140.0	-	40.0	40.0	74.0	129.1	55.2
	Cournot with arbitrage	0) Original	202.1	202.1	158.3	392.2	170.2	-	-	-	74.0	116.1	42.2
		a) Certif existing	197.2	197.2	150.7	383.5	81.6	80.0	-	-	74.0	112.3	38.4
		b) New producers	202.1	202.1	158.3	392.2	170.2	-	-	-	74.0	116.1	42.2
		c) Certif new prod	207.6	207.6	166.8	362.0	140.0	-	40.0	40.0	74.0	120.4	46.4
25MW cap$_{1,2}$	Perfect competition	0) Original	312.5	250.0	281.1	490.6	353.1	-	-	-	25.0	153.1	128.1
		a) Certif existing	307.7	245.2	273.6	482.0	264.5	*80.0*	-	-	25.0	149.3	124.3
		b) New producers	312.5	250.0	281.1	490.6	353.1	-	-	-	25.0	153.1	128.1
		c) Certif new prod	307.7	245.2	273.6	482.0	264.5	*80.0*	-	x	25.0	149.3	124.3
	Cournot, no arbitrage	0) Original	199.1	175.9	187.4	330.3	232.1	-	-	-	25.0	106.2	81.2
		a) Certif existing	194.2	171.0	179.8	321.6	143.4	80.0	-	-	25.0	102.4	77.4
		b) New producers	203.3	184.3	187.4	334.5	202.7	-	-	37.8	25.0	106.2	81.2
		c) Certif new prod	206.5	189.0	187.7	323.9	179.3	-	13.9	66.1	25.0	106.3	81.3
	Cournot with arbitrage	0) Original	218.2	185.9	158.3	334.9	227.6	-	-	-	25.0	91.7	66.7
		a) Certif existing	213.3	181.0	150.7	326.2	138.9	80.0	-	-	25.0	87.9	62.9
		b) New producers	217.7	191.8	162.5	336.5	206.6	-	-	29.0	25.0	93.8	68.8
		c) Certif new prod	216.6	198.5	166.8	329.8	172.2	-	7.8	72.2	25.0	95.9	70.9

An “x” in Table 5 indicates that renewable production could be distributed otherwise between the firms, thus the solution is indeterminate. We have assumed that Firm 2 generates the renewable quantity.

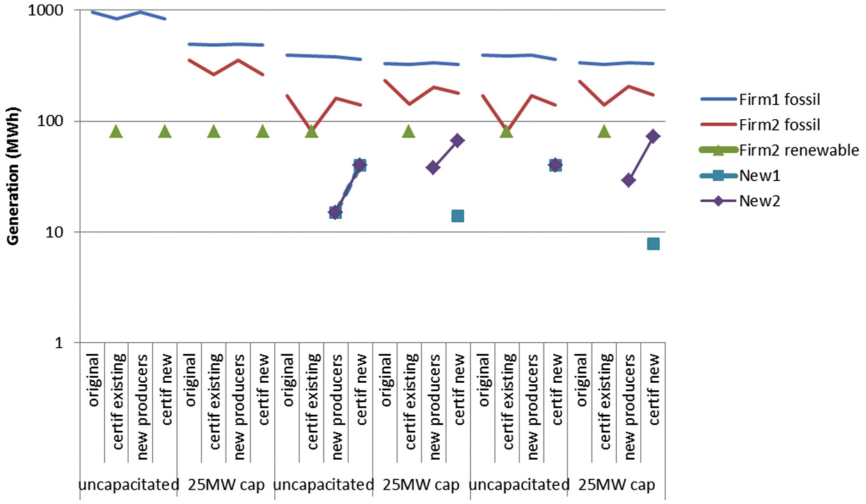

Fig. 8. Generation by case (logarithmic scale).

A.1. Sensitivity analysis of each case

We have seen that new firms will enter the market in cases with Cournot competition green certificates. In this section we investigate production decisions of existing firms if they have *lower* production costs than new firms. We find that marginal production costs must be considerably lower for existing firms to generate new renewable power.

A.1.1. Cournot competition, no arbitrage - Uncapacitated network

What if the existing firms improve their technology, and get a lower production cost? The figure below shows production by firm when we increase production cost for firm $f = 2$ in region 2 (the choice between $f = 1$ or 2 is arbitrary in this case).

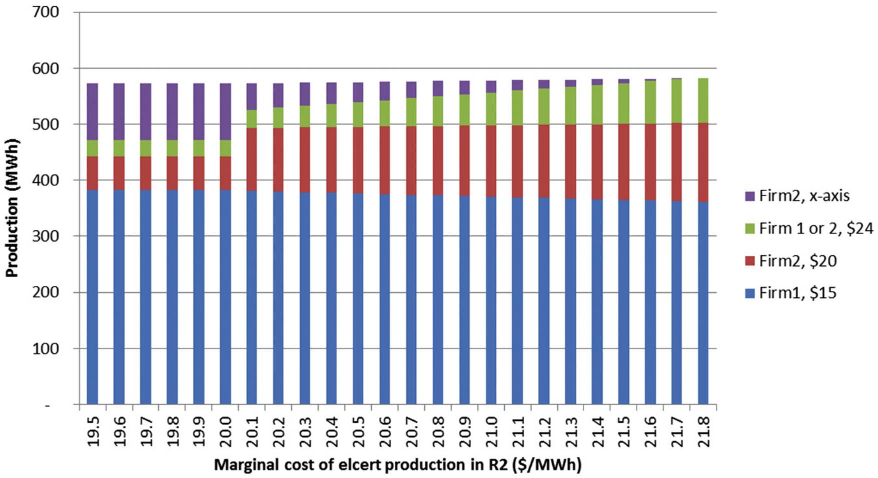

Fig. 9. Equilibrium under Cournot competition without arbitrage in uncapacitated network.

Firm $f = 2$ does not produce any renewable electricity unless the production cost is significantly lower than the new firm's cost of 24. Firm 2 starts generating renewable electricity if the cost comes below the sum of its marginal fossil generation cost and the certificate price (20 + 1.75). If the cost goes down to or below the fossil marginal cost of 20, firm 2 produces only renewable electricity, and more than the certificate quota of 80 MWh. Thus the certificate price is zero. The new firms still produce 30 MWh of renewable electricity in this situation.

Consumers pay a lower price if firm 2's marginal cost of renewable electricity is above 20:

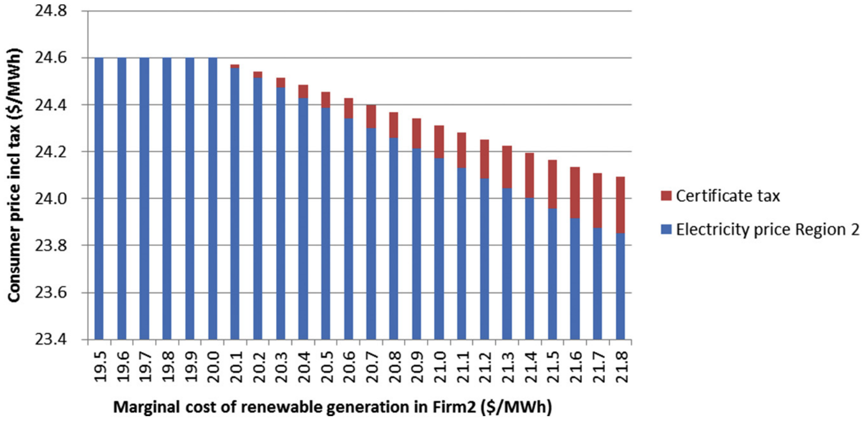

Fig. 10. Consumer price in Region 2.

When the consumer price decreases, the consumer surplus grows – but the firms lose profit and social welfare decreases (as one should expect when marginal production cost increase):

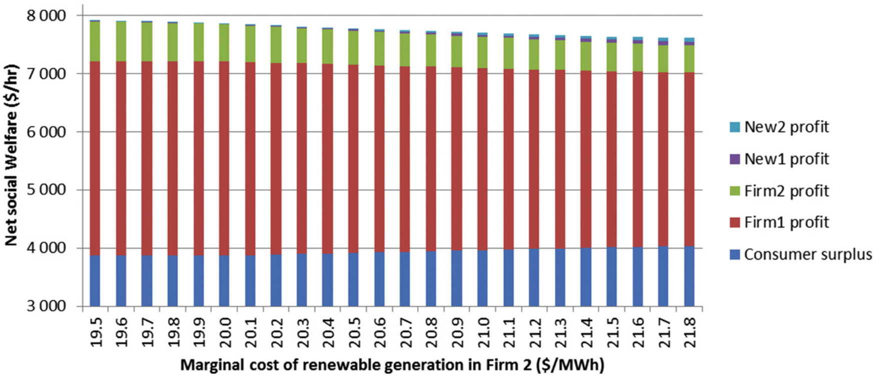

Fig. 11. Net social welfare, Cournot without arbitrage in uncapacitated network.

A.1.2. Cournot competition with arbitrage - Uncapacitated network

An existing firm (let's say firm 2) does not generate renewable electricity unless it is able to decrease its LCOE below the marginal cost of 20 plus the green certificate price of 1.75 as before. A change from the no arbitrage case is that new firms do not generate renewable electricity if the LCOE of firm 2 goes down to 20 or below.

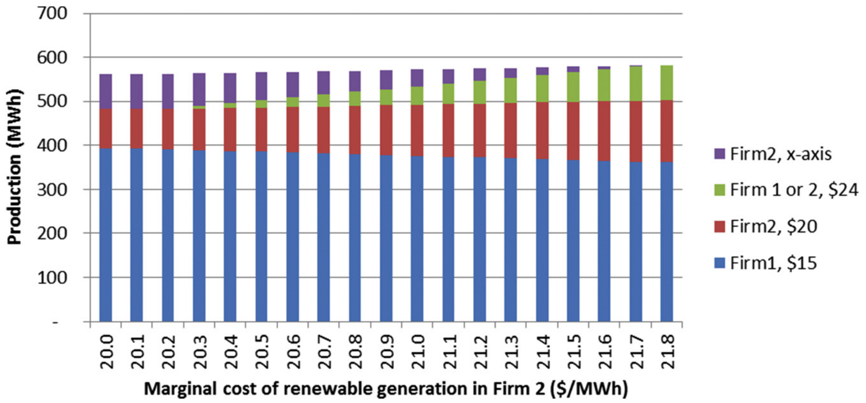

Fig. 12. Equilibrium under Cournot competition with arbitrage in uncapacitated network.

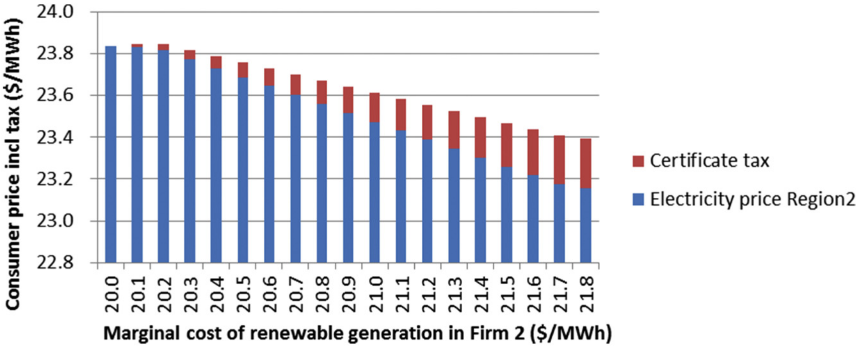

Fig. 13. Consumer price in Region 2.

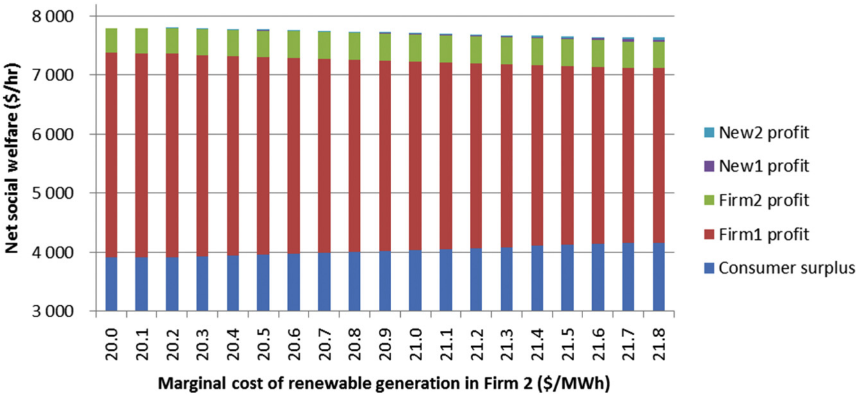

Fig. 14. Net social welfare, Cournot with arbitrage in uncapacitated network.

A.1.3. Cournot competition, no arbitrage - $LinkCapacity_{12} = 25$

How low LCOE must firm 2 have to generate renewable electricity? If firm 2 has LCOE below 21.45 it starts to generate.

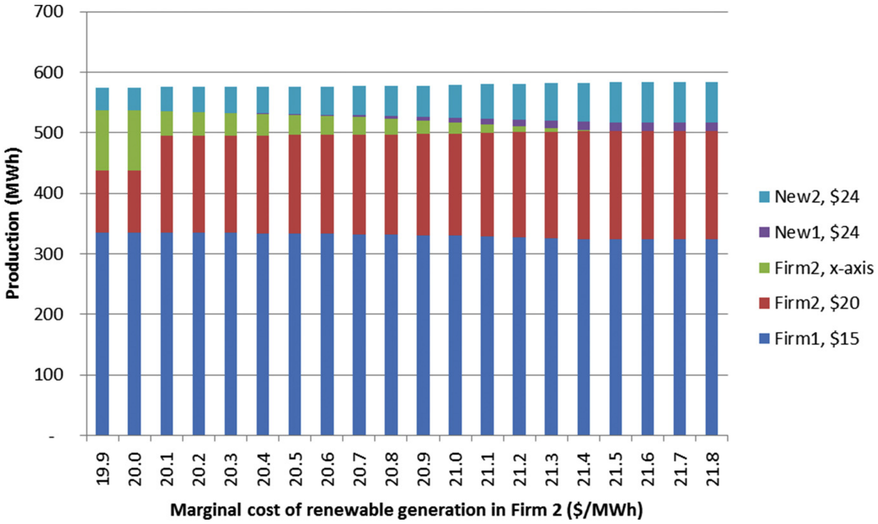

Fig. 15. Equilibrium under Cournot competition with arbitrage in capacitated network.

Again we see the paradoxical picture that consumer price decreases for an increasing firm 2 LCOE cost:

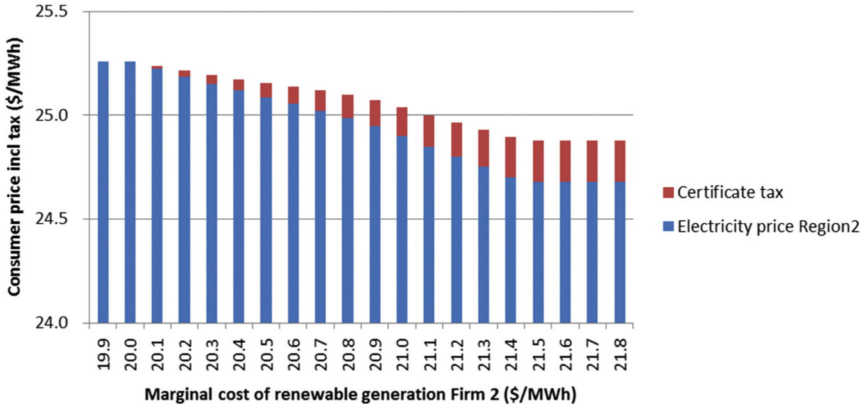

Fig. 16. Consumer price in Region 2.

Thus consumer surplus increases with increasing firm 2 LCOE cost. Existing firms lose profit to new firms in region 2 and then 1. Not surprisingly, net social welfare decreases with increasing LCOE.

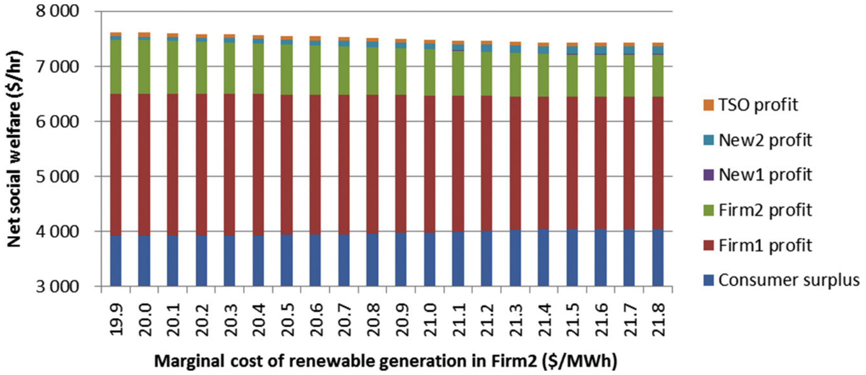

Fig. 17. Net social welfare, Cournot without arbitrage in capacitated network.

A.1.4. Cournot competition with arbitrage - $LinkCapacity_{12} = 25$

How low LCOE must firm 2 have to generate renewable electricity? If firm 2 has LCOE below 21.75 it starts to generate renewable electricity.

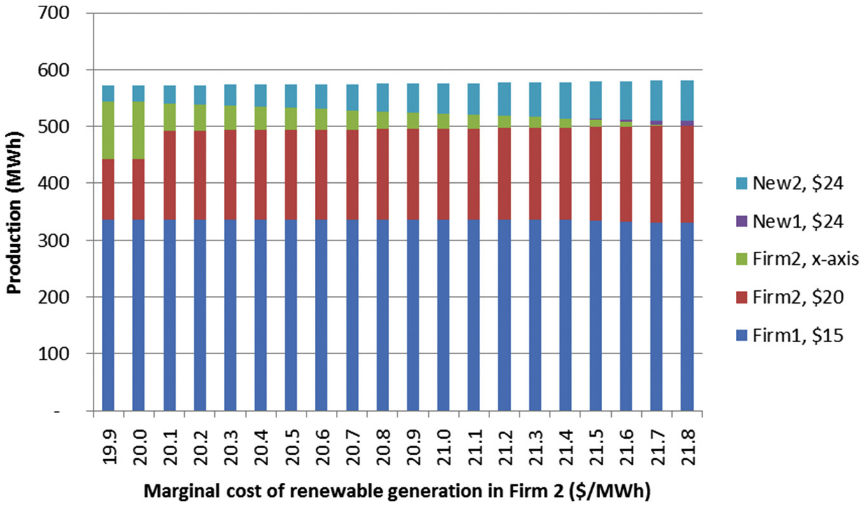

Fig. 18. Equilibrium under Cournot competition with arbitrage in capacitated network.

Consumers in Region 2 would see the following price development.

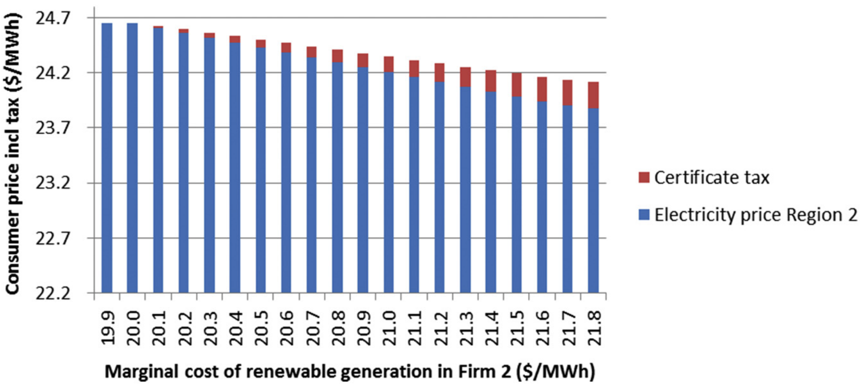

Fig. 19. Consumer price in Region 2.

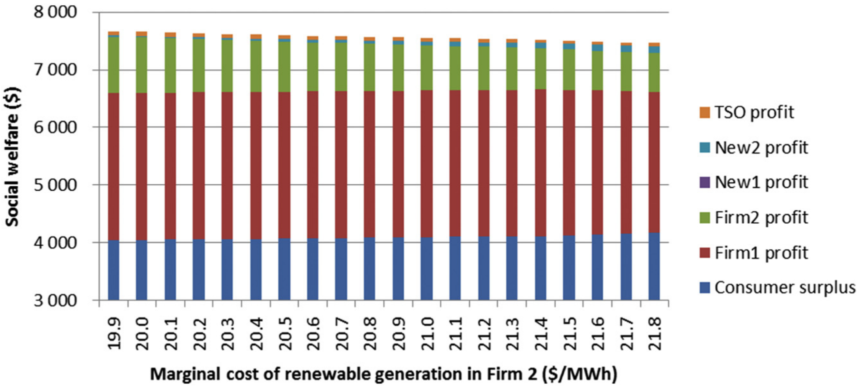

Fig. 20. Net social welfare, Cournot with arbitrage in capacitated network.

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