Internet service provision and content services: paid peering and competition between internet providers

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Internet service provision and content services: paid peering and competition between Internet providers

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Abstract We consider the relationship of Internet service providers (ISP) and content service providers (CP) in the Internet ecosystem. Currently the position of ISPs is challenged by the emergence of powerful content service providers, especially with the spreading of bandwidth demanding video services. The further investment in the network capacity may be hindered by prevailing business models that largely exclude the ISPs from sharing in the major cash flows resulting from content provision.

We develop modeling tools for evaluation of business models of ISPs and present results of analysis of two models with the potential for generation of additional cash flows for ISP: paid content peering and service differentiation. Firstly, we show that under certain conditions on the cost structure and the level of demand elasticity and uncertainty, it can be profitable for a powerful content provider to resort to paid content peering, thus transferring to the ISP a part of his content provision revenue. The resulting business model may provide substantial benefits to all major participants in this ecosystem: network providers, content and service providers and end users. After this we consider competition in the Internet provision sector and show that also in this case the paid content peering can help ISPs to expand the network capacity and at the same time increase profits of content providers. The end users benefit from the lower prices for content services. Finally, we consider the

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situation when an ISP differentiates the service offer by engaging in content provision, thus entering in direct competition with content providers.

1 Introduction

The current state of the Internet presents substantial challenges to telcos/network operators in their capacity of Internet Service Providers (ISP). These challenges are not new, see [46] for an earlier perspective, but their impact has a tendency to grow. In particular, due to the introduction and explosive growth of services that are heavy on content (like video related services) their fixed and mobile networks are experiencing substantial growth of traffic requiring more investment in the network infrastructure [30]. However, the current Internet business models direct revenue streams towards content service providers, in particular those in possession of Content Delivery Networks (CDN) and utilizing content peering. As a result, this revenue stream largely bypasses the ISPs (see, for example, [32], [29], [24]). The growth of cloud based services has a potential to aggravate this situation even more. This jeopardizes the market position of the network operators, which may result in future overall deterioration of network infrastructure due to the lack of investment, something that will be detrimental to all the actors involved in the Internet ecosystem (for in-depth exposition of the underlying Internet structure we refer to [7],[20]).

These issues have generated recently a substantial interest in academic and industrial literature, see [29], [40], [54], where one can find additional references and further discussion of policies for exchange of Internet traffic like peering. Proposals directed towards enhancement of the position of providers of Internet connectivity (ISP) involve paid content peering, when content providers (CP) share their content provision revenue with respective ISP. Different network operators consider the introduction of policies that infringe on network neutrality, but allows them to collect additional revenue by differentiation of subscription fees according to usage. For example, [45] reports that Deutsche Telecom considers a differentiation of subscription fees that will limit the usage of video services from external CPs, but not from its own video service. Similar cases of challenging the network neutrality were reported elsewhere, in particular in France and US. For discussion of economical issues of the network neutrality we refer to [1], [3], [17], [28], [35].

There are two streams of literature of relevance to this paper. Here we provide a survey of the most relevant literature, where one can find further references to related papers. The first body of literature studies interconnection economics and paid peering between two ISPs. *Peering* is an agreement between two or more ISPs to reciprocally admit the traffic generated by customers from one ISP directed towards customers of another ISP [13]. Originally such agreement does not require any reciprocal payments and implies that two networks are directly connected between themselves [44]. The alternative for an ISP is to arrange paid *transit* settlement with some Internet provider having global reach, who will transport the traffic between ISPs for some fee (for in-depth exposition of different Internet interconnection strategies see [19]). Understandably, these agreements are central for the functioning of the Internet as the global network and for this reason they attract attention of industrial analysts, academics and regulators. For example, a simple model for comparing free peering agreement and transit settlement for two ISPs is considered in [55] (see also [2]). This and other qualitative and quantitative analysis shows that ISPs will enter free peering agreements when their reciprocal traffic flows are relatively symmetrical, which is confirmed by industrial practice (see the survey in [14]). In the case of asymmetrical traffic flows, the ISP, who generates substantially smaller amount of traffic in the direction of another ISP, has an incentive to avoid peering agreement. In such case *paid peering* when the generator of larger volume of traffic pays some fee to have his traffic admitted can be an attractive alternative, as recent literature suggests.

In particular, [33] shows that paid peering can increase the incentive for bilateral and multilateral peering agreements among ISPs with asymmetric traffic flows. A non cooperative game theoretic model for analysis of paid peering between two ISPs is considered in [49]. It is shown that rational ISPs will choose paid peering also in the case when they will not settle on free peering. Both ISPs are strictly better off when compared to not peering at all. In addition, paid peering increases the incentive to invest in capacity. More precise game theoretic analysis of different interconnection regimes between two ISPs equipped with infinite capacity networks is considered in [26] where, in addition to selection of interconnection by transit, free and paid peering ISPs compete in Internet connection prices charged to heterogeneous customers. It is shown that for medium ranges of network asymmetry, paid peering dominates both alternative interconnection regimes. The authors of [14] analyze paid peering by defining the value of peering link, solving the resulting matrix game between two ISPs and calculating the optimal peering price, which yields in a certain sense a fair, optimal and stable peering agreement. They note that paid peering leads to a higher density of peering links. In [57] it is observed that peering agreement on the basis of pure traffic ratios between ISPs may be misleading and a simple game theoretic approach is advocated, which considers the economic benefits directly. Finally, the approach of cooperative game theory to profit sharing between two ISPs exchanging traffic is developed in [58].

The present paper differs from this literature in that instead of generic ISPs we focus on peering relationship between ISPs and CPs, which brings specific issues studied here. In particular, reciprocal traffic disbalance between ISP and CP is extreme and reaches several orders of magnitude, service pricing by CPs and the network capacity expansion by ISPs become important aspects to analyze. Dependence of service demand on pricing and Quality of Experience (QoE) should also be considered as well as the random nature of demand for new services, which are constantly updated. Other issues of importance are the competition between ISPs in the presence of strong CPs and vertical integration of ISPs with content provision. Besides these specific issues our paper differs by explicit consideration of CPs with commanding market power (see discussion further in this section).

Another body of literature related to this paper is the literature on economics of network neutrality. This is a vast topic, recently surveyed in [28], see also [1], [3], [17], [35]. A large part of this literature considers the relationship between ISPs and CPs as a two sided market, where an ISP serves as a platform connecting CPs with end users. From this point of view paid content peering serves as termination fee, which the CP pays to the ISP. The closest part of this literature regarding our paper investigates the effects of these fees on investment decisions by an ISP. The paper [39] considers a market consisting of one CP and several ISPs, which are the monopolists on their respective user markets. This situation is modeled as Stackelberg game with ISPs as leaders and it is found that under certain conditions ISPs incentives to invest increase in the presence of termination fees. A six stage game between the mass of CPs, two competing ISPs and a mass of consumers is considered in [43], where it is found that in the presence of payments of CPs to ISPs the ISPs investments are higher. A similar conclusion is reached in [3], [16], [5] [27] using different modeling assumptions and techniques. However, [6] [48] [34] are more nuanced, finding conditions when such payment diminishes investment by ISPs. The summary of this debate is given in [28], p. 804 as "in light of these many arguments for and against a termination fee model, the policy conclusion is not obvious".

Finally, there are papers, which similarly to our paper focus on specific relations between CP and ISP. The closest problem setting to our paper is considered in [8], [9], where the authors analyze the paid peering between ISPs and CPs. The Nash bargaining model for defining the optimal peering charges for a system consisting of a single ISP and single CP is considered in [8]. Service prices of a CP and capacity expansion by an ISP are outside of this model. These aspects are considered in [9] using a Stackelberg game with an ISP acting as a leader and several CPs as followers. Demand in both papers is considered to be known and deterministic. Interesting related development is analyzed in [11], where a CP invests directly in the ISPs network infrastructure instead of making additional payments.

The present paper adds to the previous literature in two aspects. Firstly, it analyses the relationship between a CP and an ISP from the new important angle not studied before and, secondly, it enriches the previous discussion by analysis of new market features, relevant for understanding the relationship between these actors.

1. *New angle: a negotiation process between a powerful CP and ISPs.* We explicitly investigate the situation, when the CP has user appeal and market power sufficient to dictate the peering conditions to ISPs, something that was not analyzed before. Analysis of this situation is important due to the following reasons.

Such situations represent an important aspect of industrial reality. The large CPs held substantial, even dominating market power. For example, [41] reports that the ten largest CPs amount to more than 63% of the time U.S.

users spend online. According to [47] Netflix alone accounts for about 30% of all Internet traffic in the US. Such large CPs with exclusive content do wield this market power in order to put pressure even on reasonably large ISPs in order to get favorable conditions. For example, [42] reports the words of the head of the Norwegian division of Telenor (the largest Norwegian ISP with international reach) Berit Svendsen, who said back in 2012 about their relationship with Netflix: "They do not want to pay for the capacity they use as others do, and now they use threats in order to get a free service". Similarly, describing the situation in the US [47] reports that Netflix was successful in dictating its terms (amounting to free content peering) to some ISPs: "cable operators Cablevision in the Northeast and Grande Communications in Texas have agreed to Netflix's business terms for interconnection", while Comcast resisted.

Surprisingly, the existing literature sheds very little light on this practically important situation. The closest existing research deals with modeling of the relationship between two generic ISPs with asymmetric traffic flows and establishes that paid peering in such situation leads to a peering arrangement when free peering is rejected [26], [49]. This, however, is still far from considering a relationship between a powerful CP and an ISP. The literature on network neutrality is concerned with the opposite situation: a powerful ISP, which dictates its conditions to CPs. We believe that the reason for this is that the modeling approach taken in this literature serves well for modeling the case with dominating ISP, but it is much less useful for the case of a dominating CP. The literature on network neutrality attempts to model a stylized market with reasonably timed moves of participating actors. Within this approach it is natural to assume that the ISP makes the first move by deciding the capacity expansion and possibly the termination fee followed by moves of CPs like service pricing. This is because the capacity expansion is a long term decision, while pricing is a short term one. This leads either to a noncooperative game between equally important players or a Stackelberg game with the ISP as a leader, which is an adequate tool for modeling a powerful ISP.

In order to model adequately the case of a dominating CP we depart from this approach and do not attempt to model a sequence of natural moves in a stylized market. Instead, we model a stylized *negotiation process* between a powerful CP and ISPs. In such process the natural sequence of market moves yet to make does not matter and a powerful CP takes the role of a negotiation leader, reveals first its planned decision on content peering to less powerful ISPs and receives their response in the terms of planned connection quality, which follows from their planned decision on capacity investment. After a sequence of such exchanges this process is concluded with a binding agreement. The parties to this agreement proceed with the implementation of the agreed decisions in their natural market sequence. A Stackelberg game with a CP as a leader is a natural modeling tool in this setting.

We know that such negotiations between dominating CPs and less powerful ISPs proceed in the industry, and agreements on paid contents peering and similar arrangements have been concluded. Indeed, by now the relationship between Telenor and Netflix has evolved to a strategic partnership [52]. Netflix has also concluded agreements with several other European ISPs [37] and Comcast [56]. Generally, the climate between CPs and ISPs has improved considerably [51]. So, the relevant research questions to answer are: what is going on? Why even powerful CPs conclude agreements with ISPs involving paid peering instead of forcing them to accept free peering? Apparently, these agreements are beneficial to both parties. Under what conditions this occurs? These are the questions, which we try to answer in this paper.

Thus, our main contribution consists in demonstrating that paid peering can be beneficial to both CPs and ISPs and we find conditions under which even a dominating CP should voluntarily offer a share in his profits to the ISP. Briefly, such conditions require a reasonably efficient ISP (in terms of the network maintenance and expansion costs), moderate demand uncertainty and high elasticity innovative services. The CP does this in order to facilitate the network expansion by ISPs with resulting increase in the volume of demand, which can be served with required QoE. We show this in different settings: the case of a single ISP, which enjoys a monopolistic position in its market (Section 2), the case of competition between two ISPs (Section 3), and , finally, the case when an ISP diversifies into content provision (Section 4; this case was motivated by the policy of Deutsche Telecom, reported in [45]). While the large part of previous debate on network neutrality and paid peering has seen ISPs pushing in favor of paid peering and CPs resisting this push [28], we show that the existence of this divide should not be taken for granted and explore conditions under which CPs should support ISPs in establishing the paid peering agreements.

2. *Analysis of new market features.* Our other contribution consists in the analysis of important market aspects, which were either not studied before or have been studied insufficiently. In particular, we consider a more realistic description of demand: it is assumed to be stochastic. Thus, we are able to analyze the consequences of demand variability observed in real markets and uncertainty in demand forecasts for new services. We show that the extent of demand uncertainty influences substantially the behavior of actors: they exhibit aversion to risk by cutting on the network expansion and reducing the scope of paid peering agreement. To the best of our knowledge, the effects of demand uncertainty were not studied before in the literature relevant to our topic. We also contribute to analysis of the effects of competition between ISPs, which [28] mentions in the conclusions among important topics requiring further study. We quantify the competitive advantage, which an ISP with concluded paid peering agreement has over an ISP without such agreement. The study of vertical diversification of an ISP into content provision in the context of paid peering is also among the novelties. We show that in this case the paid content peering can be beneficial to both diversified ISP and CP and, in addition, removes a part of the incentive to challenge the network neutrality by the ISP prioritizing its own service.

Additional motivation for our research comes from the position of European regulators presented in [10]. Thus, Dr. Cara Schwartz-Schilling representing BEREC (Body of European Regulators for Electronic Communications) noted in her presentation that the current regulatory framework, while it foresees imposition of an obligation to interconnect on a non-discriminatory basis, "does not provide a legal basis for mandating free peering". Her presentation as well as presentations of representatives of French and Dutch regulatory bodies mention favorably paid peering, providing some examples.

2 Paid content peering with strong content service provider

We model the relationship between a strong CP and an ISP by the leader-follower model based on the Stackelberg game. First we consider a single content service; the more evolved case of several interacting services is considered in Section 4.

For industrial examples of the situation analyzed in the section one can refer to the recent agreements between Netflix and ISP's in the US [56] and Europe. In particular in [37] it is reported that the agreement between Netflix and Orange "would give Orange a share of revenue for carrying Netflix", which is a kind of arrangement analyzed here.

The Stackelberg game presented below models a stylized negotiation process between a CP and an ISP. The strong CP takes the lead and offers to the ISP a share of its content provision revenues (which may be zero). In order to do this it predicts the future end user demand for its content provision service and, consequently, decides the pricing of its service. The possible uncertainty in demand prediction is modeled by consideration of the demand to be random with known distribution. If it possesses the knowledge about the ISPs profit model then it makes a single offer, which maximizes its profit under the predicted ISPs response in terms of capacity expansion obtained by maximizing ISPs profit. If such knowledge is lacking then we assume that the negotiation process consists of several (possibly many) offers and counteroffers, which converge to the offers in the presence of this information. In both cases we assume that both actors possess the common knowledge about the users response to the content service pricing. This negotiation process is concluded with a binding agreement about the share of the CPs profit to be transferred to the ISP and the obligation of the ISP to assure the required QoE, which requires the modeled capacity expansion. After conclusion of this agreement the actors proceed with its implementation.

In this setting the ISP can only accept the decision of the CP and react on it by the decision on capacity expansion. Thus, we model the case when the CP is in possession of dominating market power as explained in the Introduction. This is, of course, an approximation to reality where the ISP can possess some (albeit smaller) market power even facing a powerful CP. Therefore, the predictions of our analysis about acceptance of paid peering will be conservative: one can expect that it will be accepted for a wider set of model parameters.

1. *The profit model of the content provider*. We assume that the content provider maximizes its profit, which is the difference between the content pro-

vision revenue net of paid peering fraction and the costs. There are two types of costs: provision costs and opportunity costs resulting from not satisfying demand. This results in the following profit function.

$$\begin{aligned} P_{CP} &= \left(p\left(1-x\right)-c\right)\mathbb{E}_{\omega}\min\left\{W_0+W, D\left(p,\omega\right)\right\} \\ &\quad -e\mathbb{E}_{\omega}\max\left\{0, D\left(p,\omega\right)-W_0-W\right\} \end{aligned} \tag{1}$$

where p - service price, c - service provision costs, x - fraction of the revenue transferred to the connectivity provider, e - opportunity cost for not satisfied demand, W_0 - existing network capacity and W - additional capacity that may be added to existing capacity.

Since we focus on the relationship between a CP and ISPs, the structure of our profit functions is more detailed than what is usually found in the literature on network neutrality [28]. In particular here we consider the initial capacity W_0 and opportunity costs e. Considering the initial capacity W_0 is important because it establishes the distinction between the incumbent provider and the new entrant (with $W_0 = 0$). In particular, in the limiting case of infinite initial capacity as in [26] with a powerful CP the paid content peering does not occur at all. This is because we assume that the ISP can not engage in the deliberate practice of downgrading connection quality and the CP in this case will not have an incentive to share its revenue with the ISP. The intermediate values of initial capacity W_0 between zero and infinity can give an insight into the relative difference between a new entrant and a large incumbent, even though we do not study this issue in detail.

The explicit consideration of opportunity cost e is also important. This is because the failure to serve demand leads not only to immediate loss of revenue, but also to a possible loss of a future revenue stream due to churn (desertion of customers to alternative sources of content). The phenomenon of churn is a major concern of industrial actors [53], [15]. One of the reasons for this is that a small improvement in customer retention can lead to a significant increase in profit [12]. Thus, even though we do not model here the existence of other content providers explicitly, consideration of opportunity costs serves as a simple proxy for their presence.

We assume here that the amount of new demand, which can be served by the expanded ISP network with required Quality of Service (QoS), is proportional to the added capacity and we normalize the proportionality coefficient to one. Implicitly this assumption ignores the possibility that the newly added capacity can be utilized by other CPs and the end users for delivery and consumption of other content services. This effect can be approximately taken into account by considering the proportionality coefficient to be less than 1, but we think that it will not bring a substantial difference to our conclusions. One can also attempt to model the presence of other CPs and content services explicitly together with their effect on QoS. This is outside the scope of the present paper, but it represents a promising topic of our future research. In our recent paper [21] in different context we have already utilized a possible approach to this issue using queuing theory approximations, see also [5].

Demand for service at price p is denoted by $D(p,\omega)$. Besides the price, it depends on the random variable ω that describes the demand uncertainty. The exact form of this function will be described later.

Here the price p and the paid peering revenue fraction x are the decisions of the content provider that it takes in order to maximize its profit, W is the decision of the connectivity provider, demand $D(p,\omega)$ results from decisions of the service users and c, e, W_0 are parameters.

2. *The profit model of the connectivity provider.* We assume that he takes the role of the Internet Service Provider (ISP). Its profit P_{ISP} is the difference between its revenue (fixed subscription fees from customers plus the share of content provider's revenue obtained through paid peering) and its costs (network maintenance costs and network expansion costs):

$$P_{ISP} = C + px\mathbb{E}\min\left\{W_0 + W, D(p,\omega)\right\} - rW - q(W_0 + W) \qquad (2)$$

where C is subscription fees; we assume that all the user population is subscribed to the Internet for a flat fee, r - cost of the unit capacity expansion, q - cost of unit capacity maintenance.

We consider here the subscription fees C to be a fixed parameter of the model. More detailed analysis will consider the Internet subscription fees as a decision parameter of the ISP and determine their optimal value from the model. This will capture the effect of premium content services on the willingness of customers to pay possibly higher subscription fees. It will be especially relevant for the analysis of competition between ISPs in the direction presented in Section 3. This analysis will borrow from the theory of multisided platforms [18] and constitutes a possible topic of further research.

The connectivity provider maximizes his profit by choosing the level W of capacity expansion.

3. *The demand function of the user population.* The CP has to predict its future revenue in order to estimate its future profits and make an offer of revenue sharing to the ISP. For this reason it needs to have an estimate of the users demand function and how this function depends on its pricing decision and prospective QoS. Here we model explicitly only the dependence of the demand function on the price of the content services. Thus, we make a simplification assuming that the content service is delivered with constant QoS, which allows to ignore the dependence of demand on QoS. Explicit consideration of the dependence of demand on QoS or QoE is outside the scope of this paper, but can be done similar to our recent paper [21] adding another layer of complexity. The possible uncertainty in demand predictions we model considering the parameters of demand function to be random.

We proceed with a selection of demand function following the consumption theory of microeconomics [36] and empirical evidence reported in applied research. There exists substantial such evidence that the demand function for ICT (Information and Communication Technology) products and services exhibits constant elasticity γ with respect to price (see [38],[25], [22]). In such cases the demand function takes the form $D = M/p^{\gamma}$,where M is the population specific parameter that is interpreted as available budget. We modify this function in the following way

$$D(p,\omega) = \frac{M}{(a+p)^{\gamma}}(1+\omega) \tag{3}$$

where a is the opportunity price that plays the role of the stabilization parameter that prevents the demand function from excessive growth for small values of price p. The parameters a, M, γ are all uncertain and should be described by random variables with appropriate probability distributions. In order to admit the analytical treatment, we simplify the description of uncertainty here by assuming that a, M, γ are deterministic, but the demand function is multiplied by the term $1+\omega$, where ω is a random variable with the cumulative distribution function $H(\cdot)$ that has density $h(\cdot)$.

4. *An alternative interpretation of the model.* As explained above, the reference interpretation of our model is the negotiation process between a powerful CP and an ISP. However, the model can also be interpreted in terms of a sequence of actors moves in a stylized market. With such interpretation one should assure that the timing of decisions is such that the more durable decisions come first. We achieve this by assuming that the ISP does not actually install the additional capacity in its network upon receiving the offer from the CP, but rather dedicates additional capacity for transporting the CPs content from already existing network (or, just connects CPs server from CPs CDN directly to its network, which will have a similar effect). This makes the durability of capacity decision commensurate with the durability of pricing and sharing decisions of the CP. This does not preclude the ISP from actually adding a new physical capacity in a fashion uncoordinated with moves by the CP. With such interpretation the timing of decisions is the following.

i. The content provider selects the service price p and the share x of its service provision revenue to be transferred to the ISP in the framework of content peering agreement.

ii.Assuming p, both CP and ISP predict the demand for the service as in (3).

iii. Knowing its revenue share x and the demand for the service $D(p,\omega)$ up to the random variable ω with known distribution H, the ISP decides the volume W of additional network capacity to dedicate transporting of CPs content that maximizes its expected profit.

iv. Anticipating the decision principles of the users and the connectivity provider, described in items ii and iii, the content provider selects at point i his decisions x and p in such a way, as to maximize its profit, taking into account the reactions of other actors described in ii, iii.

Now we can analyze the relationship of the content provider and the ISP, following the governance just described. The analysis consists of the following steps.

1. Maximization the profit of the ISP (2), substituting there the demand function (3), this will yield the dependence $W(p,x)$ of the optimal network expansion on the decisions (p,x) of the content provider.

2. Maximization of the profit of the content provider (1), substituting there the demand function (3) and the optimal expansion function $W(p,x)$ obtained on the previous step. This yields the optimal policy (p,x) of the content provider, his best profit and the resulting profit of the connectivity provider.

The resulting optimization problems are the following.

1. *The optimal expansion program* $W(p,x)$ *of ISP:*

$$\max_{W\geq 0}\left\{px\mathbb{E}\min\left\{W_0+W,\frac{M}{a+p^{\gamma}}\left(1+\omega\right)\right\}-\left(r+q\right)W\right\} \tag{4}$$

Compared to (2) we have omitted here the constant components of revenue and costs that do not depend on decision W of the ISP. Due to simplifying assumptions about the demand uncertainty, it is possible to derive its explicit solution.

Theorem 1 *The solution* $W(p,x)$ *of problem (4) is this:*

$$W\left(p,x\right)=\max\left\{0,\frac{M}{a+p^{\gamma}}\left(1+\quad H^{-1}\left(1-\frac{r+q}{px}\right)\right)-W_0\right\}$$

2. The optimal pricing p^* and paid content peering share x^* of the content provider*. They are obtained by substituting (3),(5) into (1) and solving the resulting optimization problem:*

$$\max_{p,x}\left(p\left(1-x\right)-c+e\right)\mathbb{E}\min\left\{W_0+\max\left\{0,\frac{M}{a+p^{\gamma}}\times\right.\right.$$

$$\left.\left.\left(1+H^{-1}\left(1-\frac{r+q}{px}\right)\right)-W_0\right\},\frac{M}{a+p^{\gamma}}\left(1+\tau\right)\right\}-\frac{Me}{a+p^{\gamma}} \tag{5}$$

$$p\geq 0, 0\leq x\leq 1.$$

The proof of Theorem 1 is obtained by the analytical solution of optimization problem formulated above. This problem (5) does not admit explicit solution like problem (4). We obtain the dependence of the actors' profits and policies on significant parameters by solving this problem numerically.

2.1 Results of the numerical analysis

We have solved the optimization problem from Theorem 1 numerically for different values of parameters and provide below a sample of representative results.

2.1.1 Free versus paid peering: efficiency of connectivity provider and uncertainty of demand

The paid peering is beneficial to content provider if the ISP is efficient enough in terms of its expansion costs and the demand uncertainty is moderate.

One way to see this is to observe dependence of the profit of the content provider on the service price on Figures 1,2. On Figure 1 for comparison we show also free peering. It is the lowest curve, which left part is thick dashed and the right part is thick solid. Besides, on this figure we show the case of reference expansion costs (thick solid line), case of expansion costs being 0.6 of the reference (thin solid line) and the case of expansion costs being 1.6 of the reference (thin dashed line). All the cases coincide in the region of high service prices, because in this region the service prices diminish demand to the values where no network expansion is necessary.

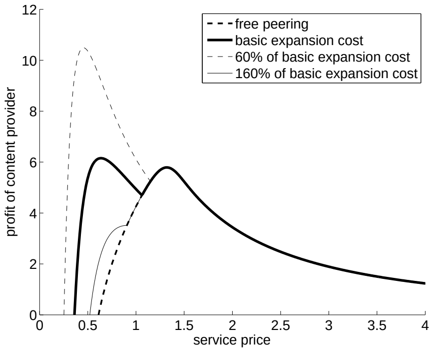

Fig. 1. Effect of expansion costs on adoption of paid peering

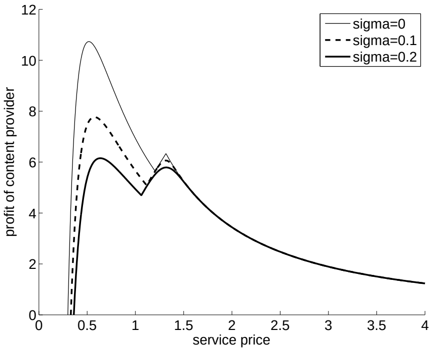

Fig. 2. Effect of demand uncertainty on adoption of paid peering

We see that the free peering curve has a familiar bell like shape, where the profit grows at first with increase in price, reaches maximum and then declines. The paid peering curve for the reference network expansion cost has a qualitatively different camel like shape, with two maxima. One maximum is found in the region of high service prices and free peering, but another one is found in the region of low prices. It shows that the paid peering allows to price the service aggressively to stimulate demand and to incentivate the ISP to expand the network capacity in order to provide the necessary bandwidth. Whether the paid peering will be adopted depends on which of the two maxima is higher. In the case of the reference network expansion costs the paid peering maximum is somewhat higher (about 6%) and paid peering will be adopted. However, if the network expansion costs grow, we see from Figure 2 that the paid peering maximum sinks and at some point becomes smaller than the free peering maximum, which leads to abandoning of paid peering. For high network expansion costs the paid peering maximum becomes an insignificant bulge. On the contrary, when these costs decrease, the paid peering becomes strongly beneficial for the CP. Thus, the more efficient the ISP is in expanding the network, the easier for it is to induce the CP to adopt the paid peering.

A similar pattern we see on Figure 2, where the ISP's expansion costs are kept constant, but the demand uncertainty is different for different curves. It is measured by the standard deviation of ω from (3) for the demand function (referenced as sigma on Figure 2). One can see again the camel like curves, which describe the dependence of the CP's profit on service price. The paid peering maximum in the region of low prices strongly depends on the level of uncertainty. The larger uncertainty the smaller is this maximum until at some level of uncertainty the paid peering becomes less beneficial than free peering.

2.1.2 The effect of demand uncertainty

High demand uncertainty induces risk averse behavior of the content provider, caution with pricing and less interest towards paid peering.

We have studied the dependence of the actor's profits and policies on the different problem parameters: costs c, e, r, q, initial available capacity W_0, demand parameters a, M, demand elasticity γ and variability σ. We show here these dependencies on the demand uncertainty/variability σ.

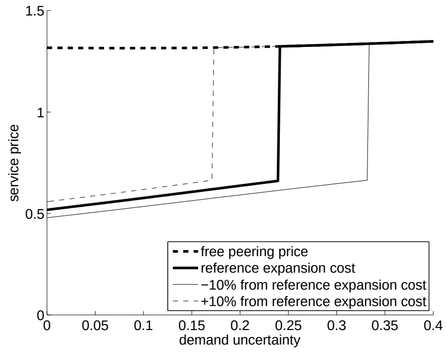

Fig. 3. Dependence of service price on demand uncertainty

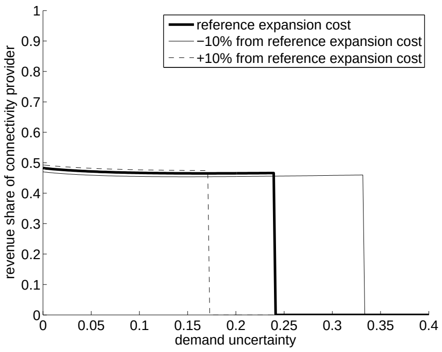

Fig. 4. Dependence of ISP's revenue share on demand uncertainty

Figures 3-5 show the dependence of optimal content provider's policy on the demand uncertainty. When the uncertainty increases the content provider tries to hedge increasing risk by rising the price for its services (Figure 3). This has an effect of decreasing the demand and, consequently, also decreasing the demand variability and uncertainty as can be seen from the demand function (3). For low to moderate levels of uncertainty the content provider utilizes the paid peering because it is profitable to him to induce the ISP to expand the network capacity. In this uncertainty range the revenue share accorded to the ISP is approximately constant and substantial (Figure 4).

After the level of uncertainty passes a certain threshold the risk inherent in expanding capacity becomes too high and the content provider rises his price substantially to limit the demand to already existing capacity, thus denying the ISP from any share of his revenue.

The profit of the content provider decreases with increasing uncertainty (Figure 5). This is due to the combined effect of two causes. Firstly, the increase of price due to the effort to reduce risk leads to decreasing demand that has as a consequence contracted profits. Secondly, even for the constant demand the profit will decline with the increase of uncertainty. This is because in order to serve the same percentage of demand the content provider needs more capacity when the variation of demand increases. If, instead it lets the percentage of served demand to go down then it gets penalized by the opportunity costs. At the same time it gets the same or declining revenue because it get paid for the actual volume of service. To the contrary, the profit of ISP grows because it gets incentivated more in order to install more capacity per unit of served demand. This is accompanied also by growth of its return on investment measured by the ratio of the profit and expansion costs. But, this happens only in the region of paid peering. After the content provider switches to the free peering the profit of the ISP abruptly disappears and the profit of the content provider continues to decline, albeit more slowly, because no part of its revenue goes to the ISP.

2.1.3 The dependences of profits, prices and revenue shares on demand elasticity

Here we show that content peering is beneficial in the case of innovative high elasticity services.

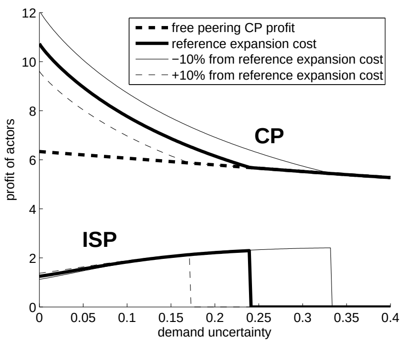

Fig. 5. Dependence of actors' profits on demand uncertainty

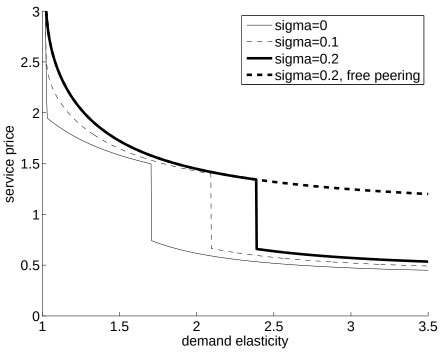

Fig. 6. Dependence of service price on demand elasticity

Demand elasticity γ in the context of ICT services can be related to the relationship between basic, established, traditional services and innovative new services. For traditional services that cater to very basic communication needs viewed as indispensable, the demand elasticity is low. For example, there is considerable evidence in the literature that demand elasticity for the basic fixed net telephony is only marginally larger than 1. For new, innovative services that serve discretionary interests, like video on demand, demand elasticity can be high, exceeding 2 or more [31].

Figures 6-8 show how the optimal policies of the content provider change with increasing demand elasticity. If it caters to the basic services in the low to medium elasticity range then it sets the price relatively high (Figure 6). There is no need for paid peering in this elasticity range because the existing capacity is sufficient for the service provision. While the elasticity increases and the service consumption becomes more discretionary, the optimal price gradually drops. When the elasticity crosses a certain threshold, the content provider becomes more profitable by adopting the paid peering in order to stimulate the ISP to install more capacity, and hence obtain the capability to drop the price substantially and stimulate the demand. After the substantial initial drop the price continues to decrease slowly as the elasticity grows. The share of revenue accorded to the ISP starts from a relatively high level on crossing the threshold to paid peering and continues to increase slowly afterwards (Figure 7).

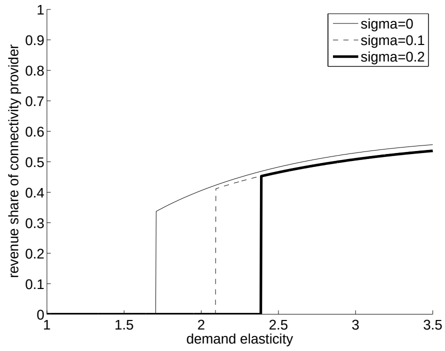

Fig. 7. Dependence of ISP's profit share on demand elasticity

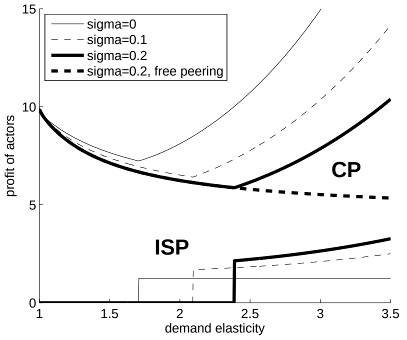

Fig. 8. Dependence of actor's profits on demand elasticity

The profit of the content provider shown on Figure 8 decreases with increasing elasticity in the region of low to medium elasticities. This is because the decline of prices is not offset sufficiently by increase of volume that remains capped by already installed capacity W_0 due to free peering, and hence the absence of ISP incentives to expand the capacity. When the content provider switches to paid peering the profit starts to grow with increasing elasticity because ever more capacity is becoming available. Also the profit of the ISP grows with increasing elasticity in the case of paid peering, even though not as steep as the profit of the content provider, while its return on investment decreases due to ever larger volume of capacity required to install.

3 Several competing ISPs

Let us consider now the case when a user has a choice between several ISPs, but the switching to another ISP entails switching costs. As before, we consider the Content Provider (CP), which offers high quality bandwidth intensive content service S, like real time video. This CP can decide to enter paid peering agreements with some (or all) of ISPs. As we have seen in the previous sections, such an agreement will induce an ISP to expand the network capacity available for delivery of service S in order to maintain required Quality of Service (QoS). This may induce some of the users served by ISPs without such agreements to move to ISPs with paid peering agreements. Thus, the ISPs, which neglect to increase their capacity for delivery of service S may experience increased *churn* in favor of ISPs, which engage in capacity expansion. This may serve as an incentive to increase capacity even for ISPs, which do not have a paid peering agreement with the CP. In the rest of this section we develop a model, which allow us to analyze these effects.

3.1 Profit models of service providers

We assume that the user population is served by I ISPs indexed by $i = 1 : I$, which provide a set of basic Internet services to the population of subscribers. At the beginning the ISP i has U_i subscribers and charges a fixed price C_i per subscription for Internet connection net of Internet provision costs. In addition to delivery of the basic Internet services, it dedicates the network capacity W_{0i} for delivery of the service S with required QoS and can decide to expand this capacity by additional amount W_i at cost r_i per unit of capacity.

The CP charges price p_i and experiences costs c_i for the unit of its service and has potential demand $D_i\left(p_i, \omega\right)$ from customer population U_i at this price, where ω are some random parameters as before. It may decide to deliver a share x_i of resulting revenue $p_i D_i$ to the ISP in order to induce it to expand the capacity for provision of service S by some amount W_i. If part of the demand is not satisfied, the CP experiences opportunity costs e, which are composed from the immediately lost revenue and from potentially lost revenue due to possible transfer of dissatisfied customers to alternative ways of satisfaction of the need satisfied by the service S. After the capacity expansion occurs a fraction ν_{ij} of the user population U_i may decide to move from ISP i to ISP j and bring with them demand $D_{ij}\left(p_j, \omega\right)$. Then the total demand for the service S from customers of ISP i will be

$$\bar{D}_i\left(p_i, \omega\right) = D_i\left(p_i, \omega\right) + \sum_{j=1, j\neq i}^{I} \left(D_{ji}\left(p_i, \omega\right) - D_{ij}\left(p_i, \omega\right)\right) \tag{6}$$

which will be generated by the population of customers

$$\bar{U}_i = U_i \left(1 - \sum_{j=1, j\neq i}^{I} \nu_{ij}\right) + \sum_{j=1, j\neq i}^{I} \nu_{ji} U_j. \tag{7}$$

We shall denote by v_i the total *churn* experienced by ISP i :

$$v_i = \sum_{j=1, j \neq i}^{I} \nu_{ij},\ v_i \leq 1.$$

In these notations the profits of actors are expressed as follows.

Profit of the CP:

$$\begin{aligned} P_{CP} = & \sum_{i \in I}^{I} \left(p_i \left(1 - x_i\right) - c_i\right) \mathbb{E}_\omega \min \left\{W_{0i} + W_i, \bar{D}_i \left(p_i, \omega\right)\right\} \\ & - e \sum_{i \in I}^{I} \mathbb{E}_\omega \max \left\{0, \bar{D}_i \left(p_i, \omega\right) - W_{0i} - W_i\right\} \end{aligned} \tag{8}$$

Profit of the ISP i:

$$P_{ISP}^{i} = C_i \bar{U}_i + p_i x_i \mathbb{E}_\omega \min \left\{W_{0i} + W_i, \bar{D}_i \left(p_i, \omega\right)\right\} - r_i W_i \tag{9}$$

3.2 Demand and churn model

In order to define the values of ν_{ij} and $D_{ij}\left(p_i, \omega\right)$ from (6),(7) it is necessary to develop a more detailed model of the users behavior compared to Section 2, where we have assumed that demand has a constant price elasticity (3). Let us start with the deterministic case without demand uncertainty. We shall consider here that the users are rational economic agents, whose behavior is described by consumption theory of microeconomics [36]. Following this theory, we shall consider first a single user from population U_i, whose decisions about selection of ISP and consumption of service S is governed by utility function of the following form

$$\varphi_i \left(M, p_{ij}, d\right) = \psi_i \left(M_i, d\right) - p_{ij} d - \delta_{ij} \tag{10}$$

where d is demand, generated by this user and p_{ij} is the total user cost of user from population U_i, who switches to ISP j, associated with consumption of unit of service S. In particular, we can have

$$p_{ij} = a_i + p_j \tag{11}$$

where a_i is the internal user's cost associated with consumption of unit of service S and and p_j is its price charged by the CP to the customers of the ISP j. Besides, δ_{ij} is the switching cost experienced by a user for changing ISP from i to j. There is no switching cost if a user from population U_i stays with ISP i : $\delta_{ii} = 0$. The function $\psi_i\left(M, d\right)$ describes the benefit resulting from consumption of amount d of service S experienced by a user from population U_i. We assume that the marginal benefit is a continuous function of consumption d and monotonously tends to zero with increasing d :

$$d_1 > d_2 \ \Rightarrow \ \frac{\partial \psi_i \left(M_i, d_1\right)}{\partial d} < \frac{\partial \psi_i \left(M_i, d_2\right)}{\partial d}, \ \lim_{d \to \infty} \frac{\partial \psi_i \left(M_i, d\right)}{\partial d} = 0. \tag{12}$$

The population parameter M_i describes nonhomogeneity of population U_i. In particular, M_i can define the total budget in terms of money and other resources, allocated by a user from the population U_i to consumption of service S. We assume that M_i is distributed according to known density $h_i\left(M_i\right)$:

$$\int h_i\left(M_i\right) dM_i = 1,\ h_i\left(M_i\right) \geq 0. \tag{13}$$

Thus, $h_i\left(M_i\right) dM_i$ is the fraction of user population U_i with the population parameter between M_i and $M_i + dM_i$.

Example 1 . Budget parameter M_i. Suppose that M_i defines the budget allocated by a user from the population U_i for consumption of the service S. Let us assume that the population U_i is representative of the total population of a given country and that this budget equals a fraction τ of the household income. Then the distribution $h_i\left(M_i\right)$ is obtained from the national statistics of the household income for this country. For example, data reported in [4] show, that the income of the US households is approximated reasonably well by unimodal distribution with piecewise linear density $h(y)$, if we neglect the households in the top 5% bracket, which yields:

$$h_i(M_i) = \begin{cases} 0 & if \quad \begin{array}{c} M_i < 0 \vee \\ M_i > \tau\left(1 + k_M\right)\bar{M} \end{array} \\ \frac{2}{(k_M+1)\tau \bar{M}^2} M_i & if \quad 0 \leq M_i \leq \tau \bar{M} \\ \frac{2}{(k_M+1)\tau \bar{M}} \left(1 + \frac{1}{k_M} - M_i \frac{1}{k_M \tau \bar{M}}\right) & if \ \tau \bar{M} \leq M_i \leq \tau\left(1 + k_M\right)\bar{M} \end{cases} \tag{14}$$

Here $\bar{M}$ is the maximal point of the density of household income. This distribution is skewed to the right with $k_M \simeq 5$.

We focus here on the utility of consumption of the service S and on its availability with required QoS to customers of different ISPs. For this reason we do not include in utility function (10) the utility of a general Internet service, while the differences between subscription fees C_i are included in the switching costs δ_{ij}.

Assuming that the users decide their consumption by maximizing their utility function (10) we obtain the individual demand function $d_i\left(M, p_{ij}\right)$ of a user from population U_i by solving the following optimization problem:

$$\begin{aligned} d_i\left(M_i, p_{ij}\right) &= \arg\max_{d \geq 0} \left[\psi_i\left(M_i, d\right) - p_{ij} d\right], \\ \beta_i\left(M_i, p_{ij}\right) &= \psi_i\left(M_i, d_i\left(M_i, p_{ij}\right)\right) - p_{ij} d_i\left(M_i, p_{ij}\right) \end{aligned} \tag{15}$$

The solution of this problem exists and is unique due to (12). Here $\beta_i\left(M_i, p_{ij}\right)$ is the maximal net benefit obtainable by the user with population parameter M_i from consumption of the service S, which we assume to be positive.

This general approach for defining demand functions can be specified for the case of demand with constant price elasticity $\gamma \geq 1$ considered in Section 2, as the following theorem shows.

Theorem 2 Demand with constant price elasticity. *Suppose that*

$$\psi_i\left(M_i, d\right) = \begin{cases} \frac{1}{1-\frac{1}{\gamma}} M_i^{\frac{1}{\gamma}} d^{1-\frac{1}{\gamma}} & if\ \gamma \neq 1 \\ M_i \ln d & if\ \gamma = 1 \end{cases} . \tag{16}$$

Then

$$d_i\left(M_i, p_{ij}\right) = \frac{M_i}{p_{ij}^{\gamma}},\ \beta_i\left(M_i, p_{ij}\right) = \begin{cases} \frac{1}{\gamma-1} \frac{M_i}{p_{ij}^{\gamma-1}} & if\ \gamma \neq 1 \\ M_i\left(\ln \frac{M_i}{p_{ij}} - 1\right) & if\ \gamma = 1 \end{cases}$$

This theorem is proved by substitution of (16) into (15) and solving the problem (15).

Having obtained the individual demand function $d_i\left(M_i, p_{ij}\right)$ from (15) we can now obtain the demand of population U_i for the service S as follows:

$$D_i\left(p_{ii}\right) = U_i \int d_i\left(M_i, p_{ii}\right) h_i\left(M_i\right) dM_i \tag{17}$$

where $h_i\left(M_i\right)$ is taken from (13). This is the demand $D_i\left(p_i, \omega\right)$ from (6) faced by ISP i from its own customers in the case when there are no random parameters and dependence of demand on p_i occurs through the total consumption costs p_{ij} from (11).

Let us now derive expressions for transferred demand $D_{ij}\left(p_j, \omega\right)$ from population U_i to ISP j and churn ν_{ij} from ISP i to ISP j, which are needed in profit expressions (8),(9). First of all, we shall assume that the switching costs δ_{ij} are substantial and the difference between consumption costs p_{ij} for different j is limited, such that

$$\beta_i\left(M_i, p_{ii}\right) > \beta_i\left(M_i, p_{ij}\right) - \delta_{ij}$$

for all i and j. This means that consumers will not have an incentive to change ISP only due to the difference in consumption costs for the service S. However, a consumer from population U_i will have an incentive to move to ISP j if ISP i can not assure the delivery of the service S with required QoS due to insufficient network capacity $W_{0i} + W_i$ while ISP j has an extra capacity and

$$\beta_i\left(M_i, p_{ij}\right) - \delta_{ij} > 0.$$

Therefore the largest fraction of user population U_i, which can be *potentially* interested in moving from ISP i to ISP j is

$$\eta_{ij} = \int\limits_{\mu_{ij}\left(p_{ij}\right)} h_i\left(M_i\right) dM_i,\ \mu_{ij}\left(p_{ij}\right) = \left\{M_i \mid \beta_i\left(M_i, p_{ij}\right) > \delta_{ij}\right\}$$

The fraction of total demand, which is not satisfied by ISP i is

$$\rho_i = \max\left\{0, \frac{D_i\left(p_{ii}\right) - W_{0i} - W_i}{D_i\left(p_{ii}\right)}\right\}.$$

We assume that ISP i treats equally all its customers independent of their population parameter M_i. Therefore the fraction of customers, which are not satisfied with QoS of the service S will be equal for any group of customers with any specific ranges of the population parameter. Thus, the largest potential churn from ISP i to ISP j is

$$\bar{\nu}_{ij} = \rho_i \eta_{ij} = \max\left\{0, \frac{D_i\left(p_{ii}\right) - W_{0i} - W_i}{D_i\left(p_{ii}\right)}\right\} \int\limits_{\beta_i\left(M_i, p_{ij}\right) > \delta_{ij}} h_i\left(M_i\right) dM_i$$

This is the fraction of population U_i, which is willing to move from ISP i to ISP j. Not all of them will actually move to ISP j because this ISP may lack the necessary capacity to accommodate everybody and because the unsatisfied users from U_i may have more than one ISP with spare capacity to choose from. It is necessary to introduce some rule, which will resolve this ambiguity of multiple choices. An example of such rule follows. We assume that the recipient ISPs treat all willing customers from any population equally irrespective of their population parameters. In addition, we assume that $\beta_i\left(M_i, p_{ij}\right)$ increases monotonically with M_i. This is the case, for example, when the conditions of Theorem 2 are satisfied. Then we have if $\mu_{ij}\left(p_{ij}\right) \neq \varnothing$:

$$\mu_{ij}\left(p_{ij}\right) = \left\{M_i \mid M_i > M_{ij}\left(p_{ij}\right)\right\}, \ \beta_i\left(M_{ij}\left(p_{ij}\right), p_{ij}\right) = \delta_{ij}$$

If $\mu_{ij}\left(p_{ij}\right) \neq \varnothing$ then we take $M_{ij}\left(p_{ij}\right) = \infty$.

Algorithm 3 The priority of the minimal population parameter.

1. Initialization. *At the start select the set I^- of ISPs, who have customers willing to move to some other ISP and the set I^+ of ISPs who have the possibility to accommodate new customers:*

$$I^- = \left\{i \mid \exists j : \bar{\nu}_{ij} > 0\right\}, \ I^+ = \left\{i \mid \bar{W}_i = W_{0i} + W_i - D_i\left(p_{ii}\right) > 0\right\}.$$

Take $D_{ij}\left(p_{ij}\right) = 0$, $\nu_{ij} = 0$, $\bar{\rho}_i = \rho_i$, $i, j \in I$, $i \neq j$.

2. Generic step. *By its beginning the algorithm obtained current sets I^- and I^+. Proceed with the following actions.*

i. For each $i \in I^-$ select an arbitrary $j\left(i\right) \in I^+$ such that $M_{ij(i)}\left(p_{ij(i)}\right) = \min_j M_{ij}\left(p_{ij}\right)$. For each $j \in I^+$ let us denote by I_j the set of all such $i \in I^-$ for which $j = j(i)$.

ii. For each $j \in I^+$ compute demand $\hat{D}_{ij}$ generated by customers from populations $i \in I_j$ potentially moving to ISP j :

$$\hat{D}_{ij} = \bar{\rho}_i U_i \int\limits_{M_i \geq M_{ij}\left(p_{ij}\right)} d_i\left(M_i, p_{ij}\right) h_i\left(M_i\right) dM_i \tag{18}$$

iii. For each $j \in I^+$ compute the newly arrived demand $D_{ij}\left(p_{ij}\right)$ and churn coefficients ν_{ij} for $i \in I_j$, corresponding to $D_{ij}\left(p_j, \omega\right)$ and ν_{ij} from (8),(9) for the case when there is no uncertainty and dependence from p_j occurs through dependence on p_{ij} from (11). Two cases are possible.

a. ISP j can accommodate all potential demand $\hat{D}_{ij}$ from (18), $i \in I_j$. Then

$$\sum_{i \in I_j} \hat{D}_{ij} \leq \bar{W}_j. \tag{19}$$

In this case take $\nu_{ij} = \bar{\rho}_i \eta_{ij},\ D_{ij}(p_{ij}) = \hat{D}_{ij}$, for $i \in I_j,\ I^- := I^- \backslash I_j$,

$$\bar{W}_j := \bar{W}_j - \sum_{i \in I_j} \hat{D}_{ij}$$

and $I^+ := I^+ \backslash \{j\}$ if (19) is satisfied with equality.

b. ISP j can not accommodate all potential demand $\hat{D}_{ij}$, then (19) is not satisfied. In this case compute

$$\zeta_j = \frac{\bar{W}_j}{\sum_{i \in I_j} \hat{D}_{ij}}$$

and take $\nu_{ij} = \bar{\rho}_i \zeta_j \eta_{ij},\ D_{ij}(p_{ij}) = \zeta_j \hat{D}_{ij}$ for $i \in I_j$, $\bar{\rho}_i := (1 - \zeta_j)\bar{\rho}_i$ for $j \in I$, $I^+ := I^+ \backslash \{j\}$.

iv. Prune the set I^- excluding from it all i for which $\bar{\nu}_{ij} = 0$ for all $j \in I^+$.

v. If I^- or I^+ is empty then stop. Else proceed with step 2i.

Observe that on each step of this algorithm at least one element is subtracted from one of the sets I^- or I^+. Therefore this algorithm will produce the assignment of nonsatisfied users to new ISPs in finite number of steps. Some of the nonsatisfied users may remain such continuing to be assigned to original ISPs.

Thus, the description of deterministic model is completed. The case of demand uncertainty is considered similarly to how it was done in Section 2, see (3). We take in (6):

$$D_i(p_i, \omega) = D_i(p_{ii})(1 + \omega_i),\ D_{ij}(p_i, \omega) = D_{ij}(p_{ij})(1 + \omega_{ij}),\ i, j \in I$$

where ω_i, ω_{ij} are random variables with known distributions having support $[-1, \infty]$.

Example 2 . Two ISPs. Suppose that there are just two ISPs, $i = 1, 2$, the distribution of population parameters M_i follows (14) and the conditions of Theorem 2 are satisfied. Then

$$D_i(p_{ii}) = U_i \frac{2 + k_M}{3} \frac{\tau \bar{M}}{p_{ii}^{\gamma}},\ \hat{D}_{ij} = \frac{\rho_i \lambda_{ij} U_i}{p_{ij}^{\gamma}}$$

$$D_{ij}(p_{ij}) = \begin{cases} 0 & if\ \Gamma \\ \min\left\{\hat{D}_{ij}, W_{0i} + W_i - D_j(p_{jj})\right\} & if\ \neg\Gamma \end{cases}$$

$$\nu_{ij} = \begin{cases} 0 & if \quad \Gamma \\ \rho_i \eta_{ij} & if\ \neg\Gamma \ \wedge \ \left[W_{0j} + W_j - D_j\left(p_{jj}\right) \geq \hat{D}_{ij}\right] \\ \rho_i \eta_{ij} \frac{W_{0j}+W_j-D_j(p_{jj})}{\hat{D}_{ij}} & if\ \neg\Gamma \ \wedge \ \left[W_{0j} + W_j - D_j\left(p_{jj}\right) < \hat{D}_{ij}\right] \end{cases}$$

$$\Gamma = \left[D_i\left(p_{ii}\right) \leq W_{0i} + W_i\right] \ \vee \ \left[D_j\left(p_{jj}\right) \geq W_{0j} + W_j\right]$$

$$\eta_{ij} = \begin{cases} b_1^{ij} \text{ if } & M_{ij}\left(p_{ij}\right) \leq \tau \bar{M} \\ b_2^{ij} \text{ if } \tau\bar{M} \leq & M_{ij}\left(p_{ij}\right) \leq \tau b_0 \bar{M} \\ 0 \text{ if } & M_{ij}\left(p_{ij}\right) \geq \tau b_0 \bar{M} \end{cases}$$

$$\lambda_{ij} = \begin{cases} b_3^{ij} \text{ if } & M_{ij}\left(p_{ij}\right) \leq \tau \bar{M} \\ b_4^{ij} \text{ if } \tau\bar{M} \leq & M_{ij}\left(p_{ij}\right) \leq \tau b_0 \bar{M} \\ 0 \text{ if } & M_{ij}\left(p_{ij}\right) \geq \tau b_0 \bar{M} \end{cases}$$

$$M_{ij}\left(p_{ij}\right) = \sigma_{ij}\left(\gamma - 1\right) p_{ij}^{\gamma-1}, \ b_0 = 1 + k_M,$$

$$b_1^{ij} = 1 - \frac{M_{ij}^2\left(p_{ij}\right)}{\tau^2 \bar{M}^2 b_0}, \ b_2^{ij} = \frac{b_0}{k_M} - \frac{2M_{ij}\left(p_{ij}\right)}{\tau k_M \bar{M}}\left(1 - \frac{M_{ij}\left(p_{ij}\right)}{2\tau\bar{M}b_0}\right)$$

$$b_3^{ij} = \frac{\tau\bar{M}\left(b_0+1\right)}{3} - \frac{2}{3b_0}\frac{M_{ij}^3\left(p_{ij}\right)}{\tau^2\bar{M}^2}$$

$$b_4^{ij} = \frac{1}{3\tau k_M b_0 \bar{M}}\left(b_0^3\tau^2\bar{M}^2 + M_{ij}^2\left(p_{ij}\right)\left(\frac{2}{\tau\bar{M}}M_{ij}\left(p_{ij}\right) - 3b_0\right)\right).$$

We shall use this example for the case study in the next section.

3.3 Case study: A newcomer ISP competes against an incumbent ISP: the effect of paid content peering

In this section we use the model of competition between ISPs developed in the previous sections in order to analyze the situation when a newcomer ISP challenges an established ISP by offering content packages of superior QoS and entering into agreements with content providers, including paid content peering. For example, this describes the ISP competition between traditional providers of telecommunication services and electricity companies like NTE in Norway or ENEL in Italy. These companies extend fiber to homes through which they offer Internet services packaged with telephony, television and video on demand. More specifically, we analyze the evolution of the Internet market in some geographical location consisting of the following stages.

Stage 1. *Initial state.* The user population is served by the incumbent ISP 1, who charges fixed subscription fees for Internet connection. The customers access the bandwidth intensive content services of a CP through the network of ISP 1. The CP does not share its content provision revenue with ISP 1.

Stage 2. *Arrival of newcomer ISP.* The newcomer ISP 2 arrives without network capacity and customer base of its own. However, it decides to set up its network with additional network capacity, which can be used by former customers of ISP 1 for accessing the attractive bandwidth intensive content services of the CP with reasonable QoS. At first, the incumbent ISP 1 does not react on arrival of competition and as a result loses a substantial part of its market share to ISP 2. Many former customers of ISP 1 decide to switch to ISP 2 attracted by availability of the CP services. Meanwhile, the CP does not share its content provision revenue with the ISPs, but experiences substantial increase in its profit due to competition between ISP 1 and ISP 2.

Stage 3. *The incumbent ISP expands its network.* Observing its diminishing market share, the ISP 1 decides to expand its network capacity in order to allow more users to have access to the CP services with reasonable QoS. This tactics proves successful in limiting the expansion of ISP 2 and even driving it completely from the market. Thus, ISP 1 can recover the lost market share, but it still can not bring its profit back to the level where it was in times when ISP1 was the monopolist. The CP sees its profit increasing further due to increased competition, but it still does not share its content provision revenue with the ISPs.

Stage 4. *The newcomer ISP enters into paid peering agreement with the CP.* Observing a strong response from ISP 1, the ISP 2 enters into a paid content peering agreement with the CP, which allow it to expand its network further and wrestle again a substantial part of the market share from ISP 1. This agreement increases again the profit of the CP due to the expansion of the market for its services. However, ISP 2 manages only to increase its market share and expand its network without noticeable increase of its profit.

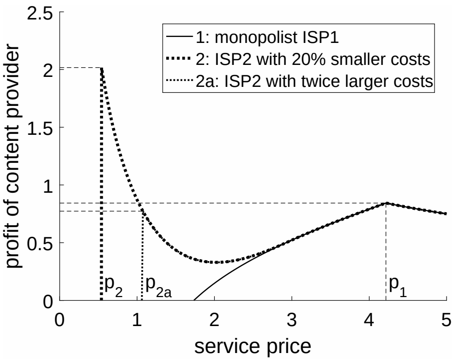

Fig. 9. Profits of the CP before and after arrival of ISP 2

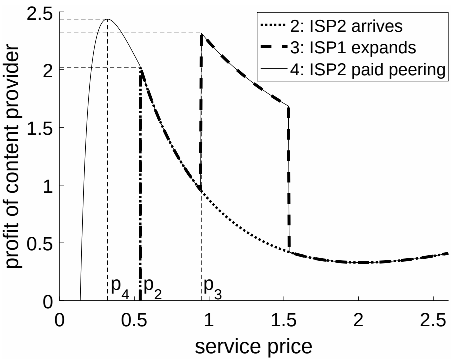

Fig. 10. Changes in profits of the CP with further market evolution

The results of this analysis are shown on Figures 9-12, which present the dependence of profits and market shares of different actors on the service price $p_j, j = 1, 2$ from (11), which is considered to be equal for customers of both ISPs: $p_j = p$. Also internal service consumption costs $a_i = a$ and Internet subscription prices $C = C_i$ were taken equal for customers of both ISPs. The curves on these figures are numbered according to numbering of stages above. We assume that the CP delivers an attractive and innovative content service, such that a substantial part of customers may be willing to change ISP in the favor of the provider, which assures better QoS.

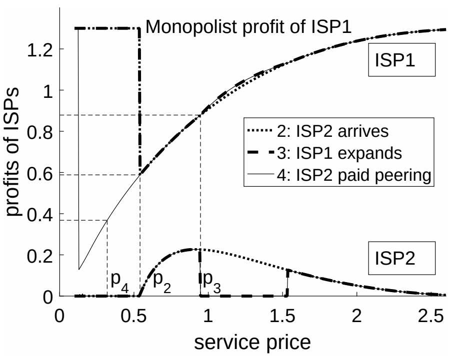

Fig. 11. Changes in profits of ISPs with market evolution

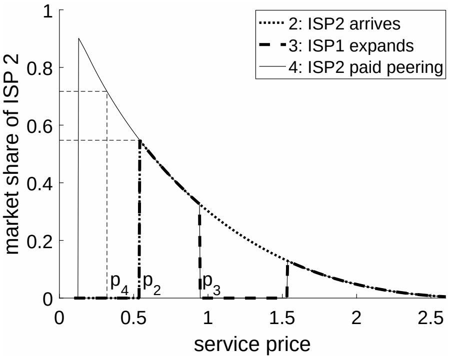

Fig. 12.Changes in market share of ISP 2 with market evolution

We have observed, that in the presence of both ISPs the Nash equilibrium between them in the volumes W_i of network expansion may not exist. For this reason we have considered the two stage Stackelberg game. Similar to Section 2 we assume that the CP enjoys a strong negotiation position and announces to the ISPs its decision on the service price p and shares of the content provision revenue x_i to be transferred to each of the ISPs (if any). The incumbent ISP 1 acts on this information by selecting its network expansion program W_1. This information becomes available to the newcomer ISP 2, who on its basis decides its own network expansion W_2.

The market before and after arrival of newcomer ISP 2 (comparison of Stage 1 and Stage 2). Figure 9 compares the CP profits during Stage 1 and Stage 2 before and after the arrival of newcomer ISP 2. When ISP 1 enjoys monopolistic position and high profits and in the absence of competition and paid peering it does not have any incentive to expand its network. In this case the dependence of the CP profit on price (curve 1) exhibits a familiar bell like shape with the maximum p_1 in the region of high prices. The picture changes substantially with the arrival of newcomer ISP 2. We assume that it is identical to ISP 1, except that it has not its own customer base yet and starts to develop its network from zero, hoping to convince the customers of ISP 1 to change the Internet provider. In addition, it has 20% smaller network expansion costs compared to ISP 1. Due to additional network capacity delivered by ISP 2 the profit curve of the CP assumes the second maximum in the region of low prices (curve 2 on Figures 9,10). Now the CP can choose between low price p_2 with substantially higher demand and the old high price p_1. Which of two maximums will be higher, depends on the network expansion costs by ISP 2. As shown on Figure 9, if ISP 2 has 20% smaller network expansion costs compared to ISP 1 then the low price p_2 yields substantially higher profit to the CP and therefore should be preferred. In this case ISP 2 manages to win slightly over half of the customers of ISP 1 (curve 2 on Figure 12). The former monopolist ISP 1 loses over the half of its profit (curve 2 with the profit of ISP 1 on Figure 11). However, the substantial market share does not transforms into high profits for ISP 2. In fact, if the CP chooses the optimal price p_2 then all the revenue of ISP 2 is consumed by the network expansion and its profit is zero (curve 2 with the profit of ISP 2 on Figure 11). Of course, ISP 2 may extract profit by offering additional services to its now substantial customer base, but this aspect remains outside our model here.

The picture is the opposite if ISP 2 is inefficient in expanding its network having too high the network expansion costs. The curve 2a on Figure 9 shows the CP profit when ISP 2 has twice larger the network expansion costs compared to ISP 1. The profits are identical to the case 2 for service price exceeding p_{2a}, but they drop dramatically for lower prices. In this case the best low region price p_{2a} yields lower profit for the CP than the original high price p_1 when the demand is fully accommodated by ISP 1. In this case the inefficient ISP 2 does not manage to enter the market at all.

Summarizing, the arrival of competition in the Internet provision can be very beneficial to content providers and users also in the absence of paid content peering when the new arrival can set up its network efficiently. However, the total profit of the Internet provision sector drops substantially. This conclusion by different means was obtained also in [29]. Of course, this happens in the case when the switching costs for changing provider are not excessively high.

Incumbent ISP 1 expands its network (Stage 3). When ISP 1 has enjoyed monopoly power it did not have any incentive to expand its network accommodating larger demand for content services of the CP. Now, with the arrival of competition from ISP 2 and substantial churn in favor of the newcomer, the ISP 1 has no choice but expand its network, even though it is less efficient than ISP 2 in doing this. The resulting CP profit curve (curve 3 on Figure 10) has three maximums at p_1, p_2 and p_3. Which of them is higher depends on the relationship between the network expansion costs of both providers. In the case shown on Figure 10 the maximum yielded by p_3 is higher than the other two and should be selected by the CP. It gets even higher profits than on Stage 2 due to sharpened competition between the ISPs, still without paid content peering. With this pricing the ISP 2 manages to recover back its customers and drive ISP 2 out of the market. However, it manages to recover only less than half of its lost monopolistic profit (curve 3 with profits of ISP 1 on Figure 11) because of the need to expand its network.

Newcomer ISP 2 strikes a paid peering agreement with the CP (Stage 4). Facing the threat to be driven out of the market, ISP 2 convinces the CP to resort to paid content peering to enable increased network capacity and hence the possibility to accommodate larger demand for the CP's content services with reasonable QoS. The profit curve of the CP is the curve 4 on Figure 10. It also has three maximums in price points p_1, p_3 and p_4 as the curve 3 from the previous stage. However, the maximum at point p_2 on curve 3 is replaced by the higher maximum at point p_4 in the region of smaller prices and larger demand, which dominates the other two maxima. Thus, the paid peering allows to the CP to obtain even higher profits. It allows ISP 2 to expand its network further and to get even higher market share compared to its arrival at Stage 2 (curve 4 on Figure 12). The profit of ISP 1 drops dramatically and constitutes less than the third of its original monopolistic profit (curve 4 with profits of ISP 1 on Figure 11). Still, the large market share of ISP 2 does not yield large profits because all the revenue was consumed by the network expansion (curve 4 with profits of ISP 2 on Figure 11).

Summary of findings.

1. Competition in Internet provision can be very beneficial to providers of high bandwidth innovative content services and for consumers of such services. However, it drives down the profits of the Internet provision sector.

2. Paid content peering can also in the case of competition between ISPs increase the profits of content providers by subsidizing the network expansion of the ISPs. However, in the competitive landscape the benefits go mainly to content providers and end users. Still, it can provide the competitive edge to the ISPs, which can resort to it.

3. The outcome of competition and the effect of paid peering depends very much on the degree of efficiency of the ISPs in expansion of their networks.

4 Content peering and service differentiation

In this section we consider the situation when the ISP offers several Internet provision services, which differ by the connection speeds and, consequently, differ by Quality of Experience (QoE) for the customers, which consume video services with high bandwidth requirements. More specifically, the connection speed decreases substantially for the basic connection package users after they exceed a specified download limit. Connection options without download limits are also available, but for higher price. In addition, the ISP provides his own content service in competition with existing video content providers, but this service is exempt from bounds on downloading. This policy of the ISP challenges the principles of network neutrality because it treats differently the data streams generated by similar services of different origin. It is similar to the policy announced recently by Deutsche Telekom regarding its own video service versus rival services like YouTube from Google, as described in [45].

We describe this situation by considering the population of customers to which three services s_i, $i = 1 : 3$ are offered. Each service is composed from two components, which together create the QoE for the end user: *content* and *connectivity.* Content can be provided by the ISP as well as the CP, while the connectivity is provided only by the ISP. More specifically:

- Service s_1, with content provided by the CP. It is available to subscribers of the basic Internet connectivity package for a flat price C_1 with the high speed V_1 until download limit $\bar{d}$ is reached and the low speed V_2 beyond this limit.

- Service s_2 with the same content as in s_1 provided by the CP. It is available to subscribers to the enhanced Internet connectivity package for a flat price $C_2 > C_1$ with high speed V_1 irrespective of download quantity.

- Service s_3 with competing content to s_1, s_2 provided by the ISP. It is available to subscribers to the basic connectivity package for a flat price C_1, but the high speed V_1 is kept for this particular service without any download limit.

We analyze this setting using the approach followed in Section 2. First of all, we derive the demand function of user population similar to (3) in the new considerably more complex setting. Compared to Section 2, we have to describe how the users select between three services (subsection 4.1). Then in 4.2 we obtain the actor's profit functions, profit maximization problems and formulate the Stackelberg game with CP as a leader. Finally, in 4.3 we perform a numerical analysis of this game and show that under appropriate circumstances the profits of the actors increase with simultaneously decreasing incentive for ISP to challenge the network neutrality.

4.1 Service selection by a single subscriber

Let us consider first services s_2 and s_3 taken in isolation. Suppose that p is a price that the respective providers charge for the unit of content measured in bandwidth. Similarly to Section 2 we assume that demand d, generated by a single subscriber, has a constant elasticity dependence on the service price

$$d_i(p) = \frac{M}{(a+p)^{\gamma}}, \ i = 2, 3 \tag{20}$$

which conforms well with empirical data [38]. Here $a < 1$ is an opportunity cost for customer, associated with consumption of the service unit, M is proportional to the income of subscriber, and $\gamma = \gamma_1$ for service s_2 and $\gamma = \theta$ for service s_3. We assume that elasticity γ describes the QoE, that is, the larger γ the better is the QoE. Indeed, with larger γ the consumption grows faster with the decrease in the service price and the limit consumption with $p = 0$ is higher, while for small γ the consumption will be low even for small prices. Thus, a service with larger γ is more attractive to consumers than a service with smaller γ. Let us assume further that QoE for service s_2 is higher than QoE for s_3. Indeed, they are provided with the same connection speed and one can expect that content of s_2 is in average superior to content of s_3 because content provision is a core business of the CP. Thus, we assume that $\gamma_1 > \theta$.

Let us consider now service s_1. Until demand is smaller than $\bar{d}$, it is the same service as s_2, therefore its demand function is described by (20) with $\gamma = \gamma_1$. After demand exceeds $\bar{d}$ the connection speed drops, leading to substantially inferior QoE. Therefore we describe the demand function of s_3 when demand exceeds $\bar{d}$ by (20) with $\gamma = \gamma_2$, $\gamma_2 < \theta < \gamma_1$. This yields the following demand function

$$d_1(p) = \begin{cases} \frac{M}{(a+p)^{\gamma_1}} \text{ if } p \geq \left(\frac{M}{\bar{d}}\right)^{\frac{1}{\gamma_1}} - a \\ \frac{M}{(a+p)^{\gamma_2}} - M^{1-\frac{\gamma_2}{\gamma_1}} \bar{d}^{\frac{\gamma_2}{\gamma_1}} + \bar{d} \text{ otherwise} \end{cases} \tag{21}$$

These demand functions are shown on Figure 13.

So far we have considered these services in isolation. The next step is to describe how a subscriber selects between these services, depending on their respective prices. Let us assume that the consumer subscribes for just one of the services s_i, $i = 1 : 3$ and follow again the approach of consumption theory of microeconomics [36] as in Section 3. Thus, we associate with the consumption of service s_i the individual utility function of a consumer $\varphi_i(p, d)$. He selects the amount d of a service to consume by maximizing this utility function with respect to d for a given unit price p. Similarly to Section 3, for a risk neutral consumer this utility function can be further structured as follows:

$$\varphi_i(p, d) = \psi_i(d) - (a + p)\, d - C \tag{22}$$

where $\psi_i(d)$ is utility of consumption of amount d of service s_i, $(a + p)\, d$ is the cost of amount d of the service and C is the subscription fee. Demand function $d_i(p)$ is obtained from (22) by maximizing $\varphi_i(p, d)$ with respect to d. Substituting demand function $d_i(p)$ into $\varphi_i(p, d)$ we obtain the maximal consumer utility $\beta_i(p) = \varphi_i(p, d(p))$ associated with consumption of service s_i at price p. Having these functions for each service s_i, we can obtain the demand of the consumer for service s_i by the following rule.

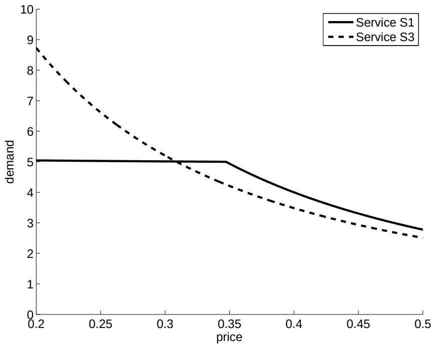

Fig. 13. Demand functions for services $s_1$and $s_3$with $\gamma_1 = 2$, $\gamma_2 = 0.1$, $\theta = 1.8$,$M = 1$,$a = 0.1$,$\bar{d} = 5$

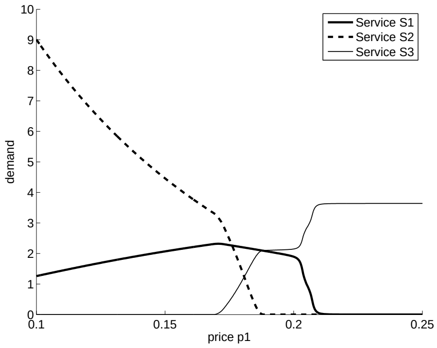

Fig. 14. Dependence of population demand functions on price $p_1$with $p_2 = 0.2$,$\gamma_1 = 2$,$\gamma_2 = 0.1$,$\theta = 1.8$,$a = 0.1$,$\bar{d} = 5$

Consumption of services by a single consumer. Suppose that services s_1, s_2 are offered at unit price p_1 and service s_3 is offered at unit price p_2. Then

- Find the highest value among $\beta_1(p_1), \beta_2(p_1), \beta_3(p_2)$, suppose that it is attained for service s_k.

- The demand d_k, for service s_k will be $d_k(p_1)$ if $k = 1, 2$ and $d_k(p_2)$ if $k = 3$. The demand for services s_i, $i \neq k$ is zero.

Observe that this operation of taking maximum between three utilities makes demand $d_i = d_i(p, C, M)$ for service s_i dependent on both prices $p = (p_1, p_2)$ and both subscription fees $C = (C_1, C_2)$.

In order to implement this rule we need to know expressions for utilities $\beta_i(p)$ and these are obtained from expressions for $\varphi_i(p, d)$. These expressions are obtained taking into account that demand functions $d_i(p)$ from (20),(21) are obtained by maximization of (22). Similarly to Theorem 2 from Section 3 we get

$$\beta_i(p) = \begin{cases} \frac{1}{\gamma - 1} \frac{M}{(a+p)^{\gamma-1}} - C & \text{if} \quad \gamma \neq 1 \\ M\left(\ln \frac{M}{a+p} - 1\right) - C & \text{otherwise} \end{cases} \tag{23}$$

with $(\gamma, C) = (\gamma_1, C_2)$ for $i = 2$ and $(\gamma, C) = (\theta, C_1)$ for $i = 3$.

Functions $\psi_1(d), \beta_1(p)$ are obtained similarly to (21) by gluing together at point $p = \left(\frac{M}{d}\right)^{\frac{1}{\gamma_1}} - a$ pieces of functions (23) with $\gamma = \gamma_1, \gamma_2$.

Demand functions of population of subscribers. We obtain these functions similarly to (17) from Section 3 by integrating the individual demand function with respect to distribution of population parameter M. For example, the distribution from Example 1 can be used for this purpose. Figure 14 shows an example of dependence of demand functions on p_1 for fixed p_2, which can be obtained through numerical integration.

4.2 Profit maximization problems for actors

We assume here that the share of fixed subscription equal to the share of not satisfied demand is lost. Then the satisfied demands D_i^+ and respective subscription shares S_i^+ can be expressed as follows

$$D_1^+ = \min\{D_1, \max\{0, w - D_2 - D_3\}\}, \; S_1^+ = \frac{D_1^+}{D_1} S_1$$

$$D_2^+ = \min\{D_2, \max\{0, w - D_3\}\}, \; S_2^+ = \frac{D_2^+}{D_2} S_2$$

$$D_3^+ = \min\{D_3, w\}, \; S_3^+ = \frac{D_3^+}{D_3} S_3$$

Similarly, we define non satisfied potential demand and missing subscription shares for $i = 1 : 3$ as

$$D_i^- = D_i(p, C) - D_i^+(p, C, w), \; S_i^- = S_i(p, C) - S_i^+(p, C, w)$$

Let us define the content provision costs and the opportunity costs due to not meeting potential demand and possible churn:

c_1 - provision cost for the CP for content of services s_1, s_2;

c_2 - provision cost for the ISP for content of service s_3;

e_1 - opportunity cost for the CP for not meeting potential demand for services s_1, s_2;

e_2 - opportunity cost for the ISP for not meeting potential demand for service s_3;

g_i - opportunity cost for the ISP for not meeting subscriptions for service s_i.

The service provision revenue of the CP is

$$R_{CP} = R_{CP}\left(p, C, W\right) = p_1\left(D_1^+ + D_2^+\right)$$

and we assume that share x of this revenue the CP transfers to the ISP in the context of paid content peering. Then the profit of the content provider is expressed as follows

$$P_{CP} = \left(p_1\left(1-x\right) - c_1\right)\left(D_1^+ + D_2^+\right) - e_1\left(D_1^- + D_2^-\right) \tag{24}$$

The revenue of the ISP consists of the revenue for provision of service s_3, the subscription revenue and the transfer of revenue from CP:

$$R_{ISP} = p_2 D_3^+ + C_1\left(S_1^+ + S_3^+\right) + C_2 S_2^+ + p_1 x\left(D_1^+ + D_2^+\right)$$

and its profit is equal to revenue minus provision, opportunity, expansion and maintenance costs:

$$P_{ISP} = R_{ISP} - c_2 D_3^+ - e_2 D_3^- - \sum_{i=1}^{3} g_i S_i^- - rW - q\left(W_0 + W\right) \tag{25}$$

System governance. In order to evaluate the possible impact of the paid content peering, we assume again that the CP moves first, exercising his superior market power, selects price p_1 for his content and share x (if any) of his content provision revenue that he voluntarily transfers to the ISP. The ISP responds by selecting capacity expansion program $W = W(x, p_1)$, price for his content $p_2 = p_2(x, p_1)$ and subscription fee $C_2 = C_2(x, p_1)$ by solving the profit maximization problem

$$\max_{W, p_2, C_2} P_{ISP}\left(x, p, C, W\right). \tag{26}$$

Anticipating these decisions of the ISP, the CP selects its decisions (p_1, x) by maximizing its profit

$$\max_{p_1, x} P_{CP}\left(x, p, C, W(x, p_1)\right) \mid p_2 = p_2(x, p_1),\ C_2 = C_2(x, p_1) \tag{27}$$

Thus, this is again the leader-follower Stackelberg game [50].

4.3 Numerical analysis: the actor's profits and network neutrality

We have solved the problems (26)-(27) for different values of problem parameters. The patterns obtained in the simpler case of a single service from Section 2 were confirmed and additional patterns emerged, regarding the impact of paid content peering on the degree of network neutrality. The main findings are the following.

1. The ISP has an incentive to extract additional subscription fee for allowing customers to have the similar QoE for content provision service of the CP as for his own content service. Thus, in the absence of regulation the network neutrality will be challenged (see Figure 15). However, the extent of violation of network neutrality can be reduced substantially by resort to paid content peering. The difference between subscription fees can be halved, as shown on Figure 15.

2. This increase of grade of network neutrality happens in parallel with increase in profit for both actors. One can see on Figure 16 that profit of the CP increases substantially with the share of content provision revenue accorded to the ISP in the range of 0.3-0.4, compared to the absence of such share.

5 Conclusion and future work

We have developed several game theoretical models of Stackelberg type for analysis of relationship between a powerful content provider and Internet service providers. These models were used for analysis of paid versus free content peering. We have shown that paid content peering can be mutually beneficial to content and connectivity providers even when the content provider has the market power to force the connectivity provider to accept the free content peering. In the case of competition in Internet provision the paid peering can bring a competitive advantage for the ISPs, which have such agreements with content providers. We have provided an insight as to when this will happen: efficient enough connectivity providers, not excessively high demand uncertainty/variability and innovative new services with high price elasticity. We have shown also that paid peering removes part of the incentive to challenge the principle of network neutrality in the case when the ISP diversifies into content provision.

The future work will involve the precise modeling of the influence of Quality of Experience on user's demand in the context of paid peering. Further interesting and important issues to explore are the competition between content providers, welfare analysis of paid content peering and analysis of competition between ISPs viewed as multisided platforms. Future studies should also include the evaluation of different business models for ISPs, taking into account varying among countries economic and regulatory conditions [23].

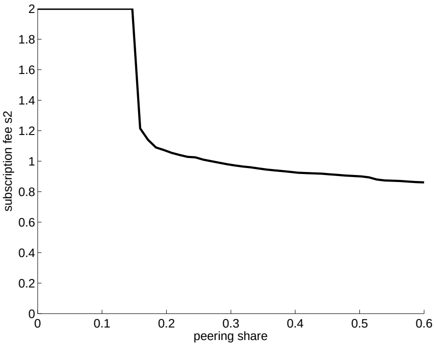

Fig. 15. Dependence of subscription fee for service $s_2$on paid peering share

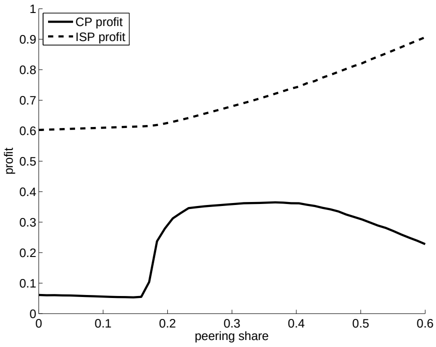

Fig. 16. Dependence of actor's profit on paid peering share

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