The association between engineering students' self-reported mathematical identities and average grades in mathematics courses

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Arguments have been made that one purpose of learning mathematics successfully is for students to develop mathematical identities. Thus, since students are frequently evaluated with grades in university mathematics courses, a relevant question is how mathematical identities are associated with average grades. This study has measured engineering students' mathematical identities and compared these measures with grades in university mathematics courses, and a Welch's ANOVA conclude that the mean average grade amongst students with high mathematical identities is significant, and about one grade higher than students with low mathematical identities. Moreover, the variance is greater amongst students with low mathematical identities, which indicates a strong association between mathematical identity and average grade only when mathematical identities are high.

Keywords: Mathematical identity, Rasch, ANOVA.

Introduction

The transfer of mathematical knowledge from university to the world of work seems problematic. Specifically, evidence has been provided that "attainment" in university mathematics courses is poorly transferred. One example is an experiment that illustrated how 17 students and researchers all failed a mathematics examination they had previously passed, even the students who had recently passed the original exam with an "A" (Rystad, 1993). Moreover, selected studies illustrate how the mathematics is often hidden in "black-boxes" (e.g. Williams & Wake, 2007) in the world of work, and consequently, arguments have been made that the world of work seeks more general mathematical characteristics than what is typically assessed in standard exams (e.g. Hoyles, Wolf, Molyneux-Hodgson, & Kent, 2002). On a general note of education, Wenger (1998) argued that learning is about developing identities in communities of practice. In general, over the last decades, there has been an increased attention towards the construct of identity, and mathematical identity in particular (e.g. Axelsson, 2009; Black et al., 2010; Wenger, 1998). Thus, if the world of work seeks general characteristics of working mathematically, a relevant question is how mathematical attainment in university mathematics courses, as represented by average grades, is associated with mathematical identity. This paper addresses this question.

This study has examined the association between self-reported mathematical identities and average grades in university mathematics courses. From a Rasch calibrated instrument, previously validated in Kaspersen (2015), the students were categorised as having a "low," "medium," or "high" mathematical identity, and the paper will illustrate how the mean average grade of students with high mathematical identities was significant and about one grade higher than students with low mathematical identities. Moreover, the variance amongst students with low mathematical identities was higher than amongst students with high mathematical identities, although the difference was not significant (p=0.06). The paper concludes that high mathematical identities are associated with high average grades in university mathematics courses. However, the same conclusion is not true amongst students with lower mathematical identities.

Theoretical framework

The construct of identity suffers from a lack of consensus on general philosophical issues (Cote & Levine, 2014). Specifically, identity is defined differently across different studies and paradigms, such as "a certain kind of person" (Gee, 2000, p. 99), "those narratives about individuals that are reifying, endorsable and significant" (Sfard & Prusak, 2005, p. 44), and "self-perceived mathematical knowledge, ability, motivation and anxiety" (Axelsson, 2009, p. 387).

This lack of consensus is typical in pre-paradigmatic fields (Kuhn, 1970). Unlike firm paradigmatic fields where well-established theories tend to guide the analyses, research in pre-paradigmatic areas has a more dialectical relationship between data and theory (Kuhn, 1977). This description is a fair representation of how the theoretical perception in this study was chosen. That is, no ready-made theory was chosen on pure faith. Rather, a definition of identity was established that was consistent with measurement (i.e., consistent to conclude some persons to have stronger mathematical identities than others), yet, influences by fragments of multiple existing theories. The following theoretical perspective and a wider discussion on practical significance has been provided in more detail in Kaspersen, Pepin, and Sikko (2017).

On another note, we do not regard theories as mirrors of some true reality. Thus, we do not believe that some theories *are* true, and that others *are* false. When we propose the following theoretical perspective, therefore, we are not refusing other perspectives, for instance, a narrative view on identity. Rather, we claim that *if* we choose the following perspective, then the practical consequence is that mathematical identity can be measured.

The perspective of mathematical identity relies on two assumptions. First, we assume that identity (originated from the Latin *idem*) is about sameness and distinction. As such, the position in this study juxtaposes perspectives that consider persons to have their unique identity. That is, persons are indeed unique. However, they can be defined as identical with respect to a set of characteristics, just like mathematical objects can be identified by certain characteristics while remaining unique on others. Moreover, since there exists an infinite number of characteristics, identities have a varying degree of complexity. That is, mathematical identity can be binary, linear, or multidimensional, and we argue that there is no ontological limit to the number of dimensions. Consequently, there exists no set of criteria that dictates when researchers have arrived at the final dimension. Hence, the choice of complexity can be nothing but pragmatic, and in this study, we have chosen a one-dimensional perspective on mathematical identity, whereby persons are distinguished on a continuum from having a low to having a high mathematical identity within the engineering education context.

Furthermore, if we accept that persons participate and contribute in multiple activities, a consequence is that each person has multiple identities, a position that is shared by many authors, for example Black and colleagues (2010) who, inspired by Leont'ev (1981), presented the idea of "leading identity." Since there is no limit to how many ways persons can be distinguished, we argue that there exists no limit to the number of identities, although the number of identities that individuals are consciously aware of is likely to be finite. Moreover, in this study, we take no definite position on the relationship between identities. Thus, when we later will conclude that selected persons have (more or less) the same mathematical identity, we do not make claims about how these are related to the multiplicity of identities–for instance, whether they are central/leading or peripheral identities.

Second, we assume that identity is relational by nature. That is, persons can be concluded to be identical relative to a set of characteristics, only if the structure of these characteristics is person-independent. Thus, in quantitative studies, we reject the assumption that persons with the same score on some test or questionnaire are identical unless statistical evidence is provided that the items stay invariant across relevant subgroups. Hence, there likely exist contexts that are so different that comparisons of identities across these contexts do not make sense. Consequently, we argue that the methods that are applied to capture identities should also capture the level of invariance.

In conclusion, we define mathematical identity to be *where persons position themselves relative to the social structure of being mathematical within the activity in which they participate and contribute*. From a one-dimensional perspective, "the social structure of being mathematical" is a person-independent set of characteristics and their internal structure (i.e., their relative distance) that distinguishes persons on a continuum from having a "low" to having a "high" mathematical identity. "Where persons position themselves" is persons' positions relative to the social structure.

Method

To test the relationship between engineering students' self-reported mathematical identities and average grade in mathematics courses, a convenience sample consisting of Norwegian engineering students (*N*=361) was selected. 47 students attended an "Introductory course in mathematics," 71 students attended a "Calculus 2" course, 113 attended a "Calculus 3" course, 11 a "Cryptography" course, and 119 were students from a variety of courses in their normalised final year of education. The participants responded to a Rasch-calibrated instrument (Rasch, 1960), previously validated in Kaspersen (2015), that measures persons on a continuum from having a low to having a high mathematical identity relative to 20 uni-dimensional characteristics. The items in the instrument were collected from three sources: the literature, other related instruments, and from persons contributing in mathematical activities (e.g., students and lecturers). The validation of the instrument will not be discussed in depth, as details can be found in Kaspersen (2015). The person reliability, analogous to Cronbach's alpha, was 0.87. Moreover, from principal component analysis of residuals, the instrument was found to be sufficiently uni-dimensional for the purpose of measurement with a 1.99 unexplained variance (in Eigenvalue units) in a second contrast. Furthermore, the mean of the squared standardised residuals (outfit mnsq) and the information-weighted version (infit mnsq) (see e.g., Bond & Fox, 2003, p. 238 for a detailed description) indicated a sufficient data-model fit, with Item 6 and Item 15 as the most underfitting items (Table 1).

Rasch measurement requires additivity, uni-dimensionality, and invariance, and the probability of an observation is a function of the difference between a person's measure and a characteristic's measure (e.g. Wright & Stone, 1979). Thus, most response strings follow a Guttman-like structure with most deviations around the measure of the person. Consequently, persons with approximately the same measures, except those with large misfit, have, not only the same measures but also approximately the same combination of self-reported characteristics (and thus concluded to be identical with respect to these characteristics).

After the validation of the instrument, the respondents were categorised as having either low (measures lower than -1), medium (measures between -1 and 1), or high (measures above +1) mathematical identities (all measures are in logit units). The distance from the "low"/"medium" to the "medium"/"high" thresholds was about the same distance as one response category. Consequently, persons with "high" mathematical identities were expected to respond at least one category higher on each characteristic than persons with "low" mathematical identities. Subsequently, a one-way ANOVA was conducted to compare the association between mathematical identity and the self-reported average grade in mathematics courses at the University (from grade F=1 to grade A=15). However, since the Levene's (1960) test barely accepted the null hypothesis of homogeneity of variances (p=0.06), and the sample sizes across categories were unequal, the Welch's ANOVA was chosen since it is more robust to unequal sample size and variance.

Moreover, the assumption of normality was violated, and the grades were ordinal as opposed to interval measures. Since Welch's ANOVA assumes normal and interval measures, 10,000 simulations were made in R (R Core Team, 2015) to assess how these violations affected the robustness of the analysis. To ease this part of the analysis, we considered a transformed data set which had no difference in the mean across groups but was otherwise identical to ours–the assumptions of Welch's ANOVA were violated equally in the empirical study and the simulated studies. This transformation eased the interpretation since we could compare the results with the statistical ideal situation (perfectly normal interval data, equal sample size and variance). If our data set was as good as the ideal situation, we would expect the Welch's ANOVA to show a significant difference in about 5% of the simulations.

Specifically, from the empirical data frame, M, a new data frame, M', was made whereby each grade in the medium and high groups was shifted so that the mean of all three categories in M' were equal (i.e., keeping the sample sizes and distributions, but aligning the means). From M', 10,000 data frames, $M_1 - M_{10,000}$, were randomly sampled whereby the sample sizes in the three groups were equal to the original M. Subsequently, Welch's ANOVA was conducted on each simulated data frame. Since the result showed that 5.2% of the p-values in the simulations were less than .050, it was concluded to ignore violations of Welch's ANOVA's assumptions since they had only a trivial negative effect on the robustness.

Result

Mathematical identities

Due to the Guttman-like response strings, a rough interpretation of Table 1 is that most students with low mathematical identities (measures lower than -1) agreed with characteristics much lower than -1, and disagreed with those much higher than -1. That is, students with low mathematical identities often keep trying when they get stuck, but they rarely study proofs until they make sense (to them), they rarely like to discuss mathematics, they rarely derive formulas, etc. Likewise, students with medium mathematical identities (measures between -1 and 1) frequently keep trying, connect new and existing knowledge, and can explain why their solutions are correct, but rarely take the initiative to learn more than expected, rarely take the time to find better methods, etc. Students with high mathematical identities (measures above +1) agree with most characteristics in the instrument. A more thorough discussion is discussed in Kaspersen, Pepin, and Sikko (2017).

Item statistics: Measure order

Measure	INFIT MNSQ	OTFIT MNSQ	Item
1.91	.81	.83	1. Takes time to find better methods
1.58	1.08	.99	2. Takes the initiative to learn more
1.24	.91	.86	3. Thinks of times when methods don't work
.55	1.22	1.20	4. Struggles with putting problems aside
.51	1.05	1.07	5. Derives formulas
.45	1.36	1.37	6. (x) Likes to be told exactly what to do
.41	.96	.95	7. New ideas lead to trains of thoughts
.32	1.05	1.05	8. Likes to discuss math
.20	1.07	1.07	9. Makes his/her own problems
.05	.99	.99	10. Studies proofs until they make sense
.04	.86	.88	11. Moves back and forth between strategies
–.10	.87	.86	12. Tries to understand formulas/algorithms
–.20	.72	.74	13. Considers different possible solutions
–.26	1.03	1.05	14. Pauses and reflects
–.38	1.32	1.31	15. Finding out why methods do not work
–.47	.86	.86	16. Wants to learn more things
–.77	1.20	1.20	17. Visualises problems
–1.19	.71	.76	18. Can explain why solutions are correct
–1.83	.83	.88	19. Connects new and existing knowledge
–2.05	1.02	1.06	20. Keeps trying

Note. Item 6 was negatively coded

Items in their entirety in https://www.researchgate.net/publication/309740755_math_identity_questionnaire

Table 1: Characteristics of mathematical identities amongst Norwegian Engineering students

Moreover, it is evident from Table 1 how the identities in this study were situated amongst the engineering student context. That is, persons with measures, say, around 0.5 in other contexts would be identical to engineering students with the same measures, only if the same set of characteristics were proven to be invariant (i.e., calibrated to have the same structure) in both contexts.

The relationship between self-reported mathematical identities and average grade

Figure 1 illustrates the relationship between self-reported mathematical identity and average grade in university mathematics courses. The Welch's ANOVA showed that the association between mathematical identity and self-reported average grade was significant, $F(2, 110.79)=31.966$, $p=0.000$. Moreover, the mean of the self-reported average grade amongst students with high mathematical identities was about one grade higher than those with low mathematical identities. The Games-Howell test showed that the difference was significant between all groups with low-medium as the least significant ($p=0.001$).

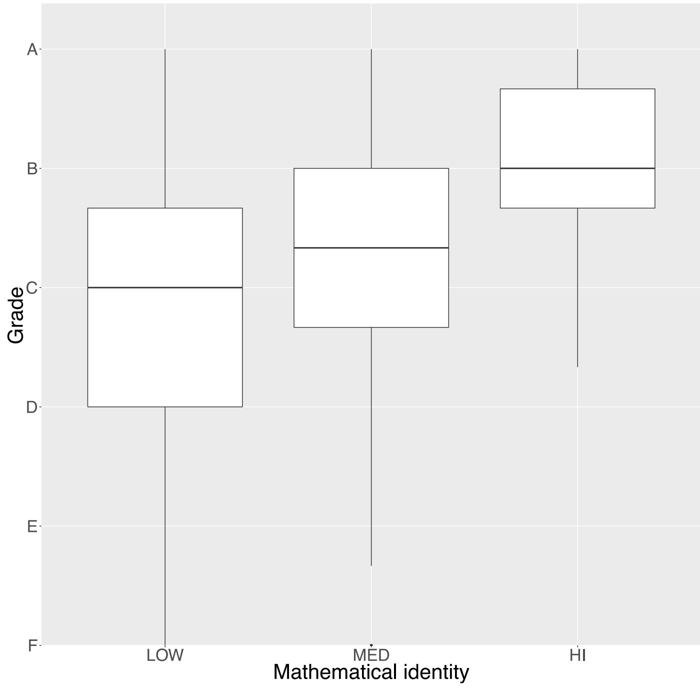

Figure 1: The relationship between self-reported mathematical identity and average grade in university mathematics courses

The unequal variance is also illustrated in Figure 1. Specifically, the variances decreased with the increase of mathematical identity. That is, high mathematical identities are associated with high self-reported average grade. However, there seems to be no limit to how low mathematical identities students can have and still get high grades.

Conclusion and discussion

In this paper, we have argued that the average grade in university mathematics courses amongst students with high mathematical identities is about one grade higher than amongst students with low mathematical identities, and the difference is significant. Moreover, we have shown that the variance of self-reported average grades amongst students with low mathematical identities is higher than amongst students with high mathematical identities. That is, students with high identities get, for the most, high grades. However, the grades of students with lower identities are more uncertain.

We have in this study examined the association, and not the causal relationship, between self-reported mathematical identities and average grades, and therefore we argue that the significance of the result is that it points the direction for future research. Specifically, we suggest future research to address the following:

First, replicates of this study should seek more precise measures. That is, the precisions of the mathematical identity measures can be improved by including more response categories (as long as they are sufficiently validated) and more items, particularly near the "gaps" (e.g., between 0.5 and 1.2 logits). Moreover, the precision of the average grade would most likely be improved if self-reported average grades were substituted with actual average grades.

Second, future research should seek a more causal relationship between identities and grades. Specifically, this study does not conclude that an increase in mathematical identity infers an increase in attainment.

Third, future research could study the significance of mathematical identity versus the significance of attainment. For instance, students can be categorised as having “low identities and low grades,” “low identities and high grades,” or “high identities and high grades,” and subsequently studied with respect to other variables, for example, in the transition from university to the world of work.

Fourth, we argue that future research can transfer the design of this study to other samples and forms of testing students’ attainment. For example, relationships between mathematical identity and measures on international standardised tests, such as PISA and TIMSS, can be tested. Accordingly, we argue that future research can nuance the debate on the significance of these tests. If some districts/countries are “teaching to the test,” then one might hypothesise that a relatively great proportion of students in these districts/countries are in the “top left corner”–that is, students with low mathematical identities, yet, high measures of attainment.

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