'Walking a graph' – primary school students' experimental session on functions and graphs

Svein Arne Sikko, Øistein Gjøvik, Jardar Cyvin, Maria I.M. Febri and Ragnhild L. Staberg

Norwegian University of Science and Technology (NTNU), Norway; svein.a.sikko@ntnu.no, oistein.gjovik@ntnu.no

In this paper we report an experimental activity involving working with functions and graphs in a grade 6 class in a Norwegian primary school. We argue that working with graph loggers in the form of an echo sound system enhances student conception of mathematical graphs.

Keywords: Mathematical concepts, mathematics activities, mathematics education, educational technology.

Background and literature review

The research reported in this paper was part of the EU FaSMEd project[1], which brings together seven European countries and South Africa, researching the use of formative assessment and technology in mathematics and science education. Part of each country's work has included case study interventions in ordinary classrooms in close cooperation with school teachers. In this paper, we focus on a particular set of lessons concerning functions and graphs, and how one particular type of technology was used to increase student activity and engagement, leading to enhanced learning.

The function concept is generally regarded as a difficult concept for students to grasp (Dreyfus & Eisenberg, 1982; Sajka, 2003; Sierpinska, 1992). Dreyfus and Eisenberg (1982) point out that the function concept is not a single concept by itself, but has several aspects and sub-concepts associated with it. DeMarois and Tall (1999) connect this to the learning of functions, saying that "for many students, the complexity of the function concept is such that the making of direct links between all the different representations is a difficult long-term task" (p.264). Also in Norway, national and international tests have shown that mathematical functions is a problematic topic area. It took humanity several thousand years of mathematical activity until functions were introduced in the 17th century; and even then it took time to place functions on a solid foundation within mathematics. It is therefore not surprising that pupils struggle with functions and how to respond to this struggle has been addressed in various ways. During the "New Math" movement in the 1960-70s, it was believed that school mathematics should resemble research mathematics, and attempts were made to introduce functions during the first years of primary school. Eicholz, Martin, Brumfiel and Shanks (1963) did exactly that. This American textbook was translated into several languages, including Norwegian. The pendulum turned away from this with the "back to basics" movement, and, in Norway, functions were moved to the secondary school. As a consequence, most research on students' understanding of functions and graphs is conducted with secondary school students. In this paper, we report on work done in primary school.

[1]FaSMEd = Improving Progress for Lower Achievers through Formative Assessment in Science and Mathematics Education, see https://research.ncl.ac.uk/FaSMEd/

According to Duval (2006) we can only gain access to mathematical objects by semiotic representations. Janvier (1984) distinguished between four representations of functions: situations; graphs; tables; and formulae; and how to work with transitions between these. Duval (2006) stresses that “What matters is not representations but their transformation” (p. 107). That is, when learning about a mathematical concept, students deal with a representation of the object, and the main difficulty is to change between different representations of the same object. Duval distinguishes between conversions and treatments. Here, treatments take place between the same registers (e.g., changing y = 2x from one particular form to another), while conversions take place between registers (e.g., reading a table and using the numbers within it to interpret a situation). The latter seems to be far more difficult, while the former is the most common activity format in school. Consequently, Janvier’s framework helps teachers to focus on the change of registers rather than algebraic manipulation alone. To support the learning of functions, many different kinds of digital tools have been developed and used. Regardless of any particular view on learning outcomes from using technological tools, it is important to realize that use of technology is more than the introduction of new tools. In a survey on mathematics teachers’ use of technology in England, Bretscher (2014) found that while ICT might contribute to change, the direction of this change was as likely to be towards “more teacher-centered practices rather than encouraging more student-centered practices” (p. 43). The tools used in our study are a particular type of data loggers. These are mainly used in science, but we claim that inclusion of these types of tools may be beneficial also in the mathematics classroom, and may contribute to more student-centered practices. According to Newton (2010) “data-logging methods involve the use of electronic devices to sense, measure and record physical parameters in experimental settings.” (p. 1247). Measurements and results of the logging can be displayed on a computer screen, either subsequently or simultaneously. We used this type of technology to study students’ early understanding of graphs. The use of motion experiments in the learning of functions has been studied by several authors. Nemirovsky (2003) conjectures that "mathematical abstractions grow to a large extent out of bodily activities" (p. 106). Arzarello and Robutti (2004) claim that students can grasp mathematical concepts through meaningful sensory-motor experiences if they are encouraged to communicate and have the necessary support (p. 308). Arzarello, Pezzi and Robutti (2007) point out that teachers can use new technology to design experiences for students “where graphs can be presented in a dynamical and genetic way” (p. 135). Robutti (2009) conducted research on time-distance graphs with kindergarten children using motion sensors and calculators, finding that even very young children were able to make connections between the movements they made in front of the sensor and the graph sketched by a calculator.

The research question addressed in this paper is: How can a primary school teacher use data logger technology to enhance primary school students’ engagement and conceptual knowledge about function graphs?

Method

The teaching experiment was carried out in a grade 6 class (students around 11 years old) in a primary school in Norway. The number of students in the school is close to 600, and the number of teachers around 35. The participating teacher has background from general teacher education, with specialization in mathematics and history. At the time of the experimental sessions, he had been working as a teacher for 7 years, the last three years at the school in question. He had been teaching mainly mathematics, and also some science. During his participation in the FaSMEd project he was teaching the same group of students, which began in grade 5 and continued with the same group of students into grade 6. There were 31 students in his class, 15 girls and 16 boys.

The theme of the teaching sessions was time-distance graphs. Several technological tools and software had been introduced to the participating teachers at FaSMEd meetings at the university. Teaching material from the FaSMEd toolkit that addressed time-distance graphs had also been introduced. The planning of the experimental lessons began at this meeting. The teacher would give one lesson introducing the students to graphs and to the connection between graphs and real life situations. Working with mathematical graphs connecting situations and graphical representations is usually not done in Norwegian primary schools. According to the national curriculum, functions and graphs is not a specified learning goal for students until after grade 10. This would therefore be the first time this teacher had worked with students in primary school on graphs. Because of this, he wanted to first pilot the lesson on a small group of students that he considered high achieving and with an interest in mathematics. Subsequently the lesson was repeated with a group of students considered to be lower achievers.

The technology used was two echo sounder devices developed by Pasco. This was chosen as the entry level for using it was not too high, and therefore the teacher considered it could be experimented with grade 6 students. It facilitated students to walk back and forth in relation to a logging device, such that a graph was immediately drawn on the computer screen indicating how near they were the device during a time lapse of ten seconds. The immediate live update of the graph distinguishes this activity from most regular science data logging activities. The computer was loaded with an app with premade tasks that were presented to the students. When students walked in front of the echo device, the computer would give a live display of graph in a time – position coordinate system. The tasks[2] were a mix of practical tasks: “Walk a graph”, and open-ended questions about interpretation of the graphs from the “walks”. All the results were saved and could be used by the teacher for assessment and feedback to the students. These data were e.g. used by the teacher at the end of the sessions to determine which student groups should present their work in a plenary. Students were chosen deliberately to give good examples of graphs made and how to interpret them.

Data sources collected during the experiment include a) observation sheets from two sessions; b) audio recordings from two sessions, from teacher pre- and post-interviews, post-lesson reflections, interviews and q-sorting activities with students; c) video recordings from two sessions; d) transcriptions of audio and video recordings; e) photos taken at sessions and of student work; f) files and screen shots from PC during student activity; g) teacher lesson plans for two sessions.

After the “walking a graph” activity, students were interviewed in a q-sorting[3] activity; i.e. they were presented with a set of statements printed on cards and asked to sort the cards according to

[2] The echo sound activity used some tasks taken from the software bundled with the Pasco software. Instructions for using the software and tasks were translated into Norwegian by the FaSMEd team. Some additional tasks were added.

[3] https://en.wikipedia.org/wiki/Q_methodology

whether they agreed, disagreed or were undecided about, the statement on the card. This activity was carried out in groups of 3 to 4 students.

Findings

The echo sound activity made this lesson stand out from an ordinary mathematics lesson. One student said,

> Student A: It was very different (…) In maths lessons we never move, we sit at our seat; except sometimes we go out to do measurements, but that is always during summer.

The tasks were also considered different to normal mathematics exercises on two accounts. First, students were not used to doing mathematics tasks using computers. Second, in the classroom they usually have to compute things, whereas in these lessons

> Student B: There were word problems and we had to do things.

The q-sort revealed that students generally agreed to statements that connected mathematics to real life. E.g., students agreed to the statements “Mathematics helps us to understand our surroundings” and “Mathematics is used in everyday life”, whereas they disagreed with the statements “Mathematics is only for the classroom, not for real life outside school”, “I can do without mathematics” and “Mathematics is not relevant for my future life”. The q-sorting activities were completed around two weeks after the time-distance graph lesson. We may therefore claim that there is some evidence indicating that the lesson had made students aware of, or strengthened their awareness of, connections between mathematics in school and real life situations that can be described by mathematics or where mathematics is used. Students agreed that “Mathematics is important”, claiming that

> Student C: We use it all the time. Everywhere. In the shop. (…) On trains. Airplanes. The bus.

It seems that these groups of students held positive attitudes towards mathematics, and that they were able to see mathematics as relevant for themselves and for real life situations. During the echo sound activity, the students had to relate what they were doing, i.e. the way they were walking in front of the sensor to the graph the software would display on the PC screen. We can see evidence that students were able to connect the pace of their walking to the slope of the graph:

> Student E: It rises earlier because you walk faster.

This relates the time (horizontal axis), distance from the sensor (vertical axis) to speed (how steep the curve is), a fundamental relationship in understanding time distance graphs, and a fundamental relationship in physics, and of course in everyday life. There were several student utterances showing the same kind of understanding:

> Student F: It will be more slanted the faster you walk. So you start slow, then you walk faster.

The teacher asked another student how you can find from the graph where you walked faster. The student said that

> Student H: You can see, because, first it is quite slanted, and then it goes straight up.

Students also developed understanding of the fact that a graph does not have to start at the origin. When trying to walk in a way that would produce a W as graph:

Student I: You have to start far away [from the sensor] because then it goes downwards and then it goes upwards and then it goes downwards.

We see here that they understand that a graph can cross anywhere on the vertical axis, and the relationship between distance from the sensor and time passed. Their descriptions and discussions did not use mathematical vocabulary or concepts. Rather, they described what they saw in everyday terms, which shows that they are able to change registers and not only operate within the same register. These examples show how such an activity helps students in the process of conversion. The activity offers two aspects of working with graphs. On the one hand, students had to translate a given situation into a movement in front of the echo sound device, observe the graph being plotted on the computer screen and adapt their movement to change the graph as needed. On the other hand, students would interpret a graph plotted on the screen into what kind of movement that this would correspond to. In the interviews, students said the tasks in this lesson were more challenging than the mathematics tasks they normally work with, e.g., in that they had to explain how they did things. Being challenging is not really a bad thing, and students said they found the sessions to have been great fun and exciting. They claimed that they had learned a lot about graphs. During q-sorting, students who agreed to the statement “I can better understand when I use the technology tools in our mathematics lessons” also agreed that the statement referred to learning about graphs:

Student E: I learned a lot about graphs and how they change with the computers

When the lesson was repeated with students that were considered to be lower achievers, the setup remained unchanged, making it more relevant for comparison. Notably, we found that it was not possible to distinguish any big differences between the first session with higher achievers and the second session with lower achievers.

Teacher: It was indeed very similar (…) maybe these were a bit slower. And I would be tougher, push the others more. (..) But I think they were clever, they were good at cooperating, learning on each other. (…) It shows that if you have open and good tasks, you have a lot of differentiation included.

The activity prompted student communication and discussion. The teacher found that students who normally keep quiet were engaged in discussion.

Teacher: In particular, some of the girls in the last group, they were talking, usually they are very quiet. Now they talked, without me having to point at them, prompting them; now they gave their opinion (…) And I was positively surprised at how easy it was for them, to listen to each other’s arguments.

It was obvious that, even if these types of activities with graphs are common in Norwegian primary schools, it had not been too difficult and that this is a topic that could easily have been done with the whole class. The teacher said that

Teacher: I think, interpreting graphs, it could have been done quite easily. (…) I think this might be more fun in primary than in lower secondary school. They still find it exciting with graphs (...) they are more curious and less biased.

In the interviews, all students said that they had enjoyed taking part in the project and performing the lessons with graphs.

Student: In my opinion everything was good (…) We learned a lot about graphs.

Discussion and conclusions

The echo sound graph plotting activity was very useful to give the students hands-on experience in using modern technology and use their own physical movements to create something to talk about mathematically. Acquiring experience with new technologies can be an educational goal in itself, and in particular, echo sound technology is not common in the classroom, but it is well known in other aspects of life. In the interviews, students claimed this was an important part of what they had learned and which distinguished these lessons from ordinary mathematics lessons. In traditional data logging experiments, students might see the data collection and the data analysis as two separate entities as these are separated in time (Barton, 1997). In our experiment, the gap between the collection of the data and the displayed graph is narrowed down to practically zero. In this respect, this activity also resembles working with dynamical graph tools, like GeoGebra[4]. These software tools allow students to explore graphs by manipulating parameters within designated bounds, while walking a graph changes freely the look of a graph only limited by the range of the echo sound device. This is more in line with work by e.g. Arzarello and Robutti (2004) and Robutti (2009).

When looking at the Janvier table we see that what the students had engaged in was making a transition between a situation and a graph. However, the typical sketching activity proposed by the Janvier framework when working with functions usually has a different feel than in this experiment. Not only is the sketching part of the activity itself done in a kinesthetic manner. There is also a dual aspect in that the students continually interpret the graph whilst the graph is sketched by the program on the screen. This way we can say that students work simultaneously with two elements of Janvier's framework, giving further evidence that changing registers is a difficulty that can be overcome by giving appropriate teaching materials.

The kinesthetic part of the activity, the walking, is in itself an important aspect of the experiment. As it turned out, the designated low achievers were able to perform well and display great enthusiasm during the session. This can be related to the way learning through movement can be an alternative approach to put students in a receptive state, ready for learning. Learning through actually moving your body is rarely an aspect of mathematics lessons, but can certainly encourage engagement, as seen in this experiment.

The type of activity exemplified in this experiment is completely devoid of focus on algorithms or procedural performance in the form of computations. Students do not know in advance how to solve the problems presented, and so focus is on developing conceptual knowledge about function graphs. From their statements, we also see that they relate mathematical concepts, like slope, to real world experiences, like speed. This is similar to findings in Robutti (2009, p. 68). A well founded conceptual understanding of functions and graphs in a time-distance setting will contribute to better understanding of functions on a more general level. When students encounter functions at higher

[4] http://www.geogebra.org

grades, their conceptual foundation will make it easier to grasp other aspects and algorithmic approaches to functions.

The literature suggests that teachers need support of different kinds in order to conduct teaching with new technologies in a meaningful manner. For example, building on a large teacher survey in Singapore, Tan, Hedberg, Koh and Seah (2006) suggest that teachers need support from laboratory technicians, data logger training, and instructional material to use data loggers effectively. In our case, none of these were present. We do however acknowledge the collaborative effort between teacher and researchers as instrumental to the success of the sessions. It is also important to stress that learning is not an automatic outcome from playing with technological tools, no matter how sophisticated the tools are. The role of the teacher is instrumental in bringing about learning, as highlighted by Clark-Wilson, Robutti and Sinclair (2014, p.396).

Acknowledgment

The FaSMEd project has received funding from the European Union Seventh Framework Programme under grant agreement n° 612337.

References

Arzarello, F., & Robutti O. (2004). Approaching functions through motion experiments. *Educational Studies in Mathematics*, *57*(3), 305–308.

Arzarello, F., Pezzi, G., & Robutti, O. (2007). Modelling body motion: an approach to functions using measuring instruments. In W. Blum, P.L. Galbraith, H.-W. Henn & M. Niss (Eds.), *Modelling and applications in mathematics education* (pp. 129–136). New York, NY: Springer.

Barton, R. (1997). Does data logging change the nature of children's thinking in experimantal work in science? In B. Somekh & N. Davis (Eds.), *Using information technology effectively in teaching and learning: studies in pre-service and in-service teacher education* (pp. 63–72). London: Routledge.

Bretscher, N. (2014). Exploring the quantitative and qualitative gap between expectation and implementation: A survey of English mathematics teachers' uses of ICT. In A. Clark-Wilson, O. Robutti, N. Sinclair (Eds.), *The mathematics teacher in the digital era: An international perspective on technology focused professional development* (pp. 43–70). Dordrecht: Springer.

Clark-Wilson, A., Robutti, O., & Sinclair, N. (2014). Summary and suggested uses for the book. In A. Clark-Wilson, O. Robutti, N. Sinclair (Eds.), *The mathematics teacher in the digital era: An international perspective on technology focused professional development* (pp. 395–401). Dordrecht: Springer. doi:10.1007/978-94-007-4638-1_17

DeMarois, P., & Tall, D. (1999). Function: Organizing Principle or Cognitive Root? In O. Zaslavzky (Ed.), *Proceedings of the 23rd Conference of the IGPME vol. 2* (pp. 257–264). Haifa, Israel: Israel Institute of Technology.

Dreyfus, T., & Eisenberg, T. (1982). Intuitive functional concepts: A baseline study on intuitions. *Journal for Research in Mathematics Education*, *13*(5), 360–380.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational studies in mathematics*, *61*, 103–131. doi: 10.1007/s10649-006-0400-z

Eicholz, R. E., O'Daffer, P. G., Brumfield, C. F., & Shanks, M. E. (1963). *Elementary School Mathematics*. Reading, MA: Addison-Wesley Publishing.

Janvier, C. (1984). Translation processes in mathematics education. In C. Janvier (Ed.*), Problems of representation in the teaching and learning of mathematics*. Hillsdale, NJ: Erlbaum

Nemirowsky, R. (2003). Three Conjectures concerning the relationship between body activity and understanding mathematics. In N.A. Pateman, B.J. Dougherty, & J.T. Zilliox (Eds.), *Proceedings of PME 27* (pp. 105–109). Honolulu: HI: College of Education, University of Hawaii.

Newton, L. (2000) Data-logging in practical science: research and reality. *International Journal of Science Education*, *22*(12), 1247–1259. doi:10.1080/09500690075003624

Robutti O. (2006). Motion, Technology, Gesture in Interpreting Graphs. *The International Journal for Technology in Mathematics Education, 13*(3), 117–126.

Robutti, O. (2009). Space-time representations in young children. Thinking through gestures in motion experiments. In C. Andersen, N. Scheuer, M. P. Pérez Echeverria, & E. V. Teubal (Eds.), *Representational systems and practices as learning tools* (pp. 59–75). Rotterdam: Sense publishers.

Sajka, M. (2003). A secondary school student's understanding of the concept of function - A case study. *Educational Studies in Mathematics*, *53*(3), 229–254. doi:10.1023/A:1026033415747

Sierpinska, A. (1992). On understanding the notion of functions. In E. Dubinsky & G. Harel (Eds.), *The concept of function. Aspects of epistemology and pedagogy* (pp. 25–58). Washington, DC: Mathematical Association of America.

Tan, K. C. D., Hedberg, J. G., Koh, T. S., & Seah, W. C. (2006). Datalogging in Singapore schools: supporting effective implementations. *Research in Science & Technological Education*, *24*(1), 111–127. doi:10.1080/02635140500485431