# Conflict and cooperation in an age structured fishery 

Asle Gauteplass, Anders Skonhoft

Published in

Fisheries Research
Volume 203, July 2018, Pages 35-45
https:/ / doi.org/10.1016/j.fishres.2017.08.020

# Conflict and cooperation in an age structured fishery 

Asle Gauteplass and Anders Skonhoft<br>Department of Economics<br>Norwegian University of Technology and Science NTNU

## Abstract

The literature on 'fish wars', where agents engage in non-cooperative exploitation of single fish stocks or interacting fish stocks is well established, but age and stage structured models do not seem to have been handled within this literature. In this paper we study a game where two agents, or fishing fleets, compete for the same fish stock, which is divided into two harvestable age classes. The situation modelled here may be representative for many fisheries, such as the Norwegian North Atlantic cod fishery where the coastal fleet targets old mature fish while the trawler fleet targets young mature fish. We analyse the game under different assumptions about the underlying information available to each fleet and the actions of the agents. The outcomes of the games are compared to the optimal cooperative solution. The paper provides several results, which differ in many respects from what are found in biomass models. The analysis is supported by numerical examples.

Key words: Fishery economics, age model, conflicts, optimal exploitation

JEL Classification: Q22, Q58

## 1.Introduction

Marine fisheries are frequently a source of international conflicts and often characterized by suboptimal resource management. Fish stocks spread across vast distances, and are often present both in the high seas and within the exclusive economic zones of one or more countries at the same time. Many fish species are also highly migratory, travelling along coastlines and up and down rivers, spending much of their lifetime outside of the breeding grounds, and are hence subject to harvest from different agents at different points in time. A particular aspect of this situation is that different age categories of the same stock frequently reside within the economic zones of different countries. In this case, different fleets do not strictly speaking aim for the same fish, but they nevertheless affect each other's harvest and profit through the biological interaction of the stock. A similar situation may also occur between fleets that are distinguished not by nationality, but by different gear, thus aiming for different age categories of the same stock. This situation, which is not adequately handled within the existing literature on biomass models and sequential fishing, is not uncommon. Examples include the Norwegian North Atlantic cod that feeds in the Barents region, thus subject to harvest by trawlers, but where the old mature fish migrates along the Norwegian cost to spawn, there being exploited by small scale coastal fishing vessels. This fishery has been extensively studied, see e.g., see e.g. Sumaila (1997) and Armstrong (1999). Other examples in the same vein include the Southern bluefin tuna that spends its immature phase along the coast of Australia, but then migrates to the high seas in the Indian Ocean. Similar descriptions apply to the Canada halibut and the North Sea herring, and in general to anadromous species, such as salmon that spawns in rivers but lives most of its life in the open sea. These are some of the world's most valuable fisheries.

The literature on 'fish wars', where agents engage in non-cooperative games of exploiting a fish stock, has grown large since the seminal contributions of Munro (1979) and Levhari and Mirman (1980). A survey is provided by Kaitala and Lindroos (2007). For our purpose, the literature on 'sequential' fishing, where agents alternate in exploiting a common stock that migrates between economic zones, is of particular relevance. Hannesson (1995) studies the possibility for selfenforcing agreements in such a sequential fishery, and McKelvey (1997) expands the framework to consider the possibility of side payments. Laukkanen (2001) shows that the effectiveness of trigger strategies to maintain a cooperative equilibrium is undermined when stock recruitment is
subject to stochastic shocks. However, these studies all employ biomass models, implicitly assuming that the fish caught in one area is identical to the fish caught in another. Age structured models, on the other hand, are still scarce in the economic literature, as noted by Skonhoft et al. (2012). The seminal book on bioeconomic modeling by Clark (1990) treats the Beverton-Holt model to some extent (Beverton and Holt 1957), but puts main emphasis on biomass models. Important contributions by Reed (1980), Charles and Reed (1985) and Getz and Haight (1989) have subsequently enhanced the economic understanding of the exploitation of age structured fish stocks. In a more recent contribution, Tahvonen (2009) presents a thorough study of the optimal harvesting of age structured stocks, under the assumption of non-selective gear. See also Tahvonen (2010) for a general survey, and Quaas et al. (2013). Very few studies address age structured stocks in a game theoretic setting. One example is Lindroos (2004) who examines the benefit of cooperation in the Norwegian spring-spawning herring fishery. Two other notable examples that both study the North Atlantic Norwegian cod fishery mainly through numerical analysis include Sumaila (1997) and Diekert et al. (2010). Sumaila analyses the difference in profitability between a trawler fleet and a coastal fleet, and demonstrates several results that concur with the findings in the present paper. Specifically, the observation that the least profitable fleet in a cooperative harvesting scenario, which typically may be the trawler fleet that targets the smaller fish, may have a strategic advantage in a non-cooperative situation due to the biological interaction of the stock. Thus, the least profitable fleet may be able to drive the other fleet entirely out of business, with large consequences for overall profit. The age structure of the fishery thus gives rise to a noncooperative game that is even more harmful than the standard one found in biomass models. Diekert et al. (2010) assume symmetric players, i.e. two trawler fleets, that compete both through mesh size and fishing effort. They show that a non-cooperative solution implies 'fishing down the size categories', and that the outcome of a non-cooperative open loop equilibrium is both far from the cooperative optimum and close to the status quo situation in terms of profit and stock size.

In the present study we do not attempt to accurately describe a particular fishery, but to analyze a stylized situation where different age categories of a fish stock reside within two different economic zones, or management areas. The exploitation of the stock is modeled as a game between two fleets that aim for different cohorts, but nevertheless affect each other's profitability through the biological interaction of the stock. We derive analytical results characterizing the equilibrium
solutions under different management regimes. First, overall optimality is addressed, which under certain conditions also can be interpreted as a cooperative equilibrium with side payments. Second, we discuss the situation where both fleets are unable to organize internally and hence exhibit myopic behavior, and derive conditions for one of the fleets to be excluded from the fishery in this case. Third, the situation where one fleet is uncoordinated and the other behaves as a single entity is studied. It is shown that, depending on parameter values, both coexistence and exclusion is possible in all different scenarios. The results are subsequently illustrated with a numerical example.

The paper is organized as follows. In the next section 2, the population model with two harvestable age classes is formulated. In section 3 we analyze the optimal harvest regime under cooperation Section 4 presents the non-cooperative solution where we first focus on myopic exploitation. Additionally, we also study a Stackelberg solution where one the agent is myopic while the other one has a long-term management view. In section 5 some numerical illustrations are provided. Section 6 concludes the paper.

## 2. Population model and harvest

For analytical tractability, we use a population model consisting of only three cohorts; recruits (juvenils) $X_{0, t}($ year $<1)$, young mature fish $X_{1, t}(1 \leq$ year $<2)$ and old mature fish $X_{2, t}($ $2 \leq$ year ). Young and old mature fish are both harvestable, but the juveniles are not subject to fishing mortality. While recruitment is endogenous and density dependent, natural mortality is assumed fixed and density independent for all three age classes. The population is measured just before spawning, and in the single period of one year, three events take place in the following order; first, recruitment and spawning, then fishing and finally natural mortality.

The number of juveniles is governed by the recruitment function
(1) $X_{0, t}=R\left(X_{1, t} X_{2, t}\right)$,
where $R(0,0)=0$ and $\partial R / \partial X_{i, t}=R_{i}{ }^{\prime}>0$, together with $R_{i}{ }^{\prime \prime}<0(i=1,2)$. The number of young mature fish follows next as
(2) $X_{1, t+1}=s_{0} X_{0, t}$,
where $s_{0}$ is the fixed natural survival rate. Finally, the number of old mature fish is described by
(3) $X_{2, t+1}=s_{1}\left(1-f_{1, t}\right) X_{1, t}+s_{2}\left(1-f_{2, t}\right) X_{2, t}$,
where $0 \leq f_{1, t}<1$ and $0 \leq f_{2, t}<1$ are the fishing mortalities, or harvest rates, of the young and old mature stage, respectively, while $0<s_{1}<1$ and $0<s_{2}<1$ are the natural survival rates. When combining Eqs. (1) and (2) we have
(4) $\quad X_{1, t+1}=s_{0} R\left(X_{1, t}, X_{2, t}\right)$.

Eqs. (3) and (4) represent a reduced form model in two age-classes, where both equations are first order difference equations.

The population equilibrium for fixed fishing mortalities $f_{i, t}=f_{i}$ is defined by $X_{i, t+1}=X_{i, t}=X_{i}$ ( $i=1,2$ ) such that Eq. (3) holds as
(3') $\quad X_{2}=s_{1}\left(1-f_{1}\right) X_{1}+s_{2}\left(1-f_{2}\right) X_{2}$,
and Eq. (4) as
(4') $\quad X_{1}=s_{0} R\left(X_{1}, X_{2}\right)$.
(3') is identified as the spawning constraint while (4') is the recruitment constraint. An interior equilibrium holds for $0 \leq f_{1}<1$ only; that is, not all the young mature fish can be harvested. An interior equilibrium is shown in Figure 1, where the recruitment function is specified as the Beverton-Holt function (see numerical section 5). Based on this function, the recruitment constraint describes the number of mature fish as a positive, increasing, and convex function of the number of young mature fish. Taking the differential of Eq. (4') yields $d X_{2} / d X_{1}=\left(1-s_{0} R_{1}{ }^{\prime}\right) / s_{0} R_{2}{ }^{\prime}>0$. An increasing recruitment function therefore requires $s_{0} R_{1}{ }^{\prime}<1$ which holds for all positive values of $X_{2}$ with our Beverton-Holt function. Higher fishing mortalities shift down the spawning constraint (3') and hence lead to smaller stocks, while higher natural survival rates work in the opposite direction. The ratio of old to young mature fish is given by the slope of the spawning constraint, $X_{2} / X_{1}=s_{1}\left(1-f_{1}\right) /\left(1-s_{2}\left(1-f_{2}\right)\right)$. Therefore, none of the parameters pertaining to the recruitment function influence the equilibrium fish ratio, while it is evident that lower fishing mortalities of both age classes increase the proportion of old mature fish.

Figure 1 about here

Two fishing fleets exploit the fish stock, and each fleet targets a particular age class of the fish. As explained in the introduction, this harvesting scenario fits reality in many instances, either because of differences in gear selection, and/ or because the two age classes reside in different fishing zones. In most instances, the catches are composed of specimens from different cohorts and there is hence 'bycatch' irrespective of the fact that the fleets might be able to influence their catch composition. For example, the mesh size may be increased, or other gears may be adopted to leave the younger and smaller fish less exploited (see, e.g., Beverton and Holt 1957 and Clark 1990, and the more recent Singh and Weninger 2009). However, here we neglect bycatch and assume perfect targeting, where fleet one targets the young mature fish (stock one) while fleet two targets the old mature fish (stock two). We choose a specific production function in our analysis, the so-called Baranov function (see, e.g., Quinn 2003) defined as
(5) $\quad H_{i, t}=X_{i, t}\left(1-e^{-q_{i} E_{i, t}}\right) ;(i=1,2)$,
where $H_{i, t}$ is the harvest of fleet $i$ at time $t$ (in \# of fish), $E_{i, t}$ is the fishing effort, interpreted as, e.g., the number of standardized fishing vessels, and $q_{i}$ is the productivity, or 'catchability', parameter (1/effort). The Spence function exhibits decreasing marginal effort productivity. Notice also that with this harvesting function, the fishing mortalities can never reach one for a finite amount of effort, and extinction of the population is hence not possible within our modelling framework.

With the fishing mortality rate defined as $f_{i, t}=H_{i, t} / X_{i, t}(i=1,2)$, the mature age class growth Eq. (3) becomes

$$
\text { (6) } \quad X_{2, t+1}=s_{1} e^{-q_{1} E_{1, t}} X_{1, t}+s_{2} e^{-q_{2} E_{2, t}} X_{2, t}
$$

while $e^{-q_{1} E_{i, t}}$ is interpreted as the escapement rate of the stock after harvesting and ( $\left.1-e^{-q_{1} E_{i, t}}\right)$ hence represents the fishing mortality, or harvest rate.

## 3. Exploitation I: Cooperation

### 3.1 The optimal program

We start by looking at the cooperative solution where the maximum present-value profit of both fleets is determined jointly. As we wish to focus on biological interaction, we assume that the fleets do not interfere with each other through market mechanisms (but see e.g., Quaas and Requate 2013). The fish prices are thus assumed not to be influenced by the size of the catches, and they are constant through time. Therefore, with $p_{2}>p_{1}$ as the fixed fish prices (Euro/fish) and $c_{i}$ as the unit effort cost (Euro/effort), also assumed to be fixed, $\pi_{t}=p_{1} X_{1, t}\left(1-e^{-q_{1} E_{1, t}}\right)-c_{1} E_{1, t}+p_{2} X_{2, t}\left(1-e^{-q_{2} E_{2, t}}\right)-c_{2} E_{2, t}$ describes the current total profit. The constraints of this problem are the biological equations (4) and (6). In addition, the initial stock sizes, $X_{i, 0}$, are assumed known.

The Lagrangian of this present-value maximizing problem may be written as

$$
\begin{aligned}
L=\sum_{t=0}^{\infty} \rho^{t}\{ & p_{1} X_{1, t}\left(1-e^{-q_{1} E_{1, t}}\right)-c_{1} E_{1, t}+p_{2} X_{2, t}\left(1-e^{-q_{2} E_{2, t}}\right)-c_{2} E_{2, t} \\
& \left.\quad-\rho \lambda_{t+1}\left[X_{1, t+1}-s_{0} R\left(X_{1, t} X_{2, t}\right)\right]-\rho \mu_{t+1}\left[X_{2, t+1}-s_{1} e^{-q_{1} E_{1, t}} X_{1, t}-s_{2} e^{-q_{2} E_{2, t}} X_{2, t}\right]\right\}
\end{aligned},
$$

where $\lambda_{t}>0$ and $\mu_{t}>0$ are the shadow prices of the biological constraints (4) and (6), respectively, and $\rho=1 /(1+\delta)$ is a discount factor with $\delta \geq 0$ as the discount rate. Following the KuhnTucker theorem the first order necessary conditions (with $X_{i, t}>0, i=1,2$ ) are
(7) $\quad \partial L / \partial E_{1, t}=p_{1} q_{1} X_{1, t} e^{-q_{1} E_{1, t}}-c_{1}-\rho \mu_{t+1} s_{1} q_{1} X_{1, t} e^{-q_{1} E_{1, t}} \leq 0 ; \quad E_{1, t} \geq 0, t=0,1,2, \ldots$,
(8) $\quad \partial L / \partial E_{2, t}=p_{2} q_{2} X_{2, t} e^{-q_{2} E_{2, t}}-c_{2}-\rho \mu_{t+1} s_{2} X_{2, t} e^{-q_{2} E_{2, t}} \leq 0, \quad E_{2, t} \geq 0, \quad t=0,1,2, \ldots$,
(9) $\quad \partial L / \partial X_{1, t}=p_{1}\left(1-e^{-q E_{1, t}}\right)-\lambda_{t}+\rho \lambda_{t+1} s_{0} R_{1}{ }^{\prime}+\rho \mu_{t+1} s_{1} e^{-q_{1} E_{1, t}}=0, \quad t=1,2,3, \ldots$,
and

$$
\begin{equation*}
\partial L / \partial X_{2, t}=p_{2}\left(1-e^{-q_{2} E_{2, t}}\right)+\rho \lambda_{t+1} s_{0} R_{2}^{\prime}-\mu_{t}+\rho \mu_{t+1} s_{2} e^{-q_{2} E_{2, t}}=0, \quad t=1,2,3, \ldots \tag{10}
\end{equation*}
$$

The interpretation of the control conditions (7) and (8) is straightforward. Condition (7) states that the fishing effort of fleet 1 should take place up to the point where the marginal profit is equal to, or below, the economically, $\rho$, and biologically, $s_{1}$, discounted marginal biomass loss of the immature stage, as evaluated by the shadow price of the biological constraint (6). Condition (8) is analogous for the old mature stock. Eqs. (9) and (10) steer the shadow price values. Rewriting Eq.
(9) as $\lambda_{t}=p_{1}\left(1-e^{-q_{1} E_{1, t}}\right)+\rho \lambda_{t+1} s_{0} R_{1}{ }^{\prime}+\rho \mu_{t+1} s_{1} e^{-q_{1} E_{1, t}}$, it is seen that the number of young mature fish should be maintained such that the recruitment shadow price equalizes the marginal harvest value plus its growth contribution to recruitment and the old mature stage, as evaluated by their shadow prices with biological and economic discounting taken into account. Eq. (10) can be given a similar interpretation.

The control conditions (7) and (8) may be rewritten as
(7') $\quad \frac{p_{1}}{s_{1}}\left(\frac{X_{1, t} e^{-q_{1} E_{1, t}}-c_{1} / p_{1} q_{1}}{X_{1, t} e^{-q_{1} E_{1, t}}}\right) \leq \rho \mu_{t+1} ; E_{1, t} \geq 0, \quad t=0,1,2, \ldots$
and
(8') $\frac{p_{2}}{s_{2}}\left(\frac{X_{2, t} e^{-q_{2} E_{2, t}}-c_{2} / p_{2} q_{2}}{X_{2, t} e^{-q_{2} E_{2, t}}}\right) \leq \rho \mu_{t+1} ; E_{2, t} \geq 0, \quad t=0,1,2, \ldots$,
respectively. These equations reveal that the survival rates $s_{i}$ and the economic parameters $p_{i}, q_{i}$ and $c_{i}(i=1,2)$ alone determine the optimal harvesting priority. Fertility plays no direct role. Therefore, although the recruitment function certainly impacts on the optimal harvest of the two stocks, its properties are not observed directly in the conditions characterizing the optimal harvesting policy. This is stated as:

Result 1: Fertility and differences in fertility among the harvestable year classes have no direct effect on the harvesting priority.

This result is similar to what is obtained by Reed (1980), but in a model where the maximum sustainable yield (MSY) is maximized and hence no economic parameters are included.

As we have $p_{2}>p_{1}$ and the natural survival rates do not differ too much, we may suspect that harvest of the old mature age class should be given priority if the harvest cost of fleet 1 exceeds that of fleet 2. That is, $E_{1, t}=0$ and $E_{2, t}>0$, if the harvest cost discrepancy $c_{1} / q_{1}>c_{2} / q_{2}$ holds. In the opposite situation with $c_{1} / q_{1}<c_{2} / q_{2}$, an interior solution with $E_{1, t}>0$ and $E_{2, t}>0$ can be a possible optimal outcome. Altogether, when the possibility of no harvesting at all is ignored, the
optimal harvest policy comprises the three possibilities; Case i) with $E_{1, t}>0$ and $E_{2, t}>0$, Case ii) with $E_{1, t}>0$ and $E_{2, t}=0$, and Case iii) with $E_{1, t}=0$ and $E_{2, t}>0$. Case i) is the interior solution and in contrast to Skonhoft et al. (2012) it is a possible option here as the Lagrangian is strictly concave in the control variables because of decreasing marginal effort productivity. This is stated as:

Result 2: Optimal harvesting under full cooperation may involve harvesting of both stocks, stock 1 only or stock 2 only.

Combining ( $7^{\prime}$ ) and ( $8^{\prime}$ ) and assuming the interior solution Case i) gives the condition

$$
\frac{p_{1}}{s_{1}}\left(\frac{X_{1, t} e^{-q_{1} E_{1, t}}-c_{1} / p_{1} q_{1}}{X_{1, t} e^{-q_{1} E_{1, t}}}\right)=\frac{p_{2}}{s_{2}}\left(\frac{X_{2, t} e^{-q_{2} E_{2, t}}-c_{2} / p_{2} q_{2}}{X_{2, t} e^{-q_{2} E_{2, t}}}\right)=\rho \mu_{t+1}>0
$$

which states that share of the escapement of each stock above its zero marginal profit level $c_{i} / p_{i} q_{i}$ is equal among the two stocks, when weighted by the price-to-survival ratio $p_{i} / s_{i}$. The stock that has the highest price-to-survival ratio will have the smallest escapement share above its zero marginal profit level, and can be said to be harvested more aggressively. Therefore, with equal survival rates and a higher market price for the old mature stock, stock 2 should be harvested more intensively than stock 1, which is a result in accordance with previous studies (i.e. Diekert et. al., 2010, Skonhoft et al. 2012). In the special case where $c_{1} / p_{1} q_{1}=c_{2} / p_{2} q_{2}$, the escapement in terms of number of fish is simply higher for the stock with the lower price-to-survival ratio. Still with an interior solution, Eqs. (7') and ( $8^{\prime}$ ) may also be written as

$$
\frac{1}{s_{1}}\left(p_{1}-\frac{c_{1}}{q_{1} X_{1, t} e^{-q_{1} E_{1, t}}}\right)=\frac{1}{s_{2}}\left(p_{2}-\frac{c_{2}}{q_{2} X_{2, t} e^{-q_{2} E_{2, t}}}\right)
$$

The content in the brackets expresses the marginal profit. Therefore, we may state:

Result 3: In the cooperative solution with joint harvest of both stocks, the ratio between marginal profit at the end of the harvesting season and the own stock survival rate is equal between the two fleets at every point in time.

Note that this is an equation that holds at every point in time and hence indicates a fixed relationship between the escapement of the two fishable stocks also outside the steady state. It is independent of discounting and all parameters pertaining to the recruitment function. Notice also that if the price - survival ratio is equal among the two stocks, i.e., $p_{1} / s_{1}=p_{2} / s_{2}$, the escapement ratio will be given as $X_{1, t} e^{-q_{1} E_{1, t}}=\frac{c_{1} q_{2} s_{2}}{s_{1} q_{1} c_{2}} X_{2, t} e^{-q_{2} E_{2, t}}$. Through the spawning constraint (6), we then find $X_{2, t+1}=s_{2}\left(\frac{q_{2} c_{1}+q_{1} c_{2}}{q_{1} c_{2}}\right) e^{-q_{1} E_{2, t}} X_{2, t}$. In a steady state with $X_{2, t+1}=X_{2, t}$, the effort use of fleet 2 is then determined by cost and survival parameters alone and is hence independent of the recruitment relationship and discounting. All dynamic considerations are addressed by adjusting the effort of fleet 1 only.

When still assuming the interior solution Case i) with fishing of both fleets, the optimality condition for each age class can be rewritten in terms of the optimal escapement $X_{i, t} e^{-q_{i} E_{i, t}}$ as a function of the economic parameters and the shadow price of stock 2 as

$$
\begin{equation*}
X_{i, t} e^{-q_{i} E_{i, t}}=\frac{c_{i} / q_{i}}{p_{i}-\rho s_{i} \mu_{t+1}}, i=1,2 \tag{11}
\end{equation*}
$$

With $\rho s_{i} \mu_{t+1}=0$, that is, when either the discount factor or the shadow price of the spawning constraint is zero, myopic adjustment results where both age classes are harvested down to their zero marginal profit levels $c_{i} / p_{i} q_{i}$ each year (more details section 4.2 below).

In Case iii) with $E_{1, t}=0$ and $E_{2, t}>0$ combination of conditions ( $8^{\prime}$ ) and ( $9^{\prime}$ ) yields

$$
\frac{1}{s_{2}}\left(p_{2}-\frac{c_{2}}{q_{2} X_{2, t} e^{-q_{2} E_{2, t}}}\right)>\frac{1}{s_{1}}\left(p_{1}-\frac{c_{1}}{q_{1} X_{1, t}}\right)<\rho \mu_{t+1}
$$

Notice that this case with $E_{1, t}=0$ may be an optimal solution even if positive profit is possible for fleet 1 . As indicated above, we may suspect that this case can be an optimal option when the harvest cost discrepancy $c_{1} / q_{1}>c_{2} / q_{2}$ is 'high'. In this situation, we hence find that the marginal net benefit of letting the young mature fish stay one more year in the ocean exceeds that of the marginal
natural mortality loss. This condition is seen even more clearly if we assume cost free harvest. The above relationship then simply reads $p_{2} / s_{2}>p_{1} / s_{1}$.

Case ii) with $E_{1, t}>0$ and $E_{2, t}=0$ gives in a similar manner

$$
\frac{1}{s_{1}}\left(p_{1}-\frac{c_{1}}{q_{1} X_{1, t} e^{-q_{1} E_{1, t}}}\right)>\frac{1}{s_{2}}\left(p_{2}-\frac{c_{2}}{q_{2} X_{2, t}}\right)<\rho \mu_{t+1}
$$

The interpretation of this condition is parallel as above, and may only be an optimal option if the discrepancy $c_{2} / q_{2}>c_{1} / q_{1}$ is 'high'. If we again assume cost free harvest, this is not a possible solution as long as $p_{2} / s_{2}>p_{1} / s_{1}$ holds.

### 3.2 Steady state analysis

In a steady state with the optimal harvesting policy as Case i), the biological constraints read (4'), and:
(6') $\quad X_{2}=s_{1} e^{-q_{1} E_{1}} X_{1}+s_{2} e^{-q_{2} E_{2}} X_{2}$,
such that the escapement rates $e^{-q_{i} E_{i}}$, or fishing mortalities $f_{i}=\left(1-e^{-q_{i} E_{i}}\right)(i=1,2)$, are constant through time. In Case ii) and Case iii), the spawning constraint (6) becomes $X_{2}=s_{1} e^{-q_{1} E_{1}} X_{1}+s_{2} X_{2}$ and $X_{2}=s_{1} X_{1}+s_{2} e^{-q_{2} E_{2}} X_{2}$, respectively. As already explained, the slope of the spawning constraint indicates the fishing pressure. However, it is difficult to draw general conclusions about the differences of this slope between our three different harvest options. Therefore, harvest option Case i) may be either more aggressive or less aggressive than Case ii), and so on. However, rewriting the spawning constraint in Case i) as $\quad X_{2} / X_{1}=\frac{s_{1} e^{-q_{1} E_{1}}}{1-s_{2} e^{-q_{2} E_{2}}}$ indicates that more effort of both fleets contributes to reducing the slope of the spawning constraint and hence leads to smaller stocks and a lower ratio of stock 2 compared to stock 1 in biological equilibrium. See Figure 1. The same happens with Case ii) or Case iii) as the optimal harvest options.

We may expect that the steady state exploitation of each stock increases with small upward shifts in own price and catchability coefficient, and decreases with higher unit costs. We may also expect
that a lower discount factor ${ }^{\rho}$ (i.e., a higher discount rate $\delta$ ) will increase the harvesting pressure of both stocks. However, except that we know that $\rho=0$ yields myopic exploitation and lower stock sizes (see also section 4.2 below), the comparative static effects are generally difficult to assess. This will be so for parameter shifts within the various harvesting schemes, but also when changes in the biological and economic environment give a switch between the different schemes. Numerical section 5 below demonstrates several comparative static results.

### 3.3. Dynamic properties

Above some properties of possible steady states with a constant number of fish through time was analyzed. As the profit is non-linear in the controls, economic theory suggests that fishing should be adjusted through some kind of saddle-path dynamics to lead the fish population to steady state. However, the gradual adjustment may not be a regular one in our age-structured fish population because control of the fish population may lead to corner solutions where one of the age classes is left unexploited. The age structure may for example imply that the population could be above that of the optimal steady state level for one age-class and at the same time lower than the optimal steady state for the other age-class. That is, some degree of under- or overshooting due to the ageclass formulation, but also because of the discrete time formulation, may be present. Section 5 below demonstrates the dynamics numerically.

## 4. Exploitation II: Non-cooperation

### 4.1 The setting

We now consider the situation where the two fleets are owned and managed by separate agents that exploit the fish stocks in a non-cooperative manner. We choose to focus on two situations that we believe to be quite realistic. In the first scenario both fleets behave as myopic agents, thus maximizing instantaneous profit without taking their own impact on the next period's stock into account. This represents a decentralized decision environment, where each individual vessel owner neglects its own impact on the standing biomass. The other scenario under consideration here is where fleet 1 is coordinated and behaves as a sole owner, while fleet two is myopic. This can be viewed as a Stackelberg game with fleet 1 as the leader and fleet 2 as the follower. We compare the steady state outcomes of these two harvesting schemes, both with each other and with the cooperative solution.

### 4.2 Myopic exploitation

### 4.2.1 Optimality conditions

We first consider a myopic solution, where both agents maximize their respective current profit while taking the stock sizes as given. The number of vessel owners in the two fleets may be large, and myopic behavior may result from open access dynamics. However, it may also be realistic with a small number of agents. Indeed, as shown by Clark (1980), myopic behavior may occur even with only two agents, in a continuous time setting. It may be noted here, however, that due to the discrete nature of the system positive profit is still present in the fishery because the stock is able to renew itself between the harvesting seasons. For fleet 1 where the current profit reads $\pi_{1, t}=p_{1} X_{1, t}\left(1-e^{-q_{1} E_{1, t}}\right)-c_{1} E_{1, t}$, we find the myopic profit maximizing condition as
(12) $\quad \partial \pi_{1, t} / \partial E_{1, t}=p_{1} q_{1} X_{1, t} e^{-q_{1} E_{1, t}}-c_{1} \leq 0 ; \quad E_{1, t} \geq 0, t=0,1,2, \ldots$,
while

$$
\begin{equation*}
\partial \pi_{2, t} / \partial E_{2, t}=p_{2} q_{2} X_{2, t} e^{-q_{2} E_{2, t}}-c_{2} \leq 0 ; \quad E_{2, t} \geq 0, \quad t=0,1,2, \ldots \tag{13}
\end{equation*}
$$

is for fleet 2. These two conditions together with the biological constraints (4) and (6) thus determine the effort use, the stock sizes, and the dynamic interaction between the two agents. As already indicated (section 3 above), conditions (12) and (13) coincide with conditions (7) and (8) in the cooperative solution if the discount factor is set to zero.

### 4.2.2 Steady state analysis

Harvest is profitable if and only if marginal profit exceeds marginal cost for zero effort; that is, $p_{i}-c_{i} / q_{i} X_{i, t}>0$, or $X_{i, t}>c_{i} / p_{i} q_{i}(i=1,2)$. We then find $X_{i, t} e^{-q_{i} E_{i, t}}=c_{i} / p_{i} q_{i}$ with $E_{i, t}>0$ so that escapement equals the zero marginal profit stock level. When this holds for both agents, Case i) prevails. Inserting these conditions into the spawning constraint (6) yields $X_{2, t+1}=s_{1} c_{1} / p_{1} q_{1}+s_{2} c_{2} / p_{2} q_{2}$. In a steady state, the above zero effort marginal profit condition $X_{2}>c_{2} / p_{2} q_{2}$ then implies $s_{1} c_{1} / p_{1} q_{1}+s_{2} c_{2} / p_{2} q_{2}>c_{2} / p_{2} q_{2}$. Therefore, we find that $\frac{p_{2}}{p_{1}}>\frac{\left(1-s_{2}\right)}{s_{1}}\left(\frac{c_{2} / q_{2}}{c_{1} / q_{1}}\right)$ must hold if both fleets should be in operation. As an example, assume that
$c_{1} / q_{1}=c_{2} / q_{2}$ holds. The above inequality then demands $\frac{p_{2}}{p_{1}}>\frac{\left(1-s_{2}\right)}{s_{1}}$. With $s_{1}=s_{2}>0.5$ this condition is thus for sure satisfied as the market value of old mature fish is higher than that of the young. In Case ii) with $X_{1, t}>c_{1} / p_{1} q_{1}$ and $X_{2, t}<c_{2} / p_{2} q_{2}$, and hence no fishing of fleet 2, the steady state spawning constraint reads $X_{2}=\frac{s_{1}}{1-s_{2}} \frac{c_{1}}{p_{1} q_{1}}$. The condition $X_{2}<c_{2} / p_{2} q_{2}$ now implies $\frac{p_{2}}{p_{1}}<\frac{\left(1-s_{2}\right)}{s_{1}}\left(\frac{c_{2} / q_{2}}{c_{1} / q_{1}}\right)$. With identical fleet costs, this harvesting scheme is therefore not a possible option when $s_{1}=s_{2} \geq 0.5$. These observations are stated as:

Result 4: In a myopic non-cooperative setting the possibility for fleet 2 to be in the fishery depends only on the price and cost parameters, along with the survival rates of the two mature stocks.

The steady state effects of parameter changes on effort use and stock sizes are generally as expected. For each fleet that is in operation, we find that effort decreases with $c_{i} / p_{i} q_{i}$, for any given size of the stock. In Case i) where the spawning constraint reads $X_{2}=s_{1} c_{1} / p_{1} q_{1}+s_{2} c_{2} / p_{2} q_{2}$ , $X_{2}$ is affected positively by increased cost/price ratio of fleet 2 targeting this stock. However, the old mature stock is also positively affected by a higher cost/price ratio of fleet 1 . As there is a positive relationship between $X_{1}$ and $X_{2}$ through the recruitment constraint, which in this Case i) reads $X_{1}=s_{0} R\left(X_{1}, s_{1} c_{1} / p_{1} q_{1}+s_{2} c_{2} / p_{2} q_{2}\right)$, we find similar effects also for stock 1 . Therefore, a higher cost/price ratio of fleet 2 also shifts up the size of stock 1 . For Case ii), where the stocks are defined through $\quad X_{2}=\frac{s_{1}}{\left(1-s_{2}\right)}\left(c_{1} / p_{1} q_{1}\right)$ and $X_{1}=s_{0} R\left(X_{1}, s_{1} c_{1} / p_{1} q_{1}\left(1-s_{2}\right)\right)$, and Case iii) with $X_{2}=s_{2} \frac{c_{2}}{p_{2} q_{2}}+s_{1} X_{1}$ and $X_{1}=s_{0} R\left(X_{1}, s_{2} \frac{c_{2}}{p_{2} q_{2}}+s_{1} X_{1}\right)$ the same results prevail. This is stated as:

Result 5: In the myopic fishery game, a higher cost/price ratio for fleet 1 not only increases the steady state young mature fish stock, but also the old mature stock targeted by fleet 2, and vice versa.

Higher survival rates $s_{1}$ and $s_{2}$ also shift up the spawning constraint in all cases, and hence lead to higher stocks of both categories of fish. The same happens with the biological parameters that increase the spawning productivity, as these changes shift the recruitment constraint outwards (see section 5 below).

### 4.2.3 Comparing with cooperative solution

The suspected result is that non-cooperative myopic harvesting yields a higher exploitation pressure than when the exploitation is steered by long-term cooperation. In what follows, this is demonstrated for the steady state solutions where we compare case for case. However, notice that this comparison excludes the possibility that the myopic game solution and the cooperative solution for the same parameter values may lead to different steady state cases. In the cooperative solution Case i) with harvest of both fleets, the spawning constraint reads
$X_{2}=s_{1} X_{1} e^{-q_{1} E_{1}}+s_{2} X_{2, t} e^{-q_{2} E_{2}}$. From the control conditions (7) and (8) it is also evident that we find $X_{i} e^{-q_{i} E_{i}}=c_{i} / p_{i} q_{i}+\Delta_{i}$, with $\Delta_{i}>0(i=1,2)$, when $\rho>0$. Therefore, the old mature stock size can be described as $X_{2}=s_{1}\left(c_{i} / p_{i} q_{i}+\Delta_{1}\right)+s_{2}\left(c_{2} / p_{2} q_{2}+\Delta_{2}\right)$ through the spawning constraint. When comparing with $X_{2}=s_{1} c_{1} / p_{1} q_{1}+s_{2} c_{2} / p_{2} q_{2}$ from the Case i) myopic solution, it is then evident that the old mature stock size will be larger in the cooperative solution than in the myopic game solution. The size of the young mature stock will accordingly be larger as well.

In Case ii) $X_{1} e^{-q_{1} E_{1}}=c_{1} / p_{1} q_{1}+\Delta_{1}$ together with $E_{2}=0$ describes the optimal control conditions in the cooperative solution. The spawning constraint may therefore now be written as $X_{2}=s_{1}\left(c_{i} / p_{i} q_{i}+\Delta_{1}\right)+s_{2} X_{2}$, or $X_{2}=\frac{s_{1}}{\left(1-s_{2}\right)}\left(c_{i} / p_{i} q_{i}+\Delta_{1}\right)$. Comparing with the Case ii) myopic solution $X_{2}=\frac{s_{1}}{\left(1-s_{2}\right)}\left(c_{1} / p_{1} q_{1}\right)$ it is again evident that the size of the mature stock will be lower in the myopic solution than in the cooperative solution. Therefore, the size of the young mature stock will be larger in the cooperative solution as well. We find the same outcomes in Case iii). These observations are stated as:

Result 6: In steady state, the fish stocks will be more heavily exploited in the myopic game solution than in the cooperative solution within all three possible harvesting scenarios.

Notice that nothing is inferred about the effort use in the above comparison between the myopic non-cooperative and cooperative solution. We may suspect that higher stocks may be followed by lower effort use for both fleets in the cooperative solution. However, as shown in the numerical section 5 , this will not necessarily be the case.

### 4.2.4 Dynamics

Finally, we consider the dynamics in the myopic game situation where we again analyze case for case. In Case i) the spawning constraint reads $X_{2, t+1}=s_{1} c_{1} / p_{1} q_{1}+s_{2} c_{2} / p_{2} q_{2}$. Therefore, starting with an old mature stock $X_{2,0}$, it jumps to $s_{1} c_{1} / p_{1} q_{1}+s_{2} c_{2} / p_{2} q_{2}=X_{2,1}$ in period 1 and stays at this level for the rest of the game; that is, $X_{2, t}=X_{2,1}=X_{2}$ for all $t=2,3,4 \ldots$. The corresponding dynamics for the young mature stock is found through the recruitment constraint (4) as $X_{1, t+1}=s_{0} R\left(X_{1, t}, X_{2}\right)$ for $t=1,2,3, \ldots$. For the given initial value $X_{1,0}$ this describes a non-linear first order difference equation and yields a stable equilibrium when $s_{0} R_{1}{ }^{\prime}<1$. With the Beverton Holt recruitment function this stability condition will be satisfied (section 2 below).

In Case ii) where fleet 2 is unprofitable, the linear difference equation $X_{2, t+1}=s_{1} c_{1} / p_{1} q_{1}+s_{2} X_{2, t}$ describes the spawning constraint. Accordingly, $X_{2}=s_{1} c_{1} / p_{1} q_{1}\left(1-s_{2}\right)$ yields the steady state of the old mature stock. The young mature stock dynamics is then found through $X_{1, t+1}=s_{0} R\left(X_{1, t}, X_{2, t}\right)$ with a recursive link from the evolvement of the old mature stock. The equilibrium is locally stable, which is confirmed by calculating the Jacobian matrix $J=\left(\begin{array}{cc}\left(s_{0} R_{1}^{\prime}-1\right) & s_{0} R_{2}^{\prime} \\ 0 & -\left(1-s_{2}\right)\end{array}\right)$ where we find det $J>0$ and $\operatorname{Tr} J<0$ when $s_{0} R_{1}^{\prime}<1$.

In Case iii) with unprofitable harvest of fleet 1 , the spawning constraint reads $X_{2, t+1}=s_{1} X_{1, t}+s_{2} c_{2} / p_{2} q_{2}$. Therefore, the jointly interacting equations $X_{1, t+1}=s_{0} R\left(X_{1, t}, X_{2, t}\right)$ and
$X_{2, t+1}=s_{1} X_{1, t}+s_{2} c_{2} / p_{2} q_{2}$ now describe the fish stock dynamics The Jacobian matrix of this system is $J=\left(\begin{array}{cc}\left(s_{0} R_{1}^{\prime}-1\right) & s_{0} R_{2}^{\prime} \\ s_{1} & -1\end{array}\right)$, still with with $\operatorname{Tr} J<0$. We also now find Det $J=\left[1-s_{0}\left(R_{1}{ }^{\prime}+s_{1} R_{2}{ }^{\prime}\right)\right]>0$ because the recruitment constraint intersects with the spawning constraint $X_{2, t+1}=s_{1} X_{1, t}+s_{2} c_{2} / p_{2} q_{2}$ in equilibrium from below (Figure 1). These observations are stated as:

Result 7: The dynamics of the myopic game solutions are locally stable in all three possible harvesting scenarios.

### 4.3 Stackelberg solution

### 4.3.1 Optimality conditions

We now assume that only one of the two fleets is myopic and maximizes profit each year without considering the future. At least for fleet 2 this may be a rather realistic case as the coastal fishery typically consists of many small vessels, and where the owners are not sufficiently organized to behave strategically so as to affect the harvest decision of fleet 1 . In what follows, we thus choose to focus on the situation where fleet 2 is the myopic player. As all strategic considerations then belong to fleet 1 , and although we assume simultaneous moves, the model can be considered as a Stackelberg game with fleet 1 as the dominant and leading player. Fleet 2 thus adjusts passively to the behavior of fleet 1 while fleet 1 takes fleet 2 's optimal adjustment into account before forming its own harvest decision.

The game is solved by backwards induction where we first solve the problem of fleet 2 in stage two. Fleet 2 maximizes current profit $\pi_{2, t}=p_{2} X_{2, t}\left(1-e^{-q_{2} E_{2, t}}\right)-c_{2} E_{2, t}$ while taking the stock size $X_{2, t}$ as given. This gives the same first order condition as Eq. (13) with $p_{2} q_{2} X_{2 . t} e^{-q_{2} E_{2, t}}-c_{2} \leq 0$. The Lagrangian of agent 1's maximization problem is then accordingly formulated as

$$
\begin{aligned}
L_{1}= & \sum_{t=0}^{\infty} \rho^{t}\left\{p_{1} X_{1, t}\left(1-e^{-q_{1} E_{1, t}}\right)-c_{1} E_{1, t}-\rho \lambda_{1, t+1}\left[X_{1, t+1}-s_{0} R\left(X_{1, t}, X_{2, t}\right)\right]_{473}^{472}\right. \\
& -\rho \mu_{1, t+1}\left[X_{2, t+1}-s_{1} e^{-q_{1} E_{1, t}} X_{1, t}-s_{2} e^{-q_{2} E_{2, t}} X_{2, t}\right]-\psi_{t}\left[p_{2} q_{2} X_{2, t} e^{-q_{2} E_{2, t}}-4 q_{2} 4\right\}
\end{aligned}
$$

The biological shadow prices now reflect that the biological constraints are viewed from the perspective of agent 1 , while the new shadow price $\psi_{t} \geq 0$ takes into account the harvest restriction imposed upon agent 1 due to the myopic harvesting activity of agent (fleet) 2 . We have $\psi_{t}>0$ when fleet 2 operates, and $\psi_{t}=0$ otherwise.

The necessary first order conditions for maximum for agent 1 are

$$
\begin{equation*}
\partial L_{1} / \partial E_{1, t}=p_{1} q_{1} X_{1 . t} e^{-q_{1} E_{1, t}}-c_{1}-\rho \mu_{1, t+1} s_{1} q_{1} X_{1 . t} e^{-q_{1} E_{1, t}} \leq 0 ; \quad E_{1, t} \geq 0, \quad t=0,1,2, \ldots, \tag{15}
\end{equation*}
$$

$$
\begin{equation*}
\partial L_{1} / \partial X_{1, t}=p_{1}\left(1-e^{-q_{1} E_{1, t}}\right)-\lambda_{1, t}+\rho \lambda_{1, t+1} s_{0} R_{1}{ }^{\prime}+\rho \mu_{1, t+1} s_{1} e^{-q_{1} E_{1, t}}=0, \quad t=1,2,3, \ldots, \tag{16}
\end{equation*}
$$

and

$$
\begin{equation*}
\partial L_{1} / \partial X_{2, t}=\rho \lambda_{1, t+1} s_{0} R_{2}{ }^{\prime}-\mu_{1, t}+\rho \mu_{1, t+1} s_{2} e^{-q_{2} E_{2, t}}-\psi_{t} p_{2} q_{2} e^{-q_{2} E_{2, t}}=0, t=1,2,3, \ldots \tag{17}
\end{equation*}
$$

Conditions (15) and (16) are similar to conditions (7) and (9) in the cooperative solution, respectively. On the other hand, Eq. (17) differs from Eq. (10) because of the inclusion of the new shadow price reflecting the harvest constraint imposed from agent 2, but also because the marginal harvest value of the old mature fish stock is absent. Both these factors work in the direction of a lower shadow price of the old mature stock. This is more clearly observed when rewriting Eq. (17) as $\mu_{1, t}=\rho \lambda_{1, t+1} s_{0} R_{2}{ }^{\prime}+\rho \mu_{1, t+1} s_{2} e^{-q_{2} E_{2, t}}-\psi_{t} p_{2} q_{2} e^{-q_{2} E_{2, t}}$ and comparing with $\mu_{t}$ in the cooperative solution.

Assume that $\psi_{t}>0$ holds and hence that fishing is profitable also for fleet 2. Optimal escapement of the young mature stock is given from condition (15), and depends positively on $\mu_{1 . t+1}$. But if $\mu_{1, t}$ decreases with $\psi_{t}$ as indicated by Eq. (17), effort from fleet 1 will be higher in the Stackelberg case than under cooperation. Further, as $\mu_{t}>0$ still holds because the spawning constraint must bind, fleet 1 effort is lower than under myopic adjustment. Hence, fleet 1 will not overfish, in the
sense that it operates with negative marginal profit, to keep fleet 2 out of business. This will hold in in the transitional dynamics phase and in steady state. The dynamics are studied more closely in the numerical section 5 .

### 4.3.2 Steady state analysis

We now assume two possible exploitation schemes in the Stackelberg steady state solution; Case i) with harvest of both fleets and Case ii) with $E_{2}=0$ and $E_{1}>0$. In both cases Eq. (15) reads $\frac{p_{1}}{s_{1}}\left(\frac{X_{1, t} e^{-q_{1} E_{1, t}}-c_{1} / p_{1} q_{1}}{X_{1, t} e^{-q_{1} E_{1, t}}}\right)=\rho \mu_{t+1}>0$ with the same interpretation as in the cooperative solution. In both these cases we also find the same spawning constraints as in the cooperative solution. However, again it is difficult to say which of these two cases that give the highest exploitation pressure. On the other hand, it is possible to prove that the Stackelberg solution yields higher stock sizes compared to the myopic game situation where we again compare case for case. In Case i) in the Stackelberg solution we find $X_{1} e^{-q_{1} E_{1}}=c_{1} / p_{1} q_{1}+\Delta_{1}$, where again $\Delta_{1}$ represents a positive number, together with $X_{2 . t} e^{-q_{2} E_{2, t}}=c_{2} / p_{2} q_{2}$. Therefore, the spawning constraint in Case i) in the Stackelberg solution may be written as $X_{2}=s_{1} c_{1} / p_{1} q_{1}+s_{1} \Delta_{1}+s_{2} c_{2} / p_{2} q_{2}$. Comparing with $X_{2}=s_{1} c_{1} / p_{1} q_{1}+s_{2} c_{2} / p_{2} q_{2}$ in the myopic solution, it is then evident that the spawning constraint in the Stackelberg game will be located above the spawning constraint in the myopic game solution in this Case i). Hence, both mature stocks will be higher. We find the same outcome in Case ii). This is stated as:

Result 8: In a steady state both mature stocks will be more heavily exploited in the myopic game than in the Stackelberg game in harvesting schemes Case i) and Case ii).

## 5. Numerical illustration

### 5.1 Data and functional forms

The above theoretical reasoning will now be illustrated numerically. As our theoretical model is somewhat stylized, we do not aim to provide an accurate empirical description of a particular fishery. However, the parameter values used here are meant to give a reasonable description of the workings of the model. The baseline survival rates for the three age categories are set to $s_{0}=0.6$
and $s_{1}=s_{2}=0.7$ which may concur with average estimates for the North Atlantic Norwegian cod fishery (see, e.g., Sumaila 1997). As indicated, the recruitment function is specified as the Beverton-Holt function $\quad R\left(X_{1, t} X_{2, t}\right)=\frac{\alpha\left(\gamma X_{1, t}+X_{2, t}\right)}{\beta+\left(\gamma X_{1, t}+X_{2, t}\right)} \quad$ with $\quad \alpha=1,500 \quad$ as the scaling parameter (\# of 1,000 fish) and $\beta=500$ as the shape parameter (\# of 1,000 fish). Because it is conventionally assumed that fertility is positively related to the weight of the fish (e.g., Getz and Haigh 1989, p. 154), we impose higher fertility for the old mature fish than for the young by including the relative fertility parameter $\gamma=0.5$ as the baseline value. When solving Eq. (3') and $(4$ ') in absence of harvest and with the Beverton-Holt function, these baseline parameter values imply that the steady state stocks equal $X_{1}=s_{0} \alpha-\beta /\left[\gamma+s_{1} /\left(1-s_{2}\right)\right]=723$ and $X_{2}=s_{1} X_{1} /\left(1-s_{2}\right)=1687 \quad(\#$ of 1,000 fish $)$. We also have $R_{1}^{\prime}=\gamma \alpha \beta /\left(\beta+\gamma X_{1}+X_{2}\right)^{2}<1$ everywhere in the range $\left\{X_{1}, X_{2}\right\} \in\left\{\left[s_{0} \alpha-\beta / \gamma, \infty\right],[0, \infty]\right\}$, which are the stock values that satisfy the spawning constraint that ensures stability under myopic harvesting. In addition we find that $R_{2}^{\prime}=R_{1}^{\prime} / \gamma>R_{1}^{\prime}$, reflecting higher fertility for the old mature stock. The impact of changes in these parameters can be understood in light of Figure 1 above, where, for instance, higher spawning productivity through increased values of $\alpha$ and $\gamma$ shift the recruitment constraint outwards.

As for the economic parameters, we set $p_{1}=2$ (Euro/fish), $p_{2}=3$ (Euro/fish), and $c_{1}=c_{2}=10$ (Euro/effort). We further set $q_{2}=0.01$ (1/effort) while we assume $q_{1}=0.03$ to reflect that the fleet that targets the young mature fish (typically a trawler fleet) may have higher catchability than the fleet targeting the old mature fish (typically small coastal vessels). Together these imply the zero marginal profit stock levels as $c_{1} / p_{1} q_{1}=167$ and $c_{2} / p_{2} q_{2}=333$ (\# of 1,000 fish), which are well below the steady state stock levels in absence of harvest, meaning that profit is possible for both fleets individually. The discount rate is assumed to be $\delta=0.04$, implying $\rho=1 /(1+\delta)=0.9615$. We first present results with the baseline parameter values and subsequently demonstrate the implications of changes in the biological, economic and technological conditions through varying the fertility parameter, the discount rate, and the catchability parameter for fleet one.

### 5.2 Results baseline parameters ${ }^{1}$

We start with presenting the basic dynamic results. Figure 2 demonstrates first the development of the two stocks under the three management scenarios; cooperation, myopic behavior by both fleets and the Stackelberg game where fleet 1 optimizes and fleet 2 adjusts passively (denoted Stackelberg1). The solid lines show (pre harvest) stock sizes $X_{i, t}$, the dashed lines show escapement $X_{i, t} e^{-q_{i} E_{i, t}}$, and the dotted lines show the zero marginal profit stock levels, $X_{i}=c_{i} / p_{i} q_{i} \quad(i=1,2)$. As is seen, the stocks stabilize quickly towards a steady state after an initial impulse harvest. This happens for both stocks under all three management scenarios. For the old mature stock in the myopic non-cooperative solution, this is just as expected from the theoretical analysis. Also, just as shown in sections 4.2 and 4.3, the steady state stock sizes are larger under cooperation than in the other scenarios, and escapement is kept well above the zero marginal profit level for both stocks. In the myopic scenario, both stocks are harvested down to their zero marginal profit levels each year, while the Stackelberg solution only differs slightly from the myopic case, in that the leader maintains a somewhat higher young mature stock. We have also run the various scenarios with different initial situations, and we find the dynamic to be ergodic, that is, unique steady states are approached under different initial conditions.

Figure 2 about here

Figure 3 shows the development of effort over the same harvesting period. For our baseline parameter values, we find that Case i), with fishing effort of both fleets, represents the optimal fishing scheme in the cooperative solution as well as in the two non-cooperative solutions. In the myopic solution, it was shown that $\frac{p_{2}}{p_{1}}>\frac{\left(1-s_{2}\right)}{s_{1}}\left(\frac{c_{2} / q_{2}}{c_{1} / q_{1}}\right)$ must hold if both fleets should be in operation and this holds for the baseline parameter values despite the substantially higher catchability coefficient of fleet 1 . In the cooperative solution, it is optimal with higher effort use of fleet 2 , targeting the old mature stock, than of fleet 1 . This is because the old mature stock commands a higher price per fish, and that this price effect dominates the cost effect from the

[^0]higher catchability of fleet 1 . In the two non-cooperative solutions, we find the opposite pattern. The reason is that the high effort of fleet 1 with correspondingly low levels of both stocks renders the old mature stock barely profitable under the baseline parameter values.

Figure 3 about here

### 5.3 Steady state and sensitivity analysis

We now examine the sensitivity of the solutions obtained to changes in certain parameter values where we focus on the steady state. Table 1 shows first the detailed steady state outcomes with baseline parameter values, and where profit is included as well. As already seen, the optimal cooperative solution implies higher effort from fleet 2 than from fleet 1 while the opposite happens in the two non-cooperative solutions. On the other hand, we find a larger steady state old mature stock than young mature stock in the cooperative solution and the opposite in the non-cooperative solutions. Both total steady state profit and the profit accruing to fleet 2 are substantially higher in the cooperative solution than in the other scenarios. However, fleet 1 individually obtains higher profit in the non-cooperative scenarios, where fleet 1 effort is higher than fleet 2 . The benefits from cooperation must therefore be shared in some way between the two fleets such that fleet 1 finds it profitable to stay in the cooperation. Otherwise, a prisoner's dilemma-like situation will result where none of the fleets find it rational to cooperate. The cooperative solution is thus not stable without side payments. The outcomes do not differ much between the wholly myopic solution and the Stackelberg 1 situation where fleet 1 acts as the leader. As also can be seen from Table 1, the Stackelberg solution yields a somewhat lower total profit than the myopic solution. This may seem surprising, but remember that we report steady state profit, and not present-value profit. Therefore, this result, depending among other on the choice of discount rate, could be reversed if net present value instead was reported ${ }^{2}$.

Table 1 about here

[^1]Next, Figures 4-6 show how the steady state values of the stocks and efforts in the cooperative solution are affected by changes in the catchability of fleet 1 , the discount rate and the fertility parameter, respectively. In Figure 4, the fleet 1 catchability coefficient $q_{1}$ is varied in the range from 0.02 to 0.05 . For low levels of $q_{1}$, not surprisingly, we obtain Case iii) where only fleet 2 is in operation and escapement of the young mature stock equals the pre harvest stock level. Escapement of the old mature stock is kept above the zero marginal profit level. Increasing $q_{1}$ to about 0.027 leads to Case i) where both fleets are in operation, and further increase leads to a gradual more fleet 1 effort while the effort of fleet 2 is reduced correspondingly. The steady state level of both stocks are reduced. For $q_{1}>0.047$, we finally obtain Case ii) with only fleet 1 in operation, and the escapement of the young mature stock approaches the zero marginal profit level

Figure 4 about here

Figure 5 demonstrates the steady state relationship between the discount rate, varied from $\delta=0$ to $\delta=0.25$ (implying the discount factor is varied from 1 to 0.8 ), and the state stocks and efforts. It is seen that, for a low discount rate a corner solution with Case iii) where only fleet 2 is utilized is optimal. Increasing the discount rate leads as expected to smaller stocks and to a gradual shift towards targeting also the young mature stock, and thus we obtain Case i). Therefore, while a higher discount rate reduces both stocks, the effort effect is somewhat surprisingly ambiguous as fleet leffort use increases while fleet 2 effort reduces.

Figure 5 about here

Figure 6 finally shows the effect of changes in the fertility parameter $\gamma$ on the optimal steady state stocks and efforts. The relative fertility of the young mature stock is varied from 0 to 1 . The baseline value is $\gamma=0.5$, and with $\gamma=1$ both stocks have equally high fertility. With a very low value of $\gamma$, it is not beneficial with harvest of fleet 1 and Case iii) represents the optimal cooperative solution. Increased fertility of the young mature stock leads gradually to higher effort of fleet 1 and hence a stronger targeting of the young mature stock. The pre-harvest level of the young mature stock increases with fertility, but escapement is reduced for both stocks.

Figure 6 about here

## 6. Concluding remarks

In this paper, we have considered a simple formulation of a 'complete' age structured fishery model with a harvest trade-off among two harvestable and mature age classes, and where recruitment is endogenously determined. These two harvestable age classes are targeted by two separate fishing fleets where we assume perfect fishing selectivity. The fishing is governed by the Baranov catch function, and the fishing prices and effort costs are assumed fixed. Three dynamic different harvest scenarios are studied. First, we analyze the cooperative solution where the two fleets act so to maximize the joint present value harvesting profit. Next, we consider two scenarios where the two fleets are managed by separate agents exploiting the fish stocks in a non-cooperative manner. We start by analyzing the situation where both fleets behave as myopic agents, thus maximizing current profit without taking own impact on next period's stocks into account. The other non-cooperative scenario is where fleet 1 is coordinated and behaves as a sole owner maximizing present value profit, while fleet 2 is myopic. This can be viewed as a Stackelberg game with fleet 1 as the leader and fleet 2 as the follower.

In the cooperative solution, we find that fertility and differences in fertility among the harvestable and mature year classes have no direct effect on the harvesting priority. Moreover, we demonstrate that the optimal harvesting may involve harvesting of both stocks, or only stock 1 , or only stock 2 . Typically, stock 2 only will be exploited when the higher fish price of this age class is accompanied with lower harvesting effort costs. In the cooperative solution when both stocks are exploited we also find that the stock with the highest price-to-survival rate can be said to harvested more aggressively. In the non-cooperative myopic situation it is shown that the possibility for fleet 2 to be in the fishery depends only on the price and cost parameters together with the survival rates of the two mature stocks. In steady state, we also find that the fish stocks will be more heavily exploited in the game solutions than in the cooperative solution. Overfishing of both stocks will therefore take place when the exploitation is uncoordinated. When comparing the Stackelberg solution and the myopic solution, it is also shown that the steady state stocks will be more heavily exploited in the myopic game than in the Stackelberg game. Therefore, coordinated management
is needed to omit economic losses, and where the quota management should be related to the different harvestable age classes, and not the total harvested biomass.

The theoretical reasoning is supplemented with some numerical illustrations. Under the baseline parameter scheme, we find, that fleet 1 obtain a higher profit in the non-cooperative solutions than in the cooperative solutions. The cooperative solution is thus not stable without side payments. We also find, somewhat surprisingly, that the non-cooperative myopic solution yields a higher total profit than the non-cooperative Stackelberg solution. This is surprising because one of the fleets has long-term considerations in the Stackelberg solution. However, we also find that this outcome hinges upon the choice of discount rate. Comparing the cooperative solution for different levels of harvest productivity shows that there will be a switch between the different harvesting schemes. For example, not surprisingly, fleet 2 only will be in operation if the productivity of fleet 1 is 'low'. Changing the discount rate and fertility also demonstrates switches among the different harvesting schemes, and where we find that while a higher discount rate reduce both stocks the effort effect is ambiguous.

## References

Armstrong, C. W. (1999). Sharing a fish resource - bioeconomic analysis of an applied allocation rule. Environmental and Resource Economics 13, 75-94.

Beverton, R. and Holt, S. (1957). On the dynamics of exploited fish populations. Blackburn Press.
Charles, A. T. and Reed, W. J. (1985). A bioeconomic analysis of sequential fisheries: competition, coexistence, and optimal harvest allocation between inshore and offshore fleets. Canadian Journal of Fisheries and Aquatic Sciences 42, 952-962.

Clark, C. W. (1980). Restricted access to common-property fishery resources: a game-theoretic analysis. In Dynamic optimization and mathematical economics (pp. 117-132). Springer US.

Clark, C. W. (1990). Mathematical bioeconomics: the optimal management of renewable resources. John Wiley \& Sons, Inc.

Diekert, F. K., Hjermann, D., Nævdal, E. and Stenseth, N. C. (2010). Spare the young fish: optimal harvesting policies for North-East Arctic cod. Environmental and Resource Economics 47, 455475.

Fischer, R. D. and Mirman, L. J. (1992). Strategic dynamic interaction: fish wars. Journal of Economic Dynamics and Control 16, 267-287.

Getz, W. and Haight, R. (1989). Population harvesting. Princeton University Press
Hannesson, R. (1995). Sequential fishing: cooperative and non-cooperative equilibria. Natural Resource Modeling 9, 51-59.

Kaitala, V., and Lindroos, M. (2007). Game theoretic applications to fisheries. In Handbook of operations research in natural resources (pp. 201-215). Springer US.

Laukkanen, M. (2001). A bioeconomic analysis of the northern Baltic salmon fishery: coexistence versus exclusion of competing sequential fisheries. Environmental and Resource Economics 18, 293-315.

Levhari, D. and Mirman, L. J. (1980). The great fish war: an example using a dynamic CournotNash solution. The Bell Journal of Economics, 322-334.

Lindroos, M. (2004). Sharing the benefits of cooperation in the Norwegian spring-spawning herring fishery. International Game Theory Review 6, 35-53

Mesterton-Gibbons, M. (1993). Game-theoretic resource modeling. Natural Resource Modeling 7, 93-147.

McKelvey, R. (1997). Game-theoretic insights into the international management of fisheries. Natural Resource Modeling 10, 129-171.

Munro, G. R. (1979). The optimal management of transboundary renewable resources. Canadian Journal of Economics, 355-376.

Quaas, M. and Requate, T. (2013). Sushi or fish fingers? Seafood diversity, collapsing fish stocks, and multispecies fishery management. The Scandinavian Journal of Economics 115, 381-422

Quaas, M., Requate, T., Ruckes, K., Skonhoft, A., Vestergaard, N. and Voss, R. (2013). Incentives for optimal management of age-strcutured fish populations. Resource and Energy Economics 35, 113-134

Quinn, T. (2003). Ruminations on the development and future of population dynamic models in fisheries. Natural Resource Modeling 16, 341 - 392

Reed, W. J. (1980). Optimum age-specific harvesting in a nonlinear population model. Biometrics 36, 579-593.

Singh R. and Weninger, Q (2009). Bioeconomics of scope and the discard problem in multiplespecies fisheries. Journal of Environmental Economics and Management 58, 72-92

Skonhoft, A., Vestergaard, N. and Quaas, M. (2012). Optimal harvest in an age structured model
with different fishing selectivity. Environmental and Resource Economics 51, 525-544.
Sumaila, U. R. (1997). Cooperative and non-cooperative exploitation of the Arcto-Norwegian cod stock. Environmental and Resource Economics 10, 147-165.

Tahvonen, O. (2009). Economics of harvesting age-structured fish populations. Journal of Environmental Economics and Management 58, 281-299.

Tahvonen, O. (2010). Age-structured optimization models in fisheries bioeconomics: a survey. In Boucekkine R. et al. (eds): Optimal control of age structured populations in economy, demography and the environment. Routledge, pp. 140-173


Figure 1. Biological equilibrium with fixed fishing mortalities.


Figure 2. Stock sizes over time (in \# of 1,000 fish). Cooperation, myopic behavior by both fleets and the Stackelberg game with fleet 1 as leader (Stackelberg1).


Figure 3. Fishing effort over time. Cooperation, myopic behavior by both fleets and the Stackelberg game with fleet 1 as leader (Stackelberg1).


Figure 4. Steady state stocks and efforts cooperative solution. Variation of fleet 1 catchability coefficient $q_{1}$ (basline value $q_{1}=0.03$ ).


Figure 5. Steady state stocks and efforts cooperative solution. Variation of the discount rate $\delta$ (baseline value $\delta=0.04$ ).


Figure 6. Steady state stocks and efforts cooperative solution. Variation of the fertility parameter $\gamma$ (baseline value $\gamma=0.5$ ).

Table 1. Steady state stocks, effort and profit. Cooperation, myopic behavior by both fleets and the Stackelberg game with fleet 1 as leader (Stackelberg1).

|  | Cooperative solution | Non-cooperative myopic | Stackelberg1 |
| :---: | :---: | :---: | :---: |
| $X_{1}(\# \text { of } 1,000$ <br> fish) | 614 | 489 | 501 |
| $X_{2}(\# \text { of } 1,000$ <br> fish | 767 | 350 | 376 |
| $E_{1}$ (effort) | 7 | 36 | 30 |
| $E_{2}$ (effort) | 25 | 5 | 12 |
| $\pi_{1}(1,000$ Euro) | 140 | 246 | 252 |
| $\pi_{2}(1,000$ Euro $)$ | 475 | 40 | 7 |
| $\pi$ (1,000 Euro) | 615 | 286 | 259 |


[^0]:    ${ }^{1}$ The optimization was performed with the fmincon solver in MATLAB release 2016b.

[^1]:    ${ }^{2}$ Indeed, this actually happens with the baseline discount rate $4 \%(\delta=0.04)$. Results can be obtained from the authors upon request. We have also studied the Stackelberg game with fleet 2 as the leader, and where we find that this solution with the baseline parameter values also yields lower total steady state profit than the myopic solution. Results from this game can also be obtained from the authors.

