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Optimale risikojusterte strategier basert på tekniske handelsregler: Glidende Gjennomsnitt og Relativ Styrkeindeks

En studie på S&P 500

Risk-adjusted optimal strategies based on the Moving Average Crossover and the RSI technical trading rules

A study on the S&P 500

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Abstract

This article tests the performance of optimized technical trading rules for the SPDR S&P 500 exchange traded fund in the period 2000 to 2016. To avoid data snooping, the optimization is performed on simulated time series from three different models. Two technical trading rules are optimized in this article: The dual moving average crossover and the Relative Strength Index (RSI). For both trading rules, the performance of four different trading strategies are evaluated. To measure the performance of the strategies we use the Sharpe ratio as assessment criteria. A brute-force optimization algorithm is used since the Sharpe ratio is a function of non-continues parameters. After finding optimal parameters for each strategy based on the simulated price series, these strategies are back-tested on historical data. The results for the moving average crossover rule are mixed. Some of the trading strategies provide a higher Sharpe ratio and higher returns than a buy-and-hold strategy, but these returns are not significantly greater than the buy-and-hold returns. For the RSI rule the optimized strategies generate few trading signals on historical data, and several of the strategies do not generate any signal during the whole trading period. Some of the strategies obtain a higher Sharpe ratio compared to the buy-and-hold, but this is caused by holding a risk-free position during the trading period. No strategy provides positive excess returns, and for the strategies generating negative excess returns, they are not significantly different from zero. Our results are consistent with the weak form market efficiency for the S&P 500 index during the time period 2000 to 2016.

Sammendrag

I denne artikkelen testes optimerte tekniske handelsstrategier på det børshandlede fondet SPDR S&P 500 i perioden 2000-2016. For å unngå problemet med «data-snooping» er de tekniske handelsstrategiene optimert på simulerte tidsserier fra tre ulike prosesser. Vi vil i denne artikkelen optimere handelsstrategier basert på to ulike handelsregler: Glidende Gjennomsnitt og Relativ Styrkeindeks (RSI). Risikojustert avkastning, målt ved Sharpe ratio, brukes for å evaluere handelsstrategiene. For å finne de optimale parameterne for en enkel handelsstrategi bruker vi en «*brute force*» optimeringsalgoritme. En av årsakene til at denne optimerings-metoden brukes er at Sharpe ratio er en funksjon av ikke kontinuerlige parametere. For begge handelsreglene er fire ulike handelsstrategier optimert. Etter å ha funnet optimale parametere tester vi de optimerte handelsstrategiene på historiske data. Resultatene fra de optimerte handelsstrategiene basert på glidende gjennomsnitt gir blandede resultater på historiske data. Noen av de optimerte strategiene har høyere Sharpe ratio og oppnår meravkastning utover en kjøp-og-hold strategi i indeksen i perioden, men denne meravkastningen er ikke funnet signifikant. De optimerte handelsstrategiene basert på RSI regelen genererer få signaler på historiske data, og noen av strategiene genererer ikke signaler i det hele tatt. Noen av strategiene oppnår svært god risikojustert avkastning, men dette skyldes at de i lengre perioder er ute av markedet og gir en risikofri avkastning. Ingen av strategiene basert på RSI gir meravkastning utover en kjøp-hold strategi. I følge våre resultater er S&P 500 effisient på svak form i perioden 2000-2016.

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1. Introduction

The proposition of efficient financial markets has had a central position in financial literature since the 1950´s. Notable work by Eugene Fama has been important in the development of the «Efficient Market Hypothesis» (EMH). According to Fama (1970), "*in an efficient capital market, stock prices will fully reflect all available information*". However, Fama (1970) considered three different types of information subsets stock prices could reflect. In the weak form market efficiency stock prices reflect all historical information, in the semi-strong form they will in addition reflect all publicly available information, and price adjustments associated with public information happen quickly. The strong form market efficiency applies when stock prices also reflect «insider» information. According to Fama (1970) the following conditions are sufficient, but not necessary, to assure efficient capital markets: there are no transaction costs when trading securities, all available information is available for free to all market participants and all market participators agree on the implications of current information for the current price and distribution of future prices for each security.

Most of the previous studies on the EMH operate with the following market equilibrium expression for the expected price of security j at time $t + 1$:

$$E\left[\tilde{p}_{j,t+1}|\Phi_t\right] = \left(1 + E\left[\tilde{r}_{j,t+1}|\Phi_t\right]\right)p_{j,t} \text{ ,}$$

where Φ_t is the information set at time t, $p_{j,t}$ is the price of security j at time t and $E\left[\tilde{r}_{j,t+1}|\Phi_t\right]$ is the expected return for security j at time $t + 1$ given the information set at time t. Fama (1970) refers to the equilibrium as the «fair game» model based on the assumptions that i) *"the conditions of market equilibrium can be stated in terms of expected returns",* and (ii) *"the information Φ is fully utilized by the market in forming equilibrium expected returns and thus current prices"*. The last assumption prevents technical trading rules from generating excess returns over equilibrium expected returns by exploiting the set of information at a given time (Φ_t). According to the EMH, technical trading rules should not in expectation provide greater returns than buying-and-holding the security.

Other studies discussing the EMH, like Alexander (1961), consider stock prices to follow a random walk process. Fama (1970) regarded the random walk model for stock prices as an extension of the fair game model. Cootner (1964) stated that "*the conditional expectation of tomorrow's price, given today's price, is today's price*". If stock prices follow a random walk, any new information arriving tomorrow is independent of the price today, and "*period-to-period price changes of a stock should be random movements, statistically independent of one* *another"* (Cootner, 1964). Even if there is statistically significant evidence for dependence in series of price changes, studies, such as Alexander (1961) and Fama and Blume (1966), show that it cannot be used to make profitable predictions of the future. These studies concluded that technical trading rules can result in excess return over a buy-and-hold strategy, but even minimal transaction costs would cause the excess returns to disappear. Technical analysis is built on the assumption that prices move in trends, which are established from changing attitudes of traders towards economic, political and psychological factors (Murphy, 1999). Technical analysis is therefore often regarded as a study of human psychology, and the reactions of investors to changing market conditions (Murphy, 1999). Another assumption, supplementing the first one, is the fact that «history repeats itself». Chart patterns, identified in the past, reveal the bullish or bearish psychology of the market. Since certain patterns have worked in the past, it is reasonable to assume they will continue to work in the future. The patterns are based on the study of human psychology, which does not tend to change (Murphy, 1999). Following mentioned assumptions, technical trading is implemented by first identifying a trend, and then follow it until it shows signs of reversing. While others, for instance Timmermann and Granger (2004) argue that investors constantly searching for predictable patterns, affect prices in their attempt to exploit trading opportunities. In their view, forecasting patterns are unlikely to persist, because of the self-destructive effect occurring when investors apply these patterns.

Many within the academic community remain sceptical towards the usefulness of technical analysis, despite its widespread acceptance and adoption by the financial industry. A common argument against technical analysis is that there is no economic theory backing the methods applied by technical analysts. Menkhoff (2010) investigated how extensive the use of technical analysis is in the fund managing industry. The study included 692 fund managers from the following countries: China, Italy, the US, Germany and Thailand. Menkhoff (2010) found that 87 % of the survey's respondents used technical analysis to some extent as an information source, and 18 % preferred technical analysis compared to fundamental. For short-term horizon, the study shows that technical analysis is the most preferred investment tool.

Previous research by Brock et. al. (1992), Bessembinder and Chan (1995) and Metghalchi et. al. (2012) find positive results for the profitability and the forecasting power of technical trading rules. In these studies, the returns are not risk-adjusted. According to Neely (2003), judging trading rules only in the terms of excess raw returns over equilibrium expected returns is not sufficient to conclude whether they violate the EMH or not. Jensen (1978) emphasizes that the correct interpretation of the EMH must be expressed in terms of the potential risk-adjusted excess returns. Based on information reflected in the stock prices, the risk-adjusted excess returns should not exceed the transaction costs of trading on that information. Risk adjustment is important, because some trading strategies might be out of the market and are therefore less risky than the buy-and-hold-strategy.

The motivation behind this paper is to test the performance of popular technical trading rules on the S&P 500 index, and implicitly test if the index is weak form efficient. Our research differs from other studies by using an ETF instead of the raw index (except for instance Metghalchi et. al., 2012), applying more than one or two trading strategies (Brock et. al., 1992; Bessembinder and Chan, 1995; Metghalchi et. al., 2012) and testing optimized rather than common rules. Our paper aims at determining optimal trading strategies, performed on two of the most popular technical trading rules: the moving average crossover and the Relative Strength Index (RSI), using the Sharpe ratio as assessment criteria. We use a «brute-force» optimization algorithm, meaning going through all possible solutions within some constraints, and thereby locating the optimal solution. The advantage of using «brute-force» over a genetic algorithm is that we are sure we find the global optimum when there is more than one local optimum. To get a more realistic approach, we use the SPDR S&P 500 Exchange Traded Fund Trust, which tracks the S&P 500 index. The ETF provides an opportunity to trade in the underlying index as if it had been a stock.

To identify optimal trading strategies, it is not sufficient to use one or view historical price paths, but rather simulate many possible price paths, based on the already observed daily closing prices of the SPDR S&P 500 during years 2000-2016. Since not knowing the stochastic process for the S&P 500 returns, we estimate three different time series models to simulate index returns. In our study we use Random Walk, AR (1) and exponential GARCH (EGARCH) model. The same kind of models are used in the previous studies (Brock et. al., 1992; Kwon and Kish, 2002; Marshall and Cahan, 2005). Based on the simulated returns, we obtain simulated index prices, which are used for the trading purpose.

For each model, we simulate 5000 paths, each with a horizon of 2000 daily returns to make results robust. Assuming 250 trading days in a year, each simulated path has a length of 8 years. We apply 4 trading strategies, based on the moving average crossover rule with and without a 1 % band and the Relative Strength Index (RSI). In the case of the moving average crossover rule, we optimize lengths of short and long moving average lag. When it comes to the RSI rule, optimization parameters are number of days in a look-back period and values of horizontal decision bands. Our approach is to find a solution with the highest Sharpe ratio across 5000 simulations for each trading strategy. The optimal solution will be evaluated by comparing its Sharpe ratio with the Sharpe ratio obtained from a buy-and-hold strategy on the same set of simulated price series. Optimized solutions will also be back-tested on the original price series, and adjusted for transaction costs.

The paper is organized as follows. The next section describes technical analysis, focusing especially on the moving average and the RSI rules. Section 3 provides a brief review of the previous studies, dividing them into early and modern studies. In Section 4 we present the data and methodology. The results are presented in Section 5, followed by summary and ideas for further research in the final section.

2. Technical Analysis

Murphy (1999) describes technical analysis as the study of market action, primarily using charts to forecast future price trends. In general case, the trend can be defined as the direction of the market. Murphy (1999) emphasizes that markets will usually not move in a straight line in any direction. Instead market moves are often represented in terms of «zigzags» series with obvious tops and bottoms. Murphy (1999) underlines that it is the direction of those tops and bottoms that defines market trend. An ascending pattern of tops and bottoms represents an uptrend, while a descending pattern indicates a downtrend. The market is defined as trendless if there are horizontal tops and bottoms. Murphy (1999) introduces support and resistance levels, defined as previous lows and highs. For an uptrend to continue each support and resistance level must be higher than the one preceding it, and the opposite when it comes to a downtrend. The resistance levels in an uptrend operate as «pauses» in that uptrend. If a horizontal trend in support or resistance levels appears, it is an early warning about a trend reversal, or at least the shift to a sideways trend. When a trend reversal occurs, support and resistance levels switch their roles and become the opposite of each other. Murphy (1999) warns that technical analysis is «designed» for trending markets, rather than markets without a trend. When the market moves sideways, the best decision is staying out of the market, while an uptrend or a downtrend can provide profitable trading returns.

Park and Irwin (2007) argue that a possible theoretical explanation behind technical trading profits can be stated by the «Noisy Rational Expectations Model». Two strong assumptions are taken when it comes to the standard model of market efficiency: participants are rational, and they have homogeneous beliefs when interpreting information. The current price does not fully reveal all available information under the Noisy Rational Expectations Model. Prices adjust slowly to a set of new information, because of the «noise» in the current equilibrium price, due to heterogeneous beliefs regarding this information. Thus, profitable opportunities might occur when using technical trading. Murphy (1999) has the opposite angle when he approaches technical trading, pointing out that the technical analysis is based on the following condition: all the aspects that can possibly affect the price are reflected in the market price. The statement is very close to the EMH. Murphy (1999) states that "*if the fundamentals are reflected in the market price, then the study of those fundamentals becomes unnecessary*". Technical analysis is indirectly about studying fundamental analysis, which is the underlying force of supply and demand of stocks. What investors should do is to study the price action, reflecting shifts in supply and demand (Murphy, 1999). The charts themselves do not cause bull and bear markets, and only if the mentioned condition about the market price is fulfilled it makes sense to study price charts (Murphy, 1999). Both technical and fundamental analysis attempt to solve the same problem: determination of the direction for stock prices. Fundamental analysis studies the cause of market movements, while technical analysis studies the effect. This is also the reason why these investment approaches often conflict with each other, but despite the differences there is also a lot of overlap. Technical analyst uses a wide variety of tools to look for profitable investment opportunities. Different kinds of the indicators are used to identify trends or to evaluate if a stock is overbought or oversold. In this paper, we focus on two of these indicators: the moving average crossover and the Relative Strength Index (RSI).

2.1. The Moving Average

The purpose of moving average rules is to identify a trend, and then track it until a new trend is revealed. Since moving average rules are used as a trend following device, they do not predict the appearance of a trend. They will rather reveal a trend only after its arrival, implicitly meaning that a moving average trend line does not work in terms of forecasting market prices (Murphy, 1999).

The most used moving average is the «simple moving average» (SMA). A stock or stock index N-day simple moving average is calculated by taking an equally weighted arithmetic mean of the stock's N last prices at a given time (t). A security's SMA can mathematically be expressed as follows:

$$SMA_t = \frac{1}{N}\sum_{i=0}^{N-1} P_{t-i}$$

To ensure that a heavier weighting is given to more recent prices, some analysts use the «exponential moving average» (EMA), calculated as follows:

$$EMA_t = [\alpha \cdot (P_t - EMA_{t-1})] + EMA_{t-1} \,,$$

where α is the weight value between 0 and 1, while the initial exponential moving average (EMA_1) is the n-day simple average of stock prices.

The moving average rule, called the «double crossover method», is applied by creating two moving averages of different lengths. When the short moving average crosses above the long one, the underlying investment is assumed to be in an upward trend. If the short moving average moves below the long one, the stock is assumed to be in a downward trend, and is expected to decrease further. According to Brock et. al. (1992), the idea behind the moving average crossover rule is to smooth out volatile price series. The length of the moving average represents the time lag in prices. The shorter a moving average is, the more sensitive it is to the current price (Murphy, 1999). The use of a very sensitive moving average will generate more trades and thus higher transaction costs. In addition, there is a greater risk of false signals, because of the short-term price movement, which causes «noise». The advantage of using the shorter moving average is that the trend signal appears earlier, because of increased sensitivity (Murphy, 1999).

Another moving average trading rule, that aims to utilize trends in the stock market is the «Moving Average Convergence Divergence» (MACD). Two exponential moving averages are used, and the MACD is calculated by subtracting the long moving average from the short moving average. If the MACD increases above zero from below an uptrend signal is generated, while the cross from above means the beginning of a downtrend period. The MACD differs from the crossover method, because it is based on the «oscillator» technique, which is an alternative to the trend-following approaches. The oscillator is represented by a flat horizontal band, in the case of the MACD the oscillator is a zero line. The advantage of using the oscillator technique is that it works on trendless markets, which is not the case for most of trend-following systems (Murphy, 1999). The oscillator is also useful when it comes to trending markets, by alerting about short term market extremes, and warning when a trend is losing the «momentum» before it is confirmed by the price itself (Murphy, 1999). The momentum for a security is generally defined as price difference for a fixed time interval:

$$M = P_t - P_{t-n} ,$$

where P_t is the closing price today, and P_{t-n} is the closing price n days ago. If prices are rising, and the momentum line is above the oscillator, it means that uptrend is under acceleration. The momentum line is always ahead of the movement in prices, providing forecast power for market prices (Murphy, 1999). In case of the MACD the momentum line is presented as a subtraction of the long exponential moving average from the short one.

2.2. The Relative Strength Index

The Relative Strength Index (RSI), based on the oscillator technique, attempts to decide if a stock is overbought or oversold. The RSI, developed and presented by J. Welles Wilder, Jr. in 1978, is built on the momentum concept. The purpose behind the development of the RSI is to overcome some major problems associated with the momentum approach (Wilder, 1978). The first problem is erratic movement within momentum line configuration, caused by major shifts in the closing price n days ago (Wilder, 1978). For instance, if using a 10-day momentum, the changes in a momentum line can be erroneously big, though the price «today» is close to the «yesterday's». The reason is large rise or fall in a 10-day lagged price. According to Wilder (1978) some «smoothing» must be provided to overcome this issue. The second problem is that a momentum does not have a constant range, making it difficult for comparison purposes (Wilder, 1978). The solution to these problems is the development of the standardized indicator, the RSI, calculated as follows:

$$RSI_t = 100 - \left(\frac{100}{1 + RS_t}\right),$$

where RS is the average of last n day's absolute gains divided by the average of last n day's absolute losses. In the RSI, the way it was originally made by Wilder (1978), the number of days (n) in a look-back period is set to 14. Wilder (1978) emphasizes that the first averages of gains and losses are calculated as simple n-day average. Thereafter average gain and loss are computed with a modified moving average, using the following formula:

$$\overline{Gain_t} = \frac{\overline{Gain_{t-1}} \cdot (n-1) + Gain_t}{n}$$

$$\overline{Loss_t} = \frac{\overline{Loss_{t-1}} \cdot (n-1) + |Loss_t|}{n},$$

where $RS_t = \frac{\overline{Gain_t}}{\overline{Loss_t}}$.

The RSI solves earlier mentioned problems by providing necessary smoothing, and creating a constant range between 0 and 100. Although Wilder (1978) used 14 days as the look-back period, analysts also operate with 5, 7, 9, 21 and 28 days. The shorter period, the more sensitive the RSI becomes, and the wider its amplitude (Murphy, 1999). When applying the original RSI, there are two oscillators, the upper and the lower horizontal bands. Wilder (1978) operates with 70 as the upper band, and 30 as the lower one. If a stock has an RSI above 70, it is assumed that the stock is overbought, whilst an RSI below 30 indicates that a stock is oversold.

3. Literature Overview

3.1. Early Studies

A wide variety of technical trading rules were investigated in the early literature. Alexander (1961 & 1964), Fama and Blume (1966) and Sweeney (1986) all investigated the profitability of different «filter» rules. A buy signal occurs when the price rises above a given percent from the most recent high, while a sell signal is generated when the closing price falls a given percent from its most recent low. One of the most known studies on filter rules is by Fama and Blume (1966). They tested Alexander's filter rules on daily closing prices of 30 individual securities listed on the Dow Jones Industrial Average (DJIA). Their study showed that only some small filters generated higher profits than the buy-and-hold strategy. After transaction costs were accounted for, none of the Alexander's filter rules would have been profitable. Early studies on moving average trading rules, such as Van Horne and Parker (1967 & 1968), James (1968), and studies applying the RSI like Jensen and Benington (1970) conclude that these rules are not profitable in stock markets.

While many of the earlier studies find that technical trading rules are not able to predict price movements in stock markets, technical trading rules applied to foreign exchange markets and future markets can earn net profits (Park and Irwin, 2007). For instance, Leuthold (1972) showed that filter rules would give a profit after considering transaction costs. Sweeney (1986) found that long positions based on small filter rules would give net transaction cost profits in all the 10 foreign exchanges tested.

There are a couple of factors that cast doubt over the findings of earlier research. Common for the early studies is that they only investigate a small number of technical trading rules. Most of the studies do not apply statistical tests for the returns obtained from the trading rules. Many of the studies also do not include the aspect of riskiness in trading strategies. A strategy can provide excess return over a benchmark, but the excess return might be a result from taking on more risk, rather than a breach on the market efficiency hypothesis (Park and Irwin, 2007). Bias from data snooping is also a flaw that can affect the reliability of some of the earlier studies. Profitable trading rules might be identified, but could just as well be a result of luck, rather than having predictive power (Jensen, 1967).

3.2. Modern Studies

Lukac et al. (1988) is one of the first modern studies, in which 12 technical trading systems have been optimized on price series from 12 futures markets over 1975-1984. The method used is a 3-year re-optimization method. Trading rules were optimized during a three-year in-sample period, while a year after this period was considered as an out-of-sample period. Parameters generating the largest profit over a three-year period were used for the following year's trading. Then a new three-year in-sample period, including the previous out-of-sample year, was used to find optimal trading rules. These parameters were again tested on a new one-year out-of-sample period, and so on. Based on the assumption that the Capital Asset Prising Model (CAPM) holds, Jensen's α is used as an approach to determine the significance of risk-adjusted returns. The findings in this study show that four out of twelve trading systems generated significant positive net transaction costs risk-adjusted return. This study concludes that some futures markets might have been inefficient during the sample period.

The study by Brock et. al. (1992) is considered as one of the most important works on technical trading (Park and Irwin, 2007). The reason is the finding of strongly consistent, positive results about the forecasting power of technical trading rules over a long period of time. Two simple and popular trading rules, the moving average crossover and the «Trading Range Break-Out» (TRB) were tested on daily Dow Jones Index data from 1897 to 1986. Several variations of the moving average crossover rule were used by Brock et. al. (1992): 1-50, 1-150, 5-150, 1-200 and 2-200. When it comes to the TRB rule, a buy signal is provided when the price goes above the resistance level, and a sell signal when the price goes below the support level. Brock et. al. (1992) found that the mean index return on buy days was higher than the mean index return on sell days for both trading rules. The mean index return on sell days was in addition negative. Furthermore, index returns on buy days were less volatile than on sell days. Brock et. al. (1992) was the first paper to applicate the model-based bootstrap method, since t-ratios are assumed to build on the normal, stationary and time independent, but doubtfully realistic, distribution of stock returns. The returns from Dow series were simulated, and the same set of trading rules was applied to the simulated series. Then, the trading returns obtained from the simulated series were compared with the actual trading returns. Various models were used to simulate returns: Random Walk, AR (1), GARCH-M and EGARCH. Brock et. al. (1992) found that actual trading profits were not consistent with any of the processes lying behind the models used for simulation.

Bessembinder and Chan (1995) examined whether technical analysis could predict stock price movements in Asian markets by using the same trading rules as Brock et. al. (1992). They found that trading rules had more explanatory power in emerging Asian markets, such as Malaysia, Thailand and Taiwan, than in the more developed markets of Hong Kong, Japan and Korea. Bessembinder and Chan (1995) argued that even if technical trading rules provided higher returns than a buy-and-hold strategy, a stock market can still be efficient. One explanation behind the surplus return is transaction costs. These costs are higher in the case of technical trading rather when keeping a long position in the stock market. Two cases were considered: in the first case, it was assumed that an investor reacted the same day a trading signal appeared, while in the other case a one-day trading lag was included. The last case is due that investors require time to react to the signal, and because of the nonsynchronous trading bias. Break even transaction costs, which would eliminate gains from technical trading, were estimated to be 1,57 % for the first case, and 1,34 % for the other one. These costs were less than the actual transaction costs in Hong Kong, Japan and Korea, while higher than those in Malaysia, Thailand and Taiwan. Profits from technical trading were unlikely to appear in developed markets, while seemed possible in emerging markets. However, Bessembinder and Chan (1995) did not control for the relative riskiness of the technical trading strategies tested. To evaluate the forecast power of trading rules, indices return means and variances on buy and sell days were compared. The results were similar to those, obtained by Brock et. al. (1992). The mean index return conditional on buy signals exceeded the mean index return conditional on sell signals for 53 of 60 cases. The variance for index returns on sell days was in addition higher than on buy days for the «variable-length moving average» (VMA) rule, meaning no constraints when it comes to the length of a holding period**,** in Hong Kong, Japan, Korea and Malaysia.

Allen and Karjalainen (1999) implemented a genetic algorithm in their attempt to find a technical trading rule giving excess return over a simple buy-and-hold strategy. In their study, daily prices of the S&P 500 index from 1928 to 1995 were used. The strategy consisted of either being long in the market or out of the market earning a risk-free rate of return. They estimated trading rules in ten different time periods, each with a length of 6 years, using the strategy's ability to create excess return over the buy-and-hold strategy as the fitness criteria. They thereby tested the performance of the strategy on the remaining historical data, which was not used in the estimation procedure. Only one estimation period provided trading rules, which generated net cost profits out-of-sample. The profits obtained in their study were not adjusted for risk. Their study supports market efficiency for the S&P 500.

The research by Neely (2003), built on the study of Allen and Karjalainen (1999), concluded that most technical trading rules would not result in the risk-adjusted net cost excess return for the S&P 500, in the period 1929 to 1995. The period was divided into ten sub samples, where each in-sample period was 5 years and each out-of-sample period was 2 years. Neely (2003) stated that the most optimal trading rule based on genetic programming should be used rather than common rules. In only two of the ten periods the optimal trading rule gained excess return, with no evidence of significant results, over the buy-and-hold strategy when tested in the out-of-sample period. Whilst in the other periods, the optimal trading rule produced results roughly equal to the buy-and-hold strategy using the Sharpe ratio as the assessment criteria. Their study also supports market efficiency for the S&P 500.

Metghalchi et. al. (2012) tested the profitability of two trading strategies across several types of moving average rules on 16 different European ETF stock indices. The strategies were applied across following rules: the simple moving average crossover, the «increasing moving average» (IMA) and the «Arnold and Rahfeldt's autoregressive moving average» (ARMA). When it comes to the IMA rule, the only difference from the simple moving average crossover rule is that the long moving average must, as an additional requirement, have a positive slope to provide a buy signal. The ARMA rule generates a buy signal if the current price is above both long and short moving averages, and a sell signal if the price is below them. Following the simple moving average crossover rule, a one day moving average was used as a proxy for the short moving average, which corresponds to the raw price of the index. As for the long moving average the following number of days were approached: 20, 50, 100 and 200 days. Metghalchi et. al. (2012) concluded that all three moving average rules provided significant excess returns over the buy-and-hold strategy, even after considering data snooping effects and transaction costs. Most profitable was the increasing moving average rule, which obtained significant excess returns in all the 16 stock indices tested. The simple moving average crossover and the Arnold and Rahfeldt's moving average rules provided significant excess returns in 10 and 11 of the European stock indices tested, respectively.

4. Data and Methodology

4.1. Data

To optimize trading strategies, we use simulated price series based on the closing prices of the SPDR S&P 500 ETF Trust, listed on NYSE Arca. Our sample consists of 4435 daily returns from Datastream for the price level of the Trust ranging from the beginning of 2000 to the end of 2016. First issued in 1993 by the State Street Global Advisors, the SPDR S&P 500 ETF Trust is the oldest of the exchange-traded funds tracking the S&P 500. The index consists of five hundred selected U.S. stocks from twenty-five separate industry groups.

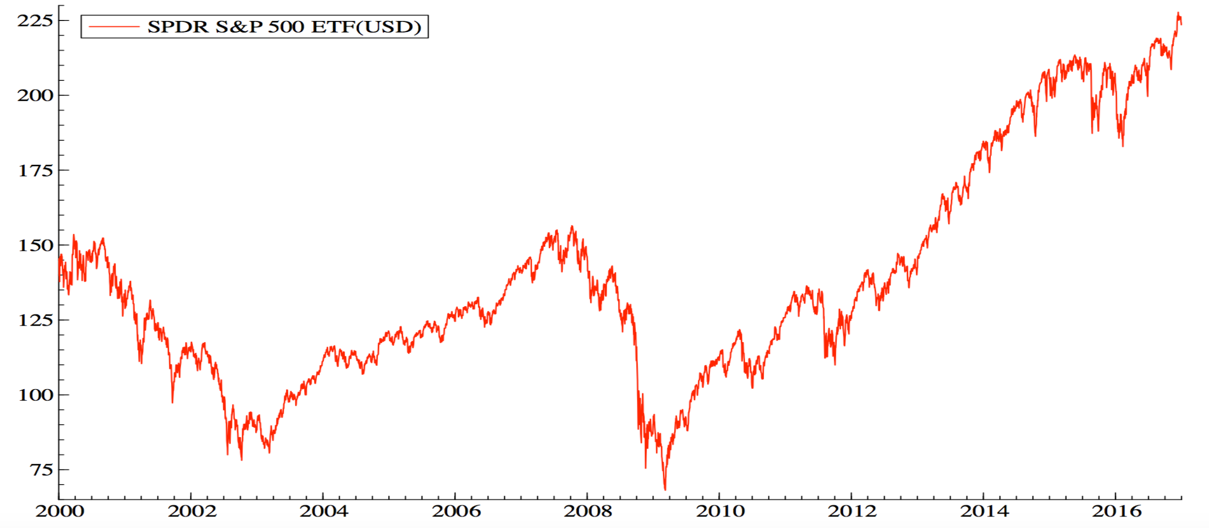

Figure 1. Prices SPDR S&P 500

In the years following 2000 the S&P 500 experienced a sharp decline largely caused by the «dot.com bubble». The overvaluation of many stocks in the IT-sector caused the index to fall greatly. During the financial crisis in 2008 the S&P 500 also decreased greatly until its bottom in March 2009. In the years following the financial crisis the index increased without any major drawbacks. After Standard & Poor´s decreased the credit rating of the US in late 2011, the U.S. stock market fell, but recovered quite quickly. The low oil prices in the beginning of 2016 caused the S&P 500 to fall, but it recovered quickly as well, and all in all the index rose in 2016.

Descriptive Statistics for Prices of SPDR S&P 500								
Mean	Std. Dev.	Skewness	Excess Kurtosis	Min.	Max.	Median	Obs.	Jarque Bera
137,77	36,038	0,7645	-0,2871	68,11	277,76	129,61	4435	447,30

Table 1. Descriptive statistics SPDR S&P 500 Prices 2000-2016

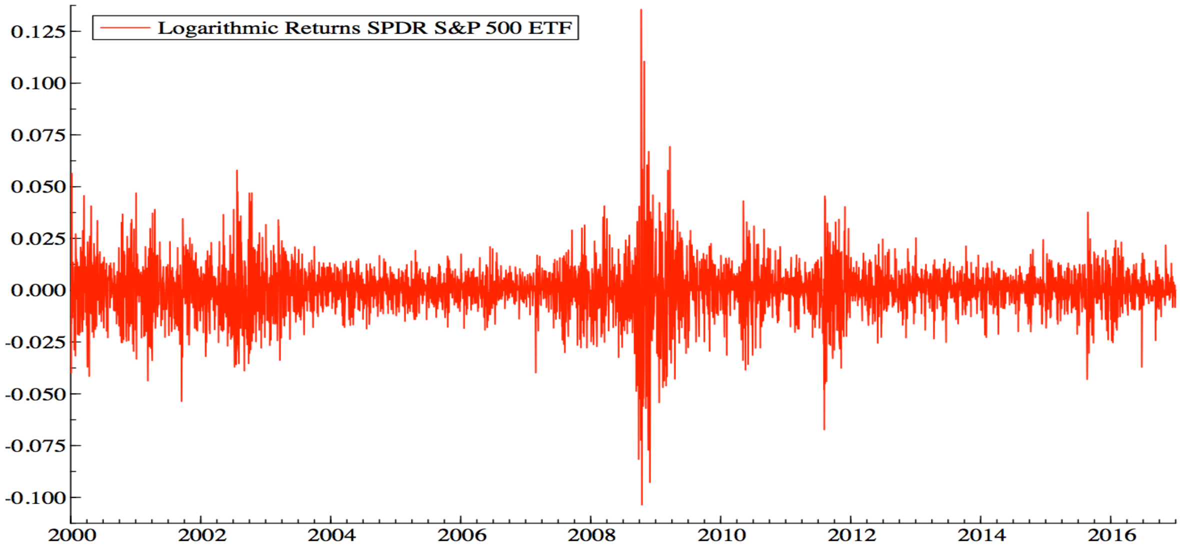

Figure 2. Returns SPDR S&P 500

From Figure 2 we can see increased volatility for returns in periods where there was great uncertainty in the financial markets. The most apparent case of volatility clustering occurred late in 2008, during the financial crises.

Descriptive Statistics for Returns of SPDR S&P 500								
Mean	Std. Dev.	Skewness	Excess Kurtosis	Min.	Max.	Median	Obs.	Jarque Bera
$9{,}6933 \cdot \mathbf{10^{-5}}$	0,0123	-0,0194	10,479	-0,1036	0,1356	0,000263	4434	20286

Table 2. Descriptive statistics SPDR S&P 500 Returns 2000-2016

The return series is computed by taking the natural logarithm of the current price level divided by the previous price level (ln $(\frac{P_t}{P_{t-1}})$). The time series of returns has a mean of approximately zero, it is also negatively skewed and fat tailed. The Jarque Bera statistic concludes that returns are not normally distributed since the tests statistic exceeds the critical value of 5,99.

4.2. Return Series Modelling

To optimize trading strategies for a given trading rule we use prices obtained from simulated return series, rather than historical SPDR S&P 500 prices. Optimizing on historical data will certainly provide profitable trading strategies in-sample, but they are unlikely to be robust when testing them out-of-sample. Three different simulation models, Random Walk, AR (1) and EGARCH, are estimated on historical returns. For each model, we simulate 5000 time series, each with a horizon of 2000 returns.

Despite usage of same models, as in studies by Brock et. al. (1992), Kwon and Kish (2002) and Marshall and Cahan (2005), the approach in our paper differs. In mentioned studies trading returns obtained from the original series are compared to trading returns obtained from the simulated return series. Our approach is to evaluate each trading strategy by comparing risk-adjusted trading returns, obtained from simulated series, with risk-adjusted buy-and-hold returns, obtained from the same set of simulated series.

The mentioned previous studies apply a bootstrap methodology. Residuals in simulation models are obtained from observed series and then randomly re-sampled. This is done to overcome the weaknesses of t-tests, because distributions for financial returns often are leptokurtic, autocorrelated and heteroscedastic (Park and Irwin, 2007). However, as pointed out by Maddala and Li (1996), Ruiz and Pascual (2002), if the observed return series is misspecified in bootstrapped simulation model or is highly complex, the use of the model-based bootstrap methodology can provide inconsistent estimates. Therefore, we do not apply the model-based bootstrap methodology in this paper. In the next sections we present each model, used for simulation of index returns.

4.2.1. Random Walk

We use the following model for a random walk process:

$$\ln(P_t) = \ln(P_{t-1}) + \varepsilon_t \,,$$

where $\ln(P_t)$ is the natural logarithm of the price for the SPDR S&P 500 at time t, $\ln(P_{t-1})$ is the natural logarithm of the price at time $t-1$, while ε_t is a shock that represents the impact of new information, assumed to follow an i.i.d. process, i.e. $\varepsilon_t \sim i.i.d.(0, \sigma^2)$.

The equation can be transformed, in terms of returns, as follows:

$$\ln(P_t) - \ln(P_{t-1}) = \varepsilon_t$$

$$\ln\left(\frac{P_t}{P_{t-1}}\right) = \varepsilon_t \,,$$

where the left side in the equation is the return for the SPDR S&P 500 at time t. A random walk process is not stationary because it contains a unit root. An augmented Dickey-Fuller test shows that the random walk process is stationary in first differences, the returns series for the SPDR S&P 500 are therefore stationary. The results for the Dickey-Fuller tests can be found in Appendix 1.1. A model for a random walk process can be viewed as a special case of an ARIMA model, specified as an ARIMA (0,1,0) model without a constant where $\ln(P_t)$ is the dependent variable.

4.2.2. AR (1)

Based on estimated parameters in Table 3, the following AR (1) model is used to simulate 5000 different return series:

$$r_t = 0{,}00011 - 0{,}06586 \cdot r_{t-1} + \varepsilon_t \,,$$

where ε_t is assumed to follow a Gaussian distribution.

	Coefficient	Std. Error	t-value	Probability
r_{t-1}	-0,06586	0,01497	-4,40	0,00
Constant	0,00011	0,000184	0,611	0,541

Table 3. Estimated parameters in the AR (1) model

The null hypothesis of no autocorrelation in the residuals from the AR (1) model is rejected by performing a Ljung-Box test as shown in Appendix 2.1. Time varying volatility in the residuals can be detected by performing a Ljung-Box test on the squared residuals from the autoregressive model. As shown in Appendix 2.1 the residuals from the AR (1) model are heteroscedastic. Since the residuals are correlated and have time varying volatility we use robust standard errors corrected for both autocorrelation and heteroscedasticity (Newey and West, 1987). Table 4 provides estimated parameters in AR (1) model, but now with robust standard errors corrected for both autocorrelation and heteroscedasticity (HAC-SE). Although the robust standard error for coefficient r_{t-1} is greater than the standard error from the ordinary least squares estimation, the coefficient is still significant.

	Coefficient	HAC-SE	t-value (HAC-SE)
r_{t-1}	-0,06586	0,020285	-3,1579
Constant	0,00011	0,000169	0,6642

Table 4. Estimated parameters in the AR (1) model with robust standard errors

4.2.3. EGARCH

The EGARCH is an extension of the original GARCH model, allowing for asymmetric effects on the conditional volatility from positive and negative shocks (Nelson, 1991). The EGARCH specification can capture the negative correlation between current returns and future volatility, which is present in the return series for many stocks, contrary to Bollerslev´s original GARCH specification. This is known as the «leverage effect», first introduced by Fisher Black in 1976. Nelson (1991) proposed the following specification to model the conditional variance:

$$\log\widehat{\sigma_t^2} = \alpha_0 + \sum_{i=1}^{p}\beta_i \cdot \log\widehat{\sigma_{t-\iota}^2} + \sum_{j=1}^{q}\alpha_j\left[\frac{|\varepsilon_{t-j}|}{\widehat{\sigma}_{t-j}} - E\left\{\frac{|\varepsilon_{t-j}|}{\widehat{\sigma}_{t-j}}\right\}\right] + \sum_{j=1}^{q}\gamma_j\left(\frac{\varepsilon_{t-j}}{\widehat{\sigma}_{t-j}}\right),$$

where $\widehat{\varepsilon_t} = \widehat{\sigma_t}z_t$ and $z_t{\sim}i.i.d\ (0,1)$.

Estimating the conditional volatility on a log-linear form ensures that the estimated value of the conditional volatility is positive. This allows the estimated coefficients to be negative contrary to the original formulation of the GARCH model. The coefficients $\gamma_1, \gamma_2 ... \gamma_q$ make up the sign effect, and are therefore typically negative, while the coefficients $\alpha_1, \alpha_2 ... \alpha_q$ determine the size effect and are typically positive.

4.2.3.1. Estimation of the EGARCH Model

To estimate the most appropriate EGARCH model we will first specify the ARMA model with the best fit for the historical return series. Plausible candidates for simulating returns are AR (1), AR (2), ARMA (1,1) or MA (1). All these models provide significant parameters when estimated with maximum likelihood. Although the condition of serially uncorrelated residuals and constant variance are breached, as discussed below, these issues can be solved by estimating the conditional variance. Table 5 shows the Log Likelihood, Schwarz-, Hannan-Quinn- and Akaike information criteria for the mentioned models of the conditional mean. The information criteria are described in Appendix 3.1.

Model	Log Likelihood	Schwarz IC	Hannan-Quinn IC	Akaike IC
AR (1)	13228,205	-5,9610	-5,9638	-5,9654
AR (2)	13235,985	-5,9626	-5,9664	-5,9684
ARMA (1,1)	**13239,095**	***-5,9641***	***-5,9678***	***-5,9698***
MA (1)	13229,420	-5,9616	-5,9644	-5,9659

Table 5. Log Likelihood and information criteria for ARMA models

All three information criteria select the ARMA (1,1) as the best model. Performing a Ljung-Box test on the squared residuals from the ARMA (1,1) model, as shown in Appendix 3.2, indicates Arch effects amongst the residuals. Since the squared residuals are not heteroscedastic we will also model the conditional variance. By fitting different EGARCH variations we find that EGARCH (1,1) and EGARCH (1,2) provide significant parameters in the equation for the conditional variance. The standard errors are assumed to be normally distributed. Table 6 shows the Log Likelihood and information criteria for these two EGARCH models.

Model	Log Likelihood	Schwarz IC	Hannan-Quinn IC	Akaike IC
EGARCH (1,1)	14343	-6,4544	-6,4619	-6,4660
EGARCH (1,2)	14363	***-6,4595***	***-6,4689***	***-6,4740***

Table 6. Log Likelihood and information criteria for EGARCH models

All three information criteria select the EGARCH (1,2) model as the preferred model as shown in Table 6. First, we estimate the ARMA (1,1) - EGARCH (1,2) model since it has the highest Log Likelihood and each information criteria prefers this model. The EGARCH (1,2) provides the following equation for the conditional variance:

$$log\ \hat{\sigma}^2 = -0{,}1706 + 0{,}9823 \cdot \log \hat{\sigma}^2_{t-1} - 0{,}0915 \cdot \frac{|\varepsilon_{t-1}|}{\hat{\sigma}_{t-1}} + 0{,}2127 \cdot \frac{|\varepsilon_{t-2}|}{\hat{\sigma}_{t-2}} - 0{,}2705 \cdot \frac{\varepsilon_{t-1}}{\hat{\sigma}_{t-1}} + 0{,}1302 \cdot \frac{\varepsilon_{t-2}}{\hat{\sigma}_{t-2}}$$

Since the coefficient for the first Arch lag and the coefficient for the second leverage lag have opposite signs than expected we rather estimate the conditional volatility with an EGARCH (1,1) model.

By plotting the quantiles of the standardized residuals against the standardized normal distribution we can see if the standardized residuals are normally distributed or not. Figure 3 shows that the residuals are negatively skewed and it is a thicker tail than implied by the normal distribution. The standardized residuals have a skewness of -0,5127 and an excess kurtosis of 1,6835. We therefore assume that z_t is drawn from a t-distribution to assess the problem with fat tails.

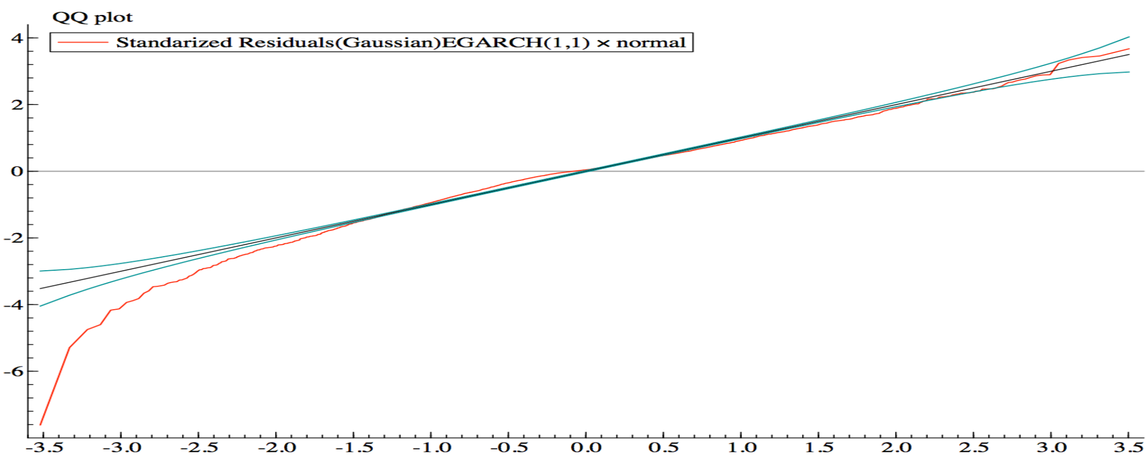

Figure 3. Quantiles of the standardized residuals versus the standard normal quantiles

Model	Log Likelihood	Schwarz IC	Hannan-Quinn IC	Akaike IC
EGARCH (1,1) (t-innovations)	14424	-6,4892	-6,4976	-6,5022

Table 7. Log Likelihood and information criteria for the EGARCH (1,1) with t-innovations

A likelihood ratio test, described in Appendix 3.3, concludes that the EGARCH model with t-innovations has the best fit for our data. We will therefore use the following EGARCH model:

$$\hat{r}_t = 0{,}000639948 - 0{,}753901 \cdot \hat{r}_{t-1} + \ 0.734934 \cdot \varepsilon_{t-1} + \varepsilon_t$$

$$log\ \hat{\sigma}^2 = -0{,}153869 + 0{,}984037 \cdot \log \hat{\sigma}_{t-1}^2 + 0{,}104674 \cdot \frac{|\varepsilon_{t-1}|}{\hat{\sigma}_{t-1}} - 0{,}158091 \cdot \frac{\varepsilon_{t-1}}{\hat{\sigma}_{t-1}},$$

where $\hat{\varepsilon_t} = \hat{\sigma_t} z_t$ and z_t is t-distributed with 6,8369 degrees of freedom. Figure 4 shows the distribution of the residuals from the EGARCH model compared to a theoretical t-distribution with 6,83 degrees of freedom. From figure 4 we can see that there are less residuals in the right tail than implied by the t-distribution. Ideally the residuals should have followed a skewed t-distribution, but MATLAB does not include this distribution.

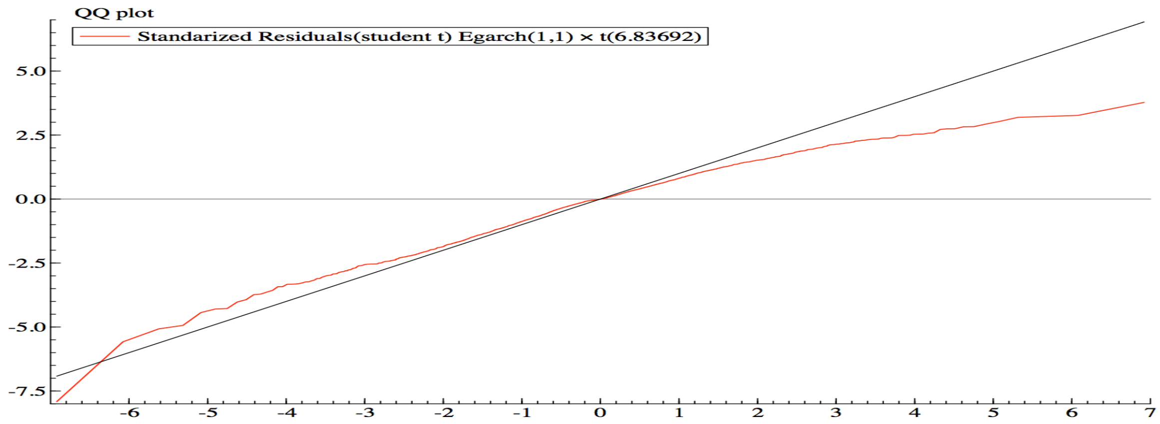

Figure 4. Quantiles of the standardized residuals following the t-distribution versus quantiles of a t-distribution with 6,8369 degrees of freedom

In GARCH models we test the standardized residuals for autocorrelation rather than the raw residuals. Ljung-Box test in Appendix 3.4 barely accepts the null hypothesis of no serial correlation amongst the standardized residuals. A Ljung-Box test on the squared standardized residuals concludes with no remaining Arch effects amongst the residuals. The test statistic is shown in Appendix 3.4.

4.3. Description of Trading Procedures and Return Computations

To form the simulated price series, simulated returns from the models in Section 4.2 are raised to Euler's number and multiplied by the previous price level, assuming 100 as the initial value of the index. Based on obtained 5000 index price series for each model, we apply 4 trading strategies, performed on the moving average crossover rule with and without a 1 % band, and the RSI rule. It is important to distinguish the definition of a «trading rule» and a «trading strategy». The first one provides a trading signal, «buy» or «sell», while a trading strategy defines the position an investor takes for a given signal. Days following a buy (sell) signal are classified as buy (sell).

Following the moving average crossover rule, a buy signal is emitted when the short simple moving average breaks the long one from below, while a sell signal appears when the short simple moving average breaks the long one from above. The occurrence of a signal can be expressed as follows:

$$SMA_t(s) > SMA_t(l) \text{ and } SMA_{t-1}(s) < SMA_{t-1}(l) = \text{«buy»}$$

$$SMA_t(s) < SMA_t(l) \text{ and } SMA_{t-1}(s) > SMA_{t-1}(l) = \text{«sell»}$$

Considering earlier research (Brock et. al., 1992; Bessembinder and Chan, 1995; Fifield et. al., 2005; Metghalchi et. al., 2012) we will examine the moving average crossover rule both with and without a 1 % band. For a rule with a band a buy (sell) signal is emitted when the short moving average exceeds (is less than) the long moving average by 1 %. The occurrence of a signal can be expressed as below:

$$MA_t(s) > 1{,}01 \cdot MA_t(l) \text{ and } MA_{t-1}(s) < 1{,}01 \cdot MA_{t-1}(l) = \text{«buy»}$$

$$MA_t(s) < 0{,}99 \cdot MA_t(l) \text{ and } MA_{t-1}(s) > 0{,}99 \cdot MA_{t-1}(l) = \text{«sell»}$$

According to Brock et. al. (1992), the introduction of a band will lead to reduced number of buy and sell signals. That way one will eliminate «whiplash» signals that may occur when the values of the short and long moving averages are similar. With a band of zero all days will be classified as either buys or sells (Bessembinder and Chan, 1995). The only exception might be the days in the beginning of a trading period, because of the requirement that a buy (sell) signal first occurs when the short moving average breaks the long from below (above). For a rule with a band, not all days will be classified as buy or sell days, in addition to those in the beginning of a trading period. When the two following conditions are met, neither a buy nor sell signal applies:

$$MA_t(s) < MA_t(l) \cdot 1{,}01$$

$$MA_t(s) > MA_l(l) \cdot 0{,}99$$

When it comes to the RSI rule, upper and lower horizontal bands are oscillators, providing buy and sell signals. According to Wilder (1978) and Murphy (1999) a move below the lower band warns of an oversold market condition for a given security, while a move above the upper band indicates an overbought condition. Among various approaches of the RSI rule (Wong et. al., 2003), we operate with the simplest form. A buy signal is emitted when the RSI crosses the lower oscillator from above, while a sell signal appears when the RSI crosses the upper oscillator from below. The occurrence of a signal is expressed as follows:

$$RSI_t < lower\ band\ and\ RSI_{t-1} \geq lower\ band = «buy»$$

$$RSI_t > upper\ band\ and\ RSI_{t-1} \leq upper\ band = «sell»$$

As the case with the moving average crossover rule, all days are considered as buy or sell, except the days before the first trading signal is provided.

Below, we present trading strategies applying for both rules used in this paper. Some assumptions are taken, applicable for each strategy. An investor holds a long position in the index when trading period days are neither buy nor sell days, as it provides a better comparison with a buy-and-hold strategy. The second assumption is that the borrowing and lending rates are the same (Brock et. al., 1992). As a proxy for the risk-free interest rate we use the 3 month US T-BILL security market rate. Daily observations of the annualized rate have been collected from Datastream and then transformed to daily rates with the following formula:

$$(1 + r_t)^{\frac{1}{360}} - 1$$

In the optimization and back-testing procedures, the average of daily interest rates is used as a proxy for the risk-free interest rate. The last assumption is that an investor trades the same day a signal is emitted, by observing the prices a few minutes prior to the day's closing and makes the trading order at approximately the closing index price (Neely, 2003; Metghalchi et. al., 2012).

The first strategy is «Leverage Money», and is the same strategy used by Brock et. al. (1992), Bessembinder and Chan (1995) and Metghalchi et. al. (2012). Upon a buy signal, an investor borrows in the money market to double the investment in the index. Upon a sell signal, the strategy requires closing of the buy position in the index and investing in a risk-free asset. Following Bessembinder and Chan (1995) a pre-transaction cost trading return is computed as below:

$$TR_t = 2 \cdot R_t - r_t$$

$$TR_t = r_t\ ,$$

where TR_t is the trading return on day t given a buy signal, R_t is the index return on day t and r_t is the mean daily interest rate on day t given a sell signal.

The second strategy, «Long Money», has been applied by Neely (2003) and Metghalchi et. al. (2012). Following this strategy, an investor takes a long position in the index when the trading rule emits a buy signal, while goes long in the money market upon a sell signal earning the risk-free rate of return. The returns, obtained from the strategy, are computed as follows:

$$TR_t = R_t$$
$$TR_t = r_t$$

The third strategy, «Long Short», is based on the idea, that an investor is always in the stock market, taking either a long or short position in the index, obtaining R_t on buy days and $-R_t$ on sell days. The fourth strategy, «Money Short», is the opposite of the Long Money strategy, meaning investment in a risk-free asset upon buy signals, and taking a short position in the index upon sell signals. The strategy provides r_t upon buy days and $-R_t$ upon sell days.

Each strategy will be evaluated in terms of profit by first taking obtained trading returns (TR_t) and creating a value index as follows:

$$V_t = V_{t-1} \cdot e^{TR_t}$$

with a start value (V_0) of 100 on the first day of a trading period. The total return for a trading period is computed as below:

$$TR_N = \frac{V_N - V_0}{V_0},$$

where N stands for the length of a trading period, while V_N is the index value on the last day of the trading period. As mentioned by Neely (2003), a strategy with large total raw returns might also be the riskiest strategy, therefore we use risk-adjusted trading returns to evaluate trading strategies. For the strategies which are out of the market on specific signals, the volatility of trading returns might be less than the volatility of returns for the buy-and-hold strategy. A common measure on risk-adjusted returns is the Sharpe ratio (Sharpe, 1966). It has been used by Neely (2003) as assessment criteria in terms of risk-adjusted profitability for trading strategies. The ex-ante Sharpe ratio will be measured as follows:

$$Sharpe\ ratio = \frac{\mathrm{E}[TR_N]}{\sigma_{TR_N}},$$

where $\mathrm{E}[TR_N]$ is the expected total return for the value index and σ_{TR_N} is the standard deviation of the value index's total returns. Although Sharpe (1966) originally subtracted the risk-free rate from the expected return, in this paper the Sharpe ratio will be calculated as stated above.

The transaction costs will not be accounted for in the optimization procedure. When the optimized strategies are back-tested on historical SPDR S&P 500 data we assume a one-way transaction cost of 0,30 %, which equals the transaction cost using Deutsche Bank as a broker (Deutsche Bank AG, 2017). We assume, as Neely (2003), that the transaction costs are the same for the whole trading period. (Deutsche Bank AG, 2017).

4.4. Pseudo Codes

In this section we present pseudo codes, linked to programming codes in MATLAB. The pseudo codes show the process behind the moving average crossover without a band and the RSI rules, in addition to establishment and evaluation of trading strategies based on those rules. For the moving average crossover rule, the first 199 days in each simulated price series are not traded on, that is to assure that the short and long moving averages are compared on the equal basis. For the RSI rule, there are no trades for the first 34 days in each simulated price series.

4.4.1. Pseudo Code Moving Average Crossover Rule

1. The long and short moving averages
LOAD 'simulated prices'
long moving average = calculate simple moving average [simulated prices, **length of moving average (SMA_L)**]
short moving average = calculate simple moving average [simulated prices, **length of moving average (SMA_S)**]
2. Trading signals
IF $short\ moving\ average_t$ > $long\ moving\ average_t$ AND
$short\ moving\ average_{t-1}$ < $long\ moving\ average_{t-1}$
TRUE: $signal_t$ = 1
IF $short\ moving\ average_t$ < $long\ moving\ average_t$
$short\ moving\ average_{t-1}$ > $long\ moving\ average_{t-1}$
TRUE: $signal_t$ = -1
3. Buy and sell days
FOR trading days 200 to 2000
FOR all 5000 simulated price paths
IF $signal_t$ = 1 AND $signal_{t+1}$ = 0
TRUE: $signal_{t+1}$ = 1
IF $signal_t$ = -1 AND $signal_{t+1}$ = 0
TRUE: $signal_{t+1}$ = -1
DELETE last signal since it is not used for trading
4. Trading strategy returns
LOAD 'simulated index returns'
FOR trading days 200 to 2000
FOR all 5000 simulated price paths
IF $signal_t$ = 1
TRUE: $trading\ returns_t$ = buy day returns for a given trading strategy
IF $signal_t$ = -1
TRUE: $trading\ returns_t$ = sell day returns for a given trading strategy
IF $signal_t$ = 0
TRUE: $trading\ returns_t$ = simulated index returns
5. Evaluation of trading strategy
The initial investment is 100 for all 5000 simulations
value index = cumulative product [initial investment; $e^{trading\ returns_t}$]
total returns = final return for each simulation
mean final return = mean [total returns]
standard deviation = Std. Dev. [total returns]

Sharpe ratio = $\frac{mean\ final\ return}{standard\ deviation}$

SAVE Program 'The moving average crossover rule:
Leverage Money/ Long Money/ Long Short/ Money Short strategy
Random Walk/ AR (1)/ EGARCH model'

4.4.2. Pseudo Code RSI Rule

1. Daily gain and loss
LOAD 'simulated prices'
FOR days 1 to 2000
FOR all 5000 simulated price paths
IF $simulated\ prices_t$ > $simulated\ prices_{t-1}$
TRUE: $Gain_t$ = $simulated\ prices_t$ – $simulated\ prices_{t-1}$
IF $simulated\ prices_t$ < $simulated\ prices_{t-1}$
TRUE: $Loss_t$ = $simulated\ prices_{t-1}$ – $simulated\ prices_t$
2. Computing the RSI
$\overline{Gain_t}$ = **modified moving average** of gains for a given look-back period (**MMA**)
$\overline{Loss_t}$ = **modified moving average** of losses for a given look-back period (**MMA**)
FOR trading days 35 to 2000
FOR all 5000 simulated price paths
RSI_t = 100 – (100 / (1 + ($\overline{Gain_t}$ / $\overline{Loss_t}$)))
3. Trading signals
FOR trading days 35 to 2000
FOR all 5000 simulated price paths
IF RSI_t < **lower band (*lb*)** AND RSI_{t-1} >= **lower band (*lb*)**
TRUE: $Signal_t$ = 1
IF RSI_t > **upper band (*ub*)** AND RSI_{t-1} <= **upper band (*ub*)**
TRUE: $Signal_t$ = -1
4. Buy and sell days
FOR trading days 35 to 1999
FOR all 5000 simulated price paths
IF $Signal_t$ = 1 AND $Signal_{t+1}$ = 0
TRUE: $Signal_{t+1}$ = 1
IF $Signal_t$ = -1 AND $Signal_{t+1}$ = 0
TRUE: $Signal_{t+1}$ = -1
5. Trading strategy returns
LOAD 'simulated index returns'
FOR trading days 35 to 2000
FOR all 5000 simulated price paths
IF $Signal_t$ = 1
TRUE: $trading\ returns_t$ = buy day returns for a given trading strategy
IF $Signal_t$ = -1
TRUE: $trading\ returns_t$ = sell day returns for a given trading strategy
IF $Signal_t$ = 0
TRUE: $trading\ returns_t$ = simulated index returns
6. Evaluation of trading strategy
The initial investment is 100 for all 5000 simulations
value index = cumulative product [initial investment; $e^{trading\ returns_t}$]
total returns = final return for each simulation
mean final return = mean [total returns]
standard deviation = Std. Dev. [total returns]
Sharpe ratio = $\frac{mean\ final\ return}{standard\ deviation}$

SAVE Program 'The RSI rule:
Leverage Money/ Long Money/ Long Short/ Money Short strategy
Random Walk/ AR (1)/ EGARCH model'

4.5. Brute-force Optimization

The aim of the optimization algorithm is to find the parameters for a given trading rule that maximizes the mean final risk-adjusted return across 5000 simulations, dependent on the trading strategy. Therefore, we test all possible combinations of parameters within some pre-specified restrictions for both rules. By performing a brute force optimization, we obtain the Sharpe ratio for each combination of parameters for a given trading rule. This is done because we, before executing the optimization algorithm, have no prior knowledge of which combinations are likely to provide best solutions.

For the moving average crossover rule, the final risk-adjusted return is a function of number of days in the long moving average ($\mathrm{SMA_L}$) and the number of days in the short moving average ($\mathrm{SMA_S}$) as shown in Section 4.4.1. The optimization problem for the moving average crossover rule is as follows:

$$\max \frac{\overline{TR}_N(\mathrm{SMA_L}, \mathrm{SMA_S})}{\sigma_{TR_N}(\mathrm{SMA_L}, \mathrm{SMA_S})}$$

subject to the constraints: $41 \leq \mathrm{SMA_L} \leq 200$

$$1 \leq \mathrm{SMA_S} \leq SMA_L - 40$$

For the trading strategies based on the moving average crossover rule we have imposed the following constraints. The number of days in the longer of the two moving averages ranges from 41 to 200 in steps of one. For each length of the long moving average a series of short moving averages are computed with lengths ranging from 1 day to 40 days less than the length of the corresponding long moving average.

For the Relative Strength Index, the final risk-adjusted return is a function of the number of days the modified moving average is calculated over (MMA) and the values of lower (lb) and upper (ub) bands as shown in Section 4.4.1. The optimization problem for the RSI rule is expressed as follows:

$$\max \frac{\overline{TR}_N(\mathrm{MMA}, \mathrm{lb}, \mathrm{ub})}{\sigma_{TR_N}(\mathrm{MMA}, \mathrm{lb}, \mathrm{ub})}$$

subject to the constraints: $5 \leq MMA \leq 35$

$20 \leq lb \leq 40$ for each value of MMA

$40 \leq ub \leq 60$ for each combination of MMA and lb

For the RSI, the number of days in a look-back period, for which the modified moving average is calculated for, ranges from 5 to 35. For each length of the modified moving average the value of the lower band varies from 20 to 40, while the value of the upper band is between 60 and 80.

The optimization problems above are non-linear, because the Sharpe ratio is not a linear function of decision variables ($SMA_L, SMA_S, MMA, lb, ub$). The optimization problems are also non-differentiable, since the decision variables consist of only integer values and are therefore not continuous variables. Gradient-based optimization methods can therefore not be applied. Figure 5 illustrates the problem of finding the local optimum among Sharpe ratios in for instance the Leverage Money strategy, based on the moving average crossover rule and simulations from the Random Walk model. The number of days in the long moving average is kept constant at 200, while the length of the short moving average varies from 1 to 160 days. Figure 5 shows that the Sharpe ratio is non-monotonic function of mentioned decision variables. An optimization based on a genetic algorithm might therefore find a local rather than the global maximum.

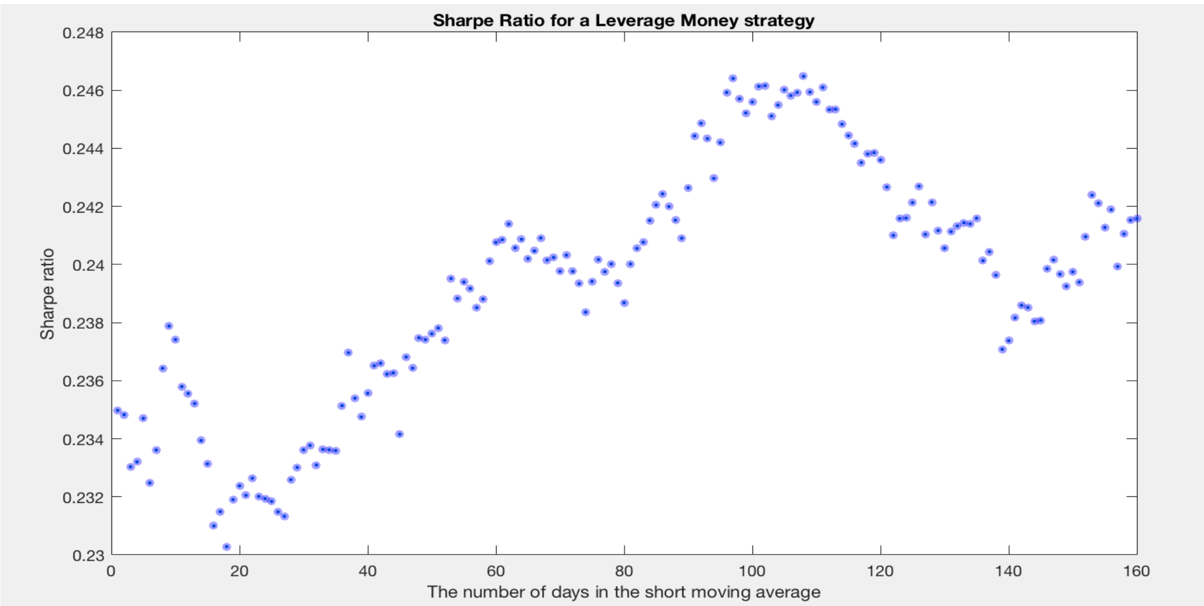

Figure 5. Sharpe Ratios for the Leverage Money strategy with 200 days in the long moving average and various lengths of the short moving average

4.5.1. Brute-force Optimization Algorithm for the Moving Average Crossover Rule

For the moving average crossover rule, the optimization algorithm provides the results for a total of 12880 different combinations. The optimization algorithm used in MATLAB can be expressed in terms of a pseudo code as follows:

```
counter = 0
FOR length of the long moving average = 41 to 200 days with a step of 1
FOR length of the short moving average = 1 to 40 days less than the long moving average with a step of 1
        counter = counter + 1
        LOAD Program 'The moving average crossover rule:
```

```
        Leverage Money/ Long Money/ Long Short/ Money Short strategy
        Random Walk/ AR (1)/ EGARCH model'
For a given combination of short and long moving average:
        mean final return (counter)
        standard deviation (counter)
        Sharpe ratio (counter)
```

4.5.2. Brute-force Optimization Algorithm for the RSI Rule

For the RSI rule, 13671 different combinations are obtained from the optimization process. A pseudo code linked to the optimization algorithm is expressed below:

```
counter = 0
FOR number of days in a look-back period = 5 to 35 days with a step of 1
FOR value of the lower band = for 20 to 40
FOR value of the upper band = for 60 to 80
        counter = counter + 1
        LOAD Program 'The RSI rule:
        Leverage Money/ Long Money/ Long Short/ Money Short strategy
        Random Walk/ AR (1)/ EGARCH model'
For a given combination of look-back period, lower and upper band:
        mean final return (counter)
        standard deviation (counter)
        Sharpe ratio (counter)
```

5. Results

In this section, we present the optimized solutions for the moving average crossover and the RSI rules, based on the 4 different trading strategies: «Leverage Money», «Long Money», «Long Short» and «Money Short». We also present the results obtained from the back-testing the optimized strategies on historical SPDR S&P 500 data, considering transaction costs and applying statistical tests. The result section is divided in terms of trading rules, where we first introduce the optimized strategies and the back-testing for the moving average crossover rule, and afterwards for the RSI. For both rule, the optimized strategies are presented for each of the simulation models.

5.1. The Moving Average Crossover Rule

5.1.1. Optimized Trading Strategies

Optimal Combi-nation (MA_S, MA_L)	Total Return	Annual Return	Excess Return (over B&H)	Annual Excess Return	Std. Dev. (Total Return)	Annual Std. Dev.	Sharpe ratio	Annual Sharpe ratio
Optimized strategies for the Random Walk model:								
Lev. Money (3,43)	30,90%	3,81%	17,52%	2,26%	117,37%	43,72%	0,2633	0,0871
Lev. Money b. (2,43)	27,65%	3,44%	14,27%	1,87%	107,35%	39,98%	0,2576	0,0862
Long Money (1,41)	10,98%	1,46%	-2,4%	-0,34%	42,84%	15,96%	0,2562	0,0912
Long Money b. (3,47)	12,01%	1,59%	-1,37%	-0,19%	47,68%	17,76%	0,2519	0,0893
Long Short (65,107)	14,94%	1,95%	1,56%	0,21%	61,68%	22,97%	0,2422	0,0849
Long Short b. (3,47)	15,67%	2,04%	2,29%	0,32%	63,81%	23,77%	0,2456	0,0859
Money Short (1,41)	12,28%	1,62%	-1,10%	-0,15%	43,90%	16,35%	0,2797	0,0991
Money Short b. (1,49)	13,53%	1,78%	0,15%	0,021%	47,36%	17,64%	0,2857	0,1007
Buy-and-Hold (B&H)	13,38%	1,76%	-	-	63,08%	23,50%	0,2121	0,0748
Optimized strategies for the AR (1) model:								
Lev. Money (91,131)	60,55%	6,79%	23,74%	2,99%	146,99%	54,75%	0,4119	0,1240
Lev. Money b. (159,200)	54,93%	6,26%	18,13%	2,34%	135,05%	50,30%	0,4068	0,1245
Long Money (160,200)	25,17%	3,16%	-11,64%	-1,70%	51,85%	19,31%	0,4854	0,1638
Long Money b. (140,183)	29,74%	3,68%	-7,07%	-1,01%	59,56%	22,18%	0,4992	0,1658
Long Short (160,200)	17,16%	2,22%	-19,65%	-2,99%	61,46%	22,89%	0,2792	0,0970
Long Short b. (160,200)	25,62%	3,21%	-11,19%	-1,63%	66,05%	24,60%	0,3878	0,1307

Money Short (160,200)	3,15%	0,43%	-33,65%	-5,53%	37,97%	14,14%	0,0831	0,0305
Money Short b. (154,194)	16,42%	2,13%	-20,38%	-3,11%	47,87%	17,83%	0,3431	0,1196
Buy-and-Hold (B&H)	36,81%	4,44%	-	-	69,50%	25,89%	0,5296	0,1717
Optimized strategies for the EGARCH model:								
Lev. Money (2,42)	174,68%	15,05%	62,68%	6,98%	157,51%	58,67%	1,1089	0,2565
Lev. Money b. (4,44)	166,33%	14,56%	54,34%	6,21%	142,05%	52,91%	1,1709	0,2751
Long Money (2,69)	70,85%	7,71%	-41,14%	-7,09%	49,71%	18,52%	1,4253	0,4166
Long Money b. (2,45)	78,14%	8,34%	-33,86%	-5,57%	55,25%	20,58%	1,4143	0,4053
Long Short (159,200)	63,55%	7,06%	-48,44%	-8,78%	82,26%	30,64%	0,7725	0,2305
Long Short b. (160,200)	89,36%	9,26%	-22,64%	-3,50%	87,70%	32,66%	1,0189	0,2835
Money Short (109,200)	10,38%	1,38%	-101,61%	-	54,79%	20,41%	0,1895	0,0676
Money Short b. (160,200)	36,52%	4,41%	-75,48%	-17,72%	49,89%	18,58%	0,7319	0,2375
Buy-and-Hold (B&H)	111,99%	10,99%	-	-	82,61%	30,77%	1,3557	0,3571

Table 8. The performance of optimized strategies based on simulated time series from three different models. The optimal combination of lengths for the short and long moving averages are shown in parenthesis in Column 1. The total return in Column 2 stands for the final mean return across simulated price series, without accounting for transaction costs. Since we operate with 1802 trading days, and assume 250 trading days a year, the trading period spans over 7,208 years. The annual return in Column 3 is therefore computed as follows: $(1 + Total\ Return)^{\frac{1}{7,208}} - 1$. *The excess return in Column 4 is the difference between the total return from Column 2 and the total buy-and-hold return for the specific simulation model. The annual excess return is presented in Column 5, by using the same formula as for the annual return. The standard deviation in Column 6 is calculated on basis of total returns across 5000 simulations, and annualized by multiplying the standard deviation with* $7{,}208^{-0,5}$, *presented in Column 7. The Sharpe ratio and the annual Sharpe ratio are shown in Columns 8 and 9.*

In Table 8, we summarize the performance, in terms of both raw and risk-adjusted returns for each optimized trading strategy. Table 8 is divided into three parts, providing the results for the Random Walk, the AR (1) and the EGARCH models. All optimized trading strategies from the Random Walk model obtain higher risk-adjusted returns than a buy-and-hold strategy for the same set of simulations. Although three of the strategies generate negative excess return, their standard deviation is lower than the standard deviation for the buy-and-hold strategy, which is always in the market. Some of the trading strategies are out of the market on buy or sell days, earning the risk-free return, which will lower their standard deviation. In contrast to the results obtained from the Random Walk model, none of the optimized trading strategies based on the AR (1) model obtain higher risk-adjusted returns than a buy-and-hold strategy on the same set of simulations. Although the Leverage Money strategies generate positive excess return, they also contain more risk than the buy-and-hold strategy, and therefore have less risk-adjusted returns. For the optimized strategies obtained from the EGARCH model, the Leverage Money strategies are the only strategies generating positive excess return, but they do not obtain a higher Sharpe ratio than the buy-and-hold strategy for the same set of simulations. The Long Money strategies are the only strategies with a higher Sharpe ratio than the buy-and-hold for time series simulated by the EGARCH model.

The optimal lengths of the moving averages are all small for trading strategies obtained from simulations based on the Random Walk model, except for one strategy. For the AR (1) model the optimal lengths for the short and long moving averages are both long. Several of the strategies choose the longest combination (160, 200) tested. The lengths of the moving averages vary from small to large for the simulations based on the EGARCH model.

5.1.2. Predictive Power on Historical SPDR S&P 500 Data

Optimal Combi-nation (MA_S, MA_L)	Number of Buy and Sell Days	Mean Return Buy Days	Std. Dev. (Buy Days)	Mean Return Sell Days	Std. Dev. (Sell Days)	Buy>0	Sell>0	Buy-Sell
Back-testing of the optimized strategies for the Random Walk model:								
Lev. Money (3,43)	2595 buy 1621 sell	0,0029% (-0,3208)	0,8361%	0,0231% (0,2705)	1,6525%	0,5129	0,5219	-0,0202% (-0,4569)
Lev. Money b. (2,43)	2039 buy 1199 sell	0,0066% (-0,1658)	0,8094%	0,0248% (0,2473)	1,8312%	0,5110	0,5196	-0,0182% (-0,3255)
Long Money (1,41)	2604 buy 1614 sell	-0,0007% (-0,4634)	0,8412%	0,0288% (0,3974)	1,6507%	0,5131	0,5217	-0,0295% (-0,6665)
Long Money b. (3,47)	2065 buy 1207 sell	0,0114% (0,0222)	0,8167%	0,0148% (0,070)	1,8287%	0,5119	0,5145	-0,0033% (-0,06)
Long Short (65,107)	2614 buy 1612 sell	0,0188% (0,3167)	0,8474%	-0,0011% (-0,2659)	1,6479%	0,5249	0,5037	0,0199% (0,4499)
Money Short (1,41)	2604 buy 1614 sell	-0,0007% (-0,4634)	0,8412%	0,0288% (0,3974)	1,6507%	0,5131	0,5217	-0,0295% (-0,6665)
Money Short b. (1,49)	2116 buy 1232 sell	0,0037% (-0,2798)	0,8134%	0,0261% (0,2795)	1,8017%	0,5123	0,5170	-0,0072% (-0,4140)
Back-testing of the optimized strategies for the AR (1) model:								
Lev. Money (91,131)	2624 buy 1582 sell	0,0222% (0,4441)	0,8825%	-0,0060% (-0,3755)	1,6245%	0,5225	0,5070	0,0282% (0,6356)
Lev. Money b. (159,200)	1472 buy 803 sell	0,0127% (0,0593)	0,9053%	-0,0316% (-0,6405)	1,7999%	0,5353	0,4956	0,0442% (0,6523)
Long Money (160,200)	2716 buy 1481 sell	0,0226% (0,4620)	0,8845%	-0,0065% (-0,3698)	1,6592%	0,5269	0,4990	0,0291% (0,6272)
Long Money b. (140,183)	1561 buy 847 sell	0,0152% (0,1488)	0,8672%	-0,0240% (-0,5439)	1,7816%	0,5272	0,4888	0,0043% (0,6014)
Long Short b. (160,200)	1439 buy 776 sell	0,0193% (0,2763)	0,9058%	-0,0321% (-0,6326)	1,8190%	0,5379	0,4961	0,0514% (0,7387)
Money Short b. (154,194)	1459 buy 802 sell	0,0122% (0,0427)	0,9059%	-0,0269% (-0,5784)	1,7690%	0,5326	0,4938	0,0390% (0,5838)
Back-testing of the optimized strategies for the EGARCH model:								
Lev. Money (2,42)	2608 buy 1609 sell	0,0068% (-0,1629)	0,8350%	0,0169% (0,1337)	1,6579%	0,5153	0,5183	-0,0101% (-0,2277)
Lev. Money b. (4,44)	2016 buy 1189 sell	0,0112% (0,0138)	0,8209%	0,0245% (0,2440)	1,8174%	0,5104	0,5231	-0,0133% (-0,2382)

Long Money (2,69)	2641 buy 1517 sell	0,0124% (0,0636)	0,8132%	0,0111% (0,0055)	1,6923%	0,5146	0,5221	0,0013% (0,0281)
Long Money b. (2,45)	2059 buy 1212 sell	0,0067% (-0,1604)	0,8090%	0,0333% (0,4047)	1,8176%	0,5109	0,5190	-0,0266% (-0,4820)
Long Short (159,200)	2723 buy 1474 sell	0,0208% (0,3932)	0,8839%	-0,0034% (-0,3029)	1,6627%	0,5259	0,5007	0,0242% (0,5209)
Money Short (109,200)	2681 buy 1516 sell	0,0230% (0,4813)	0,8714%	-0,0066% (-0,3760)	1,6578%	0,5256	0,5020	0,0296% (0,6467)
	Daily Return	Daily Std. Dev.						
Buy-and-Hold (B&H)	0,0109%	1,2191%						

Table 9. Standard test results for the predictive power of the optimized moving average combinations for the different trading strategies back-tested on historical SPDR S&P 500 data. The trading period ranges from 2000 to 2016 and consists of 4236 days. Strategies with the same combination of short and long moving averages are not presented repeatedly, because they give the same results. The number of buy and sell days during the sample are presented in Column 2. Column 3 shows the mean daily return for the SPDR S&P 500 on buy days, while Column 5 shows the mean daily index return on sell days. The mean returns do not necessary equal the returns from the trading strategies, but rather the index return conditional on signals. The values in parenthesis, in Columns 3 and 5, are t-values for a two-tailed T-test. Columns 4 and 6 provide the standard deviation for the index returns on days, where a buy (sell) signal is given. Columns 7 and 8 present the fraction of positive index returns on buy (sell) days. Last column provides the difference in mean daily index return on buy and sell days, where the value in parenthesis is the t-value for a two-tailed T-test testing if the difference is significantly different from zero. T-values in the table are calculated as shown in appendix 4.1.

Table 9 shows the predictive power of each optimal combination of moving averages on historical SPDR S&P 500 data. According to Brock et. al. (1992) technical trading rules have predictive power if the mean daily index return on buy (sell) days differ from the mean daily index return for the whole sample. The mean daily index return should also be positive (negative) for days' conditional on buy (sell) signal. The reason is that an investor for most of the trading strategies, besides the Money Short in our case, will take a long position in the index on buy days, and a short position or stay out of the market on sell days. If a technical trading rule has predictive power, the mean daily index return on buy days will differ from the mean daily index return on sell days, obtaining a positive difference. Predictive power is also due to the fraction of positive index returns higher for buy than sell days. If a technical trading rule does not provide predictive power, the fraction of positive index returns will be the same for both buy and sell days. Brock et. al. (1992) found that several popular combinations of moving averages had significant predictive power on the Dow Jones index. The similar results were obtained by Bessembinder and Chan (1995) concluding with predictive power of moving average crossover rule in emerging Asian markets. Metghalchi et. al. (2012) concluded with predictive power for the same rule on several European ETF stock indices.

After back-testing the optimal combinations of the moving averages on historical SPDR S&P 500 data, our results conclude that the moving average crossover rule does not have predictive power on the index. None of the combinations provide signals that give significantly different mean daily index return on buy (sell) days from the mean daily index return for the whole trading period. In addition, mean daily index return on buy days is not significantly different from the mean daily index return on sell days for all optimized combinations. Although not providing significant results, the optimal combinations obtained from the AR (1) model has the best predictive power on historical data by obtaining positive (negative) mean daily index return on buy (sell) days and providing the lowest fraction of positive index returns on sell days applicable to all combinations. The negative mean daily index return on sell days are especially noteworthy. As stated by Brock et. al. (1992), the predictive power on sell days will either reflect changes resulting from an equilibrium model or market inefficiency. However, as pointed out by Brock et. al. (1992), it is difficult to imagine that an equilibrium model will provide negative returns for a long time period.

The optimal combinations obtained from the Random Walk model have the least predictive power, since only one combination provides negative mean daily index return on sell days. In addition, two combinations give negative mean daily index return on buy days, and in none of the cases a fraction of positive index returns on sell days is lower than 0,50. The results obtained from the EGARCH model are mixed, 4 out of 8 optimal combinations provide negative mean daily index return on sell days. For the same combinations, the fraction of positive index returns is 0,50 or lower on sell days.

Combinations of moving averages with a 1 % band obtain as expected fewer buy and sell days, since days that neither are buy nor sell may occur during the trading period after the first signal has occurred. Brock et. al. (1992) found that the introduction of a band increased the difference between mean daily index return on buy and sell days when using the same combinations of moving averages for the Dow Jones index. In our paper, it is more difficult to compare results with and without a band, since combinations with a band differ from the combinations without a band. Only the combination of 160 days for the short moving average and 200 days for the long moving average is chosen both with and without a band in the brute-force optimization process. The spread between mean daily index return on buy and sell days is greater by including a band. In 6 out of 12 cases the introduction of a band provides more positive difference between the mean daily index return on buy and sell days for strategies with a band.

Our results support findings in earlier studies as Brock et. al. (1992), Bessembinder and Chan (1995) and Metghalchi et. al. (2012) when it comes to the variance of index returns on buy and sell days. Mentioned studies reported greater standard deviation for index returns on sell days than for index returns on buy days for the indices studied. Our paper provides similar results, the standard deviation for index returns is higher on sell days than on buy days for all optimized combinations.

5.1.3. Back-tested Results for Optimized Trading Strategies

Optimal Combi-nation (MA_S, MA_L)	Total Return	Annual Return	Excess Return (over B&H)	Annual Excess Return	Mean Daily Return	Std. Dev. (Daily Returns)	Annual Std. Dev.	Annual Sharpe ratio
Back-testing of the optimized strategies for the Random Walk model:								
Lev. Money (3,43)	12,50%	0,70%	-45,97%	-3,57%	0,0028%	1,3138%	20,7735%	0,0336
Lev. Money b. (2,43)	29,74%	1,55%	-28,73%	-1,98%	0,0061%	1,2180%	19,2576%	0,0804
Long Money (1,41)	6,90%	0,394%	-51,56%	-4,19%	0,0016%	0,6694%	10,5836%	0,0373
Long Money b. (3,47)	39,85%	1,99%	-18,62%	-1,21%	0,0079%	0,7306%	11,5533%	0,1730
Long Short (65,107)	64,33%	2,98%	5,87%	0,34%	0,0117%	1,2191%	19,2762%	0,1543
Long Short b. (3,47)	10,90%	0,61%	-47,56%	-3,74%	0,0024%	1,2192%	19,2770%	0,0318
Money Short (1,41)	-28,57%	-1,97%	-87,03%	-11,36%	-0,0079%	1,0254%	16,2124%	-0,1213
Money Short b. (1,49)	-15,43%	-0,98%	-73,89%	-7,62%	-0,0040%	1,0751%	16,9996%	-0,0579
Back-testing of the optimized strategies for the AR (1) model:								
Lev. Money (91,131)	197,75%	6,65%	139,29%	5,28%	0,0258%	1,3958%	22,070%	0,3014
Lev. Money b. (159,200)	138,81%	5,27%	80,35%	3,54%	0,0206%	1,3140%	20,7755%	0,2537
Long Money (160,200)	86,36%	3,74%	27,90%	1,46%	0,0147%	0,7238%	11,4440%	0,3270
Long Money b. (140,183)	101,55%	4,22%	43,09%	2,14%	0,0166%	0,9230%	14,5977%	0,2894
Long Short (160,200)	92,22%	3,93%	33,76%	1,73%	0,0154%	1,2191%	19,2755%	0,2040
Long Short b. (160,200)	160,92%	5,82%	102,45%	4,25%	0,0226%	1,219%	19,2737%	0,3021
Money Short (160,200)	17,51%	0,96%	-40,95%	-3,06%	0,0038%	0,9922%	15,6877%	0,0610
Money Short b. (154,194)	117,84%	4,70%	59,38%	2,79%	0,0184%	1,0970%	17,3454%	0,2711
Back-testing of the optimized strategies for the EGARCH model:								
Lev. Money (2,42)	37,90%	1,91%	-20,56%	-1,35%	0,0076%	1,3155%	20,7995%	0,0921
Lev. Money b. (4,44)	43,12%	2,14%	-15,34%	-0,98%	0,0085%	1,2335%	19,5038%	0,1096
Long Money (2,69)	43,15%	2,14%	-15,32%	-0,98%	0,0085%	0,6789%	10,7345%	0,1993
Long Money b. (2,45)	11,64%	0,65%	-46,82%	-3,66%	0,0026%	0,7357%	11,6317%	0,0561
Long Short (159,200)	75,32%	3,37%	16,85%	0,92%	0,0132%	1,2191%	19,2759%	0,1748

Money Short (109,200)	17,79%	0,97%	-40,68%	-3,03%	0,0039%	1,0027%	15,8548%	0,0612
Money Short b. (160,200)	110,78%	4,50%	52,32%	2,51%	0,0176%	1,0988%	17,3734%	0,2590
Buy-and-Hold (B&H)	58,46%	2,75%	-	-	0,0109%	1,2191%	19,2763%	0,1429

Table 10. The performance of optimized strategies, back-tested on historical SPDR S&P 500 data during the trading period of 4236 days in years 2000-2016. The Long Short band strategy has the same combination of moving averages for the simulations based on the AR (1) and EGARCH model, and is therefore only reported once in the table.

In Table 10, we present the performance of optimized trading strategies, obtained from simulation models, on historical SPDR S&P 500 data. To evaluate the risk for a trading strategy we find the standard deviation for daily trading returns. For optimized trading strategies, provided by the Random Walk model, only two strategies obtain a higher annual Sharpe ratio than the buy-and-hold strategy. These strategies are the Long Money band and the Long Short. Though the Long Money band strategy generates a negative annual excess return (-1,21 %) compared to the buy-and-hold, it is the best strategy in terms of annual risk-adjusted returns. The Long Short is the only strategy that generates positive annual excess return (0,34 %). Since the Long Short and the buy-and-hold strategies are always in the market, their volatility will be almost the same. Strategies that are out of the market on either buy or sell signals will have a lower volatility than the buy-and-hold strategy. As expected, the Long Money and the Money Short are the least risky strategies. The riskiest strategies are the Leverage Money strategies. Though they spend time out of the market on sell days, a double long position on buy days makes these strategies the riskiest. The introduction of a band makes the Leverage Money strategy less risky, obtaining the standard deviation close to the standard deviation for the buy-and-hold. For the strategies obtained from the Random Walk model the inclusion of a band provides a higher risk-adjusted return compared to the strategies without a band, besides the Long Short strategy.

Seven out of eight optimized strategies provided by the AR (1) model obtain positive excess return and a higher Sharpe ratio than the buy-and-hold. The best strategy is the Long Money, providing an annual Sharpe ratio of 0,3270, which is also the highest Sharpe ratio obtained on historical data by any trading strategy in the sample. The Leverage Money strategy provides the highest annual excess return (5,28 %) across all the strategies provided by simulation models, but it is also the riskiest, obtaining the highest annual standard deviation (22,07 %). As Neely (2003) pointed out, although a technical trading rule can achieve greater returns than the buy-and-hold strategy, the Efficient Market Hypothesis is not violated if the strategy has less risk adjusted returns than the buy-and-hold strategy. The Leverage Money and the Long Money strategies become worse when a band is included. Four out of eight strategies, provided by the EGARCH model, have a better Sharpe ratio than the buy-and-hold. The best strategy is the Long Short band, providing a Sharpe ratio of 0,3021, which is in fact the second-best across all the strategies in the sample. The combination of moving averages for this strategy is the same as for the AR (1) model. Only the Long Money strategy becomes worse when including a band in the strategies obtained from simulations based on the EGARCH model. Only the Money Short strategy is better applying a band for all three cases across simulation models.

When comparing the strategies' back-tested performance with the performance obtained in the optimization procedure, the differences are noticeable. For the simulations based on the Random Walk model, all the strategies beat the Sharpe ratio for the buy-and-hold in the optimization, while in the back-testing only two of the strategies do the same. The results are opposite for the AR (1) model, none of the optimized strategies provided a better Sharpe ratio than the buy-and-hold in the optimization, while seven out of eight strategies do it in the back-testing. For the EGARCH model the performance of the optimal strategies in the optimization and the back-testing procedures are more coordinated. Two strategies beat the risk-adjusted returns for the buy-and-hold in the optimization, while four strategies do the same when back-tested. Therefore, simulated returns based on the EGARCH model seem to approach actual SPDR S&P 500 returns better than the case for the other two models. Nevertheless, it shows the difficulty of modeling stock returns. Another issue is that the Sharpe ratios for different combinations of moving averages are similar in value, which makes it hard to distinguish between them and get stable results in terms of an optimal solution. The optimal combination for the lengths of the moving averages might change if optimized on a new set of simulated data.

Optimal Combination (MA_S, MA_L)	Number of Buy Signals	Number of Sell Signals	Total Return	Excess Return (over B&H)	Annual Excess Return	Annual Sharpe
Back-testing of the optimized strategies for the Random Walk model: (after transaction costs)						
Lev. Money (3,43)	107	106	-40,44%	-98,91%	-23,39%	-0,1450
Lev. Money b. (2,43)	139	98	-68,93%	-127,39%	-	-0,3477
Long Money (1,41)	175	174	-62,37%	-120,83%	-	-0,5296
Long Money b. (3,47)	103	78	-13,43%	-71,89%	-7,22%	-0,0743
Long Short (65,107)	18	18	33,21%	-25,25%	-1,70%	0,0885
Long Short b. (3,47)	103	78	-57,01%	-115,47%	-	-0,2532
Money Short (1,41)	175	174	-74,85%	-133,32%	-	-0,4826
Money Short b. (1,49)	178	130	-93,35%	-151,81%	-	-0,8744
Back-testing of the optimized strategies for the AR (1) model: (after transaction costs)						
Lev. Money (91,131)	15	15	172,94%	114,48%	4,61%	0,2766
Lev. Money b. (159,200)	15	7	109,28%	50,81%	2,45%	0,2144
Long Money (160,200)	12	12	73,94%	15,48%	0,85%	0,2902
Long Money b. (140,183)	15	8	92,11%	33,64%	1,73%	0,2692
Long Short (160,200)	12	12	67,44%	8,98%	0,51%	0,1602
Long Short b. (160,200)	15	6	142,79%	84,33%	3,68%	0,2789
Money Short (160,200)	12	12	9,67%	-48,79%	-3,87%	0,0348
Money Short b. (154,194)	15	8	80,87%	22,41%	1,20%	0,2052
Back-testing of the optimized strategies for the EGARCH model: (after transaction costs)						
Lev. Money (2,42)	126	125	-34,86%	-93,32%	-14,76%	-0,1201
Lev. Money b. (4,44)	94	72	-47,59%	-106,05%	-	-0,1926
Long Money (2,69)	94	93	-18,07%	-76,53%	-8,20%	-0,1089
Long Money b. (2,45)	131	98	-38,60%	-97,06%	-18,80%	-0,2470
Long Short (159,200)	10	10	56,43%	-2,03%	-0,12%	0,1388
Money Short (109,200)	8	8	12,60%	-45,86%	-3,56%	0,0443
Money Short b. (160,200)	15	6	79,26%	20,80%	1,12%	0,2017
Buy-and-Hold (B&H)	-	-	58,46%	-	-	0,1429

Table 11. The net cost performance of optimized strategies, back-tested on historical SPDR S&P 500 data during the trading period of 4236 days in years 2000-2016. The Long Short band strategy has the same combination of moving averages for the simulations based on the AR (1) and EGARCH model, and is therefore only reported once in the table.

In Table 11, we present the back-tested performance of same optimized strategies, but now adjusted for transaction costs. As pointed out by Bessembinder and Chan (1995), technical trading rules providing higher returns than the buy-and-hold strategy is not necessary consistent with an inefficient stock market. Frequent trading with high transaction costs might justify surplus return provided by technical analysis. Jensen (1978) emphasized that the Efficient Market Hypothesis is violated if risk-adjusted surplus returns exceed the transaction costs of trading.

For optimized strategies, obtained from the Random Walk model, only the Long Short strategy provides a positive Sharpe ratio, but it is not higher than for the buy-and-hold. The annual excess return turns from positive to negative (-1,70 %) when transaction costs are accounted for. The Long Short was in fact the only strategy that earned positive annual pre-cost excess return. The Long Money band strategy, which was the best strategy before adjusting for transaction costs now provides a negative net cost Sharpe ratio (-0,0743). The performances of optimized strategies, obtained from the AR (1) model are like before transaction costs. All strategies, besides the Money Short strategy, provide positive annual excess returns and a higher Sharpe ratio than the buy-and-hold strategy. The Long Money strategy is the best strategy obtained from the AR (1) model, and the best across all the strategies in the sample, in terms of risk-adjusted returns providing a Sharpe ratio of 0,2902. The strategy was also the best in the sample before accounting for transaction costs. For optimal strategies, based on the EGARCH model, only the Long Short band and the Money Short band provide positive annual after-cost excess returns. These strategies also obtain a better Sharpe ratio than the buy-and-hold. The Long Short band, obtained from the EGARCH model, is in fact the second-best strategy for the whole sample as it was before accounting for transaction costs, providing a Sharpe ratio of 0,2789.

In total, 8 strategies across the whole sample obtain positive annual excess net returns and Sharpe ratios higher than the buy-and-hold. Common to these strategies is that they all have few trading signals and thus lower transaction costs. This is not surprising, since optimal combinations of moving averages consist of long moving averages. As pointed by Murphy (1999), the use of very sensitive (short) moving averages generates more trades and thus higher transaction costs.

Optimal Combination (MA_S, MA_L)	**Mean daily excess return**	**Std. Dev. (daily excess return)**
Back-testing of the optimized strategies for the Random Walk model:		
Lev. Money (3,43)	-0,0231% (-1,2270)	1,2254%
Lev. Money b. (2,43)	-0,0385% (-2,1829) *	1,1468%
Long Money (1,41)	-0,0156% (-0,9941)	1,0195%
Long Money b. (3,47)	-0,0143% (-0,9393)	0,9888%
Long Short (65,107)	-0,0041% (-0,1312)	2,0339%
Long Short b. (3,47)	-0,0308% (-1,0185)	1,9678%
Money Short (1,41)	-0,0435% (-1,3148)	2,1512%
Money Short b. (1,49)	-0,0749% (-2,3740) *	2,0518%
Back-testing of the optimized strategies for the AR (1) model:		
Lev. Money (91,131)	0,0128% (0,6891)	1,2123%
Lev. Money b. (159,200)	0,0066% (0,4501)	0,9495%
Long Money (160,200)	0,0022% (0,1459)	0,9811%
Long Money b. (140,183)	0,0045% (0,3714)	0,7966%
Long Short (160,200)	0,0013% (0,0432)	1,9621%
Long Short b. (160,200)	0,0101% (0,4211)	1,5569%
Money Short (160,200)	-0,0087% (-0,2710)	2,0861%
Money Short b. (154,194)	0,0031% (0,1247)	1,6302%
Back-testing of the optimized strategies for the EGARCH model:		
Lev. Money (2,42)	-0,0210% (-1,1117)	1,2287%
Lev. Money b. (4,44)	-0,0261% (-1,5023)	1,1316%
Long Money (2,69)	-0,0156% (-0,9941)	1,0195%

Long Money b. (2,45)	-0,0224% (-1,4753)	0,9874%
Long Short (159,200)	-0,0003% (-0,0101)	1,9615%
Money Short (109,200)	-0,0081% (-0,2498)	2,1011%
Money Short b. (160,200)	0,0270% (0,1152)	1,6440%

*Table 12. Standard test results for the excess return obtained from historical SPDR S&P 500 data during the back-testing period of 4236 days in years 2000-2016. Test statistics (t-values) are shown in parenthesis, * denotes significance at the 5 % level for a two-tailed test. The Long Short Band strategy has the same combination of moving averages in the simulations based on the AR (1) and the EGARCH models, therefore it is only reported once in Table 12. T-values are calculated as shown in appendix 4.2.*

For all the optimized strategies in the sample we test if the excess returns are significantly different from zero. In Table 12, we present the results for statistical tests of significance for mean daily excess return. We find that only two optimized trading strategies, both obtained from the Random Walk model and providing negative excess returns, have the mean daily excess return different from zero. For all other optimized strategies, we cannot reject the null hypothesis that the mean daily excess return is zero. Positive excess returns, provided by 8 optimized strategies, are therefore not significant.

5.2. The RSI Rule

5.2.1. Optimized Trading Strategies

The results for the RSI rule can be found in Section 5 of the Appendix. All the optimized trading strategies based on the Random Walk model obtain a higher risk-adjusted return than a buy-and-hold strategy on the same simulated time series, although only the Long Short strategy provides positive excess returns. All optimized strategies obtained from the Random Walk model, except the Long Short, have substantially less standard deviation than the buy-and-hold strategy.

For the simulations based on the AR (1) model, all optimized strategies have a greater Sharpe ratio than the buy-and-hold, except the Long Short strategy, which has the same Sharpe ratio. All the strategies provide lower total returns than the buy-and-hold strategy for the simulated time series. Optimization performed on simulations, obtained from the AR (1) model, provides the same optimal combinations as the Random Walk model for the Leverage Money and the Money Short strategies.

Only the Long Money strategy has a higher Sharpe ratio than the buy-and-hold strategy for the simulations based on the EGARCH model. All the optimized trading strategies have negative excess returns and a risk close to the risk for the buy-and-hold. The performance of optimized trading strategies on simulated data can be found in Table 16 in Appendix 5.

5.2.2. Back-tested Results for Optimized Trading Strategies

When back-testing the optimized trading strategies on historical data, we find that not all strategies generate buy or sell signals during the trading period. The strategies that provide trading signals only generate either buy or sell signals. Strategies that generate neither a buy nor sell signal will always hold a long position, and are therefore equal to the buy-and-hold strategy. None of the strategies that provide buy or sell signals have significant different mean daily index return on buy and sell days from the mean daily index return for the sample period. The results are shown in Table 17 in Appendix 5.

Back-testing of the optimized trading strategies, based on the Random Walk model, shows that no strategy obtains positive excess returns. Although, three optimized strategies obtain a higher Sharpe ratio than the buy-and-hold, while the Long Short strategy has the same Sharpe ratio as the buy-and-hold. The Money Short is the best strategy in terms of risk-adjusted returns across all simulated models, but as shown in Table 17 it only provides buy signals. Since an investor earns the risk-free return on buy days, the high Sharpe ratio is caused by a low standard deviation rather than large returns. The results for the optimized strategies, based on the AR (1) model, are similar as for the Random Walk model. Three optimized strategies provide a Sharpe ratio higher than for the buy-and-hold, while the Long Short obtains the same Sharpe ratio, but the excess return for all strategies is negative. For the optimized strategies, based on the EGARCH model, all obtain the same Sharpe ratio as for the buy-and-hold, with zero annual excess return. The optimized strategies for all models are not adjusted for transaction costs. This is because the strategies maintain either a buy or sell signal for the whole trading period or no signal at all, and transaction costs are therefor negligible. The performance of optimized strategies on historical data are presented in Table 18 in Appendix 5. Negative excess returns, provided by all back-tested optimized trading strategies, are not significantly different from zero, as shown in Table 19 in Appendix 5.

6. Summary and Further Ideas

In this article, we optimized and back-tested four different strategies, based on two technical trading rules: the moving average crossover and the RSI. For the moving average crossover rule the lengths of the short and long moving averages were found to maximize the Sharpe ratio. For the RSI rule the number of days in the look-back period, and values for the lower and upper bands were used as optimization parameters. The trading strategies were optimized on the simulated series of SPDR S&P 500 ETF, using Random Walk, AR (1) and EGARCH models, and then back-tested on historical data, where the trading period was extended and transaction costs were accounted for.

As Brock et. al. (1992), Bessembinder and Chan (1995) and Metghalchi et. al. (2012) we found the volatility of index returns for days' conditional on a sell signal to be higher than for days' conditional on a buy signal when back-testing the optimized strategies based on the moving average crossover rule. We also found that the mean daily index return on buy (sell) days was not significantly different from the mean daily index return for the whole trading period. Furthermore, the difference between means on buy and sell days was not significantly different from zero. Though optimized strategies, obtained from the AR (1) model, provided positive mean daily index return on buy days and negative mean daily index return on sell days, pointing in the direction of predictive power on historical data, none of the means were significantly different from the mean daily index return for the whole trading period.

After adjusting for transaction costs, 8 optimized strategies for the moving average rule provided higher annual return and a greater Sharpe ratio than the buy-and-hold strategy when back-tested. Common for these strategies is that the optimal combinations consisted of two long moving averages, generating fewer signals and lower transaction costs. Statistical tests on excess returns, provided by these strategies, showed that the surplus returns were not significantly different from zero. The same apply for negative excess returns, provided by remaining trading strategies. Therefore, we conclude with weak form efficiency for the S&P 500 index in the years 2000-2016. Our findings are consistent with Allen and Karjalainen (1999), who concluded that technical trading rules did not outperform the buy-and-hold strategy for the S&P 500 index in the period 1928-1995.

Optimized strategies for the RSI rule provided no trading signal at all or only either buy or sell signals when back-tested. None of the back-tested optimized strategies generated positive excess returns, though all the strategies obtained either the same or higher Sharpe ratio as the buy-and-hold strategy. For instance, one trading strategy had abnormally high Sharpe ratio, because only buy signals were generated during the whole period of trading, providing a risk-free return. Negative excess returns, provided by the strategies, were not significantly different from zero.

An issue encountered in this paper is that a trading strategy might achieve a greater Sharpe ratio than the buy-and-hold position, but this can be caused by taking a risk-free position for long time periods. The excess return for the strategy will be negative at the end of the trading period. Based solely on the risk-adjusted returns, represented by Sharpe ratio, a strategy might outperform the buy-and-hold, but not necessary in terms of the raw returns. Trading strategies could have been optimized by using the excess return over the buy-and-hold strategy as assessment criteria. Although this would have solved the issue of strategies taking risk-free positions for long time periods and outperforming the buy-and-hold, another issue could have arisen: the optimized strategies might be much riskier than the buy-and-hold strategy. Another flaw with using a Sharpe ratio as a measure of risk is that standard deviation can be misleading when the returns are not normally distributed. Negatively skewed return distributions will have a greater probability for high losses than normal distributions, which make returns riskier then implied by their standard deviation. The Sortino ratio and the Value at Risk measure the downside risk for a strategy, and could have been estimated to supplement the Sharpe ratio when optimizing and evaluating strategies.

Transaction costs could have been included when optimizing trading strategies. The optimal parameters might no longer be optimal after adjusting for transaction costs. From our results, we see that the strategies with combinations consisting of two short moving averages create more signals, and therefore have higher transaction costs. By including a cost of trading the optimized parameters would have created fewer trading signals and thus lower transaction costs. Technical trading rules might also have better predictive power in less developed stock markets than for the S&P 500 index. Bessembinder and Chan (1995) found that technical trading rules would not be profitable in developed Asian markets like Hong Kong, Japan and Korea, but could be profitable in emerging markets of Malaysia, Thailand and Taiwan.

In this paper, trading strategies were built on the moving average crossover rule both with and without a 1 % band. The results for the strategies with a band compared to the corresponding strategies without a band were mixed. By including a band half of the strategies have improved. In further research the size of the band could have been optimized instead of using the given size of 1 %. Optimizing the size of a band, in addition to the number of days in the moving averages, requires substantial computing power, which we did not have access to until late in the work of this paper. Furthermore, other technical trading rules could be investigated and optimized. Although our results are consistent with the market efficiency for the S&P 500 index, we have only tested two rules.

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Appendix

1. Random Walk

Random Walk model	
Number of observations	4434
Mean	$9{,}693 \cdot 10^{-5}$
Standard Deviation	0,01228
Residual Sum of squares	0,66822

Table 13. Random Walk model summary

1.1. Tests for Stationarity

A time series is assumed to be weakly stationary if its mean, variance and covariance are all time independent. Mathematically the conditions can be expressed as follows:

$$E[y_t] = E[y_{t-s}] = \mu$$
$$E[(y_t - \mu)^2] = E[(y_{t-s} - \mu)^2] = \sigma_y^2$$
$$E[(y_t - \mu)(y_{t-s} - \mu)] = E\big[(y_{t-j} - \mu)(y_{t-j-s} - \mu)\big] = \gamma_s$$

A breach of the stationary restrictions will cause statistical inference measures to be invalid. An augmented Dickey-Fuller test can be used to test if a time series is stationary or contains a unit root (Dickey and Fuller, 1979). The equation below can be used to test if a time series without a constant and trend is stationary:

$$\Delta y_t = \gamma y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + \varepsilon_t$$

The following Dickey-Fuller regression is used to test if log prices are stationary. Two lags of Δy_t are included to ensure no serial correlation in the error terms.

$$\Delta \ln(P_t) = \gamma \ln(P_{t-1}) + \beta_1 \Delta \ln(P_{t-1}) + \beta_2 \Delta \ln(P_{t-2})$$

Under the null hypothesis γ equals zero and the time series contains a unit root, while under the alternative hypothesis γ is less than zero, which implies stationarity. Formally this can be expressed as:

$$H_0\colon \gamma = 0$$
$$H_1 : \gamma < 0$$

The null hypothesis is not rejected because the t-value for γ is 0,6103 while the critical value is -2,57 with a significance level of 1 %.

Stationarity in first differences has been confirmed by performing the following Dickey-Fuller regression:

$$\Delta\Delta\ln(P_t) = \gamma\Delta\ln(P_{t-1}) + \beta_1\Delta\Delta\ln(P_{t-1}) + \beta_2\Delta\Delta\ln(P_{t-2})$$

This regression gives a t-value of -41,47 for γ. The return series is therefore stationary, since the critical value is -2,57 with a significance level of 1 %.

1.2. Log Normal Distribution

Although the distribution of stock returns often has positive excess kurtosis and are skewed, we will assume stock returns are normally distributed when simulating returns with the Random Walk model. The model is estimated on basis of continuously compounded returns. The same also apply for the other two models used in this paper. The price at a given time will be computed by multiplying the previous price by Euler's number raised to the returns in the current period. Euler's number raised to a normally distributed variable will follow a log-normal distribution. Even though a variable is normally distributed with a mean of zero, the same variable will have a positive drift when Euler's number is raised to the variable. The expected value of a log-distributed variable (X) with zero mean is $E[X] = e^{\frac{1}{2}\cdot\sigma^2}$, where σ^2 is the variance of the normally distributed variable. The buy-and-hold for the simulated returns from the Random Walk model will therefore increase during the period.

2. AR (1)

We assume the error term is normally distributed with a mean of zero, constant variance and uncorrelated with each other:

$$E[\varepsilon_t] = E[\varepsilon_{t-1}] = 0$$

$$Variance(\varepsilon_t) = E[\varepsilon_t^2] = E[\varepsilon_{t-1}^2] = \sigma^2$$

$$Covariance(\varepsilon_t, \varepsilon_{t-s}) = cov(\varepsilon, \varepsilon_{t-j-s}) = 0\ for\ all\ j\ and\ s.$$

AR (1) model	
R^2	0,00435
Adjusted R^2	0,00412
Number of observations	4434
Mean	0,00011
Standard Deviation	0,01224
F (1, 4431) [p-value]	19,35 [0.000]
Residual sum of squares	0,66370

Table 14. AR (1) model summary

Descriptive Statistics Residuals AR (1)								
Mean	Standard Deviation	Skewness	Excess Kurtosis	Minimum	Maximum	Median	Obs.	Jarque Bera
0,00	0,01224	-0,09553	10,396	-0,10473	0,13385	0,00034	4433	19971

Table 15. Descriptive statistics for the residuals from the AR (1) model

Descriptive statistics for the residuals from the AR (1) model, as presented in Table 15, shows that the residuals are negatively skewed and have heavier tails than implied by the normal distribution. A perfectly normally distributed variable will have a skewness and excess kurtosis of zero. A Jarque Bera test can be used to conclude if the skewness and excess kurtosis for a sample exceeds what is implied by a standard normal distribution. The Jarque Bera test statistic is Chi-squared distributed with two degrees of freedom and is computed as follows:

$$JB = \frac{T}{6}(Skew^2 + \frac{EKurtosis^2}{4})$$

The Jarque Bera concludes that the residuals are not normally distributed, since the test statistic exceeds the critical value of 5,99.

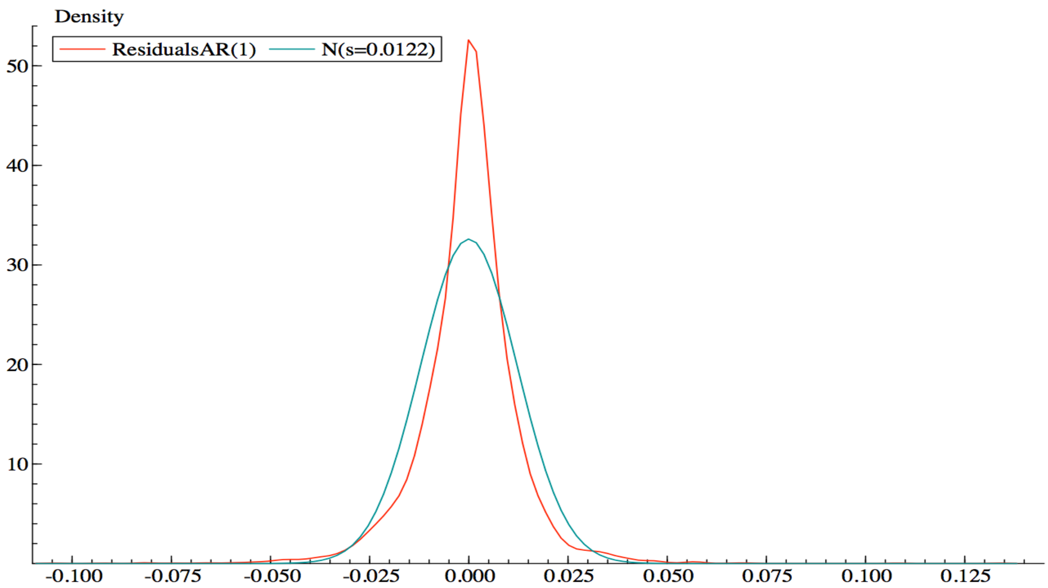

Figure 6. Distribution of the residuals from the AR (1) model

2.1. Tests for Autocorrelation and Heteroscedasticity

Autocorrelation and heteroscedasticity will not cause bias in the estimation of the regression coefficients in the AR (1) model, but might cause the coefficients' standard errors to be over- or underestimated, which make T-tests and F-tests invalid.

Autocorrelation in the residuals can be detected by performing a Ljung-Box test on the residuals. The Ljung-Box test statistic is calculated as follows:

$$LB(s) = T(T+2)\sum_{j=1}^{s}\frac{r_j^2}{T-j} \ ,$$

where T is the number of observations, r_j^2 is the squared autocorrelation and s is the number of autocorrelation coefficients included. Under the null hypothesis, the test statistic is Chi-squared distributed with $s - p$ degrees of freedom where p is the order of the AR model estimated.

Conducting a Ljung-Box test with 20 lags of the residuals from the AR (1) model provides a test statistic of 58,25. The 5 % critical value with 19 degrees of freedom is 30,14. The null hypothesis of no autocorrelation in the residuals is therefore rejected.

A Ljung-Box test on the squared residuals rejects the null hypothesis of homoscedastic error terms. The test statistic is 5136,4, while the 5 % critical value with 19 degrees of freedom is 30,14.

3. EGARCH

3.1. Information Criteria

Information criteria can be used to compare the fit of different ARMA models. The model with the lowest information criteria fits the sample best. The information criteria used in this paper are the Akaike, Schwarz and Hannan-Quinn. The difference between information criteria is the degree of penalization from estimated parameters. By estimating more parameters, the fit of the model will increase, but the information criteria will be penalized. The Schwarz information criteria penalizes the most for extra estimation of parameters, and will therefore prefer more parsimonious models, while the AIC penalizes the least for the number of estimated parameters. The information criteria are estimated as follows:

$$AIC = \frac{-2\ Log\ Likelihood + 2 \cdot k}{T}$$

$$SBC = \frac{-2\ Log\ Likelihood + k \cdot \log(T)}{T}$$

$$HQ = \frac{-2\ Log\ Likelihood + 2 \cdot k \cdot \log(\log(T))}{T},$$

where k is the number of parameters, including the dependent variable, and T is the sample size.

3.2. Autocorrelation for Squared Residuals ARMA (1,1)

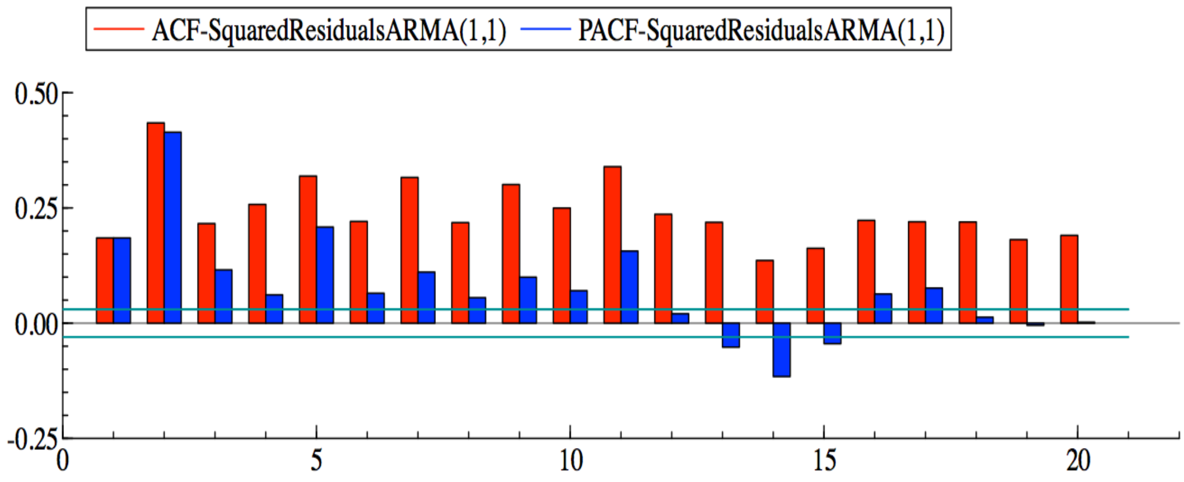

Figure 7. ACF and PACF for the squared residuals from the ARMA (1,1) model

The autocorrelation function for the squared residuals from the ARMA (1,1) model, as shown in Figure 7, indicates Arch effects in the residuals. Performing a Ljung-Box test with 20 lags gives a test statistic of 5630,6, while the critical value with 18 degrees of freedom is 28,86. We can therefore reject the null hypothesis of no serial correlation amongst the squared residuals.

3.3. Log Likelihood Test

A likelihood ratio test can be used to determine if the model with Gaussian or t-distributed residuals fits our data best. The model with t-innovations has one more parameter than the model with Gaussian innovations, and is therefore an unrestricted version of the model with Gaussian innovations. Model with more parameters will have equal or greater Log Likelihood compared to model with less parameters. Under the null hypothesis, the Log Likelihood from two models are equal, while the alternative hypothesis states that the model with the most parameters has the highest Log Likelihood. The test statistic is Chi-squared distributed with a degree of freedom equal to the difference in number of estimated parameters in the models compared. The test statistic (D) is calculated as follows:

$$D = 2 \cdot (\log L_{UR} - \log L_R)$$

Performing a likelihood ratio test gives a test statistic of 162,71, while the 5 % critical value is 3,84, meaning that the null hypothesis is rejected.

3.4. ACP and PACF for Residuals and Squared Residuals from the ARMA-EGARCH Model

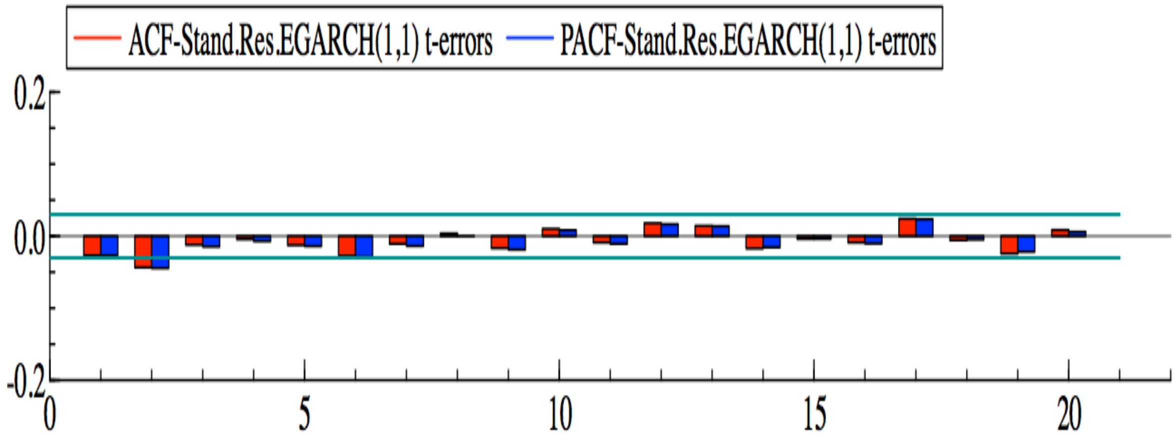

Figure 8. ACF and PACF for standardized residuals from the ARMA (1,1)-EGARCH (1,1) model

Plotting the autocorrelation and partial autocorrelation functions for the standardized residuals, as shown in Figure 8, provide few significant spikes. Performing a Ljung-Box test with 20 lags gives a test statistic of 28,6843 with a corresponding p-value of 0,0524. The null hypothesis about no serial correlation is therefore barely not rejected.

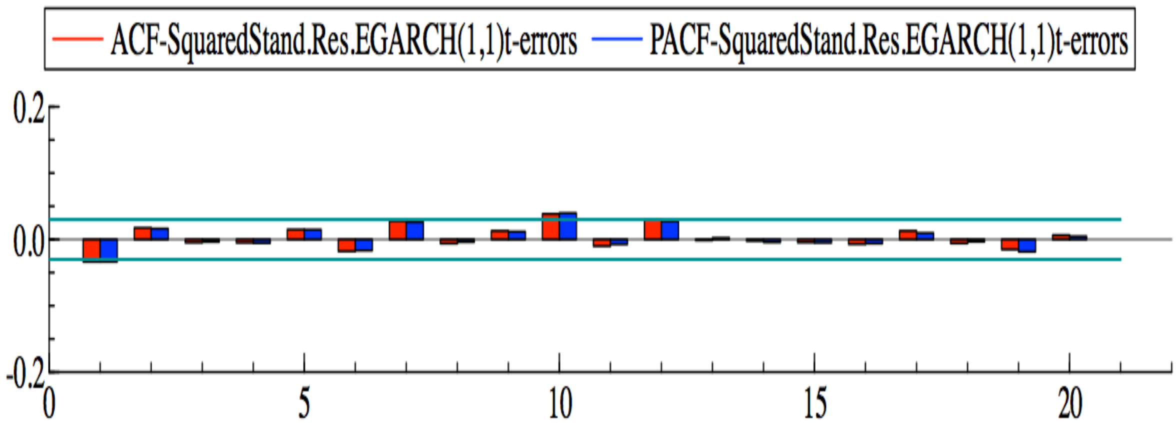

Figure 9. ACF and PACF for squared standardized residuals from the ARMA (1,1)-EGARCH (1,1) model

Plotting the autocorrelation and partial autocorrelation functions for the squared standardized residuals, as presented in Figure 9, shows if there are any remaining ARCH effects that the model does not capture. Performing a Ljung-Box test with 20 lags gives a test statistic 27,0023 and a corresponding p-value of 0,0790. We can therefore conclude that an EGARCH (1,1) is sufficient to capture the Arch effects in the residuals.

4. Statistical Tests for Back-testing of Trading Strategies

4.1. T-tests for Predictive Power

When testing if the mean daily index return on buy days differs from the mean daily index return for the whole sample, the following test statistic is used:

$$t = \frac{\mu_B - \mu}{\sqrt{\frac{Var(R_B)}{N_B - 1} + \frac{Var(R)}{N - 1}}},$$

where μ_B is the mean daily index return on buy days, μ is the mean daily index return for the whole sample, $Var(R_B)$ is the variance of index returns on buy days, $Var(R)$ is the variance of index returns on all days, N_B is the number of buy days and N is the total number of days in the back-testing trading period. When testing if the mean daily index return on sell days differs from the mean daily index return for the whole sample, the mean daily index return on sell days and the variance of index returns on sell days are substituted in the equation above.

To test if the mean daily index return on buy days differs from the mean daily index return on sell days the following test statistic is used:

$$t = \frac{\mu_B - \mu_S}{\sqrt{\frac{Var(R_B)}{N_B - 1} + \frac{Var(R_S)}{N_S - 1}}},$$

where μ_S is the mean daily index return on sell days, $Var(R_S)$ is the variance of index returns on sell days, N_S is the number of sell days. The other variables are defined as in the first test statistic formula.

4.2. T-tests for Excess Return

To test if the mean daily excess return is significantly different from zero the following t-statistic is applied:

$$t = \frac{\bar{X}_{ex}}{\sqrt{Var(\bar{X}_{ex})/(N - 1)}},$$

where $\bar{X}_{ex}$ is the mean daily excess return, while N is the total number of returns obtained in the trading period. The null hypothesis is that the mean daily excess return is zero, while the alternative hypothesis is that the mean daily excess return is different from zero. The critical value for a two-tailed T-test with a significance level of 5 % is 1,96.

5. Results for the RSI Rule

Optimal Combination (MMA, lb, ub)	Total Return	Annual Return	Excess Return (over B&H)	Annual Excess Return	Std. Dev. (Total Return)	Annual Std. Dev.	Sharpe ratio	Annual Sharpe ratio
Optimized strategies for the Random Walk model:								
Lev. Money (24;20,60)	7,16%	0,88%	-7,47%	-0,98%	11,65%	4,15%	0,6148	0,2126
Long Money (22;20,60)	7,18%	0,88%	-7,46%	-0,98%	10,33%	3,68%	0,6949	0,2403
Long Short (26;20,79)	15,19%	1,81%	0,56%	0,07%	67,13%	23,93%	0,2263	0,0758
Money Short (21;40,80)	10,13%	1,23%	-4,50%	-0,58%	12,91%	4,60%	0,7851	0,2682
Buy-and-Hold (B&H)	14,64%	1,75%	-	-	66,87%	23,84%	0,2189	0,0735
Optimized strategies for the AR (1) model:								
Lev. Money (24;20,60)	8,16%	1%	-32,51%	-4,87%	9,23%	3,29%	0,8837	0,3044
Long Money (19;20,60)	7,73%	0,95%	-32,94%	-4,95%	8,27%	2,95%	0,9351	0,3226
Long Short (35;40,80)	40,50%	4,42%	-0,17%	-0,02%	75,30%	26,84%	0,5379	0,1645
Money Short (21;40,80)	10,67%	1,30%	-30%	-4,43%	13,02%	4,64%	0,8194	0,2793
Buy-and-Hold (B&H)	40,67%	4,43%	-	-	75,58%	26,94%	0,5381	0,1645
Optimized strategies for the EGARCH model:								
Lev. Money (35;20,80)	125,15%	10,87%	-1,56%	-0,20%	95,31%	33,98%	1,3130	0,3198
Long Money (31;40,80)	124,87%	10,85%	-1,84%	-0,24%	91,27%	32,54%	1,3682	0,3334
Long Short (33;40,80)	124,06%	10,80%	-2,65%	-0,34%	91,69%	32,69%	1,3531	0,3303
Money Short (35;20,80)	118,82%	10,46%	-7,89%	-1,04%	94,44%	33,67%	1,2582	0,3108
Buy-and-Hold (B&H)	126,71%	10,96%	-	-	92,62%	33,02%	1,3681	0,3320

Table 16. The performance of the optimized strategies, based on simulated time series from the different models, for the trading period of 1967 days. The optimal combination of lengths for a look-back period, lower and upper bands are shown in parenthesis in Column 1.

Optimal Combination (MMA, lb, ub)	Number of Buy (Sell) Days	Mean Return Buy Days	Std. Dev. Buy Days	Mean Return Sell Days	Std. Dev. Sell Days	Buy>0	Sell>0	Buy - Sell
Back-testing of the optimized strategies for the Random Walk model:								
Lev. Money (24;20,60)	0 buy 4377 sell	-	-	0,0091% (-0,0889)	1,2106%	-	0,5154	-
Long Money (22;20,60)	0 buy 4379 sell	-	-	0,0096% (-0,0694)	1,2105%	-	0,5156	-
Long Short (26;20,79)	0 buy 0 sell	-	-	-	-	-	-	-
Money Short (21;40,80)	4397 buy 0 sell	0,0112% (-0,0077)	1,2227%	-	-	0,5156	-	-
Back-testing of the optimized strategies for the AR (1) model:								
Long Money (19;20,60)	0 buy 4341 sell	-	-	0,0075% (-0,1498)	1,2053%	-	0,5160	-
Long Short (35;40,80)	0 buy 3546 sell	-	-	0,0240% (0,4704)	1,1567%	-	0,5257	-
Back-testing of the optimized strategies for the EGARCH model:								
Lev. Money (35;20,80)	0 buy 0 sell	-	-	-	-	-	-	-
Long Money (31;40,80)	4234 buy 0 sell	0,0111% (-0,0136)	1,2192%	-	-	0,4960	-	-
Long Short (33;40,80)	4234 buy 0 sell	0,0111% (-0,0136)	1,2192%	-	-	0,5165	-	-
	Daily Return	Daily Std. Dev.						
Buy-and-Hold (B&H)	0,011%	1,2224%						

Table 17. Standard test results for the predictive power of the optimized solutions for the RSI rule on historical data. The trading period ranges from 2000 to 2016 and consists of 4401 days. Test statistics (t-values) are shown in parenthesis. Strategies with the same combination of the look-back period, lower and upper bands are not presented repeatedly, because of the same results.

Optimal Combination (MMA, lb, ub)	Total Return	Annual Return	Excess Return (over B&H)	Annual Excess Return	Mean Daily Return	Std. Dev. (Daily Return)	Annual Std. Dev.	Annual Sharpe
Back-testing of the optimized strategies for the Random Walk model:								
Lev. Money (24;20,60)	34,66%	1,71%	-30,53%	-2,05%	0,0068%	0,1177%	1,8607%	0,9163
Long Money (22;20,60)	31,70%	1,58%	-33,49%	-2,29%	0,0063%	0,1148%	1,8146%	0,8689
Long Short (26;20,79)	65,20%	2,89%	0%	0%	0,0114%	1,2224%	19,3274%	0,1497
Money Short (21;40,80)	22,63%	1,17%	-42,56%	-3,10%	0,0046%	0,0181%	0,2855%	4,0829
Back-testing of the optimized strategies for the AR (1) model:								
Long Money (19;20,60)	44,48%	2,11%	-20,71%	-1,31%	0,0084%	0,1743%	2,7558%	0,7665
Long Short (35;40,80)	65,20%	2,89%	0%	0%	0,0114%	1,2224%	19,3274%	0,1497
Back-testing of the optimized strategies for the EGARCH model:								
Lev. Money (35;20,80)	65,20%	2,89%	0%	0%	0,0114%	1,2224%	19,3274%	0,1497
Long Money (31;40,80)	65,20%	2,89%	0%	0%	0,0114%	1,2224%	19,3274%	0,1497
Long Short (33;40,80)	65,20%	2,89%	0%	0%	0,0114%	1,2224%	19,3274%	0,1497
Money Short (35;20,80)	65,20%	2,89%	0%	0%	0,0114%	1,2224%	19,3274%	0,1497
Buy-and-Hold (B&H)	65,20%	2,89%	-	-	0,0114%	1,2224%	19,3274%	0,1497

Table 18. The performance of the optimized strategies, approached on historical SPDR S&P 500 data during the back-testing period of 4401 days. Same combinations are provided for the Leverage Money and the Money Short strategies obtained from the Random Walk and the AR (1) models, therefore the results for these strategies are not presented twice.

Optimal Combination (MMA, lb, ub)	Mean daily excess return	Std. Dev. (daily excess return)
Back-testing of the optimized strategies for the Random Walk model:		
Lev. Money (24;20,60)	-0,0046% (-0,2484)	1,2167%
Long Money (22;20,60)	-0,0051% (-0,2753)	1,2170%
Money Short (21;40,80)	-0,0068% (-0,3605)	1,2222%
Back-testing of the optimized strategies for the AR (1) model:		
Long Money (19;20,60)	-0,0030% (-0,1637)	1,2099%

Table 19. Standard test results for the excess return obtained from the historical SPDR S&P 500 data during the back-testing period of 4401 days. Test statistics (t-values) are shown in parenthesis.

6. MATLAB Codes

In this section we show the most important codes used in this paper. The optimized Long Short strategy, based on simulated returns from the Random Walk model will be used as an example.

6.1. Moving Average Crossover Rule: Long Short Strategy Random Walk

```
load('SimReturnsRW') % load 5000 saved simulated series of daily returns
with a time horizon of 2000 days

% Obtaining simulated prices
SimRet=exp(SimReturnsRW); % simulated returns are raised to Euler's number
A=100*ones(1,5000); % create matrix consisting of hundreds with dimension
1X5000
Y=[A;SimRet]; % matrix consisting of hundreds in the first row and  returns
raised to Euler's number
SimPricesRW=cumprod(Y); % obtaining prices by multiplying simulated returns
raised to Euler's number with the previous price level assuming 100 as the
initial value

% Moving average calculation
MA_long=tsmovavg(SimPricesRW,'s',m,1); % m = length of the long moving
average
MA_short=tsmovavg(SimPricesRW,'s',n,1); % n = length of the short moving
average

% Trading signals
S=size(MA_long); % define the size of the "Signal" matrix
Signal=zeros(S);
for i=1:S(1)
    for j=1:S(2)
        if MA_short(i,j)>MA_long(i,j) && MA_short(i-1,j)<MA_long(i-1,j)
            Signal(i,j)=1; % if the short moving average cuts the long
moving average from below, a buy signal(=1) is given
        end
        if MA_short(i,j)<MA_long(i,j) && MA_short(i-1,j)>MA_long(i-1,j)
            Signal(i,j)=-1; % if the short moving average cuts the long
moving average from above, a sell signal(=-1) is given
        end
    end
end

for i=200:2000 % the first 199 days are not considered as trading days
    for j=1:5000
        if Signal(i,j)==1 && Signal(i+1,j)==0
            Signal(i+1,j)=1; % if a buy signal occurs, days following after
the signal are defined as buy days until appearance of a sell signal
        end
        if Signal(i,j)==-1 && Signal(i+1,j)==0
            Signal(i+1,j)=-1; % if a sell signal occurs, days following
after the signal are defined as sell days until appearance of a buy signal
        end
    end
end

Signal(2001,:)=[]; % cut the last signal, since it is not used for trading
purpose

% Trading returns based on the Long Short strategy
T=size(Signal); % define the size of the "Trading returns" matrix
```

```
tradingreturns=zeros(T);
for i=1:T(1)
    for j=1:T(2)
        if Signal(i,j)==1
            tradingreturns(i,j)=SimReturnsRW(i,j); % holding long position
in the index on buy days
        end
        if Signal(i,j)==-1
            tradingreturns(i,j)=-1*SimReturnsRW(i,j); % holding short
position in the index on sell days
        end
        if Signal(i,j)==0
            tradingreturns(i,j)=SimReturnsRW(i,j); % holding long position
in the index for days before the first signal has occured
        end
    end
end

tradingreturns=tradingreturns(200:2000,:); % cut trading returns for the
first 199 days

% Evaluation Long Short strategy
value=cumprod([100*ones(1,5000);exp(tradingreturns)]); % create Value index
with initial investment of 100 and calculate the value for each day by
multilplying the previous day's value with trading returns raised to Euler's
number
average=mean(value,2); % compute mean value of the index across 5000
simulations for each day
lastaverage=average(1802,1); % find the mean value of the index across 5000
simulations at the end of trading period

totalret=(value(1802,:)-100)/100; % compute the final return for each
simulation
stdev=std(totalret); % compute the standard deviation of total returns for
5000 simulations

finalret=(lastaverage-100)/100; % compute the mean total return across 5000
simulations
sharpe=finalret/stdev; % compute mean Sharpe ratio across 5000 simulations
```

6.2. RSI Rule: Long Short Strategy Random Walk

```
% Computing daily gain and loss
load('SimReturnsRW')% load simulated returns
load('SimPricesRW') % load prices obtained from simulated returns

RWUps=zeros(2000,5000);
RWDowns=zeros(2000,5000);
for i=1:2000
    for j=1:5000
        if SimPricesRW(i+1,j)>SimPricesRW(i,j)
            RWUps(i,j)=SimPricesRW(i+1,j)-SimPricesRW(i,j); % calculate
daily gain
        end
        if SimPricesRW(i+1,j)<SimPricesRW(i,j)
            RWDowns(i,j)=SimPricesRW(i,j)-SimPricesRW(i+1,j); % calculate
daily loss
        end
    end
end
```

```
% Modified moving average
averageUPS=tsmovavg(RWUps,'m',n,1); % n = length of the modified moving
average
averageDOWNS=tsmovavg(RWDowns,'m',n,1);

% RSI
RSI=zeros(2000,5000);
for i=1:2000
    for j=1:5000
        RSI(i,j)=100-(100/(1+(averageUPS(i,j)/averageDOWNS(i,j))));
% calculate the RSI value for each day
    end
end

% Signal
Signal=zeros(2000,5000);
for i=1:2000
    for j=1:5000
        if RSI(i,j)<b && RSI(i-1,j)>=b % b = value of the lower band
            Signal(i,j)=1; % 1 = buy signal
        end
        if RSI(i,j)>s && RSI(i-1,j)<=s % s = value of the upper band
            Signal(i,j)=-1; % -1 = sell signal
        end
    end
end

for i=35:1999 % the first 34 days are not traded on, and the last signal is
not used
    for j=1:5000
        if Signal(i,j)==1 && Signal(i+1,j)==0
            Signal(i+1,j)=1;
        end
        if Signal(i,j)==-1 && Signal(i+1,j)==0
            Signal(i+1,j)=-1;
        end
    end
end

% Returns Long Short strategy
T=size(Signal);
tradingreturns=zeros(T);
for i=1:T(1)
    for j=1:T(2)
        if Signal(i,j)==1
            tradingreturns(i,j)=SimReturnsRW(i,j);
        end
        if Signal(i,j)==-1
            tradingreturns(i,j)=-1*SimReturnsRW(i,j);
        end
        if Signal(i,j)==0
            tradingreturns(i,j)=SimReturnsRW(i,j);
        end
    end
end

tradingreturns=tradingreturns(35:2000,:); % cut trading returns for the
first 34 days

% Evaluation Long Short strategy
value=cumprod([100*ones(1,5000);exp(tradingreturns)]);
average=mean(value,2);
lastaverage=average(1967,1);
```

```
totalret=(value(1967,:)-100)/100;
stdev=std(totalret);

finalret=(lastaverage-100)/100;
sharpe=finalret/stdev;
```

6.3. Moving Average Crossover Rule: Brute-force Optimization Algorithm

```
counter=0;

for m=40:1:200      % constraints for the long moving average
     for n=1:1:m-40 % constraints for the short moving average


            counter=counter+1;
            LongShortRW;
            LASTAVERAGE(counter)=lastaverage;
            FINALRET(counter)=finalret;
            STDEV(counter)=stdev;
            SHARPE(counter)=sharpe;
            M(counter)=m;
            N(counter)=n;
            RESULTS=[N;M;LASTAVERAGE;FINALRET;STDEV;SHARPE];
       end
end
```

6.4. RSI Rule: Brute-force Optimization Algorithm

```
counter=0;

for n=5:1:35
       for b=20:1:40
           for s=60:1:80

            counter=counter+1;
            RSILongShortRW;
            LASTAVERAGE(counter)=lastaverage;
            FINALRET(counter)=finalret;
            STDEV(counter)=stdev;
            SHARPE(counter)=sharpe;
            N(counter)=n;
            B(counter)=b;
            S(counter)=s;
            RESULTS=[N;B;S;LASTAVERAGE;FINALRET;STDEV;SHARPE];
       end
       end
end
```

6.5. Moving Average Crossover Rule: Predictive Power on Historical Data

```
load('SPDRPrices'); % load historical data for the SPDR S&P 500 from 2000-
2016
returnsSPDR=xlsread('SPDRReturns'); % load historical returns for the SPDR
S&P 500

MA_long=tsmovavg(SPDRPrices,'s',107,1); % number of days in the long moving
average

MA_short=tsmovavg(SPDRPrices,'s',65,1); % number of days in the short
```

```
moving average

S=size(MA_long);
Signal=zeros(4435,1);

% this for loop shows when buy and sell signals occur
for i=1:4435
    for j=1
        if MA_short(i,j)>MA_long(i,j) && MA_short(i-1,j)<MA_long(i-1,j)
            Signal(i,j)=1;
        end
        if MA_short(i,j)<MA_long(i,j) && MA_short(i-1,j)>MA_long(i-1,j)
            Signal(i,j)=-1;
        end
    end
end

% this for loop shows for how long positions are held
for i=200:4434    % trading starts at day 200
    for j=1
        if Signal(i,j)==1 && Signal(i+1,j)==0
            Signal(i+1,j)=1;
        end
        if Signal(i,j)==-1 && Signal(i+1,j)==0
            Signal(i+1,j)=-1;
        end
    end
end

Signal(4435)=[]; % the last signal is redundant

% this for loop shows the returns from the Long Short strategy
for i=1:4434
    for j=1
        if Signal(i,j)==1
            tradingreturns(i,j)=returnsSPDR(i,j);
        end
        if Signal(i,j)==-1
            tradingreturns(i,j)=-1*returnsSPDR(i,j);
        end
        if Signal(i,j)==0
            tradingreturns(i,j)=returnsSPDR(i,j);
        end
    end
end

% number of buys and sells
Signal2=Signal(200:4434,1);

for i=1:4235
    for j=1
        if Signal2(i,j)==1
            Numberbuy(i,j)=1;
        end
        if Signal2(i,j)==-1
            Numbersell(i,j)=1;
        end
    end
end

Nbuy=sum(Numberbuy)
Nsell=sum(Numbersell)
```

```
Neutraldays=4235-nnz(Signal2)

returnsSPDR2=returnsSPDR(200:4434,1)

% t-tests
% this for loop calculates the index returns on buy and sell days
for i=1:4235
    for j=1
        if Signal2(i,j)==1
            returnsbuydays(i,j)=returnsSPDR2(i,j);
        end
        if Signal2(i,j)==-1
            returnsselldays(i,j)=returnsSPDR2(i,j);
        end
    end
end

totalreturnsbuydays=sum(returnsbuydays);

totalreturnselldays=sum(returnsselldays);
sizebuydays=size(returnsbuydays)

averagereturnbuydays=totalreturnsbuydays/Nbuy;

averagereturnselldays=totalreturnselldays/Nsell;
sizeselldays=size(returnsselldays)

% this for loop finds the squared deviation from the mean for buy and sell
days
for i=1:sizebuydays(1)
    for j=1
        if Signal2(i,j)==1
            Squareddeviationbuydays(i,j)=(returnsbuydays(i,j)-
averagereturnbuydays)^2;
        end
    end
end

for i=1:sizeselldays(1)
    for j=1
         if Signal2(i,j)==-1
            Squareddeviationselldays(i,j)=(returnsselldays(i,j)-
averagereturnselldays)^2;
        end
    end
end

returnsBuyandHold2=returnsSPDR(200:4434,1); % returns from the buy-and-hold
strategy
meanreturnBH=mean(returnsBuyandHold2); % mean daily return buy-and-hold
standarddeviationBuyandHold=std(returnsBuyandHold2); % standard deviation
for the buy-and-hold returns

varianceBuyandHold=standarddeviationBuyandHold^2; % variance for the buy-
and-hold returns

% variance and standard deviation for index returns on buy days
sumsquaresbuydays=sum(Squareddeviationbuydays);
variancebuydays=sumsquaresbuydays/(Nbuy-1);
standarddeviationbuydays=sqrt(variancebuydays);
```

```
% variance and standard deviation for index returns on sell days
sumsquaresselldays=sum(Squareddeviationselldays);
varianceselldays=sumsquaresselldays/(Nsell-1);
standarddeviationselldays=sqrt(varianceselldays);

% comparing mean index return on buy days to mean buy-and-hold day return
meanbuyMinusMeanBuyandHold=averagereturnbuydays-meanreturnBH; % numerator
pooledvarianceBuyandBuyandHold=sqrt((variancebuydays/(Nbuy-
1)+varianceBuyandHold/(4235-1)));                              % denominator
TvaluemeanBuymeanBuyandHold=meanbuyMinusMeanBuyandHold/pooledvarianceBuyand
BuyandHold

% comparing mean index return on sell days to mean buy-and-hold day return
meansellminusmeanBuyandHold=averagereturnselldays-meanreturnBH;
pooledvarianceSelldaysandbuyandHolddays=sqrt((varianceselldays/(Nsell-
1)+varianceBuyandHold/(4235-1)));
TvalueMeanSellmeanbuyandHold=meansellminusmeanBuyandHold/pooledvarianceSell
daysandbuyandHolddays

% comparing mean index return on buy days to mean index return on sell days
meanBuyminusmeanSell=averagereturnbuydays-averagereturnselldays;
pooledvarianceBuyminusSell=sqrt(variancebuydays/(Nbuy-
1)+varianceselldays/(Nsell-1));
TvalueBuyminussell=meanBuyminusmeanSell/pooledvarianceBuyminusSell

% fraction of positive index returns on buy and sell days
for i=1:4235
    j=1;
    if Signal2(i,j)==1 && returnsSPDR2(i,j)>0
        buyDaysGreaterThanZero(i,j)=1;
    end
end

numberBuyhits=sum(buyDaysGreaterThanZero)
fractionBuydaysGreaterThanZero=sum(buyDaysGreaterThanZero)/Nbuy

for i=1:4235
    for j=1;
        if Signal2(i,j)==-1 && returnsSPDR2(i,j)>0
            sellDaysGreaterThanZero(i,j)=1;
        end
    end
end
numbersellhits=sum(sellDaysGreaterThanZero)
fractionSelldaysGreaterThanZero=sum(sellDaysGreaterThanZero)/Nsell
```

6.6. Moving Average Crossover Rule: Net Cost Performance on Historical Data

```
load('SPDRPrices');
returnsSPDR=xlsread('SPDRReturns');

MA_long=tsmovavg(SPDRPrices,'s',200,1); % number of days in the long moving
average

MA_short=tsmovavg(SPDRPrices,'s',159,1); % number of days in the short
moving average

S=size(MA_long);
Signal=zeros(4435,1);
```

```
% this for loop shows when buy and sell signals occur
for i=1:4435
    for j=1
        if MA_short(i,j)>MA_long(i,j) && MA_short(i-1,j)<MA_long(i-1,j)
            Signal(i,j)=1;
        end
        if MA_short(i,j)<MA_long(i,j) && MA_short(i-1,j)>MA_long(i-1,j)
            Signal(i,j)=-1;
        end
    end
end

% shows how many buy and sell signals have been generated
numberbuysell=Signal(200:4434,:);
numberbuysell1=unique(numberbuysell)
numberbuysell2=[numberbuysell1,histc(numberbuysell(:),numberbuysell1)]

% this for loop shows for how long positions are held
for i=200:4434    % trading starts at day 200
    for j=1
        if Signal(i,j)==1 && Signal(i+1,j)==0
            Signal(i+1,j)=1;
        end
        if Signal(i,j)==-1 && Signal(i+1,j)==0
            Signal(i+1,j)=-1;
        end
    end
end

Signal(4435)=[]; % the last signal is redundant
Signal2=Signal(200:4434,1);

transactioncost=zeros(4235,1)
% since going from a long to a short position in the index requires closing
the current and taking the new postion, transactions costs are doubled
for i=1:4234
   for j=1
    if Signal2(i,j)==1 && Signal2(i+1,j)==-1
        transactioncost(i,j)=-0.006;
    end
    if Signal2(i,j)==-1 && Signal2(i+1,j)==1
        transactioncost(i,j)=-0.006;
    end
   end
end

for i=1:4434
    for j=1
        if Signal(i,j)==1
            tradingreturns(i,j)=returnsSPDR(i,j);
        end
        if Signal(i,j)==-1
            tradingreturns(i,j)=-1*returnsSPDR(i,j);
        end
        if Signal(i,j)==0
            tradingreturns(i,j)=returnsSPDR(i,j);
        end
    end
end


% evaluation of trading strategy
```

```
tradingreturns=tradingreturns(200:4434,:);
tradingreturnsaftertransactionscosts=tradingreturns+transactioncost

value=cumprod([100;exp(tradingreturnsaftertransactionscosts)]); % shows how
the investment would have evolved with transactions costs

totalreturnstrading=(value(4236,:)-100)/100; % trading returns after
transaction costs

stdevtradingreturns=std(tradingreturns);

annualizedreturntrading=(1+totalreturnstrading)^(250/4236)-1;

annualizedstandarddeviation=stdevtradingreturns*sqrt(250);

annualizedsharpe=annualizedreturntrading/annualizedstandarddeviation;

% Buy-and-hold
finalreturnBuyandHold=(SPDRPrices(4435,1)-
SPDRPrices(200,1))/SPDRPrices(200,1)

% excess returns
excessreturns=totalreturnstrading-finalreturnBuyandHold % shows the excess
return from the Long Short strategy compared to the buy-and-hold strategy

annualizedexcessreturns=(1+excessreturns)^(250/4236)-1 % calculates the
annualized excess return

% t-tests for difference in returns from trading and buy-and-hold returns

returnsSPDR2=returnsSPDR(200:4434,1)

dailyExcessreturn=tradingreturnsaftertransactionscosts-returnsSPDR2 % daily
excess returns for the Long Short strategy

standarddeviationDailyExcessreturn=std(dailyExcessreturn)

varianceDailyexcessreturn=std(dailyExcessreturn)^2

averageDailyExcessreturn=mean(dailyExcessreturn)

denominator=((varianceDailyexcessreturn)/(4235-1))^0.5

Tvalue=averageDailyExcessreturn/denominator % t-statistic concluding if the
excess return is different from zero
```