An Iterative LMI Approach to Controller Design and Measurement Selection in Self-Optimizing Control

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Abstract— **Self-optimizing control focuses on minimizing loss for processes in the presence of disturbances by holding selected controlled variables at constant set-points. The loss can further be reduced by controlling measurement combinations to constant values. Two methods for finding appropriate measurement combinations are the Null-space and the Exact local method. Both approaches offer sets with an infinite number of solutions that give the same loss. Since self-optimizing control is mainly concerned with minimizing the steady-state loss, little attention has been put on the dynamic performance when selecting measurement combinations.**

In this work, an iterative LMI approach is used to find a measurement combination and PI controllers for the Null-space or the Exact local method. The measurement combination and the controllers are designed such that, the dynamic response is improved when the process is facing disturbances.

I. INTRODUCTION

Control systems for chemical plants are large-scale systems, often consisting of hundreds or even thousands of control loops. In theory, a single, centralized controller would be optimal for such a large scale system, as it would allow all information to be available for the calculation of all control variables. In practice, however, the system is too large for the design of such an 'ideal' controller, and instead, the control system is typically divided into several layers that address different time scales. The systematic procedure for control structure design, including what to control and how to pair the variables is usually referred to as plantwide control [1], [2]. Without going into any greater detail, it first requires defining and solving a scalar cost function. The resulting solution should give the desired steady-state operation when using the degrees of freedom that are available. Once this has been established, the final step is to implement the optimal policy. This involves deciding on the pairing of the manipulated variables (MVs) and the controlled variables (CVs), together with designing their controllers. For processes with active constraints at the optimal operating point, it is recommended to use some of the MVs to keep the CVs close to these constraints. Therefore, the selection of the remaining unconstrained MVs and CVs becomes a reduced space problem. Determining on how to best use the remaining unconstrained degrees of freedom is not a trivial task but has been quite successfully addressed in the works of [3] and [4], who coined the term self-optimizing control.

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Self-optimizing control focuses on finding controlled variables when kept at constant set-points results in near optimal steady-state economic operation in spite of disturbances. Besides using single measurements as the controlled variables, selecting linear combinations of measurements will further improve the self-optimizing control performance. Two methods that achieve this are the Exact local method [5], [6] and the Null-space method [7]. Both the Null-space and the Exact local method offers sets, consisting of an infinite number of possibilities for choosing the measurement combinations. All solutions give the same loss from a steady-state perspective, but the dynamic response depends on how the measurement combinations are selected. Therefore, it would be preferable to find a combination that also improves the dynamic performance. However, the resulting closed-loop system is not just dependent on the measurement combination, but also on the feedback controllers.

The proportional integral (PI) controller is by far the most commonly used controller in the process industries due to its simplicity and robust performance [8]. With progress in numerical methods, new convex optimization methods have been developed for designing controllers. However, for restricted-order controllers (e.g., PI/PID controller) the optimization problems tend to become non-convex in the controller parameter space. They are usually solved by employing heuristics or intelligent methods [9], [10]. A loop shaping method was proposed in [11], by specifying bounds on the phase and gain margins.

These methods often aim to minimize some common control performance criterion, e.g., the integrated absolute error (IAE). However, in self-optimizing control (SOC) minimizing IAE may not be optimal. Instead, the SOC variables should ideally, when subjected to disturbances, drive the process to the new optimal operating point while minimizing deviations in variables with large economic impact (e.g., the active constraints). Therefore, it might be better to recast it as an optimization problem for finding, e.g., the H_2 optimal static output feedback (SOF) controller. Contrary to full state-feedback or full-order controllers, which can be solved using Linear Matrix Inequalities (LMI), structured static output feedback generally results in Bilinear Matrix Inequalities (BMI) and remains an open problem [12], [13]. They are often solved to a local optimum by iteratively fixing some variables and solving the resulting LMI.

The present work is based on [14], [15], and a method is proposed for simultaneously selecting the measurement combinations and PI controller parameters for the self-optimizing control variable. The results are illustrated on a binary distillation column model, showing improved dynamic performance while maintaining the same steady-state loss based on the self-optimizing control principle.

The paper is organized as follows. Section II quotes the notations used in this paper together with the Elimination Lemma. The concept of self-optimizing control is briefly described in section III, and then the main method is presented in section IV. The proposed method is applied to a distillation column model in section V, and finally, a conclusion is given in section VI.

II. Preliminaries

Let $\mathbb{R}^{n\times m}$ denote the set of $n \times m$ real matrices. For a matrix A, its transpose is denoted A^T, and A^{-1} denotes its inverse. The symbols $He\{A\}$ indicates $A^T + A$ and $A^\perp$ denotes any matrix of maximum rank that satisfies $A^\perp A = 0$. The identity and the null matrix of dimension $n\times m$ are given by $I_{n\times m}$ and $0_{n\times m}$. The notation $A \prec 0$, $A \succ 0$ means the matrix is positive and negative definite respectively. Finally, let's recall the Elimination lemma:

Lemma 2.1: [16] *For $B \in \mathbb{R}^{n\times l}$, $C \in \mathbb{R}^{m\times n}$, and $Q = Q^T \in \mathbb{R}^{n\times n}$, the following conditions are equivalent:*

1) $\exists\ X \in \mathbb{R}^{l\times m}$ *such that* $Q + He\{BXC\} \prec 0$.
2) $B^\perp Q B^{\perp T} \prec 0$ *and* $(C^T)^\perp Q (C^T)^{\perp T} \prec 0$.

III. Self-optimizing Control

Self-optimizing control is when an acceptable loss is achieved with constant set-points without the need to reoptimize for disturbances [3]. More precisely, the aim is to select controlled variables rather than determining optimal set-points. By using the available degrees of freedom (u), the goal is to minimize the constrained cost function (J), in order to find the optimal operating point for the process. Typically, J defines the economic cost of the process and can often be expressed as

$$J = \text{feed cost} + \text{utilities cost} - \text{product value}.$$

For specified disturbances (d), the optimization problem can be formulated as,

$$\min_{x,u} \quad J\Big(x, u, d\Big) \tag{1}$$

subject to:

$$f(x, u, d)) = 0 \tag{2}$$

$$g(x, u, d)) \leq 0 \tag{3}$$

$$y = f_y(x, u, d) \tag{4}$$

where $x \in \mathbb{R}^{nx}$, $u \in \mathbb{R}^{nu}$, and $d \in \mathbb{R}^{nd}$ are the states, inputs, and disturbances respectively. The equality constraints are represented by $f(\cdot)$ and contain the steady-state model equations; the inequality constraints in $g(\cdot)$ defines the limits of the operation, and the available measurements are given by y. The solution to the optimization problem usually results in some of the constraints being active, i.e., $g_i(x, u, d) = 0$. To achieve optimal operation at steady-state, the variables related to the active constraints should be controlled and kept as close as possible to their optimal set-points. Stabilizing the plant and controlling the active constraints, therefore, requires a corresponding number of degrees of freedom. This results in a reduced space optimization problem:

$$\min_{u} J^*\Big(u, d\Big) \tag{5}$$

Here, the model equations and active constraints, are implicitly included in J^*. What remains is to determine which of unconstrained variables (c) should be kept constant by using the remaining degrees of freedom, in order to minimize loss. To quantify the loss resulting from keeping the selected controlled variables at constant values, methods for calculating the worst case and average loss were derived in [5] and [17].

A. Optimal Measurement Combination

Rather than selecting single measurements for the unconstrained optimization problem in (5), a further reduction in loss can be obtained by selecting the control variables as optimal linear measurement combinations $c = Hy$. The matrix $H \in \mathbb{R}^{nu\times ny}$ defines the measurement combinations, and $y \in \mathbb{R}^{ny}$ is a subset of the available measurements. Two methods for computing H are the Null-space [7] and Exact local method [5], [6].

1) The Null-space method: Under the assumption that implementation error (e.g., measurement noise) can be neglected and that $ny \geq nu + nd$ independent measurements are available, then [7] proposed the Null-space method for selecting a measurement combination. This results in a zero local loss by choosing H such that,

$$HF = 0 \tag{6}$$

where F is the sensitivity matrix for the optimal deviations in the measurements (∂y^{opt}) with respect to changes in the disturbances (∂d):

$$F = \frac{\partial y^{opt}}{\partial d} \tag{7}$$

The matrix F can be obtained analytically, but it is often easier to compute numerically, by optimizing the nonlinear steady-state model of the plant for selected disturbances.

2) The Exact Local method: Besides the assumption of having no measurement noise is unrealistic in practice; it also requires that the number of measurements used exceeds the sum of inputs and disturbances, which can become very large. Based on the Exact local method, an explicit solution which gives an optimal trade-off between rejecting disturbances and implementation errors can be found using:

$$H^* = (G^y)^T (YY^T)^{-1} \tag{8}$$

Where $Y = \begin{bmatrix} FW_d & W_n \end{bmatrix}$ with W_d and W_n representing the magnitudes of the disturbances and implementation errors respectively. The optimal solution H^* in (8) was shown in [4] to be non-unique and for any non-singular matrix D,

$$H = DH^* \tag{9}$$

results in the same loss as the solution given by (8). Therefore, both the Null-space and the Exact local method have an infinite number of solutions for H that satisfies (6) or (9) and thus gives the same steady-state operation.

IV. STATIC OUTPUT FEEDBACK CONTROL

In this section, the method for selecting the measurement combination and designing the PI controllers is presented. The method is based on the two-step procedure for static output feedback controller design proposed by [14]. While their method has been used in several applications [18], [19], it has to the authors' knowledge not been used for simultaneous measurement selection and controller design.

A. Process model

Consider a system described by the continuous linear time-invariant state-space model,

$$\dot{x}(t) = A_x x(t) + B_u u(t) \tag{10}$$

$$y(t) = C_{yx} x(t) \tag{11}$$

where $x \in \mathbb{R}^{nx}$, $u \in \mathbb{R}^{nu}$ and $y \in \mathbb{R}^{ny}$ are the states, inputs, and measurements respectively. The aim is to find a measurement combination matrix H and design decentralized PI controllers of the form:

$$u(t) = k_p \ H \ y(t) + k_i \ H \int_0^t y(\tau) \ d\tau \tag{12}$$

The system in (10) and (11) can be augmented to include the integrating states from the decentralized PI controllers:

$$\bar{A}_x = \begin{bmatrix} A_x & B_u \\ 0_{nu\times nx} & 0_{nu\times nu} \end{bmatrix}, \ \bar{B}_u = \begin{bmatrix} B_u & 0_{nx\times nu} \\ 0_{nu\times nu} & I_{nu\times nu} \end{bmatrix},$$

$$\bar{C}_{yx} = \begin{bmatrix} C_{yx} & 0_{ny\times nu} \end{bmatrix}$$

The closed loop system with self-optimizing control and decentralized PI controllers can thus be given by,

$$\begin{cases} \dot{\bar{x}}(t) = \bar{A}_x \bar{x}(t) + \bar{B}_u \bar{u}(t) + \bar{B}_d \bar{d}(t) \\ \bar{z}(t) = \bar{C}_{zx} \bar{x}(t) + \bar{D}_{zu} \bar{u}(t) \\ \bar{y}(t) = \bar{C}_{yx} \bar{x}(t) \end{cases} \tag{13}$$

where $\bar{x} \in \mathbb{R}^{(nx+nu)}$, $\bar{u} \in \mathbb{R}^{2nu}$, $\bar{y} \in \mathbb{R}^{ny}$, $\bar{d} \in \mathbb{R}^{nd}$, and $\bar{z} \in \mathbb{R}^{nz}$ are the augmented state, control input, measurement output, disturbance, and controlled output vectors respectively. The control input $\bar{u}(t)$ can thus be given by,

$$\bar{u}(t) = K \, \Gamma \, H \, \bar{C} \, \bar{x}(t) \tag{14}$$

where

$$K = \mathrm{diag}(k_{p_1}, \cdots, k_{p_{nu}}, k_{i_1}, \cdots, k_{i_{nu}},), \tag{15}$$

$$\Gamma = \begin{bmatrix} I_{nu\times nu} \\ I_{nu\times nu} \end{bmatrix} \tag{16}$$

B. Stabilization

A new parametrization for static output feedback control was introduced in [14] by including slack variables to reduce the conservativeness. By defining the linear function,

$$M(P) = \begin{bmatrix} 0_{2nu\times 2nu} & \bar{B}_u^T P \\ P\bar{B}_u & \bar{A}_x^T P + P\bar{A}_x \end{bmatrix} \tag{17}$$

a necessary and sufficient condition for determining if a controller for a measurement combination is stable can thus be given by the following theorem.

Theorem 4.1: There exists stabilizing PI controllers with the measurement combination $H \in \mathbb{R}^{nu\times ny}$ iff there exists a stabilizing state feedback matrix $K_{SF} \in \mathbb{R}^{2nu\times(nx+nu)}$, a diagonal matrix $X = K^{-1} \in \mathbb{R}^{2nu\times 2nu}$ and a matrix $P = P^T \in \mathbb{R}^{(nx+nu)\times(nx+nu)} \succ 0$ such that

$$M(P) + He\left\{ \begin{bmatrix} I_{2nu\times 2nu} \\ -K_{SF}^T \end{bmatrix} \begin{bmatrix} X & -\Gamma H \bar{C}_{yx} \end{bmatrix} \right\} \prec 0 \tag{18}$$

Proof: Similar to the proof in [15], the closed-loop system is stable if the matrix $\bar{A}_x + \bar{B}_u K \Gamma H \bar{C}_{yx}$ is *Hurwitz*, i.e., there exists a matrix $P = P^T \succ 0$ satisfying the inequality,

$$(\bar{A}_x + \bar{B}_u K \Gamma H \bar{C}_{yx})^T P + P(\bar{A}_x + \bar{B}_u K \Gamma H \bar{C}_{yx}) \prec 0 \tag{19}$$

This can be formulated as:

$$\begin{bmatrix} \bar{C}_{yx}^T H^T \Gamma^T K^T & I_{2nu\times 2nu} \end{bmatrix} M(P) \begin{bmatrix} K\Gamma H \bar{C}_{yx} \\ I_{2nu\times 2nu} \end{bmatrix} \prec 0 \tag{20}$$

$$\begin{bmatrix} I_{2nu\times 2nu} \\ -\bar{C}_{yx}^T H^T \Gamma^T K^T \end{bmatrix}^{\perp} M(P) \begin{bmatrix} I_{2nu\times 2nu} \\ -\bar{C}_{yx}^T H^T \Gamma^T K^T \end{bmatrix}^{\perp T} \prec 0 \tag{21}$$

According to Lemma 2.1, the above expressions can be shown to be equivalent to,

$$M(P) + He\left\{ \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} \begin{bmatrix} I_{2nu\times 2nu} & -K\Gamma H \bar{C}_{yx} \end{bmatrix} \right\} \prec 0 \tag{22}$$

with the matrices $Z_1 \in \mathbb{R}^{2nu\times 2nu}$ and $Z_2 \in \mathbb{R}^{(nx+nu)\times 2nu}$. Factorizing Z_1 gives,

$$M(P) + He\left\{ \begin{bmatrix} I_{2nu\times 2nu} \\ Z_2 Z_1^{-1} \end{bmatrix} \begin{bmatrix} Z_1 & -Z_1 K \Gamma H \bar{C}_{yx} \end{bmatrix} \right\} \prec 0 \tag{23}$$

which is equal to (18) with $Z_1 = X$ (or $Z_1 = K^{-1}$) and $K_{SF}^T = -Z_2 Z_1^{-1}$. Finally, it can be shown that pre- and post-multiplying (18) with $\begin{bmatrix} K_{SF}^T & I_{2nu\times 2nu} \end{bmatrix}$ and its transpose respectively satisfies:

$$\begin{bmatrix} K_{SF}^T & I_{2nu\times 2nu} \end{bmatrix} M(P) \begin{bmatrix} K_{SF} \\ I_{2nu\times 2nu} \end{bmatrix} \prec 0 \tag{24}$$

$$(\bar{A}_x + \bar{B}_u K_{SF})^T P + P(\bar{A}_x + \bar{B}_u K_{SF}) \prec 0 \tag{25}$$

■

C. H_2 Optimal Control Synthesis

As previously mentioned, self-optimizing control does not take the dynamic performance into account. Therefore, the aim is not only to stabilize the closed-loop system but also to reduce the dynamic impact disturbances have.

The H_2 optimal control problem consists of minimizing the H_2 norm of the closed-loop system from exogenous disturbance signals $\bar{d}(t)$ to the controlled output signals $\bar{z}(t)$. This can be done by re-parametrizing (17) as in [14]:

$$N(P) = M(P) + \begin{bmatrix} \bar{D}_{zu} & \bar{C}_{zx} \end{bmatrix}^T \begin{bmatrix} \bar{D}_{zu} & \bar{C}_{zx} \end{bmatrix} \tag{26}$$

The globally optimal solution for the following theorem gives the H_2 optimal K and H that satisfies the self-optimizing control solution for the Null-space or the Exact local method.

Theorem 4.2: The H_2 optimal solution for a PI controller, with the parameters given by $K = X^{-1}$ and the measurement combination H for the SOC variable, is given by:

$$J_{k,1} = \min_{K_{SF},P,X,H} \operatorname{trace}(\bar{B}_d^T P \bar{B}_d) \tag{27}$$

$$\text{subject to: } P \succ 0 \tag{28}$$

$$N(P) + He\left\{ \begin{bmatrix} I_{2nu \times 2nu} \\ -K_{SF}^T \end{bmatrix} \begin{bmatrix} X & -\Gamma H \bar{C}_{yx} \end{bmatrix} \right\} \prec 0 \tag{29}$$

$$X = \operatorname{diag}(x_1 \dots x_{2nu}) \tag{30}$$

$$HF = 0 \qquad \text{(Null-space method)} \tag{31}$$

$$H = DH^* \qquad \text{(Exact local method)} \tag{32}$$

This results in a non-convex BMI, and thus an optimal global solution can't be guaranteed. However, an iterative algorithm can be used to find a local optimum:

Algorithm

1) Initialize, choose a stabilizing state feedback gain K_{SF}.
2) For a fixed K_{SF} solve the LMI:
$$J_{k,1} = \min_{P,X,H} \operatorname{trace}(\bar{B}_d^T P \bar{B}_d) \tag{33}$$
subject to: (28), (29), (30) and (31) or (32)
3) Fix X and H at the values obtained in step 2 and solve the LMI:
$$J_{k,2} = \min_{P,K_{SF}} \operatorname{trace}(\bar{B}_d^T P \bar{B}_d) \tag{34}$$
subject to: (28) and (29)
4) If $J_{k,1} - J_{k,2} < \epsilon$ stop, else update K_{SF} and repeat step 2 and 3

The controller parameters can be obtained from $K = X^{-1}$.

Since this algorithm only ensures convergence towards a local minimum, it is heavily dependent on the initial choice of the state feedback gain K_{SF}. However, choosing K_{SF} as the H_2 optimal state feedback gain often seems to give satisfactory solutions. Otherwise, e.g. [20] can be applied to generate a set of stabilizing state feedback gains.

V. CASE STUDY: DISTILLATION COLUMN

The proposed method was applied to the "column A" distillation column model [21], where a binary mixture is separated that has a relative volatility of $\alpha = 1.5$. The distillation column has 41 stages, which includes the reboiler and the condenser. The stages are counted from the bottom with the reboiler as stage 1 and with the feed at stage 21. For the distillation column, the feed is assumed to be given. Thus it has four degrees of freedom; bottoms flow rate (B), distillate flow rate (D), reflux flow rate (L) and vapor boilup (V). The distillate boilup and bottom flow rate are used to stabilize the two liquid levels in the condenser and the reboiler. This results in the LV configuration shown in Fig. 1 where the two remaining degrees of freedom are:

$$u = \begin{bmatrix} L & V \end{bmatrix}^T \tag{35}$$

The objective is to get a top product with 99% light component (1% heavy) and a bottom product with 1% light component, i.e., the cost function is,

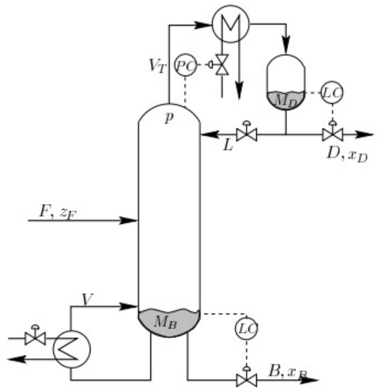

Fig. 1: A typical distillation column with LV configurations.

$$J = \left(\frac{x_H^{top} - x_H^{top,s}}{x_H^{top,s}} \right)^2 + \left(\frac{x_L^{btm} - x_L^{btm,s}}{x_L^{btm,s}} \right)^2 \tag{36}$$

where the specifications are denoted with the subscript, s.

As composition often is difficult to measure, they will be controlled indirectly using the temperatures inside the column. It is assumed that the temperatures $T_i(^oC)$ on each stage i can be calculated using the linear function [21]:

$$T_i = 0x_{L,i} + 10x_{H,i} \tag{37}$$

The main disturbances considered are changes in feed flow rate (F), feed composition (z_F) and feed liquid fraction (q_F).

The controlled variable $c_{ref,1}$ is used as a reference for the simulations, where x_D and x_B are controlled directly, resulting in zero steady-state loss. The PI controllers presented in [21] are used for $c_{ref,1}$, which has been demonstrated to have good disturbance rejection properties.

A. Case 1: Exact local method

The selection of controlled variables for indirect control of the compositions for the distillation example was investigated in [22]. Based on the Exact local method, a Mixed Integer Quadratic Programming (MIQP) approach was used to select the best subsets of the available measurements ($T_1 \cdots T_{41}$). This resulted in the following controlled variable when using three measurements:

$$c_{ref,2} = H^* \begin{bmatrix} T_{12} \\ T_{30} \\ T_{31} \end{bmatrix}, H^* = \begin{bmatrix} -0.0369 & 0.6449 & 0.6572 \\ -1.2500 & 0.2051 & 0.1537 \end{bmatrix}$$

To illustrate, the ease of controlling $c_{ref,2}$, the author of [22] implemented two PI controllers, that were tuned using the SIMC method [23]. However, it should be possible to find a different measurement combination together with PI controllers that further improves the transient response, without affecting the steady-state loss.

By using the proposed algorithm for the Exact local method, a new controlled variable and PI parameters are obtained with the new measurement combination $H = DH^*$. This resulted in the controlled variable, c_{EL} which can be seen in Table I together with its PI parameters.

Controlled variables	PI Parameters	H_2 norm
$c_{ref,1} = \begin{bmatrix} x_D \\ x_B \end{bmatrix}$	$k_p = 26.1,\ k_i = 6.94$ $k_p = -37.5,\ k_i = -11.33$	0.0152
$c_{ref,2} = \begin{bmatrix} 0.0369\ T_{12} - 0.6449\ T_{30} - 0.6572\ T_{31} \\ 1.2500\ T_{12} - 0.2051\ T_{30} - 0.1537\ T_{31} \end{bmatrix}$	$k_p = 0.59,\ k_i = 0.074$ $k_p = 0.73,\ k_i = 0.091$	0.0135
$c_{EL} = \begin{bmatrix} 0.2669\ T_{12} - 0.5106\ T_{30} - 0.5097\ T_{31} \\ 1.0000\ T_{12} - 0.3018\ T_{30} - 0.2636\ T_{31} \end{bmatrix}$	$k_p = 2.12,\ k_i = 0.46$ $k_p = 1.32,\ k_i = 0.29$	0.0125
$c_{NS,1} = \begin{bmatrix} 0.3481\ T_{11} - 0.2482\ T_{12} + 0.5944\ T_{30} - 0.8111\ T_{31} \\ 1.0000\ T_{11} - 0.7548\ T_{12} - 0.1458\ T_{30} + 0.1817\ T_{31} \end{bmatrix}$	$k_p = 17.00,\ k_i = 1.66$ $k_p = 19.91,\ k_i = 2.81$	0.0059
$c_{NS,2} = \begin{bmatrix} 0.0804\ T_{11} - 0.0065\ T_{12} - 0.0320\ T_{14} + 0.3007\ T_{30} - 0.4057\ T_{31} \\ 0.9872\ T_{11} - 1.0000\ T_{12} + 0.1721\ T_{14} - 0.2087\ T_{30} + 0.2504\ T_{31} \end{bmatrix}$	$k_p = 36.01,\ k_i = 3.89$ $k_p = 46.20,\ k_i = 10.10$	0.0046
$c_{NS,3} = \begin{bmatrix} 0.4723\ T_{11} - 0.4653\ T_{12} + 0.0779\ T_{14} - 0.0886\ T_{30} + 0.1209\ T_{31} - 0.3957\ T_{37} \\ 0.7826\ T_{11} - 0.6552\ T_{12} + 0.0439\ T_{14} + 0.6016\ T_{30} - 0.8721\ T_{31} + 1.0000\ T_{37} \end{bmatrix}$	$k_p = 31.09,\ k_i = 10.81$ $k_p = 35.06,\ k_i = 6.17$	0.0027

TABLE I: Controlled variables and PI parameters.

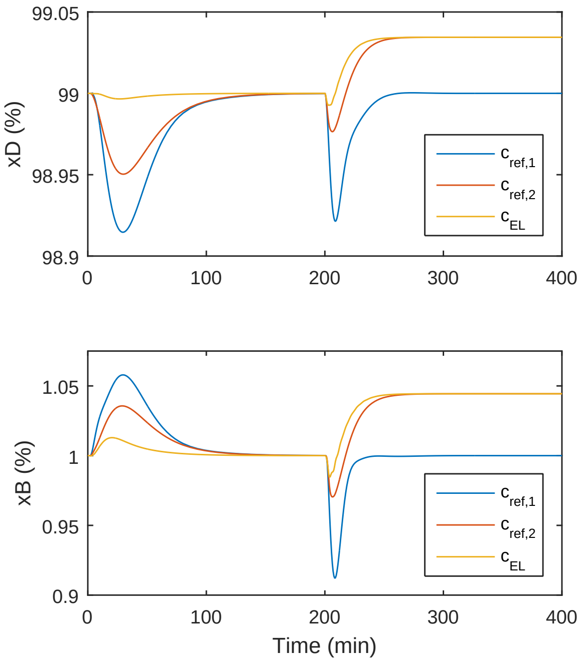

Fig. 2: Case 1: deviations in x_D and x_B, for a $+10\%$ filtered ($\frac{1}{25s+1}$) step change in F after 1 min and a -20% filtered ($\frac{1}{10s+1}$) step change in q_F after 200 min.

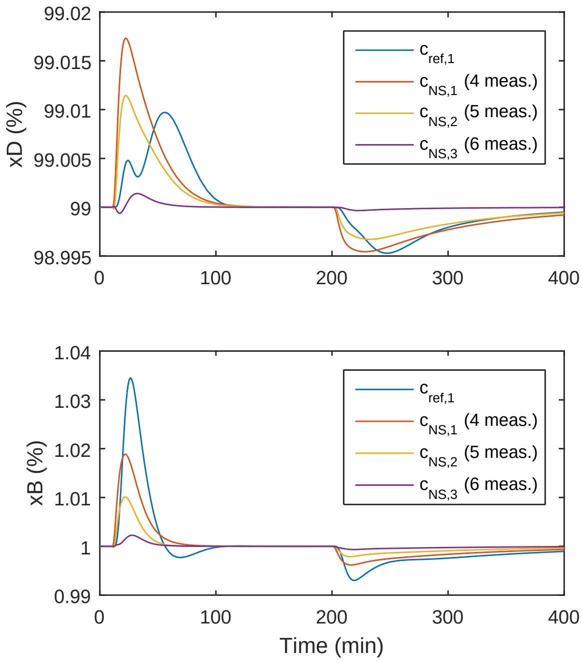

Fig. 3: Case 2: deviations in x_D and x_B, for a $+20\%$ filtered ($\frac{1}{15s+1}$) step change in z_F after 10 min and a -20% filtered ($\frac{1}{100s+1}$) step change in z_F after 200 min.

To demonstrate the improvements in the transient response, dynamic simulations were performed. The disturbances are a $+10\%$ filtered ($\frac{1}{25s+1}$) step change in F at 1 min and a -20% filtered ($\frac{1}{10s+1}$) step change in q_F at 200 min. The result can be seen in Fig. 2 which shows a clear improvement in the transient behavior for the proposed c_{EL} compared to $c_{ref,2}$. Furthermore, both c_{EL} and $c_{ref,2}$ have the same steady-state loss since their respective x_D and x_B converges to the same values.

B. Case 2: Null-space method

The Null-space method requires that the number of independent measurements is greater or equal the sum of the number inputs and disturbances. Out of the three disturbances F, z_F and q_F, changes in feed flow F have no steady-state effect on the cost, and thus four independent measurements are required. Adding more measurements can minimize the effect of measurement noise and thus further reduce the steady-state loss [24]. This often comes at the expense of a more complex control structure and can make the controller design more challenging. However, this also has the potential to improve the dynamic response when using the proposed algorithm, since extra measurements are just treated as additional degrees of freedom.

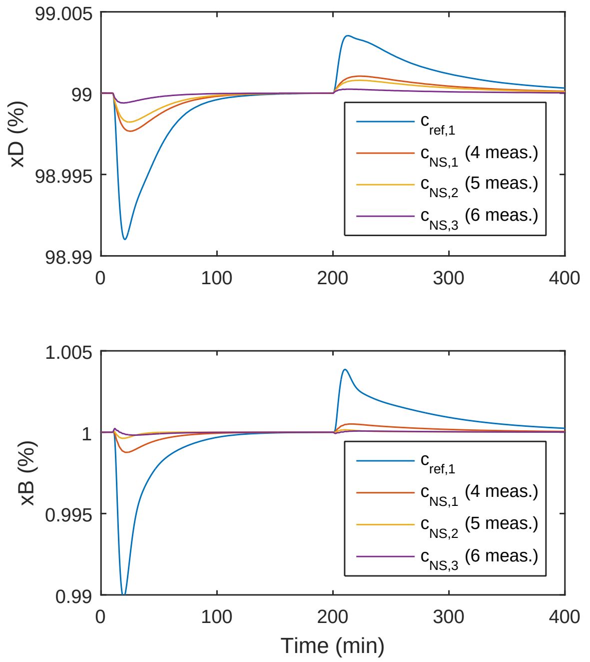

Fig. 4: Case 2: deviations in x_D and x_B, for a -5% filtered $(\frac{1}{25s+1})$ step change in q_F after 10 min and a $+5\%$ filtered $(\frac{1}{75s+1})$ step change in q_F after 200 min.

To illustrate this, following controlled variables are used,

$$\begin{aligned} c_{NS,1} =& H_1 \begin{bmatrix} T_{11} & T_{12} & T_{30} & T_{31} \end{bmatrix}^T \\ c_{NS,2} =& H_2 \begin{bmatrix} T_{11} & T_{12} & T_{14} & T_{30} & T_{31} \end{bmatrix}^T \\ c_{NS,3} =& H_3 \begin{bmatrix} T_{11} & T_{12} & T_{14} & T_{30} & T_{31} & T_{37} \end{bmatrix}^T \end{aligned}$$

where H_1, H_2, H_3, and controller parameters are to be determined. The controlled variables ($c_{NS,1}$, $c_{NS,2}$ and $c_{NS,3}$) and their PI parameters obtained using the proposed algorithm are shown in Table I together with their corresponding H_2 norm. The H_2 norm gets reduced when more measurements are added, which should indicate improvements in the dynamic response. This is confirmed by the simulation shown in Fig. 3 and 4, where the controlled variables with more measurements achieve better disturbance rejection.

More measurements can easily be implemented using the proposed method and further improves the dynamic performance. However, the increased risks of sensor failure and the cost of obtaining the measurements have to be considered.

VI. Conclusion

In this work, the transient behavior in the concept of self-optimizing control was considered. The main idea is to find measurement combinations together with PI controllers that minimize the dynamic impact from disturbances while satisfying the self-optimizing control conditions. The proposed method iteratively solves an LMI where the measurement combinations and controller parameters are computed simultaneously. The method was tested on a distillation case study, where simulations for the resulting control system showed an improved transient response, while maintaining the desired steady-state performance.

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