A Decision Support Model for Routing and Scheduling a Fleet of Fuel Supply Vessels

Marielle Christiansen[a], Kjetil Fagerholt[a],

Nikolaos P. Rachaniotis[b], Ingeborg Tveit[a] and Marte Viktoria Øverdal[a]

[a]Department of Industrial Economics and technology Management, Norwegian University of Science and Technology, Norway
[b]Department of Economics, Democritus University of Thrace, Greece

Abstract. We consider a real fuel supply vessel routing and scheduling problem faced by a Hellenic oil company with a given fleet of fuel supply vessels used to supply customer ships outside Piraeus Port. The supply vessels are loading fuel at refineries in the port area before delivering it to a given set of customer ships within specified time windows. A customer ship may place orders of more than one fuel type, and all orders placed by a customer ship do not have to be serviced by the same vessel, meaning customer splitting is possible. Fuel transported to the customer ships is allocated to compartments on board the supply vessels, and fuels of different types cannot be mixed in the same compartment. The objective is to design routes and schedules for the supply vessels while maximizing the company's profit. We propose a mixed-integer programming (MIP) model for the problem and provide a computational study based on real instances.

Keywords: Maritime transport, routing, scheduling, split loads

1 Introduction

Maritime transportation planning problems have attracted considerable attention in the literature in the last decades; see the surveys by Christiansen et al. (2013). However, even though fuel refilling is an important task for ships entering ports, the planning problem considered in this paper, where incoming customer ships are supplied with fuel by a given fleet of specialized fuel supply vessels, has, to the authors' knowledge, not been studied previously in the Operations Research literature. As a case study, we consider a Hellenic oil company operating in the broader area of Pireaus Port illustrated in Figure 1. The figure also shows where incoming customer ships anchor, waiting to be supplied by the company's fuel supply vessels within given agreed time windows. The supply vessels load at refineries in the inner part of the port before supplying the customer ships. The refineries offer different types of fuel, and a customer ship can order quantities of several fuel types to be delivered within the same time window. Fuel transported to the customer ships is allocated to compartments on board the supply vessels, and fuels of different types cannot be

adfa, p. 1, 2011.

mixed in the same compartment. Some customer ships are mandatory to service, while other customer ships are optional and can be supplied if the company has the available capacity. The planning problem consists of determining routes and schedules for the fleet of supply vessels such that the profit is maximized and all mandatory customer ships are serviced within their time windows. The vessels can perform more than one voyage during the planning horizon. The problem also includes determining which optional customers to service, as well as allocating the different types of fuel to separate compartments within the supply vessels, which add substantial complexity to the problem. The problem can be considered as a version of the multi-trip vehicle routing problem with time windows, see for example Nguyen et al. (2013) and Cattaruzza et al. (2014). In addition, we have a multi-compartment routing problem, see for instance Mendoza et al. (2010).

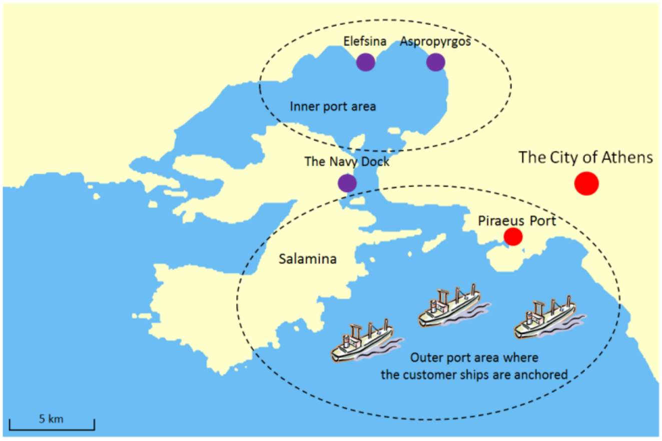

Fig. 1. Map of Piraeus port area

The fuel supply business in Piraeus Port has long traditions, and the business is to a large extent characterized by manual efforts in determining routes and schedules for the fuel supply vessels. However, many complicating factors and the large amount of money involved indicate that some decision support could be of good use.

The objective of this paper is to introduce this planning problem to the research community, and to propose a mixed-integer-programming (MIP) model for the problem that can support decision-making.

The outline of the remaining of the paper is as follows: Section 2 presents the planning problem in more detail followed by a MIP model in Section 3. Computational results are reported and discussed in Section 4, while concluding remarks are provided in Section 5.

2 Problem description

A given heterogeneous fleet of supply vessels is used to supply customer ships anchored in a port area. The customer ships place orders of different fuel types. The supply vessels load all fuel types at refineries. Since the distances within the refineries are almost negligible for this particular case study, we assume that the refineries can be modeled as a single depot. In the start of a planning horizon, some vessels may not be available for loading until some specified time. After finishing loading at the depot, the supply vessels start sailing to the customer ships. The sailing time between the depot and the customer ships is dependent on the hour of the day because sailing is not allowed in the area of the navy dock at night time, and vessels that would like to sail between the inner and outer port area in this period must sail around Salamina Island (Figure 1). The sailing times between different customer ships are assumed independent of time and which customer ships the vessels sail between. Loading time at the depot is assumed independent of vessel and loading quantity. The depot has a berth capacity, which implies that a maximum number of vessels may load simultaneously at a time. Figure 2 illustrates the customer ships, the supply vessels and the depot. The vessels may wait at a customer ship or at the depot before operation starts.

A vessel's voyage starts with loading at the depot, continue with sailing to and servicing the customer ships before returning empty to the depot. Within the planning horizon, a vessel may perform more than one voyage. Hence, every time a vessel loads at the depot, it also starts a new voyage. In Figure 2, vessel 1 executes two voyages, while vessel 2 performs only one.

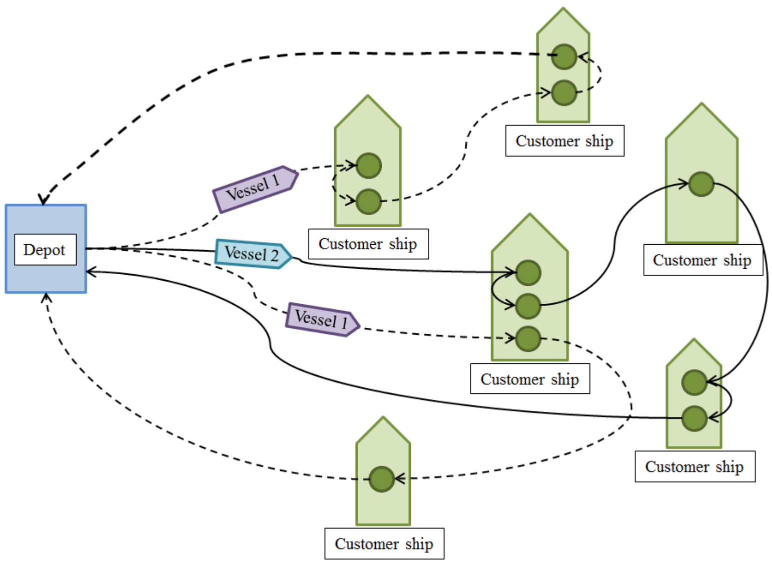

Fig. 2. Illustration of the customer ships, vessels and the depot. The customer ships' demands range from one to three different orders. One of the customer ships is serviced by both vessels, while the other customer ships are operated by only one vessel. Vessel 1 sails two voyages, while vessel 2 only sails one voyage.

A customer ship may place orders of different fuel types to be delivered at the same time. Each customer ship states a time window in which all its orders must be serviced. All orders at a customer ship do not need to be serviced by the same supply vessel, but if they are, the operation of the orders must happen continuously. If several vessels are servicing different orders at the same customer ship, there is an upper time limit between starting the first and the last order operations. In addition, only one vessel may service a customer ship at a time. The supply vessels are obliged to service contract customers, while spot customers can be serviced if the supply vessels have sufficient capacity. There are given quantities for the contract orders, while the spot orders' quantities are flexible within given upper and lower limits specified by the customers. The company must operate either all or none of a given spot customer's orders.

The supply vessels have a different number of compartments with given capacities where the fuels are loaded. A compartment may carry several fuel types, but it may only contain one fuel type at a time. The same fuel type may be carried in several compartments at the same supply vessel, and large orders may be split between compartments. Moreover, if different customer ships order the same fuel type, the orders may be allocated to the same compartment.

The planning problem consists of determining routes and schedules for the fleet of supply vessels such that the profit is maximized and all mandatory customer ships are serviced within their time windows. The profit equals the revenue through operation of contract and spot customers subtracted fixed daily costs and variable sailing costs. The problem also includes determining which optional customers to service, as well as allocating the different types of fuel to separate compartments within the supply vessels.

3 Mathematical model

In this section, we propose a MIP model for the problem. Section 3.1 introduces some modelling choices and definitions that are used in the mathematical model. Section 3.2 describes the notation used, while the objective function and the constraints of the mathematical model are presented in Section 3.3.

3.1 Modeling approach

We have chosen to develop a discrete time model due to the time dependent sailing time between the inner and outer port area. With discrete time representation, the planning horizon is divided into time periods of equal lengths.

Nodes are introduced to describe the orders placed by the customer ships. A node, a customer node and an order represent the same, and the terms may be used interchangeably. In addition to the nodes representing the orders, we include a depot node and a dummy end node. The depot node represent both refineries, while the dummy end node represent a fictive node where the vessels end up after operating all scheduled nodes in the planning horizon.

The vessel may execute multiple voyages during the planning horizon. In the mathematical model the numbering of voyages is related to each supply vessel.

The time window of a customer ship is defined by two parameters. One parameter represents the start of the time window and is the first time period a vessel may start servicing one of the customer ship's nodes. The other parameter represents the end of the time window, meaning that this is the last possible time period for servicing the customer ship. Notice that the end of the time window here is defined as the time period where operations must be finished, while in most relevant literature the time windows are defined as time periods which service may start.

3.2 Notation

Indices

v	supply vessel
i, j	Node
0	the depot node
d	the dummy end node
u	customer ship
f	fuel type
c	Compartment
m	Voyage
t	time period

Sets

$\mathcal{V}$	supply vessels
$\mathcal{N}$	customer nodes
$\mathcal{N}^T$	Nodes (total), $\mathcal{N} \cup \{0\} \cup \{d\}$
$\mathcal{U}$	customer ships
$\mathcal{U}^C \subseteq \mathcal{U}$	contract customer ships
$\mathcal{U}^O \subseteq \mathcal{U}$	spot customer ships
$\mathcal{N}_u \subseteq \mathcal{N}$	nodes that belong to customer ship u
$\mathcal{F}$	fuel types
$\mathcal{F}_c \subseteq \mathcal{F}$	fuel types allowed on compartment c
$\mathcal{C}_v$	compartments on supply vessel v
$\mathcal{M}_v$	voyages for vessel v
$\mathcal{T}$	time periods
$\mathcal{T}^{DAY} \subseteq \mathcal{T}$	time periods that represent a day's first time period. For example, when the planning horizon starts with time period 0 and one time period represent one hour, time periods 0, 24, 48 etc. are time periods in the set.
$\mathcal{S}^x$	possible combinations of (v, i, j, m, t) for variable x_{vijmt}

$\mathcal{S}^y$ possible combinations of (v, i, m, t) for variable y_{vimt}

$\mathcal{S}^w$ possible combinations of (v, i, m, t) for variable w_{vimt}

Parameters

T^{SA}_{vijt} sailing time when vessel v sails directly between nodes i and j when arriving at node j in time period t

T^{SD}_{vijt} sailing time when vessel v sails directly between node i and j when departing node j in time period t

T^{O}_{vi} vessel v's operating time at node i

$\underline{T}_u$ start of time window of customer ship u

$\overline{T}_u$ end of time window of customer ship u

T^{D}_{u} maximum time difference between start of operation of the first and last node at customer ship u

T^{M}_{v} the minimum time a vessel may use on any voyage

T^{E}_{v} the earliest time vessel v is available for operation

H number of time periods within 24 hours

B berth capacity of the depot

D_{if} demanded quantity of fuel type f for contract node i

$\underline{D}_{if}$ minimum accepted quantity of fuel type f for spot node i

$\overline{D}_{if}$ maximum accepted quantity of fuel type f for spot node i

Q_{cv} load capacity of compartment c on vessel v

C^{F}_{v} fixed daily cost of using vessel v

C^{S}_{v} sailing cost per time period with vessel v

R_f revenue per quantity delivered of fuel type f

Variables

x_{vijmt} 1, if vessel v starts sailing in time period t from node i directly to node j on voyage m/ 0, otherwise

y_{vimt} 1, if vessel v starts operating node i in time period t on voyage m / 0, otherwise

w_{vimt} 1, if vessel v is waiting in time period t at node i on voyage m/ 0, otherwise

γ_{vum} 1, if vessel v operates all nodes at customer ship u on voyage m/ 0, otherwise

z_u 1, if spot customer ship u is operated/ 0, otherwise

δ_{vt} 1, if vessel v is utilized the day that start with time period t/ 0, otherwise

k_{vfcm} 1, if compartment c on vessel v is allocated to fuel type f on voyage m/ 0, otherwise

l_{vijfcm} quantity of fuel type f in compartment c of vessel v when sailing

	directly from node i to j on voyage m
q_{vifm}	delivered quantity of fuel type f to spot node i by vessel v on voyage m

3.3 Model

Objective function

The objective function (1) represents the company's profit. It comprises the revenue from operating spot orders, variable sailing costs and daily fixed costs of using the vessels. The revenue from the contract orders is not included in the objective function since they can be considered as fixed. By including daily fixed costs in this way, the model will strive towards solutions where the vessels are busy some days, and are doing nothing other days. This is assumed to be practical in the real case problem, as long breaks in the utilization of a vessel allow for necessary repairs and time off.

$$max\,\Pi = \sum_{m\in\mathcal{M}_v}\sum_{f\in\mathcal{F}}\sum_{i\in\mathcal{N}}\sum_{v\in\mathcal{V}} R_f q_{vifm} - \sum_{t\in\mathcal{T}}\sum_{v\in\mathcal{V}} C_v^F \delta_{vt} - \sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}_v}\sum_{j\in\mathcal{N}^T}\sum_{i\in\mathcal{N}^T}\sum_{v\in\mathcal{V}} C_v^S T_{vijt}^{SD} x_{vijmt} \quad (1)$$

Flow constraints

The flow or routing constraints are given as follows:

$$\sum_{t=\underline{T}_u}^{\bar{T}_u - T_{vi}^O}\sum_{m\in\mathcal{M}_v}\sum_{v\in\mathcal{V}} y_{vimt} = 1 \qquad \forall i\in\mathcal{N}_u, u\in\mathcal{U}^C \quad (2)$$

$$\sum_{t=\underline{T}_u}^{\bar{T}_u - T_{vi}^O}\sum_{m\in\mathcal{M}_v}\sum_{v\in\mathcal{V}} y_{vimt} - z_u = 0 \qquad \forall i\in\mathcal{N}_u, u\in\mathcal{U}^O \quad (3)$$

$$\sum_{t\in\mathcal{T}} y_{v0mt} \le 1 \qquad \forall v\in\mathcal{V}, m\in\mathcal{M}_v \quad (4)$$

$$\sum_{\tau=0}^{t-T_v^M} y_{v0(m-1)\tau} - y_{v0mt} \ge 0 \qquad \forall v\in\mathcal{V}, m\in\mathcal{M}_v, t\in\mathcal{T} \quad (5)$$

$$y_{vim(t-T_{vi}^{O})} = \sum_{j \in \mathcal{N}^T} x_{vijmt} \qquad \forall v \in \mathcal{V}, i \in \mathcal{N} \cup \{0\}, m \in \mathcal{M}_v, t \in \mathcal{T} \tag{6}$$

$$\sum_{j \in \mathcal{N} \cup \{0\}} x_{vjim(t-T_{vjit}^{SA})} + w_{vim(t-1)} = y_{vimt} + w_{vimt} \qquad \forall v \in \mathcal{V}, i \in \mathcal{N}, m \in \mathcal{M}_v, t \in \mathcal{T} \tag{7}$$

$$\sum_{j \in \mathcal{N}} x_{vj0m(t-T_{vj0t}^{SA})} + w_{v0m(t-1)} = y_{v0(m+1)t} + w_{v0mt} \qquad \forall v \in \mathcal{V}, m \in \mathcal{M}_v, t \in \mathcal{T} \tag{8}$$

$$\sum_{t \in \mathcal{T}} y_{v01t} - \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}_v} \sum_{j \in \mathcal{N}} x_{vjdmt} = 0 \qquad \forall v \in \mathcal{V} \tag{9}$$

$$\sum_{\tau=t}^{t+(H-1)} \sum_{m \in \mathcal{M}_v} \sum_{i \in \mathcal{N} \cup \{0\}} (y_{vim\tau} + \sum_{j \in \mathcal{N} \cup \{0\}} x_{vijm\tau}) - H\delta_{vt} \leq 0 \qquad \forall v \in \mathcal{V}, t \in \mathcal{T}^{DAY} \tag{10}$$

$$x_{vijmt} \in \{1,0\} \qquad \forall (v,i,j,m,t) \in \mathcal{S}^x \tag{11}$$

$$y_{vimt} \in \{1,0\} \qquad \forall (v,i,m,t) \in \mathcal{S}^y \tag{12}$$

$$w_{vimt} \in \{1,0\} \qquad \forall (v,i,m,t) \in \mathcal{S}^w \tag{13}$$

$$z_u \in \{1,0\} \qquad \forall u \in \mathcal{U}^O \tag{14}$$

$$\delta_{vt} \in \{1,0\} \qquad \forall v \in \mathcal{V}, t \in \mathcal{T}^{DAY} \tag{15}$$

Constraints (2) ensure that every contract node is serviced only once, by one vessel on one voyage. The constraints also control that the customer nodes are serviced within their time windows. Constraints (3) hold for the nodes at the spot customer ships. If these nodes are serviced, each node can only be serviced by one vessel on one voyage within its time window. They also ensure that either all or none of the nodes for a given spot customer ship are serviced. Furthermore, constraints (4) make sure that the vessels operate at the depot at most once on each voyage. Constraints (5) control that a vessel cannot start a new voyage if it has not ended the previous voyage. The constraints also ensure that the previous voyage takes at least time T_v^M, which is the minimum time any vessel may use on a voyage. In constraints (6), it is described that when a vessel has finished servicing a node, it must start sailing to a customer node, the depot node or the dummy end node. Even when the same supply vessel is servicing two different nodes belonging to the same customer ship, it must start sailing after operating the first node. The sailing times between the nodes will in that case be zero.

Since the sailing time between nodes at the same customer ship is zero, sailing variables and operating variables may equal 1 in the same time periods. Constraints (7) make sure that a vessel either starts waiting or operating at a customer node when the vessel arrives at the node. Moreover, if a vessel waits at a node in a time period, it is restricted to either operate or wait at the node in the following time period. Constraints (8) are equivalent to the previous constraints, but concern the depot node. They make sure that when a vessel arrives at the depot, it must either start loading at the depot for a new voyage or wait at the depot on the current voyage. If a vessel waits at the depot in a time period, it may start operating on a new voyage or keep waiting on the current voyage in the next time period. Constraints (9) control that every vessel, if it is used at all, executes the fictive sailing to the dummy end node once during the planning horizon. Constraints (10) ensure that the variable δ_{vt} equals 1 if a given vessel is utilized the day which starts with time period t. Waiting is not included, since it is possible to wait at the depot which in practice corresponds to not utilizing the vessel. Finally, the binary restrictions for the variables are given in (11)-(15).

Time constraints

The time or schedule constraints are as follows:

$$\gamma_{vum} - \frac{\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}_u} y_{vimt}}{|\mathcal{N}_u|} \leq 0 \qquad \forall v \in \mathcal{V}, u \in \mathcal{U}, m \in \mathcal{M}_v \tag{16}$$

$$\gamma_{vum} - \frac{\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}_u} y_{vimt}}{|\mathcal{N}_u|} \geq \frac{1-|\mathcal{N}_u|}{|\mathcal{N}_u|} \qquad \forall v \in \mathcal{V}, u \in \mathcal{U}, m \in \mathcal{M}_v \tag{17}$$

$$\left(y_{vimt} - y_{v(i+1)m\left(t+T^O_{vi}\right)}\right) + \gamma_{vum} \leq 1 \qquad \forall v \in \mathcal{V}, i \in \mathcal{N}_u, u \in \mathcal{U}, m \in \mathcal{M}_v, t \in \mathcal{T} \tag{18}$$

$$y_{vimt} + y_{v'jn\left(\tau+T^D_u\right)} \leq 1 \qquad \forall v, v' \in \mathcal{V}, i, j \in \mathcal{N}_u | i \neq j, u \in \mathcal{U}, m \in \mathcal{M}_v, n \in \mathcal{M}_{v'}, t, \tau \in \mathcal{T} | \tau > t \tag{19}$$

$$\sum_{\tau=\max\{0,t-T^O_{v0}+1\}}^{t} \sum_{m\in\mathcal{M}_v} \sum_{v\in\mathcal{V}} y_{v0m\tau} \leq B \qquad \forall t \in \mathcal{T} \tag{20}$$

$$\sum_{\tau=\max\{0,t-T^O_{vi}+1\}}^{t} \sum_{m\in\mathcal{M}_v} \sum_{i\in\mathcal{N}_u} \sum_{v\in\mathcal{V}} y_{vim\tau} \leq 1 \qquad \forall u \in \mathcal{U}, t \in \mathcal{T} \tag{21}$$

$$\gamma_{vum} \in \{1,0\} \qquad \forall v \in \mathcal{V}, u \in \mathcal{U}, m \in \mathcal{M}_v \tag{22}$$

Constraints (16) and (17) force the γ_{vum} variables to 1 if all nodes at the same customer ship are serviced by the same vessel. Constraints (18) further control that the nodes at such customer ships are operated continuously. Note that the constraints assume that the nodes are serviced in a specific order, which reduces symmetry. When a vessel services all nodes at a customer ship, this must happen on the same voyage, since continuous operation by the same vessel will never happen on two different voyages. If a customer ship is serviced by more than one vessel, constraints (19) narrow the time span where the nodes at the customer ship can be operated. It is desirable that the nodes of a customer ship are operated continuously without any waiting in between. Since the operating times vary with vessel and the fact that the operating sequence of the nodes are not known a priori, these constraints give some possibilities for waiting in between. All operation of nodes at a given customer ship must start within an interval, T_u^D, calculated from the vessels' operating times at the customer ship. Constraints (20) ensure that in any time period, the company cannot have more than *B* vessels loading at the depot. In addition, a customer ship can only be operated by one vessel at the time. Constraints (21) take care of this. Finally, the binary requirements for the γ_{vum} variables are given in (22).

Load constraints

The load management on board the ships is taken into account by the following constraints:

$$\sum_{c\in\mathcal{C}_v}\sum_{j\in\mathcal{N}\cup\{0\}} l_{vjifcm} - \sum_{t\in\mathcal{T}} D_{if}y_{vimt} - \sum_{c\in\mathcal{C}_v}\sum_{j\in\mathcal{N}^T} l_{vijfcm} = 0 \qquad \forall v\in\mathcal{V}, i\in\mathcal{N}_u, u\in\mathcal{U}^C, f\in\mathcal{F}, m\in\mathcal{M}_v \tag{23}$$

$$\sum_{c\in\mathcal{C}_v}\sum_{j\in\mathcal{N}\cup\{0\}} l_{vjifcm} - q_{vifm} - \sum_{c\in\mathcal{C}_v}\sum_{j\in\mathcal{N}^T} l_{vijfcm} = 0 \qquad \forall v\in\mathcal{V}, i\in\mathcal{N}_u, u\in\mathcal{U}^O, f\in\mathcal{F}, m\in\mathcal{M}_v \tag{24}$$

$$\sum_{t\in\mathcal{T}} \underline{D}_{if}y_{vimt} \le q_{vifm} \le \sum_{t\in\mathcal{T}} \overline{D}_{if}y_{vimt} \qquad \forall v\in\mathcal{V}, i\in\mathcal{N}_u, u\in\mathcal{U}^O, f\in\mathcal{F}, m\in\mathcal{M}_v \tag{25}$$

$$\sum_{c\in\mathcal{C}_v}\sum_{f\in\mathcal{F}_c} l_{vijfcm} - \sum_{t\in\mathcal{T}}\sum_{c\in\mathcal{C}_v} Q_{vc}x_{vijmt} \le 0 \qquad \forall v\in\mathcal{V}, i\in\{0\}, j\in\mathcal{N}, m\in\mathcal{M}_v \tag{26}$$

$$\sum_{c\in\mathcal{C}_v}\sum_{f\in\mathcal{F}_c} l_{vijfcm} - \sum_{t\in\mathcal{T}}(\sum_{c\in\mathcal{C}_v} Q_{cv} - \sum_{f\in\mathcal{F}_c} D_{if})x_{vijmt} \le 0 \qquad v\in\mathcal{V}, u\in\mathcal{U}^C, i\in\mathcal{N}_u, j\in\mathcal{N}^T, m\in\mathcal{M}_v \tag{27}$$

$$\sum_{c\in\mathcal{C}_v}\sum_{f\in\mathcal{F}_c} l_{vijfcm} - \sum_{t\in\mathcal{T}}(\sum_{c\in\mathcal{C}_v} Q_{cv} - \sum_{f\in\mathcal{F}_c} \underline{D}_{if})x_{vijmt} \leq 0 \qquad v\in\mathcal{V}, u\in\mathcal{U}^O, i\in\mathcal{N}_u, j\in\mathcal{N}^T, m\in\mathcal{M}_v \tag{28}$$

$$\sum_{f\in\mathcal{F}_c} k_{vfcm} \leq 1 \qquad v\in\mathcal{V}, c\in\mathcal{C}_v, m\in\mathcal{M}_v \tag{29}$$

$$l_{vijfcm} - \min\{Q_{vc}, \sum_{k\in\mathcal{N}_u|u\in\mathcal{U}^C} D_{kf} + \sum_{k\in\mathcal{N}_u|u\in\mathcal{U}^O} \overline{D}_{kf}\} k_{vfcm} \leq 0 \qquad \forall v\in\mathcal{V}, i,j\in\mathcal{N}\cup\{0\}, f\in\mathcal{F}_c, c\in\mathcal{C}_v, m\in\mathcal{M}_v \tag{30}$$

$$\sum_{c\in\mathcal{C}_v}\sum_{f\in\mathcal{F}_c}\sum_{j\in\mathcal{N}} l_{vjifcm} = 0 \qquad \forall i\in\{0\}\cup\{d\}, v\in\mathcal{V}, m\in\mathcal{M}_v \tag{31}$$

$$k_{vfcm} \in \{1,0\} \qquad \forall v\in\mathcal{V}, f\in\mathcal{F}_c, c\in\mathcal{C}_v, m\in\mathcal{M}_v \tag{32}$$

$$l_{vijfcm} \geq 0 \qquad \forall v\in\mathcal{V}, i\in\mathcal{N}\cup\{0\}, j\in\mathcal{N}\cup\{0\}, f\in\mathcal{F}_c, c\in\mathcal{C}_v, m\in\mathcal{M}_v \tag{33}$$

$$q_{vifm} \geq 0 \qquad \forall v\in\mathcal{V}, i\in\mathcal{N}, m\in\mathcal{M}_v, f\in\mathcal{F} \tag{34}$$

The difference in load within a supply vessel's compartments before and after operating a customer node equals the demanded fuel quantity of the node. This is ensured by constraints (23) and (24) for contract and spot nodes, respectively. Constraints (25) ensure that the quantity delivered to the spot nodes are within the upper and lower limits. The load variables, l_{vijfcm}, can be denoted as arc-load flow variables. Agra et al. (2013) describe the advantages of having arc-load flow variables instead of more common load variables, where the latter do not include a destination node j. They state that using the arc-flow load variables strengthen the model. Constraints (26)-(28) control that the l_{vijfcm} variables are assigned non-zero values only if the given vessel, v, sails directly between nodes i and j, and that the compartments' capacity limits are not exceeded. Constraints (29) ensure that only one fuel type is allocated to a compartment on each voyage. The constraints also make sure that a compartment is only loaded with a fuel type that it is allowed to carry. Constraints (30) control that the arc-flow load variables only take values for combinations of fuel type and compartment if the fuel type is actually allocated to that compartment. To facilitate the reading, we introduce constraints (31) to ensure that the vessels do not carry any load when returning to the depot or the dummy end node. Finally, the binary and non-negativity requirements for the variables related to loading are given in (32)-(34).

We have tested several types of valid inequalities to strengthen the linear relaxation of the model. Neither clique nor cover inequalities improved the solution process, so they are not included. Instead we include the promising valid inequalities (35) ensuring that a spot node i cannot be operated by a vessel v if the vessel is not utilized the day the node has its time window.

$$q_{vifm} - D_{if}\delta_{vt} \leq 0 \qquad \forall v \in \mathcal{V}, i \in \mathcal{N}_u,, u \in \mathcal{U}^O, f \in \mathcal{F}, m \in \mathcal{M}_v, \; t \in \mathcal{T}^{DAY} | t \leq \underline{T}_u < (t+H) \qquad (35)$$

4 Computational study

This section presents a computational study performed on a number of test instances generated from real data from the company. The model described in the previous section has been implemented in Mosel and solved using the commercial optimization software Xpress v7.3 64-bit on an HP DL 165 G6 computer with two AMD Opteron 24312 4.0 GHz processors, 24 GB of RAM and running on a Linux operating system. Section 4.1 describes the test instances, while computational results are presented and discussed in Section 4.2.

4.1 Test instances

The test cases are generated based on data regarding customer ships and their fuel orders provided by the company. The shipping company's vessel fleet consisted of three vessels. Information regarding the vessels' compartments, load capacities, costs and pumping rates was also given. The vessels have between 5 and 7 compartments and the total load capacities are in the range of approximately 1300 to 3000 m^3. The pumping rates vary between 180 to 320 m^3/h.

Since the sailing times for this problem are small compared to the operating times at the customer ships and the depot (3 to 12 hours), we have approximated the sailing times between customer ships and between the depot and the customer ships to one hour. Exceptions are the sailing time between the depot and the customer ships during night time, which is four hours because of the navy dock closure, and the time to the dummy end node, which is set to 0. Taking these sailing times into account, we have chosen to use a time discretization of 1 hour.

The customer ships had between one and three different orders each. Most customer ships had a wide time window specifying service within a given day (i.e. during a period of 24 time periods). However, some of the ships had requested morning deliveries where the deliveries had to be made between 7 am in the morning and 2 pm on the given day.

The number of time periods to include in the planning horizon was set to the end time of the latest time window of the customer ships: $|\mathcal{T}| = \max_{(i \in \mathcal{N})} \bar{T}_u$. This varied between 48 and 96 hours (i.e. two to four days). The start of the planning horizon was set to $t = 0$. Since the vessels were already engaged in fuel deliveries (from the previous planning period), they were given different times for when they became available. Vessel 1 became available for loading at the depot from time period $t = 17$, meaning $T_1^E = 17$, while vessels 2 and 3 were available from $t = 7$ and $t = 0$, respectively.

Table 1. Test cases with varying number of customer ships that have placed orders on different days

Test Case	# Ships day 1	# Ships day 2	# Ships day 3	# Total ships	# Time periods
4_4_0	4	4	0	8	72
3_3_2	3	3	2	8	96
10_0_0	10	0	0	10	48
5_5_0	5	5	0	10	72
6_6_0	6	6	0	12	72
4_4_4	4	4	4	12	96

Table 1 shows the different test cases used in the computational study. Since we did not have any data whether the customer orders were mandatory or optional, we assumed all were the latter in our tests.

4.2 Computational experiments and results

Table 2 shows the best obtained solutions within a time limit of 10,000 seconds running time. The table shows the results from running the model without and with the valid inequalities (35).

Table 2. Test results from testing the basic mathematical model on test cases with spot nodes. The numbers in front of the'/' show the results obtained without the valid inequalities (35), while the numbers after the '/' are with.

Test Case	After 10,000 seconds		
	Objective Function Values	Best Bounds	Gaps (%)
4_4_0	2492/2492	2513/2514	0.8/0.9
3_3_2	2483/2490	2498/2505	0.6/0.6
10_0_0	2659/2950	2989/2990	11.0/1.3
5_5_0	2329/777	2990/2987	22.1/74.0
6_6_0	1011/1032	2490/3488	59.4/70.4
4_4_4	2148/3257	3476/3476	38.2/6.3

As shown in the table, we were not able to find proven optimal solutions within the given time limit. This gives a good indication of the problem's complexity. However, for four of the six test cases, we are able to find solutions with reasonably small gaps of 6.3 % and less. We may also note that the model with the valid inequalities yields a better solution to four of the test cases, while the one without provides a better solution only on test case 5_5_0. The model without the valid inequalities is however best at finding better upper bounds on four of the six test cases.

Since we experienced large difficulties in solving the problem, we also tested two simplified versions of the model. In the first version, denoted *NoCS*, we remove the possibility of customer splitting. The motivation behind this is that there are only very few cases where the company actually performs a customer splitting (which is something that our results also showed). Furthermore, reducing this possibility will reduce the model's complexity, and one might actually obtain better solutions within the given time limit despite that the feasible space is reduced.

In the second simplified version of the model, denoted *ES50_NoCS*, we additionally eliminated the complicating stowage of orders to separate compartments in the vessels. This means that we assume in the model that all orders can be mixed in a single compartment only considering the total capacity. Because of this we risk that the solutions obtained are not feasible with respect to the real stowage problem. Therefore, we have tested this with running the simplified model with reduced ship capacities, which makes it easier to find feasible stowage plans for the solutions obtained. Actually, tests showed that we had to reduce the capacity of the ship to as little as 50 % of the original capacity to achieve solutions that always where feasible with respect to the real allocation requirements. This feasibility check was done by solving a stowage model for the given route of each ship, which becomes a much simpler problem to solve.

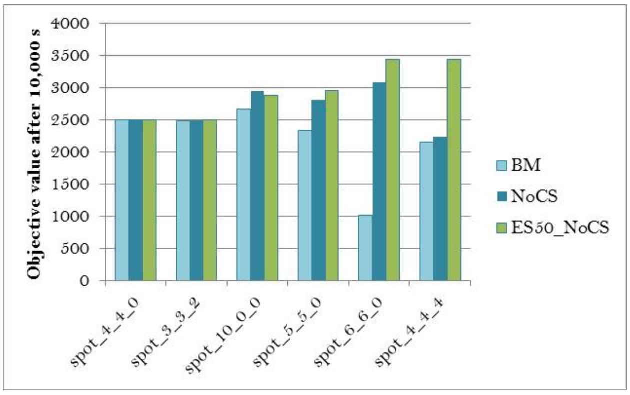

Fig. 3. Comparing the objective values after 10,000 seconds for the basic model (BM), without customer splitting (NoCS), and without stowage and customer splitting (ES50_NoCS)

Figure 3 compares the objective values for the two simplified model versions with the original one (without the valid inequalities). We can see that for the two smallest test cases, the three model versions gave similar results. However, for the four other test cases both simplified models gave better results. The model without both customer splitting and stowage (ES50_NoCS) gave best results for all test cases except for 10_0_0. If we compare the best results obtained with the simplified model version with the best upper bound provided in Table 2, it can be shown that these solutions are in fact very close to the optimal solutions. This indicates that these simplification strategies can be a good way to provide good decision support to a decision maker in a real planning situation when limited time is available. To respond to spot order inquiries, there is typically need for a solution within approximately 10 minutes. The ES50_NoCS model with neither stowage nor customer splitting provided solutions within this time limit. In addition, the model can be solved during the night to produce a schedule for the next days. The other two models can be useful for this purpose.

5 Concluding remarks

We have proposed a mixed integer programming model for a combined fuel supply vessel scheduling and fuel allocation problem. Three versions of the model have been evaluated using data for a real-life problem; a basic model which comprises all relevant aspects as shown in Section 3, a simplified version of the model without customer splitting, and another simplified version without stowage and customer splitting.

Test results showed that the two simplified models provided significantly better solutions within a time limit of 10,000 seconds compared to the basic model.

The fuel supply vessel routing and scheduling problem is important for the shipping industry and includes several challenges for the research community. In order to solve even larger instances of the problem in limited time, interesting future research could be to integrate some heuristic elements to the solution method presented, develop advanced metaheuristics for the problem or methods based on branch-and-price (see e.g. Barnhart et al., 1998).

6 References

1. Agra A., Christiansen M., and Delgado A. (2013). Mixed integer formulations for a short sea fuel oil distribution problem. Transportation Science, 47(1), 108-124.
2. Barnhart C., Johnson E.L., Nemhauser G.L., Savelsbergh M.W.P., and Vance P.H. (1998). Branch-and-price: Column generation for solving huge integer programs. Operations Research, 46(3), 316-329.
3. Cattaruzza D., Absi N., Feillet D., and Vigo D. (2014). An iterated local search for the multi-commodity multi-trip vehicle routing problem with time windows. Computers & Operations Research, 51, 257-267.
4. Christiansen M., Fagerholt K., Nygreen B., and Ronen D. (2013). Ship routing and scheduling in the new millennium. European Journal of Operational Research, 228(3), 467-483.
5. Mendoza, J. E., Castanier, B., Guéret, C., Medaglia, A. L., and Velasco, N. (2010). A memetic algorithm for the multi-compartment vehicle routing problem with stochastic demands. Computers & Operations Research, 37(11), 1886–1898.
6. Nguyen P.K., Crainic T.G., and Tolouse, M. (2013). A tabu search for time-dependent multi-zone multi-trip vehicle routing problem with time windows. European Journal of Operational Research, 231(1), 43-56.