Compressor Surge Control Design Using Linear Matrix Inequality Approach

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***Abstract*—A novel design for active compressor surge control system (ASCS) using linear matrix inequality (LMI) approach is presented and including a case study on piston-actuated active compressor surge control system (PAASCS). The non-linear system dynamics of the PAASCS is transformed into linear parameter varying (LPV) system dynamics. The system parameters are varying as a function of the compressor performance curve slope. A compressor surge stabilization problem is then formulated as a LMI problem. Solving the LMI problem results in a feedback control gain for the compressor surge stabilization and stability proof of the closed loop system in the whole compressor operating area. Simulation results show that the designed surge control system is able to stabilize compressor surge. Significant improvement of the control system performance is achieved by combining the LMI approach and linear quadratic regulator (LQR).**

I. INTRODUCTION

Compressor operation at lower mass flows is limited by compressor surge. Compressor surge is an aerodynamic instability and results in axisymmetric oscillations of the compressor mass flow and the compressor pressure. This phenomenon is indicated by fluctuations at the compressor mass flow, at the compressor discharged pressure, and at the compressor flow temperature, and followed by vibrations on the rotating parts. The vibrations reduce the reliability of the rotating parts and large amplitude vibrations lead to compressor damage, especially to the compressor blades and bearing.

A method for stabilizing compressor using a state feedback control and an active element (actuator) has been introduced by Epstein et al. [1]. The method is known as active compressor surge control. Several studies on active compressor surge control using different control design methods by including linear and non-linear control methods for different actuators have been presented, and are summarized in [2]–[4]. Most of the active surge control studies uses the Greitzer compression model to represent the dynamics of compressor states (pressure and mass flow). The Greitzer compressor model is a model of compression system which is able to predict transient compressor states during compressor surge including compressor mass flow and compressor [5].

Physical observations show that compressor produced pressure (compressor discharged pressure) has a non-linear relation to the compressor mass flow which is usually described in a compressor map. A compressor map is commonly provided by the compressor manufacturer. However, the compressor map may also be obtained through a compressor performance test by following steps: operate the compressor for several operating points, record the mass flow and pressure data, and do a curve fitting to approximate the whole operating points. An approximation of compressor performance for a constant compressor speed has been introduced in [6], while for the varying compressor speeds has been presented in [7].

The main goal of an active compressor surge control system design is to make the closed loop compressor system to operate stable in the surge area. Previously, the surge control was designed base on linear control approach [8]–[10]. Using the linear control approach, the closed loop system achievies locally asymptotically stable such that the stabilized compressor surge area is limited. In order to make the closed loop compressor system stable in the whole surge region, the closed loop system must be globally asymptotically stable (GAS). A necessary condition for GAS is the existence of a Lyapunov function. Simon and Valavani applied Lyapunov-based control for an active surge control system using close-coupled valve [11]. Krstic et al. introduced a non-linear control design method known as backstepping in [12]. The backstepping method provides a systematic procedure to find state feedback and Lyapunov function simultaneously. Several works on active surge control using backstepping have been presented afterwards [13]–[16]. However, the backstepping may result in a complicated state feedback and can be difficult to be implemented [17]. Two general state feedback control laws for compressor surge control have been derived using Lyapunov-based control method and guarantee the GAS of the closed loop system [18]. However, applying the Lyapunov-based control method is not straightforward and in general, it is not possible to do performance adjustment base on a cost function as in optimal control.

A non-linear system can be approximated by a linear system through a linearization around an operating point and therefore linear control methods are applicable. This approximation is only valid for a limited region around the operating point as basis in the linearization. Therefore, linearization at several operating points is required to cover the whole operating area. This will result in several linear systems and the parameters

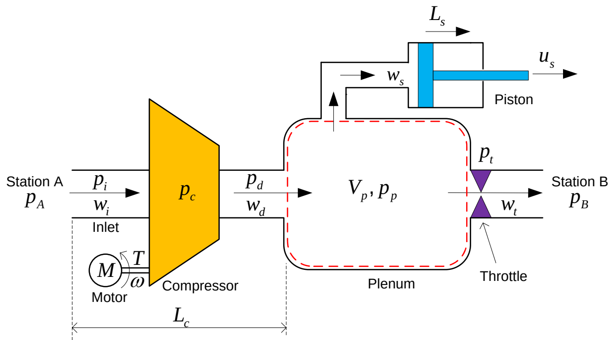


Fig. 1. COMPRESSION SYSTEM EQUIPPED PAASCS.

are varying for each operating point. Linear matrix inequalities (LMI)-based control is one of the powerful control design methods for such linear parameter varying system [19], [20]. The LMI-based control is a convex optimization base on Lyapunov stability condition. A Lyapunov function candidate, which is a positive definite function and the time derivative should be negative definite, is formulated as an LMI problem [21]. The LMI problem is solved to obtain a positive definite matrix such that the time derivative of the Lyapunov candidate is negative definite. The LMI solution can be obtained using available computational tools, for an example the YALMIP Toolbox [22].

A piston-actuated active surge control system (PAASCS) is a method to stabilize compressor surge by dissipating the downstream compressor energy using a piston [17]. This paper is presenting an application of LMI-based control method in an active compressor surge system with a case study on PAASCS. The goal is to obtain a state feedback control to stabilize the whole compressor operating area and the control system has a performance. The non-linear dynamics of PAASCS is transformed into a linear parameter varying system, and the compressor surge stabilization is formulated as an LMI problem.

II. Piston Actuated Active Surge Control System

A piston actuated active surge control system (PAASCS) is an active surge control system utilizing a piston as an actuator to stabilize compressor surge. The piston generates mass flow to manipulate the compressor downstream pressure in order to stabilize compressor surge. PAASCS has been introduced in [17] and the model is shown in Figure 1. The model was developed base on the Greitzer compressor model [5] and all assumptions in the Greitzer model are adopted. It is assumed that pressures at station A (p_A) and at station B (p_B) are equivalent to the ambient pressure. All pressures in the system are measured relative to the ambient pressure. Heat transfer in the system is neglected. Pressure drop along the inlet line is neglected such that the inlet pressure (p_i) is equal to p_A. There is no mass storage in the compressor such that the inlet mass flow (w_i) is equal to the compressor discharge mass flow (w_d).

A plenum is a model of downstream control volume. The plenum volume (V_p) can be representing a volume of pipeline and/or vessel. It is assumed that the pressure in the plenum (p_p) is uniformly distributed in the plenum space. A throttle is used to adjust the outlet mass flow (w_t). The throttle generates pressure drop (p_t) between plenum and the outlet. A piston is connected to the plenum for generating piston mass flow (w_s). The piston mass flow is used to manipulate the plenum pressure to maintain the system to be stable.

Dynamic equations of the system are given as follows [18]:

$$\dot{w}_i = \frac{A_c}{L_c}\left[p_c - p_p\right] \quad (1)$$

$$\dot{p}_p = \frac{a_0^2}{V_p}\left[w_i - w_t - w_s\right] \quad (2)$$

where A_c is the inlet cross section area, L_c is the effective length of the inlet, a_0 is the speed of sound, and V_p is the plenum volume.

The inlet mass flow dynamics is a function of the pressure difference between the compressor discharge and the plenum. Pressure at the compressor discharge is a result of energy conversion from mechanical into pneumatic. It is a function of the compressor mass flow and the compressor speed as commonly shown in a compressor map. The compressor map usually consists of a plot of the compressor produced pressure against the compressor mass flow for several compressor speeds. However, we consider only on a constant compressor speed in this study.

A compressor performance at a constant speed can be approximated by a qubic function [6]:

$$p_c = p_{c_0} + \frac{H}{2}\left[2 + 3\left(\frac{w_i}{W} - 1\right) - \left(\frac{w_i}{W} - 1\right)^3\right] \quad (3)$$

where p_{c_0} is the shut-off value of the axisymmetric characteristic, W is the semi-width of the cubic axisymetric compressor characteristic and H is the semi-hight of the cubic axisymetric compressor characteristic, consults [6] for more detailed definition.

III. Linear Parameter Varying System Representation

Define system states for the PAASCS as follows:

$$x_1 = w_i - w_t \quad (4)$$

$$x_2 = p_c - p_p \quad (5)$$

and substituting into (1) and (2) results in:

$$\dot{x}_1 = k_1 x_2 - \dot{w}_t \quad (6)$$

$$\dot{x}_2 = k_m k_1 x_2 - k_2 x_1 + k_2 w_s \quad (7)$$

where $k_1 = \frac{A_c}{L_c}$, $k_2 = \frac{a_0^2}{V_p}$ and $k_m = \frac{dp_c}{dw_i}$. The variable k_m represents the slope of compressor performance curve which is varying at each operating point.

A state space form of the PAASCS dynamics is given as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & k_1 \\ -k_2 & k_m k_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ k_2 \end{bmatrix} w_s + \begin{bmatrix} -1 \\ 0 \end{bmatrix} \dot{w}_t. \quad (8)$$

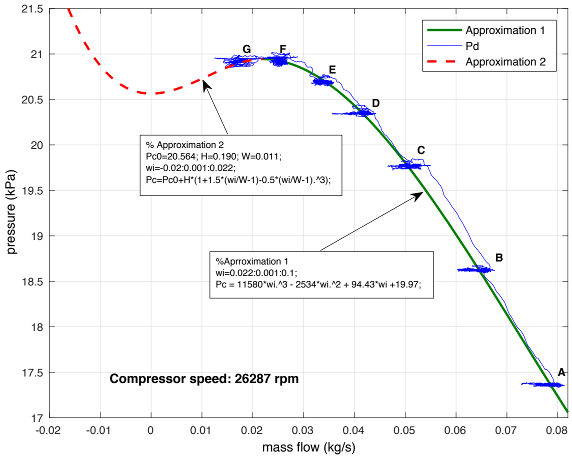


Fig. 2. Compressor performance curve obtained through a performance test [23].

The equation (8) can be expressed by:

$$\dot{x} = A(k_m)x + Bw_s + D\dot{w}_t, \tag{9}$$

where $x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$, $A(k_m) = \begin{bmatrix} 0 & k_1 \\ -k_2 & k_m k_1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & k_2 \end{bmatrix}^T$, and $D = \begin{bmatrix} -1 & 0 \end{bmatrix}^T$. The variable x is the system states, w_s is the system input, and w_t is the system disturbance. The $\dot{w}_t$ is the rate change of compressor outlet flow and the value is assumed to be bounded as the outlet valve has usually slow dynamics.

Eigenvalues of the open loop system (8) are given by:

$$s_{1,2} = \frac{k_m k_1 \pm \sqrt{(k_m k_1)^2 - 4k_1 k_2}}{2}, \tag{10}$$

where the real parts of eigenvalues indicate the system stability. Because the value of k_1 is constant and $k_1 > 0$, the compressor system stability depends on the value of k_m, which is the compressor performance curve slope. Therefore, the compressor operates stable at along compressor performance curve at negative slope ($k_m < 0$) and unstable at the positive slope ($k_m > 0$). The unstable compressor operating along the positive slope is known as surge. For a compressor performance curve described in (3), the curve slope is given by:

$$k_m = \frac{3}{2W} - \frac{3}{2}\left(\frac{w_i}{W} - 1\right)^2, \tag{11}$$

which is a function of the compressor mass flow. The slope is varying in a range of $k_m^- \leq k_m \leq k_m^+$, where k_m^+ is the maximum slope and k_m^- is the minimum slope. The maximum slope is $k_m^+ = \frac{3H}{2W}$ achieved at mass flow $w_i = W$, while the minimum slope is $k_m^- = \frac{2p_p}{w_o}$ [18].

Therefore, the system dynamics in (8) or (9) is a linear parameter varying (LPV) system as the system matrix A is a function of k_m. From now we use notation A instead of $A(k_m)$ in the interest of simplicity.

IV. LPV System Stabilization

A PAASCS applies a piston to generate mass flow for stabilizing compressor surge. The generated mass flow is represented by the system input, w_s, in (9). In order to design a controller, define a state feedback:

$$w_s = Kx \tag{12}$$

where K is a control gain matrix. The closed loop system of (9) is given as follows:

$$\dot{x} = [A + BK]\,x + D\dot{w}_t. \tag{13}$$

It is required to find K such that the closed loop system matrix is asymptotically stable. It is achieved iff $[A + BK]$ is Hurwitz. The asymptotic stability of a system is guaranteed by the existence a Lyapunov function $V(x)$, where: $V(x) > 0$ and $\dot{V}(x) < 0$. For (13), define a Lyapunov function candidate

$$V(x) = x^T P x \tag{14}$$

where P needs to satisfy the following conditions:

$$P > 0 \tag{15}$$

$$A^T P + PA + K^T B^T P + PBK < 0. \tag{16}$$

Equation (16) is a bilinear matrix inequality (BMI) because it has multiplication of two unknown variables, P and K. Solving a BMI problem is difficult, and it is recommended to convert a BMI problem into a linear matrix inequalities (LMI) problem by the following steps [21]:

a. Define $Y = P^{-1}$ and do pre- and post- multiplication to (16) such that it results in:

$$YA^T + AY + YK^T B^T + BKY < 0. \tag{17}$$

Inequality (17) is still a BMI.

b. Define $L = KY$ and substitute into (17) such that it results in:

$$YA^T + AY + L^T B^T + BL < 0. \tag{18}$$

Inequality (18) is a LMI.

Because the compressor map slope is varying in a certain range, we need to define two LMIs which represents the extreme operating region:

$$YA_1^T + A_1 Y + L^T B^T + BL < 0 \tag{19}$$

$$YA_2^T + A_2 Y + L^T B^T + BL < 0 \tag{20}$$

where $A_1 = A(k_m^-)$ for the minimum slope and $A_2 = A(k_m^+)$ for the maximum slope, respectively. Both LMIs are then solved simultaneously to find a positive definite matrix Y and a matrix L such that the both LMIs in (19) and (20) are satisfied. The YALMIP Toolbox together with Matlab is one of the available software to solve the problem. Commands for solving the problem are given as follows:

```
Y = sdpvar(2,2);
L = sdpvar(1,2,'full');
F = [Y > 0];
F = [F, [Y*A1'+A1*Y+Bs*L+L'*Bs'] < 0];
F = [F, [Y*A2'+A2*Y+Bs*L+L'*Bs'] < 0];
solvesdp(F,-trace(Y))
```

TABLE I
PAASCS PARAMETERS [23]

Parameter	Value	Unit	Parameter	Value	Unit
a_0	340	m/s	V_p	0.12	m^3
L_c	0.8	m	A_c	0.0038	m^2
ρ	1.2041	kg/m^3	A_s	0.0314	m^2
m_s	2	kg			

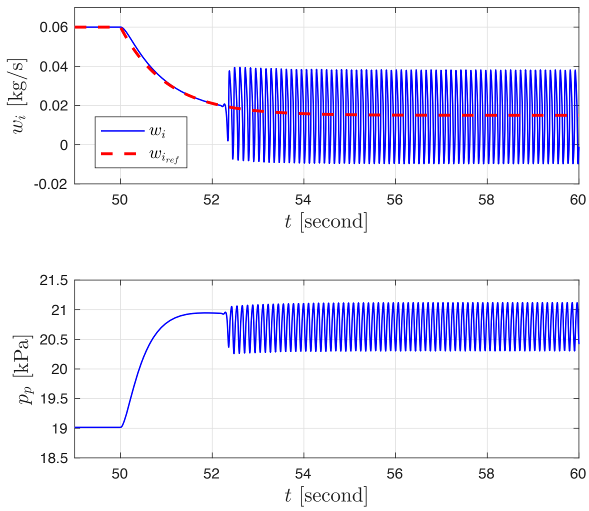


Fig. 3. COMPRESSION SYSTEM STATES WITHOUT SURGE CONTROL.

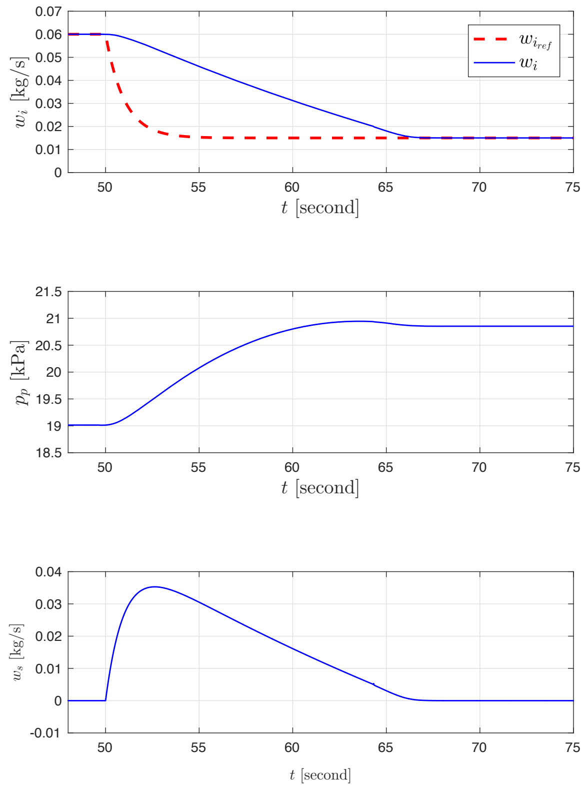


Fig. 4. CLOSED LOOP TIME RESPONSE.

While the matrices Y and L are found, the matrices P and K are obtained by $P = Y^{-1}$ and $K = LY^{-1}$, respectively.

A simulation is done to evaluate the closed loop system performance using parameters data in Table I and a compressor performance curve shown in Figure 2. The simulation scenario is given as follows. A compressor is initially operating steady at mass flow 0.06 kg/s and then the operating point is changed at t = 50 seconds by reducing the mass flow to 0.015 kg/s which is crossing the compressor surge line.

The simulation results are shown in Figure 3. It is shown that the open loop compressor system experiences oscillation in plenum pressure and inlet mass flow, and is known as compressor surge. Moreover, the compressor surge is known as a deep surge as the compressor mass flow is reversed (negative mass flow). While Figure 4 shows simulation results of the closed loop system using the designed control law, the system operates stable in the desired operating point, which means that the PAASCS is able to stabilize compressor surge. However, the closed loop system has a long settling time and the piston mass flow is very high compare to the compressor mass flow which is not practical. A piston with a large diameter and fast movement is required to generate the high piston mass flow. The performance of the closed loop system is therefore needed to be improved.

V. LMI-LQR METHOD

Considering the simulation results in the previous section, the closed loop system performance needs to be improved. Using the LMI formulation in (16), we can not do any performance adjustment as the matrices A and B are given from the plant, and the matrices P and K are obtained through a computational process using the YALMIP Toolbox. Performance adjustment bases on a cost function is commonly done in optimal control theory, for example linear quadratic regulator (LQR). Fortunately, combination of LQR method and LMI method for control system design (LMI-LQR) has been presented in [19], [21], [24], [25] and summarized as follows. The LMI-LQR gives an opportunity to adjust the performance of a closed loop system designed using LMI. The concept of LMI-LQR is described as follows. A LQR problem for a system:

$$\dot{x} = Ax + Bu \tag{21}$$

is basically to find a control gain K such that a states feedback control $u = Kx$ minimizes a cost function:

$$J = \frac{1}{2}\int_0^\infty \left(x^TQx + u^TRu\right) dt, \tag{22}$$

where $Q \geq 0$ and $R > 0$. Using $u = Kx$, the closed loop system of (21) is given by:

$$\dot{x} = [A + BK]\,x \tag{23}$$

and the cost function (22) can be expressed as:

$$J = \frac{1}{2}\int_0^\infty x^T \left(Q + K^T RK\right) xdt. \quad (24)$$

Assuming (23) is asymptotically stable by existing a Lyapunov function:

$$V(x) = x^T Px > 0 \quad (25)$$

then time derivative of $V(x)$ along the system trajectories (23) is given by:

$$\dot{V}(x) = x(A^T P + PA + K^T B^T P + PBK)x < 0 \quad (26)$$

where $P > 0$. The negative definiteness of (26) can be reinforced by defining:

$$\dot{V}(x) < -x^T \left(Q + K^T RK\right) x < 0. \quad (27)$$

A time integration from 0 to ∞ of (27) will result in:

$$V(\infty) - V(0) < -\int_0^\infty x^T \left(Q + K^T RK\right) xdt \quad (28)$$

or

$$x^T(\infty)Px(\infty) - x^T(0)Px(0) < -\int_0^\infty x^T \left(Q + K^T RK\right) xdt. \quad (29)$$

Since (23) is asymptotically stable then $x(\infty)$ is equal to zero such that (29) becomes:

$$x^T(0)Px(0) > \int_0^\infty x^T \left(Q + K^T RK\right) xdt. \quad (30)$$

Equation (30) shows upper bound of the cost function (24), such that the cost function will be minimum by minimizing the matrix P:

$$\begin{array}{ll} \min\limits_{P,K} & x^T(0)Px(0) \\ s.t & \dot{V}(x) < -x^T \left(Q + K^T RK\right) x. \end{array} \quad (31)$$

Expressing the minimization constraint along the trajectories of system (23), the (31) becomes:

$$\begin{array}{ll} \min\limits_{P,K} & x^T(0)Px(0) \\ s.t & (A + BK)^T P + P(A + BK) + (Q + K^T RK) < 0. \end{array} \quad (32)$$

Since the initial condition $x(0)$ is given, it can be eliminated from (32) and the minimizing problem becomes:

$$\begin{array}{ll} \min\limits_{P,K} & tr(P) \\ s.t & (A + BK)^T P + P(A + BK) + Q + K^T RK < 0. \end{array} \quad (33)$$

Equation (33) is a non-convex optimization problem where the constraint is BMI. A transformation is required to transform the BMI into LMI as done in the previous section. Define $Y = P^{-1}$ and do pre- and post- multiplications to the optimization constraint of (33) such that the constraint becomes:

$$YA^T + AY + YK^T B^T + BKY + YQY + YK^T RKY < 0. \quad (34)$$

Define $L = KY$ and substituting it into (34) results in:

$$YA^T + AY + L^T B^T + BL + YQY + L^T RL < 0. \quad (35)$$

By using Schur complement, (35) can be expressed as:

$$\begin{bmatrix} (AY + BL)^T + (AY + BL) & Y & L^T \\ Y & -Q^{-1} & 0 \\ L & 0 & -R^{-1} \end{bmatrix} < 0. \quad (36)$$

which is in a LMI. A detail explanation of Schur complement is given in the Appendix. Equation (33) is therefore expressed as a convex optimization as follows:

$$\begin{array}{ll} \max\limits_{Y,L} & tr(Y) \\ s.t & \begin{bmatrix} (AY + BL)^T + (AY + BL) & Y & L^T \\ Y & -Q^{-1} & 0 \\ L & 0 & -R^{-1} \end{bmatrix} < 0. \end{array} \quad (37)$$

Applying the LMI-LQR method in PAASCS design is presented as follows. Define weighting cost function matrices $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $R = 10$. YALMIP Toolbox is used to solve the LMI by the following commands:

```
Q=eye(2);
R=10;
Y = sdpvar(2,2);
L = sdpvar(1,2,'full');
F = [Y >= 0];
F = [F, [-A1*Y-Bs*L + (-A1*Y-Bs*L)' Y L';...
         Y inv(Q) zeros(2,1);...
         L zeros(1,2) inv(R)] > 0];
F = [F, [-A2*Y-Bs*L + (-A2*Y-Bs*L)' Y L';...
                 Y inv(Q) zeros(2,1);...
                 L zeros(1,2) inv(R)] > 0];
solvesdp(F,-trace(Y))
K = double(L)*inv(double(Y));
```

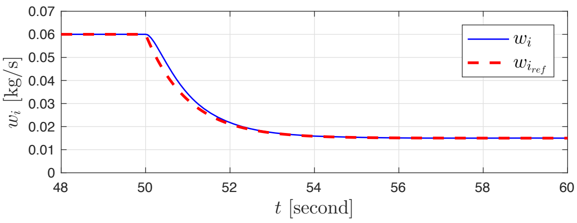
and running the code results in a new control gain K. Simulation of PAASCS using the new control gain for the same system parameters and simulation scenario results in an improvement of the closed loop system performance as shown in Figure 5. Simulation of the PAASCS designed using LMI-LQR results in stabilized compressor surge where the control system requires much less piston mass flow and much faster system response than the PAASCS designed using LMI only.

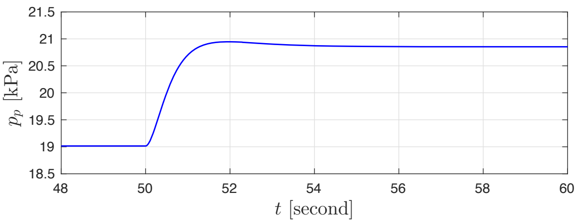
VI. Conclusion

A control design of an active compressor surge control system using LMI method has been presented. The LMI method provides an analytic solution to obtain a control gain and a stability proof. A study case of applying the LMI method for the PAASCS resulted in a linear state feedback control which is able to stabilize the compressor surge. The control system performance is improved significantly by combining the LMI method and LQR method. This method is very systematically for nonlinear control system design by approaching a nonlinear system as a linear parameter varying system and the control system design problem is expressed as a LMI problem. The LMI solution is obtained directly using the available software.

Acknowledgment

The authors acknowledge the financial support of Siemens Oil and Gas Solutions Offshore through the Siemens-NTNU collaboration project in a period of 2009-2013.





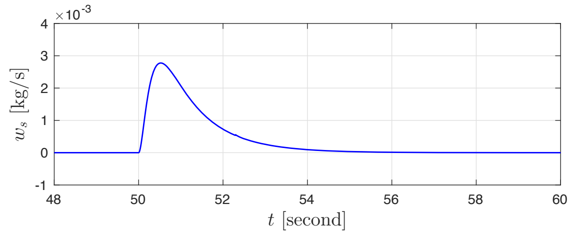


Fig. 5. CLOSED LOOP TIME RESPONSE LQR.

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APPENDIX

APPENDIX: LMI-LQR DERIVATION

$$
\begin{gathered}
(A+BK)^TP+P(A+BK)<-(Q+K^TRK)\\
A^TP+K^TB^TP+PA+PBK+Q+K^TRK<0\\
X(A^TP+K^TB^TP+PA+PBK+Q+K^TRK)X<0\\
XA^T+XK^TB^T+AX+BKX+XQX+XK^TRKX<0\\
XA^T+L^TB^T+AX+BL+XQX+XK^TRKX<0\\
XA^T+AX+L^TB^T+BL+XQX+L^TRL<0\\
XA^T+AX+L^TB^T+BL+\begin{bmatrix} X & L^T \end{bmatrix}\begin{bmatrix} Q & 0\\ 0 & R\end{bmatrix}\begin{bmatrix} X\\ L\end{bmatrix}<0
\end{gathered}
$$

By using Schur complement it can be represented as:

$$
\underbrace{XA^T+AX+L^TB^T+BL}_{S_{11}}+\underbrace{\begin{bmatrix} X & L^T \end{bmatrix}}_{S_{12}}\underbrace{\begin{bmatrix} Q & 0\\ 0 & R\end{bmatrix}}_{S_{22}}\underbrace{\begin{bmatrix} X\\ L\end{bmatrix}}_{S_{21}}<0
$$

and then

$$
\begin{bmatrix} XA^T+AX+L^TB^T+BL & X & L^T\\ X & -Q^{-1} & 0\\ L & 0 & -R^{-1}\end{bmatrix}<0.
$$

Schur complement of s_{22} in S:

$$
S=\begin{bmatrix} s_{11} & s_{12}\\ s_{21} & s_{22}\end{bmatrix}<0 \Leftrightarrow s_{11}-s_{12}(s_{22})^{-1}s_{21}<0
$$