Constitutive Model for Long-Term Behavior of Saturated Frozen Soil

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ABSTRACT

In this paper a two stress-state variables model with solid phase stress and cryogenic suction is introduced for simulating the long-term behavior of frozen soils. The solid phase stress is introduced as the combined stress of soil grains and ice. The model is able to represent the behavior of frozen soils in different temperatures and ice contents with a single set of parameters. It is also able to simulate the transition between frozen and unfrozen states. The so-called over-stress methodis applied for simulating the elastic-viscoplastic deformation of the material due to variation of solid phase stress, while an elastic-plastic formulation is applied for the suction induced deformation. In unfrozen state, the model becomes a conventional elastic-viscoplasticcritical state model. The model is implemented in PLAXIS and results are compared with the available testdata and reasonable agreement is achieved.

INTRODUCTION

Interest in understanding the mechanical behavior of frozen soils has been increasingly growing due to the need for engineering activities in permafrost, seasonally frozen grounds and artificial ground freezing.Long-term observation of ground deformation under engineering structures in warm permafrost regions, e.g. Qinghai-Tibetan, indicates that thawingdeformation is not the only source ofsettlement,but also creep of frozen soil should be considered as another significant source(Jilin et al. 2007).

Saturated frozen soil is a natural particulate composite, composed of solid grains, ice and unfrozen water. Defining the relevant stress state variables is essential for developing constitutive models for such a material. Total stress based models have been widely used in the literature to describe the mechanical behavior of frozen soils (Arenson and Springman 2005; Qi et al. 2013; Wang et al. 2014; Xu 2014). However, working with total stress, description of soil behavior in the presence of unfrozen water will face some significant difficulties which are not clearly addressed. Beside the total stress approach,Zhang et al. (2016) proposed an effective stress based rheological model for simulating long-term deformation of frozen soils.

As a more efficient method, the two stress-state approachwas proposed by Nishimura et al. (2009) for developing an elastic-plastic model for saturated frozen soil. They employed the net stress (i.e. the excess of total stress over ice pressure) and the cryogenic suction as the relevant stress variables.This approach is not fully consistent with the microscopic description of the freezing/thawing processes. According to Wettlaufer and Worster (2006), there exists two types of mechanisms controlling the behavior regarding ice content and temperature variations: curvature-induced premelting and interfacial premelting mechanisms (Fig. 1).The former is the result of the surface tension and acts very similar to the capillary suction by bonding the grains together. This mechanism has been effectively considered in this model by noting the analogy between the capillary and cryogenic suctions. The interfacial mechanism is the result of disjoining pressure and tends to widen the gap by sucking in more water.This mechanism has not been taken into account in the model proposed by Nishimura et al. (2009).

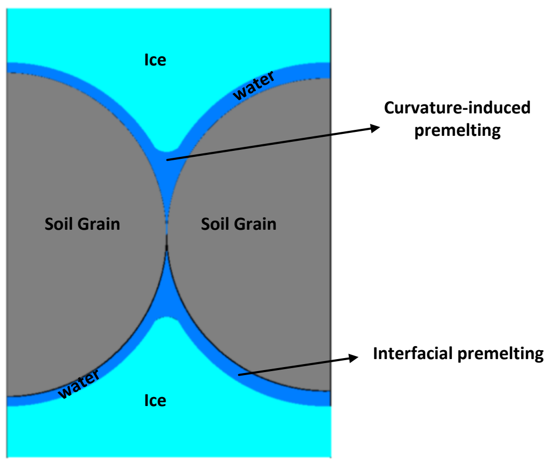

Figure 1. Schematic representation of the curvature induced premelting and interfacial premelting mechanisms(Ghoreishian Amiri et al. 2016A)

Ghoreishian Amiri et al. (2016A)proposed another two stress variables model by introducing the solid phase stress and the cryogenic suction as the relevant stress variables. The solid phase stress is defined as the combined stress in the soil grains and ice. Thus, in contrast with the model proposed by Nishimura et al. (2009), the contribution of the ice phase in carrying the shear stress of the system was implicitly considered in this model. Indeed, the model was consistent with the micromechanical description of the behavior due to variation of ice content and temperature; i.e. curvature-induced premelting and interfacial premelting mechanisms. The elastic-viscoplastic version of this model has been introduced by Ghoreishian Amiri et al. (2016B)and reasonable results were reported.

In this paper, the elastic-viscoplastic model proposed by Ghoreishian Amiri et al. (2016B)is modified and implemented in PLAXIS using an implicit integration scheme. The proposed model is applied to simulate a long-term plate loading experiment conducted by Zhang et al. (2016)in a warm permafrost region in Qinghai-Tibetan. The simulation results are validated against the experimental data and reasonable agreement is achieved.

MODEL FORMULATION

The solid phase stress; i.e. the combined stress in the soil grains and ice; and the cryogenic suction are considered as the relevant stress variables

$$\boldsymbol{\sigma}^* = \boldsymbol{\sigma} - s_w p_w \boldsymbol{I} \tag{1}$$

$$S = p_i - p_w = -\rho_i l \ln\frac{T}{T_0} \tag{2}$$

where $\boldsymbol{\sigma}^*$ is the solid phase stress, s_w is the unfrozen water saturation (i.e. the ratio of the volume of unfrozen water on the volume of frozen and unfrozen water), $\boldsymbol{I}$ denotes the unit tensor, S is the cryogenic suction, p_w and p_i denote the pressure of water and ice phases, respectively, ρ_i indicates the density of ice, l is the latent heat of fusion, T stands for temperature on the thermodynamic scale and T_0 is the freezing/thawing temperature of water/ice at a given pressure.

A strain increment is assumed to be decomposed into two major parts due to variations of solid phase stress and suction. In this model, elastic-viscoplastic behavior is considered for the deformation due to variation of solid phase stress, while elastic-plastic behavior is assumed for the suction induced deformation. Thus, any strain increment, $d\boldsymbol{\varepsilon}$, can be additively decomposed into the following parts

$$d\boldsymbol{\varepsilon} = d\boldsymbol{\varepsilon}^{me} + d\boldsymbol{\varepsilon}^{se} + d\boldsymbol{\varepsilon}^{mvp} + d\boldsymbol{\varepsilon}^{sp} \tag{3}$$

where $d\boldsymbol{\varepsilon}^{me}$ and $d\boldsymbol{\varepsilon}^{mvp}$ are the elastic and viscoplastic parts of the strain due to the solid phase stress variation, $d\boldsymbol{\varepsilon}^{se}$ and $d\boldsymbol{\varepsilon}^{sp}$ are the elastic and plastic parts of the strain due to the cryogenic suction variation, respectively.

Elastic Response. The elastic part of the strain due to the solid phase stress variation can be calculated based on the equivalent elastic parameters of the mixture

$$G = (1-s_i)G_0 + \frac{s_i E_f}{2(1+\nu_f)} \tag{4}$$

$$K = (1-s_i)\frac{(1+e)p^*}{\kappa_0} + \frac{s_i E_f}{3(1-2\nu_f)} \tag{5}$$

where

$$E_f = E_{f_{ref}} - E_{f_{inc}}\left(T - T_{ref}\right) \tag{6}$$

G and K are the equivalent shear modulus and bulk modulus of the mixture, respectively, G_0 and κ_0 stand for the shear modulus and the elastic compressibility coefficient of the soil in an unfrozen state, respectively, p^* is the solid phase mean stress, E_f and ν_f denote the Young's modulus and Poisson's ratio of the soil in the fully frozen state, respectively, s_i is the ice saturation, $E_{f_{ref}}$ is the value of E_f at the reference temperature (T_{ref}) and $E_{f_{inc}}$ denotes the rate of change in E_f with temperature.

The elastic part of the strain due to suction variation is calculated as

$$d\boldsymbol{\varepsilon}^{se} = \frac{\kappa_s}{3(1+e)} \times \frac{dS}{(S+p_{at})}\boldsymbol{I} \tag{7}$$

where κ_s is the compressibility coefficient due to suction variation within the elastic region, e is the void ratio and p_{at} is the atmospheric pressure.

Reference, Dynamic and Yield Surfaces. Based on the overstress method, instead of the common yield surface, reference and dynamic surfaces are defined to formulate the model in the elastic-viscoplastic context. The inelastic deformation can be calculated based on the distance of the reference and dynamic surfaces. The loading collapse (LC) reference surface is defined as

$$F_r = \left(p_r^* + p_{tr}^*\right)\left[\left(p_r^* + p_{tr}^*\right)s_w^m - \left(p_{yr}^* + p_{tr}^*\right)\right] + \left(\frac{q_r^*}{M}\right)^2 \tag{8}$$

where

$$p_{yr}^* = p_c^*\left(\frac{p_{y_{0r}}^*}{p_c^*}\right)^{\frac{\lambda_0-\kappa}{\lambda-\kappa}} \tag{9}$$

$$\lambda = \lambda_0\left[(1-r)\exp(-\beta S) + \mathrm{r}\right] \tag{10}$$

and $\boldsymbol{\sigma}_r^*$ indicates a stress point on the LC reference surface, M stands for the slope of the critical state line, p_{tr}^* shows the apparent cohesion of the soil, p_c^* indicates the reference stress, $p_{y_{0r}}^*$ is the reference preconsolidationstress for unfrozen condition, κ denotes the compressibility coefficient of the system within the elastic region, λ_0 represents the compressibility coefficient for the unfrozen state along virgin loading, r is a constant related to the maximum stiffness of the soil (for infinite cryogenic suction), β is a parameter controlling the rate of change in soil stiffness with suction, and m is a parameter controlling the strength of the material with ice saturation.

The LC dynamic surface should always pass through the current stress sate (p^*, q^*, S) and also keep a similar shape to the LC reference surface. The LC dynamic surface is defined as

$$F_d = \left(p^* + p_{td}^*\right)\left[\left(p_d^* + p_{td}^*\right)s_w^m - \left(p_{yd}^* + p_{td}^*\right)\right] + \left(\frac{q^*}{M}\right)^2 = 0 \tag{11}$$

where

$$p_{yd}^* = R \, . \, p_{yr}^* \tag{12}$$

$$p_{td}^* = R \, . \, p_{tr}^* \tag{13}$$

p^* and q^* are the solid phase mean stress and deviatoric stress, respectively, and R indicates the similarity ratio of LC dynamic and LC reference surfaces. Similar to Ghoreishian Amiri et al.

(2016A) and Ghoreishian Amiri et al. (2016B), the grain segregation line is defined to simulate the plastic deformation due to suction variation.

$$F = S - S_{seg} = 0. \quad (14)$$

where S_{seg} is the threshold value of suction for grain segregation phenomenon.

Hardening Rules. The following hardening rules are considered in this model

$$\frac{dp^*_{y_{0r}}}{p^*_{y_{0r}}} = \frac{1+e}{\lambda_0 - \kappa_0} d\varepsilon_v^{mvp} + \frac{1+e}{\lambda_0 - \kappa_0} d\varepsilon_v^{sp} \quad (15)$$

$$\frac{dS_{seg}}{S_{seg} + p_{at}} = -\frac{1+e}{s_w(\lambda_s + \kappa_s)} d\varepsilon_v^{sp} - \frac{1+e}{(\lambda_s + \kappa_s)}\left(1 - \frac{S}{S_{seg}}\right) d\varepsilon_v^{mvp} \quad (16)$$

$$\frac{dp^*_{tr}}{p^*_{tr}} = -k_{t_1}\frac{dS}{p^*_{tr}}\left(H(-p^*_{tr}) \times H(dS)\right) - k_{t_2}.d\varepsilon_q^{mvp} \quad (17)$$

where

$$H(x) = \begin{cases} 0 & x \le 0 \\ 1 & x > 0 \end{cases} \quad (18)$$

k_{t_1} and k_{t_2} are model parameters, and λ_s is the compressibility coefficient for an increase in suction across the virgin state.

Flow Rules. The following flow rules are employed to calculate the viscoplastic and plastic portion of strain due to variation of solid phase stress and cryogenic suction, respectively

$$d\boldsymbol{\varepsilon}^{mvp} = \mu_0 \frac{\lambda_0(\lambda - k)}{\lambda(\lambda_0 - k_0)}\left\langle R^N \right\rangle \frac{\partial Q_1}{\partial \boldsymbol{\sigma}^*}.dt = \Delta\lambda_1 \times \frac{\partial Q_1}{\partial \boldsymbol{\sigma}^*} \quad (19)$$

$$d\boldsymbol{\varepsilon}^{sp} = -d\lambda \frac{\partial F}{\partial S}\boldsymbol{I} \quad (20)$$

where

$$N = N_0 + b_1 \times S - b_2 \times s_i \quad (21)$$

$$Q_1 = s_w^{\gamma}\left[p^* - \frac{p^*_{yd} + p^*_{td}}{2}\right]^2 + \left(\frac{q^*}{M}\right)^2 \quad (22)$$

$\langle \; \rangle$ is the Macaulay brackets, dt is the time increment, μ_0 is the fluidity of unfrozen soil at the reference strain rate, N_0 is the strain rate coefficient of unfrozen soil, b_1, b_2 and γ are three model parameters, and λ and λ_1 are the plastic multiplayers.

It should be noted that the present model is similar to the model presented by Ghoreishian Amiri et al. (2016B), but with different reference, dynamic and plastic potential surfaces. In the present model, by changing the shapes of these surfaces with respect to unfrozen water saturationit could properly consider the behavior from a porous type material (unfrozen state) to a non-porous material (fully frozen state). Figures 2 shows the shape of the reference and dynamic surfaces of the present model in $p^* - q^*$ plane, and for different states of the material.

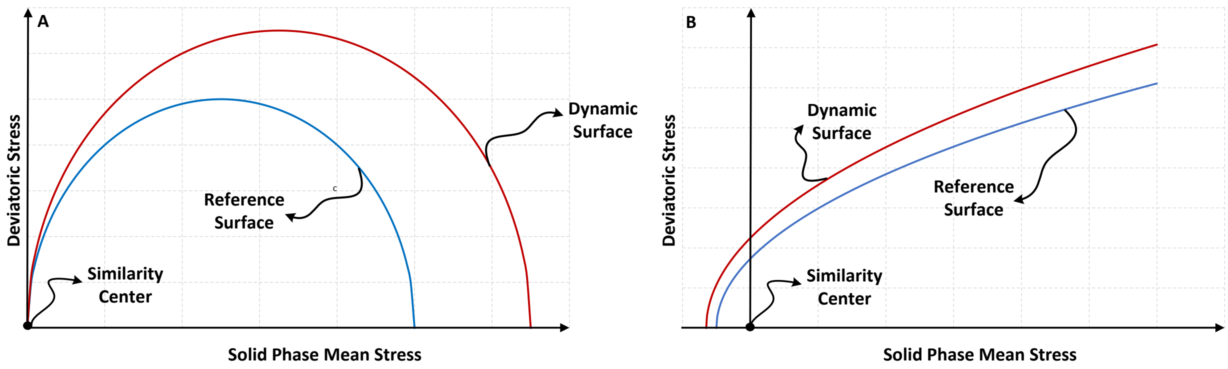

Figure 2. Reference and dynamic surfaces of the model for: A) unfrozen state; B) fully frozen state of the material with m=1

NUMERICAL SIMULATION

The model is implemented in PLAXIS and applied to simulate the results from the in situ plate loading experiments reported in Zhang et al. (2016). Figure 3 shows the geometry of the model and the hydro-thermal properties of the soil layers.The mechanical parameters of the soil layers that are used to simulate the experiment are listed in table 1. More details for the initial and boundary conditions, and also for seasonal temperature variation can be found in Zhang et al. (2016).

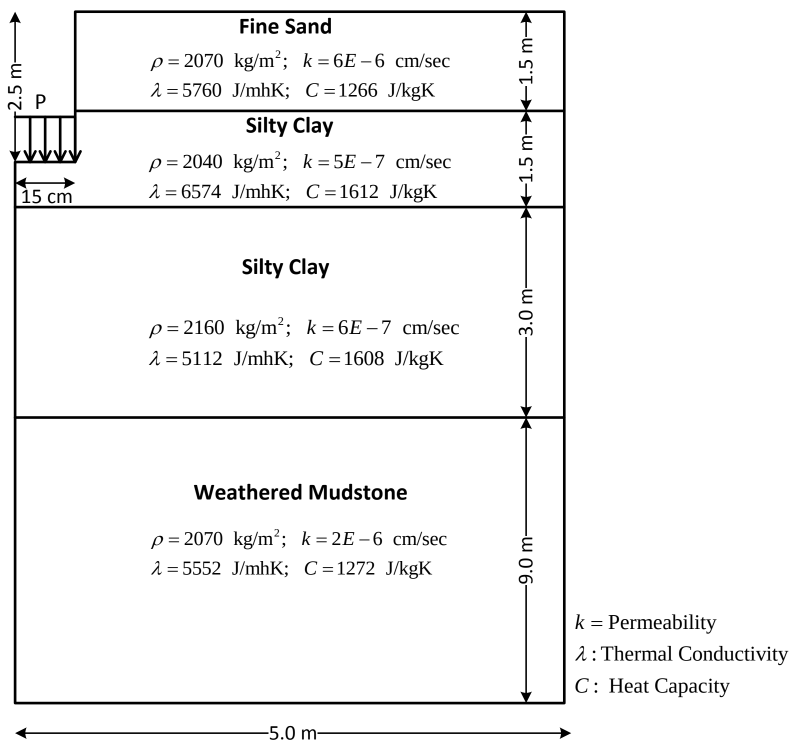

Figure 3. Geometry of the model

Table 1. Mechanical parameters of the soil layers

Parameter	Units	Fine Sand	Silty Clay 1	Silty Clay 2	Mudstone
E_{fRef}	Pa	160E6	140E6	150E6	190E6
E_{fIncr}	Pa/K	15E6	10E6	10E6	5E6
ν_f	-	0.48	0.45	0.46	0.48
G_0	Pa	4E6	5E6	8E6	15E6
κ_0	-	0.01	0.02	0.02	0.01
p^*_c	Pa	7000	5000	5000	5000
λ_0	-	0.07	0.2	0.2	0.09
γ	-	0.01	0.1	0.05	0.1
m	-	1	1	1	1
K_{t1}	-	0.25	0.35	0.25	0.45
M	-	1.1	0.85	0.95	1.25
λ_s	-	0.4	0.4	0.4	0.4
κ_s	-	8E-3	8E-3	8E-3	8E-3
r	-	0.59	0.49	0.49	0.57
β	Pa^{-1}	0.1E-6	0.15E-6	0.15E-6	0.12E-6
K_{t2}	-	2.5	1.5	1.5	1
μ_0	s^{-1}	0.01E-6	0.08E-6	0.1E-6	0.1E-9
N_0	-	30	25	25	35
b_1	Pa^{-1}	2E-6	1E-6	1E-6	0.03E-3
b_2	-	5	7	7	2
p^*_{y0r}	Pa	40E3	55E3	1E5	1.5E5
S_{seg}	Pa	15E6	15E6	15E6	15E6

In this experiment, the settlement and ground temperature were measured and reported for 8 years. From 2006 to 2014, the platform was step-loaded three times: 0.09 MPa, 0.19 MPa and 0.29 MPa for 3, 1 and 4 years, respectively. Figure 4 shows a comparison of the measureddata and model prediction for the long-term settlement of the plate. As shown in the figure, the creep rate increases with increasing load and temperature.

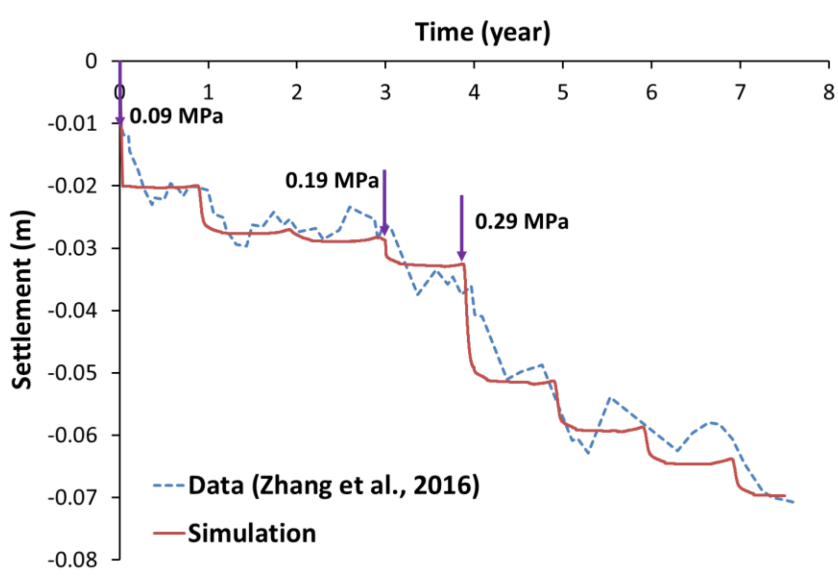

Figure 4. Comparison of the calculated and measured settlement results

CONCLUSION

In this paper a two stress-state elastic-viscoplastic model was proposed to simulate the long-term behavior of frozen soils. The unfrozen water saturation dependency of the reference, dynamic and plastic potential surfaces of the model provides the flexibility for simulatingthe behavior from a porous type material, i.e. unfrozen state, to a non-porous material, i.e. fully frozen state.The model was validated against the results of a long-term plate loading experiment and reasonable agreement was achieved.

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