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REGULARIZED METHODS OF NOISY SIGNALS DIFFERENTIATION IN REAL TIME

One particular case of solving the problem of interpreting observations is considered, when the instrument function of the measuring transducer is a pure integrator. In this case, the problem of interpretation is reduced to differentiating the output signal of the measuring transducer.

Key words: *noisy signal, numerical differentiation, Volterra integro-power series, regularization.*

Introduction. As examples of the problems of interpretation of this class, we can calculate the speed along the measured path, calculate the radiation intensity from the measured dose of radiation, and a number of other problems. The procedure of stable numerical differentiation is also used in the construction of mathematical models of nonlinear dynamic objects in the form of the Volterra integro-power series. The problem of stable differentiation of a noisy signal is also of independent importance, for example, when constructing regulators with a differential component in automatic control systems.

Thus, the problem is differentiating stable signal $y(t)$ under the presence of noise and measurement errors. The main difficulty is in the instability of the solution $x(t) = y'(t)$, which is manifested due to the presence of interference in the initial information and errors in the performance of certain operations in accordance with the adopted algorithm. The problem of differentiation of a noisy signal $y(t)$ belongs to the class of ill-posed problems. Traditional approach to solving these problems is based on the use of approximation methods, in particular, on the use of regularization methods [1] by which the integral is solved

$$\int_0^t x(\tau) d\tau = y(t) - y(0). \quad (1)$$

Without loss of generality, we further assume $y(0) = 0$.

Let us consider methods of approximate stable differentiation of a noisy signal in real time, which make it possible to improve the accuracy of the solution of the problem mentioned. The proposed methods are simi-

lar to the method of regularization of A.N. Tikhonov with regularizers of zeroth, first and second orders.

Let us assume there is a known approximation $y_\delta(t)$ of the function $y(t)$, to be differentiated $y_\delta(t) = y(t) + \eta(t)$, where $\eta(t)$ — stationary random process with limited spectral density and zero expectation and function $y(t)$ has N limited derivatives $|y^{(n)}| < M$, where $n = L, 2 \dots N$, $M > 0$ and is a sufficiently smooth function of time.

It is known that the solution obtained by direct differentiation of $y_\delta(t)$ does not satisfy the conditions of correctness. [2] We consider the differentiation problem in the integral formulation

$$\int_0^t x_\delta(\tau) d\tau = y_\delta(t), \quad (2)$$

where $x_\delta(t) = x(t) + \mu(t)$, $\mu(t)$ — random process that occurs at the output due to the presence of interference $\eta(t)$ in the input signal.

We consider the construction of a family of regularizing operators for the differentiation problem and compare them in accuracy. According to [1] instead of (2) a Volterra integral equation of the second kind can be solved

$$\alpha x_\delta(t) + \int_0^t x_\delta(\tau) d\tau = y_\delta(t). \quad (3)$$

In [1] it was shown that with a suitable choice of α algorithm defined by equation (3) is regularizing.

As a result of applying the Laplace transform, we obtain

$$\alpha \cdot X_\delta(p) + \frac{1}{p} X_\delta(p) = Y_\delta(p), \quad (4)$$

where $X_\delta(p) \stackrel{.}{=} x_\delta(t)$, $Y_\delta(p) \stackrel{.}{=} y_\delta(t)$.

Adding member $\alpha x_\delta(t)$ to (2) corresponds to the Tikhonov regularization method with zeroth order regularizer [4].

Similarly, the regularization method with first and second order regularizes small terms are added to the equation (2) that are the function's $x_\delta(t)$ first and second derivatives.

Let us consider the following equations

$$\alpha \cdot \frac{p}{p+k} X_\delta(p) + \frac{1}{p} \cdot X_\delta(p) = Y_\delta(p), \quad (5)$$

$$\alpha \cdot \frac{p^2}{p^2 + k_1 p + k_2} X_\delta(p) + \frac{1}{p} \cdot X_\delta(p) = Y_\delta(p). \quad (6)$$

The presence of complete polynomials in the denominator of the added terms is due to the stability requirements of these systems in the sense of Lyapunov. Let's note, that equation (5) corresponds to the dynamic method of regularization of ill-posed problems [1].

Consider the process of passing a random signal $\eta(t)$ by differentiating the devices described by equations (4)–(6). Dispersion of the output signal for each device is determined by expressions

$$D_{X_1} = \frac{D_y}{2\alpha} \cdot \frac{1}{1+\alpha\beta}, \quad (7)$$

$$D_{X_2} = \frac{D_y}{2\alpha} \cdot \frac{k+\beta-\alpha k^2}{k+\beta+\alpha k^2}, \quad (8)$$

$$D_{X_3} = \frac{D_y}{2\alpha} \cdot \frac{\left[(\beta k_1 + k_2)(2\alpha k_2 + \beta + k_1) + (1 + \alpha\beta)(\alpha k_1^2 + \alpha k_2^2 - \beta k_2) \right]}{(\alpha k_2 - k_1) \left[\beta(k_1 + \beta + \alpha\beta^2) + k_2 \right]}. \quad (9)$$

This implies a theorem is valid for the algorithms (4)–(6).

Theorem. When $D_y \rightarrow 0$, exists $\alpha(D_y) = 0(D_y^p)$, such that $D_y/\alpha D_y \rightarrow 0$, and consequently, $D_{X_i} \rightarrow 0$ for any $0 < p < 1, i = 1, 2, 3$.

The theorem is qualitative in nature and indicates that all of the methods defined by the equations (4)–(6) have regularizing properties.

Obtained estimates (7)–(9) are characterized by an unbiased solution error that occurs due to the presence of random errors in the input signal.

Let us determine the value of the solution's offset error arising as a result of replacement of the exact equation (2) by approximate ones (4)–(6) for each of the algorithms.

Let us assume that the current operation input $y(t)$ is a slowly varying function of time, i.e. we will neglect the transient component of the solution process and consider only the forced motion of the system. This assumption is possible when all the roots of the characteristic equation for the system (4)–(6) are real, negative and maximum by modulus. By direct substitution, it can be shown that when

$$k = \frac{1}{4\alpha}, \quad k_1 = \frac{1}{3\alpha}, \quad k_2 = \frac{1}{27\alpha^2} \quad (10)$$

the influence of the transient component can be neglected, if the signal range of useful input $y(t)$ is limited in the segment: $|\omega| < 1/\alpha$ for equation (4), $|\omega| < 1/(2\alpha)$ for equation (5), $|\omega| < 1/(3\alpha)$ for equation (6).

Assuming that at the system input exact signal $y(t)$ works for each case from the equations (4)–(6), we find the output image $X(p)$ and the

image of the system error $E(p) = X(p) - X_T(p)$ by subtracting the equation of the unperturbed system $X_T(p) = pY(p)$:

$$E_1(p) = \frac{\alpha p^2}{\alpha p + 1} \cdot Y(p), \quad (11)$$

$$E_2(p) = \frac{\alpha p^3}{\alpha p^2 + p + k} \cdot Y(p), \quad (12)$$

$$E_3(p) = \frac{\alpha p^4}{\alpha p^2 + p^2 + k_1 p + k_2} \cdot Y(p). \quad (13)$$

We develop obtained transfer functions into a series by increasing powers of complex value p

$$E(p) = \left[c_0 + c_1 p + \frac{c_2}{2!} p^2 + \dots \right] \cdot Y(p), \quad (14)$$

converging for small values of p , ie, for sufficiently large values of time t , which corresponds to the settled process of output change for a given input form.

Moving in (14) to the original, we get the formula for the steady-state error

$$\varepsilon_{ycm}(t) = c_0 y(t) + c_1 \frac{dy(t)}{dt} + c_2 \cdot \frac{1}{2!} \cdot \frac{d^2 y(t)}{dt^2} + \dots \quad (15)$$

The values of c_0, c_1, c_2 are errors coefficients and are determined by the general rule of the development into the Taylor series.

$$c_0 = [W_\varepsilon(p)]_{p=0}, \quad c_1 = \left[\frac{dW_\varepsilon(p)}{dp} \right]_{p=0}, \quad c_2 = \left[\frac{d^2 W_\varepsilon(p)}{dp^2} \right]_{p=0}, \dots, \quad (16)$$

where $W_\varepsilon(p) = E(p)/Y(p)$.

We calculate coefficients for each case: for equation (4) $c_0 = c_1 = 0$, $c_2 = 21\alpha$; for equation (5) $c_0 = c_1 = c_2 = 0$, $c_3 = 31 \frac{\alpha}{k}$; for equation (6) $c_0 = c_1 = c_2 = c_3 = 0$, $c_4 = 41 \frac{\alpha}{k_2}$.

Thus, in equation (6) if the input signal $y(t)$ derivatives $\frac{d^n y(t)}{dt^n} = 0$ for $n \geq 4$, then the settled error will be absent. If $\frac{d^4 y(t)}{dt^4} \neq 0$, then a settled error will appear

$$|\varepsilon_{ycm}| \leq \frac{\alpha}{k_2} \cdot \max_t \left| \frac{d^4 y(t)}{dt^4} \right|. \quad (17)$$

It should be noted that the replacement of the original equation (2) by approximate equations (4)–(6) increases the system's astatism order, respectively for 1, 2 and 3 units. Increasing the order of astatism naturally leads to an improvement in the accuracy characteristics of the system.

Obviously, it is possible in principle to further increase the order of the system's astatism, but for each specific case it is necessary to analyze the costs associated with the complication of the calculation scheme and the resulting effect of increasing the accuracy of differentiation.

The total average quadratic error in the differentiation of the signal arriving in the mixture with noise is determined by the expression

$$\bar{\varepsilon}_\varepsilon^2 = D_x + \varepsilon_{ycm}^2. \quad (18)$$

Function $\bar{\varepsilon}_\varepsilon^2$ in the case when $\varepsilon_{ycm} \leq 0$, $D_x \leq 0$ has a minimum value, which is achieved for the regularization parameter α , satisfying equation

$$\frac{d\bar{\varepsilon}_\varepsilon^2}{d\alpha} = 0. \quad (19)$$

Conclusions. Thus, it is possible, with a priori information about the values of the input signal $y(t)$ derivatives, its spectral range, as well as the interference dispersion value D_y , to select one of the proposed relations (4)–(6) in the solution of a noisy signal differentiation problem, in which to determine the optimal value of the regularization parameter α considering (19). It should be noted that the quality of the solution depends on a priori information about the reliability of the solution ($y(t)$ derivatives values) and interference (noise dispersion D_y). In the limit, if there is a fully reliably source data, proposed methods allow to obtain accuracy-optimal solution of the problem.

Note also that in view of the Volterra properties conservation of proposed algorithms to approximate a continuous sustainable differentiation noisy signal can solve the task in real time by applying the method of direct modeling [3].

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Рассмотрен один частный случай решения задачи интерпретации наблюдений, когда аппаратная функция измерительного преобразователя является чистым интегратором. При этом задача интерпретации сводится к дифференцированию выходного сигнала измерительного преобразователя.

Ключевые слова: *зашумлений сигнал, численое дифференцирование, интегростепенной ряд Вольтерра, регуляризация.*

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МАСШТАБНІ АТАКИ НА ДЕЦЕНТРАЛІЗОВАНІ СИСТЕМИ, ЩО ПОБУДОВАНІ НА ОДНОРАНГОВИХ ПІРИНГОВИХ МЕРЕЖАХ

Представлені сценарії масштабних атак на децентралізовані системи на прикладі криптовалюти Bitcoin, які основані на вразливостях протоколу BGP та надмірній централізації першого рівня архітектури даних систем.

Ключові слова: *децентралізована система, Bitcoin, BGP, однорангова пірингова мережа, механізм досягнення консенсусу.*

Вступ. Найбільш популярною децентралізованою системою на даний момент є криптовалюта Bitcoin, ринкова капіталізація якої на початок 2017 року складала понад 18 мільярдів доларів США [1]. Популярність децентралізованих систем пояснюється відсутністю централізованого органа управління, відкритістю та можливістю збереження конфіденційності. Переважна більшість таких систем будується на єдиній архітектурі, яка показана на рис. 1.



Рис. 1. Загальна архітектура децентралізованих систем