Instantaneous Frequency Tracking of Harmonic Distortions for Grid Impedance Identification based on Kalman Filtering

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***Abstract*—This paper presents a non-invasive approach for real-time grid impedance identification based on the principle of instantaneous frequency tracking of voltage and current harmonics already present in the system, by the use of Kalman Filter (KF). The KF based technique is compared to the FFT frequency domain approach highlighting the advantages and disadvantages of each method. The main limitation of the KF-based identification lies in its intrinsic model based nature, which in this case will limit its scope to the system steady state and to stationary signals. The KF based approach presented in this paper, can be useful as a basis for real-time grid stability assessment based on the impedance identification.**

I. Introduction

Knowledge of the source and load impedances in an electrical grid is essential to estimate the stability of the grid using the impedance-based method with the generalized Nyquist stability criterion, first described in [1], with an application for AC systems in [2]. The impedance-based method requires, however, the injection of small signals in a wide range of frequencies to obtain a good estimate of the stability of the grid. This frequency scanning technique does not allow for a real-time estimation of the impedances and stability. The real-time identification of the impedances of the grid can be relevant considering the non-stationary environment where the impedance may vary with the operating conditions and with parameter variations over time [3]. This paper is proposing a non-invasive method to identify the impedances of the grid based on the use of the information of the distortions already present in the current and voltage waveforms, without resorting to any small-signal injection.

Modern electrical systems are including more and more non-linear loads that are a frequent source of harmonic distortions. Electrical grids that are already affected by distorted voltages and currents can readily benefit from a non-invasive approach of impedance identification that makes use of the information provided by such distortions and can lead to a straightforward grid stability assessment in time-domain. The tracking, at a given node, of each of the harmonic components present in the voltage and current, can enable the identification of the grid impedances in a non-invasive way, by simply calculating the ratio $V(\omega)/I(\omega) = Z(\omega)$ at each harmonic distortion.

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The commonly applied methods relying on frequency scanning [4], generally require assumptions to be made about the signal such as the signal being stationary and periodic, where the sampling frequency is equal to the number of samples multiplied by the fundamental frequency, the Nyquist sampling theorem holds and each frequency is an integer multiple of the fundamental frequency which will be the inverse of the window length [5].

Harmonics estimation in the time-domain using Kalman theory will track the phase and amplitude of the harmonics of a distorted signal (voltage and current) using state space form and has the potential for real-time processing due to the relatively low computational load. The use of KF for this problem is proposed in [5] using a linear KF as first described in [6]. These methods conclude that KF theory can estimate accurately harmonic components even during high power system disturbance conditions.

In the following, this paper will present how a linear KF can be used for combined harmonic identification and impedance estimation based on a linear state-space model and best fitting curve.

II. Harmonic Identification by Kalman Filter

The method for harmonic detection builds on the work from [7] where an Adaptive Kalman Filter (AKF) is introduced. In [8] a method for tracking the angular frequency ω for a Smart Grid is proposed by following the positive and negative sequence fundamental frequency for a three-phase system. By restating the problem to track all harmonics and assume that the angular frequency is known, tracking of positive and negative sequence harmonics can be done with inspiration from the work in that paper. The development for such a tracking is given in this section.

Given a three-phase signal in the abc frame in (1), where $\omega(t)$ is the angular frequency which is assumed to be known, $A_a(t), A_b(t), A_c(t)$ are the amplitude for each of the phases and ϕ_a, ϕ_b, ϕ_c are the initial phase angles for each phase. $\omega = 2\pi f$ is the angular frequency, and one defines $\theta_i(t) = \omega t + \phi_i$ for each phases $i \in \{a, b, c\}$.

$$S_a(t) = A_a(t)cos(\omega(t)t + \phi_a(t)) \tag{1a}$$
$$S_b(t) = A_b(t)cos(\omega(t)t + \phi_b(t)) \tag{1b}$$
$$S_c(t) = A_c(t)cos(\omega(t)t + \phi_c(t)) \tag{1c}$$

In a balanced system there is a relationship between the phase angles by stating them as time-invariant given by $\phi_b = \phi_a - \frac{2\pi}{3}$ and $\phi_c = \phi_a + \frac{2\pi}{3}$, and for the amplitudes $A_a = A_b = A_c = A$. In an unbalanced system, no such assumptions can be made, and one ends up with six variables for each time instant by stating the system in the *abc* frame for tracking the components of such a signal. To reduce the number of variables, the unbalanced three-phase system is transformed into three balanced systems named the positive, negative and zero sequence by using the Fortescue Theorem [9]. The system of the three balanced systems is given by

$$S_a = S_{a,p} + S_{a,n} + S_{a,0} \tag{2a}$$
$$S_b = S_{b,p} + S_{b,n} + S_{b,0} \tag{2b}$$
$$S_c = S_{c,p} + S_{c,n} + S_{c,0}. \tag{2c}$$

The unbalanced $\mathbf{S_{abc}} = [\mathbf{S_a}, \mathbf{S_b}, \mathbf{S_c}]^\mathbf{T}$ is now stated as 3 balanced systems $\mathbf{S_p} = [\mathbf{S_{a,p}}, \mathbf{S_{b,p}}, \mathbf{S_{c,p}}]^\mathbf{T}$, $\mathbf{S_n} = [\mathbf{S_{a,n}}, \mathbf{S_{b,n}}, \mathbf{S_{c,n}}]^\mathbf{T}$ and $\mathbf{S_0} = [\mathbf{S_{a,0}}, \mathbf{S_{b,0}}, \mathbf{S_{c,0}}]^\mathbf{T}$. By using the phase sequence one can state this problem in time domain form in (3) by using $\theta_p = \omega t + \phi_p$ and $\theta_n = \omega t + \phi_n$.

$$\begin{bmatrix} S_a(t) \\ S_b(t) \\ S_c(t) \end{bmatrix} = \mathbf{S_p} \begin{bmatrix} cos(\theta_p(t)) \\ cos(\theta_p(t) - \frac{2\pi}{3}) \\ cos(\theta_p(t) + \frac{2\pi}{3}) \end{bmatrix} + \mathbf{S_n} \begin{bmatrix} cos(\theta_n(t)) \\ cos(\theta_n(t) + \frac{2\pi}{3}) \\ cos(\theta_n(t) - \frac{2\pi}{3}) \end{bmatrix} + \mathbf{S_0} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{3}$$

This signal is transformed using the Clarke transform to the $\alpha\beta0$ stationary frame with the transformation defined in (4).

$$\mathbf{T}_{\alpha\beta0} = \begin{bmatrix} \frac{2}{3} & \frac{-1}{3} & \frac{-1}{3} \\ 0 & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{4}$$

The three-phase signal in $\mathbf{S}_{\alpha\beta\mathbf{0}}$ is obtained by using the defined transformation (5).

$$\mathbf{S}_{\alpha\beta\mathbf{0}}(\mathbf{t}) = \mathbf{T}_{\alpha\beta0}\mathbf{S_{abc}}(\mathbf{t}) \tag{5}$$

This transformation is useful because if the system is balanced, then the 0 component will always be reduced to zero. Applying the $T_{\alpha\beta0}$ transformation on the system in (3) one obtains the following.

$$\mathbf{S}_{\alpha\beta\mathbf{0}} = \mathbf{T}_{\alpha\beta0}\mathbf{S_{abc}} = \mathbf{T}_{\alpha\beta\mathbf{0}}\mathbf{S_p} \begin{bmatrix} cos(\theta_p) \\ sin(\theta_p) \\ 0 \end{bmatrix} + \mathbf{T}_{\alpha\beta\mathbf{0}}\mathbf{S_n} \begin{bmatrix} cos(\theta_n) \\ -sin(\theta_n) \\ 0 \end{bmatrix} \mathbf{T}_{\alpha\beta\mathbf{0}}\mathbf{S_0} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

By inspection of (6) one sees that all the zero components are zero, because of the assumption that the Fortescue's theorem holds. The reduced model with only the α and β component is shown in (8) using the reduced Clarke Transform as shown in (7):

$$\mathbf{T}_{\alpha\beta} = \begin{bmatrix} \frac{2}{3} & \frac{-1}{3} & \frac{-1}{3} \\ 0 & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \end{bmatrix} \tag{7}$$

$$S_{\alpha\beta} = \mathbf{T}_{\alpha\beta}\mathbf{S_{abc}} = \mathbf{T}_{\alpha\beta}\mathbf{S_p} \begin{bmatrix} cos(\theta_p) \\ sin(\theta_p) \end{bmatrix} + \mathbf{T}_{\alpha\beta}\mathbf{S_n} \begin{bmatrix} cos(\theta_n) \\ -sin(\theta_n) \end{bmatrix} \tag{8}$$

By remembering that $\theta_p = \omega t + \phi_o$ and $\theta_n = \omega t + \phi_n$ and using the sum and difference formulas for trigonometric functions: $\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$ and $\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$ one can write the (8) as (9). Here all the harmonics of the system are included into one equation. By stating the fundamental angular frequency of the grid as ω other harmonics, but not limited to natural numbers i, is a harmonic with angular frequency $\omega i = i\omega$. Here $A_{i,p}$ and $A_{i,n}$ are the magnitudes of the positive and negative sequence respectively for harmonic i. $\phi_{i,p}$ and $\phi_{i,n}$ are the initial phase angles for the negative and the positive sequence for harmonic i.

$$\begin{bmatrix} \mathbf{S}_\alpha(\mathbf{t}) \\ \mathbf{S}_\beta(\mathbf{t}) \end{bmatrix} = \sum_{i \in h_p} \begin{bmatrix} \cos(i\omega t) & -\sin(i\omega t) \\ \sin(i\omega t) & \cos(i\omega t) \end{bmatrix} \begin{bmatrix} A_{i,p}\cos(\phi i, p) \\ A_{i,p}\sin(\phi_{i,p}) \end{bmatrix} + \sum_{i \in h_n} \begin{bmatrix} \cos(i\omega t) & -\sin(i\omega t) \\ -\sin(i\omega t) & -\cos(i\omega t) \end{bmatrix} \begin{bmatrix} A_{i,n}\cos(\phi_{i,n}) \\ A_{i,n}\sin(\phi_{i,n}) \end{bmatrix} \tag{9}$$

$\mathbf{S_i}\cos(\phi_\mathbf{i})$ and $\mathbf{S_i}\sin(\phi_\mathbf{i})$ in (9) are written as $x_{i,1}$ and $x_{i,2}$ for making the formulation suitable for KF. Using this formulation one has separated the time-varying ωt and the slow varying part of magnitude and initial phase angle. The state-space and measurement equations are given in (10) and (11) which estimate the states $x_{i,1}$ and $x_{i,2}$ in which states have information about the phase and amplitude of each harmonic i analyzed by the KF.

$$y[k] = \sum_{h \in h_p} \begin{bmatrix} \cos(i\omega\Delta tk) & -\sin(i\omega\Delta tk) \\ \sin(i\omega\Delta tk) & \cos(i\omega\Delta tk) \end{bmatrix} \begin{bmatrix} x_{i,1} \\ x_{i,2} \end{bmatrix}_k + \sum_{i \in h_n} \begin{bmatrix} \cos(i\omega\Delta tk) & -\sin(i\omega\Delta tk) \\ -\sin(i\omega\Delta tk) & -\cos(i\omega\Delta tk) \end{bmatrix} \begin{bmatrix} x_{i,1} \\ x_{i,2} \end{bmatrix}_k \tag{10}$$

$$\begin{bmatrix} x_{i,1} \\ x_{i,2} \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_{i,1} \\ x_{i,2} \end{bmatrix}_k \tag{11}$$

A. Instantaneous Frequency Tracking

In contrast to the frequency domain approach of the FFT, the Kalman filter identification operates directly in the time domain, and there is no lag from measurements to the estimation result. Using frequency domain approach there is a minimum lag of one fundamental period before the estimation has meaningful data. In KF based estimation, the states are estimated using an adaptive KF in linear state space form with a random walk for the states which corresponds to the α and β signal magnitude at each time instant. The tuning of a KF is a difficult task when tracking signals with sudden changes; hence, an adaptive KF filter based on [7] is implemented. For adaptively changing the model error covariance $\mathbf{Q}$, depending on the errors and variations in the system, the update law for an adaptive Kalman filter algorithm is given in (12).

$V_i \cos(\phi_i)$ and $V_i \sin(\phi_i)$ in (9) are written as $x_{i,1}$ and $x_{i,2}$ for making the formulation suitable for KF. Using this formulation one has separated the time-varying ωt and the slow varying part of magnitude and initial phase angle. The state space and measurement equations are given in (10) and (11), which estimate the states $x_{i,1}$ and $x_{i,2}$ in which states have information about the phase and amplitude of each harmonic i analyzed by the KF.

$$\hat{\mathbf{x}}^{-}[k] = \mathbf{A}\hat{x}[k-1] \tag{12a}$$
$$\mathbf{P}^{-}[k] = \mathbf{A}\mathbf{P}[k-1]\mathbf{A}^{T} + \mathbf{Q}[k-1] \tag{12b}$$
$$\mathbf{K}[k] = \mathbf{P}^{-}[k]\mathbf{C}[k]^{T}(\mathbf{C}[k]\mathbf{P}^{-}[k]\mathbf{C}[k]^{T} + \mathbf{R})^{-1} \tag{12c}$$
$$\hat{x}[k] = \hat{x}^{-}[k] + \mathbf{K}[k](y[k] - \mathbf{C}[k]\hat{x}^{-}[k]) \tag{12d}$$
$$\hat{w}[k] = \mathbf{K}[k](y[k] - \mathbf{C}[k]\hat{x}^{-}[k]) \tag{12e}$$
$$\mathbf{Q}[k] = \sum_{i} \hat{w}_i[k]^2 \tag{12f}$$
$$\mathbf{P}[k] = (\mathbf{I} - \mathbf{K}[k]\mathbf{C}[k])\mathbf{P}^{-}[k] \tag{12g}$$

In the adaptive Kalman filter, one can define the size of the system N as twice the number of positive and negative harmonics. Then the $\mathbf{A}$ is a $N \times N$ system matrix, $\mathbf{C}$ is a $2 \times N$ measurement matrix. For the Kalman filter, the $\mathbf{Q}$ is a $N \times N$ adaptively updated covariance matrix, which is initialized to the zero matrix. $\mathbf{R}$ is a 2×2 measurement error matrix which needs to be tuned by measurement noise.

With the states estimated, the phasor information of the positive and negative sequence are obtained by calculating the magnitude and phase. Furthermore, the impedance could be calculated by voltage over current phasor division of $\mathbf{V_i}$ and $\mathbf{I_i}$ for each harmonic i estimated by

$$A_i = \sqrt{(x_{i,1}^2 + x_{i,2}^2)} \tag{13a}$$
$$\theta_i = arctan(\frac{x_{i,2}}{x_{i,1}}) \tag{13b}$$

In the system described in Fig.1, three-phase voltage and current measurements in the abc-frame, $\mathbf{V_{abc}}$ and $\mathbf{I_{abc}}$ are sampled. To ease the extraction of the phase and amplitude of the phasors, the Clarke transform is used to obtain the signal in the $\alpha\beta\gamma$ frame with the γ component omitted.

B. Comparison to the Sequence Analyzer in MATLAB

The KF based identification results shown in Fig. 2 and 4 are compared to a Fourier-based method [10] where a waveform

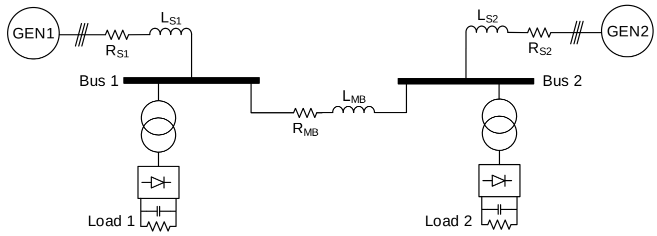

Fig. 1: Power Grid model with harmonic distortions due to the presence of the non-linear loads

$x(t)$ can be approximated by estimating all harmonics with angular frequency $\omega_k = k\frac{2\pi}{T}, k \in \{1, 2, ..\}$ and T is the sliding window length. This estimation method will provide a dynamic phasor with high accuracy when the harmonic is an integer multiple of the inverse of the window function length, which in most applications is one period of the fundamental frequency.

C. Prerequisites for Application of the Method

For the proposed method to function correctly, the assumption of neglecting zero sequences has to hold. If not, this could easily be added to the algorithm by changing the Clarke transform. For the method to measure impedances at multiple frequencies, the system must be distorted with harmonic frequencies. This assumption holds when using line commuted rectifiers which distort the grid with higher harmonic frequencies above the fundamental frequency. One of the limitation with this is that impedance below the fundamental frequency is not detected. In the Kalman measurement equation, (10), it is assumed that the waveform can be structured into harmonic sine functions with a given frequency and phase angle. If the signal does not have this characteristic, this equation has to be changed accordingly.

III. Simulation and Analysis

The method is verified by using two models, one simple grid model where a source impedance of the line and a generator are identified and a more complex system with a Modular-Multilevel Converter (MMC).

A. Power Grid and Measured Signals

The system under investigation in this paper is a shipboard power system with two generators and two loads based on 6-pulse diode rectifiers. The 6-pulse diode rectifier will distort the AC side of the converter currents with higher order harmonics because of the non-linear relation between voltages and currents [11]. These harmonic currents will propagate through the electrical grid and distort the source voltage for other grid-connected equipment. N_d-pulse diode rectifiers will create harmonic distortions in the positive sequence in the harmonic components in the set $h_p = \{1, N_dk+1|k = 1, 2, ...\}$ and in the negative sequence $h_n = \{N_dk - 1|k = 1, 2, ...\}$.

A simulation model is built in MATLAB Simulink using the SimPowerSystems library components. The first step is to estimate the phasor of the harmonics for the currents and voltages. The voltage is measured at Bus 1 and the current through Generator 1 in Fig. 1. The grid is simulated for 100ms before the estimation starts for the grid to stabilize. The signal is filtered by a low-pass filter with cut-off frequency equal to 13th harmonic component.

The grid is simulated for 50ms with a sampling time of 1×10^{-5} μs. This signal is low-pass filtered using a Finite Impulse Response filter and down-sampled according to the Nyquist sampling theorem. The grid has a nominal frequency of 50Hz.

B. Harmonics Identification

The KF based method in Fig. 2 and the sequence analyzer in Fig. 3 give both very similar results when tracking components of periodic signals. The KF has slightly faster response as it does not need one fundamental period before meaningful results are obtained. When adding a non-characteristic harmonic, the Fourier method is not able to track the harmonics with sufficient accuracy. This is because the signal is not an integer multiple of the inverse of the window length. However, the KF based method is unaffected by the presence of non-characteristic harmonics, resulting in a good identification as shown in Fig. 4 and Fig. 5. This is simulated by injecting two non-characteristic harmonics with frequencies of 75Hz and 175Hz, (1.5 and 3.5 harmonics) in the grid from Generator 1.

Both methods can track non-stationary signals, but the KF based method has a slightly faster response compared to the Fourier method with dynamics faster than the inverse of the window length. These results are seen in Fig. 10 with amplitude variation of the fundamental frequency component in addition to the 5th and the 13th order harmonics with no time-variation. In this case, the non-stationary signal is not generated by the grid, but artificially generated signals so that the time variation is better viewed instead of the transient after a step change in the grid.

For tracking of non-stationary signals in systems with dynamics, the assumption that there are only frequency components at the given frequencies does not hold outside of the steady state. The derivatives of such dynamical system are not equal to zero in the transient response. One can by this, argue that the method is a steady state method. In steady state, the impedance is simply a function of the imaginary angular frequency $j\omega$ for linear systems $Z(j\omega)$, but outside the steady state the impedance is a function of the complex variable s, $Z(s)$. This gives a smooth band of frequencies, which means that this is a steady state method where the signals are assumed to vary slowly.

C. Impedance Estimation of MMC Converter

The method is verified using a MATLAB Simulink model of a MMC as shown in Fig. 6 with the setup in Table I The MMC behaves as a voltage source at the ac terminal. In Fig. 7 the estimated impedance as a time series is shown and the impedance at a given time step is shown in Fig. 8. Using this model, the method can identify the impedance at each higher harmonic component and converges quickly. The impedance of each component is verified by doing an FFT analysis and also compared to the analytically obtained impedance.

TABLE I: System Specification

MMC setup	
$A_{gen1} = A_{gen2}$	690V
f_{nom}	50Hz

IV. Real-Time Impedance Identification

Based on the identified impedance values for each given harmonic component, the Least Squares method is used to best fit the identified points to an impedance curve following the technique described in [12]. The proposed method utilizes the harmonics present in the system to obtain all the operating points in each time step. By modeling the portion of the grid under investigation as a pure series R-L branch in series with a voltage source with component only for the fundamental frequency, the impedance is $Z = R_g + j\omega L_g$ and is equal for each harmonic where R_g and L_g is the sum of impedance in the generator and the distribution lines [13]. The estimated impedance magnitudes and phases on frequency are presented in Fig. 9 where the estimations from the Kalman filtering, the estimated impedance from the Least Squares method and the analytical expression for the impedance are shown. The result is promising when assuming an impedance model of the grid. For doing this estimation in real-time, a recursive least squares estimator could be used to fit the parameters to the measured impedance.

V. Discussions and Conclusion

A non-invasive method based on Kalman filter for tracking magnitude and phase of harmonics is proposed in this paper. The method can track harmonics and relate those to the real grid parameters with good accuracy. The method yields good results in the steady state for systems where certain frequencies are naturally occurring but comes short for estimating harmonics during transient response.

By using the proposed method, the positive and negative sequence harmonics are detected with the Kalman based harmonics tracker for systems in the steady state. To estimate the parameters correctly, based on the harmonics, a model of the impedance in the steady state has to be known. From this basic assumption, it is evident that the method will not yield accurate results in the presence of non-stationary signals, as for the case of the existence of an oscillatory instantaneous frequency.

From the measured harmonics, one can estimate the parameters specifying the impedance of a system by fitting them in real-time using a recursive least squares estimator. When the impedance can be parametrized from for instance linear elements, such as a resistor, capacitor, and inductor, this method can be used to estimate the impedance in real-time based on the harmonics already present in the grid in a non-invasive way.

From the estimated parameters of the impedance for different

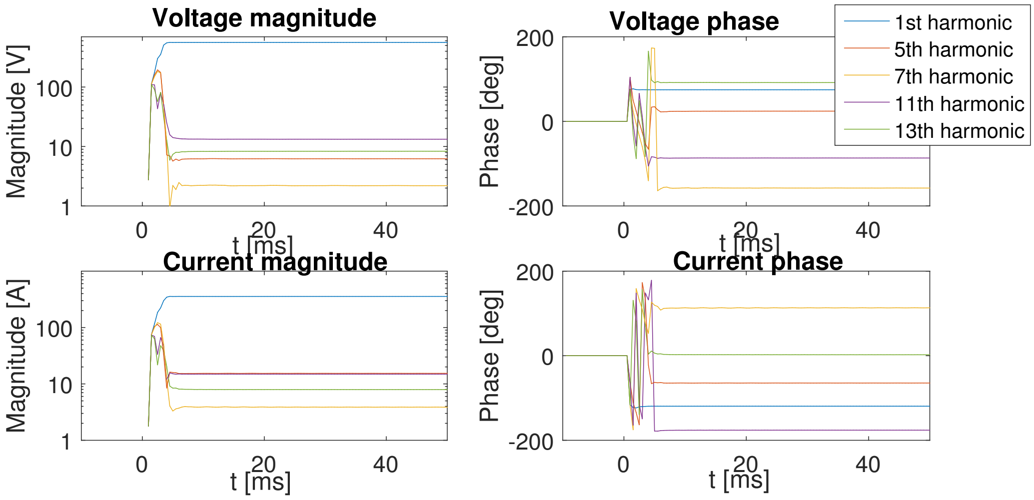

Fig. 2: Current and voltage harmonics estimated with Kalman filter

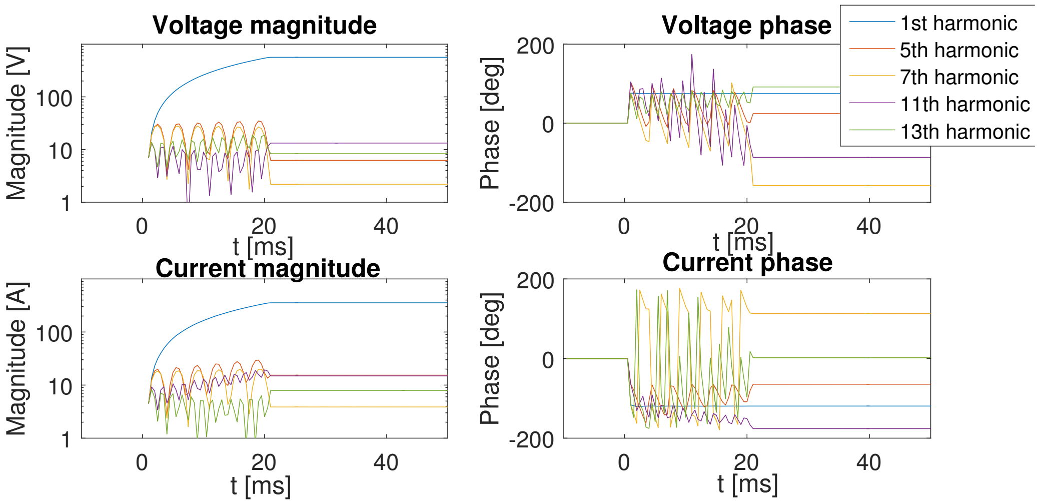

Fig. 3: Current and voltage harmonics estimated with Sequence Analyzer in MATLAB Simulink

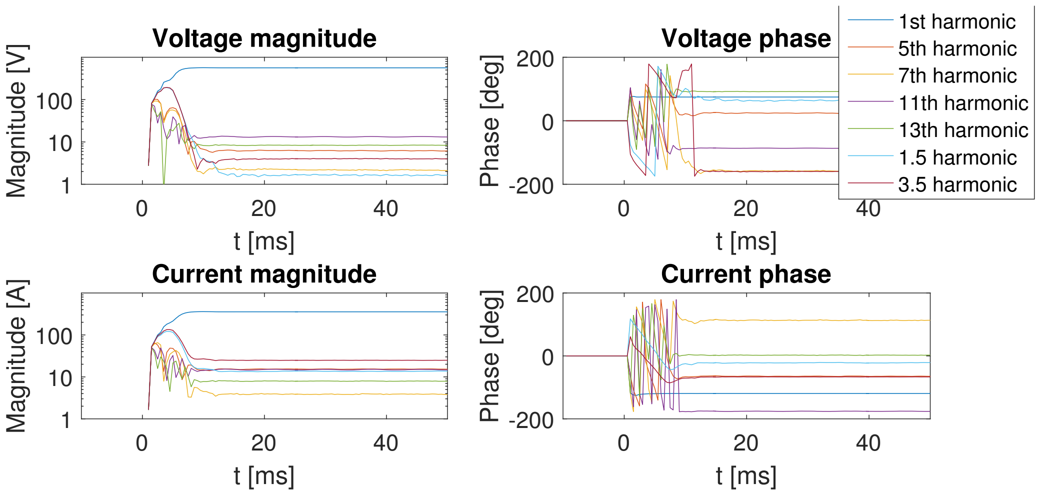

Fig. 4: Non-characteristics harmonics current and voltage phasor estimated with Kalman filter

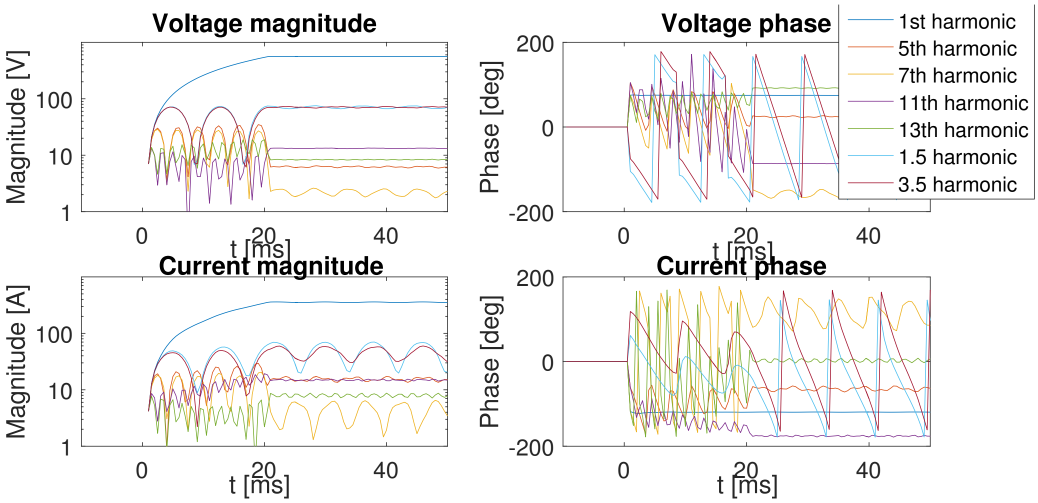

Fig. 5: Non-characteristics harmonics current and voltage phasor estimated with Sequence Analyzer in MATLAB Simulink

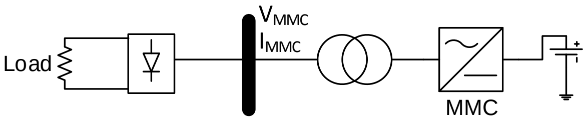

Fig. 6: Souce-Load System with MMC as source and Diode Rectifier as load

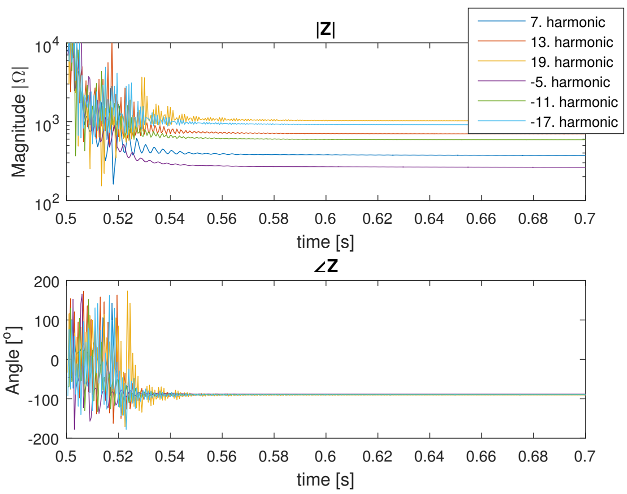

Fig. 7: Estimated impedance of the MMC with Kalman filter

parts of a grid, an stability analysis will be possible with the use of the Nyquist stability criterion. The extension of this research from impedance estimation to the stability analysis will be the subject of our next paper.

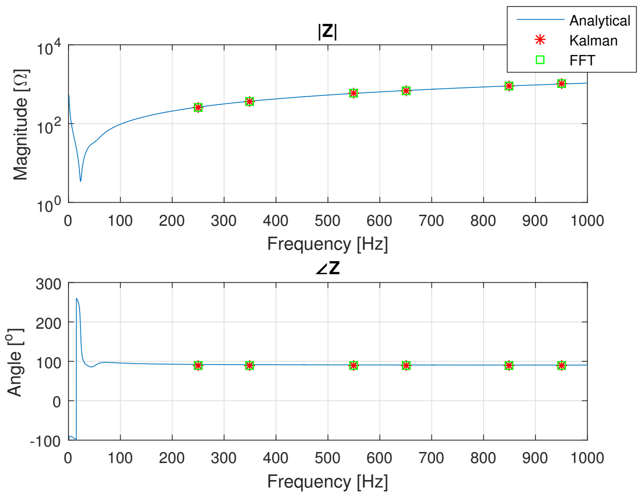

Fig. 8: Estimated impedance of the MMC with Kalman filter at a time step compared to

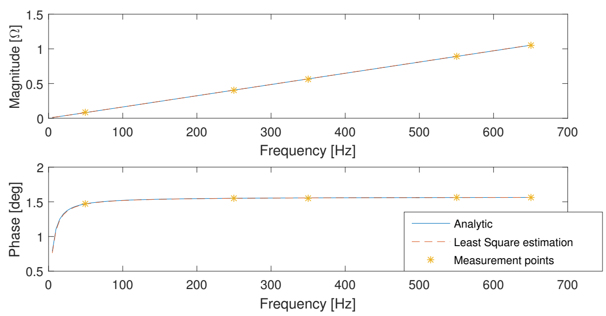

Fig. 9: Estimated impedance with Least Squares compared to analytically calculated impedance

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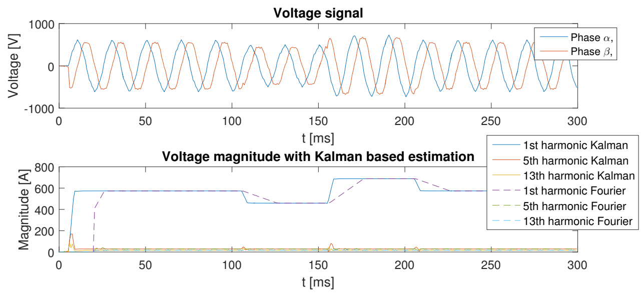

Fig. 10: Generated non-stationary signal, Kalman harmonics tracking compared to Fourier based tracking

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