Voltage Stability Monitoring using a Modified Thevenin Impedance

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***Abstract* – This paper presents a method for voltage stability monitoring based on the maximum power transfer to the load. The only required information is the system topology, the data from PMUs and the operational status of synchronous generators. With this information, the Thevenin impedance seen from a load bus can be estimated and by using the Thevenin theorem for maximum power transfer an impedance based stability index can be established. Since the Thevenin impedance is influenced by the operational conditions of the synchronous generators, it is consequently not a constant value and differs from the short circuit impedance of the bus. It is emphasized that the method requires only the information of the considered subsystem. Therefore, since the computation requirement is insignificant, the algorithm can be used for online monitoring. The validation of the approach is achieved by simulating a simple transmission system.**

***Index Terms* – PMU, Thevenin impedance, voltage stability, influence of generator capability**

I. INTRODUCTION

The combination of increasing energy consumption and the boost of renewable energy sources requires a paradigm shift regarding operation of transmission and distribution grids in Europe. It becomes necessary to be more flexible and better informed of the system status in power system operations, especially due to the unbundling of the former vertically integrated energy supply companies into independent energy production and transmission companies. As a result no direct information link between power grid status and the control of energy production units exists anymore. Altogether, this leads to the need of a real-time identification of the transmission limits and given operation margins to be able to react fast enough to maintain a sufficient security of supply.

Voltage stability becomes more and more a limiting factor, since generation and load centers are often connected with long and heavily loaded transmission lines. Most of previous research on voltage stability focused on off-line solutions such as the continuation power flow [1] or the analysis of the sensitivity of the Jacobian matrix [2]. The increasing use of phasor measurement units (PMUs) makes real-time approaches to the voltage stability monitoring possible. Most analyses concentrate on wide area monitoring of voltage stability, such as the real-time dynamics monitoring system [3] or wide area monitoring implementations [4]. A different method, more suitable for distribution grids, uses PMU-data to estimate the Thevenin impedance and based on that calculating the maximum loadability at a given node [5]. The main advantage of this application is that only the measured data of the PMUs and the topology of the examined subsystem must be known. Furthermore, due to the detailed consideration of any concerned bus, it can be enhanced to a protection system.

However, there is a lack of analyses related to performance and validity of the Thevenin impedance based methods. So far, the Thevenin impedance seen from the load bus was assumed to be the equivalent impedance in short circuit calculation. This assumption is only valid, if none of the generators reach a limiting parameter. The algorithm proposed in this paper offers a solution on how to include the influence of these limits on the Thevenin impedance and therefore increase the accuracy method to monitor voltage stability. Similar to [5], this approach requires the topology of the concerned subsystem and the data of PMUs in the concerned buses. Additional information about the actual operational status and limiting parameters of the generators are required.

The rest of this paper is organized as follows. Section II focuses on the basic idea of online voltage stability monitoring based on the Thevenin impedance and the required algorithm to calculate the Thevenin impedance in a general power system. The main generator parameters affecting the Thevenin impedance and the proposed method to take them into account are described in section III. Section IV shows the variation of the Thevenin impedance caused by generator limits and simulation results. The drawn conclusion is stated in section V.

II. REVIEW OF ONLINE VOLTAGE STABILITY MONITORING BASED ON THE THEVENIN IMPEDANCE

The basic idea of the proposed method is to use the available data of the network topology, SCADA systems and measurements from PMUs to reduce the complex voltage instability estimation to a maximum power transfer problem, which can be solved by knowing the equivalent impedance of the network and the load impedance.

According to circuit theory, a network can be represented by an equivalent Thevenin voltage source E_{Th} and an equivalent Thevenin impedance Z_{Th}. The load can be considered as the impedance Z_L, which is determined by the ratio of the load voltage V_L and the load current I_L. These values are measured by PMUs at the load bus or are provided by SCADA/EMS and state estimator systems. For the resulting two-bus system, shown in Fig. 1, the maximum apparent power transfer is reached when the magnitudes of load impedance and Thevenin impedance are equal.

$$|Z_L| = |Z_{Th}| \tag{1}$$

This correlation can be used for the Impedance Stability Index (ISI) established in [6],

$$ISI = \frac{|Z_L|}{|Z_{Th}|} \tag{2}$$

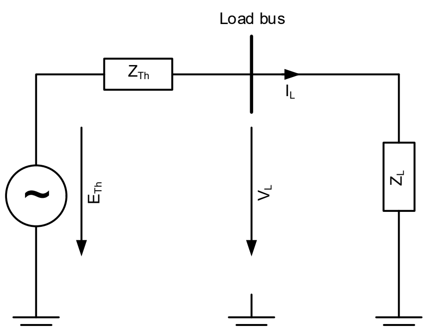

Fig. 1 *Thevenin equivalent circuit*

It can be noted that according to [5] and [7] the equivalent voltage E_{Th} and the maximum load ability S_{max} can be estimated, but these values are not necessarily needed as the method proposed in this paper focuses on an evaluation of the ISI. With the condition for the maximum power transfer (*1*) the loadability limit is given for ISI equals 1. A further increase of the load, causing ISIs value to fall below 1, might lead to a voltage collapse for power or current controlled loads, where the actual voltage collapse mechanism depends on the composition of the load.

The implementation of the ISI for an online monitoring leads to the necessity of a fast and simple computation of the equivalent impedance seen from each load bus. Reference [5] proposes a solution for calculating the equivalent impedance for any bus directly from the complete admittance matrix *Y* of a general n-bus network. *Y* includes the impedances of all serial elements describing the grid topology and the load impedances. The shunt impedances of the generators are neglected since it is assumed that the voltage controller is able to keep the generator bus voltage at the rated value. The actual grid topology with all necessary switching status must be provided by state estimators or SCADA systems.

$$Y = \begin{bmatrix} Y_{11} & \cdots & Y_{1n} \\ \vdots & \ddots & \vdots \\ Y_{1n} & \cdots & Y_{nn} \end{bmatrix} \tag{3}$$

By inversion of Y, the impedance matrix Z of the system is defined.

$$Z = Y^{-1} = \begin{bmatrix} Z_{11} & \cdots & Z_{1n} \\ \vdots & \ddots & \vdots \\ Z_{1n} & \cdots & Z_{nn} \end{bmatrix} \tag{4}$$

The elements in the main diagonal of Z are the equivalent impedance Z_{Th} parallel to the load impedance $Z_{L,k}$ connected to the concerned bus. Therefore, Z_{Th} of the bus *k* is calculated as

$$Z_{Th,k} = \frac{Z_{L,k} \cdot Z(k,k)}{Z_{L,k} - Z(k,k)} \tag{5}$$

This impedance only represents the network accurately, if all generators are operated within their capability borders. In this case, the voltage regulator maintains the rated voltage at the bus and the generator can be seen as an ideal voltage source. However, generators generally reach their operational limits before a voltage collapse occurs; these limits must be taken into account since they cause a deviation between the calculated Thevenin impedance Z_{Th} and the actual equivalent impedance. The proposed method implements the limitations with an algorithm, which adds a variable impedance at each generator bus. This impedance depends on the operational status and limiting parameters of the corresponding generator.

III. Main Factors Affecting the Thevenin Impedance with Regards to Voltage Stability

The Thevenin impedance merely describes the system accurately as long as generators function as ideal voltage sources. In this case, the load contribution only depends on the connecting impedances between the buses. At the instant the limit of a generator is reached, the load contribution changes. The limit can be interpreted as additional requirement for the source representing the generator, e.g. maximum current. The proposed method models these conditions as additional impedance, whose magnitude is calculated to fulfil them for the maximum load transfer from the generator to the network.

The main cause of voltage instability is a lack of reactive power. Therefore, the parameters defining the operational border of an under-excited generator do not have any effects on the voltage stability. The concerning parameters limiting the reactive power and influencing the active power distribution between several generators are the restricted active power, the maximum armature current and the maximum excitation. Fig. 2 presents the section of the generator capability diagram showing the operational borders caused by these parameters.

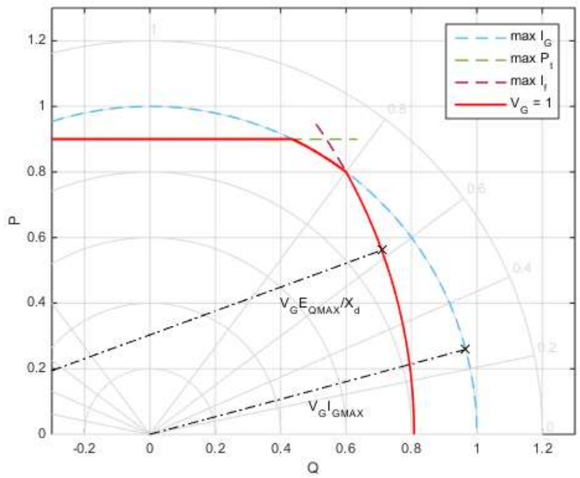

Fig. 2 *Generator capability diagram*

The basic idea of the method proposed in this paper is to model the influence of the limits as an additional shunt impedance for each generator.

A. Dispatch of active power

Similar to [5], a simple network consisting of a generator represented by an ideal voltage source and a shunt impedance connected to a load impedance is used to derive the calculation of the additional impedance. This configuration equals the equivalent Thevenin circuit, Fig. 1, with Z_{Th} replaced with Z_G and E_{Th} with V_G.

Since the reactive power has no influence on this limitation, a pure active power load is assumed. Therefore, Z_L becomes R_L. With Z_G split into its real part R_G and imaginary part X_G, the load power P_L is

$$P_L = |V_G|^2 \frac{R_L}{R_L^2 + 2R_L R_G + R_G^2 + X_G^2} \tag{6}$$

For a certain rated voltage of the generator V_{rated} the maximum active load power occurs under the following condition

$$R_L = \sqrt{R_G^2 + X_G^2} = |Z_G| \tag{7}$$

and holds the value

$$P_L = \frac{|V_G|^2}{4\,|Z_G|} \tag{8}$$

However, the network assumed for this derivation can be interpreted as a synchronous generator feeding active power into a grid represented by Z_L. Since the active power is limited by the capability of the generator, the shunt impedance Z_G must be replaced by the impedance *Z'* to fulfill the Thevenin based voltage stability criteria stated in Section II at maximum active power of the generator. The magnitude of *Z'* is calculated by transforming (8) and assuming that the load power P_L equals the maximum active power of the generator $P_{G,max}$ and the generator voltage equals the rated voltage V_{rated} of the bus.

$$|Z'| = \frac{V_{rated}^2}{4\,P_{G,max}} \tag{9}$$

It is noted that the maximum active power can take on any value up to the rated power of the turbine according to the settings of the turbine controller.

B. Armature current limiter

The current-based limitations can be better examined by transforming the network used in the previous section into its Norton equivalent, Fig. 3. The generator is then represented as a current source parallel to Z_G.

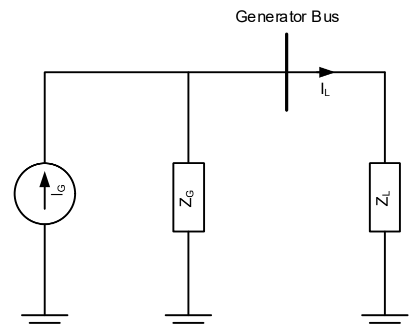

Fig. 3 *Equivalent Norton network of a generator feeding a load impedance*

The implementation of the armature current limiter follows a similar idea as the one for an active power dispatch. The maximum load power is calculated as a function of Z_G and the current I_G in the first step.

$$|S_L| = \frac{|I_G|^2 |Z_G|}{4} \tag{10}$$

Since a network can be seen as a synchronous generator feeding an arbitrary load power into the grid, Z_G can be replaced by *Z'* to fit the requirement of maximum armature current at the instant of the maximum load ability at the bus. Therefore, I_G becomes the rated armature current of the generator $I_{G,rated}$ and the apparent load power S_L the rated apparent power of the generator $S_{G,rated}$.

$$|Z'| = \frac{4\,S_{G,rated}}{I_{G,rated}^2} \tag{11}$$

In the per unit system this equation can be simplified to a voltage and current correlation.

$$|Z'| = \frac{4\,V_{rated}}{I_{G,rated}} \tag{12}$$

C. Overexcitation limiter

The calculation of *Z'* for this limitation follows exactly the same idea when armature current limiter is considered. However, since the armature current depends on the actual power composition of the generator, see Fig. 2, magnitude of the impedance *Z'* is not constant. Therefore, the actual armature current I_G must be provided by PMU or SCADA to calculate *Z'*.

$$|Z'| = \frac{4\,V_{rated}}{I_G} \tag{13}$$

In summary, the operation limits are modelled with a shunt impedance *Z'* for each generator. The magnitude of *Z'* depends on the operational status of the generator and replaces the generator shunt impedance.

IV. Implementation into a Thevenin Impedance Based Voltage Stability Monitoring

The limits discussed in Section III can be used to estimate the Thevenin impedance, which forms an indicator for voltage stability monitoring. This task is fulfilled by adding the impedance *Z'* into the admittance matrix *Y*. As mentioned in Section II, *Y* is not containing any generator shunt impedance. This can be interpreted in terms of section III as $Z_G = 0$ and leads to the following magnitudes of *Z'*.

$$|Z'| = \begin{cases} 0 & \textit{no limits reached} \\ \dfrac{V_{rated}^2}{4P_{G,max}} & \textit{active power dispatch} \\ \dfrac{4\,V_{rated}}{I_G} & \textit{over excitation limit} \\ \dfrac{4\,V_{rated}}{I_{G,rated}} & \textit{armature current limit} \end{cases} \qquad (14)$$

Z' is added into the network as an additional shunt impedance at the generator bus, as shown in Fig. 4. In a network with an arbitrary number of generators the corresponding *Z'* must be calculated for each generator and included at the associated generator bus.

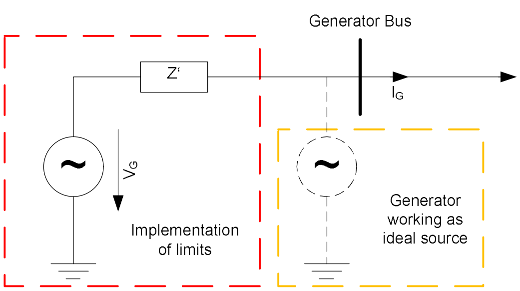

Fig. 4 Implementation of Z' into the network

The Thevenin impedance adjusted to the influence of the limiters Z'_{Th} is calculated as described in Section II. Due to the additional impedances at the generator buses the magnitude of Z'_{Th} is higher than the original Thevenin impedance's magnitude.

However, since the method should be used to detect problems in voltage stability issues before a voltage collapse occurs, it is necessary to implement the adjustments before any limit is actually reached. This is achieved by comparing the actual values of the limiting parameters active power, excitation current and armature current with SCADA systems or PMUs with their maximum values. The difference between the maximum and actual value in percent related to the maximum value is used by an algorithm for deciding the corresponding limit for calculating the magnitude of Z'. The selection process of the algorithm is illustrated in Fig. 5.

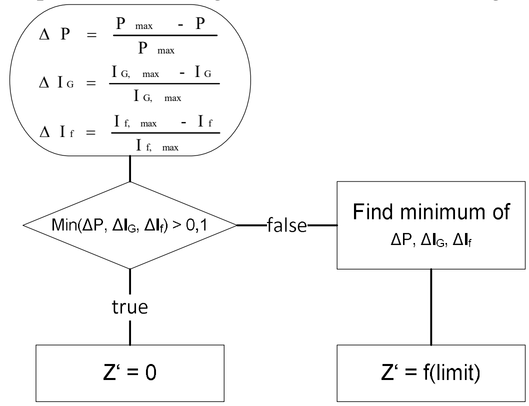

Fig. 5 *Selection process for calculation of Z'*

The operation of the generator with constant active power is a special case. The actual active power will equal the rated/maximum active power, if a perfect controller is assumed. According to the selection process, *Z'* is calculated for active power dispatch no matter the operational condition of the generator. To avoid that, the algorithm uses an offset to the difference of rated value and actual value of the active power.

V. Analysis of the Thevenin Impedance

The proposed method to properly estimate the equivalent impedance and its use in voltage stability monitoring is tested on a simple grid shown in Fig. 6. The generator SG_{ref} represents the transmission network and its parameters are chosen in such a way that the operational limits are not reached before the voltage collapse. The second generator SG_1 is assumed much smaller, to assure that it will definitely reach its limits to demonstrate the influence on the Thevenin impedance.

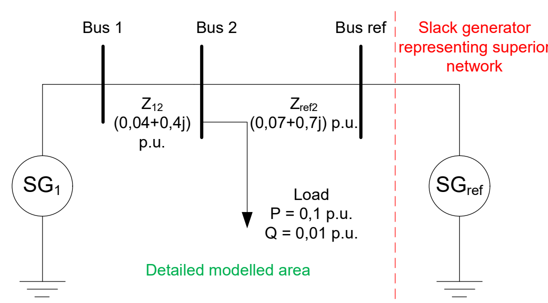

Fig. 6 *Simulated network*

The synchronous generators are represented by their 5^{th} order model and are introduced into the network as a current injection. The load is integrated as an admittance at the load bus and the network topology is represented as an admittance matrix. To simulate the behavior of the power controlled load, the magnitude of the load admittance is regulated by a PI controller. The implementation in SIMULINK offers the needed flexibility in the parameterization.

The evaluation of the proposed method and the voltage stability is done by increasing the active and reactive load power, starting at the values stated in Fig. 6. To show the influence of the generator limitations, the voltage stability is observed by using the ISI based on the original and the adjusted Thevenin impedance. Moreover, the voltage stability is checked up by using the $d\Delta Q/dV$ and the dQ_G/dQ_L criteria, which are described in [8], [9] and [10, pp. 305-310].

A global per unit system is used for the simulations, whose base magnitudes are 100MVA and 110kV. The rated terminal voltage of the generators is assumed as 110kV to simplify the calculations.

The original Thevenin impedance Z_{Th} seen from the load bus is

$$Z_{Th} = Z_{12}||Z_{ref2} = \frac{Z_{12} \cdot Z_{ref2}}{Z_{12} + Z_{ref2}} \tag{15}$$

The adjusted Thevenin impedance Z'_{Th} is calculated as

$$Z'_{Th} = (Z' + Z_{12})||Z_{ref2} = \frac{(Z' + Z_{12}) \cdot Z_{ref2}}{(Z' + Z_{12}) + Z_{ref2}} \tag{16}$$

whereby Z' has a variable magnitude according to the method described in Section III. Since the impedance of generators and transmission lines are mainly inductive, Z' is assumed as inductance without a resistive part. Adopting the per unit system in the calculation of Z', it becomes

$$Z' = \begin{cases} 0 & \textit{no limits reached} \\ \dfrac{j}{4P_{G,max}} & \textit{active power dispatch} \\ \dfrac{4\,j}{I_G} & \textit{over excitation limit} \\ \dfrac{4\,j}{I_{G,rated}} & \textit{armature current limit} \end{cases} \tag{17}$$

The limiting parameters of the generators can be seen in TABLE I. The over excitation limit is given by the field voltage instead of the field current, since it is a better way to implement it into the used model. The simulation is run with several values of the maximum active power of SG_1 to examine the influence of the load disposition between SG_1 and SG_{ref}.

TABLE I LIMITING PARAMETERS GENERATORS

Limiting parameters	SG_1	SG_{ref}
$P_{G,max}$	0,01 - 0,07	3,1
$I_{G,rated}$	0,15	3,2
$E_{f,rated}$	1,3561	5

The following figures result for simulation with a maximum active power $P_{G,max}$ = 0,03 p.u. of SG_1. Fig. 7 shows the field voltage E_f and armature current I_G of SG_1 over time in the upper diagram. In the lower diagram the impedances seen at the Bus 2 and the load power S_{load} are plotted. Since SG_1 is operated with constant active power, the Z'_{Th} is adjusted according to active power dispatch from the beginning. After t = 50 s S_{load} is increased with a rate of (0,002 + j0,002) p.u. E_f and I_G start to rise and the load impedance to fall. At the time marked with *t1*, E_f hits its limit and stays constant till the AVR reduces it as a reaction of reaching the maximum armature current, marked with *t2*. The visible change in the adjusted Thevenin impedance Z'_{Th} at the marker *t3* happens shortly before E_f is at its maximum, since the margin indicating the necessary adjusment is allready reached then. The vertical dashed lines indicate the instant of ISI = 1 and consequently the loss of stability, the green one shows the adjusted Thevenin impedance and the red one the original Thevenin impedance. The detailed transition of Z'_{Th} is shown in Fig. 8.

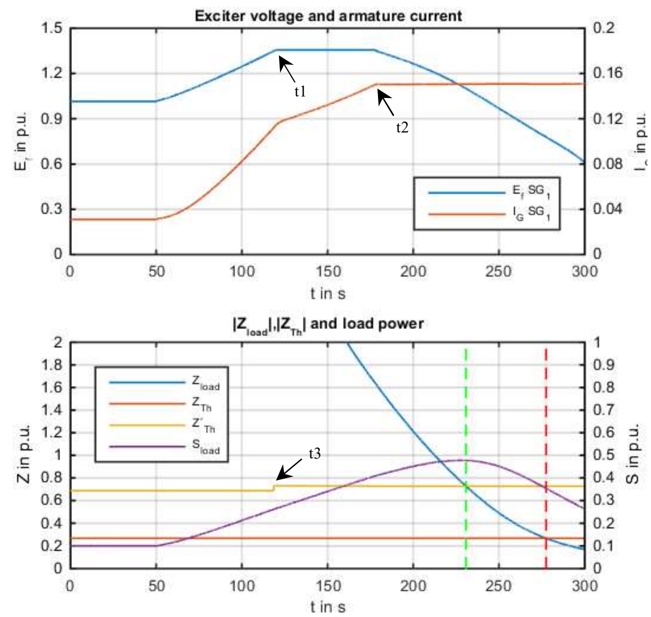

Fig. 7 *Time course field voltage, armature current, impedances at the load bus and apparent load power, active power SG_1 0,03 p.u.*

The adjusted impedance seen at the load bus is calculated according to (16) and (17). The abrupt change in its value at approximately *t = 120s* is caused by the transition from active power dispatch to the over excitation limit. From this time on Z'_{Th} is decreasing, since the armature current is still rising. The overtake from over excitation limit to armature current limit does not cause a jump in the impedances value, since the actual armature current, defining the adjusted impedance for the excitation limit, is equal to the rated (maximum) armature current, defining the adjusted impedance for the armature current limit.

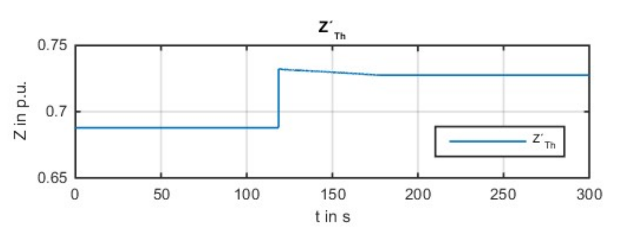

Fig. 8 Transition of the adjusted Thevenin impedance Z'_{Th}, active power SG_1 0,03 p.u.

Fig. 9 shows the nose curve used to estimate the voltage stability. The markers indicate the operation condition at the time instant of ISI = 1, which should be located at the nose point of the curve since it marks the stability border. The green marker, tagged to the adjusted impedance, marks an accurate result, whereas the red one, associated with the original impedance, marks an already instable point of operation.

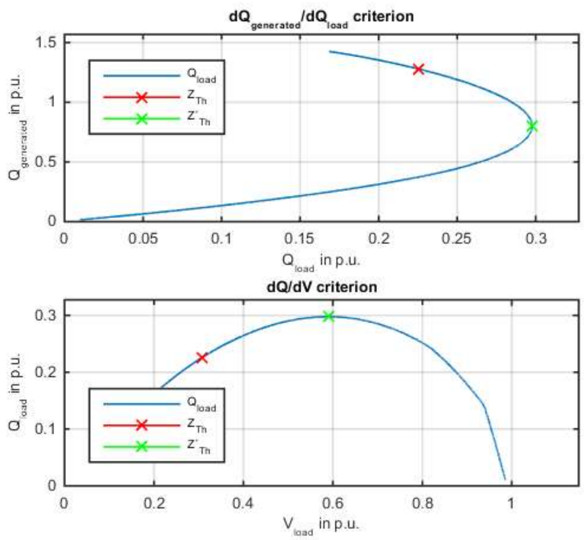

Fig. 9 *Evaluation of voltage stability criteria, the markers refer to the actual operational condition at the instant of ISI = 1, active power SG_1 0,03 p.u.*

To evaluate the accuracy of the proposed method, the deviation between the apparent load at the instant of ISI = 1 and the actual maximum apparent power is used.

$$\varepsilon = \frac{S_{max} - S(ISI = 1)}{S_{max}} \quad (18)$$

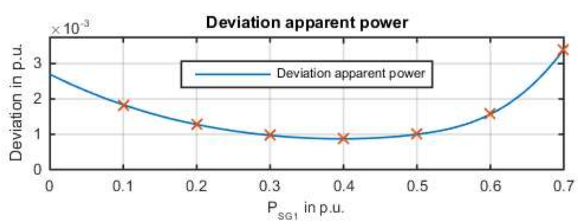

Fig. 10 *Deviation in apparent power at the instant of ISI = 1*

Since apparently a small deviation appears, the value of the ISI at the instant of maximum apparent load power is also evaluated and can be used to assess the value of the adjusted impedance. Values above the dashed line, correlating with the theoretical ISI for maximum load power, imply that the actual Thevenin impedance is higher than the calculated one.

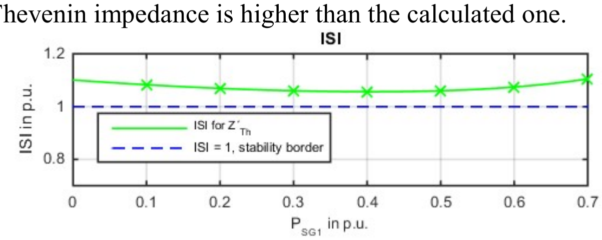

Fig. 11 *ISI at the instant of maximum apparent load power*

VI. Conclusion and Discussion

This paper presents a method for online voltage stability monitoring by using the Thevenin impedance seen from the concerned load bus. The Thevenin impedance is calculated based on the available information of the system topology. Since the Thevenin impedance is influenced by the operating conditions of the generators in the system and must be adjusted accordingly, additional PMU measurements at the generator buses and information from the SCADA systems are necessary.

The results of the simulation show a significant improvement in the accuracy of estimating the point where the voltage stability limit is reached. The ISI for the estimated instance of the voltage collapse is slightly above the theoretical border of 1. That means, the estimated stability border is already at an instable operation point. But as can be seen, the error in ISI is quite small and with adding a security margin, i.e. 10 %, this method is suitable for detecting occurring problems in voltage stability before the system collapses.

It is noted, that so far the adjustments have only been tested in a simple topology and no measurements in actual grids have been taken.

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