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ORIGINAL RESEARCH ARTICLE

Wave-induced bottom shear stress estimation in shallow water exemplified by using deep water wind statistics

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Summary The paper provides a simple and analytical method which can be used to give estimates of the wave-induced bottom shear stress for very rough beds and mud beds in shallow water based on wind statistics in deep water. This is exemplified by using long-term wind statistics from the northern North Sea, and by providing examples representing realistic field conditions. Based on, for example, global wind statistics, the present results can be used to make estimates of the bottom shear stress in shallow water.

1. Introduction

Simple and effective descriptions of transport mechanisms in operational estuarine, coastal and ocean circulation models are often required, in which the bottom shear stress represents an important component in finite water depths. In estuarine and coastal zones, at shallow and intermediate water depths, the water particle movements induced by surface waves have a strong effect in the entire water column from the surface to the bottom of the sea. The flow in this region is generally induced by surface waves and currents, where the bottom wave boundary layer is a thin flow region at the seabed dominated by friction arising from the bottom roughness. The wave boundary layer flow determines the bottom shear stress, which affects, e.g. the sediment transport and assessment of the stability of scour protections in the marine environment. The boundary layer flow regime is most commonly rough turbulent, although the flow regime over mud beds is mostly laminar and smooth turbulent depending on the bottom sediments and wave activity.

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The rough turbulent flow regime considered here corresponds to very rough beds. The results in this flow regime are relevant for assessing, e.g. the stability of scour protection in the coastal environment for relative large stone sizes compared to the near-bed random wave activity.

Laminar flow near mud beds, where clays and silt are referred to as mud, is of practical interest. The movement of mud within coastal and estuarine waters might have large economical and ecological impact in the development of new engineering works and maintenance of existing installations, e.g. related to necessary routine dredging required for ports' accessibility to shipping. The capability to predict the movement of the mud is also essential to understand the distribution of certain pollutants adsorbed to mud, as cohesive sediments are often contaminated. It appears that organic (polychlorinated biphenyl (PCBs), etc.) pollutants adhere easily to the clay particles and organic materials of the sediments. The results for laminar flow are relevant for assessing erosion and deposition of mud beneath random waves.

Further details on the background and complexity of the flow, as well as reviews of the problems are found in the textbooks of, e.g. Nielsen (1992), Fredsøe and Deigaard (1992), Soulsby (1997), Whitehouse et al. (2000), Winterwerp and van Kesteren (2004).

The purpose of this study is to demonstrate how wind statistics in deep water can be used to provide the wave-induced bottom shear stress in shallow water. Results are given for the bottom shear stress beneath random surface waves at beds with very large roughness and for laminar flow applied to mud beds, and are primarily based on the previous work by Myrhaug and Holmedal (2010) who provided the seabed shear stress spectrum for very rough beds and for laminar flow. Examples of results representing realistic field conditions are given.

2. Bottom shear stress beneath random waves in shallow water

2.1. Spectrum of bottom shear stress

Following Myrhaug and Holmedal (2010) (hereafter referred to as MH10) the bottom shear stress spectrum for laminar flow in shallow water ($kh \ll 1$) is obtained as (see Eq. (A12) in the Appendix)

$$S_{(\tau/\rho)(\tau/\rho)}(\omega, h) = \frac{\nu_f \omega^3}{(kh)^2} \frac{\omega^2 h}{2g} S_{\zeta\zeta}(\omega). \tag{1}$$

Here τ is the bottom shear stress, ρ is the fluid density, ω is the cyclic wave frequency, h is the water depth, ν_f is the kinematic viscosity of the fluid, k is the wave number determined from the dispersion relationship $\omega^2 = gk \tanh kh$ which in shallow water reduces to $\omega^2 = k^2 gh$, g is the acceleration due to gravity, and $S_{\zeta\zeta}(\omega)$ is the deep water wave spectrum.

The bottom shear stress spectrum for rough turbulent flow over a bed with very large roughness in shallow water is obtained as (see Eq. (A16) in the Appendix)

$$S_{(\tau/\rho)(\tau/\rho)}(\omega, h) = \frac{c^2 z_0^2 \omega^4}{4(kh)^2} \frac{\omega^2 h}{2g} S_{\zeta\zeta}(\omega). \tag{2}$$

Here z_0 is the average bottom roughness, and c is a constant with the two values 9 and 18 reflecting that c depends strongly on the geometry of the large roughness elements (see the Appendix for more details). The first term on the right hand side of Eqs. (1) and (2) represents the square of the magnitude of the transfer function between the bottom shear stress τ/ρ and the free surface elevation ζ; the second term represents the depth correction factor in shallow water, i.e. a correction factor which is used to transform the deep water wave spectrum $S_{\zeta\zeta}(\omega)$ to shallow water (see the Appendix for more details).

By substituting $k^2 = \omega^2/gh$, Eqs. (1) and (2) are rearranged, respectively, to

$$S_{(\tau/\rho)(\tau/\rho)}(\omega, h) = \frac{1}{2} \nu_f \omega^3 S_{\zeta\zeta}(\omega); \quad \text{laminar}, \tag{3}$$

$$S_{(\tau/\rho)(\tau/\rho)}(\omega, h) = \frac{1}{8} (cz_0)^2 \omega^4 S_{\zeta\zeta}(\omega); \quad \text{rough}. \tag{4}$$

Thus the shear stress spectra in shallow water are given in terms of the deep water wave spectrum, and it should be noted that the dependence on the water depth disappears. Overall this is a consequence of transforming the waves from deep to shallow water and using the bed shear stress for laminar flow (Eqs. (A3) and (A8)) and for very rough beds (Eqs. (A3) and (A13)).

2.2. Laminar flow

The zeroth spectral moment of the bottom shear stress spectrum for laminar flow is obtained from Eq. (3) as

$$m_{0\tau/\rho} = \int_0^\infty S_{(\tau/\rho)(\tau/\rho)}(\omega, h) d\omega = \frac{\nu_f}{2} m_3, \tag{5}$$

where m_3 is the third wave spectral moment in deep water, i.e. the nth spectral moment of the deep water wave spectrum is defined as

$$m_n = \int_0^\infty \omega^n S_{\zeta\zeta}(\omega) d\omega; \quad n = 0, 1, 2, 3, 4, \ldots \tag{6}$$

Thus, from Eq. (5) the significant value of the bottom shear stress height is obtained as

$$H_{s\tau/\rho} = 4\sqrt{m_{0\tau/\rho}} = 2\sqrt{2\nu_f m_3}. \tag{7}$$

2.3. Very rough beds

The zeroth spectral moment of the bottom shear stress spectrum for very rough beds is obtained from Eq. (4) as

$$m_{0\tau/\rho} = \frac{1}{8} (cz_0)^2 m_4, \tag{8}$$

where m_4 is the fourth spectral moment of the wave spectrum in deep water defined in Eq. (6). The most common model wave spectra are proportional to ω^{-5} for large ω, and thus m_4 does not exist. However, m_4 can be expressed in terms of the spectral moments m_0, m_1 and m_2, and the spectral bandwidth parameters ε and ν are given in terms of the spectral moments of the deep water wave spectrum as (see e.g. Tucker and Pitt, 2001)

$$\varepsilon^2 = 1 - \frac{m_2^2}{m_0 m_4}, \tag{9}$$

$$\nu^2 = \frac{m_0 m_2}{m_1^2} - 1. \tag{10}$$

For a narrow-band wave process $\nu = \varepsilon/2$ (Longuet-Higgins, 1975), which gives

$$m_4 = \frac{m_1^2 m_2^2}{m_0(5m_1^2 - 4m_0 m_2)}. \tag{11}$$

Thus, from Eq. (8) the significant value of the bottom shear stress height is obtained as

$$H_{s\tau/\rho} = 4\sqrt{m_{0\tau/\rho}} = \sqrt{2} c z_0 \sqrt{m_4}, \tag{12}$$

where m_4 is given in Eq. (11).

3. Examples of results for a Phillips wave spectrum and wind statistics in deep water

Two examples are included to illustrate the applicability of the results for practical purposes using data typical for field conditions: a water depth with $h = 3$ m consisting of very rough beds with bed roughness $z_0 = 0.0094$ m representative for cobble according to Soulsby (1997, Fig. 4), and mud beds with median grain diameter $d_{50} = 0.03$ mm representative of medium silt according to Soulsby (1997, Fig. 4).

First, the common features of the two examples will be described. Examples of results are given by choosing the Phillips deep water wave spectrum for which analytical expressions can be obtained (see e.g. Holthuijsen, 2007; Tucker and Pitt, 2001)

$$S(\omega) = \alpha \frac{g^2}{\omega^5}, \quad \omega \geq \omega_p = \frac{g}{U_{10}}, \tag{13}$$

where $\alpha = 0.0081$ is the Phillips constant, ω_p is the spectral peak frequency, and U_{10} is the mean wind speed at the 10 m elevation above the sea surface. By using the definition of the spectral moments in Eq. (6) it follows that the significant wave height in deep water is

$$H_s = 4\sqrt{m_0} = 2\sqrt{\alpha} \frac{g}{\omega_p^2}. \tag{14}$$

Then it follows from Eqs. (11) and (12) for very rough beds that

$$H_{s\tau/\rho} = 2\sqrt{\alpha} g c z_0; \quad \text{rough}. \tag{15}$$

Moreover, for laminar flow it follows from Eq. (7) that

$$H_{s\tau/\rho} = 2\sqrt{2\nu_f \alpha g U_{10}}; \quad \text{laminar}. \tag{16}$$

Furthermore, the Phillips spectrum transformed to shallow water becomes

$$S_{\zeta\zeta}(\omega, h) = \frac{\omega^2 h}{2g} S_{\zeta\zeta}(\omega) = \alpha \frac{gh}{2\omega^3}; \quad \omega \geq \omega_p = \frac{g}{U_{10}}, \tag{17}$$

which gives the following significant wave height in shallow water:

$$H_{sh} = 4\sqrt{m_{0h}} = \frac{2\sqrt{\alpha g h}}{\omega_p}; \quad \text{shallow water}, \tag{18}$$

where $m_{0h} = \int_0^\infty S_{\zeta\zeta}(\omega, h) d\omega$.

It is noted that for very rough beds (Eq. (15)) $H_{s\tau/\rho}$ is independent of U_{10}, while for laminar flow (Eq. (16)) $H_{s\tau/\rho}$ depends on U_{10}. Consequently, the results for laminar flow can be obtained from available wind statistics for an ocean area, e.g. from a long-term distribution of U_{10}.

Different parametric models for the cumulative distribution function (*cdf*) or the probability density function (*pdf*) of U_{10} are given in the literature, see e.g. a recent review in Bitner-Gregersen (2015). In the present example the *cdf* of U_{10} given by Johannessen et al. (2001) is used to demonstrate the application of the results. This *cdf* is based on wind measurements covering the years 1973—1999 from the northern North Sea. The database consists of composite measurements from the Brent, Troll, Statfjord and Gullfaks fields as well as the weather ship Stevenson. Model data from the Norwegian hindcast archive (WINCH, gridpoint 1415) have been filled in for periods where measured data were missing. Thus a 25-year-long continuous time series has been used (see Johannessen et al., 2001 for more details), upon which the *cdf* of the 1-h values of U_{10} is described by the two-parameter Weibull model

$$P(U_{10}) = 1 - \exp\left[-\left(\frac{U_{10}}{\theta}\right)^\beta\right]; \quad U_{10} \geq 0, \tag{19}$$

with the Weibull parameters:

$$\theta = 8.426, \quad \beta = 1.708. \tag{20}$$

Now the long-term statistics of $H_{s\tau/\rho}$ for laminar flow can be derived by using this *cdf* of U_{10}. Statistical quantities of interest are, e.g. the expected value of $H_{s\tau/\rho}$, $E[H_{s\tau/\rho}]$, and the variance of $H_{s\tau/\rho}$, $Var[H_{s\tau/\rho}]$, which for laminar flow will be proportional to $E[U_{10}^{1/2}]$ and $Var[U_{10}^{1/2}]$, respectively. This requires calculation of $E[U_{10}^n]$, which for a Weibull distributed quantity is given by (Bury, 1975)

$$E[U_{10}^n] = \theta^n \Gamma\left(1 + \frac{n}{\beta}\right), \tag{21}$$

where Γ is the gamma function, and n is a real number. Furthermore (Bury, 1975):

$$Var[U_{10}^n] = E[U_{10}^{2n}] - (E[U_{10}^n])^2. \tag{22}$$

Moreover, the results are further exemplified by the deep water wave conditions corresponding to the expected value of U_{10}, i.e. given by $E[U_{10}] = 7.5$ m s^{-1} according to Eqs. (20) and (21). Based on this it follows that:

- spectral peak frequency $\omega_p = g/E[U_{10}] = 1.308$ rad s^{-1}, corresponding to the spectral peak period $T_p = 2\pi/\omega_p = 4.8$ s,
- significant wave height in deep water, $H_s = 2\sqrt{\alpha}g/\omega_p^2 = 1.03$ m,
- significant wave height in shallow water, $H_{sh} = 2\sqrt{\alpha g h}/\omega_p = 0.75$ m,
- k_p from the shallow water dispersion relationship corresponding to ω_p, $k_p = \omega_p/\sqrt{gh} = 0.241$ rad m^{-1},

- peak near-bed orbital displacement amplitude, $A_p = H_{sh}/(2k_p h) = 0.52$ m.

3.1. Example 1: Very rough beds

Now it follows that

- $A_p/z_0 = 55 < 300$, i.e. being in the range of the data of both Myrhaug et al. (2001) and Dixen et al. (2008) (see the Appendix and MH10 for more details). Thus both $c = 9$ and $c = 18$ are used in this example for very rough beds.

Then it follows from Eq. (15) that

$$H_{s\tau/\rho} = 2\sqrt{\alpha} g z_0 \begin{cases} 9 \\ 18 \end{cases} = \begin{cases} 0.15\,\mathrm{m^2\,s^{-2}} \\ 0.30\,\mathrm{m^2\,s^{-2}} \end{cases}. \tag{23}$$

The critical shear stress for movement of the bottom material is given by Soulsby (1997) as

$$\left(\frac{\tau}{\rho}\right)_c = 0.055(s-1) g\, d_{50}, \tag{24}$$

where $s = 2.65$ is the sediment density to fluid density ratio taken as for quartz sand and $d_{50} = 12z_0$. This example gives $(\tau/\rho)_c = 0.10\ \mathrm{m^2\,s^{-2}}$, showing that the bottom material will be exposed to erosion for both values of c. However, in other cases it might be that the two values of c will give different results. In such cases the user has to make the best jugdement based on the location and situation considered.

3.2. Example 2: Mud beds

The given flow conditions are:

- density of water, $\rho = 1027\ \mathrm{kg\,m^{-3}}$,
- kinematic viscosity of water at temperature 10°C and salinity 35‰, $\nu_f = 1.36 \times 10^{-6}\ \mathrm{m^2\,s^{-1}}$.

Now it follows that:

- the wave Reynolds number, $Re = U_p A_p/\nu_f = \omega_p A_p^2/\nu_f = 2.6 \times 10^5$, i.e. the flow is in the laminar flow regime $Re \lesssim 3 \times 10^5$ (Soulsby, 1997) and $U_p = \omega_p A_p$.

Then it follows from Eqs. (16), (20) and (21) that

$$E[H_{s\tau/\rho}] = 2\sqrt{2\nu_f \alpha g}\, E[U_{10}^{1/2}] = 0.00243\ \mathrm{m^2\,s^{-2}}, \tag{25}$$

and from Eqs. (16), (20), (21) and (22) that the standard deviation (equal to the square root of the variance) of $H_{s\tau/\rho}$ is

$$\sigma[H_{s\tau/\rho}] = 2\sqrt{2\nu_f \alpha g}\, \sigma[U_{10}^{1/2}] = 0.00079\ \mathrm{m^2\,s^{-2}}. \tag{26}$$

Thus the interval corresponding to the mean value $\pm$ one standard deviation is given by $(0.00164\ \mathrm{m^2\,s^{-2}}, 0.00322\ \mathrm{m^2\,s^{-2}})$.

By following the examples in Whitehouse et al. (2000) (i.e. example 4.2 for erosion and example 8.1 for deposition), the critical bottom shear stress for erosion is $\tau_e = 0.197\ \mathrm{N\,m^{-2}}$ and for deposition it is $\tau_d = 0.08\ \mathrm{N\,m^{-2}}$. Thus it follows that $\tau_e/\rho = 0.00019\ \mathrm{m^2\,s^{-2}}$ and $\tau_d/\rho = 0.000078\ \mathrm{m^2\,s^{-2}}$, showing that the bed is exposed to erosion for this flow condition.

In general, mud beds exhibit cohesive properties and the details of the flow can only be understood by including a number of complex transport mechanisms; see e.g. Whitehouse et al. (2000) and Winterwerp and van Kesteren (2004) for further details. The flow over muds is not necessarily laminar, but will depend on the wave Reynolds number Re, which can be large enough corresponding to turbulent flow over smooth (or mud) beds, i.e. $Re \gtrsim 3 \times 10^5$. Furthermore, the results in Fredsøe and Deigaard (1992, Fig. 2.13) can be used to distinguish between laminar and turbulent flow for different combinations of grain size and Reynolds number. Further details including formulas which can be used for practical purposes are given in Soulsby (1997, Ch. 4.5).

4. Comments

Finally some comments are given on the present method versus common practice in coastal engineering. For calculating bottom shear stress for random waves in shallow water common practice would be to start from available data on joint statistics of H_s and T_p within directional sectors at a nearby offshore location; then to transform these by using a wave simulation model to obtain joint statistics of H_s and T_p at the relevant location; then to use this information as input for calculating the bottom shear stress. Alternatively, this paper provides a simple analytical method giving first estimates of random wave-induced bottom shear stress for very rough beds and mud beds from observed deep water sea surface wind statistics with an example based on in situ data obtained from the Northern North Sea. The Phillips deep water wave frequency spectrum is used to relate wind to waves together with the narrow-band and shallow water assumptions. Thus an analytical estimate of the associated bottom shear stresses is obtained. The narrow-band assumption is justified since the waves with the frequencies close to the spectral peak frequency are the most energetic contributing to the bed shear stresses in shallow water. Such simple methods are useful to be able to quickly make estimates which can be used for comparison and verification of more complete computational methods, as well as in situations when time and access to computational resources are limited (under e.g. field conditions). Moreover, it might also serve as a first inexpensive estimate of the quantities of interest before eventually applying more work-intensive computational tools.

5. Summary and conclusions

A simple analytical method which can be used to give estimates of the random wave-induced bottom shear stress for very rough beds and mud beds in shallow water based on wind conditions in deep water is provided. Results are exemplified by using long-term wind statistics from the northern North Sea and by giving examples representing realistic field conditions. The example calculations demonstrate that the sea bottom material will be exposed to erosion both for very rough beds and mud beds. The results should represent a useful tool for assessment of, e.g. the stability of scour protections in coastal environments where the stone size is large compared to the near-bed wave activity, as well as assessment of erosion and deposition of mud. The method should also represent a useful representation of the bottom shear stress often required in operational estuarine, coastal and ocean circulation models based on, for example, available global wind statistics.

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Appendix. Spectrum of seabed shear stresses in shallow water

Here a brief summary of the theoretical background from MH10 is given.

Consider an oscillatory wave boundary layer flow where the motion is horizontally uniform in the direction along the seabed. By using complex notation the free stream velocity outside the wave boundary layer is

$$u(t) = Ue^{i\omega t}, \tag{A1}$$

where U is the near-bed orbital velocity amplitude, t is the time, ω is the cyclic wave frequency, and $i = (-1)^{1/2}$ is the complex unity. The seabed shear stress is

$$\tau(t) = \tau_m e^{i(\omega t + \varphi)}, \tag{A2}$$

where φ is the phase angle between $\tau(t)$ and $u(t)$, and τ_m is the maximum seabed shear stress given by

$$\frac{\tau_m}{\rho} = \frac{1}{2} f_w U^2, \tag{A3}$$

where ρ is the fluid density and f_w is the wave friction factor.

Now $u(t)$ can be expressed in terms of the free surface elevation $\zeta(t)$ as:

$$u(t) = \frac{\omega\zeta(t)}{\sinh kh}; \quad \zeta(t) = \zeta_A e^{i\omega t}, \tag{A4}$$

where ζ_A is the wave amplitude, h is the water depth, and k is the wave number determined from the dispersion relationship for linear waves as given in Section 2.1. From Eqs. (A1) and (A4) it follows that

$$U = \frac{\omega\zeta_A}{\sinh kh}. \tag{A5}$$

The wave spectrum in finite water, $S_{\zeta\zeta}(\omega, h)$, can be obtained by multiplying the deep water wave spectrum, $S_{\zeta\zeta}(\omega)$, with a depth correction factor, $\Psi(\omega, h)$, as

$$S_{\zeta\zeta}(\omega, h) = \Psi(\omega, h) S_{\zeta\zeta}(\omega). \tag{A6}$$

In shallow water ($kh \ll 1$) (Holthuijsen, 2007, Section 8.3.2)

$$\Psi(\omega, h) = \frac{\omega^2 h}{2g}. \tag{A7}$$

First, using laminar flow as a reference case, the wave friction factor and the phase angle are given as, respectively,

$$f_w = 2Re^{-0.5}; \quad Re = \frac{UA}{\nu_f}, \tag{A8}$$

$$\varphi = \frac{\pi}{4}. \tag{A9}$$

Here Re is the Reynolds number associated with the wave motion, $A = U/\omega$, and ν_f is the kinematic viscosity of the fluid. By using Eqs. (A1), (A3), (A4), (A8) and (A9), Eq. (A2) takes the form

$$\frac{\tau(t)}{\rho} = \frac{\sqrt{\nu_f\omega}\,\omega}{\sinh kh} \zeta_A e^{i(\omega t + (\pi/4))}. \tag{A10}$$

Then the Response Amplitude Operator (RAO) = ratio between the amplitude of the seabed shear stress, $\tau(t)/\rho$, and the amplitude of the free surface elevation, $\zeta(t)$, in shallow water is obtained as (i.e. corresponding to the magnitude of the transfer function)

$$RAO = \frac{(\nu_f\omega)^{1/2}\omega}{kh}. \tag{A11}$$

Thus the seabed shear stress spectrum for laminar flow is given as

$$S_{(\tau/\rho)(\tau/\rho)}(\omega, h) = \frac{\nu_f\omega^3}{(kh)^2} \frac{\omega^2 h}{2g} S_{\zeta\zeta}(\omega). \tag{A12}$$

For rough turbulent flow the transfer function between the free surface elevation and the seabed shear stress at beds with very large roughness can be found analytically. In this roughness regime the wave friction factor is given as

$$f_w = c\left(\frac{A}{z_0}\right)^{-1}, \tag{A13}$$

where c is a constant. This variation of f_w was found by Sleath (1984) and Dixen et al. (2008) based on theoretical considerations; Myrhaug et al. (2001) and Dixen et al. (2008) found experimental evidence of this behaviour based on laboratory tests. The data used by Myrhaug et al. (2001) were in the range $1 \lesssim A/z_0 \lesssim 300$ with a bed roughness of nominal 1.0 cm diameter granite chippings. The Dixen et al. (2008) data were in the range $20 \lesssim A/z_0 \lesssim 90$ with a bed roughness using two kinds of stones as well as ping-pong balls. Myrhaug et al. (2001) found that $c = 18$. Dixen et al. (2008) proposed the friction factor $f_w = 4.86(A/z_0)^{-0.8}$ obtained as best fit to data in the range $6 < A/z_0 < 300$ (based on their own data plus other data; see Fig. 14 in their paper). However, by considering the scatter of the data in the range $6 < A/z_0 < 90$, the difference between using this friction factor or Eq. (A13) with $c = 9$ is not significant. Thus a friction factor proportional to $(A/z_0)^{-1}$ is used here to serve the purpose of demonstrating how this can be used to determine the seabed shear stress spectrum for very rough beds. The two different values of c (i.e. 9 and 18) suggest that c might depend strongly on the geometry of the large roughness elements. The phase angle, φ, is not known for the Myrhaug et al. (2001) data; φ was found to be in the range 19—23° for the Dixen et al. (2008) data. Further details are given in MH10.

By combining Eqs. (A1), (A3), (A4), $A = U/\omega$ and (A13), Eq. (A2) takes the form of

$$\frac{\tau(t)}{\rho} = \frac{cz_0\omega^2}{2\sinh kh} \zeta_A e^{i(\omega t + \varphi)}, \tag{A14}$$

and the RAO between the free surface elevation and the seabed shear stress in shallow water is obtained as

$$RAO = \frac{cz_0\omega^2}{2kh}. \quad \text{(A15)}$$

Thus the seabed shear stress spectrum for rough turbulent flow over a bed with very large roughness is given as

$$S_{(\tau/\rho)(\tau/\rho)}(\omega, h) = \frac{c^2 z_0^2 \omega^4}{4(kh)^2} \frac{\omega^2 h}{2g} S_{\zeta\zeta}(\omega). \quad \text{(A16)}$$

It should be noted that only the waves with wavelengths longer than approximately two times the water depth will give wave activity at the seabed. Thus, by using the deep water dispersion relationship $\omega^2 = gk$, this means that the waves at the seabed have frequencies below $(\pi g\, h^{-1})^{1/2}$.

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