APPLICATION OF WAVE RUNUP AND WAVE RUNDOWN FORMULAE BASED ON LONG-TERM VARIATION OF WAVE CONDITIONS

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Abstract

The purpose of this note is to demonstrate that it is possible to use the proposed formulae in Blenkinsopp et al. (2016) to estimate runup and rundown based on long-term variation of wave conditions by using the Myrhaug and Fouques (2010) joint distribution of significant wave height and Iribarren number for a sea state. Examples of application are given for typical field conditions including a procedure of determining the 100-years return period values for the runup and rundown and the corresponding values of significant wave height and Iribarren number.

Keywords: Wave runup; wave rundown; significant wave height; Iribarren number; bivariate distributions; contour lines

1. Introduction

The paper by Blenkinsopp et al. (2016) presented results on wave runup and overwash on a prototype-scale sand barrier using data obtained from laboratory experiments. They found that the extreme runup ($R_{2\%}$) as well as the extreme lower limit of the swash zone, defined by the extreme rundown ($R_{d2\%}$) scaled well with the deep water Iribarren number. Furthermore, they also made a thorough literature review of the topic; see their paper for more details.

They presented the following formulae for estimation of the 2% exceedance value of runup maxima (Eqs. (23) and (24) in the original paper)

$$R_{2\%} = 1.165 H_s \xi_p^{0.77} \tag{1}$$

$$R_{2\%} = (0.39 + 0.795\xi_p) H_s \tag{2}$$

as well as Eq. (25) in the original paper for the estimation of the 2% exceedance value of rundown maxima

$$R_{d2\%} = (0.21 - 0.44\xi_p) H_s \tag{3}$$

Here H_s is the significant wave height in deep water, and ξ_p is the deep water Iribarren number (also denoted as the surf parameter) defined as

$$\xi_p = m \left(\frac{H_s}{\frac{g}{2\pi} T_p^2} \right)^{-1/2} \tag{4}$$

where $m = \tan\alpha$ is the slope with an angle α with the horizontal, T_p is the spectral peak period, and g is the acceleration due to gravity. It should be noted that H_s may represent

$H_{1/3}$, obtained as the mean of the (1/3)rd highest wave heights from a time series, or H_{m0}, obtained from a wave spectrum (see e.g. Tucker and Pitt (2001)).

Eqs. (1) to (3) were obtained by investigating the performance of existing parameterizations of extreme runup maxima and rundown maxima based on both laboratory and field data. The parameterizations were compared with their own data, and they found that two of the parameterizations of extreme runup maxima developed using small scale laboratory data, i.e. those of Mase (1989) and Hedges and Mase (2004), performed best. Thus, Eqs. (1) and (2) are modified versions of the Mase (1989) and Hedges and Mase (2004) models, respectively. Overall, both equations cover the same ranges of beach slopes (0.088 to 0.154) and the deep water Iribarren numbers (1 to 2.9).

Eqs. (1) to (3) can be represented as

$$R_2 = (a + b\xi_p^c)H_s \tag{5}$$

where $a = 0$, $b = 1.165$, $c = 0.77$ represent Eq. (1); $a = 0.39, b = 0.795, c = 1$ represent Eq. (2); $a = -0.21, b = 0.44, c = 1$ represent $(-R_{d2\%})$ in Eq. (3), i.e. the rundown is taken as positive if it is below the mean water level.

2. Application of the Myrhaug and Fouques (2010) joint distribution of H_s and ξ_p with examples

Myrhaug and Fouques (2010) (hereafter referred to as MF10) provided a joint probability density function (*pdf*) of H_s and ξ_p given as

$$p(H_s, \xi_p) = p(\xi_p \mid H_s)p(H_s) \tag{6}$$

where $p(H_s)$ is the marginal *pdf* of H_s given by a combined lognormal and Weibull distributions (see Eq. (2) in MF10), and $p(\xi_p \mid H_s)$ is the conditional *pdf* of ξ_p given H_s given by the log-normal *pdf*

$$p(\xi_p \mid H_s) = \frac{1}{\sqrt{2\pi}\sigma\xi_p} \exp\left[-\frac{(\ln \xi_p - \mu)^2}{2\sigma^2} \right] \tag{7}$$

Here μ and σ^2 are the conditional mean value and the conditional variance, respectively, of $\ln \xi_p$ given by

$$\begin{aligned} \mu &= \ln\left[m\left(\frac{2\pi}{g}H_s\right)^{-1/2} \right] + a_1 + a_2 H_s^{a_3} \\ (a_1, a_2, a_3) &= (1.780, 0.288, 0.474) \end{aligned} \tag{8}$$

$$\begin{aligned} \sigma^2 &= b_1 + b_2 e^{b_3 H_s} \\ (b_1, b_2, b_3) &= (0.001, 0.097, -0.255) \end{aligned} \tag{9}$$

It should be noted that H_s is in metres in Eqs. (8) and (9).

Statistical properties of R_2 (from which the statistical properties of $R_{2\%}$ and $R_{d2\%}$ can be obtained) can be derived using the joint *pdf* of H_s and ξ_p, e.g., giving the joint *pdf* of R_2 and H_s. This is obtained from Eq. (5) by a change of variables from (H_s, ξ_p) to (H_s, R_2), which takes the form

$$p(H_s, R_2) = p(R_2 \mid H_s) p(H_s) \tag{10}$$

It should be noted that this change of variables only affects $p(\xi_p \mid H_s)$ since $\xi_p = \left[(R_2 - aH_s)/\mathrm{b}H_s\right]^{\frac{1}{c}}$, yielding a lognormal *pdf* of R_2 given H_s in the form (by using the Jacobian $|\partial\xi_p / \partial R_2| = (R_2 - aH_s)^{\frac{1}{c}-1} / c(bH_s)^{\frac{1}{c}}$)

$$p(R \mid H_s) = \frac{1}{\sqrt{2\pi}\sigma_R R} \exp\left[-\frac{(\ln R - \mu_R)^2}{2\sigma_R^2} \right] \tag{11}$$

where $R = R_2 - aH_s$, and μ_R and σ_R^2 are the conditional mean value and the conditional variance, respectively, of $\ln R$, given by

$$\mu_R = c\mu + \ln(bH_s); \sigma_R^2 = (c\sigma)^2 \quad (12)$$

where μ and σ^2 are given in Eqs. (8) and (9), respectively.

The cumulative distribution function (*cdf*) of R given H_s is obtained from

$$P(R \mid H_s) = \Phi\left[\frac{\ln R - \mu_R}{\sigma_R}\right] \quad (13)$$

where Φ is he standard Gaussian *cdf* given by

$$\Phi(v) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{v} e^{-t^2/2} dt \quad (14)$$

The expected value of R given H_s is given by (Bury, 1975)

$$E[R \mid H_s] = \exp(\mu_R + \frac{1}{2}\sigma_R^2) \quad (15)$$

The standard deviation of R given H_s is given by (Bury, 1975)

$$\sigma[R \mid H_s] = \left[(e^{\sigma_R^2} - 1)\exp(2\mu_R + \sigma_R^2)\right]^{1/2} \quad (16)$$

From this it follows that (since $R = R_2 - aH_s$)

$$E[R_2 \mid H_s] = E[R \mid H_s] + aH_s \quad (17)$$

$$\sigma[R_2 \mid H_s] = \sigma[R \mid H_s] \quad (18)$$

2.1 Example 1. Conditional statistical values of runup and rundown within a sea state

Examples of results are given for

- Significant wave height in deep water, $H_s = 7.5\,\mathrm{m}$
- Slope of barrier, $\mathrm{m} = 1/10$.

Substitution of these values in Eqs. (8) and (9) gives $\mu = -0.558$ and $\sigma^2 = 0.01533$, respectively, which substituted in Eq. (12) and combined with Eqs. (15) to (18) give the following results for:

Wave runup from Eq. (1)

$$E[R_{2\%} \mid H_s = 7.5\text{m}] = 5.71\text{m}$$

$$\sigma[R_{2\%} \mid H_s = 7.5\text{m}] = 0.55\text{m}$$

Wave runup from Eq. (2)

$$E[R_{2\%} \mid H_s = 7.5\text{m}] = 6.36\text{m}$$

$$\sigma[R_{2\%} \mid H_s = 7.5\text{m}] = 0.43\text{m}$$

Wave rundown from Eq. (3) (by taking into account the sign contained in Eq. (3))

$E[R_{d2\%} \mid H_s = 7.5\text{m}] = 0.33\text{m}$ (i.e. the rundown is below the mean water level)

$$\sigma[R_{d2\%} \mid H_s = 7.5\text{m}] = 0.24\text{m}$$

Thus, the present results demonstrate how long-term variation of wave conditions can be used to make assessments of wave runup and wave rundown within a sea state by using the formulas presented by the authors.

2.2 Example 2. Application of n-years return period contour lines

It should also be noted that the joint distribution in Eq. (10) can be used to determine the n-years return period contour lines. Estimates of the extreme wave runup and the extreme wave rundown can then be obtained by using the results corresponding to the n- (e.g. 1-, 10-, 100-) years return period contour lines (see MF10 for more details).

An alternative use of the n-years return period contour lines will be discussed in the following by using the results in MF10, i.e. more specifically, exemplified by utilizing the information of the 100-years contour line given in Fig. 1 reproduced from Fig. 8 in MF10.

Fig. 1 shows the 1-year and the 100-years return period contour lines of H_s and $\hat{\xi}_p = \xi_p / m$ represented by the inner and outer contours, respectively. It should be noted that these results are based on a joint *pdf* of significant wave height and spectral peak period obtained as best fit to data from wave measurements made in the Northern North Sea during a 29 year period (see MF10 for more details).

Firstly, consider Eq. (1); solving this equation for H_s expressed in terms of both the runup and the surf parameter gives:

$$H_s = 0.858 R_{2\%} \xi_p^{-0.77}$$

For a given value of the runup, this gives a curve in the (H_s, ξ_p) plane. The value of the runup which implies that this curve will have a tangent point with the 100-years contour can now be determined iteratively. The coordinates of the corresponding tangent point can also be found. This is shown graphically in Fig. 1 by means of the dashed line (note that the abscissa axis has been switched from ξ_p to ξ_p / m based on the slope m = 1/10; hence there is a scaling by a factor of 10 for the two variables in the present case). The value of the runup $R_{2\%}$, which gives a tangent point, is found to be 10.1m (i.e. corresponding to the 100-years return period value of the runup), and the coordinates of the tangent point are evaluated as ($\xi_p / m = 5.6$, H_s= 14.0m), or ($\xi_p = 0.56$, H_s= 14.0m). It is seen that the 100-years return period value of the runup is governed by H_s in the sense that the tangent point is located very close to the maximum value of H_s along the 100-year contour line.

Secondly, the same procedure is applied to the runup as expressed by Eq. (2); solving this equation for H_s gives:

$$H_s = R_{2\%} / (0.39 + 0.795 \xi_p)$$

This produces the full line in Fig. 1, corresponding to a value of the runup $R_{2\%}$ which is equal to 11.64m. The coordinates of the tangent point are quite similar to those based on Eq. (1), i.e. (ξ_p/m = 5.4, H_s = 14.2m), or (ξ_p = 0.54, H_s= 14.2m). Also now the 100-years return period value is governed by the significant wave height.

Thirdly, by applying the same procedure on the rundown in Eq. (3); solving this equation for H_s gives:

$$H_s = R_{d2\%} / (-0.21 + 0.44\xi_p)$$

It is noted that this expression has a singular point at ξ_p= 0.21/0.44 = 0.48. This implies that for smaller values of ξ_p there is an "unphysical" branch of the asymptotic curve which is not relevant and that should not be applied for the present purpose. The value of the run-down $R_{d2\%}$, which implies that the level curve has a tangent point at the 100-years contour is found to be 1.74m. The coordinates of the tangent point are evaluated as (ξ_p/m = 13.0, H_s = 4.8m), or (ξ_p = 1.30, H_s= 4.8m). The 100-years value is presently not governed by H_s; instead, the tangent point is located at combined and intermediate values of ξ_p/m and H_s.

A similar procedure can be used for other n-years return period contour lines to determine
the runup and rundown values together with the corresponding values of significant wave height
and Iribarren number on given slopes.

3. Summary

It has been demonstrated how the Blenkinsopp et al.'s (2016) extreme runup and extreme rundown formulae together with the Myrhaug and Fouques (2010) joint *pdf* of significant wave height and Iribarren number for sea states can be used to estimate extreme runup and extreme rundown based on long-term wave conditions. Examples of application for typical field conditions are given including: (1) estimating conditional expected values and standard deviations for a given sea state; (2) a procedure to determine the 100-years return period values and the corresponding values of significant wave height and Iribarren number. The present analytical method can be used to estimate wave runup and wave rundown for sea states based on available wave statistics.

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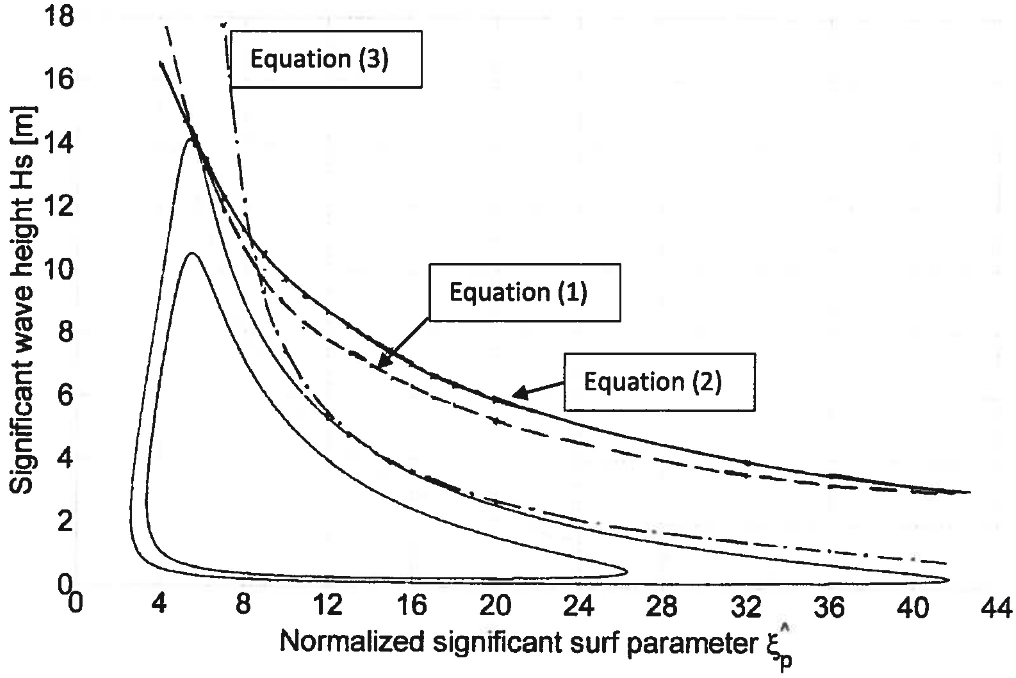

Fig. 1 1-year and 100-years contour lines of H_s and $\hat{\xi}_P = \xi_P / m$; 1-year (inner curve); 100-years (outer curve). Tangent lines to the 100-years contour line represent: Eq. (1) (dashed); Eq. (2) (full); Eq. (3) (dashed-dotted).