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Energy Procedia 23 (2012) 226 – 235

Energy Procedia

TRONDHEIM CCS CONFERENCE

Pipeline flow modelling with source terms due to leakage: The straw method

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Abstract

A numerical method for calculating the leakage flow rate through a crack in a pressurized pipeline is presented. Calculations of the flow inside, and leakage from, a pipeline with a running crack are performed. For the case of single-phase flow, the flow through the crack can also be calculated using choked-flow theory. The two methods are compared and identical results obtained. The advantage of the present method is that it does not rely on analytical expressions for the flow through the crack, and it is therefore thought to be applicable for two-phase flow, including phase transition. Hence it may be of use in the development of coupled fluid-structure models for the assessment of running ductile fracture in carbon dioxide transport pipelines.

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Keywords: CO_2 transport; flow through a crack; choked flow; fluid dynamics; thermodynamics

1. Introduction

Pipelines are a common and convenient way of transporting natural gas, and with the increasing interest in carbon capture and storage (CCS) technology, pipeline transport will also become an important link between the capture and storage sites of CO_2. As for natural gas, a rupture of a CO_2 pipeline can cause serious accidents as well as economic losses and must be avoided. In order to control and predict the risk of accidental failure, such as a running fracture initiated e.g. by damage to the pipeline by a third party, the fracture properties of the pipe materials have long been a subject of study. A semi-empirical model based on research at the Battelle memorial institute in the 1970s [1] where the fluid flow

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and the material structure behaviour are assumed to be uncoupled processes, is traditionally used for the assessment of running ductile fractures. New pipeline materials have, however, motivated the search for improved models, and it is natural to consider a coupling of the fluid and structure processes [2]. Moreover, the thermodynamic properties of CO_2 are different from those of natural gas at the relevant conditions for pipeline transport. It is not clear how e.g. phase change and a large heat capacity will influence the fracture mechanics. Further, various impurities will be present in the transported CO_2, and even small amounts will change the properties compared to pure CO_2 [3,4]. Therefore, a flexible framework is required with respect to the employed equations of state. For hydrocarbon mixtures, the Peng–Robinson equation of state is appropriate, and was used by Oke et al. [5] in a study of a fluid model for pipeline puncture.

In Berstad et al. [6], a coupled fluid-structure model was presented and tested by comparisons to full-scale experiments of running ductile fractures in steel pipelines. In this model, the effect of the leakage of the fluid through the crack opening is included in the one-dimensional fluid equations as source terms. The emerging fluid pressure is then coupled back into the structure part of the model as a load. In order to evaluate the source terms in the fluid part, the fracture is modelled by a sequence of orifices and the leakage is assumed to be an isentropic process. By using the ideal-gas equation of state and the choked-flow theory it is possible to derive analytical expressions for them. The model agreed well with the experiments, but is still restricted to the ideal gas case and therefore premature to be applied for CO_2 at typical operating conditions. A generalization of the model to handle other equations of state may follow two paths, either an analytical approach where the source terms are derived explicitly as in [6], or a numerical approach in which the source terms are evaluated using the flow solver. It is the latter that will be studied here, and it will be referred to as “the straw method”. In this study, we assume the fracture opening to be given, so that the structure part of the model can be ignored, and the focus is on a one-dimensional fluid model inflicted by a radial leakage. Also, this study intends to prove the concept of the straw method, therefore we consider only single-phase, ideal gas, for which we have analytical results. However, this method is developed to be easily generalized to two-phase mixtures.

In the following, the governing equations of the one-dimensional (1D) fluid model will be presented in Section 2, and Section 3 then gives an introduction to the straw method. Section 4 explains the implementation of the straw method for a pipe filled with pressurized gas. Further, Section 5 presents some numerical simulations of a pipe depressurization due to a running fracture. Finally, Section 6 concludes the paper.

2. Governing equations

It was shown in Berstad et al. [6] that the leakage of the fluid in a fractured pipeline, illustrated in Fig. 1, can be incorporated into the Euler equations, as source terms written on the right-hand side:

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}=-\zeta_e, \tag{1}$$

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho u^2)}{\partial x}+\frac{\partial p}{\partial x}=-u\zeta_e, \tag{2}$$

$$\frac{\partial E}{\partial t}+\frac{\partial\left[(E+p)u\right]}{\partial x}=-(E_e+p_e)\frac{1}{\rho_e}\zeta_e. \tag{3}$$

Herein, p, ρ and u are the pressure, density and the x-directed velocity, respectively, and $E = \rho(e + u^2/2)$ is the total energy per unit volume, with e being the internal specific energy. The general source term ζ_e is the leakage mass flow rate and can be written

$$\zeta_e = \rho_e u_e \frac{2r_e}{A}, \tag{4}$$

where the subscript e indicates that the quantities are taken in the crack opening through which the fluid may escape. This opening is given by its width $2r_e(x)$, whereas the cross section of the main pipe is A. For simple equations of state, analytical expressions can be derived for the escape quantities [6], but in general, numerical methods must be used.

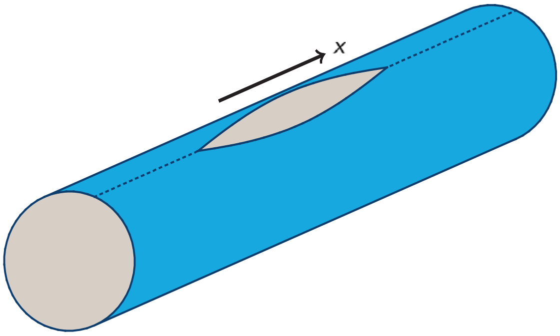

Fig. 1. A section of the pipeline with a fracture running along the x-direction.

3. The straw method

The straw method takes the fracture geometry as given. The main challenge is to evaluate the flow rate through the fracture. For a two-phase flow with a "black box" equation of state, an attempt to develop analytical expressions for choked flow may lead to intractable expressions, let alone the inclusion of phase change. An alternative idea is then to let the flow rate be evaluated by a numerical solver analogously to what happens inside the pipe. We assume here that the fracture along the pipe can be modelled by a sequence of transversal tubes, whose length is the thickness of the pipe steel. These tubes are plugged into the main pipe, and their cross-sections represent the crack opening (see Fig. 2). The fluid dynamics in the tubes, as well as in the pipe, is solved as one-dimensional conservation laws averaged over the cross-section. By inserting one tube in each of the fractured pipe cells, we obtain a discretization of the fracture along the pipe. The variation of the fracture width is represented by adjusting the tube diameters at each time step. The propagation of the fracture is accounted for by adding new tubes along the pipe.

The inlet flows in the transversal sub-tubes become mass, momentum and internal-energy source terms for the flow in the main pipe. To simplify, we assume that the flow in the pipe is quasi-stationary with regard to the flow through the fracture, therefore we let the sub-tubes reach steady-state flow between the pipe and outside pressures at all time steps. Particular attention has to be given to the boundary conditions for the sub-tubes, depending on whether the outflow is choked or not.

In the present paper, we validate the approach by applying the straw method to a single-phase pure gas. We can thus compare the results to those obtained by a method using the analytical expressions from the choked-flow theory.

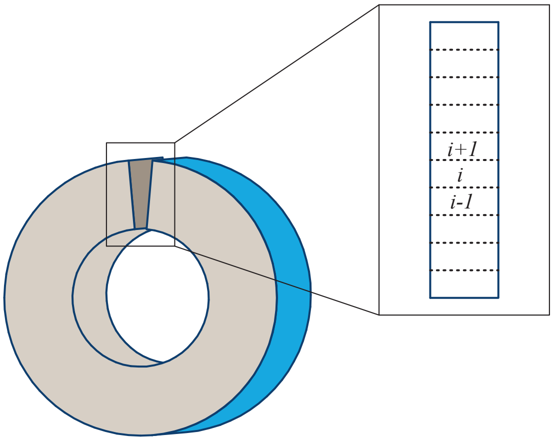

Fig. 2. A small section of the pipe where the exaggerated pipe wall is partly open due to a crack. The flow rate through the opening is evaluated separately by modelling the crack as a sequence of small tubes, or straws, transversal to the main pipe as indicated. Each tube is discretized into finite volumes, *i*.

4. Implementation of the straw method

This section begins by explaining how the flow through the crack is modelled, and how it is implemented in the numerical framework. Then the boundary conditions that apply to the straw are detailed.

4.1. Modelling choices

Fig. 3 (a) illustrates the leakage flow through a crack in a pipe, where the crack is thought of as an orifice. Ahead of the orifice, the fluid inside the pipe is accelerated, while the pressure decreases. The streamlines are contracting and all leading to the orifice. Downstream of the orifice, the fluid is expanding and decelerating, while mixing with the outside air [7]. We assume that the flow ahead of the orifice is isentropic. This supposes that heat exchange can be neglected, which is acceptable due to the high velocity of the gas. This also supposes that we can neglect the viscosity. In this situation, the flow is similar to that in a convergent nozzle leading to the orifice, see Fig. 3 (b). However, downstream of the crack, the mixing with the outside air makes the isentropic assumption invalid. We therefore decide not to model the diverging part. Instead, we let the outflow boundary condition govern the release pressure. Fig. 3 (c) shows the pressure profile along the leakage flow. Two regimes can be distinguished. In the subcritical regime, which the pressure at the orifice is equal to the atmospheric pressure. The velocity across the orifice is then subsonic. In the supercritical regime, also called choked flow, the pressure at the orifice becomes independent of the atmospheric pressure. The velocity across the orifice is then exactly equal to the sound speed in the fluid. This duality may be problematic when the pressure ratio is supercritical, because the outflow pressure at the orifice is not known *a priori*. This may cause convergence problems that the boundary conditions have to handle.

In the present work, the straw is of constant cross-section. When there is no phase change or friction, the steady state is reached when the pressure profile is flat. The flow ahead of the orifice has to be accounted for in the inlet boundary condition. Actually, in this work the straw is only present to connect the two boundary conditions, see Fig. 4.

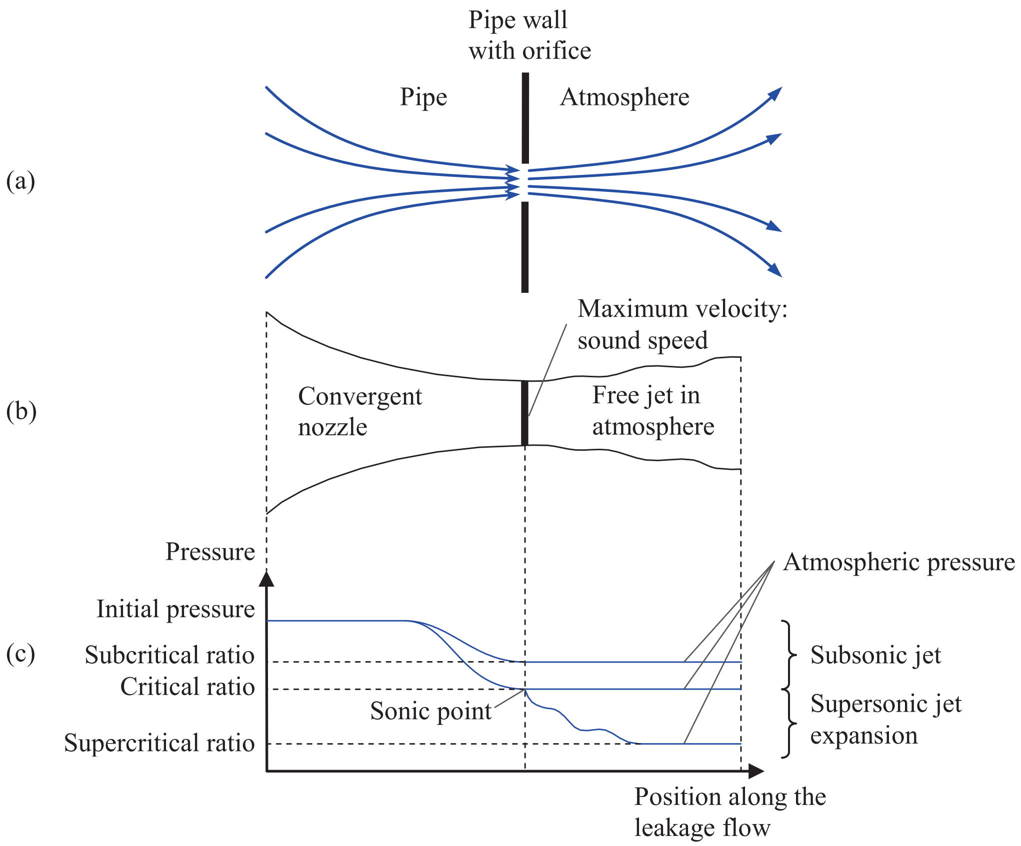

Fig. 3. (a) Real situation of a leak across an orifice in the pipe wall; (b) Model of the leakage flow using a convergent nozzle and a free jet in atmosphere; (c) Pressure profile in the adopted model, depending on the ratio of the internal pressure to the surrounding (atmospheric) pressure.

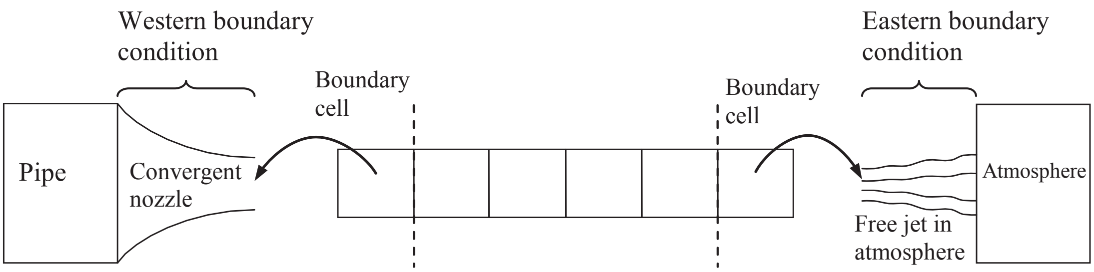

Fig. 4. Practical implementation of the straw method.

4.2. Straw inlet boundary condition

The convergent flow is accounted for by the inlet boundary condition. Since the model contains three equations, three quantities have to be taken care of to fully describe the state of the fluid at the straw boundary. Depending on the flow regime, these quantities are either specified to be equal to their values in the pipe, or extrapolated from the straw. In the straw method, before the steady state is attained, we expect either a subsonic or a supersonic outflow from the pipe, or a reversed subsonic inflow.

In an isentropic flow, the entropy s is a conserved quantity since we can write

$$\rho\frac{\partial s}{\partial t}+\rho u\frac{\partial(s)}{\partial x}=\rho\frac{Ds}{Dt}=0. \tag{5}$$

Next, we need a quantity characterizing the energy. We start from the energy equation

$$\frac{\partial E}{\partial t}+\frac{\partial\left(E+p\right)u}{\partial x}=0, \tag{6}$$

in which we add and subtract the time derivative of the pressure

$$\frac{\partial\left(E+p\right)}{\partial t}+\frac{\partial\left(E+p\right)u}{\partial x}-\frac{\partial p}{\partial t}=0. \tag{7}$$

In steady state, we recover a conservation equation

$$\rho\frac{\partial\left(h+u^2/2\right)}{\partial t}+\rho u\frac{\partial\left(h+u^2/2\right)}{\partial x}=\rho\frac{D\left(h+u^2/2\right)}{Dt}=0. \tag{8}$$

Therefore the second chosen quantity is what we can call the total enthalpy per unit volume, $H=\rho\left(h+u^2/2\right)$, where h is the specific enthalpy. The third chosen quantity is the momentum, ρu.

Table 1 shows which variables are specified or extrapolated depending on the flow regime. In the subsonic regime, the effect of this boundary condition is to convert the mechanical potential energy of the pressure into kinetic energy. The total enthalpy, which contains all the potential energy but the mechanical potential energy, is conserved along the convergent nozzle. On the other hand, the straw imposes the momentum and indirectly the kinetic energy. Together with the entropy, from the pipe or from the straw depending on the direction of the flow, we can determine the thermodynamic state of the fluid as well as its velocity.

Table 1. Straw inlet boundary conditions.

	Specified to be equal to its value in the pipe	Extrapolated from the straw
Supersonic outflow from pipe	s, H, ρu	-
Sonic/subsonic outflow from pipe	s, H	ρu
Subsonic inflow to pipe	H	s, ρu

4.3. Straw outlet boundary condition

The outlet boundary condition has to account for the discharge to the atmosphere. Most of the time, the flow leaving the straw is either exactly sonic or subsonic. Due to the fact that we do not model the jet in the atmosphere, the release pressure is not necessarily the atmospheric pressure. As shown in Fig. 3 (c), if the pressure ratio is subcritical, atmospheric pressure applies at the orifice. However, if the ratio is supercritical, the release pressure at the orifice is different from the atmospheric pressure. In calculations, this may cause the pressure to oscillate between the atmospheric pressure and the orifice pressure.

To cover all the possible situations, subsonic or supersonic outflow, and subsonic inflow, we again choose three variables to specify or extrapolate. Here we use the natural primitive variables: the density, the velocity and the pressure. Table 2 shows how the variables are either specified or extrapolated. In the supersonic regime, the pressure is extrapolated from the straw, and therefore the atmosphere does not have any effect on the flow in the straw, as expected. In the subsonic regime, the straw always decides over the discharge velocity, whereas the density comes from the side from which the fluid is flowing.

Table 2. Straw outlet boundary conditions.

	Specified to be equal to the atmospheric value	Extrapolated from the straw
Supersonic outflow from pipe	-	ρ, u, p
Sonic/subsonic outflow from pipe	p	ρ, u
Subsonic inflow to pipe	ρ, p	u

As can be seen in Table 2, the pressure is imposed or not, depending on whether the flow is sonic or supersonic. Now, when the flow is choked, the velocity at the straw end should be exactly sonic, while the pressure in the straw and in the atmosphere will be very different from each other (Fig. 3 (c)). Therefore numerically, the slightest oscillation of velocity around the sonic point will cause a large variation in pressure, thus hindering convergence. Since we are only interested in the steady state in the straw, we can speed up convergence by correcting the imposed boundary pressure, as long as the correction term vanishes in steady state. We define λ as the difference between the flow velocity and the sound speed. For a constant C that we choose equal to 0.1s/m, the outlet boundary pressure is defined as

$$p_{boun} = \begin{cases} p_{straw} & \text{if } \lambda \geq 0 \text{ (simple extrapolation)} \\ p_{straw} + \left(p_{straw} - p_{atm}\right) C\lambda & \text{if } \lambda < 0 \text{ and } \left|C\lambda\right| < 1 \\ p_{boun} = p_{atm} & \text{if } \lambda < 0 \text{ and } \left|C\lambda\right| > 1 \end{cases} \tag{9}$$

Note that the boundary pressure p_{boun} is a continuous function of λ, the atmospheric pressure p_{atm}, and the straw pressure p_{straw}. Further, in steady choked flow, λ=0, and we recover the simple extrapolation. In steady subsonic flow, $p_{atm}= p_{straw}$, so that the outflow pressure is the atmospheric pressure. Therefore the correction term does not have any effect in steady state.

5. Numerical results

Numerical tests have been performed for a pipe filled with methane at 122 bar, closed at both extremities. The pipe is 12m long, has a diameter of 0.261m and is divided into 400 cells. After 2ms, a crack is initiated in the middle of the pipe, propagating at a constant velocity of 100m/s in both directions. The crack is shaped like a sinus function (cf. Fig. 1) and its width at maximum opening is 0.2m. We let it evolve during 30ms.

The numerical method used is the multi-stage (MUSTA) centred method with four cells and four substeps [8,9] with forward Euler time steps. The source terms are solved with first order time splitting.

Fig. 5 shows the evolution of the pressure in the pipe at approximately 1m (x=4.995m) and 3m (x=2.985m) from where the crack is initiated. We see that the depressurization begins later for the red curve, further from the crack, than for the green curve. This is due to the depressurization wave travelling from the crack towards the extremities of the pipe. The kink in the red curve is due to the reflection wave from the closed end of the pipe. There is also a smaller kink in the green curve at t=0.0012s, due to the crack tip passing at the corresponding position. The black squares – denoted "ref" – are the results using the choked flow theory. We see that the results are practically identical, thus showing that the straw method in one dimension gives the same results as the analytical theory to reasonable accuracy.

Fig. 6 shows the pressure profile in the tube at four different times. The depressurization starts in the middle of the pipe, where the crack is initiated. Then decompression waves begin to propagate towards the extremities, faster than the crack progresses. The crack tips are located at the two kinks in the pressure profile, e.g., at about 4 and 8 m in the last graph. Further, there is a ridge in the pressure profile in the middle of the pipe. Although this ridge probably exists in reality, its size may be exaggerated in this method. The ridge is due to the fluid flowing towards the middle of the pipe and colliding with the fluid from the other side. In reality, the fluid is deflected transversally towards the crack, but in one-dimensional flow, the fluid is constrained to move longitudinally. Therefore it has to stop first, thus building up pressure, before it is accelerated again transversally in the straws. However, the total enthalpy is conserved in both cases. Further, since this always happens in a region of the pipe where there already is a crack, it does not impact the crack-propagation problem.

6. Concluding remarks

We have presented a numerical method to evaluate the leakage flow rate through a fracture in a pipe. The method has been applied to single-phase flow, where a direct comparison to choked-flow theory is possible. The two methods were compared for a case with a running fracture. The results obtained were identical to plotting accuracy; therefore it is insignificant whether the leakage flow is a sequence of orifice flows or of straw flows in the single-phase case. Hence, we deduce from the results of Berstad et al. [6] that our approach is valid.

This method was developed with the extension to two-phase flows in mind. Such an extension is future work, but is believed to be reasonably straightforward, since it does not rely on finding analytical expressions for the flow through the fracture. The only model-specific part is the adaptation of the boundary conditions of the straw, following the same principles of release in the atmosphere at the outflow, and conservation of entropy, total enthalpy and momentum at the inflow. The present method is

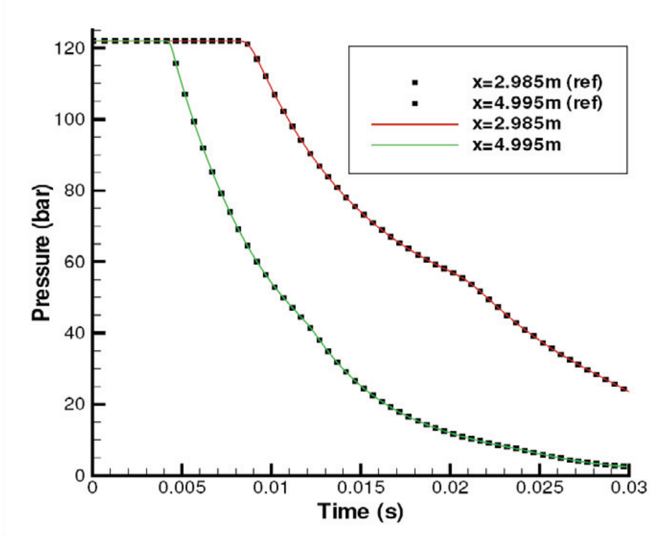

Fig. 5. Evolution of the pressure in the pipe at two positions.

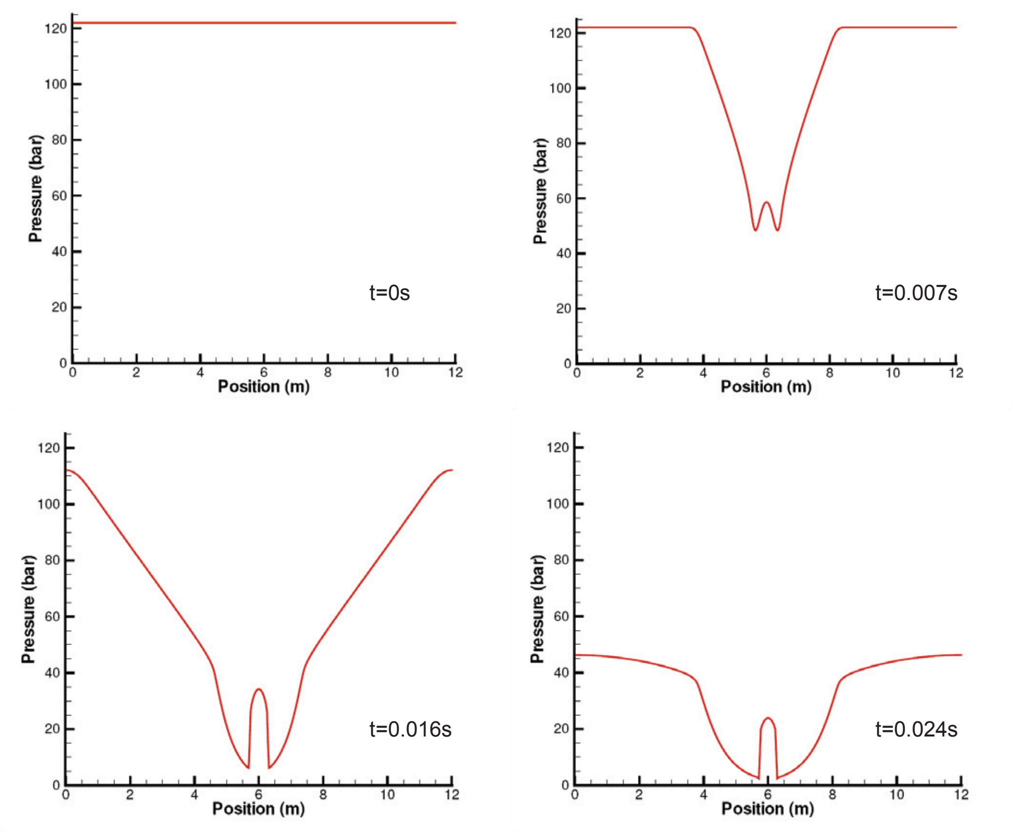

Fig. 6. Pressure profile in the pipe at four different times.

hoped to be of use in the development of coupled fluid-structure models for the assessment of running ductile fracture.

In the single-phase case, the flow in the straw itself is uniform, everything happening in the boundary conditions, which is why orifices and straws give indistinguishable results. The aim with the straw method for two-phase flow is to let the straw account for phase change due to the depressurization across the fracture. This is an important aspect, since the sound speed may change significantly when the gas volume fraction is changing. Since the flow rate is limited by the sound speed, phase change across the crack will have an effect on the flow rate. This modelling choice will have to be assessed by comparison to experimental results with two-phase flows.

Compared to previous work [5,6], the present method offers a high flexibility with respect to the flow model, for example single-phase flow or two-phase flow with or without phase change, friction or heat exchange in the pipe and across the fracture. And last but not least it can naturally handle any thermodynamical routine, either analytical or "black box".

Acknowledgements

This publication has been produced with support from the BIGCCS Centre, performed under the Norwegian research program *Centres for Environment-friendly Energy Research (FME)*. The authors acknowledge the following partners for their contributions: Aker Solutions, ConocoPhilips, Det Norske Veritas, Gassco, Hydro, Shell, Statkraft, Statoil, TOTAL, GDF SUEZ and the Research Council of Norway (193816/S60).

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