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From Rock Scissor Paper to study and modeling of Chinese Five Elements

Evolutionary Game Theory

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Master's Thesis

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This Thesis work has been assigned by and written for Professor Sheng Li at Shanghai Jiaotong University, which is relevant in his research in game theory. The thesis work was already pre-studied in October, when I had the pleasure to discuss very much with Professor Li in Shanghai. The writing process of thesis was mainly carried out while working in US, with Professor Li sincerely helped me with advice through e-mails.

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Abstract

In this thesis, we have developed a model to study the behavior of an extended version of evolutionary game "Rock-Scissor-Paper", that is "Five Elements". The origin of five elements are from "Chinese five elements" in which five different types of elements compete with each other in a similar way as in a SIR model. Elements are affecting each other in two different ways, either to compete with each other or to help some particular elements in resisting their superior elements. The competition and cooperation is linked in circle which initiates a steady state. The analytical expressions (mean field) and simulation results have been presented. We have made some basic simulations of the original "Rock-Scissor-Paper" in order to build a better understanding of this kind of evolutionary game, and to extend from three elements to five elements. One of the real examples of five elements we have studied is a game called "Rock-Paper-Scissor-Lizard-Spock", which is a direct extension from the "Rock-Scissor-paper". Cooperation in the game is expressed in form of either direct cooperation/help from different elements as in "Chinese Five Elements" or indirect help caused by direct competition. Based on the simulations, some characteristic behaviors of five elements have been found. Reaction rates and different competition probabilities have proven to be the critical part in the game.

Table of Contents

Chapter 1

Introduction

1.1 Foreword

We live in a world full of games and competitions. In order to find the optimal way to win, under a certain degree of limited resources, mathematical game theory analysis support to other research directions or practical analysis has been increasingly developed and applied. It's much developed in economics to find some typical behaviors in economy. It has been further applied to sociological, political and physiological problems for explaining general behaviors in a systematic way.

Let's first consider the definition of a game. A game is a competition where participants are fighting under limited resources, using different strategies to gain the highest possible final outcome for their own. It is consisted of a set of players, a set of strategies available for them to choose and a set of payoffs generated by the outcomes of strategies. In real world, games exist everywhere, in the competition between companies fighting for the dominance of the markets, in wars fought between countries, in video games where players are using different strategies and the available resources to win a simple round and so on. All of them may develop and use different strategies in order to win. And game theory is, as expressed by Roger B. Myerson, "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers."[1] The complexities and variations of the strategies and decision-makings cause the game theory to be more interesting, attracting and or course challenging.

The history of game theory can be dated as early as back to the time of Darwin where he made some informal evolutionary game theoretic statements. In 1930s, Ronald Fisher used game theoretic analysis to study animal behaviors and this predates the name "Game Theory". Evolutionary game theory, being developed on the basis of game theory, is perhaps the most common and useful physical way of considering non-stationary systems like evolutionary mechanical systems, the whole theory background has been much influenced by the theoretical and statistical physical view, in addition to complex network principle. Today, with the increasing branches under the evolutionary game theory, the whole of physical view is becoming opening up widely and game theory gradually becomes mature.

The network principle a very important factor connected to the game theory. Each individual, are in different networks, such as the various biological networks in the human body, the information network in our everyday, transport network, ecosystem network, and so on. The study of the basic properties of these networks is necessary, while in the physical field, complex network's structure and the statistical properties, the largest degree of correlation, the network's study also included space among the chaotic nature of the inquiry, the network research on the complex system of the phase transition point, the mean

field and its derivation and much more, are all interesting and important characteristics which can be combined with game theory.

In Biophysics, evolutionary game theory is also a hot spot concerning problems in ecology. In an ecosystem, the participants' preferences and purpose for the maintenance of biological diversity is one of the basic problems in ecology, and this problem is related to the cycle between species' interactions and the hierarchy. Therefore researching on the cycle of interactions between the the species' individual space and the organizational structure regarding how to maintain the species' diversity is of great practical significance.

Evolutionary game theory is providing a great understanding to interactions between the individual species and the the organization as a whole. "Rock, scissor, paper" simply represent three species in an ecosystem, if species' diversity has been maintained, the system is in a steady state after three species coexist; If only one species survive, species diversity has been destroyed. "Rock scissor paper" game reflects an ecosystem in a cycle of mutual predation, and formed a small ecosystem with mutual constraint, to live together.

1.2 Purpose

The purpose of this project is to first consider the game of "rock scissor and paper" for recreating the basic law for a game system with mutual predation(which also implies mutual survival), to a more complex game influenced by the Ancient Chinese Five elements game, where the original three-node system has been extended to a five-node system.

In 1996, researchers discovered a real ecosystem "rock paper scissors" game applied to the color shift of lizards in California[2]. And then the three subspecies of E. coli was also found to follow the same game principle. The biologists found that there are a total of three kinds of E. coli in nature, their mutual constraint and symbiotic interaction is observed between the three E. coli in accordance to the relationship between a "rock - scissor - paper", viewing the E.coli interactions as the theoretical background to extend it to a more complex system is providing an induction way to find a universal law for a evolutionary game theory in the same kind.

Based on the "rock - scissor - paper", we will recreate the scheme of a 3-node game, then go on to simulate the Five Elements as a more complex game.

For the ecosystem as a whole, it can simply use the complex networks principle such in rock - scissor - paper game simulation. In reality, the individuals all have a chance to interact with distant individuals. By adding a probability P in the evolution of individual game to a certain individual implies a fixed game, that the steady state will disappear. The computer simulation results will be displayed to show: when P is greater than a critical value, the diversity of species disappear after a certain evolution time, and only a single specie would survive, after adding parameters and make adjustments later in this article. Within a certain interval of the fixed probability, the number of a certain specie would also change periodically dependent on P. This probability change expresses a non-uniform game which could also be found as exceptional examples in the real world due to some species' evolving. The same scheme will be applied to five elements to see the survival of different elements, in order to find an universal rule.

Chapter 2

Theoretical Background

2.1 Classic Game Theory

The traditional game theory is a research actually based on rational assumption of individual mutual interaction, it is built on the basis of assumptions regarding the game structure. The development of game theory is therefore a systematic breakthrough from the limitations set by the assumptions.

In classical game theory, we can assume that the participants' goal are in accordance with the favored strategy, and its objective is therefore mutual dependent. Mutual dependence means that the every participant's decision will be affected by other participants, and his behavior will also affect others in a game. Because of this mutual dependence, results of the game is simultaneously dependent on each participant's decision, nobody can take control of the event or the outcome by his own, and nobody stays alone as an isolated outcast. Mutual dependence causes the participants to compete with each other during the game, but the competition is not everything in the game theory. Taking an example of dividing a cake, although a little bit of non-even divide could cause an unfairness, causing a potential competition to occur between two people, but on the other hand, it's also possible to increase the size of the cake to be much bigger so that the divided two parts would increase simultaneously, thus a cooperation relation is established. In the classical game theory, the cooperation and competition is the the focus point.

The classic game theory states that the participants are completely rational, this is the basic assumption for the classical game theory. And entirely rationality for participants is an universal sense. However, here the reference of rationality seems to be an obscurely defined concept. Extreme views can even believe that the concept of perfect rationality is apparently rational behavior of tautology and decided by the rules of the game, but the conduct of the game based on the assumption of rationality is again, rational. How to define the reasonableness of the central recurring theme in the development of game theory through years is certainly not an accidental phenomenon. In fact, any rational definition is a negative definition, that the rational participants did not explain what should be done, and the main concern is that they should not do. In addition, the theory also pointed out, rational participants are not necessarily in the game to be dominant, because the entire rationality may cause to exclude the possibility of cooperation, such as the problem of the classic Prisoner's Dilemma. To fix these shortcomings of classical game theory is dependent on improvement in balancing the rational extent and strategy equilibrium. In the history of the game theory development, it's already been a lot of improvements from the development of the traditional game, which is static with open information, to the development of dynamic games without full access to information.

After the Nash-era, the development of the game theory is focused on the rationality level and the improvements of the strategy. During this time, a lot of games have been developed, like hawk-dove game, prisoners' dilemma and so on. In these games, the option is to select cooperation or betrayal, for their own sake, like in prisoner's dilemma:

Participant 2/participant 1	Cooperate(C)	Not Cooperate(D)
Cooperate(C)	(R,R)	(R,T)
Not Cooperate(D)	(T,S)	(P,P)

Table 2.1: An example of Prisoners' dilemma scheme

In this game, R,T,S,P can represent the outcome of different strategies or the strategy sets, they satisfies $T > R > P > S$ and $2R > T + S$.

Although classical games have achieved a great success through these developments, but it's still restraint to many limitations, its development and implementations sometimes demand improper assumptions, therefore new breakthroughs must be added in order to achieve more developments of the classical game theory.

2.2 Evolutionary Game Theory

Research objects of evolutionary game is a group or a set of objects which are time-dependent and evolving in time. The aim for this game model is to study the time dependence and the dynamic behavior of this group, in order to explain and understand how the group is behaving is as it is and what results their current behavior. Through evolutionary game theory, we can see that the group evolutionary behavior is constituted by randomness and other perturbations, in other words, sudden change. At the same time, because of the selective rule set by the game's constraints, periodicity will also occur. The most evolutionary game theories can predict and explain group behaviors based on this selection rule and its process, groups usually select with according to their habits, their favored strategy, but this process also contains the possibility of sudden change, a phase transition, thus causing to new evolutions and new characteristics.

In the field of biology, evolutionary game theory is applied to discuss the biological evolutions through modeling and simulation. For example, when the biological fitness of a particular phenotype is dependent on its population distribution frequency, evolutionary game theory will become a way of thinking about biological evolution from the perspective of the phenotype. Another example, in a comparative study of morphological evolution of the bird's wing and the same distribution of birds behavior in order to figure out the shape of the wings, the atmospheric conditions of life in which these birds with different wing shapes caused by the difference of the lift and drag. It is also necessary to check if wing feathers contribute to constraints also included in the range of considerations, because if one compares bats with ancient pterosaurs, this constraint is clearly different. However, there is no need to consider the behavior of other members of the population. However, the evolution of biological distribution relationship mainly depends on the same kind of

biological action, because distribution and finding the right spouse, to avoid competition for resources, and jointly prevent predators between biological factors are closely related.

Evolution as a historical process, it can be said that the result of a series of events which led to this theory, are difficult to reproduce. This process can produce two categories of theory, that is, universal theory and particularity theory, evolutionary game theory belongs to the second category, it assumes that evolution from natural selection within populations, while game theory is an effective tool to construct this theory makes it able to explain the specific evolution sober, and to identify the power of natural selection that will lead to the evolution of specific (genetic) characteristics. People sometimes think that the particularity of the second category can not be verified, because history does not repeat itself, and that certain factors do not change, then re-test results to check if they really change. But this ignores an important point, is the explanation of the causal relationship needs to be built on the basis of the test.

Evolutionary game theory integrates the idea of rational economics and evolutionary biology, and that the game players are no longer modeled as super-rational, is trivial with the biological evolution. The selected the equilibrium is the equilibrium achieved through the function to obtain equilibrium, therefore, history, rules and the details during the processing of equilibrium would influence the different equilibriums in a game. So evolutionary game theory is also having an important impact in economics, some results can explain economical phenomena, such as "rock-scissor-paper" can be found in similar economic cases.

Another different point between evolutionary game theory and traditional game theory is that evolutionary game theory has chosen to abandon perfectly rational assumptions, such as the ideological foundation of Darwin's theory of biological evolution and the Lamarckian genetic theory, starting from a system theory, considering the groups' behavior as an adjustable process which reflects dynamic, in which the relationship between the behavior of each individual and the groups separately characterize the formation mechanism, as well as from individual behavior to group behavior can be related to various factors into, a evolutionary game model, constitutes a macro-economic model with micro-based, and therefore be able to more realistically reflect the diversity and complexity of the group behaviors, and can provide a theoretical basis for the macro-control group behavior.

In evolutionary game theory, actors are assumed to be programmed following a given behavior, their understandings of rules for the economical conduct or some successful understanding of behavioral strategies can be constantly improved and revised in the process of evolution. Successful strategy is to imitate, in order to establish some general "rules" and "systems" following the behavior of the main action standards. In these general rules, the actors get the "satisfaction" in the form of income, but this process requires a relative long time, to establish. Evolutionary game theory states that time is an irreversible state, what happens in the past time and in the next time is totally asymmetric, therefore, the state of evolution is closely related to the initial state. Random (mutations) factors play a key role in the evolution process, mutation is often seen as a process of trial and error. The perpetrator will try a variety of different behavioral strategies, and will generally have some adjustments following different times.

Evolutionary game theory draws an important principle of "Evolutionary Stability Strategy", or ESS. A strategy is the performance of an act, which is the individual performance

by a phenotype in a known situation, thereby the actions taken in this environment. If every player is set to take this strategy, under the conditions of natural selection there would not be any mutation strategy to violate this population's current common strategy. The concept of using "strategy", is derived from studying the behavior of animals. This idea can be equally applied to any kind of variations in the phenotype, "strategy" word can also be replaced by the performance of phenotype, a strategy may be the state of plant growth, the relative age of parents to have children and the number of their children. According to ESS, we can see a lot of individual organisms to sacrifice their own interests for the collective interest considerations.

2.3 Achievements and Applications of Evolutionary Game Theory

To say something about the future development and application of evolutionary game theory, the inspiration is firstly from the evolution dynamics, replication dynamics as a tool to calculate a strategy in the percentage of game populations,in order to express the game participants' behavior and status. Later mathematicians improved this evolutionary dynamic based on this tool. The initial breakthrough of the evolutionary game theory comes from the study of evolution dynamics with focus concern mainly on the characteristics of a system in a long term period of time. It also includes several aspects, such as periodicities and fixed points, the system's stability, existence of the chaotic state, as well covering static concept, such as the Nash equilibrium, evolutionary stability related to the dynamic predictions. Another development of evolutionary game theory incurred in three aspects for further improvements, such as the deeper understanding of bounded rationality mechanisms, innovation of kinetic mechanism, balanced choice for multiple Nash equilibrium cases, these parts will be the future exploration of the evolution dynamics and the development foundation.

At the same time, in evolutionary biology, evolutionary biologists Nowak and May introduced the game to the space lattice[3], they found that the inquiry on the spatial structure to provide answer to the problems which troubled people for a long time, is necessary, through repeated number of games, for instance the well-known problem in the prisoner's dilemma, we can see that the strategy of cooperation can exist in the space topology frame and by taking a step forward to upgrade Prisoner's outcome to a matrix , finding and directing to the weaker prisoner's Dilemma, then T = b, R = 1, P = S = 0, and satisfies $1 < b < 2$.

Evolutionary game theory related in physics research has generated a fruitful of results recently. For physicians it's important to apply the evolution of the game dynamics to statistical mechanics, and through this to develop the theory of dynamic evolution. In addition, along with the development of traditional non-linear and statistical mechanics, hot topics as complex network has also been greatly studied, and added a large number of applications for game theory. For example, a qualitative leap into this area much recognized is the contribution by two physicians Szabo and Toke in 1998[4], their work in the two-dimensional plane lattice Ising model along with the Fermi mechanism introduced into

the game space. Their discovery from space topology, that each grid between adjacent connected to each other to form a tight cluster in order to resist the "attack" from the betrayer, at the same time, they introduced phase transitions, percolation and mean-field approximation into the game space. In recent years, based on their work, new theory such as the spatial game theory has been established, more realistic problems like spatial topology network have also been increasingly studied with positive outcome.

Evolutionary game theory has been increasingly applied to other kind of studies, some practical,this causes more people to pay attention to its theoretical and practical significance. From the "Tragedy of the Commons" proposed by Hardin to the recent popular social problems concerned by everyone, like the public transportation system, citizens' health insurance, environmental issues, the distribution and redistribution of resources, and biological diversity etc, are all attracting attention, these problems are closely linked with mankind. These realities is no doubt the reality of human society, games played between human beings and the nature. How to optimize the treatment of these issues is worth humanity itself to ponder. At the same time, the evolutionary game theory in the development and improvement of the sociology and economics properly handle these problems to provide a good theoretical support, in the future, people might be able to get more from evolutionary game in order explore better solutions to solve these kinds of problems.

We have entered the "Post-Darwin" era, he proposed series of biological evolution theories which do not fully explain the common biological phenomena in our daily lives, such as cooperation, because his theory is based on appropriate survival, rather than altruistic cooperation. So, how to break through the traditional Darwinian theory to explain these complex biological phenomena, evolutionary game theory becomes the best tool for understanding based on mathematical reasoning and proof. In addition, when exploring network problems, game theory also provides one significant way to consider these problems. In addition to the traditional method of constructing the network, improvements and reestablishing through the principle of evolutionary game theory open up new ways for understanding. Finally, evolutionary game theory can explain certain phenomena of biological evolution successfully, and it is better than the classic game theory for explaining and analyzing the realistic problems in economics and management related issues.

2.4 Network Theory

While Evolutionary game theory is mostly combined with time, we would also introduce the basic network theory into the system. As common simulations are carried out on a mash, which is commonly known as graph, due to network's property of consisting of objects with connections.

In a graph, the objects are called nodes, and the connections between objects are called links. The links may be directed, that means that the affection does not always go in either ways, sometimes perhaps only from one object to another. There are also un-directional links, which can travel in both directions. Links can also have weights assigned to them, often expressed as probability, in order to indicate the cost of using each link or the link's capacity. An example of a un-directional network of people is given here in the figure 2.1

Figure 2.1: A figure of a network example consisting of 17 nodes, occupied by persons, and numbers next to each person is its degree k

As figure 2.1 showed, each node has *degree, k*, defined as the number of connections to each node from other nodes. In figure 2.1, there are nodes with 1,2,3 or 5 degrees. The degree distribution *p(k)* is important when considering from each node and for the network's properties. Degree variation brings great difference in networks and difficulties in analyzing them. Real-world networks can often be heterogeneous, that means that they have large degree variations. On a graph made for simulation of game theories, the degree variation could contribute to different outcome of game results. This is the reason why homogeneous networks are better for simulation of game theories, which is often carried out on a 2D lattice.

Another important definition is the shortest path, commonly used between two nodes. This is the shortest distance between them by traversing the network. In a network where all links have a unit weight, the shortest path is equal to the lowest number of links used for traveling from one node to the other. One important property of network to be taken into account is the average distance between nodes increases slowly when increasing the number of nodes. This is the small-world effect. This effect proves be essential to the time dependence in game theory.

2.5 EGT and Rock-Scissor-Paper

In 1996, B.Sinervo and C.M.Lively for the first time found the similar game as "Rock-Scissor-Paper" in real ecology system[2], where the male side-blotched lizards can develop a throat-color multi-morphism as they matures in order to defend territories and their female partners. Males with orange throats are aggressive and defend large territories(Group 1). Males with dark blue throats are less aggressive and defend smaller territories(Group 2). Males with prominent yellow strips on their throats(Group 3) are "sneakers" and do not defend territories. Group 1-lizards use their aggression to snatch females from Group 2-lizards(rock-scissor), group 2-lizards defend their females from group 3-lizards(scissor-paper), and group 3-lizards sneaks to the large region guarded by the group 1-lizards to find females for breeding since group 1-lizards' territories are too large to be fully watched(paper-rock). These actions taken between lizards in their breeding strategies reflect the "Rock -Scissor-Paper" game and causes periodical changes in number of each species, thus fluctuations. B.Sinervo and co found that, the numbers of different kind lizards periodically fluctuate during their observation from 1990 to 1995, that the dominating specie varies from time to time, therefore not causing one particular specie to breed too much. This "rock-scissor-paper" strategy set is considered as a way to conserve the steady stability of this ecological system and B.Sinervo also found similar behaviors among European lizards.

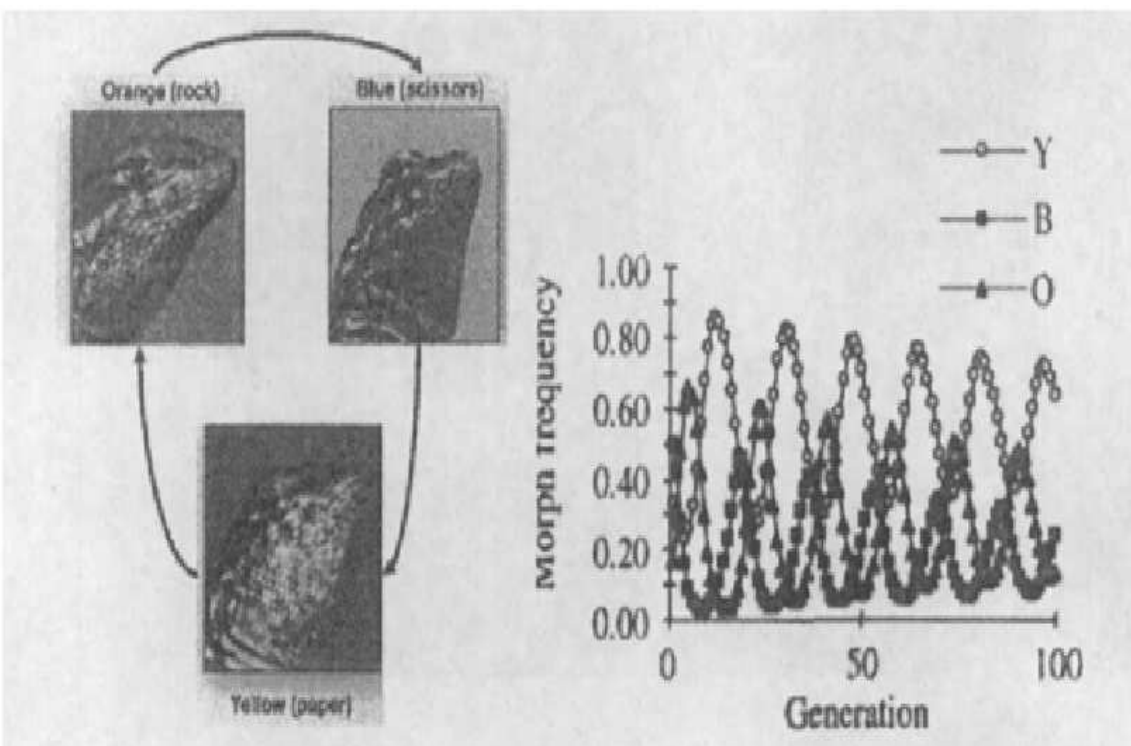

Figure 2.2: lizards develop rock-scissor-paper breeding strategy, picture taken from [2]

E.coli bacteria is another living creature which has the same kind of phenomena. There are three kinds of E.coli bacteria, toxin type, sensitive type and resistant type. Their way of

survival is their strains to secrete colicin, in order to kill other types of E.coli. To compensate colicin, a mutation of the some kinds of collicin can occur, thus making them immune against the attacking colicin, and also some E.coli which do not process this ability, can not survive. B.Kerr and co[5] studied the different evolutionary behaviors of E.coli bacteria under different conditions and discovered that only the partial interaction between individual E-coli cause the total survival, or two of the kinds would be eliminated and only one kind would survive under the isolated condition. Although the initial state was randomly decided, after three days of experiments, self-organizing pattern formation in the bottom of the experimenting container appeared, and the boundary between the various strains of subspecies changes over time. According to the abundance of the three subspecies strains in image showing the change of the numbers, we can see that the abundance of various strains of E.coli remain basically stable, cyclical, and the shifting. However, after the change of conditions, immediately when stirred in the flask and uniformly conditions utensil, the steady state is broken, the S-type strain disappeared after some times. This results indirectly implies that, the partial interaction of "rock-scissor-paper" game promotes and strengthens biological diversity, whereas the total interaction would hinder the biological diversity.

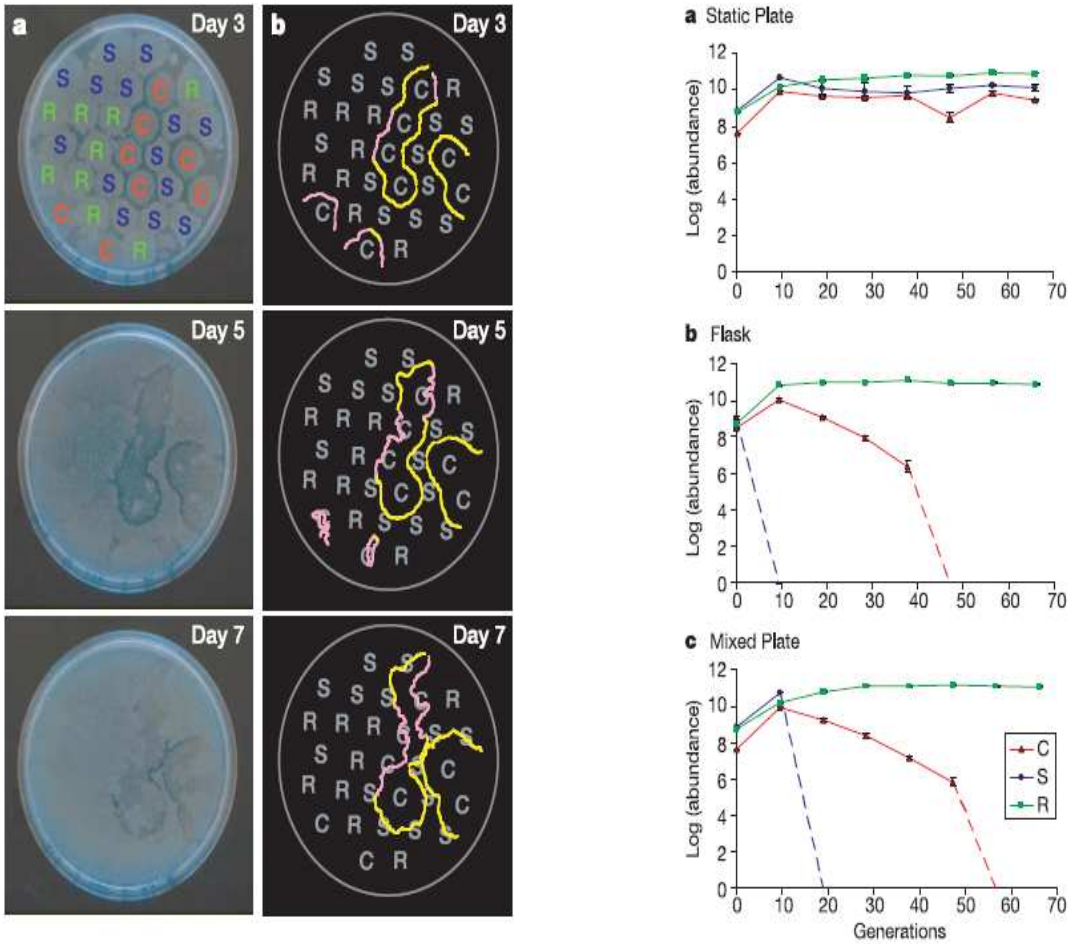

Figure 2.3: Test pictures from the experiment of E.coli interactions, taken from[5]

2.6 Chinese Five Elements

The principle of "Chinese Five Elements" is combined to the Chinese tradition of Taiji, which is a social belief to taoists. The five elements express the natural basics which constitute to our life. Just like "rock-scissor-paper", the five elements form a ring of mutual predation, but as an extended version of the former 3-node system, five elements can also have mutual generation, it means that a certain element can strengthen another and this generation cycle is also formed as a chain. This was an ancient philosophical concept used to explain the composition and phenomena of the physical universe. In traditional Chinese medicine the theory of five elements is used to interpret the relationship between the physiology and pathology of the human body and the natural environment. According to the theory, the five elements are in constant move and change, and the interdependence and mutual restraint of the five elements explain the complex connection between material objects as well as the unity between the human body and the natural world. This is a very good example stressing the principle of evolutionary game theory, therefore attracting our

attention to look closely into this.

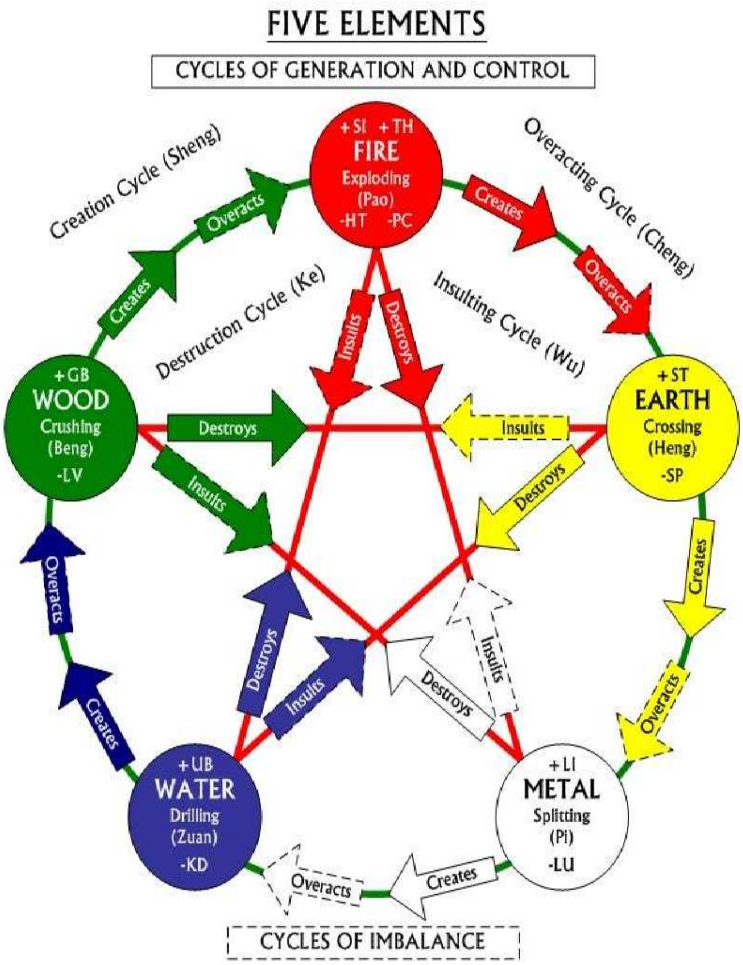

Figure 2.4: Five elements principle, also called Wuxing, picture taken from[7]

The five elements are Metal, Water, Wood, Fire and Earth. The cycles can be seen in figure 2.4. We have in the predation cycle that Metal defeats Wood, Wood defeats Earth, Earth defeats Water, Water defeats Fire, Fire defeats Metal. In the generation cycle, Metal generates Water, Water generates Wood, Wood generates Fire, Fire generates Earth, Earth generates Metal. The generation term can be referred as when the generator is nearby one element, this element is strengthened, more immune against its predator. But every three elements connected to each other can again be seen as a "rock-scissor-paper", for example, Fire defeats Metal, Metal generates Water, Water defeats Metal. Although one generation term is combined into this, it makes the predation system to more complicated than the original "rock-scissor-paper", this is a more steadier game, since that the generated one would make up to the defeated one, therefore a more harmonic spatial system.

In traditional Chinese medicine, the visceral organs, as well as other organs and tissues, have similar properties to the five elements; they interact physiologically and pathologically as the five elements do. Through similarity comparison, different phenomena are attributed

to the categories of the five elements. Based on the characteristics, forms, and functions of different phenomena, the complex links between physiology and pathology as well as the interconnection between the human body and the natural world are explained.

The five elements emerged from an observation of the various groups of dynamic processes, functions and characteristics observed in the natural world. The aspects involved in each of the five elements are follows:

Fire: draught, heat, flaring, ascendancy, movement.
Wood: germination, extension, softness, harmony, flexibility.
Metal: strength, firmness, killing, cutting, cleaning up.
Earth: growing, changing, nourishing, producing.
Water: moisture, cold, descending, flowing.

In order to understand this system, evolutionary game theory principle is applied, on the basis of the "rock-scissor-paper" model, a similar simulation method will be implemented later for study the Five Elements.

A real example of five elements is the extended game of "rock-scissor-paper", which is called the "Rock-paper-scissors-lizard-Spock", where the "lizard" is formed by a hand as a sock-puppet-like mouth, and "spock" is formed as a Star Trek Vulcan salute. This game was mentioned in the famous American tv show "Big Bang Theory", which has also been used to understand the transitivity of climate change policy choice[8]. The rules for this game are following:

Scissor: cuts paper, decapitates lizard
Paper: covers rock, disproves spock
Rock: breaks scissor, crushes lizard
Lizard: poisons spock, eats paper
Spock: smashes scissor, vaporizes rock

This game deviates from the regular elements game somehow, as the different gestures are not exactly improving some's power by helping some as in the five elements, game. There are in total ten ways to pair the five gestures, a picture showing the game rules:

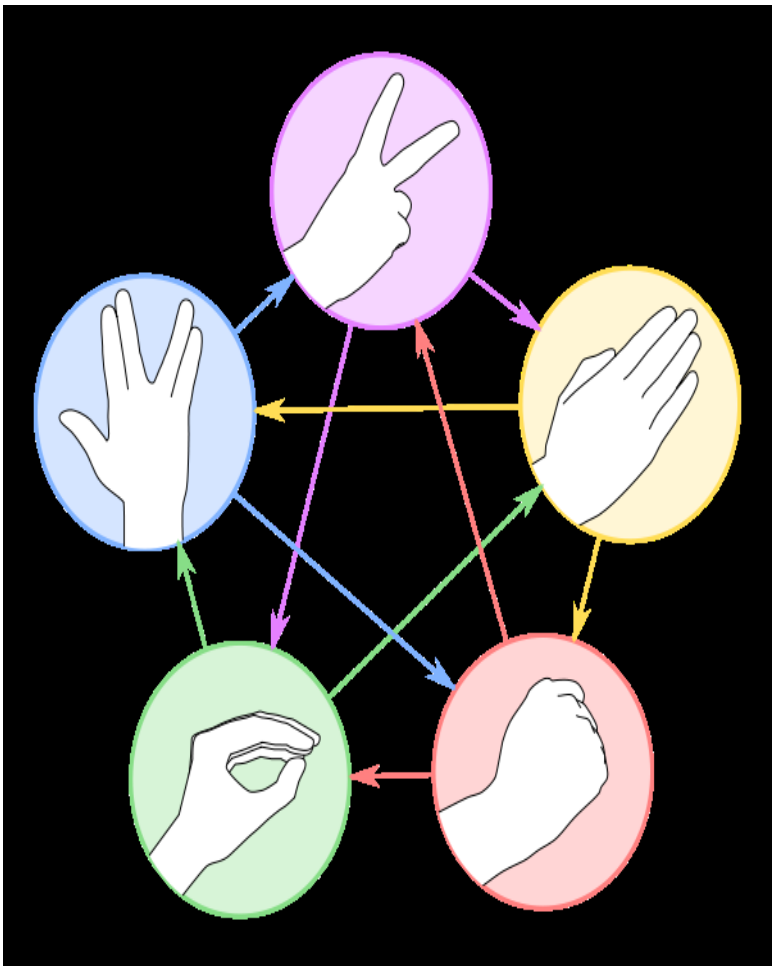

Figure 2.5: Game rule of Rock-paper-scissors-lizard-Spock

The game of "Rock-Paper-Scissors-Lizard-Spock" game is somehow easier than the five elements, as it can be considered as a game of direct competition contributing to indirect cooperation.

We will later try to discuss and compare this game with the five elements, try to find similarities with the original five elements.

Chapter 3

Implementation

3.1 Main Principle

The biological diversity is a focal problem in the study of ecology. A model of "Rock-Scissor-paper" from real ecology can provide a better understanding of biological diversity. Let's assume a system with A, B, C three species, these three species generate an ecological chain of mutual predation and interaction, denoting as K_a, K_b and K_c:

$$\begin{aligned} A + B - -K_a - - > A + A \\ B + C - -K_b - - > B + B \\ A + C - -K_c - - > C + C \end{aligned} \tag{3.1}$$

This is in accordance with Lotka-Volterra equations. Now let's construct the outcome matrix:

Strains of E.Coli	Wins against	Loses against
Killer	Sensitive	Resistant
Sensitive	Resistant	Killer
Resistant	Killer	Sensitive

Table 3.1: Outcome matrix

Expressed for numerical simulation:

	Killer	Sensitive	Resistant
Killer	(1,1)	(2,0)	(0,2)
Sensitive	(0,2)	(1,1)	(2,0)
Resistant	(2,0)	(0,2)	(1,1)

Table 3.2: Score register of outcome

When the evolutionary system has achieved the steady state, if all three species still survive, this implies the steady state of biological diversity. On the other hand, if one of the species would die out, bringing the other one to follow the same path, thus the biological diversity is broken. Our initial assumption is that the E.coli bacteria are just like "rock-scissor-paper" to compete with each other. With computer simulations and a random starting point, the game will be set on in order to get a steady final result.

In 2007, Reichenbach and co used this method[6], implementing with the principle of mean field method, to simulate the final outcome, resulting either steady state for all species or that only one would survive. The following equations can be obtained for illustrating process:

$$\begin{aligned} \dot{N_A} &= N_A(K_a \cdot N_B - K_c \cdot N_C) \\ \dot{N_B} &= N_B(K_b \cdot N_C - K_a \cdot N_A) \\ \dot{N_C} &= N_C(K_c \cdot N_A - K_b \cdot N_C) \end{aligned} \tag{3.2}$$

During the numerical simulation, since the interaction is taken randomly between the points on the game matrix, if time is much bigger than the total number of E.Coli, t>>N, an extinction is eager to occur. Also due to the nature of "rock-scissor-paper", the extinction of one specie would bring another specie into extinction quickly as well, therefore only one specie would remain. Therefore, this mean field theory is only suited for the initial state of the game, and this article will also be focusing on the relative early stage of the game.

3.2 Construction of the models

Using the mean field equations given above, we will only consider the percentage of different species, thus transforming the equations to, setting V as the total number, or the volume:

$$\begin{aligned} \dot{N_A(i,t)}/V &= N_A(i,t)/V(K_a \cdot N_B(i,t) - K_c \cdot N_C(i,t)) \\ \dot{N_B(i,t)}/V &= N_B(i,t)/V(K_b \cdot N_C(i,t) - K_a \cdot N_A(i,t)) \\ \dot{N_C(i,t)}/V &= N_C(i,t)/V(K_c \cdot N_A(i,t) - K_b \cdot N_B(i,t)) \end{aligned} \tag{3.3}$$

Here i denotes each element at node i, due to the square lattice we have chosen to simulate the model, the degree would be in most cases be k=4, except for those on the edges would be 3.

Letting $\dot{N_A(i,t)}/V$=$(N_A(i,t+1) - N_A(i,t))/V$, $\dot{N_B(i,t)}/V$=$(N_B(i,t+1) - N_B(i,t))/V$ and $\dot{N_C(i,t)}/V$=$(N_C(i,t+1) - N_C(i,t))/V$. N_A,N_B,N_C are all integer particle numbers, Implementing with the well known SIR-model:

Algorithm 1: Implementation of the SIR model for the above equations

Input: K_a,K_b,K_c, $N_A(0)$, $N_B(0)$, $N_C(0)$, t_{end}
Output: $N_A(t_{end})$, $N_B(t_{end})$,$N_C(t_{end})$
t=0;
While t<t_{end} DO
t=t+1;
Reaction:
$N_A(t)$=N_A(t-1)+N_A(t-1)($K_a N_B$(t-1)-$K_c N_C$(t-1))
$N_B(t)$=N_B(t-1)+N_B(t-1)($K_b N_C$(t-1)-$K_a N_A$(t-1))
$N_C(t)$=N_C(t-1)+N_C(t-1)($K_c N_A$(t-1)-$K_b N_B$(t-1))
end

This implementation gives the final number of each species after generating t_{end} rounds.

The game rule is implemented in a much simple way, randomly set 1,2,3 to denote different particles on the 2D array for expressing rocks, scissor and papers:

Algorithm 2: Rock-Scissor-Paper Game Rule

Input a(t),b(t)
Output a(t+Δt),b(t+Δt)
While a(t) ≠ b(t) DO
If a(t)=1,
b(t)=2,
b(t+Δt)=1, a(t+Δt)=1
Elseif a(t)=2,
b(t)=3,
b(t+Δt)=2,a(t+Δt)=2
Elseif a(t)=3,
b(t)=1,
b(t+Δt)=3,a(t+Δt)=3
end

In order to see biological diversity to be broken up, we can change the default K_a,K_b and K_c by means of probabilities P_i, it means that the predating specie wouldn't have the full chance to eliminate the weak specie, therefore causing imbalance to the steady state.

Algorithm 3: Implementation of the SIR model with fixed predation probability

Input P_1,P_2,P_3,K_a,K_b,K_c,$N_A(0)$,$N_B(0)$,$N_C(0)$,t_{end}
Output $N_A(t_{end})$,$N_B(t_{end})$, $N_C(t_{end})$
While t<t_{end} DO
t=t+1;
Reaction:
$N_A(t)$=$N_A(t-1)$+$N_A(t-1)(P_1K_aN_B(t-1)$-$P_3K_cN_C(t-1))$
$N_B(t)$=$N_B(t-1)$+$N_B(t-1)(P_2K_bN_C(t-1)$-$P_1K_aN_A(t-1))$
$N_C(t)$=$N_C(t-1)$+$N_C(t-1)(P_3K_cN_A(t-1)$-$P_2K_bN_B(t-1))$
end

With all these given algorithms and equations, we can simulate the "Rock-Scissor-Paper" game!

3.3 Extension to the Five Elements

From the above sections, we have already a model for the "rock-scissor-paper" game. In order to simulate Five Elements as an extension of the original 3-node game, we need to add extra nodes and links. The original outcome matrix would now be:

	Metal	Water	Wood	Fire	Earth
Metal	(1,1)	(1,2*)	(2,0)	(0,2)	(2*,1)
Water	(2*,1)	(1,1)	(1,2*)	(2,0)	(0,2)
Wood	(0,2)	(2*,1)	(1,1)	(1,2*)	(2,0)
Fire	(2,0)	(0,2)	(2*,1)	(1,1)	(1,2*)
Earth	(1,2*)	(2,0)	(0,2)	(2*,1)	(1,1)

Table 3.3: Score register of outcome in Five Elements

Here for the generation cycle, we use that the strengthened element is increased, therefore not afraid of the predation element for the time being, while the generator stays the same. Now illustrating the elements as A,B,C,D,E and with their corresponding Ka,Kb,Kc,Kd and Ke. As for the generator, there will be no changes, thus, this link will not provide any changes to the number of generators,for the strengthened element, there is practically no change to the number of element, but it stays temporarily immune to its predator, therefore, no final results extracted, only that the rule is affecting our final result. This is the reason for marking the strengthened element as 2* in the outcome table.

Comparing to the "Rock-paper-scissors-lizard-Spock" game, the outcome marix would be like this:

	Rock	Paper	Scissor	Lizard	Spock
Rock	(0,0)	(-1,1)	(1,-1)	(1,-1)	(-1,1)
Paper	(1,-1)	(0,0)	(-1,1)	(-1,1)	(1,-1)
Scissor	(-1,1)	(1,-1)	(0,0)	(1,-1)	(-1,1)
Lizard	(-1,1)	(1,-1)	(-1,1)	(0,0)	(1,-1)
Spock	(1,-1)	(-1,1)	(1,-1)	(-1,1)	(0,0)

Table 3.4: Score register of outcome in Rock-paper-scissors-lizard-Spock

For the following implementations, we will focus on the general five elements model.

The mean field equations are as following:

$$\begin{aligned}
\dot{N}_A/V &= N_A/V(K_a \cdot N_C - K_d \cdot N_D) \\
\dot{N}_B/V &= N_B/V(K_b \cdot N_D - K_e \cdot N_E) \\
\dot{N}_C/V &= N_C/V(K_c \cdot N_E - K_a \cdot N_A) \\
\dot{N}_D/V &= N_D/V(K_d \cdot N_A - K_b \cdot N_B) \\
\dot{N}_E/V &= N_E/V(K_e \cdot N_B - K_c \cdot N_C)
\end{aligned} \tag{3.4}$$

and the corresponding extended algorithms are:

Algorithm 4: Implementation of the SIR model for Five Elements

Input: K_a,K_b,K_c,K_d,K_e,$N_A(0)$, $N_B(0)$, $N_C(0)$, $N_D(0)$, $N_E(0)$, t_{end}
Output: $N_A(t_{end})$, $N_B(t_{end})$,$N_C(t_{end})$,$N_D(t_{end})$, $N_E(t_{end})$
t=0;
While t<t_{end} DO
t=t+1;
Reaction:
$\dot{N_A}(t)$=$N_A(t-1)(K_aC(t-1)$-$K_dN_D(t-1)$+$K_e(N_E(t-1)\cup N_A(t-1)$-$N_D(t-1)\cup N_A(t-1))$
$\dot{N_B}(t)$=$N_B(t-1)(K_bD(t-1)$-$K_eN_E(t-1)$+$K_a(N_A(t-1)\cup N_B(t-1)$-$N_E(t-1)\cup N_B(t-1))$
$\dot{N_C}(t)$=$N_C(t-1)(K_cE(t-1)$-$K_aN_A(t-1)$+$K_b(N_B(t-1)\cup N_C(t-1)$-$N_A(t-1)\cup N_C(t-1))$
$\dot{N_D}(t)$=$N_D(t-1)(K_dA(t-1)$-$K_bN_B(t-1)$+$K_c(N_C(t-1)\cup N_D(t-1)$-$N_B(t-1)\cup N_D(t-1))$
$\dot{N_E}(t)$=$N_E(t-1)(K_eB(t-1)$-$K_cN_C(t-1)$+$K_d(N_D(t-1)\cup N_E(t-1)$-$N_C(t-1)\cup N_E(t-1))$
end

Here, the $\cup$ symbol expresses the definition of neighborhood, the generation contribution would only provide assistance if the predation element is also a neighbor to the strengthened element as well as the generator. For considering the model with the fixed predation probability, some extra complications are added to the algorithm 3 just as algorithm 4 did to algorithm 1.

Algorithm 5: Implementation of the SIR model for Five Elements with Fixed probability

Input: P_1,P_2,P_3,P_4,P_5,K_a,K_b,K_c,K_d,K_e,$N_A(0)$, $N_B(0)$, $N_C(0)$, $N_D(0)$, $N_E(0)$, t_{end}
Output: $N_A(t_{end})$, $N_B(t_{end})$,$N_C(t_{end})$,$N_D(t_{end})$, $N_E(t_{end})$
t=0;
While t<t_{end} DO
t=t+1;
Reaction:
$\dot{N_A}(t)$=$N_A(t-1)(P_1K_aN_C(t-1)$-$P_4K_dN_D(t-1)$+$P_5K_e(N_E(t-1)\cup N_A(t-1)$-$N_D(t-1)\cup N_A(t-1))$
$\dot{N_B}(t)$=$N_B(t-1)(P_2K_bN_D(t-1)$-$P_5K_eN_E(t-1)$+$P_1K_a(N_A(t-1)\cup N_B(t-1)$-$N_E(t-1)\cup N_B(t-1))$
$\dot{N_C}(t)$=$N_C(t-1)(P_3K_cN_E(t-1)$-$P_1K_aN_A(t-1)$+$P_2K_b(N_B(t-1)\cup N_C(t-1)$-$N_A(t-1)\cup N_C(t-1))$
$\dot{N_D}(t)$=$N_D(t-1)(P_4K_dN_A(t-1)$-$P_2K_bN_B(t-1)$+$P_3K_c(N_C(t-1)\cup N_D(t-1)$-$N_B(t-1)\cup N_D(t-1))$
$\dot{N_E}(t)$=$N_E(t-1)(P_5K_eN_B(t-1)$-$P_3K_cN_C(t-1)$+$P_4K_d(N_D(t-1)\cup N_E(t-1)$-$N_C(t-1)\cup N_E(t-1))$
end

3.4 Model Ideas and Extension

With all implementations, we are now ready ready to simulate the "rock-scissor-paper" game and the "five elements". As the "rock-scissor-paper" game is a relative simple system, we just simulate it on a 25x25 2D array, denoting the A,B,C as rock, scissor and paper with different colors, expressing equivalently to the toxin killer, sensitive and resistant E.Coli bacteria. While five elements will be simulated on bigger array.

The game rule is common for both games, as each element on the 2D array can only interact with its neighbors, this expresses the "rock-scissor-paper" corrosion phenomena, it means, that for the winner, it conquers its new space if it wins.

On the boundary nodes, they have less neighbors, they will be randomly assigned an initial value first, when randomly picking points to incite competitions, these boundary nodes will be avoided.

The results will be displaying the final outcome of different elements, their phase transition point in order to decide the critical probability for completely destroying the biological diversity.

Our study will be based on discrete time reaction.

Chapter 4

Results and discussions

4.1 Rock-Scissor-Paper

4.1.1 Steady State - No fixed probability

We will make a simulation of the steady state, a normal state without any biological diversity breaking down. Starting by randomly displace some 1,2,3 to denote the different elements(Rock, Scissor, Paper), using the principle from the original outcome matrix, and simulate randomly, get the 2D map of the final stand:

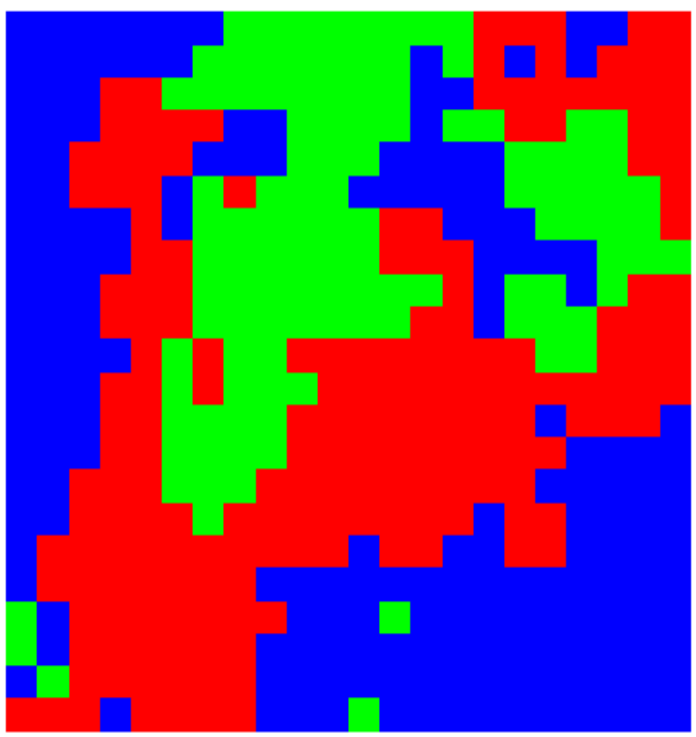

Figure 4.1: Final stand of "Rock-Scissor-Paper" competitions, after t_{end}=100000, V=25x25

And the graphical distribution:

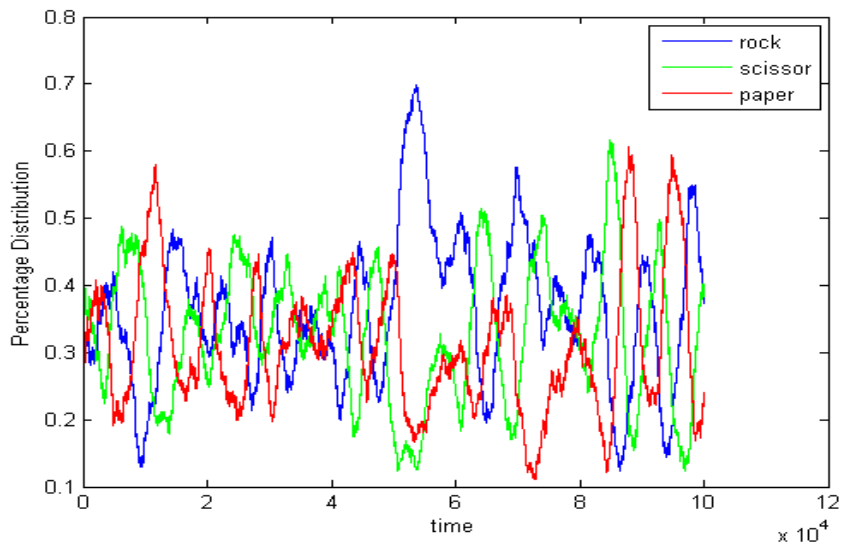

Figure 4.2: Distribution of "Rock-Scissor-Paper" plotted against time, after t_{end}=100000,V=25x25

As the steady state is a harmonic interaction, unless the time is large enough, it's unusual for an extinction to occur. So we will take a closer look at what happens if we fix the predation probability, causing non-equally weighted state.

4.1.2 Non-steady state

Non-equally weighted state means that the three elements' respective winning probability is fixed, this principle is in accordance to a realistic scenario. Under the non-equally weighted state, we implement three parameters, through adjusting and controlling the parameters, to develop a game without equally weighted winning probability. The propose of using these parameters are to control the corrosion rates between the different elements, for example, let the probability of A defeating B become 0.8, then, the corrosion rate will be 80% of its original, this is clearly noted in the last chapter.

The reason for the similarity with real life scenario is that in nature, it's almost impossible to have equally weighted corrosion rates in the predation chain, nor any absolute predation or generation can be found, for example, we all know that a lion is a predator for a goat, but still, under a possibility, a goat can avoid getting eaten by the lion under some circumstances, or if the lion really wants to eat a goat. The implementation of the fixed probability is therefore in well accordance with real life.

Adjusting only one fixed probability

Under this condition, we only change one probability. Choosing P_1 as the fixed probability, while P_2 and P_3 still remain the same. Now let's stepwise adjust the probabilities:

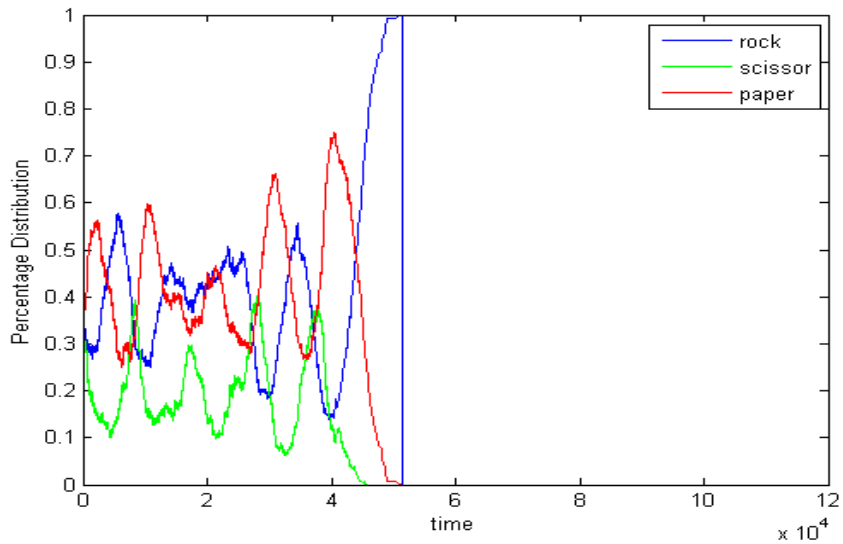

Figure 4.3: Distribution of "Rock-Scissor-Paper" plotted against time, P_1=0.01, V=25x25

When the fixed probability P_1 limits rock to defeat/conquer scissor, it means originally that A has the minimum chance to corrode scissor. The final result turns to be that rock is the final winner after all, and the competition went relatively fast to the end. This is mainly because that, scissor is favoring to defeat paper in this process, and paper is also favoring to defeat rock. With the fixed possibility, due to large amount of paper to be killed by scissor, rock is losing its enemies and turned out to be winning against scissors at the last.

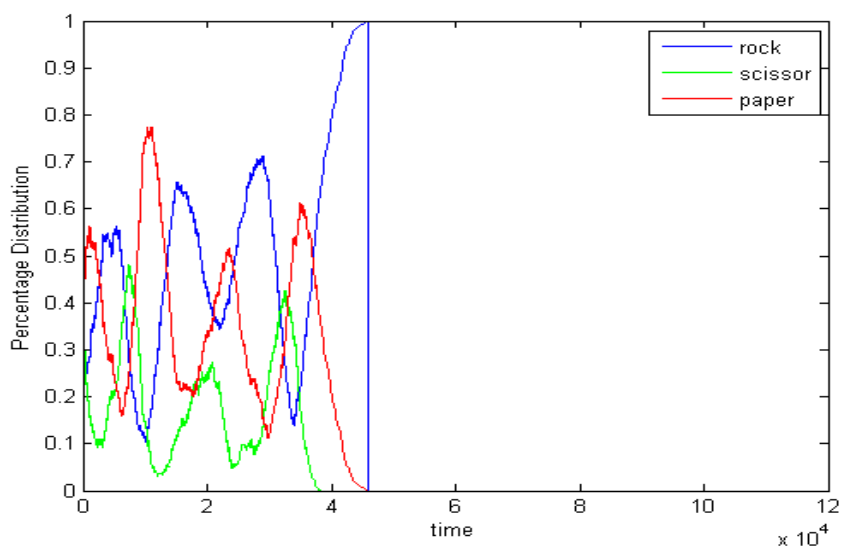

Figure 4.4: Distribution of "Rock-Scissor-Paper" plotted against time, P_1=0.1, V=25x25

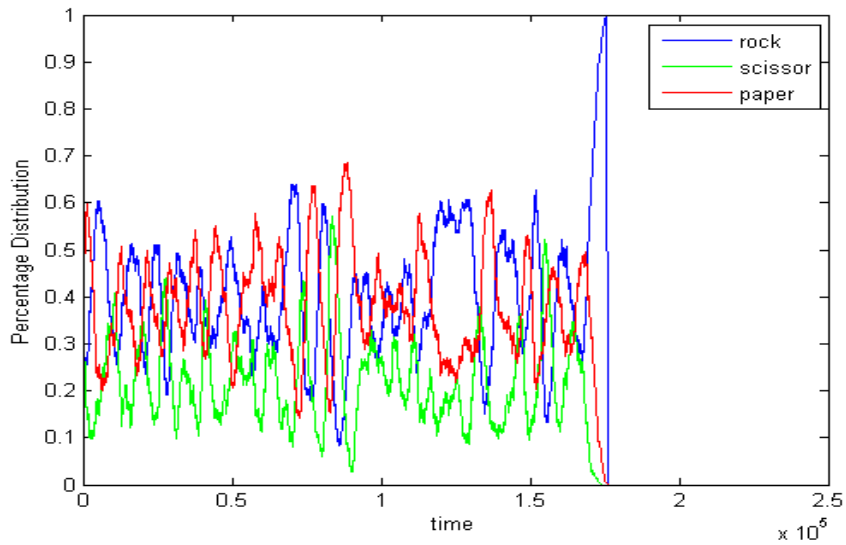

Figure 4.5: Distribution of "Rock-Scissor-Paper" plotted against time, P_1=0.2, V=25x25

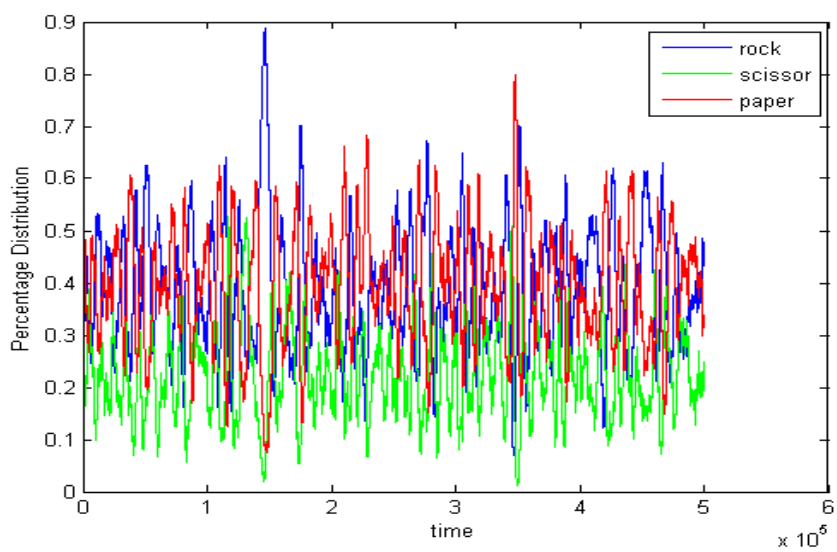

Figure 4.6: Distribution of "Rock-Scissor-Paper" plotted against time, P_1=0.3, V=25x25

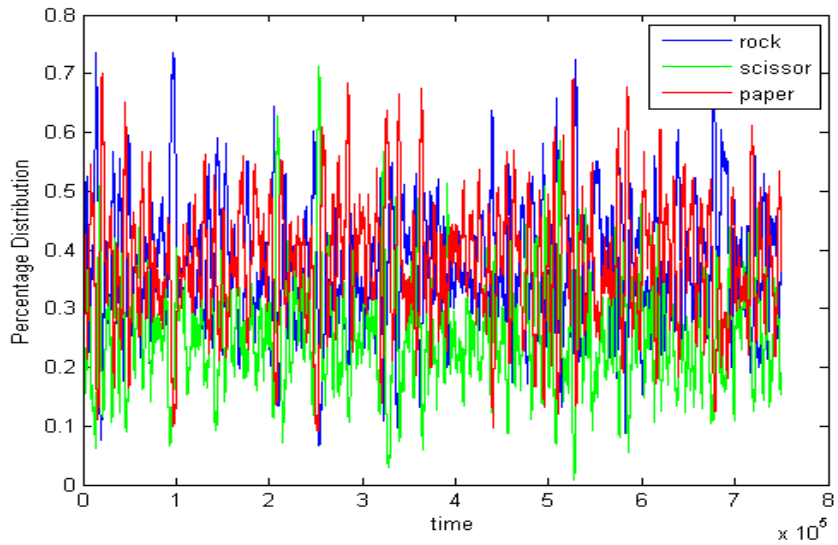

Figure 4.7: Distribution of "Rock-Scissor-Paper" plotted against time, P_1=0.5, V=25x25

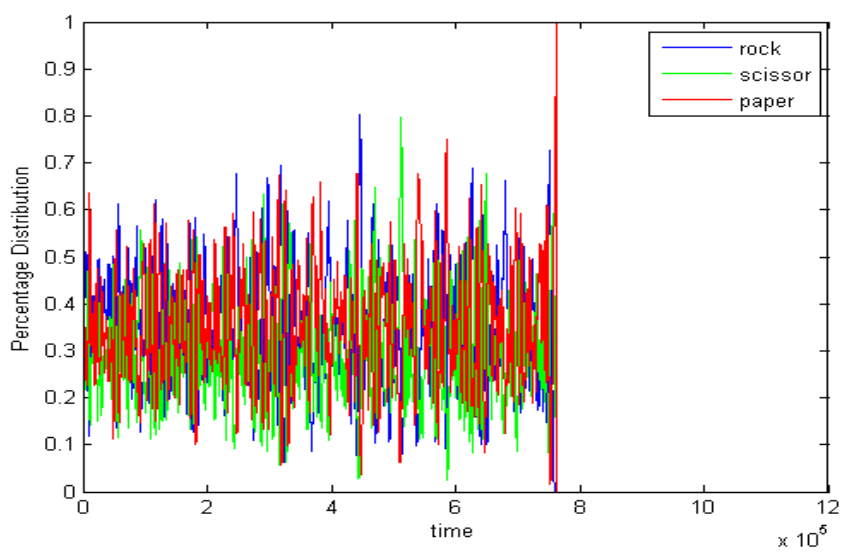

Figure 4.8: Distribution of "Rock-Scissor-Paper" plotted against time, P_1=0.8, V=25x25

Increasing the corrosion probability is causing increased number of paper, when P_1 is nearing 0.3 in figure 4.6, the three elements remain very stable under the given time domain, given that there is not much fluctuation of the numbers. When P_1 is 0.5, only rock and paper are exchanging their leadership role without scissor fluctuating together with them. In figure 4.8, we have a state turning into favor of paper,

Adjusting more fixed probabilities

In this scenario, we can control all the three parameter, first to check if the lowering the same portion of all probabilities.

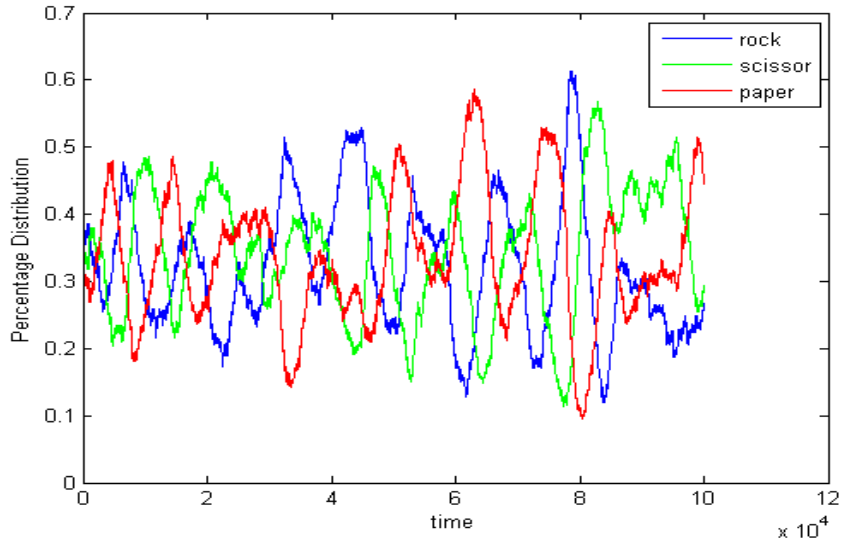

Figure 4.9: Distribution of "Rock-Scissor-Paper" plotted against time, P_1=P_2=P_3=0.5, V=25x25

Figure 4.9 gives almost the same result as a steady state with all probabilities equal to 1 as in figure 4.2, the reason is that the fixed possibilities are dominating much over the randomness in the game that causes the same effect just as for no fixed possibilities.

If we adjust only two of the fixed probabilities, it's expected to cause a whole different result. Adjusting some of the fixed probabilities and get:

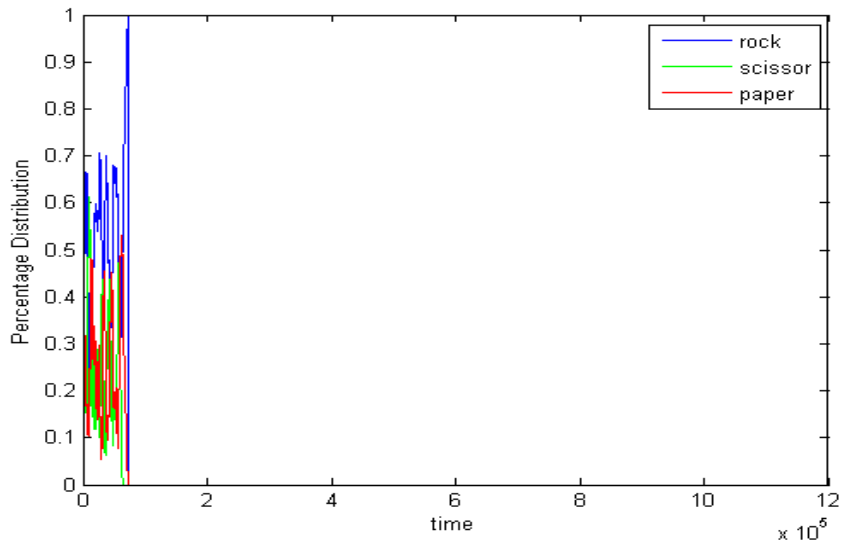

Figure 4.10: Distribution of "Rock-Scissor-Paper" plotted against time, P_1=P_3=0.2,P_2=1, V=25x25

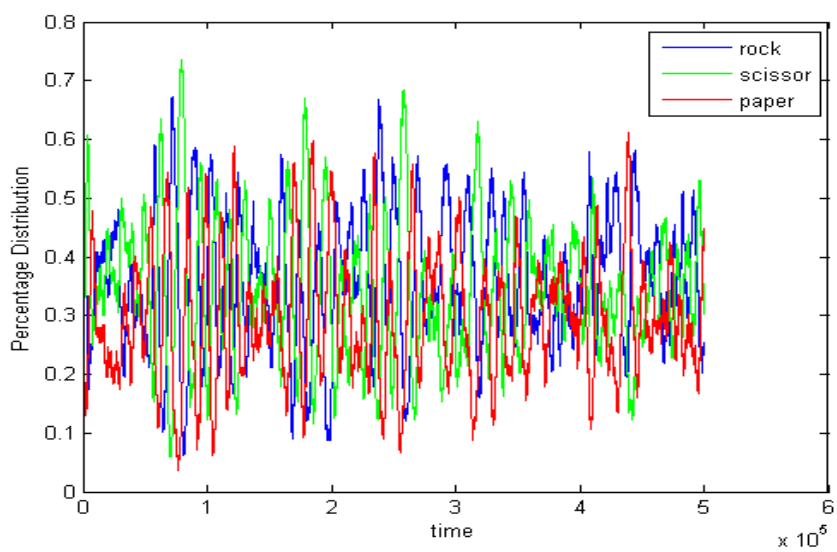

Figure 4.11: Distribution of "Rock-Scissor-Paper" plotted against time, P_1=P_2=0.3,P_3=0.1, V=25x25

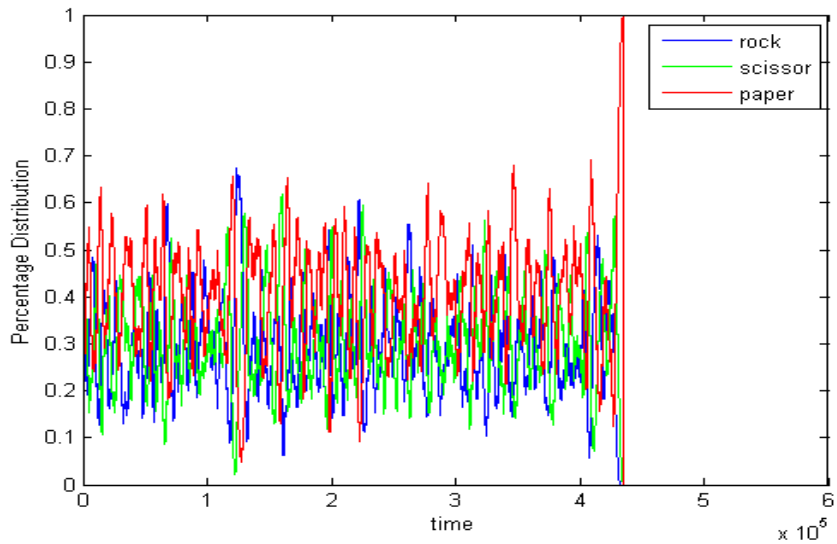

Figure 4.12: Distribution of "Rock-Scissor-Paper" plotted against time, P_1=P_2=0.4,P_3=1, V=25x25

From figure 4.10, we clearly see the corrosion process is much dependent on the probabilities, as, the two elements following each other not as willing as Scissor, to corrode its corresponding weak opponent, this gives the rising of Scissor to corrode much of Paper, less Paper to corrode Rock, and then enough Rock to corrode Scissor at last.

When adjusting the probabilities as in figure 4.11, we can maintain a temporary stability of the elements' distribution. But the dominating element remains the one coming before the highest corrosion probability. In this case, we have all elements compete in the interval of assumed critical points, therefore the stability is maintained in the given time frame.

In figure 4.12, since both Rock and Scissor have a corrosion rate bigger than the assumed critical values, we see that they are not low enough to cause Scissor to win after the previous analogy, therefore Paper ended up winning.

Considering the graphs in figure 4.10 and 4.11, we have that a phase transition has occurred in the region of $P_i \in [0.1, 0.3]$.

The fixed probability phenomena can again be proved analytically, considering the equation set from table 3.2 and transforming to the derivative form:

$$\begin{aligned}
\frac{N_{A,k}(t) - N_{A,k}(t-1)}{N_{A,k}(t-1)} &= P_1 K_a N_{B,k}(t-1) - P_3 K_c N_{C,k}(t-1) \\
\frac{N_{B,k}(t) - N_{B,k}(t-1)}{N_{B,k}(t-1)} &= P_2 K_b N_{C,k}(t-1) - P_1 K_a N_{A,k}(t-1) \\
\frac{N_{C,k}(t) - N_{C,k}(t-1)}{N_{C,k}(t-1)} &= P_3 K_c N_{A,k}(t-1) - P_2 K_b N_{B,k}(t-1)
\end{aligned} \tag{4.1}$$

Redefining the reaction rate:

$$\begin{aligned} K_a N_A(t) &= \mu_1 \\ K_b N_B(t) &= \mu_2 \\ K_c N_C(t) &= \mu_3 \end{aligned} \tag{4.2}$$

Here we declared the reaction rates for the losing terms, but each of the K_i will earn the same amount of the winning terms, this means, for example that, $\mu_3 = K_c N_A = K_c N_c$ in total amount, as this denotes that for element C, $K_c N_A$ is the gaining term, but for A, $K_c N_c$ is the loss term. This denotation will be used later study on five elements as well.

For general solutions, dividing V_k, and redefine the time derivative[1], the mean field equations become like:

$$\begin{aligned} \partial_t \rho_{A,k}(t) &= -\rho_{A,k}(t) + \mu_1 \rho_{B,k}(t) - \mu_3 \rho_{C,k}(t) \\ \partial_t \rho_{B,k}(t) &= -\rho_{B,k}(t) + \mu_2 \rho_{C,k}(t) - \mu_1 \rho_{A,k}(t) \\ \partial_t \rho_{C,k}(t) &= -\rho_{C,k}(t) + \mu_3 \rho_{A,k}(t) - \mu_2 \rho_{B,k}(t) \end{aligned} \tag{4.3}$$

We have trivial solutions: $\rho_A = \rho, \rho_B = \rho_C = 0$, $\rho_B = \rho, \rho_A = \rho_C = 0$ and $\rho_C = \rho, \rho_A = \rho_B = 0$, but they are of little interests to us here. To find the nontrivial solutions, we use equations 4.3, set $\partial_t \rho_{A,k}(t) = \partial_t \rho_{B,k}(t) = \partial_t \rho_{A,k}(t) = 0$ and get:

$$\begin{aligned} \rho_{A,k}(t) &= \mu_1 \rho_{B,k}(t) - \mu_3 \rho_{C,k}(t) \\ \rho_{B,k}(t) &= \mu_2 \rho_{C,k}(t) - \mu_1 \rho_{A,k}(t) \\ \rho_{C,k}(t) &= \mu_3 \rho_{A,k}(t) - \mu_2 \rho_{B,k}(t) \end{aligned} \tag{4.4}$$

This is a general solution for the "Rock-Scissor-Paper" game. To find the the critical values, or the phase transition points, we can easily use equations 4.4 to decide the critical points for each element:

$$\begin{aligned} \bar{\rho_{A,k}} &= \frac{\mu_1}{\mu_3} \\ \bar{\rho_{B,k}} &= \frac{\mu_2}{\mu_1} \\ \bar{\rho_{C,k}} &= \frac{\mu_3}{\mu_2} \end{aligned} \tag{4.5}$$

Therefore we have a equation set to describe the critical points for "rock-scissor-paper", we observe that this is independent on node degree k.

[1] $\partial_t N_{A,k}$(t)=$N_i(t+2)$-$N_i(t)$

Considering for the cases with varying probability P_1, and holding $P_2 = P_3 = 1$:

$$\begin{aligned}
\partial_t \rho_{A,k}(t) &= -\rho_{A,k}(t) + P_1\mu_1\rho_{B,k}(t) - \mu_3\rho_{C,k}(t) \\
\partial_t \rho_{B,k}(t) &= -\rho_{B,k}(t) + \mu_2\rho_{C,k}(t) - P_1\mu_1\rho_{A,k}(t) \\
\partial_t \rho_{C,k}(t) &= -\rho_{C,k}(t) + \mu_3\rho_{A,k}(t) - \mu_2\rho_{B,k}(t)
\end{aligned} \tag{4.6}$$

For $P_1 \to 0$, we have: $\partial_t \rho_{A,k}(t) \approx -\rho_{A,k}(t) - \mu_3\rho_{C,k}(t)$ and $\partial_t \rho_{B,k}(t) \approx -\rho_{B,k}(t) + \mu_2\rho_{C,k}(t)$, while $\partial_t \rho_{C,k}(t)$ stays the same. This is easily proven that while B(Scissor) is gaining all the time, with C(Paper) both gaining and losing to B(Scissor) while B(Scissor) is not losing against A(Rock) at all, C(Paper) gets killed before B gets eliminated by A(Rock) at the end. Using the equations 4.5, we could also see that when $\mu_1 \to 0$, we get the critical point $\bar{\rho_B} \to \infty$, here we could have two cases, that ρ_B continuing to grow towards ∞ or $\bar{\rho_B}$ can be scaled by a large enough μ_2, if μ_2 gets larger, then $\rho_C \to 0$ very quick, after this corrosion from B to C have been completed, A corrode B.

Using the results we have generated from tests, simulating the dependency between P_1 and the time for A(t) to win the competition, since P_1 is dominating over the randomness in the game, one could also try to describe the randomness in the game which is an unavoidable part. Szolnoki and Szab [9]have previously done simulations in order to show the strong random dependency in the system, considering the same size of the network of a square lattice, and focusing on the winner, a plot of $\rho_A - P_1$ is driven from our simulation results in order to show how well the fixed probability is ruling over the randomness in the game:

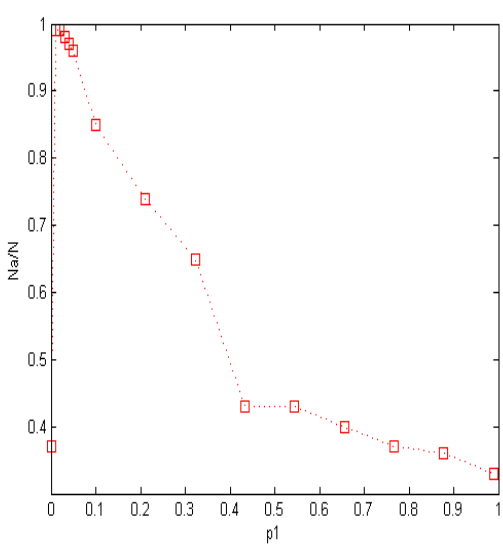

Figure 4.13: $\rho_{A,k}$-P_1

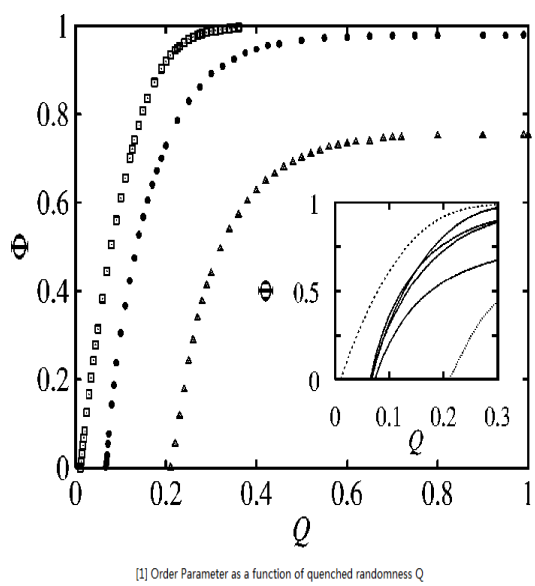

Figure 4.14: Φ-Q

On the figure 4.13, we have at given time frame, $\rho_{A,k}$ as function of P_1, averaging over 450 samples(30 simulations on each probability), V=$25x25$ square lattice, and the figure 4.14 expresses Order parameter of distribution as function of the quenched randomness Q for different kinds of lattices, taken directly from [9]

We see that the dependency on the fixed probability is almost inverse as in figure 4.14, where the deviation in the origin at(0,0) from the results are caused by initiation of the graph. As in [9] the initial distribution of different objects are set to be minimum in order to automatically spread over the whole graph, and in our simulation, it's been preset to be randomly distributed but close to a equally distribution of rock, paper and scissor elements. This somehow shows the fixed probability could be strong enough to defeat the strong randomness in the game, therefore causing the whole game and network to be fixed.

Finally, we could observe in figure 4.15 a dependency on the lattice size that the critical point tends to increase when competing on a bigger array.

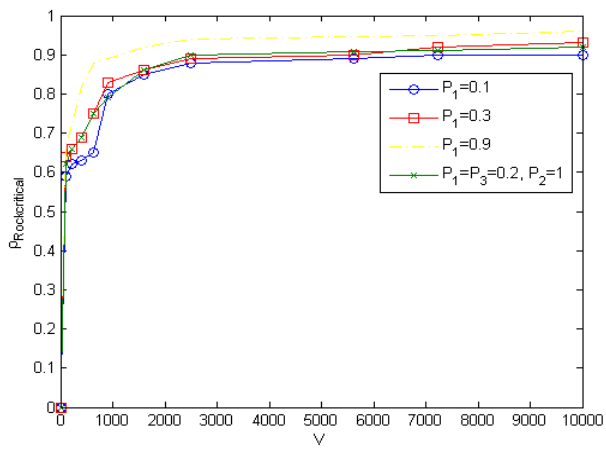

Figure 4.15: Simulation results of critical points of rock simulated on different sizes of square lattices, 30 samples per lattice, by 10 different probabilities

For the this game purely consisting of competition, the rise to critical point is not very dependent on the lattice size, seeing that the critical density for rock converges to a constant value when closing big world approximation, in other word when $V \to \infty$. We also observe that the global winning probabilities somehow regulate the critical densities when emerging from small world to big world approximation.

Summary

In this section, we observe that through adjusting the winning probability of Rock defeating Scissor reflects that Rock, Scissor and Paper are caused by evolutions through time. When the probability P_i is small, it does not direct point to absolute defeat, but depending on other competitor's action. In this mutual predation case, it's very similar to a real life scenario when a company A is no longer deploying a strategy to challenge its competitor B while the competitor B thinks it's dominating and challenge other competitor C which is really causing problems for company A. When C is defeated, A has lost its real threat, then continuing to defeat B.

When the global probability is between 0.1 and 0.3, there is a phase transitional point, this point is causing the original fluctuating instability among the bacteria to a more stable

state. This is for both adjusting one single fixed probability or for adjusting two together.

4.2 Five Elements

4.2.1 Steady State

Now we will make some simulations of the five elements game, because of the increasing elements in the game, increased nodes and links, we will consider this in a larger array. Some test simulations give the following results:

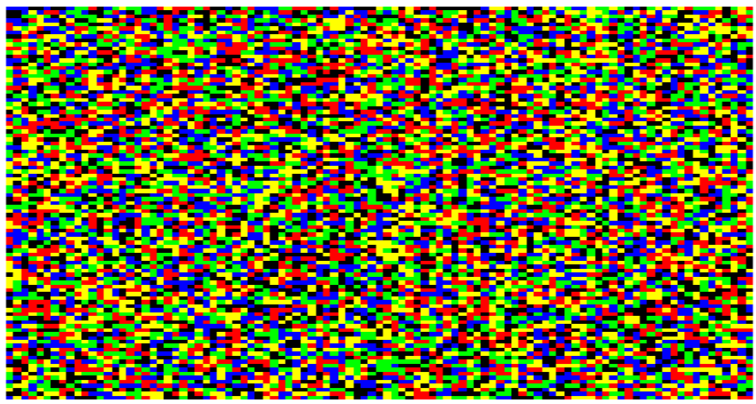

Figure 4.16: Initial random distribution of five elements, V=100x100

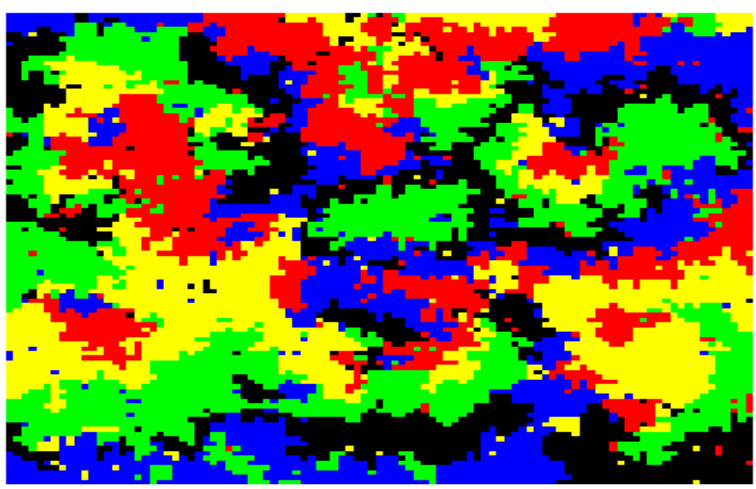

Figure 4.17: Final distribution of five elements after 1000000 rounds, V=100x100

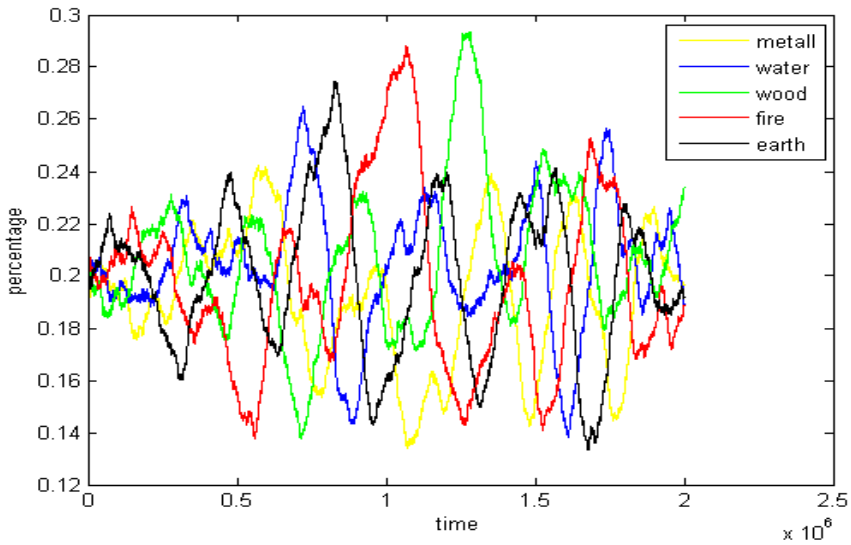

Figure 4.18: Percentage of five elements plotted against time, V=100x100

Here, we have Wood to be the dominating element in the final time domain, another final result is returning a different scenario:

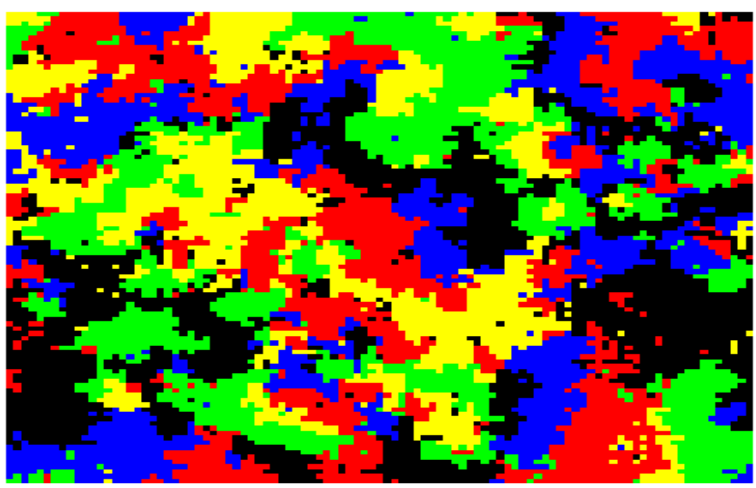

Figure 4.19: Another final distribution of five elements after 1000000 rounds, V=100x100

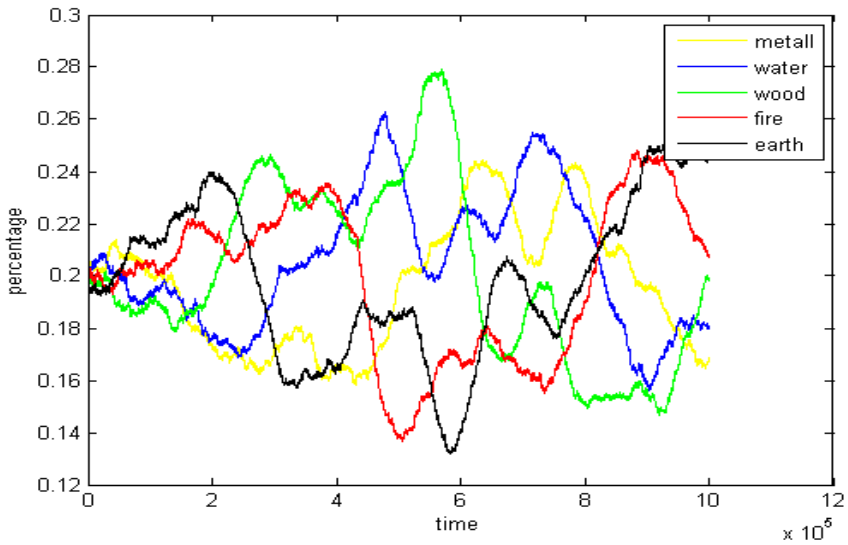

Figure 4.20: Another percentage of five elements plotted against time, V=100x100

Because of the large scale of randomness, a major difference between these two results is the fluctuating level, it seems like in figure 4.18, the competition is more intense than in figure 4.20.

4.2.2 Non-steady State

In this case, adjusting fixed probabilities are much more complicated than in "rock-scissor-paper". Here we can adjust one single probability in four different ways just for one node. If adjusting a lot of probabilities at the same time, one could get easily confused. So we will mainly focus on adjusting p1, the probability for Metal to corrode Wood. We will stepwise adjust P_1 and see the different results:

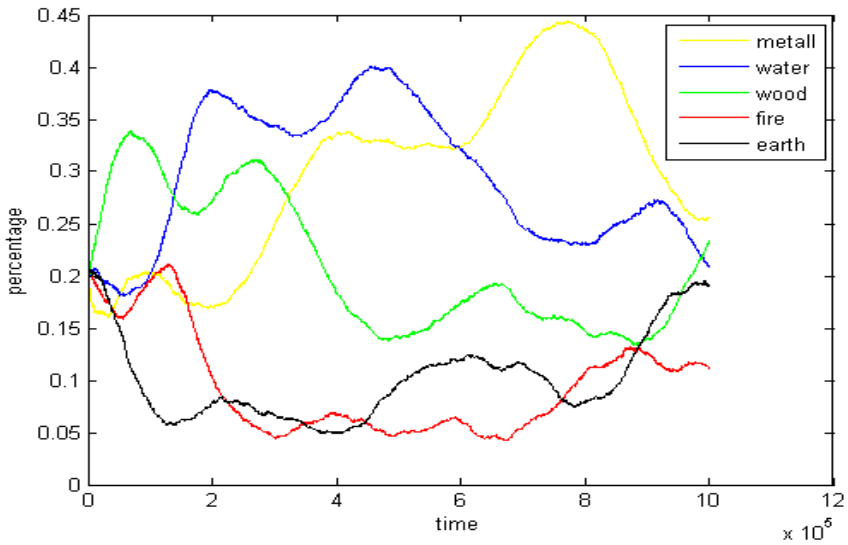

Figure 4.21: Percentage of five elements plotted against time, P_1=0.01, V=100x100

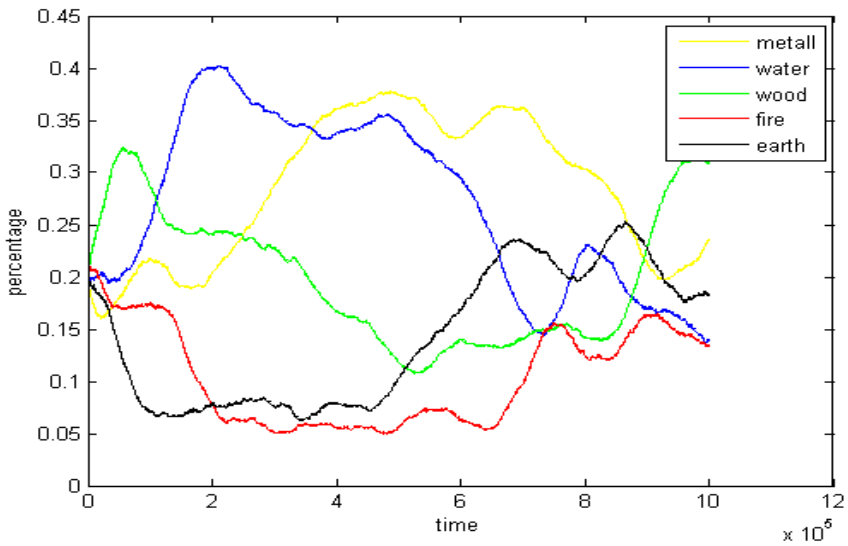

Figure 4.22: Percentage of five elements plotted against time, P_1=0.1, V=100x100

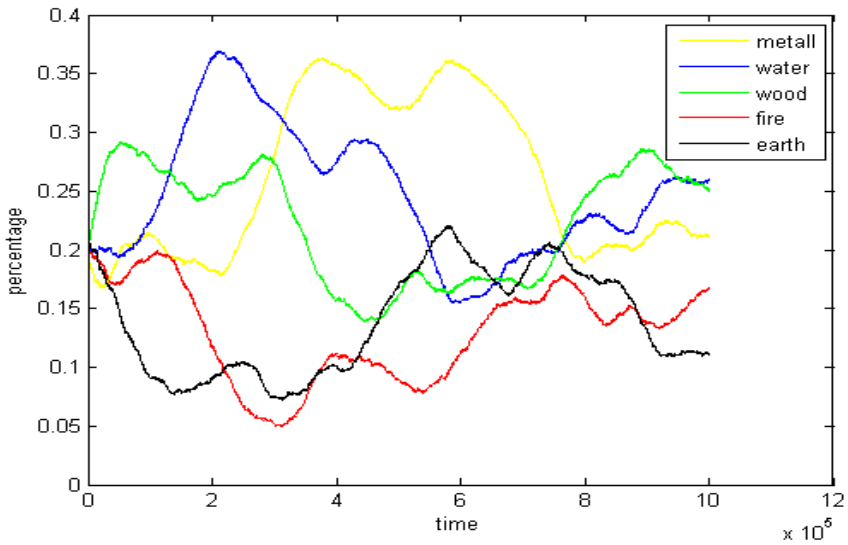

Figure 4.23: Percentage of five elements plotted against time, P_1=0.2, V=100x100

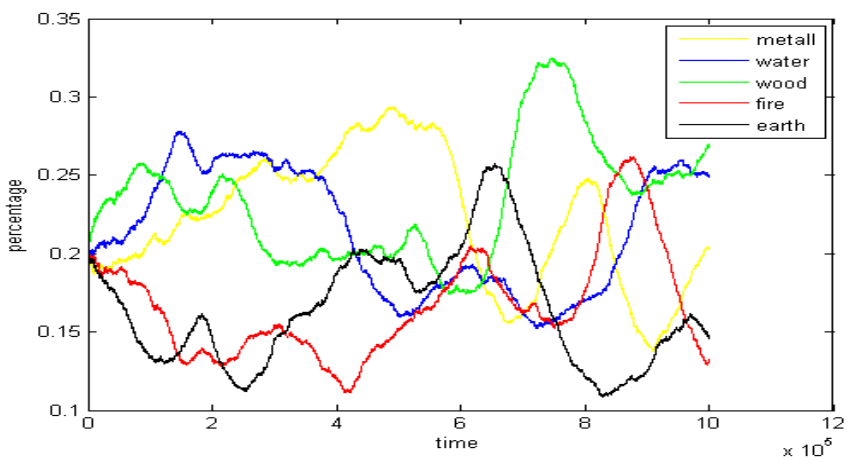

Figure 4.24: Percentage of five elements plotted against time, P_1=0.5, V=100x100

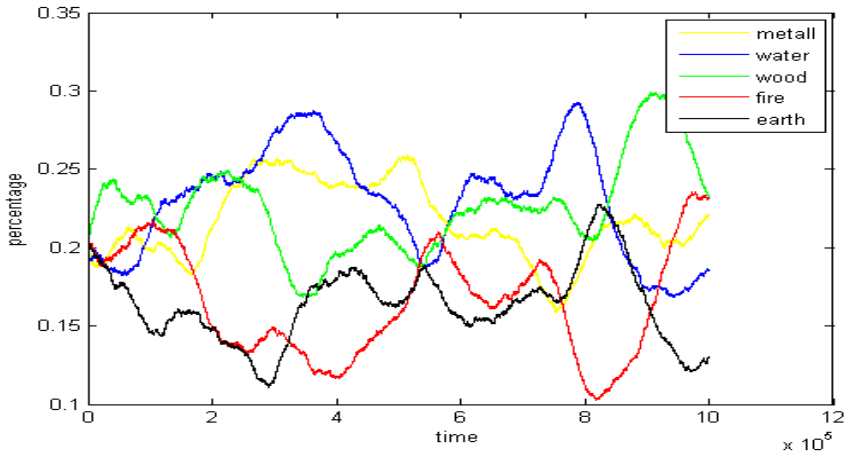

Figure 4.25: Percentage of five elements plotted against time, P_1=0.55, V=100x100

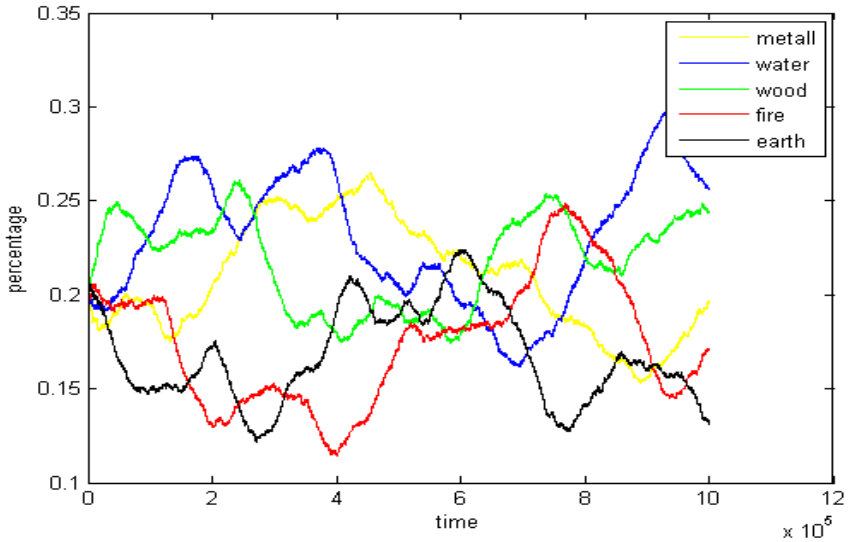

Figure 4.26: Percentage of five elements plotted against time, P_1=0.6, V=100x100

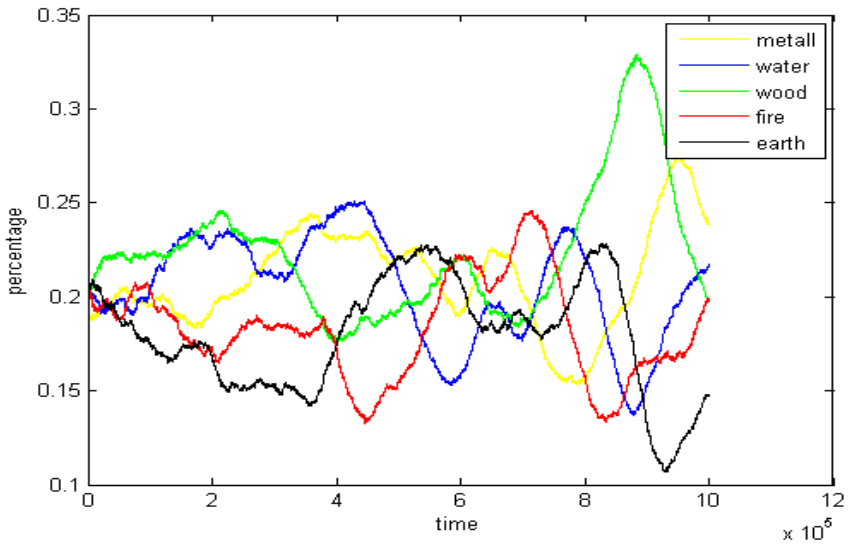

Figure 4.27: Percentage of five elements plotted against time, P_1=0.8, V=100x100

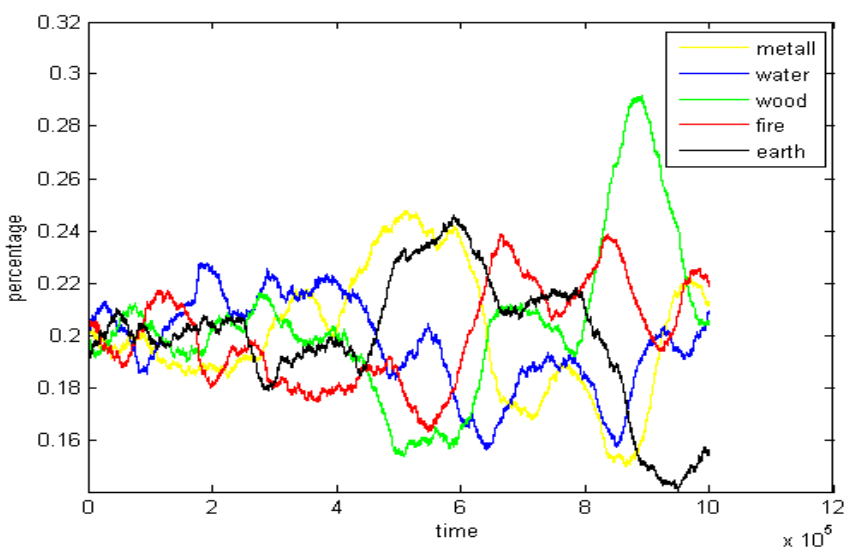

Figure 4.28: Percentage of five elements plotted against time, P_1=0.99, V=100x100

As observed from figure 4.21 to figure 4.26, with the adjustment of P_1, the probability for metal to corrode wood is fixed, the outcome of metal distribution has significantly fallen. By intuition, one would probably expect that metal would "die" out soon enough, but although with the low P_1 in the beginning, metal did not die out just as what happened earlier in the "rock-scissor-paper" game. Although not turned out to be the winner with increasing P_1, but still limiting its succeeding elements coming after it. Looking closer to the different equations in table 3.3, we could analyze more of the behaviors under the

circumstance of fixed probabilities. First transforming into density form, just as we did in previous section:

$$\begin{aligned}
\rho_{A,k}(t) &= \frac{N_{A,k}(t)}{V_k} \\
\rho_{B,k}(t) &= \frac{N_{B,k}(t)}{V_k} \\
\rho_{C,k}(t) &= \frac{N_{C,k}(t)}{V_k} \\
\rho_{D,k}(t) &= \frac{N_{D,k}(t)}{V_k} \\
\rho_{E,k}(t) &= \frac{N_{E,k}(t)}{V_k}
\end{aligned} \tag{4.7}$$

Where

$$\sum_{A}^{E} \rho_{i,k} = \rho = \frac{N_{i,k}}{V_k} \tag{4.8}$$

Where V = A+B+C+D+E, is the total number of elements. Simplifying and renaming to:

$$\begin{aligned}
\partial_t \rho_{A,k}(t) &= -\rho_{A,k}(t) + P_1\mu_1\rho_{C,k}(t) - P_4\mu_4\rho_{D,k}(t) + P_5\mu_5(\rho_{EA,k}(t) - \rho_{DA,k}(t)) \\
\partial_t \rho_{B,k}(t) &= -\rho_{B,k}(t) + P_2\mu_2\rho_{D,k}(t) - P_5\mu_5\rho_{E,k}(t) + P_1\mu_1(\rho_{AB,k}(t) - \rho_{EB,k}(t)) \\
\partial_t \rho_{C,k}(t) &= -\rho_{C,k}(t) + P_3\mu_3\rho_{E,k}(t) - P_1\mu_1\rho_{A,k}(t) + P_2\mu_2(\rho_{BC,k}(t) - \rho_{AC,k}(t)) \\
\partial_t \rho_{D,k}(t) &= -\rho_{D,k}(t) + P_4\mu_4\rho_{A,k}(t) - P_2\mu_2\rho_{B,k}(t) + P_3\mu_3(\rho_{CD,k}(t) - \rho_{BD,k}(t)) \\
\partial_t \rho_{E,k}(t) &= -\rho_{E,k}(t) + P_5\mu_5\rho_{B,k}(t) - P_3\mu_3\rho_{C,k}(t) + P_4\mu_4(\rho_{DE,k}(t) - \rho_{CE,k}(t))
\end{aligned} \tag{4.9}$$

Where for the different $\rho_{ij} = i(t-1) \cup j(t-1)$, and simplifying to simplified form where μ represents the reaction probability. Depending on the different elements, the different μ_i is also different and contributes to the final outcome at time t. The general nontrivial solutions for the "Chinese five elements" are:

$$\begin{aligned}
\rho_{A,k}(t) &= \mu_1\rho_{C,k}(t) - \mu_4\rho_{D,k}(t) + \mu_5(\rho_{EA,k}(t) - \rho_{DA,k}(t)) \\
\rho_{B,k}(t) &= \mu_2\rho_{D,k}(t) - \mu_5\rho_{E,k}(t) + \mu_1(\rho_{AB,k}(t) - \rho_{EB,k}(t)) \\
\rho_{C,k}(t) &= \mu_3\rho_{E,k}(t) - \mu_1\rho_{A,k}(t) + \mu_2(\rho_{BC,k}(t) - \rho_{AC,k}(t)) \\
\rho_{D,k}(t) &= \mu_4\rho_{A,k}(t) - \mu_2\rho_{B,k}(t) + \mu_3(\rho_{CD,k}(t) - \rho_{BD,k}(t)) \\
\rho_{E,k}(t) &= \mu_5\rho_{B,k}(t) - \mu_3\rho_{C,k}(t) + \mu_4(\rho_{DE,k}(t) - \rho_{CE,k}(t))
\end{aligned} \tag{4.10}$$

Here, all P_i are default set as 1. For these solutions we could use the same analogy from

the last section to express the critical points for each different element:

$$
\begin{aligned}
\rho_{\bar{A},k} &= \frac{\mu_1 + \mu_5 \rho_{EA,k}}{\mu_4 + \mu_5 \rho_{DA,k}} \\
\rho_{\bar{B},k} &= \frac{\mu_2 + \mu_1 \rho_{AB,k}}{\mu_5 + \mu_1 \rho_{EB,k}} \\
\rho_{\bar{C},k} &= \frac{\mu_3 + \mu_2 \rho_{BC,k}}{\mu_1 + \mu_2 \rho_{AC,k}} \\
\rho_{\bar{D},k} &= \frac{\mu_4 + \mu_3 \rho_{CD,k}}{\mu_2 + \mu_3 \rho_{BD,k}} \\
\rho_{\bar{E},k} &= \frac{\mu_5 + \mu_4 \rho_{DE,k}}{\mu_3 + \mu_4 \rho_{CE,k}}
\end{aligned}
\tag{4.11}
$$

While the "rock-scissor-paper" are purely dependent on reaction rates, or adjustment of reactions rates caused by the corrosion probabilities P_i, the critical phases for the five elements are dependent on the cooperation terms expressed as ρ_{ij} for $i \neq j, i = A..E, j = A..E$. Without these cooperation terms that contributes virtual score[2], the whole game would just be like "rock-scissor-paper" with linear fractional dependency on the reaction rates. Observing that ρ_{ij} is dependent on μ_i and μ_j, we could express the dynamical change in ρ_{ij}, for $\rho_{\bar{A},k}$, we could find a certain dependency on the anti-cooperation term expressed by $\rho_{DA,k}$

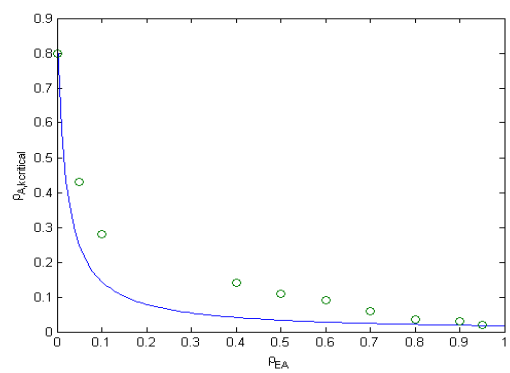

Figure 4.29: Plot of $\rho_{\bar{A},k}$ as function of anti-cooperation term $\rho_{DA,k}$, the line is the theoretical result expressed as a part of equations 4.11, and the dots are the simulation results generated as a mean of 300 simulations, 30 per adjusted P_{DA} as a global fixed probability to fix $\rho_{DA,k}$, V=100x100

From figure 4.29, we could see the dramatic change by adjusting the anti-cooperation term $\rho_{DA,k}$, without the enemy neighboring effect, metal could easily win, but with the increasing enemy neighboring effect, the critical points for metal drops down dramatically, ending to be defeated. Our simulation data is in some deviation compared to the theoretical results, but somehow follows the pattern.

[2]virtual scores as mentioned in table 3.3 in section 3.

Consider from the results generated earlier in figures 4.21 to 4.28, letting $P_1 \to 0$, this is equivalent to $\mu_1 \to 0$, considering from the above equation set 4.10:

$$\begin{aligned}
\rho_{A,k}(t) &= -\mu_4\rho_{D,k}(t) + \mu_5(\rho_{EA,k}(t) - \rho_{DA,k}(t)) \\
\rho_{B,k}(t) &= \mu_2\rho_{D,k}(t) - \mu_5\rho_{E,k}(t) \\
\rho_{C,k}(t) &= \mu_3\rho_{E,k}(t) + \mu_2(\rho_{BC,k}(t) - \rho_{AC,k}(t)) \\
\rho_{D,k}(t) &= \mu_4\rho_{A,k}(t) - \mu_2\rho_{B,k}(t) + \mu_3(\rho_{CD,k}(t) - \rho_{BD,k}(t)) \\
\rho_{E,k}(t) &= \mu_5\rho_{B,k}(t) - \mu_3\rho_{C,k}(t) + \mu_4(\rho_{DE,k}(t) - \rho_{CE,k}(t))
\end{aligned} \tag{4.12}$$

In this case, we see that the three first equations from equation set 4.18 has been cut down to fewer terms, rearranging and gather the gain and loss terms:

$$\rho_{A,k}(t) = \mu_5\rho_{EA,k}(t) - (\mu_4\rho_{D,k}(t) + \mu_5\rho_{DA,k}(t)) \tag{4.13}$$

$$\rho_{B,k}(t) = \mu_2\rho_{D,k}(t) - \mu_5\rho_{E,k}(t) \tag{4.14}$$

$$\rho_{C,k}(t) = (\mu_3\rho_{E,k}(t) + \mu_2(\rho_{BC,k}(t)) - \mu_2\rho_{AC,k}(t) \tag{4.15}$$

$$\rho_{D,k}(t) = (\mu_4\rho_{A,k}(t) + \mu_3\rho_{CD,k}(t)) - (\mu_2\rho_{B,k}(t) + \mu_3\rho_{BD,k}(t)) \tag{4.16}$$

$$\rho_{E,k}(t) = (\mu_5\rho_{B,k}(t) + \mu_4\rho_{DE,k}(t)) - (\mu_3\rho_{C,k}(t) + \mu_4\rho_{CE,k}(t)) \tag{4.17}$$

From all these equations, we find that the equation of densities describing motion is following the logarithmic pattern. Which is in coherence with results found earlier in [12, 13] that found the average loop of MC simulations on the random regular graph increases with InN.

Using the same method as earlier in "rock-scissor-paper", we will try to find a dependency between P_1 and ρ_{Metal} in order to see if we can find a global transition probability point, simulating, averaging and get:

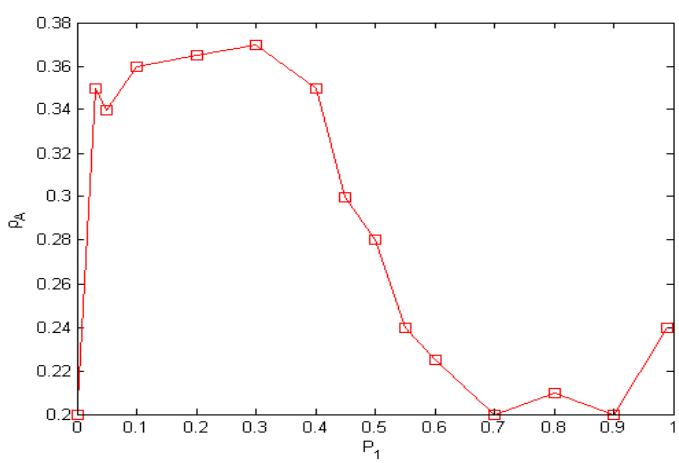

Figure 4.30: At a given time frame, $\rho_{A,k}$ as function of P_1, averaging over 450 samples(30 simulations on each probability), V=$100x100$ square lattice

We could observe that the global fixed probability P_1 does not have so strong impact on the final outcome for metal at the given time frame. This is underlined by the strong dependency on the neighboring effect from element D and element E, which is already demonstrated in figure 4.29.

Observing the dependency of Metal on the lattice size in figure 4.31, we see a similar pattern as shown in "rock-scissor-paper", therefore underlining its similar competition nature and the dependency on the lattice size.

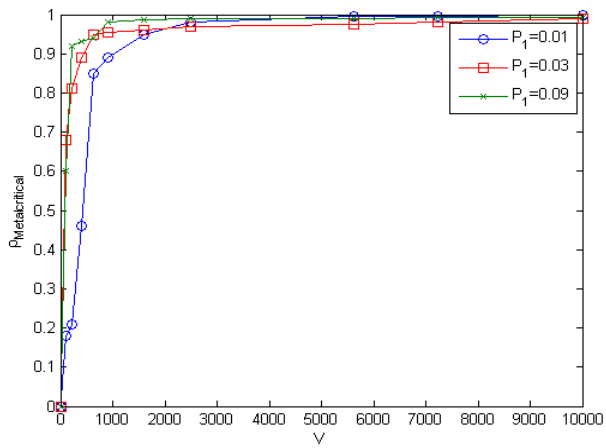

Figure 4.31: Simulation results of critical points of metal simulated on different sizes of square lattices, 30 samples per lattice, by 10 different probabilities, $t_{end} = 1000000$

Compared to "rock-scissor-paper", we see in figure 4.31 has a higher critical density than in figure 4.15, this is due to the more complex nature of "Five elements", resulting absorbing states to be more unachievable in real big world. The dependencies on the probability is eventually less than on the size of the array.

4.2.3 Summary

The study of "Chinese Five Elements" shows that the this system somehow follows a similar pattern as the "Rock-Scissor-Paper", but extended to direct competition and direct cooperation. The competition terms are not the deciding factors any more, as the cooperation term could contribute far more effect to the change in critical points for different elements. At the same time, cooperation terms dependent both on reaction rates expressing the competition and the nodal degree due to the neighboring effect, the cooperation terms are dynamical and changes from time to time and from node to node. This underlines the complex system and behavior in the "Chinese Five Elements". With competition and cooperation striving against each other, the critical points can be achieved in far longer time than in the "Rock-Scissor-Paper", therefore is similar to nature's long term behavior.

4.3 Five elements and Rock-Paper-Scissor-Lizard-Spock

The best example of five elements to be found in reality is this wonderful game from the American Television "Big Bang Theory". Given it's based on "rock-scissor-paper" mechanism, and only extended with two more different elements. We will try to do some simulation of this game and compare it with the Chinese version of Five Elements. Basically the equations of motion will be in the same form, the only thing that differs is the outcome matrix which give rise to a different result. Renaming the original density functions to:

$$\begin{aligned}
\rho_{Rock,k}(t) &= \frac{N_{Rock,k}(t)}{V_k} \\
\rho_{Paper,k}(t) &= \frac{N_{Paper,k}(t)}{V_k} \\
\rho_{Scissor,k}(t) &= \frac{N_{Scissor,k}(t)}{V_k} \\
\rho_{Lizard,k}(t) &= \frac{N_{Lizard,k}(t)}{V_k} \\
\rho_{Spock,k}(t) &= \frac{N_{Spock,k}(t)}{V_k}
\end{aligned} \tag{4.18}$$

In the case of "Rock-Paper-Scissor-Lizard-Spock", there will be no global neighborhood effect like in the five elements. So the equations of motions is simpler:

$$\begin{aligned}
\partial_t \rho_{rock,k}(t) &= -\rho_{rock,k}(t) + \mu_1(\rho_{Scissor,k}(t) + \rho_{Lizard,k}(t)) - (\mu_2 \rho_{Paper,k}(t) + \mu_5 \rho_{Spock,k}(t)) \\
\partial_t \rho_{Paper,k}(t) &= -\rho_{Paper,k}(t) + \mu_2(\rho_{Rock,k}(t) + \rho_{Spock,k}(t)) - (\mu_3 \rho_{Scissor,k}(t) + \mu_4 \rho_{Lizard,k}(t)) \\
\partial_t \rho_{Scissor,k}(t) &= -\rho_{Scissor,k}(t) + \mu_3(\rho_{Paper,k}(t) + \rho_{Lizard,k}(t)) - (\mu_1 \rho_{Rock,k}(t) + \mu_5 \rho_{Spock,k}(t)) \\
\partial_t \rho_{Lizard,k}(t) &= -\rho_{Lizard,k}(t) + \mu_4(\rho_{Paper,k}(t) + \rho_{Spock,k}(t)) - (\mu_1 \rho_{Rock,k}(t) + \mu_3 \rho_{Scissor,k}(t)) \\
\partial_t \rho_{Spock,k}(t) &= -\rho_{Spock,k}(t) + \mu_5(\rho_{Rock,k}(t) + \rho_{Scissor,k}(t)) - (\mu_2 \rho_{Scissor,k}(t) + \mu_4 \rho_{Lizard,k}(t))
\end{aligned} \tag{4.19}$$

The general nontrivial solutions are:

$$\begin{aligned}
\rho_{Rock,k}(t) &= \mu_1(\rho_{Scissor,k}(t) + \rho_{Lizard,k}(t)) - (\mu_2 \rho_{Paper,k}(t) + \mu_5 \rho_{Spock,k}(t)) \\
\rho_{Paper,k}(t) &= \mu_2(\rho_{Rock,k}(t) + \rho_{Spock,k}(t)) - (\mu_3 \rho_{Scissor,k}(t) + \mu_4 \rho_{Lizard,k}(t)) \\
\rho_{Scissor,k}(t) &= \mu_3(\rho_{Paper,k}(t) + \rho_{Lizard,k}(t)) - (\mu_1 \rho_{Rock,k}(t) + \mu_5 \rho_{Spock,k}(t)) \\
\rho_{Lizard,k}(t) &= \mu_4(\rho_{Paper,k}(t) + \rho_{Spock,k}(t)) - (\mu_1 \rho_{Rock,k}(t) + \mu_3 \rho_{Scissor,k}(t)) \\
\rho_{Spock,k}(t) &= \mu_5(\rho_{Rock,k}(t) + \rho_{Scissor,k}(t)) - (\mu_2 \rho_{Scissor,k}(t) + \mu_4 \rho_{Lizard,k}(t))
\end{aligned} \tag{4.20}$$

Using equations 4.20, combining the simulation results, we have a basis to describe the game of "Rock-Paper-Scissor-Lizard-Spock".

The critical points can also be obtained by using same analogy from earlier:

$$\begin{aligned}
\rho_{\bar{Rock},k} &= \frac{2\mu_1}{\mu_2+\mu_5} \\
\rho_{\bar{Paper},k} &= \frac{2\mu_2}{\mu_3+\mu_4} \\
\rho_{\bar{Scissor},k} &= \frac{2\mu_3}{\mu_1+\mu_5} \\
\rho_{\bar{Lizard},k} &= \frac{2\mu_4}{\mu_1+\mu_3} \\
\rho_{\bar{Spock},k} &= \frac{2\mu_5}{\mu_2+\mu_4}
\end{aligned} \tag{4.21}$$

Here we also see that the critical points of "rock-scissor-paper-lizard-spock", just like its original "rock-scissor-paper", are independent on node degree k.

Initial simulation of this game gives a similar scenario as the original five elements:

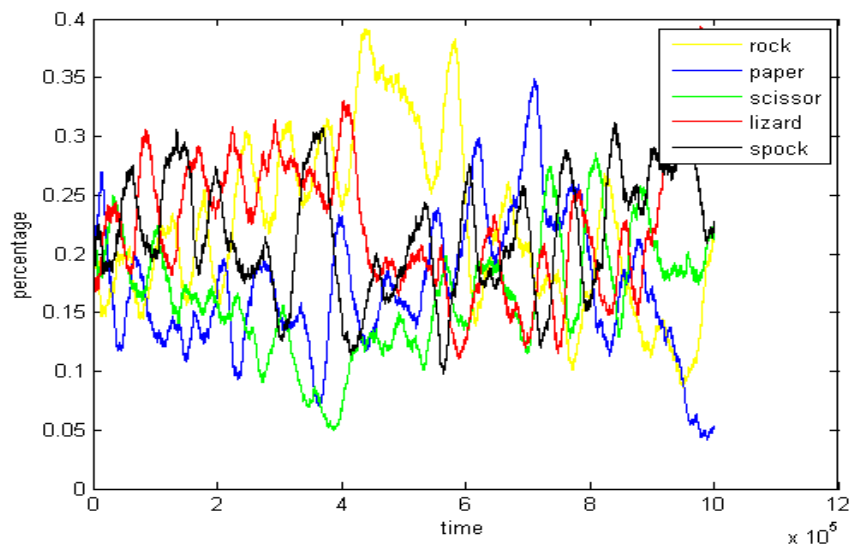

Figure 4.32: Initial Simulation of "Rock-Paper-Scissor-Lizard-Spock", V=50x50

Comparing this to figure 4.18, we see that the competition form is very similar, due to the strong competitiveness in the game.

4.3.1 Non-steady competition

Assuming in the beginning that the by fixing the probability P_1, namely the variable μ_1, we will get a similar outcome as in figures 4.21 to 4.28. However, unlike in the five elements, μ_1 is not only effecting one element this time, as all the elements in the "Rock-Paper-Scissor-Lizard-Spock" have the ability to defeat two of the corresponding elements in the system.

Our work is to identify if the difference in direct competition in this scenario is much different than the previous neighborhood effect caused by cooperation between elements.

Following the same steps and adjusting P_1 stepwise, a round of simulation results is generated:

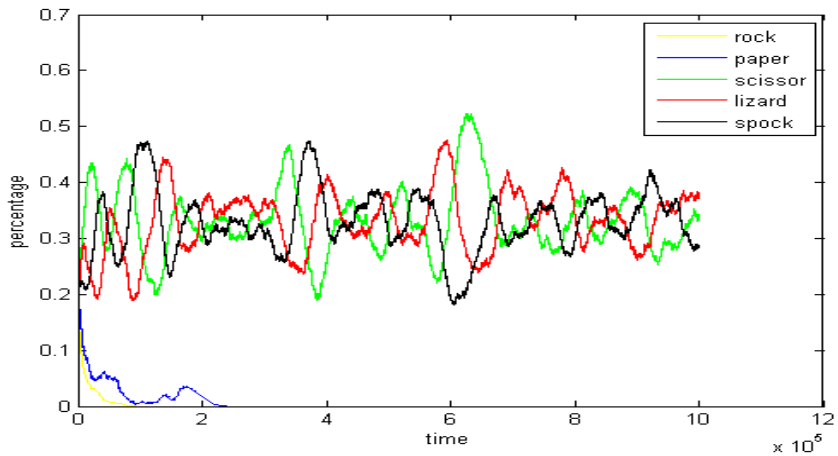

Figure 4.33: Simulated distribution of "rock-paper-scissor-lizard-spock" against time t with P_1=0.01, V=50x50

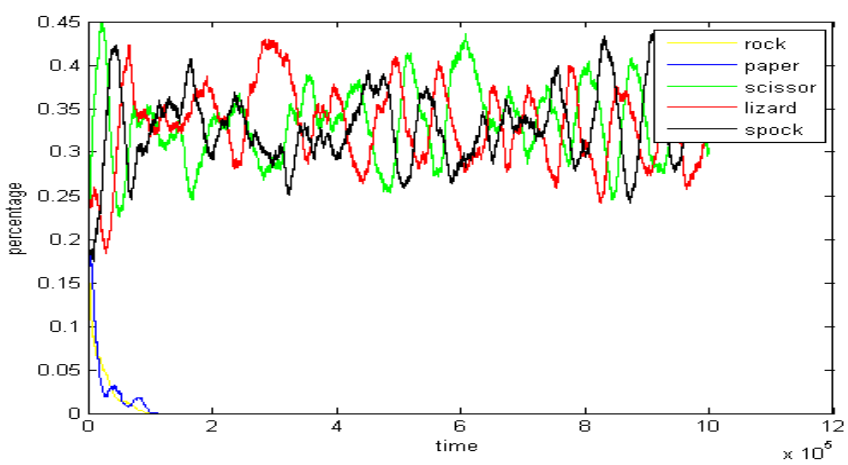

Figure 4.34: Simulated distribution of "rock-paper-scissor-lizard-spock" against time t with P_1=0.1, V=50x50

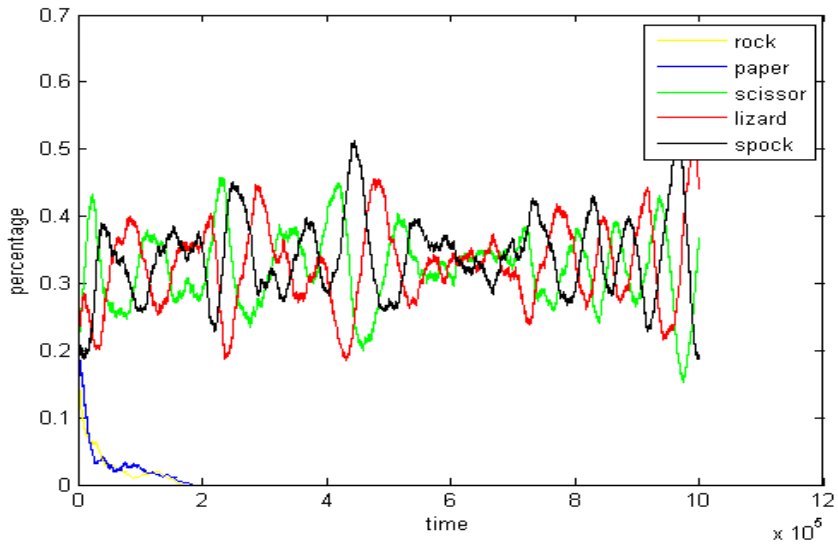

Figure 4.35: Simulated distribution of "rock-paper-scissor-lizard-spock" against time t with P_1=0.2, V=50x50

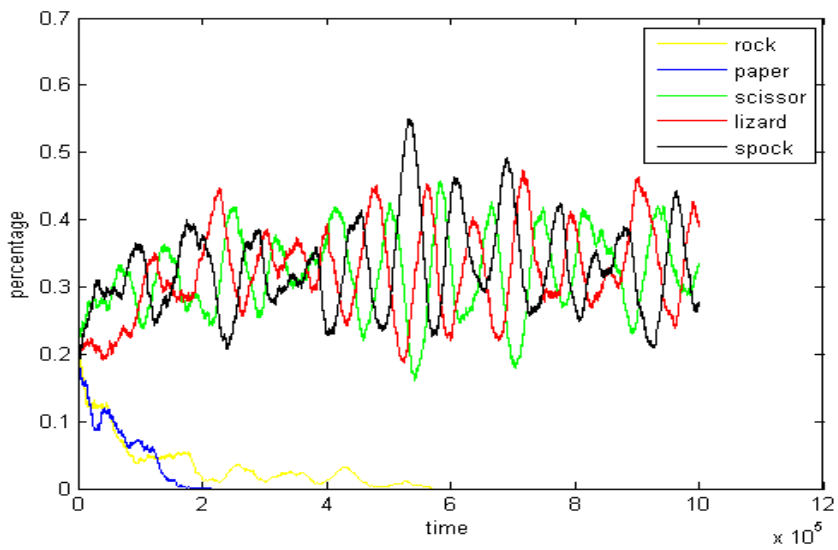

Figure 4.36: Simulated distribution of "rock-paper-scissor-lizard-spock" against time t with P_1=0.5, V=50x50

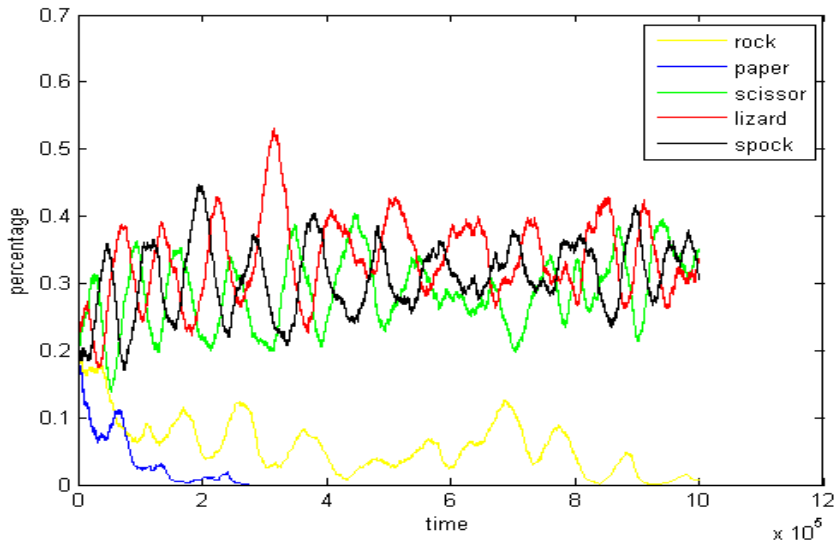

Figure 4.37: Simulated distribution of "rock-paper-scissor-lizard-spock" against time t with P_1=0.55, V=50x50

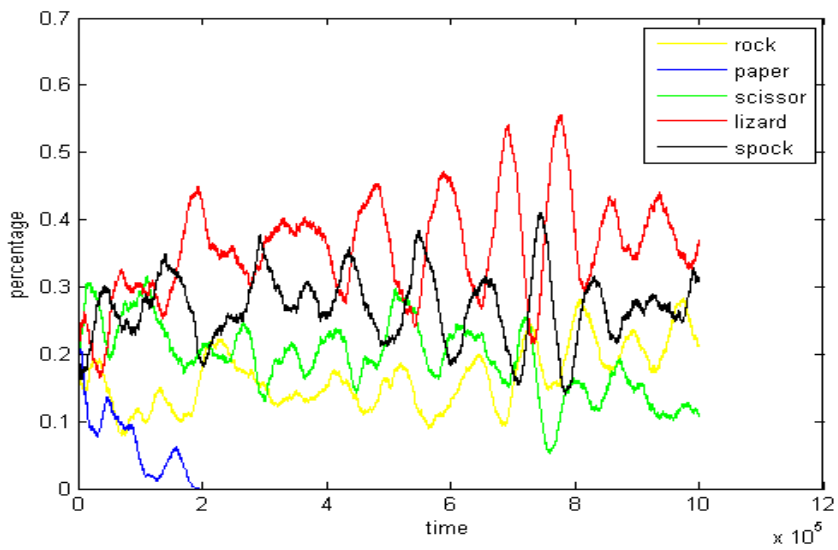

Figure 4.38: Simulated distribution of "rock-paper-scissor-lizard-spock" against time t with P_1=0.6, V=50x50

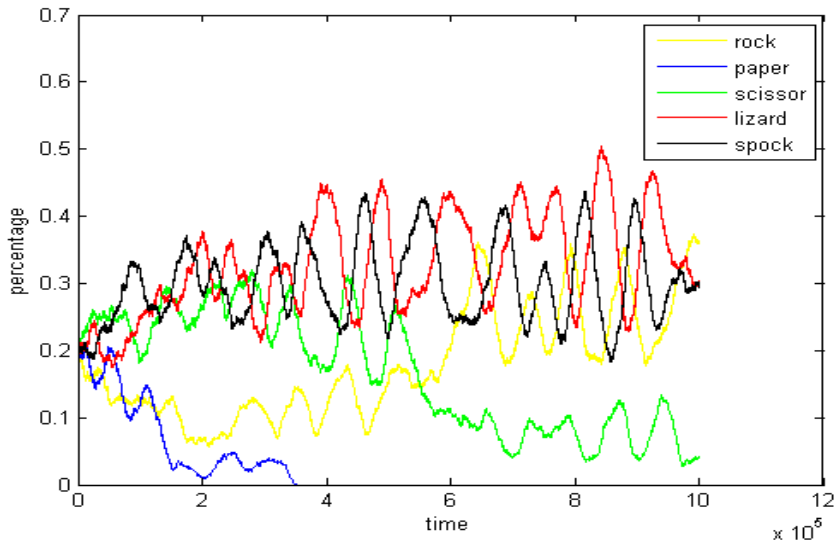

Figure 4.39: Simulated distribution of "rock-paper-scissor-lizard-spock" against time t with P_1=0.8, V=50x50

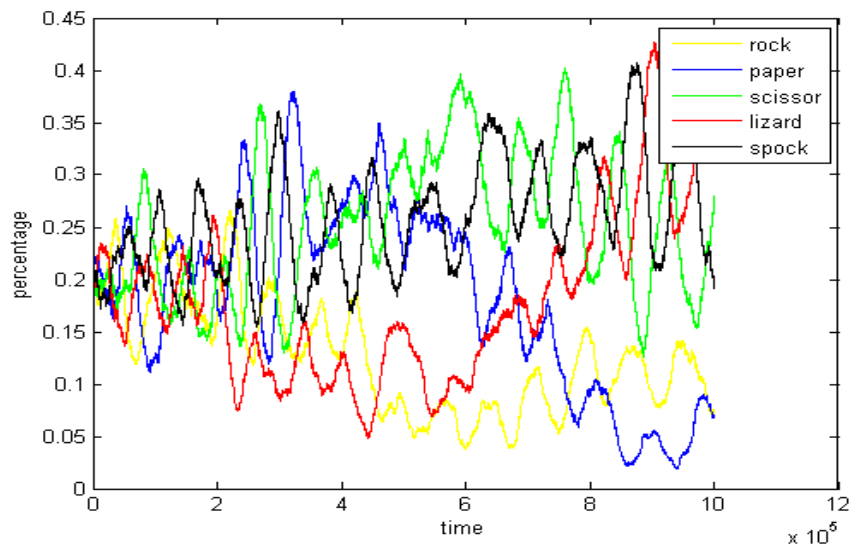

Figure 4.40: Simulated distribution of "rock-paper-scissor-lizard-spock" against time t with P_1=0.99, V=50x50

From figures 4.33 to 4.40, we see an obvious difference in the organization behavior compared to the original five elements. With significant lower global winning probability P_1 for rock to defeat scissor and lizard, we observe that both rock and paper get defeated pretty quickly when P_1 is small. We can look at this intuitively first, we have that:

Rock: no longer breaks scissor nor crushes lizard
Scissor: cuts paper quickly
Paper: covers rock, gets beaten up by double as much as rock
Lizard: eats paper quickly
Spock: smashes scissor, vaporizes rock

This easy intuition tells us that since paper has two enemies which were supposed to be defeated by rock in order to keep in balance is no longer protected. Therefore when $P_1 \to 0$, paper should have been defeated as the first element type. Without paper, spock loses one of its enemies, and has more power to defeat rock and causes rock to die out. While this happened, we still have a three elements game in form as earlier "rock-scissor-paper" remaining:

Rock: defeated
Scissor: decapitates lizard
Paper: defeated
Lizard: poisons spock
Spock: smashes scissor

Therefore "rock-paper-scissor-lizard-spock" is intuitively proven as a direct extension from the original "rock-scissor-paper" game and we could observe a steady competition when rock and paper are defeated.

Consider the equations of motion from , letting $\mu_1 \to 0$, we get the following equations:

$$\begin{aligned}
\rho_{Rock,k}(t) &= -(\mu_2\rho_{Paper,k}(t) + \mu_5\rho_{Spock,k}(t)) \\
\rho_{Paper,k}(t) &= \mu_2(\rho_{Rock,k}(t) + \rho_{Spock,k}(t)) - (\mu_3\rho_{Scissor,k}(t) + \mu_4\rho_{Lizard,k}(t)) \\
\rho_{Scissor,k}(t) &= \mu_3(\rho_{Paper,k}(t) + \rho_{Lizard,k}(t)) - \mu_5\rho_{Spock,k}(t) \\
\rho_{Lizard,k}(t) &= \mu_4(\rho_{Paper,k}(t) + \rho_{Spock,k}(t)) - \mu_3\rho_{Scissor,k}(t) \\
\rho_{Spock,k}(t) &= \mu_5(\rho_{Rock,k}(t) + \rho_{Scissor,k}(t)) - (\mu_2\rho_{Scissor,k}(t) + \mu_4\rho_{Lizard,k}(t))
\end{aligned} \tag{4.22}$$

Here we see that $\rho_{Paper,k}(t)$ and $\rho_{Spock,k}(t)$ become:

$$\begin{aligned}
\rho_{Paper,k}(t) &= \mu_2(-\mu_2\rho_{Paper,k}(t) - \mu_5\rho_{Spock,k}(t) + \rho_{Spock,k}(t)) - (\mu_3\rho_{Scissor,k}(t) + \mu_4\rho_{Lizard,k}(t)) \\
\rho_{Spock,k}(t) &= \mu_5(-\mu_2\rho_{Paper,k}(t) - \mu_5\rho_{Spock,k}(t) + \rho_{Scissor,k}(t)) - (\mu_2\rho_{Scissor,k}(t) + \mu_4\rho_{Lizard,k}(t))
\end{aligned} \tag{4.23}$$

Rearranging and get:

$$\begin{aligned}
\rho_{Paper,k}(t) &= \frac{\rho_{Spock,k}(\mu_2 - \mu_2\mu_5) - (\mu_3\rho_{Scissor,k} + \mu_4\rho_{Lizard,k})}{1+\mu_2^2} \\
\rho_{Spock,k}(t) &= \frac{\rho_{Scissor,k}(\mu_5 - \mu_2) - \mu_5\mu_2\rho_{Paper,k} - \mu_4\rho_{Lizard,k}}{1+\mu_5^2}
\end{aligned} \tag{4.24}$$

Assuming that the distribution of the rest three elements will remain approximately stable and equally distributed. Getting common denominator and we easily observe that equation 4.23 goes to negative, since the time derivative of $\rho_{Rock,k}$is negative and with Scissor and Lizard both eating up paper and at the same time Lizard helps to hold Scissor stable by defeating Spock.

Due to the higher order of reaction rates shown in equations 4.24, we see that the first term converges to zero much faster than the later terms, thus causing Paper to be negative very soon, therefore eliminating paper caused by the elimination of rock.

At last, we will look at adjusting P_1 and ρ_{Rock} should have some dependencies with each other, simulating, averaging and get:

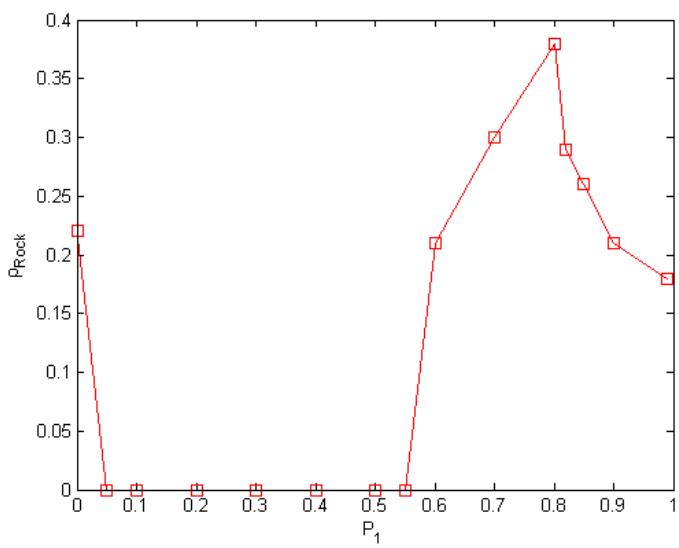

Figure 4.41: At a given time frame, $\rho_{Rock,k}$ as function of P_1, averaging over 450 samples(30 simulations on each probability), V=50x50 square lattice

4.3.2 Going back to three elements

Continuing from the last subsection and we will carry on to adjust one more winning probability, based on the already adjusted P_1 to see if we could get similar behavior just as in the original "Rock-Scissor-Paper" game. Our intuition tells us that a different scenario is expected. Choosing to adjust P_3, the probability for Scissor to defeat paper and lizard. Considering the boundary conditions, when $P_1 \rightarrow 0$, we will have a 3 elements' system after a short while. The following simulation results are generated for different P_3:

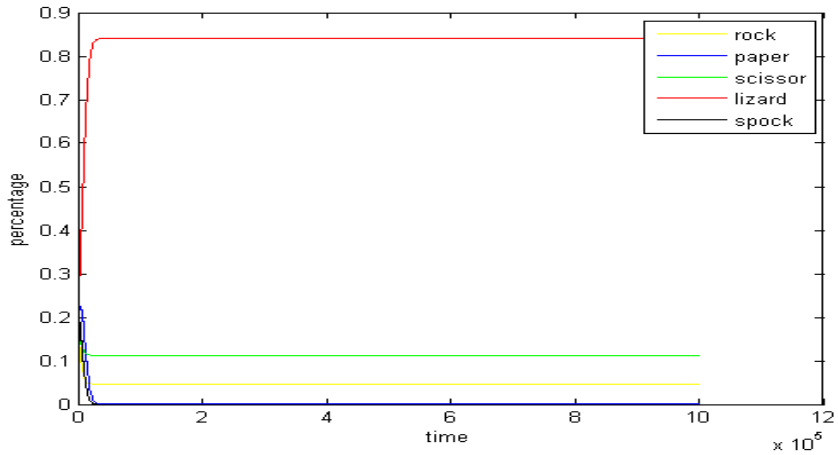

Figure 4.42: Simulated distribution of "rock-paper-scissor-lizard-spock" against time t with $P_1 \to 0$, P_3=0.01, V=50x50

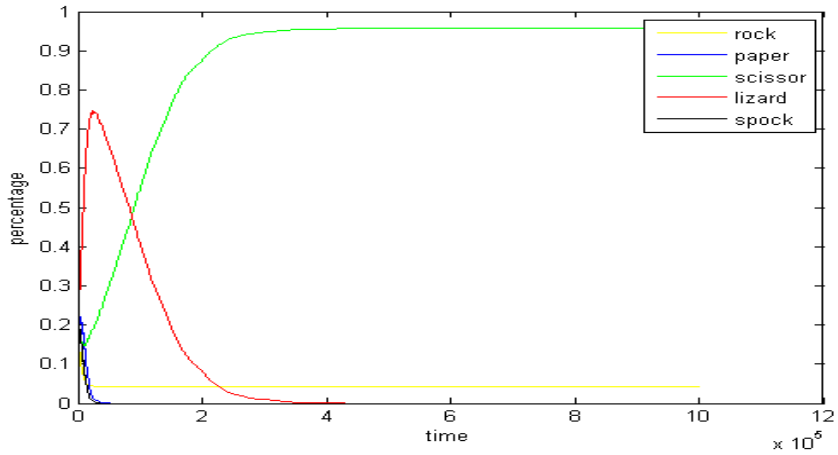

Figure 4.43: Simulated distribution of "rock-paper-scissor-lizard-spock" against time t with $P_1 \to 0$, P_3=0.1, V=50x50

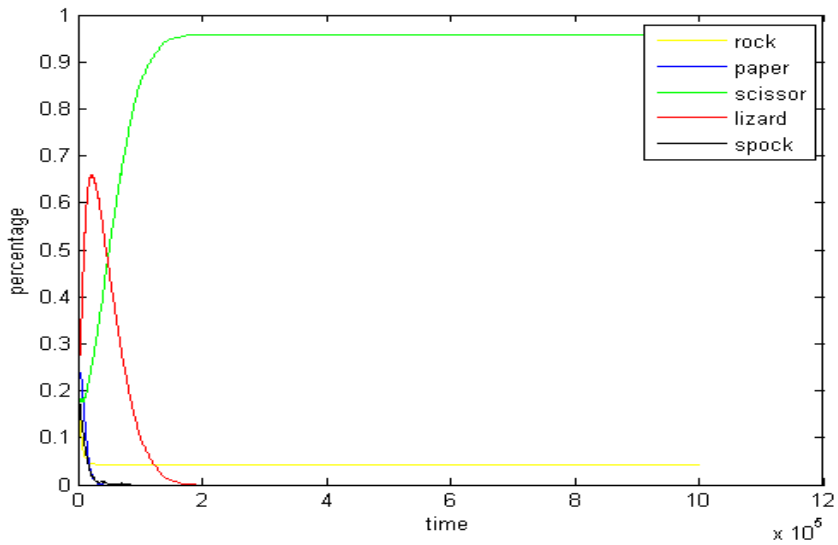

Figure 4.44: Simulated distribution of "rock-paper-scissor-lizard-spock" against time t with $P_1 \to 0$, P_3=0.2, V=50x50

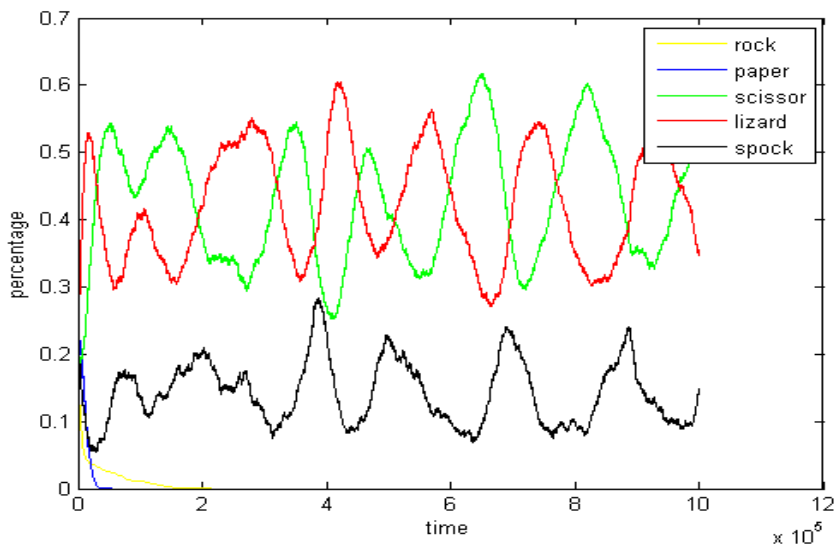

Figure 4.45: Simulated distribution of "rock-paper-scissor-lizard-spock" against time t with $P_1 \to 0$, P_3=0.4, V=50x50

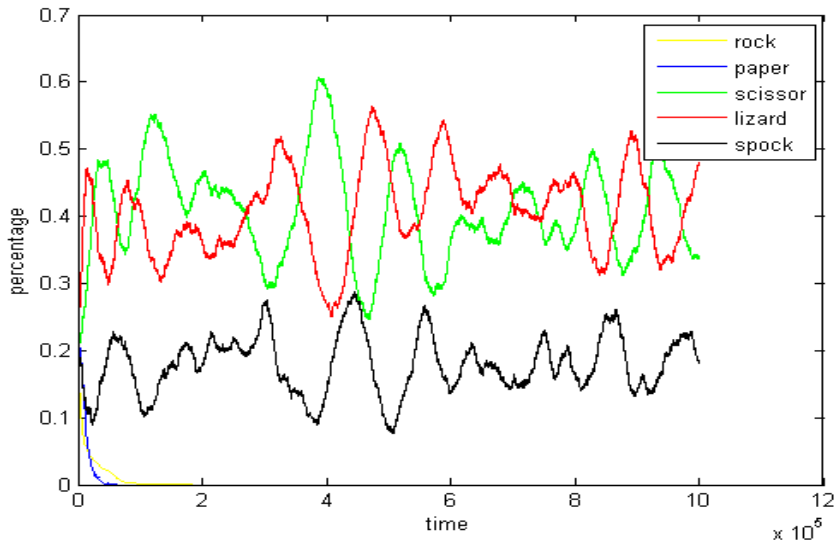

Figure 4.46: Simulated distribution of "rock-paper-scissor-lizard-spock" against time t with $P_1 \to 0$, P_3=0.5, V=50x50

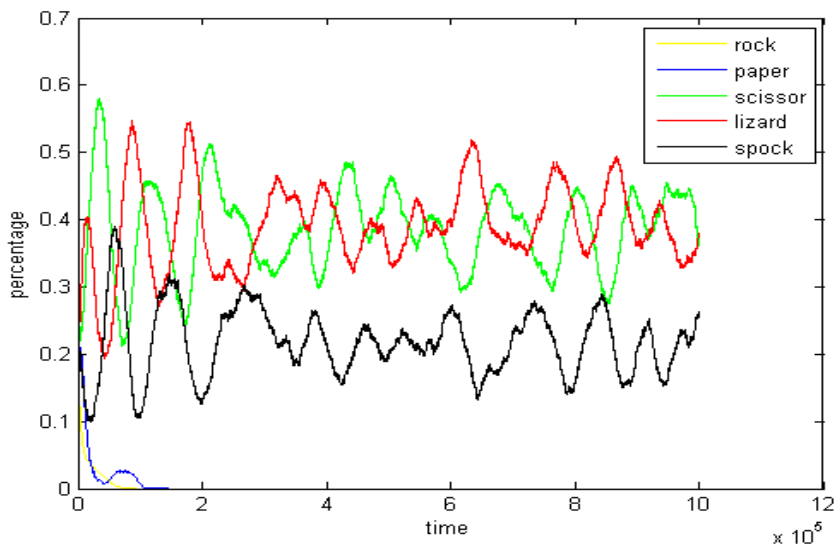

Figure 4.47: Simulated distribution of "rock-paper-scissor-lizard-spock" against time t with $P_1 \to 0$, P_3=0.6, V=50x50

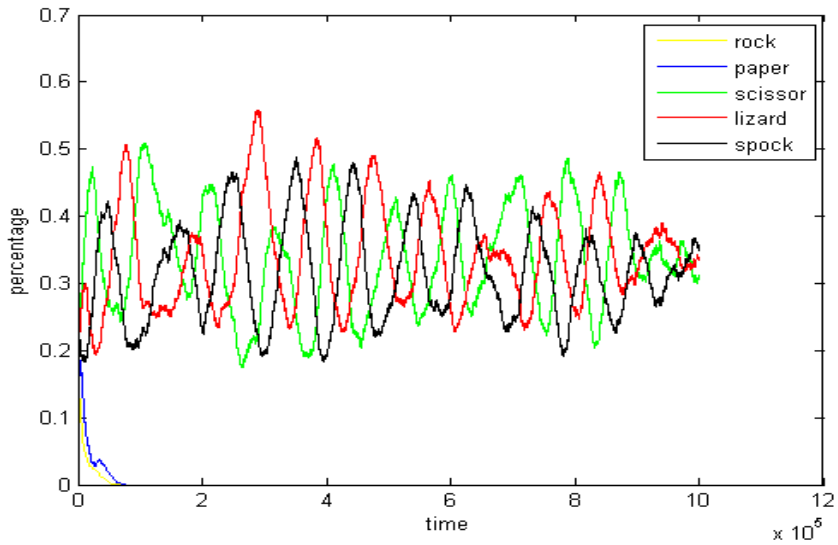

Figure 4.48: Simulated distribution of "rock-paper-scissor-lizard-spock" against time t with $P_1 \to 0$, P_3=0.95, V=50x50

From the figures 4.42 to 4.44, we see clearly a match in behavior compared to figures 4.3 to 4.4, we see that when keeping the winning probability of a certain element low, at a level of less than 0.2, the element itself turn out to be the winner in a three elements competition. In the original "Rock-Scissor-Paper" the Rock is the case, and in the extended "Rock-Paper-Scissor-Lizard-Spock" Scissor is the winner. At the same time we also observe that Scissor still wins quickly at probability around 0.2, which in previous "Rock-Scissor-Paper" Rock has stopped the winning behavior in the given time frame. This is caused by the fixed probability P_1, which delays the P_3 to win as quick as what happened to rock before in the original three elements game.

4.3.3 Summary

In this section, we have proved that "Rock-Paper-Scissor-Lizard-Spock" is a game in the form of five elements, but is more a direct extended version of "Rock-Scissor-Paper" due to the winning behavior. While following the same pattern as the five elements at steady state, if one of the elements end up being defeated, it will drag one corresponding to be the follower so a new steady state consisting of three remaining elements would keep the game steady in relative short time frame. Unlike the "Chinese five elements", "Rock-Paper-Scissor-Lizard-Spock" is a direct competition game, this means that the neighborhood support does not exist, while competition is the main factor that decides the self organization behavior.

4.4 Network impact and extended mean field analysis

Theoretically we have four stationary states exists for "rock-scissor-paper" and six stationary states exists for the five elements. On the macroscopic view, the corrosion rate and

fixed possibility is deciding how well the different elements are competing with each other. On the microscopic view, we have to extend it into a network problem by considering the inner game and what happens at each node.

With the absorbing states (where $N_i(t_end) = N_{total} = V$ for i=A,B,C,D,E) and for the corresponding different elements, and when all elements are equally distributed, 1/3 for "rock-scissor-paper" and 1/5 for the five elements. In order to understand and use the network principle into the game system, we will apply the node probability function and pair approximation.

Pair approximation is inspired by the nature of evolutionary Game theory where one could closely simulate the hierarchy of link impact from node to node that would effect the final outcome, for the "rock-scissor-paper" game, there have been done investigation of three cyclically dominated strategies on a random graph[10]. This model exhibits transitions when varying the parameters of payoff matrix and it is underlined that the neighborhood strategies depends on neighborhood. Let's denote different node with specific number, k_j, as the specific node.[3]

In five elements, the definition of neighborhood is even more important than to the "rock-scissor-paper" since neighbors for the resisting element could contribute to help the invasion from any enemy elements to a particular one. Let's for example denote a couple of nodes as k_1 and k_2, thus the probability $p_2(k_1, k_2)$[4] defines the probability of finding two nearest neighbor(linked) nodes. In this case, the derivation of motion for all quantities have to take contribution of all the elementary invasion processes into account, details can be found from[11]. Theoretically, we would assume that although the equation of motion involve the number of neighbors, the pair approximation can not distinguish the difference between different structures of lattices. For large N, the local structure will become tree-like due to strong heritage system brought by the pair approximation. Another theory that contradicts this is the original theory of network, which says that the degree of network is the dominating contribution to the outcome, as in [14] have shown, the reaction kernel Γ which is degree dependent, and contributes to the local variations.

Although we have mentioned earlier that the pair approximation is not capable of describing the local self-organizing pattern due to cyclic invasions. This could and should be eliminated by choosing larger clusters, meaning that not only considering from the individual links, but seeing more links gathered together in a group, and then do *cluster expansion*. This is an advanced way of studying collective behavior. In this way we can determine all the possible configuration possibilities at each cluster, the simplest cluster one could choose is a four link approximation with probability $p_4(k_1, k_2, k_3, k_4)$ on a 2x2 cluster assumed to be translation invariant on our square lattice for competition.

let's first take a look at a minimized square network:

[3]this is to not interfere with the definition of the node degree k

[4]Note that all local probabilities are addressed in small letters, while global probabilities are addressed as capital letters

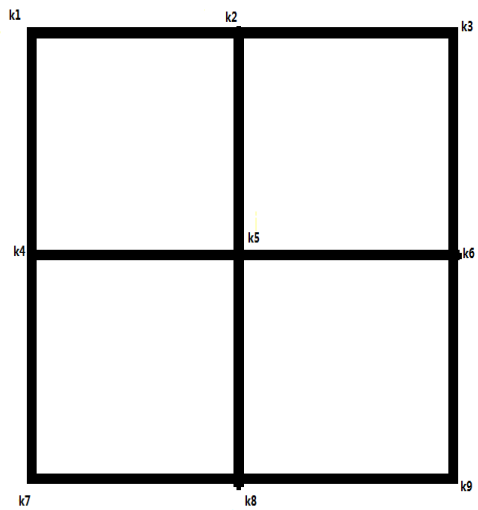

Figure 4.49: A simple example of a square lattice with 9 nodes, considering 4 nodes forming clusters

Here if an invasion from k_4 to k_5 would cause some effects to all the four four-nodes configuration probabilities involved. Knowing the conservation of probability at each node. Let's use this notation for different configuration: Defining the node k_9 as the end node, and considering it in its own cluster, the local node probability at k_9 following the spatial effect caused by the invasion from k_4 to k_5 can be defined as:

$$p_9(k_1, ..., k_9) = \frac{p_4(k_1, k_2, k_4, k_5)p_4(k_2, k_3, k_5, k_6)}{p_2(k_2, k_5)p_2(k_4, k_5)} \cdot \frac{p_4(k_4, k_5, k_7, k_8)p_4(k_5, k_6, k_8, k_9)}{p_2(k_5, k_6)p_2(k_5, k_8)} p_1(k_5) \tag{4.25}$$

And the configuration probabilities is defined as:

$$p_1(k_1) = \sum_{k_2} p_2(k_1, k_2) = \sum_{k_2} p_2(k_2, k_1) \tag{4.26}$$

Based on our initial condition of undirected links. And for a 2-nodes configuration:

$$p_2(k_1, k_2) = \sum_{k_1,k_2} p_4(k_1, k_2, k_3, k_4) = \sum_{k_3,k_4} p_4(k_3, k_4, k_1, k_2) = \sum_{k_3,k_4} p_4(k_1, k_3, k_2, k_4) = \sum_{k_3,k_4} p_4(k_3, k_1, k_4, k_2) \tag{4.27}$$

Considering the probability as master equation, we could describe the cluster effect for the

time derivative of local probability$p_4(k_1, k_2, k_3, k_4)$ as:

$$
\begin{aligned}
\partial_t p_4(k_1, k_2, k_3, k_4, t) = -p_c \sum_{k_x} p_4(k_1, ..., k_4) p_1(k_x) (\sum_{j=1,2,3,4} \Gamma(k_x, k_j) \\
+p_c \sum_{k_x} p_4(k_x, k_2, k_3, k_4) p_1(k_1) \Gamma(k_1, k_x) \\
+p_c \sum_{k_x} p_4(k_1, k_x, k_3, k_4) p_2(k_2) \Gamma(k_2, k_x) \\
+p_c \sum_{k_x} p_4(k_1, k_2, k_x, k_4) p_3(k_3) \Gamma(k_3, k_x) \\
+p_c \sum_{k_x} p_4(k_1, k_2, k_3, k_x) p_4(k_4) \Gamma(k_4, k_x) \\
-\frac{1-p_c}{4} \sum_{k_5,...,k_9} p_9(k_1, ..., k_9) \Gamma(k_2, k_4) \\
-\frac{1-p_c}{4} \sum_{k_5,...,k_9} p_9(k_1, ..., k_9) \Gamma(k_1, k_3) \\
-\frac{1-p_c}{4} \sum_{k_5,...,k_9} p_9(k_1, ..., k_9) \Gamma(k_6, k_2) \\
-\frac{1-p_c}{4} \sum_{k_5,...,k_9} p_9(k_1, ..., k_9) \Gamma(k_6, k_1) \\
+\frac{1-p_c}{4} \sum_{k_x,k_5,...,k_9} p_9(k_1, k_2, k_5, k_3, k_x, k_6, k_7, k_8, k_9) \delta(k_2, k_4) \Gamma(k_2, k_x) \\
+\frac{1-p_c}{4} \sum_{k_x,k_5,...,k_9} p_9(k_5, k_1, k_2, k_6, k_x, k_4, k_7, k_8, k_9) \delta(k_1, k_3) \Gamma(k_1, k_x) \\
+\frac{1-p_c}{4} \sum_{k_x,k_5,...,k_9} p_9(k_5, k_6, k_7, k_1, k_x, k_8, k_3, k_4, k_9) \delta(k_6, k_2) \Gamma(k_6, k_x) \\
+\frac{1-p_c}{4} \sum_{k_x,k_5,...,k_9} p_9(k_5, k_6, k_7, k_8, k_x, k_2, k_9, k_3, k_4) \delta(k_6, k_1) \Gamma(k_6, k_x) \\
+...
\end{aligned}
\tag{4.28}
$$

Here $\delta(k_x, k_y)$ denotes the Kronecker delta and $\Gamma(k_x, k_y)$ is the constraint of local spatial effect which is expressed as:

$$
\Gamma(k_x, k_y) = \begin{cases} 1 & \text{if } k_y - 1 = k_x mod 3 \\ 0 & \text{otherwise} \end{cases}
$$

The terms proportional to p_c describe the contributions coming from other invasions from arbitrary distance while the contributions from one of the four nearest-neighbor nodes are proportional to $\frac{1-p_c}{4}$. The missing terms in equation 4.28 is quite straightforward by using the same hierarchy. Using these theoretical data and only keep the nearest cluster

spatial effect, we can compare the global corrosion probability with the local spatial nodal probability and see if the local terms bring a significant effect to the game as a whole, cause changes to the self organization pattern.

Solving the master equation above for finding a stationary probability for one single node, we set the left hand side equal to 0 and get:

$$\begin{aligned} p_(k_x) = & \sum_{k_x} \sum_{i=1}^{4} p_4(k_x, k_2, k_3, k_4) p_i(k_i) \Gamma(k_i, k_x) \\ & - \frac{1-p_c}{4p_c} \sum_{k_5,\ldots,k_9} \sum_{i,j,i\neq j} p_9(k_1, ..k_9) \Gamma(k_i, k_j) \\ + \frac{1-p_c}{4p_c} & \sum_{k_x,k_5,\ldots,k_9} \sum_{i,j=x,i\neq j} p_9(k_1, ..k_9) \delta(k_i, k_j) \Gamma(k_i, k_x) \end{aligned} \tag{4.29}$$

The first part of equation 4.29 are simple the gain from the first four nearest neighbors coming into one point, the second term denotes the term that loses to other 5 nodes, while the last term denotes the gain caused by other neighbors from the rest of the 5 nodes.

Seeing probability of different node taking contributions from all neighboring nodes in a nine-nodal cluster.

At this point, we need to introduce the node degree k into the system. The classical way to express the nodal probability impact is:

$$\mu_i = \sum_{k'} p_(k') \rho_{i,k'}(t) \tag{4.30}$$

where i in the equation above is for different element, and k' denotes a virtual node that absorbs all the contributions for each cluster. Here we have a reaction rate that takes each node contribution into account and make it to a global reaction rate for different elements i.

Applying this reaction rate to the "rock-scissor-paper" and the "Chinese Five Elements", we could observe some similar behaviors but with different outcome. On the figure 4.50, we have the outcome of distribution of "Rock-Scissor-Paper" after applying the new reaction rates following the network impact, we could observe that due to this pure competition state, the system follows the same pattern as before but with high fluctuation between elements, on the figure 4.51, we have the outcome of distribution of "Chinese Five Elements", where we see that the network impact causing more fluctuation and intense competition between elements than previously shown in figures 4.21 to 4.28.

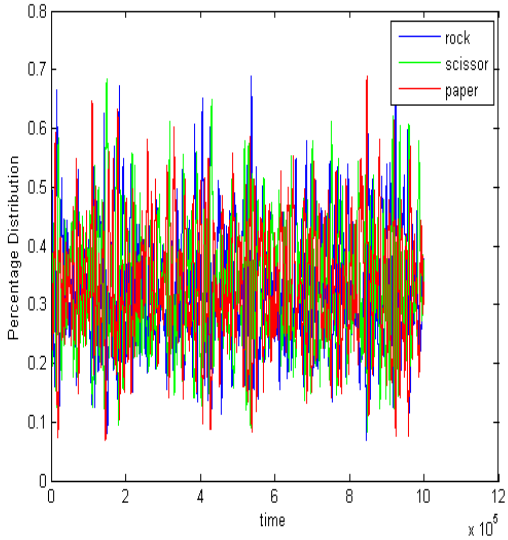

Figure 4.50: "Rock-Scissor-Paper"

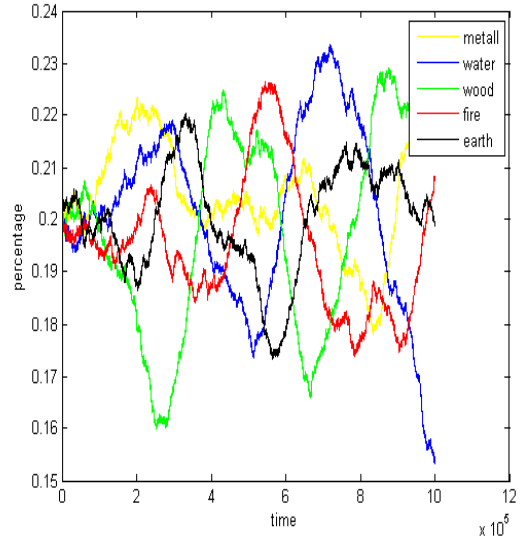

Figure 4.51: "Chinese Five Elements"

"Rock-Scissor-Paper" simulated on V=25x25, and "Chinese Five Elements" simulated on V=100x100, $t_{end} = 1000000$

Using the simulation results, we could observe in figures 4.52 that the critical points for rock and metal in their respective games have dropped lower than previously. Local probabilities contributing to reaction rate are changing the system more harmoniously rather than global competition. Due to the cluster effect, we have local probabilities no longer concentrated with individual elements, but rather in a 9-elements' cluster as a sub-group probability, causing lower critical densities for rock and metal in our simulation.

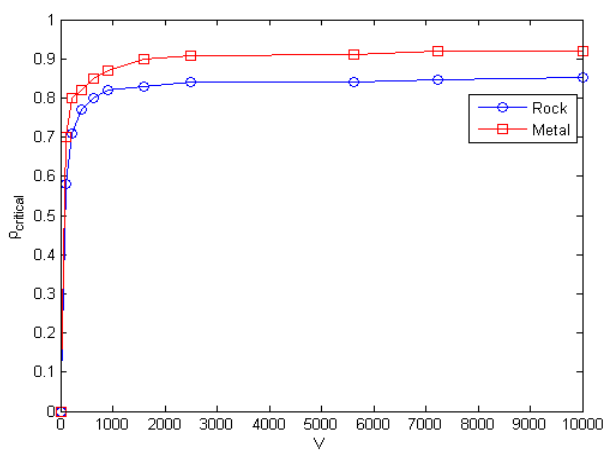

Figure 4.52: Simulation results of critical points of metal simulated on different sizes of square lattices, 30 samples per lattice, by 2 different elements from different games, $t_{end} = 1000000$, with . p_c=0.5, $P_{rock} = 0.1$ and $P_{metal} = 0.01$

With lower critical point, this means that the competition hardness is not really causing big global changes. This is comparable to real life happenings, where internal battle in the company might cause disharmony, but the real threat would be the challenge and battle from outside.

4.4.1 Summary

We could observe some changes after applying the network impact on the two different games, while the game patterns stay the same, the intensity in the games have increased somehow. This phenomena could be explained as the quick nodal probability exchanges between in each round, that causes a variations to reaction rates, thus in this system, we could less easily experience a sudden breakdown compared to what happened earlier by considering the global winning probabilities in order to fix the reaction rates. Here we rather have some change locally, meaning no longer a everyone fights everyone game system, no longer a certain reaction is destined to fight all of its weaker or stronger enemies.

Chapter 5

Conclusion

In this thesis, we have provided a classical way to simulate the rock-scissor-paper game. We found its use and similarity with real life E.coli bacteria survival behavior. This included the study of rock-scissor-paper phenomena in the bacteria nature, building the simulation model and generation of results for finding characteristic behaviors of the three elements in a theoretical way. By adjusting the corrosion probability in different cases successfully proved some evolutionary game properties of the system, and giving an interval for possible phase transitions. The findings are in accordance with major findings and simulated results are in similar pattern as researchers have done previously. We have found that the stability of species would occur under a certain corrosion probability in a range of 0.1 to 0.3 for either one or two fixed probabilities. This stable state probability is previously found by other ecologists in related study of diffusion.

In order to understand a more complicated system, The Ancient Chinese Five Elements, which is an extended variation of rock-scissor-paper, an extended and improved model has been built in order to find some characteristic behaviors. A brand new analytical expression is formulated to give an insight in what's happening in the game. Five Elements, constitutes of both a virtual score and a real score in the outcome matrix, is an unusual game. They are formed, indirectly, by different circles of three varying system which have almost the same property as the rock-scissor-paper. The complicated system in the Five Elements make it very difficult to adjust all the combinations of fixed possibility, therefore, only some single fixed possibility has been adjusted in order to extend it to a general understanding of stability in the system.

The Five Elements, being a network of up to four nodes with more varying elements, is more unstable, compared to the original rock-scissor-paper. The instability is caused by, more or less, its virtual links, which makes the generation cycle. An understanding of this phenomena is still not present in physics, as the principle of Five Elements is from an ancient mythology. We could observe from our results that adjusting global corrosion probabilities did not affect that much to the game as it did in the Rock-Scissor-Paper, meaning that the randomness in the Five Elements is very high that exceed the scale for the game to be completely controlled. Cooperation causes competition to be more random. At some scale, at some scale, fixing one of the corrosion probabilities does adjust the final outcome in the given relative short time frame small world simulation, but this case could easily be neglected in real world.

A real example of Five elements have been studied in this thesis, that is "Rock-Paper-Scissor-Lizard-Spock", which originated from the famous American TV "Big Bang Theory". We found combination of both five elements behaviors and three elements behavior in this funny game. This game is a sole extension from the original "Rock-Scissor-Paper", but

added with two more elements which causes it to be a great candidate to express the five elements behavior. Through simulations we found that on the steady state, the game behaves just like the "Chinese Five Elements", but when fixing the one of the corrosion probabilities, the game tends to fall back to its original "Rock-Scissor-Paper" form, which is caused by its cyclic competition nature that directly extended from the three elements game. Transitional points also lie in close range as for "Rock-Scissor-Paper".

A trial of considering the game as a network by adding cluster effect has been simulated, due to the strong reactions rates that determine the game outcome, the local nodal probabilities' effect on the game as a whole could be neglected due to its high orders. We could also conclude by asserting that in a real reaction-diffusion system, strong reaction will be the domination force in deciding the game nature.

Chapter 6

Future Work

The most challenging part in this thesis work is that no one has ever studied the Chinese Five elements before, due to its similarities to "Rock-Scissor-Paper", we managed to work out a simple model using classic mean field theory to explain this system better. This game was not quite considered as a game for just a while ago, until the introduction of a real example was shown in public as late as in January 2013. The "Chinese Five elements" remain a big myth in Chinese culture and folklore, being an important part of Taoism, but really difficult to connect with science. This way of explaining nature is very mythic to the Western, when a lot of Chinese people consider this as the reason for nature's development and the balance of the force in Universe, this game need to be studied more, not only by Chinese scientists, but also by Western who have a more neutral viewpoint.

The biggest constraint in this work was caused by the computer simulation limitation, due to the limited memory, simulation time could not be looked into a larger scale, meaning we could only focus on the small world effect in a relative short time frame. This limitation caused strong difficulty for simulating the last part where we tried to apply local cluster effect from nodal probabilities. With a better memory, more data could be generated and simulation time could also be extended.

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