

# On the equivalence and impact on stability of impedance modelling of power electronic converters in different domains

Atle Rygg, Marta Molinas, Chen Zhang and Xu Cai

**Abstract**—Small-signal analysis of power electronic converters and systems is often carried out by impedance-based methods. At the core of these methods lies the impedance modelling, which can either be obtained through analytical calculations, simulations or measurements. The impedance models can be obtained into two main domains - the  $dq$ -domain and the sequence domain. In the  $dq$ -domain the impedance model is a 2x2 matrix, while in the sequence domain it is composed by the positive and negative sequence impedance. Recently, a third domain called the modified sequence domain was defined as an extension to the sequence domain, but also with clear similarities to the  $dq$ -domain. The objective of this paper is to unambiguously relate to each other the impedances in these three domains, and to show how this equivalence translates into their respective stability assessments. It is also proven that the sequence domain impedance has the same marginal stability condition as the  $dq$ -domain impedance matrix.

The three-phase Voltage Source Converter is used as an example converter in this paper, as its impedance model in all three domains is well established and reported by previous research. The results in this paper shows that the modified sequence domain model can be derived from the  $dq$ -domain model (and vice versa), and that the stability analysis will be identical in these two domains. It is also shown how the original sequence domain model can be derived from the two other models through a model reduction. However, a small discrepancy between the two Nyquist plots is observed in the presence of components such as phase lock loop or DC-link control.

**Keywords**— $dq$ -domain, Impedance Modeling, Power Electronic Systems, Sequence Domain, Stability Analysis.

## I. INTRODUCTION

Impedance models of power electronic converters are useful for control dynamics and stability studies, as well as for harmonic resonance analysis. Previous works have derived models of e.g. three-phase Voltage Source Converters (VSC) [1]-[4], Modular Multilevel Converters [5] [6] and single-phase VSCs [7]-[9]. Some examples of applications of impedance-based models for small signal stability analysis can be found in [10]-[16].

The two-level three-phase VSC is a widespread converter technology utilized in a many applications as well as power

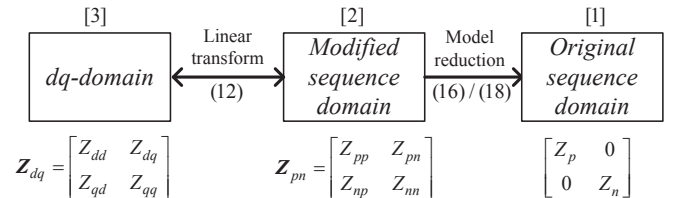


Fig. 1: Illustration of the transformation of models presented in the paper. Benchmark analytical models are indicated by references, while the relevant equation numbers are indicated below the arrows

levels. Impedance modeling of the three-phase VSC is normally performed in either the  $dq$ -domain or in the sequence domain. The  $dq$ -domain was first applied in [17], and the resulting impedance model is a 2x2 matrix that relates the  $d$ - and  $q$ -axis voltages to the corresponding currents. Examples of  $dq$  impedance model applications can be found in the three-phase VSC is performed in e.g. [3][4][18][19].

Sequence domain analysis is the other main approach to obtain an impedance model. The method is presented in [20], and a widely applied model is derived in [1].

Recently, a third domain called the *Modified Sequence Domain* was introduced for obtaining an impedance model which combines the  $dq$ - and sequence domains. It has been shown that the  $dq$ -impedance matrix is equivalent to a 2x2 modified sequence domain impedance matrix where the positive and negative sequence are shifted by two times the fundamental frequency [21]. The analytical VSC-model in the modified sequence domain was derived in [2]. Application of the modified sequence domain to low frequency interactions in wind farms is performed in [22], while a VSC-HVDC system is analyzed in [23].

The contribution of the present paper is on the method for relating analytical impedance models in the  $dq$ -domain to their respective equivalents in the sequence domain. The methodology is illustrated in Fig. 1. The  $dq$ -model is first transformed into the modified sequence domain by the linear transformation (12) derived in section III. The modified sequence domain matrix can be used to obtain the original sequence domain impedances by another transformation. A frequency shift is also required in this step. This is achieved by the relations (16) or (18) derived in section IV. Note that this procedure is not reversible, i.e. the modified sequence domain impedance

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$$\begin{aligned}
Z_p^L(s) &= \frac{K_m V_{dc} [H_i(s - j\omega_1) - jK_d] G_i(s) + sL}{1 - (\frac{C_1}{2} e^{j\phi_{c1}} + [H_i(s - j\omega_1) - jK_d] \frac{I_1}{2} e^{j\phi_{i1}}) \cdot T_{PLL}(s - j\omega_1) G_v(s) \frac{K_m V_{dc}}{V_1}} \\
Z_n^L(s) &= \frac{K_m V_{dc} [H_i(s + j\omega_1) + jK_d] G_i(s) + sL}{1 - (\frac{C_1}{2} e^{-j\phi_{c1}} + [H_i(s + j\omega_1) + jK_d] \frac{I_1}{2} e^{-j\phi_{i1}}) \cdot T_{PLL}(s + j\omega_1) G_v(s) \frac{K_m V_{dc}}{V_1}}
\end{aligned} \quad (1)$$

matrix cannot be obtained from the original sequence domain impedances.

The presented methodology is general and extends a previous work by the same authors [21]. In the present paper additional expressions are derived, and the method is applied to the three-phase VSC as an example. The objective of this example is to establish an unambiguous relation between the analytical  $dq$ -model in [3] with the modified sequence domain model in [2], and also the sequence domain model in [1].

## II. IMPEDANCE MODELS OF THE THREE-PHASE VSC

The three-phase VSC is used as an example converter for the presented methodology. A schematic of the converter and the control system is given in Fig. 2.  $G_v$  and  $G_i$  represent the combined effect of transducer delay, lowpass filter and Analog-to-Digital conversion for voltage and current measurement, respectively. The delay due to Pulse-Width-Modulation (PWM) is also included in these transfer functions in the same way as in [1]. The output filter is simplified to the series inductor  $L$  in order to be consistent with the modeling in [1] and [2]. Details of the Phase Lock Loop (PLL) and the current controller are given in Fig. 3. The basic synchronous reference frame PLL is considered with  $H_{PLL}(s) = \left( k_{p,pll} + \frac{k_{i,pll}}{s} \right) \frac{1}{s}$ . The current controller applies a PI-controller  $H_i(s) = k_p + \frac{k_i}{s}$  with decoupling term  $K_d$ . A constant dc-link voltage  $V_{dc}$  is assumed in the modeling in the same way as the reference models listed in the next paragraph.

The following three impedance models will be used as basis for the comparison in this paper:

- A The sequence domain model from [1]
- B The  $dq$ -domain model from [3]
- C The modified sequence domain model from [2]

These three models are now presented in separate subsections along with a brief discussion on their respective impedance domains.

### A. Impedance Model by Harmonic linearization in the sequence domain

Sequence domain impedances are widely adopted for stability analysis and harmonic resonance analysis. In the present paper this domain will also be referred to as the *original* sequence domain, in order to clearly distinguish from the *modified* sequence domain. By applying harmonic linearization, a subsystem can be characterized by its positive and negative sequence impedance defined as [20]:

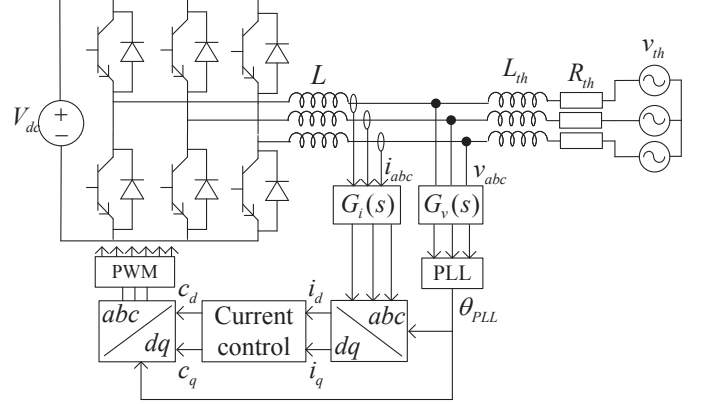


Fig. 2: Grid-connected converter with PLL and current controller

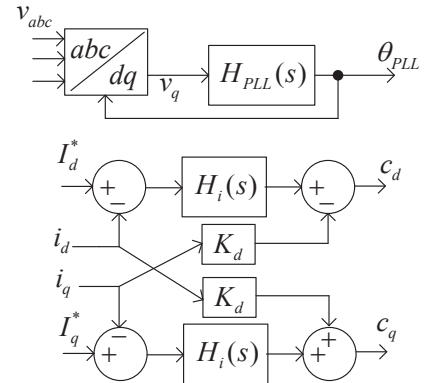


Fig. 3: PLL (upper) and current controller (lower)

$$\begin{aligned}
Z_p(s) &= \frac{V_p(s)}{I_p(s)} \\
Z_n(s) &= \frac{V_n(s)}{I_n(s)}
\end{aligned} \quad (2)$$

where subscripts  $p$  and  $n$  denotes positive and negative sequence, respectively. The analytical model of the grid-connected VSC with  $dq$ -domain current controller was derived in [1]. This model is given in (1). Superscript  $L$  indicates that this is the *load* subsystem (e.g. the VSC in Fig. 2), whereas the Thevenin grid equivalent is the *source* subsystem.  $K_m$  is the modulator gain, while  $I_1 e^{j\phi_{i1}} = I_{d0} + jI_{q0}$  is the fundamental

$$\begin{aligned} \mathbf{Z}_{dq}^L(s) &= (Z_{out}^{-1} + G_{id}G_{del}[-G_{ci} + G_{dei}]G_{PLL}^i + G_{PLL}^d)K)^{-1} \cdot (I + G_{id}G_{del}[G_{ci} - G_{dei}]K) \\ \mathbf{Y}_{dq}^L(s) &= (I + G_{id}G_{del}[G_{ci} - G_{dei}]K)^{-1} \cdot (Z_{out}^{-1} + G_{id}G_{del}[-G_{ci} + G_{dei}]G_{PLL}^i + G_{PLL}^d)K \end{aligned} \quad (3)$$

$$\begin{aligned} \mathbf{Y}_{pn}^L(s) &= \begin{bmatrix} Y_p(s) & J_n(s) \\ J_p(s) & Y_n(s) \end{bmatrix} \\ Y_p(s) &= \frac{1 - \frac{K_m V_{dc}}{V_1} G_v(s + j\omega_1) T_{PLL}(s) (\frac{I_1}{2} H_i(s) e^{j\phi_{i1}} + \frac{C_1}{2} e^{j\phi_{c1}})}{(s + j\omega_1)L + K_m V_{dc} G_i(s + j\omega_1) H_i(s)} \\ Y_n(s) &= \frac{1 - \frac{K_m V_{dc}}{V_1} G_v(s - j\omega_1) T_{PLL}(s) (\frac{I_1}{2} H_i(s) e^{-j\phi_{i1}} + \frac{C_1}{2} e^{-j\phi_{c1}})}{(s - j\omega_1)L + K_m V_{dc} G_i(s - j\omega_1) H_i(s)} \\ J_p(s) &= \frac{\frac{K_m V_{dc}}{V_1} G_v(s - j\omega_1) G_v(-j\omega_1)^2 T_{PLL}(s) (\frac{I_1}{2} H_i(s) e^{-j\phi_{i1}} + \frac{C_1}{2} e^{-j\phi_{c1}})}{(s - j\omega_1)L + K_m V_{dc} G_i(s - j\omega_1) H_i(s)} \\ J_n(s) &= \frac{\frac{K_m V_{dc}}{V_1} G_v(s + j\omega_1) G_v(j\omega_1)^2 T_{PLL}(s) (\frac{I_1}{2} H_i(s) e^{j\phi_{i1}} + \frac{C_1}{2} e^{j\phi_{c1}})}{(s + j\omega_1)L + K_m V_{dc} G_i(s + j\omega_1) H_i(s)} \end{aligned} \quad (4)$$

(stationary) current in the specific operation point. Similarly,  $C_1 e^{j\phi_{c1}} = C_{d0} + jC_{q0}$  is the fundamental (stationary) duty cycles provided from the current controller. The angles  $\phi_{i1}$  and  $\phi_{c1}$  are referred to the fundamental phase A terminal voltage. The transfer function  $T_{PLL}$  in (1) is derived in [1] as:

$$T_{PLL}(s) = \frac{V_1 H_{PLL}(s)}{1 + V_1 H_{PLL}(s)} \quad (5)$$

where  $V_1$  is the amplitude of the fundamental (stationary) terminal voltage in the specific operation point.

Note that the voltage feed-forward ( $K_f(s)$  in [1]) is omitted since it is not included in [3] or [2], and the objective of the present paper is to compare these three impedance models, from the stability point of view.

### B. Impedance Model in the dq-domain

The following model can be used to describe the small-signal dynamics of a power electronic system in the dq-domain [17]:

$$\begin{bmatrix} V_d(s) \\ V_q(s) \end{bmatrix} = \mathbf{Z}_{dq}(s) \begin{bmatrix} I_d(s) \\ I_q(s) \end{bmatrix} = \begin{bmatrix} Z_{dd}(s) & Z_{dq}(s) \\ Z_{qd}(s) & Z_{qq}(s) \end{bmatrix} \begin{bmatrix} I_d(s) \\ I_q(s) \end{bmatrix} \quad (6)$$

The analytic impedance model in dq-domain for the grid-connected converter in Fig. 2 has been derived in e.g. [3] and [4]. The model in [3] is used as basis for comparison in the present paper, and is given in (3).

The variables in (3) are generally a function of the Laplace operator  $s$ , but  $s$  is omitted in order to have a more compact representation. The model is composed by the following

transfer functions and matrices:

$$\begin{aligned} G_{id} &= -V_{dc} \begin{bmatrix} sL & -\omega_1 L \\ \omega_1 L & sL \end{bmatrix}^{-1} & Z_{out} &= \begin{bmatrix} sL & -\omega_1 L \\ \omega_1 L & sL \end{bmatrix} \\ G_{PLL}^d &= \frac{T_{PLL}(s)}{V_1} \begin{bmatrix} 0 & -C_{q0} \\ 0 & C_{d0} \end{bmatrix} & G_{dei} &= \begin{bmatrix} 0 & -K_d \\ K_d & 0 \end{bmatrix} \\ G_{PLL}^i &= \frac{T_{PLL}(s)}{V_1} \begin{bmatrix} 0 & I_{q0} \\ 0 & -I_{d0} \end{bmatrix} & G_{ci} &= \begin{bmatrix} H_i(s) & 0 \\ 0 & H_i(s) \end{bmatrix} \end{aligned} \quad (7)$$

Note that the nomenclature in [3] has been slightly modified in order to adapt with the nomenclature in [1]. Specifically,  $G_{PLL}(s)$  in [3] is equal to  $\frac{T_{PLL}(s)}{V_1}$  in [1]. Also, the output filter resistance is neglected to be consistent with [1]. A small difference in the modeling between [1] and [3] lies in the representation of measurement filter and transducer delay. In [3] the delay transfer matrix  $G_{del}$  and the filter transfer matrix  $K$  in [3] are acting on the dq-domain current and voltages, while in [1] the corresponding transfer functions  $G_v(s)$  and  $G_i(s)$  are acting directly on the phase ( $abc$ ) signals. This will result in some differences in the resulting impedance model, see Appendix B for more details. In order to minimize the differences between the models, the following assumption is made:

$$G_{del}(s) \cdot K(s) = G_v(s)I = G_i(s)I \quad (8)$$

where  $I$  is the 2x2 identity matrix. Consequently, the delay and filter transfer functions for current and voltage in [1] are assumed equal:  $G_i = G_v$ .

### C. Impedance Model in the Modified sequence domain - a bridge between dq-and sequence domain

The term *modified sequence domain* was introduced in [21] as an extension to the well established harmonic linearization in the sequence domain [20]. The same domain has been

applied by others in [2],[22]-[24]. It is also remarked that the 2x2 modified impedance domain matrix has similarities to the method involving complex transfer matrices in [25], especially concerning the definition of  $dq$  unsymmetric systems. Other works relating the  $dq$  and sequence domains to each other can be found in [26] and [27].

The modified sequence domain extends the original sequence domain by letting the terminal equivalent also include the coupling between  $s + j\omega_1$  and  $s - j\omega_1$ . This coupling is caused by e.g. PLL, DC-link dynamics and control, power controllers and synchronous machine rotor saliency. The coupling is formally defined as  $dq$  unsymmetric systems in [25] and as mirror frequency coupling in [21]. See appendix C for more details. The impedance matrix  $\mathbf{Z}_{pn}$  is capturing the coupling as follows:

$$\begin{aligned} \begin{bmatrix} V_p(s + j\omega_1) \\ V_n(s - j\omega_1) \end{bmatrix} &= \mathbf{Z}_{pn}(s) \begin{bmatrix} I_p(s + j\omega_1) \\ I_n(s - j\omega_1) \end{bmatrix} \\ &= \begin{bmatrix} Z_{pp}(s) & Z_{pn}(s) \\ Z_{np}(s) & Z_{nn}(s) \end{bmatrix} \begin{bmatrix} I_p(s + j\omega_1) \\ I_n(s - j\omega_1) \end{bmatrix} \end{aligned} \quad (9)$$

where subscript  $p$  denotes positive sequence, and subscript  $n$  denotes negative sequence. It is seen from the definition that the off-diagonal elements  $Z_{pn}$  and  $Z_{np}$  are a direct measure of the above mentioned coupling. Note that the voltages and currents are referred to  $s + j\omega_1$  and  $s - j\omega_1$ , while the matrix itself is referred to  $s$ . This choice is made in order to obtain the linear transform derived in section III. It is remarked that the matrix  $\mathbf{Z}_{pn}(s)$  is referred to the  $dq$ -domain, since  $s = 0$  leads to positive sequence components at fundamental frequency. Consequently, a frequency shift is needed when relating the modified sequence domain matrix to the original sequence domain impedances defined in (2). This frequency shift is discussed in section IV.

In [2] the admittance matrix of the grid-connected converter was derived in the modified sequence domain. The model is repeated in (4).

Note that the frequency was referred to the positive sequence in the original publication [2], and that the substitution  $s \rightarrow s + j\omega_1$  is applied to all elements in (4) in order to be compatible with the definition used in the present paper (9). The notation is similar to the model in [1] with the following exceptions. The current controller decoupling term  $K_d$  is neglected in [2]. Also, the transfer function  $TF_{PLL}(s)$  in [2] is equal to  $V_1 T_{PLL}(s)$  in [1]. Finally, in order to be consistent with [1], the modulator gain  $K_m$  is assumed to be a pure gain, while all dynamics are assumed to be included in  $G_v(s)$  and  $G_i(s)$ .

### III. RELATION BETWEEN IMPEDANCE MODELS IN $dq$ - AND MODIFIED SEQUENCE DOMAIN

By combining Parks transform with the symmetric components transform the following relations can be derived [21]:

$$\begin{aligned} \begin{bmatrix} V_p(s + j\omega_1) \\ V_n(s - j\omega_1) \end{bmatrix} &= \frac{\sqrt{3}}{\sqrt{2}} \begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \begin{bmatrix} V_d(s) \\ V_q(s) \end{bmatrix} \\ \begin{bmatrix} I_p(s + j\omega_1) \\ I_n(s - j\omega_1) \end{bmatrix} &= \frac{\sqrt{3}}{\sqrt{2}} \begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \begin{bmatrix} I_d(s) \\ I_q(s) \end{bmatrix} \end{aligned} \quad (10)$$

The relation between  $dq$ -domain and modified sequence domain impedance matrices can then be found by combining (6), (9) and (10):

$$\begin{aligned} \begin{bmatrix} V_p(s + j\omega_1) \\ V_n(s - j\omega_1) \end{bmatrix} &= \mathbf{Z}_{pn}(s) \begin{bmatrix} I_p(s + j\omega_1) \\ I_n(s - j\omega_1) \end{bmatrix} \\ \frac{\sqrt{3}}{\sqrt{2}} \begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \begin{bmatrix} V_d(s) \\ V_q(s) \end{bmatrix} &= \mathbf{Z}_{pn}(s) \frac{\sqrt{3}}{\sqrt{2}} \begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \begin{bmatrix} I_d(s) \\ I_q(s) \end{bmatrix} \\ \begin{bmatrix} V_d(s) \\ V_q(s) \end{bmatrix} &= \begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix}^{-1} \mathbf{Z}_{pn}(s) \begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \begin{bmatrix} I_d(s) \\ I_q(s) \end{bmatrix} \end{aligned} \quad (11)$$

The last relation can be written with matrix notation as:

$$\begin{aligned} \mathbf{Z}_{dq} &= A_Z^{-1} \mathbf{Z}_{pn} A_Z \\ \mathbf{Z}_{pn} &= A_Z \mathbf{Z}_{dq} A_Z^{-1} \\ A_Z &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \end{aligned} \quad (12)$$

where the transformation matrix  $A_Z$  is unitary since its inverse is equal to its complex conjugate transpose. It is known from linear algebra that eigenvalues are invariant when multiplied with unitary matrices as in (12). Consequently, a stability analysis by e.g. the Generalized Nyquist Criterion will be identical in the  $dq$ -domain and the modified sequence domain. This is verified by simulations in Fig. 7.

The linear impedance transform in (12) has been applied to the analytical  $dq$ -domain model in (3), and the derivation is presented in appendix B. The result is then a modified sequence domain matrix. Note that the admittance is considered instead of the impedance in order to be compatible with [2]. This result is compared with the modified sequence domain analytical model (4), and the following equivalence was established:

$$A_Z \mathbf{Y}_{dq}^L(s) A_Z^{-1} = \mathbf{Y}_{pn}^L(s) \quad (13)$$

Note that a few minor differences in notation and assumptions were identified in order to reach this result, as discussed in appendix B. Hence, the model in [2] has been derived based on the  $dq$ -model from [3]. This completes the first step in the methodology from Fig. 1.

### IV. RELATION BETWEEN IMPEDANCE MODELS IN ORIGINAL AND MODIFIED SEQUENCE DOMAIN

This section explains the second step in the methodology in Fig. 1. The main assumption in this section is that the grid equivalent does not contain the coupling between  $s + j\omega_1$  and  $s - j\omega_1$ . By the definitions in [25] the grid is then assumed  $dq$  symmetric. In other words,  $Z_{pn}^S = Z_{np}^S = 0$ , where superscript  $S$  denotes the source subsystem (i.e. the grid equivalent). This is a widely applied assumption, and will simplify the expressions significantly. In [21] the more general expressions taking into account  $dq$  asymmetry in both subsystems were derived.

The impedance  $Z_p$  is obtained by assuming positive sequence shunt current injection  $I_{inj,p}$  in the interface point

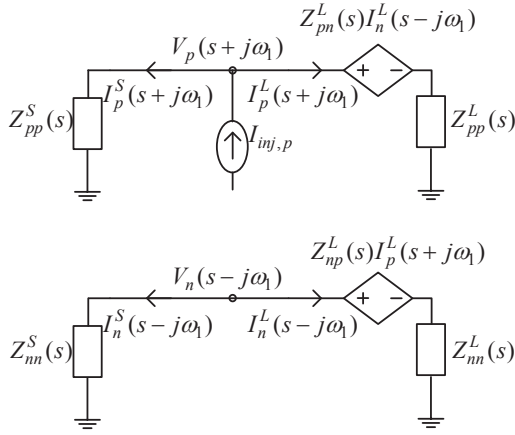


Fig. 4: Illustration of equations (14) needed to obtain the model reduction from modified to original sequence domain (16)

between source and load subsystem [21]. This gives the following set of equations:

$$\begin{aligned}
 V_p(s + j\omega_1) &= I_p^L(s + j\omega_1)Z_{pp}^L(s) + I_n^L(s - j\omega_1)Z_{pn}^L(s) \\
 V_n(s - j\omega_1) &= I_p^L(s + j\omega_1)Z_{np}^L(s) + I_n^L(s - j\omega_1)Z_{nn}^L(s) \\
 V_p(s + j\omega_1) &= I_p^S(s + j\omega_1)Z_{pp}^S(s) \\
 V_n(s - j\omega_1) &= I_n^S(s - j\omega_1)Z_{nn}^S(s) \\
 I_n^S(s - j\omega_1) &= -I_n^L(s - j\omega_1) \\
 Z_p^L(s + j\omega_1) &= \frac{V_p(s + j\omega_1)}{I_p^L(s + j\omega_1)} \quad (14)
 \end{aligned}$$

A circuit is presented in Fig. 4 to illustrate (14). A similar set of equations can be used to also obtain the negative sequence impedance  $Z_n^L(s - j\omega_1)$ . Solving these two sets of equations gives:

$$\begin{aligned}
 Z_p^L(s + j\omega_1) &= Z_{pp}^L(s) - \frac{Z_{pn}^L(s)Z_{np}^L(s)}{Z_{nn}^S(s) + Z_{nn}^L(s)} \\
 Z_n^L(s - j\omega_1) &= Z_{nn}^L(s) - \frac{Z_{np}^L(s)Z_{pn}^L(s)}{Z_{pp}^S(s) + Z_{pp}^L(s)} \quad (15)
 \end{aligned}$$

The next step is to shift the frequency axis in order to be compatible with (2). This is done by substituting  $s \rightarrow s - j\omega_1$  for  $Z_p^L$ , and  $s \rightarrow s + j\omega_1$  for  $Z_n^L$ . The resulting original sequence domain impedance is then:

$$\begin{aligned}
 Z_p^L(s) &= Z_{pp}^L(s - j\omega_1) - \frac{Z_{pn}^L(s - j\omega_1)Z_{np}^L(s - j\omega_1)}{Z_{nn}^S(s - j\omega_1) + Z_{nn}^L(s - j\omega_1)} \\
 Z_n^L(s) &= Z_{nn}^L(s + j\omega_1) - \frac{Z_{np}^L(s + j\omega_1)Z_{pn}^L(s + j\omega_1)}{Z_{pp}^S(s + j\omega_1) + Z_{pp}^L(s + j\omega_1)} \quad (16)
 \end{aligned}$$

Models obtained by these relations are referred to as *accurate models in the original sequence domain* [28]. It is proven in appendix D that this model has the same marginal stability condition as the *dq* and modified sequence domain models. An important and undesirable property in (16) is that parts of the source (grid) subsystem impedance appears in the expression for the load (converter) subsystem impedance. This appears counter-intuitive as one would expect the converter impedance model to only depend on the converter parameters and the system operation point. This apparent contradiction is a consequence of the effect referred to as *mirror frequency coupling* in [21] and *dq unsymmetric systems* in [25]. Note that the effect vanishes in case the off-diagonal elements in  $Z_{pn}^L$  are zero. A simplified and more applicable version of (16) can be derived if *one* of the two following assumptions are satisfied:

- The grid is relatively strong, i.e. the term  $|Z_{nn}^S(s)|$  is small compared with  $|Z_{nn}^L(s)|$ , and  $|Z_{pp}^S(s)|$  is small compared with  $|Z_{pp}^L(s)|$
- The converter is close to *dq* symmetric, i.e.  $Z_{pn}^L(s) \approx Z_{np}^L(s) \approx 0$

The simplified expressions are then:

$$\begin{aligned}
 Z_p^L(s) &\approx \frac{Z_{pp}^L(s - j\omega_1)Z_{nn}^L(s - j\omega_1) - Z_{pn}^L(s - j\omega_1)Z_{np}^L(s - j\omega_1)}{Z_{nn}^L(s - j\omega_1)} \\
 Z_n^L(s) &\approx \frac{Z_{pp}^L(s + j\omega_1)Z_{nn}^L(s + j\omega_1) - Z_{np}^L(s + j\omega_1)Z_{pn}^L(s + j\omega_1)}{Z_{pp}^L(s + j\omega_1)} \quad (17)
 \end{aligned}$$

By observing that both numerators in (17) are the determinant  $D_{pn}^L$  of  $Z_{pn}$ , the expressions can be written as:

$$\begin{aligned}
 Z_p^L(s) &\approx \frac{D_{pn}^L(s - j\omega_1)}{Z_{nn}^L(s - j\omega_1)} = \frac{1}{Y_{pp}^L(s - j\omega_1)} \\
 Z_n^L(s) &\approx \frac{D_{pn}^L(s + j\omega_1)}{Z_{pp}^L(s + j\omega_1)} = \frac{1}{Y_{nn}^L(s + j\omega_1)} \quad (18)
 \end{aligned}$$

where  $Y_{pp}^L$  and  $Y_{nn}^L$  are the diagonal elements in the matrix  $\mathbf{Y}_{pn}^L = (\mathbf{Z}_{pn}^L)^{-1}$ . Equation (18) completes the methodology in Fig. 1 as the original sequence impedances are obtained from the modified sequence domain matrix. If these equations are used, the model is referred to as a *reduced model in the original sequence domain* [28].

The simplified relation (18) can be verified for the three-phase VSC analytical models by comparing the diagonal elements in (21) with the expressions derived in [1] given in (1). By visual inspection it is concluded that these two expressions complies with (18). The only exception is the argument of the filter transfer functions  $G_v(s)$  and  $G_i(s)$ , this is due to the different modeling discussed in section II-B and appendix B. It is therefore concluded that the sequence domain impedance model derived in [1] has been obtained based on the *dq*-model from [3].

## V. IMPEDANCE MODELS COMPARISON

### A. Comparison of matrix impedance models

In addition to the *dq*- and modified sequence domain impedance models extensively discussed in this paper, a few

other related models are proposed in recent research:

- Phasor-based 2x2 impedance matrix [29]
- Modified sequence domain based transfer matrix including also DC-terminals [23]
- Modified sequence domain impedance matrix referred to phase domain frequency [30]

In [29] an impedance model based on phasors is derived, and the equivalence with the  $dq$ - and modified sequence domain matrix is proven. The previous work by the same authors [23] extends the modified sequence domain definition to also include DC-terminals. This is useful when analyzing systems where an AC/DC converter is the interface between AC- and DC- power systems.

In [30] a sequence domain impedance matrix is derived that is slightly different than the modified sequence domain matrix. Actually, the modified sequence domain matrix is an intermediate step in their derivation, as can be seen by equation (31) in [30]. The final step is to frequency shift the elements in the matrix, e.g. equations (32)-(34) in [30]. Consequently, the matrix elements are all referred to the phase domain frequency and not to the  $dq$ -domain frequency.

When comparing all 2x2 matrix based impedance models, there is no obvious practical benefit with one method or the other as:

- The analytical models are very similar, and equally challenging to derive. The exception is the  $dq$ -domain model, see the discussion below.
- The procedure for establishing the matrices from frequency sweeps is close to identical
- Methods for stability analysis and the associated interpretation are equal

A small disadvantage with the  $dq$ -domain impedance model compared with the other models is the fact that the magnitude of its off-diagonal elements is normally larger than the other models. This makes interpretation and stability analysis more challenging, as neglecting the off-diagonal elements for simplicity may not be possible.

However, a small advantage with the  $dq$ -domain impedance model is that the analytical models are normally easier to derive than the other domains. In the authors' opinion, a good approach is to derive models in the  $dq$ -domain, and then transform them into the modified sequence domain when performing the stability analysis.

### B. Comparing SISO vs. MIMO impedance models

Interpretation of the different elements in the modified sequence domain impedance matrix is more challenging than the original sequence impedance due to the off-diagonal elements. The off-diagonal elements can be viewed as a measure of the mirror frequency coupling in the system. A large magnitude is equivalent to a strong coupling. When the magnitude is small, modified and original sequence domain impedances are close to identical. Contributors to off-diagonal elements are e.g. PLL, DC-link dynamics and synchronous machine saliency.

In most cases, a rough indication of the stability can be obtained by considering the diagonal elements only. By

applying the reduced model (18), it is clear that the diagonal elements in the modified sequence domain admittance matrix equals the original sequence domain admittance. Consequently, considering the diagonal elements only is a valid approach for simplified analysis. For example, a negative real part in the diagonal elements indicates potential for stability issues.

A challenge with the 2x2 impedance matrices in the  $dq$ - and sequence domains is that they are referred to a certain reference frame, typically the local terminal voltage. As different matrices are referred to different reference frame in the system, basic circuit operations such as series connection is not directly applicable. They will require a coordinate transformation in order to align them to the same global reference frame to be able to apply the rules for series or parallel combination of impedances. This is elaborated in [31].

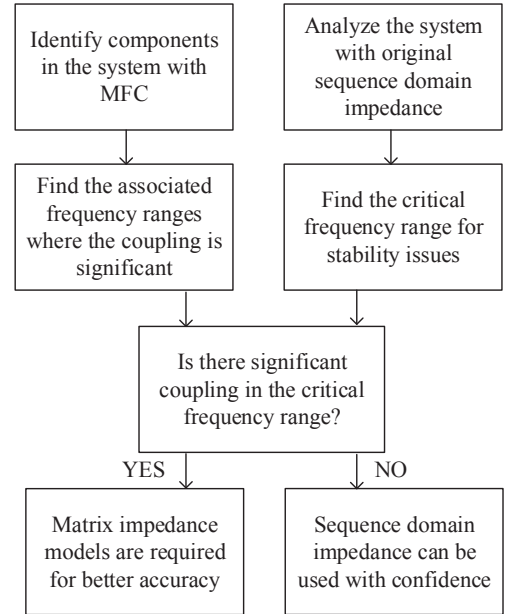


Fig. 5: Proposed method for evaluating when original sequence domain impedances give same stability analysis results as matrix impedance models

### C. Methods to evaluate accuracy of SISO impedance models

Several methods are derived in previous works for evaluating the error in neglecting off-diagonal elements of 2x2 impedance matrices:

- $AC_{index}$  in [19] based on the  $dq$ -domain impedance matrix
- Decoupling norm  $\epsilon$  in [32]
- Diagonal dominance based criteria in [23]

All these norms are based on the minor-loop gain matrix  $\mathbf{L} = \mathbf{Z}_S \mathbf{Y}_L$ . This is required for an accurate estimate of the error, due to the coupling between source and load impedance

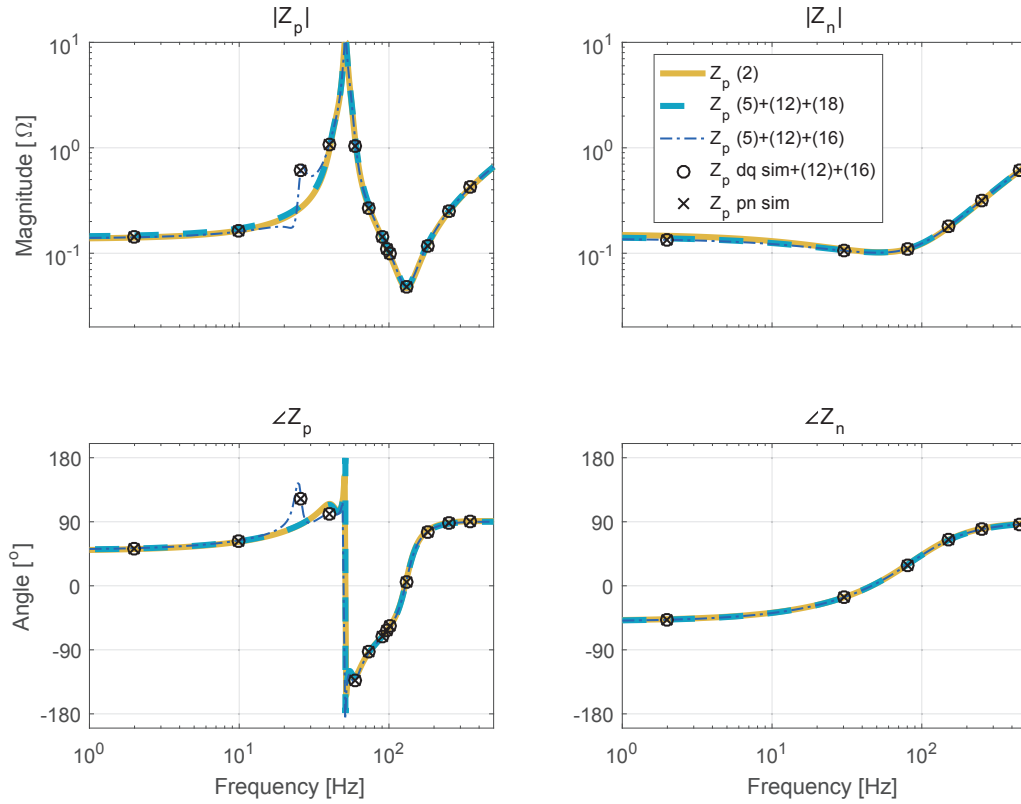


Fig. 6: Verification of impedance relations. Lines are analytical expressions, while circles and crosses are obtained by frequency sweep simulation. Relevant equation numbers are indicated in the figure legend.

given by (16). Relating the above three methods to the modified sequence domain yields:

$$AC_{index,pn} = \left| \frac{(L_{pp} - L_{nn})^2}{4L_{np}L_{pn}} \right| > 1$$

$$\epsilon_{pn} = \frac{1}{2} \left( L_{pp} - L_{nn} - \sqrt{(L_{pp} - L_{nn})^2 + 4L_{np}L_{pn}} \right) \quad (19)$$

The diagonal dominance based criteria is satisfied if the matrix is either row or column dominant, i.e.:

$$\text{EITHER } |L_{pp}| - |L_{pn}| > 0 \quad , \quad |L_{nn}| - |L_{np}| > 0$$

$$\text{OR } |L_{pp}| - |L_{np}| > 0 \quad , \quad |L_{nn}| - |L_{pn}| > 0 \quad (20)$$

The method from [19] has been redefined in the modified sequence domain in (19). All variables in (19) and (20) are a function of  $s$ .  $AC_{index,pn}$  and  $\epsilon_{pn}$  have similar structure, but the advantage of  $\epsilon_{pn}$  is that it directly expresses the resulting error in the Nyquist plot as a function of frequency (both magnitude and angle). The advantage with the diagonal dominance method is that the expressions are simpler. The method is also more conservative. A drawback with this method is that no phase angles are taken into account, so it is considered less accurate.

The three methods are equally challenging to apply, since the minor loop gain matrix  $L_{pn}$  needs to be computed. Therefore, none of the methods apply if the system is modeled by original sequence domain impedance only. In addition, both source and load subsystems are evaluated together, hence is it not possible to evaluate the error by considering one subsystem independently.

A method which is simpler to apply, and which does not require evaluation by matrix impedance models, is proposed in Fig. 5. The frequency ranges associated with Mirror Frequency Coupling (MFC) can be identified based on the system data. The frequency ranges associated with stability issues is estimated based on the original sequence domain analysis. If there is no MFC in the critical frequency range for stability issues, one can conclude that the reduced models in the original sequence domain are accurately predicting the stability and stability margin.

#### D. Equivalence validation

A comparison of original sequence domain impedance models is presented in this section. The system in Fig. 2 is analyzed with the parameter values given in Appendix A. Average converter model is applied in the frequency sweep simulations as this will give best match between the sweep

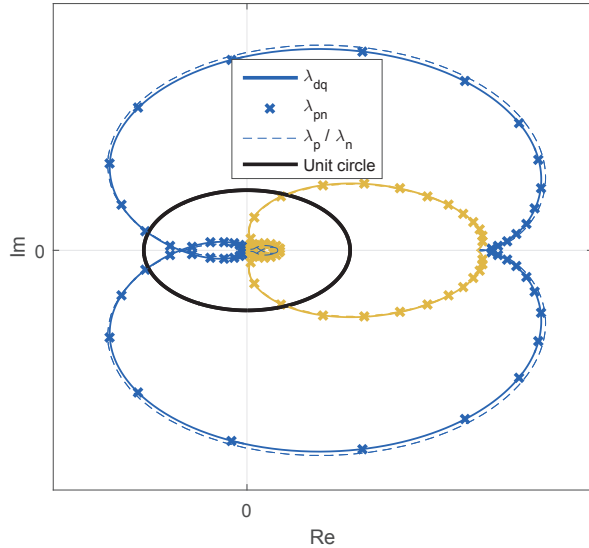


Fig. 7: Nyquist plot in  $dq$ -domain, modified sequence domain and original sequence domain. The two lines per domain represent the two eigenvalue traces

and corresponding analytic equations. It is remarked that the grid impedance  $Z_{th}$  is relatively large, with short circuit ratio equal to 3.

The following five approaches are compared in Fig. 6:

- The original sequence domain analytical model (1)
- The  $dq$ -domain analytical model (3) transformed by (12) and (16), i.e. the accurate model in the original sequence domain.
- The  $dq$ -domain analytical model (3) transformed by (12) and (18), i.e. the reduced model in the original sequence domain.
- Frequency sweep simulation used to obtain the  $dq$ -impedance, then transformed by (12) and (16)
- Frequency sweep simulation used to directly obtain the sequence domain impedance

The  $dq$  impedance sweep is obtained by the methodology explained in [33]. The results of this validation case study indicate that all five approaches arrive at very similar impedance plots, in both magnitude and angle. The only notable difference is seen in the positive sequence impedance between 20 and 40 Hz. The model in (1) gives identical result as the reduced model (18), and this is expected since their analytic expressions are the same. The accurate model (16) gives identical results as the model obtained by the two frequency sweeps. This is also expected since the frequency sweeps takes into account the coupling between  $s + j\omega_1$  and  $s - j\omega_1$  in the same way as (16). However, the small discrepancy by applying (18) instead of (16) is close to negligible. It has been verified that all plots are identical if the grid impedance  $Z_{th}$  is zero, but this plot is omitted due to space constraints.

## VI. IMPACT OF ACCURACY OF IMPEDANCE MODELLING FOR STABILITY ANALYSIS

Impedance models are often used to perform stability analysis through the Nyquist Criterion (NC) or Generalized Nyquist Criterion (GNC) [34]. How to obtain the Nyquist curves in the three domains is explained in [21]. Although the Nyquist plots were compared between the three domains in a previous work [21], it is repeated here in light of the analytical expressions derived. It was proven in [21] that the  $dq$ -domain and modified sequence domain always give the exact same Nyquist plots. This is a property of the transform in (12). However, applying the original sequence domain gives slightly different results.

The Nyquist plot comparison is presented in Fig. 7. Each domain has two eigenvalue curves. It is clear that the  $dq$ - and modified sequence domain matrices give the exact same result, and a very small difference is seen when the original sequence impedance is applied (1). Even if the original sequence domain model correctly accounts for the frequency coupling *inside* each subsystem, it does not fully capture the coupling *between* the subsystems. This will impact the stability analysis, but in this specific case analysis the difference is negligible. Based on initial empirical experience it seems that the original sequence domain can be used in stability analysis with confidence, but additional research is needed to strengthen this statement. Furthermore, in the opinion of the authors, the convenience of having two independent single-input-single-output models (2) in contrast to a 2x2 matrix (6) can generally justify a small discrepancy in the stability analysis.

## VII. CONCLUSIONS

This paper has demonstrated how impedance models in different domains can be derived from each other. More specifically, the  $dq$ -model and modified sequence domain model can be related by a linear transform, while the original sequence domain model can be obtained from the modified sequence domain matrix. The presented relations are generally applicable for all converters and systems, while the three-phase VSC has been used as an example in this paper. The impedance models from [1], [2] and [3] have been related to each other by the proposed method.

The  $dq$ - and modified sequence domain matrix are equivalent in terms of stability and eigenvalues. It is also proven that the original sequence domain has the same marginal stability condition as the matrix models if the *accurate* models are used (16). However, when using the reduced models (18), some information may be lost. More specifically, it was shown that the reduced models are not fully capturing the system-level coupling between  $s + j\omega_1$  and  $s - j\omega_1$ . Such coupling is caused by e.g. PLL, DC-link voltage controller and synchronous machine saliency. While these aspects can impact the stability analysis result, the deviation is expected to be small in most cases.

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$$\begin{aligned}
\mathbf{Y}_{pn,transf}(s) &= A_Z \mathbf{Y}_{dq}^L(s) A_Z^{-1} = \begin{bmatrix} Y_{pp,transf}(s) & Y_{pn,transf}(s) \\ Y_{np,transf}(s) & Y_{nn,transf}(s) \end{bmatrix} \\
&= A_Z (I + G_{id} G_{del} [-G_{ci} + G_{dei}] K)^{-1} \cdot (Z_{out}^{-1} + G_{id} G_{del} ([-G_{ci} + G_{dei}] G_{PLL}^i + G_{PLL}^d) K) A_Z^{-1} \\
&= A_Z (Z_{out} - V_{dc} G_v [-G_{ci} + G_{dei}])^{-1} \cdot (I - V_{dc} G_v ([-G_{ci} + G_{dei}] G_{PLL}^i + G_{PLL}^d)) A_Z^{-1} \\
&= (A_Z Z_{out} A_Z^{-1} - V_{dc} G_v A_Z [-G_{ci} + G_{dei}] A_Z^{-1})^{-1} \cdot (I + V_{dc} G_v A_Z ([-G_{ci} + G_{dei}] A_Z^{-1} A_Z G_{PLL}^i A_Z^{-1} + A_Z G_{PLL}^d A_Z^{-1})) \\
&= \left( \begin{bmatrix} (s + j\omega_1)L & 0 \\ 0 & (s - j\omega_1)L \end{bmatrix} + V_{dc} G_v(s) \begin{bmatrix} H_i(s) - jK_d & 0 \\ 0 & H_i(s) + jK_d \end{bmatrix} \right)^{-1} \\
&\cdot \left( I - V_{dc} G_v(s) \frac{T_{PLL}(s)}{V_1} \left( \begin{bmatrix} -H_i(s) + jK_d & 0 \\ 0 & -H_i(s) - jK_d \end{bmatrix} \frac{I_1}{2} \begin{bmatrix} -e^{j\phi_{i1}} & e^{j\phi_{i1}} \\ e^{-j\phi_{i1}} & -e^{-j\phi_{i1}} \end{bmatrix} + \frac{C_1}{2} \begin{bmatrix} e^{j\phi_{c1}} & -e^{j\phi_{c1}} \\ -e^{-j\phi_{c1}} & e^{-j\phi_{c1}} \end{bmatrix} \right) \right) \\
Y_{pp,transf}(s) &= \frac{1 - \frac{V_{dc}}{2V_1} G_v(s) T_{PLL}(s) ([H_i(s) - jK_d] I_1 e^{j\phi_{i1}} + C_1 e^{j\phi_{c1}})}{(s + j\omega_1)L + V_{dc} G_v(s) [H_i(s) - jK_d]} \\
Y_{pn,transf}(s) &= \frac{\frac{V_{dc}}{2V_1} G_v(s) T_{PLL}(s) ([H_i(s) - jK_d] I_1 e^{j\phi_{i1}} + C_1 e^{j\phi_{c1}})}{(s + j\omega_1)L + V_{dc} G_v(s) [H_i(s) - jK_d]} \\
Y_{np,transf}(s) &= \frac{\frac{V_{dc}}{2V_1} G_v(s) T_{PLL}(s) ([H_i(s) + jK_d] I_1 e^{-j\phi_{i1}} + C_1 e^{-j\phi_{c1}})}{(s - j\omega_1)L + V_{dc} G_v(s) [H_i(s) + jK_d]} \\
Y_{nn,transf}(s) &= \frac{1 - \frac{V_{dc}}{2V_1} G_v(s) T_{PLL}(s) ([H_i(s) + jK_d] I_1 e^{-j\phi_{i1}} + C_1 e^{-j\phi_{c1}})}{(s - j\omega_1)L + V_{dc} G_v(s) [H_i(s) + jK_d]} \tag{21}
\end{aligned}$$

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## APPENDIX

### A. Parameter values used in simulations

$V_{th} = 690 \text{ V LL-RMS}$	$S_{base} = 1 \text{ MW}$
$f_n = 50 \text{ Hz}$	$L = 227 \mu\text{H}$
$L_{th} = 505 \mu\text{H}$	$R_{th} = 15.7 \text{ m}\Omega$
$k_p = 2.42 \cdot 10^{-4} \text{ p.u./A}$	$k_i = 0.0726 \text{ p.u.}/(\text{As})$
$k_{p,pll} = 0.1775 \text{ rad}/(\text{Vs})$	$k_{i,pll} = 8.88 \text{ rad}/(\text{Vs}^2)$
$V_{dc} = 1127 \text{ V}$	$G_i(s) = G_v(s) = \frac{1}{1+5 \cdot 10^{-4}s}$
$I_d^* = I_{d0} = 100 \text{ A}$	$I_q^* = I_{q0} = 100 \text{ A}$
$K_d = 1.27 \cdot 10^{-4} \text{ p.u./A}$	

TABLE I: Parameter values applied in the simulation case study

### B. Transforming the dq-model into modified sequence domain

By applying the impedance transform (12) to the *dq*-model (3), an analytical model in the modified sequence domain is derived in (21).

Before comparing (21) with (4), the following assumptions are made:

- $G_v(s) = G_i(s)$ , in order to adapt with [3]
- The PWM-delay is integrated into  $G_v$  similar to [1] and [3]. The parameter  $K_m$  is then a constant gain, and is set equal to 1 to be consistent with [3].
- The filter and delay transfer function  $G_v(s)$  is close to unity at fundamental frequency
- The current controller decoupling term  $K_d$  is set to zero in [2].

A final difference between (21) and (4) is the argument in the filter and delay transfer functions  $G_i(s)$  and  $G_v(s)$ . They are acting on *dq*-domain signals in [3], and on *abc*-signals in [2] and [1]. Consequently,  $G_v$  and  $G_i$  in [2] will not have the same response in the off-diagonal elements in  $\mathbf{Z}_{pn}$  in the two models. If this modeling choice is disregarded, the following equivalence can be established by visual comparison:

$$\mathbf{Y}_{pn,transf}(s) = \mathbf{Y}_{pn}^L \quad (22)$$

In other words, the model derived in [2] is obtained by transforming the model in [3] through (12).

### C. dq symmetric systems and mirror frequency decoupled systems

In [25] a system was defined as *dq symmetric* if the following condition is satisfied:

$$Z_{dd} = Z_{qq} \quad Z_{dq} = -Z_{qd} \quad (23)$$

It can be shown that (23) is equivalent to  $Z_{pn} = Z_{np} = 0$  by applying (12). This property was defined as *mirror frequency decoupled* system in [21], as there is no coupling between  $s + j\omega_1$  and  $s - j\omega_1$ . Hence, a *dq symmetric* system is the same as a *mirror frequency decoupled* system. It was shown in [21] that in a *dq symmetric* system the original sequence domain impedance model has the same Nyquist plot as the *dq*-domain impedance matrix.

#### D. Marginal stability equivalence proof

Consider the minor loop gain in the modified sequence domain when the source subsystem is MFD:

$$\mathbf{L}_{pn} = \mathbf{Z}_{pn}^S \mathbf{Y}_{pn}^L = \begin{bmatrix} Z_{pp}^S & 0 \\ 0 & Z_{nn}^S \end{bmatrix} \begin{bmatrix} Y_{pp}^L & Y_{pn}^L \\ Y_{np}^L & Y_{nn}^L \end{bmatrix} \quad (24)$$

where "(s)" is omitted from all variables to save space. The eigenvalues of  $\mathbf{L}_{pn}$  are given by the equation

$$\det(\lambda \mathbf{I} - \mathbf{L}_{pn}) = 0 \quad (25)$$

According to GNC, the marginal stability condition is given by  $\lambda = -1$ , as the Nyquist Curve then passes exactly through the critical point  $(-1, 0)$ . Inserting this into (25) gives the following expression after some calculations:

$$D_L + D_S + Z_{pp}^S Z_{nn}^L + Z_{nn}^S Z_{pp}^L = 0 \quad (26)$$

where  $D_L$  and  $D_S$  are the determinants of  $\mathbf{Z}_{pn}^L$  and  $\mathbf{Z}_{pn}^S$ , respectively.

Using the accurate model in the original sequence domain (16), we find the eigenvalues directly as the ratio of source and load impedance since both matrices are decoupled:

$$\lambda_p = \frac{Z_{pp}^S}{Z_{pp}^L - \frac{Z_{pn}^L Z_{np}^L}{Z_{nn}^S + Z_{nn}^L}} \quad (27)$$

$$\lambda_n = \frac{Z_{nn}^S}{Z_{nn}^L - \frac{Z_{np}^L Z_{pn}^L}{Z_{pp}^S + Z_{pp}^L}}$$

Setting  $\lambda_p$  and  $\lambda_n$  equal to  $-1$  and expanding gives:

$$\begin{aligned} -Z_{pp}^L Z_{nn}^S - Z_{pp}^L Z_{nn}^L + Z_{pn}^L Z_{np}^L &= Z_{pp}^S Z_{nn}^S + Z_{pp}^S Z_{nn}^L \\ -Z_{nn}^L Z_{pp}^S - Z_{nn}^L Z_{pp}^L + Z_{np}^L Z_{pn}^L &= Z_{nn}^S Z_{pp}^S + Z_{nn}^S Z_{pp}^L \end{aligned} \quad (28)$$

By comparing (26) with (28) we see that the expressions are identical. Hence, it is proven that the stability analysis in the modified sequence domain predicts marginal stability if and only if the analysis by accurate models in the original sequence domain predicts marginal stability.

In [28] a similar proof is derived for a more general system where the source subsystem has non-zero off-diagonal elements.



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