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Capacity Expansion in Power Markets - a Real Options Approach

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Problem statement

This master thesis adopts a real options approach to analyze marginal investments in competitive power markets with heterogeneous technologies and time-varying demand. The aim of the thesis is to provide regulators and policymakers with decision support when developing new policies.

Preface

This master thesis is written within the field of Financial Engineering at the Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology (NTNU).

We would like to acknowledge and thank our supervisors, Professor Stein-Erik Fleten at NTNU and Associate Professor Trine Krogh Boomsma at University of Copenhagen. Their inputs and encouragements have been vital for the completion of this thesis. We would also extend our gratitude to Postdoc. Christian Skar at NTNU for help gathering data and valuable discussions. Additionally, we would like to thank Associate Professor Ruud Egging at NTNU for sharing his expertise within equilibrium modeling. Lastly, we will thank our families for being supportive and understanding.

Trondheim, June 10th, 2017

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Abstract

This thesis explores generation capacity expansions in power markets using a real options approach. Our framework considers several features of actual power markets, including time-varying demand and generation technologies with different cost characteristics. We propose a capacity expansion model where the market clearing is determined by equilibrium modelling. Further, we compute the marginal value of additional generation capacity, solve a series of optimal stopping problems and simulate the operation and investment decisions of all market participants over a horizon of several decades.

The thesis consists of two papers applying the framework in different settings. In the first paper, "Electricity Capacity Expansion in a Cournot Duopoly", we examine a duopoly where base and peak load power plants are available to the investor and compare it to a market governed by a central planner. We find that both imperfect competition in the market and a time-varying demand boost peak load investments. In a numerical example, we notice that welfare losses from producers exercising market power exceed 10 %.

The second paper, "A real options approach to generation capacity expansion in imperfectly competitive power markets", is our main contribution to the thesis. In this article, we expand our framework to include renewables, rationing and firms with different levels of market power, and we apply it to the German power market. We find that the need for peak load power plants increases with the share of renewables. Without monetary incentives, gas-fired power plants are not profitable for merchant investors. When all types of gas-fired power plants receive capacity payments, investments in combined cycling power plants occur at the expense of investments in renewables. Hence, we argue that only peaking power plants should receive capacity payments. This will reduce rationing while making renewable investments profitable.

Sammendrag

Denne masteroppgaven utforsker kapasitetsekspansjoner i kraftmarkeder ved hjelp av realopsjoner. Rammeverket inkluderer flere aspekter ved faktiske kraftmarkeder, som tidsvarierende etterspørsel og produksjonsteknologier med ulike kostnadskarakteristikker. Vi foreslår en kapasitetsekspansjonsmodell hvor markedsløsningen bestemmes med likevektsmodellering. Videre beregner vi verdien av en ekstra enhet kapasitet og finner optimal kapasitetsekspansjon med dynamisk programmering før vi simulerer drifts- og investeringsbeslutninger for alle markedsdeltagere over flere tiår.

Oppgaven består av to artikler som bruker rammeverket vårt i ulike kontekster. I den første artikkelen studerer vi et duopol hvor kraftverk med grunnlastkapasitet og spisslastkapasitet er tilgjengelig for investerende selskaper, og vi sammenligner duopolet med et marked regulert av en sentralplanlegger. Vi kommer frem til at både ufullkommen konkurranse i markedet og tidsvarierende etterspørsel gir økte investeringer i spisslastkapasitet. I et numerisk eksempel finner vi et velferdstap i duopolet på over 10 % sammenlignet med det regulerte markedet.

Den andre artikkelen er hovedbidraget vårt til masteroppgaven. Der utvides rammeverket introdusert ovenfor til å også inneholde fornybare energikilder, rasjonering og selskaper med ulik grad av markedsmakt. Her observerer vi at behovet for spisslastkratverk øker ettersom andelen fornybare energikilder øker. Uten finansielle virkemidler er ikke gasskraftverk lønnsomme å investere i. Når alle typer gasskraftverk mottar kapasitetsutbetalinger, vil investeringer i grunnlastkraftverk gjøres på bekostning av fornybare investeringer. Derfor argumenterer vi for at kun spisslastkraftverk bør motta kapasitetsutbetalinger. Disse reduserer rasjonering samtidig som fornybare investeringer forblir lønnsomme.

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Introduction

The need to respond to climate changes has enforced increasing investments in renewables the last decades. Several sources of renewable energy, such as wind and solar power, depend heavily on the weather and are uncontrollable. To ensure the reliability of supply, a proportion of the total dispatch should be controllable. In this thesis, we examine power markets including both controllable and non-controllable generation technologies. We aim to provide policymakers with decision support on how to incentivize renewable investments while keeping a portion of the dispatch controllable.

The main research topic addressed in this master thesis is capacity investments in the power sector. We address this by adopting a real options approach to analyze marginal investments in a portfolio of power generating technologies when firms have different levels of market power. A series of sub-questions follows:

1. To what extent does market structure affect investments in additional capacity?

2. How does investments in renewables and conventional power plants develop over the next decades?

3. How should regulators and policymakers respond to a larger amount of investments in non-controllable generation technologies?

The aim of the thesis is to contribute to a better understanding of the investment behavior in power markets. We extend the work of [1] and employ a real options approach for marginal capacity investments in the power sector when demand is time-varying. We examine power markets where the electricity price is endogenous and differentiated both between each year and within the year. Further, the framework incorporates generation technologies with different costs. We also consider the development of social welfare and welfare distribution over time. Thus, the approach may be used by regulators to develop energy policies. The thesis consists of the two papers "Electricity Capacity Expansion in a Cournot Duopoly" [2] and "A real options approach to generation capacity expansion in imperfectly competitive power markets" [3].

The first paper [2] is the authors contribution to the EEM17 conference in Dresden. It is accepted, and its content has been presented at the conference. The article will also be published in the IEEE Xplore database. The paper compares the investment behavior of two firms in a Cournot duopoly to a central planner's when two categories of power plants are available; base and peak load power plants. We notice higher peak load investments in the duopoly than in the market governed by a central planner. Further, we examine the effect of analyzing power markets without time-varying demand within the year and find that this underestimates investments in peak load capacity. The foundation for this paper was laid in the writing of the project thesis, authored by Brøndbo and Storebø, and delivered as part of the course TIØ4550 Financial Engineering, Specialization Project at NTNU.

The second paper [3] is the main contribution to the thesis and will be submitted to a scientific journal. It proposes a real options approach on how to model capacity ex-

pansions in the power sector where firms possess different levels of market power. We assume that the power market consists of a few Cournot firms and a host and price-taking firms. Further, we incorporate non-controllable energy sources and rationing. We apply our framework to the German power market and find that investments in solar, wind and biomass will grow after 2030. A high share for non-controllable renewables leads to larger fluctuations in the power price. By installing peaking power plants, price spikes are limited. Moreover, we find that a regulator reduces rationing by introducing capacity payments. We argue that capacity payments to technologies with low investment and high marginal costs have benefits that balance the disadvantages of a high share of renewables.

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Electricity Capacity Expansion in a Cournot Duopoly

Helene K. Brøndbo, Axel Storebø, Stein-Erik Fleten, Trine Krogh Boomsma

Abstract—This paper adopts a real options approach to analyze marginal investments in power markets with heterogeneous technologies and time-varying demand. We compare the investment behavior of two firms in a Cournot duopoly to a central planner's when two categories of power plants are available; base and peak load power plants. We find that producers exercise market power and the prices increase. Furthermore, the peak load plants become relatively more valuable and the share of installed peak load capacity exceeds the peak load share in a perfectly competitive market. In a numerical example, we show that this results in welfare losses above 10 %, and significantly larger reduction in the consumer surplus. Further, we examine the effect of analyzing power markets without time-varying demand and find that this underestimates investments in peak load capacity.

I. Introduction

Expansion of capacity in power systems is on the agenda, both in developing countries, where demand is growing, and in industrialized countries, where concerns about climate change is a driving force. Following the deregulation of European power markets in the last decades, the authorities' focus on maximizing social welfare has been replaced by the companies' aim to maximize their profits. Several mergers and acquisitions have resulted in markets with few suppliers having significant market shares. Despite the clear need for a better understanding of capacity expansions in power markets with actors in possession of market power, there is a limited amount of academic research addressing this.

Investments in power equipment are capital intensive, and the equipment is difficult to sell once it is installed, particularly when considering the whole industry at the same time. Hence, we assume investments to be irreversible. Furthermore, capacity expansions are rarely now-or-never decisions. The investment can be delayed until the company has more information about the uncertain demand. These assumptions suit real options problems well. Treating capacity expansions in the power sector with a real options approach provides flexibility to the investor because it takes the value of waiting into account while the investment is considered irreversible.

We take as a starting point the set-up of [1]. This paper introduces a real options capacity expansion model for power generation under perfect competition. The combination of real options and a social welfare perspective is also found in [3] and [4]. The framework of [1] includes heterogeneous technologies, and power is treated as a differentiated product by dividing the year into load segments, where the power demand is different in each segment. On this basis, we develop a real options capacity expansion problem under a Cournot duopoly. This makes us able to compute social welfare losses

in settings with market power relative to a market governed by a central planner.

This paper aims to contribute to the literature by implementing the particularities described below. A number of articles consider capacity expansion by real options, e.g. [2], [13], [14] and [15]. However, these approaches consider only one technology or heterogeneous technologies. We study capacity expansions for electricity technologies that may differ in both operational and investment costs. For instance, peak load plants typically have higher operational costs but lower investment costs than base load plants.

We cast the capacity expansion problem as a canonical real options problem. Such models fit well into a stochastic setting while allowing for an endogenous electricity price when the level of capacity is held constant. Canonical real options models consider a sequence of marginal capacity expansions instead of a single. Furthermore, the value of the capacity expansion and the optimal expansion path are determined simultaneously. Canonical real options theory is mainly used in markets with monopoly and perfect competition due to assumptions about homogeneous companies and symmetric technologies. By assuming myopia, however, we apply it to a diverse portfolio of technologies.

Myopia implies that each investment in incremental capacity is the last one over the time horizon, and holds for electricity capacity expansions with one technology or several technologies with identical cost characteristics. Although myopia does not necessarily hold for our capacity expansion problem, we use this as an assumption to facilitate a solution. We argue that myopia is an acceptable approximation because of the way profit maximizing firms act. In deciding whether the next investment is attractive, this is assumed to be the last one. As time passes and the electricity demand increases, a new investment might be undertaken, despite the earlier belief that the previous investment was the last one.

Electricity is treated as a differentiated product both between years and within each year. Pindyck [12] argues that long-term development of electricity prices follows a geometric Brownian motion. This view is supported in [9], [10] and [11] among others. Hence, we model the long-term fluctuations in electricity demand as a geometric Brownian motion. Additionally, our real options approach for electricity capacity expansion considers the fluctuations in short-term demand by dividing each year into a set of load segments, where the electricity demand differs between each segment. Short-term fluctuations of the electricity price in the load segment is modeled by an inverse demand function. This way, the electricity price depend on the dispatch in the market and is thus endogenous as argued in e.g. [5] and [13].

According to [5], [6], [7] and [8], power markets may be considered Cournot oligopolies. Hence, we use a partial equi-

librium model to describe the market structure in the capacity expansion model. Here, the firms extract market power and their investment decisions depend on actions of the competitors as well as economic variables. At each point in time, the firms decide their production and investments simultaneously with a view to long-term market shares and profitability. For simplicity, we assume a power market consisting of two firms in this paper.

The paper is structured as follows. Section 2 introduces a framework for capacity expansion in duopolies. Section 3 presents a numerical example and studies capacity investments, surpluses and welfare losses in a Cournot duopoly compared to a perfectly competitive market. The section also looks at the effect of modeling the power market with and without time-varying demand. Section 4 concludes.

II. CAPACITY EXPANSION MODEL

A. Instantaneous Profit of a Duopolistic Firm

We model the electricity demand shock process Y_t as a geometric Brownian motion

$$dY_t = \mu Y_t dt + \sigma Y_t dz_t, \tag{1}$$

in which μ is the deterministic drift, $\sigma > 0$ is the standard deviation and dz_t is the increment of a Wiener process. Electricity cannot be stored, and is thus a differentiated product. As a result, the electricity price is time-varying. This is modelled by dividing each year into $d(\mathbf{L})$ load segments with different electricity demand. Power generation depends on the load, which is the energy demand per unit of time. We use linear inverse demand functions $D_l(Q_l)$ to find the electricity prices, in which Q_l is the total dispatch in each load segment $l \in \mathbf{L}$.

Hence, we let the electricity price depend on both the inverse demand function $D_l(Q_l)$ and the exogenous and multiplicative shock process Y_t

$$P_l = Y_t D_l(Q_l) = Y_t(A_l - b_l Q_l), \ \forall l \in \mathbf{L}. \tag{2}$$

We assume a Cournot duopoly consisting of two firms, Firm 1 and Firm 2. Firm 1 is in possession of power plants using technology 1, and Firm 2 is in possession power plants using technology 2. Due to Cournot assumptions, the investment approach is symmetric for both firms. Hence, we only show the investment approach of Firm 1. The generators available to Firm 1 have a capacity K_1. The produced electricity by firm 1 in load segment l is $q_{1,l}$. Hence, K_1 is the maximal value of $q_{1,l}$. Operational and maintenance costs of technology 1 are given in terms of the installed capacity K_1. Hence, OMC_1 is the operational and maintenance cost per unit of installed capacity of technology 1 for Firm 1. The unit production cost for each technology 1 is denoted c_1. The cost of investing in one additional capacity unit of technology 1 is denoted I_1. We assume that the cost occurs instantaneously after an investment decision and that the additional capacity is available immediately after the investment. The revenues in load segment l are given as the product of the price function in (2) and the amount of sold electricity by Firm 1 in each

load segment $l \in \mathbf{L}$. Thus, Firm 1 finds its instantaneous profit from the optimization problem

$$\pi_1(Y_t, K_1, K_2) = \max_{q_{1,l}} \sum_{l=1}^{d(\mathbf{L})} \tau_l \bigg[P_l(Y_t, Q_l) q_{1,l}$$
$$- c_1 q_{1,l} \bigg] - OMC_1 K_1 \tag{3}$$

s.t.

$$q_{1,l} \geq 0, \qquad\qquad \forall l \in \mathbf{L} \tag{4}$$
$$q_{1,l} \leq K_1, \qquad\qquad \forall l \in \mathbf{L} \tag{5}$$
$$Q_l = q_{1,l} + q_{2,l}, \qquad\qquad \forall l \in \mathbf{L} \tag{6}$$

where τ_l is the duration of load segment l. (4) and (5) constrain the electricity produced by firm 1 $q_{1,l}$ not to exceed its upper limit K_1 or fall below its lower limit 0. (6) states that in each load segment the total dispatch Q_l is the sum of the dispatches of Firm 1 and Firm 2. Due to a downward sloping inverse demand curve, (3) is concave. Combined with linear constraints makes the problem convex. Thus, the problem can easily be solved numerically and has a solution.

The inverse demand function $D_l(Q_l)$ proves the profits from each technology 1 and 2 to be non-additively separable. When the dispatch $q_{1,l}$ increases, the inverse demand function $D_l(Q_l)$ decreases. By holding Y_t fixed, a larger dispatch results in reduced electricity prices. Holding the inverse demand function $D_l(Q_l)$ fixed, an increase in Y_t results in a larger instantaneous profit $\pi_1(Y_t, K_1, K_2)$. Changes in K_1 and K_2 also effect the profit flow $\pi_1(Y_t, K_1, K_2)$ through setting an upper limit on the electricity generation.

We find the welfare losses in the duopoly by comparing the welfare in the duopoly with the welfare under perfect competition. Social welfare is the sum of the producer and the consumer surplus, $\psi(Y_t, K_1, K_2) = \pi(Y_t, K_1, K_2) + cs(Y_t, q_{1,l}, q_{2,l})$. The producer surplus equals the profit of the producers $\pi(Y_t, K_1, K_2) = \pi_1(Y_t, K_1, K_2) + \pi_2(Y_t, K_1, K_2)$. The consumer surplus cs is given by

$$cs(Y_t, K_1, K_2) = \sum_{l=1}^{d(\mathbf{L})} \tau_l \bigg\{ \int_0^{Q_l} P_l(Y_t, x_l) dx_l$$
$$- P_l(Y_t, Q_l)(Q_l) \bigg\}. \tag{7}$$

Thus, social welfare is given by

$$\psi(Y_t, K_1, K_2) = \sum_{l=1}^{d(\mathbf{L})} \tau_l \bigg\{ \int_0^{Q_l} P_l(Y_t, x_l) dx_l$$
$$- \sum_{k \in \mathbf{K}} c_k q_k \bigg\} - \sum_{k \in \mathbf{K}} OMC_k K_k. \tag{8}$$

The social welfare is maximized when solving (3)-(6) using (8) as the objective function.

B. Value of Capacity Expansion

Investments are assumed irreversible and incremental over an infinite time horizon. Future cash flows are discounted with the exogenous annual rate ρ. In year zero, the demand shock is Y_0 and the installed capacity of Firm 1 is $K_{1,0}$. For every demand shock in each time interval Y_t, Firm 1 expands its capacity to $K_{1,t}$ at the per unit investment cost I_1 that maximizes its expected value. This implies that at each point in time, Firm 1 adapts its capacity to the demand. $F_1(Y, K_1, K_2)$ represents the value of all optimal capacity expansions of Firm 1. When Firm 1 has no other assets except from its generation capacity, $F_1(Y, K_1, K_2)$ is equivalent to the value of Firm 1. $K_{1,t}$ is the installed capacity of technology 1 at time t so that $K_{1,t} \leq K_{1,t+dt}$. Thus, the value of capacity expansion is

$$F_1(Y_t, K_1, K_2) = \max_{K_{1,t}} \mathbf{E}\left[\int_0^\infty \pi_1(Y_t, K_{1,t}, K_{2,t})e^{-\rho t}dt - \int_0^\infty I_1 e^{-\rho t}dK_{1,t}\right]. \quad (9)$$

Firm 1 invests in new capacity to maximize its expected value over an infinite time horizon. The first term on the right-hand side of the equality represents all future expected discounted profits of Firm 1. The second term on the right-hand side is the total expected discounted investment costs from capacity investments. Hence, we integrate over every point in time t to find the value of installed capacities $F_1(Y, K_1, K_2)$.

C. Optimal Stopping Problem

If the properties of myopia hold, the stochastic control problem can be converted to an optimal stopping problem. We propose a regression $\bar{\pi}_1$ in (10) to express the instantaneous profit analytically as a function of Y_t, K_1 and K_2. This particular expression is chosen for the real options problem to have an analytical solution.

$$\bar{\pi}_1(Y, K_1, K_2) = \sum_{i,j=1}^{d(\gamma),d(\alpha)} b_{1,ij}Y^{\gamma_i}K_1^{\alpha_j} + \sum_{i,j,l=1}^{d(\gamma),d(\lambda),d(\lambda)} c_{12,ijl}(\gamma_i)Y^{\gamma_i}K_1^{\lambda_j}K_2^{\lambda_l} - OMC_1K_1. \quad (10)$$

The first term of the regression shows the profit flow from technology 1. The regression coefficients $b_{1,ij}$ describe how changes in the capacity of technology 1 effect the instantaneous profit flow for a given shock process Y_t. Since new installed capacity has a positive effect on the profits, $b_{k,ij} \geq 0$. Both synergies between technologies and the impact of the other firm's capacity are captured in the regression coefficients $c_{12,ijl}$. The coefficients are positive if the technology synergies outweigh the lower price caused by the other players installed capacity, and negative otherwise. Negative coefficients may cause several roots of (18). γ, α and λ are positive base vectors of dimensions $d(\gamma)$, $d(\alpha)$ and $d(\lambda)$ used to describe changes in $\bar{\pi}(Y, K_1, K_2)$ with respect to Y, K_1 and K_2. We constrain the base vectors in the regression like [1]. We set γ_i by $0 < \gamma_i < \beta_1$ $\forall i$, where β_1 represents the positive solution of the fundamental quadratic equation $\beta_1 = (\frac{1}{2} - \frac{\mu}{\sigma^2}) + \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2\rho}{\sigma^2}}$. This is done to increase the likelihood of getting a unique investment trigger. To ensure concavity and non-increasing return to scale, we establish $0 < \alpha_j < 1$, $\forall j$, $0 < \lambda_j < 1$ $\forall j$ and $\lambda_i + \lambda_j \leq 1$ when $i \neq j$.

We convert the stochastic control problem to an optimal stopping problem and introduce the convenience yield $\delta = \rho - \mu$ to simplify. The convenience yield of electricity is interpreted as the relative benefit of delivering the commodity earlier rather than later, according to [16]. Then the Bellman equation of a marginal capacity of Firm 1 is stated

$$\frac{1}{2}\sigma^2 Y^2 \frac{\partial^3 F_1(Y, K_1, K_2)}{\partial K_1 \partial Y^2} + (\rho - \delta)Y_1 \frac{\partial^2 F_1(Y, K_1, K_2)}{\partial K_1 \partial Y_1} - \rho\frac{\partial F_1(Y, K_1, K_2)}{\partial K_1} + \frac{\partial \bar{\pi}_1(Y, K_1, K_2)}{\partial K_1} = 0, \quad (11)$$

with boundary conditions

$$\frac{\partial F_1(0, K_1, K_2)}{\partial K_1} = 0, \quad (12)$$

$$\frac{\partial F_1(Y_1^*, K_1, K_2)}{\partial K_1} = I_1, \quad (13)$$

$$\frac{\partial^2 F_1(Y_1^*, K_1, K_2)}{\partial K_1 \partial Y_1} = 0. \quad (14)$$

(12) ensures that the value of Firm 1's value of the option to invest in new capacity is zero when the demand shock equals zero. (13) and (14) are respectively the value matching and the smooth pasting conditions for an incremental investment in new capacity. The solution to 11 is given by

$$F_1(Y, K_1, K_2) = A_1 Y^{\beta_1} + \sum_{i,j=1}^{d(\gamma),d(\alpha)} \bar{b}_{1,ij}Y^{\gamma_i}K_1^{\alpha_j} + \sum_{i,j,l=1}^{d(\gamma),d(\lambda),d(\lambda)} \bar{c}_{12,ijl}(\gamma_i)Y^{\gamma_i}K_1^{\lambda_j}K_2^{\lambda_l} - \frac{OMC_1K_1}{\rho}. \quad (15)$$

where β_1 is given by the positive root of the quadratic equation and $\bar{b}_{1,ij}$ and $\bar{c}_{12,ijl}$ are given by

$$\bar{b}_{1,ij}(\gamma) = \frac{b_{1,ij}}{\rho - \mu\gamma_i - \frac{1}{2}\sigma^2 + \gamma_i(\gamma_i - 1)}, \quad (16)$$

$$\bar{c}_{12,ijl}(\gamma) = \frac{c_{12,ijl}}{\rho - \mu\gamma_i - \frac{1}{2}\sigma^2 + \gamma_i(\gamma_i - 1)}. \quad (17)$$

By using (12)-(14), the myopic investment trigger for Firm 1 is therefore the solution of (18) with respect to Y_1^*

$$\sum_{i=1}^{d(\gamma)} Y_1^{*\gamma_i} \left(\frac{\beta_1 - \gamma i}{\beta_1}\right)\left\{\sum_{j=1}^{d(\alpha)} \alpha_j \bar{b}_{1,ij}(\gamma_i)K_1^{\alpha_j-1} + \sum_{u=1,u\neq k}^{d(K)} \sum_{j,k=1}^{d(\lambda),d(\lambda)} \bar{c}_{12,ijl}(\gamma_i)K_2^{\lambda_j}K_1^{\lambda_l-1}\right\} = I_1 + \frac{OMC_1}{\rho}. \quad (18)$$

It is optimal to invest when $Y_t > Y_1^*$, and Firm 1 thus invests until Y_1^* reaches Y_t at each point in time. The identical procedure is completed for Firm 2. Firm 2 finds its trigger Y_2^* and invests until Y_2^* reaches Y_t in every time step. We emphasize that represents a simultaneous Cournot duopoly capacity expansion game where two firms are investing in new capacity in order to maximize their value over an infinite time horizon.

III. RESULTS

A. Base Case

We demonstrate our approach by presenting an illustrative example. We examine a Cournot duopoly where one firm is in possession of base load power plants and the other firm is in possession of peak load power plants. The demand is split into six load segments such that $d(\mathbf{L}) = 6$. All parameters are defined in Appendix A, table II to IV. We compare the duopoly to a market governed by a central planner in possession of both base and peak load power plants. Thus, we investigate the welfare effect of imperfect competition.

TABLE I.   SOCIAL WELFARE

	Perfect competition	Duopoly
Discounted social welfare [M€]	282 450	193 410
Discounted producer surplus [M€]	121 940	128 130
Discounted consumer surplus [M€]	160 500	65 587
Value of the firm, F [M€]	-	F_{base} = 99 860 F_{peak} = 21 958
Discounted social welfare net of investment costs [M€]	209 220	184 930
Percentage loss inn discounted social welfare subtracted for investment costs	-	11.6 %

Table I presents the discounted total surplus, the discounted producer surplus and the discounted consumer surplus. As expected, the discounted social welfare and consumer surplus of the central planner exceeds that of the Cournot duopoly. This is due to central planner's aim to maximize the social welfare. Firms of the duopoly consider only the producer surplus when deciding to invest and consequently have significantly larger producer surpluses than the central planner. The firms hold back capacity to increase prices. Hence, the social planner invests in more capacity.

The central planner has flexibility to invest in both base and peak load capacity as well as to choose the amount of electricity generated by each technology. The duopolistic firms, on the other hand, have to invest in and generate electricity by
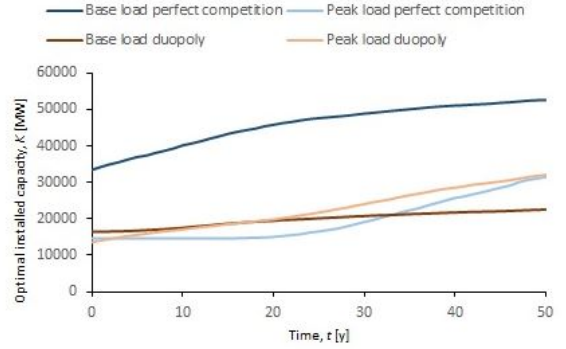

Fig. 1.   Optimal investments in base and peak load capacity

the one technology that is available to them. Additionally, both of the duopolistic firms aim to maximize their own profit. As illustrated in Figure 1, the duopolistic firm possessing peak load capacity has a higher rate of investment than the firm possessing base load capacity. In spite of this, the value of the firm in the position of base load capacity exceeds the value of the firm in the position of peak load capacity. This is a result of the lower operational costs provided by the base load capacity which leads to a contribution margin that outweighs the lower investment cost provided by the peak load capacity.

When subtracting the discounted investment costs from the discounted social surplus, the difference between the discounted social welfare in perfect competition and a duopoly is reduced. This indicates high investment costs to be a major investment barrier for firms operating under market power. The last line in Table I presents the percentage losses in total discounted surplus adjusted for investment costs. The welfare loss is 11.6 % in the duopoly. Although significant, the social losses are modest compared to the losses for the consumers of 59.1 %. This demonstrates that consumers are the ones who suffer from producers exercising market power.

B. Capacity Expansions and Time-Varying Demand

In actual power markets, the demand varies over the year. Traditional real options models do not capture this. To quantify the effect of a modeling different load segments, we compare peak load investments when the electricity demand is time-varying and when it is fixed throughout the year. We use weighted averages of the variables in Table IV to compute new values of A and b for the new load segments in the inverse demand function, presented in Table V.

Figure 2 illustrates the peak load investments with 1, 2 and 6 load segments. We observe that peak load investments increase with the number of load segments. An economical interpretation is as follows. In periods with high demand, the high electricity price results in a high contribution margin on peak load generation. Due to the minor investment costs on peak load capacity, it is sufficient with short periods of high demand for peak load to be profitable. This effect is not captured when demand is assumed constant throughout the year, i.e. when $d(\mathbf{L}) = 1$. When each year contains
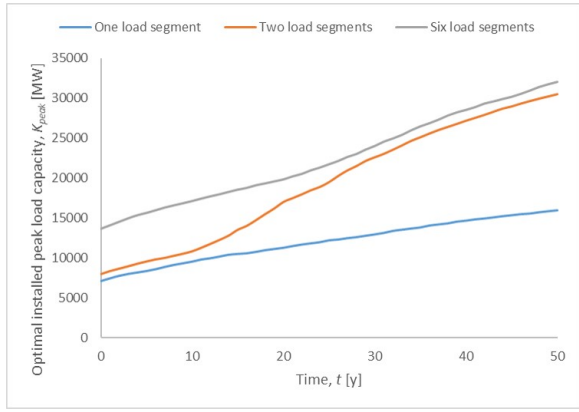
Fig. 2. Investments in peak load capacity with one, two and six load segments

2 load segments, the effect of the peak load contribution margin is captured to some extent. We witness additional peak load investments compared to when it is 1 load segment, but the peak load investments are not as high as with 6 load segments and take some time to catch up. Notice that the firms always invest in additional peak load capacity due to a positive contribution margin.

IV. CONCLUSION

We have adopted a real options approach to analyze marginal investments in peak and base load generation capacity. We study capacity expansion within a Cournot duopoly and a market governed by a central planner, and we compare optimal capacity installations for base and peak load power plants. Our approach considers several features of the real world power markets, including heterogeneous technologies, endogenous electricity prices, time-varying electricity demand, and markets with imperfect competition. We find that with imperfect competition the installed capacity increases with the number of firms in the market. In particular, imperfect competition may boost peak load investments at the expense of a loss in social welfare, explained mainly by a substantial loss in consumer surplus. We also observe that fluctuations in the electricity demand over the year enhance peak load investments.

Our capacity expansion framework may provide decision support to both policymakers and private investors. It is important for policymakers to ensure a certain capacity and flexibility to cover the electricity demand and a certain number of firms in order to avoid market power. We show how increased competition leads to a higher installed capacity and lower electricity prices, which result in smaller welfare losses.

V. ACKNOWLEDGEMENTS

APPENDIX A
INPUT DATA FOR EXAMPLE

TABLE II.   POWER PLANT PROPERTIES

Technology index, k	$Base$	$Peak$
Marginal cost, c [€/MWh]	5	65
O. & M. cost, OMC [€/MWy]	100 000	20 000
Investment cost, I [€/MW]	3 000 000	80 000
Initial capacities $K_{t=0}$ [MW]	15 000	5 000

TABLE III.   INPUT PARAMETERS

Y_0	μ	σ	ρ	β_1	N	$d(\Omega)$	T	Δt	$\Delta\kappa$
1	0.02	0.03	0.1	4.62	50	50	$50y$	$1y$	500 MW

TABLE IV.   DEMAND DATA

Load Segment l	1	2	3	4	5	6
Duration, τ [h]	10	40	310	4400	3000	1000
Max. demand, A_l	900	180	165	120	90	60
Slope, b_l	0.007	0.0014	0.0014	0.0014	0.0015	0.0020

TABLE V.   DEMAND DATA, ONE AND TWO LOAD SEGMENTS

	One Load segment	Two load segments	
Load Segment l	1	1	2
Duration, τ [h]	8760	360	8400
Max. demand, A_l	105.63	187.08	1102.14
Slope, b_l	0.00151	0.00156	0.00151

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A real options approach to generation capacity expansion in imperfectly competitive power markets

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Abstract

This paper proposes a real options approach to generation capacity expansion in imperfectly competitive power markets. Our framework incorporates time-varying demand, firms with different levels of market power and heterogeneous technologies, including renewables, base load and peak load. We apply our model to the German power market and show that investments in peak load capacity supplement a high renewable penetration. In particular, the availability of peak load generation serves to avoid rationing and stabilizes the fluctuations in the electricity price. In the absence of incentive mechanisms for peak load investments, it is not profitable to invest in this technology. On the one hand, capacity payments boost peak load investments. On the other hand, however, excessive capacity payments crowd out renewable investments. We find that capacity payments in the order of 7.2 % of the investment cost to peaking power plants prevent rationing without reducing investments in renewables.

Keywords: Capacity expansion, Competitive power markets, Germany, Real options

1. Introduction

Real options models have been applied to power markets since the eighties, see e.g. [1], [2] and [3]. Their flexibility compared advantageously with the capacity expansion approaches of the time. The need to respond to climate changes has enforced increasing investments in renewable energy sources during the last decades. Combined with structural changes of the power sector, the increasing renewable penetration has altered the functioning of power markets. Hence, we revisit the former real options capacity expansions models to explore their potential in a new context.

Investments in power equipment are capital intensive, and the equipment is difficult to sell once it is installed. Hence, we assume investments to be irreversible. Furthermore, capacity expansions are rarely now-or-never decisions [4]. The investment can be delayed until the company has more information about the uncertain demand. These assumptions

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form the basis for real options problems. Treating capacity expansions in the power sector with a real options approach takes into account the value of waiting while the investment is considered irreversible, see e.g. [5], [6], [7], [8] and [9].

Traditional real options models consider the value and timing of a single asset investment opportunity. Such models often allow for closed form solutions. However, their assumptions fit poorly with actual power markets. As advocated by [10], such models rely on numerous simplifications including an exogenous electricity price. More specifically, they exclude strategic interaction between firms through the impact of dispatch on the price. According to [10], the generation of electricity indeed creates a feedback effect on the electricity price. When the generation becomes high relative to the demand, the electricity price approaches zero and capacity investments are no longer attractive.

To capture the characteristics of actual power markets, we may cast capacity expansion problems as either a single optimization problem or a canonical real options problem. The former type of models is examined by e.g. [11] and [12]. As opposed to a single investment, such models allow for an endogenous electricity price, but become intractable when the models are extended to a stochastic setting in continuous time. Canonical real options models, on the other hand, fit well into a stochastic setting while allowing for an endogenous electricity price [13]. Such models consider a sequence of incremental capacity expansions, the value of which are determined by a stochastic control problem [10].

The drawback of canonical real options models is that they rely on the assumption of myopia to be mathematically tractable [14]. Myopia implies that the investment in additional capacity is assumed to be the last one over the time horizon. As argued by [15], the assumption of myopia requires symmetric generating technologies and homogeneous firms in the power market. However, different generating technologies have varying investment, maintenance and marginal costs, and firms differ in terms of their level of market power. Myopia may therefore not hold for our problem. In spite of this, we use it as an assumption to facilitate a solution. We argue that myopia is an acceptable approximation because of many firms' behavior. If the next investment is attractive, this is assumed the last one. When time passes and the electricity demand increases, a new investment might be undertaken, despite the earlier belief of the previous investment to be the last.

Earlier works on modeling the electricity price include the papers [16], [17], [18] and [19]. These papers model an exogenous long-term electricity price as a geometric Brownian motion. [18] argues that modelling the long-term evolution of commodity prices using geometric Brownian motion results in small errors. To include price feed-back, [10] suggest an extension for which an inverse demand curve is subject to a shock driven by a geometric Brownian motion.

Following the deregulation of European electricity markets in the last decades, power is produced by private companies, which aim to maximize their profits. Given the change of market structure, [20] consider power markets as oligopolies. This view is supported by e.g. [12], [11] and [21]. Several mergers and acquisitions have resulted in markets with few suppliers with the opportunity to exercise market power in addition to a number of small firms [22]. Thus, we create a framework where the firms have different levels of market power. Following the existing literature, we cast the problem of simultaneous dispatch of the

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firms as an equilibrium problem. This can be solved as a quadratic program when assuming a linear inverse demand function [11].

An increasing share of renewables in power systems may lead to a high level of supply risk [23]. Additionally, low variable costs of renewables make investments in gas-fired generation increasingly unprofitable [24]. Nevertheless, a high renewable penetration entails the need for flexible buffer capacity, as pointed out by [25]. This paper evaluates investments in gas-fired power plants with a real options approach and finds that capacity payments are an effective measure to new gas-fired generation projects. Furthermore, [26] claims that the power-producing technology with the lowest fixed costs and the best ramping properties should receive capacity payments. Currently, this is gas-fired power plants in Germany. This view is supported by e.g. [27] and [25]. [28] argue that these generators are the fossil fuel generation technology with smallest greenhouse gas emissions. At the same time, [29] points out that one should be careful when designing capacity remuneration mechanisms (CRMs) for the conventional power plants not to crowd out renewables.

Despite the clear need for a better understanding of capacity expansions in power markets and market power, there is a limited amount of academic research addressing this. By an extension of [10], we propose a capacity expansion model using real options and dynamic programming. Further, we apply our model to the German power market. This paper aims to contribute to the literature by implementing the following particularities in a real options context:

1. We treat electricity as a differentiated product, both between years and within each year. Fluctuations in short-term demand and supply are modelled by dividing each year into segments, using the same procedure as [30], where the distribution of demand and non-controllable supply is different in each segment.

2. We model a wide portfolio of technologies that differs in marginal, maintenance and investment costs. We also categorize the technologies into controllable and non-controllable power sources. Controllable power plants can produce electricity until all installed capacity is in use or a limit on power generation is reached. For non-controllable power plants, generation is determined by their normalized production times their installed capacity.

3. We aim to model an actual power market where the firms have different levels of market power. All firms are divided into two categories; firms with and without market power. We model the former as Cournot firms, such that each firm can affect the electricity price through their own dispatch while taking the dispatch of the other firms as given. In contrast, the firms without market power are price takers.

4. Plant revenues are determined by an auction that takes form of an equilibrium problem to handle the feedback effect of additional capacity. To obtain an analytical solution to the capacity expansion, we employ a regression that determines the incremental profits accruing from one unit of additional capacity.

The paper is structured as follows. Chapter 2 introduces our capacity expansion approach. The model parameters are determined in Chapter 3. In Chapter 4, we conduct

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an illustrative example, and in Chapter 5 we perform a case study on the German power market. Chapter 6 concludes.

2. The capacity expansion model

2.1. Electricity demand

When a firm decides whether to expand capacity, having possible price feedback effects in mind, the electricity demand is the most uncertain variable. In a multi-decade analysis, one needs to capture the long-term uncertainty in the power price, i.e. the pieces of news and developments that have a permanent effect on supply and demand in the power sector. Hence, we let the long-term fluctuations in the power price in each year t be subject to a industry-wide demand shock Y_t that follows the geometric Brownian motion

$$dY_t = \mu Y_t dt + \sigma Y_t dz_t \tag{1}$$

in which μ is the deterministic drift, $\sigma > 0$ is the standard deviation and dz_t is the increment of a Wiener process.

Electricity cannot be stored and is thus a differentiated product within the year as well as between years. As a result, we model the short-term impact of the production on the price, i.e. the downward-sloping inverse demand function. This is done for every load segment $l \in \mathbf{L}$, where $\mathbf{L}$ is a set of load segments. We assume the demand to be fixed within the load segment. Further, we denote the inverse demand function $D_l(Q_l)$ in load segment l when total power production is Q_l. As argued by [4], we assume the electricity price to be determined by the product of the stochastic industry-wide uncertainty Y_t and the inverse demand function $D_l(Q_l)$. The electricity price is thus

$$P_l(Y_t, Q_l) = Y_t D_l(Q_l). \tag{2}$$

2.2. Optimal dispatch

The optimal dispatch problem is solved for a given year and profit is instantaneous. All parameters except the electricity price and dispatch are assumed fixed. As advocated in the introduction, capacity systems have heterogeneous generating technologies. A technology is denoted by k and is a part of the set of technologies $\mathbf{K}$. Further, we introduce firm $f \in \mathbf{F}$, where $\mathbf{F}$ is the set of all firms. Firm f has capacity $K_{f,k}$ of technology k. The capacity of each technology k and firm f, $K_{f,k}$, is an element in the capacity matrix K. We split the set of technologies $\mathbf{K}$ into two subsets $\mathbf{K_{nc}}$ and $\mathbf{K_c}$ to handle the generation constraints. Subset $\mathbf{K_{nc}}$ represent non-controllable generation technologies and subset $\mathbf{K_c}$ represent controllable generation technologies.

We model the uncertainty in non-controllable generation and the correlation with electric load with the load and production duration curves. In particular, we form load and production segments based on historical load and capacity factor levels with the same procedure as [30]. We assume two non-controllable technologies, wind power and solar power, such that $d(\mathbf{K_{nc}}) = 2$. The wind and solar power generation duration is modeled using $d(\mathbf{W})$

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generation duration segments for wind and $d(\mathbf{S})$ generation duration segments for solar. The set $w \in \mathbf{W}$ represents the wind generation duration segments and the set $s \in \mathbf{S}$ represents the solar generation duration segments. Further, we introduce the set $h = (l, w, s) \in \mathbf{H}$ that represents a combination of a load, a wind power and a solar power segment. This way, we divide each year into $d(\mathbf{H}) = d(\mathbf{L}) \times d(\mathbf{W}) \times d(\mathbf{S})$ segments.

Let $q_{f,k,h}$ be the dispatch of firm f from technology k in segment h and $q_{f,h}$ be the total dispatch of firm f in segment h, such that $q_{f,h} = \sum_{k=1}^{d(\mathbf{K})} q_{f,k,h}$ for each $h \in \mathbf{H}$. Q_h is the aggregated dispatch of all firms in segment h so that $Q_h = \sum_{f=1}^{d(\mathbf{F})} q_{f,h}$. Furthermore, let $q_{-f,h}$ be the aggregated dispatch of every firm except firm f such that $q_{-f,h} = \sum_{f'=1 f' \neq f}^{d(\mathbf{F})} q_{f',h}$.

Every firm $f \in \mathbf{F}$ is profit-maximizing. We assume $d(\mathbf{F}) - 1$ Cournot firms with market power, and a number of price-taking firms with the same non-strategic operational behavior. They are aggregated into one competitive fringe $f = d(\mathbf{F})$ to avoid separate handling of many identical firms. The competitive fringe assumes its dispatch to not affect the price, such that

$$\frac{\partial P_h(Y, Q_h)}{\partial q_{d(\mathbf{F}),h}} = 0 \qquad h \in \mathbf{H}. \tag{3}$$

The Cournot firms consider the feedback effect of their dispatch on the price, which implies

$$\frac{\partial P_h(Y, Q_h)}{\partial q_{f,h}} \leq 0 \qquad h \in \mathbf{H}, \ f = 1, \ldots, d(\mathbf{F}) - 1. \tag{4}$$

The objective of firm f in its optimal dispatch problem is its profit maximization in (5). OMC_k is the operation and maintenance cost per unit of installed capacity of technology k and the total capacity related costs are $\sum_k OMC_k K_{f,k}$. The unit production cost for technology k is denoted c_k. This results in the variable costs $\sum_k c_k q_{f,k,h}$ of firm f in segment h. The revenues in segment h are given as the product of the price function in (2), firm f's dispatch in segment h, $q_{f,h}$, and the duration of the segment τ_h.

Generation from non-controllable energy sources is stated in (6), where $Z_{k,h}$ is the normalized production by technology k in segment h. The constraint implies that the non-controllable production of technology k by firm f in segment h equals the normalized production times the capacity that firm f has installed of technology k. Equation (7) and (10) constrain the generation of the technologies in the controllable subset $\mathbf{K_c}$ to not exceed its upper limit $K_{f,k}$ or fall below its lower limit of 0.

The controllable technologies $k \in \mathbf{K_c}$, may have generation limited by the available amount of energy. Thus, we introduce constraint(8), where firm f's generation by technology k is restricted by a maximal production $E_{f,k}^{max}$. Constraint (9) aggregates the dispatch from each technology $k \in \mathbf{K}$ for firm f.

Optimal dispatch problem of firm f

$$\pi_f(Y, K) = \max_{q_{f,k,h}} \sum_{h=1}^{d(\mathbf{H})} \tau_h \left[P_h(Y, q_{f,h} + q_{-f,h}) q_{f,h} - \sum_{k=1}^{d(\mathbf{K})} c_k q_{f,k,h} \right] - \sum_{k=1}^{d(\mathbf{K})} OMC_k K_{f,k} \quad (5)$$

s.t.

$$q_{f,k,h} = Z_{k,h} K_{f,k} \quad (\mu_{f,k,h}) \qquad\qquad k \in \mathbf{K_{nc}}, h \in \mathbf{H} \qquad (6)$$

$$q_{f,k,h} \leq K_{f,k} \quad (\lambda_{f,k,h}) \qquad\qquad k \in \mathbf{K_c}, h \in \mathbf{H} \qquad (7)$$

$$\sum_{h=1}^{d(\mathbf{H})} \tau_h q_{f,k,h} \leq E_{f,k}^{max} \quad (\delta_{f,k}) \qquad\qquad k \in \mathbf{K_c} \qquad (8)$$

$$\sum_{k=1}^{d(\mathbf{K})} q_{f,k,h} = q_{f,h} \qquad\qquad h \in \mathbf{H} \qquad (9)$$

$$q_{f,k,h} \geq 0 \qquad\qquad k \in \mathbf{K_c}, h \in \mathbf{H}. \qquad (10)$$

The variables $\mu_{f,k,}$, $\lambda_{f,k,l}$ and $\delta_{f,k}$ are the Lagrangian multipliers of (6), (7) and (8).

The optimal dispatch problems of each firm $f \in \mathbf{F}$ (5)-(10) are linked through their generation, which effect the electricity price. In some segments, the total dispatch may not be able to cover demand. Hence, we introduce the variable $q_{r,h}$, representing the total rationing in segment h. Rationing occurs at a cost to society c_r per unit of rationing. As rationing is costly, the regulator strives to balance the system to avoid it. Hence, we introduce the regulator's rationing problem

$$\chi(Y, K) = \max_{q_{r,h}} - \sum_{h=1}^{d(\mathbf{H})} \tau_h c_r q_{r,h}, \; q_{r,h} \geq 0 \quad h \in \mathbf{H}. \qquad (11)$$

To link the rationing problem with the optimal dispatch problems, we introduce a joint market constraint stating that the aggregated dispatch of all firms $f \in \mathbf{F}$ should exceed the lower bound Q_h^{min} in segment h. Rationing occurs otherwise. The power price is still determined by the inverse demand function (2) and is considerably lower than the cost of rationing. Furthermore, the variable ν_h is the Lagrangian multiplier of the constraint.

$$\sum_{f=1}^{d(\mathbf{F})} q_{f,h} + q_{r,h} \geq Q_h^{min} \quad (\nu_h) \qquad\qquad h \in \mathbf{H}. \qquad (12)$$

Together the optimal dispatch problems, the rationing problem and (12) form an equilibrium problem for the entire market, which can be solved as a complementarity problem. We state the KKT-conditions of the interlinked optimization problems in (13)-(19). Condition (20) defines the auxiliary variables $q_{f,k}$.

KKT-conditions of the market equilibrium problem

$$0 \leq q_{f,k,h} \perp \left(-\tau_h \left[\frac{\partial}{\partial q_{f,k,h}} P_h(Y_t, q_{f,h} + q_{-f,h}) q_{f,h} - c_k \right] + \lambda_{f,k,h} + \delta_{f,k} - \nu_h \right) \geq 0$$
$$f \in \mathbf{F}, k \in \mathbf{K_c}, h \in \mathbf{H} \qquad (13)$$

$$-\tau_h \left[\frac{\partial}{\partial q_{f,k,h}} P_h(Y_t, q_{f,h} + q_{-f,h}) q_{f,h} - c_k \right] + \mu_{f,k,h} - \nu_h = 0$$
$$f \in \mathbf{F}, k \in \mathbf{K_{nc}}, h \in \mathbf{H} \qquad (14)$$

$$0 \leq q_{r,h} \perp (\tau_h c_r - \nu_h) \geq 0 \qquad\qquad\qquad\qquad h \in \mathbf{H} \qquad (15)$$

$$0 \leq \lambda_{f,k,h} \perp (K_{f,k} - q_{f,k,h}) \geq 0 \qquad\qquad f \in \mathbf{F}, k \in \mathbf{K_c}, h \in \mathbf{H} \qquad (16)$$

$$K_{f,k} Z_{k,h} - q_{f,k,h} = 0 \quad (\mu_{f,k,h} \text{ is unrestricted}) \qquad f \in \mathbf{F}, k \in \mathbf{K_{nc}}, h \in \mathbf{H} \qquad (17)$$

$$0 \leq \delta_{f,k} \perp (E_{f,k}^{max} - \sum_{h=1}^{d(\mathbf{H})} \tau_h q_{f,k,h}) \geq 0 \qquad\qquad f \in \mathbf{F}, k \in \mathbf{K_c} \qquad (18)$$

$$0 \leq \nu_h \perp (\sum_{f=1}^{d(\mathbf{F})} q_{f,h} + q_{r,h} - Q_h^{min}) \geq 0 \qquad\qquad f \in \mathbf{F}, k \in \mathbf{K_c} \qquad (19)$$

$$\sum_{k=1}^{d(\mathbf{K})} q_{f,k,h} = q_{f,h} \qquad\qquad\qquad\qquad f \in \mathbf{F}, h \in \mathbf{H}. \qquad (20)$$

To ensure that the complementarity problem has a solution, the optimal dispatch problem of firm f in (5)-(10) must be convex, i.e. the objective (5) must be concave and the constraints (6)-(10) linear. With a downward-sloping inverse demand curve, the objective (5) is concave. Combined with linear constraints, this makes each of the interlinked optimization problems in the equilibrium problem convex. Then, the complementarity problem can be solved with the KKT-conditions to find an optimal solution.

We introduce the linear inverse demand curve

$$D_h(Q_h) = A_h - b_h Q_h \qquad (21)$$

in which A_h is the constant and b_h is the slope of the curve. As pointed out by [11] and [21], Cournot-equilibrium problems with linear inverse demand functions, may be casted as a single convex quadratic optimization problem. In (22)-(28), we find the optimal dispatch

in the market by using convex quadratic optimization. The constraints stated in (23)-(27) equals the constraints (6)-(10), (28) defines the auxiliary variable Q_h and (29) is the non-negativity constraints. It can easily be proven that the KKT-conditions of (22)-(29) equals the complementarity problem stated in (13)-(20).

Optimal dispatch problem for the market

$$\phi(Y, K) = \max_{q_{f,k,h}} \sum_{h=1}^{d(\mathbf{H})} \tau_h \left[Y_t \left(A_h Q_h - \frac{1}{2} b_h Q_h^2 - \frac{1}{2} b_h \sum_{f=1}^{d(\mathbf{F})-1} q_{f,h}^2 \right) \right.$$
$$\left. - \sum_{k=1}^{d(\mathbf{K})} c_k q_{f,k,h} - c_r q_{r,h} \right] - \sum_{f=1}^{d(\mathbf{F})} \sum_{k=1}^{d(\mathbf{K})} OMC_k K_{f,k} \quad (22)$$

s.t.

$$q_{f,k,h} = Z_{k,h} K_{f,k} \quad (\mu_{f,k,h}) \qquad\qquad f \in \mathbf{F}, k \in \mathbf{K_{nc}}, h \in \mathbf{H} \qquad (23)$$
$$q_{f,k,h} \leq K_{f,k} \quad (\lambda_{f,k,h}) \qquad\qquad f \in \mathbf{F}, k \in \mathbf{K_c}, h \in \mathbf{H} \qquad (24)$$
$$\sum_{h=1}^{d(\mathbf{H})} \tau_h q_{f,k,h} \leq E_{f,k}^{max} \quad (\delta_{f,k}) \qquad\qquad f \in \mathbf{F}, k \in \mathbf{K_c} \qquad (25)$$
$$\sum_{f=1}^{d(\mathbf{F})} q_{f,h} + q_{r,h} \geq Q_h^{min} \quad (\nu_h) \qquad\qquad h \in \mathbf{H} \qquad (26)$$
$$\sum_{k=1}^{d(\mathbf{K})} q_{f,k,h} = q_{f,h} \qquad\qquad f \in \mathbf{F}, h \in \mathbf{H} \qquad (27)$$
$$\sum_{f=1}^{d(\mathbf{F})} q_{f,h} = Q_h \qquad\qquad h \in \mathbf{H} \qquad (28)$$
$$q_{f,k,h} \geq 0 \qquad\qquad f \in \mathbf{F}, k \in \mathbf{K_c}, h \in \mathbf{H}. \qquad (29)$$

The terms $A_l Q_h - \frac{1}{2} b_l Q_h^2$ equal the objective of a welfare maximization problem and $\frac{1}{2} b_l \sum_{f=1}^{d(\mathbf{F})-1} q_{f,h}^2$ accounts for the market power exertion. Each Cournot firm assumes the dispatch of its competitors to be fixed and behave as a monopolist on its residual demand function. If $\mathbf{F}$ consists of one Cournot firm, the objective function in (22) equals a monopolist maximizing its profit.

2.3. Value of capacity expansions

Firm f adapts its capacity to the demand at each point in time t. Hence, we denote the matrix of installed capacities at time t, K_t, where $K_{t,f,k}$ is firm f's installed capacity of technology k at time t. $F_f(Y_t, K_t)$ represents the value of all optimal capacity expansions of firm f, and is determined by firm f's stochastic control problem. When firm f has no

other assets except its generation capacity, $F_f(Y_t, K_t)$ is equivalent to the value of firm f. The value of all capacity expansions of firm f is

$$F_f(Y_t, K_t) = \max_{K_{t,f,k}} \mathbf{E}\left[\int_0^\infty \pi_f(Y_t, K_t)e^{-\rho t}dt - \sum_{k=k}^{d(\mathbf{K})} \int_0^\infty I_k e^{-\rho t}dK_{t,f,k} \right] f \in \mathbf{F} \qquad (30)$$

where ρ is the annual discount rate of future cash flows and I_k is the investment cost of technology k. Firm f invests in new capacity to maximize its expected value over an infinite time horizon. The first term on the right-hand side of the equality represents all future expected discounted profits of firm f. The second term on the right-hand side is the total expected discounted investment costs from capacity investments. Hence, we integrate over every point in time t to find the value of installed capacities of firm f.

The welfare gain from capacity investments equals the total discounted social welfare from the capacity expansions, where the instantaneous social welfare ψ is the sum of the producer and the consumer surplus

$$\psi(Y_t, K_t) = \sum_{h=1}^{d(\mathbf{H})} \tau_h \left[\int_0^{Q_h} P_h(Y_t, x_h)dx_h - \sum_{f=1}^{d(\mathbf{F})}\sum_{k=1}^{d(\mathbf{K})} c_k q_{f,k,f} - c_r q_{r,h} \right]$$
$$- \sum_{f=1}^{d(\mathbf{F})}\sum_{k=1}^{d(\mathbf{K})} OMC_k K_{f,k}. \quad (31)$$

Then the welfare of all capacity expansions over infinite time horizon is

$$W(Y_t, K_t) = \max_{K_{t,f,k}} \mathbf{E}\left[\int_0^\infty \psi(Y_t, K_t)e^{-\rho t}dt - \sum_{f=1}^{d(\mathbf{F})}\sum_{k=1}^{d(\mathbf{K})} \int_0^\infty I_k e^{-\rho t}dK_{t,f,k} \right]. \qquad (32)$$

2.4. Optimal investment strategy

Capacity expansion problems are often casted as stochastic control problems. The complexity of our model implies that the stochastic control problems (30) and (32) cannot be solved analytically. However, by applying real options theory, we may state the capacity expansion problem as an optimal stopping problem if the properties of myopia hold. When assuming myopia, the option value for firm f to invest marginally in technology k, $V_{f,k}(Y_t, K_t)$, is stated

$$V_{f,k}(Y_t, K_t) = \max_\tau \mathbf{E}\left[\int_\tau^\infty \frac{\partial \pi_{f,k}(Y_t, K_t)}{\partial K_{t,f,k}} e^{-\rho t}dt - I_k e^{-\rho \tau} \right] \approx \frac{\partial F_f(Y_t, K_t)}{\partial K_{t,f,k}}$$
$$f \in \mathbf{F}, k \in \mathbf{K} \quad (33)$$

where τ is the timing of the investment. Equation (33) implies that firm f's option value of investing marginally in additional capacity in technology k equals the profits from the optimally timed investment net of the investment cost.

To obtain the optimal stopping problem, we derive the corresponding Bellman equation by dynamic programming. The Bellman equation (34) states that the return of firm f's option to invest marginally $V_{f,k}$ over a time step dt equals the sum of the profit of a marginal increase in capacity over dt, $\frac{\partial \bar{\pi}_{f,k}}{\partial K_{t,f,k}}$, and the expected change in $V_{f,k}$ over dt, $\mathbf{E}[dV_{f,k}]$. Thus, the Bellman equation of the optimal capacity expansion problem is

$$\rho V_{f,k}(Y_t, K_t)dt = \frac{\partial \pi_f(Y_t, K_t)}{\partial K_{t,f,k}}dt + \mathbf{E}[dV_{f,k}(Y_t, K_t)] \quad f \in \mathbf{F}, k \in \mathbf{K}. \tag{34}$$

We expand the right-hand side of (34) by using Ito's lemma to obtain the partial differential equation of the optimal stopping problem. Further, we introduce the convenience yield $\delta = \rho - \mu$ to simplify the exposition. The convenience yield of electricity is interpreted as the relative benefit of delivering the commodity earlier rather than later.

Optimal stopping problem of firm f in posession of technology k

$$\frac{1}{2}\sigma^2 Y_t^2 \frac{\partial^2 V_{f,k}(Y_t, K_t)}{\partial Y_t^2} + (\rho - \delta)Y_t \frac{\partial V_{f,k}(Y_t, K_t)}{\partial Y_t} - \rho V_{f,k}(Y_t, K_t)$$

$$+ \frac{\partial \pi_{f,k}(Y_t, K_t)}{\partial K_{t,f,k}} = 0 \quad f \in \mathbf{F}, k \in \mathbf{K} \tag{35}$$

with boundary conditions

$$V_{f,k}(0, K_t) = 0 \qquad f \in \mathbf{F}, k \in \mathbf{K} \tag{36}$$

$$V_{f,k}(Y_t^*, K_t) = I_k \qquad f \in \mathbf{F}, k \in \mathbf{K} \tag{37}$$

$$\frac{\partial V_{f,k}(Y_t^*, K_t)}{\partial Y_t^*} = 0 \qquad f \in \mathbf{F}, k \in \mathbf{K}. \tag{38}$$

Equation (35) is the Bellman equation of the problem, and (36) ensures that the option to invest is zero when the demand shock Y_t is zero. (37) and (38) are the value matching and the smooth pasting conditions for an incremental investment in new capacity.

In order to obtain an analytical solution of the optimal stopping problem, we propose the regression $\bar{\pi}_{f,k}$ for firm f using technology k in (39) to express the instantaneous profit from the optimal dispatch optimization problem analytically as a function of Y_t and K_t.

$$\bar{\pi}_{f,k}(Y_t, K_t) = \sum_{i=1}^{d(\gamma)} b_{f,k,i}(K_t)Y_t^{\gamma_i} - OMC_k K_{t,f,k} \qquad f \in \mathbf{F}, K \in \mathbf{K}. \tag{39}$$

The first term of the regression shows firm f's profit flow from technology k. The regression coefficients $b_{f,k,i}(K_t)$ describe how changes in the capacity of firm f with technology k affect

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the instantaneous profit flow for a given shock process Y_t. New installed capacity of firm f has a positive impact on firm f's instantaneous profit, and additional installed capacity of all other firm reduces the electricity price and thus the instantaneous profit of firm f regardless of choice of technology k. Both effects are captured in the regression coefficients $b_{f,k,i}(K_t)$. The coefficients are positive if the income from additional capacity outweighs the lower price caused by the other firms' installed capacity, and negative otherwise. The parameter γ is a positive base vector of dimensions $d(\gamma)$ used to describe changes in $\bar{\pi}_{f,k}(Y_t, K_t)$ with respect to Y_t. We constrain γ_i by $0 < \gamma_i < \beta_1 \ \forall i$ to obtain an unique investment trigger in the regression with the same procedure as [10]. β_1 is given by the positive root of the quadratic equation

$$\beta_1 = (\frac{1}{2} - \frac{\mu}{\sigma^2}) + \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2\rho}{\sigma^2}}. \tag{40}$$

The investment problem of firm f and technology k has the homogeneous solution $V_{f,k,h}(Y_t, K_t) = A_{t,f,k}(K_t)Y_t^{\beta_1}$. The particular solution of the Bellman equation (35) is the underlying value of the capacity expansion; the firm's profit flow from the marginal investment.

$$V_{f,k,p}(Y_t, K_t) = \sum_{i=1}^{d(\gamma)} \bar{b}_{f,k,i}(\gamma_i, K_t)Y_t^{\gamma_i} - \frac{OMC_k}{\rho} \qquad f \in \mathbf{F}, k \in \mathbf{K} \tag{41}$$

where the coefficients $\bar{b}_{t,f,k,i}$ are given by

$$\bar{b}_{f,k,i}(\gamma_i, K_t) = \left[\frac{\partial b_{f,k,i}(K_t)}{\partial K_{t,f,k}}\right] \frac{1}{\rho - \mu\gamma_i - \frac{1}{2}\sigma^2 + \gamma_i(\gamma_i - 1)} \quad f \in \mathbf{F}, k \in \mathbf{K}. \tag{42}$$

The solution of the Bellman equation is the sum of the homogeneous and the particular solution $V_{f,k}(Y_t, K_t) = V_{f,k,h}(Y_t, K_t) + V_{f,k,p}(Y, K)$. Consequently, firm f's option value of investing in technology k is given by

$$V_{f,k}(Y_t, K_t) = A_{f,k}(K_t)Y_t^{\beta_1} + \sum_{i=1}^{d(\gamma)} \bar{b}_{f,k,i}(\gamma_i, K_t)Y_t^{\gamma_i} - \frac{OMC_k}{\rho} \quad f \in \mathbf{F}, k \in \mathbf{K}. \tag{43}$$

By applying the value matching and smooth pasting conditions, we obtain the investment trigger equation of each technology k and firm f

$$\sum_{i=1}^{d(\gamma)} (Y_{t,f,k}^*)^{\gamma_i} \left(\frac{\beta_1 - \gamma_i}{\beta_1}\right) \bar{b}_{f,k,i}(\gamma_i, K_t) = I_k + \frac{OMC_k}{\rho} \qquad f \in \mathbf{F}, k \in \mathbf{K}. \tag{44}$$

$Y_{t,f,k}^*$ is the optimal investment trigger at time t of the firm f in technology k. As long as $Y_t > Y_{t,f,k}^*$, firm f invest marginally in new capacity with production technology k. When $Y_{t,f,k}^*$ reaches Y_t, firm f no longer invest in additional capacity.

2.5. Numerical solution procedure

It is not possible to obtain an analytical solution to our capacity expansion model. Hence, we propose a semi-analytical heuristic to solve the problem numerically. In doing this, we no longer operate in continuous time. Time is discretized using an analysis period of S years. For each year, we solve the optimal dispatch problem for a host of Monte Carlo simulations to obtain the corresponding Lagrange multipliers. We replace the regression (39) with a non-negative least square regression of the marginal value of investing over an infinite time horizon

$$\bar{V}_{f,k,p}(Y_t, K_t) = \sum_{k=1}^{d(\mathbf{K})} \sum_{i=1}^{d(\gamma)} \bar{b}_{f,k,i}(\gamma_i, K_t) Y_t^{\gamma_i} - \frac{OMC_k}{\rho} \qquad f \in \mathbf{F} \tag{45}$$

$$\bar{b}_{f,k,i}(\gamma_i, K_t) \geq 0 \quad f \in \mathbf{F}, k \in \mathbf{K}, i = 1, \ldots, d(\gamma) \tag{46}$$

where (46) is introduced to ensure a unique solution to (44). This way, we find the impact on the value of a marginal increase in the installed capacity of technology k. We divide the heuristic into an inner and an outer layer. In the former, we solve the optimal stopping problem to find investment triggers, given the current capacities. In the latter, we approximate the value of the stochastic control problems (30) and (32) by computing the expected value of the capacity expansions for the firms and the society.

In the inner layer, we solve the optimal stopping problem and compute investment triggers. We perform Monte Carlo simulations to find the Lagrange multipliers of the capacity constraints in the optimal dispatch problem over the next T years. The Lagrange multiplies are found on a Y_0^{grid}, where initial values of demand shock vary. The matrix of installed capacities at time t, K_t, is held fixed over $t = 0, \ldots, T$ years due to the assumption of myopia. Further, we use the Lagrange multipliers to compute the expected marginal discounted value of an additional unit of capacity at each point on the Y_0^{grid}. Next, we perform the non-negative least square regression stated in (45) on the marginal values with Y_0 as the explanatory variable to find the regression coefficients $\bar{b}_{f,k,i}(\gamma_i, K_t)$. Finally, we solve the trigger equations (44) simultaneously.

The outer layer iterates over the analysis period of $s = 1, \ldots, S$ years and $\omega = 1, \ldots, \Omega$ scenarios, where the demand shock Y_s follows a geometric Brownian motion. At each point in time s for each scenario ω, we run the inner layer in order to find the investment triggers of firm f with production technology k, $Y_{s,f,k}^*$. If the demand shock at time s, Y_s, exceeds the investment trigger of firm f of technology k, $Y_{s,f,k}^*$, firm f invest marginally in capacity with production technology k. If this is valid for several production technologies, firm f invests in the technology with the lowest investment trigger. Between each investment, the installed capacity K_t is updated. At each point in time s, we run the inner layer and find investment triggers until it is no longer optimal to invest. Then we move on to $s + 1$ and repeat the procedure until the end of the analysis period S. Finally, we find the expected value of all capacity expansions of each firm f by averaging the values of investment over the Ω scenarios.

3. Determining model parameters

3.1. Annual growth and volatility in long-term electricity demand

To decrease greenhouse gas emissions in Germany by $85-90\%$ by 2050 as targeted in the Energiwende [31], the German energy consumption must be moderated. Knopf [32] argues that the German energy consumption may be reduced by 4.35 % from 2010 to 2030, which constitutes a decline of 0.29% per year. Thus, we let the drift of the long-term electricity demand be $\mu = -0.29\%$.

The annual volatility is estimated from the yearly power consumption in 2006–2015. The annual volatility σ found over the years $t = 1, \ldots, T$ is calculated with the formula

$$\sigma = \sqrt{\frac{1}{T-1} \sum_{t=1}^{T} \left((\ln Q_t - \ln Q_{t-1}) - \frac{1}{T} \sum_{t=1}^{T} (\ln Q_t - \ln Q_{t-1}) \right)^2} \qquad (47)$$

where Q_t is the annual dispatch of year t. The formula is valid for log-normal distributions, including the geometric Brownian motion representing the long-term fluctuations in demand. The resulting volatility in long-term electricity demand is $\sigma = 2.3\%$. Table 1 shows the yearly power consumption data used for estimating the yearly volatility σ. We have used power consumption data provided by [33].

Table 1: Load consumption in Germany

Year	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Load (GWh)	559078	555899	557162	526857	547422	544267	539867	530558	529369	520607

3.2. Inverse demand function

We calibrate the parameters of the inverse demand function in each load segment l by linear regressions with data on hourly load provided by [33] and hourly electricity prices for the German and Austrian day-ahead market from [34]. The resulting parameters of the inverse demand functions are stated in Figure 2.

Table 2: Inverse demand function parameters

Load segment l	1	2	3	4	5
A_l	362	211	173	159	116
b_l	-0.0040	-0.0023	-0.0023	-0.0028	-0.0026

3.3. Required rate of return

We deal with the price risk when estimating the drift and the volatility of the long-term electricity demand, and we assume that no other risk factors are involved. In reality, some risk factors including technical and political risk may influence the required rate of return. Technical risk includes that components may not operate as assumed and will have to be

replaced at other rates than assumed, and the political risk entails the risk of government changes in carbon taxes and subsidies. This affects the profitability of the different power producing technologies. The technical and political risks are considered minor. Hence, we use a discount rate of 4 % as suggested by [35] as the approximated discount factor.

3.4. Load and production segments

In order to find the load and production segments, we use hourly normalized load and production profiles from 2010-2014. Data on hourly electric load consumption is provided by [33], and the data on hourly solar and wind power production profiles are provided by [36] and [37]. Based on the input data, we have produced the duration curves presented in Figure 1.

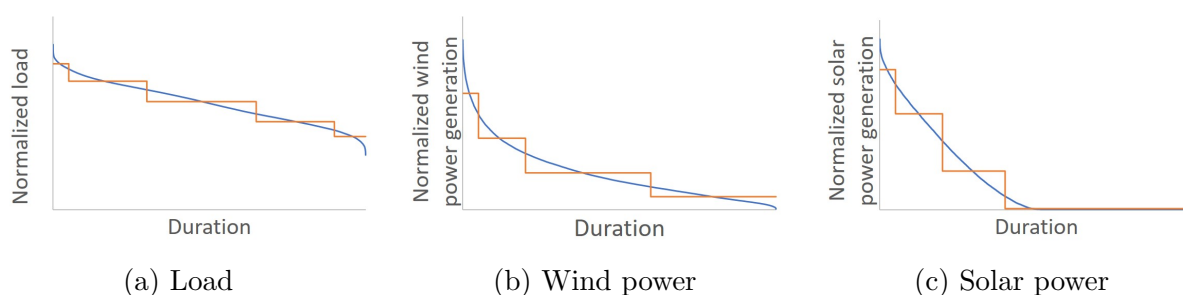(a) Load   (b) Wind power   (c) Solar power

Figure 1: Duration curves and segments for load, wind power generation and solar power generation.

We divide the load duration curve into 5 load segments, and the solar and wind production duration curve into 4 segments each. The resulting load and production segments are presented in Table 3, Table 4 and Table 5.

Table 3: Load segments

Load segment	1	2	3	4	5
Number of hours	2191	10956	15338	10956	54382
Average normalized load	0.4533	0.3992	0.3355	0.2732	0.2273

Table 4: Wind power production segments

Wind power production segment	1	2	3	4
Number of hours	2191	6574	17530	17530
Average normalized production	0.6022	0.3698	0.1901	0.0689

Table 5: Solar power production segments

Solar power production segment	1	2	3	4
Number of hours	2191	6574	8765	26294
Average normalized production	0.5911	0.4047	0.1625	0.0039

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3.5. Technology data

In Appendix A and Appendix B we present and discuss exogenous input data on costs and installed capacity. Further, we let the cost of rationing be 10 000 €/MWh, which is within the range stated in [38]. The renewable energy subsidy system in Germany is changing rapidly and governmental auctions for new renewable projects has been won without subsidies [39]. Thus, apply our framework without using subsidies.

3.6. Model implementation

We have implemented our model in Matlab 2016a [40]. The optimal dispatch problem is solved using the Gurobi 7.0.2 quadratic optimization solver [41]. The optimal stopping problem is solved on a Y^{grid} with 7 values and intervals of 0.1, as shown in Table B.19. We perform a set of Monte Carlo simulations over T years at each point on the Y_0^{grid}. The stochastic control problem is evaluated using 500 simulations of Ω, which is less than the ideal number of simulations. This is done limit the computation time, that exceeds 24 hours in the German case study. The model is solved using five computation nodes and the following hardware: Lenovo NeXtScale, 2 x Intel E5-2643v3, 3.4 GHz, 512 Gb RAM.

4. Illustrative example

We examine a power market with two Cournot firms and a competitive fringe over 10 years. All firms have access to wind, solar coal and CCGT power plants. Input data on initial installed capacity, cost, discount rate, beta and geometric Brownian motion parameters are presented in Appendix A. Further, we use parameters estimated in Section 3. For simplicity, we assume the energy constraints in (25) and the rationing constraints in (27) are never binding.
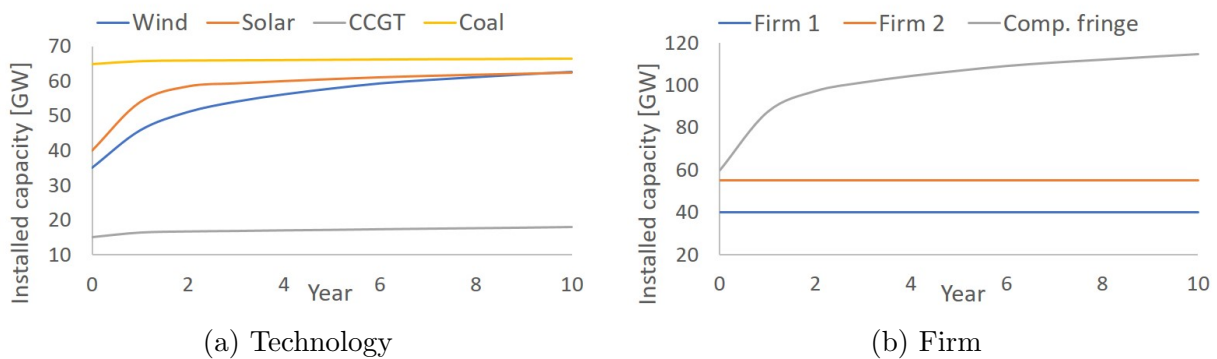
(a) Technology   (b) Firm

Figure 2: Development of installed capacity divided by technology and by firm.

Figure 2a illustrates the technology mix in the power market. The drift of 1 % limits the capacity investments. Almost all new investments occur during the first year of the analysis period, which indicates that the initial capacity is too small. Further, we note that 92 % of the investments in the analysis period are in solar and wind power plants. Investments in

such power plants are considered attractive despite their non-controllable dispatch because they have no marginal costs.

Investments in OCGT and coal-fired generators are present, but to a minor extent. Coal-fired power plants are base load plants with low marginal costs and high investment cost, while CCTG power plants have opposite cost characteristics and are thus peak load plants. Due to the high marginal cost of CCGT plants, their contribution margin is considerably smaller than that of coal-fired plants. Consequently, the marginal value of additional CCGT capacity is small compared to the marginal value of additional capacity in coal-fired power plants.

However, the operation of CCGT power plants is profitable in the segments with high load compared to non-controllable production. In segments with high demand and a modest renewable dispatch, CCGT plants play a key role in keeping the prices down. If the duration of these segments is sufficiently short, the incentive to invest in peak load capacity vanishes. Then one might observe periods of power shortage. Nevertheless, we note that the duration of these segments is sufficient to keep CCGT plants profitable.

In Figure 2b, we observe that the competitive fringe increases its market share of capacity over the next 10 years. In contrast to traditional oligopolistic behavior, we find that the Cournot firms do not try to defend their market shares. This is because they are better off with smaller market shares and higher prices than maintaining their market share at the cost of lower prices. Additionally, if the Cournot firms exercise a high level of market power, the power price increases until it is advantageous for new firms to enter the market and for existing price-taking firms to generate additional electricity and thus expand their capacity [4]. The competitive fringe, on the other hand, is a price-taker. Increasing its dispatch is not assumed to influence the price, and its marginal value of additional capacity exceeds that of a Cournot firm.

5. Case study of the German power market

Through the epochal transformation Energiewende, Germany aims to revamp their energy sector by reducing carbon emissions by 90% by 2050 compared to 1990 levels [31]. To meet their ambitious emission targets, German power-producing firms intend to install new renewable capacity to substitute the conventional power plants of today. Thus, we apply our capacity expansion model to the German power market using an analysis period from 2017 to 2040 to examine which investments competitive firms find attractive. We assume the four largest firms in the German power sector, i.e. RWE, Uniper, Vattenfall and EnBW to be Cournot firms. The rest of the power-producing firms are considered price-takers. Exogenous input data are presented in Chapter 3 and Appendix B.

Although our model does not account for time varying costs, we let costs change over the analysis period. Investment costs of solar and wind power are expected to decline over the next decades. Additionally, carbon prices and thus the marginal costs of conventional power plants are estimated to rise over the same period. For technologies with decreasing costs, the value of waiting is larger than the model predicts and vice versa. An underestimated value of waiting results in earlier investments than optimal and the effect is opposite for an

overestimated value of waiting. Moreover, we no longer assume the lifetime of power plants to be infinite, which results in a smaller marginal value of additional capacity compared to a model assuming infinite lifetime.

5.1. Base Case

Figure 3a illustrates the amount of installed capacity of different generation technologies. No investments occur before 2022 despite power plants reaching the end of their lifetime. This indicates that Germany has a sufficient level of installed capacity today. This view is supported by the reduced capacity in conventional power plants over the analysis period. In spite of this, the total installed capacity increases due to comprehensive investments in renewables. With a higher share of renewables, the installed capacity must increase since solar and wind power plants have a lower normalized generation than conventional base load power plants.
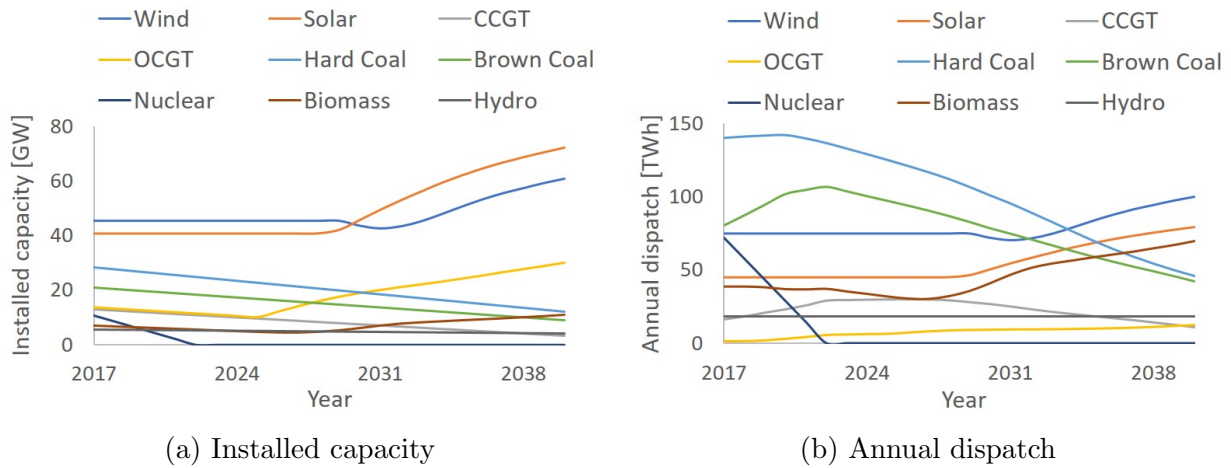

(a) Installed capacity   (b) Annual dispatch

Figure 3: Development of installed capacity and annual dispatch of the generation technologies, 2017–2040.

After 2030, investments in solar and wind power plants are boosted despite their non-controllable dispatch. This may be explained by their reduced investment costs and a rising carbon price making renewables profitable relative to conventional power plants. However, one may argue that these investments are overestimated because the value of waiting for a lower investment cost or a higher carbon price is not fully captured in our model. Nevertheless, we do not notice investments solar and wind power before 2030. No capacity is installed in the conventional technologies covering base load, i.e. hard coal, brown coal and nuclear. Nuclear power plants are actively phased out, and the increased marginal cost of the other technologies due to the rising carbon price makes them uncompetitive compared to renewables and OCGT power plants.

The installed capacity in renewables and base load power plants is sufficiently high to avoid power shortage in most of the segments before 2030. OCGT power plants have low investment costs and high marginal costs and are thus considered peak load power plants and are well suited to cover small periods of shortage. In periods with high load and

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modest renewable generation, they have a positive contribution margin and are consequently attractive to invest in. As investments in renewable capacity advance over the analysis period, the peak load investments rise as well. With a high share of renewables, fluctuations in dispatch and price increases. To complement non-controllable renewables, one needs flexible power plants that can ramp up and down quickly, i.e. peak load power plants. They are inexpensive to install and generate electricity at a positive contribution margin when the power price is high. Therefore, OCGT power plants are well suited to cover power demand net of non-controllable generation.

To reach their climate targets, Germany should have no more than 60 GW of non-renewable power by 2040, most of it in gas-fired power plants [31]. The resulting gas-fired capacity in the model is 34 GW in 2040, while the capacity of coal-fired power plants amounts to 21 GW. Although the coal-fired capacity exceeds the targeted level in 2040 [31], no investments in coal-fired power plants occur. To obtain an adequate scale-down of coal-fired power plants, a market regulator may introduce additional phase-out incentives. Except for the high levels of capacity in coal-fired power plants, our approach returns a capacity mix close to the one sought in Germany. This target is almost reached without including any subsidies or capacity mix policy targets. Our findings also support the view on conventional base load power plants being substituted by renewables as non-controllable base load and additional peak load power plants as their backup [31].

The dispatch is presented in Figure 3b. It declines 22.2 % over the analysis period. Given the capacity mix in Figure 3a, generation from peaking power plants is necessary to avoid shortage. As time passes, the share of controllable capacity with high marginal cost increases. Demand net of renewables and conventional base load is covered by peaking power plants, which have a high marginal cost. This shifts the market equilibrium to one with a lower dispatch at a higher price. The slightly negative drift in demand may also cause a declining dispatch.

We notice that the wind and solar dispatch escalate in line with investments and that more power is generated by biomass as time passes. This is not surprising, as solar and wind power have no marginal costs and biomass has the lowest marginal cost except solar and wind power after 2030. Additionally, hydropower plants have no marginal cost, but their dispatch is constrained and their investment cost is high. We see that the coal-fired power plants have a high normalized generation as they produce a considerable share of the total dispatch despite their modest share of installed capacity. This happens because the coal-fired power plants have lower marginal costs than gas-fired power plants.

The remaining demand after all renewable and coal power plants are utilized is covered by the gas-fired power plants, first with CCGT and then with OCGT power plants. CCGT power plants have higher investment cost and lower marginal cost than OCGT power plants and do thus cover the medium load. OCGT power plants, on the other hand, operates as a strategic reserve in the market and do only generate electricity when all other power plants are in use. Moreover, we find consistency between our results and the policy targets in Energiwende. Our approach indicates that 70 % of the total dispatch is generated by renewables by 2040 compared to the target 67 % [42].
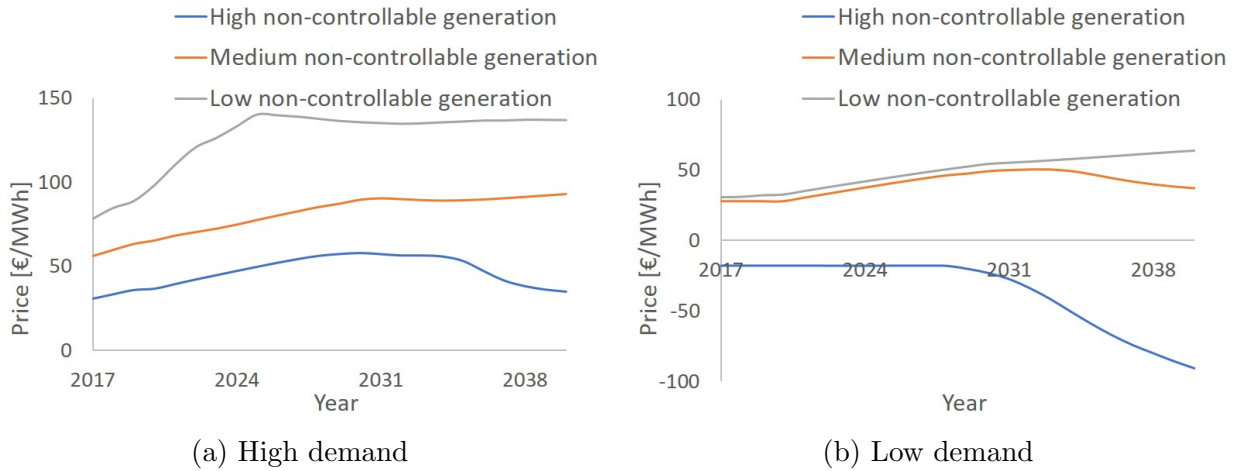
(a) High demand

(b) Low demand

Figure 4: Power prices in selected load segments when non-controllable dispatch is high, medium and low, 2017–2040.

In Figure 4, power prices are presented for selected segments with varying load and renewable generation. We notice that an increased share of renewables leads to lower power prices when the renewable normalized production is high, and higher prices otherwise. Additionally, the increase of non-controllable wind and solar power plants results in larger fluctuations in the power price between segments. Neither of our findings are surprising. The market clearing returns a low price when dispatch is high, and the supply of renewables is inflexible because we assume every solar and wind power plant to have the same normalized generation, respectively.
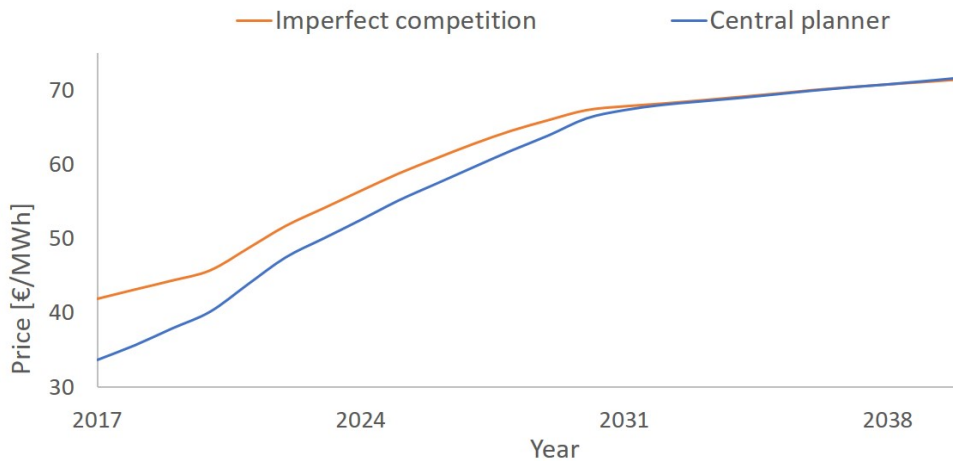

Figure 5: Average power prices in the imperfectly competitive market and the market governed by a central planner, 2017–2040.

Next, we compare power prices for an imperfectly competitive market to a regulated one, where investments and dispatch are determined by a benevolent central planner. In contrast to competitive firms, the central planner aims to maximize the total welfare gain

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from capacity investments. Consequently, the regulated market gains a higher social welfare and consumer surplus than the imperfectly competitive market at the expense of producer surplus. In Figure 5, we see that for the first 13 years of the analysis period, the average power price is lower in the regulated market than in the imperfectly competitive market. In the latter, firms in possession of market power control a high share of the installed controllable capacity. They exhibit market power by withholding dispatch to keep the power price and the producer surplus high, especially during peak periods. After 2030, a significant part of the base load capacity with low marginal cost has been phased out and the Cornout firms have lost market power. As a result, the average price in the imperfectly competitive market converges to the average price in the regulated.

5.2. Capacity payments

In the optimal dispatch problem for the market (22)-(29), we included the objective of a regulator to avoid rationing by forcing this authority to pay the cost of rationing. This affects the marginal value of additional capacity for the firm through the joint market constraint (12) although firms do not carry the cost of rationing. For comparison, we present capacity investment and dispatch in a market excluding the perspective of a regulator in Figure 6. We observe that the installed capacity in, and dispatch from, gas-fired power plants are reduced significantly compared to when the objective of the regulator is included in the optimal dispatch problem. Not surprisingly, we notice a significant level of rationing within 2040.

(a) Installed capacity
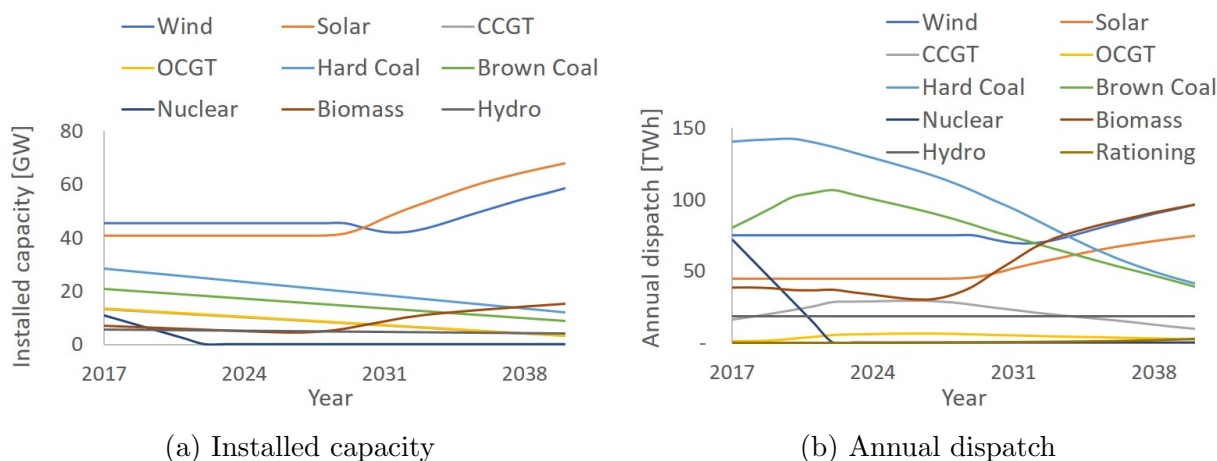
(b) Annual dispatch

Figure 6: Installed capacity and annual dispatch in the market without a regulator, 2017–2040.

To avoid rationing, the regulator must incentivize investments in peak load capacity, e.g. by introducing capacity payments. We argue for a capacity payment of 31 000 €/MW in Appendix C. Higher levels make peak load power plants profitable even with a minor generation, and lower levels leads to minor peak load investments compared to what is optimal for the regulator, as shown in Table C.20 and C.21. It is reasonable for the regulator to make capacity payments to gas-fired power plants as these have the lowest emissions

and easily can be ramped up and down. Figure 7a present the installed capacity when both OCGT and CCGT power plants receive capacity payments. This leads to excessive investments in CCGT plants at the expense of renewables. Now we notice a renewable dispatch share of 42 % in 2040.

With capacity payments for installed capacity of OCGT as illustrated in Figure 7, we find a capacity mix similar to the one presented in Figure 3a. However, we do not manage to replicate the perspective of a regulator by introducing capacity payments, as the renewable dispatch is 67 % in 2040 and rationing still is present, but to a minor extent. Nevertheless, we argue that introducing capacity payments of 31 000 €/MW, i.e. 7.2 % of the investments cost of OCGT power plants for installed capacity returns results closest to the base case where the objective of avoiding rationing is included. Additionally, we obtain a renewable dispatch share equal to the target of 67 % in Energiwende.

(a) OCGT and CCGT   (b) OCGT
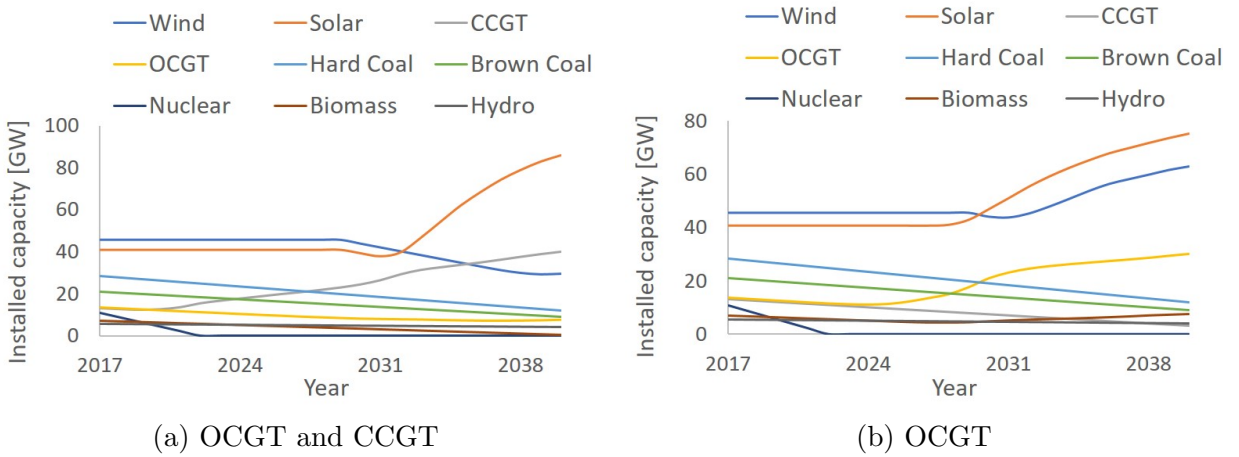
Figure 7: Installed capacity with capacity payments of 31 000 €/MW, 2017–2040.

6. Concluding remarks

This paper examines how competitive firms with different levels of market power invest in additional capacity in a real options context. Our framework considers several features, including heterogeneous technologies, an endogenous electricity price and time-varying demand. The aim is to provide regulators and policymakers with a better understanding of the investment behavior in imperfectly competitive power markets.

In an illustrative example, we find that the large firms use their market power to keep prices up instead of defending their market shares. Increasing their dispatch reduces the power price. Thus, their incentive to invest in additional capacity is small compared to price-taking firms. If too much market power is exercised, new firms will enter the market and the intensity of competition is boosted.

Furthermore, we have applied our framework to the German power market while considering the objective of competitive firms and a regulator. We find that a high renewable penetration results in large price fluctuations. However, investments in peak load capacity

provide a controllable dispatch that limits price spikes. Although the installed capacity in coal-fired power plants diminishes over the analysis period, their utilization is high. The regulator should introduce incentives to curtail carbon emitting dispatch from coal-fired power plants, e.g. emission performance standards or increasing carbon prices. Furthermore, we find that capacity payment is a helpful instrument to avoid high levels of rationing and that the design of the capacity payments affects the capacity mix. We argue that capacity payments to technologies with low investment and high marginal costs have benefits that balance the disadvantages of a high share of renewables.

Although our framework finds that capacity payments to peak load power plants help to complete the targets in Energiwende, we should keep in mind that several properties of the German power market are left out, e.g. offshore wind power plants, batteries, cross-border interconnections and carbon capture and storage. Extensions of our framework are possible in several directions. One may extend the optimal stopping problem to allow for time dependent costs. However, this might be at the expense of being able to obtain an analytical solution of the optimal stopping problem. Another possible extension is to quantify the inaccuracy of assuming myopia when firms and technologies differ in terms of market power and costs. The equilibrium model may also be extended to include a power grid with several nodes, where our framework can be used to examine a regulator's optimal grid investment behavior.

Appendix A. Input parameters of illustrative example

Table A.6: Cost data

	Wind	Solar	CCGT	Coal
Fuel cost [€/MWh]			41.20	11.61
Emission cost [€/MWh]			7.62	21.71
Marginal cost, c [€/MWh]	0	0	48.83	33.32
O. & M. cost, OMC [€/MWy]	38 000	17 000	18 000	42 000
Investment cost, I [€/MW]	760 000	650 000	730 000	2 380 000

Operation and maintenance costs and investment costs are extracted from [43]. For solar and wind power, we use data from 2030. Fuel and emission costs are extracted from Table B.13. [43] summarizes cost data from several sources. Validation of the different sources presented in [43] is outside the scope of this paper.

Table A.7: Initial installed capacity

	Wind	Solar	CCGT	Coal
Firm 1 [MW]	0	0	0	40 000
Firm 2 [MW]	15 000	20 000	10 000	10 000
Competitive fringe [MW]	20 000	20 000	5 000	15 000

Table A.8: Parameters for demand shock process, discount rate and β_1

Y_0	μ	σ	ρ	β_1	S	T
1	0.01	0.03	0.04	3.58	10	50

Table A.9: Y^{grid} and γ_i

Y_0^{grid}	$Y_s - 0.3, Y_s - 0.2, \ldots, Y_s + 0.3$
γ_i	$0.27, 0.54, 0.81, \ldots, 3.51$

Appendix B. Input data case study Germany

All costs are stated for 2017, 2020, 2030 and 2040. We use linear interpolation to find costs between these years. After 2040, we assume all costs to be fixed at 2040 levels. Validation of data sources are outside the scope of this paper.

Table B.10: Investment costs [€/kW]

Technology	2017	2020	2030	2040
Wind	980	860	760	660
Solar	980	800	650	500
Biomass	2 350	2 400	2 300	2 220
Hydro	1 600	1 600	1 570	1 570
Nuclear	6 480	6 480	6480	6480
Hard coal	1 940	1 940	1 940	1940
Brown coal	2 380	2 380	2 380	2 380
OCGT	430	430	430	430
CCGT	730	730	730	730

Investment costs are extracted from [43]. We assume all wind power to be onshore and assume that all hydropower plants are impoundment facilities. There are few unused reservoirs in Germany today. Hence, we use cost data for refurbishing existing hydropower plants with reservoirs.

Table B.11: Fixed operation and maintenance costs [€/kW]

Technology	2017	2020	2030	2040
Wind	44	41	38	37
Solar	18	17	17	17
Biomass	63	63	63	63
Hydro	65	65	65	65
Nuclear	198	198	198	198
Hard coal	54	54	54	54
Brown coal	42	42	42	42
OCGT	17.5	17.5	17.5	17.5
CCGT	18	18	18	18

Operation and maintenance costs are extracted from [43].

Table B.12: Marginal costs [€/MWh]

Technology	2017	2020	2030	2040
Wind				
Solar				
Biomass	30.8	31.4	33.5	34.6
Hydro				
Nuclear	7.1	7.1	7.1	7.1
Hard coal	28.0	28.0	52.4	64.6
Brown coal	23.6	23.6	48.1	60.3
OCGT	67.1	67.1	84.4	93.0
CCGT	44.8	44.8	60.1	66.2

Marginal costs are extracted from [44] and [45], as well as the input data presented in table B.13.

Table B.13: CO_2-prices, gas prices, hard coal prices, lignite fuel costs and plant deficiencies

	2017	2020	2030	2040
CO_2-prices [€/tCO_2]	20	20.0	60.0	80.0
Gas price [€/MMBut]	7.2	7.2	7.2	7.2
Hard coal [€/t]	60.6	60.6	60.6	60.6
Fuel cost of brown coal plants [€/MWh$_{el}$]	11.6	11.6	11.6	11.6
Efficiency				
Hard coal	47 %	47 %	47 %	47 %
OCGT	42 %	42 %	42 %	42 %
CCGT	59 %	59 %	59 %	59 %

CO_2-prices, gas prices and coal prices are based on a path between the carbon neutral scenario and the 4 degree scenario presented in [46]. The efficiency of the different plants and fuel cost of brown coal plants are found in [44].

Table B.14: Initial installed capacity [MW]

Technology	RWE	Uniper	Vattenfall	EnBW	Comp. fringe	Total
Wind	0	0	19	400	44580	45510
Solar	0	0	0	0	40850	40850
Biomass	0	0	40	109	6911	7060
Hydro	0	1895	2600	1095	0	5590
Nuclear	6482	0	0	2712	1606	10800
Hard coal	4098	3200	2800	3430	14850	28380
Brown coal	12756	900	164	393	6687	20900
OCGT	662	1813	911	416	9236	13038
CCGT	2523	1528	0	0	9576	13627

Total installed capacity in Germany is found in [47] and [48]. The initial installed capacity of RWE, Uniper, Vattenfall and EnBW is found in [49], [50], [51], [52] and [48].

Table B.15: Minimum demand in the differen load segments [MWh/h]

Load segment	1	2	3	4	5
Q_l^{min}	55 000	40 000	30 000	20 000	15 000

Table B.16: Energy constrained technologies

	RWE	Uniper	Vattenfall	EnBW	Comp. fringe	Total
Hydro, E_k^{max} [TWh/y]	0	6.47	8.88	3.74	0	19.09

We assume that hydropower is the only technology with a binding energy constraint. Furthermore, we assume that the amount of energy in the different reservoirs are distributed equally to the installed capacity. The total energy availability is found in [53].

Table B.17: Technical lifetime and annual phase-out rate of the different technologies

Technology	Lifetime [y]	Phase-out rate for existing plants
Wind	25	*0.0400
Solar	25	*0.0400
Biomass	25	0.0400
Hydro	40	0.0125
Nuclear	60	**0.0167
Hard coal	40	0.0250
Brown coal	40	0.0250
OCGT	30	0.0333
CCGT	30	0.0333

Technical lifetime is found in [43]. The phase-out rate is estimated with the formula *Annual phase-out* $rate = \frac{1}{Lifetime}$.

*A considerable share of the wind and solar investments are made within the last few years [47]. Thus, we assume that no wind and solar plants are phased out before 2030.

**Germany has decided to phase out nuclear power by 2022 [54]. For simplicity, we assume a linear phase-out of installed capacity between 2017 and 2022.

Table B.18: Simulation parameters

Y_0	μ	σ	ρ	β_1	S	T	*Utilization rate*
1	-0.0029	0.023	0.04	13.92	23	40	0.76

Power plants can not be utilized at all times. Thus, we define a utilization rate reflecting the down time of conventional power plants and the desire to have back-up capacity installed. The utilization rate is calibrated by dividing maximum observed demand for one hour in Germany in 2016 by the total installed controllable capacity.

Table B.19: Y^{grid} and γ_i

Y_0^{grid}	$Y_s - 0.3, Y_s - 0.2, \ldots, Y_s + 0.3$
γ_i	1, 2, 2, $\ldots$, 13

Appendix C. Determining the level of capacity payment

When implementing capacity payments, the reliability of supply is a major concern [27]. Thus, we argue that capacity payments should be determined such that the level of installed peak load capacity, i.e. gas plant capacity, approximates the capacity found in section 5.1. Table C.20 presents the installed capacity of gas-fired power plants, the share of renewables in 2040, the aggregated rationing from 2017 to 2040 found in section 5.1 and the first paragraph of section 5.2, i.e. when no capacity policy is implemented. Table C.21 shows the effect of different capacity payment schemes. We do not examine capacity payments above 31 000 €/MW, as these make OCGT power plants profitable even with minor operation.

Table C.20: Installed capacity of gas-fired power plants, renewable dispatch in 2040, and aggregated rationing from 2017 to 2040 under different policy schemes

	Base case	No capacity policy
Gas capacity [GW]	34	6
Renewable dispatch	70 %	73 %
Rationing 2017-2040 [TWh]	0	16

Table C.21: Installed capacity of gas-fired power plants, renewable dispatch in 2040, and aggregated rationing from 2017 to 2040 under different policy schemes

	25 000 €/MW		28 000 €/MW		31 000 €/MW	
	Gas	OCGT	Gas	OCGT	Gas	OCGT
Gas capacity [GW]	35	18	40	26	48	33
Renewable dispatch	48 %	71 %	45 %	69 %	42 %	67 %
Rationing 2017-2040 [TWh]	5	9	2	5	1	2

Table C.21 shows that capacity payments for all types of gas-fired power plants results in high investments in gas-fired power plants at the expense of renewables. Further, we find that a capacity payment of 31 000 €/MW to OCGT power plants results in a small level of rationing and an installed capacity of gas-fired power plants close to the base case, where the objective of a regulator is included. Therefore, we argue for capacity payments of 31 000 €/MW to OCGT power plants.

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Appendix

A Numerical solution heuristic

The stochastic control problems stated in the preceding papers cannot be solved analytically. Hence, the results presented in the papers are based on a semi-analytical heuristic. To provide the reader with a better understanding of our results, we complete an in-depth description of our solving procedure. The heuristic is based on two layers of Monte Carlo simulations in discrete time. The first layer of simulations obtains a solution to the optimal stopping problem. Based on the solutions to a series of optimal stopping problems, the second layer of simulations approximates a solution to the stochastic control problem. Section A.1 presents the procedure for solving the optimal stopping problem, and section A.2 provides a procedure for approximating a solution to the stochastic control problem.

A.1 Optimal stopping problem

Consider an initial capacity K_0, a set of firms $\mathbf{F}$, a set of available technologies $\mathbf{K}$ and a set of segments $\mathbf{H}$. Further, assume that every investment has an infinite lifetime. The procedure described below then allows us to find the optimal investment triggers for each firm and technology.

1. Assume that the current shock level are Y_0. Define a time grid $T^{grid} = (0, t_1, t_2, \ldots, T)$ and a set of scenarios $\mathbf{N} = (1, 2, \ldots, N)$. For each scenario in $\mathbf{N}$, let Y_t sample from a Geometric Brownian Motion starting at Y_0. This means that one creates N samples of the stochastic process Y_t over T^{grid}. These paths can be described as $(Y_{0,n}, Y_{t_1,n}, \ldots, Y_{T,n})$ for $n = 1, \ldots, N$.

2. At each point on the T^{grid} in every simulation, solve the optimal dispatch problem for the entire market and compute the Lagrangian multipliers of the capacity constraints of each firm f using each available technology k in each segment h, $\lambda_{f,k,h}(Y_{t,n}, K_0)$. The Lagrangian multiplier represents the marginal value of additional capacity of technology k in load segment h for firm f.

3. For each scenario in $\mathbf{N}$, compute firm f's marginal value $M_{f,k,n}(Y_0, K_0)$ of additional capacity of technology k. The marginal value represents the value of having

one additional unit of capacity over time T.

$$M_{f,k,n}(Y_0, K_0) = \sum_{t=0}^{T} \sum_{h=1}^{d(\mathbf{H})} \lambda_{f,k,h}(Y_{t,n}, K_0) e^{-\rho t}$$
$$- \frac{OMC_k}{\rho}, \qquad f \in \mathbf{F}, k \in \mathbf{K} \quad \text{(A.1)}$$

By averaging over the N scenarios, one determines the expected value of the marginal investment starting at Y_0. This value is a good approximation to the particular solution of the partial differential equation stating the optimal stopping problem.

$$V_{f,k,p}(Y_0, K_0) \approx \mathbf{E}\left[M_{f,k,n}(Y_0, K_0)\right] =$$
$$\hat{V}_{f,k}(Y_0, K_0) - \frac{OMC_k}{\rho}, \qquad f \in \mathbf{F}, k \in \mathbf{K} \quad \text{(A.2)}$$

where

$$\hat{V}_{f,k}(Y_0, K) = \mathbf{E}\left[\sum_{t=0}^{T} \sum_{l=1}^{d(\mathbf{L})} \lambda_{f,k,l}(Y_{t,n}, K_0) e^{-\rho t} \right] =$$
$$\frac{1}{N} \left[\sum_{n=1}^{N} \sum_{t=0}^{T} \sum_{l=1}^{d(\mathbf{L})} \lambda_{f,k,l}(Y_{t,n}, K_0) e^{-\rho t} \right], \qquad f \in \mathbf{F}, k \in \mathbf{K}. \quad \text{(A.3)}$$

4. The preceding procedure is repeated for every point of a grid of initial values of Y so that $Y_0^{grid} = (Y_0^1, Y_0^2, \ldots, Y_0^G)$. Consequently, we find $\hat{V}_{f,k}(Y_0^g, K_{0,k})$, where $g = 1, \ldots, G$ is the number of points on the Y_0^{grid}.

5. For firm f with technology k, one performs a non-negative least square regression $\bar{R}_{f,k}$ as an estimate of $\hat{V}_{f,k}$ using a power function of Y_0 as the explanatory variable, where $\gamma_i < \beta_1, i = 1, \ldots d(\gamma)$.

$$\bar{R}_{f,k}(Y) = \sum_{i=1}^{d(\gamma)} a_{f,k,i} Y^{\gamma_i}, \qquad f \in \mathbf{F}, k \in \mathbf{K} \quad \text{(A.4)}$$

$$a_{f,k,i} \geq 0 \qquad f \in \mathbf{F}, k \in \mathbf{K}, i = 1, \ldots, d(\gamma). \quad \text{(A.5)}$$

6. When assuming myopia and using the regression coefficients from the regressions in equation (A.4), one solves the optimal stopping problem. The investment trigger for firm f with technology k is given by the solution of equation (A.6) with respect to $Y_{f,k}^*$.

$$\sum_{i=1}^{d(\gamma)} a_{f,k,i} Y_{f,k}^{*\,\gamma_i} \left(\frac{\beta_1 - \gamma_i}{\beta_1}\right) = I_k + \frac{OMC_k}{\rho}, \qquad f \in \mathbf{F}, k \in \mathbf{K}. \quad \text{(A.6)}$$

A.2 Stochastic control problem

After solving the optimal stopping problem, we find an approximate solution to the stochastic control problem. This procedure finds the optimal investment path, the expected discounted social welfare, profits and investment costs.

1. Assume that all the investment triggers $Y^*_{f,k}$ has been computed. Firm f invests marginally $\Delta\kappa$ in technology k if $Y^*_{f,k} < Y_t$ and $Y^*_{f,k} < Y^*_{f',k}$, $f' = 1, \ldots, d(\mathbf{F})$, $f' \neq f$. If no investments are optimal, jump to step 3.

2. After a marginal investment is made, additional investments might be optimal. Thus, one repeats the procedure described in appendix A.1 and in step 1 until no further investments are optimal.

3. After finding the optimal installed capacity at time t, one computes the instantaneous social welfare ψ, the instantaneous consumer surplus cs and the instantaneous producer surplus π. If rationing occurs, compute the instantaneous cost of rationing as well.

4. Create a second time grid $S^{grid} = (0, 1, \ldots, S)$, where S is the analysis period we use to solve the optimal stopping problem. For each point on the S^{grid}, repeat the procedure described in section A.1. For each time repeating the procedure, update the T^{grid} described in section A.1, step 1, so that the first point on the T^{grid} equals the current point on the S^{grid}. Invest marginally with the procedure in step 1 and compute the surpluses as described in step 3.

5. Create a second set of scenarios $\Omega = (1, 2, \ldots, d(\Omega))$ to compute the expected discounted surpluses. Repeat the entire problem for every scenario in Ω.

6. Compute the expected discounted social welfare, the expected discounted producer surplus, the expected discounted consumer surplus and the expected discounted investment costs. If rationing occurs, compute the expected discounted cost of rationing as well.

B   Real options derivations

In this appendix, we derive the value of the option to invest, the particular solution to the optimal stopping problem and the trigger equation. We use the notation in [3]. Corresponding derivations are valid in [2], but they have a different notation.

B.1   The value of the investment option

Due to the assumption of myopia, we have approximated the solution to a stochastic control problem using an optimal stopping problem. The optimal stopping problem is derived using dynamic programming and Ito's Lemma. We start out by the Bellman equation.

$$\rho V_{f,k}(Y_t, K_t)dt = \frac{\partial \pi_{f,k}(Y_t, K_t)}{\partial K_{t,f,k}}dt + \mathbf{E}[dV_{f,k}(Y_t, K_t)] \quad f \in \mathbf{F}, k \in \mathbf{K}. \quad \text{(B.1)}$$

Ito's Lemma implies

$$dV_{f,k}(Y_t, K_t) = \frac{\partial V_{f,k}(Y_t, K_t)}{\partial t}dt + \frac{\partial V_{f,k}(Y_t, K_t)}{\partial Y_t}dY_t$$
$$+ \frac{\partial^2 V_{f,k}(Y_t, K_t)}{\partial Y_t^2}(dY_t)^2 \quad f \in \mathbf{F}, k \in \mathbf{K}. \quad \text{(B.2)}$$

As $V_{f,k}$ is independent of t, we know that $\frac{\partial V_{f,k}}{\partial t} = 0$. Furthermore we know that Y_t follows a geometric Brownian motion. Hence, we re-write equation (B.2).

$$dV_{f,k}(Y_t, K_t) = \frac{\partial V_{f,k}(Y_t, K_t)}{\partial Y_t}(\mu Y_t dt + \sigma Y_t dz)+$$
$$\frac{\partial^2 V_{f,k}(Y_t, K_t)}{\partial Y_t^2}(\frac{1}{2}\sigma Y_t^2 dt) \quad f \in \mathbf{F}, k \in \mathbf{K} \quad \text{(B.3)}$$

$$\mathbf{E}[dV_{f,k}(Y_t, K_t)] =$$
$$\left(\mu Y_t \frac{\partial V_{f,k}(Y_t, K_t)}{\partial Y_t} + \frac{1}{2}\sigma Y_t^2 \frac{\partial^2 V_{f,k}(Y_t, K_t)}{\partial Y_t^2}\right)dt \quad f \in \mathbf{F}, k \in \mathbf{K}. \quad \text{(B.4)}$$

When substituting (B.4) into the Bellman equation in (B.1), we get

$$\rho V_{f,k}(Y_t, K_t)dt = \frac{\partial \pi_{f,k}(Y_t, K_t)}{\partial K_{t,f,k}}dt$$
$$+ \left(\mu Y \frac{\partial V_{f,k}(Y_t, K_t)}{\partial Y_t} + \frac{1}{2}\sigma Y_t^2 \frac{\partial^2 V_{f,k}(Y_t, K_t)}{\partial Y_t^2}\right) \quad f \in \mathbf{F}, k \in \mathbf{K}. \quad \text{(B.5)}$$

$$\frac{1}{2}\sigma Y_t^2 \frac{\partial^2 V_{f,k}(Y_t, K_t)}{\partial Y_t^2} + \mu Y \frac{\partial V_{f,k}(Y_t, K_t)}{\partial Y_t} - \rho V_{f,k}(Y_t, K_t)$$
$$+ \frac{\partial \pi_{f,k}(Y_t, K_t)}{\partial K_{t,f,k}} = 0, \quad f \in \mathbf{F}, k \in \mathbf{K}. \quad \text{(B.6)}$$

We then use the convenience yield to substitute $\mu = \rho - \delta$ into (B.6). This leaves us with the Bellman equation from the optimal stopping problem in [3].

$$\frac{1}{2}\sigma^2 Y_t^2 \frac{\partial^2 V_{f,k}(Y_t, K_t)}{\partial Y_t^2} + (\rho - \delta)Y_t \frac{\partial V_{f,k}(Y_t, K_t)}{\partial Y_t} - \rho V_{f,k}(Y_t, K_t)$$
$$+ \frac{\partial \pi_{f,k}}{\partial K_{t,f,k}}(Y_t, K_t) = 0 \quad f \in \mathbf{F}, k \in \mathbf{K}, \quad \text{(B.7)}$$

with boundary conditions

$$V_{f,k}(0, K_t) = 0 \qquad f \in \mathbf{F}, k \in \mathbf{K} \qquad \text{(B.8)}$$

$$V_{f,k}(Y_t^*, K_t) = I_k \qquad f \in \mathbf{F}, k \in \mathbf{K} \qquad \text{(B.9)}$$

$$\frac{\partial V_{f,k}(Y_t^*, K_t)}{\partial Y_t^*} = 0 \qquad f \in \mathbf{F}, k \in \mathbf{K}. \qquad \text{(B.10)}$$

Equation (B.8) ensures that the option to invest in new capacity is zero when the value of the capacity expansion is zero. (B.9) and (B.10) are the value-matching and the smooth-pasting conditions for an incremental investment in new capacity. Solving equation (B.7) with respect to $V_{f,k}(Y_t, K_t)$ gives

$$V_{f,k}(Y_t, K_t) = A_{1,f,k}(K_t)Y_t^{\beta_1} + A_{2,f,k}(K_t)Y_t^{\beta_2} + V_{f,k,p}(Y_t, K_t) \quad f \in \mathbf{F}, k \in \mathbf{K} \quad \text{(B.11)}$$

where $V_{f,k,p}(Y_t, K_t)$ is the particular solution to the optimal stopping problem. From the boundary condition in equation (B.8) we know that $A_{2,f,k}(K_t) = 0$. Thus, the value of the investment option can be stated

$$V_{f,k}(Y_t, K_t) = A_{1,f,k}(K_t)Y_t^{\beta_1} + V_{f,k,p}(Y_t, K_t) \quad f \in \mathbf{F}, k \in \mathbf{K}. \qquad \text{(B.12)}$$

$V_{f,k,p}(Y_t, K_t)$ is derived in Appendix B.2.

B.2 The particular solution to the optimal stopping problem

To obtain an analytical solution to the optimal stopping problem, we employ the following regression as an estimate of $\pi_f(Y_t, K_t)$.

$$\bar{\pi}_{f,k}(Y_t, K_t) = \sum_{i=1}^{d(\gamma)} b_{f,k,i}(K_t)Y_t^{\gamma_i} - OMC_k K_{t,f,k} \qquad f \in \mathbf{F}, k \in \mathbf{K}. \qquad \text{(B.13)}$$

Hence, we state firm f's marginal profit from technology k with respect to $K_{t,f,k}$

$$\frac{\partial \bar{\pi}_{f,k}(Y_t, K_t)}{\partial K_{t,f,k}} = \sum_{i=1}^{d(\gamma)} \frac{\partial b_{f,k,i}(K_t)}{\partial K_{t,f,k}}Y_t^{\gamma_i} - OMC_k \qquad f \in \mathbf{F}, k \in \mathbf{K}. \qquad \text{(B.14)}$$

The particular solution equals the expected marginal profit over an infinite time horizon. As Y_t follows a geometric Brownian motion, we find $V_{f,k,p}(Y_t, K_t)$

$$V_{f,k,p}(Y_t, K_t) = \sum_{i=1}^{d(\gamma)} \int_0^\infty \mathbf{E}[Y^{\gamma_i}] e^{-\rho t} dt \left[\frac{\partial b_{f,k,i}(K_t)}{\partial K_{t,f,k}} \right] -$$
$$\int_0^\infty OMC_k e^{-\rho t} dt \quad f \in \mathbf{F}, k \in \mathbf{K} \quad (B.15)$$

$$V_{f,k,p}(Y_t, K_t) = \sum_{i=1}^{d(\gamma)} \int_0^\infty Y^{\gamma_i} e^{[\gamma_i \mu + \frac{1}{2}\sigma^2 \gamma_i(\gamma_i - 1)] - \rho t} dt \left[\frac{\partial b_{f,k,i}(K_t)}{\partial K_{t,f,k}} \right] -$$
$$- \frac{OMC_k}{\rho} \quad f \in \mathbf{F}, k \in \mathbf{K} \quad (B.16)$$

$$V_{f,k,p}(Y_t, K_t) = \sum_{i=1}^{d(\gamma)} \frac{Y^{\gamma_i}}{\rho - \mu\gamma_i - \frac{1}{2}\sigma^2\gamma_i(\gamma_i - 1)} \left[\frac{\partial b_{f,k,i}(K_t)}{\partial K_{t,f,k}} \right]$$
$$- \frac{OMC_k}{\rho} \quad f \in \mathbf{F}, k \in \mathbf{K}. \quad (B.17)$$

This corresponds to

$$V_{f,k,p}(Y_t, K_t) = \sum_{i=1}^{d(\gamma)} \bar{b}_{f,k,i}(\gamma_i, K_t) Y_t^{\gamma_i} - \frac{OMC_k}{\rho} \quad f \in \mathbf{F}, k \in \mathbf{K} \quad (B.18)$$

where the coefficients $\bar{b}_{f,k,i}$ are given by

$$\bar{b}_{f,k,i}(\gamma_i, K_t) = \left[\frac{\partial b_{f,k,i}(K_t)}{\partial K_{t,f,k}} \right] \frac{1}{\rho - \mu\gamma_i - \frac{1}{2}\sigma^2 + \gamma_i(\gamma_i - 1)} \quad f \in \mathbf{F}, k \in \mathbf{K}. \quad (B.19)$$

B.3 Investment triggers

We have shown that the the value of the option to invest marginally can be stated as

$$V_{f,k}(Y_t, K_t) = A_{1,f,k}(K_t) Y_t^{\beta_1} + \sum_{i=1}^{d(\gamma)} \bar{b}_{f,k,i}(\gamma_i, K_t) Y_t^{\gamma_i}$$
$$- \frac{OMC_k}{\rho} \quad f \in \mathbf{F}, k \in \mathbf{K}. \quad (B.20)$$

We use the value matching and smooth pasting conditions to to obtain an analytical expression of the trigger equation. Due to the assumption of myopia, we hold K_t constant when finding the triggers. The analytical expressions for the value matching and smooth pasting conditions are given by respectively

$$V_{f,k}(Y_t^*, K_t) = A_{1,f,k}(K_t)Y_t^{\beta_1} + \sum_{i=1}^{d(\gamma)} \bar{b}_{f,k,i}(\gamma_i, K_t)Y_t^{\gamma_i}$$

$$-\frac{OMC_k}{\rho} = I_k \quad f \in \mathbf{F}, k \in \mathbf{K} \quad \text{(B.21)}$$

and

$$\frac{\partial V_{f,k}(Y_t^*, K_t)}{\partial Y_t^*} = \beta_1 A_{1,f,k}(K_t)Y_t^{\beta_1-1}$$

$$+ \sum_{i=1}^{d(\gamma)} \gamma_i \bar{b}_{f,k,i}(\gamma_i, K_t)Y_t^{\gamma_i-1} = 0 \quad f \in \mathbf{F}, k \in \mathbf{K}. \quad \text{(B.22)}$$

The smooth pasting equation is rephrased to

$$A_{1,f,k}(K_t) = -\frac{\sum_{i=1}^{d(\gamma)} \gamma_i \bar{b}_{f,k,i}(\gamma_i, K_t)Y_t^{\gamma_i-1}}{\beta_1 Y_t^{\beta_1-1}} \quad f \in \mathbf{F}, k \in \mathbf{K} \quad \text{(B.23)}$$

and substituted into the value-matching equation to find the investment trigger equation

$$\sum_{i=1}^{d(\gamma)} (Y_{t,f,k}^*)^{\gamma_i} \left(\frac{\beta_1 - \gamma_i}{\beta_1} \right) \bar{b}_{f,k,i}(\gamma_i, K_t) = I_k + \frac{OMC_k}{\rho} \quad f \in \mathbf{F}, k \in \mathbf{K}. \quad \text{(B.24)}$$

C Optimal dispatch and the KKT-conditions

In [3] we stated the optimal dispatch problem of each firm as well as the regulators rationing problem as an equilibrium problem. By using the KKT-conditions of the equilibrium problem, we stated that the market clearing could be determined through a single optimization problem. In this appendix, we derive the KKT-conditions of the equilibrium problem and the single optimization problem and show that these are equal. In doing so, we also prove that the two problems return the same result.

A non-linear maximization problem with non-negativity constraints can be stated as

$$\max_{q_{f,k,h}} f(x) \tag{C.1}$$

$$\text{s.t.} \quad g_i(x) \leq 0 \quad (\alpha_i) \quad\quad i = 1, \ldots, m \tag{C.2}$$

$$h_j(x) = 0 \quad (\beta_j) \quad\quad j = 1, \ldots, n \tag{C.3}$$

$$x \geq 0 \tag{C.4}$$

The general formulation of the KKT conditions of this maximization problem is

$$0 \leq \left[-\nabla f(x) + \sum_{i=1}^{m} \alpha_i \nabla g_i(x) + \sum_{j=1}^{n} \beta_j \nabla h_j(x) \right] \perp x \geq 0 \tag{C.5}$$

$$0 \leq \alpha_i \perp -g_i(x) \geq 0 \quad\quad i = 1, \ldots, m \tag{C.6}$$

$$h_j(x) = 0 \quad\quad j = 1, \ldots, n. \tag{C.7}$$

We define x, α, β, $g(x)$ and $h(x)$ using variables, constants, parameters and sets defined in [3].

$$x = \begin{cases} q_{f,k,h}, & f \in \mathbf{F}, k \in \mathbf{K}, h \in \mathbf{H} \\ q_{r,h} & h \in \mathbf{H} \end{cases} \tag{C.8}$$

$$\alpha = \begin{cases} \lambda_{f,k,h} & f \in \mathbf{F}, k \in \mathbf{K_c}, h \in \mathbf{H} \\ \delta_{f,h} & f \in \mathbf{F}, h \in \mathbf{H} \\ \nu_h & h \in \mathbf{H} \end{cases} \tag{C.9}$$

$$\beta = \mu_{f,k,h} \quad f \in \mathbf{F}, k \in \mathbf{K_{nc}}, h \in \mathbf{H} \tag{C.10}$$

$$g(x) = \begin{cases} q_{f,k,h} - K_{f,k}, & f \in \mathbf{F}, k \in \mathbf{K_c} h \in \mathbf{H} \\ \sum_{h=1}^{d(\mathbf{H})} q_{f,k,h} - E_{f,k}^{max} & k \in \mathbf{K_c} \\ -\sum_{f=1}^{d(\mathbf{F})} q_{f,h} - q_{r,h} + Q_h^{min} & h \in \mathbf{H} \end{cases} \tag{C.11}$$

$$h(x) = q_{f,k,h} - Z_{k,h} K_{f,k} \quad f \in \mathbf{F}, k \in \mathbf{K_{nc}}, h \in \mathbf{H} \tag{C.12}$$

Additionally, we define the auxiliary variables

$$\sum_{k=1}^{d(\mathbf{K})} q_{f,k,h} = q_{f,h} \quad f \in \mathbf{F}, h \in \mathbf{H} \tag{C.13}$$

and

$$\sum_{f=1}^{d(\mathbf{F})} q_{f,h} = Q_h \quad h \in \mathbf{H}. \tag{C.14}$$

Substituting (C.11) and (C.12) into (C.6) and (C.7) gives us (16)-(19) from [3]. We define $f^E(x)$ as the objective functions in the equilibrium problem

$$f^E(x) \quad = \quad \begin{cases} \sum_{h=1}^{d(\mathbf{H})} \tau_h \left[P_h(Y, q_{f,h} + q_{-f,h}) q_{f,h} - \sum_{k=1}^{d(\mathbf{K})} c_k q_{f,k,h} \right] \\[2em] \qquad\qquad\qquad\qquad - \sum_{k=1}^{d(\mathbf{K})} OMC_k K_{f,k} \quad f \in \mathbf{F} \\[2em] - \sum_{h=1}^{d(\mathbf{H})} \tau_h c_r q_{r,h} \end{cases} \tag{C.15}$$

such that

$$\nabla f^E(x) = \left[\frac{\partial f^E(x)}{q_{f,k,h}} \bigg|_{k \in \mathbf{K_c}} \quad \frac{\partial f^E(x)}{q_{f,k,h}} \bigg|_{k \in \mathbf{K_{nc}}} \quad \frac{\partial f^E(x)}{q_{r,h}} \right]$$
$$f \in \mathbf{F}, h \in \mathbf{H} \tag{C.16}$$

$$\nabla f^E(x) = \left[\begin{matrix} \tau_h [\frac{\partial}{\partial q_{f,k,h}} P_h(Y_t, q_{f,h} + q_{-f,h}) q_{f,h} - c_k]|_{k \in \mathbf{K_c}} \\ 0 \\ \tau_h [\frac{\partial}{\partial q_{f,k,h}} P_h(Y_t, q_{f,h} + q_{-f,h}) q_{f,h} - c_k]|_{k \in \mathbf{K_{nc}}} \quad 0 \\ 0 \qquad\qquad -\tau_h c_r \end{matrix} \right] \quad f \in \mathbf{F}, h \in \mathbf{H}. \tag{C.17}$$

Substituting (C.17), (C.11) and (C.12) into (C.5) gives us (13)-(15) from [3]. With a linear inverse demand function, we have that

$$P_h(Y_t, q_{f,h} + q_{-f,h}) = Y_t(A_h - b_h Q_h) \quad h \in \mathbf{H}. \tag{C.18}$$

For the Cournot firms, we state

$$\frac{\partial}{\partial q_{f,k,h}} P_h(Y_t, q_{f,h} + q_{-f,h}) q_{f,k} = Y_t(A_h - 2b_h q_{f,k})$$
$$f = 1, \ldots, d(\mathbf{F}) - 1, h \in \mathbf{H}, \tag{C.19}$$

and for the competitive fringe we state

$$\frac{\partial}{\partial q_{f,k,h}} P_h(Y_t, q_{f,h} + q_{-f,h}) q_{f,k} = Y_t(A_h - b_h q_{f,k}) \quad f = d(\mathbf{F}), h \in \mathbf{H}. \quad \text{(C.20)}$$

Equations (C.19) and (C.20) differs as the competitive fringe is a price-taker and assumes its dispatch to not affect the price. This is reflected in the term $b_h q_{f,k}$. By substituting (C.19) and (C.20) into the elements of (C.17), we write

$$\frac{\partial f^E(x)}{q_{f,k,h}}\bigg|_{k \in \mathbf{K}} = \begin{cases} \tau_h[Y_t(A_h - 2b_h q_{f,k}) - c_k] & f = 1, \ldots, d(\mathbf{F}) - 1, h \in \mathbf{H} \\ \tau_h[Y_t(A_h - b_h q_{f,k}) - c_k] & f = d(\mathbf{F}), h \in \mathbf{H} \end{cases} \quad \text{(C.21)}$$

After stating the algebraic expressions for all elements of $\nabla f^E(x)$, we prove that $\nabla f^E(x) = \nabla f^Q(x)$, where $f^Q(x)$ is the objective function of the quadratic optimization problem, stated in (C.22). This is sufficient to prove that the two problems are equal, as the constraints $g(x)$ and $h(x)$ are identical for the two problems.

$$f^Q(x) = \sum_{h=1}^{d(\mathbf{H})} \tau_h \bigg[Y_t(A_h Q_h - \frac{1}{2} b_h Q_h^2 - \frac{1}{2} b_h \sum_{f=1}^{d(\mathbf{F})-1} q_{f,h}^2)$$
$$- \sum_{k=1}^{d(\mathbf{K})} c_k q_{f,k,h} - c_r q_{r,h} \bigg] - \sum_{f=1}^{d(\mathbf{F})} \sum_{k=1}^{d(\mathbf{K})} OMC_k K_{f,k}. \quad \text{(C.22)}$$

We can state

$$\nabla f^Q(x) = \left[\frac{\partial f^Q(x)}{q_{f,k,h}}\bigg|_{k \in \mathbf{K_c}} \quad \frac{\partial f^Q(x)}{q_{f,k,h}}\bigg|_{k \in \mathbf{K_{nc}}} \quad \frac{\partial f^Q(x)}{q_{r,h}} \right]$$
$$f \in \mathbf{F}, h \in \mathbf{H}. \quad \text{(C.23)}$$

where

$$\frac{\partial f^Q(x)}{q_{f,k,h}}\bigg|_{k \in \mathbf{K}} = \begin{cases} \tau_h[Y_t(A_h - 2b_h q_{f,k}) - c_k] & f = 1, \ldots, d(\mathbf{F}) - 1, h \in \mathbf{H} \\ \tau_h[Y_t(A_h - b_h q_{f,k}) - c_k] & f = d(\mathbf{F}), h \in \mathbf{H} \end{cases} \quad \text{(C.24)}$$

and

$$\frac{\partial f^Q(x)}{q_{r,h}} = -\tau_h c_r \quad h \in \mathbf{H} \quad \text{(C.25)}$$

We notice that all the elements of $\nabla f^E(x)$ and $\nabla f^Q(x)$ are identical. Hence, we have proven that the equilibrium problem and the quadratic optimization problem are equivalent.