Models of gas-liquid two-phase flow in drilling for control and estimation applications

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Abstract

Most model-based control and estimation techniques put limitations on the structure and complexity of the models to which they are applied. This has motivated the development of simplified models of gas-liquid two-phase flow in drilling for control and estimation applications. This paper reviews the literature for such models. The models are categorized in terms of complexity and the physical interpretation of the simplifications employed. A simulation study is used to evaluate their ability to qualitatively represent dynamics of 3 different gas-liquid scenarios encountered in drilling, based on which conclusions are drawn.

Keywords: Two-phase flow, Drilling, Well control, Kick Handling, Managed Pressure Drilling, Underbalanced Drilling, Automatic Control, Estimation, Soft Sensing, Distributed Parameter Systems, PDE

1. Introduction

Drilling for hydrocarbons is the process of creating a wellbore, sometimes extending several thousand meters into the ground, until it reaches an oil or gas reservoir (Fig. 1). There is a multitude of risks and challenges associated with this process, not least in regards to controlling the distributed pressure in the well within the constraints imposed by the operation.

Dealing with these challenges has entailed an increasing drive for automation in many aspects of drilling (Thorogood et al., 2010; Godhavn, 2011). Simultaneously, a goal of improved drilling efficiency is pursued through reducing nonproductive time, optimizing operations, and detecting and avoiding incidents before adverse consequences occur (Cayeux et al., 2013). The trend for drilling deeper and more complex wells (Lukawski et al., 2014) is also a driver for automation as an enabling technology, allowing for continued exploration of difficult and mature reservoirs.

Following the demand of the drilling industry, high fidelity simulators of the drilling process have been developed. Applications of these include training of drilling personnel and real time decision support (Petersen et al., 2008; Rommetveit et al., 2004). At the same time, automated control systems for controlling various aspects of the drilling process have been developed and are gradually being accepted by the industry (Santos et al., 2007).

Modern approaches to process monitoring, optimization and control promise to enhance robustness and performance of such automation through the merger of process knowledge encoded in mathematical models with real-time measurements from the process. By intelligently combining predictions from the mathematical model with information from multiple sensors one can estimate unmeasured quantities, optimize automatic control procedures, predict future behavior, and plan countermeasures for unwanted incidents. Such design techniques, often referred to as model based estimation and control (Åström and Murray, 2010; Anderson and Moore, 1990), require a mathematical model with the right balance between complexity and fidelity: i.e. the complexity must be limited to facilitate the use of established mathematical analysis and design techniques, while the qualitative response of the process is retained.

Models that strike the right balance between complexity and fidelity are sometimes referred to as fit-for-purpose models, and have been employed in control (Stamnes et al., 2011a) and monitoring (Willersrud, 2015) of drilling processes in one-phase flow regimes. Obtaining such simplified models becomes significantly more difficult for gas-liquid two-phase dynamics due to the significant complexity and distributed nature of multiphase pipe-flow (Aarsnes et al., 2014b, 2016). This makes the reduction to fit-for-purpose models for scenarios such as gas-kick incidents, and underbalanced operations, challenging.

Consequently, several different approaches have been suggested in the literature, ranging from using complicated high-order numerical schemes with advanced multiphase-flow models to simplified low-order or black-box step response representations. The present paper presents a review of these models used for designing control and estimation/monitoring algorithms of gas-liquid two-phase dynamics encountered in drilling. The survey will focus on the models used and not the methods in themselves.

1.1. Components of a simulation model

To structure the following discussion, it is useful to identify the distinct components which make up a complete simulation model. The three components are *Mathematical structure*, *Closure Relations* and the *Numerical Scheme* and they are summarized in Table 1.

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Nomenclature

c	Sound speed
$m = \alpha_\ell \rho_\ell$	Liquid mass
$n = \alpha_\mathrm{g} \rho_\mathrm{g}$	Gas mass
q	Volumetric flow-rate
t	Time, independent variable
v	Velocity
v_∞	Slip relation drift velocity
w	Mass flow-rate
x	Position, independent variable
C_0	Slip relation profile parameter
F	Frictional pressure loss
G	Gravitational pressure loss
P	Pressure
T	Temperature
V	Volume
α	Volume fraction
β	Bulk modulus
γ	Adiabatic index
ρ	Density
μ	Chemical potential
$\mathcal{J}, \mathcal{K}, \mathcal{M}, \mathcal{H}$	Relaxation coefficients

Subscripts

a	Lumped annulus parameter
c	At or exiting through the choke
d	Lumped drill string parameter
i	Interface
M	Mixture
ℓ	Liquid phase
g	Gas phase
bit	Entering the annulus from the drill string
inj	Injected into the drill string
res	Entering the annulus from the reservoir

Abbreviations

DFM	Drift Flux Model
BHP	Bottom-Hole Pressure: $p_{bh} = P(0)$
LOL-model	Low Order Lumped-model
MPD	Managed Pressure Drilling
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
UBD	Under-Balanced Drilling
WHP	Well-Head Pressure: $p_a = P(L)$

Table 1: The three components of a complete simulation model

Mathematical Structure	**Closure Relations**	**Numerical Scheme**
• PDE or ODE	• Slip law	• Numerical accuracy
• Hyperbolic or Parabolic PDE	• Equation of State	• Numerical stability/robustness
• Number of equations	• Frictional pressure loss	• Implementation complexity
• Stiffness	• Other source terms	• Solution speed

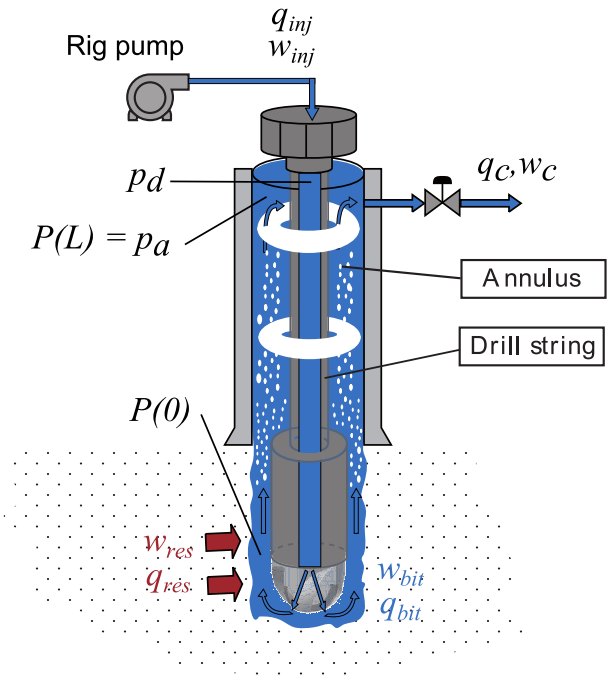

Figure 1: Schematic of the system under consideration.

The complexity of a model is mainly determined by its mathematical structure. This is the type and number of dynamical equations needed to describe the model. Determining the mathematical structure of the model also determines, crucially in our case, the mathematical tools and the model based estimation and control algorithms which can be employed with it.

The *closure relations* that are used will necessarily depend on the mathematical structure of the model. When a model is simplified, the closure relations will often also have to be modified to accommodate for the simplification, typically in such a way as to retain the steady state profile. Closure relations can also be chosen and *tuned* based on experiments or measurements, and consequently, given a mathematical structure, the accuracy of the model will mostly be determined by the value and form of the closure relations chosen.

The final component to a simulation model is the *numerical scheme*. This is the way the mathematical equations are approximated in order for them to be solved numerically. The solution procedures that can be utilized have varying degrees of accuracy and solution speeds. Crucially for our purposes are the numerical stability, robustness and complexity in implementation of the scheme chosen.

1.2. A coarse classification and outline

To structure the paper we split the models found in the literature into the three broad categories according to their overarching mathematical structure.

1.2.1. High fidelity models:

This category encompasses models which are designed to be highly accurate and have a high degree of predictive power over a wide range of application scenarios. They are often used for training, analysis and planning of operations and are not created for the application to a specific scenario or use with mathematical algorithms. They are thus not fit-for-purpose models.

1.2.2. Drift Flux Models (DFM):

We use this category to denote a set of distributed (i.e. PDE) models which are very popular in the literature due to their reasonable accuracy and relative simplicity compared to the high fidelity models. They are essentially simplifications of these models in that they require significantly fewer equations and they can be rigorously derived from the high fidelity models through the process of relaxation of dynamics explained in Section 2.1. The DFMs have reduced predictive power compared to High fidelity models, but high accuracy can be retained through selection of closure relations adapted to the considered scenario. Estimation and control results do exist for these models, although their distributed nature often makes these results non-standard and limited to specialists.

1.2.3. Simplified ODE models:

This category refer to models which can be represented with low order ODEs. This means that the models are unable to represent the full richness of dynamics inherent in distributed models, and consequently they are highly specialized with a limited range of operation. Their simplicity, however, makes them very well suited for the design of algorithms, often allowing for the use of highly effective and well established design procedures.

1.2.4. Outline

This paper will consider each of these categories, review their basic mathematical structure and their uses in control and estimation applications. Then, in section 5 a brief review of numerical schemes is presented. Finally, in Section 6, a simulation study is performed to evaluate the ability of the models to qualitatively capture the dynamics of drilling scenarios involving gas-liquid flow.

1.3. Note on the equations

The equations given in this paper to represent the models are not exhaustive, but are meant to give a general idea of the model properties and structure. The reader is referred to the specific original literature for the full, rigorous, model representations.

2. High fidelity models

In one space dimension, the PDE formulation of dynamic distributed models may be written in a highly general form

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{A}(\mathbf{U}, x, t)\frac{\partial \mathbf{U}}{\partial x} = \mathbf{D}(\mathbf{U}, x, t)\frac{\partial^2 \mathbf{U}}{\partial x^2} + \mathbf{Q}(\mathbf{U}, x, t). \quad (1)$$

Herein, each term affects the *mathematical structure* of the model in its own unique way. In particular:

- $\boldsymbol{U}(x,t)$ represents the vector of unknowns, i.e. the independent variables needed to represent the physical state at each point in space and time. The dimension of the vector

$\boldsymbol{U}$ depends on the level of detail and complexity we want represented in the model. As the dynamical equations represent transport of conserved quantities in physical space, the components of $\boldsymbol{U}$ would typically represent density, momentum and energy. Through variable transformations, the equations could also be expressed in terms of more directly observable physical quantities such as temperature or pressure (Fjelde and Karlsen, 2002).

- $\boldsymbol{A}$ encodes information that propagates with a finite speed across the computational domain, and represents transport effects such as convection or momentum transfer through pressure. The fundamental underlying physical structure of the model is encoded here. Furthermore, this term has a dominating effect on the the velocity of the high-frequency waves in the system (Solem et al., 2015).

- $\boldsymbol{D}$ represents irreversible diffusive effects in the flow direction, such as viscosity, or heat or mass diffusion. Loss of information due to upscaling may be one mechanism behind terms in this form. If the symmetric part of $\boldsymbol{D}$ is positive definite, then $\boldsymbol{D}$ will have a stabilizing effect on the system, acting more strongly on higher frequencies.

  For the discretized equations used in practical simulations, this term is often small compared to the artificial numerical diffusion (Bruce Stewart and Wendroff, 1984).

- $\boldsymbol{Q}$ represents source terms, i.e. exchanges between the state and the environment, or exchanges between separate state variables at each point. Interactions with the environment may include friction terms, gravity terms and heat transfer. Exchanges between state variables may include heat, volume, mass and momentum transfer.

  In the limit that $\boldsymbol{Q}$ acts infinitely fast, we obtain a *reduced model* where the information represented by $\boldsymbol{Q}$ is incorporated into the matrix $\boldsymbol{A}$ (Flåtten and Lund, 2011; Lund, 2012). This model reduction, termed *relaxation*, is illustrated in Figure 2 where the Baer-Nunziato model (Baer and Nunziato, 1986) is the starting point.

  This term has a dominating effect on the velocity of the longest wavelengths in the system (Solem et al., 2015).

2.1. The Baer-Nunziato model

The physical structure of a flow model is mainly determined from the description of how the physical variables are transported, and the process to approach full thermodynamic equilibrium.

Following the classical approach of Baer and Nunziato (1986), we may then write the foundation for one-dimensional two-phase flow modeling as a set of hyperbolic partial differential equations in the following form (Linga, 2015):

- Volume advection:

$$\frac{\partial \alpha_\mathrm{g}}{\partial t} + v_p \frac{\partial \alpha_\mathrm{g}}{\partial x} = \mathcal{J}(P_\mathrm{g} - P_\ell), \tag{2}$$

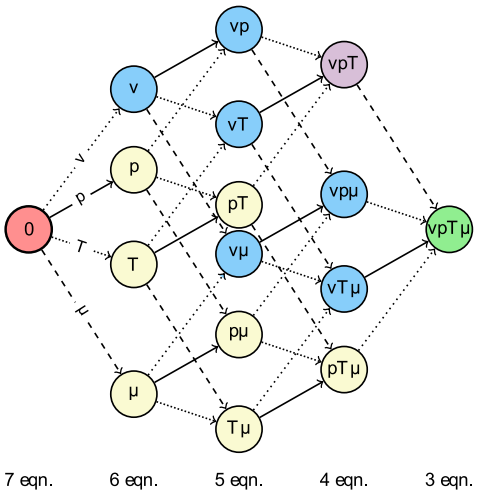

Figure 2: The 4-dimensional hypercube representing the model hierarchy of two-fluid relaxation models. Each vertex represents a unique model in the hierarchy where the leftmost one is the seminal formulation of Baer and Nunziato (1986). The edges correspond to relaxation processes in the variables p (pressure), T (temperature), μ (chemical potential) and v (velocity). Models with a v relaxation are typically referred to as *Drift Flux Models*. Taken from (Linga, 2015).

- Mass conservation:

$$\frac{\partial}{\partial t}\left(\rho_\mathrm{g}\alpha_\mathrm{g}\right) + \frac{\partial}{\partial x}\left(\rho_\mathrm{g}\alpha_\mathrm{g}v_\mathrm{g}\right) = \mathcal{K}(\mu_\ell - \mu_\mathrm{g}), \tag{3}$$

$$\frac{\partial}{\partial t}\left(\rho_\ell\alpha_\ell\right) + \frac{\partial}{\partial x}\left(\rho_\ell\alpha_\ell v_\ell\right) = \mathcal{K}(\mu_\mathrm{g} - \mu_\ell), \tag{4}$$

- Momentum balance:

$$\frac{\partial}{\partial t}\left(\rho_\mathrm{g}\alpha_\mathrm{g}v_\mathrm{g}\right) + \frac{\partial}{\partial x}\left(\rho_\mathrm{g}\alpha_\mathrm{g}v_\mathrm{g}^2 + \alpha_\mathrm{g}P_\mathrm{g}\right) - p_\mathrm{i}\frac{\partial \alpha_\mathrm{g}}{\partial x} = v_\mathrm{i}\mathcal{K}(\mu_\ell - \mu_\mathrm{g}) + \mathcal{M}(v_\ell - v_\mathrm{g}), \tag{5}$$

$$\frac{\partial}{\partial t}\left(\rho_\ell\alpha_\ell v_\ell\right) + \frac{\partial}{\partial x}\left(\rho_\ell\alpha_\ell v_\ell^2 + \alpha_\ell P_\ell\right) + p_\mathrm{i}\frac{\partial \alpha_\mathrm{g}}{\partial x} = v_\mathrm{i}\mathcal{K}(\mu_\mathrm{g} - \mu_\ell) + \mathcal{M}(v_\mathrm{g} - v_\ell), \tag{6}$$

- Energy balance:

$$\frac{\partial E_\mathrm{g}}{\partial t} + \frac{\partial}{\partial x}\left(E_\mathrm{g}v_\mathrm{g} + \alpha_\mathrm{g}P_\mathrm{g}v_\mathrm{g}\right) - p_\mathrm{i}v_p\frac{\partial \alpha_\mathrm{g}}{\partial x} = -p_\mathrm{i}\mathcal{J}(P_\ell - P_\mathrm{g}) + \left(\mu_\mathrm{i} + \frac{1}{2}v_\mathrm{i}^2\right)\mathcal{K}(\mu_\ell - \mu_\mathrm{g}) + v_p\mathcal{M}(v_\ell - v_\mathrm{g}) + \mathcal{H}(T_\ell - T_\mathrm{g}), \tag{7}$$

$$\frac{\partial E_\ell}{\partial t} + \frac{\partial}{\partial x}\left(E_\ell v_\ell + \alpha_\ell P_\ell v_\ell\right) + p_\mathrm{i}v_p\frac{\partial \alpha_\mathrm{g}}{\partial x} = -p_\mathrm{i}\mathcal{J}(P_\mathrm{g} - P_\ell) + \left(\mu_\mathrm{i} + \frac{1}{2}v_\mathrm{i}^2\right)\mathcal{K}(\mu_\mathrm{g} - \mu_\ell) + v_p\mathcal{M}(v_\mathrm{g} - v_\ell) + \mathcal{H}(T_\mathrm{g} - T_\ell). \tag{8}$$

Herein, the terms representing interactions with the environment and diffusive terms are not included. In the context of (1), the left-hand sides of (2)–(8) contribute to the $\partial_t \boldsymbol{U} + \boldsymbol{A}\partial_x \boldsymbol{U}$ terms and the right-hand sides contribute to the $\boldsymbol{Q}$ term.

Furthermore, α_k, ρ_k, v_k, P_k, μ_k and T_k represent, respectively, the volume fraction, density, velocity, pressure, chemical potential and temperature of each phase and the interface $k \in \{\mathrm{g}, \ell, \mathrm{i}\}$, and v_p the volume advection velocity. However, the model is equally valid regardless of whether g and ℓ represent gases, liquids or solids.

The relaxation coefficients $\mathcal{J}, \mathcal{K}, \mathcal{M}$ and $\mathcal{H}$ are all non-negative so as to induce

1. volume transfer towards the phase with the highest pressure,
2. mass transfer towards the phase with the lowest chemical potential,
3. momentum transfer towards the phase with the lowest velocity and
4. heat transfer towards the phase with the lowest temperature.

The model has been extended to three phases, herein care must be taken to ensure compliance with the second law of thermodynamics (Hérard, 2007).

This highly general model is unnecessarily complex for the drilling applications considered in this paper. Significant model reduction is desirable, and one main path of achieving this model reduction is the process of *relaxation*, where the dynamical equations representing non-equilibrium processes are replaced by static equilibrium assumptions. For the Baer-Nunziato model, such model reductions have been studied extensively in for instance (Flåtten and Lund, 2011; Linga, 2015; Lund, 2012).

In this respect, a highly important and relevant concept is the *subcharacteristic condition* (Chen et al., 1994; Solem et al., 2015). Simply put, it states that every reduction of the model through relaxation will reduce the velocity of information propagation in the system, sometimes to a very large degree (Flåtten and Lund, 2011; Lund, 2012). This effect must be considered when evaluating the validity of simplified models for control applications. Another main theme is the discarding of the energy equations, motivated by the assumption that temperature transients have no significant effect on the macroscopic flow dynamics on the time scale of interest. This fundamental simplification will be described in more detail below.

2.2. Discarding the energy equation

Fundamental thermodynamics dictates that density is a state function $\rho(P, T)$. In the context of the Baer-Nunziato model, the pressure dynamics is normally dominated by the mass equations and the temperature dynamics by the energy equations. In the limit $\mathcal{H} \to \infty$ of the Baer-Nunziato model, the energy equations may be replaced by one single energy equation describing the evolution of the common temperature $T = T_\mathrm{g} = T_\ell$.

Further, there are many situation of interest where a temperature distribution $T(x, t) = T(x)$ is known (to sufficient approximation) a priori. This can occur for instance when

1. The flow may be assumed to be always in thermal equilibrium with the environment.
2. The temperature boundary conditions do not change significantly over time, and the heat exchange with the environment is not sensitive to the flow transients.
3. The temperature is measured and can be treated as an exogenous variable.

If phase transitions are important for our application, the dynamics may easily be very sensitive to $T(x, t)$, Otherwise, in situations where any of 1.–3. are valid we may replace the energy PDE with the given approximate temperature distribution $T(x)$, and the density may now be obtained as

$$\rho(P, T) = \rho(P, T(x)) = \rho(P, x). \tag{9}$$

Herein, we remark that this simplification can at best be fully valid only for slow dynamics. The sound speed (giving the propagation velocity of pressure waves) is generally given as

$$c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s \tag{10}$$

However, from (9) we see that the corresponding value $\bar{c}$ in models with no energy equations will be more akin to

$$\bar{c}^2 = \left(\frac{\partial P}{\partial \rho}\right)_T, \tag{11}$$

and hence from fundamental thermodynamic stability

$$\bar{c} \leq c. \tag{12}$$

Hence, we may expect that reducing the model through discarding the energy equation will tend to *under-predict* the velocity of pressure propagation.

2.3. Examples from literature

High fidelity models used for flow assurance and production design include the OLGA (Bendiksen et al., 1991) and LedaFlow (Danielson et al., 2011) commercial simulators. These are both 3-phase flow 1D models, imposing instantaneous pressure equilibrium in all phases. LedaFlow has the option of solving separate energy equations for each phase, and for both simulators the energy equations can be discarded as described in the previous section.

LedaFlow also incorporates additional mass equations to allow for the possibility of the phases being dispersed in each other.

Although not focusing on drilling applications, the commercial real time multiphase flow simulator FlowManager$^{\mathrm{TM}}$ Dynamic (Holmås and Løvli, 2011) is worthy of mention. This is a tool used for on-line surveillance and optimization of subsea oil and gas production. The underlying model consists of 3 mass conservation equations (oil, gas and water), 1 total momentum conservation equation and 1 total energy equation.

Other high-fidelity multiphase flow simulators for drilling include WeMod, developed by the International Research Institute of Stavanger (IRIS) based on a R&D program started in the

early 2000's in collaboration with Petrobras (Rommetveit and Lage, 2001), and a simulator due to Sintef Petroleum Research (Bjørkevoll et al., 2010; Petersen et al., 2008).

Due to their complexity, the application of high-fidelity models to model based control and estimator design is limited to the use with general, high order, numerical schemes. A promising application is the flow metering and estimation of states and parameters in production based on indirect measurements. This application is sometimes referred to as *soft sensing*, see e.g. (Gryzlov, 2011; de Kruif et al., 2008; Lorentzen et al., 2010). Herein, a popular approach is using so-called particle filters, where an ensemble of simulations are performed in parallel with the uncertain states and parameters perturbed. This ensemble is used to develop statistics of the uncertainties which can then be combined with measurements to produce estimates. This approach is still being refined with more sophisticated particle filters being applied to more complicated multiphase models (Luo et al., 2014; Lorentzen et al., 2014, 2015).

3. Drift Flux Models (DFM)

Probably the most widely used model in the literature on two-phase flow in drilling is the 3-PDE drift-flux model

$$\frac{\partial}{\partial t}\left(\alpha_{\mathrm{g}}\rho_{\mathrm{g}}\right)+\frac{\partial}{\partial x}\left(\alpha_{\mathrm{g}}\rho_{\mathrm{g}}v_{\mathrm{g}}\right)=\Gamma \tag{13}$$

$$\frac{\partial}{\partial t}\left(\alpha_{\ell}\rho_{\ell}\right)+\frac{\partial}{\partial x}\left(\alpha_{\ell}\rho_{\ell}v_{\ell}\right)=-\Gamma \tag{14}$$

$$\begin{aligned}&\frac{\partial}{\partial t}\left(\alpha_{\ell}\rho_{\ell}v_{\ell}+\alpha_{\mathrm{g}}\rho_{\mathrm{g}}v_{\mathrm{g}}\right)+\frac{\partial}{\partial x}\left(\alpha_{\ell}\rho_{\ell}v_{\ell}^{2}+\alpha_{\mathrm{g}}\rho_{\mathrm{g}}v_{\mathrm{g}}^{2}+P\right)\\&=-F-G,\end{aligned} \tag{15}$$

where the source terms are: the mass transfer term $\Gamma = \mathcal{K}\left(\mu_{\ell}-\mu_{\mathrm{g}}\right)$ often assumed zero, viscous pressure loss F and gravitational pressure loss $G=\rho_M g\sin\theta$.

This model is equivalent to the four eqn. vpT node in Fig. 2 with, additionally, the energy equation discarded. The two different fields for mass transport (13) and (14) are required since typically the two phases are represented by two or more different components[1]. This model may be obtained from the Baer-Nunziato system by discarding the energy equations and imposing velocity and pressure equilibrium. Herein, velocity equilibrium does not necessarily imply $v_{\mathrm{g}}=v_{\ell}$: Due to averaging and external interactions, it is stated in a more general form known as the *slip relation*:

$$v_{\mathrm{g}}-v_{\ell}=\Phi(\boldsymbol{U}), \tag{16}$$

i.e. the relative velocity between the phases is determined by the local full flow configuration.

The slip relation is one of three *closure relations* needed to complete the model.

[1] e.g. nitrogen and water are different components (Crowe et al., 2011).

3.1. Closure relations

When choosing closure relations for the DFM, two different approaches are taken in the literature. The more complicated approach, sometimes referred to as mechanistic models, predicts local flow regimes and then uses different correlations depending on the regime predicted, see e.g. Fjelde et al. (2003); Perez-Tellez et al. (2003). This typically entails solving implicit relations using an iterative numerical procedure.

The alternative approach is to use flow-pattern independent closure relations which allow a for simpler solution procedure.

3.1.1. Slip relation

For the case of two phase flow in inclined pipes, several flow-pattern independent correlations have been suggested. Recent comparisons of such correlations can be found in (Bhagwat and Ghajar, 2014; Choi et al., 2012).

A highly simple correlation is the *Zuber-Findlay slip law* (Zuber and Findlay, 1965), which determines the relative velocity between the phases. Writing the mixture velocity as $v_M=\alpha_{\mathrm{g}}v_{\mathrm{g}}+\alpha_{\ell}v_{\ell}$ the classical formulation is:

$$v_{\mathrm{g}}=C_0 v_M+v_{\infty}, \tag{17}$$

where C_0 is called the profile parameter and v_{∞} the drift velocity (Zuber and Findlay, 1965; Bhagwat and Ghajar, 2014). This is most naturally interpreted in the context of bubbly flows.

A mechanistic approach for finding correlations, which considers the annular geometry and upward flow encountered in drilling, was developed by Lage and Time (2000); Lage et al. (2000); Lage and Time (2002), which uses 5 different flow patterns *Bubble, Slug, Churn, Annular* and *dispersed bubble*. This approach was further studied by Perez-Tellez (2003); Perez-Tellez et al. (2003) and employed on Iranian field data by Ashena and Moghadasi (2010).

Closure relations for counter current flow was considered by Taitel and Barnea (1983); Hasan et al. (1994).

3.1.2. Viscous pressure loss

A frequently used structure for the relation of viscous pressure loss is

$$F=f\rho_M v_M|v_M|, \tag{18}$$

with f sometimes set as a constant friction coefficient dependent on viscosity and flow geometry, and other times given by the multiphase Reynolds number.

In mechanistic models (18) is used for bubbly flow while other, modified, relations are used for other flow regimes (Lage and Time, 2002).

3.1.3. Equation of state

When the energy equation have been discarded, the *equation of state* is reduced to a relation between the principal variables and the pressure. Assuming the conserved quantities $n=\rho_{\mathrm{g}}\alpha_{\mathrm{g}}$ and $m=\alpha_{\ell}\rho_{\ell}$ to be our principal variables, the simplest relation

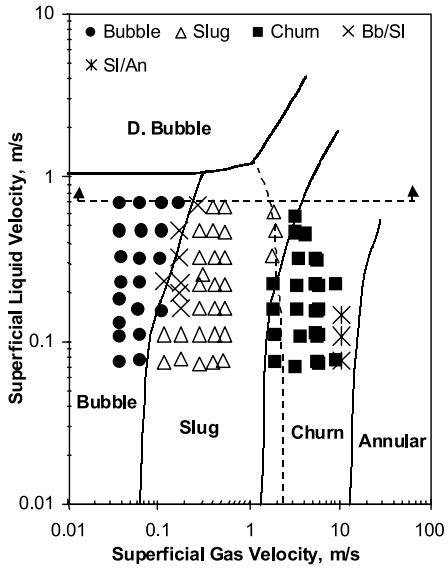

Figure 3: Flow pattern map for air/water of upward flow in an annulus, by Lage and Time (2002).

available is obtained by using the ideal gas law and assuming incompressible liquid

$$P = \frac{nc_{\mathrm{g}}^2}{1 - \frac{m}{\rho_\ell}}, \tag{19}$$

where c_{g}, c_ℓ is the sound speed of the gas and liquid phase respectively. With a compressible liquid, but still using the ideal gas law, the relation becomes more involved. From (Evje and Fjelde, 2002; Aarsnes et al., 2014b):

$$\alpha_{\mathrm{g}} = \frac{1}{2} - \frac{\frac{c_G^2}{c_L^2}n + m + \sqrt{\Delta}}{2\rho_{L,0}}, \tag{20a}$$

$$\Delta = \left(\rho_{L,0} - \frac{c_G^2}{c_L^2}n + m\right)^2 + 4\frac{c_G^2}{c_L^2}\rho_{L,0}, \tag{20b}$$

$$P = \begin{cases} \left(\frac{m}{1-\alpha_{\mathrm{g}}} - \rho_{L,0}\right)c_L^2, & \text{if } 1-\alpha_{\mathrm{g}} > \epsilon \\ \frac{n}{\alpha_{\mathrm{g}}}c_G^2, & \text{otherwise.} \end{cases} \tag{20c}$$

3.2. Model range of operation

Some models have limited ranges of operation due to unboundedness introduced by simplifying assumptions: A model assuming incompressible liquid becomes invalid when gas disappears as the distributed pressure becomes undefined, see (19).

The slip relation can also limit the applicability of the model. By inspecting (17) we see that

$$C_0 < \frac{1}{\alpha_{\mathrm{g}}} \tag{21}$$

must be enforced. Representing the slip in annular flow regimes has proven to be challenging Lage and Time (2002), and it is often recommended to use a two-fluid model (i.e. a model without a relaxed momentum transfer) in such a case (Masella et al., 1998).

3.3. Application to estimation and control

Despite its relative complexity, the DFM has seen some use to state and parameter estimation.

The mechanistic model of (Lage et al., 2000; Lorentzen et al., 2001) was used with an ensemble Kalman filter to tune coefficients in the slip-law and frictional pressure loss in (Lorentzen et al., 2003, 2001), and for estimating reservoir parameters in (Vefring et al., 2003). In Vefring et al. (2002) a similar approach was used, but with a least squares fitting procedure in place of the ensemble Kalman filter.

A similar approach to reservoir characterization was in Nygaard et al. (2007a) implemented using an Unscented Kalman Filter (UKF). This was combined with a Nonlinear Model Predictive Control (NMPC) scheme to automatically control BHP during UBD connections.

Results on the simpler Drift Flux Model, without flow regime predictions, include using an Unscented Kalman Filter in (Nikoofard et al., 2015), and an extended Kalman filter in Bloemen et al. (2006) and Aarsnes et al. (2014a).

Finally, emerging results from the field of control and estimation of hyperbolic systems are starting to reach a degree of sophistication that enables working with (linearized versions of) the DFM directly (Di Meglio and Aarsnes, 2015). In Di Meglio et al. (2014) an adaptive observer was designed that could estimate the states and a boundary reflection coefficient of a so-called $n + 1$ hyperbolic system (n characteristics moving right and 1 moving left), of which the DFM, (13)–(15), is an instance (Di Meglio, 2011). The result was consequently applied to estimating the reservoir pressure in UBD. In Di Meglio et al. (2013) a controller was derived for the $n + 1$ system, which was generalized to disturbance attenuation in (Hasan, 2014) and to $n + m$ systems in (Hu et al., 2015). Results to be appear present further extensions such as estimating multiple boundary parameters simultaneously and only using topside measurements (Anfinsen et al., 2016a,b).

3.4. Reduced Drift Flux model

In attempting to reduce the complexity of the two-phase flow models, a particular approach has been proposed where the distributed pressure dynamics are relaxed, thus neglecting the fast pressure waves (Di Meglio, 2011), while keeping the dynamics of the slow gas propagation.

This approach was used by Taitel et al. (1989) where the resulting model was described by a single transient PDE of the liquid continuity, obtained by assuming incompressible liquid, and a set of steady state relations. The closure relations are flow regime dependent, making the model mechanistic. The approach was expanded upon by Taitel and Barnea (1997), where the assumption of incompressible liquid was dropped, yielding two transient equations. A similar model was investigated by Masella et al. (1998), here called the No Pressure-Wave (NPW) model. In this particular incarnation, the resulting equations are (13)–(14) but in place of (15) the static relation

$$\frac{\partial P}{\partial x} = -\frac{\partial \alpha_\ell \rho_\ell v_\ell^2 + \alpha_{\mathrm{g}}\rho_{\mathrm{g}} v_{\mathrm{g}}^2}{\partial x} - F - G, \tag{22}$$

is used to relate the steady-state pressure drop to the acceleration term plus a frictional and gravitational source term.

Further, the DynaFloDrill (Rommetveit et al., 2005) simulator is based on this approach, solving for 7 mass equations (for oil, water, gas, mud and cuttings) coupled to a static pressure balance where convective momentum terms are neglected.

A similar model is used by Choi et al. (2013), except that here the steady-state pressure drop is found from a fitted correlation given as

$$\frac{2}{\rho_\ell(\alpha_\ell v_\ell)^2}\frac{\partial P}{\partial x} = -\frac{2G}{\rho_\ell(\alpha_\ell v_\ell)^2} + A\left(\frac{\alpha_\ell\rho_\ell v_\ell}{\alpha_g\rho_g v_g}\sqrt{\frac{\rho_g}{\rho_\ell}}\right)^B, \tag{23}$$

where A and B are tuning coefficients.

A 1-equation PDE, coupled with a ODE, based on the Lagrangian formulation of (Gavrilyuk and Fabre, 1996) is used in (Aarsnes et al., 2015) to estimate reservoir parameters during a kick incident. An alternate version of this model is derived in (Ambrus et al., 2015), where the idea of (Taitel et al., 1989) is expanded upon to derive a PDE-ODE model on the form

$$\dot{p}_a = \frac{\beta_a}{V_a}(q_{inj} - q_c + q_{res} + A\int_0^L E_G \mathrm{d}x), \tag{24}$$

$$E_G = \frac{\alpha_g(1 - C_0\alpha_g)\alpha_\ell}{\gamma}\frac{1}{P}\left(\frac{\partial P}{\partial t} + v_g\frac{\partial P}{\partial x}\right). \tag{25}$$

$$\frac{\partial \alpha_g}{\partial t} + v_g\frac{\partial \alpha_g}{\partial x} = E_G, \tag{26}$$

The ODE, (24), represents the slow pressure mode of the annulus with the last term of (24), given by (25), accounting for the total gas expansion in the well. In (25) γ is the adiabatic index. The gas propagation dynamics is given by (26). The pressure profile must then be given by a closure relation, and approaches such as (22) or (23) can be used.

4. Simplified ODE models

In this section we consider ODE-models which do not attempt to capture the distributed nature of the system through high order spatial discretizations, but lump the dynamics into one, or a few, control volumes.

4.1. 1-phase model

A popular, simple, liquid only model of the drilling hydraulics from the literature is due to (Kaasa et al., 2012), and is given by

$$\dot{p}_a = \frac{\beta_a}{V_a}(q_{bit} + q_{res} - q_c), \tag{27}$$

$$\dot{p}_d = \frac{\beta_d}{V_d}(q_{inj} - q_{bit}), \tag{28}$$

$$\dot{q}_{bit} = \frac{1}{M}(p_{bd} - p_{ad}), \tag{29}$$

$$p_{bd} \equiv p_d + G_d - F_d(q_{bit}), \tag{30}$$

$$p_{ad} \equiv p_a + G_a + F_a(q_{bit}). \tag{31}$$

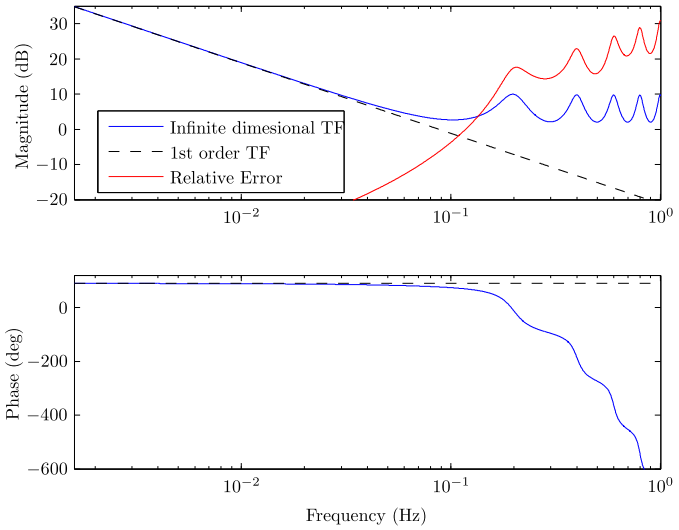

Figure 4: Comparison of the transfer function of a distributed PDE model (i.e. the DFM (13)–(15) with liquid only) vs the 1st order approximation (32), with input q_c and p_{bh} (normalized amplitude). The effect would be similar when discarding the distributed pressure dynamics in the 2-phase case, i.e. comparing the DFM (13)–(15) with the reduced DFM (24)–(26).

Here M is essentially a tuning constant giving the relaxation time of the pressure between the control volumes representing the annulus a and drill-string d. For many applications, however, the volume of the drill-string can be assumed small which allows for the further approximation

$$\dot{p}_a = \frac{\beta_a}{V_a}(q_p + q_{res} - q_c). \tag{32}$$

This is equivalent to the pressure dynamics in the reduced DFM (24), i.e. when the gas dynamics have been discarded.

This formulation and variations thereof have, due to their simplicity, seen a wide range of applications as a dynamic model for both estimation (Stamnes et al., 2011a,b; Kaasa et al., 2011), fault detection (Willersrud et al., 2013, 2014a,b) and control (Godhavn, 2011; Asgharzadeh Shishavan et al., 2015). As mentioned, the model does not capture two-phase flow, but has still been applied to kick detection applications (Willersrud et al., 2015; Zhou et al., 2011).

The effect of ignoring the spatially distributed dynamics for this model was studied in (Aarsnes et al., 2012), where it is noted that models of the same type (i.e. lumped models) with a higher order spatial discretizations can be obtained, as suggested in (Landet et al., 2012, 2013). The deciding factor for whether the distributed pressure dynamics are required is the frequency range for which the model is required to be valid. This can be observed in Fig. 4, which compares the transfer functions of a PDE representation of the pressure dynamics vs the 1st order ODE of (32). This comparison is also broadly valid for 2-phase flow, i.e. in the comparison between the full and the reduced DFM.

4.2. Lagrangian model

An interesting approach to representing the effect of Gas-Liquid flow of a single bubble in the annulus, while still re-

taining the simplicity of an ODE model, is the Lagrangian formulation proposed by Hauge et al. (2013); Hauge (2013) (cf. Gavrilyuk and Fabre (1996); Aarsnes et al. (2015)). Here the dynamics are given through the states L_1, V_G representing the bubble position and size:

$$\dot{L}_1 = -\frac{2(q_c - q_{bit})}{A_p \alpha_{dist}} + v_g \tag{33}$$

$$\dot{V}_G = q_c - q_{bit}. \tag{34}$$

where the first term in (33) accounts for the gas expansion with α_{dist} being a distribution parameter set to 0.21, and v_G found from a slip law. Equation (34) is obtained by assuming the liquid incompressible.

This model was shown to be adept at tracking a bubbles position and expansion in a kick scenario, and was used to estimate these values in the paper. However, due to the Lagrangian formulation tracking a single bubble, it is unable to handle the continuous influx encountered in underbalanced operations.

4.3. Low order lumped models

A range of low order lumped models have seen success in modeling for control of severe slugging in two-phase production risers, such as Di Meglio et al. (2009); Eikrem et al. (2008); Esmaeil and Skogestad (2011); Storkaas et al. (2003).

Such models have been applied to UBD in the literature, in particular the model suggested by Nygaard and Nævdal (2005), which shares several features with the "slugging models". Using as states n, m representing total gas and liquid mass in the drill-string and annulus denoted by the subscripts a, d respectively, the model is given as:

$$\dot{n}_d = w_{g,inj} - w_{g,bit} \tag{35}$$

$$\dot{m}_d = w_{\ell,inj} - w_{\ell,bit} \tag{36}$$

$$\dot{n}_a = w_{g,bit} + w_{g,res} - w_{g,c} \tag{37}$$

$$\dot{m}_a = w_{\ell,bit} + w_{\ell,res} - w_{\ell,c} \tag{38}$$

where the mass-rates $w_{G,bit}, w_{L,bit}$ are found through considering a pressure balance over the bit. The model lumps the spatial dynamics into a drill string (35)–(36) and an annular (37)–(38) control volume, but is unable to represent the gas distribution profile within the control volumes.

In (Nygaard and Nævdal, 2005) this model was used to tune a PI controller for use in a connection scenario during UBD, and a similar model with the drill string dynamics discarded (thus retaining only (37)–(38)) was applied to estimation in (Nikoofard et al., 2014a,b) and control in (Nikoofard et al., 2013).

4.3.1. ODE-models from identification techniques

Due to the high level of complexity encountered when deriving two-phase models from 1st principles, using black box or step response models are an attractive approach, The downside is that it might be difficult to identify and handle situations when the two-phase system exhibit changes in qualitative response, see Aarsnes et al. (2016).

Model identification techniques were used to obtain 1st-order delay models for control of UBO using MPC by Pedersen and Godhavn (2013), with extensions in Pedersen et al. (2015). This is similar to the technique used in (Nygaard et al., 2007b) where a NMPC controller was designed based on identified time constants. Here it was concluded that this approach was significantly simpler to implement and analyze than the NMPC controller of (Nygaard et al., 2007a), designed using the full DFM, and, furthermore, outperformed this controller in the scenario considered.

The approach of identifying time constants of the response was also taken by Carlsen et al. (2013) where the identification was used to design automatic controllers evaluated in dynamic kick handling scenario.

5. Numerical Schemes

For control and estimation purposes, an essential feature of the simulation is the numerical procedure to resolve the underlying model. The choice of numerical scheme may influence the accuracy, stability and computational speed of the ultimate predictions.

Numerical methods in use typically build on established methods for classical CFD, and may be divided into two main lines:

- *Pressure-based methods*. These are typically variations of the SIMPLE method (Patankar and Spalding, 1972), where the pressure is solved for as a primary variable. Herein, a staggered grid is typically used, where the pressure is resolved at the cell centers and velocities are resolved at the cell interfaces.

- *Density-based methods* (Roe, 1981). These methods typically guarantee mass conservation directly, and commonly formulated on the simpler collocated grids. However, they are prone to introducing pressure oscillations for low Mach number flows (Guillard and Viozat, 1999).

Methods may be further classified into *explicit* (forward Euler time discretization) and *implicit* (backward Euler time discretization) schemes.

Explicit schemes are simpler to implement than implicit schemes, and their fast computation time per iteration facilitates using smaller time-steps making them well suited for representing fast dynamics such as pressure waves. Explicit schemes, however, are typically bound[2] by the Courant-Friedrichs-Lewy (CFL) condition limiting time-steps according to:

$$|\lambda| \frac{\Delta t}{\Delta x} \leq 1, \tag{39}$$

where Δt is the time step Δx is the grid cell-size and λ is the velocity of the characteristic being resolved. This restriction is very limiting in the case of multi-phase flow since Δt must be designed to allow for the worst-case value of λ which in most cases is the sound velocity of liquid $\sim$ 1000 m/s, much greater

[2]There are exceptions known as explicit large time step schemes.

than the other transport phenomena of interest (Lorentzen and Fjelde, 2005).

Consequently, when computational efficiency is essential, implicit schemes are normally superior due to their potential for large time steps. An alternative is the semi-implicit schemes which resolve the slow characteristics, related to mass propagation, explicitly while the fast pressure characteristics are resolved implicitly.

Both the OLGA (Bendiksen et al., 1991) and LedaFlow (Danielson et al., 2011) numerical schemes are pressure-based and implicit. The FlowManager^{TM} scheme due to Evje and Flåtten (2006); Holmås and Løvli (2011) is essentially a density-based implicit method, incorporating some ideas from the classical pressure-based schemes. WeMod, developed by IRIS, uses a semi-implicit scheme with a slopelimiter technique which can be categorized as a finite element pressure-based method (Lorentzen and Fjelde, 2005).

Among the explicit schemes, the density-based AUMSV scheme (Evje and Fjelde, 2002) has proved useful for the drift-flux model (Udegbunam et al., 2014), as it provides an efficient combination of simplicity, accuracy and robustness. Consequently it is used in the Straume^{TM} Hydraulic Simulator developed by Kelda Drilling Controls (2015).

6. Simulation Study

In this section we will attempt to evaluate and compare the ability of the models covered in the preceding survey to qualitatively recreate the dynamics of three different gas-liquid flow scenarios encountered in drilling. The parameters are identical over the three scenarios except for the parameter changes indicated in Table 3.

6.1. Models used

In the simulations study, a total of 6 models are used for the kick scenario and 5 for the two UBD scenarios. These are summarized in Table 2. Here we use OLGA as reference to illustrate the dynamics that we would like to recreate with the simpler models.

At this point we again refer to Table 1 and note that we are only investigating the ability of the different mathematical structures represented to capture the qualitative transient behavior of the scenario. That is, we do not evaluate quantitative performance of the models, as this is mainly determined by which closure relations are used and these are beyond the scope of this paper.

*6.2. **Scenario 1**: Dynamically handled gas kick*

In this scenario, see Fig. 5 (a), a well is drilled using MPD at BHP = 260 bar when a high pressure zone of 270 bar is encountered. This causes a gas influx for the next 10 minutes, at which point the choke opening is decremented, increasing back-pressure and attenuating the kick. The gas is then circulated out.

6.2.1. Model performance

The drift flux models, Fig. 5 (b), are able to give a good qualitative representation of the kick incident. It is unclear, however, how to enable the reduced DFM to capture a shut-in incident with liquid back-flow. This can limit the applicability of this model in some instances.

The LOL model fails due to its inability to account for the gas position as it percolates through the well, see Fig. 5 (c). The Lagrangian model is able to amend this by using a state to track the position of the gas bubble. This model work well for single bubble dynamics, i.e. when the duration of the influx is limited, however, it stops working when the gas reaches the choke.

Finally, the 1-phase model does not include 2-phase dynamics and so only captures the changes in flow-rate and choke opening. We note that this might still be enough for certain control applications, although one has to handle the large changes in the effective bulk modulus that then occurs when gas enters the well.

*6.3. **Scenario 2**: UBD connection*

This scenario recreates the dominating dynamics of a connection in an underbalanced well by ramping down the liquid injection rate, keeping it at zero flow for 20 minutes, and then ramping it back up again, see Fig. 6 (a). This causes the gas in the well to start replacing the liquid causing lower bottom-hole pressure and increased gas influx consequently resulting in an increase in the WHP.

6.3.1. Model performance

Again we conclude that the drift flux models show satisfactory performance, see Fig. 6 (b). The majority of the discrepancies that can be observed between the high-fidelity models and the DFMs are due to difficulty to accurately replicate the wellhead pressure resulting from the highly varying multi-phase flow-rates through the choke.

Due to the inability of the lumped model to capture the gas distribution accurately, the hydrostatic pressure difference over the well had to be significantly modified, see Fig. 6 (c). A reasonable closure relation to deal with this issue can likely be developed with some effort. Given that this is fixed, however, the LOL model gives a reasonable representation of the response of the bottom-hole pressure, and consequently the increased gas-influx and its corresponding increase of WHP.

*6.4. **Scenario 3**: UBD operating envelope*

This scenario is inspired by the fact that an underbalanced gas-well exhibits very different dynamics when operating at different draw-downs, see (Aarsnes et al., 2016). At 2 hour intervals the choke opening is decremented and the well allowed to reach steady-state. As the wells nears the balance point, the WHP exhibits a characteristic inverse response, and then goes overbalanced toward the very end of the simulation, see Fig. 7 (a).

Table 2: Ability of models used in the simulation study to qualitatively represent dynamics in given scenario.

			Scenario Suitability		
Model	Equations	Reference	Scenario 1	Scenario 2	Scenario 3
High fidelity models:					
OLGA	-	(Bendiksen et al., 1991)	Reference model		
Drift flux models:					
Full DFM	(13)–(15)	(Lage et al., 2000)	Good	Good	Good*
Red DFM	(24)–(26)	(Ambrus et al., 2015)	Good†	Good	Good*
ODE models:					
LOL mod	(35)–(36)	(Nygaard and Nævdal, 2005)	Poor	Fair*	Fair*
1-ph mod	(32)	(Kaasa et al., 2012)	Poor	Poor	Poor
Lagrangian	(33)–(34)	(Hauge et al., 2013)	Good	N/A	N/A

*Requires extra modifications to closure relations to fit specific scenario.
†Difficulty with shut-in and liquid back-flow.

6.4.1. Model performance

In this scenario the DFMs give a largely correct representation of the behavior, and they are able to capture the qualitative effects of the different operating regimes (Aarsnes et al., 2016), see Fig. 7 (b):

1. Intuitive regime
2. Non-intuitive regime
3. Unstable/slugging regimes close to the balance point.
4. Overbalanced regime.

There is a discrepancy, however, in regard to at which points the qualitative response of the models change. This is determined in large part by the closure relations, hence these should be tuned/adapted to the observed response or selected with care.

The lumped model is also able to represent the difference between the intuitive and non-intuitive response, but again the pressure drop over the well requires tuning to obtain this, see Fig. 7 (c).

7. Conclusion

This paper have considered the topic of obtaining models representing gas-liquid flow dynamics encountered in drilling for the purpose of model based estimation and control design. We have used the framework that a complete simulation model is made up of a *mathematical structure*, *closure relations* and a *numerical scheme*, and argued that while the qualitative behavior of the model to a large degree is determined by the mathematical structure, the quantitative accuracy is primarily given by the closure relations.

For model based controller and estimator design it is desirable to have simpler models than what is obtained from general 1st principle considerations. Significant simplifications necessarily entail removing equations, thus altering the mathematical structure. This can be interpreted physically as imposing instantaneous equilibrium of certain dynamics, which yields static relations in place of the dynamic equations removed.

Simplifications reviewed in this paper employed imposing equilibriums:

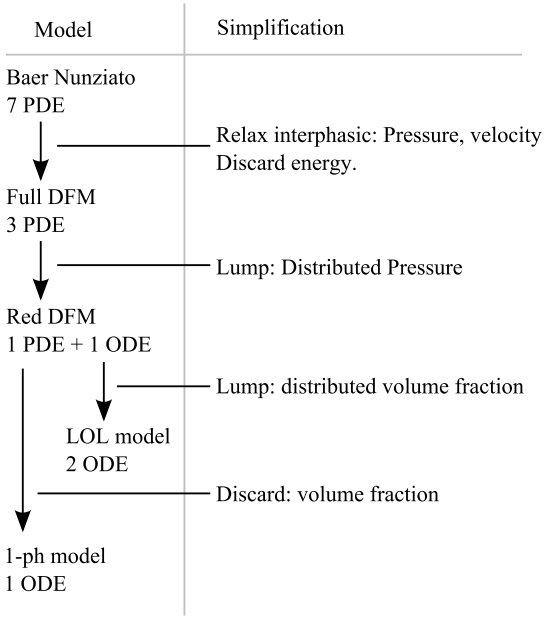

Figure 8: Relation between the models considered in this paper.

- Between two phases: interphasic relaxation.
- Between two or more spatial locations: lump dynamics.
- Overall: discard dynamics.

Using these processes one can extend the hierarchy of relaxation models derived from the Baer-Nunziato model, shown in Fig. 2. Specifically, the models considered in this paper follows the simplification processes as shown in Fig. 8.

The literature survey combined with a simulation study that was performed indicates that quite significant simplifications can be undertaken without removing essential qualitative behavior required to represent two-phase drilling dynamics. The optimal blend between accuracy and simplicity for several applications seems to be somewhere in between the classical 3 PDE drift flux model, and the lumped ODE models. The reduced drift flux model that has seen renewed interest in recent

years is consequently very promising.

The main challenges to be dealt with in future work is to adapt closure relations to simplified models and investigate how these can be effectively combined with estimation techniques. Dealing directly with PDEs for controller and estimator design in a robust manner also remains a challenge due to the complexity of most such proposed solutions.

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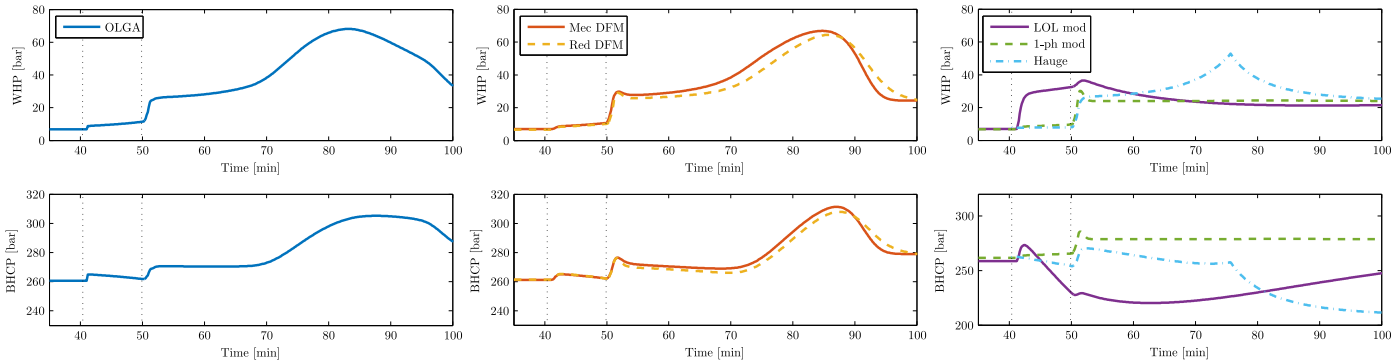
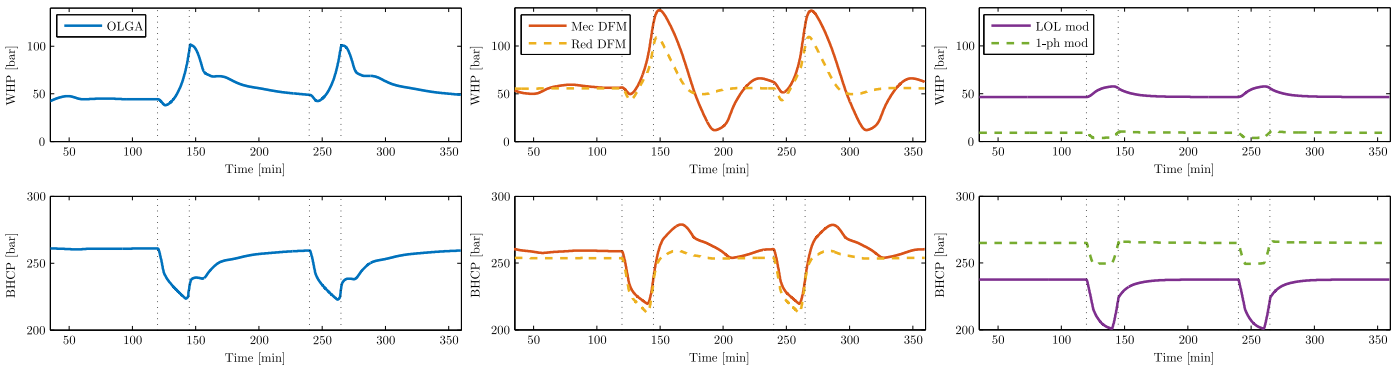
Table 3: Simulation scenarios

Scenario 1: MPD gas kick			**Scenario 2: UBD connection**		**Scenario 3: UBD operating envelope**	
Time:	Reservoir-Pressure:	Choke-opening:	Time:	Rig pump:	Time:	Choke opening:
0 minutes	200 bar	$Z = 6\%$	0 minutes	$q_L = 13.33$kg/s	0 minutes	$Z = 20\%$
40 minutes	270 bar	$Z = 6\%$	120 minutes	$q_L = 0$kg/s	120 minutes	$Z = 12\%$
50 minutes	270 bar	$Z = 3\%$	140 minutes	$q_L = 13.33$kg/s	240 minutes	$Z = 8\%$
			240 minutes	$q_L = 0$kg/s	360 minutes	$Z = 5\%$
			260 minutes	$q_L = 13.33$kg/s	480 minutes	$Z = 3.5\%$



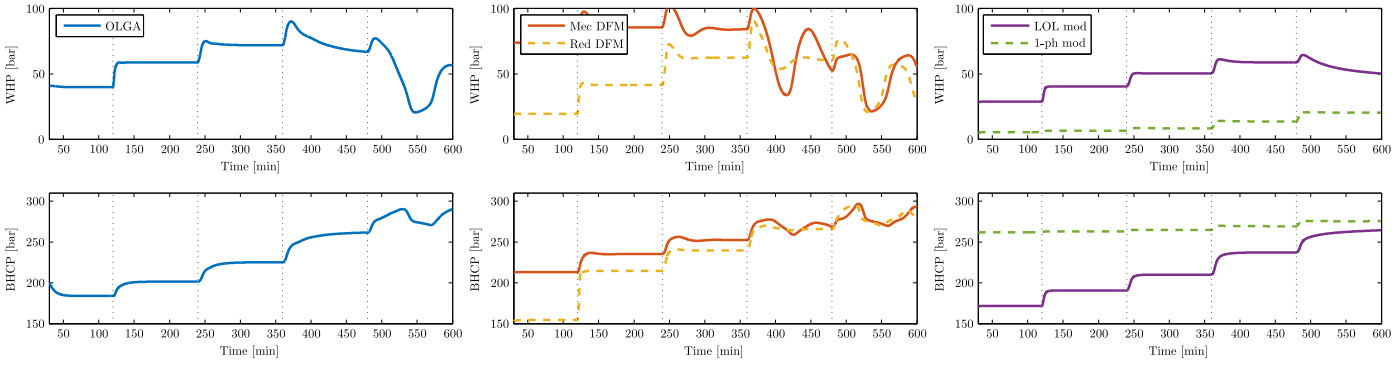
(a) (b) (c)

Figure 5: Pressure trends for **Scenario 1**.



(a) (b) (c)

Figure 6: Pressure trends for **Scenario 2**.



(a) (b) (c)

Figure 7: Pressure trends for **Scenario 3**.