Fuel Optimal Thrust Allocation in Dynamic Positioning

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Abstract: This paper is focused on the thrust allocation algorithm, which is a part of a Dynamic Positioning (DP) system in marine vessels with diesel-electric power system. In this paper the focus is on using the thrust allocation to make the diesel generators on board the vessel work more fuel efficiently, by reducing the total fuel consumption of all online diesel generators. A static model for the fuel consumption of a diesel generator as a function of its produced power is derived from data, and this model is used to create a convex Quadratic Programming (QP)-problem which finds the most fuel efficient thrust allocation solutions. The simulation scenarios shown in this paper typically give a fuel reduction of a rather common Platform Supply Vessel (PSV) of up to 2% of its maximum possible fuel consumption. The fuel optimization can be implemented as a standard QP-problem by recalculation of its cost function weights based on linear and quadratic model approximations at the current operation point.

1. INTRODUCTION

In today's marine industry there are many operations e.g; pipelay operations, dredging, crane barge operations, station keeping, drilling, anchor handling etc, that are performed at low speeds and requires the vessel to maintain heading and/or position. In order to achieve this the vessel is equipped with thrusters such that longitudinal and latitudinal thrust forces can be produced at all times, and a Dynamic Positioning (DP) system to control them.

The three main parts of the DP control system is a state estimator, a positioning controller and the thrust allocation algorithm. The positioning controller calculates forces in surge, sway and yaw needed to maintain position and heading, while the thrust allocation algorithm takes the vector containing these forces, and calculates thrust and direction for each active thruster. The thrust allocation algorithm is the focus of this paper.

The thrust allocation problem, because most DP vessels are over-actuated, is usually solved as an optimization problem, searching for solutions within the thrusters physical limitations, while minimizing some user-defined criterion. Thrust allocation has been an active area of research for the past two decades, and the criterion which is minimized are usually produced thrust or consumed power by the thrusters, while taking physical constraints like azimuth turn rate, forbidden zones, maximum thrust capacity etc. into account.

A paper which has a general view of the control allocation problem is Johansen and Fossen [2013]. They present constrained and unconstrained optimization problems to solve the allocation problem, and the criterion to be minimized is usually some penalty related to the use of actuators or violation of constraints.

Fossen and Johansen [2006] is a survey of control allocation methods for marine vessels. They introduce optimization problems which solves the thrust allocation problem with respect to physical constraints on the thrusters and a constraint which specifies that the DP-command should be obtained. The criterion which is minimized, is a penalty on slack variables on the DP-command constraint, since there might be situations where it is just not possible to obtain the DP commanded total thrust, and with a hard constraint in these situations the optimization problem would have no feasible solution. In addition to the slack variables, there are different criteria which also include either the produced thrust, or the consumed power by the thrusters. Thrust allocation algorithms that seek to minimize the consumed power of the thrusters are seen in e.g, Jenssen and Realfsen [2006],Leavitt [2008],Larsen [2012],Wit [2009], Ruth [2008],Veksler et al. [2012a],Johansen et al. [2004] and Veksler et al. [2012b].

Veksler et al. [2012b] and Veksler et al. [2012a] as well as minimizing consumed power by the thrusters, presents a method to reduce load variations on the bus by dynamically biasing the thrusters. Two methods for reducing frequency and load variations in the power distribution are discussed in Mathiesen et al. [2012].

The authors have not found anything in the literature where the thrust allocation algorithm explicitly includes the fuel consumption of online diesel generators in the cost function. Instead, thrust allocation algorithms in the literature tend to minimize the power consumed by the thrusters and some take the load conditions on the bus into consideration. Radan [2008] and Hansen [2000], has some discussion on the fuel-optimal operation conditions of a diesel generator plant on a vessel, but they both discuss it from a Power Management System (PMS) point of view.

Aithal [2010],Widd [2012] and Guzzella and Onder [2010], to name a few, discuss diesel engines fuel consumption and emissions. The diesel generator models presented in Aithal [2010] and Widd [2012] seem however too complex to use in a thrust allocation algorithm.

This paper investigates the possibility to use the thrust allocation algorithm in such a way that the fuel consumption of the online diesel generators on each power bus will be minimized. A simple model of the fuel consumption of a diesel generator as a function of its produced power is derived from sampled data, and incorporated in an optimization problem which is used to solve the thrust allocation problem. The fuel consumption of the diesel generators on each power bus is formulated as a quadratic function of produced thrust and minimized, while making sure that thrusters operate within their physical limitations and that the DP-command is obtained if possible. There is also some discussion on the implication of variations in the produced power by a diesel generator and its fuel consumption, sooting and NO_x emission. The results are illustrated with DP class 2 operations of a typical PSV.

2. FORMULATING THE FUEL OPTIMAL THRUST ALLOCATION PROBLEM

The thrust allocation problem can be formulated as shown in the optimization problem (1), where all the variables and symbols used in this paper are described in Table 1.

$$\min_{\mathbf{u}\epsilon\mathbb{R}^{2n},\mathbf{s}\epsilon\mathbb{R}^{3}} \mathbf{s}^T\mathbf{Q}\mathbf{s} + f(\cdot) \tag{1a}$$

s.t

$$\boldsymbol{\tau_c} - \mathbf{B}\mathbf{u} - \mathbf{s} = \mathbf{0} \tag{1b}$$

$$\mathbf{u}_{min} \leq \mathbf{u} \leq \mathbf{u}_{max} \tag{1c}$$

$$\boldsymbol{\Delta}\mathbf{u}_{min} \leq \boldsymbol{\Delta}\mathbf{u} \leq \boldsymbol{\Delta}\mathbf{u}_{max} \tag{1d}$$

$$\boldsymbol{\alpha}_{min} \leq \boldsymbol{\alpha} \leq \boldsymbol{\alpha}_{max} \tag{1e}$$

$$\boldsymbol{\Delta\alpha}_{min} \leq \boldsymbol{\Delta\alpha} \leq \boldsymbol{\Delta\alpha}_{max} \tag{1f}$$

$$\sum_{i=1}^{n} \mathbf{M}_{ji} p_{T_i} \leq p_{bus_j}^{avail} - p_{ext_j} \quad \forall j \in \{1, 2, \ldots, m\} \tag{1g}$$

This problem will minimize the slack variables $\mathbf{s}$ while making sure that constraint (1b) is satisfied, meaning the DP-command is obtained if possible, and the thrusters operate within their physical limitations by constraint (1c)-(1f). Constraint (1g) makes sure the thrusters does not consume more power than available on the bus.

Since the thrust allocation problem is usually over-actuated, there usually exists many solutions that satisfies the constraints given by (1b)-(1g). This gives us some freedom in choosing which of the solutions we would prefer, and we do this with the help of the function $f(\cdot)$ in (1a). In this paper $f(\cdot)$ will primarily be used to describe the fuel consumed by the online generators. This will lead to thrust allocation solutions that tries to have the generators consume as little fuel as possible while satisfying constraints.

It is beneficial if the thrust allocation problem can be formulated as a convex QP-problem, since these are well known and relatively easy to solve numerically. This implies that constraints (1c)-(1g) have to be linear, and suitable reformulations can be found in the literature, e.g Larsen [2012] and Ruth [2008]. In addition to linear constraints, a QP-problem also needs a convex quadratic objective function. This means that when designing $f(\cdot)$ it has to be quadratic and convex in the decision variables given by $\mathbf{u}$.

Letter	*Description*
$\mathbf{s}$	Slack variables that relaxes constraint (1b). This is the constraint which specifies that the thrusters should obtain the DP-command given by $\boldsymbol{\tau_c}$
$\mathbf{u}$	Vector containing each thrusters forces in both surge and sway direction. $\mathbf{u} = \left(\mathbf{u}_1\ \mathbf{u}_2\ \ldots\ \mathbf{u}_{2n}\right)^T$, $\mathbf{u}_i = \left(u_{i,surge}\ u_{i,sway}\right)^T$
$\Delta\mathbf{u}$	Change in thrust from one time-step to the next
$\Delta\mathbf{u}_{min}/\Delta\mathbf{u}_{max}$	Maximum allowed thrust reduction/increase from on time-step to the next.
$\boldsymbol{\alpha}$	Azimuth direction, given by $atan2\left(u_{i,sway}, u_{i,surge}\right)$ for thruster number i
$\Delta\boldsymbol{\alpha}$	Change in thrust direction from one time-step to the next
$\Delta\boldsymbol{\alpha}_{min}/\Delta\boldsymbol{\alpha}_{max}$	Maximum allowed thrust angle reduction/increase from one time-step to the next.
$\mathbf{Q}$	Symmetric positive weighting matrix, used to put a cost on the use of slack variables $\mathbf{s}$. Reducing the values of the slack variables has the highest priority, so the weights in $\mathbf{Q}$ should be such that the cost of the first term in (1a) is larger than the second term.
$\mathbf{B}$	Control allocation matrix. Maps the $2n$ dimensional thrust vector $\mathbf{u}$, to the 3 dimensional $\boldsymbol{\tau}$-vector.
$\mathbf{M}$	$m \times n$ matrix with 1's and 0's stating which thruster is connected to which bus.
$\mathbf{E}$	$l \times l$ matrix describing which generator supplies which bus, and the load sharing between the generators connected to the same bus.
$\boldsymbol{\tau_c}$	Requested generalized forces from the DP-controller. $\boldsymbol{\tau_c} = \left(F_{surge}\ F_{sway}\ M_{yaw}\right)^T$.
$f(\cdot)$	User defined function relating cost to produced thrust, consumed power by the thrusters, consumed fuel by the generators, load variations on the bus etc.
T_i	Thrust produced by thruster number i
$T_{i,prev}$	Thrust produced by thruster number i at the previous time-step
p_{T_i}	Power consumed by thruster number i
$p_{T_i}^Q$	Quadratic approximation of the power consumed by thruster number i
p_{G_k}	Power generated by generator number k
$p_{G_k}^Q$	Power generated by generator number k expressed quadratically in the decision variables $\mathbf{u}$.
q_{G_k}	Fuel rate by generator number k
$q_{G_k}^Q$	Fuel rate by generator number k expressed as a quadratic function of the thruster forces $\mathbf{u}$.
$p_{bus_j}^{avail}$	Total available power on bus number j.
p_{ext_j}	Power consumed by external consumers other than the thrusters.
n	Number of thrusters.
m	Number of buses.
l	Number of generators.
μ_k, ρ_k, γ_i	Scaling factors.

Table 1. Table explaining the notation and symbols used in this paper

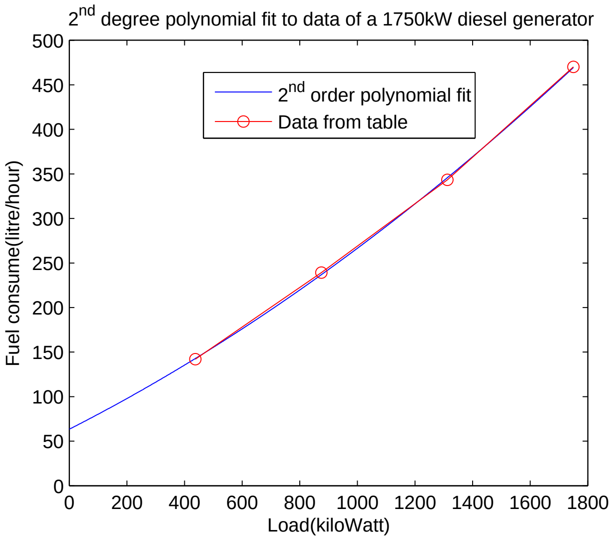

Fig. 1. This figure shows (2) fitted to the data from DieselServiceAndSupply [2013] of a diesel generator rated to $1750kW$

2.1 Static fuel consumption model

The model for the fuel consumption of a generator as a function of its produced power is found by fitting a polynomial of degree 2 to the data given by the table in DieselServiceAndSupply [2013]. This gives us the model

$$q_{G_k} = h_k(p_{G_k}) = a_2 p_{G_k}^2 + a_1 p_{G_k} + a_0 \tag{2}$$

where a_2,a_1 and a_0 are found by fitting the polynomial to the data, and their values will depend on the size of the generator the polynomial is chosen to fit. Fig.1 shows (2) fitted to the data of a diesel generator rated to $1750kW$.

2.2 Defining the cost function

Assuming symmetric load sharing among the generators on each bus we can set up the following set of linear equations.

$$\mathbf{E}\boldsymbol{p_G} = \begin{pmatrix} \boldsymbol{p_{bus}} \\ \mathbf{0}_{(l-m)\times 1} \end{pmatrix} \tag{3}$$

We know that the load on bus number j is given by $p_{bus_j} = \sum_{i=1}^{n} \mathbf{M}_{ji} p_{T_i} + p_{ext_j}$, and by using this together with (3) we can set up the load generated by generator number k as shown in (4).

$$p_{G_k} = \sum_{j=1}^{m} \sum_{i=1}^{n} (\mathbf{E}^{-1})_{kj} \mathbf{M}_{ji} p_{T_i} + p_{ext_j} \tag{4}$$

We notice now that if we chose $f(\cdot)$ to be linear in the fuel consumption, linearise (2) wtr to p_G, make p_G linear in p_T and p_T quadratic in $u_{i,surge}$ and $u_{i,sway}$, $f(\cdot)$ will be quadratic in the decision variables ($u_{i,surge}$ and $u_{i,sway}$).

From Fossen [2002] we know that the power consumed by thruster number i is given by $p_{T_i} = T_i^{3/2}$ and $T_i = \sqrt{u_{i,surge}^2 + u_{i,sway}^2}$, which is obviously not quadratic in the decision variables ($u_{i,surge}$ and $u_{i,sway}$). Several quadratic approximation of this expression are used in the literature e.g Ruth [2008] and Johansen et al. [2004]. In this paper the approximation derived in Ruth [2008] will be used.

$$p_{T_i} \approx p_{T_i}^Q = \frac{T_i^2}{\sqrt{|T_{i,prev}|}} \tag{5}$$

Inserting (5) into (4) gives

$$\begin{aligned} p_{G_k} \approx p_{G_k}^Q &= \sum_{j=1}^{m} \sum_{i=1}^{n} (\mathbf{E}^{-1})_{kj} \mathbf{M}_{ji} p_{T_i}^Q + p_{ext_j} \\ &= \sum_{j=1}^{m} \sum_{i=1}^{n} (\mathbf{E}^{-1})_{kj} \mathbf{M}_{ji} \frac{u_{i,surge}^2 + u_{i,sway}^2}{\sqrt{|T_{i,prev}|}} + p_{ext_j} \end{aligned} \tag{6}$$

Equation (6) now describes the power produced by generator number k expressed quadratically in the decision variables ($u_{i,surge}$ and $u_{i,sway}$).

Linearising (2) wtr to load, around the produced power from the previous time-step using first-order Taylor expansion gives

$$q_{G_k} \approx q_{G_k}^Q = h_k(p_{G_k,prev}) + \frac{dh_k}{dp_{G_k}}(p_{G_k,prev}) \cdot (p_{G_k} - p_{G_k,prev}) \tag{7}$$

where the superscript ^{Q} indicates that the fuel consumption is quadratic in the decision variables after the linearization.

We can now use (7) to approximate $f(\cdot)$ as a linear function of the fuel consumption, namely $f(q_{G_k}^Q)$ as shown in (8).

$$f(q_{G_k}^Q) = \sum_{k=1}^{l} \frac{1}{\mu_k} q_{G_k}^Q \tag{8}$$

This function is used when solving problem (1) to generate most of the results in this paper. Combining (6)-(7), the cost is quadratic in $\mathbf{u}$.

3. RESULTS

The dynamics of the vessel will not be considered, and it is assumed that as long as the DP-command $\boldsymbol{\tau_c}$ is obtained by the thrust allocation algorithm, the vessel will maintain its position and heading reference, Fossen [2002]. Even though the vessel dynamics are not considered, the thruster-/generator- and busbar layout of the vessel are needed. The dimensions for the relevant equipment are also needed in order to get simulations with realistic values. In this paper the specification for the DP class 2 PSV Bourbon Tampen will be used: One $883kW$ tunnel thrusters fwd, one $883kW$ azimuth thruster fwd, two $2500kW$ azipull thrusters aft, two busbars and four $1825kW$ diesel generators. We assume that the vessel operates with open bus-bar.

All the simulations shown in this paper are done over 200 seconds, and the thrust allocation algorithm is executed one time pr second.

In order to save space, the plot that shows that the thrust allocation algorithm obtains the DP-command $\boldsymbol{\tau_c}$ have not been included, however the numerical values are given in kiloNewton(kN).

3.1 Minimizing power vs Minimizing fuel

In this subsection, the problem (1) with (9) as the objective function is compared to the problem (1) with (10) as the objective function with a handful of simulations.

$$J_1 = \mathbf{s}^T\mathbf{Q}\mathbf{s} + \sum_{k=1}^{l} \frac{1}{\mu_k} q_{G_k}^{Q} \quad (9)$$

$$J_2 = \mathbf{s}^T\mathbf{Q}\mathbf{s} + \sum_{i=1}^{n} \frac{1}{\gamma_i} p_{T_i}^{Q} \quad (10)$$

J_1 is the objective function in the fuel minimizing problem, while J_2 is the objective function in the power minimizing problem.

In all the simulations, the second term in J_1 that puts a penalty on the use of fuel, is scaled such that gives a cost between 0-100.

The second term in J_2 that puts a penalty on the power consumed by the thrusters is simulated with two different methods of scaling. One method scales the term such that the thrusters are penalised by a percentage of their total power consumption. The the second method penalises the thrusters equally in the amount of power they consume. In the simulations the second term in J_2 is also scaled such that it returns values between 0-100.

We notice from the results shown in Fig. 2 that the fuel consumption is reduced and that the sum of power consumed by thrusters gets smaller. At first glance this might seem odd, since power was minimized the first 100 seconds! However, because of the scaling of the second term in J_2, the total sum of consumed power by the thrusters is not what is minimized. The scaling of the second term in J_2 used in this simulation prefers the thrusters to consume percentage-wise equal amounts of power. This can be seen in the top-right plot of Fig. 2 where in the first 100 seconds the thrusters work more equally, whereas in the last 100 seconds they separate more. The comparison of the fuel minimizing objective function, with the power minimizing objective function that has this kind of scaling is included in this paper, because it appears to be a common way to scale the second term in J_2.

If we change the scaling of the second term in J_2 such that the sum of total consumed power is minimized, we get the result shown in Fig. 3. Not surprisingly, both the fuel consumption of the generators and power consumption of the thrusters stay almost constant throughout the simulation. We notice that the fuel consumption of each generator, and the power consumption by each thruster changes slightly when the objective function changes. This does have an effect on the total fuel consumption, but it is so small that it cannot be seen in this plot. From the results shown in Fig. 3 one might think that if one minimizes the sum of consumed power by all the thrusters, the fuel consumption will be minimized as well. This however, may not be the case if there are non-zero external load on the buses.

In Fig. 4 both the fuel consumption and power consumed by the thrusters are reduced after the objective function switch. In Fig. 5 however, we notice that the fuel consumption goes down as expected, but the total power consumed

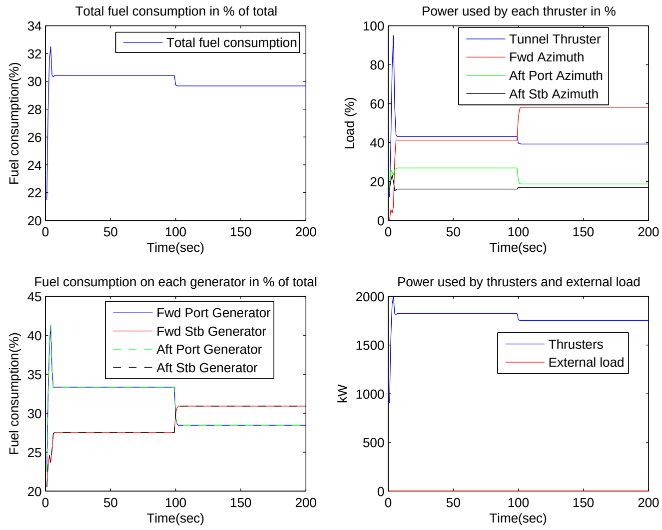

Fig. 2. This simulation is done with $\boldsymbol{\tau_c} = (100\ 200\ 0)^T$ and zero external load. Power consumed by the thrusters is minimized in the first 100 seconds while the last 100 seconds fuel consumption by the generators is minimized. In the first 100 seconds, power consumed by thruster number j is scaled with the inverse of its maximum possible power consumption. $\sim 1\%$ fuel consumption reduction.

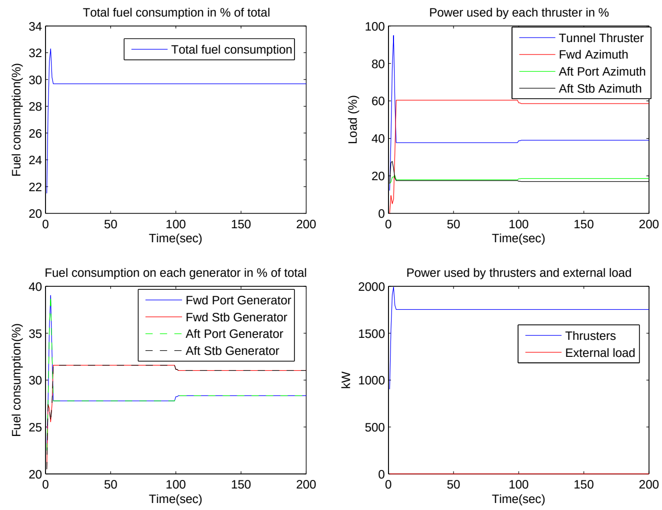

Fig. 3. This simulation is done with $\boldsymbol{\tau_c} = (100\ 200\ 0)^T$ and zero external load. Power consumed by the thrusters is minimized in the first 100 seconds while the last 100 seconds fuel consumption by the generators is minimized. In the first 100 seconds, the power consumption by each thruster is scaled independently of its size. $\sim 0\%$ fuel consumption reduction.

by the thrusters actually increases. This is because of the external load that is present on the port bus bar and the non-linear relationship between thrust, power and fuel. The non-linear relationship between fuel consumption and power production of a generator dictates that the generator gets more efficient at high loads which can be seen from Fig. 1.

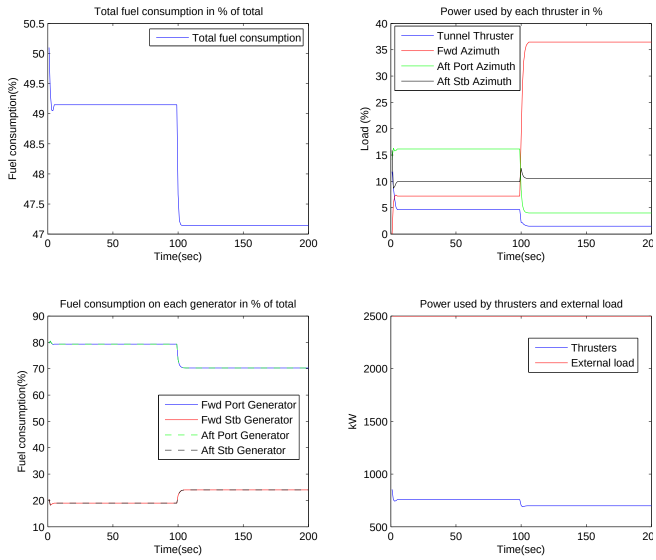

Fig. 4. This simulation is done with $\boldsymbol{\tau_c} = (100\ 50\ 0)^T$ external load of 2500kW on the port bus bar. Power consumed by the thrusters is minimized in the first 100 seconds while in the last 100 seconds fuel consumption, by the generators is minimized. In the first 100 seconds, power consumed by thruster number j is penalised with the inverse of its maximum possible power consumption. $\sim$ 2% fuel consumption reduction.

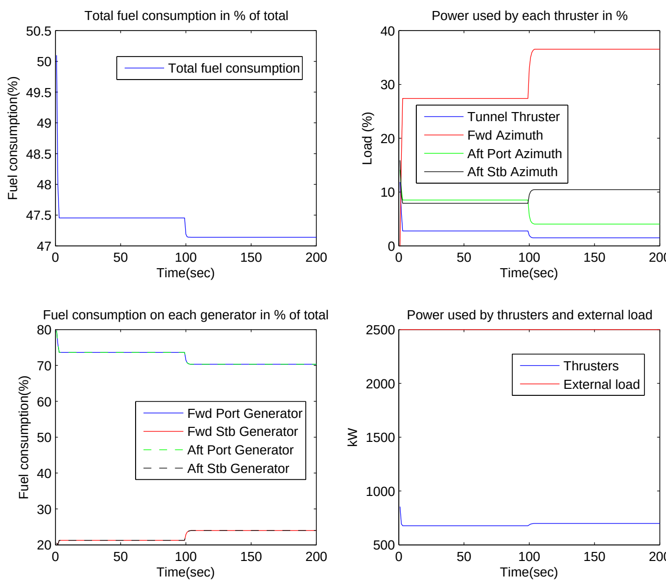

Fig. 5. This simulation is done with $\boldsymbol{\tau_c} = (100\ 50\ 0)^T$ external load of 2500kW on the port bus bar. Power consumed by the thrusters is minimized in the first 100 seconds while the last 100 seconds fuel consumption by the generators is minimized. In the first 100 seconds, the power consumption by each thruster is scaled independently of its size. $\sim$ 0.5% fuel consumption reduction.

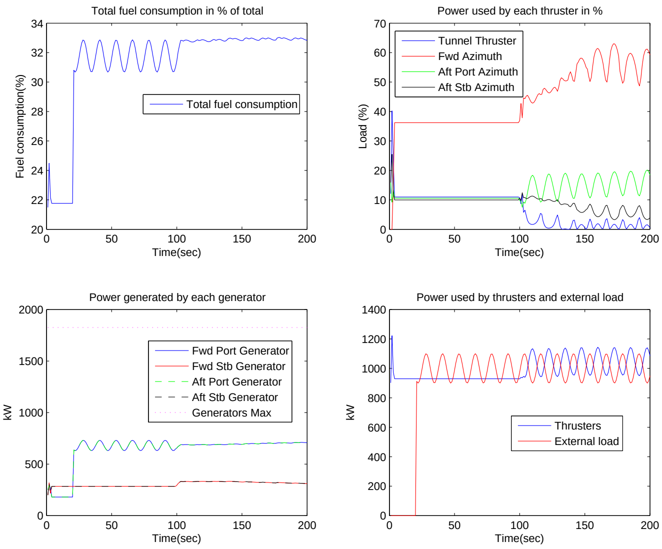

Fig. 6. This simulation is done with $\boldsymbol{\tau_c} = (100\ 100\ 0)^T$ and after 20 seconds an external load on the port bus bar that varies periodically around 1000kW with amplitude of 100kW and a frequency of 0.08Hz is switched on. The first 100 seconds of this simulation minimizes power consumed by the thrusters. After 100 seconds the objective function is switched to (11) which penalises changes in the load on each generator from one time-step to the next.

3.2 *Reducing load variations on the generators*

In this section simulations of problem (1) with (11) as the objective function are considered.

$$J_3 = \mathbf{s}^T\mathbf{Q}\mathbf{s} + \sum_{k=1}^{l} \frac{1}{\rho_k}\dot{p}_{G_k} + \sum_{k=1}^{l} \frac{1}{\mu_k} q_{G_k}^{Q} \tag{11}$$

The 2^{nd} term in J_3 penalizes variations in power produced by the diesel-generators, and the 3^{rd} term in J_3 is included such that fuel is minimized when there are no load variations present on the busbar.

In Fig. 6 we see that thrusters operate at a steady state, while the load on the generators oscillate proportionally to the external load variations for the first 100 seconds. After the objective function switch, we notice that the thrusters starts counteracting the varying external load on the bus, such that the generated load by the generators evens out.

Reducing the load variations will lead to less wear and tear on the generators, and will cause less frequency variations on the bus, which if large enough, can cause a black out. The trade off however, is that the wear and tear on the thrusters will increase which becomes evident by looking at the power consumed by the thrusters in Fig. 6.

How the fuel consumption behaves during the simulation can be seen in the top-left plot of Fig. 6. Since the fuel consumption is based on a static model, the fuel consumption in situations where the dynamics of the diesel generator is excited will not be correctly represented by the model in this paper. The model is a good approximation for the fuel consumption in steady-state situations, or situations where the load varies slowly enough to not excite the dynamics and large transients on the generator. Hence the real fuel consumption may actually be larger during the first 100 seconds.

In the bottom-left plot of Fig. 6 we notice that the generators connected to the bus with the oscillating external load also has oscillations in their load before the load variation reduction is switched on, which is expected. After the load variation reduction is switched on, the oscillation on these generators are reduced significantly, while the load on the two other generators increase. This means that the mean power produced by the generators are higher after the load variation reduction is switched on. Variations in the power produced by a generator may lead to incomplete combustion which implies higher fuel consumption and more soot pollution. Also, the higher the mean load of the generators are, the more NO_x will be produced. So before the load variation reduction is switched on, the fuel consumption and soot formation will be high, while NO_x production will be low. After the load variation reduction is switched on however, the generators will operate with smaller load variations which implies lower fuel consumption and soot production, but since the mean load increases, then the NO_x production will go up as well. According to Realfsen [2009], the generators have to work above a specific percentage of their maximum capacity in order for cleaning of NO_x to be done by a Selective Catalytic Reduction (SCR)-filter. One might find oneself in a situation then, where by reducing the load variations on the bus and increasing the mean load such that NO_x cleaning can be done, one will effectively reduce wear, fuel consumption, sooting and NO_x emission.

4. CONCLUSION

The fuel optimization can be implemented within the conventional framework of quadratic programming-based thrust allocation, by recalculating the cost function and constraints based on linear and quadratic model approximations at the current operation point.

The fuel minimizing thrust allocation have found solutions that uses less fuel than the power minimizing thrust allocation, except in special situations where there are no external loads. If there are no external load, the fuel minimizing thrust allocation and the power minimizing thrust allocation will produce solutions that will make the generators consume the same amount of fuel. One could say that the fuel minimizing thrust allocation finds solutions that consumes less or equal amounts of fuel as the power minimizing thrust allocation.

When it comes to the load variation reduction, it is a trade-off between two factors. Not reducing load variations leads to increased wear and tear on the generators, higher fuel consumption and more sooting. Reducing the load variations will lead to more wear and tear on the thrusters and higher NO_x emission.

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