Evaluation of Lyapunov-based Adaptive Observer using Low-Order Lumped Model for Estimation of Production Index in Under-balanced Drilling

Amirhossein Nikoofard * **Tor Arne Johansen** *
Glenn-Ole Kaasa **,*

* *Center for Autonomous Marine Operations and Systems, Department of Engineering Cybernetics, Norwegian University of Science and Technology, 7491 Trondheim,Norway (e-mail: Amirhossein.nikoofard@ntnu.no, Tor.arne.johansen@itk.ntnu.no).*
** *Kelda Drilling Controls (email:gok@kelda.no).*

Abstract: A distributed drift-flux model and a low-order lumped model describing a multiphase (gas-liquid) flow in the well during Under-Balanced Drilling (UBD) has been presented. This paper presents a novel nonlinear adaptive observer to estimate the total mass of gas and liquid in the annulus and production constant of gas and liquid from the reservoir into the well during UBD operations. Furthermore, it describes a joint unscented Kalman filter to estimate parameters and states for both the distributed drift-flux and lumped model by using real-time measurements of the choke and the bottom-hole pressures. The performance of the adaptive observers are evaluated for typical drilling scenarios. The results show that all adaptive observers are capable of identifying the production index, although the adaptive observers based on the low-order lumped model achieves better convergence rate than adaptive observer based on the drift-flux model. The results show that the LOL model is sufficient for the purpose of estimating the production parameters.

Keywords: Low-order lumped model, Estimation, Simplified drift-flux model, Lyapunov stability and Unscented Kalman Filter.

1. INTRODUCTION

Since the number of depleted formations and cost of field exploration and development has increased, there has been increasing interest in new technology and automation of drilling process which can improve drilling efficiency and increase oil recovery for the past two decades. Automatic under balanced drilling (UBD) has the potential to both improve hydrocarbon recovery and decrease drilling problems such as differential sticking, invasive formation damage and lost circulation. Modeling, estimation and model based control are important parts of automatic UBD.

Lyapunov based adaptive observers and the Kalman filter are widely used for the estimation of state and parameters. A Lyapunov based adaptive observer is generally designed as Luenberger type observer for the state combined with an appropriate adaptive law to estimate the unknown parameters (Ioannou and Sun (1996)).

Due to the complexity of the multi-phase flow dynamics of a UBD well coupled with a reservoir, the modeling, estimation and control of UBD operations is still considered an emerging and challenging topic in drilling automation. Nygaard et al. (2006) compared and evaluated the performance of the extended Kalman filter, the ensemble Kalman filter and the unscented Kalman filter based on a low order model to estimate the states and the production index (PI) in UBD operation. Lorentzen et al. (2003) designed an ensemble Kalman filter based on a drift-flux model to tune the uncertain parameters of a two-phase flow model in the UBD operation. Vefring et al. (2003, 2006) compared and evaluated the performance of the ensemble Kalman filter and an off-line nonlinear least squares technique utilizing the Levenberg-Marquardt optimization algorithm to estimate reservoir pore pressure and reservoir permeability during UBD while performing an excitation of the bottom-hole pressure. Both methods are capable of identifying the reservoir pore pressure and reservoir permeability. In Nygaard et al. (2007), a finite horizon nonlinear model predictive control in combination with an unscented Kalman filter was designed for controlling the bottom-hole pressure based on a low order model developed in Nygaard and Nævdal (2006), and the unscented Kalman filter was used to estimate the states, and the friction and choke coefficients. A Nonlinear Moving Horizon Observer based on a low-order lumped model (LOL) was designed for estimating the total mass of gas and liquid in the annulus and geological properties of the reservoir during UBD operation for pipe connection procedure in Nikoofard et al. (2014b). Aarsnes et al. (2014b) introduced a novel simplified drift-flux model and estimation of the distributed multiphase dynamics during

UBD operation. This model used a specific empirical slip law without flow-regime predictions. The estimation algorithm separates slowly varying parameters and potentially more quickly changing parameters such as the PI. Fast changing parameters are estimated online simultaneously with the states of the model, but other parameters are calibrated infrequently and offline. Nikoofard et al. (2014a) designed Lyapunov-based adaptive observer, a recursive least squares estimator and a joint unscented Kalman filter based on a LOL model to estimate states and parameters during UBD operations by using the total mass of gas and liquid as measurements calculated from pressure measurements using a model. The performance of the adaptive estimators were compared and evaluated for pipe connection procedure using a simple simulation model. In this paper, states and parameters are estimated by directly using real-time measurements of the choke and the bottom-hole pressures and using an extension of the observer in Nikoofard et al. (2014a)

The purpose of the paper is to evaluate the LOL model for estimation of PI in UBD employing an adaptive observer that uses the bottom hole and choke pressure measurements. This paper describes the design of a novel Lyapunov-based adaptive observer and the joint unscented Kalman filter for estimating the total mass of gas and liquid in the annulus and production constant of gas and liquid from the reservoir into the well based on a LOL model for UBD operation. In addition, it explains the design of joint unscented Kalman filter to estimate states and parameters for the simplified drift-flux model by using real-time measurements of the choke and the bottom-hole pressures. The adaptive observers are compared with each other in terms of rate of convergence and accuracy.

This paper consists of the following sections: Section 2 describes the basics concept of the UBD process. The modeling section presents a LOL and simplified drift-flux model based on mass and momentum balances for UBD operation. Section 4 explains the Lyapunov-based adaptive observer and joint UKF methods for simultaneously estimating the states and model parameters from real-time measurements. In the section 5, simulation results are provided for state and parameter estimation. Finally, the conclusion of the paper is presented.

2. UNDER BALANCED DRILLING

In drilling operations, the drilling fluid is pumped down through the drill string and the drill bit (see Figure 1). The annulus is sealed with a rotating control device (RCD), and the drilling fluid exits through a controlled choke valve, allowing for faster and more precise control of the annular pressure. The drilling fluid carries cuttings from the drill bit to the surface. In conventional (over-balanced) drilling, or managed pressure drilling (MPD), the pressure in the well is kept greater than pressure of reservoir to prevent influx from entering the well (Nikoofard et al. (2014c, 2013)). But in UBD operations, the pressure of the well is kept below the pressure of the reservoir, allowing formation fluid flow into the well during the drilling operation.

The pump flow rate, choke valve and density of the drilling fluid (mud) are the various inputs used to adjust the

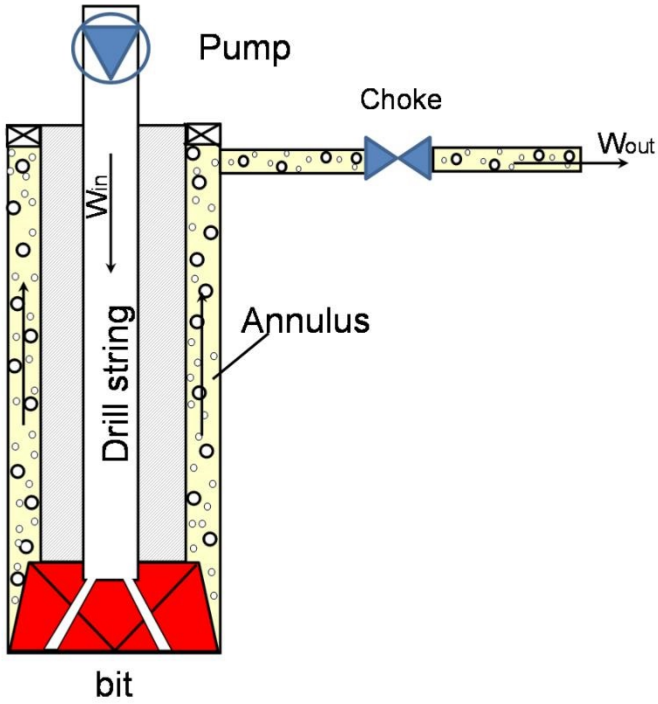

Fig. 1. Schematic of an UBD system

pressure in the well-bore. The choke valve is the most common input used to regulate the pressure in the annulus during MPD and UBD operations. Some states for a dynamic model of multi phase flow in the well can not be measured directly or have a delay or low measurement frequency, and some parameters may be varied only during drilling. So, states and parameters for the dynamic model of multi phase flow in the well must be estimated.

3. MODELING

Due to the existence of multiphase flow (i.e. oil, gas, water, drilling mud and cuttings) in the system, the modeling of the system is challenging. Multiphase flow can be modeled by a distributed model or a simplified model. A distributed model is capable of describing the gas-liquid behavior along the annulus in the well. The simplified LOL model is based on some simplifying assumptions, and considers only the gas-liquid behavior at the drill bit and the choke system. The LOL model is very similar to the two-phase flow model found in (Nygaard and Nævdal (2006); Aamo et al. (2005)). In the simplified drift-flux model and the LOL model, the drilling fluid, oil, water, and rock cuttings are lumped into the liquid phase.

3.1 Simplified drift-flux model

There are two common methods for modeling distributed multiphase flow in UBD operations. The most general and detailed method is called a two-fluid model. This method uses four partial differential equations (PDE's) for conservation of mass and momentum in each phase. The two-fluid model is difficult to solve both analytically or numerically, because the source terms reflecting interphase drag are stiff and this can lead to significant problems in the numerical computation (Evje and Fjelde (2002)). Due to the complexity of the two-fluid model, the drift-flux model is derived by merging the momentum equations of both phases (gas/liquid) into one equation. Therefore, difficult phase interaction terms cancel out, and the missing information in the mixture momentum equation must be replaced by a slip equation which gives a relation between the phase velocities. The mechanistic models use different relations between the phase slip velocities and pressure loss terms for different flow patterns (Lorentzen et al. (2003); Lage et al. (2000)). These models need to predict flow patterns at each time step. In this paper, a simplified drift-flux model (DFM) is used. The simple DFM uses a specific empirical slip law, without flow-regime predictions, but which allows for transition between single and two phase flows. The isothermal simple DFM formulation of the conservation of mass and momentum balance are given by Aarsnes et al. (2014a)

$$\frac{\partial m}{\partial t} + \frac{\partial m v_l}{\partial x} = 0, \tag{1}$$

$$\frac{\partial n}{\partial t} + \frac{\partial n v_g}{\partial x} = 0, \tag{2}$$

$$\frac{\partial (m v_l + n v_g)}{\partial t} + \frac{\partial (P + m v_l^2 + n v_g^2)}{\partial x} = -(m+n) g \cos \Delta\theta - \frac{2 f (m+n) v_m |v_m|}{D}. \tag{3}$$

where the mass variables are defined as follows

$$m = \alpha_l \rho_l, \qquad n = \alpha_g \rho_g$$

where $k = l, g$ denoting liquid or gas, ρ_k is the phase density, and α_k is the volume fraction satisfying

$$\alpha_l + \alpha_g = 1. \tag{4}$$

Further v_k denotes the velocities, and P the pressure. All of these variables are functions of time and space. We denote $t \geq 0$ the time variable, and $x \in [0, L]$ the space variable, corresponding to a curvilinear abscissa with $x = 0$ corresponding to the bottom hole and $x = L$ to the outlet choke position. In the momentum equation (3), the term $(m+n) g \cos \Delta\theta$ represents the gravitational source term, g is the gravitational constant and $\Delta\theta$ is the mean angle between gravity and the positive flow direction of the well, while $-\frac{2f(m+n)v_m|v_m|}{D}$ accounts for frictional losses. The closure relations , boundary conditions and discretization schemes for this model can be found in (Aarsnes et al. (2014a)).

3.2 LOL model

The simplified LOL model equations for mass of gas and liquid in the annulus are derived from mass and momentum balances as follows (Nikoofard et al. (2014a))

$$\dot{m}_g = w_{g,d} + w_{g,res}(m_g, m_l) - \frac{m_g}{m_g + m_l} w_{out}(m_g, m_l) \tag{5}$$

$$\dot{m}_l = w_{l,d} + w_{l,res}(m_g, m_l) - \frac{m_l}{m_g + m_l} w_{out}(m_g, m_l) \tag{6}$$

where m_g and m_l are the total mass of gas and liquid, respectively. The liquid phase is assumed incompressible, and ρ_l is the liquid mass density. The gas phase is compressible and occupies the volume left free by the liquid phase. $w_{g,d}$ and $w_{l,d}$ are the mass flow rates of gas and liquid from the drill string, and $w_{g,res}$ and $w_{l,res}$ are the mass flow rates of gas and liquid from the reservoir. The total mass outflow rate is

$$w_{out} = K_c Z \sqrt{\frac{m_g + m_l}{V_a}} \sqrt{p_c - p_{c0}} \tag{7}$$

where K_c is the choke constant. Z is the control signal to the choke opening, taking its values on the interval $(0, 1]$. The total volume of the annulus is denoted by V_a. p_{c0} is the constant downstream choke pressure (atmospheric). The choke pressure is denoted by p_c, and derived from ideal gas equation

$$p_c = \frac{RT}{M_{gas}} \frac{m_g}{V_a - \frac{m_l}{\rho_l}} \tag{8}$$

where R is the gas constant, T is the average temperature of the gas, and M_{gas} is the molecular weight of the gas. The bottom-hole pressure is given by the following equation

$$p_{bh} = p_c + \frac{(m_g + m_l) g \cos(\Delta\theta)}{A} + \Delta p_f \tag{9}$$

where Δp_f is the friction pressure loss in the well

$$\Delta p_f = K_f (w_{g,d} + w_{l,d})^2 \tag{10}$$

and K_f is the friction factor.

3.3 Reservoir flow

The mass flow from the reservoir into the well for each phase is modeled by a linear relation

$$w_{g,res} = \begin{cases} K_g(p_{res} - p_{bh}), & \text{if } p_{res} > p_{bh} \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

$$w_{l,res} = \begin{cases} K_l(p_{res} - p_{bh}), & \text{if } p_{res} > p_{bh} \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

where p_{res} is the known pore pressure in the reservoir, and K_g and K_l are the production constants of gas and liquid from the reservoir into the well, respectively. Reservoir parameters could be evaluated by seismic data and other geological data from core sample analysis. Still, local variations of reservoir parameters such as the production constants of gas and liquid may be revealed only during drilling. So, it is valuable to estimate the partial variations of some of the reservoir parameters while drilling is performed (Nygaard et al. (2006)).

4. ESTIMATION ALGORITHM

In this section, first an adaptive observer to estimate states and parameters in UBD operation for the LOL model is derived. Then, the joint unscented Kalman filter is presented for both the distributed and LOL models. The measurements and inputs of models are summarized in Table 1. The production constant of gas (K_g) and liquid (K_l) from the reservoir into the well are unknown and must be estimated. K_g and K_l are defined by θ_1 and θ_2, respectively.

4.1 Lyapunov-based adaptive observer

A full-order state observer for the system (5)-(6) is

Table 1. Measurements and inputs

Variables	Measurement/Input
Choke pressure (p_c)	Measurement
Bottom-hole pressure (p_{bh})	Measurement
Drill string mass flow rate of gas ($w_{g,d}$)	Input
Drill string mass flow rate of liquid ($w_{l,d}$)	Input
Choke opening (Z)	Input

$$\dot{\hat{m}}_g = w_{g,d} + \hat{w}_{g,res}(\hat{\theta}_1) - \frac{\hat{m}_g}{\hat{m}_g + \hat{m}_l}\hat{w}_{out}(\hat{m}_g, \hat{m}_l) + k_1(p_{bh} - \hat{p}_{bh}) \tag{13}$$

$$\dot{\hat{m}}_l = w_{l,d} + \hat{w}_{l,res}(\hat{\theta}_2) - \frac{\hat{m}_l}{\hat{m}_g + \hat{m}_l}\hat{w}_{out}(\hat{m}_g, \hat{m}_l) + k_1(p_{bh} - \hat{p}_{bh}) \tag{14}$$

where

$$\hat{w}_{g,res} = \hat{\theta}_1(p_{res} - p_{bh}) \tag{15}$$

$$\hat{w}_{l,res} = \hat{\theta}_2(p_{res} - p_{bh}) \tag{16}$$

$$\hat{w}_{out} = K_c Z\sqrt{\frac{\hat{m}_g + \hat{m}_l}{V_a}}\sqrt{p_c - p_{c0}} \tag{17}$$

$$\hat{p}_{bh} = p_c + \frac{(\hat{m}_g + \hat{m}_l)g\cos(\Delta\theta)}{A} + \Delta p_f \tag{18}$$

and $\frac{k_1}{A} = l_1$ has to be chosen sufficiently large positive. $\hat{m}_g$ and $\hat{m}_l$ are estimates of states m_g and m_l.Defining the state estimation errors $e_1 = m_g - \hat{m}_g$ and $e_2 = m_l - \hat{m}_l$, $\hat{\theta}_1$ and $\hat{\theta}_2$ are estimates of parameters $\theta_1 = K_g$ and $\theta_2 = K_l$.

$$\dot{\hat{\theta}}_1 = q_1(p_{res} - p_{bh})e_1 \tag{19}$$

$$\dot{\hat{\theta}}_2 = q_2(p_{res} - p_{bh})e_2 \tag{20}$$

Since the total mass of gas and liquid in the well could not be measured directly, they are computed by solving a series of nonlinear algebraic equations with measuring the choke and the bottom-hole pressures. m_g^c and m_l^c are calculated by measurements of the choke and the bottom-hole pressures and an inversion of the equations (8) and (9)

$$m_l^c = \frac{1}{1 - \frac{p_c M_{gas}}{RT\rho_l}}\left(\frac{p_{bh} - p_c - \Delta p_f}{g\cos(\Delta\theta)} - \frac{p_c M_{gas} V_a}{RT}\right) \tag{21}$$

$$m_g^c = \frac{p_c M_{gas}(V_a - \frac{m_l^c}{\rho_l})}{RT} \tag{22}$$

The adaptation laws (19)-(20) can be implemented by using $e_1 = m_g^c - \hat{m}_g$ and $e_2 = m_l^c - \hat{m}_l$. The error dynamics can be written as follows

$$\dot{e_1} = (\theta_1 - \hat{\theta}_1)(p_{res} - p_{bh}) - \left(\frac{m_g}{m_g + m_l}w_{out} - \frac{\hat{m}_g}{\hat{m}_g + \hat{m}_l}\hat{w}_{out}\right) - l_1 g\cos(\Delta\theta)(e_1 + e_2) \tag{23}$$

$$\dot{e_2} = (\theta_2 - \hat{\theta}_2)(p_{res} - p_{bh}) - \left(\frac{m_l}{m_g + m_l}w_{out} - \frac{\hat{m}_l}{\hat{m}_g + \hat{m}_l}\hat{w}_{out}\right) - l_1 g\cos(\Delta\theta)(e_1 + e_2) \tag{24}$$

Let $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$, $\tilde{\theta}_2 = \theta_2 - \hat{\theta}_2$, and the Lyapunov function candidate for adaptive observer design be defined as

$$V(e, \tilde{\theta}) = \frac{1}{2}(e_1^2 + e_2^2 + q_1^{-1}\tilde{\theta}_1^2 + q_2^{-1}\tilde{\theta}_2^2) \tag{25}$$

where q_1 and q_2 are positive tuning parameters. It is easy to check that $V(e, \tilde{\theta})$ is positive definite and can be made decrescent. From (23) and (24), the time derivative of $V(e, \tilde{\theta})$ along the trajectory of the error dynamics is

$$\begin{aligned}\dot{V}(e, \tilde{\theta}) =& -l_1 g\cos(\Delta\theta)(e_1 + e_2)^2 - \frac{e_1^2 w_{out}}{m_g + m_l} \\ &+ \tilde{\theta}_1\left[(p_{res} - p_{bh})e_1 + q_1^{-1}\dot{\tilde{\theta}}_1\right] + \tilde{\theta}_2\left[(p_{res} - p_{bh})e_2 + q_2^{-1}\dot{\tilde{\theta}}_2\right] \\ &- \frac{\hat{m}_l e_2(w_{out} - \hat{w_{out}})}{\hat{m}_g + \hat{m}_l} - \frac{e_2^2 w_{out}}{m_g + m_l} - \frac{\hat{m}_g e_1(w_{out} - \hat{w_{out}})}{\hat{m}_g + \hat{m}_l} \\ &+ \frac{\hat{m}_l e_2(e_1 + e_2)w_{out}}{(m_g + m_l)(\hat{m}_g + \hat{m}_l)} + \frac{\hat{m}_g e_1(e_1 + e_2)w_{out}}{(m_g + m_l)(\hat{m}_g + \hat{m}_l)}\end{aligned} \tag{26}$$

The detail calculations of the derivative of the Lyapunov function is presented in Appendix.A:

$$\begin{aligned}\Longrightarrow \dot{V}(e, \tilde{\theta}) <& -l_1 g\cos(\Delta\theta)(e_1^2 + e_2^2) \\ &- \frac{w_{out}(\ e_1^2 + e_2^2)}{m_g + m_l + \sqrt{m_g + m_l}\sqrt{\hat{m}_g + \hat{m}_l}} \\ &- e_1 e_2\left(\ 2l_1 g\cos(\Delta\theta) - \frac{w_{out}(\frac{\sqrt{\hat{m}_g + \hat{m}_l}}{\sqrt{m_g + m_l}})}{m_g + m_l + \sqrt{m_g + m_l}\sqrt{\hat{m}_g + \hat{m}_l}}\right)\end{aligned} \tag{27}$$

By choosing l_1 sufficiently large, then

$$0 \leq \left(\ 2l_1 g\cos(\Delta\theta) - \frac{w_{out}(\frac{\sqrt{\hat{m}_g + \hat{m}_l}}{\sqrt{m_g + m_l}})}{m_g + m_l + \sqrt{m_g + m_l}\sqrt{\hat{m}_g + \hat{m}_l}}\right) < 2l_1 g\cos(\Delta\theta) \tag{28}$$

and this gives

$$\begin{aligned}\dot{V}(e, \tilde{\theta}) <& -l_1 g\cos(\Delta\theta)(e_1^2 + e_2^2) + 2l_1 g\cos(\Delta\theta)|e_1||e_2| \\ &- \frac{w_{out}(\ e_1^2 + e_2^2)}{m_g + m_l + \sqrt{m_g + m_l}\sqrt{\hat{m}_g + \hat{m}_l}}\end{aligned} \tag{29}$$

By using Young's inequality $2|e_1||e_2| \leq e_1^2 + e_2^2$, gives

$$\dot{V}(e, \tilde{\theta}) < -\frac{w_{out}(\ e_1^2 + e_2^2)}{m_g + m_l + \sqrt{m_g + m_l}\sqrt{\hat{m}_g + \hat{m}_l}} \leq 0 \tag{30}$$

which implies that all signals $e_1, e_2, \tilde{\theta}_1, \tilde{\theta}_2$ are bounded. From (23,24) and $e_1, e_2, \tilde{\theta}_1, \tilde{\theta}_2 \in \mathcal{L}_\infty$, $\dot{e_1}, \dot{e_2}$ are bounded. It follows by using Barbalat's lemma that e_1, e_2 converge to zero. If $(p_{res} - p_{bh})$ is persistently exciting, i.e, $\int_t^{t+T}(p_{res} - p_{bh})^2(\tau)\ \mathrm{d}\tau \geq \alpha$ for some $\alpha, T > 0$ and $\forall t \geq 0$, then the parameter estimates will also converge to their true values (Ioannou and Sun (1996)). Thus according to theorem 4.9 in Khalil (2002), the adaptive observer system is globally asymptotically stable if the persistency excitation condition is satisfied. There must be flow from the reservoir to satisfy persistence exciting condition.

4.2 Joint Unscented Kalman Filter

The UKF technique has been developed to work with non-linear systems without using a linearization of the model (Julier et al. (2000); Julier and Uhlmann (2004)). The UKF estimates the mean and covariance matrix of the estimation error with a minimal set of sample points (called sigma points) around the mean by using a deterministic sampling approach known as the unscented transform. The nonlinear model is applied to propagate uncertainty of sigma points instead of using a linearization

Table 2. PARAMETER VALUES FOR WELL AND RESERVOIR

Name	LOL	Unit
Reservoir pressure (p_{res})	278.35	bar
Collapse pressure (p_{coll})	155	bar
Well total length (L_{tot})	2530	m
Drill string outer diameter (D_d)	0.0889	m
Annulus inner diameter (D_a)	0.1548	m
Liquid flow rate ($w_{l,d}$)	13.33	kg/s
Gas flow rate ($w_{g,d}$)	0	kg/s
Liquid density (ρ_l)	1000	kg/m^3
Production constant of liquid (K_l)	0	kg/bar
Production constant of liquid (K_g)	0.1	kg/bar
Gas average temperature (T)	285.15	K
Average angle ($\Delta\theta$)	0	rad
Choke constant (K_c)	0.0096	m^2

of the model. So, this method does not need to calculate the explicit Jacobian or Hessian. More details can be found in (Julier and Uhlmann (2004); Simon (2006)).

The augmented state vector is defined by $x^a = [X, \theta]$. The state-space equations for the the augmented state vector at time instant k is written as:

$$\begin{bmatrix} X_k \\ \theta_k \end{bmatrix} = \begin{bmatrix} f(X_{k-1}, \theta_{k-1}) \\ \theta_{k-1} \end{bmatrix} + q_k$$
$$= f^a(X_{k-1}, \theta_{k-1}) + q_k \tag{31}$$

where $q_k \sim N(0, Q_k)$ is the zero mean Gaussian process noise and model error. The number of states that must be estimated by the joint UKF is equal to three times of the number of spatial discretization cells in the drift-flux model. In the following simulations, it is assumed that only the choke and the bottom-hole pressure are measured.

5. SIMULATION RESULTS

The parameter values for the simulated well and reservoir are summarized in Table 2. The measurements have been synthetically generated by using the simplified drift-flux model tuned by OLGA dynamic multiphase flow simulator (Aarsnes et al. (2014a)).

A discretization of the time and space variables is required for using numerical methods. The PDE of the drift-flux model are discretized by using a finite volumes method for both the joint UKF and simulation. 6 cells were used for the spatial discretization. A measurement sampling period of 10 seconds were used and the model was run with time steps of 10 seconds. The parameter values for the nonlinear adaptive observer and UKF for both models are summarized in Table 3. The initial values for the estimated

Table 3. Parameter Values for model and estimators

Parameter	Value	Parameter	Value
q_1	5×10^{-12}	l_1	0.009
κ_{LOL}	0	κ_{DFM}	0
α_{LOL}	0.001	α_{DFM}	0.00001
β_{LOL}	2	β_{DFM}	2

production constant of gas is ($\hat{K_g} = 0.14$ kg/s/bar). The initial values for the estimated and real states and parameters are as follows

$$m_g = 3377, \quad m_l = 6461.9, \quad \hat{m_g} = 4045.2, \quad \hat{m_l} = 7754.3$$

This paper uses the same model parameters as Aarsnes et al. (2014a), considering UBD operation of a vertical well drilled into a dry gas reservoir (i.e. $W_{l,res} = K_l = 0$). Since the gas reservoir produces gas, adaptive observers only estimate production constant of gas. It should be noted that the adaptive observers are able to estimate production constant of gas and liquid simultaneously. The scenario in this simulation is as follows. First drilling in a steady-state condition is initiated with the choke opening of 3.5 % , then the choke is opened to 3.7 % at 1 hour. After 2 hours, the choke is closed to 3.6 %. In this simulation, the choke and the bottom-hole pressure measurements are corrupted by zero mean additive white noise with the following covariance matrix

$$R = \begin{bmatrix} 500^2 & 0 \\ 0 & 500^2 \end{bmatrix} (Pa^2)$$

Figures 2 and 3 show the simulated and estimated total mass of gas and liquid, respectively. Simulation results demonstrated satisfactory performance of nonlinear adaptive observer to estimate total mass of gas and liquid. Since the choke pressure in LOL model during transient time has a small error, estimation of total mass of gas and liquid with joint UKF has a small bias during transient time. The estimation of the production constant of gas from the reservoir into the well is shown in Figure 4. The production constant of gas is identified correctly by all estimators. In estimation of production constant of gas from the reservoir into the well, adaptive observers based on LOL model have better convergence rate than UKF based on DFM.

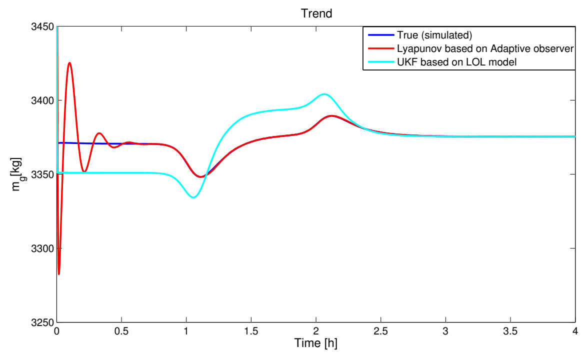

Fig. 2. Estimation of total mass of gas

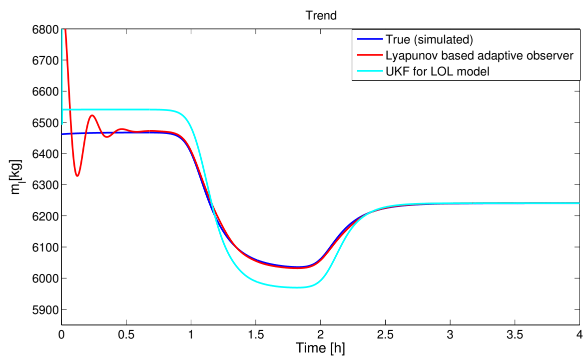

Fig. 3. Estimation of total mass of liquid

Simulation time of adaptive observers based on LOL model executes at least 100 times faster than joint UKF based on simple DFM.

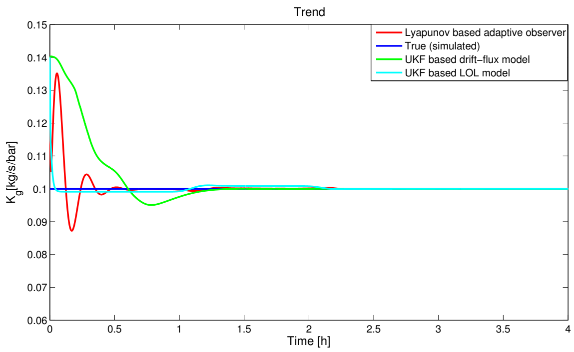

Fig. 4. Actual value and estimated production constant of gas

6. CONCLUSION

This paper describes Lyapunov-based adaptive observer and joint UKF based on LOL model to estimate states and parameters during UBD operations. Moreover, it presents the joint UKF to estimate states and production constant of gas based on the simplified DFM by using real-time measurements of the choke and the bottom-hole pressures. The results show that all adaptive observers are capable of identifying the production index. Adaptive observers based on LOL model shows faster convergence rate than the joint UKF based on DFM, and are computationally simpler. The results suggest that the LOL model may be preferable for the purpose of estimating production constants, although further validation in a wider range of scenarios and study of robustness is recommended.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the financial support provided to this project through the Norwegian Research Council and Statoil ASA (NFR project 210432/E30 Intelligent Drilling). We would like to thank Florent Di Meglio, Ulf Jakob Aarsnes, and Agus Hasan for their contribution to the modeling and simulation data.

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Appendix A. CALCULATION OF DERIVATIVE OF THE LYAPUNOV FUNCTION

$$
\begin{aligned}
\dot{V}(e,\tilde{\theta}) &= -l_1 g\cos(\Delta\theta)(e_1+e_2)^2 + \frac{e_1e_2w_{out}}{m_g+m_l} \\
&- \frac{\hat{m}_g e_1 K_c Z\sqrt{\frac{p_c-p_{c0}}{V_a}}(\sqrt{m_g+m_l}-\sqrt{\hat{m}_g+\hat{m}_l})}{\hat{m}_g+\hat{m}_l} \\
&- \frac{\hat{m}_l e_2 K_c Z\sqrt{\frac{p_c-p_{c0}}{V_a}}(\sqrt{m_g+m_l}-\sqrt{\hat{m}_g+\hat{m}_l})}{\hat{m}_g+\hat{m}_l} \\
&- \frac{e_1^2 w_{out}}{m_g+m_l} + \frac{\hat{m}_g e_1^2 w_{out}}{(m_g+m_l)(\hat{m}_g+\hat{m}_l)} \\
&+ \frac{\hat{m}_l e_2^2 w_{out}}{(m_g+m_l)(\hat{m}_g+\hat{m}_l)} - \frac{e_2^2 w_{out}}{m_g+m_l} \\
\Longrightarrow \dot{V}(e,\tilde{\theta}) &= -l_1 g\cos(\Delta\theta)(e_1+e_2)^2 + \frac{e_1e_2w_{out}}{m_g+m_l} \\
&- \frac{\hat{m}_g e_1 K_c Z\sqrt{\frac{p_c-p_{c0}}{V_a}}(e_1+e_2)}{(\hat{m}_g+\hat{m}_l)(\sqrt{m_g+m_l}+\sqrt{\hat{m}_g+\hat{m}_l})} \\
&- \frac{\hat{m}_l e_2 K_c Z\sqrt{\frac{p_c-p_{c0}}{V_a}}(e_1+e_2)}{(\hat{m}_g+\hat{m}_l)(\sqrt{m_g+m_l}+\sqrt{\hat{m}_g+\hat{m}_l})} \\
&- \frac{\hat{m}_l e_1^2 w_{out}}{(m_g+m_l)(\hat{m}_g+\hat{m}_l)} - \frac{\hat{m}_g e_2^2 w_{out}}{(m_g+m_l)(\hat{m}_g+\hat{m}_l)} \\
\Longrightarrow \dot{V}(e,\tilde{\theta}) &= -l_1 g\cos(\Delta\theta)(e_1+e_2)^2 + \frac{e_1e_2w_{out}}{m_g+m_l} \\
&- \frac{\hat{m}_g e_1^2 K_c Z\sqrt{\frac{p_c-p_{c0}}{V_a}}}{(\hat{m}_g+\hat{m}_l)(\sqrt{m_g+m_l}+\sqrt{\hat{m}_g+\hat{m}_l})} \\
&- \frac{\hat{m}_l e_2^2 K_c Z\sqrt{\frac{p_c-p_{c0}}{V_a}}}{(\hat{m}_g+\hat{m}_l)(\sqrt{m_g+m_l}+\sqrt{\hat{m}_g+\hat{m}_l})} \\
&- \frac{e_1e_2 K_c Z\sqrt{\frac{p_c-p_{c0}}{V_a}}}{(\sqrt{m_g+m_l}+\sqrt{\hat{m}_g+\hat{m}_l})} \\
&- \frac{\hat{m}_l e_1^2 w_{out}}{(m_g+m_l)(\hat{m}_g+\hat{m}_l)} - \frac{\hat{m}_g e_2^2 w_{out}}{(m_g+m_l)(\hat{m}_g+\hat{m}_l)} \\
\Longrightarrow \dot{V}(e,\tilde{\theta}) &= -l_1 g\cos(\Delta\theta)(e_1+e_2)^2 + \frac{e_1e_2w_{out}}{m_g+m_l} \\
&- \frac{\hat{m}_g e_1^2 w_{out}}{(\hat{m}_g+\hat{m}_l)(m_g+m_l+\sqrt{\hat{m}_g+\hat{m}_l}\sqrt{m_g+m_l})} \\
&- \frac{\hat{m}_l e_2^2 w_{out}}{(\hat{m}_g+\hat{m}_l)(m_g+m_l+\sqrt{\hat{m}_g+\hat{m}_l}\sqrt{m_g+m_l})} \\
&- \frac{e_1e_2 w_{out}}{(m_g+m_l+\sqrt{\hat{m}_g+\hat{m}_l}\sqrt{m_g+m_l})} \\
&- \frac{\hat{m}_l e_1^2 w_{out}}{(m_g+m_l)(\hat{m}_g+\hat{m}_l)} - \frac{\hat{m}_g e_2^2 w_{out}}{(m_g+m_l)(\hat{m}_g+\hat{m}_l)}
\end{aligned}
$$

$$
\begin{aligned}
\Longrightarrow \dot{V}(e,\tilde{\theta}) &= -l_1 g\cos(\Delta\theta)(e_1+e_2)^2 \\
&- \frac{\hat{m}_g\ e_1^2\ w_{out}}{(m_g+m_l+\sqrt{m_g+m_l}\sqrt{\hat{m}_g+\hat{m}_l})(\hat{m}_g+\hat{m}_l)} \\
&+ \frac{e_1e_2\ w_{out}\sqrt{m_g+m_l}\sqrt{\hat{m}_g+\hat{m}_l}}{(m_g+m_l+\sqrt{m_g+m_l}\sqrt{\hat{m}_g+\hat{m}_l})(m_g+m_l)} \\
&- \frac{\hat{m}_l\ e_2^2\ w_{out}}{(m_g+m_l+\sqrt{m_g+m_l}\sqrt{\hat{m}_g+\hat{m}_l})(\hat{m}_g+\hat{m}_l)} \\
&- \frac{\hat{m}_g e_2^2 w_{out}}{(m_g+m_l)(\hat{m}_g+\hat{m}_l)} - \frac{\hat{m}_l e_1^2 w_{out}}{(m_g+m_l)(\hat{m}_g+\hat{m}_l)} \\
\Longrightarrow \dot{V}(e,\tilde{\theta}) &< -l_1 g\cos(\Delta\theta)(e_1+e_2)^2 \\
&- \frac{e_1^2\ w_{out}}{m_g+m_l+\sqrt{m_g+m_l}\sqrt{\hat{m}_g+\hat{m}_l}} \\
&+ \frac{e_1e_2\ w_{out}\sqrt{m_g+m_l}\sqrt{\hat{m}_g+\hat{m}_l}}{(m_g+m_l+\sqrt{m_g+m_l}\sqrt{\hat{m}_g+\hat{m}_l})(m_g+m_l)} \\
&- \frac{e_2^2\ w_{out}}{m_g+m_l+\sqrt{m_g+m_l}\sqrt{\hat{m}_g+\hat{m}_l}} \\
\Longrightarrow \dot{V}(e,\tilde{\theta}) &< -l_1 g\cos(\Delta\theta)(e_1+e_2)^2 \\
&- \frac{w_{out}(\ e_1^2+e_2^2-e_1e_2\frac{\sqrt{\hat{m}_g+\hat{m}_l}}{\sqrt{m_g+m_l}})}{m_g+m_l+\sqrt{m_g+m_l}\sqrt{\hat{m}_g+\hat{m}_l}}
\end{aligned}
$$