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Product Upgrading Decisions under Uncertainty in a Durable Goods Market

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Problem description

This paper studies investment behavior of firms deciding when to introduce an upgrade in a durable goods market under uncertain product testing and risk of malfunction

Preface

This thesis was written during the Spring of 2017 as the final part of a five-year Master of Science degree at the Norwegian University of Science and Technology (NTNU). The degree specializes in Financial Engineering at the Department of Industrial Economics and Technology Management. We would like to thank our supervisors Verena Hagspiel and Maria Lavrutich at the department for excellent guidance throughout the project. We would also like to thank Kuno Huisman and Peter Kort at Tilburg University for useful insights and feedback.

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Sammendrag

Denne oppgaven omhandler investeringsadferd for selskaper som står overfor en beslutning om når en oppgradert produktversjon skal lanseres i et marked for varige produkter. Selskapet velger både investeringstidspunkt og pris, samtidig som det avveier risikoen for produktfeil og tilbakekalling av solgte enheter. Mer spesifikt ønsker oppgaven å vise hvilke insentiver et selskap kan ha til å introdusere den oppgraderte versjonen tidlig og dermed akseptere høyere risiko for produktfeil. Selskapet kan redusere denne risikoen ved å gjennomføre produkttester av uviss varighet. Vi viser at villigheten til lansere den oppgraderte versjonen tidlig med betydelig risiko for produktfeil er større når (*i*) etterspørselen for den eksisterende versjonen har blitt svekket, (*ii*) kvaliteten og mengden potensielle kunder for den oppgraderte versjonen er høy eller (*iii*) prosessen for testing er treg. Videre viser vi at tilstedeværelsen av en innovatør i markedet, som allerede har lansert en ny og risikabel produktversjon, gjør at etterhengeren lanserer et mer pålitelig produkt. Denne effekten er sterkere når innovatørens produkt har en større sannsynlighet for feil.

Product Upgrading Decisions under Uncertainty in a Durable Goods Market

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Abstract

This paper studies investment behavior of firms deciding when to introduce an upgrade in a durable goods market. The firm chooses both the investment timing and the price of the upgrade while facing the risk of the upgrade experiencing a serious malfunction and requiring a complete recall. More specifically, the paper aims to show what incentives a firm may have to introduce an upgrade early and accept a higher malfunction risk. The firm can reduce this risk by performing product tests of uncertain duration. We show that the willingness to introduce an upgrade early with significant malfunction risk is larger when (*i*) the demand for the existing version has weakened, (*ii*) the quality and stock of potential customers for the upgrade is high or (*iii*) the testing process is slow. Furthermore, we find that the presence of an innovator with a risky product in the market for the upgrade makes the laggard release a more reliable product. This effect is stronger when the innovator's product is more likely to malfunction.

[1]We would like to thank Verena Hagspiel and Maria Lavrutich at the department for excellent guidance throughout the project. We would also like to thank Kuno Huisman and Peter Kort for useful insights.

1 Introduction

Samsung introduced Note 7, the annual upgrade of the Samsung Galaxy Note smartphone series, in the fall of 2016. Customers immediately reported incidents of the battery overheating, causing the Note 7 to catch fire or even explode. Samsung ended up recalling all sold Note 7 units and refunding the customers, essentially losing all revenue from the smartphone[2]. The example of Samsung is only the most recent among the many product upgrades that have been recalled as a result of serious malfunction. Other examples include self-igniting water heaters from OSO sold in Norway [3] and F-250 pickup trucks manufactured by Ford that would roll unintended while in park [4]. Typically, these products are durable goods. Durable goods are consumer goods that do not wear out fast or have to be repurchased for several years[5]. In general, product upgrades are critical for producers of durable goods to be successful. Failing to deliver innovative upgrades may lead to a substantial loss in demand, as evident by Nokia's drop from 50% to under 5% market share since the introduction of the iPhone in 2007 [6]. This paper investigates the incentives that drive producers of durable goods to introduce a product upgrade with risk of malfunction.

Although durable goods expenditure accounted for roughly $1.3 trillion in the USA alone in 2016 [7], the research on investment decisions for producers in durable goods markets has gained limited attention in the literature. Interestingly, most of the durable goods literature ignores the concept of product upgrades although durable goods markets are typically characterized by products that improve over time. Levinthal and Purohit (1989) were the first to address the upgrading problem and studied the decision of a monopolist in a two-period model under the case of both separate and joint production. They found that the profits from separate production, meaning that the existing version is phased out when the upgrade is introduced, gives unambiguously higher profits than joint production. Furthermore, they found that a buy-back policy can make joint production most profitable when there is a substantial difference in quality between the two versions. We consider a separate production model like the one presented in Levinthal and Purohit (1989) and extend it by introducing a testing phase of uncertain duration and a possibility of product malfunction for the upgraded version.

Fudenberg and Tirole (1998) present a more thorough analysis of the upgrading problem, focusing on what happens in the presence of a second-hand market. They assume that new and used products are imperfect substitutes because the quality of the products deteriorate over time. Consequently, the monopolist has incentive to lower the durability to make used products less competitive to the upgrade. Contrary to Fudenberg and Tirole (1998), we will ignore second-hand markets to more accurately evaluate the effects of a testing phase and risk of product malfunction requiring a total recall.

A topic that has received significant attention in the durable goods literature is the time inconsistency problem. This problem was first addressed by Coase (1972) and arises because durable goods sold in the future affects the future value of units sold today. Coase (1972) argued that unless a monopolist is able to pre-commit to a price, the consumer's expectations of future price reductions will instantly lower the price of the durable good to marginal cost. He also argued that leasing would avoid the problem, which was later confirmed by Bulow (1982). Common topics in subsequent durable goods literature include the robustness of Coase's time inconsistency problem and ways to overcome it. A more recent contribution on the latter is Hahn (2006)

[2] http://www.samsung.com/us/note7recall/
[3] https://www.nrk.no/telemark/flere-tusen-brannfeller-star-igjen-1.11897142
[4] http://www.reuters.com/article/us-ford-motor-recall-idUSKBN1733Q5
[5] http://www.investopedia.com/terms/d/durables.asp
[6] https://www.statista.com/statistics/263438/market-share-held-by-nokia-smartphones-since-2007/
[7] http://www.investopedia.com/terms/d/durables.asp

who found that introducing a stripped-down version of a durable good would in fact mitigate the time inconsistency problem. Waldman (1996) analyzed a model for the upgrading problem similar to Fudenberg and Tirole (1998) and showed that product upgrade introductions are subject to a time inconsistency problem like the one presented in Coase (1972). They found that this gives the firm an incentive to make the existing version obsolete upon introduction of the upgrade. These papers assume that firms are not able to pre-commit to a price. In reality however, many firms, like Apple, have a credible reputation for not reducing the price of existing products until a new version is introduced. We therefore assume that the firm is in fact able to pre-commit to a price for the upgrade.

Firms considering to introduce an upgraded version of a durable good have to consider the uncertainty involved. The risk of malfunction may to some degree be mitigated by performing product tests, but testing is itself often an uncertain process. Investment under uncertainty is typically modeled by a real options approach, for which Dixit and Pindyck (1994) represent some of the earlier literature. They start out with a basic real options model where the payoff from investing follows a geometric Brownian motion and present a solution approach to the optimal stopping problem. In later chapters, they extend this basic model to include several real world applications, like options to mothball or abandon projects after investment, and allowing the firm to invest in a certain capacity. However, they do not consider the specific characteristics of the good. More specifically, they do not distinguish between consumable and durable goods, even though they have different demand structures. We follow a similar solution approach as Dixit and Pindyck (1994) and extend their work by considering the specific features of a durable good.

In our model, we consider uncertainty in the testing phase for the upgrade, which is a type of technological uncertainty. Grenadier and Weiss (1997) were among the first to apply a real options approach to investment under technological uncertainty. They consider the optimal investment strategy of a firm confronted with a sequence of technological innovations and apply a geometric Brownian motion to model technological progress. This modeling assumption implies that the technology level can actually decrease over time, which is hard to defend with intuition. The recent norm has therefore been to instead model technological progress with a Poisson process with positive jumps, like Farzin et al. (1998), who consider optimal timing of technology adoption in a dynamic programming framework. In our model, we apply a Poisson process to model the evolution of the testing phase.

Several papers on investment under uncertainty use examples from the durable goods industry to motivate their research, without taking the specific features of durable goods into account. Recent examples include Lavrutich et al. (2016), who consider entry deterrence and hidden competition, and Hagspiel, Kort, et al. (2016), who consider investment in a production facility with flexibility to scale the capacity. To motivate their work, both papers refer to the car industry. Lavrutich et al. (2016) use Apple as an example of a hidden competitor developing their own electric car, while Hagspiel, Kort, et al. (2016) present volume flexibility as a key strategy implemented by car manufacturers to cope with demand uncertainty. Although the market for cars is clearly a durable goods market, this is not accounted for in the authors' respective demand functions.

The contribution of this paper is two-fold. First, it extends the literature on investment decisions in durable goods markets by including a testing phase and a risk of having to recall a malfunctioned product. We propose to model the arrival of a malfunction event as an inhomogeneous Poisson process, which, to the best of our knowledge, has not been done before. Second, it extends the literature on investment under uncertainty by developing a model that accounts for the specific features of durable goods. We find that a firm's incentive to introduce a risky upgrade in the market may be explained by three reasons. First, an incentive to release a risky upgrade arises when the profits from selling the existing version has dropped low. Second, if the upgrade is of good quality and has a high expected initial stock of potential customers, the expected value of the investment is high. This gives incentive to invest early even if significant risk of malfunction is present. Finally, a slow testing process may drive the firm to gamble on a risky upgrade rather than wait for more sub-tests to complete. We also find that the presence of an innovator with a product that may itself malfunction in the market for the upgrade, makes the laggard firm invest later and in a more reliable product. This delaying effect is stronger if the innovator did limited product testing before it released its product, but decreases over time as the product is sold without malfunctioning.

The rest of the paper is organized as follows. In Section 2, we solve the upgrading problem for a monopolistic firm, and in Section 3 we consider an innovator-laggard model. In both models we consider two cases: *(1)* the firm must refrain from investment until the upgrade has gone through a testing phase with no uncertainty, and *(2)* the firm can invest at any stage of an uncertain testing process, at the risk of malfunction. We also compare an analytical solution approach suggested in Hagspiel, Huisman, et al. (2016) to a numerical solution, and argue that the analytical approach is erroneous. Section 4 summarizes the findings and concludes. The proofs of all propositions can be found in the appendix.

2 Monopoly

Consider first a risk-neutral and profit-maximizing monopolist producing a single durable good. The firm makes a decision on when, or if, it should introduce an upgraded version of the product, and which price to set for it. The technology for the upgrade needs to go through an exogenous testing phase affecting the risk of malfunction. In Section 2.1, the duration of the testing phase is assumed to be deterministic and finish at time t_n[8]. The firm does not invest before the testing phase is completed, but can then adopt the technology and introduce the upgrade by paying a fixed investment cost I. Later, in Section 2.2, we relax this assumption and allow the firm to introduce the upgrade at any stage of a stochastic testing process. Upon introduction of the upgrade, a stock of potential customers, hereafter referred to as customer potential, arises for the new version and is expected to be Q_2^0.

The firm is currently selling the existing version at the price P_1, in a market with a customer potential Q_1^0. Since we do not consider the decision to introduce this version, P_1 is treated as a fixed parameter. In other words, P_1 is the price the firm committed to when the existing version was introduced. Furthermore, we let $Q_i(t)$ denote the remaining customer potential for version i. Subscript 1 represents the existing version, while subscript 2 represents the upgrade to be introduced. For most durable goods, each customer would normally not buy more than one unit of the product. For instance, a single person would rarely purchase more than one iPhone 7. The problem is therefore modeled such that a customer can buy at most one unit of each version of the product. The customer potential therefore reduces over time as more units are sold, and the dynamics are given by equation (2.1)

$$dQ_i(t) = -q_i(t)dt, i = 1, 2, \tag{2.1}$$

where $dQ_i(t)$ denotes the instantaneous change in the customer potential for version i over time period dt and $q_i(t)$ denotes the instantaneous demand for version i. As stated in the introduction we also assume that the existing version becomes completely obsolete upon introduction of the

[8]The time-lag from technology adoption to market release is included in t_n

upgrade, implying that Q_1^0 drops to zero. This allows us to investigate the effects of a testing phase and product malfunction risk on the investment decision in more detail.

The instantaneous demand is a linear function as given in equation (2.2)

$$q_i(t) = Q_i(t) - \eta_i P_i, i = 1, 2, \tag{2.2}$$

where η_i in the demand function is a price penalty factor determined by the quality of the product. Typically, one would expect the upgrade to be of better quality than the existing version and therefore the relation $\eta_1 > \eta_2$ to hold. However, Hahn (2006) showed that firms have incentives to introduce a stripped-down lower quality version of a durable good to mitigate the time inconsistency problem. Our modeling approach provides flexibility to capture both situations.

2.1 Deterministic product testing

Following Dixit and Pindyck (1994), we assume that the firm's operating costs are equal to zero without loss of generality. Future cash flows are discounted at the exogenous rate r. The present value of future cash flows, denoted $V(\tau, P_2)$, if the firm introduces the upgrade at time τ and price P_2 is then equal to

$$V(\tau, P_2) = \int_0^{t_n} P_1 q_1(t) e^{-rt} dt + \int_{t_n}^{\tau} P_1 q_1(t) e^{-rt} dt + \int_{\tau}^{\infty} P_2 q_2(t-\tau) e^{-rt} dt - I e^{-r\tau}. \tag{2.3}$$

From equation (2.3) it is clear that the value $V(\tau, P_2)$ consists of two parts. The first two terms correspond to the value of selling the existing version until τ, while the last two terms correspond to the value of selling the upgrade from τ onwards. The firm faces a trade-off in deciding the introduction time τ. On one hand, additional revenue from the existing version represents a benefit of waiting, and gives the firm an incentive to delay the introduction of the upgrade. On the other hand, delaying the introduction leads to a heavier discounting on the revenue from the upgrade. This effect represents a cost of waiting and gives the firm an incentive to hasten the introduction.

The firm's objective is to maximize the present value of future cash flows by optimally deciding the time of introduction and price for the upgrade. Finding the maximized present value of the firm's profits, denoted V^* and hereafter referred to as the firm value, translates to the following optimization problem

$$V^* = \int_0^{t_n} P_1 q_1(t) e^{-rt} dt + \max_{\tau} \left[\int_{t_n}^{\tau} P_1 q_1(t) e^{-rt} dt + \max_{P_2} \left[\int_{\tau}^{\infty} P_2 q_2(t-\tau) e^{-rt} dt \right] - I e^{-r\tau} \right]. \tag{2.4}$$

This optimization problem can be solved backwards in two steps. First, we derive the optimal price to maximize the present value of profits after the introduction. Second, we derive the optimal investment timing to maximize the profits over the whole period of consideration. The first step corresponds to solving the inner maximization in equation (2.4). We find the maximized present value of profits after introduction, denoted V_2^*, by solving

$$V_2^* = \max_{P_2} \left[\int_{\tau}^{\infty} P_2 q_2(t-\tau) e^{-rt} dt \right]. \tag{2.5}$$

Applying the first order condition for optimality to equation (2.5), we find that the optimal price for the upgrade, P_2^*, is given by

$$P_2^* = \frac{Q_2^0}{2\eta_2}. \tag{2.6}$$

An increase in the initial customer potential leads to an increase in the demand for the upgrade and allows the firm to charge a higher price, implying that P_2^* is increasing in Q_2^0. Similarly, an increase in the penalty factor from lower product quality leads to a reduction in demand, which implies that P_2^* is decreasing in the price penalty factor η_2. Given the optimal price for the upgrade, equation (2.5) is reduced to

$$V_2^* = \frac{(Q_2^0)^2}{4\eta_2(1+r)} e^{-r\tau}. \tag{2.7}$$

An increase in the initial customer potential for the upgrade increases the demand for and therefore the revenue from the upgrade, implying that V_2^* is increasing in Q_2^0. An increase in the price penalty factor resulting from lower product quality reduces revenue from the upgrade, implying that V_2^* is decreasing in η_2. The term $e^{-r\tau}$ discounts the present value of the revenue at the time of introduction back to today. Equation (2.7) also allows us to derive the net present value of the upgrade evaluated at the time of introduction as $\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I$.

We continue with the second step of the optimization problem from equation (2.4) and derive the optimal investment timing to maximize the present value of profits over the entire investment horizon. The optimal timing of investment, denoted τ^*, is presented in Proposition 2.1.

Proposition 2.1 *The optimal time to introduce the upgrade is given by*

$$\tau^* = \begin{cases} t_n & \hat{\tau^*} < t_n, \\ \hat{\tau^*} & \hat{\tau^*} \geq t_n, \\ \to \infty & \frac{(Q_2^0)^2}{4\eta_2(1+r)} - I \leq 0, \end{cases} \tag{2.8}$$

where

$$\hat{\tau^*} = \ln \left[\frac{P_1(Q_1^0 - \eta_1 P_1)}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right)} \right]. \tag{2.9}$$

The optimal ivestment timing is given by one of three strategies, depending on the parameter values. In the case where $\hat{\tau^*} < t_n$, it is optimal for the firm to introduce the upgrade as soon as the testing is completed. Hence, the optimal time to introduce the upgrade is t_n.

In the case where $\hat{\tau^*} \geq t_n$, the firm prefers to delay the introduction of the upgrade even after the testing is completed. This implies that at the time testing is completed, the benefit of additional revenue by delaying is greater than the cost of heavier discounting on the revenue from the upgrade. The optimal strategy for the firm is therefore to delay the introduction of the upgrade until the benefit and cost of waiting are equal, represented by $\hat{\tau^*}$ in equation (2.9) of Proposition 2.1.

The case $\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I \leq 0$ represents the situation when introducing the upgrade is unprofitable, i.e the investment has a negative net present value. It will thus never be optimal for the firm to invest in the upgrade, implying that τ^* goes to infinity.

When $\tau^* = t_n$ or $\tau^* \to \infty$, the optimal time of introduction can be considered constant and insensitive to changes in the model parameters. When the optimal investment timing is instead given by $\hat{\tau^*}$, we derive the parameter sensitivities. The results are presented in Proposition 2.2.

Proposition 2.2 *The sensitivity of the optimal investment timing when $\tau^* = \hat{\tau^*}$ with respect to the model parameters Q_1^0, P_1, η_1, Q_2^0, η_2, I, r is presented in Table (1)*

Sensitivity	Value
$\frac{\partial \hat{\tau^*}}{\partial Q_1^0}$	> 0
$\frac{\partial \hat{\tau^*}}{\partial \eta_1}$	< 0
$\frac{\partial \hat{\tau^*}}{\partial P_1}$	> 0 when $P_1 < \frac{Q_1^0}{2\eta_1}$ < 0 when $P_1 > \frac{Q_1^0}{2\eta_1}$
$\frac{\partial \hat{\tau^*}}{\partial Q_2^0}$	< 0
$\frac{\partial \hat{\tau^*}}{\partial \eta_2}$	> 0
$\frac{\partial \hat{\tau^*}}{\partial I}$	> 0
$\frac{\partial \hat{\tau^*}}{\partial r}$	> 0 when $\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I < rI$ < 0 when $\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I > rI$

Table 1: The introduction timing's sensitivity to the model parameters

An increase in the demand for the existing version results in a higher benefit of waiting and therefore delays investment. This implies that $\hat{\tau^*}$ is increasing in Q_1^0 and decreasing in η_1. Similarly, increasing the profits from the upgrade increases the cost of waiting and therefore hastens investment. This implies that $\hat{\tau^*}$ is decreasing in initial customer potential Q_2^0 and increasing in the price penalty factor η_2 and the investment cost I.

The effect of changing the price of the existing version is ambiguous. We show in Appendix B that the term $\frac{Q_1^0}{2\eta_1}$ is actually the optimal price $P_1^*(Q_1^0)$ given the customer potential Q_1^0. When P_1 is close to P_1^*, staying in the existing market is more attractive and investment is delayed. This implies that $\hat{\tau^*}$ is decreasing in P_1 when $P_1 > \frac{Q_1^0}{2\eta_1}$ and increasing in P_1 when $P_1 < \frac{Q_1^0}{2\eta_1}$. However, recall that the firm pre-commits to a price for its products. The initially optimal pre-commitment price for the existing version was $\tilde{P}_1 = \frac{\tilde{Q}_1^0}{2\eta_1}$, where $\tilde{Q}_1^0$ was the initial customer potential. The customer potential decreases over time as units are sold, implying that $\tilde{P}_1 > \frac{Q_1^0}{2\eta_1}$. Thus, if the firm did pre-commit to the optimal price $\tilde{P}_1$, the optimal investment timing $\hat{\tau^*}$ is decreasing in P_1.

An increase in the discount rate results in two contradicting effects on the optimal timing. The first effect is that the present becomes relatively more important, which reduces the value of waiting. This gives the firm an incentive to introduce the upgrade earlier. The second effect is that the value of investing, given by the net present value of the upgrade, decreases. This makes the upgrade relatively less attractive, and gives the firm incentive to introduce its upgrade later. Hence, the overall effect on the investment timing depends on which of these effects are most significant. It turns out that the first effect dominates and investment is hastened when the net present value of the upgrade is above rI, while the second effect dominates and the investment is delayed otherwise. Furthermore, there exists a threshold discount factor $\tilde{r} = \sqrt{\frac{{Q_2^0}^2}{4\eta_2 I}} - 1$ for which an increase above $\tilde{r}$ delays investment, and an increase below $\tilde{r}$ hastens investment. For large Q_2^0 and low η_2 and I, the region below $\tilde{r}$ is large, while the region above $\tilde{r}$ is large in the opposite case. The ambiguous relation between the discount rate and the optimal investment timing is illustrated in Figure 1.

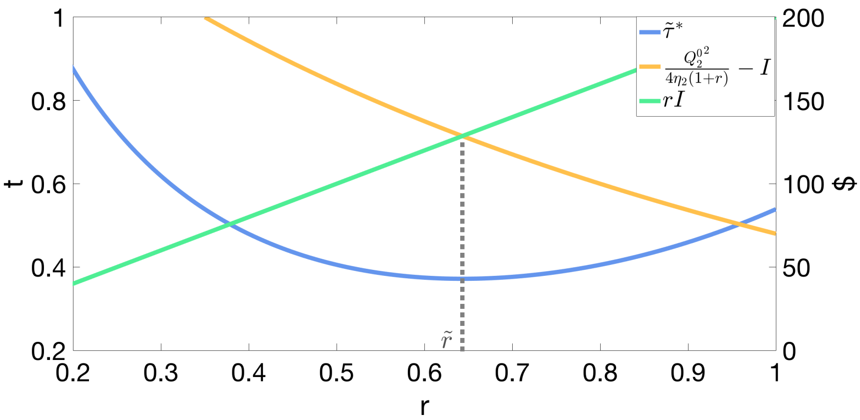


Figure 1: An increase in the discount rate has an ambiguous effect on the investment timing. ($Q_1^0 = 100$, $P_1 = 3$, $\eta_1 = 20$, $Q_2^0 = 180$, $\eta_2 = 15$, $I = 200$)

Given the optimal investment timing, we can derive the optimal value of the firm. The result is stated in Proposition 2.3.

Proposition 2.3 *The value of the firm, $V^*(\tau^*)$, is given by*

$$V^*(\tau^*) = \begin{cases} \frac{P_1(Q_1^0-\eta_1 P_1)}{1+r} + \mathrm{e}^{-\mathrm{rt_n}} \left(\left(\frac{Q_2^{0^2}}{4\eta_2(1+r)} - I \right) - \frac{P_1(Q_1^0-\eta_1 P_1)}{1+r} \mathrm{e}^{-\mathrm{t_n}} \right) & \tau^* = t_n, \\ \frac{P_1(Q_1^0-\eta_1 P_1)}{1+r} + \frac{\frac{Q_2^{0^2}}{4\eta_2(1+r)} - I}{1+r} \left(\frac{P_1(Q_1^0-\eta_1 P_1)}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right)} \right)^{-r} & \tau^* = \hat{\tau^*}, \\ \frac{P_1(Q_1^0-\eta_1 P_1)}{1+r} & \tau^* \to \infty. \end{cases} \tag{2.10}$$

The firm value takes one of three forms depending on the optimal investment time τ^*. Common for all three is the term $\frac{P_1(Q_1^0-\eta_1 P_1)}{1+r}$, which represents the value of selling the existing version forever. For $\tau^* = t_n$, where it is optimal for the firm to invest immediately upon completion of the testing phase, the term $\mathrm{e}^{-\mathrm{rt_n}} \left(\frac{Q_2^{0^2}}{4\eta_2(1+r)} - I \right)$ represents the discounted value of the profits from the upgrade. The subtraction of $\mathrm{e}^{-(\mathrm{r}+1)\mathrm{t_n}} \frac{P_1(Q_1^0-\eta_1 P_1)}{1+r}$ corrects for the fact that the existing version is discontinued once the upgrade has been introduced at t_n.

In the case where $\tau^* = \hat{\tau^*}$, the second term represents the value added by introducing the upgrade at $\hat{\tau^*}$ compared to selling the existing version forever. In the third case, $\tau^* \to \infty$, the firm never introduces the upgrade and there is no additional value above selling the existing version forever.

We continue to derive the dependency of the firm value to the model parameters, as laid out in Proposition 2.4.

Proposition 2.4 *The sensitivity of the firm value with respect to the parameters Q_1^0, P_1, η_1, Q_2^0, η_2, I, r is presented in Table (2)*

Sensitivity	Value
$\frac{\partial V^*(\tau^*)}{\partial Q_1^0}$	> 0
$\frac{\partial V^*(\tau^*)}{\partial \eta_1}$	< 0
$\frac{\partial V^*(\tau^*)}{\partial P_1}$	> 0 when $P_1 < \frac{Q_1^0}{2\eta_1}$ < 0 when $P_1 > \frac{Q_1^0}{2\eta_1}$
$\frac{\partial V^*(\tau^*)}{\partial Q_2^0}$	≥ 0
$\frac{\partial V^*(\tau^*)}{\partial \eta_2}$	≤ 0
$\frac{\partial V^*(\tau^*)}{\partial I}$	≤ 0
$\frac{\partial V^*(\tau^*)}{\partial r}$	< 0

Table 2: The firm value's sensitivity to the model parameters

An increase in the value from selling the existing version will increase the overall value of the firm, so $V^*(\tau^*)$ is increasing in the customer potential Q_1^0 and decreasing in the price penalty factor η_1. We observe the same ambiguous relationship to the price of the existing product as we did for the optimal investment timing in Proposition 2.2. This implies that $V^*(\tau^*)$ is increasing in P_1 when $P_1 < \frac{Q_1^0}{2\eta_1}$, and decreasing in P_1 when $P_1 > \frac{Q_1^0}{2\eta_1}$. Hence, the firm value is decreasing in P_1 if the price that was committed to at the introduction of the existing version was chosen optimally to be $\tilde{P}_1$.

Similar to the existing version, an increase in the value of selling the upgrade increases the overall value of the firm, implying that $V^*(\tau^*)$ is increasing in the initial customer potential Q_2^0 and decreasing in the price penalty factor η_2 and the investment cost I. For the case $\tau^* \to \infty$, the firm never introduces the upgrade, and the firm value is unaffected by changes in Q_2^0, η_2 and I.

Furthermore, an increase in the discount rate reduces the value of future cash flows, which implies that $V^*(\tau^*)$ is decreasing in r.

2.2 Stochastic product testing and risk of malfunction

As the example with the Samsung Galaxy Note 7 in the introduction illustrated, producers of durable goods may choose to introduce a product before it is thoroughly tested for malfunction risks. We therefore relax the assumption that the firm refrains from undertaking the investment until testing is completed. Instead, we introduce a testing level θ to describe the amount of testing that has been completed. Overall product testing is often considered a series of independent sub-tests; a smartphone test could for instance consist of battery testing, CPU testing and screen responsiveness testing. Hence, a Poisson jump process is a suitable modeling approach to describe the development in the testing phase. The process for θ is therefore given as in equation (2.11).

$$d\theta = \begin{cases} u & Prob = \lambda dt, \\ 0 & Prob = 1 - \lambda dt. \end{cases} \tag{2.11}$$

The jump size u defines the impact of a single sub-test, while λ determines the expected duration of each sub-test by $E[duration] = \frac{1}{\lambda}$. In reality, each sub-test arrival would likely have a random impact. In this case, u can be interpreted as the expected impact of each sub-test arrival instead of the actual impact. Hence, this modification will not affect the implications of the model, and we will henceforth assume a deterministic jump size.

Samsung ended up recalling all sold units of their Galaxy Note 7 and essentially lost the entire revenue from the project. We therefore assume that a malfunction results in the firm losing the entire value generated by the product, including the revenue up until the malfunction. However, if a malfunction only resulted in a partial recall, the incentive to introduce the upgrade with malfunction risk would only be stronger and the results in this paper would still be valid. Furthermore, we want the marginal effect of completing sub-tests to be decreasing in θ, because firms often test the most vital parts of the product first. This makes an exponentially decreasing relation between the testing level θ and the probability of experiencing a malfunction suitable, which also captures the real world fact that a product will never be completely free of malfunction risk. The probability of experiencing a malfunction should also decrease when the product has been for sale over time without malfunctioning. We therefore suggest that the arrival of a malfunction event can be modeled by a single arrival inhomogeneous Poisson process, given as

$$dV_2 = \begin{cases} -V_2 & Prob = \phi(t)dt, \\ 0 & Prob = 1 - \phi(t)dt, \end{cases} \tag{2.12}$$

where

$$\phi(t) = \alpha e^{-(\theta + \alpha t)}, \alpha > 0, \tag{2.13}$$

is the inhomogeneous arrival rate of a product malfunction, and V_2 is the total revenue from the upgrade. The intensity parameter α determines how quickly the probability of experiencing a malfunction decreases with the time it has been for sale. The instantaneous probability of experiencing a malfunction within the next period dt is denoted $p(t)$ and given by $p(t) = \phi(t)dt$. Further, we assume that the existing version has been for sale for a long time, and the probability of it malfunctioning is therefore zero.

For the monopolistic firm in this section, the time at which the upgrade malfunctions after introduction does not matter, because it results in a total recall and all revenue is lost. Hence, the firm only considers the accumulated probability of ever experiencing a malfunction after the upgrade is released, hereafter referred to as the disaster probability and denoted p_d. The disaster probability is calculated by integrating the instantaneous probability from zero to infinity and is given by $p_d(\theta) = e^{-\theta}$. Figure 2 shows how the disaster probability relates to the testing level.

An important characteristic of the disaster probability is that it is independent of the intensity parameter α. This is because α does not affect the overall probability of experiencing a malfunction, but only the expected time at which it will happen. It is therefore irrelevant for the monopolistic firm in this section.

The firm's objective is to maximize the expected present value of future cash flows, by optimally deciding the testing level θ and the time τ to introduce the upgrade, as well as its price. This optimization problem is formulated in equation (2.14).

$$\max_{\tau, P_2} \mathbf{E}_\theta \left[V_1(\tau) + (V_2(\theta) - I)\, e^{-r\tau} \right], \tag{2.14}$$

The term $V_1(\tau)$ represents the revenue from selling the existing product up until the time of investment τ, and $(V_2(\theta) - I)\, e^{-r\tau}$ represents the discounted value of selling the new product

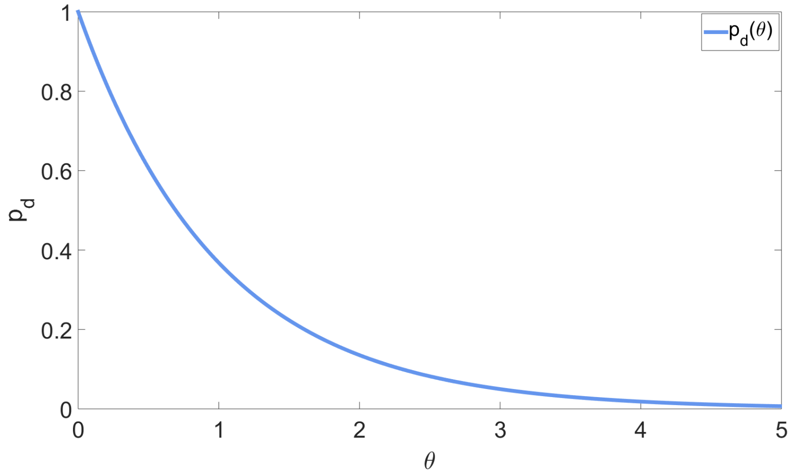


Figure 2: The marginal effect of testing is diminishing.

from the investment time onwards. The optimization problem corresponds to an optimal stopping problem of deciding the testing level θ at which the firm is willing to give up the remaining customer potential $Q_1(t)$ in the existing market to undertake the investment. Since the trade-off depends on the remaining customer potential and not time directly, we prefer to derive the solution in terms of Q_1 and θ instead of τ and θ.

The optimal stopping problem is solved in three steps. First, we derive the value of the firm in the stopping region, i.e. when it is optimal to invest immediately. Second, we derive the value of the firm in the continuation region, i.e. the region in which the firm prefers to stay in the existing market. Finally, we find the testing threshold θ for which the firm is indifferent between investing and staying in the continuation region.

The value of the firm in the stopping region, denoted $V_2(\theta)$, is equal to the expected present value of the revenue from the upgrade and calculated as follows

$$V_2(\theta) = p_d(\theta)V_2\left[\theta|\ disaster\right] + \left(1 - p_d(\theta)\right)V_2\left[\theta|\ no\ disaster\right]. \tag{2.15}$$

When we assume that a malfunction requires all sold units to be recalled, the revenue in the event of a disaster is zero. Recall from section 2.1 that the present value of the revenue from the upgrade when there is no malfunction risk is given by $\frac{{Q_2^0}^2}{4\eta_2(1+r)}$. Hence, the value of the firm in the stopping region is given as

$$V_2(\theta) = \frac{{Q_2^0}^2}{4\eta_2(1+r)}\left(1 - e^{-\theta}\right). \tag{2.16}$$

In the continuation region, the value of the firm is derived by applying Itô's Lemma and the Bellman equation. This results in a partial differential equation for the continuation value, denoted $F(\theta, Q_1)$, given as

$$rF(\theta, Q_1) = P_1(Q_1 - \eta_1 P_1) - (Q_1 - \eta_1 P_1)\frac{\partial F(\theta, Q_1)}{\partial Q_1} + \lambda\left[F(\theta + u, Q_1) - F(\theta, Q_1)\right]. \tag{2.17}$$

The first term on the right hand side, $P_1(Q_1 - \eta_1 P_1)$, represents the instantaneous profits from selling the existing version in the next period dt. The second term, $-(Q_1 - \eta_1 P_1)\frac{\partial F(\theta,Q_1)}{\partial Q_1}$, represents the decrease in value over the next period dt from the reduction in customer potential Q_1. The last term, $\lambda [F(\theta + u, Q_1) - F(\theta, Q_1)]$, represents the expected value of a sub-test arriving in the next period dt.

The optimal stopping problem can now be solved by deriving the testing level $\theta^*(Q_1)$ at which the firm is indifferent between investing and staying in the existing market. If the testing level is above the threshold, i.e. $\theta > \theta^*(Q_1)$, it is optimal for the the firm to introduce the upgrade immediately. If the testing level is below the threshold, i.e. $\theta < \theta^*(Q_1)$, it is optimal for the firm to delay the introduction of the upgrade and drain the customer potential in the existing market further. Therefore, the optimal strategy for the firm is given by the threshold function $\theta^*(Q_1)$, dividing the (θ, Q_1)-plane into a continuation region and a stopping region.

The threshold function $\theta^*(Q_1)$ is challenging to derive analytically because the optimal stopping problem is two-dimensional. Consequently, the available literature on analytical solutions to such problems is limited. The only contribution that offers a solution approach so far is the working paper of Hagspiel, Huisman, et al. (2016). In the following sub-section, we solve the optimal stopping problem by applying their solution approach. However, we argue that their approach is erroneous and provides an incorrect solution. We therefore present a numerical solution both to prove this, and to investigate the characteristics of the optimal investment timing and the firm value.

2.2.1 Analytical solution approach

The working paper of Hagspiel, Huisman, et al. (2016) considers an incumbent firm in a declining market due to technology development and pressure on continuous innovation. The firm is already producing and selling an established product and has the option to make an innovating move in order to develop a more technologically viable product. This will increase the firm's demand and, in turn, result in higher revenue. Hagspiel, Huisman, et al. (2016) propose that a longer waiting time before commitment will result in a more advanced development with better products, as the technology level and manufacturing expertise is expected to increase in time. Hence, the firm must weigh the benefits of continuing to sell the established product while waiting for further technology developments, against committing to the current innovation level and release a better product with a higher demand. In conformity with Hagspiel, Huisman, et al. (2016), we consider a two-dimensional problem with a Poisson-process and hence, their solution approach can be adopted.

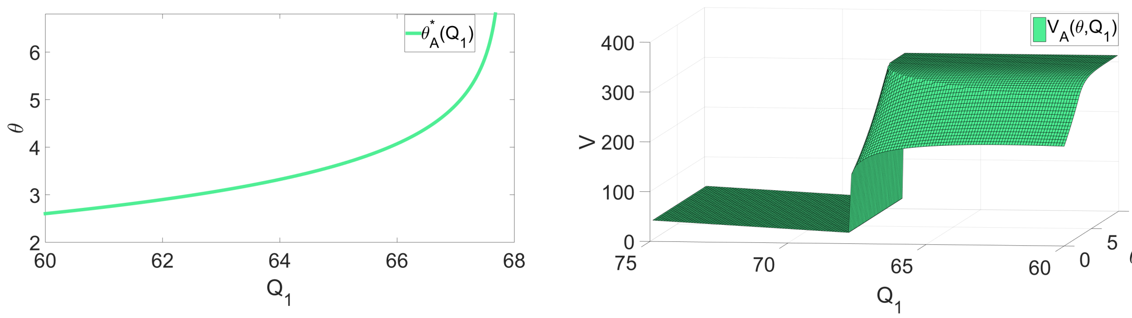
Hagspiel, Huisman, et al. (2016) solve the optimal stopping problem by splitting the continuation region into two parts. The split is such that in the second part, it is optimal for the firm to invest if a technology increment arrives now. In the first part on the other hand, the firm prefers to stay in the existing market even if a technology increment arrives. Each part of the continuation region requires the solution of a first-order inhomogeneous partial differential equation, both solved by guessing a general form with undetermined coefficients. The coefficients of the guessed solutions are then determined by applying value matching conditions between the two parts, and between the second part and the stopping region at the investment threshold θ^*. The problem is, however, that the coefficients they derive are themselves functions of the underlying variables. It turns out that the optimal values derived using the above approach do not solve the corresponding partial differential equations, indicating that the proposed solution is incorrect. In order to verify this, we have applied the approach of Hagspiel, Huisman, et al. (2016) to our model and present the resulting firm value, $V_A(\theta, Q_1)$, and testing threshold, $\theta^*_A(Q_1)$, in Proposition 2.5.

Proposition 2.5 *The analytical value of the firm, $V_A(\theta, Q_1)$, and investment threshold, $\theta^*_A(Q_1)$, are given as*

$$V_A(\theta, Q_1) = \begin{cases} \frac{Q_2^{0\,2}}{4\eta_2(1+r)}\left(1-e^{-\theta}\right) - I & \theta \geq \theta^*, \\ \frac{\lambda}{\lambda+r}\left(\frac{Q_2^{0\,2}}{4\eta_2(1+r)}\left(1-e^{-(\theta+u)}\right) - I\right) + \frac{P_1(Q_1-\eta_1 P_1)}{\lambda+r+1} & \theta^*-u < \theta < \theta^*, \\ \left(\frac{\lambda}{\lambda+r}\right)^{\frac{\theta^*-\theta}{u}}\left(\frac{Q_2^{0\,2}}{4\eta_2(1+r)}\left(1-e^{-\theta^*}\right) - I\right) + \frac{P_1(Q_1-\eta_1 P_1)}{1+r}\left(1-\left(\frac{\lambda}{\lambda+r+1}\right)^{\frac{\theta^*-\theta}{u}}\right) & \theta < \theta^* - u, \end{cases} \tag{2.18}$$

$$\theta^*_A(Q_1) = \ln\left[\frac{\frac{Q_2^{0\,2}}{4\eta_2(1+r)}\left(1-\frac{\lambda}{\lambda+r}e^{-u}\right)}{\left(\frac{Q_2^{0\,2}}{4\eta_2(1+r)} - I\right)\left(1-\frac{\lambda}{\lambda+r}\right) - \frac{P_1(Q_1-\eta_1 P_1)}{\lambda+r+1}}\right]. \tag{2.19}$$

To illustrate the shortcomings of these results, both the firm value and the threshold are graphed in Figure 3.



(a) Investment timing.

(b) Firm value.

Figure 3: Results from the analytical solution.
($P_1 = 3$, $\eta_1 = 20$, $Q_2^0 = 180$, $\eta_2 = 15$, $I = 200$, $r = 0.05$, $\lambda = 2$, $u = 0.2$)

We have from figure 3a that $\theta^* \to \infty$ for $Q_1 \approx 68$. Figure 3b shows a large drop in the firm value with increasing Q_1 around this region. This is counter-intuitive, as a higher market potential and, thus, higher demand should increase the firm value. Moreover, the firm value is completely unaffected by the testing level θ for $Q_1 > 68$, which seems erroneous as well. All $Q_1 > 68$ lies within the region $\theta < \theta^*(Q_1) - u$ of the firm value, so we investigate this region further. When $\theta^* \to \infty$, the only remaining term in this case is $\frac{P_1(Q_1-\eta_1 P_1)}{1+r}$. This is equal to the present value of the revenue generated by selling the existing version forever and never introduce the upgrade. Hence, the value function ignores the option to release the upgrade at a later point in time, which has value even though it is not optimal to exercise it immediately. We therefore conclude that the analytical solution vastly undervalues the option to release the upgrade in the continuation region, in particular when $\theta^*(Q_1) \to \infty$.

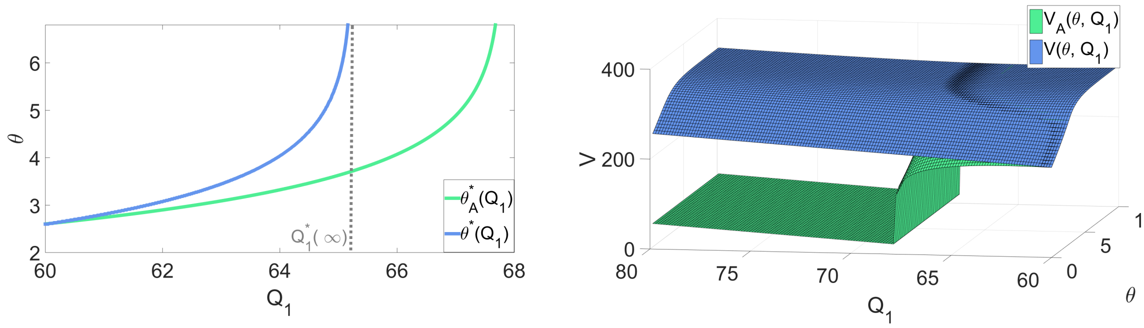
Another indication of a mistake in the solution approach is the overall shape of the firm value surface. Q_1 is a continuous variable and the firm value should therefore be continuous and differentiable in Q_1 over its entire domain. Figure 3b shows that the firm value is indeed continuous for all Q_1, but it is not smooth for $Q_1 \approx 68$.

2.2.2 Numerical solution approach

As an alternative approach to the one suggested by Hagspiel, Huisman, et al. (2016), we have developed a numerical model to derive the optimal investment threshold as well as the firm value. The numerical model builds on the method of finite differences. This is a tool commonly used to solve partial differential equations, and it is carried out by creating a point-grid and approximating a solution by iterating over the entire domain, starting from some boundary condition. The value in the stopping region has an analytical expression given in equation (2.16) and does not require a numerical solution. For the continuation region on the other hand, we translate equation (2.17) to the following finite difference scheme

$$F(j,i) = \frac{P_1(Q_1(i) - \eta_1 P_1) + (Q_1(i) - \eta_1 P_1)\frac{F(j,i-1)}{dQ_1} + \lambda F(j + \frac{u}{d\theta}, i)}{r + \lambda + \frac{Q_1(i) - \eta_1 P_1}{dQ_1}}. \tag{2.20}$$

Here, i is the index in Q_1 and j is the index in θ. The step-size in Q_1 is dQ_1 and the step-size in θ is $d\theta$. More details on the numerical solution approach and the boundary conditions applied are presented in Appendix C. Figure 4 graphs the numerical results, $\theta^*(Q_1)$ and $V(\theta, Q_1)$, together with the analytical results,$\theta^*_A(Q_1)$ and $V_A(\theta, Q_1)$, and gives a foundation for comparison.



(a) Investment timing.

(b) Firm value.

Figure 4: Comparison of numerical and analytical solutions.
($P_1 = 3$, $\eta_1 = 20$, $Q_2^0 = 180$, $\eta_2 = 15$, $I = 200$, $r = 0.05$, $\lambda = 2$, $u = 0.2$)

The two thresholds are equal for Q_1 relatively close to $\eta_1 P_1$. This is because the value of exploiting the existing market is negligible for such values, and the investment problem can be evaluated as a simple call option on the upgrade without any cost of giving up the existing version. However, when Q_1 is increased sufficiently above $\eta_1 P_1$ and the existing market becomes more valuable, the analytical threshold is significantly below the numerical one. This implies that the analytical model undervalues the benefit of draining the existing market or overvalues the investment for large Q_1.

Another significant difference is the Q_1^* for which θ^* goes to infinity, i.e. for what Q_1 the firm does not invest immediately regardless of the testing level. If we let $\theta \to \infty$, we can ignore the malfunction risk and apply the results from Proposition 2.1 to derive Q_1^*. Immediate investment corresponds to $\hat{\tau}^* = 0$, which by rearranging gives

$$Q_1^*(\theta \to \infty) = \frac{r\left(\frac{Q_2^{0\,2}}{4\eta_2(1+r)} - I\right)}{P_1} + \eta_1 P_1. \tag{2.21}$$

With the parameter values in Figure 4, we get $Q_1^*(\theta \to \infty) = 65.24$. This verifies the numerical threshold function, as it approaches infinity when Q_1 approaches 65.24. The analytical threshold on the other hand approaches 3.7 and is thus proven incorrect.

Both solutions yield the same firm value for small customer potentials and high technology levels. This is because investment is then triggered in both cases, and the firm values are equal to the common stopping value. Outside the stopping region, the value given by the analytical approach is very different from the numerical value, especially for $Q_1 > 68$, as shown in Figure 4b. This is because the analytical firm value ignores the value of the option to introduce the upgrade later, as previously discussed.

2.3 Characteristics of the solution to the optimal stopping problem

In what follows, we discuss the optimal strategy of the firm and how investment may be triggered. Remember that the threshold $\theta^*(Q_1)$ divides the (Q_1,θ)-plane into a continuation region where the firm prefers to stay in the existing market, and a stopping region where the firm introduces the upgrade immediately. Figure 5 illustrates the two regions, in addition to an example path for entering the stopping region.

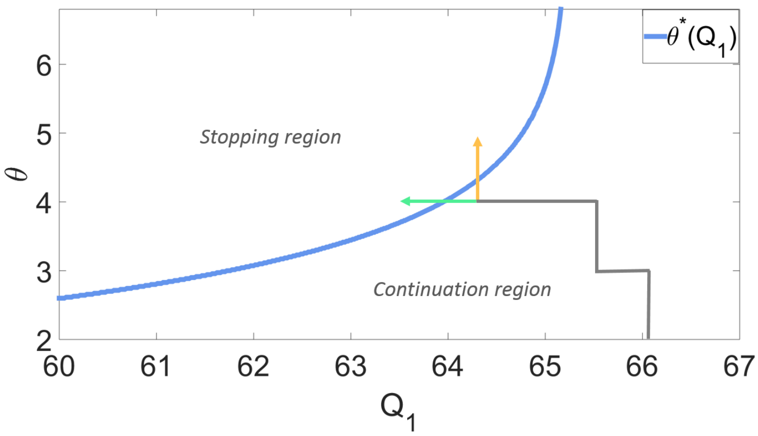
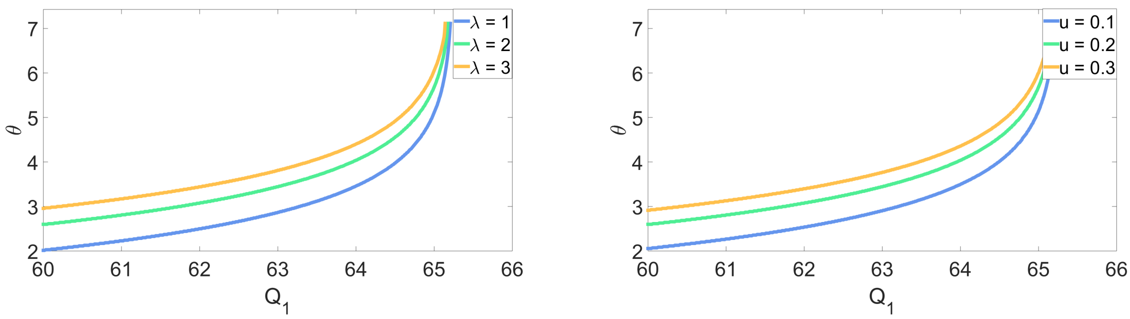


Figure 5: The investment threshold $\theta^*(Q_1)$ divides the (Q_1,θ) plane into a stopping region and a continuation region.
$(P_1 = 3,\ \eta_1 = 20,\ Q_2^0 = 180,\ \eta_2 = 15,\ I = 200,\ r = 0.05,\ \lambda = 2,\ u = 0.2$

The firm may transition from the continuation region into the stopping region for two reasons. The first reason is a continuous decrease in Q_1, leading into the stopping region horizontally from the right, as illustrated by the green line. This suggests that the remaining customer potential for the existing version has become sufficiently small to trigger investment at the current testing level. The second reason that could trigger investment is the completion of another sub-test, i.e. a discrete jump in θ leading into the stopping region vertically, as illustrated by the orange line. This could for instance correspond to the completion of battery testing for a smartphone.

The main motivation for this paper was to investigate the factors that may drive a firm to release an upgrade with significant risk of malfunction. In what follows, we investigate how the the investment threshold depends on the parameters of the testing process, i.e. λ and u. The sensitivity to other model parameters are through numerous numerical experiments found to be the same as in Proposition 2.1.

An increase in the impact of completing a single sub-test will increase the benefit of waiting until the next sub-test is completed, implying that $\theta^*(Q_1)$ is increasing in u. Similarly, an increase in the arrival rate results in a shorter expected duration for each sub-test and increases the benefit

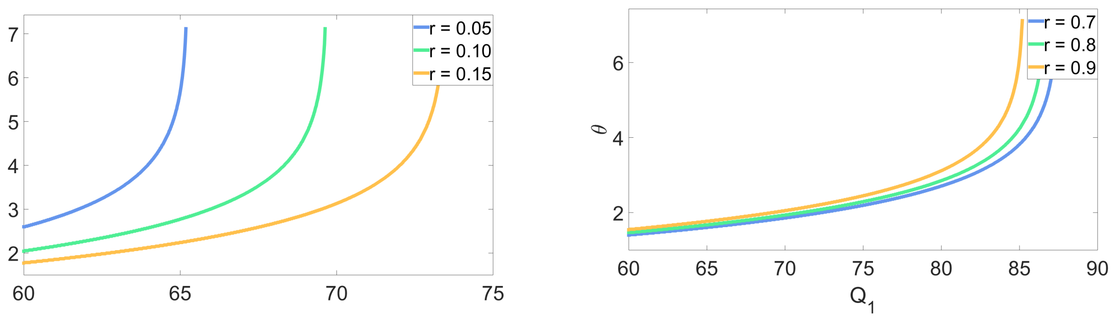


(a) $\theta^*(Q_1)$ is increasing in λ. (b) $\theta^*(Q_1)$ is increasing in u.

Figure 6: A faster testing process increases the investment threshold.
$(P_1 = 3,\ \eta_1 = 20,\ Q_2^0 = 180,\ \eta_2 = 15,\ I = 200,\ r = 0.05,\ \lambda = 2,\ u = 0.2)$

of waiting for the next sub-test to be completed. This implies that $\theta^*(Q_1)$ is also increasing in λ. From this we conclude that firms are more willing to accept malfunction risks if the testing process is slow.

A change in the discount rate has a similar ambiguous effect to what was found in the deterministic model. On one hand, it makes the present more important, which gives incentive to hasten the introduction of the upgrade. On the other hand, it makes the upgrade relatively less valuable, which gives incentive to delay the introduction. The threshold discount factor $\tilde{r}$ that splits the regions of which the first and second effect dominates exists also in this case, but does not have an analaytical expression. We therefore limit the analysis to illustrate the non-monotonic relation between the discount rate and the test threshold in Figure 7.

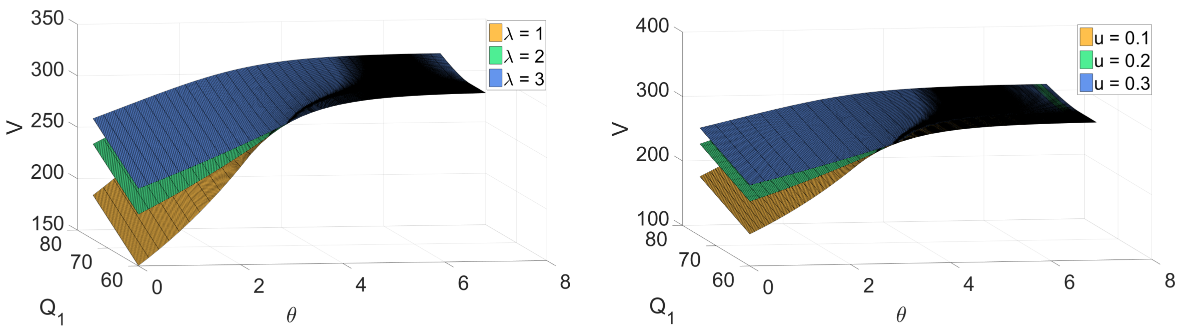


(a) $\theta^*(Q_1)$ is decreasing in r for low r. (b) $\theta^*(Q_1)$ is increasing in r for high r.

Figure 7: An increase in the discount rate has an ambiguous effect on the investment timing.
$(P_1 = 3,\ \eta_1 = 20,\ Q_2^0 = 180,\ \eta_2 = 15,\ I = 200,\ \lambda = 2,\ u = 0.2)$

In addition to investigating how the optimal investment timing depends on the parameters of the testing process, it is interesting to see how they affect the firm value. The results of this numerical analysis are shown in the Figure 8.

An increase in the arrival rate reduces the expected duration of each sub-test. This will make the testing level increase faster and therefore the failure probability decline faster. The firm value $F(\theta, Q_1)$ is therefore increasing in the arrival rate λ. An increase in the impact of the sub-tests would also make the testing level increase faster. Hence, the firm value $F(\theta, Q_1)$ is increasing in the jump size u. The sensitivity to other model parameters are unchanged by the relaxation of the assumption that the firm must refrain from investment until testing is completed. We therefore refer to Proposition 2.4 for the sensitivity to the other parameters.



(a) $V(\theta, Q_1)$ is increasing in λ.

(b) $V(\theta, Q_1)$ is increasing in u.

Figure 8: A faster testing process increases the firm value.
($P_1 = 3$, $\eta_1 = 20$, $Q_2^0 = 180$, $\eta_2 = 15$, $I = 200$, $r = 0.05$, $\lambda = 2$, $u = 0.2$)

An interesting observation from Figure 8 is that the firm value is insensitive to changes in the testing level for high θ. This follows from our modeling assumption that the marginal benefit of testing is decreasing, as discussed under the introduction of the disaster probability in Section 2.2. For large level of θ, the disaster probability approaches zero, and further testing will therefore not increase the firm value. Since the firm value does not increase as more testing is finished, the firm value is insensitive to the speed of the testing process, i.e. λ and u, for such large θ. The surface plots in Figure 8 therefore merge for large θ.

3 Innovator-laggard model

In reality, producers of durable goods usually operate in a competitive environment, which may affect the decision to release an upgrade. For instance, Samsung does not operate as a monopolist in the market for smartphones, but faces strong competition from Apple in particular. In this section, we therefore extend the monopoly model to incorporate the aspect of competition. More specifically, we consider an innovator-laggard problem, suitable for markets where two actors are dominating, like Apple and Samsung in the smartphone market.

Consider two risk-neutral and profit-maximizing firms that are both selling a similar durable good. One of the firms, hereafter referred to as the innovator, has already released an upgraded version of the product and is selling it in a new market. The other firm, hereafter referred to as the laggard, is currently selling an older version of the product in the existing market. The laggard makes a decision on when, or if, it should introduce an upgraded version of the product and enter the new market in competition with the innovator. If it chooses to invest, the laggard can set the price for the upgrade optimally. Similar to the monopoly model, the technology for the upgrade goes through a testing phase. In Section 3.1 we assume the testing phase is deterministic and finishes at time t_n. The firm does not invest until the testing phase is completed, but can then introduce the upgrade by paying a fixed investment cost I. In Section 3.2, we relax this assumption and allow the firm to introduce the upgrade at any stage of a stochastic testing process.

The laggard is selling the existing version at a price P_1^L which, as in the monopoly model, is treated as a fixed parameter. Let $Q_1(t)$ denote the remaining customer potential in the existing market. The customer potential decreases as more units are sold, and the dynamics are given by equation (3.1)

$$dQ_1(t) = -q_1^L(t)dt, \tag{3.1}$$

where $q_1^L(t)$ is the instantaneous demand for the existing version. To focus on the effects of the testing phase and product malfunction risk, we assume no competition across different versions of the product. The laggard is therefore acting as a monopolist in the existing market, and $q_1^L(t)$ is the same as $q_1(t)$ in equation (2.2).

The initial customer potential for upgraded products, Q_2^0, is divided between those initially loyal to the innovator's product, $(1-\nu)Q_2^0$, and those initially loyal to the laggard's product, νQ_2^0. The share of the potential that prefers the laggard's product does not arise until the laggard introduces the upgrade. Customers in this market may switch from being loyal to one firm to the other, depending on the relative prices of the innovator's product, P_2^I, and the laggard's product, P_2^L. The degree of loyalty among the customers in the market depends on the characteristics of the goods sold. For example, Ensslen et al. (2016) finds that customers buying electric vehicles are less sensitive to relative prices than customers buying fossil fueled vehicles. We therefore introduce a disloyalty factor γ to capture the degree of brand loyalty in the market.

Let $Q_2^L(t)$ denote the remaining customer potential in the new market loyal to the laggard's product at time t. The customer potential is decreasing as more units are sold and follows the dynamics in equation (3.2)

$$dQ_2^L(t) = -q_2^L(t)dt, \tag{3.2}$$

where $q_2^L(t)$ is the instantaneous demand for the upgrade in the new market. We propose the following structure function for the instantaneous demand

$$q_2^L(t) = Q_2^L(t) - \eta_2 P_2^L + \gamma(P_2^I - P_2^L). \tag{3.3}$$

The first two terms are similar to the monopoly demand in equation (2.2) and depends on the quality of the product, η_2, the price, P_2^L, and the remaining customer potential $Q_2^L(t)$. The third term, $\gamma(P_2^I - P_2^L)$, is the additional demand the laggard can achieve by stealing from or losing customers to the innovator, representing the competition effect.

3.1 Deterministic product testing

The value of the laggard that introduces the upgrade at time τ has similar form as in equation (2.3). Applying the demand functions from equations (2.2) and (3.3), the value of future cash flows, denoted $V_L(\tau, P_2^L)$, if the laggard invests at time τ and price P_2^L is given as

$$V_L(\tau, P_2^L) = \int_0^{t_n} P_1^L q_1^L(t) e^{-rt} dt + \int_{t_n}^{\tau} P_1^L q_1^L(t) e^{-rt} dt + \int_{\tau}^{\infty} P_2^L q_2^L(t-\tau) e^{-rt} dt - I e^{-r\tau}. \tag{3.4}$$

The value $V_L(\tau, P_2^L)$ in equation (3.4) consists of two parts. The first two terms represent the value of selling the existing product as a monopolist until time τ, while the last two terms represent the value of investing at τ and selling the upgrade onwards. The firm faces a similar trade-off in deciding the investment timing as in the monopoly case, between draining the existing market and reducing discounting on the revenue from the upgrade.

The objective of the laggard is to maximize its present value of future cash flows by optimally deciding the time of introduction and price of the upgrade. The maximized present value of future profits, denoted V_L^*, is found by solving the following optimization problem:

$$V_L^*(\tau, P_2) = \int_0^{t_n} P_1^L q_1^L(t) e^{-rt} dt + \max_{\tau} \left[\int_{t_n}^{\tau} P_1^L q_1^L(t) e^{-rt} dt + \max_{P_2^L} \left[\int_{\tau}^{\infty} P_2^L q_2^L(t-\tau) e^{-rt} dt \right] - I e^{-r\tau} \right]. \tag{3.5}$$

We derive the optimal investment timing, denoted τ_L^*, in the same steps as in Section 2.1. We therefore only summarize the steps and present the results, while the detailed calculations are placed in Appendix C. First, we derive the present value of the revenue after introduction at time τ, denoted V_2^{L*}, given as

$$\begin{aligned} V_2^{L*} &= \max_{P_2^L} \left[\int_{\tau}^{\infty} q_2^L(t-\tau) P_2^L e^{-rt} dt \right], \\ &= P_2^{L*} \frac{\nu Q_2^0 - \eta_2 P_2^{L*} + \gamma(P_2^I - P_2^{L*})}{1+r} e^{-r\tau}, \end{aligned} \tag{3.6}$$

where P_2^{L*} is the optimal price for the upgrade. Second, we derive this optimal price by applying the first-order condition for optimality, resulting in

$$P_2^{L*} = \frac{\nu Q_2^0 + \gamma P_2^I}{2(\eta_2 + \gamma)}. \tag{3.7}$$

The prices of the innovator's and the laggard's products are strategic complements, moving in the same direction. As in the monopoly model, the optimal price is increasing in the customer potential, i.e. ν and Q_2^0 and decreasing in η_2. When the customers are completely loyal to their preferred product, i.e. when $\gamma = 0$, no competition effect is present, and the optimal price is the same as in the monopoly case. It is also worth noticing that even though the laggard has a second-mover advantage in determining the price, it may choose a price higher than the innovator's. More specifically, it chooses a higher price if the innovator's price is below $\bar{P}_2^I = \frac{\nu Q_2^0}{2\eta_2 + \gamma}$. When $\gamma = 0$ and no competition is present, $\bar{P}_2^I$ approaches the optimal monopoly price with customer potential νQ_2^0.

Finally, we derive the optimal investment timing τ_L^* by maximizing the expected present value of the profits over the entire investment horizon. This result is presented in Proposition 3.1, along with the firm value $V_L^*(\tau_L^*)$.

Proposition 3.1 *The optimal time to introduce the upgrade, τ_L^*, and the corresponding firm value, $V_L^*(\tau_L^*)$, is given by*[9]

$$\tau_L^* = \ln \left[\frac{P_1^L(Q_1 - \eta_1 P_1^L)}{r\left(P_2^{L*} \frac{\nu Q_2^0 - \eta_2 P_2^{L*} + \gamma(P_2^I - P_2^{L*})}{1+r} - I \right)} \right], \tag{3.8}$$

$$V_L^*(\tau_L^*) = \frac{P_1^L(Q_1 - \eta_1 P_1^L)}{1+r} + \frac{P_2^{L*} \frac{\nu Q_2^0 - \eta_2 P_2^{L*} + \gamma(P_2^I - P_2^{L*})}{1+r} - I}{1+r} \left(\frac{P_1^L(Q_1 - \eta_1 P_1^L)}{r\left(P_2^{L*} \frac{\nu Q_2^0 - \eta_2 P_2^{L*} + \gamma(P_2^I - P_2^{L*})}{1+r} - I \right)} \right)^{-r}, \tag{3.9}$$

where

$$P_2^{L*} = \frac{\nu Q_2^0 + \gamma P_2^I}{2(\eta_2 + \gamma)}. \tag{3.10}$$

[9] The same three cases as in Proposition 2.1 apply, but we exclude the cases when $\tau^* = t_n$ and $\tau^* \to \infty$ as they are similar to the monopoly case.

An increase in the innovator's price P_2^I increases the demand for and, thus, the revenue from the upgrade. This increases the value of introducing the upgrade and makes the laggard invest earlier. When the laggard's price is lower than the innovator's, i.e. $P_2^E < P_2^I$, an increase in the disloyalty factor γ increases the revenue from the upgrade and hastens investment. Furthermore, a higher market share ν also makes investing in the upgrade more attractive and makes the laggard introduce it earlier [10].

3.2 Stochastic product testing and risk of malfunction

In this section we extend the duopoly model to include the risk of product malfunction. Previously, we have argued that the occurrence of a product malfunction in the real world often results in a total product recall for the exposed firm. In case of such an event, the customers who initially preferred the recalled product now have to buy substitutes from another brand. When two similar firms are operating in a duopoly, this would imply that all customers would purchase from the producer of the remaining product or not at all. Hence, if either the innovator's or the laggard's product malfunctions, the other firm will become a monopolist and, thus, receive the entire initial customer potential Q_2^0 of the new market.

Suppose the innovator has just introduced its upgrade in the new market at a testing level θ^I. Let $p^I(t)$ denote the instantaneous probability of the innovator experiencing a product malfunction within the next period dt. As suggested in Section 2.2, this probability is given by

$$p^I(t) = \phi^I(t)dt = \alpha e^{-(\theta^I + \alpha t)} dt. \tag{3.11}$$

The innovator's disaster probability, i.e. the probability that its product will ever malfunction given that it has not up until time t, is denoted $p_d^I(t)$ and given as $p_d^I(t) = e^{-(\theta^I + \alpha t)}$. Unlike in the monopoly model, α now affects the strategy of the laggard, as it determines how quickly the innovator's risk probabilities $p^I(t)$ and $p_d^I(t)$ reduce over time. Figure 9 shows how the arrival rate, $\phi^I(t)$, of the innovator's product malfunction reduces over time for different α.

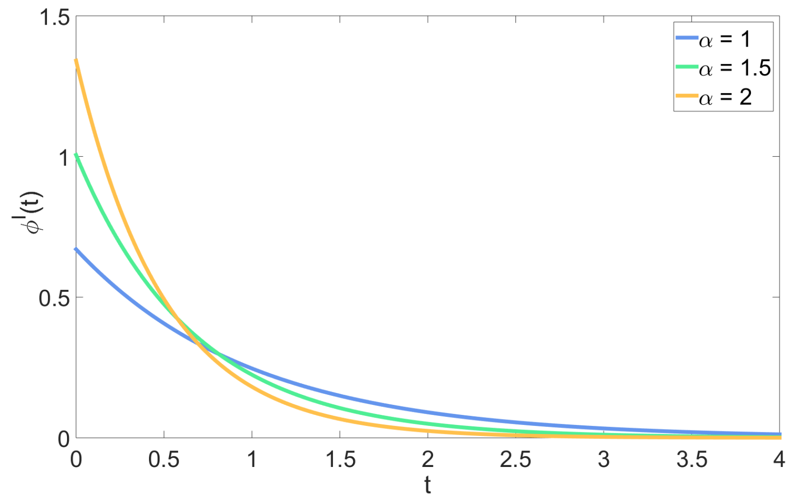


Figure 9: Higher intensity factor α decreases $\phi^I(t)$ faster ($\theta^I = 0.4$)

[10] The sensitivities to the model parameters that are common remain the same, so we refer to Propositions 2.2 and 2.4 for η_1, η_2, P_1^L, Q_2^0, I.

A higher α yields a higher arrival rate of product malfunction for small values of t that will converge to zero fast when t is increased. A high α would therefore be suitable for products that are expected to malfunction shortly after release, if ever. Such products could for instance be smartphones or computers. Lower values of α result in a more evenly distributed arrival rate, making it suitable for products that are more susceptible to malfunction over time, and not only in the very first period after release. Such products could for instance include car brakes or vacuum cleaners.

We prefer to determine the strategy in terms of remaining customer potential Q_1 rather than time t, as in Section 2.2. However, time has a direct effect on the innovator's risk probabilities and must be considered in this section. The deterministic relation between customer potential in the existing market and time, from equation (2.1), allows us to represent time in terms of Q_1 and Q_1^0, by relation (3.12), and still solve the problem in terms of Q_1 and θ.

$$t = \ln\left(\frac{Q_1^0 - \eta_1 P_1^L}{Q_1 - \eta_1 P_1^L}\right) \tag{3.12}$$

Applying this to $p^I(t)$ and $p_d^I(t)$, we find the probabilities in terms of remaining customer potential as

$$p^I(Q_1) = \alpha e^{-\theta^I}\left(\frac{Q_1 - \eta_1 P_1^L}{Q_1^0 - \eta_1 P_1^L}\right)^{\alpha} dt, \tag{3.13}$$

$$p_d^I(Q_1) = e^{-\theta^I}\left(\frac{Q_1 - \eta_1 P_1^L}{Q_1^0 - \eta_1 P_1^L}\right)^{\alpha}. \tag{3.14}$$

The objective of the laggard is to maximize the expected present value of future cash flows, by optimally deciding the time τ and testing level θ for the upgrade, as well as the price, P_2^L. This optimization problem is formulated in equation (3.15)

$$\max_{\tau, P_2^L} \mathbf{E}_\theta\left[V_1(\tau) + \left(V_2(\theta,\tau) - I\right)e^{-r\tau}\right]. \tag{3.15}$$

The terms have the same interpretation as in equation (2.14), but V_2 is now also a function of time, because it depends on the innovator's risk probabilities. This optimization problem corresponds to an optimal stopping problem, which can be solved by finding the testing level θ for which the laggard is indifferent between introducing the upgrade and staying in the continuation region.

We start by describing the value of the laggard in the stopping region. After the upgrade is introduced to the new market, there are four scenarios to potentially materialize: (1) neither firms' products malfunction, (2) only the innovator's product malfunctions, (3) only the laggard's product malfunctions and (4) both firms' products malfunction. Each outcome has a probability determined by the current testing level θ of the laggard, the testing level θ^I at which the innovator's product was introduced, and for how long the innovator has been active. We denote the only two non-zero revenue outcomes (1) and (2) as $\bar{d}^I$ and d^I respectively. The value of the laggard in the stopping region, denoted $V_2(\theta, Q_1)$, is equal to the expected present value of the revenue from the upgrade, as presented in equation (3.16).

$$V_2(\theta, Q_1) = \left(1 - p_d^L(\theta)\right) p_d^I(Q_1)\mathbf{E}[V_2|d^I] + \left(1 - p_d^L(\theta)\right)\left(1 - p_d^I(Q_1)\right)\mathbf{E}[V_2|\bar{d}^I]. \tag{3.16}$$

Here, $\mathbf{E}[V_2|d^I]$ and $\mathbf{E}[V_2|\bar{d}^I]$ are the expected present values of the revenue given that the innovator's product malfunctions and not, respectively. Evaluating the conditional expressions results in the stopping value in Proposition 3.2.

Proposition 3.2 *The value of the laggard in the stopping region, $V_2(\theta, Q_1)$, is given as*

$$\begin{aligned} V_2(\theta, Q_1) = & \left(1 - p_d^L(\theta)\right) p_d^I(Q_1) P_2^L \left[\frac{\nu Q_2^0 - \eta_2 P_2^L + \gamma(P_2^I - P_2^L)}{1+r} \left(\frac{\alpha}{\alpha} - \frac{\alpha}{\alpha+r+1} + \frac{\alpha}{\alpha+r+2} \right) \right. \\ & \left. + \frac{(1-\nu)Q_2^0 - \gamma(P_2^I - P_2^L)}{1+r} \frac{\alpha}{\alpha+r+1} \right] \\ & + \left(1 - p_d^L(\theta)\right) \left(1 - p_d^I(Q_1)\right) \left[P_2^L \frac{\nu Q_2^0 - \eta_2 P_2^L + \gamma(P_2^I - P_2^L)}{1+r} \right] \end{aligned} \tag{3.17}$$

where

$$p_d^I(Q_1) = e^{-\theta^I} \left(\frac{Q_1 - \eta_1 P_1^L}{Q_1^0 - \eta_1 P_1^L} \right)^\alpha, \tag{3.18}$$

$$p_d^L(\theta) = e^{-\theta}. \tag{3.19}$$

The first part represents the expected value if only the innovator's product malfunctions and the laggard at some time becomes a monopolist in the new market. The second part represents the expected value if neither products malfunction. A high disaster probability p_d^I of the innovator gives the monopoly outcome the highest weight, while a low p_d^I gives the duopoly outcome the highest weight. Since the monopoly outcome is more attractive, the stopping value is strictly decreasing in the innovator's testing level θ^I. The innovator's disaster probability is also larger for large Q_1 before the product becomes established, and the stopping value is therefore higher for large Q_1. The effect of γ on the stopping value is ambiguous and depends on the relative prices P_2^L and P_2^I. More specifically, a highly competitive market, i.e. high γ, increases the stopping value when the innovator's price is high relative to the laggard's.

When the innovator's product is certain to malfunction, i.e. $p_d^I = 1$, and is expected to do so immediately, i.e. $\alpha \to \infty$, the laggard becomes a monopolist immediately after investment. Hence, the stopping value of the laggard approaches the monopoly value from equation (2.16). If the innovator's product is certain to never malfunction, i.e. $p_d^I = 0$, the laggard will never become a monopolist in the new market. The stopping value is then equal to the duopoly revenue in equation (3.6), weighted by the probability that the laggard's product does not malfunction.

Following Proposition 3.2, we derive the optimal price, $P_2^{L^*}$, for the laggard to set for the upgrade. The result is presented in Lemma 3.1.

Lemma 3.1 *The optimal price, $P_2^{L^*}$, to charge by the laggard is given by*

$$P_2^{L^*}(Q_1) = \frac{\nu Q_2^0 + \gamma P_2^I + p_d^I(Q_1) \frac{\alpha}{\alpha+r+1} \left[(1-\nu)Q_2^0 - \gamma P_2^I - \frac{\nu Q_2^0 + \gamma P_2^I}{\alpha+r+2} \right]}{2(\eta_2 + \gamma) - 2 p_d^I(Q_1) \frac{\alpha}{\alpha+r+1} \left[\gamma + \frac{\eta_2 + \gamma}{\alpha+r+2} \right]} \tag{3.20}$$

The optimal price exists on a continuum between the deterministic duopoly price in equation (3.10) and the deterministic monopoly price in equation (2.6). A high probability of becoming a monopolist, i.e. high p_d^I, and short expected time until it happening, i.e. high α, yields a price that is closer to the monopoly price. On the other hand, a low probability of becoming a monopolist or a long expected time until it happening, yields a price closer to the duopoly price.

Furthermore, the prices are strategic complements moving in the same direction. However, the sensitivity to P_2^I is only significant if a duopoly outcome is likely and the competition effect appears, which is heavily influenced by the disaster probability p_d^I.

As in the deterministic case, the laggard has a second-mover advantage, but may still choose to set a higher price than the innovator. Now, the threshold price, $\tilde{P_2^I}$, also depends on the likelihood of becoming a monopolist and is given by

$$\tilde{P_2^I} = \frac{\nu Q_2^0 + p_d^I(Q_1)\frac{\alpha}{\alpha+r+1}\left[(1-\nu)Q_2^0 - \frac{\nu Q_2^0}{\alpha+r+2}\right]}{2\eta_2 + \gamma - p_d^I \frac{\alpha}{\alpha+r+1}\left[\gamma + \frac{2\eta_2+\gamma}{\alpha+r+2}\right]}. \tag{3.21}$$

In the extreme case where the innovator never experiences a product malfunction, i.e. when $p_d^I = 0$, $\tilde{P_2^I}$ approaches the threshold price from the deterministic case. If, on the other hand, the innovator's product malfunctions immediately, i.e. $\alpha \to \infty$ and $p_d^I = 1$, the laggard chooses the optimal monopolist price and $\tilde{P_2^I}$, thus, approaches $\frac{Q_2^0}{2\eta_2}$.

We now turn to the value in the continuation region. If the laggard decides not to invest, there are four possible outcomes over the next period dt: (1) the innovator's product malfunctions and another sub-test is completed, (2) the innovator's product malfunctions and no sub-test is completed, (3) the innovator's product does not malfunction and another sub-test is completed and (4) the innovator's product does not malfunction and no sub-test is completed. Considering these outcomes and the instantaneous probabilities of each, we derive the following partial differential equation for the firm value $F_d(\theta, Q_1)$ in the continuation region

$$\begin{aligned} rF_d(\theta, Q_1) = {} & P_1^L(Q_1 - \eta_1 P_1^L) - (Q_1 - \eta_1 P_1^L)\frac{\partial F(\theta, Q_1)}{\partial Q_1} \\ & + \lambda\left[F_d(\theta+u, Q_1) - F_d(\theta, Q_1)\right] + ae^{-(\theta^I+\alpha t)}\left[F_m(\theta, Q_1) - F_d(\theta, Q_1)\right]. \end{aligned} \tag{3.22}$$

F_m denotes the value of the laggard if the innovator experiences a malfunction. The term involving $[F_m(\theta, Q_1) - F_d(\theta, Q_1)]$ therefore represents the additional value of becoming a monopolist in the next period dt, weighted by the instantaneous probability $p^I(Q_1)$ of the innovator experiencing a malfunction in this period. The remaining terms have the same interpretation as in equation (2.17).

As in the monopoly case, this partial differential equation has to be solved numerically. Let i be the indexed step of the customer potential Q_1 and j be the indexed step of the testing level θ. Furthermore, let $d\theta$ be the grid step in θ and dQ_1 be the grid step in Q_1. Equation (3.22) can then be translated into the following finite difference scheme for the value of the laggard in the continuation region:

$$F_d(j,i) = \frac{P_1^L(Q_1(i) - \eta_1 P_1^L) + (Q_1(i) - \eta_1 P_1^L)\frac{F(j,i-1)}{dQ_1} + \lambda F_d(j + \frac{u}{d\theta}, i) + \alpha e^{-\theta^I}\left(\frac{Q_1(i)-\eta_1 P_1^L}{Q_1^0 - \eta_1 P_1^L}\right)^{\alpha} F_m(j,i)}{r + \lambda + \alpha e^{-\theta^I}\left(\frac{Q_1(i)-\eta_1 P_1^L}{Q_1^0-\eta_1 P_1^L}\right)^{\alpha} + \frac{Q_1(i)-\eta_1 P_1^L}{dQ_1}} \tag{3.23}$$

We continue to solve the optimal stopping problem by deriving the testing level $\theta_d^*(Q_1)$ for which the firm is indifferent between introducing the upgrade and continuing to sell the existing version. The threshold is found by using equation (3.23) for the value in the continuation region, and equation (3.2) for the value in the stopping region. The optimal strategy for the laggard if the innovator's product has not malfunctioned at current Q_1 is then given by $\theta_d^*(Q_1)$, dividing the (θ, Q_1)-plane into a continuation region and a stopping region. We also derive the investment threshold given that the innovator has experienced a malfunction at current Q_1, denoted $\theta_m^*(Q_1)$, by the numerical procedure in Section 2.2.2. A new region then arises, behaving as a stopping region if the innovator's product has malfunctioned, and a continuation region if it has not. The three different regions are presented in Figure 10.

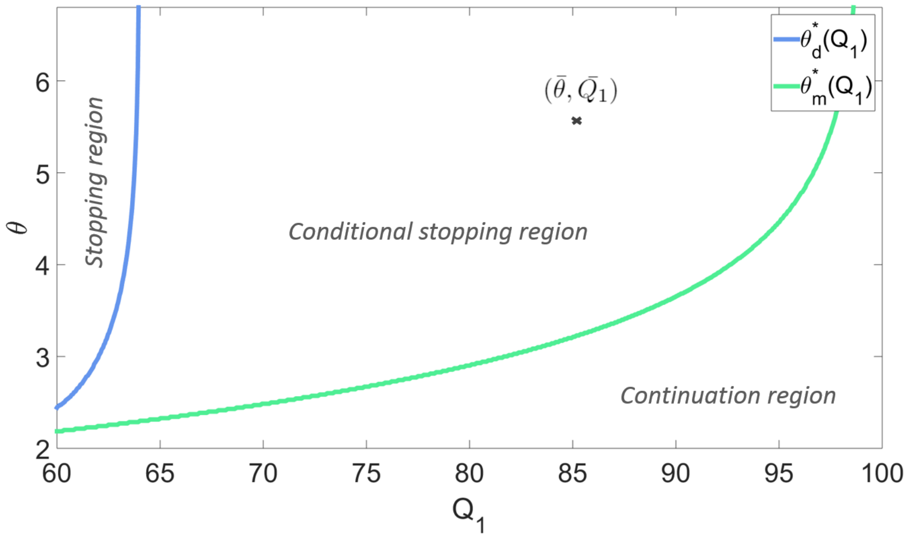


Figure 10: The investment thresholds θ_m^* and θ_d^* divide the (Q_1,θ)-plane into three distinct strategic regions[11].
($P_1^L = 3$, $\eta_1 = 20$, $Q_2^0 = 400$, $\eta_2 = 15$, $I = 200$, $r = 0.05$, $\lambda = 2$, $u = 0.2$, $P_2^I = 8$, $\gamma = 5$, $\nu = 0.5$, $\alpha = 1.5$, $\theta^I = 0.4$)

In the monopoly case, investment may be triggered by a continuous decrease in Q_1 or a discrete jump in the testing level θ, taking the firm from the continuation region into the stopping region. Now, investment may also be triggered by the innovator experiencing a malfunction in the conditional stopping region. If the innovator experiences a malfunction for such (θ, Q_1), exemplified by the point $(\bar{\theta}, \bar{Q_1})$, the region becomes a stopping region, and the laggard introduces the upgrade.

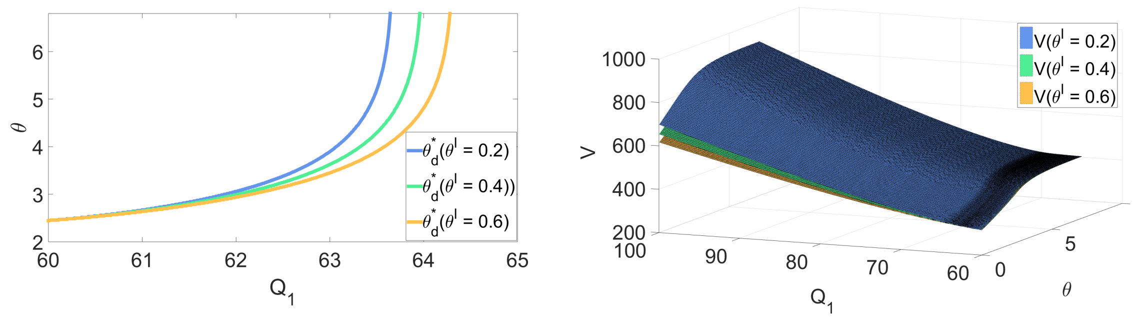
The difference between $\theta_d^*(Q_1)$ and $\theta_m^*(Q_1)$ in Figure 10 shows that the laggard requires a higher testing level to trigger investment when a risky innovator is present. This is because it is more valuable for the laggard if the innovator's product malfunctions before the laggard has introduced its upgrade. The laggard is then able to set the optimal monopoly price and reap the full benefit of becoming a monopolist. On the other hand, if the innovator's product malfunctions after the investment is made, the laggard has to operate in the new market with the pre-committed price. This gives an incentive to stay in the continuation region and observe the innovator's product. We therefore conclude that the laggard has a greater value of staying in the existing market when a risky innovator is present in the new market and, thus, introduces the upgrade with a lower risk of malfunction. For the rest of this paper, we will refer to this as the "delaying effect".

The main motivation for this paper was to investigate the factors that drive a firm to release an upgrade with significant risk of malfunction. In the monopoly model, we studied how the characteristics of the testing process affected the strategy. In the remainder of this section, we

[11]Numerous numerical experiments confirm that this is the general shape for all parameter values.

focus on how the presence of a risky innovator affects the investment decision. In particular, we focus on the threshold $\theta_d^*(Q_1)$, assuming the innovator's product has not malfunctioned.

The degree of which the innovator affects the optimal strategy depends on how risky its product is, i.e. the testing level θ^I at which it was released. Figure 11 illustrates this for values of θ^I equal to 0.2, 0.4 and 0.6.



(a) Lower θ^I delays investment.

(b) Lower θ^I increases firm value.

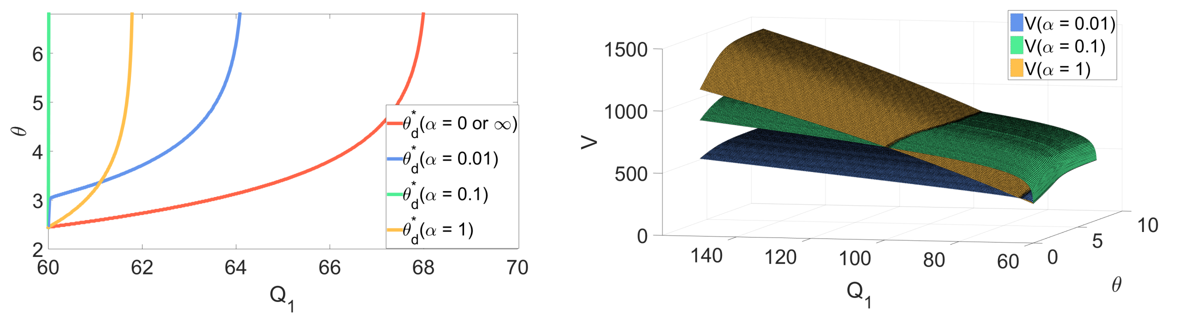
Figure 11: Sensitivity to the innovator's testing level θ^I.
($P_1^L = 3$, $\eta_1 = 20$, $Q_2^0 = 400$, $\eta_2 = 15$, $I = 200$, $\lambda = 2$, $r = 0.05$, $u = 0.2$, $P_2^I = 8$, $\gamma = 5$, $\nu = 0.5$, $\alpha = 1.5$)

If the innovator released its product before it was thoroughly tested, implying low θ^I, it is more likely to malfunction. In this case, the laggard has a larger incentive to stay in the existing market and observe the innovator's product. Hence, the investment threshold $\theta_d^*(Q_1)$ is decreasing in θ^I due to the decreased delaying effect, as illustrated in Figure 11a. We conclude that the laggard is less willing to accept malfunction risk for its own product if the innovator's product has not been thoroughly tested and is likely to malfunction.

If the innovator's product was released at a low testing level, the laggard is more likely to become a monopolist and has a higher firm value. The laggard's value is therefore strictly decreasing in θ^I, as shown in Figure 11b. However, the figure also shows that the firm values converge at low Q_1 for all θ^I. This is because the innovator's product has then been sold for a long time without experiencing malfunction, and the probability of it ever malfunctioning has decreased to zero irrespective of the initial testing level.

The intensity parameter α has two effects on the optimal investment timing. The first effect is that it determines how quickly the probability of the innovator's product malfunctioning approaches zero and, thus, the likelihood of the laggard becoming a monopolist at a given Q_1. The second effect is that it determines the expected time until the innovator's product malfunctions. Hence, it affects the expected discounting on the benefit of becoming a monopolist and, thus, the expected present value of it. The influence of α on the strategy and the laggard's value is illustrated in Figure 12 for values of α equal to 0.01, 0.1 and 1, as well as the extreme cases $\alpha = 0$ and $\alpha \to \infty$ for the strategy.

Figure 12a shows that increasing α has a non-monotonic effect on the investment timing. This finding arises because the two effects discussed work in opposite directions when α is increased: it decreases the probability of the innovator's product malfunctioning, but increases the expected present value of becoming a monopolist. The ambiguous relation becomes clearer when a fixed Q_1 is considered, as illustrated for $Q_1 = 60.6$ in Figure 13. For low α, the dominating effect of increasing α is that the expected present value of becoming a monopolist increases. This increases the delaying effect, and the laggard requires a higher testing threshold $\theta_d^*(Q_1)$ to trigger investment. For high α, the dominating effect of increasing α further is that the probability of



(a) α has an ambiguous effect on the investment timing.

(b) High α implies higher (lower) firm value at high (low) Q_1.

Figure 12: Sensitivity to the intensity parameter α.
($P_1^L = 3$, $\eta_1 = 20$, $Q_2^0 = 400$, $\eta_2 = 15$, $I = 200$, $r = 0.05$, $\lambda = 2$, $u = 0.2$, $P_2^I = 8$, $\gamma = 5$, $\nu = 0.5$, $\theta^I = 0.4$)

becoming a monopolist decreases. This reduces the delaying effect, and the laggard is willing to invest at a lower $\theta_d^*(Q_1)$. For an intermediate region of α, both the expected present value and the probability of becoming a monopolist is high, making the delaying effect strong enough that the laggard is unwilling to invest at any testing level θ.

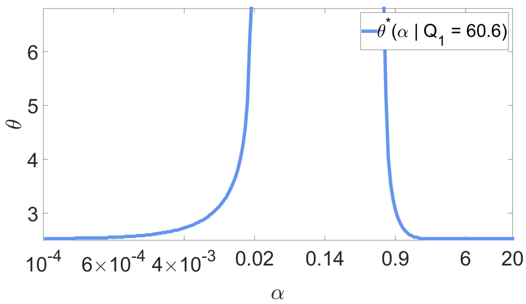


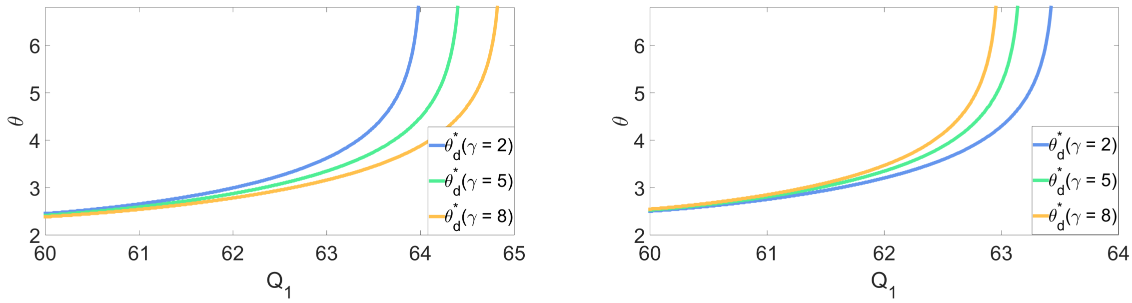
Figure 13: The intensity parameter α has a non-monotonic effect on the investment timing.
($P_1^L = 3$, $\eta_1 = 20$, $Q_2^0 = 400$, $\eta_2 = 15$, $I = 200$, $r = 0.05$, $\lambda = 2$, $u = 0.2$, $P_2^I = 8$, $\gamma = 5$, $\nu = 0.5$, $\theta^I = 0.4$)

A consequence of the non-monotonic effect of increasing α is that the investment thresholds for the extreme cases $\alpha = 0$ and $\alpha \to \infty$ are equal. In fact, when $\alpha = 0$, the heavy expected discounting on the benefit of becoming a monopolist makes it worthless, while for $\alpha \to \infty$, the probability of becoming a monopolist is zero for all Q_1 where the laggard considers investment. Hence, in both cases, the risky innovator yields no delaying effect and does not affect the strategy. Both extremes therefore have the same investment threshold, illustrated by the red line in Figure 12a.

Figure 12a also shows a drop in the threshold going from $Q_1 = 60 + dQ_1$ to 60, for $\alpha = 0.1$ and $\alpha = 0.01$. This is because α is so low, that the probability of the innovator experiencing a malfunction is significant even at $Q_1 = 60 + dQ_1$. At the same time, α is not yet low enough to erode the value of becoming a monopolist. Hence, the risky innovator yields a significant delaying effect even for $Q_1 = 60 + dQ_1$. However, when the customer potential drops to 60, the exponential relation between Q_1 and time implies that the innovator has been active for an infinite amount of time. The probability of the innovator's product malfunctioning then drops to zero regardless of α, and the threshold depends solely on the testing process. Thus, the thresholds at $Q_1 = 60$ are equal for all values of the intensity parameter. This effect is also present in the value of the laggard in Figure 12b, most clear for $\alpha = 0.1$. The fact that the innovator may malfunction has a high value at $Q_1 = 60 + dQ_1$, which vanishes completely at $Q_1 = 60$. We conclude that for certain intensity parameters, the laggard is not willing to accept any risk of malfunction unless the innovator has malfunctioned, and, thus, delays investment forever.

The ambiguous effect of increasing α is present in the value of the laggard as well. For high Q_1, the dominating effect is that large α decreases the expected time until the innovator experiences a malfunction, which increases the firm value. For low Q_1, the dominating effect is that large α implies a lower probability of the innovator experiencing a malfunction, which decreases the firm value. The values therefore intersect at a line where the effects are equal, as illustrated in Figure 12b. Generally, we find that the laggard's value is increasing in α for high Q_1 and decreasing in α for low Q_1.

The strategy of the laggard also depends on the competitiveness of the duopoly market, determined by the disloyalty parameter γ. The degree of loyalty heavily influences the value of acting in duopoly with the innovator, and its impact on the investment timing is shown in Figure 14.



(a) High γ hastens investment if P_2^I is high. ($P_2^I = 10$)

(b) High γ delays investment if P_2^I is low. ($P_2^I = 4$)

Figure 14: Sensitivity to the competition parameter γ.
($P_1^L = 3$, $\eta_1 = 20$, $Q_2^0 = 400$, $\eta_2 = 15$, $I = 200$,$r = 0.05$, $\lambda = 2$, $u = 0.2$, $\nu = 0.5$, $\alpha = 1.5$, $\theta^I = 0.4$)

A market with customers that have low product loyalty, i.e. high γ, allows the firm with lower price to steal more customers from the competitor. Hence, a higher γ increases the value of investing, as long as the laggard chooses a price lower than the innovator's. However, we showed in equation (3.21) that although the laggard has a second-mover advantage in determining the price, it sets a higher price than the innovator if $P_2^I < \tilde{P_2^I}$. A higher γ will then decrease the value of investing, and the laggard requires a higher testing level to trigger investment. The threshold $\theta_d^*(Q_1)$ is therefore decreasing in γ for $P_2^I > \tilde{P_2^I}$ and increasing for $P_2^I < \tilde{P_2^I}$. From this, we conclude that if the market for the upgrade is highly competitive and the innovator's price is high relative to the laggard's, the laggard is more willing to accept malfunction risk for its upgrade.

The share of potential customers for the upgraded versions who are initially loyal to the laggard, ν, also influences the value of entering into a duopoly with the innovator. Increasing the market share increases the demand for the laggard's product, which implies a higher value of investing. Hence, the testing threshold $\theta_d^*(Q_1)$ that triggers investment is lower for large ν, and the laggard is more willing to accept malfunction risk for its product.

Finally we find a similar ambiguous relation between the discount rate and the optimal investment timing as in the monopoly model. This effect is most clear when we fix Q_1 and graph the investment timing as a function of r, as presented in Figure 15.

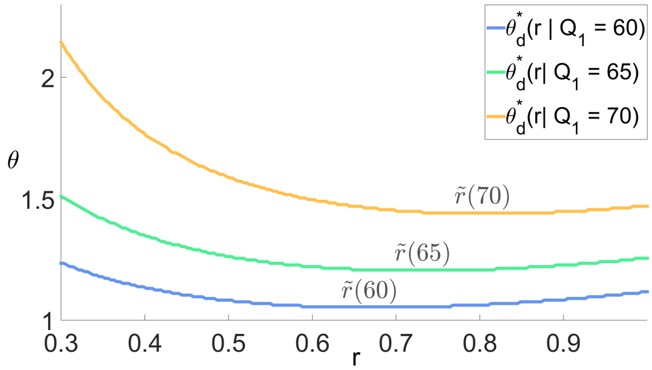


Figure 15: The discount effect r has an ambiguous effect on the optimal strategy. ($P_1^L = 3$, $\eta_1 = 20$, $Q_2^0 = 400$, $\eta_2 = 15$, $I = 200$, $\lambda = 2$, $u = 0.2$,$P_2^I = 8$, $\gamma = 5$, $\nu = 0.5$, $\alpha = 1.5$, $\theta^I = 0.4$)

Recall that in the monopoly model, increasing r had two effects on the investment timing; *(i)* the present becomes relatively more important, and *(ii)* the net present value from the upgrade decreases. In this section, the investment timing is in addition affected by the delaying effect. This effect arises from the relative benefit of becoming a monopolist before introducing the upgrade versus after, which does not depend on the discount rate. Hence, isolating the delaying effect of the innovator, an increase in r only results in the present becoming relatively more important and, thus, hastens investment. We therefore find that for high Q_1, where the delaying effect of the innovator influences the strategy more heavily, effect *(i)* dominates for a larger range of r. The threshold rate $\tilde{r}$ is then increasing in Q_1, as illustrated in Figure 15.

4 Conclusions

This paper examines a firm facing an investment decision on when to introduce an upgraded version of an existing durable good. The upgrade is subject to a risk of malfunction that depends on the progress of an uncertain testing process. We consider both the case when the firm has monopoly rights in the market for the upgrade, and when an innovator is already present in the new market. The paper extends the durable goods literature by introducing the concepts of product testing and malfunction risks. Furthermore, it extends the literature on investment under uncertainty by developing a model that accounts for the specific features of durable goods.

We find that a monopolistic firm's incentive to introduce a risky upgrade in the market may be explained by three reasons. First, an incentive to release the risky upgrade arises when the profits from selling the existing version has dropped. Second, if the upgrade is of high quality and has a high expected initial stock of potential customers, the expected value of the investment is large. This gives incentive to invest early even if significant malfunction risk is present. Third, a slow testing process may drive the firm to gamble on a risky upgrade rather than wait for more sub-tests to complete. We also find that the presence of an innovator with a risky upgrade gives the laggard an incentive to stay longer in the market for the existing version and observe the innovator's product. Hence, the laggard requires a higher testing level to justify investment and is less willing to accept malfunction risk. This may have been one of the reasons why Nokia waited too long to enter the smartphone market in 2007 and ended up losing the entire mobile phone market to innovating companies like Apple and Samsung[12]. The delaying effect is stronger if the innovator did limited product testing before it released its product, but decreases over time as the innovator's product has been sold without malfunctioning. Another interesting finding is that a change in the discount rate has an ambiguous effect on the optimal timing of the upgrade introduction, both in the monopoly and the duopoly case.

In what follows, we present several suggestions for further research. First of all, we assume that a second-hand market does not exist. Including an active second-hand market would make the model more suitable for a real world case, as companies like Norwegian Finn.no and American Craigslist and eBay have made it easier than ever for customers to sell their used durable goods. Second, the paper only considers the case of separate production, i.e. firms discontinue the previous version when introducing an upgrade. In reality, most firms continue to sell the old version at a discounted price after the upgrade is introduced. The relaxation of this assumption would, however, come at the cost of added complexity from the time inconsistency problem introduced in Coase (1972). Finally, one could allow for cross competition between the markets like in Arslan et al. (2009) to make the model more realistic.

An interesting extension for the duopoly model would be to consider a leader-follower model similar to Steg and Thijssen (2015) instead of an innovator-laggard model. To continue the smartphone example used throughout the paper, companies like Samsung and Apple are continuously releasing upgraded versions of their products. It would therefore be interesting to investigate the incentives to be the first-mover in the transition from one generation of smartphones to the next, and whether a preemption game would appear. However, such an extension would require modeling of two distinct Poisson processes, one for each company, as it would be unreasonable to argue that they have common testing. This would complicate the model drastically and was therefore left out of scope.

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Appendix A

Parameter List		
Parameter	Symbol	Example Values
Interest rate	r	0.05
Initial customer potential in existing market	Q_1^0	150
Initial customer potential in new market [Duo (Mon)]	Q_2^0	400 (180)
Remaining customer potential in existing market	Q_1	N/A
Remaining customer potential in new market	Q_2	N/A
Price of product in existing market	P_1/P_1^L	3
Price of product in new market.	P_2	N/A
Price of product in new market for innovator.	P_2^I	8
Price of product in new market for laggard.	P_2^L	N/A
Price penalty factor for product in existing market	η_1	20
Price penalty factor for product in new market	η_2	15
Customer disloyalty parameter	γ	5
Initial fraction of customers loyal to laggard in new market	ν	0.5
Investment cost	I	200
Arrival rate of completed sub-tests	λ	2
Poisson jump size	u	0.2
Testing level at which innovator has invested	θ^I	0.4
Intensity parameter	α	1.5

Appendix B

B.1 Deterministic monopoly problem

B.1.1 Derivation of demand relations

To find an expression for the demand $q(t)$ to insert into the value function in equation (2.4), we solve the following differential equations for $i = 1, 2$:

$$dQ_i(t) = -q_i(t)dt = -(Q_i - \eta_i P_i)dt, i = 1, 2, \tag{B.1}$$

with the boundary conditions $Q_1(0) = Q_1^0$ and $Q_2(0) = Q_2^0$. The solutions are

$$Q_1(t) = \eta_1 P_1 + (Q_1^0 - \eta_1 P_1)e^{-t}, \tag{B.2}$$
$$Q_2(t) = \eta_2 P_2 + (Q_2^0 - \eta_2 P_2)e^{-t}. \tag{B.3}$$

Inserted into the expressions for instantaneous demand as given by equation (2.2), this gives

$$q_1(t) = (Q_1^0 - \eta_1 P_1)e^{-t}, \tag{B.4}$$
$$q_2(t) = (Q_2^0 - \eta_1 P_1)e^{-t}. \tag{B.5}$$

B.1.2 Proof of Proposition 2.1

Derivation of optimal price

We start with equation B.4 and B.5 insert these into the value function in (2.4). The expression for firm value then becomes

$$V^* = \int_0^{t_n} (Q_1^0 - \eta_1 P_1)e^{-t} P_1 e^{-rt} dt + \max_\tau \left[\int_{t_n}^{\tau} (Q_1^0 - \eta_1 P_1)e^{-t} P_1 e^{-rt} dt - Ie^{-r\tau} + \max_{P_2} \left[\int_\tau^\infty (Q_2^0 - \eta_2 P_2)e^{-(t-\tau)} P_2 e^{-rt} dt \right] \right]. \tag{B.6}$$

Applying the first-order condition for maximization on

$$\max_{P_2} \left[\int_\tau^\infty P_2 (Q_2^0 - \eta_2 P_2)e^{-(t-\tau)} e^{-rt} dt \right] \tag{B.7}$$

gives

$$\frac{d}{dP_2} \left[P_2 \frac{Q_2^0 - \eta_2 P_2}{1+r} e^{-r\tau} \right] = 0. \tag{B.8}$$

Solving equation (B.8) yields the optimal price P_2^* as presented in equation (2.6). Inserting this into (B.7) gives the optimal value V_2^* as presented in equation (2.7).

Derivation of optimal investment timing τ^*

We continue by inserting the expression for V_2^* in the inner maximization in (B.6) and apply the first-order condition for maximization on the outer maximization with respect to the investment timing τ

$$\frac{d}{d\tau} \left[P_1 \frac{(Q_1^0 - \eta_1 P_1)}{1+r} \left[e^{-(1+r)t_n} - e^{-(1+r)\tau} \right] - Ie^{-r\tau} + \frac{(Q_0^2)^2}{4\eta_2(1+r)} e^{-r\tau} \right] = 0. \tag{B.9}$$

This gives the following equation for the optimal timing

$$P_1\left(Q_1^0-\eta_1 P_1\right)e^{-(1+r)\tau^*}-r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)e^{-r\tau^*}=0 \tag{B.10}$$

and results in the expression for $\hat{\tau^*}$ in the case $\tau^*=\hat{\tau^*}$ in Proposition 2.1.

Proof that $\hat{\tau^*}$ is an optimum

Applying the convexity condition to confirm optimality requires interpretation of the second derivative. Instead, we determine the regions of which the value increases and decreases in τ by evaluating the sign of the expression in equation B.10.

First, we consider the region of which the firm value increases in τ

$$\frac{\partial V(\tau)}{\partial \tau}>0, \tag{B.11}$$

$$P_1(Q_1^0-\eta_1 P_1)e^{-(1+r)\tau}-r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)e^{-r\tau}>0, \tag{B.12}$$

$$\tau<\ln\left[\frac{P_1(Q_1-\eta_1 P_1)}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)}\right]=\hat{\tau^*}. \tag{B.13}$$

This implies that $\frac{\partial V(\tau)}{\partial \tau}>0$, i.e. it is beneficial to invest at a later time then τ, when $\tau<\hat{\tau^*}$.

Similarly, we consider the region of which the firm value decreases in τ

$$\frac{\partial V(\tau)}{\partial \tau}<0 \tag{B.14}$$

$$P_1(Q_1^0-\eta_1 P_1)e^{-(1+r)\tau}-r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)e^{-r\tau}<0 \tag{B.15}$$

$$\tau>\ln\left[\frac{P_1(Q_1-\eta_1 P_1)}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)}\right]=\hat{\tau^*} \tag{B.16}$$

This implies that $\frac{\partial V(\tau)}{\tau}<0$, i.e. it is beneficial to invest earlier than τ, when $\tau>\hat{\tau^*}$. Altogether, we find that it is optimal to invest later when $\tau<\hat{\tau^*}$ and earlier when $\tau>\hat{\tau^*}$ and therefore, that the optimal investment timing is in fact $\hat{\tau^*}$.

B.1.3 Optimal price P_1

We interpret P_1 as a parameter, so the following only applies for the purpose of sensitivity analysis as discussed in Proposition 2.2 and 2.4. We apply the first-order condition for maximization w.r.t. the price P_1 to the the firm value when $\tau^*=\hat{\tau^*}$ in Proposition 2.3. This gives the equation

$$\frac{\partial V(\hat{\tau^*},P_1)}{\partial P_1}=\frac{Q_1^0-2\eta_1 P_1}{1+r}\left[1-\left(P_1\frac{(Q_1^0-\eta_1 P_1)}{r\left(\frac{{Q_2^0}^2}{4\eta_2(1+r)}-I\right)}\right)^{-(1+r)}\right]=0. \tag{B.17}$$

This has the candidate solutions $P_1^{c1} = \frac{Q_1^0}{2\eta_1}$ and P_1^{c2} that is the solution to

$$1 - \left(\frac{P_1^{c2}(Q_1^0 - \eta_1 P_1^{c2})}{r\left(\frac{Q_2^{0\,2}}{4\eta_2(1+r)} - I\right)} \right)^{-(1+r)} = 0.$$

We evaluate the convexity of the value with respect to the price. The second-order derivative is given by

$$\begin{aligned}\frac{\partial^2 V(\hat{\tau^*}, P_1)}{\partial P_1^2} &= \frac{-2\eta_1}{1+r}\left[1 - \left(P_1 \frac{(Q_1^0 - \eta_1 P_1)}{r\left(\frac{Q_2^{0\,2}}{4\eta_2(1+r)} - I\right)}\right)^{-(1+r)}\right] \\ &+ \frac{Q_1^0 - 2\eta_1 P_1}{1+r}\left[(1+r)\frac{Q_1^0 - 2\eta_1 P_1}{r\left(\frac{Q_2^{0\,2}}{4\eta_2(1+r)} - I\right)}\left(\frac{P_1(Q_1^0 - \eta_1 P_1)}{r\left(\frac{Q_2^{0\,2}}{4\eta_2(1+r)} - I\right)}\right)^{-(2+r)}\right].\end{aligned} \tag{B.18}$$

For the candidate solution P_1^{c1}, the second term goes to zero while the first term is negative. Hence, P_1^{c2} is a maximum. For the candidate solution P_1^{c2}, the first term becomes becomes zero, while the second term is positive. Hence, P_1^{c2} is a minimum.

We therefore conclude that the optimal price for the remaining customer potential Q_1^0 is $\frac{Q_1^0}{2\eta_1}$.

B.1.4 Proof of Proposition 2.2

For the case where $\hat{\tau^*} < \infty$, the net present value of undertaking the investment must be positive. This requires that $\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right) \geq 0$. We also require that the demand $\left(Q_1^0 - \eta_1 P_1\right) e^{-t}$ for the existing version cannot be negative, as this would imply that the optimal time of introduction has already passed. In the proofs that follow, we refer to these as the base assumptions.

Sensitivity to the initial stock of potential customers for the existing product, Q_1^0

$$\frac{\partial \hat{\tau^*}}{\partial Q_1^0} = \frac{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right)}{P_1(Q_1^0 - \eta_1 P_1)} P_1. \tag{B.19}$$

This is > 0 under the base assumptions.

Sensitivity to the initial stock of potential customers for the new product, Q_2^0

$$\begin{aligned}\frac{\partial \hat{\tau^*}}{\partial Q_2^0} &= \frac{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right)}{P_1(Q_1^0 - \eta_1 P_1)} \frac{-P_1(Q_1^0 - \eta_1 P_1)}{\left(r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right)\right)^2} \frac{2rQ_2^0}{4\eta_2(1+r)} \\ &= \frac{-2Q_2^0}{4\eta_2(1+r)\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right)}.\end{aligned} \tag{B.20}$$

This is < 0 under the base assumptions.

Sensitivity to the price of the existing product, P_1

$$\frac{\partial \hat{\tau^*}}{\partial P_1} = \frac{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right)}{P_1(Q_1^0 - \eta_1 P_1)} \left(Q_1^0 - 2\eta_1 P_1\right). \tag{B.21}$$

Hence, $\frac{\partial \hat{\tau^*}}{\partial P_1} > 0$ when $P_1 < \frac{Q_1^0}{2\eta_1}$, and $\frac{\partial \hat{\tau^*}}{\partial P_1} < 0$ when $P_1 > \frac{Q_1^0}{2\eta_1}$.

Sensitivity to the price penalty factor for the existing product, η_1

$$\frac{\partial \hat{\tau^*}}{\partial \eta_1} = \frac{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right)}{P_1(Q_1^0 - \eta_1 P_1)}(-P_1^2). \tag{B.22}$$

This is < 0 under the base assumptions.

Sensitivity to the price penalty factor for the new product, η_2

$$\begin{aligned}\frac{\partial \hat{\tau^*}}{\partial \eta_2} &= \frac{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right)}{P_1(Q_1^0 - \eta_1 P_1)} \frac{-P_1(Q_1^0 - \eta_1 P_1)}{\left(r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right)\right)^2} \frac{-r{Q_2^0}^2}{(4\eta_2(1+r))^2} 4(1+r) \\ &= \frac{{Q_2^0}^2}{4\eta_2^2(1+r)\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right)}.\end{aligned} \tag{B.23}$$

This is > 0 under the base assumptions.

Sensitivity to the investment cost, I

$$\frac{\partial \hat{\tau^*}}{\partial I} = \frac{-r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right)}{P_1(Q_1^0 - \eta_1 P_1)} \frac{-P_1(Q_1^0 - \eta_1 P_1)}{\left(r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right)\right)^2} = \frac{1}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right)}. \tag{B.24}$$

This is > 0 under the base assumptions.

Sensitivity to the discount rate, r

$$\begin{aligned}\frac{\partial \hat{\tau^*}}{\partial r} &= \frac{-r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right)}{P_1(Q_1^0 - \eta_1 P_1)} \frac{P_1(Q_1^0 - \eta_1 P_1)}{\left(r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right)\right)^2} \left(\frac{4\eta_2 (Q_2^0)^2\left[(1+r) - r\right]}{16\eta_2^2(1+r)^2} - I\right) \\ &= -\frac{1}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right)} \left(\frac{(Q_2^0)^2}{4\eta_2(1+r)^2} - I\right).\end{aligned} \tag{B.25}$$

This expression is negative when

$$\frac{{Q_2^0}^2}{4\eta_2(1+r)^2} - I > 0, \tag{B.26}$$

$$\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right) > rI, \tag{B.27}$$

and positive otherwise.

B.1.5 Proof of Proposition 2.3

Derivation of firm value

The value of the firm is given by equation (2.4). We evaluate the integrals by inserting the results from equation (2.7) and (B.4). This gives the following expression for the firm value, given investment timing τ.

$$\begin{aligned} V(\tau) = & \frac{P_1(Q_1^0 - \eta_1 P_1)}{1+r}\left[1 - e^{-(1+r)t_n} + e^{-(1+r)t_n} - e^{-(1+r)\tau}\right] \\ & + \frac{(Q_2^0)^2}{4\eta_2(1+r)} e^{\tau} e^{-(1+r)\tau} - I e^{-r\tau} \end{aligned} \tag{B.28}$$

Collecting terms, rearranging and factoring out $e^{-r\tau}$ gives:

$$V(\tau) = \frac{P_1(Q_1^0 - \eta_1 P_1)}{1+r} + e^{-r\tau}\left[\left(\frac{{Q_2^0}^2}{4\eta_2(1+r)} - I\right) - \frac{P_1(Q_1^0 - \eta_1 P_1)}{1+r} e^{-\tau}\right] \tag{B.29}$$

Inserting for $\tau = t_n$, $\tau = \hat{\tau^*}$ and $\tau \to \infty$ gives the results in Proposition 2.3

B.1.6 Proof of Proposition 2.4

Apart from the assumptions presented in the proofs for Proposition 2.3, we require the optimal investment timing to be greater than or equal to 0. This implies that $\left(\frac{P_1(Q_1^0 - \eta_1 P_1)}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right)}\right) > 1$, and we extend the reference to the base assumptions to include this.

Sensitivity to the remaining stock of potential customers for product 1, Q_1^0

$$\begin{aligned} \frac{\partial V}{\partial Q_1^0} &= \frac{P_1}{1+r} + \frac{\frac{{Q_2^0}^2}{4\eta_2(1+r)} - I}{1+r}(-r)\left(\frac{P_1(Q_1^0 - \eta_1 P_1)}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right)}\right)^{-(1+r)} \frac{P_1}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right)} \\ &= \frac{P_1}{1+r}\left(1 - \left(\frac{P_1(Q_1^0 - \eta_1 P_1)}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)} - I\right)}\right)^{-(1+r)}\right). \end{aligned} \tag{B.30}$$

This is > 0 under the base assumptions.

Sensitivity to the initial stock of potential customers for product 2, Q_2^0

$$\begin{aligned}
\frac{\partial V}{\partial Q_2^0} &= \frac{2Q_2^0}{4\eta_2(1+r)^2}\left(\frac{P_1(Q_1^0-\eta_1 P_1)}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)}\right)^{-r} \\
&\quad + \frac{\frac{{Q_2^0}^2}{4\eta_2(1+r)}-I}{1+r}(-r)\left(\frac{P_1(Q_1^0-\eta_1 P_1)}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)}\right)^{-(1+r)} \frac{-P_1(Q_1^0-\eta_1 P_1)}{\left(r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)\right)^2}\frac{2r{Q_2^0}^2}{4\eta_2(1+r)} \\
&= \left(\frac{P_1(Q_1^0-\eta_1 P_1)}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)}\right)^{-r}\frac{2Q_2^0}{4\eta_2(1+r)^2}\left(1+2r^2Q_2^0\right).
\end{aligned} \tag{B.31}$$

This is >0 under the base assumptions. In the case $\tau^* \to \infty$, the sensitivity is 0.

Sensitivity to the price of the existing product, P_1

$$\begin{aligned}
\frac{\partial V}{\partial P_1} &= \frac{Q_1^0-2\eta_1 P_1}{1+r} - \frac{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)}{1+r}\left(\frac{P_1(Q_1^0-\eta_1 P_1)}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)}\right)^{-(1+r)}\frac{Q_1^0-2\eta_1 P_1}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)} \\
&= \frac{Q_1^0-2\eta_1 P_1}{1+r}\left[1-\left(\frac{P_1(Q_1^0-\eta_1 P_1)}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)}\right)^{-(1+r)}\right].
\end{aligned} \tag{B.32}$$

Hence, $\frac{\partial V}{\partial P_1} > 0$ when $P_1 < \frac{Q_1^0}{2\eta_1}$, and $\frac{\partial V}{\partial P_1} < 0$ when $P_1 > \frac{Q_1^0}{2\eta_1}$.

Sensitivity to the demand factor of product 1, η_1

$$\begin{aligned}
\frac{\partial V}{\partial \eta_1} &= \frac{-P_1^2}{1+r} + \frac{\frac{{Q_2^0}^2}{4\eta_2(1+r)}-I}{1+r}(-r)\left(\frac{P_1(Q_1^0-\eta_1 P_1)}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)}\right)^{-(1+r)}\frac{-P_1^2}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)} \\
&= \frac{P_1^2}{1+r}\left(\left(\frac{P_1(Q_1^0-\eta_1 P_1)}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)}\right)^{-(1+r)}-1\right).
\end{aligned} \tag{B.33}$$

This is <0 under the base assumptions.

Sensitivity to the demand factor of product 2, η_2

$$\frac{\partial V}{\partial \eta_2} = -\frac{(Q_2^0)^2}{4\eta_2^2(1+r)^2}\left(\frac{P_1(Q_1^0-\eta_1 P_1)}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)}\right)^{-r} - \frac{(Q_2^0)^2(Q_1^0-\eta_1 P_1)P_1}{4\eta_2^2(1+r)^2\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)}\left(\frac{P_1(Q_1^0-\eta_1 P_1)}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)}\right)^{-(1+r)}. \tag{B.34}$$

Both terms are clearly negative under the base assumptions, making the sensitivity <0. Note that in the case $\tau^* \to \infty$, the derivative is 0.

Sensitivity to the investment cost, I

$$\begin{aligned}\frac{\partial V}{\partial I} &= -\frac{1}{1+r}\left(\frac{P_1(Q_1^0-\eta_1 P_1)}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)}\right)^{-r} - \frac{1}{1+r}\left(\frac{P_1(Q_1^0-\eta_1 P_1)}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)}\right)^{-(1+r)} \frac{P_1(Q_1^0-\eta_1 P_1)}{\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I} \\ &= -\left(\frac{P_1(Q_1^0-\eta_1 P_1)}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)}\right)^{-r}.\end{aligned} \tag{B.35}$$

This is < 0 under the base assumptions. Note that in the case where $\tau^* \to \infty$, the derivative is 0.

Sensitivity to the discount rate, r
This proof is more complex and requires several steps.

$$\begin{aligned}\frac{\partial V}{\partial r} =& -\frac{P_1(Q_1^0-\eta_1 P_1)}{(1+r)^2} + \frac{\frac{-4\eta_2 {Q_2^0}^2}{(4\eta_2(1+r))^2}(1+r) - r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)}{(1+r)^2}\left(\frac{P_1(Q_1^0-\eta_1 P_1)}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)}\right)^{-r} \\ &+ \frac{\frac{{Q_2^0}^2}{4\eta_2(1+r)}-I}{1+r}\left(\frac{P_1(Q_1^0-\eta_1 P_1)}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)}\right)^{-r}\left[-\ln\left(\frac{P_1(Q_1^0-\eta_1 P_1)}{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)}\right)\right. \\ &\left. -r\frac{r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)}{P_1(Q_1^0-\eta_1 P_1)}\frac{-P_1(Q_1^0-\eta_1 P_1)}{\left(r\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)\right)^2}\left(\left(\frac{(Q_2^0)^2}{4\eta_2(1+r)}-I\right)+r\frac{-4\eta_2 {Q_2^0}^2}{(4\eta_2(1+r))^2}\right)\right].\end{aligned} \tag{B.36}$$

We now introduce the simplification $NPV_2 = \frac{{Q_2^0}^2}{4\eta_2(1+r)} - I$. Inserting for NPV_2 and collecting terms gives

$$\begin{aligned}\frac{\partial V}{\partial r} =& -\frac{P_1(Q_1^0-\eta_1 P_1)}{(1+r)^2} - \frac{(1+r)NPV_2+I}{(1+r)^2}\left(\frac{P_1(Q_1^0-\eta_1 P_1)}{rNPV_2}\right)^{-r} \\ &- \frac{NPV_2}{1+r}\left(\frac{P_1(Q_1^0-\eta_1 P_1)}{rNPV_2}\right)^{-r}\left[\ln\left(\frac{P_1(Q_1^0-\eta_1 P_1)}{rNPV_2}\right) + \frac{1}{NPV_2}\left(NPV - r\frac{NPV_2+I}{1+r}\right)\right].\end{aligned} \tag{B.37}$$

When we evaluate the expression, it is clear that only one term can be positive, namely:

$$-\frac{NPV_2}{1+r}\left(\frac{P_1(Q_1^0-\eta_1 P_1)}{rNPV_2}\right)^{-r}\frac{1}{NPV_2}(-r)\frac{NPV_2+I}{1+r} = r\frac{NPV_2+I}{(1+r)^2}\left(\frac{P_1(Q_1^0-\eta_1 P_1)}{rNPV_2}\right)^{-r}. \tag{B.38}$$

However, this is dominated by

$$-\frac{(1+r)NPV_2+I}{(1+r)^2}\left(\frac{P_1(Q_1^0-\eta_1 P_1)}{rNPV_2}\right)^{-r}, \tag{B.39}$$

and we can therefore conclude that $\frac{\partial V}{\partial r} < 0$.

B.2 Stochastic monopoly problem

B.2.1 Explanation of probability terms

We now show that equation (3.11) represents the probability of the innovator's product malfunctioning within the next time period dt. Since the arrival rate $\phi(t)$ is continuous in time, the probability of an event occurring in the next time period dt can be determined by integrating the function from time t to $(t + dt)$.

$$\int_t^{t+dt} \phi(s)ds = \int_t^{t+dt} \alpha e^{-(\theta+\alpha s)}ds = -\left[e^{-(\theta+\alpha s)}\right]_t^{t+dt} = e^{-(\theta+\alpha t)}\left(1 - e^{-\alpha dt}\right). \tag{B.40}$$

The term $e^{-\alpha dt}$ can be approximated by use of a Taylor expansion as follows:

$$e^{-\alpha dt} \approx 1 - \alpha dt + \frac{(-\alpha dt)^2}{2} + ... + \frac{(-\alpha dt)^n}{n!}. \tag{B.41}$$

Terms with order of 2 and higher will approach zero fast as $dt \to 0$. Hence, they are neglected. Using the first two terms from the Taylor expansion as an approximation for $e^{-\alpha dt}$, equation (B.40) simplifies to $\alpha e^{-(\theta+\alpha t)}dt$. Hence, equation (3.11) is verified.

B.2.2 Determining the firm value in the continuation region

The firm value in the region of continuation is given from two solution approaches: analytical and numerical. Both approaches are presented here.

In general, the dynamics of the firm value can be found from Itô's Lemma:

$$E[dF(\theta, Q_1)] = -(Q_1 - \eta_1 P_1)\frac{\partial F(\theta, Q_1)}{\partial Q_1} + \lambda dt\left(F(\theta + u, Q_1) - F(\theta, Q_1)\right) + o(dt) \tag{B.42}$$

The Bellman equation is derived from fundamentals of dynamic programming and states that the total return on an investment opportunity over a period dt must equal its instantaneous profit plus the expected change in value over this period. The equation is given as follows:

$$rF = \pi + \frac{1}{dt}\mathbf{E}\left[dF\right] \tag{B.43}$$

The differential equation governing the continuation region of the investment problem can then be derived by combining equation B.42 and B.43, which is the same as presented in equation (2.17).

Analytical approach

We follow in the steps of Hagspiel, Huisman, et al. (2016) and divide the continuation region in two parts. In the first part of the continuation region, the firm will not invest even if a sub-test is completed in the next step. In the second part, the firm will invest upon test completion. Hence, by denoting $\theta^*(Q_1)$ the optimal test threshold for a given level of Q_1, the first part of the continuation region is defined as $\theta < \theta^*(Q_1) - u$, and the second part of the continuation region is defined as $\theta^*(Q_1) - u < \theta < \theta^*(Q_1)$.

Second part of continuation region

The firm will here invest immediately upon completion of the next sub-test. From this follows that

$F(\theta + u, Q_1) = V_2(\theta + u) - I$, where $V_2(\theta)$ is given by equation (2.16). Inserting into equation (2.17) gives:

$$(r + \lambda)F(\theta, Q_1) = P_1(Q_1 - \eta_1 P_1) - (Q_1 - \eta_1 P_1)\frac{\partial F(\theta, Q_1)}{\partial Q_1} + \lambda\left(V(\theta + u, Q_1) - I\right). \quad \text{(B.44)}$$

This is a first-order inhomogeneous PDE, and the solution can be found by combining a homogeneous and a particular solution. We first find the homogeneous solution governed by the following equation:

$$(r + \lambda)F_H(\theta, Q_1) = -(Q_1 - \eta_1 P_1)\frac{\partial F(\theta, Q_1)}{\partial Q_1}. \quad \text{(B.45)}$$

To simplify the expressions, we use the substitution $Z = (Q_1 - \eta_1 P_1)$, as all terms involving Q is of this form. We guess on a solution to the equation above of the following form:

$$F(\theta, Z) = A_0 Z^{\beta_0}. \quad \text{(B.46)}$$

Substitution of this into the homogeneous equation (B.45) gives

$$(r + \lambda)A_0 F(\theta, Z) = -Z A_0 \beta_0 Z^{\beta_0 - 1}, \quad \text{(B.47)}$$

which leads to

$$\beta_0 = -(\lambda + r). \quad \text{(B.48)}$$

By applying this and substituting back for Z, we conclude that the solution to the homogeneous equation is given by

$$F_H(\theta, Q_1) = A_0(Q_1 - \eta_1 P_1)^{-(\lambda + r)}. \quad \text{(B.49)}$$

Again using the Z simplification, we rely on the method of undetermined coefficients and guess a particular solution of the form

$$F_P(\theta, Z) = \gamma_0 Z + \gamma_1 e^{-(\theta + u)} + \gamma_2, \quad \text{(B.50)}$$

which leads to

$$\begin{aligned} \gamma_0 &= \frac{P_1}{\lambda + r + 1}, \\ \gamma_1 &= \frac{-\lambda}{\lambda + r} V_2, \\ \gamma_2 &= \frac{\lambda(V_2 - I)}{\lambda + r}. \end{aligned} \quad \text{(B.51)}$$

Backwards substituting for Z and combining the homogeneous and particular solutions, we conclude that the firm value in the second part of the continuation region is given by

$$F(\theta, Q_1) = A_0(Q_1 - \eta_1 P_1)^{-(\lambda+r)} + \frac{P_1(Q_1 - \eta_1 P_1)}{\lambda + r + 1} + \frac{\lambda}{\lambda + r}\left(V_2(\theta + u, Q_1) - I\right). \tag{B.52}$$

First part of continuation region

In this region the firm will not invest in the next period even if new technology arrives, i.e. $\theta < \theta^*(Q_1) - u$. From equation (2.17) we then have the following expression

$$(r + \lambda)F(\theta, Q_1) = P_1(Q_1 - \eta_1 P_1) - (Q_1 - \eta_1 P_1)\frac{\partial F(\theta, Q_1)}{\partial Q_1} + \lambda F(\theta + u, Q_1). \tag{B.53}$$

As in the second part of the continuation region, this is a first-order inhomogeneous partial differential equation, and the solution is found in a similar manner. We first consider the homogeneous part:

$$(r + \lambda)F(\theta, Q_1) = -(Q_1 - \eta_1 P_1)\frac{\partial F(\theta, Q_1)}{\partial Q_1} + \lambda F(\theta + u, Q_1). \tag{B.54}$$

As proposed by Hagspiel, Huisman, et al. (2016) we guess on a solution of the following form:

$$F_H(\theta, Z) = A_1 Z^{\beta_1} + A_2 \beta_2^{\theta} + A_3 Z \beta_3^{\theta}, \tag{B.55}$$

where Z is again a substitution for $Q_1 - \eta_1 P_1$.

By substituting equation (B.55) into equation (B.54) we get the following expression:

$$(r+\lambda)\left(A_1 Z^{\beta_1^{\theta}} + A_2\beta_2^{\theta} + A_3 Z\beta_3^{\theta}\right) = Z\left(\beta_1 A_1 Z^{\beta_1 - 1} + A_3\beta_3^{\theta}\right) + \lambda\left(A_1 Z^{\beta_1} + A_2\beta_2\theta + u + A_3 Z\beta_3^{\theta+u}\right). \tag{B.56}$$

By factoring out each constant A and solving for β we get the following results:

$$\begin{aligned} \beta_1 &= -r, \\ \beta_2 &= \left(\frac{\lambda}{r + \lambda}\right)^{-\frac{1}{u}}, \\ \beta_3 &= \left(\frac{\lambda}{\lambda + r + 1}\right)^{-\frac{1}{u}}. \end{aligned} \tag{B.57}$$

The solution to the homogeneous part is hence given by:

$$F_H(\theta, Z) = A_1 Z^{-r} + A_2\left(\frac{\lambda}{r + \lambda}\right)^{-\frac{\theta}{u}} + A_3 Z\left(\frac{\lambda}{\lambda + r + 1}\right)^{-\frac{\theta}{u}}. \tag{B.58}$$

We now want to find a particular solution to the following inhomogeneous equation:

$$(r + \lambda)F(\theta, Q_1) = P_1(Q_1 - \eta_1 P_1) - (Q_1 - \eta_1 P_1)\frac{\partial F(\theta, Q_1)}{\partial Q_1} + \lambda F(\theta + u, Q_1). \tag{B.59}$$

Since the inhomogeneous term depends on Q_1 only, we do not have to account for the variable θ in the particular solution. Hence we guess on a solution that is a function of Q_1 only:

$$F_P(\theta, z) = \gamma_4 Z, \tag{B.60}$$

where Z still is a substitution for $Q_1 - \eta_1 P_1$.

Inserting equation (B.60) into (B.59) and solving for γ_4 gives the following result:

$$\gamma_4 = \frac{P_1}{1+r}. \tag{B.61}$$

The particular solution can hence be written as:

$$\frac{P_1(Q_1 - \eta_1 P_1)}{1+r}. \tag{B.62}$$

The full solution to the differential equation for the first part of the continuation region is given by the homogeneous solution and the particular solution combined. This is expressed as:

$$F(\theta, Z) = A_1 Z^{-r} + A_2 \left(\frac{\lambda}{r+\lambda}\right)^{-\frac{\theta}{u}} + A_3 Z \left(\frac{\lambda}{\lambda+r+1}\right)^{-\frac{\theta}{u}} + \frac{P_1(Q_1 - \eta_1 P_1)}{1+r}. \tag{B.63}$$

Summarized firm values in the three different regions

$$F(\theta, Q_1) = \begin{cases} A_1(Q_1 - \eta_1 P_1)^{-r} + A_2 \left(\frac{\lambda}{r+\lambda}\right)^{-\frac{\theta}{u}} + A_3(Q_1 - \eta_1 P_1) \left(\frac{\lambda}{\lambda+r+1}\right)^{-\frac{\theta}{u}} + \frac{P_1(Q_1-\eta_1 P_1)}{1+r} & \theta < \theta^* - u, \\ A_0(Q_1 - \eta_1 P_1)^{-(\lambda+r)} + \frac{P_1(Q_1-\eta_1 P_1)}{\lambda+r+1} + \frac{\lambda}{\lambda+r} \left(\frac{Q_2^{0\,2}}{4\eta_2(1+r)} \left(1 - e^{-(\theta+u)}\right) - I\right) & \theta^* - u < \theta < \theta^*, \\ \frac{Q_2^{0\,2}}{4\eta_2(1+r)} \left(1 - e^{-\theta}\right) - I & \theta \geq \theta^*. \end{cases} \tag{B.64}$$

We know that when Q_1 approaches $\eta_1 P_1$ the demand of the existing version goes to 0 and the firm value should therefore be finite. Hence, we can conclude that A_0 and A_1 must be zero. Furthermore, we want the value function to be continuous in the continuation region. To achieve this we apply the value matching condition between the two parts of the continuation region at $\theta^*(Q_1) - u$.

Inserting for $\theta = \theta^*(Q_1) - u$ and equating the first and second part of the continuation region gives:

$$\begin{aligned} &A_2 \left(\frac{\lambda}{r+\lambda}\right)^{-\frac{(\theta^*-u)}{u}} + A_3(Q_1 - \eta_1 P_1) \left(\frac{\lambda}{\lambda+r+1}\right)^{-\frac{(\theta^*-u)}{u}} + \frac{P_1(Q_1 - \eta_1 P_1)}{1+r} = \\ &\frac{P_1(Q_1 - \eta_1 P_1)}{\lambda+r+1} + \frac{\lambda}{\lambda+r} \left(\frac{Q_2^{0\,2}}{4\eta_2(1+r)} \left(1 - e^{-\theta^*}\right) - I\right). \end{aligned} \tag{B.65}$$

We first collect terms dependent on Q_1:

$$A_3(Q_1 - \eta_1 P_1)\left(\frac{\lambda}{\lambda + r + 1}\right)^{-\frac{(\theta^* - u)}{u}} + \frac{P_1(Q_1 - \eta_1 P_1)}{1 + r} = \frac{P_1(Q_1 - \eta_1 P_1)}{\lambda + r + 1}. \tag{B.66}$$

Solving for the constant A_3 gives the following result:

$$A_3 = -\frac{P_1}{1 + r}\left(\frac{\lambda}{1 + \lambda + r}\right)^{\frac{\theta^*}{u}}. \tag{B.67}$$

We now look at the remaining terms:

$$A_2\left(\frac{\lambda}{r + \lambda}\right)^{-\frac{(\theta^* - u)}{u}} = \frac{\lambda}{\lambda + r}\left(\frac{Q_2^{0\,2}}{4\eta_2(1 + r)}\left(1 - e^{-\theta^*}\right) - I\right). \tag{B.68}$$

Solving for the constant A_2 gives the following result:

$$A_2 = \left(\frac{\lambda}{\lambda + r}\right)^{\frac{\theta^*}{u}}\left[\frac{Q_2^{0\,2}}{4\eta_2(1 + r)}\left(1 - e^{-\theta^*}\right) - I\right]. \tag{B.69}$$

We see that the coefficients appear to be functions of the threshold θ^*, which, according to Proposition 2.5, is itself a function of the underlying variable Q_1. As a consequence, the proposed guess on a solution to equation (B.54) is not valid. Hence, the resulting firm value in Proposition 2.5 is incorrect.

The optimal threshold function is found by applying value-matching between the second part of the continuation region and the stopping region, as described by equation (2.15).

$$\frac{(Q_1 - \eta_1 P_1)P_1}{\lambda + \rho + 1} + \frac{\lambda}{\lambda + \rho}\left(V_2\left(1 - e^{-(\theta + u)}\right) - I\right) = V_2\left(1 - e^{-\theta}\right) - I. \tag{B.70}$$

Solving for $\theta^*(Q_1)$ yields the result as stated in Proposition 2.5.

Numerical approach

As for the analytical case, we want to solve the PDE given by equation (2.17).

We apply a backward difference approximation for the differential $\frac{\partial F(\theta, Q_1)}{\partial Q_1}$, given as $\frac{\partial F(\theta, Q_1)}{\partial Q_1} \approx \frac{F(j,i) - F(j,i-1)}{dQ_1}$. Further, we set the step-sizes to dQ_1 in $Q_1(i)$- direction, and $d\theta$ in $\theta(j)$-direction. A jump then takes the firm value from $F(j,i)$ to $F(j + \frac{u}{d\theta}, i)$, and we get the finite difference scheme for the firm value in the continuation region as presented in equation (2.20). This is restated below as follows:

$$F(j,i) = \frac{P_1(Q_1(i) - \eta_1 P_1) + (Q_1(i) - \eta_1 P_1)\frac{F(j,i-1)}{dQ_1} + \lambda F(j + \frac{u}{d\theta}, i)}{r + \lambda + \frac{Q_1(i) - \eta_1 P_1}{dQ_1}}. \tag{B.71}$$

At each point (j,i), the firm value is given by the maximum value of staying in the existing market and investing, i.e. $\max\left[F(j,i), V_2(j,i) - I\right]$, where $V_2(j,i)$ is given as

$$V_2(j,i) = \frac{(Q_2^0)^2}{4\eta_2(1+r)}\left(1 - e^{-\theta(j)}\right) \tag{B.72}$$

We begin solving the problem from the boundary $Q_1(1) = \eta_1 P_1$. Here, the existing market is completely drained and all terms involving $Q_1(1) - \eta_1 P_1$ in $F(j,1)$ can be ignored. Let k be the highest index in the θ vector, and choose $\theta(k)$ high enough to trigger investment. Then, iterating downwards from $j = k$ at $i = 1$, we find $\theta^*(i = 1)$ as the last $\theta(j)$ that triggers investment.

We then iterate upwards from $i = 2$, using the full expression in equation (B.71) for $F(j,i)$ and the same initial assumption that investment is triggered at $\theta(k)$. This approach works until $\theta^*(Q_1) \to \infty$, i.e. when it is never optimal to invest. The Q_1^*, with corresponding index i^*, for which this happens, is located by finding the first i where it is optimal to wait at the first possibility, i.e. at $j = k - 1$.

Iterating onwards from $i = i^*$, we need to apply a different condition at $\theta(k)$. We therefore use the fact that the effect of testing is diminishing, since the malfunction risk is given by $e^{-\theta}$ and claim that at $\theta(k)$, the firm value $F(\theta, Q_1) = F(\theta + u, Q_1)$. Then, equation (2.17) reduces to

$$rF(\theta, Q_1) = (Q_1 - \eta_1 P_1)P_1 - (Q_1 - \eta_1 P_1)\frac{\partial F(\theta, Q_1)}{\partial Q_1}, \tag{B.73}$$

with the corresponding finite difference scheme

$$F(k,i) = \frac{P_1(Q_1(i) - \eta_1 P_1) + (Q_1(i) - \eta_1 P_1)\frac{F(k,i-1)}{dQ_1}}{r + \frac{Q_1(i) - \eta_1 P_1}{dQ_1}}. \tag{B.74}$$

This is then used to find the firm value at the boundary $\theta(k)$ for $i > i^*$. The MatLab script for the numerical model is lengthy and therefore excluded from the paper, but is available upon request.

Appendix C Innovator-laggard

C.1 Deterministic product testing

C.1.1 Derivation of demand relations

We first formulate the differential equations for the remaining market potential in both the new and the existing market. This is accomplished by combining equation (3.1) and (3.2) for the dynamics of market potential in the existing market and new market respectively with their corresponding demand functions, given by equation (2.2) and (3.3). This results in the following differential equations

$$dQ_1(t) = -(Q_1(t) - \eta_1 P_1^L)dt \tag{C.1}$$
$$dQ_2^L(t) = -\left(Q_2^L(t) - \eta_2 P_2^L + \gamma(P_2^I - P_2^L)\right)dt, \tag{C.2}$$

where equation (C.1) governs the dynamics of customer potential for the laggard in the existing market (monopoly) and equation (C.2) governs the dynamics of customer potential for the laggard in the new market (duopoly).

Equation (C.1) and (C.2) can be integrated directly, and by applying the initial conditions $Q_1(0) = Q_1^0$ and $Q_2^L(0) = \nu Q_2^0$, we get the following expressions for the customer potentials

$$Q_1(t) = (Q_1^0 - \eta_1 P_1^L)e^{-t} + \eta_1 P_1^L \tag{C.3}$$
$$Q_2^L(t) = \left(\nu Q_2^0 - \eta_2 P_2^L + \gamma(P_2^I - P_2^L)\right) e^{-t} + \eta_2 P_2^L - \gamma(P_2^I - P_2^L). \tag{C.4}$$

Inserting these back into the demand functions from equation (2.2) and (3.3), the demand functions take on the following form

$$q_1^L(t) = (Q_1^0 - \eta_1 P_1^L)e^{-t}, \tag{C.5}$$
$$q_2^L(t) = \left(\nu Q_2^0 - \eta_2 P_2^L + \gamma(P_2^I - P_2^L)\right) e^{-t}. \tag{C.6}$$

C.1.2 Proof of Proposition 3.1

Derivation of optimal price to charge by the laggard, $P_2^{L^*}$

We start out with the value function from equation (3.4). Inserting the demand relations from equation (C.5) and (C.6) gives the following expression for firm value

$$V_L{}^*(\tau, P_2^L) = \int_0^{t_n} P_1^L(Q_1^0 - \eta_1 P_1^L)e^{-(1+r)t} + \max_\tau \Bigg[\int_{t_n}^{\tau} P_1^L(Q_1^0 - \eta_1 P_1^L)e^{-(1+r)tt}dt + \tag{C.7}$$
$$\max_{P_2^L}\left[\int_\tau^\infty P_2^L\left(\nu Q_2^0 - \eta_2 P_2^L + \gamma(P_2^I - P_2^L)\right) e^{-(1+r)t+\tau} dt\right] - Ie^{-r\tau}\Bigg]$$

We let V_2^L denote the value generated by the laggard from selling the upgraded version in the new market.

$$V_2^{L^*} = \max_{P_2^L}\left[\int_\tau^\infty P_2^L\left(Q_1^L(t-\tau) - \eta_2 P_2^L + \gamma(P_2^I - P_2^L)\right) e^{-(1+r)t+\tau} dt\right]$$
$$= \max_{P_2^L}\left[P_2^L \frac{\nu Q_2^0 - \eta_2 P_2^L + \gamma(P_2^I - P_2^L)}{1+r} . e^{-r\tau}\right] \tag{C.8}$$

We apply the first-order condition of maximization with respect to P_2^L.

$$\frac{\partial}{\partial P_2^L}\left[P_2^L \frac{\nu Q_2^0 - \eta_2 P_2^L + \gamma(P_2^I - P_2^{L^*})}{1+r} e^{-r\tau}\right] = \frac{\nu Q_2^0 - 2\eta_2 P_2^{L^*} + \gamma(P_2^I - 2P_2^{L^*})}{1+r} e^{-r\tau} = 0. \tag{C.9}$$

Solving for $P_2^{L^*}$ gives the optimal price as presented in equation (3.7) and given in Proposition 3.1.

Derivation of optimal investment timing, τ_L^*

We find the optimal value generated by the laggard from selling the upgraded version in the new market by plugging the optimal price from equation (3.7) into equation (C.8). In order to determine the investment timing we solve the outer maximization problem given by the value function in equation (C.7). Inserted for the optimal value of V_2^L, we get

$$\max_{\tau_L}\left[\int_{t_n}^{\tau_L} P_1^L(Q_1^0-\eta_1 P_1^L)e^{-(1+r)t}dt + P_2^{L*}\frac{\nu Q_2^0-\eta_2 P_2^{L*}+\gamma(P_2^I-P_2^{L*})}{1+r}e^{-r\tau_L}-Ie^{-r\tau_L}\right]. \tag{C.10}$$

Integrating and calculating the first-order condition with respect to τ gives the following expression

$$P_1^L(Q_1^0-\eta_1 P_1^L)e^{-(1+r)\tau_L{}^*}+r\left(I-P_2^{L*}\frac{\nu Q_2^0-\eta_2 P_2^{L*}+\gamma(P_2^I-P_2^{L*})}{1+r}\right)e^{-r\tau_L{}^*}=0. \tag{C.11}$$

Solving for $\tau_L{}^*$ gives the optimal investment timing as presented in Proposition 3.1.

Derivation of optimal firm value, $V_L{}^*$

The optimal value of the laggard is found by integrating equation C.7 and inserting expressions for optimal price P_2^{L*} and optimal investment timing $\tau_L{}^*$. Carrying out the integrations results in the following expression for optimal firm value

$$\begin{aligned}V_L{}^*(\tau_L{}^*)=\frac{P_1^L(Q_1^0-\eta_1 P_1^L)}{1+r}\left[1-e^{-(1+r)t_n}+e^{-(1+r)t_n}-e^{-(1+r)\tau^*}\right]+\\ r\left(I-P_2^{L*}\frac{\nu Q_2^0-\eta_2 P_2^{L*}+\gamma(P_2^I-P_2^{L*})}{1+r}\right)e^{-r\tau^*}.\end{aligned} \tag{C.12}$$

Collecting terms and inserting the expression for optimal investment timing $\tau_L{}^*$ results in the optimal firm value as presented in Proposition 3.1.

C.2 Stochastic product testing and risk of malfunction

C.2.1 Proof of Proposition 3.2

We start out with the expected payoff from investment, as given by equation (3.16). Given that the innovator does not experience a product malfunction, the value of the laggard is represented by the perpetual duopoly value.

$$\mathbf{E}[V|\bar{d^I}]=P_2^L\frac{\nu Q_2^0-\eta_2 P_2^L+\gamma(P_2^I-P_2^L)}{1+r}. \tag{C.13}$$

Given that the innovator experiences a malfunction, the value of the laggard depends on the timing of this malfunction. Hence, the expected value of the laggard, given that the innovator's product malfunctions, is represented by

$$\mathbf{E}[V|d^I]=\int_0^\infty Prob(m_t|m)V(m_t)dt, \tag{C.14}$$

where m_t denotes the event of malfunction at time t, and m denotes the event of a malfunction ever occurring. Note that $Prob(m_t|m) = \frac{p^I(t+\tau)}{p_d^I(\tau)}$. This comes from the fact that $P(A|B) = \frac{P(A\cap B)}{P(B)}$, so that the probability of the innovator's product malfunctioning at time t conditional upon ever malfunctioning is equivalent to the instantaneous probability of malfunctioning at time t after investment τ, divided by the probability of the innovator ever malfunctioning.

The value of the laggard given that the innovator's product malfunctions at time t consists of two parts: 1) The laggard operates in a duopoly with the innovator up until malfunction. 2) At point of malfunction t, the laggard is positioned in a monopoly situation, keeping its remaining stock of potential customers $Q_2^L(t)$ while receiving the innovator's initial stock of loyal customers $(1-\nu)Q_2^0$. This is mathematically stated as

$$V(m_t) = \int_0^t P_2^L(Q_2^L(s) - \eta_2 P_2^L + \gamma(P_2^I - P_2^L))e^{-rs}ds + \int_t^\infty P_2^L(Q_2^L(t) + (1-\nu)Q_2^0 - \eta_2 P_2^L)e^{-s}e^{-rs}ds. \tag{C.15}$$

Evaluating the integrals and collecting terms yields the following

$$\begin{aligned} V(m_t) = {} & P_2^L \frac{\nu Q_2^0 - \eta_2 P_2^L + \gamma(P_2^I - P_2^L)}{1+r}\left(1 - e^{-(1+r)t}\right) \\ & + P_2^L \frac{(\nu Q_2^0 - \eta_2 P_2^L + \gamma(P_2^I - P_2^L))e^{-t} + (1-\nu)Q_2^0 - \gamma(P_2^I - P_2^L)}{1+r} e^{-(1+r)t}. \end{aligned} \tag{C.16}$$

Inserted into equation (C.14) gives the following expected value of the laggard conditional upon the innovator experiencing a malfunction

$$\begin{aligned} \mathbf{E}[V|d^I] = {} & \frac{1}{P_d^I(\tau)} e^{-(\theta^I + \alpha\tau)} \alpha P_2^L \left[\frac{\nu Q_2^0 - \eta_2 P_2^L + \gamma(P_2^I - P_2^L)}{1+r} \left(\frac{1}{\alpha} - \frac{1}{\alpha + r + 1} + \frac{1}{\alpha + r + 2} \right) \right. \\ & \left. + \frac{(1-\nu)Q_2^0 - \gamma(P_2^I - P_2^L)}{1+r} \frac{1}{\alpha + r + 1} \right]. \end{aligned} \tag{C.17}$$

Hence, by combining equation (C.17) and (C.13) with equation (3.16), the expected value of the laggard in the stopping region is given in Proposition 3.2.

Derivation of optimal price to charge by the laggard

From equation (3.2), the optimal price can be determined by solving the first-order condition with respect to the price P_2^L. Differentiating, collecting terms and solving for $P_2^{L^*}$ yields the optimal price in Lemma 3.1.

C.2.2 Derivation of firm value in the continuation region

We start out by formulating the dynamics of the laggard's firm value. In the continuation region, there are four possible outcomes over the next time period dt: (1) innovator's product malfunctions and another sub-test is completed, (2) innovator's product malfunctions and no sub-test is completed, (3) innovator's product does not malfunction and another sub-test is completed and (4) innovator's product does not malfunction and no sub-test is completed.

$$\begin{aligned}\mathbf{E}[dF_d(\theta,Q_1)] =& -(Q_1-\eta_1 P_1^L)\frac{\partial F_d(\theta,Q_1)}{\partial Q_1}dt + \lambda dt\left(\alpha e^{-(\theta^I+\alpha t)}dt\right)[F_m(\theta+u,Q_1)-F_d(\theta,Q_1)]+\\ &+(1-\lambda dt)\left(\alpha e^{-(\theta^I+\alpha t)}dt\right)[F_m(\theta,Q_1)-F_d(\theta,Q_1)]+\\ &+\lambda dt\left(1-\alpha e^{-(\theta^I+\alpha t)}dt\right)[F_d(\theta+u,Q_1)-F_d(\theta,Q_1)]+\\ &+(1-\lambda dt)\left(1-\alpha e^{-(\theta^I+\alpha t)}\right)[F_d(\theta,Q_1)-F_d(\theta,Q_1)]+o(dt),\end{aligned} \tag{C.18}$$

where F_m denotes the firm value of the laggard in case of being a monopolist in the market, and F_d denotes the firm value of the laggard in the duopoly.

We neglect all terms involving dt of order higher than one (because they approach zero fast as $dt \to 0$). This leaves us with the following expression for firm value dynamics

$$\begin{aligned}\mathbf{E}\left[dF_d(\theta,Q_1)\right] =& -(Q_1-\eta_1 P_1^L)\frac{\partial F_d(\theta,Q_1)}{\partial Q_1} + \lambda dt(F_d(\theta+u,Q_1)-F_d(\theta,Q_1))+\\ &+\alpha e^{-(\theta^I+\alpha t)}dt\left[F_m(\theta,Q_1)-F_d(\theta,Q_1)\right]+o(dt).\end{aligned} \tag{C.19}$$

Combining equation (C.19) with the Bellman equation from (B.43), we arrive at the PDE for firm value in the continuation region, as presented in equation (3.22).

As in the monopoly model, the PDE in the continuation region must be solved numerically. The finite difference scheme is given by equation (3.23) and restated below. At each point, the firm value is given by the maximum of waiting, $F(j,i)$ and investing, i.e. the value in Proposition 3.2 less the investment cost I.

$$F_d(j,i) = \frac{P_1^L(Q_1(i)-\eta_1 P_1^L) + \lambda F_d(j+\frac{u}{d\theta},i) + (Q_1(i)-\eta_1 P_1^L)\frac{F(j,i-1)}{dQ_1} + \alpha e^{-\theta^I}\left(\frac{Q_1(i)-\eta_1 P_1^L}{Q_1^0-\eta_1 P_1^L}\right)^{\alpha} F_m(j,i)}{r+\lambda+\alpha e^{-\theta^I}\left(\frac{Q_1(i)-\eta_1 P_1^L}{Q_1^0-\eta_1 P_1^L}\right)^{\alpha} + \frac{Q_1(i)-\eta_1 P_1^L}{dQ_1}}. \tag{C.20}$$

We solve it in the same way as in the monopoly problem, iterating upwards in i from $Q_1(1) = \eta_1 P_1^L$ and downwards in j from $\theta(k)$, where it is assumed optimal to invest. The index i^*, where $\theta^* \to \infty$, is found in the same way, but the boundary PDE at $\theta(k)$ for $i > i^*$ now takes the form

$$F_d(k,i) = \frac{P_1^L(Q_1(i)-\eta_1 P_1^L) + (Q_1(i)-\eta_1 P_1^L)\frac{F(k,i-1)}{dQ_1} + \alpha e^{-\theta^I}\left(\frac{Q_1(i)-\eta_1 P_1^L}{Q_1^0-\eta_1 P_1^L}\right)^{\alpha} F_m(k,i)}{r+\alpha e^{-\theta^I}\left(\frac{Q_1(i)-\eta_1 P_1^L}{Q_1^0-\eta_1 P_1^L}\right)^{\alpha} + \frac{Q_1(i)-\eta_1 P_1^L}{dQ_1}}, \tag{C.21}$$

to capture the fact that the innovator's product can possibly malfunction. The MatLab script for the numerical model is lengthy and therefore excluded from the paper, but it is available upon request.