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# Optimal Investment Strategy in LiceFighting Technologies 

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## Problem description

What is the optimal adoption timing of a permanent lice-fighting technology, and how much is it optimal to invest in short-term solutions, before its arrival?

## Preface

This thesis is a part of a Master of Science at the Norwegian University of Science and Technology (NTNU). The degree specialisation is in Financial Engineering at the Department of Industrial Economics and Technology Management.

We would like to thank our supervisors dr. Verena Hagspiel and dr. Maria Lavrutich for their stimulating discussion, feedback and guidance. We highly appreciate their enthusiasm and involvement in our project.

We would also like to thank consultant Henning Urke and biologist Randi Grøntvedt at INAQ AS, for helping us define the problem of this thesis, and providing us with insight into the aquaculture industry and the lice problem. A major contribution from INAQ has been the network of industry experts they have introduced us to. We thank CEO of Bjørøya Fiskeoppdrett AS, Per Anton Løfsnæs, for inviting us to visit his fish farm in Flatanger. He has shared helpful insight into the lice problem and technology investment decisions from the perspective of a small sized fish farm, and provided us with data on model input parameters. We also thank Ståle Furø from Salmar who has contributed with the perspectives of a large firm. For more general information Jon Arne Grøttum, Director of Aquaculture in Sjømat Norge, and Kjell Maroni, Head of Aquaculture in The Norwegian Seafood Research Fund FHF, have been helpful. We thank them for their contribution of information on technological development and the traffic light system

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#### Abstract

Salmon louse is a parasite feeding off the flesh and skin of the fish, and is currently one of the largest challenges for the aquaculture industry, prohibiting growth in the industry. In order to allow for growth in an environmentally sustainable way, the Norwegian government has recently issued a set of regulatory guidelines referred to as the traffic light system, set to launch in October 2017. The regulation allows for production increase in regions with low lice levels, but forces reduction of production in regions with high lice levels, thus, motivating farmers to take additional measures to fight lice. Currently, profit margins in the industry are high, and salmon farmers are therefore investing heavily in existing lice mitigating technologies in order to deal with the pressing issue. Moreover, the development license scheme launched by the Norwegian government in 2015, has boosted investments in technological innovation that have a potential to mitigate the lice problem in the future.

In this thesis we use a real options approach to solve an investment problem of an aquaculture firm. The model solves a two-fold problem. First, we find the optimal adoption timing of a future, permanent lice-fighting technology, given that such a technology can only be adopted once. Second, we find the optimal investment amount in temporary, lice-fighting solutions. In addition, we introduce two extensions to this model. In the first extension, we allow the firm to perform upgrades of the permanent technology, and solve a sequential investment problem. In the second extension, we incorporate the traffic light system into the model in order to study the effect it will have on the investment strategy of a firm. We find the optimal, additional investment amount required due to the regulation, and investigate the sensitivity of the results depending on the lice pressure in the region.

The study is a contribution to the literature on real options valuations for investment decisions in the aquaculture industry. Our main findings are as follows. (i) The high investment cost of the permanent technologies causes the firm to wait relatively long before adopting the technology. Consequently, it invests a relatively large amount in short-term solutions. (ii) The firm value increases when the firm has the opportunity to upgrade the technology, however, the cost of upgrading limits the number of upgrades. (iii) The traffic light system works as intended from the government's perspective. It will reduce production in high lice level regions, and increase production in low lice level regions. (iv) The traffic light system will have a large, negative impact on firms in regions with high lice pressure, as biological constraints limit the firms from making additional investments.


## Sammendrag

Lakselus er en parasitt som fester seg på laksefisk og spiser hud, slim og blod på fisken. Den er en av de største utfordringene for havbruksnæringen, og virker sterkt begrensende for videre økonomisk vekst. I et forsøk på å tillate vekst på en bærekraftig måte, har den norske regjeringen vedtatt et vekstsystem for havbruksnæringen omtalt som trafikklyssystemet, som trer i kraft i oktober 2017. Systemet tar sikte på å øke produksjonen i regioner med lavt lusenivå, og å redusere produksjonen i områder med høyt lusenivå. Dette skal stimulere oppdrettere til å iverksette ytterligere tiltak i kampen mot lusen. Profittmarginene i næringen er svært høye, og oppdrettsselskaper investerer derfor tungt i lusebekjempende teknologier for å holde produksjonen oppe. I 2015 vedtok myndighetene en toårig prøveordning med utviklingstillatelser. Ordningen har ført til en kraftig økning av investeringer i forskning og utvikling av teknologier som kan minimere lakselusproblemet i fremtiden.

I denne oppgaven løser vi et todelt investeringsproblem for et oppdrettsfirma ved bruk av realopsjoner. Først ser vi på det optimale tidspunktet å investere i en fremtidig, permanent løsning på lakseluslusproblemet, gitt at en slik investering kan gjøres kun én gang. Deretter finner vi det optimale beløpet å investere i midlertidige lakselusbekjempende teknologier, i påvente av den permanente teknologien. I tillegg presenterer vi to utvidelser av denne modellen. I den første utvidelsen tillater vi selskapet å gjøre oppgraderinger av den permanente teknologien, hvilket betyr at problemet vi løser blir et sekvensielt investeringsproblem. I den andre utvidelsen inkluderer vi trafikklyssystemet i modellen for å undersøke hvilken effekt dette vil ha på selskapets investeringsstrategi. Nærmere bestemt finner vi hvilket beløp det er optimalt for et oppdrettsselskap å bruke på tilleggsinvesteringer grunnet trafikklyssystemet, og i hvilken grad lusenivået i regionen påvirker investeringsmengden.

Denne studien er et bidrag til litteraturen om realopsjonsverdsettelser av investeringsbeslutninger i havbruksnæringen. Våre hovedfunn er som følger: (i) Høye investeringskostnader for den permanente teknologien gjør at selskapet venter relativt lenge før de investerer i teknologien. Som en konsekvens av dette investerer de et relativt stort beløp i midlertidige løsninger. (ii) Verdien på selskapet øker når det har muligheten til å oppgradere den permanente teknologien, men kostnaden for dette begrenser antallet oppgraderinger som blir gjort. (iii) Trafikklyssystemet virker etter hensikten fra myndighetenes side, og vil redusere produksjonen i regioner med høyt lusenivå, samt øke produksjonen i regioner med lavt lusenivå. (iv) Traffikklyssystemet vil ha sterk, negativ påvirkning på selskaper i regioner med høyt lusenivå, da biologiske begrensninger hindrer de fra å gjøre nødvendige tilleggsinvesteringer.

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## Chapter 1

## Introduction

Fish farming in Norway has existed since the 1970's. Since then, a tremendous growth in the industry, and in the Norwegian market, has led Norway to being the largest producer of salmon in the world. For the last 20 years, the market has experienced high salmon prices due to strong demand growth. At the same time, the market is facing great challenges due to lice, forcing the government to introduce stringent regulations to ensure a sustainable production. Companies currently use a combination of different technologies to control lice, in order to achieve best possible effect. However, according to INAQ, the investment decisions related to these technologies are being made rapidly and with incomplete information. This is confirmed by Per Anton Løfsnæs, CEO of Bjørøya Fiskeoppdrett AS. He claims that most firms do not follow any clear strategies when it comes to investment in lice-fighting technologies. Most of the investment decisions are based on experiences from other fish farms who have previously tested the technology.

In 2017, the Norwegian government announced a new regulation known as the traffic light system. It aims to allow production expansion in regions with low lice levels, and reduce production in regions with high lice levels. The objective is to reduce the overall impact of fish farming on the wild salmon population. This creates an incentive for farmers to take even stronger measures in the battle against lice, and adds to the complexity of the farmer's investment decision. With the possibility of a lower salmon prices in the future and the implementation of the traffic light system, more thought out investment strategies are required.

A development license scheme launched by the Norwegian government in 2015 has encouraged the development of new, disruptive technologies. This has led to a sharp increase in innovations aimed at offering a permanent solution to the lice problem, which will make the temporary investments obsolete. However, the investment strategy regarding the permanent technologies depends on the firm in question. Ståle Furø from Salmar claims that large
firms are part of developing these technologies, and will therefore not wait if such a technology arrives. On the other hand, Per Anton Løfsnæs responds that smaller firms require specific strategies for investment, as they do not have insight in the development processes. Based on this, we focus on the investment problem of small and medium sized firms.

The general problem is twofold, looking at investment strategies before and after the arrival of a disruptive, permanent technology that will reduce lice costs significantly. Before the arrival, we wish to see how much a firm should invest in upgrading their current technologies, while maximising the value of the firm, and expecting the disruptive technology. After the arrival, we wish to find the optimal adoption timing for the disruptive technology. The general problem can therefore be expressed in the following way:

What is the optimal adoption timing of a permanent lice-fighting technology, and how much is it optimal to invest in short-term solutions, before its arrival?

Before the arrival of a permanent technology, a firm will try to mitigate the lice-fighting costs by investing in upgrades of the temporary, short-term solutions. A firm can upgrade by either adding a technology to the existing combination, improving one of the technologies, or replacing one of the technologies with a more efficient method. After the arrival of a disruptive technology, it continues to develop and the firm will eventually adopt it, making investments made beforehand obsolete. In addition, the traffic light system now creates an incentive for the firm to invest additionally in non-medical delousing treatments to keep the lice level down. This is especially true for the time during the emigration of the wild salmon smolt from the rivers, when the wild salmon are most prone to infection of salmon lice from fish farms. Therefore, we have extended the problem to see how much the firm should invest in additional treatments to influence the outcome of the traffic light system.

The problem description above contains a large number of uncertainties - the largest being when, and if, any permanent solution to the lice problem arrives. Many of the technologies in the development licence applications are eligible to be the first permanent technology to arrive. If the first batches of salmon farmed in the new facility are successful, commercial adoption may happen quickly. However, if adjustments must be made and further testing is required, commercial adoption will be further down the road. Moreover, the rate at which a disruptive technology develops after the arrival is also subject to a large uncertainty, as comparable technology development processes do not exist. These uncertainties have impact on the firm's investment decision as the cost of the technology is high. Not only the investment cost of the technology itself, but also costs related to training of the staff and totally transforming the way the firm produces salmon.

In the following chapters we solve the problem presented. Chapter 2 gives the background information about the aquaculture industry in Norway, the lice problem and the regulatory environment. In Chapter 3 we present a review of the relevant literature. This will form the basis for the model solved in Chapter 4. Chapter 5 provides an overview of the parameter values used as input for the case study. The results and sensitivity analysis are presented in Chapter 6. Chapter 7 concludes and presents suggestions for further work.

## Chapter 2

## Aquaculture Industry in Norway

The first real breakthrough for the Norwegian salmon farming industry came in the first part of the 1970s. Up until this point, pioneers from the 1950s and 1960s had experimented and shared knowledge that spread quickly. Since then, the industry has experienced a tremendous growth, only disrupted by overproduction in the early 1990s and the early 2000s. Because of its cold seawater and protected coastal areas, Norway is one of the few areas suitable for salmon farming in the world. The long coastline and years of experience has made Norway the largest producer of salmon in the world. (Marine Harvest Group, 2016)

In this chapter we present information about the Norwegian market to form a basis for the thesis. We focus on the growth in the Norwegian industry, salmon lice as the largest challenge, describe the regulatory environment and finally give an overview of the lice-fighting methods and technologies.

### 2.1 Profitability and Lice Challenge

The aquaculture industry has been a highly profitable industry for many years. An important reason for this is a demand growth influenced by innovation, long-term supply contracts, and effective logistics and transportation. As a result, the salmon prices have grown, and the global supply of salmon has increased by $417 \%$ in the period from 1995-2015. The compounded annual growth rate (CAGR) was $9 \%$ in the same period. However, in the recent years, the growth has diminished. In the period 2005-2014 the CAGR was $6 \%$, and in 2015 the expected annual growth from 2015-2020 was estimated to be $3 \%$. A similar trend can be found in the Norwegian market. The CAGR was $7 \%$ for Norway in the period from 1995-2015, but also here the growth has diminished in the recent years (Marine Harvest Group, 2016). One of the major reasons for the diminishing growth is that the industry has reached a level
of production where biological boundaries are being pushed. The increased concentration of fish farms means that lice and diseases spread more quickly, making both farms and wild salmon more vulnerable and limiting growth.

Salmon lice are parasites feeding off the skin, slime and blood of the fish, causing wounds and infections. The lice lives in salt water, in the upper layers of the sea. When the eggs hatch, the lice travel passively with the currents for many kilometres in search of hosts to infect. After infection, the lice feed and develop into the pre-adult and adult stage, where they become aggressive feeders and do the most harm to the fish (Norwegian Veterinary Institute, 2016).


Figure 2.1: Salmon lice Source: Marit Hommedal

The problem of salmon lice increases with both salmon production density and water temperature (Norwegian Veterinary Institute, 2016). Water temperature and currents affect the spreading of the lice and are important factors for the risk of contamination. If water temperatures are low, the lice develop slowly into the infectious stage, but in return they can spread over larger areas, with a larger probability of reaching a host. High temperatures cause the parasites to grow faster, and an increase in temperature from $10^{\circ} \mathrm{C}$ to $15^{\circ} \mathrm{C}$ can cause time from infection to grown lice to be reduced by $50 \%$ (Dalvin and Johnsen, 2015).

During the last years, the industry has experienced a large increase in costs associated with lice control. In 2015 these costs were estimated to be 3-4 billion NOK, and 5 billion NOK in 2016. The largest part of these costs come from treatment of fish to remove lice (Iversen et al., 2015).

### 2.2 Regulations from the Government

To ensure a sustainable production and avert infections spreading to wild salmon, the Norwegian Ministry of Trade, Industry and Fisheries has put a cap of 0.5 on the average number of grown female salmon lice permitted per fish in a farm. To control this, the farms have to conduct a weekly count of lice if water temperatures are higher than $4^{\circ} \mathrm{C}$. Otherwise, counting has to be done every other week. If a farm exceeds the maximum allowed number of lice per fish it is forced to slaughter the salmon early, losing valuable increase in slaughter weight.

In addition to lice regulations, all forms of aquaculture in Norway require a licence and are subject to numerous other restrictions. A licence is given for a specific geographic location in order to prevent the density of fish farms from becoming too high in any given area. This again helps to battle outbreak of deceases, limit lice infections and prevent large accumulation of biological waste from the farming process. The two most important laws regulating the business are the Aquaculture Act of 2005 and the Food Safety Act of 2003. A licence is perpetual and assigns a maximum allowed biomass (MAB), defined as the maximum volume of fish allowed in the water at any given time. A regular, commercial, saltwater licence is for 780 tonnes, but an exception is made for licences issued for Troms and Finnmark, where farmers are facing less problems related to salmon lice as a result of lower water temperatures. These licences therefore allow 980 tonnes. In addition, each production site has additional limitations on the maximum biomass, based on the local conditions (Marine Harvest Group, 2016).

Since 2009, there has been a political consensus that environmental aspects of any expansion are important, and that any expansion must be environmentally sustainable. As a result, no ordinary licences have been awarded for saltwater fish farming due to the lice problem. There were, however, 45 green permits awarded in 2013, where the recipient had to commit to using technological or operational solutions to mitigate the risk of escapes and salmon lice (The Norwegian Ministry of Trade, Industry and Fisheries, 2016a).

To help solve the lice problems of the industry, the Norwegian Directorate of Fisheries initialised a two-year trial development licence scheme in 2015. The scheme awards development licences for projects that imply significant innovation and investments. A licence allows farmers to produce fish for purpose of developing a new technology that will benefit the industry. These can be converted into regular commercial licences at the end of a fiveyear testing period. All harvest gained from the facility during the period can be used for commercial purposes.

To further allow for growth in the industry in an environmentally sustainable way, the Norwegian government issued in January 2017 a new set of regulatory guidelines for growth in the aquaculture industry, referred to as the traffic light system. The intention of the system is to regulate the size of production in a given region based on the industry's effect on the wild fish population in the region. The system will be launched in October 2017 (Ministry of Trade, Industry and Fisheries, 2017).

### 2.2.1 Traffic Light System

According to the regulation, the coastline will be divided into 13 production regions within which the lice levels in the fish farms, and the production's effect on the wild fish population will be assessed (Havforskningsinstituttet, 2016). Depending on the results, the region is given a green, yellow or red light, as described in Table 2.1.

| Low risk/effect | Moderate risk/effect | High risk/effect |
| :--- | :--- | :--- |
| It is likely that <10\% of |  |  |
| the wild salmon population |  |  |
| dies from lice infection |  |  | | It is likely that $10-30 \%$ of |
| :--- | :--- |
| the wild salmon population |
| dies from lice infection |$\quad$| It ikely that $>30 \%$ of |
| :--- |
| dies from lice infection |

Table 2.1: Description of traffic light system outcomes and the limit for impact on the wild salmon population (Havforskningsinstituttet, 2016).

The light assigned to a region is meant to represent the risk posed to the wild fish population from salmon lice. In a red light zone where the risk is high, farmers will be forced to reduce the production by six percent. Regions with a yellow light are allowed to keep production at the same level. Regions with a green light, where the risk is low, will be allowed to expand production by six percent at a cost. By reducing the production in high-risk regions, and allowing for expansion in low risk regions, the authorities hope to reduce the overall effect of fish farming on the wild salmon population. In addition, this measure could increase the overall production capacity to ensure to ensure a profitable and competitive industry in the future

The traffic light system assumes the following two conditions under which a farmer can be allowed to expand production. (i) The risk posed to the wild salmon population in the entire region is low. (ii) The lice level in the individual farm is low and does not contribute to the infection pressure in the region. The latter implies that in the period from spring to fall, the farm must have less than 0.1 grown female lice per fish. Also, no more than one chemical treatment can be conducted during the same period (The Norwegian Ministry of Trade, Industry and Fisheries, 2016b). This way, the authorities reward especially environmentally friendly farmers. Thus, the farmers are able to increase production even if they operate in regions with high natural lice levels, or where other actors do not make a sufficient effort. The
assessments of the impact on the wild salmon are made every other year by counting the number of lice on the wild smolt emigrating from rivers. This is done in two ways according to Jon Arne Grøttum, Director of Aquaculture at Sjømat Norge: (i) By trawling selected areas and counting the number of lice on the fish retrieved. (ii) By placing a number of cages close to the river outlets to trap emigrating smolt, and counting the number of salmon lice on the smolt caught in the cages. Based on the observed number of grown female lice per smolt in the region, a traffic light is given.

Despite the exceptions made for environmentally friendly farmers, Jon Arne Grøttum claims the industry is heavily opposed to the new set of regulations. Firstly, they argue that the number of lice per wild salmon is not sufficiently correlated with the efforts made to reduce the number of lice on farmed salmon. From the salmon farmers' point of view, the traffic light therefore becomes somewhat arbitrary, and they believe it is unfair to base the opportunity of expansion on the lice count on wild salmon. Secondly, they argue that the two methods for catching wild smolt for counting can produce artificially high lice numbers. This is because fish weakened by lice infection are more prone to being caught by trawling, than healthier fish without any lice. In addition, the fish caught in cages are more prone to being infected by lice after they are caught, as they are stationary in the water. Finally, industry actors also argue that because there are many firms operating in a region, all the firms have to act collectively for their efforts to have an effect. Therefore, the regulations may cause collective punishment on all the firms in a region because one firm did not make sufficient investments.

The government controls the lice problem by regulations, and to comply with these regulations and control the lice level in the individual farm, firms have traditionally used chemicals. However, in 2007, the first sign of medical resistance was discovered - a resistance that has developed rapidly and made the industry dependent on non-chemical treatment methods. In the following section, we present the lice-fighting methods and technologies of the industry.

### 2.3 Lice-Fighting Methods and Technologies

Medical resistance and the regulations of the industry have boosted innovation of lice-fighting technologies in the recent years, especially the development licences. In the period between November 2015 and October 2016, the government received 41 applications for development licences. These technologies are additions to the recent years' large increase in technology developments (Ministry of Trade, Industry and Fisheries, 2015).

According to our advisor at INAQ, Randi Grøntvedt, existing lice-fighting technologies can be categorised into three types: preventive, continuous and immediate. Preventive technologies prevent the lice from attaching to the fish, or entering the net. Examples of these are lice skirts and the Tubenot, shown in Figures 2.2 and 2.3.


Figure 2.2: Lice Skirt Source: SINTEF (2016)


Figure 2.3: The Tubenot Source: Egersund Group (2015)

Lice skirt is a product made of plankton fabric, used to prohibit lice in the infectious stage from entering the nets. As illustrated in Figure 2.2, the skirts are placed around the nets down to a recommended depth of 10 meters for optimal coverage. The Tubenot is a net with a roof that keeps the salmon 3-24 meters below the surface, away from the infectious lice in the upper layers of the sea. The snorkel in the top of the net is a passage which the fish can swim through to get to the surface and fill up its swim bladder, as shown in Figure 2.3.

Continuous methods such as cleaner fish, continuously keep the lice level down. The most used cleaner fish are wrasse and lumpfish, shown in Figures 2.4 and 2.5.


Figure 2.4: Wrasse Source: Tonje Sørdalen


Figure 2.5: Lumpfish Source: Henrik Mundal Adreassen

The wrasse is most effective in warmer water, and the lump fish in colder water. The cleaner fish usually make up $5 \%$ of the fish in the nets, depending on the available supply and the amount of lice in the location. Despite the fact that $60 \%$ of fish farming companies use
cleaner fish, there are several challenges related to them. A large scale experiment conducted by the Norwegian Veterinary Institute showed that the mortality of cleaner fish in the nets was as high as $33 \%$ in 2014 (Nilsen et al., 2014). Further, the annual catch of 20 million cleaner fish causes the wild fish populations to decline. Local stocks are also affected by cleaner fish escaping from the nets, spreading diseases and erasing attributes when mixing with the local fish in the long run (Skiftesvik et al., 2016).

Immediate technologies are acute treatment methods for removing lice when preventive or continuous methods have failed to work. Treatments are done when the amount of lice in a net is close to, or has exceeded, the legal limit. An example of an immediate technology is the Thermolicer, illustrated in Figure 2.6. (1) The fish is pumped from the water into a closed system. (2) The saltwater is drained, filtered and (3) let out of the system. (4) \& (5) In the treatment loop, the fish is rinsed with lukewarm water with a temperature of $30-34^{\circ} \mathrm{C}$.
(6) The warm water will cause the lice to lose its grip, either due to death, or loss of muscle control. (7) The lice are then filtered away to avoid them from reinfecting the fish. (8) At the same time, the salmon is led out of the system, (9) while the warm water is put back into the warm tank for cleansing and reheating. (10) The treatment water is finally pumped back into the treatment loop (Steinsvik, 2015).


Figure 2.6: Thermolicer
Source: Steinsvik (2015)

In addition to the three categories described above, we argue that there exists a separate category for permanent technologies that are currently unavailable, but have been applied for. Permanent technologies are disruptive and preventive technologies that may be a permanent solution to the lice problem. An example is Marine Harvest's Egg developed by Hauge Aqua, which was granted four development licences in 2016. The Egg, illustrated in Figure 2.7, is a large, solid, egg-shaped construction placed $90 \%$ under water. It is assumed to solve the problem of lice and escapes because of the solid construction. The construction opens
for control of the waste, and thereby reduces the ecological footprint. The Egg allows for a much higher concentration of fish farms, and every egg is supposed to hold 1000 tonnes of salmon.


Figure 2.7: The Egg Source: Hauge Aqua (2016)

The effect of a technology on the lice level varies for each firm and each use, due to different factors such as water temperature, currents, production density, but also unknown factors. Therefore, a firm uses a combination of several technologies to achieve the best effect. According to Randi Grøntvedt, the technology packages firms currently use consist of both preventive, continuous and immediate technologies. Regardless of the lice protection, the firms are still exposed to lice, however, at a lower level. The investment problem we aim to solve in this thesis is for a farm that wants to upgrade the current technology package, while anticipating the arrival of a permanent technology.

## Chapter 3

## Literature Review

In this chapter we introduce the literature relevant for our problem. First, we give a short introduction to the traditional net present value approach for evaluating investment decisions. We argue, however, that given that the investment under consideration is subject to a significant degree of uncertainty, it is more suitable to analyse the decision using a real options approach. We therefore give a general intuition to the real options approach in Section 3.2, before moving on to technology adoption specific matters. Finally, we briefly introduce some of the literature on the economics of aquaculture.

### 3.1 Net Present Value

The net present value (NPV) rule is widely used in investment decision making. It is calculated as follows. First, the present value of expected streams of profits and expenditures are calculated using a risk-adjusted discount rate. Second, the net present value is found by subtracting the expenditures from the profits. If NPV is negative, the project should be discarded, whereas a project with positive NPV should be undertaken. There exists several drawbacks related to this criterion. First, it is inherently hard to calculate the profit streams from a project because of the uncertainty associated with it. Second, the approximation of a risk-adjusted discount rate is also difficult, and minor changes in the rate can change the investment decision. Third, the NPV rule assumes either that the investment is reversible and can be undone by recovering the expenditures, or, the investment is irreversible. The method also assumes the investment to be a now or never decision. In other words, it does not take into account the possible value of postponing the investment decision to gather information about the uncertain future. This is particularly important in the investment problems where a firm has to deal with uncertainty. For this, we can use the Real Options method.

### 3.2 Real Options

When owning a financial option, the owner has the right, but not the obligation to trade the underlying asset for a predetermined price. A call option gives the owner the right to buy the asset, whereas a put option gives the owner the right to sell the asset. The predetermined price for the underlying asset is agreed upon when the option is purchased, and is called the strike price. A real option is the real world counterpart to a financial option, where an investment opportunity is valued as the right, but not the obligation to undertake an investment (Myers, 1977). There are three important characteristics of an investment opportunity that can be valued as a real option: (i) The decision to invest must be possible to delay, (ii) there must be some degree of uncertainty about the profits of the project, and (iii) the investment must be irreversible, meaning that the decision maker can not have investment costs reimbursed by reversing the investment. As uncertain elements of the projects evolve, there exists a value of waiting for the firm to gain information. Compared to the net present value method, the real options approach takes the value of delaying the investment into account. According to Dixit and Pindyck (1994), valuing a project using NPV instead of real options could lead the decision maker to make significant errors, especially if the NPV of the project is close to zero. In this thesis we will use both the NPV rule for the investment in short-term solutions, and real options method for the optimal adoption of the permanent technology.

### 3.2.1 Solution Methods

Dixit and Pindyck (1994) present two ways of solving models for valuing real options: dynamic programming and contingent claims analysis. Dynamic programming was first introduced by Bellman (1956), and is particularly useful when dealing with uncertainty. The method uses an exogenous discount rate interpreted as the opportunity cost of capital. It is a very general tool of dynamic optimisation, and the idea is to break the entire sequence of decisions down to two components: The immediate decision and a valuation function that encapsulates all the consequences of future decisions. This makes it highly compatible with numerical computation algorithms. Contingent claims analysis, on the other hand, is based on ideas from financial economics. The idea is to find a portfolio of traded assets exactly matching the payoffs from the asset underlying the option. By a simple no-arbitrage argument, the portfolio must have a value equal to the asset underlying the option. The method makes no assumptions about the interest rate or the market, and therefore uses the risk-free rate. For this thesis, we will use dynamic programming to solve the model, precisely because of its properties in dealing with uncertainty and compatibility with numerical solving. This is also in line with Grenadier and Weiss (1995), Huisman (2000), and Farzin et al. (1998) who use dynamic programming to solve their technology adoption models.

### 3.2.2 Optimal Stopping Problem

Dixit and Pindyck (1994) present the optimal stopping problem as a particular class of a dynamic programming problem. It is used to model situations where the choice in each period is binary; either the process is stopped, resulting in a termination payoff, or the problem is continued to the next period. Let $\pi(x)$ denote the profit flow, and $\Omega(x)$ denote the termination payoff. The Bellman equation describing the value of a firm, $F(x)$, considering this investment is

$$
F(x)=\max \left\{\Omega(x), \pi(x)+\frac{1}{1+\rho} \varepsilon\left[F\left(x^{\prime}\right) \mid x\right]\right\},
$$

where $x$ is the state variable, $x^{\prime}$ is the next state variable and $\rho$ is the discount rate. For some range of values of $x$, the maximum will be found by terminating the process, and for another range the maximum is found by continuing the process. In general, these ranges can be arbitrarily spread across all values of $x$, however, for most economic applications they are divided by a threshold $x^{*}$. When $x<x^{*}$, it is optimal to wait before investing, whereas for $x \geq x^{*}$, it is optimal to undertake the investment.

### 3.3 Real Options on Technology Adoption

The real options literature on technology adoption takes into account uncertainty in the technological development, in an irreversible investment decision. In what follows, we present an overview of the contributions most relevant for our problem.

Rajagopalan (1999) models a one-time investment decision with only three technologies. The first is the one the firm uses at the beginning of the period, the second is the best technology available at the beginning of the period, and the third is a technology that becomes available at some unknown time in the future, and is better than the second. Rather than calculating the optimal investment strategy for a single set of parameters, Rajagopalan (1999) concludes on the range of parameters that leads to: (i) adoption of the second technology at once, (ii) waiting some finite time with adopting, and (iii) never adopting the second technology.

Grenadier and Weiss (1995) present four different investment strategies for a firm that holds the option to invest in two technologies. One technology arrives at the beginning of the model, referred to as the current technology, whereas the other is expected to arrive in the future and is referred to as the future technology. The state of the technological process is modelled as a geometric Brownian motion (GBM) starting at zero. If the process reaches an upper boundary, the future technology arrives. Consequently, the time until the arrival of
the future technology is determined by the time it takes the GBM to reach the upper boundary. The value of the future innovation is given by a normally distributed stochastic variable. Grenadier and Weiss (1995) further present four possible investment strategies for the firm: (i) The Compulsive strategy, where the firm invests in both the current, and the future technology. By investing in the current technology, it can upgrade to the future technology at a discount. (ii) The Buy-and-hold strategy, where the firm buys the current technology, but does not upgrade to the future technology. (iii) The Leapfrog strategy, where the firm does not buy the current technology, but leapfrogs directly to the future technology. (iv) The Laggard strategy, where the firm waits until the future technology has arrived, and then buys the current technology at a discount. As the authors point out, there is always the possibility of not doing any upgrades, but the paper is built on the assumption that an upgrade is profitable. Finally, they derive the optimal investment strategy and find the probability of a firm choosing each of the four investment strategies. The probabilities are analysed to find how choosing the different strategies depends on the expected arrival time, and the expected profitability of the future technology.

The technology process in our model cannot decline, which means that modelling the process as a GBM is unfitting for our model. Additionally, unlike Grenadier and Weiss (1995) and Rajagopalan (1999), we do not consider a specific number of technologies. Therefore, we look to Farzin et al. (1998) who present a model with an infinite number of technologies. The technology process is modelled using a Poisson process with stochastic jump size, where each jump translates to the arrival of a new technology. Using a dynamic programming approach, the authors first find the optimal investment threshold for a single switch case where the firm can undertake the investment once. Then they generalise this to include multiple switches and compare the results to an NPV model. Finally, they analyse the model's sensitivity to changes in key parameters, such as the arrival rate of technology improvements and the discount rate.

Huisman (2000) generalises the model from Farzin et al. (1998) by deriving the single switch model using three differently distributed jump sizes. The distributions he uses are degenerate, uniform and exponential distributions ${ }^{1}$. The difference between a process of constant jump size and a uniformly distributed jump size, is illustrated in Figure 3.1.

[^0]

Figure 3.1: Sample paths of a technology process with constant jump size on the left, and uniformly distributed jump size on the right (Huisman, 2000).

By comparing the results for each model, he concludes that the probability distribution of the size of the jump does not influence the outcome of the model significantly. He therefore solves the model for multiple switches using a constant jump size.

We therefore base our model on the work from Huisman (2000), as his model incorporates both an infinite number of technologies, models technological improvements using a Poisson process, and allows for multiple technology switches, which suits our problem. However, we will extend the model by adding an NPV problem with stochastic ending time, to model the investment decision before a permanent solution is launched to the market. We will also alter the model to adapt it to the investment decision of an aquaculture firm.

### 3.4 Aquaculture Economics

The majority of literature on aquaculture economics is related to production planning, such as Asche and Bjørndal (2011) who present an analysis of the salmon aquaculture industry from a market and production perspective. They consider investment in fish farms using capital budget methods. Furthermore, Asche and Guttormsen (2001), combine biology and economics by studying relative price patterns between the different sizes of salmon, and research the implications for optimal harvesting decisions and aggregation. Forsberg and Guttormsen (2006) on the other hand, combine production planning and forecasting of prices. They study how information on future prices alter the production plans of a fish farmer, and how this results in extra value for the fish farmer. However, the focus of our study is on technology, and we therefore find it more relevant to look at literature related to productivity growth due to technical change.

Sandvold (2016) presents an econometric analysis of the efficiency of Norwegian smolt production from 1988-2012, in a stochastic cost frontier framework. The study aims to investigate the effect of the firm's age on the technological efficiency. The results show that there exists a significant technical inefficiency in the smolt production, and that there seems to
be a learning-by-doing effect, as older firms perform slightly better than younger. In another study, Sandvold and Tveterås (2014) identify how the change in technology from 1988 to 2010 has reduced the production costs in the industry. Specifically, they study the productivity growth in the grow-out phase for salmon by analysing the input factor - juvenile salmon. By an econometric model, they find that technological innovation has led to a reduction of unit costs in production of salmon juveniles. Both papers look at how technological improvements have affected the production of salmon in a historical perspective. However, in our thesis we study the impact of technological innovation on investment decisions, and seek to model the future investment strategy of an aquaculture firm.

The closest literature to our contribution is Hannevik et al. (2015) who use real options approach to model the investment problem of an aquaculture firm. They develop a multifactor real options model to find the optimal adoption timing of a post-smolt production technology. The model incorporates both profit and technology uncertainty in the investment decision. Further, they model technological innovations as improvement on the production efficiency, or reduction in investment cost. They conclude that real options models uncover significant excess value compared to traditional net present value analysis. Unlike Hannevik et al. (2015) who study the investment in a post-smolt production technology specifically, we model a general setting that can be applied to a wide range of technologies. In addition, we contribute to the literature by incorporating regulatory influences on the investment decision.

## Chapter 4

## The Model

In this chapter, we present a model that analyses the investment decisions of an aquaculture firm ${ }^{1}$. The objective of the firm is to maximise its value with respect to the optimal investment amount in the upgrade of short-term solutions, and optimal investment timing in a permanent technology. Due to large variations in the size of aquaculture firms, we consider a general case where we optimise the value of the company per licence of 780 tonnes MAB.

In our model, the firm first needs to decide how much to invest in an upgrade of shortterm solutions, given that it already has several technologies in place. The upgrade can be done by either (i) adding a technology to the existing combination, (ii) by improving one of the technologies, or (iii) by replacing one of the technologies with a more efficient method. The pool of technologies the firm can choose from is assumed constant and not improving over time. When consulting Randi Grøntvedt, we find this assumption reasonable as similar technologies arriving are not offering additional efficiency. Consequently, in our model an investment in short-term solutions is undertaken at time zero, as there is nothing to gain from delaying the investment in the short-term. Delaying would only shorten the time the company can benefit from the reduced operational lice costs.

We further assume that the investment in technologies reduces the level of operational licefighting costs $c_{u}$. This is due to the increased protection against lice that results in a lower lice level, and thereby lower lice costs. This is also confirmed by industry expert Audun Iversen, which is why we choose to model the costs $c_{u}$ as a function of the investment amount, $I_{u}$. The more a firm invests in an upgrade, the lower the resulting variable costs of lice are. However, the effect of increased investment amount is diminishing, and the costs will eventually approach a lower boundary $\bar{c}_{u}$ as upgrades become redundant. Based on this, the properties required for the lice cost function $c_{u}\left(I_{u}\right)$ are: (i) $c_{u}(0)=c_{0}$; at zero investment, the cost

[^1]function equals the initial lice costs $c_{0}$, (ii) $\frac{\partial^{2} c_{u}\left(I_{u}\right)}{\partial I_{u}^{2}}>0$; the cost function is convex, (iii) the function decreases towards a lower boundary $\bar{c}_{u}$. The properties are illustrated in Figure 4.1.


Figure 4.1: Lice-fighting costs $c_{u}$, as a function of investment amount $I_{u}$. $c_{0}$ denotes the initial lice costs, and $c_{u}$ approaches the lower boundary $\overline{c_{u}}$ as $I_{u}$ becomes large.

We adopt the same functional form for the lice cost function as Majd and Pindyck (1987) and Mathews and Baroni (2013), who model the cost function as exponentially decreasing with respectively increasing production capacity and investment. The operational lice-fighting costs are therefore given by

$$
\begin{equation*}
c_{u}\left(I_{u}\right)=\bar{c}_{u}+\left(c_{0}-\bar{c}_{u}\right) e^{-\alpha I_{u}} \tag{4.1}
\end{equation*}
$$

where the parameter $\alpha$ is a cost reduction factor representing the rate at which the lice costs decrease in investment amount. Specifically, a higher $\alpha$ implies a larger decrease in $c_{u}\left(I_{u}\right)$. For simplification, we choose to model the lice-fighting costs as constant over time. In reality, the costs may change over time due to seasonal factors. However, the focus of this study is to provide general insights into the optimal investment strategy, which is why the seasonality component is considered out of scope of this thesis.

The second decision of the firm in our model is to determine the investment timing in a permanent solution. A permanent solution is defined as a technology reducing lice costs down to a fraction, $\beta$, of the initial costs, $c_{0}$. The arrival of the technology at $\tau_{\lambda_{l}}$ is stochastic, and follows exponential distribution with arrival rate $\lambda_{l}$. After the permanent solution has arrived, we assume the occurrence of technological improvements follows a Poisson process. According to Huisman (2000), the Poisson process is a natural choice when the firm has no insight in the development process. As we model a small size company, and only the largest fish farming companies do this type of $R \& D$ themselves, we find this fitting.

Let $\theta(t)$ denote the state of the technological innovation process following a Poisson process, with rate parameter $\lambda_{p}$, i.e. the technology improvement arrival rate, and jump size $u$. For purpose of simplification, $u$ is assumed to be constant, which is a reasonable approximation to the steady progress made in innovation processes. We have that $\theta_{t+d t}=\theta_{t}+d \theta$, where

$$
d \theta= \begin{cases}u & \text { with probability } \lambda_{p} d t \\ 0 & \text { with probability }\left(1-\lambda_{p}\right) d t\end{cases}
$$

The lice-fighting costs of a firm producing with a permanent technology $\theta$, are denoted $c_{p}(\theta)$. It is reasonable to assume that $c_{p}(\theta)$ has to have the same functional form as $c_{u}\left(I_{u}\right)$ from the short-term investment. Because the technology offers a reduction in lice costs, improvements of the technology will reduce lice costs further. The convexity of the curve is based on the assumption that when a new technology is launched commercially, the initial improvements will be the most effective in reducing lice-fighting costs. As the technology matures, improvements will be less effective and the reduction in lice costs will thus be decreasing. We therefore let the permanent technology lice-fighting cost be given as

$$
\begin{equation*}
c_{p}(\theta)=\beta c_{0} e^{-\theta} . \tag{4.2}
\end{equation*}
$$

This is illustrated in Figure 4.2.


Figure 4.2: Permanent technology lice-fighting costs, $c_{p}$, as a function of technology level $\theta$. $c_{0}$ denotes the initial lice-fighting costs, $\overline{c_{u}}$ is the lowest boundary of lice cost achieved by short-term investments, and $\beta c_{0}$ are the initial lice costs of the permanent technology.

Note that at arrival $(\theta=0)$, the lice-fighting costs are a fraction $\beta$ of the original lice costs, $c_{0}$. As the technology develops and $\theta$ increases, $c_{p}(\theta)$ approaches zero ${ }^{2}$.

[^2]In order to adopt the permanent technology, a firm has to pay a sunk investment cost $I_{p}$. This cost is assumed to be relatively large, as the permanent technologies being developed today are much more advanced compared to the current technologies (see Section 2.3). They are costly to develop, and require a complete change in the production process of the firm. Hence, we assume that the firm can only adopt the permanent technology once. This assumption is relaxed in Section 4.2, where we model a firm that may upgrade its permanent technology. We therefore allow for multiple technology switches at an additional, but smaller switching cost $I_{p}^{\prime}$. The firm thus faces the following trade-off. On the one hand, it has an incentive to delay investment due to the decreasing lice costs in technology level $c_{p}(\theta)$. On the other hand, the investment is hastened as the company faces larger lice costs from waiting, as $c_{u}\left(I_{u}\right)>c_{p}(\theta)$.

To find the optimal investment strategy of a firm, we solve the problem backwards by first finding the optimal adoption timing in the permanent solution for each level of $I_{u}$. Then we consider the scenario before arrival, and solve for the optimal investment amount $I_{u}^{*}$ in currently available technology to mitigate lice-fighting costs.

Lastly, in Section 4.3, we extend the single switch model to incorporate the influence of the traffic light system described in Section 2.2.1. The system regulates the production in a region every other year, based on the lice level. The firm can affect the outcome of the regulation system by making additional investments, $I_{T}$, in delousing treatments to reduce the lice level in critical periods. In addition to the optimal investment strategy, we will therefore find the optimal additional investments $I_{T}^{*}$.

### 4.1 Single Switch

Consider a firm that can only switch to a permanent technology once. Let $V$ denote the value of a salmon farm per licence. Each licence provides a profit $\pi\left(c_{0}\right)=\pi_{0}-c_{0}$, where $\pi_{0}$ are the profits net of lice-fighting costs, and $c_{0}$ are the current lice-fighting costs. In reality, the profit flow depends on the price of salmon that changes over time. Since the purpose of this model is to investigate how technological uncertainty affects the optimal investment decisions of the firm, rather than the modelling of salmon prices, we assume $\pi_{0}$ to be constant. Recall that the firm can reduce the initial lice-fighting costs $c_{0}$ down to $c_{u}\left(I_{u}\right)$ by making an upgrading investment, $I_{u}$. Let $\tau_{\theta}$ denote the adoption time after the arrival of the permanent technology at $\tau_{\lambda_{l}}$. After investing in a permanent technology at time $\tau_{\lambda_{l}}+\tau_{\theta}$, the lice costs are reduced further down to $c_{p}(\theta)$, at an investment cost $I_{p}$.

The maximisation problem of the fish farming firm is therefore given by

$$
\begin{align*}
V^{*}=\max _{I_{u}, \tau_{\theta}} \mathbb{E}_{\tau_{\lambda_{l}}, \theta} & {\left[\int_{0}^{\tau_{\lambda_{l}}}\left(\pi_{0}-c_{u}\left(I_{u}\right)\right) e^{-r s} d s-I_{u}\right.} \\
& \left.+\int_{\tau_{\lambda_{l}}}^{\tau_{\lambda_{l}}+\tau_{\theta}}\left(\pi_{0}-c_{u}\left(I_{u}\right)\right) e^{-r s} d s+\int_{\tau_{\lambda_{l}}+\tau_{\theta}}^{\infty}\left(\pi_{0}-c_{p}(\theta)\right) e^{-r s} d s-I_{p} e^{-r \tau_{\theta}}\right], \tag{4.3}
\end{align*}
$$

where r denotes the constant discount rate. The rate is exogenous and equals the opportunity cost of capital. The first term is the value from the period before the arrival of the permanent technology, whereas the last two terms are the values before and after adoption in the optimal stopping problem, respectively.

In what follows, we solve the problem in (4.3) backwards by first finding the optimal adoption time $\tau_{\theta^{*}}$, and then solving for the optimal investment amount in short-term technologies $I_{u}^{*}$.

### 4.1.1 Optimal Stopping Problem

The optimal stopping problem of a firm after the permanent technology has arrived, is given by

$$
\begin{equation*}
\max _{\tau_{\theta}} \mathbb{E}_{\theta}\left[\int_{0}^{\tau_{\theta}}\left(\pi_{0}-c_{u}\left(I_{u}\right)\right) e^{-r s} d s+\int_{\tau_{\theta}}^{\infty}\left(\pi_{0}-c_{p}(\theta)\right) e^{-r s} d s-I_{p} e^{-r \tau_{\theta}}\right] . \tag{4.4}
\end{equation*}
$$

The solution space of this problem consists of two regions. In the stopping region the firm invests and adopts a new technology, whereas in the continuation region it waits with investment. According to Proposition 2 in Chapter 2 of Huisman (2000), there exists a unique threshold $\theta^{*}$ that separates the stopping and the continuation region if the profit function is concave in $\theta$. In our model it holds that

$$
\frac{\partial^{2} \pi\left(c_{p}(\theta)\right)}{\partial \theta^{2}}=\frac{\partial^{2}\left(\pi_{0}-\beta c_{0} e^{-\theta}\right)}{\partial \theta^{2}}=-\beta c_{0} e^{-\theta}<0,
$$

which means that the profit function is concave.
Consequently, a unique threshold $\theta^{*}$ exists, and the firm faces an optimal stopping problem. Therefore, it is optimal for the firm to postpone the investment in the permanent technology for $\theta<\theta^{*}$. The firm is then producing with costs $c_{u}\left(I_{u}^{*}\right)$ from the short-term investment, and its value equals the value of the option to adopt $\theta$. For $\theta \geq \theta^{*}$, it is optimal for the firm to undertake the investment and adopt a technology. In this case, the value of the firm is the perpetual profits of the firm producing with lice costs $c_{p}\left(\theta^{*}\right)$, net of investment cost, $I_{p}$.

We implement the following procedure to solve the firm's optimal stopping problem. First, we determine the value of the firm in the stopping region. Second, we derive the value of
the firm in the continuation region. Finally, we use value matching to find the optimal investment threshold $\theta^{*}$. Because the jump size, $u$, is assumed to be constant, we can split the continuation region into two regions, similar to Huisman (2000). This is illustrated in Figure 4.3. $F_{0}\left(c_{p}(\theta), c_{u}\right)$ denotes the value of the firm in the first region, $\left\{\theta \mid \theta<\theta^{*}-u\right\}$, where investment is not optimal even after the next technology jump. $F_{1}\left(c_{p}(\theta), c_{u}\right)$ denotes the value of the firm in the second region, $\left\{\theta \mid \theta \geq \theta^{*}-u\right\}$, when investment is optimal after the next jump. Note that the value of the firm in the continuation region is a function of both $\theta$ and $c_{u}$, whereas the value in the stopping region, $F_{2}\left(c_{p}(\theta)\right)$, only depends on $\theta$. This is because the value of $c_{u}$ affects the threshold $\theta^{*}$, and determines the profit flow in the continuation region.


Figure 4.3: Firm values in different regions of the optimal stopping problem. $F_{0}\left(c_{p}(\theta), c_{u}\right)$ is the value in $0 \leq \theta \leq \theta^{*}-u, F_{1}\left(c_{p}(\theta), c_{u}\right)$ is the value in $\theta^{*}-u \leq \theta \leq \theta^{*}$, and $F_{2}\left(c_{p}(\theta)\right)$ is the value in $\theta \geq \theta^{*}$.

In the stopping region, the firm receives the profit flow $\pi_{0}-c_{p}(\theta)$ from producing with a permanent technology $\theta$. The value of the profit flow equals to

$$
V\left(c_{p}(\theta)\right)=\int_{0}^{\infty}\left(\pi_{0}-c_{p}(\theta)\right) e^{-r t} d t
$$

which is the present value of the perpetual profits. The value of the firm in the stopping region, $F_{2}\left(c_{p}(\theta)\right.$ ), is then given by

$$
\begin{equation*}
F_{2}\left(c_{p}(\theta)\right)=V\left(c_{p}(\theta)\right)-I_{p}, \tag{4.5}
\end{equation*}
$$

where $I_{p}$ is the investment cost of adopting the technology $\theta$.
In both parts of continuation region, the value of the firm, $F\left(c_{p}(\theta), c_{u}\right)$, must satisfy the Bellman equation given by

$$
\begin{equation*}
r F\left(c_{p}(\theta), c_{u}\right)=\pi\left(c_{u}\right)+\lim _{d t \rightarrow 0} \frac{1}{d t} E\left[d F\left(c_{p}(\theta), c_{u}\right)\right] \tag{4.6}
\end{equation*}
$$

The expected return on firm value equals the instantaneous profit flow plus the expected change in firm value over the period $d t$.

For the first part of the continuation region, we find the value by applying Ito's lemma to $\lim _{d t \rightarrow 0} \frac{1}{d t} E\left[d F\left(c_{p}(\theta), c_{u}\right)\right]$ and inserting into (4.6). This gives the differential equation

$$
\begin{equation*}
r F_{0}\left(c_{p}(\theta), c_{u}\right)=\pi\left(c_{u}\right)+\lambda_{p}\left[F\left(c_{p}(\theta+u), c_{u}\right)-F_{0}\left(c_{p}(\theta), c_{u}\right)\right] . \tag{4.7}
\end{equation*}
$$

Here, $\lambda_{p}$ is the probability of a jump during an infinitesimal period of time, and $F\left(c_{p}(\theta+\right.$ $\left.u), c_{u}\right)-F\left(c_{p}(\theta), c_{u}\right)$ is the change in the value of the firm if a jump occurs. In line with Huisman (2000), we find the solution to (4.7) to be given by

$$
\begin{equation*}
F_{0}\left(c_{p}(\theta), c_{u}\right)=k\left(\frac{\lambda_{p}}{r+\lambda_{p}}\right)^{-\frac{\theta}{u}}+\frac{\pi\left(c_{u}\right)\left(r+\lambda_{p}\right)}{r} \tag{4.8}
\end{equation*}
$$

where $k$ is a constant found by value matching at the boundary between the two parts of the continuation region, $\theta=\theta^{*}-u$. This solution can be verified by substitution.

We now find an expression for the value of the firm in the second part of the continuation region, where investment is optimal after the next jump. We again apply Ito's lemma to $\lim _{d t \rightarrow 0} \frac{1}{d t} E\left[d F\left(c_{p}(\theta), c_{u}\right)\right]$. Combining the result with Equation 4.6 gives

$$
\begin{equation*}
r F_{1}\left(c_{p}(\theta), c_{u}\right)=\pi\left(c_{u}\right)+\lambda_{p}\left[V\left(c_{p}(\theta+u)\right)-I_{p}-F_{1}\left(c_{p}(\theta), c_{u}\right)\right] \tag{4.9}
\end{equation*}
$$

Note that the equation differs from Equation 4.7 as the value of the firm after the next jump is now given by $V\left(c_{p}(\theta+u)\right)-I_{p}$. Rearranging gives the following value of the firm in the second part of the continuation region:

$$
\begin{equation*}
F_{1}\left(c_{p}(\theta), c_{u}\right)=\frac{\pi\left(c_{u}\right)}{r+\lambda_{p}}+\frac{\lambda_{p}}{r+\lambda_{p}}\left[V\left(c_{p}(\theta+u)\right)-I_{p}\right] . \tag{4.10}
\end{equation*}
$$

For $F\left(c_{p}(\theta), c_{u}\right)$ to be continuous at $\theta=\theta^{*}-u$, we must have $F_{0}\left(c_{p}\left(\theta^{*}-u\right), c_{u}\right)=F_{1}\left(c_{p}\left(\theta^{*}-\right.\right.$ $u), c_{u}$ ). By setting Equation 4.8 equal to Equation 4.10 and solving for $k$, we find that

$$
\begin{equation*}
k=\left(\frac{\lambda_{p}}{r+\lambda_{p}}\right)^{\frac{\theta^{*}}{u}}\left[\frac{\pi\left(c_{u}\right)}{\lambda_{p}}\left(1-\frac{\left(r+\lambda_{p}\right)^{2}}{r}\right)+V\left(c_{p}\left(\theta^{*}\right)\right)-I_{p}\right] . \tag{4.11}
\end{equation*}
$$

Summarising, the value of the firm in the stopping region and the two parts of the continuation region is given by

$$
F\left(c_{p}(\theta), c_{u}\right)= \begin{cases}k\left(\frac{\lambda_{p}}{r+\lambda_{p}}\right)^{-\frac{\theta}{u}}+\frac{\pi\left(c_{u}\right)\left(r+\lambda_{p}\right)}{r}, & \text { if } \theta<\theta^{*}-u  \tag{4.12}\\ \frac{\pi\left(c_{u}\right)}{r+\lambda_{p}}+\frac{\lambda_{p}}{r+\lambda_{p}}\left(V\left(c_{p}(\theta+u)\right)-I_{p}\right), & \text { if } \theta^{*}-u \leq \theta<\theta^{*}, \\ V\left(c_{p}(\theta)\right)-I_{p}, & \text { if } \theta \geq \theta^{*}\end{cases}
$$

The optimal investment threshold, $\theta^{*}$, can be found by solving the value-matching condition at $\theta=\theta^{*}$. From (4.12), value-matching gives

$$
\begin{equation*}
\frac{\pi\left(c_{u}\right)}{r+\lambda_{p}}+\frac{\lambda_{p}}{r+\lambda_{p}}\left(\frac{\pi\left(c_{p}\left(\theta^{*}+u\right)\right)}{r}-I_{p}\right)=\frac{\pi\left(c_{p}\left(\theta^{*}\right)\right)}{r}-I_{p} \tag{4.13}
\end{equation*}
$$

Plugging in the expressions for $\pi\left(c_{u}\right), \pi\left(c_{p}\left(\theta^{*}\right)\right)$ and $\pi\left(c_{p}\left(\theta^{*}+u\right)\right.$, and solving for the optimal investment threshold $\theta^{*}$ gives

$$
\begin{equation*}
\theta^{*}=\max \left[\ln \left(\frac{\beta c_{o} \lambda_{p}\left(e^{-u}-\frac{r}{\lambda_{p}}-1\right)}{r\left(r I_{p}-c_{u}\left(I_{u}\right)\right)}\right), 0\right] . \tag{4.14}
\end{equation*}
$$

for $r I_{p}<c_{u}\left(I_{u}\right)$. Note that if $r I_{p} \geq c_{u}\left(I_{u}\right)$, it is never optimal to invest, and $\theta^{*}$ will approach infinity. If the operational lice costs $c_{u}\left(I_{u}\right)$ from a short-term investment are sufficiently low, it would never be optimal to invest in a permanent solution. The firm would rather continue producing with costs $c_{u}\left(I_{u}\right)$ from the short-term solution forever. In what follows we assume this is not the case.

Substituting $\theta^{*}$ into $c_{p}(\theta)$ in Equation 4.2, gives

$$
\begin{equation*}
c_{p}\left(\theta^{*}\right)=\frac{r\left(r I_{p}-c_{u}\left(I_{u}\right)\right)}{\lambda_{p}\left(e^{-u}-1\right)-r} \tag{4.15}
\end{equation*}
$$

which are the optimal lice-fighting costs when operating with the permanent technology.
The number of jumps needed to reach the investment threshold equals $\theta^{*} / u$, and the average speed at which these arrive is $\lambda_{p}$. Thus the expected optimal adoption time, is equal to

$$
\begin{equation*}
\bar{\tau}_{\theta^{*}}=\mathbb{E}\left[\tau_{\theta^{*}}\right]=\frac{1}{\lambda_{p}}\left\lceil\frac{\theta^{*}}{u}\right\rceil \text {, } \tag{4.16}
\end{equation*}
$$

where $\left\lceil\frac{\theta^{*}}{u}\right\rceil$ gives the smallest integer larger, or equal to $\frac{\theta^{*}}{u}$. Given the optimal investment threshold derived in (4.14), we now solve for optimal investment amount in short-term solutions $I_{u}^{*}$.

### 4.1.2 Optimal Investment Amount

Now we solve for the optimal investment amount $I_{u}^{*}$. Recall the maximisation problem (4.3) given by

$$
\begin{aligned}
V^{*}=\max _{I_{u}, \tau_{\theta}} \mathbb{E}_{\tau_{\lambda_{l}}, \theta} & {\left[\int_{0}^{\tau_{\lambda_{l}}}\left(\pi_{0}-c_{u}\left(I_{u}\right)\right) e^{-r s} d s-I_{u}\right.} \\
& \left.+\int_{\tau_{\lambda_{l}}}^{\tau_{\lambda_{l}}+\tau_{\theta}}\left(\pi_{0}-c_{u}\left(I_{u}\right)\right) e^{-r s} d s+\int_{\tau_{\lambda_{l}+\tau_{\theta}}}^{\infty}\left(\pi_{0}-c_{p}(\theta)\right) e^{-r s} d s-I_{p} e^{-r \tau_{\theta}}\right]
\end{aligned}
$$

In what follows we denote $\mathbb{E}_{\tau_{\lambda_{l}}, \theta}$ as $\mathbb{E}$. To simplify we rewrite the firm value as

$$
\begin{equation*}
V^{*}=\max _{I_{u}, \tau_{\theta}} \mathbb{E}\left[\int_{0}^{\tau_{\lambda_{l}}+\tau_{\theta}}\left(\pi_{0}-c_{u}\left(I_{u}\right)\right) e^{-r s} d s-I_{u}+\int_{\tau_{\lambda_{l}}+\tau_{\theta}}^{\infty}\left(\pi_{0}-c_{p}(\theta)\right) e^{-r s} d s-I_{p} e^{-r\left(\tau_{\lambda_{l}}+\tau_{\theta}\right)}\right] \tag{4.17}
\end{equation*}
$$

To solve the problem we use Equation 4.14 and the fact that $\mathbb{E}\left(e^{\left.-r \tau_{\theta^{*}}\right)}=\left\lceil\frac{\theta^{*}\left(I_{u}^{*}\right)-\theta_{0}}{u}\right\rceil\right.$ from Appendix 2.D in Huisman (2000), and find that the optimal value is given by

$$
\begin{equation*}
V^{*}=\frac{\pi_{0}-c_{u}\left(I_{u}^{*}\right)}{r}-I_{u}^{*}+\frac{\lambda_{l}}{r+\lambda_{l}}\left(\frac{\lambda_{p}}{r+\lambda_{p}}\right)^{\left\lceil\frac{\theta^{*}\left(I_{u}^{*}\right)-\theta_{0}}{u}\right\rceil}\left(\frac{c_{u}\left(I_{u}^{*}\right)-c_{p}\left(\theta^{*}\right)}{r}-I_{p}\right) \tag{4.18}
\end{equation*}
$$

We find the optimal investment amount by differentiating the value of the firm with respect to the investment amount, and set it equal to zero. This is done numerically and the results are presented in Section 6.1.1.

### 4.2 Multiple Switch

To make the model more realistic, we now assume the firm has the possibility to invest in upgrades after the first investment where the technology is adopted. We therefore relax the assumption of a one-time investment in the permanent solution, and allow for several technology switches. The investment problem now consists of $n$ optimal stopping problems. To solve this problem, we have adapted Huisman (2000)'s multiple switch approach to fit the problem of the salmon farming industry. Working with Huisman (2000)'s approach, we discovered some errors in the indices of the expressions for firm value. These are corrected in our thesis.

We now let $\zeta_{i}$ denote the optimal technology level adopted by the company after the $i$-th switch for $i \in\{1, . ., n\}$. Note that because it is not possible to adopt a partial technology level, $\zeta_{i}$ is a discrete variable with increments of jump size $u$. Furthermore, $c_{p}\left(\zeta_{i-1}\right)$ now denotes the lice-fighting costs of the company from the previous switch. However, before the first
switch when $i=0$, the lice-fighting costs are $c_{u}$ from the investment $I_{u}^{*}$ in short-term solutions. The value of the firm before the first switch is therefore given by

$$
F_{1}\left(c_{p}(\theta), c_{u}\right)= \begin{cases}\left(\frac{\lambda_{p}}{r+\lambda_{p}}\right)^{\frac{\theta_{1}^{*}-\theta}{u}}\left[\frac{\pi\left(c_{u}\right)}{\lambda_{p}}\left(1-\frac{\left(r+\lambda_{p}\right)^{2}}{r}\right)+F_{2}\left(c_{p}\left(\theta_{1}^{*}\right), c_{p}\left(\theta_{1}^{*}\right)\right)-I_{p}\right]  \tag{4.19}\\ +\frac{\pi\left(c_{u}\right)\left(r+\lambda_{p}\right)}{r} & \text { if } \theta<\theta_{1}^{*}-u, \\ \frac{\pi\left(c_{u}\right)}{r+\lambda_{p}}+\frac{\lambda_{p}}{r+\lambda_{p}}\left(F_{2}\left(c_{p}(\theta+u), c_{p}(\theta+u)\right)-I_{p}\right) & \text { if } \theta_{1}^{*}-u \leq \theta<\theta_{1}^{*}, \\ F_{2}\left(c_{p}(\theta), c_{p}(\theta)\right)-I_{p} & \text { if } \theta \geq \theta_{1}^{*},\end{cases}
$$

where $I_{p}$ is the investment cost paid at the first switch when the firm adopts the permanent technology.

The last stopping problem is equal to the problem solved in Section 4.1.1. Therefore, the value of the firm before the last technology switch is given by

$$
F_{n}\left(c_{p}(\theta), c_{p}\left(\zeta_{n-1}\right)\right)= \begin{cases}\left(\frac{\lambda_{p}}{r+\lambda_{p}}\right)^{\frac{\theta_{n}^{*}-\theta}{u}}\left[\frac{\pi\left(c_{p}\left(\zeta_{n-1}\right)\right)}{\lambda_{p}}\left(1-\frac{\left(r+\lambda_{p}\right)^{2}}{r}\right)\right.  \tag{4.20}\\ \left.+V\left(c_{p}\left(\theta_{n}^{*}\right)\right)-I_{p}^{\prime}\right]+\frac{\pi\left(c_{p}\left(\zeta_{n-1}\right)\right)\left(r+\lambda_{p}\right)}{r} & \text { if } \theta<\theta_{n}^{*}-u, \\ \frac{\pi\left(c_{p}\left(\zeta_{n-1}\right)\right)}{r+\lambda_{p}}+\frac{\lambda_{p}}{r+\lambda_{p}}\left(V\left(c_{p}(\theta+u)\right)-I_{p}^{\prime}\right) & \text { if } \theta_{n}^{*}-u \leq \theta<\theta_{n}^{*}, \\ V\left(c_{p}(\theta)\right)-I_{p}^{\prime} & \text { if } \theta \geq \theta_{n}^{*},\end{cases}
$$

where $I_{p}^{\prime}$ is the switching cost. After the first investment, we assume the upgrades of the adopted, permanent technology are done at a substantially lower cost than the initial investment cost of $I_{p}$. Furthermore, for the $i$-th optimal stopping problem where $i \in\{2, . ., n-1\}$, the value of the firm before the $i$-th switch is given by

$$
F_{i}\left(c_{p}(\theta), c_{p}\left(\zeta_{i-1}\right)\right)= \begin{cases}\left(\frac{\lambda_{p}}{r+\lambda_{p}}\right)^{\frac{\theta_{i}^{*}-\theta}{u}}\left[\frac{\pi\left(c_{p}\left(\zeta_{i-1}\right)\right)}{\lambda_{p}}\left(1-\frac{\left(r+\lambda_{p}\right)^{2}}{r}\right)\right. & \text { if } \theta<\theta_{i}^{*}-u,  \tag{4.21}\\ \left.+F_{i+1}\left(c_{p}\left(\theta_{i}^{*}\right), c_{p}\left(\theta_{i}^{*}\right)\right)-I_{P}^{\prime}\right]+\frac{\pi\left(c_{p}\left(\zeta_{i-1}\right)\right)\left(r+\lambda_{p}\right)}{r} & \text { if } \theta_{i}^{*}-u \leq \theta<\theta_{i}^{*}, \\ \frac{\pi\left(c_{p}\left(\zeta_{i-1}\right)\right)}{r+\lambda_{p}}+\frac{\lambda_{p}}{r+\lambda_{p}}\left(F_{i+1}\left(c_{p}(\theta+u), c_{p}(\theta+u)\right)-I_{P}^{\prime}\right) \\ F_{i+1}\left(c_{p}(\theta), c_{p}(\theta)\right)-I_{P}^{\prime} & \text { if } \theta \geq \theta_{i}^{*} .\end{cases}
$$

Equation 4.21 is a generalisation of (4.19) that accounts for $n$ switches. It also differs from (4.20) because $F_{i}\left(c_{p}(\theta),\left(\zeta_{i-1}\right)\right)$ depends on the value before the next switch at $i+1$, which again includes the value of future switches. However, when $i=n$ there are no more switches to be made and the value of the firm is therefore deterministic.

We now find the optimal investment thresholds $\theta_{n}^{*}$ and $\theta_{i}^{*}$, by solving the value-matching conditions at $\theta=\theta_{n}^{*}$, and $\theta=\theta_{i}^{*}$. From Equations 4.20 and 4.21, value-matching gives

$$
\begin{align*}
\frac{\pi\left(c_{p}\left(\zeta_{n-1}\right)\right)}{r+\lambda_{p}}+\frac{\lambda_{p}}{r+\lambda_{p}}\left(V\left(c_{p}\left(\theta_{n}^{*}+u\right)\right)-I_{p}^{\prime}\right) & =V\left(c_{p}\left(\theta_{n}^{*}\right)\right)-I_{p}^{\prime}  \tag{4.22}\\
\frac{\pi\left(c_{p}\left(\zeta_{i-1}\right)\right)}{r+\lambda_{p}}+\frac{\lambda_{p}}{r+\lambda_{p}}\left(F_{i+1}\left(c_{p}\left(\theta_{i}^{*}+u\right), c_{p}\left(\theta_{i}^{*}+u\right)\right)-I_{p}^{\prime}\right) & =F_{i+1}\left(c_{p}\left(\theta_{i}^{*}\right), c_{p}\left(\theta_{i}^{*}\right)\right)-I_{p}^{\prime} \tag{4.23}
\end{align*}
$$

where $i \in\{1, \ldots, n-1\}$.
After substituting in for $\zeta_{i}=\left\lceil\frac{\theta_{i}^{*}}{u}\right\rceil$ for $i \in\{1, . ., n-1\}$, the set of equations (4.22) and (4.23) cannot be solved analytically. This is because $\zeta_{i}$ a discrete variable, meaning that a value that satisfies the equality cannot be found. We therefore develop an algorithm to find the solution to the problem.

Recall that $\theta$ is a discrete variable with increments of jump size $u$. Each $\theta$ represents a technology available, and to give the numerical algorithm a stopping criteria, the number of technology jumps is now limited to $N$. It is, however, essential that $N$ is set sufficiently high so that it does not constrain the technology adoption strategy. Further, let $i$ be the number of switches the firm has already done, and $n$ be the maximum number of switches a firm can do. A solution is found by exploring all possible investment strategies, comparing them and choosing the one with the highest value. This is illustrated in Figure 4.4.


Figure 4.4: All possible technology adoption strategies. $i$ is the number of switches already done, $n$ is maximum number of switches. Each $\theta$ is a technology available to the firm, and these are limited to $N$ technologies.

Figure 4.4 illustrates how the numerical algorithm explores all technology adoption strategies to find the solution to the optimal stopping problem. One path from the root node to a leaf node represents a technology adoption strategy. As can be seen from the tree, these range from the leftmost branch, where the firm never switches technology, to the rightmost branch, where the firm waits until technology $N$ becomes available and adopts it then.

To find the value of the firm in each node, we must first define a numerical approximation of the value function, similar to Huisman (2000). Let $j_{i}$ denote the number of the technology used by the firm before switch $i$, and $m_{i}$ the number of the technology the firm switches to at switch $i$. Recall that $i$ is the number of switches already done by the company. As there are no incentives to adopt an old technology, we assume the firm always adopts the best technology available after a switch. The firm's value function is given by

$$
g_{i}\left(j_{i}, m_{i}\right)= \begin{cases}\max _{\left(m_{i} \geq j_{i}\right)} f_{i+1}\left(j_{i}, m_{i}\right) & \text { if } i \in\{0, \ldots, n-1\}  \tag{4.24}\\ V\left(c_{p}\left(\theta_{j_{i}}\right)\right) & \text { if } i=n\end{cases}
$$

For $i \in\{0, \ldots, n-1\}$ we have that,

$$
\begin{equation*}
f_{i+1}\left(j_{i}, m_{i}\right)=V\left(c_{p}\left(\theta_{j_{i}}\right)\right)+\mathbb{E}\left[e^{-r\left(T_{m_{i}}-T_{j_{i}}\right)}\left(g_{i+1}\left(m_{i}, m_{i+1}\right)-V\left(c_{p}\left(\theta_{j_{i}}\right)\right)\right] .\right. \tag{4.25}
\end{equation*}
$$

The first term on the right hand side in Equation 4.25 is the value of the firm when producing with the current technology. The second term is the expected gain from upgrading from technology $j_{i}$ to $m_{i}$, where $g_{i+1}\left(m_{i}, m_{i+1}\right)$ is the value of the firm in the next switch. The expected gain is discounted by rate $r$ from the adoption time of technology $m_{i}, T_{m_{i}}$, to the adoption time of technology $j_{i}, T_{j_{i}}$. From Equation 4.24 we see that when the firm has made $n$ switches, the value of the firm is simply the value of producing with its current technology.

We again use Appendix 2.D in Huisman (2000) to find that $\mathbb{E}\left(e^{-r\left(T_{m_{i}}-T_{j_{i}}\right)}\right)=\left(\frac{\lambda_{p}}{r+\lambda_{p}}\right)^{m_{i}-j_{i}}$. Therefore,

$$
\begin{align*}
f_{i+1}\left(j_{i}, m_{i}\right) & =V\left(c_{p}\left(\theta_{j_{i}}\right)\right)+\left(\frac{\lambda_{p}}{r+\lambda_{p}}\right)^{m_{i}-j_{i}}\left[g_{i+1}\left(m_{i}, m_{i+1}\right)-V\left(c_{p}\left(\theta_{j_{i}}\right)\right]\right. \\
& =\left(1-\left(\frac{\lambda_{p}}{r+\lambda_{p}}\right)^{m_{i}-j_{i}}\right) V\left(c_{p}\left(\theta_{j_{i}}\right)\right)+\left(\frac{\lambda_{p}}{r+\lambda_{p}}\right)^{m_{i}-j_{i}} g_{i+1}\left(m_{i}, m_{i+1}\right) . \tag{4.26}
\end{align*}
$$

In order to explore all technology adoption strategies illustrated previously in Figure 4.4, the algorithm uses Equation 4.24 to calculate the value at each node of the tree, as illustrated in Figure 4.5. In each node, the algorithm calculates both the value of producing with the
current technology and the expected gain of future switches, as discussed earlier. To find the expected gain, the value of the firm in all child nodes is calculated by the algorithm, which chooses the one with the highest value. This creates a recursive call that propagates throughout the tree until the base case is reached in each leaf node. By always choosing the child node with the highest value, the optimal value of the firm considering all possible scenarios is $g_{0}(0,0)$, as this is the root node of the tree.


Figure 4.5: The numerical algorithm evaluates the value of the firm, $g_{i}\left(j_{i}, m_{i}\right)$, by creating a recursive call that maximises the value of the firm in each node. $i$ is the number of switches already done by the firm, $n$ is the maximum number of switches the firm can do, and $N$ is the number of technologies available.

More formally, we can define the optimal investment strategy by letting the technology levels $\theta_{i}$ for $i \in\{0, \ldots, N\}$ be given. Let $m_{i}^{*}$ denote the optimal number of the technology adopted in switch $i-1$, so that the switch maximises Equation 4.26. The firm must then choose the $m_{i}$ that maximises the value of a firm switching from $m_{i-1}^{*}$ to $m_{i}$. It is then optimal for the firm to adopt the technology levels $\zeta_{i}=\theta_{m_{i}^{*}}$ for $i \in\{1, \ldots, n\}$ at the time they arrive, where

$$
\begin{equation*}
m_{i}^{*}=\arg \max _{m \geq m_{i-1}^{*}}\left(f_{i-1}\left(m_{i-1}^{*}, m_{i}\right)\right), i \in\{1, \ldots, n\} . \tag{4.27}
\end{equation*}
$$

This gives a solution to the $n$ optimal stopping problems of our model. The next step however, is to determine the optimal investment amount $I_{u}^{*}$ in short-term solutions. As in the single-switch case, $I_{u}^{*}$ impacts the solutions to the optimal stopping problems. Therefore, the optimal stopping problems and the optimal investment amount must be solved simultaneously.

By expanding the maximisation problem of the single switch case in (4.3), to a multiple switch case, we find that the maximum value of the firm is given by

$$
\begin{align*}
V^{*}=\max _{I_{u}, \tau_{\left.\theta_{1}, \ldots, \tau_{\theta_{n}}\right]}} \mathbb{E} & {\left[\int_{0}^{\tau_{\lambda_{l}}+\tau_{\theta_{1}}}\left(\pi_{0}-c_{u}\left(I_{u}\right)\right) e^{-r s} d s-I_{u}\right.} \\
& +\sum_{i=1}^{n-1} \int_{\tau_{\lambda_{l}}+\tau_{\theta_{i}}}^{\tau_{\lambda_{l}}+\tau_{\theta_{i+1}}}\left(\pi_{0}-c_{p}\left(\theta_{i}\right)\right) e^{-r s} d s-I_{p} e^{-r\left(\tau_{\lambda_{l}}+\tau_{\theta_{1}}\right)}  \tag{4.28}\\
& \left.+\sum_{i=2}^{n} I_{p}^{\prime} e^{-r\left(\tau_{\lambda_{l}}+\tau_{\theta_{i}}\right)}+\int_{\tau_{\lambda_{l}}+\tau_{\theta_{n}}}^{\infty}\left(\pi_{0}-c_{p}\left(\theta_{n}\right)\right) e^{-r s} d s\right]
\end{align*}
$$

As in the single switch case, the optimal investment amount, $I_{u}^{*}$, and expected optimal adoption timing, $\bar{\tau}_{\theta_{i}^{*}}$, must be solved numerically.

### 4.3 Traffic Light System

In this extension of the single switch model we include the traffic light system in the investment problem. Recall from Section 2.2.1 that the government will award the regions either a red, yellow or green light at the end of each two-year period. A green light allows the firms to increase their production capacity by $6 \%$ at an additional cost, denoted $I_{G}$. A red light implies a capacity reduction of $6 \%$, whereas a yellow light means it stays the same. To simplify the model we assume the production capacity is regulated instantaneously after receiving a light. A firm then produces with this capacity in the following two-year period, until the next capacity adjustment is made. We further assume a firm will chose to increase its capacity if receiving a green light, regardless of the additional cost. This assumption is reasonable as production expansion is highly profitable due to high salmon prices, and the traffic light system is the only way a firm can increase its capacity.

Recall that the government evaluates two things before awarding a light: (i) The risk posed to the wild salmon population in the region, and (ii) the lice level in the individual farm. There are different opinions as to how this will affect the investment behaviour of firms. According to Ståle Furø from Salmar, the system will not change their investment behaviour, as they already are trying to keep the lice level down to a minimum without causing unacceptable mortality to the fish. On the other hand, Jon Arne Grøttum at Sjømat Norge says there are other actors that are not pushing the boundaries of fish health in the same way as Salmar. These actors will therefore have an incentive to invest in additional delousing treatment of the fish in the most critical period from April to September, when the wild smolt emigrate from the rivers. We therefore expect that a firm will change its investment behaviour due to the traffic light system. However, due to the biological constraints on fish health, we set
an upper limit $\bar{I}_{T}$ on the investment in additional treatments, and let $I_{T} \in\left\{0, \bar{I}_{T}\right\}$. Note that treatments are a part of operational lice-fighting costs and therefore the investment in additional treatments does not reduce $c_{u}\left(I_{u}\right)$.

The outcome of the system depends not only on the individual firm's actions, but also on exogenous factors. These factors are mainly the initial lice level in the region, in addition to investments made by other firms. Therefore, we assign probabilities to the different outcomes, and argue that they vary with the additional investment $I_{T}$. If the firm has made no additional investment and $I_{T}=0$, the outcome of the system depends solely on the exogenous factors, according to Henning Urke from INAQ. Additional treatments will increase the probability of receiving a green light, $P_{G}$, as the lice level in the individual farm is reduced in the critical period. Consequently, the probability of a red light, $P_{R}$, decreases towards zero in $I_{T}$. The probability of a yellow light, $P_{Y}$, may exhibit non-monotonic behaviour, which depends on the initial red and green probabilities and their sensitivity with respect to $I_{T}{ }^{3}$. For example, in Figure 4.6, the initial $P_{G}$ is relatively low whereas the initial $P_{R}$ is relatively high. For low values of $I_{T}$, the increase in the green probability is slower than the decrease in red, causing the yellow probability to increase. However, for high values of $I_{T}$, the green probability is more sensitive to changes, resulting in a decrease of the yellow probability. This gives the non-monotonic behaviour of $P_{Y}$.


Figure 4.6: The probability functions varying with the additional investment $I_{T}$ for a region with high lice levels. $P_{G}$ is the green light probability, $P_{Y}$ is the yellow light probability and $P_{R}$ is the red light probability.

We base the functional form of the probability functions on discrete choice models in consumer choice theory, more precisely the multinomial logit model of Ben-Akiva and Lerman (1985). They model the probabilities of a set of mutually exclusive and collectively exhaustive

[^3]outcomes, varying with respect to the input of the function. In our case, the set of outcomes are the set of traffic lights, and the probabilities are varying with $I_{T}$. The probabilities of the traffic light outcomes are therefore given by
\[

$$
\begin{align*}
& P_{G}\left(I_{T}\right)=\frac{e^{\gamma_{G}+\mu_{G} I_{T}}}{e^{\gamma_{G}+\mu_{G} I_{T}}+e^{\gamma_{Y}-\mu_{Y} I_{T}}+e^{\gamma_{R}-\mu_{R} I_{T}}}, \\
& P_{Y}\left(I_{T}\right)=\frac{e^{\gamma_{Y}-\mu_{Y} I_{T}}}{e^{\gamma_{G}+\mu_{G} I_{T}}+e^{\gamma_{Y}-\mu_{Y} I_{T}}+e^{\gamma_{R}-\mu_{R} I_{T}}},  \tag{4.29}\\
& P_{R}\left(I_{T}\right)=\frac{e^{\gamma_{R}-\mu_{R} I_{T}}}{e^{\gamma_{G}+\mu_{G} I_{T}}+e^{\gamma_{Y}-\mu_{Y} I_{T}}+e^{\gamma_{R}-\mu_{R} I_{T}}},
\end{align*}
$$
\]

where the scale factors $\mu_{G}, \mu_{Y}$ and $\mu_{R}$, represent the effect of $I_{T}$ on the probability of getting a green, yellow and red light, respectively. The higher the scale factors, the larger the effect of $I_{T}$ on the probabilities. The constants $\gamma_{G}, \gamma_{Y}$ and $\gamma_{R}$ determine the initial probabilities in a region, when $I_{T}=0$.

Let $P_{T}\left(I_{T}\right)$ denote a capacity change factor that regulates the profit of the firm given by

$$
\begin{equation*}
P_{T}\left(I_{T}\right)=P_{G}\left(I_{T}\right) \cdot 1.06+P_{Y}\left(I_{T}\right)+P_{R}\left(I_{T}\right) \cdot 0.94, \tag{4.30}
\end{equation*}
$$

where 1.06 and 0.94 are the expansion and reduction factors of the green and red light, respectively. The effect of the capacity regulation on the revenue and production cost is taken into account, as we regulate the profit $\pi$ in the model. However, we neglect the change in lice-fighting costs as a production regulation does not result in significant change in lice costs. This is because the largest part of these costs, such as treatments, control and maintenance are not directly dependent on the number of fish in the net (Iversen et al., 2015), and are therefore less sensitive to changes in the amount of fish.

We will now solve the single switch model by including the traffic light system, and solve for the optimal additional investment $I_{T}$. We start with a simple case where we assume regulation only is done once, in $t=2$, and only if the permanent solution has not arrived. Previously, we defined the permanent technology as a solution that significantly reduces the lice level in a net. Therefore, the arrival of a permanent technology will result in more environmentally sustainable production, which is the government's main criteria for expansion. The regulatory environment will thus change, and we therefore assume the traffic light system stops at the arrival of the permanent technology.

The optimal value of the firm is then derived taking into account the two scenarios of the permanent technology arriving in $t<2$, and in $t>2$. The maximisation problem is therefore given by

$$
\begin{align*}
V^{*}=\max _{I_{u}, I_{T}, \tau_{\theta}} \mathbb{E}\{ & \left\{{\mathbb{T}\left\{\tau_{\lambda_{l}}<2\right\}}\left[\int_{0}^{t+\tau_{\theta}}\left(\pi_{0}-c_{u}\left(I_{u}\right)\right) e^{-r s} d s+\int_{t+\tau_{\theta}}^{\infty}\left(\pi_{0}-c_{p}(\theta)\right) e^{-r s} d s\right]\right.  \tag{4.31}\\
& +\mathbb{1}_{\left\{\tau_{\lambda_{l}}>2\right\}}\left[\int_{0}^{2}\left(\pi_{0}-c_{u}\left(I_{u}\right)\right) e^{-r s} d s+\int_{2}^{t+\tau_{\theta}}\left(\pi_{0} \cdot P_{T}\left(I_{T}\right)-c_{u}\left(I_{u}\right)\right) e^{-r s} d s\right. \\
& \left.\left.+\int_{t+\tau_{\theta}}^{\infty}\left(\pi_{0} \cdot P_{T}\left(I_{T}\right)-c_{p}(\theta)\right) e^{-r s} d s-P_{G}\left(I_{T}\right) \cdot I_{G} e^{-2 r}\right]-I_{u}-I_{T}-I_{p} e^{-r\left(t+\tau_{\theta}\right)}\right\},
\end{align*}
$$

where $I_{G}$ is the cost of expansion, given a green light. Note that the profit flow only is regulated if the permanent technology has not arrived. We redefine the optimal value of the company as follows (see Appendix B.2).

$$
\begin{align*}
V^{*}=\max _{I_{u}, I_{T}, \tau_{\theta}} \mathbb{E}\{ & {\left[\int _ { 0 } ^ { 2 } \left(\int_{0}^{t+\tau_{\theta}}\left(\pi_{0}-c_{u}\left(I_{u}\right)\right) e^{-r s} d s\right.\right.} \\
& \left.\left.+\int_{t+\tau_{\theta}}^{\infty}\left(\pi_{0}-c_{p}(\theta)\right) e^{-r s} d s-I_{p} e^{-r\left(t+\tau_{\theta}\right)}\right) \lambda_{l} e^{-\lambda_{l} t} d t\right]  \tag{4.32}\\
& +\left[\int _ { 2 } ^ { \infty } \left(\int_{0}^{2}\left(\pi_{0}-c_{u}\left(I_{u}\right)\right) e^{-r s} d s+\int_{2}^{t+\tau_{\theta}}\left(\pi_{0} \cdot P_{T}\left(I_{T}\right)-c_{u}\left(I_{u}\right)\right) e^{-r s} d s\right.\right. \\
& \left.\left.+\int_{t+\tau_{\theta}}^{\infty}\left(\pi_{0} \cdot P_{T}\left(I_{T}\right)-c_{p}(\theta)\right) e^{-r s} d s-P_{G}\left(I_{T}\right) \cdot I_{G} e^{-2 r}-I_{p} e^{-r\left(t+\tau_{\theta}\right)}\right) \lambda_{l} e^{-\lambda_{l} t} d t\right] \\
& \left.-I_{u}-I_{T}\right\} .
\end{align*}
$$

For simplification we rearrange the terms and get that the value of the firm can be given by

$$
\begin{align*}
V^{*}=\max _{I_{u}, I_{T}, \tau_{\theta}} \mathbb{E}\{ & \left\{\int _ { 0 } ^ { 2 } \left(\int_{0}^{t+\tau_{\theta}}\left(\pi_{0}-c_{u}\left(I_{u}\right)\right) e^{-r s} d s\right.\right. \\
& \left.\left.+\int_{t+\tau_{\theta}}^{\infty}\left(\pi_{0}-c_{p}(\theta)\right) e^{-r s} d s-I_{p} e^{-r\left(t+\tau_{\theta}\right)}\right) \lambda_{l} e^{-\lambda_{l} t} d t\right] \\
& +\left[\int _ { 2 } ^ { \infty } \left(\int_{0}^{t+\tau_{\theta}}\left(\pi_{0}-c_{u}\left(I_{u}\right)\right) e^{-r s} d s+\int_{2}^{t+\tau_{\theta}}\left(\pi_{0} \cdot P_{T}\left(I_{T}\right)-\pi_{0}\right) e^{-r s} d s\right.\right.  \tag{4.33}\\
& +\int_{t+\tau_{\theta}}^{\infty}\left(\pi_{0}-c_{p}(\theta)\right) e^{-r s} d s+\int_{t+\tau_{\theta}}^{\infty}\left(\pi_{0} \cdot P_{T}\left(I_{T}\right)-\pi_{0}\right) e^{-r s} d s \\
& \left.\left.\left.-P_{G}\left(I_{T}\right) \cdot I_{G} e^{-2 r}-I_{p} e^{-r\left(t+\tau_{\theta}\right)}-I_{T}\right) \lambda_{l} e^{-\lambda_{l} t} d t\right]-I_{u}\right\} .
\end{align*}
$$

Simplifying the expression gives

$$
\begin{align*}
& V^{*}=\max _{I_{u}, I_{T}, \tau_{\theta}} \mathbb{E}\left\{\left[\int_{0}^{t+\tau_{\theta}}\left(\pi_{0}-c_{u}\left(I_{u}\right)\right) e^{-r s} d s-I_{u}+\int_{t+\tau_{\theta}}^{\infty}\left(\pi_{0}-c_{p}(\theta)\right) e^{-r s} d s-I_{p} e^{-r\left(t+\tau_{\theta}\right)}\right]\right. \\
&\left.+\left[\int_{2}^{\infty}\left(\int_{2}^{\infty}\left(\pi_{0} \cdot P_{T}\left(I_{T}\right)-\pi_{0}\right) d s-P_{G}\left(I_{T}\right) \cdot I_{G} e^{-2 r}-I_{T}\right) \lambda_{l} e^{\lambda_{l} t} d t\right]\right\} \tag{4.34}
\end{align*}
$$

The first bracket is similar to Equation 4.17, which is the maximisation problem of the single switch model. The last terms are the additional profits and additional costs from the traffic light system, accounted for the probability that the permanent technology did not arrive by $t=2$, and a regulation took place.

To solve the problem we again use Appendix 2.D in Huisman (2000) to find that $\mathbb{E}\left(e^{\left.-r \tau_{\theta^{*}}\right)}=\left(\frac{\lambda_{p}}{r+\lambda_{p}}\right)^{\left\lceil\frac{\theta^{*}\left(L_{u}^{*}\right)-\theta_{0}}{u}\right\rceil}\right.$, and find that the optimal value is given by

$$
\begin{align*}
V^{*} & \left.=\frac{\pi_{0}-c_{u}\left(I_{u}^{*}\right)}{r}-I_{u}^{*}+\frac{\lambda_{l}}{r+\lambda_{l}}\left(\frac{\lambda_{p}}{r+\lambda_{p}}\right)^{\left[\frac{\theta^{*}\left(I_{u}^{*}\right)-\theta_{0}}{u}\right.}\right\rceil\left(\frac{c_{u}\left(I_{u}^{*}\right)-c_{p}\left(\theta^{*}\right)}{r}-I_{p}\right)  \tag{4.35}\\
& +e^{-2\left(r+\lambda_{l}\right)} \cdot \frac{\pi_{0} \cdot P_{T}\left(I_{T}^{*}\right)-\pi_{0}}{r}-I_{G} \cdot P_{G}\left(I_{T}^{*}\right) e^{-2\left(r+\lambda_{l}\right)}-I_{T}^{*},
\end{align*}
$$

where $I_{T}^{*}$ is the optimal investment amount in additional treatments. Note that the first three terms are the value of the firm in the single switch case. The fourth term is the additional profit from the traffic light system, and the last terms are the additional costs. The discount rate for the terms related to the traffic light system account for both the discounting and the probability of a regulation actually happening. This is coherent with Dixit and Pindyck (1994) who state that if an event with arrival rate $\lambda$ occurs, we can calculate the present value of a cash flow given the event, by adding $\lambda$ to the discount rate. Note that the value and cost terms from to the traffic light system do not depend on neither $I_{u}^{*}$, nor $\theta^{*}$. Hence, the optimal investment amount $I_{u}^{*}$, and the optimal investment threshold $\theta^{*}$, are not affected by the implementation of the traffic light system in the investment problem. This taken in account, we derive a general expression for the optimal value of the firm, given a deterministic set of $R$ regulations. This is given by

$$
\begin{align*}
V^{*} & \left.=\frac{\pi_{0}-c_{u}\left(I_{u}^{*}\right)}{r}-I_{u}^{*}+\frac{\lambda_{l}}{r+\lambda_{l}}\left(\frac{\lambda_{p}}{r+\lambda_{p}}\right)^{\left\lceil\frac{\theta^{*}\left(I_{u}^{*}\right)-\theta_{0}}{u}\right.}\right\rceil\left(\frac{c_{u}\left(I_{u}^{*}\right)-c_{p}\left(\theta^{*}\right)}{r}-I_{p}\right)  \tag{4.36}\\
& +\sum_{k=1}^{R}\left[\left(\frac{\pi_{k-1} \cdot P_{T}\left(I_{T_{k-1}}^{*}\right)-\pi_{k-1}}{r}-I_{G} \cdot P_{G}\left(I_{T_{k-1}}^{*}\right)-I_{T_{k-1}}^{*}\right) e^{-2 k\left(r+\lambda_{l}\right)}\right],
\end{align*}
$$

where the additional terms related to the traffic light system are discounted with $2 k$, which is the time of regulation $k$ from time zero, as the regulation lasts for a period of 2 years. The optimal investment amounts $I_{u}^{*}$ and $I_{T_{k-1}}^{*}$ are found numerically.

## Chapter 5

## Parametrisation

In this chapter we give an overview of the parameter values used as input in the model, and the motivation behind them. It has been challenging finding the correct values to use as much of the data is scarce, or unavailable. However, some sources have been especially important in estimation of the parameters, and therefore deserve a general introduction.

Nofima, a Norwegian research institution, does extensive research on the topic of salmon lice and the cost drivers in the aquaculture industry. The report from 2015 on this subject by Iversen et al. (2015), forms part of the basis for the quantification. Furthermore, Audun Iversen, the author of the report, has provided some insight and clarification on some of the parameters. For firm-specific parameters, such as the initial lice-fighting costs and EBIT per licence, we use information from Bjørøya Fiskeoppdrett AS as example. Bjørøya Fiskeoppdrett AS is a relatively small fish farming firm with 9.5 licences operating in Flatanger, which is located in a region with relatively high lice levels.

### 5.1 Cost Parameters

## Lice-Fighting Costs

## Parameter: $c_{0}$

The parameter $c_{0}$ is the initial lice-fighting cost a firm faces per licence when operating with a technology package consisting of preventive, continuous and immediate technologies. According to the report from Nofima by Iversen et al. (2015), these costs were estimated to be 5 billion NOK for the aquaculture industry in Norway in 2015. During the last couple of years, these costs have increased rapidly, and the reason for this is twofold. The main increase
comes from delousing treatments necessary due to the medical resistance of lice. Another reason for the increase are the stringent control and reporting requirements from the authorities. As a result, the salary costs have also increased, because more work hours are spent on fulfilling these requirements and performing treatments.

We consider the latest annual report from 2016 for Bjørøya Fiskeoppdrett to estimate the lice-fighting costs. The company had lice-fighting costs of 27.391 million NOK before investments. However, the labour costs related to lice control are not part of these. Per Anton Løfsnæs, CEO of Bjørøya Fiskeoppdrett, claims that $20 \%$ of the labour costs are a part of the operational lice-fighting costs. As the labour costs for 2016 were 26.676 million NOK, another 5.335 million NOK can be added to the lice-fighting cost. Bjørøya has 9.5 licences in Norway, which gives us an estimate of $c_{0}=3.445$ million NOK/licence.

However, it is a well-known fact in the industry that the registered lice costs are only part of the reality, and that there are high unrecorded numbers. These are related to diseases, death from treatments and lost growth as a result of starving before a delousing. According to Bruarøy (2016), the lice costs estimated by Iversen et al. (2015) are in fact twice as large when taking these factors in account. Henning Urke confirms that this may hold for Bjørøya, as they have experienced both mortality and lost growth due to lice. Based on this we set the lice-fighting costs to be $c_{0}=6.890$ million NOK/licence, which is twice as much as reported in the annual report, but an estimate we find more realistic.

## Parameter: $\bar{c}_{u}$

When upgrades of short-term technologies become redundant, the reduction in lice-fighting costs from $c_{0}$ will approach a lower boundary. We denote this lower boundary $\bar{c}_{u}$. Recall from Chapter 4 that the lice-fighting costs decrease as the lice level decreases, and using different short-term solutions simultaneously can remove lice, but never completely. This is because the lice level in a net depends on a combination of several factors (see Section 2.1). In addition, the efficiency of one technology affects the performance of another and the effects last for different periods of time. This makes the estimation of $\bar{c}_{u}$ difficult, as the combined effect of the technologies are not yet documented.

A decreasing lice level will eventually make delousing treatments unnecessary, and the cost of them will decrease. The other components of the lice-fighting costs, such as maintenance and control, are not affected by a lower lice level. Therefore, we estimate the lowest possible lice cost level by assuming the investment in upgrades become redundant at the point where delousing no longer is necessary. According to Iversen et al. (2015), $45 \%$ of operational licefighting costs in 2015 were due to treatments of fish. As these increase each year, we round up the estimate and set $\bar{c}_{u}=50 \% \cdot 6.890$ million NOK/licence $=3.445$ million NOK/licence.

## Investment Cost

## Parameter: $I_{p}$

$I_{p}$ denotes the investment cost of adopting a permanent technology. We have based our assumptions about the investment cost on the technologies that have been granted development licences. However, it is difficult to find accurate values as these technologies are still under development and much of the financial information is kept secret.

According to Norwegian Directorate of Fisheries (2017), the Egg, described in Section 2.3, was granted four development licenses of 1000 tonnes MAB, and Hauge Aqua estimates the cost of each Egg to be 50 million NOK per development licence. Converted into a commercial licence of 780 tonnes MAB, the investment cost is 37.5 million NOK/licence. Cato Lyngøy, CEO at Hauge Aqua, states that this may be a reasonable estimate for the future market price of the Egg if, and when, the technology is commercialised. However, as it is uncertain which technology will be commercialised, we use the average investment cost of the four technologies that have been granted development licences so far. The other three technologies are the Offshore Fish Farming, the Submersible Cage, and the Semi-Closed System, which are described in Brakstad and Matanovic (2016). The investment cost of these are estimated to be 40 million NOK/licence, 14.357 million NOK/licence, and 14.977 million NOK/licence, respectively (Norwegian Directorate of Fisheries, 2017). Based on this, we estimate the investment cost in permanent technologies to be $I_{p}=26.700$ million NOK/licence.

## Switching Cost

## Parameter: $I_{p}^{\prime}$

$I_{p}^{\prime}$ denotes the switching cost of upgrading a permanent technology. After it is implemented, the cost of upgrading the technology will be significantly lower. This is confirmed by Cato Lyngøy who states that the Egg is constructed in a way that makes upgrading of specific parts possible at only a fraction of the initial investment cost. However, as no permanent technologies have finished testing yet, it is difficult to estimate what this cost will be. As a baseline case, we set it to be $I_{p}^{\prime}=1$ million NOK/licence, which is a small fraction of $I_{p}$, but still relatively high compared to the cost the equipment in the industry today.

## Cost Reduction Factors

## Parameter: $\alpha$

The parameter $\alpha$ is the cost reduction factor for short-term investments. It defines the rate at which $c_{u}\left(I_{u}\right)$ decreases in $I_{u}$. A higher $\alpha$ means that investments are more effective in reducing lice costs, whereas a lower $\alpha$ means that investments are less effective. $\alpha$ therefore represents the marginal decrease from investing, in percent. In estimating this parameter, we encountered the same problem of insufficient data on the effect of the technologies, as when estimating $\bar{c}_{u}$. Based on our general understanding of the cost and efficiency level of combinations of the technologies discussed in Section 2.3, we have estimated that an investment of $I_{u}=1$ million NOK will give a cost reduction of 0.3 . This implies

$$
c_{u}\left(I_{u}\right)=0.7 c_{0}=\bar{c}_{u}+\left(c_{0}-\bar{c}_{u}\right) e^{-\alpha I_{u}}
$$

which gives $\alpha \approx 1 \cdot 10^{-6}$. We therefore set $\alpha=1 \cdot 10^{-6}$. The reason for the low magnitude of $\alpha$ is that $\alpha I_{u}$ is the exponent of $e$, and $I_{u}$ is in the magnitude of millions. $I_{u}$ therefore has to be scaled down by a small factor, for the term $e^{-\alpha I_{u}}$ to be reasonable. We will check the sensitivity for this parameter in Section 6.2.4.

## Parameter: $\beta$

The parameter $\beta$ is the cost reduction factor for a permanent technology. Recall that investment in short-term upgrades would bring costs down to a lower threshold $\bar{c}_{u}$, which was estimated to be 0.5 of the initial costs $c_{0}$. This means $\beta$ has to be in the range between zero and 0.5 for the reduction in lice-fighting costs to be significant enough. Similar to the estimation of the investment cost, we review the technologies applied for in the Norwegian Government's development licence scheme. Several of the technologies would disrupt the industry if they arrived, and could lead to a significant reduction in lice level and hence lice-fighting costs. However, according to Randi Grøntvedt, it is highly unlikely that a future solution will ever reduce the lice costs to zero. We therefore believe $\beta=0.2$ to be the best approximation, as this implies a substantial reduction of initial costs.

### 5.2 Profit Parameters

## Profit before Lice-Fighting Costs

## Parameter: $\pi_{0}$

$\pi_{0}$ is the continuous profit flow per licence, measured in EBIT net of lice costs. Usually, the lice costs are included in the production costs, however, we model them separately. In the annual report from 2016 for Bjørøya, the EBIT was 154.332 million NOK, which is 16.245 million NOK/licence. For the same year we estimated the lice-fighting costs to be 6.890 NOK per licence. This gives a profit before lice costs of $\pi_{0}=23.135$ million NOK/licence.

### 5.3 Technological Parameters

## Arrival Rates

## Parameter: $\lambda_{l}$

$\lambda_{l}$ denotes the arrival rate of the permanent technology. Similar to the estimations of $\beta$ and $I_{p}$, this is also based on the applications for the development licence scheme. The technologies that have received development licences, such as the Egg, must be tested and can produce salmon while testing. A development licence permits production for 5 years, and we therefore find a reasonable estimate for the arrival of a permanent technology to be $\lambda_{l}=0.2$. However, the success of the technology is uncertain, as well as the arrival of other technologies. Therefore, we will check the sensitivity for the parameter in Section 6.2.3.

## Parameter: $\lambda_{p}$

The parameter $\lambda_{p}$ denotes the arrival rate of improvements of the permanent technology. The lice problem affects the entire industry as it limits production growth due to regulations from the government. If a technology arrives, it is profitable for both the original developer to make and sell upgrades, and for third party companies to copy and improve the technologies. Considering the development scheme received 41 applications, there appears to be a high innovation rate in the industry, and several actors are potentially able to improve the technology. Based on this we argue that the arrival rate of improvements should be relatively high after the permanent technology has arrived. Therefore, we set the arrival rate to be approximately $\lambda_{p}=1$, which translates to an average of one improvement per year. We will check the sensitivity for this parameter in Section 6.2.3.

## Jump Size

## Parameter: $u$

The parameter $u$ denotes the improvement jump size of the permanent technology. The larger the jump size $u$, the more the lice costs of the permanent technology are reduced by each arrival of improvements to the technology. Similar to the estimations of $\bar{c}_{u}$ and $\alpha$, this estimation is also complicated by the lack of data, as the technologies in question have yet to be developed. To find what we believe to be a reasonable estimate based on the development projects that have applied for licences, we have assumed that the lice cost level of the best available permanent technology will be reduced by $25 \%$ every second year. With $\lambda_{p}=1$, this means that

$$
c_{p}(\theta)=0.75 \cdot \beta c_{0}=\beta c_{0} e^{-2 u}
$$

yielding $u \approx 0.15$.

### 5.4 Financial Parameters

## Discount Rate

## Parameter: $r$

The discount rate is the opportunity cost of capital. According to Norway's most acclaimed salmon analyst, Kolbjørn Giskeødegård at Nordea Markets, the discount rate used in salmon farming companies is in the range of $8-12 \%$, depending on the size of the firm and riskiness of the project (Hannevik et al., 2015). As our model takes in account both low-risk investments in short-term solutions, and high-risk investments in permanent technologies, we set $r=10 \%$ as the average discount rate.

### 5.5 Traffic Light System Parameters

## Cost Parameters

## Parameter: $\overline{I_{T}}$

$\overline{I_{T}}$ denotes the maximum amount a firm can invest in additional treatments, when taking into account biological constraints. To influence the outcome of the traffic light system, we argued in Section 4.3 that the firm can make additional investments in delousing treatments in the most critical period of the year. However, delousing requires starving of the fish for several days prior to the treatment, resulting in a loss of growth. Additionally, the treatment in itself is rough on the fish and can cause death. The number of additional treatments a firm can do is therefore limited. Estimating this biological constraint is complicated, as it varies with both location, methods currently in use, and previously performed delousing, which is confirmed by Henning Urke. We therefore set a high estimate of 5 maximum additional treatments as the biological constraint. This is more relaxed than the realistic biological constraints, but still illustrates the effect of a modelling constraint. Based on the delousing treatments described by Astrid Buran Holan et al. (2017), an average cost of a delousing treatment is approximately $0.80 \mathrm{NOK} / \mathrm{kg}$. Because the actual average biomass per licence is difficult to estimate, we use the maximum allowed biomass of 780 tonnes. We therefore estimate $\bar{I}_{T} \approx 3$ million NOK/licence.

## Parameter: $I_{G}$

$I_{G}$ denotes the cost of expanding production if a region is awarded a green light in the traffic light system. This cost has not been determined yet, but we find it reasonable to assume that it will be close to the cost of expansion today. This is set to be 1 million NOK/licence for a commercial licence of 780 MAB (The Norwegian Ministry of Trade, Industry and Fisheries, 2015). Therefore, we estimate that $I_{G}=1$ million NOK/licence.

## Constants

## Parameter: $\gamma_{G}, \gamma_{Y}, \gamma_{R}$

The constants $\gamma_{G}, \gamma_{Y}$ and $\gamma_{R}$ determine the initial probabilities of the traffic light outcomes in a production region, given no additional investments have been made. Recall that these probabilities are determined by exogenous factor such as the lice level in the region. In regions with a high lice pressure, the initial probability of getting a red light will be high, meaning a high $\gamma_{R}$, relative to $\gamma_{Y}$ and $\gamma_{G}$. For regions with a lower lice pressure, the probability of getting a green light before any additional treatments are made is larger.

As Bjørøya Fiskeoppdrett was used as example for the firm-specific parameters, it is reasonable to estimate constants for Flatanger where Bjørøya has its production facilities. According to Henning Urke, Flatanger is a production region with relatively high lice levels. This means that the initial probability for a red light is high, and $\gamma_{R}$ should be the large, relative to $\gamma_{G}$ and $\gamma_{Y}$. As a baseline case we set $\gamma_{G}=2, \gamma_{Y}=6$ and $\gamma_{R}=7$. The estimation of these constants requires an in depth analysis of biological conditions in different geographic regions, and thus is outside the scope of this thesis. Note, however, that the model is general enough to be applied to farms with different geographical locations.

In addition to Flatanger, we also define the parameters for a firm in Hardanger and a firm in Finnmark. This is because these locations are extreme cases of high and low lice pressure, and are therefore interesting to look at when evaluating the effect of the traffic light system in Section 6.3. For the location in Finnmark where the lice pressure is very low, we set $\gamma_{G}=10$, $\gamma_{Y}=9$ and $\gamma_{R}=8$. For the location in Hardanger we set $\gamma_{G}=1, \gamma_{Y}=7$ and $\gamma_{G}=10$. Note that for Hardanger $\gamma_{G}$ is low compared to $\gamma_{Y}$ and $\gamma_{R}$ because it is practically impossible to be awarded a green light, according to Henning Urke.

Based on these constants and the scale factors parametrised in the next section, the probability functions for each of the three regions given additional investment $I_{T}$ are illustrated in Figure 5.1.


Figure 5.1: The probability functions for the outcome of the traffic light system in Finnmark, Flatanger and Hardanger, respectively.

## Scale Factors

Parameters: $\mu_{G}, \mu_{Y}, \mu_{R}$

The parameters $\mu_{G}, \mu_{Y}$ and $\mu_{R}$, scale the effect of $I_{T}$ on the probability of getting a red, yellow and green light, respectively. Henning Urke claims that if Bjørøya makes no additional investment, the outcome of the traffic light system will be red. However, with a little effort the outcome will be yellow. Hence, the effect of $I_{T}$ on $P_{R}$ is large, compared to the effect on $P_{Y}$ and $P_{G}$.

For Flatanger we let $\mu_{G}=1 \cdot 10^{-6}, \mu_{Y}=2 \cdot 10^{-6}$ and $\mu_{R}=8 \cdot 10^{-6}$. We assume $I_{T}$ to be in the magnitude of hundreds of thousands. Just as in the parametrisation of $\alpha, I_{T}$ has to be scaled down by a small $\mu$ - hence the low magnitude. For the location in Finnmark we set $\mu_{G}=2 \cdot 10^{-6}, \mu_{Y}=0.1 \cdot 10^{-6}$ and $\mu_{R}=3 \cdot 10^{-6}$. Additional treatments have a large impact on lice levels, and consequently the red and yellow probabilities decrease fast, whereas the green probability increases fast. Recall that in Equation 4.29, $\mu_{Y}$ and $\mu_{R}$ have negative signs. For the location in Hardanger we set $\mu_{G}=0.1 \cdot 10^{-6}, \mu_{Y}=2 \cdot 10^{-6}$ and $\mu_{R}=2.5 \cdot 10^{-6}$. Note that for both Finnmark and Hardanger, the scale parameters are small compared to Flatanger. This is because Finnmark and Hardanger are two extreme cases where the initial lice level is so high, or so low, that investments in additional treatments will have low effect on the probabilities of the outcomes. Again, the data for this estimation is not available, but we set the values as a baseline case. The effect of the scale parameters on the initial probabilities are illustrated in Figure 5.1.

### 5.6 Summary

The parameter values that will be used for the results in the case study, are summarised in the following table.

| Parameters | Value | Unit |
| :--- | :--- | :--- |
| $\pi_{0}$ | $23.135 \cdot 10^{6}$ | NOK/licence |
| $c_{0}$ | $6.890 \cdot 10^{6}$ | NOK/licence |
| $\bar{c}_{u}$ | $3.445 \cdot 10^{6}$ | NOK/licence |
| $I_{p}$ | $26.700 \cdot 10^{6}$ | NOK/licence |
| $I_{p}^{\prime}$ | $1.000 \cdot 10^{6}$ | NOK/licence |
| $\bar{I}_{T}$ | $3.000 \cdot 10^{6}$ | NOK/licence |
| $I_{G}$ | $1.000 \cdot 10^{6}$ | NOK/licence |
| $\lambda_{l}$ | 0.2 | - |
| $\lambda_{p}$ | 1.0 | - |
| $u$ | 0.15 | - |
| $r$ | 0.1 | - |
| $\alpha$ | $1 \cdot 10^{-6}$ | - |
| $\beta$ | 0.2 | - |
| $\gamma_{G}$ | 2 | - |
| $\gamma_{Y}$ | 6 | - |
| $\gamma_{R}$ | 7 | - |
| $\mu_{G}$ | $1 \cdot 10^{-6}$ | - |
| $\mu_{Y}$ | $2 \cdot 10^{-6}$ | - |
| $\mu_{R}$ | $8 \cdot 10^{-6}$ | - |

Table 5.1: Parameter values

## Chapter 6

## Results and Sensitivity Analysis

### 6.1 Baseline Case Results

### 6.1.1 Single Switch Results

By solving Equation 4.18 for the single switch case numerically with the parameters from Chapter 5, we find the value of a firm to be $V^{*}=193.731$ million NOK/licence. The optimal investment amount in short-term technologies that gives this firm value is $I_{u}^{*}=3.212$ million NOK/licence. The operational lice-fighting costs will thus be reduced from $c_{0}=6.890$ million NOK/licence to $c_{u}\left(I_{u}^{*}\right)=3.584$ million NOK/licence. The permanent technology adoption threshold is found to be $\theta^{*}=1.283$. This implies that the operational lice-fighting costs when operating with the permanent technology will be at $c_{p}\left(\theta^{*}\right)=0.382$ million NOK/licence.

### 6.1.2 Multiple Switch Results

Table 6.1 shows the numerical results of Equation 4.28 for $n \in\{1,2,3,4\}$ maximum number of switches, given the parameter values set in Chapter 5. Unlike the single switch results, we here present the optimal technology level to be adopted, $\zeta_{i}$, and not the optimal investment threshold, $\theta_{i}^{*}$, that can lie between two technology jumps. This is because the numerical approach does not solve for the optimal investment thresholds, but for the sequence of technologies it is optimal to adopt. Note, however, that $\bar{\tau}_{\theta_{i}^{*}}$ is the expected optimal adoption timing, and not the expected time of the optimal adoption threshold. We therefore have that $\bar{\tau}_{\theta_{i}^{*}}$ is the expected adoption time of $\zeta_{i}$ (see Equation 4.16).

| $n$ | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| $V^{*}$ | $193.731 \cdot 10^{6}$ | $193.878 \cdot 10^{6}$ | $193.880 \cdot 10^{8}$ | $193.880 \cdot 10^{8}$ |
| $I_{u}^{*}$ | $3.212 \cdot 10^{6}$ | $3.172 \cdot 10^{6}$ | $3.145 \cdot 10^{6}$ | $3.145 \cdot 10^{6}$ |
| $\zeta_{1}$ | 1.35 | 1.20 | 1.05 | 1.05 |
| $\zeta_{2}$ | - | 2.40 | 2.10 | 2.10 |
| $\zeta_{3}$ | - | - | 3.90 | 3.90 |
| $\zeta_{4}$ | - | - | - | - |
| $\bar{\tau}_{\theta_{1}^{*}}$ | 9 | 8 | 7 | 7 |
| $\bar{\tau}_{\theta_{2}^{*}}$ | - | 16 | 14 | 14 |
| $\bar{\tau}_{\theta_{3}^{*}}$ | - | - | 26 | 26 |
| $\bar{\tau}_{\theta_{4}^{*}}$ | - | - | - | - |

Table 6.1: The value of the firm $V^{*}$ in NOK/licence, the optimal investment amount $I_{u}^{*}$ in NOK/licence, the optimal technology adoption level $\zeta_{i}$ and the expected adoption time after arrival in years $\bar{\tau}_{\theta_{i}^{*}}$, for the maximum number of switches $n$. Parameters used: $\pi_{0}=23.14 \cdot 10^{6}, c_{0}=6.89 \cdot 10^{6}$, $\bar{c}_{u}=3.45 \cdot 10^{6}, I_{p}=26.70 \cdot 10^{6}, I_{p}^{\prime}=1.00 \cdot 10^{6}, \lambda_{l}=0.2, \lambda_{p}=1.0, u=0.15, r=0.1, \alpha=1 \cdot 10^{-6}, \beta=0.2$.

When relaxing the assumption of a one-time investment and allowing for $n$ switches, the added flexibility increases the firm value, as seen in Table 6.1. The intuition is that after the adoption of the permanent technology, the firm does not have to operate with the same technology forever. It can afford adopting earlier and then upgrade the solution at a later point in time. Hence, the firm will operate with lower lice costs $c_{p}\left(\zeta_{i}\right)$ for a longer period of time, which increases the firm value. However, the change in value is small because lice costs are a small fraction of the firm's profits. Consequently, an additional upgrade of lice-fighting technologies does not greatly affect the firm's value.

The optimal investment amount, $I_{u}^{*}$, decreases when we allow multiple switches, and $n \geq 2$. In the multiple switch case, the firm adopts the permanent solution earlier and the incentive to invest in short-term solutions is reduced. Hence, the firm invests less. Note that only the first adoption affects the investment amount, and not the number, nor timing of the following switches. This is because the short-term investment decision only affects the value of the firm until the adoption of the permanent technology. What happens after the adoption is not relevant when finding the optimal investment amount.

Moreover, we see from Table 6.1 that the optimal technology adoption level in the single switch case, is $\zeta_{1}=1.35$, with expected adoption in 9 years. (Recall that the expected adoption time $\bar{\tau}_{\theta_{i}^{*}}$ is given in years after arrival of the permanent technology.) When giving the firm more flexibility and allowing for $n=2$ switches, the optimal adoption level decreases to $\zeta_{1}=1.20$, and the expected adoption time to 8 years. The reason for the earlier adoption is that the incentive to wait for the technology process to reach a higher level is smaller, as it is no longer the firm's only chance at obtaining low lice costs. At the same time, the firm is eager to invest to reduce lice costs from $c_{u}\left(I_{u}^{*}\right)$ to $c_{p}\left(\zeta_{1}\right)$. The firm therefore makes the
first investment earlier to benefit from the lower lice costs. When the technology level has increased sufficiently compared to when the firm first adopted the permanent technology, it upgrades to benefit from even lower lice costs $c_{p}\left(\zeta_{2}\right)$. The upgrade is to technology adoption level $\zeta_{2}=2.40$, and is expected to be done in 16 years. The intuition holds for the decrease of $\zeta_{i}$ in the case of $n=3$ switches. Note, however, that when we further increase flexibility by allowing $n=4$ switches, the investment strategy remains unchanged from $n=3$. With an investment cost of $I_{p}=26.7$ million NOK/licence, it is never optimal for the firm to make the first adoption earlier than $\bar{\tau}_{\theta_{1}^{*}}=7$, regardless of the additional flexibility. The investment cost is then higher than the benefit from lower lice costs. Likewise, the switching cost $I_{p}^{\prime}$ of 1 million NOK/licence, constraints the second switch to be done no earlier than $\bar{\tau}_{\theta_{2}^{*}}=14$, and the thirds switch no earlier than $\bar{\tau}_{\theta_{1}^{*}}=26$.

In addition, for $n=4$, we see that the firm only makes three switches, even though it has not reached its maximum number of switches. This is simply because the benefit from upgrading is now so small that it does not make up for the switching costs. As the firm will never make any additional switches as long as $I_{p}^{\prime}=1$ million NOK/licence, we do not run any results for $n>4$.

Table 6.1 also shows that the difference in technology levels $\zeta_{i}$, and expected adoption timing $\bar{\tau}_{\theta_{i}^{*}}$ increases for each switch the firm makes. The firm waits longer to make the switch from technology $\zeta_{2}$ to technology $\zeta_{3}$, than it does when switching from $\zeta_{1}$ to $\zeta_{2}$. This might seem surprising as the switching cost $I_{p}^{\prime}$ is constant, and so is the reduction in lice costs required to justify the switch. The reason for this can be explained by recalling the lice cost function of the permanent technology, presented in the beginning of Chapter 4. As seen from Figure 4.2, when the technology process reaches a higher level, the lice costs are reduced. However, the reduction is diminishing, meaning that technological improvements become less effective in reducing lice cost as the technology improves. This implies that the number of technological improvements needed to justify the upgrade from $\zeta_{2}$ to $\zeta_{3}$ is larger than from $\zeta_{1}$ to $\zeta_{2}$. As a result, the firm waits longer after each switch.

### 6.2 Sensitivity Analysis

In this section we will perform sensitivity analyses on the results for some of the parameters estimated in Chapter 5. We examine the effects of $\lambda_{p}$ and $\lambda_{l}$ as these carry the highest uncertainties, and $\alpha, I_{p}$, and $I_{p}^{\prime}$, as these were difficult to estimate. We start by checking the values for $I_{p}$ and $I_{p}^{\prime}$ for the results of the multiple switch model. However, for $\lambda_{p}, \lambda_{l}$ and $\alpha$, we only focus on sensitivity for the single switch model. This is for the sake of tractability as the models respond similarly to changes in the parameters.

### 6.2.1 Varying Investment Cost $I_{p}$

As described in Section 5.1, the investment cost $I_{p}$ of the permanent technology is difficult to quantify. Among the applicants for development licenses there are many different technology designs, and the investment cost depends on which technology proves to be most successful. We therefore do a sensitivity analysis on the parameter to see how the multiple switch results in Table 6.1 change with $I_{p}$. In order to do this, we look at a scenario where the investment cost is lower, as we considered an expensive scenario in the baseline case. It is reasonable to assume that the technologies being developed will have a lower cost once commercialised, compared to the cost of developing them the first time. We therefore test for an investment cost that is approximately $30 \%$ lower, at $I_{p}=20$ million NOK/license. The results are presented in Table 6.2

| $n$ | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| $V^{*}$ | $196.181 \cdot 10^{6}$ | $196.714 \cdot 10^{6}$ | $196.774 \cdot 10^{6}$ | $196.774 \cdot 10^{6}$ |
| $I_{u}^{*}$ | $3.039 \cdot 10^{6}$ | $2.971 \cdot 10^{6}$ | $2.891 \cdot 10^{6}$ | $2.891 \cdot 10^{6}$ |
| $\zeta_{1}$ | 0.75 | 0.6 | 0.45 | 0.45 |
| $\zeta_{2}$ | - | 1.65 | 1.35 | 1.35 |
| $\zeta_{3}$ | - | - | 2.7 | 2.55 |
| $\zeta_{4}$ | - | - | - | 6.15 |
| $\bar{\tau}_{\theta_{1}^{*}}$ | 5 | 4 | 3 | 3 |
| $\bar{\tau}_{\theta_{2}^{*}}$ | - | 11 | 9 | 9 |
| $\bar{\tau}_{\theta_{3}^{*}}$ | - | - | 18 | 17 |
| $\bar{\tau}_{\theta_{4}^{*}}$ | - | - | - | 41 |

Table 6.2: The value of the firm $V^{*}$ in NOK/licence, the optimal investment amount $I_{u}^{*}$ in NOK/licence, the optimal technology adoption level $\zeta_{i}$ and the expected adoption time after arrival in years $\bar{\tau}_{\theta_{i}^{*}}$, for the maximum number of switches $n$. Parameters used: $\pi_{0}=23.14 \cdot 10^{6}, c_{0}=6.89 \cdot 10^{6}$, $\bar{c}_{u}=3.45 \cdot 10^{6}, I_{p}=20.00 \cdot 10^{6}, I_{p}^{\prime}=1.00 \cdot 10^{6}, \lambda_{l}=0.2, \lambda_{p}=1.0, u=0.15, r=0.1, \alpha=1 \cdot 10^{-6}$, $\beta=0.2$.

In this scenario, the optimal technology adoption level, $\zeta_{1}$, has decreased. An investment cost of $I_{p}=20$ million NOK/licence means a lower reduction in lice costs is required to justify the investment. As a result, the firm value increases when the firm benefits from lower lice costs $c_{p}\left(\zeta_{i}\right)$ for a longer period. Note that also $\zeta_{2}, \zeta_{3}$ and $\zeta_{4}$ have decreased, which is surprising as the cost of adopting these technologies is the switching cost $I_{p}^{\prime}$ that remains unchanged. This means they are affected by the earlier adoption of $\zeta_{1}$. The firm now operates with higher lice costs $c_{p}\left(\zeta_{1}\right)$ after the first adoption, as $\zeta_{1}$ is lower compared to when $I_{p}$ was larger. As the cost of waiting is higher and the firm is eager to invest, the threshold of the following switches will be lower, and the firm will invest in technologies of lower optimal adoption level.

This explains why the firm now makes all four switches in $n=4$. In the original results, the value of making the fourth switch was too small to justify the switching cost of $I_{p}^{\prime}=1.0$ million NOK/licence. However, in this case, the firm makes the switch although $I_{p}^{\prime}$ is unchanged, which implies that the value of the switch has increased. Again, this is because $c_{p}\left(\zeta_{3}\right)$ is higher as a result of reduced investment cost $I_{p}$, meaning that the cost reduction of an extra switch is sufficiently large to justify the cost. In Figure 6.1, we illustrate the effect of $I_{p}$ on the technology levels $\zeta_{i}$ for $n=3$.


Figure 6.1: Optimal technology adoption levels $\zeta_{1}, \zeta_{2}, \zeta_{3}$, after first, second and third switch, as a function of the investment cost of the permanent technology, $I_{p}$. Parameters used: $\pi_{0}=23.14 \cdot 10^{6}, c_{0}=6.89 \cdot 10^{6}$,

$$
\bar{c}_{u}=3.45 \cdot 10^{6}, I_{p}^{\prime}=1.00 \cdot 10^{6}, \lambda_{l}=0.2, \lambda_{p}=1.0, u=0.15, r=0.1, \alpha=1 \cdot 10^{-6}, \beta=0.2
$$

Using Figure 6.1 to compare the optimal technology adoption levels for $I_{p}=26.7$ million NOK/licence and $I_{p}=20$ million NOK/licence, we conclude that the optimal technology levels are much higher for $I_{p}=26.7$ million NOK/licence. This is because a higher investment cost increases the incentive to wait before adopting a technology, in order for the technology process to reach a higher level.

At some point, $I_{p}$ reaches levels where it is no longer optimal to switch the technology, or adopt at all. This is illustrated by the peaks in the figure. Recall from Equation 4.14 for the optimal adoption threshold, that the condition for adoption is $I_{p}<\frac{c_{u}\left(I_{u}\right)}{r}$. When the investment cost becomes so high that it is more optimal to operate with lice costs $c_{u}\left(I_{u}^{*}\right)$ than to undertake the investment, the firm will never adopt the permanent technology. This is illustrated by the last peak in the figure. The first and second peak show the threshold for $I_{p}$ where it is never optimal to make the third and second switch, respectively.

For $I_{p}<12$ million NOK/licence, the firm adopts immediately because the value of adopting the permanent technology is higher than the investment cost, thus, $\zeta_{1}=0$. However, the optimal technology adoption level for the second and third switch are determined by the switching cost $I_{p}^{\prime}=1$ million NOK/licence, and are therefore higher than zero.

### 6.2.2 Varying Switching Cost $I_{p}^{\prime}$

Similar to the investment cost, there is high uncertainty related to the switching cost of the permanent technology. Therefore, we will check the results of the multiple switch model for a switching cost that is half of what was estimated in Section 5.1, for the same reason we tested a lower $I_{p}$. We set $I_{p}^{\prime}=0.5$ million NOK/licence and present the results in Table 6.3.

| $n$ | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| $V^{*}$ | $193.731 \cdot 10^{6}$ | $193.924 \cdot 10^{6}$ | $193.952 \cdot 10^{6}$ | $193.952 \cdot 10^{6}$ |
| $I_{u}^{*}$ | $3.212 \cdot 10^{6}$ | $3.145 \cdot 10^{6}$ | $3.145 \cdot 10^{6}$ | $3.145 \cdot 10^{6}$ |
| $\zeta_{1}$ | 1.35 | 1.05 | 1.05 | 1.05 |
| $\zeta_{2}$ | - | 2.10 | 1.95 | 1.95 |
| $\zeta_{3}$ | - | - | 3.15 | 3.15 |
| $\zeta_{4}$ | - | - | - | 6.00 |
| $\bar{\tau}_{\theta_{1}^{*}}$ | 9 | 7 | 7 | 7 |
| $\bar{\tau}_{\theta_{2}^{*}}$ | - | 14 | 13 | 13 |
| $\bar{\tau}_{\theta_{3}^{*}}$ | - | - | 21 | 21 |
| $\bar{\tau}_{\theta_{4}^{*}}$ | - | - | - | 40 |

Table 6.3: The value of the firm $V^{*}$ in NOK/licence, the optimal investment amount $I_{u}^{*}$ in NOK/licence, the optimal technology adoption level $\zeta_{i}$ and the expected adoption time after arrival in years $\bar{\tau}_{\theta_{i}^{*}}$, for the maximum number of switches $n$. Parameters used: $\pi_{0}=23.14 \cdot 10^{6}, c_{0}=6.89 \cdot 10^{6}$, $\bar{c}_{u}=3.45 \cdot 10^{6}, I_{p}=26.70 \cdot 10^{6}, I_{p}^{\prime}=0.50 \cdot 10^{6}, \lambda_{l}=0.2, \lambda_{p}=1.0, u=0.15, r=0.1, \alpha=1 \cdot 10^{-6}, \beta=0.2$.

Many of the same effects from changing $I_{p}$ are visible for a lower $I_{p}^{\prime}$. Earlier adoption results in an increase in firm value and decrease in investment amount. Compared to the original multiple switch results in Section 6.1.2, the fourth switch is now made due to both a higher value of the switch, and a lower switching cost. Note that the optimal adoption technology level $\zeta_{1}$ reaches its minimum at $n=2$ for the same value as in the original results, $\zeta_{1}=1.05$. This is because $\zeta_{1}$ is determined by the investment cost $I_{p}$ of the first adoption of the permanent technology, and all decisions made after $\zeta_{1}$. Consequently, it is less sensitive to the switching cost $I_{p}^{\prime}$ than $\zeta_{2}$ and $\zeta_{3}$, which are directly dependent on $I_{p}^{\prime}$. Figure 6.2 illustrates the effect of $I_{p}^{\prime}$ on the optimal technology adoption level for $n=3$.


Figure 6.2: Technology levels adopted after first, second and third switch, $\zeta_{1}, \zeta_{2}$ and $\zeta_{3}$, as a function of the switching cost of the permanent technology, $I_{p}^{\prime}$. Parameters used: $\pi_{0}=23.14 \cdot 10^{6}, c_{0}=6.89 \cdot 10^{6}, \bar{c}_{u}=3.45 \cdot 10^{6}$,

$$
I_{p}=26.70 \cdot 10^{6}, \lambda_{l}=0.2, \lambda_{p}=1.0, u=0.15, r=0.1, \alpha=1 \cdot 10^{-6}, \beta=0.2
$$

Figure 6.2 confirms that $\zeta_{1}, \zeta_{2}$ and $\zeta_{3}$ increase in $I_{p}^{\prime}$. Note also that $\zeta_{1}$ is the least sensitive to changes in $I_{p}^{\prime}$, as seen in Table 6.3. However, when $\zeta_{1}$ does increase for a higher $I_{p}^{\prime}$, it is because the threshold of adoption has increased due to more costly upgrades. On the other hand, $\zeta_{3}$ is the most sensitive to change. This is because the effects of a change in $I_{p}^{\prime}$ accumulate with every switch, and are therefore higher the later the switch. Further, we compare Figure 6.2 to Figure 6.1 where $I_{p}$ was varied, and note that the optimal technology adoption levels behave differently despite increasing levels in both. When varying $I_{p}$, it is the change in the first switch that delays the second and third switch. Therefore, the difference between $\zeta_{1}, \zeta_{2}$ and $\zeta_{3}$ stays relatively similar for all $I_{p}$. However, when varying $I_{p}^{\prime}$, the difference between the technology levels increases for higher $I_{p}^{\prime}$. This is caused by the accumulating effect as a result of the second and third switch both being directly affected by $I_{p}^{\prime}$. Finally, we note that the peaks in the figure represent the situation where the upgrading cost $I_{p}^{\prime}$ has become so high that the firm will not undertake the respective switch.

### 6.2.3 Varying Arrival Rates $\lambda_{l}$ and $\lambda_{p}$

To understand the effects a change in arrival rates $\lambda_{l}$ and $\lambda_{p}$ has on our model, we present a sensitivity analysis with respect to these two variables. Recall that these analyses are only done for the single switch model. In the analysis we will focus on the optimal investment threshold $\theta^{*}$, rather than the optimal technology adoption level, $\zeta_{1}$, as $\zeta_{1}$ is a function of $\theta^{*}$.

Figures 6.3 and 6.4 display the sensitivity of the investment threshold, $\theta^{*}$, and the expected optimal investment timing, $\bar{\tau}_{\theta}^{*}$, with respect to the arrival rate of improvements of the permanent technology, $\lambda_{p}$. The analyses are done for three different arrival rates of the permanent solution. $\lambda_{l}=0.1$ translates to an arrival in ten years, $\lambda_{l}=0.2$ implies arrival in five years similar to the baseline case, and $\lambda_{l}=0.3$ means the technology arrives in approximately three years. Based on our knowledge about the technologies, we find these estimates to be realistic.


Figure 6.3: Optimal investment threshold, $\theta^{*}$, as a function of arrival time of improvements in permanent technology, $\lambda_{p}$, for different arrival rates of permanent technology, $\lambda_{l}$. Parameters used: $\pi_{0}=23.14 \cdot 10^{6}$, $c_{0}=6.89 \cdot 10^{6}, \bar{c}_{u}=3.45 \cdot 10^{6}, I_{p}=26.70 \cdot 10^{6}, u=0.15, r=0.1, \alpha=1 \cdot 10^{-6}, \beta=0.2$.

As seen from Figure 6.3, $\theta^{*}$ increases in $\lambda_{p}$. To understand this, we look at the expression for $c_{p}\left(\theta^{*}\right)$ in Equation 4.15. The optimal lice-fighting costs from the permanent solution, $c_{p}\left(\theta^{*}\right)$, decrease in $\lambda_{p}$. The intuition behind this is that if technology improvements arrive sooner, the lice costs can reach a lower level before the firm chooses to invests. Consequently, the optimal threshold $\theta^{*}$ is higher for a higher arrival rate. It is worth noting that this only holds if $I_{p}<\frac{c_{u}}{r}$. If this condition is not satisfied, the investment cost is so high compared to the operational lice-fighting costs, that an investment will never become profitable, as seen in Equation 4.14.

Note that for large $\lambda_{p}$, larger $\lambda_{l}$ means a slightly lower investment threshold. This is because a firm expecting an early arrival of the permanent solution will invest less in short-term solutions. Consequently, the firm has higher costs while waiting to adopt the permanent solution, and is therefore more eager to undertake the investment. A firm expecting a late arrival, on the other hand, will invest more in short-term solutions, and have a smaller cost of waiting. In this case, the investment threshold is higher.


Figure 6.4: Expected adoption time of permanent technology, $\bar{\tau}_{\theta^{*}}$, as a function of arrival time of improvements in permanent technology, $\lambda_{p}$, for different arrival rates of permanent technology, $\lambda_{l}$. Parameters used: $\pi_{0}=23.14 \cdot 10^{6}, c_{0}=6.89 \cdot 10^{6}, \bar{c}_{u}=3.45 \cdot 10^{6}, I_{p}=26.70 \cdot 10^{6}, u=0.15, r=0.1, \alpha=1 \cdot 10^{-6}$, $\beta=0.2$.

Figure 6.4 shows that the expected adoption timing, $\bar{\tau}_{\theta^{*}}$, decreases in $\lambda_{p}$. The intuition is that when improvements of the permanent technology arrive more often, the permanent solution becomes more attractive. Hence, the firm is more eager to invest in order to benefit from the lower lice-fighting costs sooner. We also see from Figure 6.4 that the expected adoption timing $\bar{\tau}_{\theta^{*}}$ decreases marginally in $\lambda_{l}$. The intuition is the same as for $\theta^{*}$. The low sensitivity to $\lambda_{l}$ shown in both figures, implies that the base case is very robust to changes in the arrival rate.

Note that the increase in $\lambda_{p}$ reduces the optimal expected adoption timing (Figure 6.4), but increases the optimal investment threshold (Figure 6.3). This can be interpreted as contradictory, and we therefore recall the expression for $\bar{\tau}_{\theta^{*}}$ in Equation 4.16. From the expression, we see that an increase in $\lambda_{p}$ will increase $\theta^{*}$, but at the same time decrease the whole term of $\bar{\tau}_{\theta^{*}}$. However, as the expression for $\theta^{*}$ is logarithmic, the increase in $\theta^{*}$ will have a smaller impact than the decrease of the whole term. This causes $\bar{\tau}_{\theta^{*}}$ to decrease for a higher arrival rate of permanent technology improvements, as this is the dominating effect.

In a more intuitive way, this can be explained as two contradicting incentives. On one hand, the firm is eager to reduce the time it operates with lice costs $c_{u}\left(I_{u}^{*}\right)$, when improvements arrive more often. On the other hand, it has an incentive to wait before adoption in order to let the technology reach a level that results in even lower permanent lice costs. The dominating effect is the incentive to invest earlier. This is because it is more costly to operate a longer period until adoption with $c_{u}\left(I_{u}^{*}\right)$, than to operate from $\bar{\tau}_{\theta^{*}}$ to infinity with slightly lower $c_{p}\left(\theta^{*}\right)$. The reason for why the permanent lice costs are only slightly lower for higher arrival rate $\lambda_{p}$, is the diminishing reduction in lice costs for higher technology level $\theta$.

Summarising Figures 6.3 and 6.4, a higher arrival rate of the improvements leads to the firm investing earlier in time, and at a higher technology level.


Figure 6.5: Optimal investment amount in short-term solutions, $I_{u}^{*}$, as a function of arrival time of improvements in permanent technology, $\lambda_{p}$, for different arrival rates of permanent technology, $\lambda_{l}$.
Parameters used: $\pi_{0}=23.14 \cdot 10^{6}, c_{0}=6.89 \cdot 10^{6}, \bar{c}_{u}=3.45 \cdot 10^{6}, I_{p}=26.70 \cdot 10^{6}, u=0.15, r=0.1, \alpha=1 \cdot 10^{-6}$, $\beta=0.2$.

Figure 6.5 shows that the optimal investment amount $I_{u}^{*}$ decreases in $\lambda_{p}$. If improvements arrive more often, we concluded that the firm would invest earlier, at a higher technology level $\theta^{*}$. This decreases the incentive to invest in short-term solutions, as adopting a permanent technology makes these solutions obsolete. Hence, the optimal investment amount is lower. From Figure 6.5, we also see that $I_{u}^{*}$ decreases in $\lambda_{l}$. The intuition is that if the permanent solution arrives sooner, the incentive to invest in temporary, short-term solutions decreases. This effect is strengthened if the arrival rate of permanent improvements $\lambda_{p}$ is large, making the permanent solution more attractive.

Figure 6.6 shows that the maximum firm value $V^{*}$ increases in $\lambda_{p}$. The maximum value of the firm $V^{*}$ is higher when the arrival of improvements in permanent technology is higher. This is a result of how an increase in $\lambda_{p}$ affects $\theta^{*}, \bar{\tau}_{\theta_{i}^{*}}$ and $I_{u}^{*}$. If a firm invests earlier in a permanent technology, for a higher investment threshold, it benefits from low lice costs $c_{p}\left(\theta^{*}\right)$ for a longer period. Hence, the value of the firm increases, as illustrated in Figure 6.6. We also see that $V^{*}$ increases in $\lambda_{l}$. If a permanent technology arrives sooner, the firm operates with lice costs $c_{u}\left(I_{u}^{*}\right)$ for a shorter period. Because $c_{u}\left(I_{u}^{*}\right)>c_{p}\left(\theta^{*}\right)$, this increases the firm value.


Figure 6.6: Maximum firm value, $V^{*}$, as a function of arrival time of improvements in permanent technology, $\lambda_{p}$, for different arrival rates of permanent technology, $\lambda_{l}$. Parameters used: $\pi_{0}=23.14 \cdot 10^{6}, c_{0}=6.89 \cdot 10^{6}$,

$$
\bar{c}_{u}=3.45 \cdot 10^{6}, I_{p}=26.70 \cdot 10^{6}, u=0.15, r=0.1, \alpha=1 \cdot 10^{-6}, \beta=0.2
$$

### 6.2.4 Varying Cost Reduction Factor of Short-Term Investments, $\alpha$



Figure 6.7: Optimal investment amount, $I_{u}^{*}$, and maximum firm value, $V^{*}$, as functions of the cost reduction factor for short-term investments, $\alpha$. Parameters used: $\pi_{0}=23.14 \cdot 10^{6}, c_{0}=6.89 \cdot 10^{6}$, $\bar{c}_{u}=3.45 \cdot 10^{6}, I_{p}=26.70 \cdot 10^{6}, \lambda_{l}=0.2, \lambda_{p}=1.0, u=0.15, r=0.1, \beta=0.2$.

From Figure 6.7, we observe that the optimal investment amount, $I_{u}^{*}$, is lower for a higher $\alpha$. A high cost reduction factor of the lice-fighting costs, implies that investments are more effective, allowing the firm to invest less. Consequently, the firm value also increases when the present value of cash flows increases. Note that for very low values of $\alpha$, it is never optimal to invest in short-term technologies. This is because the investment $I_{u}$ does not reduce the lice-fighting costs sufficiently to justify the investment amount. More specifically, for the parameter values set, $\alpha$ must be higher than $1.36 \cdot 10^{-7}$ for it ever to be optimal to invest in the short-term solutions.

### 6.3 Traffic Light System Results

Table 6.4 shows the results of Equation 4.36, for the deterministic set of $R \in\{1,2,3,4\}$ regulations in the traffic light system. The values displayed are found by using the parameter values estimated in Chapter 5 for Flatanger.

| $R$ | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| $V^{*}$ | $198.190 \cdot 10^{6}$ | $200.833 \cdot 10^{6}$ | $202.416 \cdot 10^{6}$ | $203.362 \cdot 10^{6}$ |
| $I_{u}^{*}$ | $3.212 \cdot 10^{6}$ | $3.212 \cdot 10^{6}$ | $3.212 \cdot 10^{6}$ | $3.212 \cdot 10^{6}$ |
| $\zeta_{1}$ | 1.35 | 1.35 | 1.35 | 1.35 |
| $I_{T_{0}}^{*}$ | $2.317 \cdot 10^{6}$ | $2.330 \cdot 10^{6}$ | $2.336 \cdot 10^{6}$ | $2.340 \cdot 10^{6}$ |
| $I_{T_{1}}^{*}$ | - | $1.915 \cdot 10^{6}$ | $1.925 \cdot 10^{6}$ | $1.931 \cdot 10^{6}$ |
| $I_{T_{2}}^{*}$ | - | - | $1.583 \cdot 10^{6}$ | $1.591 \cdot 10^{6}$ |
| $I_{T_{3}}^{*}$ | - | - | - | $1.308 \cdot 10^{6}$ |

Table 6.4: Firm value $V^{*}$ in NOK/licence in Flatanger, optimal investment amount $I_{u}^{*}$ in NOK/licence, optimal technology adoption level $\zeta_{1}$, and optimal additional investment $I_{T_{k-1}}^{*}$ for deterministic set of regulations $R$. Parameters used: $\pi_{0}=23.14 \cdot 10^{6}, c_{0}=6.89 \cdot 10^{6}, \bar{c}_{u}=3.45 \cdot 10^{6}$, $I_{p}=26.70 \cdot 10^{6}, \bar{I}_{T}=3.00 \cdot 10^{6}, I_{G}=1.00 \cdot 10^{6}, \lambda_{l}=0.2, \lambda_{p}=1.0, u=0.15, r=0.1, \alpha=1 \cdot 10^{-6}$, $\beta=0.2, \gamma_{G}=2, \gamma_{Y}=6, \gamma_{R}=7, \mu_{G}=1 \cdot 10^{-6}, \mu_{Y}=2 \cdot 10^{-6}, \mu_{R}=8 \cdot 10^{-6}$.

As the traffic light system model is an extension of the single switch model, we compare the values of Table 6.4 to the single switch results in Table 6.1 for $n=1$. As expected, the investment strategy of the firm does not change with the traffic light system (see Section 4.3). $I_{u}^{*}=3.212$ million NOK/licence, and $\zeta_{1}=1.35$, as previously. As discussed, the additional investment in delousing treatments, $I_{T_{k-1}}^{*}$, is a part of operational lice-fighting costs, and not the investment $I_{u}$. Hence, when solving for the optimal investment amount, we do not take into consideration the amount invested due to the traffic light system. In addition, recall that the traffic light system is assumed to stop when the permanent technology arrives, and thus, the system does not have impact on the adoption timing of the permanent technology.

We see that the firm values in the traffic light model are significantly larger than in the single switch model where $V^{*}=193.731$ million NOK/licence. As illustrated in Section 5.5, Flatanger is located in a region where additional investments have a relatively high impact on the outcome of the traffic light system. As a result, the traffic light system is beneficial for a firm in this region. This can also be seen by the increase in firm value for a higher number of regulations. However, the increase is declining because regulations happen every two years, and the expected arrival time of the permanent technology is estimated to be five years. As the number of regulations increases, so does the probability of arrival of the permanent technology during the next two-year period. The firm then faces a higher risk of investing $I_{T_{k-1}}^{*}$
in the beginning of a period because the arrival of the permanent solution might induce the government to stop the traffic light system. Hence, the firm might lose the payoff from the investment. For the same reason, $I_{T_{k-1}}^{*}$ decreases for each regulation $k$. Note that the values for $I_{T_{k-1}}^{*}$ in the table are discounted to time zero by $r=0.1$.

We can also see from Table 6.4 that a higher number of regulations $R$, leads the firm to make larger additional investments. For example, $I_{T_{0}}^{*}=2.317$ million NOK/licence in the case of $R=1$ regulations, compared to $I_{T_{0}}^{*}=2.330$ million NOK/licence when there are $R=2$ regulations. The accumulated gain from expanding an already regulated production gives incentive to invest more, in order to achieve a green light.

In what follows, we present the results of the traffic light system for two regions with extreme cases of lice pressure, in order to compare them with the results for Flatanger. The first region is Finnmark where the lice pressure is low, and the second region is Hardanger where the lice pressure is high. Recall Figure 5.1 in section 5.5 that illustrates the probability functions for the outcome of the system in each region. To solve for the two cases, we change the constants, $\gamma$, and the scale factors, $\mu$, according to what was presented in Section 5.5. Recall that the constants determine the initial probabilities of the outcome of the system, whereas the scale parameters determine the effect of $I_{T_{k-1}}^{*}$ on the probabilities of the outcome. We keep the lice-fighting cost and profit parameters for Bjørøya in order to get comparable results. The results are presented in Table 6.5, and in Table 6.6.

| $R$ | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| $V^{*}$ | $199.572 \cdot 10^{6}$ | $202.966 \cdot 10^{6}$ | $204.957 \cdot 10^{6}$ | $206.125 \cdot 10^{5}$ |
| $I_{u}^{*}$ | $3.212 \cdot 10^{6}$ | $3.212 \cdot 10^{6}$ | $3.212 \cdot 10^{6}$ | $3.212 \cdot 10^{5}$ |
| $\zeta_{1}$ | 1.35 | 1.35 | 1.35 | 1.35 |
| $I_{T_{0}}^{*}$ | $8.212 \cdot 10^{5}$ | $8.354 \cdot 10^{5}$ | $8.434 \cdot 10^{5}$ | $8.495 \cdot 10^{5}$ |
| $I_{T_{1}}^{*}$ | - | $6.939 \cdot 10^{5}$ | $7.054 \cdot 10^{5}$ | $7.120 \cdot 10^{5}$ |
| $I_{T_{2}}^{*}$ | - | - | $5.857 \cdot 10^{5}$ | $5.952 \cdot 10^{5}$ |
| $I_{T_{3}}^{*}$ | - | - | - | $4.939 \cdot 10^{5}$ |

Table 6.5: Firm value $V^{*}$ in NOK/licence in Finnmark, optimal investment amount $I_{u}^{*}$ in NOK/licence, optimal technology adoption level $\zeta_{1}$, and optimal additional investment $I_{T_{k-1}}^{*}$ for deterministic set of regulations $R$. Parameters used: $\pi_{0}=23.14 \cdot 10^{6}, c_{0}=6.89 \cdot 10^{6}, \bar{c}_{u}=3.45 \cdot 10^{6}$, $I_{p}=26.70 \cdot 10^{6}, \bar{I}_{T}=3.00 \cdot 10^{6}, I_{G}=1.00 \cdot 10^{6}, \lambda_{l}=0.2, \lambda_{p}=1.0, u=0.15, r=0.1, \alpha=1 \cdot 10^{-6}$, $\beta=0.2, \gamma_{G}=10, \gamma_{Y}=9, \gamma_{R}=8, \mu_{G}=2 \cdot 10^{-6}, \mu_{Y}=0.1 \cdot 10^{-6}, \mu_{R}=3 \cdot 10^{-6}$.

The low lice pressure in Finnmark is reflected in the high firm values $V^{*}$, and low additional investments $I_{T_{k-1}}^{*}$, in Table 6.5. For this region, the traffic light system is highly beneficial and allows expansion of production for relatively low additional investments.

| $R$ | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| $V^{*}$ | $187.273 \cdot 10^{6}$ | $183.703 \cdot 10^{6}$ | $181.772 \cdot 10^{6}$ | $180.727 \cdot 10^{6}$ |
| $I_{u}^{*}$ | $3.212 \cdot 10^{6}$ | $3.212 \cdot 10^{6}$ | $3.212 \cdot 10^{6}$ | $3.212 \cdot 10^{6}$ |
| $\zeta_{1}$ | 1.35 | 1.35 | 1.35 | 1.35 |
| $I_{T_{0}}^{*}$ | $3.000 \cdot 10^{6}$ | $3.000 \cdot 10^{6}$ | $3.000 \cdot 10^{6}$ | $3.000 \cdot 10^{6}$ |
| $I_{T_{1}}^{*}$ | - | $2.456 \cdot 10^{6}$ | $2.456 \cdot 10^{6}$ | $2.456 \cdot 10^{6}$ |
| $I_{T_{2}}^{*}$ | - | - | $2.011 \cdot 10^{6}$ | $2.011 \cdot 10^{6}$ |
| $I_{T_{3}}^{*}$ | - | - | - | $1.646 \cdot 10^{6}$ |

Table 6.6: Firm value $V^{*}$ in NOK/licence in Hardanger, optimal investment amount $I_{u}^{*}$ in NOK/licence, optimal technology adoption level $\zeta_{1}$, and optimal additional investment $I_{T_{k-1}}^{*}$ for deterministic set of regulations $R$. Parameters used: $\pi_{0}=23.14 \cdot 10^{6}, c_{0}=6.89 \cdot 10^{6}, \bar{c}_{u}=3.45 \cdot 10^{6}$, $I_{p}=26.70 \cdot 10^{6}, \bar{I}_{T}=3.00 \cdot 10^{6}, I_{G}=1.00 \cdot 10^{6}, \lambda_{l}=0.2, \lambda_{p}=1.0, u=0.15, r=0.1, \alpha=1 \cdot 10^{-6}$, $\beta=0.2, \gamma_{G}=10, \gamma_{Y}=9, \gamma_{G}=1, \gamma_{Y}=7, \gamma_{R}=10, \mu_{G}=0.1 \cdot 10^{-6}, \mu_{Y}=2 \cdot 10^{-6}, \mu_{R}=2.5 \cdot 10^{-6}$.

Table 6.6 for Hardanger, shows that the high lice pressure results in firm values that are drastically lower than for both Flatanger and Finnmark. Further, the firm value now also decreases for an increase of regulations. This is because the probability of receiving a red light is so high that the firm is expected to reduce its production capacity by $6 \%$ every two years. Hence, a higher number of regulations means more reductions, and a lower firm value. Note that in the results for Flatanger and Finnmark, the optimal investments in additional treatments are not constrained by the upper limit, $\bar{I}_{T}=3$ million NOK/licence. However, for Hardanger in year zero, the firm will make investments of $I_{T_{0}}^{*}=3$ million NOK/licence at the upper limit $\bar{I}_{T}$, because the expected gain from the traffic light system still is larger than the additional investment. However, if the upper limit was set to $\bar{I}_{T}=2.5$ million NOK/licence, the impact on the probabilities would be too low, and making the investment would most likely lead to additional loss. In that case, the firm would chose not to invest in additional treatments.

We now check the results for Hardanger in a scenario where we assume there are no biological constraints and thus, no upper limit on the optimal investment in additional treatments. The results are given in Table 6.7.

| $R$ | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| $V^{*}$ | $195.529 \cdot 10^{6}$ | $196.707 \cdot 10^{6}$ | $197.485 \cdot 10^{6}$ | $197.988 \cdot 10^{6}$ |
| $I_{u}^{*}$ | $3.212 \cdot 10^{6}$ | $3.212 \cdot 10^{6}$ | $3.212 \cdot 10^{6}$ | $3.212 \cdot 10^{6}$ |
| $\zeta_{1}$ | 1.35 | 1.35 | 1.35 | 1.35 |
| $I_{T_{0}}^{*}$ | $4.914 \cdot 10^{6}$ | $4.927 \cdot 10^{6}$ | $4.935 \cdot 10^{6}$ | $4.940 \cdot 10^{6}$ |
| $I_{T_{1}}^{*}$ | - | $4.043 \cdot 10^{6}$ | $4.055 \cdot 10^{6}$ | $4.061 \cdot 10^{6}$ |
| $I_{T_{2}}^{*}$ | - | - | $3.327 \cdot 10^{6}$ | $3.360 \cdot 10^{6}$ |
| $I_{T_{3}}^{*}$ | - | - | - | $2.737 \cdot 10^{6}$ |

Table 6.7: Firm value $V^{*}$ in NOK/licence in Hardanger, optimal investment amount $I_{u}^{*}$ in NOK/licence, optimal technology adoption level $\zeta_{1}$, and optimal additional investment $I_{T_{k-1}}^{*}$ for deterministic set of regulations $R$. Parameters used: $\pi_{0}=23.14 \cdot 10^{6}, c_{0}=6.89 \cdot 10^{6}, \bar{c}_{u}=3.45 \cdot 10^{6}$, $I_{p}=26.70 \cdot 10^{6}, I_{G}=1.00 \cdot 10^{6}, \lambda_{l}=0.2, \lambda_{p}=1.0, u=0.15, r=0.1, \alpha=1 \cdot 10^{-6}, \beta=0.2, \gamma_{G}=10$, $\gamma_{Y}=9, \gamma_{G}=1, \gamma_{Y}=7, \gamma_{R}=10, \mu_{G}=0.1 \cdot 10^{-6}, \mu_{Y}=2 \cdot 10^{-6}, \mu_{R}=2.5 \cdot 10^{-6}$.

Compared to when $I_{T_{k-1}}$ was constrained, the firm values are now higher and increasing for a larger number of regulations. However, the optimal additional investments are now more than twice as high as optimal investments in the case of Flatanger. This implies that the firm will have to make large additional investments in order to achieve a firm value that is only slightly higher than it would be without the traffic light system. Based on this, and the results from the more realistic scenario with biological constraints, we argue that for regions such as Hardanger that struggle with very high lice levels, the traffic light system will have a large and negative impact on the firms.

## Chapter 7

## Concluding Remarks

In this thesis, we have aimed to find the optimal adoption timing of a permanent lice-fighting technology, and the optimal investment amount in short-term, temporary solutions before it is adopted. We will now present what we consider the main findings of this study, and suggestions for further work.

Our results show four main findings: (i) The high investment cost of the permanent solution causes the firm to wait a relatively long time after the arrival of a permanent solution, before adopting it. This again causes the firm to invest a significant amount in currently available technologies in order to lower the lice-fighting cost until it is optimal to adopt the permanent technology. This is interesting, as firms currently tend to invest rapidly in new and unproven technologies once they hit the market.
(ii) When we extend the basic model to allow for sequential investments, the firm value increases because of the increased flexibility. The firm will then adopt the permanent technology earlier, and upgrade it at a lower cost. However, because of diminishing reduction of lice cost, the firm only performs a limited number of upgrades before it is no longer profitable. That said, the number of upgrades are highly dependent on the investment cost and switching cost - the smaller the costs, the more upgrades the firm performs.
(iii) The traffic light system works as intended from the government's perspective. The policy implication on production is that it will be reduced in regions currently experiencing high lice pressure, and increased in regions with low lice pressure. However, if the government also aimed to stimulate additional lice-fighting actions, our results show that this depends on the biological constraints. Firms in regions with low lice levels will make investments within the biological constraints. However, firms in high lice level regions will either make investments significantly exceeding the other regions, or refrain from investing completely to avoid additional loss.
(iv) The implication of the traffic light system on firms in high lice level regions is a significant reduction in firm value. The regulations will have a large, negative impact as the firm cannot make adequate investments in treatments to impact the outcome. However, for firms in regions with lower lice level, such as in Flatanger and Finnmark, the implication of the policy is a significant increase in firm value. Due to lower lice levels, the firms are not limited by the biological constraints, and can make additional investments to achieve a green light.

The model in our thesis is based on the work of Huisman (2000). We added a short-term solution dimension and adapted the model to fit the investment problem of an aquaculture firm. The optimal adoption time was found by solving an optimal stopping problem, and we incorporated the uncertainty in arrival and improvements of the permanent technology. We further modelled the technological innovations as improvement of the operational licefighting costs. The real options model was combined with the net present value method in order to solve for the optimal investment amount in temporary, short-term technologies. This was further extended to allow for multiple technology switches, and to incorporate the effects of the traffic light system. This allowed us to consider the effect the traffic light system has on the investment strategy and value of the firm, and to find the optimal investments in additional treatments. In addition, we analysed a case study of firms in three regions with different lice levels. Finally, we developed a numerical algorithm to solve the model. We have estimated the parameter values in collaboration with industry experts, however many of the estimates carry high uncertainty.

### 7.1 Suggestions for Further Work

## Increase in Operating Costs

All current suggestions for permanent solutions to the lice problem are far more complex structures than the pens used in the industry today. More complex structures means higher operating costs, something that is also likely to decline as the technology matures. An interesting extension to our model would therefore be to see the effect of increased operating costs when the first adoption of the permanent technology is made. These costs should decline as the technology matures, which would improve the accuracy of the model.

## Indirect Cost of Treatments Included in Lice-Fighting Costs

We suggest including a component that incorporates the death rate of fish and lost growth, as a result of treatments. As discussed in Section 5.1, there are high, unrecorded costs related to treatment of fish, as a result of diseases, death and lost growth due to starvation prior to
the delousing treatment. However, these costs can vary with the technology used, the fish health and with the number of previous treatments. To include these factors in the model, a component can be added to the lice cost function $c_{u}\left(I_{u}\right)$. It can also be used to limit the investments in additional treatments.

## Game Theoretic Model for Traffic Light

In this model, we have assumed the behaviour of other firms in the same traffic light region to be completely exogenous to the model. However, for regions with only a few, but large actors, the lice-fighting strategy of a single firm can have a great impact on the traffic light outcome of the entire region. This means that the strategies chosen by one firm in the region may affect the strategies chosen by the others. Therefore, a possible extension of the problem is to create a game theoretic model to find the optimal lice-fighting strategy for a large actor in a region with other large actors.

## Regulation of Lice-Fighting Costs

We have assumed that lice-fighting costs do not change after a traffic light regulation has occurred. This is because the lice costs are not directly dependent on the number of fish in the nets, as significant part of lice-fighting costs are fixed. However, it would be interesting to see how incorporating the regulations of the lice-fighting costs would affect the outcome of the model. If the costs are to be regulated, the lice-fighting costs, $c_{u}$ and $c_{p}$, would have to be a function of both the MAB, and the investments in short-term solutions and technology level, respectively. As a suggestion we write up the maximisation function for a single traffic light regulation below.

$$
\begin{aligned}
V^{*}=\max _{I_{u}, I_{T}, \tau_{\theta}} \mathbb{E}\{ & \mathbb{T}_{\left\{\tau_{\lambda_{l}}<2\right\}}\left[\int_{0}^{t+\tau_{\theta}}\left(\pi_{0}-c_{u}\left(I_{u}, M_{0}\right)\right) e^{-r s} d s+\int_{t+\tau_{\theta}}^{\infty}\left(\pi_{0}-c_{p}\left(\theta, M_{0}\right)\right) e^{-r s} d s\right] \\
& +\mathbb{1}_{\left\{\tau_{\lambda_{l}}>2\right\}}\left[\int_{0}^{2}\left(\pi_{0}-c_{u}\left(I_{u}, M_{0}\right)\right) e^{-r s} d s+\int_{2}^{t+\tau_{\theta}}\left(\pi_{0} \cdot P_{T}\left(I_{T}\right)-c_{u}\left(I_{u}, M_{1}\right)\right) e^{-r s} d s\right. \\
& \left.+\int_{t+\tau_{\theta}}^{\infty}\left(\pi_{0} \cdot P_{T}\left(I_{T}\right)-c_{p}\left(\theta, M_{1}\right)\right) e^{-r s} d s-P_{G}\left(I_{T}\right) \cdot I_{G} e^{-2 r}\right] \\
& \left.-I_{u}-I_{T}-I_{p} e^{-r\left(t+\tau_{\theta}\right)}\right\} .
\end{aligned}
$$

$M_{0}$ is the MAB before the regulation, and $M_{1}$ is the MAB after the regulation. Additional functions must be developed to describe the relation between the lice-fighting costs and the MAB.

## Accurate Probabilities in the Traffic Light Model

The parameters $\gamma$ and $\mu$ define the probabilities of the outcome of the traffic light system. It would therefore be interesting to estimate the parameters by an in-depth analysis of biological conditions in different geographical regions. Our suggestion would be to use the statistical grounds from the weekly lice counting performed by fish farms, and make a formal model for the probabilities.

## Volatile Salmon Prices

We assumed the profit flow per license to be constant, but in reality, it varies with the price of salmon that changes over time and is highly volatile. For further work, uncertainty in the salmon price could be incorporated into the model. This, however, comes at a cost of significant complication of the analysis.

## Appendix A

## Variables and Parameters

| Decision <br> variable | Description | Unit |
| :--- | :--- | :--- |
| $\theta^{*}$ | Optimal threshold of permanent technology level | - |
| $\bar{\tau}_{\theta^{*}}$ | Expected adoption time of permanent technology | Years |
| $I_{u}^{*}$ | Optimal investment amount in short-term technologies | NOK/licence |
| $I_{T_{k-1}}^{*}$ | Optimal additional investment in delousing treatments | NOK/licence |

Table A.1: Decision variables

| Variable | Description | Unit |
| :--- | :--- | :--- |
| $V$ | Firm value | NOK/licence |
| $f$ | Firm value | NOK/licence |
| $g$ | Firm value | NOK/licence |
| $c_{u}$ | Lice-fighting costs from short-term investment | NOK/licence |
| $c_{p}$ | Lice-fighting costs from permanent technology | NOK/licence |
| $\tau_{\lambda_{l}}$ | Expected arrival time of permanent technology | Years |
| $\zeta_{i}$ | Optimal technology adoption level of permanent technology | - |
| $P_{G}$ | Probability of green traffic light | - |
| $P_{Y}$ | Probability of yellow traffic light | - |
| $P_{R}$ | Probability of red traffic light | - |
| $P_{T}$ | Production capacity change factor | - |


| Parameter | Description | Unit |
| :--- | :--- | :--- |
| $\pi_{0}$ | Profit flow before lice-fighting costs | NOK/licence |
| $c_{0}$ | Initial lice-fighting costs | NOK/licence |
| $\bar{c}_{u}$ | Lowest boundary of lice-fighting costs from short-term | NOK/licence |
|  | investments |  |
| $I_{p}$ | Investment cost of permanent technology for single switch | NOK/licence |
| $I_{p}^{\prime}$ | Switching cost of permanent technology | NOK/licence |
| $\bar{I}_{T}$ | Maximum limit of investments in additional treatments | NOK/licence |
| $I_{G}$ | Production expansion cost | NOK/licence |
| $\lambda_{l}$ | Arrival rate of permanent technology | - |
| $\lambda_{p}$ | Arrival rate of upgrades of permanent technology | - |
| $u^{\prime}$ | Dump size of upgrades of permanent technology | - |
| $r$ | Cost reduction factor of short-term investments | - |
| $\alpha^{\prime}$ | Cost reduction factor of permanent technology | - |
| $\beta$ | Initial probability constant for green light | - |
| $\gamma_{G}$ | Initial probability constant for yellow light | - |
| $\gamma_{Y}$ | Initial probability constant for red light | - |
| $\gamma_{R}$ | Scale factor of green light probability | - |
| $\mu_{G}$ | Scale factor of yellow light probability | - |
| $\mu_{Y}$ | Scale factor of red light probability | - |

Table A.3: Parameters

| Index | Description |
| :--- | :--- |
| $i$ | Number of switches made |
| $j$ | Technology firm uses |
| $k$ | Number of regulation |
| $n$ | Maximum switches allowed |
| $N$ | Number of technologies available |
| $R$ | Deterministic set of regulations |

Table A.4: Indices

## Appendix B

## Derivations

## B. 1 Multiple Switch

Calculations of Equations (4.20), (4.19), and (4.21).

## B.1.1 First Switch

$$
F_{1}\left(c_{p}(\theta),\left(c_{u}\right)\right)= \begin{cases}k_{1}\left(\frac{\lambda_{p}}{\lambda_{p}+r}\right)^{-\frac{\theta}{u}}+\frac{\pi\left(c_{u}\right)\left(r+\lambda_{p}\right)}{r}, & \text { if } \theta<\theta_{1}^{*}-u  \tag{B.1}\\ \frac{\pi\left(c_{u}\right)}{r+\lambda_{p}}+\frac{\lambda_{p}}{r+\lambda_{p}}\left(F_{2}\left(c_{p}(\theta+u), c_{p}(\theta+u)\right)-I_{p}\right), & \text { if } \theta_{1}^{*}-u \leq \theta<\theta_{1}^{*} \\ F_{2}\left(c_{p}(\theta), c_{p}(\theta)\right)-I_{p}, & \text { if } \theta \geq \theta_{1}^{*}\end{cases}
$$

For $F_{1}\left(c_{p}(\theta), c_{u}\right)$ to be continuous at $\theta=\theta_{1}^{*}-u$, we solve the continuity condition at $\theta=\theta_{1}^{*}$ and find that

$$
\begin{equation*}
k_{1}=\left(\frac{\lambda_{p}}{\lambda_{p}+r}\right)^{\frac{\theta_{1}^{*}}{u}}\left[\frac{\pi\left(c_{u}\right)}{\lambda_{p}}\left(1-\frac{\left(r+\lambda_{p}\right)^{2}}{r}\right)+F_{2}\left(c_{p}\left(\theta_{1}^{*}\right), c_{p}\left(\theta_{1}^{*}\right)\right)-I_{p}\right] . \tag{B.2}
\end{equation*}
$$

Inserting $k_{1}$ into (B.1) gives Equation 4.19.

## B.1.2 Last Switch

$$
F_{n}\left(c_{p}(\theta),\left(c_{p}\left(\zeta_{n-1}\right)\right)= \begin{cases}k_{n}\left(\frac{\lambda_{p}}{\lambda_{p}+r}\right)^{-\frac{\theta}{u}}+\frac{\pi\left(c_{p}\left(\zeta_{n-1}\right)\right)\left(r+\lambda_{p}\right)}{r}, & \text { if } \theta<\theta_{n}^{*}-u  \tag{B.3}\\ \frac{\pi\left(c_{p}\left(\zeta_{n-1}\right)\right.}{r+\lambda_{p}}+\frac{\lambda_{p}}{r+\lambda_{p}}\left(V\left(c_{p}(\theta+u)\right)-I_{p}^{\prime}\right), & \text { if } \theta_{n}^{*}-u \leq \theta<\theta_{n}^{*}, \\ V\left(c_{p}(\theta)\right)-I_{p}^{\prime}, & \text { if } \theta \geq \theta_{n}^{*},\end{cases}\right.
$$

Similar to the simple switch case, for $F_{n}\left(c_{P}(\theta),\left(c_{p}\left(\zeta_{n-1}\right)\right)\right.$ to be continuous at $\theta=\theta_{n}^{*}-u$, we solve the continuity condition and find that

$$
\begin{equation*}
k_{n}=\left(\frac{\lambda_{p}}{\lambda_{p}+r}\right)^{\frac{\theta_{n}^{*}}{u}}\left[\frac{\pi\left(c_{p}\left(\zeta_{n-1}\right)\right)}{\lambda_{p}}\left(1-\frac{\left(r+\lambda_{p}\right)^{2}}{r}\right)+V\left(c_{p}\left(\theta_{n}^{*}\right)\right)-I_{p}^{\prime}\right] . \tag{B.4}
\end{equation*}
$$

Inserting $k_{n}$ into Equation (B.3) gives Equation 4.20.

## B.1.3 $i$-th Switch

$$
F_{i}\left(c_{p}(\theta),\left(\zeta_{i-1}\right)= \begin{cases}k_{i}\left(\frac{\lambda_{p}}{\lambda_{p}+r}\right)^{-\frac{\theta}{u}}+\frac{\pi\left(\zeta_{i-1}\right)\left(r+\lambda_{p}\right)}{r}, & \text { if } \theta<\theta_{i}^{*}-u  \tag{B.5}\\ \frac{\pi\left(\zeta_{i-1}\right)}{r \lambda_{p}}+\frac{\lambda_{p}}{r+\lambda_{p}}\left(F_{i+1}\left(c_{p}(\theta+u), c_{p}(\theta+u)\right)-I_{p}^{\prime}\right), & \text { if } \theta_{i}^{*}-u \leq \theta<\theta_{i}^{*} \\ F_{i+1}\left(c_{p}(\theta), c_{p}(\theta)\right)-I_{p}^{\prime}, & \text { if } \theta \geq \theta_{i}^{*}\end{cases}\right.
$$

For $F_{i}\left(c_{p}(\theta), c_{u}\right)$ to be continuous at $\theta=\theta_{i}^{*}-u$, we solve the continuity condition at $\theta=\theta_{i}^{*}$ and find that

$$
\begin{equation*}
k_{i}=\left(\frac{\lambda_{p}}{\lambda_{p}+r}\right)^{\frac{\theta_{i}^{*}}{u}}\left[\frac{\pi\left(\zeta_{i-1}\right)}{\lambda_{p}}\left(1-\frac{\left(r+\lambda_{p}\right)^{2}}{r}\right)+F_{i+1}\left(c_{p}\left(\theta_{i}^{*}\right), c_{p}\left(\theta_{i}^{*}\right)\right)-I_{p}^{\prime}\right] \tag{B.6}
\end{equation*}
$$

Inserting $k_{1}$ in (B.5) gives Equation 4.21.

## B. 2 Traffic Light system

## B.2.1 Indicator Functions

$$
\begin{align*}
& \mathbb{Z}_{\left\{\lambda_{\lambda_{l}}<t\right\}}=\int_{0}^{t} \lambda_{l} e^{-\lambda_{l} t} d t=1-e^{-t \lambda_{l}} \\
& \mathbb{Z}_{\left\{\lambda_{\lambda_{l}}>t\right\}}=\int_{t}^{\infty} \lambda_{l} e^{-\lambda_{l} t} d t=e^{-t \lambda_{l}} \tag{B.7}
\end{align*}
$$

## Appendix C

## Matlab Code

## Numerical Solution of the Multiple Switch Model

```
function [firmValue, optimalInvestmentAmount, optimalSwitching] =
    IuStarForMultipleSwitch(IuStart, IuStop, IuN, numberOfSwitches)
% This method is the numerical implementation of the algorithm solving
% Equation 4.28 and finds the optimal Iu within a given range.
% Inputvariables:
% IuStart = start of the range of Iu
% IuStop = end of the range of Iu
% IuN = number of Ius to test for
% numberOfSwitches = number of switches the firm may do.
testInvestments = linspace(IuStart, IuStop, IuN);
firmValues = zeros(1,IuN);
switchesMade = zeros(1, numberOfSwitches+1, IuN);
maxTechnologyJumps = 50;
%Compares firm values corresponding to all possibles Ius, finds optimal Iu.
for (i = 1:IuN)
    if numberOfSwitches > 2
        fprintf('i = %d av %d \n', i, IuN);
    end
    [firmValues(i), switchesMade(:,:,i)] = multipleSwitchGivenIu(testInvestments(i),
        0, 0, numberOfSwitches, maxTechnologyJumps);
end
optimalInvestmentAmount = testInvestments(find(firmValues == max(firmValues)));
optimalSwitching = switchesMade(:,:,find(firmValues == max(firmValues)));
firmValue = max(firmValues);
end
```

```
function [firmValue, switchesMade] = multipleSwitchGivenIu(Iu, i, j, maxSwitches,
    maxJumps)
% This method finds the optimal technology adoption strategy for a given Iu.
% InputVariables:
% Iu = Investment in short-term solutions
% i = number of technology switches already done by the firm
% j = the number of the technology the firm is currently using
% maxSwitches is the maximum number of switches the firm may do.
% maxJumps = the maximum number of jumps in the technology process
%Note that in order for the firm to produce with lice costs cu, for j=0,
%the tehcnology numbers have been adjusted so that permanent technology
%starts in j=1. This is, however, adjusted for in the cost function, so that
%an immediate adoption of the permanent solution still implies lice costs of
%cp = beta*c0.
lambdaP = 1.0; %Arrival rate of technological improvements
lambdaL = 0.2; %Arrival rate of permanent technology
r = 0.1; %Discount rate
pi0 = 23.135e+6; %Yearly profit net of lice-fighting costs
beta = 0.2; %Cost reduction factor for a permanent technology
c0 = 6.89e+6; % Initial lice-fighting cost a firm faces per licence
u = 0.15; % Improvement jump size of the technology process
Ip0 = 26.7e+6; % Investment cost of adopting a permanent technology
Ip1 = 1e+6; % Switching cost of upgrading a permanent technology
cuBar = 0.5*c0; %Lower boundry of lice-fighting costs when using short-term
    technologies
alpha = 1*10^(-6); %Cost reduction factor for short-term investments
values = zeros(1, maxJumps - j);
inputSwitchesMade = [];
%If the firm is currently using technology zero, the cost level
%depends on the investments in short-term technologies. Else, it depends
%on the technology level. Note that in order for the firm to produce with
%lice-fighting costs, cu at j=0, the technology numbers have been shifted
if (j == 0)
    costLevel = cuBar+(c0-cuBar)*exp(-alpha*Iu);
else
    costLevel = beta*c0*exp(-u*(j-1));
end
% Base case
if (i >= maxSwitches || j == maxJumps + 1)
```

```
    firmValue = perpetualProfits(pi0, costLevel, r);
    inputSwitchesMade(maxSwitches+1) = j -1 ;
    switchesMade = inputSwitchesMade;
% Recursive case
else
    for k = j:maxJumps+1
        valueOfCurrentProduction = perpetualProfits(pi0, costLevel, r);
        [firmValueBeforeInvestmentCost, tempSwitchArray] = multipleSwitchGivenIu(Iu,
                i +1,k,maxSwitches,maxJumps);
        inputSwitchesMade = [inputSwitchesMade;tempSwitchArray];
        if (j == 0 && k > 0)%The firm is making its first adoption of the permanent
        technology and must therefore pay the investment cost Ip0
        valueOfFutureChoices = firmValueBeforeInvestmentCost - Ip0;
        elseif(k == j) %The firm "switches" to the same technology as it is
        currently using
        valueOfFutureChoices = firmValueBeforeInvestmentCost;
        else %The firm is upgrading its permanent solution and therefore pays the
        lower switching cost Ipl
        valueOfFutureChoices = firmValueBeforeInvestmentCost - Ip1;
        end
        if (j == 0) %Firm has still not invested in a permanent technology and is
        therefore paying Iu in the current period
        tempValueBeforeNextAdoption = (1-(lambdaL/ lambdaL + r))*(lambdaP / (
            lambdaP + r))^(k-j))*valueOfCurrentProduction - Iu;
        else %Firm has already paid Iu
        tempValueBeforeNextAdoption = (1-(lambdaL/(lambdaL + r))*(lambdaP/(
                lambdaP + r))^(k-j))*valueOfCurrentProduction;
        end
        tempValueAfterNextAdoption = ((lambdaL/(lambdaL + r))*(lambdaP/(lambdaP + r
        ))^(k-j))*valueOfFutureChoices;
        values (k-j+1) = tempValueBeforeNextAdoption + tempValueAfterNextAdoption;
    end
    firmValue = max(values);
    currentSwitch = find(values == max(values),1,'last') - 1 + j;
    inputSwitchesMade(currentSwitch+1-j, i+1) = j - 1;
    switchesMade = inputSwitchesMade(currentSwitch+1-j, l:end);
end
```


## Numerical Solution of the Traffic Light System Model

```
function [firmValue, optimalIu, optimalIt, switchingStrategy] =
    ItStarAndIuStarForTrafficLightSystem(IuStart, IuStop, IuN, ItStart, ItStop, ItN,
    numberOfTrafficLightPeriods, location)
%This method is the numerical solution to Equation 4.36. It finds IuStar
%and ItStar given a range of Ius and Its.
% Inputvariables:
% IuStart = start of the range of Iu
% IuStop = end of the range of Iu
% IuN = number of Ius to test for
% ItStart = start of the range of It
% ItStop = end of the range of It
% ItN = number of Its to test for
% numberOfTrafficLightPeriods = Number of traffic light regulations we
% model
% location = location for the farm in consideration. May only be Flatanger,
% Hardanger or Finnmark.
r = 0.1; %Discount rate
maxSwitches = 1; %Max number
maxJumps = 30; %the maximum number of jumps in the technology process
testIts = linspace(ItStart, ItStop, ItN);
testIus = linspace(IuStart, IuStop, IuN);
firmValue = 0;
optimalIt = zeros(1, numberOfTrafficLightPeriods);
%Find optimal Iu, as Iu is independent of the traffic light system
for Iu=testIus
    tempFirmValue = longTermValueOfFirm(Iu, 0, 0, maxSwitches, maxJumps);
    if tempFirmValue > firmValue
        firmValue = tempFirmValue;
        optimalIu = Iu;
    end
end
%Reset firm value
firmValue = 0;
```

```
%Finds the optimal sequence of Its given the number of traffic light
%periods
if numberOfTrafficLightPeriods == 4
    for ItTestIndex1 = 1:ItN
    for ItTestIndex2 = 1:ItN;
        for ItTestIndex3 = 1:ItN
            for ItTestIndex4 = 1:ItN
                testItVector = [testIts(ItTestIndex1), testIts(ItTestIndex2),
                        testIts(ItTestIndex3), testIts(ItTestIndex4)];
                [tempFirmValue, switchingStrategy] = trafficLightMain(optimalIu,
                        testItVector, numberOfTrafficLightPeriods, location);
                if (tempFirmValue > firmValue)
                    firmValue = tempFirmValue ;
                        for i = 0:2:(numberOfTrafficLightPeriods-1)*2
                        optimalIt((i/2)+1) = testItVector((i/2)+1)*exp(-r*i);
                        end
                end
            end
        end
    end
    end
elseif numberOfTrafficLightPeriods == 3
    for ItTestIndex1 = 1:ItN
        for ItTestIndex2 = 1:ItN;
            for ItTestIndex3 = 1:ItN
            testItVector = [testIts(ItTestIndex1), testIts(ItTestIndex2), testIts(
                ItTestIndex3)];
            [tempFirmValue, switchingStrategy] = trafficLightMain(optimalIu,
                testItVector, numberOfTrafficLightPeriods, location);
            if (tempFirmValue > firmValue)
                firmValue = tempFirmValue ;
                for i = 0:2:(numberOfTrafficLightPeriods-1)*2
                                    optimalIt((i/2)+1) = testItVector((i/2)+1)*exp(-r*i);
                end
                    end
            end
        end
    end
elseif numberOfTrafficLightPeriods == 2
    for ItTestIndex1 = 1:ItN
        for ItTestIndex2 = 1:ItN;
        testItVector = [testIts(ItTestIndex1), testIts(ItTestIndex2)];
        [tempFirmValue, switchingStrategy] = trafficLightMain(optimalIu,
            testItVector, numberOfTrafficLightPeriods, location);
```

```
            if (tempFirmValue > firmValue)
```

            if (tempFirmValue > firmValue)
                firmValue = tempFirmValue ;
                firmValue = tempFirmValue ;
        for i = 0:2:(numberOfTrafficLightPeriods-1)*2
        for i = 0:2:(numberOfTrafficLightPeriods-1)*2
            optimalIt((i/2)+1) = testItVector((i/2)+1)*exp(-r*i);
            optimalIt((i/2)+1) = testItVector((i/2)+1)*exp(-r*i);
                end
                end
            end
            end
        end
        end
    end
    end
    elseif numberOfTrafficLightPeriods == 1
elseif numberOfTrafficLightPeriods == 1
for ItTestIndex1 = 1:ItN
for ItTestIndex1 = 1:ItN
testItVector = testIts(ItTestIndexl);
testItVector = testIts(ItTestIndexl);
[tempFirmValue, switchingStrategy] = trafficLightMain(optimalIu, testItVector,
[tempFirmValue, switchingStrategy] = trafficLightMain(optimalIu, testItVector,
numberOfTrafficLightPeriods, location);
numberOfTrafficLightPeriods, location);
if (tempFirmValue > firmValue)
if (tempFirmValue > firmValue)
firmValue = tempFirmValue ;
firmValue = tempFirmValue ;
for i = 0:2:(numberOfTrafficLightPeriods-1)*2
for i = 0:2:(numberOfTrafficLightPeriods-1)*2
optimalIt((i/2)+1) = testItVector((i/2)+1)*exp(-r*i);
optimalIt((i/2)+1) = testItVector((i/2)+1)*exp(-r*i);
end
end
end
end
end
end
end

```
end
```

```
function [firmValue, switchingStrategy] = trafficLightMain (Iu, ItVector,
    numberOfTrafficLightPeriods, location)
% This method finds the value of a firm and the optimal adoption time of
% the permanent technology for a given Iu and a given sequence of Its.
% InputVariables:
% Iu = Investment in short-term solutions
% ItVector = sequence of additional investments due to the traffic light
% system
% numberOfTrafficLightPeriods = The number of traffic light regulations we
% model
% location = location for the farm in consideration. May only be Flatanger,
% Hardanger or Finnmark.
numberOfSwitches = 1; %Number of technology switches. Set to one, as it extends the
    single switch model
pi0 = 23.135e+6; %Yearly profit net of lice-fighting costs
maxNumberOfJumps = 30; %Max number of improvements of the permanent technology
%Find the optimal switching strategy and the value of the firm without the
%traffic light system
[singleSwitchValueOfFirm, switchingStrategy] = multipleSwitchGivenIu(Iu, 0, 0,
    numberOfSwitches, maxNumberOfJumps);
inputProfit = pi0;
cumulativeTrafficLightValue = 0;
%Finds the additional value of the firm due to the traffic light system
for t = 2:2:(numberOfTrafficLightPeriods*2)
    [valueOfCurrentPeriod, outputProfit] = valueOftrafficLightPeriod(inputProfit,
        ItVector(t/2), t, location);
    inputProfit = outputProfit;
    cumulativeTrafficLightValue = cumulativeTrafficLightValue + valueOfCurrentPeriod;
end
firmValue = singleSwitchValueOfFirm + cumulativeTrafficLightValue;
end
```

```
function [firmValue, switchingStrategy] = trafficLightMain (Iu, ItVector,
    numberOfTrafficLightPeriods, location)
% This method finds the value of a firm and the optimal adoption time of
% the permanent technology for a given Iu and a given sequence of Its.
% InputVariables:
% Iu = Investment in short-term solutions
% ItVector = sequence of additional investments due to the traffic light
% system
% numberOfTrafficLightPeriods = The number of traffic light regulations we
% model
% location = location for the farm in consideration. May only be Flatanger,
% Hardanger or Finnmark.
numberOfSwitches = 1; %Number of technology switches. Set to one, as it extends the
    single switch model
pi0 = 23.135e+6; %Yearly profit net of lice-fighting costs
maxNumberOfJumps = 30; %Max number of improvements of the permanent technology
%Find the optimal switching strategy and the value of the firm without the
%traffic light system
[singleSwitchValueOfFirm, switchingStrategy] = multipleSwitchGivenIu(Iu, 0, 0,
    numberOfSwitches, maxNumberOfJumps);
inputProfit = pi0;
cumulativeTrafficLightValue = 0;
%Finds the additional value of the firm due to the traffic light system
for t = 2:2:(numberOfTrafficLightPeriods*2)
    [valueOfCurrentPeriod, outputProfit] = valueOftrafficLightPeriod(inputProfit,
        ItVector(t/2), t, location);
    inputProfit = outputProfit;
    cumulativeTrafficLightValue = cumulativeTrafficLightValue + valueOfCurrentPeriod;
end
firmValue = singleSwitchValueOfFirm + cumulativeTrafficLightValue;
end
```

```
function [value, outputProfit] = valueOftrafficLightPeriod(inputProfit, It, t,
    location)
% This method finds the value of a period in the traffic light system
% starting in time t for a given It. It also finds the expected profits at
% the end of the period
% Inputvariables:
% inputProfit = The profit the firm had after the last regulation in the
% traffic light system
% It = Additional investments due to the traffic light system
% t = The time in years at the beginning of the period
% location = location for the farm in consideration. May only be Flatanger,
% Hardanger or Finnmark.
lambdaL = 0.2; %Arrival rate of permanent technology
r = 0.1; % Discount rate
Ig = 1e+6; %Cost of expanding production if awarded a green light
%Hardanger
if stremp(location, 'Hardanger')
    greenConstant = 1; %Corresponds to gamma_G
    yellowConstant = 7; %Corresponds to gamma_Y
    redConstant = 10; %Corresponds to gamma_R
    green = 0.01e-5; %Corresponds to mu_G
    yellow = 0.2e-5; %Corresponds to mu_Y
    red = 0.25e-5; %Corresponds to mu_R
% Finnmark
elseif strcmp(location,'Finnmark')
    greenConstant = 10; %Corresponds to gamma_G
    yellowConstant = 9; %Corresponds to gamma_Y
    redConstant = 8; %Corresponds to gamma_R
    green = 0.2e-5; %Corresponds to mu_G
    yellow = 0.01e-5; %Corresponds to mu_Y
    red = 0.3e-5; %Corresponds to mu_R
% Flatanger
elseif strcmp(location,'Flatanger')
    greenConstant = 2; %Corresponds to gamma_G
    yellowConstant = 6; %Corresponds to gamma_Y
    redConstant = 7; %Corresponds to gamma_R
    green = 0.1e-5; %Corresponds to mu_G
    yellow = 0.2e-5; %Corresponds to mu_Y
    red = 0.8e-5; %Corresponds to mu_R
end
```

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```
productionIncreaseFactor = 1.06;
productionReductionFactor = 0.94;
Pg = (exp (greenConstant + green *It)/(exp(greenConstant + green*It) + exp(
    yellowConstant - yellow*It) + exp(redConstant-red*It)));
Py = (exp(yellowConstant - yellow*It)/(exp(greenConstant + green*It) + exp(
    yellowConstant - yellow*It) + exp(redConstant - red*It)));
Pr = (exp (redConstant - red*It)/(exp(greenConstant + green*It) + exp(yellowConstant -
    yellow*It) + exp(redConstant - red*It)));
greenFactor = Pg*productionIncreaseFactor;
yellowFactor = Py;
redFactor = Pr*productionReductionFactor;
outputProfit = inputProfit*(greenFactor+yellowFactor+redFactor);
value = exp(-t*(r+lambdaL ) ) *((outputProfit-inputProfit)/r) - exp(-t*(r+lambdaL)) *Ig*Pg
    - It*exp(-(t-2)*(r+lambdaL));
end
```


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[^0]:    ${ }^{1}$ A degenerate distribution implies that the jump size is constant.

[^1]:    ${ }^{1}$ The problem is described in Chapter 1.

[^2]:    ${ }^{2}$ This assumption is made for tractability, and can be relaxed to allow for positive lice costs when $\theta \rightarrow \infty$.

[^3]:    ${ }^{3}$ This is because $P_{G}, P_{Y}$ and $P_{R}$ must sum up to 1.

